The effect of competition in contests: A unifying approach*

Andrzej Baranski† Sumit Goel‡

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Abstract

We study rank-order contests with finite type spaces and establish the winner-takes-all contest as robustly optimal: it maximizes the total effort of the top q agents, for any q, under linear, concave, and even moderately convex cost functions—thereby resolving an open question in contest design. At the same time, the effect of competition is nuanced. We uncover an *interior discouragement effect*: shifting value toward better-ranked prizes may reduce effort when inefficient types are relatively likely. An experiment provides qualitative support for these findings. Methodologically, our analysis develops a novel approach based on characterizing symmetric equilibria through the probability of outperforming an arbitrary agent. The representation is broadly applicable and provides a unifying lens that reconciles contrasting results across complete-information and continuum type-space environments, for which we also establish an equilibrium convergence result.

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†NYU Abu Dhabi; a.baranski@nyu.edu

‡NYU Abu Dhabi; sumitgoel58@gmail.com; 0000-0003-3266-9035

1 Introduction

A central question in contest theory is how prize structures shape agents' incentives to exert effort, and in particular, which structures maximize effort. Under complete information, optimal contests typically feature multiple prizes, allocated in minimally competitive ways (Barut and Kovenock (1998); Fang, Noe, and Strack (2020)). By contrast, in incomplete information settings with a continuum of types, the most competitive winner-takes-all structure is frequently optimal (Moldovanu and Sela (2001); Zhang (2024)). Yet, what drives this divergence remains unclear, and it is an open question which (if any) of these findings extend to the intermediate and fundamental case of a finite type-space. This gap was also noted in a survey article by Sisak (2009), who conjectured that multiple prizes might be optimal:

> *"The case of asymmetric individuals, where types are private information but drawn from discrete, identical or maybe even different distributions, has not been addressed so far. From the results ... on asymmetric types with full information, one could conjecture that multiple prizes might be optimal even with linear costs."*

In this paper, we address this question by studying rank-order contests where ex-ante symmetric agents have private abilities drawn from a finite type-space. The finite type-space framework embeds the complete information as a special case and can approximate any continuum type-space. Thus, our analysis not only bridges a gap in the literature, but provides a unifying approach offering insights into the contrasting results in these extreme environments. Beyond its theoretical appeal, this framework is practically relevant, can accommodate richer non-parametric type-spaces, and enables experimental investigation.

We begin by characterizing the unique symmetric equilibrium of the Bayesian contest game. We show that equilibrium is in mixed strategies and exhibits a monotonic structure: different types randomize over disjoint but contiguous effort intervals, with more efficient types always outperforming less efficient ones. To overcome the analytical complexity of this mixed equilibrium, we introduce a novel representation that characterizes equilibrium effort through the ex-ante probability of outperforming an arbitrary agent. This representation is broadly applicable and may prove valuable in other environments where mixed equilibria hinder analysis. We use it to study how increasing competition by shifting value from lower-ranked prizes to better-ranked prizes affects equilibrium effort. We first fully characterize these effects under linear costs and then identify conditions under which they extend qualitatively to equilibrium under more general cost functions.

Under linear costs, we show that shifting value to the best prize always encourages effort. Intuitively, such a transformation creates strong incentives for the most efficient types, who are cheapest to incentivize, and this encouragement effect more than compensates for the discouragement it induces among less efficient types. Consequently, the winner-takes-all contest maximizes total effort under linear costs, resolving Sisak (2009)'s conjecture in the negative. Moreover, when the designer instead cares about the total effort of the top q agents, the objective effectively places greater weight on the effort of more efficient types. Since winner-takes-all disproportionately strengthens incentives precisely for these types, its optimality persists under this broader objective.

