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Pattern Recognition for Statistical Process Control Charts

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Control charts are important statistical process control tools for determining whether a process is run in its intended mode or in the presence of unnatural patterns. Patterns displayed on control charts can provide information about the process. This paper describes the development of a pattern recognition system designed to detect and analyse various patterns that can occur on statistical quality control charts. The system looks not only for simple patterns, such as trend, shift and stratification, but also for superimposed patterns, such as trend + shift. The effect of noise associated with individual patterns is also analysed. The benefits of the approach compared with the alternatives are discussed.

Keywords: Control charts; Pattern recognition; Patterns; Statistical process control

1. Introduction

Nowadays, statistical process control charts are widely used to assess the quality of products being manufactured. However, the effectiveness of the use of control charts depends largely on recognising out-of-control conditions in terms of patterns. Patterns can be classified as natural (normal) and unnatural (abnormal) [1,2]. The basic significance of a natural pattern is that it indicates a process under control. An unnatural pattern indicates a process which is out of control. When an unnatural pattern is displayed on the control charts, assignable causes must be found to explain the patterns presented [3]. Recognition of patterns on control charts, especially for complicated patterns or super-imposed patterns, can often be a complex problem. Thus, automatic pattern-recognition systems can be of considerable help to quality engineers and shop floor personnel.

This paper describes an automatic pattern recognition system which is capable of detecting a variety of both simple and super-imposed unnatural patterns from $\bar{X}$ and R charts. Pattern definition and algorithmic specifications are discussed. The response of the system to noise associated with the patterns is analysed.

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2. Definition of Patterns

In order to identify patterns from control charts, the patterns must be explicitly defined in the first place. Generally speaking, there are two ways to define patterns. First, patterns can be defined using descriptive words, e.g. trend, cycle, etc. This was the approach used by Western Electric which, as far back as 1956 [4], identified and defined the most common unnatural patterns on control charts. These were:

Cycles
Freaks
Beyond control limits
Gradual change in level
Systematic variation
Trends
Mixtures
Abnormal fluctuations

Most of these will be illustrated later. Some unnatural patterns are clearly defined by their names and may be recognised easily, others are less clear and need experience and expertise in order to recognise them. In these latter cases, a name plus an explanation is necessary to avoid confusion. For example, trend patterns are described by Walsh et al. [5] as "characterised by a strong movement of data, up or down".

On the other hand, patterns can also be defined in terms of a run of points and limits on a control chart. For example, a trend pattern is defined by Nelson as when "six consecutive points in a row steadily increase or decrease" [6,7]. Nelson provides tests, in this manner, to identify 8 patterns. A number of these refer directly to areas or zones of control charts. Fig. 1 shows a large number of control charts each divided into 6 zones (A, B, C, C, B, A). Each zone is one sigma wide. Thus the upper control limit (UCL) and the lower control limit (LCL) are set at three sigma above and three sigma below the centre-line (CL), respectively. The upper warning limit (UWL) and the lower warning limit (LWL) are at two

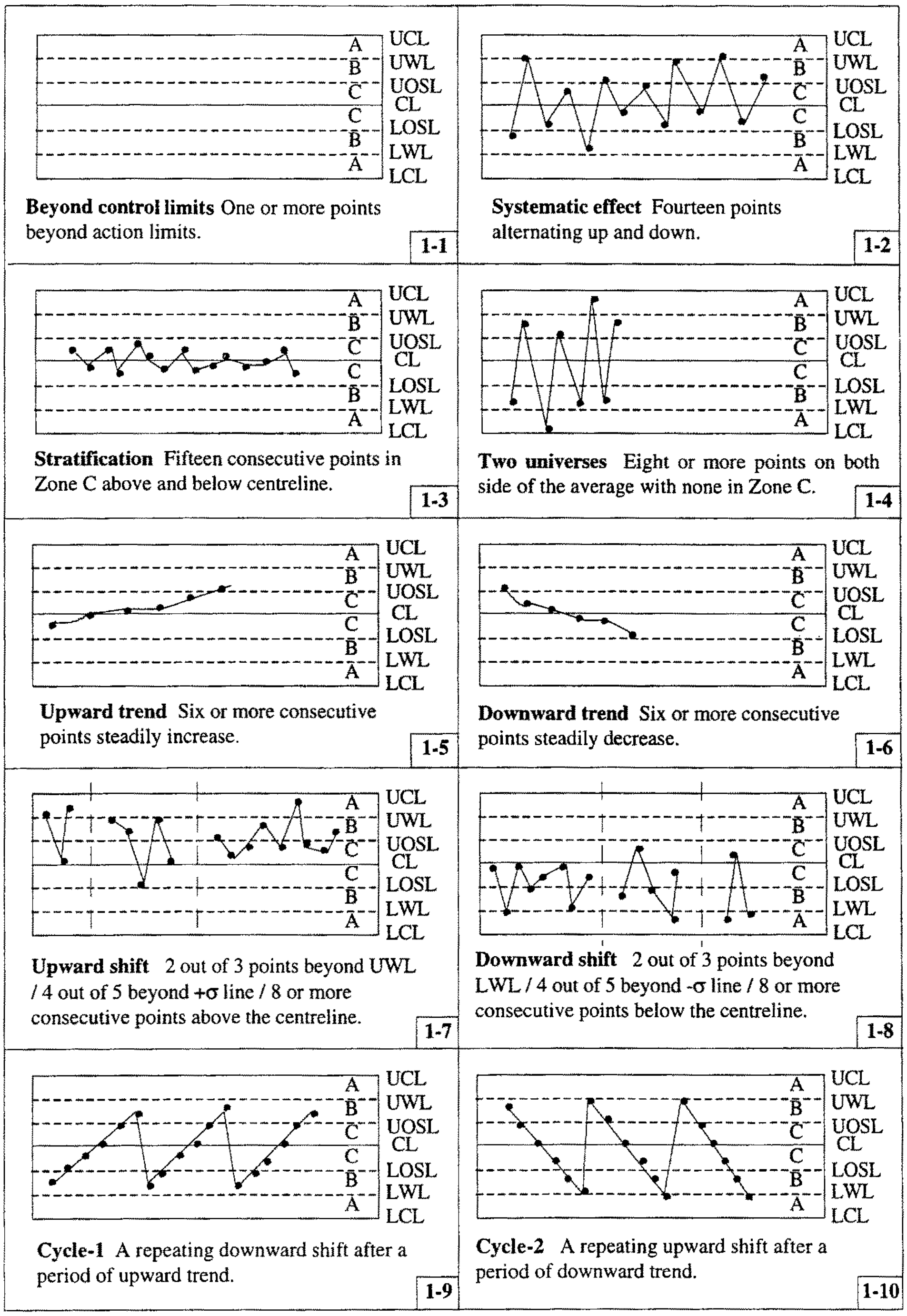
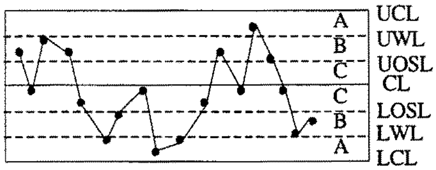


Fig. 1. Representation of pattern definitions.

sigma above and two sigma below the centre-line. The upper one-sigma limit (UOSL) and lower one-sigma limit (LOSL) are at one sigma above and one sigma below the centre-line. The points plotted on the control chart take the form of statistical values (i.e. the mean of a number of samples).

