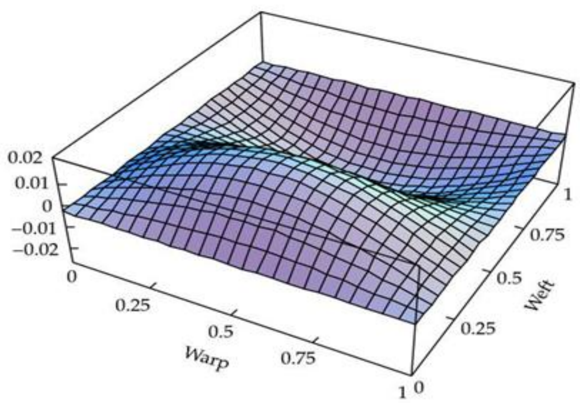
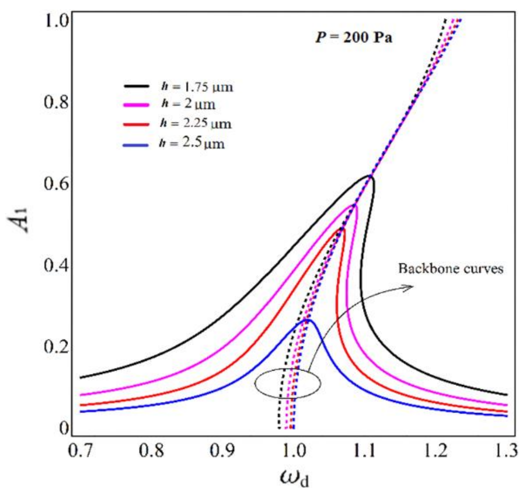
Nonlinear Vibration: Solutions by 'Harmonic Balance Method'

Sumitesh Behera
Bachelor of Technology
Mechanical Engineering
Veer Surrendra Sai University of Technology

DR Somnath Sarangi
Associate Professor
Mechanical Engineering Department
Indian Institute of Technology Patna

Indian Institute of Technology Patna, Bihta-801103





Modern machinery combining higher operating speeds with lighter elements is one of the reasons why nonlinear vibrations occur frequently. The scope of equations of motion increases tremendously since many mechanical elements show different types of nonlinearities; even more, the mathematical model representing the equation of motion can include several orders and an infinitive number of possible coefficients.

Harmonic Balance Method is a method used to calculate the steady-state response of nonlinear differential equations is a technique for finding the solutions of nonlinear ordinary differential equation by using truncated Fourier series. In this course, analysis was made on the nonlinear behaviour in the motion of the spring- mass system.

INTRODUCTION

Nonlinearities in vibration, mathematically, occur due to nonlinear functions in the equation of motion of vibration. Hence, the extraction of solution becomes quite difficult. To reach the solution harmonic balance method was used. There are also several other methods.

DURING THE COURSE

Analysis was made on:

- Undamped Duffing equation
- Duffing equation with slowly varying amplitudes
- Damped Duffing equation

using Harmonic Balance Method. Graphs were observed using MATLAB software and behaviour of the curves were studied.

Several interesting concepts were learnt. One of such concepts is the Jump Phenomenon. In words, basically, it is the sudden drop or increase in the maximum amplitude with the increase or decrease in the frequency of the oscillation respectively. The plot of this curve was the area of concern.

CONCLUSION

The aim of the study is to find the minimum frequency at which the cusp occurs in order to design the oscillator(spring) within the safety limits. The study was possible under the guidance of DR. Somnath Sarangi (Mechanical Dept.).

<u>Undamped Duffing equation:</u>

The analysis begins with an ideal equation, that is, the undamped Duffing equation:

$$\ddot{x} + x + \beta x^3 = \Gamma \cos \omega t$$

A truncated Fourier series is assumed to be the solution:

$$x(t) = a \cos \omega t + b \sin \omega t$$

Now, the second derivative of the assumed solution becomes:

$$\ddot{x}(t) = -a\omega^2 \cos \omega t - b\omega^2 \sin \omega t$$

and x^3 becomes after application of some trigonometric identities:

$$x^3 = a^3 \cos^3 \omega t + 3a^2b(\cos^2 \omega t.\sin \omega t) + 3ab^2(\cos \omega t.\sin^2 \omega t) + b^3 \sin^3 \omega t$$
$$= (3/4)a\,(a^2 + b^2) \cos \omega t + (3/4)b\,(a^2 + b^2) \sin \omega t +$$
$$(1/4)a\,(a^2 - 3b^2) \cos 3\omega t + (1/4)b\,(3a^2 - b^2) \sin 3\omega t$$

On neglecting the terms of sin $3\omega t$ and cos $3\omega t$ as the attenuation is very large with comparison to the that of the fundamental frequency and comparing the terms of sin ωt and cos ωt, following equations are obtained:

$$b\,\{(w^2 - 1) - (3/4)\beta\,(a^2 + b^2)\} = 0$$
$$a\,\{(w^2 - 1) - (3/4)\beta\,(a^2 + b^2)\} = -\Gamma$$

Considering b = 0, gives an admissible solution:

$$(3/4)\beta a^3 - (\omega^2 - 1)a - \Gamma = 0$$

The roots are given by the intersections, in the plane of a and z, of the curves:

$$z = -\Gamma, \qquad z = -(3/4)\beta a^3 + (\omega^2 - 1)a$$

Graphs of z against a has been plotted using MATLAB.