This conclusion extends to concave cost functions as well. Under concavity, marginal effort costs are decreasing, making it even cheaper to induce higher effort levels from efficient types. As a result, shifting value to the best prize further amplifies effort where it is most cost-effective, reinforcing the optimality of the winner-takes-all contest. We therefore establish that winner-takes-all maximizes the total effort of the top q agents, for any q, under both linear and concave costs. These results extend the optimality of winner-takes-all established in continuum type-space models (Moldovanu and Sela (2001)) to finite type spaces, while strengthening it to more general objectives that place greater weight on the effort of higher-performing agents.

This intuition, however, does not extend directly to convex costs. When costs are convex, marginal effort costs are increasing, so inducing additional effort at high levels becomes progressively more expensive. As a result, shifting value to the best prize need not encourage effort, since stronger incentives for the efficient types must now be weighed against rising marginal costs. Nevertheless, we show that winner-takes-all optimality persists under moderate convexity. As long as costs are not too convex, the encouragement effect of shifting value to the top prize outweighs the increase in marginal costs. Our analysis further suggests that the degree of convexity required to overturn winner-takes-all optimality shrinks as the environment approaches complete information, where, in contrast, the winner-takes-all contest actually minimizes total effort. In this sense, our analysis provides a unified perspective that reconciles contrasting results in the complete information and incomplete information environments.

Despite the winner-takes-all contest being robustly optimal, it is not generally the case that shifting value toward better-ranked prizes increases effort. We identify an *interior discouragement effect*: shifting value from lower-ranked prizes to better-ranked intermediate (but not top) prizes may reduce aggregate effort when inefficient types are sufficiently likely. Intuitively, such transformations discourage inefficient types—much like shifting value to the top prize—but the encouraging effect on efficient types is muted, relative to the top prize. This is because, as a result of this transformation, efficient types have weaker incentives to exert effort in pursuit of the best prize. Consequently, the encouragement effect on the most efficient types is dampened, and their effort may even decline. When inefficient types are sufficiently prevalent, the discouragement effect dominates and total effort falls.

Our finite type-space framework enables direct experimental testing of the model's predictions. Since implementing a continuum of types is infeasible, prior experiments have relied on large but finite type-spaces, assuming equilibrium properties extend from the continuum setting (Müller and Schotter (2010)).[1] While our convergence result justifies this assumption, our experiment employs a simple binary type-space with linear costs where the inefficient type is relatively likely. We vary the prize structure by gradually increasing competitiveness across four contests, including winner-takes-all. The results reveal an over-provision of effort, particularly by the inefficient type, but aggregate patterns broadly align with our comparative statics: winner-takes-all remains optimal, and the interior discouragement effect receives partial support. Specifically, although efforts do not decline as theory predicts, we observe no significant increase in effort when competition intensifies in the interior.

Related literature

The existing game-theoretic literature in contests has predominantly focused on the design problem in environments where the type-space is either a continuum, or a singleton (the complete information case), and the results highlight how the structure of the optimal contest can vary significantly depending on the environment. For the continuum type-space, the most competitive winner-takes-all contest has been shown to be optimal under linear or concave costs (Moldovanu and Sela (2001)), in some cases under convex costs (Zhang (2024)), with negative prizes (Liu, Lu, Wang, and Zhang (2018)), and with general archi-

[1] Rasooly and Gavidia-Calderon (2020); Swarthout and Walker (2009) illustrate how the set of equilibria can differ qualitatively between finite and continuous environments in the all-pay auction and the Groves–Ledyard mechanism, respectively.

tectures (Moldovanu and Sela (2006); Liu and Lu (2014)). In comparison, in the complete information environments, the minimally competitive budget distribution (all agents but one receive an equal positive prize) has been shown to be a feature of the optimal contest quite generally (Barut and Kovenock (1998); Letina, Liu, and Netzer (2023, 2020); Xiao (2018)). In a general framework with many agents, Olszewski and Siegel (2016, 2020) show that awarding multiple prizes of descending sizes is optimal under convex costs. Other related work has examined the effect of competition in complete information setting (Fang, Noe, and Strack (2020)), and continuum type-space setting (Goel (2025); Krishna, Lychagin, Olszewski, Siegel, and Tergiman (2025)).[2]