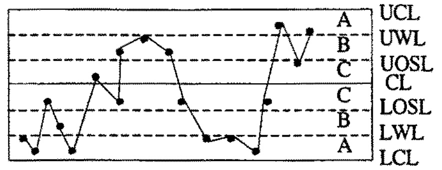
Nelson's eight tests are:

Test 1. One point beyond Zone A.
Test 2. Nine points in a row in either or both Zone Cs.
Test 3. Six points in a row steadily increasing or decreasing.
Test 4. Fourteen points in a row alternating up and down.
Test 5. Two out of three points in a row in Zone A.
Test 6. Four out of five points in Zone B.
Test 7. Fifteen points in a row in Zone C, above and below the centre-line.
Test 8. Eight points in a row on both sides of the centre-line with none in Zone C.



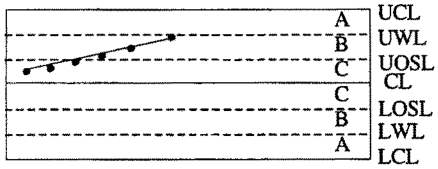
Cycle-3 Shift upward and downward alternatively.

1-11



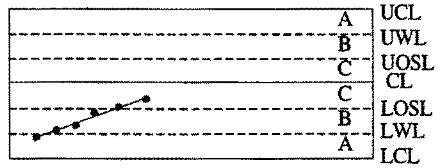
Cycle-4 Shift downward and upward alternatively.

1-12



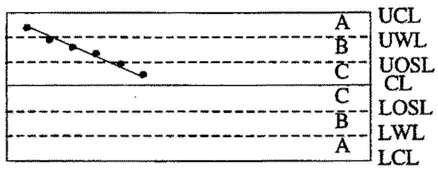
Mixture-1 Upward trend appears above the centreline.

1-13



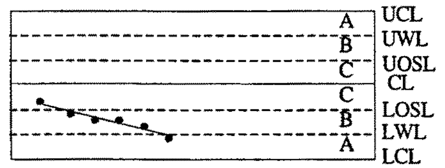
Mixture-2 Upward trend appears below the centreline.

1-14



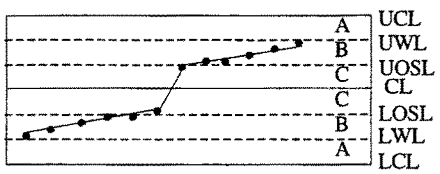
Mixture-3 Downward trend appears above the centreline.

1-15



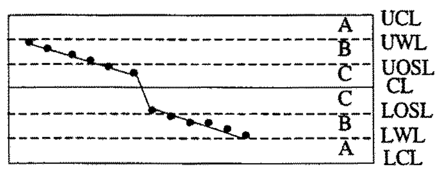
Mixture-4 Downward trend appears below the centreline.

1-16



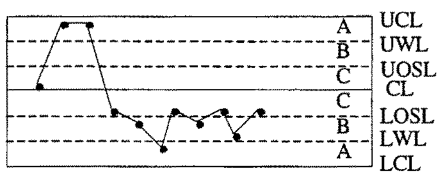
Mixture-5 Two upward trend patterns appear on both side of the centreline.

1-17



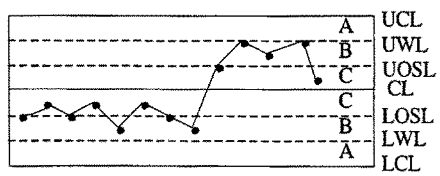
Mixture-6 Two downward trend patterns appear on both side of the centreline.

1-18



Mixture-7 Shift upward first then shift downward.

1-19



Mixture-8 Shift downward first then shift upward.

1-20

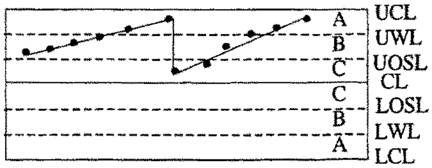
Fig. 1. Continued.

Although complex patterns of control charts are not included in the eight tests, Nelson's work provides a uniform and scientific basis for the consideration of more complicated patterns.

Both the descriptive and control chart methods of definition have their advantages. The descriptive form is generally easier to understand. The control-chart based form is more suitable and easier for pattern recognition by computers. In the work reported in this paper the Nelson approach has been used.

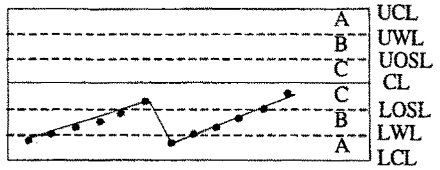
It is appropriate to note that the pattern directions are not explicitly specified in the literature. For example, trends can involve increase or decrease, shifts can be upwards or downwards. Information relating to pattern directions can be very helpful in the diagnosis of assignable causes because different directions may relate to different process situations.

In this paper, patterns will be classified as of two types: simple and superimposed and their direction will be taken into account. Based on Nelson's work, simple patterns are defined



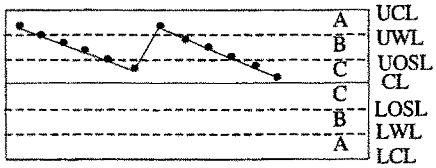
Complex-1 Upward trend and group shift down pattern appears above centreline.

1-21



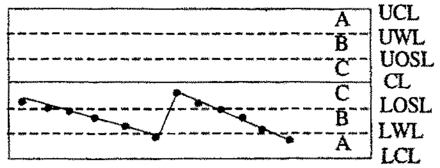
Complex-2 Upward trend and group shift down pattern appears below centreline.