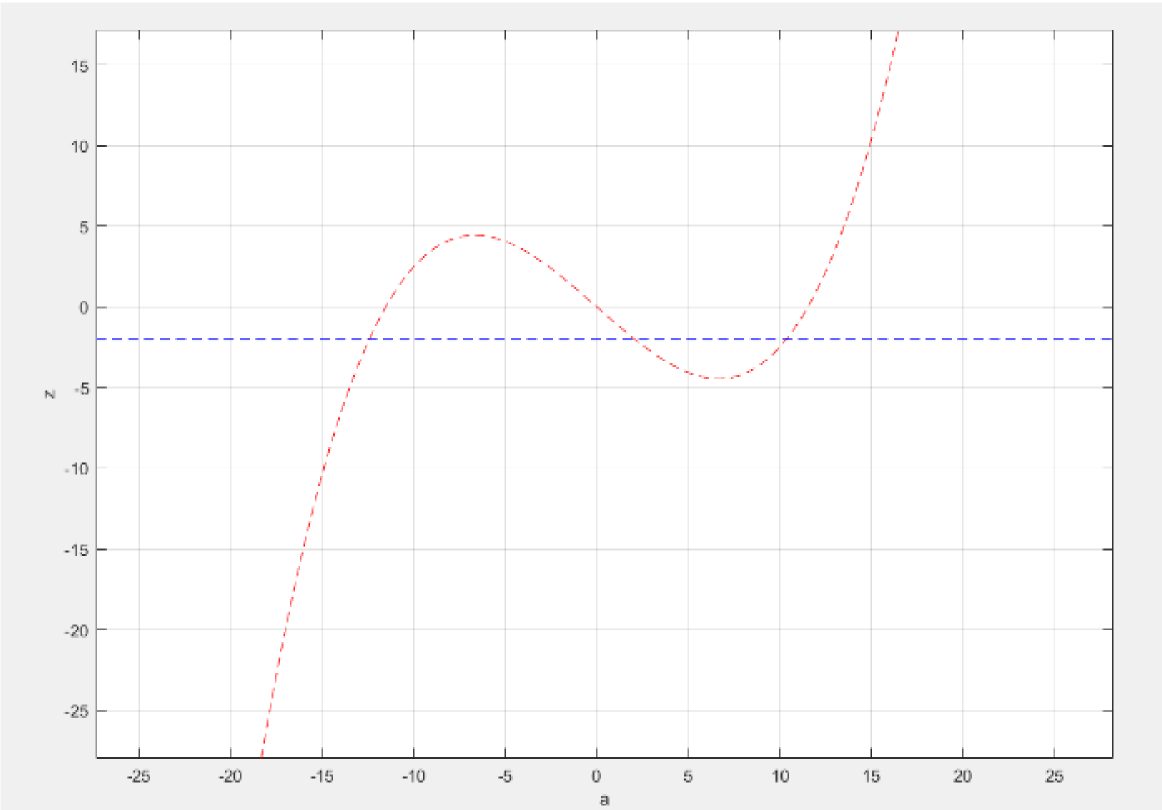


figure-1: for w < 1

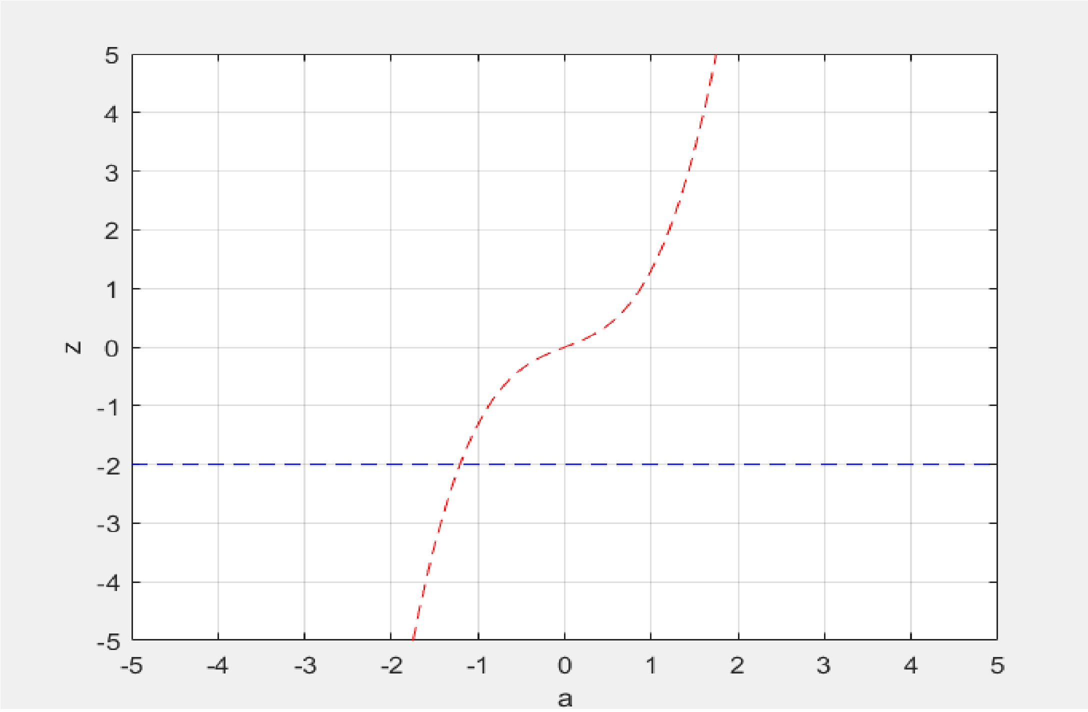


figure-2: for w > 1

Duffing equation for slowly varying amplitudes:

For slowly varying amplitudes a and b in:

$$x(t) = a(t) \cos \omega t + b(t) \sin \omega t$$

and $\dot{x}$ and $\ddot{x}$ become respectively:

$$\dot{x}(t) = (a' + \omega b) \cos \omega t + (-\omega a + b') \sin \omega t$$
$$\ddot{x}(t) = (a'' - \omega^2 a + 2\omega b') \cos \omega t + (b'' - 2\omega a' - \omega^2 b) \sin \omega t$$