There is a related literature on contests with a finite type-space, much of which assumes binary type-spaces or a small number of agents and focuses on characterizing equilibrium properties under correlated or asymmetric types. Siegel (2014) establishes the existence of a unique equilibrium under general distributional assumptions. With correlated types, Liu and Chen (2016) show that the symmetric equilibrium may be non-monotonic when the degree of absolute correlation is high, Rentschler and Turocy (2016) highlight the possibility of allocative inefficiency in equilibrium, while Tang, Fu, and Wu (2023) and Kuang, Zhao, and Zheng (2024) explore the impact of reservation prices and information disclosure policies, respectively. With asymmetric type distributions, Szech (2011) shows that agents may benefit from revealing partial information about their private types, while Chen (2021) characterizes equilibrium outcomes for varying levels of signal informativeness.[3]

There is a long tradition of studying contests through incentivized laboratory experiments (see Dechenaux, Kovenock, and Sheremeta (2015) for a survey). Perhaps the most closely related work is Müller and Schotter (2010), who provide evidence broadly consistent with the predictions of Moldovanu and Sela (2001): winner-takes-all is optimal under linear costs,

[2] In early work, Glazer and Hassin (1988) highlight the distinction between the two environments by solving the problem in some special cases. Other related studies include Schweinzer and Segev (2012); Drugov and Ryvkin (2020) who examine the budget allocation problem under different contest success functions. For general surveys of the literature in contest theory, see Corchón (2007); Sisak (2009); Konrad (2009); Vojnović (2015); Fu and Wu (2019); Chowdhury, Esteve-González, and Mukherjee (2023); Beviá and Corchón (2024).

[3] Other related work has studied imperfectly discriminating contests (Ewerhart and Quartieri (2020)), contests with altruistic or envious types (Konrad (2004)), and common value all-pay auctions with private asymmetric information (Einy, Goswami, Haimanko, Orzach, and Sela (2017)). There is also some work in mechanism design and auction design with finite type-spaces (Maskin and Riley (1985); Jeong and Pycia (2023); Vohra (2012); Lovejoy (2006); Doni and Menicucci (2013); Elkind (2007)).

whereas splitting prizes is favored under convex costs. Two other studies, Barut, Kovenock, and Noussair (2002) and Noussair and Silver (2006), examine all-pay auctions with private valuations, a setting strategically equivalent to all-pay contests with private costs. All three experiments employ large finite type-spaces to approximate continuum-type equilibria, an assumption that our convergence result formally justifies. Like us, these studies observe substantial overbidding relative to theoretical benchmarks. A key difference, however, is that low-valuation (or less efficient) agents in their settings tend to underbid, whereas in our experiment, inefficient types significantly overbid, driving much of the aggregate over-provision of effort.[4]

2 Model

Game

A *contest environment* is a tuple $(N+1, \Theta, p)$, where

- $N+1$ is the number of agents,
- $\Theta = \{\theta_1, \ldots, \theta_K\}$ is a finite set of types, with $\theta_1 > \cdots > \theta_K$, and
- $p = (p_1, \ldots, p_K)$ is a probability distribution over Θ.

Each of the $N+1$ agents has a private type $\theta \in \Theta$, which represents the agent's marginal cost of exerting effort. The types are drawn independently according to p. We let $P_k = p_1 + \cdots + p_k$.

A *contest* $v = (v_0, \ldots, v_N)$ assigns a prize value to each rank, with $v_0 \leq \cdots \leq v_N$ and $v_0 < v_N$.

Given a contest environment $(N+1, \Theta, p)$, contest v, and their private types, the agents simultaneously choose their effort. They are ranked according to their efforts, with ties broken uniformly at random, and awarded the corresponding prizes. An agent who outperforms exactly $m \in \{0, \ldots, N\}$ out of the N other agents is awarded the prize v_m. If an agent of type $\theta \in \Theta$ wins prize v_m after exerting effort $x \geq 0$, their vNM utility is

$$v_m - \theta x.$$

[4]In other related work, Brookins and Ryvkin (2014) compares behavior under complete and incomplete information in Tullock contests.