1-22



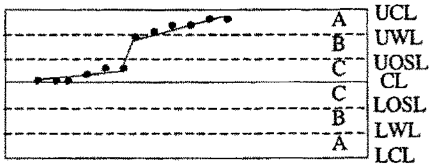
Complex-3 Downward trend and group shift up pattern appears above centreline.

1-23



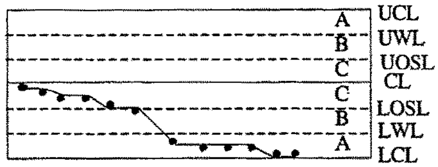
Complex-4 Downward trend and group shift up pattern appears below centreline.

1-24



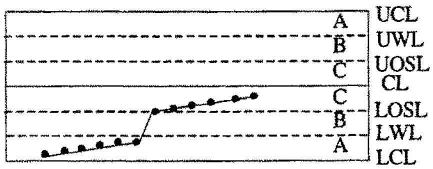
Complex-5 Upward trend and group shift up patterns appear above centreline.

1-25



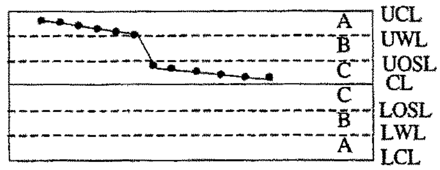
Complex-6 Downward trend and group shift down patterns appear below centreline.

1-26



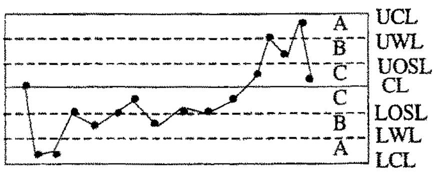
Complex-7 Upward trend and group shift up patterns appear below the centreline.

1-27



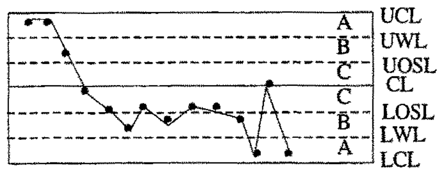
Complex-8 Downward trend and group shift down patterns appear above the centreline.

1-28



Multiple shift upward.

1-29



Multiple shift down.

1-30

Fig. 1. Continued.

as shown in Figs. 1–1 to 1–8. The eight simple patterns are listed as follows:

Beyond control limits. One or more points beyond control limits.

Systematic effects. Fourteen or more consecutive points alternating up and down.

Stratification. Fifteen or more consecutive points in Zone C above and below the centre-line.

Two universes. Eight or more consecutive points on both sides of the centre-line with none in Zone C.

Upward trend. Six or more consecutive points steadily increase.

Downward trend. Six or more consecutive points steadily decrease.

Upward shift. Two out of three points in a row above the upper warning limit/four out of five points in a row above the 1-sigma line/eight or more consecutive points in a row above the centre-line.

Downward shift. Two out of three points in a row below the lower warning limit/four out of five points in a row below the 1-sigma line/eight or more consecutive points in a row below the centre-line.

When combinations of two or more simple patterns occur frequently in a process, they produce superimposed patterns [8]. A superimposed pattern is a combination of two or more simple patterns. The superimposed patterns can be further divided into four subtypes, i.e. cycle, mixture, complex and multiple shift depending on their characteristics. Twenty-two superimposed patterns are illustrated in Figs. 1–9 to 1–30, using the subtype terms. These terms are now described.

Cycle patterns are formed by trends and shifts which repeat. They always have a repetition after a period of time. Cycle-1 (Fig. 1–9), for example, indicates a repeating downward shift after a period of positive trend.

Mixture patterns relate to situations where two process variations occur, without repetition, at the same time, e.g. Mixture-1 – positive bias and upward trend appear simultaneously. Or one variation occurs after another, e.g. Mixture-7 – an upward shift followed by a downward shift.

Complex patterns involve at least three process variations, e.g. Complex-1 – upward trend and downward shift appears above the centre-line.

Multiple shift patterns concern more than two shifts in one direction, either upward or downward.

There are twenty-two superimposed patterns which are combinations of the simple patterns, such as shifts and trends. These twenty-two patterns together with eight simple patterns are considered to be an adequate number for most pattern recognition tasks, being based as they are on Western Electric's and Nelson's work.

3. Pattern Recognition

Most of the research work on pattern recognition for control charts involves the application of either neural networks [9–11] or expert systems [12,13]. Neural network-based pattern recognisers consist of a number of interconnected processing elements that are usually called neurons. Each neuron receives a number of input signals that are assigned a relative weight. The effective input to the neuron is the weighted sum of the input signals. The output of a neuron is determined by an activation function. The output may be sent to another neuron or to itself as input signal through interconnections. The significant characteristics and difficulties associated with the use of neural networks for pattern recognition are:

Neural networks need to be trained to recognise and distinguish between patterns. This requires the use of large numbers of samples. The performance of a neural network based pattern recogniser depends to a large extent on the characteristics of the training samples. This can be a problem for superimposed patterns because these contain the elements of their constituents. Thus, superimposed patterns require a neural network to recognise two phenomena at once whereas they are usually trained to distinguish between phenomena.

The information contained in neural networks is implicit and virtually inaccessible. A problem arises in justifying how a particular conclusion is reached.

There is no systematic way to select an appropriate topology and structure for a neural network-based pattern recogniser.

The expert system-based pattern recognisers classify patterns from control charts based on rules provided by a domain expert. These recognition rules will have some similarity to the algorithmic specifications given in this paper. The significant characteristics and difficulties associated with the use of expert system-based pattern recognition are:

Each rule is stand alone but will invariably depend on other rules to be satisfied. The modularity of an expert system is such that it must be possible to add new rules, which in theory, should not affect existing rules. While this can be done, the interaction between rules becomes increasingly difficult to anticipate. The superimposed patterns will pose a problem for expert systems, for the same reason as they pose problems for the neural networks.

The information can easily be updated or modified, but, for the reasons just given, this is not always as helpful as it might seem.

The information encapsulated in the recognition procedure has an explicit nature and it is usually possible to find out how a conclusion has been reached. It is not possible, however, to check if other equally valid conclusions could have been reached.

The rules and/or recognition procedures used in expert system-based pattern recognisers have to be elicited from a human expert. This is usually a complex and time-consuming process.

A disadvantage of both expert-system based and neural network-based pattern-recognition systems is that they appear to be capable only of detecting simple patterns such as trend, shift and cycle. This statement is made based on a study of the available literature which does not identify any pattern recognition systems which can recognise superimposed patterns defined as a combination of simple patterns [9–11]. The reason for this may be explained by the problems just discussed.