However, after neglecting a" and b" terms, $\ddot{x}$ becomes:

$$\ddot{x}(t) = (-\omega^2 a + 2\omega b') \cos \omega t + (-2\omega a' - \omega^2 b) \sin \omega t$$

Solving for x^3 gives:

$$x^3(t) = (a^3/4) (\cos 3\omega t + 3\cos \omega t) + (b^3/4) (3\sin \omega t + \sin 3\omega t) + (3/2)\, a^2 b \sin\omega t + (3/4)\, a^2 b (\sin 3\omega t - \sin \omega t) + (3/2)\, ab^2 \cos \omega t - (3/4)\, ab^2 (\cos 3\omega t + \cos \omega t)$$

Neglecting the terms in cos $3\omega t$ and sin $3\omega t$, x^3 becomes:

$$x^3(t) = (3/4)a\,(a^2 + b^2)\cos\omega t + (3/4)b\,(a^2 + b^2)\sin\omega t$$

On putting the values of x, $\dot{x}$, $\ddot{x}$ in the undamped Duffing equation, following result is found:

$$[2\omega b' - a\,\{(\omega^2 - 1) - (3/4)\beta\,(a^2 + b^2)\}]\cos\omega t + [-2\omega a' - b\,\{(\omega^2 - 1) - (3/4)\beta\,(a^2 + b^2)\}]\sin\omega t = \Gamma\cos\omega t$$

Matching the coefficients of sin ωt and cos ωt leads to the following **autonomous system:**

$$a' = -(1/2\omega)b\,\{(\omega^2 - 1) - (3/4)\beta\,(a^2 + b^2)\} \equiv \mathbf{A(a, b)}$$
$$b' = (1/2\omega)a\,\{(\omega^2 - 1) - (3/4)\beta\,(a^2 + b^2)\} + (\Gamma/2\omega) \equiv \mathbf{B(a, b)}$$

Putting initial conditions assuming a'(0) to be very small:

$$a(0) = x(0)$$
$$b(0) = \dot{x}(0)/\omega$$

The phase plane for a, b in the system is called the **van der Pol plane.**
The equilibrium points, given by A(a, b) = B(a, b) = 0, represent the steady periodic solutions.
The other paths correspond to non-periodic solutions.

Damped Duffing equation:

Stepping forward to more practical case by introducing damping force into the equation:

$$\ddot{x} + k\dot{x} + x + \beta x^3 = \Gamma\cos\omega t, \quad k > 0$$

The equilibrium points are given by:

$$b\,\{\omega^2 - 1 - (3/4)\beta\,(a^2 + b^2)\} + k\omega a = 0$$
$$a\,\{\omega^2 - 1 - (3/4)\beta\,(a^2 + b^2)\} - k\omega b = -\Gamma$$

After squaring and adding these equations, following equations are obtained:

$$r^2\,\{k^2\omega^2 + (\omega^2 - 1 - (3/4)\beta r^2)^2\} = \Gamma^2$$
$$r = \sqrt{(a^2 + b^2)}$$

The Jump Phenomenon:

Continuing, the last two equations obtained:

$$r^2 \{k^2\omega^2 +(\omega^2 – 1 – (¾)\beta r^2)^2\} = \Gamma^2, \qquad (k > 0, \omega > 0, \Gamma > 0)$$

For $\beta < 0$ and $\rho = -\beta r^2$, $\gamma = \Gamma\sqrt{(-\beta)}$:

$$\gamma^2 = G(\rho) = \rho \{k^2\omega^2 + (\omega^2 - 1 + (¾)\rho)^2\}$$

Since the above equation has four variables, assuming k to be constant and taking several combinations of ω and Γ, $G(\rho)$ becomes:

$$G(\rho) = (9/16)\rho^3 + (3/2)(\omega^2 - 1)\rho^2 + \{k^2\omega^2 + (\omega^2 - 1)^2\}\rho$$

For $\rho=0$ and $\rho \to \infty$, values of $G(\rho)$ respectively are:

$$G(\rho) = 0 \text{ and } G(\rho) \to \infty$$

So, it can be concluded that at one positive real root exists in between. Now, for some parameter values, there will be 3 real roots if $G'(\rho) = 0$ has two distinct solutions for $\rho \geq 0$.

$$\rho_1 , \rho_2 = (8/9) (1 - \omega^2) \pm (4/9) \sqrt{[(1 - \omega^2)^2 - 3k^2\omega^2]}$$

Only real roots are desired, hence:

$$\sqrt{[(1 - \omega^2)^2 - 3k^2\omega^2]} \geq 0$$

Also, from the equations:

$$G(\rho) = (9/16)\rho^3 + (3/2)(\omega^2 - 1)\rho^2 + \{k^2\omega^2 + (\omega^2 - 1)^2\}\rho$$
$$G'(\rho) = (27/16)\rho^2 + 3(\omega^2 - 1)\rho + \{k^2\omega^2 + (\omega^2 - 1)^2\}$$

Following conditions are obtained:

$$\rho_1\rho_2 = (16/27) \{(1 - \omega^2)^2 + k^2\omega^2\} > 0$$
$$\rho_1 + \rho_2 = 16/9 (1 - \omega^2)$$