The Bayesian game induced by v is strategically equivalent to the game induced by the contest w where $w_m = v_m - v_0$ for all $m \in \{0, \ldots, N\}$. We normalize $v_0 = 0$, and focus on contests in the set

$$\mathcal{V} = \{v \in \mathbb{R}^{N+1} : v_0 \leq v_1 \leq \cdots \leq v_N \text{ where } 0 = v_0 < v_N\}.$$

Equilibrium

We focus on symmetric Bayes-Nash equilibria. A symmetric Bayes-Nash equilibrium is a strategy profile in which all agents use the same strategy—a mapping from types to (possibly random) effort—such that, for every agent and every type $\theta \in \Theta$, the prescribed effort maximizes expected utility given that all other agents follow the same strategy. We denote a symmetric Bayes-Nash equilibrium by $(X_1, X_2, \ldots, X_K)$, where $X_k \sim F_k$ is the effort exerted by an agent of type θ_k. We further denote by $X \sim F$ the ex-ante equilibrium effort of an arbitrary agent, so that for any $x \in \mathbb{R}$, $F(x) = \sum_{k=1}^{K} p_k F_k(x)$. Accordingly, the expected equilibrium effort of an arbitrary agent is

$$\mathbb{E}[X] = \sum_{k=1}^{K} p_k \mathbb{E}[X_k].$$

Competition

We are interested in examining how increasing competitiveness of a contest influences the expected equilibrium effort. As is standard in the literature (Fang, Noe, and Strack (2020); Goel (2025)), we define a contest $v \in \mathcal{V}$ as being *more competitive* than $w \in \mathcal{V}$ if the prizes are more unequal, measured using the Lorenz order, i.e.,

$$\sum_{i=0}^{m} v_i \leq \sum_{i=0}^{m} w_i \text{ for all } m \in \{0, 1, \ldots, N\},$$

with equality for $m = N$.

Importantly, if v is more competitive than w, v can be obtained from w through a sequence of transfers from lower-ranked prizes to higher-ranked prizes. The marginal effect of such a transfer—from prize m' to prize m, with $m > m'$—on expected equilibrium effort is captured by

$$\frac{\partial \mathbb{E}[X]}{\partial v_m} - \frac{\partial \mathbb{E}[X]}{\partial v_{m'}}.$$

Our objective is to understand how this effect may depend on the pair of prizes, the structure of the contest, and the underlying contest environment.

We further discuss implications to the classical design problem of allocating a fixed budget $V \in \mathbb{R}_+$ across prizes to maximize effort. Notice that among all contests $w \in \mathcal{V}$ that distribute the entire budget, the winner-takes-all contest $v = (0, 0, \ldots, 0, V)$ is the most competitive, while the contest that allocates the budget equally among all but the worst-performing agent $v = (0, \frac{V}{N}, \ldots, \frac{V}{N})$ is the least competitive.

Notation

Suppose an agent outperforms each of the other N agents independently with probability $t \in [0, 1]$. Let

$$H^N_m(t) = \binom{N}{m} t^m (1-t)^{N-m}$$

denote the probability that the agent outperforms exactly m out of N agents. Define

$$H^N_{\leq m}(t) = \sum_{i=0}^{m} H^N_i(t) \text{ and } H^N_{\geq m}(t) = \sum_{i=m}^{N} H^N_i(t),$$

as the probabilities that the agent outperforms at most m and at least m agents, respectively. Finally, given a contest $v \in \mathcal{V}$, define

$$\pi_v(t) = \sum_{m=0}^{N} v_m H^N_m(t),$$

which is the expected prize value obtained by the agent under v.

3 Equilibrium

In this section, we characterize symmetric Bayes-Nash equilibria of the Bayesian game.