For the research reported in this paper, it was decided to investigate whether an algorithmic approach would provide a better alternative to the neural-network and expert-system approaches, and whether it could also handle the superimposed patterns needed.

An algorithmic approach requires all the patterns to be specified mathematically. It requires the patterns to be distinguishable and for a logical sequence to exist to provide for the organisation of the algorithm.

3.1 Mathematical Pattern Specifications

The words and representative forms of the 30 patterns, shown in Fig. 1, were found to be expressible mathematically and examples of these for two simple and one superimposed patterns are now given in the format of mathematical pattern specifications, using the characters specified in the notation.

Pattern 5. Upward Trend

If $x_i < x_{i+1} < x_{i+2} < x_{i+3} < \ldots < x_{i+j}$ $(i = 1,2, \ldots, k-5)$

and if $j \geq 6$

then ptn = "Upward trend"

This specification refers to the definition "six or more consecutive points steadily increase" and can be easily implemented in a computer program.

Pattern 7. Upward Shift

$$\begin{aligned}
&\text{UWL} < x_i \leq \text{UCL} \cap \text{UWL} < x_{i+1} \leq \text{UCL} \cap x_{i+2} \leq \text{UWL} \\
&\quad \text{or} \\
\text{If} \quad &\text{UWL} < x_i \leq \text{UCL} \cap x_{i+1} \leq \text{UWL} \cap \text{UWL} < x_{i+2} \leq \text{UCL} \\
&\quad \text{or } (i = 1,2, \ldots, k-2) \\
&x_i \leq \text{UWL} \cap \text{UWL} < x_{i+1} \leq \text{UCL} \cap \text{UWL} < x_{i+2} \leq \text{UCL} \\
&\quad \text{or}
\end{aligned}$$

or

$$\begin{aligned}
&\text{LCL} \leq x_i \leq \text{UOSL} \cap \text{UOSL} < x_{i+1} \leq \text{UCL} \cap \text{UOSL} \\
&\quad < x_{i+2} \leq \text{UCL} \cap \text{UOSL} < x_{i+3} \leq \text{UCL} \cap \text{UOSL} \\
&\quad < x_{i+4} \leq \text{UCL} \qquad \text{or} \\
&\text{UOSL} < x_i \leq \text{UCL} \cap \text{LCL} \leq x_{i+1} \leq \text{UOSL} \cap \text{UOSL} \\
&\quad < x_{i+2} \leq \text{UCL} \cap \text{UOSL} < x_{i+3} \leq \text{UCL} \cap \text{UOSL} \\
&\quad < x_{i+4} \leq \text{UCL} \qquad \text{or} \\
&\text{UOSL} < x_i \leq \text{UCL} \cap \text{UOSL} < x_{i+1} \leq \text{UCL} \cap \text{LCL} \\
&\quad \leq x_{i+2} \leq \text{UOSL} \cap \text{UOSL} < x_{i+3} \leq \text{UCL} \cap \text{UOSL} \\
&\quad < x_{i+4} \leq \text{UCL} \qquad \text{or} \\
&\text{UOSL} < x_i \leq \text{UCL} \cap \text{UOSL} < x_{i+1} \leq \text{UCL} \cap \text{UOSL} \\
&\quad < x_{i+2} \leq \text{UCL} \cap \text{LCL} \leq x_{i+3} \leq \text{UOSL} \cap \text{UOSL} \\
&\quad < x_{i+4} \leq \text{UCL} \qquad \text{or} \\
&\text{UOSL} < x_i \leq \text{UCL} \cap \text{UOSL} < x_{i+1} \leq \text{UCL} \cap \text{UOSL} \\
&\quad < x_{i+2} \leq \text{UCL} \cap \text{UOSL} < x_{i+3} \leq \text{UCL} \cap \text{LCL} \\
&\quad \leq x_{i+4} \leq \text{UOSL} \qquad \text{or}
\end{aligned}$$

or

$$\text{CL} < \bigcap_{i}^{i+j} x_{i+j} \leq \text{UCL} \quad (i = 1,2, \ldots, k-j, \ \ 8 \leq j \leq k)$$

then ptn = "Upward shift"

It can be seen that this specification includes three sections joined by a logical operator "OR". The three sections reflect the definition for upward shift: "2 out of 3 points above upper warning limit or 3 out of 5 points above σ limit or 8 consecutive points above centre-line".

Pattern 13. Mixture-1

If $x_i < x_{i+1} < x_{i+2} < x_{i+3} < \ldots < x_{i+j}$ $(i = 1,2, \ldots, k-5)$

and if $j \geq 6$

and if $x_i > \text{CL}$

then ptn = "Mixture-1"

This specification refers to the definition "upward trend appears above the centre-line".

In this system, all the recognition algorithms are explicitly specified in ways similar to the examples above. The values of the variables for use in the specifications are obtained from the statistical calculations when developing control charts. These values are required to be accurate, thus the individual patterns can be classified to indicate the out-of-conditions accurately. This requires that the definition of patterns has to be on a scientific basis provided by Nelson's tests.

3.2 Software Implementation

Based on the mathematical/algorithmic specifications, a pattern-recognition system, which runs under MS-Windows environment, has been developed using Borland C++ 4.5, an object-oriented programming language. The system is designed to accept experimental data and perform statistical analysis, control charting and pattern recognition. The pattern recogniser analyses the control chart data for patterns. The process of pattern recognition is logically performed with regard to the pattern specification and follows the sequence illustrated in Fig. 2. From Fig. 2, it can be seen that the system looks for patterns in a carefully devised sequence and by matching the pattern specifications with the values plotted on the control chart. Once an unnatural pattern is detected, the system displays the pattern and indicates the starting point of the pattern. Examples of the operation of the system and the outputs are given later in the paper.

4. Noise and Noise Tolerance

The pattern definitions specify the particular relationships or shapes formed by the points plotted on the control charts. Every pattern has its own specific relationships that mean it can be identified and distinguished from others. These relationships are described in such a way that the nominal position of the points can vary within certain limits and the patterns still exist. For example a trend is usually not a straight line, rather the points involved in a trend pattern may lie on or above or below a straight line. The variation of the points involved in a pattern from the nominal can be termed as *pattern noise*. The pattern will effectively be destroyed as a pattern if the noise is too great. The allowable variation can be referred to as *noise tolerance*. To establish and test the sensitivity of the algorithmic approach to pattern recognition, the noise tolerance has been determined to find allowable values with resepect to different patterns. The full discussion and analysis of noise tolerance in relation to the recognition of all patterns is a topic beyond the scope of this paper; however, an example of evaluating the noise tolerance for trend patterns follows.