For ρ_1 and ρ_2 to be positive real numbers:

$$0 < \omega < 1$$
$$\omega^2 + k\omega\sqrt{3} - 1 > 0$$

Combinedly, it can be interpreted as:

$$0 < \omega < (½) \{\sqrt{(3k^2 + 4)} - k\sqrt{3}\}$$

Therefore, for three real distinct roots of $G(\rho) = \gamma^2$:

$$G(\rho_2) < G(\rho_1)$$
$$\omega < (½) \{\sqrt{(3k^2 + 4)} - k\sqrt{3}\}$$

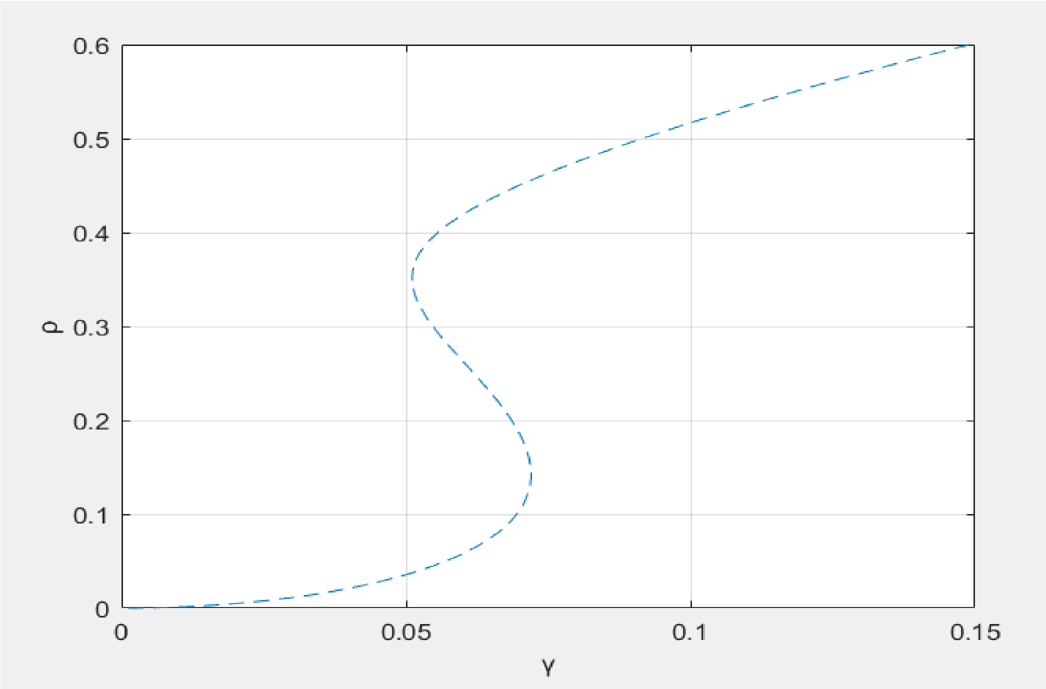


figure- 3: Amplitude ρ versus γ for k = 0.1 and ω = 0.85

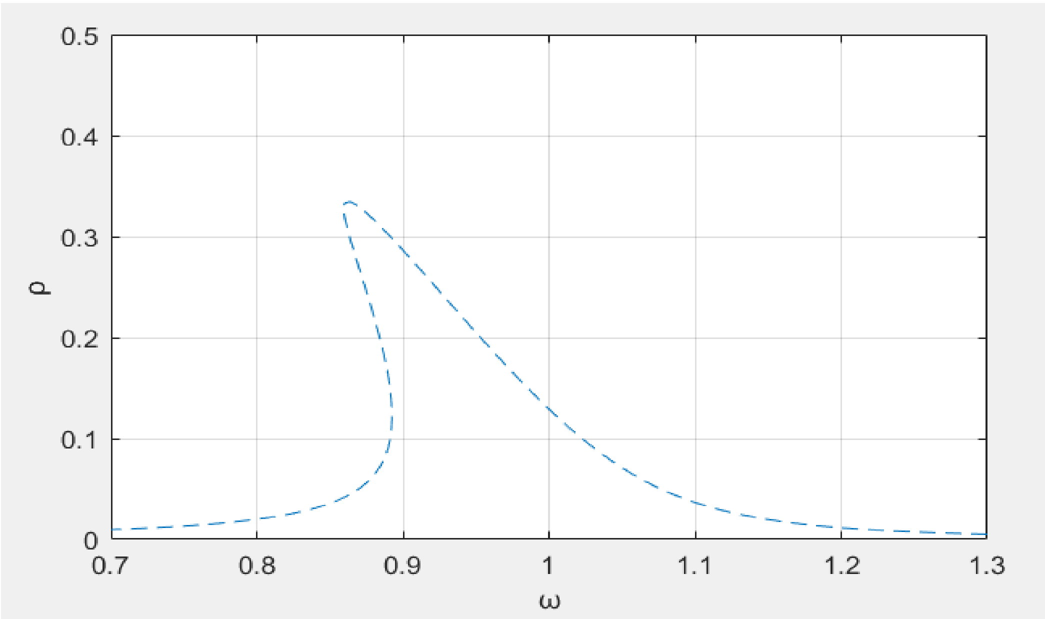


figure- 4: Amplitude ρ versus ω for k = 0.1 and γ = 0.05

An Example of Nonlinear Oscillator:

Conservative nonlinear oscillatory systems can often be modeled by potentials having a rational form for the potential energy. An example is the nonlinear oscillator modeled by:

$$d^2y/d\tau^2 + (\alpha y^3)/(\beta + \gamma y^2) = 0$$

for which the parameters (a, b, γ) are non-negative. Defining $y = \sqrt{(\beta/\gamma)}$ and $\tau = \sqrt{(\gamma/\alpha t)}$, so the equation is reduced to the following non-dimensional equation:

$$\ddot{x} + x^3(1+x^2)^{-1} = 0 \qquad \ldots(i)$$

where overdots denote differentiation with respect to time t. For small x, the equation of motion is that of a Duffing-type nonlinear oscillator (i.e., $\ddot{x} + x^3 \sim 0$), while for large x, the second equation of motion approximates that of a linear harmonic oscillator (i.e., $\ddot{x} + x \sim 0$). Hence, the second equation is referred as the Duffing-harmonic oscillator. The restoring force in the equation is the same for both negative and positive amplitudes. The second can be rewritten as:

$$(1+x^2)\,\ddot{x} + x^3 = 0 \qquad \ldots(ii)$$

An analytic approximation to the periodic solution of Eq. (3) can be calculated by the use of the method of harmonic balance [4]. Assume that the angular frequency of the Duffing-harmonic oscillator is o, which is unknown to be further determined. The first approximation is:

$$x(t) = A\cos(\omega t)$$

as it satisfies the initial conditions:

$$x(0) = A, \quad \dot{x}(0) = 0$$

On putting value of x(t) in the equation "(ii)" and similarly, further, neglecting the higher order harmonics:

$$[(1+(^3/_4)A^2) + (^3/_4)A^2].A\cos(\omega t) = 0$$

Equating coefficient of cos(ωt) to zero leads to:

$$\omega = \sqrt{[(^3/_4)A^2 / (1+(^3/_4)A^2)]}$$

The main objective of this short communication is to solve equation "(i)" instead of equation "(ii)" by applying the method of harmonic balance. It can be easily shown that:

$$(A\cos\Theta)^3/(1 + A\cos\Theta) = b_1\cos\Theta + b_3\cos\Theta + \ \ldots$$

where,

$$b_1 = \int_0^{\pi} \frac{(A\cdot\cos\theta)^3\cdot\cos\theta}{1 + (A\cdot\cos\theta)^2}\cdot d\theta$$

$$= \frac{2\cdot A}{\pi}\int_0^{\pi}\left((\cos\theta)^2 - \frac{(\cos\theta)^2}{1 + (A\cdot\cos\theta)^2}\right)\cdot d\theta$$

$$= A - \frac{2}{\pi\cdot A}\int_0^{\pi}\left(1 - \frac{1}{1 + (A\cdot\cos\theta)^2}\right)\cdot d\theta$$

$$= A - \frac{2}{A} + \frac{1}{\pi\cdot A}\int_0^{\pi}\left(\frac{1}{1 + \frac{A^2}{2} + \frac{A^2}{2}\cdot\cos x}\right)\cdot dx$$

$$= A - \frac{2}{A} + \frac{2}{\pi\cdot A}\int_0^{\pi}\left(\frac{1}{1 + \frac{A^2}{2} + \frac{A^2}{2}\cdot\cos 2\theta}\right)\cdot d\theta$$

We know that:

$$\int_0^{2\pi}\frac{dx}{a + b\cdot\cos x} = \frac{2\pi}{\sqrt{a^2 - b^2}}$$

So, b_1 becomes:

$$= A + \frac{2}{A}\left(\left(1 + A^2\right)^{-\frac{1}{2}} - 1\right)$$

Putting the value of x(t) in equation (ii), the simplified equation becomes:

$$(-\omega^2 A + b_1)\cos\omega t + \text{higher-order harmonics} = 0$$

Further, solving for ω gives:

$$\omega = \sqrt{\frac{b_1}{A}} = \sqrt{1 + \frac{2}{A^2}\left(\frac{1}{\sqrt{1 + A^2}} - 1\right)}$$

Final Task Equation:

During the end period, the following equation was provided:

$$\left(\mu+\frac{\nu(\gamma+1)}{3\lambda(t)^4}\right)\left(\frac{d^2}{dt^2}\lambda(t)\right)-\frac{\nu\gamma\left(\frac{d}{dt}\lambda(t)\right)^2}{2\lambda(t)^4}+\alpha\left(\lambda(t)-\frac{1}{\lambda(t)^2}\right)-\frac{\beta(1+\phi\sin(\omega t))^2}{\lambda(t)^3}+(-1)^{c+1}$$

On simplification, the equation becomes:

$$\left(\mu\lambda(t)^4+\frac{\nu(\gamma+\lambda(t)^4)}{3}\right)\left(\frac{d^2}{dt^2}\lambda(t)\right)-\frac{\nu\gamma\left(\frac{d}{dt}\lambda(t)\right)^2}{2}+\alpha\left(\lambda(t)^5-\lambda(t)^2\right)-\beta(1+\phi\sin(\omega t))^2\lambda(t)+(-1)^{c+1}\lambda(t)^4$$

Equating the above equation to zero(0) and assuming following solution for the above equation:

$$\lambda(t)=A\cos(\omega t)+B\sin(\omega t)+C\cos(2\omega t)+D\sin(2\omega t)$$

Putting the value of lambda(t) and expanding gives a very lengthy solution. Therefore, for plotting the graph of amplitude against frequency, only coefficients of trigonometric functions with angles ωt and 2ωt are considered.

For sin(ωt), the coefficients are:

$$-\frac{5}{24}A^4\nu B\omega^2-\frac{5}{8}A^4\mu B\omega^2-\frac{5}{12}A^2B^3\nu\omega^2-\frac{5}{4}A^2B^3\mu\omega^2-\frac{1}{3}B\gamma\nu\omega^2-B\beta+\frac{5}{8}B^5\alpha+\frac{5}{4}A^2B^3\alpha$$
$$+\frac{5}{8}A^4\alpha B-\frac{5}{8}B^5\mu\omega^2-\frac{5}{24}B^5\nu\omega^2-\frac{3}{4}B\beta\phi^2$$

Similarly, for cos(ωt):

$$\frac{5}{8}A^5\alpha-A\beta+\frac{5}{4}A^3B^2\alpha+\frac{5}{8}AB^4\alpha-\frac{5}{24}A^5\nu\omega^2-\frac{1}{4}A\beta\phi^2-\frac{5}{8}A^5\mu\omega^2-\frac{5}{4}A^3B^2\mu\omega^2$$
$$-\frac{5}{12}A^3B^2\nu\omega^2-\frac{5}{8}AB^4\mu\omega^2-\frac{5}{24}AB^4\nu\omega^2-\frac{1}{3}A\gamma\nu\omega^2$$

for sin(2ωt):

$$-\frac{5C^4D\mu\omega^2}{2}-\frac{5C^4D\nu\omega^2}{6}-\frac{4D\gamma\nu\omega^2}{3}+\frac{15\alpha A^2C^2D}{4}-\frac{21A^2D^3\mu\omega^2}{2}-\frac{3(-1)^cA^2CD}{2}$$
$$+\frac{45\alpha A^2B^2D}{8}-\frac{21B^2D^3\mu\omega^2}{2}-\frac{7B^2D^3\nu\omega^2}{2}-\frac{5B^4D\mu\omega^2}{2}-\frac{5B^4D\nu\omega^2}{6}-\frac{5C^2D^3\nu\omega^2}{3}-\frac{5\alpha AB^3C}{4}$$
$$+\frac{5\alpha A^3BC}{4}-\frac{3(-1)^cABC^2}{2}+\frac{3(-1)^cB^2CD}{2}-\frac{5A^4D\mu\omega^2}{2}+\frac{15\alpha B^2C^2D}{4}-\frac{7A^2D^3\nu\omega^2}{2}$$
$$-5C^2D^3\mu\omega^2-\frac{5A^4D\nu\omega^2}{6}-\frac{9(-1)^cABD^2}{2}+\frac{15\alpha B^2D^3}{4}+\frac{25\alpha B^4D}{16}+\frac{25\alpha A^4D}{16}-\frac{5D^5\mu\omega^2}{2}$$
$$-\frac{5D^5\nu\omega^2}{6}+\frac{5\alpha C^2D^3}{4}-\frac{\beta\phi^2D}{2}-A^3B(-1)^c-A\beta\phi+\frac{5\alpha C^4D}{8}-AB\alpha+\frac{15\alpha A^2D^3}{4}-AB^3(-1)^c$$

$$-\beta D+\frac{5\alpha D^5}{8}-\frac{2A^3BC\nu\omega^2}{3}+\frac{AB\gamma\nu\omega^2}{2}-\frac{21B^2C^2D\mu\omega^2}{2}-\frac{7B^2C^2D\nu\omega^2}{2}-2A^3BC\mu\omega^2$$
$$+2AB^3C\mu\omega^2+\frac{2AB^3C\nu\omega^2}{3}-\frac{21A^2C^2D\mu\omega^2}{2}-\frac{7A^2C^2D\nu\omega^2}{2}-9A^2B^2D\mu\omega^2-3A^2B^2D\nu\omega^2$$

and for cos(2ωt):

$$B\beta\phi+\frac{15\alpha B^2CD^2}{4}-\frac{7B^2CD^2\nu\omega^2}{2}-\frac{5\alpha AB^3D}{4}-\frac{7A^4C\mu\omega^2}{2}-\frac{7A^4C\nu\omega^2}{6}-3(-1)^cABCD$$
$$-2A^3BD\mu\omega^2-\frac{2A^3BD\nu\omega^2}{3}-\frac{7A^2C^3\nu\omega^2}{2}-\frac{7B^4C\mu\omega^2}{2}-\frac{7B^4C\nu\omega^2}{6}-\frac{21A^2C^3\mu\omega^2}{2}$$
$$-\frac{21B^2C^3\mu\omega^2}{2}-\frac{7B^2C^3\nu\omega^2}{2}+\frac{A^2\gamma\nu\omega^2}{4}-\frac{B^2\gamma\nu\omega^2}{4}-\frac{21B^2CD^2\mu\omega^2}{2}-\frac{5C^5\nu\omega^2}{6}-\frac{5C^5\mu\omega^2}{2}$$
$$+\frac{5\alpha C^3D^2}{4}+\frac{5\alpha CD^4}{8}+\frac{5\alpha A^3BD}{4}+\frac{15\alpha B^2C^3}{4}+\frac{15\alpha A^2C^3}{4}+\frac{35\alpha A^4C}{16}+2AB^3D\mu\omega^2$$
$$+\frac{2AB^3D\nu\omega^2}{3}-\frac{21A^2CD^2\mu\omega^2}{2}-\frac{7A^2CD^2\nu\omega^2}{2}+\frac{15\alpha A^2CD^2}{4}-\beta C+\frac{5\alpha C^5}{8}+\frac{B^4(-1)^c}{2}$$
$$-\frac{A^4(-1)^c}{2}+\frac{35\alpha B^4C}{16}-\frac{\beta\phi^2C}{2}+\frac{3(-1)^cB^2D^2}{4}+\frac{9(-1)^cB^2C^2}{4}-\frac{3(-1)^cA^2D^2}{4}-\frac{5CD^4\mu\omega^2}{2}$$
$$-\frac{5CD^4\nu\omega^2}{6}-\frac{4C\gamma\nu\omega^2}{3}-5C^3D^2\mu\omega^2-\frac{5C^3D^2\nu\omega^2}{3}+\frac{15\alpha A^2B^2C}{8}-\frac{9(-1)^cA^2C^2}{4}-\frac{\alpha A^2}{2}$$
$$+\frac{B^2\alpha}{2}-3A^2B^2C\mu\omega^2-A^2B^2C\nu\omega^2$$

Considering the coefficients of the mentioned trigonometric functions equal to zero, for different values of frequency, different amplitudes are obtained. The following plot is achieved:

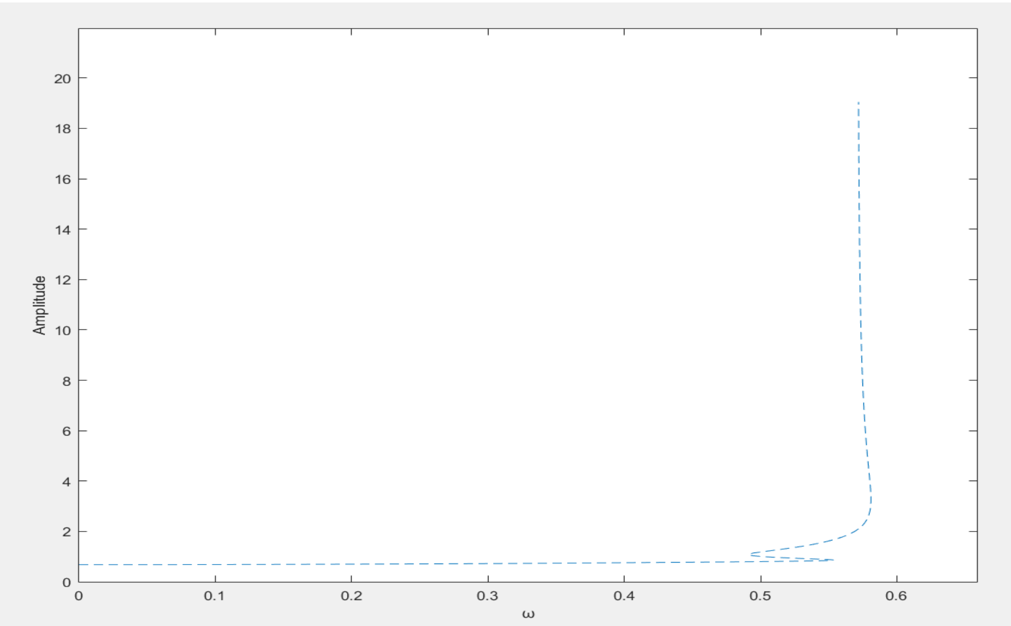


figure- 5: for α = 1, β = 0.1, μ = 0.5, ν = 0.8, ϕ = 0.5, γ = 1, c = 1

MATLAB Codes (for reference):

Figure- 1:

```
B = -0.01; w = 0.01; G = 2;
f1 = @(a,z) (3/4).*B.*a.^3 - (w.^2 - 1).*a + z;
fimplicit(f1,'--r')
hold on
f2 = @(a,z) z + G;
fimplicit(f2,'--b')
xlabel('a')
ylabel('z')
hold off
grid on
```

Figure- 2:

```
B= -1; w = 0.01; G = 2;
f1 = @(a,z) (3/4).*B.*a.^3 - (w.^2 - 1).*a + z;
fimplicit(f1,'--r')
hold on
f2 = @(a,z) z + G;
fimplicit(f2,'--b')
xlabel('a')
ylabel('z')
hold off
grid on
```

Figure- 3:

```
k = 0.1; w = 0.85;
f = @(g,r) -9/16*r.^3 -3/2*(w.^2 - 1).*r.^2 -(k.^2*w.^2 +(w.^2- 1).^2).*r +(g).^2;
fimplicit(f,'--',[0 0.15 0 0.6])
xlabel('γ')
ylabel('ρ')
grid on
```

Figure- 4:

```
k = 0.1; g = 0.05;
f = @(w,r) -9/16*r.^3 -3/2*(w.^2 - 1).*r.^2 -(k.^2*w.^2 +(w.^2- 1).^2).*r +(g)^2;
fimplicit(f,'--',[0.7 1.3 0 0.5])
grid on
xlabel('ω')
ylabel('ρ')
```

<u>Figure- 5:</u>

<u>Code of function “continuation”:</u>

```
function continuation(fun, N)
global bigX continuation_function
continuation_function = fun;
 for n=1:N
    xg = 2*bigX(:,end)-bigX(:,end-1);
    xg = newton('continuation_side',xg);
    bigX = [bigX,xg];
 end
```

<u>Code of function “continuation_side”:</u>

```
function z = continuation_side(x)
global bigX sub continuation_function
sub = x(end);
z = feval(continuation_function,x(1:end-1));
Delta_S = norm(bigX(:,1)-bigX(:,2));
z=[z;norm(bigX(:,end)-x)-Delta_S];
end
```

<u>Code of function “newton”:</u>

```
function x=newton(fun,x)
ep=1e-7;
n=length(x);
e=eye(n)*ep;
f0 = feval(fun,x);
tol=1e-11;
 iter=0;
while (iter < 60)*(norm(f0)>tol)
   iter=iter+1;
   D=zeros(n);
   for k=1:n
      D(:,k)=(feval(fun,x+e(:,k))-f0)/ep;
   end
   x=x-D\f0;
   f0=feval(fun,x);
end
if iter==60
   disp('did not converge')
end
```