An upward trend pattern is defined such that six points in a run must increase one by one, i.e. the second is bigger

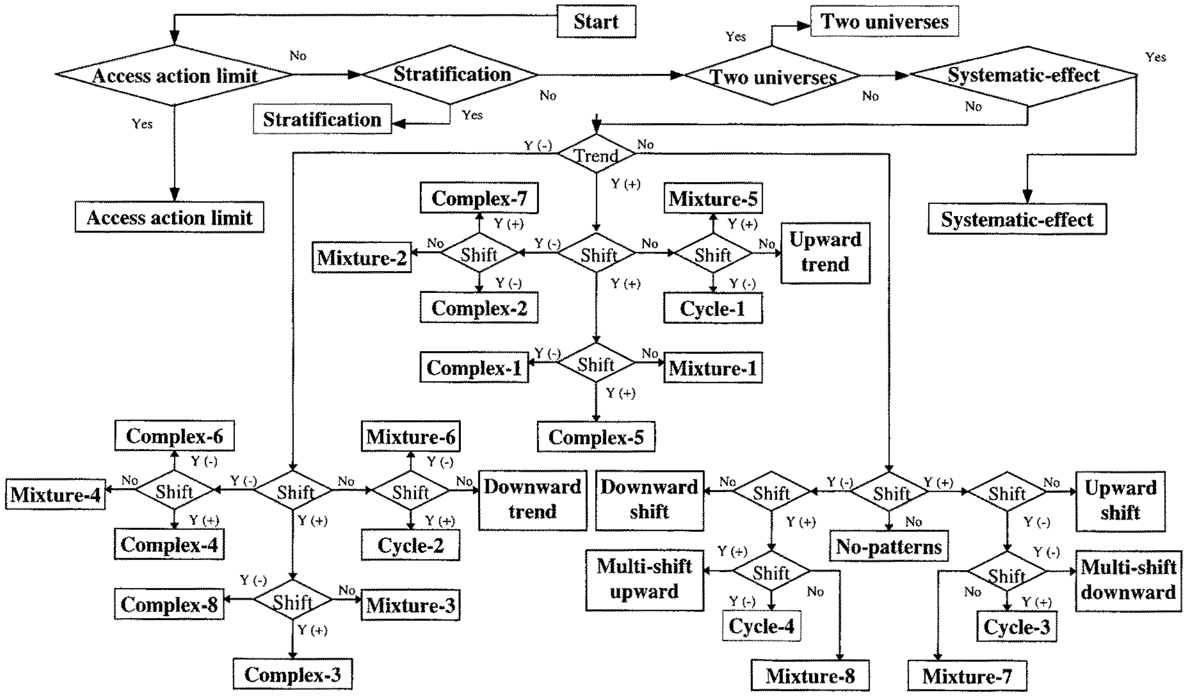


Fig. 2. The sequence of the pattern recognition system. +, upward; −, downward.

than the first, the third is bigger than the second, and so forth. Similarly, downward trend is defined so that six points in a run must decrease one by one. If any point in a run of six points does not satisfy these conditions, the patterns will not be identified. However, the degree of increment or decrement can vary from point to point. This variation is the noise in the trend patterns. The noise tolerance can be calculated by imposing a noise series on the nominal patterns.

Suppose the trend patterns specified by the following expressions satisfy two straight lines:

$$x_i < x_{i+1} < x_{i+2} < \ldots < x_{i+j} \quad \text{(for upward trend)}$$

$$x_i > x_{i+1} > x_{i+2} > \ldots > x_{i+j} \quad \text{(for downward trend)}$$

where

$i = 0,1, \ldots, k-j, \;\; 6 \le j \le k$

(k = total number of samples)

When imposing noise on the upward trend pattern, the following relationship for the patterns to be maintained must be satisfied:

$$x_i + N_i < x_{i+1} + N_{i+1} < x_{i+2} + N_{i+2} \tag{1}$$

$$(i = 1, 2, \ldots, k-j, \;\; 6 \le j \le k)$$

hence

$$x_i + N_i - x_{i+1} < N_{i+1} < x_{i+2} + N_{i+2} - x_{i+1} \tag{2}$$

$$(i = 1, 2, \ldots, k-j, \;\; 6 \le j \le k)$$

Thus, the noise tolerance, N_T, for the point x_{i+1} for an upward trend pattern is:

$$N_T = (x_{i+2} + N_{i+2} - x_{i+1}) - (x_i + N_i - x_{i+1})$$
$$= (x_{i+2} + N_{i+2}) - (x_i + N_i) \tag{3}$$
$$(i = 1,2, \ldots, k-l, \; l \ge 5)$$

If the elements in equation (3) are re-organised, the following noise tolerance applies:

$$N_T = (x_{i+2} - x_{i+1}) + (x_{i+1} - x_i) + (N_{i+2} - N_i) \tag{4}$$
$$(i = 1,2, \ldots, k-l)$$

For the same reason, the noise tolerance for the downward trend is as follows:

$$N_T = (x_i - x_{i+1}) + (x_{i+1} - x_{i+2}) + (N_i - N_{i+2}) \tag{5}$$
$$(i = 1,2, \ldots, k-l)$$

It is interesting to note that the values $x_{i+2} - x_{i+1}$ and $x_{i+1} - x_i$ in equation (4) represent the increment of an upward trend pattern and the values $x_i - x_{i+1}$ and $x_{i+1} - x_{i+2}$ in (5) represent the decrement of a downward trend pattern. Generally speaking, this increment or decrement varies from point to point. However, in an ideal situation the increment or the decrement from point to point can become a constant. This constant is normally called the slope. In mathematical terms, the noise tolerances for the upward shift and shift downward patterns can be described by equations (6) and (7), respectively.

$$N_T = 2slope + \zeta \text{ (for upward trend)}$$
$$i = 1,2, \ldots, k-l \tag{6}$$

$$N_T = 2slope - \zeta \text{ (for downward trend)} \tag{7}$$
$$i = 1,2, \ldots, k-l$$

where

$$\zeta = N_{i+2} - N_i \qquad slope = x_{i+2} - x_{i+1} = x_{i+1} - x_i$$
for equation (6)

$$\zeta = N_i - N_{i+2} \qquad slope = x_{i+1} - x_{i+2} = x_i - x_{i+1}$$
for equation (7)

Hence, the noise tolerances for trend patterns can be expressed as a single equation:

$$N_T = 2slope \pm \zeta \quad (i = 1,2, \dots, k - l) \tag{8}$$

Where "+" applies to upward trend patterns and "−" applies to the downward patterns.