Code of function “junk_1”:

```
function z = junk_1(x)
global  sub
omega = sub;

A = x(1); B = x(2); C = x(3); D = x(4);

alpha = 1; mu = 0.5; nu = 0.8; beta = 0.1; phi = 0.5; gamma = 1; c = 1;

z = -5/24*A^4*nu*B*omega^2 - 5/8*A^4*mu*B*omega^2 -
5/12*A^2*B^3*nu*omega^2 - 5/4*A^2*B^3*mu*omega^2 -
1/3*B*gamma*nu*omega^2 - B*beta + 5/8*B^5*alpha + 5/4*A^2*B^3*alpha +
5/8*A^4*alpha*B - 5/8*B^5*mu*omega^2 - 5/24*B^5*nu*omega^2 -
3/4*B*beta*phi^2;

z = [z; 5/8*A^5*alpha - A*beta + 5/4*A^3*B^2*alpha + 5/8*A*B^4*alpha -
5/24*A^5*nu*omega^2 - 1/4*A*beta*phi^2 - 5/8*A^5*mu*omega^2 -
5/4*A^3*B^2*mu*omega^2 - 5/12*A^3*B^2*nu*omega^2 -
5/8*A*B^4*mu*omega^2 - 5/24*A*B^4*nu*omega^2 -
1/3*A*gamma*nu*omega^2];

z = [z; -(5*A^4*D*mu*omega^2)/2 + (15*alpha*B^2*C^2*D)/4 -
(2*A^3*B*C*nu*omega^2)/3 - (5*D^5*mu*omega^2)/2 - 5*C^2*D^3*mu*omega^2
+ A*B*gamma*nu*omega^2/2 + (2*A*B^3*C*nu*omega^2)/3 -
(5*D^5*nu*omega^2)/6 - (21*A^2*D^3*mu*omega^2)/2 - (3*(-1)^c*A^2*C*D)/2 +
(45*alpha*A^2*B^2*D)/8 - (5*A^4*D*nu*omega^2)/6 -
(7*A^2*D^3*nu*omega^2)/2 - (21*A^2*C^2*D*mu*omega^2)/2 +
(5*alpha*C^2*D^3)/4 - A^3*B*(-1)^c - A*beta*phi - A*B*alpha - A*B^3*(-1)^c -
(5*B^4*D*mu*omega^2)/2 - (5*B^4*D*nu*omega^2)/6 - beta*D -
(5*C^2*D^3*nu*omega^2)/3 - (5*alpha*A*B^3*C)/4 + (5*alpha*C^4*D)/8 -
(7*A^2*C^2*D*nu*omega^2)/2 - beta*phi^2*D/2 + (15*alpha*A^2*D^3)/4 +
(25*alpha*B^4*D)/16 + (25*alpha*A^4*D)/16 + (3*(-1)^c*B^2*C*D)/2 -
(21*B^2*D^3*mu*omega^2)/2 - (7*B^2*D^3*nu*omega^2)/2 -
(7*B^2*C^2*D*nu*omega^2)/2 - (5*C^4*D*mu*omega^2)/2 -
(5*C^4*D*nu*omega^2)/6 - (4*D*gamma*nu*omega^2)/3 +
(15*alpha*A^2*C^2*D)/4 - 2*A^3*B*C*mu*omega^2 + 2*A*B^3*C*mu*omega^2 -
9*A^2*B^2*D*mu*omega^2 - 3*A^2*B^2*D*nu*omega^2 + (5*alpha*D^5)/8 - (9*(-
1)^c*A*B*D^2)/2 + (15*alpha*B^2*D^3)/4 + (5*alpha*A^3*B*C)/4 - (3*(-
1)^c*A*B*C^2)/2 - (21*B^2*C^2*D*mu*omega^2)/2];

z = [z; (35*alpha*B^4*C)/16 - alpha*A^2/2 + B*beta*phi - beta*phi^2*C/2 - 3*(-
1)^c*A*B*C*D - 2*A^3*B*D*mu*omega^2 - (2*A^3*B*D*nu*omega^2)/3 -
(7*A^2*C^3*nu*omega^2)/2 + B^2*alpha/2 - (7*B^4*C*mu*omega^2)/2 -
```

```
(7*B^4*C*nu*omega^2)/6 - (21*A^2*C^3*mu*omega^2)/2 -
(21*B^2*C^3*mu*omega^2)/2 + (15*alpha*B^2*C^3)/4 + (5*alpha*C^5)/8 + B^4*(-
1)^c/2 + (3*(-1)^c*B^2*D^2)/4 + (9*(-1)^c*B^2*C^2)/4 +
(2*A*B^3*D*nu*omega^2)/3 + (5*alpha*C*D^4)/8 + (5*alpha*A^3*B*D)/4 -
(7*B^2*C^3*nu*omega^2)/2 + A^2*gamma*nu*omega^2/4 + (5*alpha*C^3*D^2)/4
- (5*C^5*nu*omega^2)/6 - (5*C^5*mu*omega^2)/2 + 2*A*B^3*D*mu*omega^2 -
A^4*(-1)^c/2 - (5*C*D^4*nu*omega^2)/6 - (3*(-1)^c*A^2*D^2)/4 -
(5*C*D^4*mu*omega^2)/2 - beta*C - (5*C^3*D^2*nu*omega^2)/3 -
(21*A^2*C*D^2*mu*omega^2)/2 - (7*A^2*C*D^2*nu*omega^2)/2 +
(15*alpha*A^2*C*D^2)/4 - 5*C^3*D^2*mu*omega^2 - (9*(-1)^c*A^2*C^2)/4 -
B^2*gamma*nu*omega^2/4 - (21*B^2*C*D^2*mu*omega^2)/2 -
(5*alpha*A*B^3*D)/4 - (7*A^4*C*mu*omega^2)/2 - (7*A^4*C*nu*omega^2)/6 +
(15*alpha*B^2*C*D^2)/4 - (7*B^2*C*D^2*nu*omega^2)/2 -
(4*C*gamma*nu*omega^2)/3 + (15*alpha*A^2*B^2*C)/8 + (15*alpha*A^2*C^3)/4
+ (35*alpha*A^4*C)/16 - 3*A^2*B^2*C*mu*omega^2 - A^2*B^2*C*nu*omega^2];
end
```

<u>Main Code (to be run):</u>

```
global sub bigX
sub = 0;
newton('junk_1',[1;0;0;0])

bigX = [ans;sub]
sub = 0.01
newton('junk_1',[1;0;0;0])
bigX = [bigX,[ans;sub]]

continuation('junk_1',2000)
w = bigX(5,:);
Amp = sqrt(bigX(1,:).^2 + bigX(2,:).^2 + bigX(3,:).^2 + bigX(4,:).^2 );
plot(w,Amp,'--')
```