Equation (8) indicates that the noise tolerance for trend patterns is directly related to the slope of the trend and the adjacent difference. It shows that trend patterns with bigger slopes are able to accept more noise than trend patterns with smaller slopes.

Figure 3 is a graphical illustration of the noise tolerance which is denoted as vertical lines. It can be seen from the figure that the point can be anywhere on the vertical line but not off it. That is the essence of the noise for trend patterns.

5. Pattern Simulation

In order to test the output of the pattern recognition system, numerous sets of data containing individual patterns are required. In the absence of actual useful industrial data for assessing the pattern recognition system, a simulator based on Monte Carlo techniques has been devised for generating raw data for assessment. The simulator aims to mimic the actual data in operation, storing the data in the same manner as the format required by control charts.

As mentioned earlier, all the complex patterns are formed by superimposing of shifts and trends. It is, therefore, relatively easy to generate different patterns. The key point is to generate shift and trend patterns. Subsequently, any other patterns can be created by the combinations of shifts and trends.

Upward shift and downward shift patterns indicate assignable process conditions under which the process average shifts upward or downward. This is automatically reflected on control charts in that some segment of the data will be plotted above or below the centre-line. Therefore, the variate for upward shift and downward shift patterns can be mathematically expressed as the following segment functions:

$$X = \begin{cases} \mu + \sigma\left(\sum_{i=1}^{12} r_i - 6\right) \pm C & \text{when } \begin{array}{l} l \le i \le m, \\ 0 \le l \le m \le k \end{array} \\ \mu + \sigma\left(\sum_{i=1}^{12} r_i - 6\right) & \text{otherwise} \end{cases} \tag{9}$$

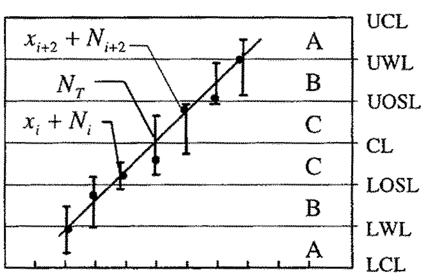


Fig. 3. Trend pattern on $\overline{X}$ chart.

where $+C$ applies to the upward shift pattern and $-C$ to the downward shift.

It is recommended that the value of C should be $-2.5\sigma \le C \le 2.5\sigma$, because any values beyond these limits may result in the X_i value simulated being greater than the value of UCL or smaller than the value of LCL. Normally, satisfactory results can be achieved with the value $C = \pm\,\sigma$.

Upward trend and downward patterns can be simulated using the following relationships:

$$X = \begin{cases} \pm Ax_i + B & \text{when } \begin{array}{l} l \le i \le m, \\ 0 \le l \le m \le k \end{array} \\ \mu + \sigma\left(\sum_{i=1}^{12} r_i - 6\right) & \text{otherwise} \end{cases} \tag{10}$$

where A is the slope and B is a constant.

For pattern simulation purposes, it is suggested that $A = 0.1 \sim 0.5 \times \sigma$. Since the width of the control chart is $6 \times \sigma$ and the length of trend patterns is at least six points, a bigger A will enable the points to be plotted beyond the control limits. When $A = 0.1 \times \sigma$, quite satisfactory results are obtained. The trend pattern becomes a horizontal line when A is equal to zero. $+A$ applies to an upward trend pattern and $-A$ applies to a downward trend.

The parameter B is very important in trend pattern simulation. It affects the position of the trend pattern on control charts.

When $+A$ Applies. If $-\sigma \le B \le \sigma$, the pattern, in the form of an upward trend, will be plotted around the centre-line. If $B > \sigma$, the trend pattern would be plotted above the centre-line, which means the process mean shifts upward. This is a pattern called Mixture-1 – upward shift + upward trend. If $B < -\sigma$, it indicates that an upward trend pattern and a downward shift pattern is exhibited at the same time. This is a pattern called Mixture-2 – downward shift + upward trend.

When $-A$ Applies. If $-\sigma \le B \le \sigma$, the pattern, in the form of a downward trend, will be plotted around the centre-line. If $B > \sigma$, the trend pattern would be plotted above centre-line, which means the process mean shifts upward. This is a pattern called Mixture-3 – upward shift + downward trend. If $B < -\sigma$, it indicates that a downward trend and a downward shift happen at the same time. This pattern is called Mixture-4 – downward shift + downward trend.

6. Examples

Several example applications demonstrating "Cycle-1", "Upward trend", "Multiple downward shift", "Mixture-7" and "Downward shift" patterns displayed on $\overline{X}$ chart have been created via simulation techniques. The example applications are illustrated in Figs. 4 to 10. The figures are plotted using thirty test results each. The expected mean and standard deviation is about 30 and 3. Each figure contains two parts,

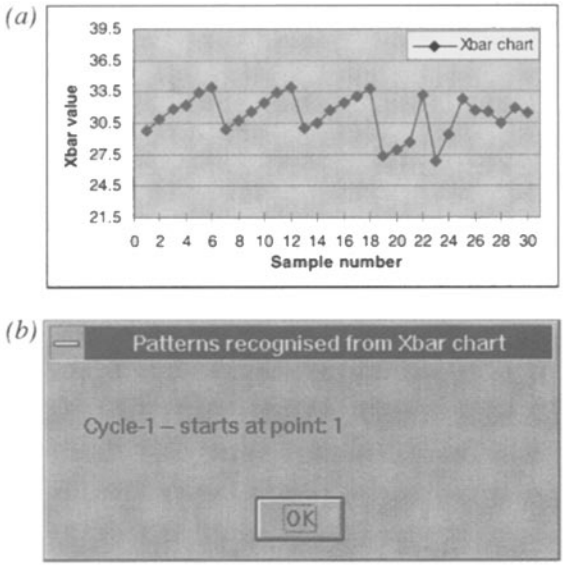


Fig. 4. Cycle-1 pattern identified. (*a*) The pattern on $\overline{X}$ chart. (*b*) Recognition display window.

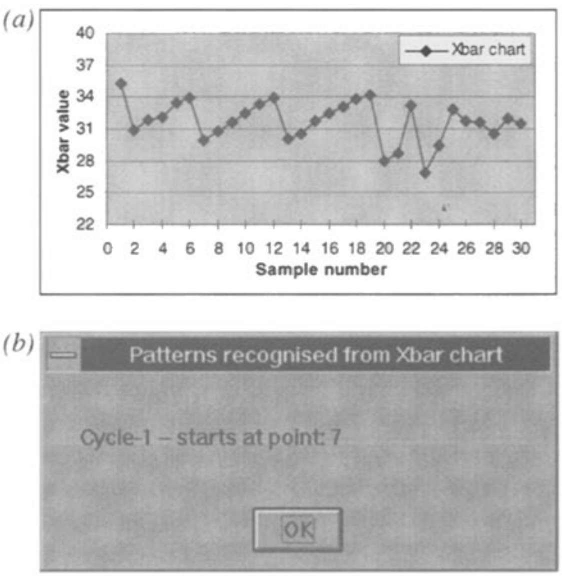


Fig. 5. A Cycle-1 pattern starting at point 7. (*a*) Cycle-1 pattern on $\overline{X}$ chart. (*b*) Recognition result display window.

(*a*) and (*b*). Part (*a*) shows the plotted control charts and part (*b*) is a window used to display the recognition results.

Cycle-1 is a superimposed pattern formed by at least two upward trend patterns. As can be seen in Fig. 4, a Cycle-1 pattern starting at point one is identified on the $\overline{X}$ chart. This pattern contains three trend patterns, one appears after another. Therefore, it has the characteristics of cycles. Each trend pattern contains at least six consecutive points increasing one by one. If the increasing points are less than six, the system will not treat them as a trend although they are increasing one by one. Points 19 to 22 on the chart are not recognised as an upward trend because there are only four increasing points. Similarly, points 25 to 28 are not recognised

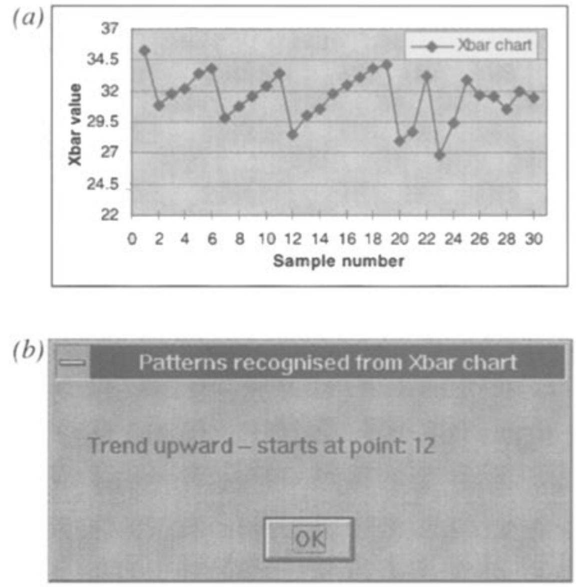


Fig. 6. An upward trend pattern recognised. (*a*) Upward trend pattern on $\overline{X}$ chart. (*b*) Recognition result display window.

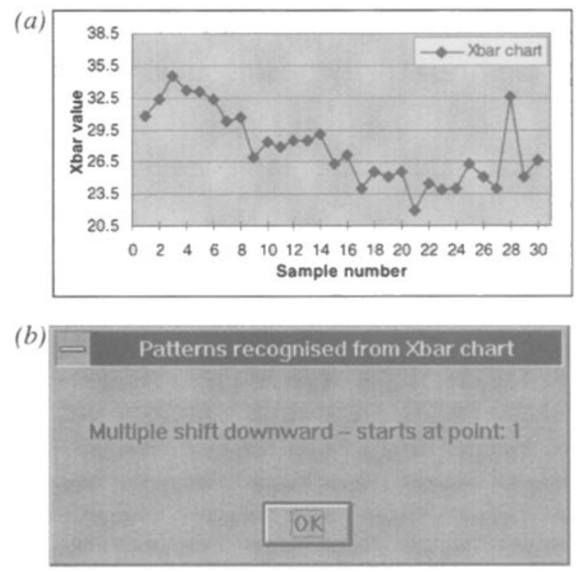


Fig. 7. A multiple downward shift pattern. (*a*) The pattern on $\overline{X}$ chart. (*b*) Recognition result display window.

as a downward trend because there are only four decreasing points. When trend patterns are recognised, the recogniser will automatically count the number of trend patterns. If more than two trend patterns occur one after another, the system will indicate cycle patterns.

If the patterns displayed in Fig. 4 are disturbed by noise such that the first point, for example, cannot be considered part of the rising trend, then the first trend will not be recognised by the system because it has less than the required six points. This condition is illustrated in Fig. 5, an example of a Cycle-1 pattern which contains two upward trend patterns, that is one pitch of a cycle. Fig. 5(*b*) indicates that

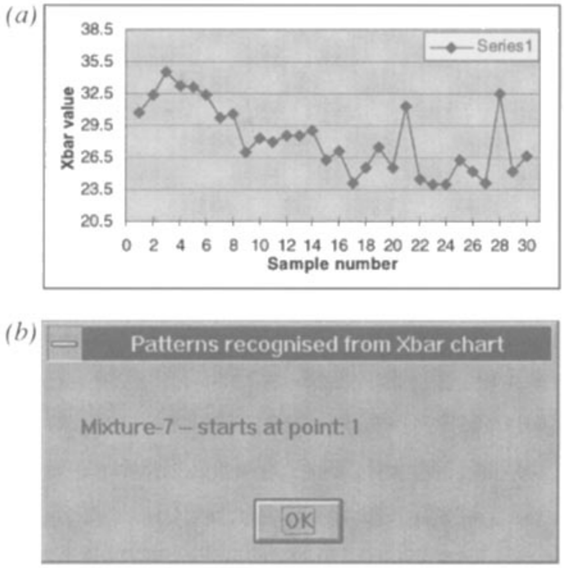


Fig. 8. Mixture-7 pattern identified. (*a*) The pattern on $\overline{X}$ chart. (*b*) Recognition display window.

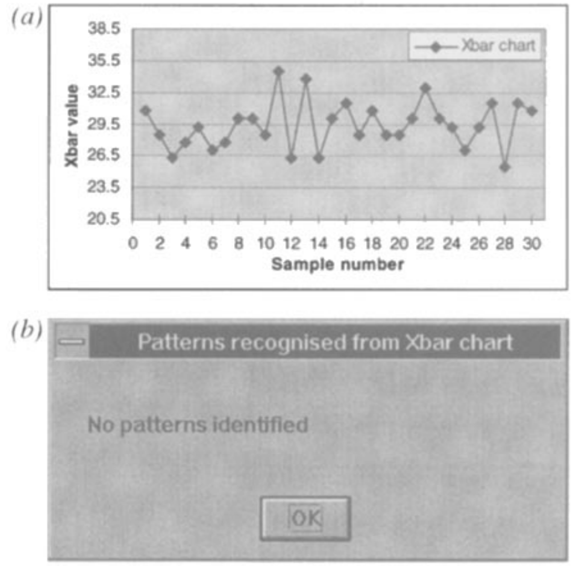


Fig. 10. No unnatural patterns identified. (*a*) $\overline{X}$ chart. (*b*) Display window.

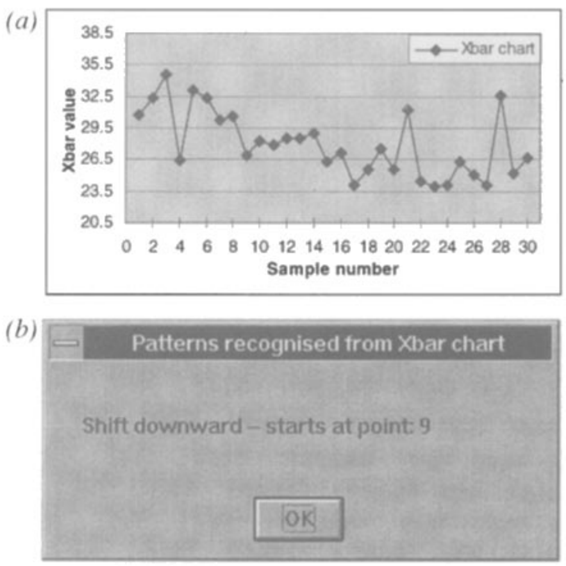


Fig. 9. A downward shift pattern recognised. (*a*) The pattern on $\overline{X}$ chart. (*b*) Recognition result display window.

Cycle-1 starts from point 7. It can be clearly seen that points 2 to 6 are not counted because there are only five points. However, points 13 to 19 are counted because there are seven increasing points.

If only one trend pattern is identified, the system will indicate a trend pattern, as can be seen in Fig. 6. As Fig. 6(*b*) indicates, the upward trend pattern starts at point 12. That trend contains eight consecutive increasing points. Points 2 to 11 are not treated as trends.

A multiple downward trend pattern which consists of three shifts is illustrated in Fig. 7. In this case, a group containing more than eight points shifts downward two or more times. Each shift in this case should consist of at least eight points otherwise the shift cannot be recognised. The multiple downward shift pattern starts from the first point on the chart. It can be seen from Fig. 7(*a*) that there are three sections in the multiple downward pattern, that is, points 1 to 8 plotted above the centre-line, points 9 to 16 below the centre-line and points 17 to 27 below the lower warning limits. The first and second sections each comprise eight consecutive points. The third section, however, contains eleven points, i.e. more than eight points. If the third section is disturbed by noise which would result in point 21 being above the centre-line, as shown in Fig. 8(*a*), then the third section no longer contains at least eight points. Hence, the pattern recogniser will classify the condition as Mixture-7.

Also in Fig. 8, Mixture-7 starting at point 1 comprises two sections, i.e. the points 1 to 8 shifting upward and points 9 to 20 shifting downward. The upward shift consists of eight consecutive points above the centre-line and the downward shift consists of 12 consecutive points below it. If the first section in Fig. 8(*a*) has fewer than eight points, the upward shift is disturbed and will not be recognised. This situation is shown in Fig. 9(*a*). The pattern recogniser, however, still recognises the downward shift pattern as illustrated in Fig. 9(*b*). This downward shift pattern starts at point 9 and contains twelve points.

The pattern recognition system can also indicate the condition that no unnatural patterns are identified (see Fig. 10).

7. Concluding Remarks

This paper has described a mechanism and algorithms based on mathematical analysis for supporting statistical process control activities. The mechanism and algorithms use the statistical values associated with control charts. A pattern-recognition system for process control charts has been developed which runs under the MS-Windows environment.

The pattern-recognition system looks for both simple patterns and superimposed patterns to indicate process abnormalities. The system views the simple patterns, such as shift and trend as components which may be combined together to form a superimposed pattern, such as mixture, cycle and complex. Once a superimposed pattern is identified, the simple patterns which form the superimposed pattern are just treated as components rather than independent patterns. Only when they are not a part of a superimposed pattern, will the simple patterns be classified as patterns in their own right.

Each pattern has an explicit definition associated with a minimum number of points of a run. If the number of points in a run is smaller than the minimum number, the run will not be identified as a pattern or a part of a pattern. If the number of points is equal to or greater than the minimum number, the run is classified as a pattern or a part of a pattern.

The pattern recognition system has been tested including the determination of noise tolerances, that is, when patterns lose their characteristics and therefore cannot be recognised as patterns. The system has been shown to be capable of recognising not only when one portion of a superimposed pattern loses its characteristics, but that the remaining portions may be classified as another pattern by the system with respect to the characteristics displayed. The overall system performance related to the effect of pattern noise and the noise tolerance associated with individual patterns have been evaluated using simulated data.

Like expert system-based pattern recognition, the pattern-recognition algorithms and procedures encapsulated in this system are explicit in nature. Unlike expert systems, this system is automatic in execution in identifying patterns from control charts and in displaying the recognition results. Some knowledge of statistical control concepts is required to understand the information provided by the system.

This system has also been argued to have benefits when compared with neural networks-based pattern recognisers. Its performance for classifying patterns depends on the mathematical algorithms, using the sample values and the statistical values associated with control charts, whilst neural networks' performance depends on training samples. For this reason, the recognition results reached by this system will be more reliable than that reached by neural networks. This is an advantage over neural networks which is especially relevant for superimposed patterns.

The system has been assessed via the application of simulation techniques. It shows considerable potential for use in real statistical process control applications.

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Notation

N_i	*i*th value of the noise series
N_T	noise tolerance
x_i	*i*th data item from a number sequence
r_i	seed for random number simulation
ζ	adjacent difference
σ	standard deviation
μ	mean of the data
A	slope of a straight line
B	constant
C	constant
i	indexing integer
j	indexing integer
k	total number of samples
l	starting point of a pattern on control chart
m	ending point of a pattern on control chart
n	size of samples
ptn	pointer to the pattern identified
slope	slope for trend patterns
X	normally distributed variate arising from simulation
CL	centre-line
LCL	lower control limit
LOSL	lower one-sigma limit
LWL	lower warning limit
UCL	upper control limit
UOSL	upper one-sigma limit
UWL	upper warning limit