To begin, we establish a robust structural property: different agent types mix over contiguous intervals, with more efficient types choosing higher effort than less efficient ones.[5]

[5] Wang (1991) studies common-value auctions with finite signals and finds a similar equilibrium structure.

Lemma 1. *If $(X_1, \ldots, X_K)$ is an equilibrium, then there exist boundary points*

$$0 = b_0 < b_1 < \cdots < b_K$$

such that, for each k, X_k is continuously distributed on $[b_{k-1}, b_k]$.

In words, agents of the least efficient type θ_1 mix over the interval $[0, b_1]$, agents of type θ_2 mix over $[b_1, b_2]$, and so on, up to agents of the most efficient type θ_K, who mix over $[b_{K-1}, b_K]$. The supports are disjoint across types because moving from one effort level to another generates the same change in expected prize for all types, while the change in cost is type-dependent. Consequently, two distinct types cannot both be indifferent between the same pair of effort levels, implying that their supports can intersect only at boundary points. As a result, more efficient agents outperform less efficient agents. Mixing arises solely because agents may compete against others of the same type.

The mixing distributions must satisfy the indifference conditions. Notice that if a type-θ_k agent chooses effort $x_k \in [b_{k-1}, b_k]$, it outperforms an arbitrary agent with probability

$$P_{k-1} + p_k F_k(x_k).$$

Since the agent must be indifferent across all such effort levels, the equilibrium distribution F_k must satisfy, for all $x_k \in [b_{k-1}, b_k]$,

$$\pi_v\big(P_{k-1} + p_k F_k(x_k)\big) - \theta_k x_k = u_k, \tag{1}$$

where u_k denotes the equilibrium utility of type θ_k. While Equation (1) uniquely pins down F_k—and one can verify that the resulting distributions constitute an equilibrium—its implicit form makes the equilibrium analytically difficult to study.

We develop an alternative representation of the equilibrium that allows us to circumvent these analytical difficulties. Specifically, given a symmetric equilibrium $(X_1, X_2, \ldots, X_K)$, define a mapping

$$X_v : [0, 1] \to \mathbb{R}_+$$

that assigns to each probability $t \in [0, 1]$ of outperforming an arbitrary agent the corresponding equilibrium effort. From Lemma 1, probability $t = 0$ corresponds to zero effort, $t = 1$ corresponds to maximal effort b_K, and probability $t = P_k$ maps to the effort b_k. This

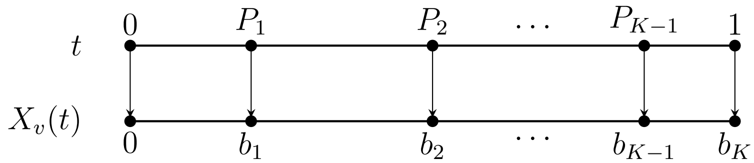


Figure 1: Symmetric equilibrium in terms of probability of outperforming an arbitrary agent.

is illustrated in Figure 1.

In general, the indifference condition in Equation (1) can be restated directly in terms of this representation. Specifically, the equilibrium effort schedule must satisfy, for all $t \in [P_{k-1}, P_k]$,

$$\pi_v(t) - \theta_k X_v(t) = u_k. \tag{2}$$

To complete the characterization, it remains to determine the equilibrium utilities. The following iterative argument, initiated by $b_0 = 0$, uniquely pins down utilities for all types:

$$\begin{aligned}
b_0 = 0 \Rightarrow u_1 &= 0 && \text{(type-}\theta_1\text{ agent's utility at } b_0) \\
\Rightarrow\ b_1 &= \frac{\pi_v(P_1) - u_1}{\theta_1} && (X_v(P_1) \text{ from Equation (2) using } u_1) \\
\Rightarrow\ u_2 &= \pi_v(P_1) - \theta_2 \cdot b_1 && \text{(type-}\theta_2\text{ agent's utility at } b_1) \\
\Rightarrow\ b_2 &= \frac{\pi_v(P_2) - u_2}{\theta_2} && (X_v(P_2) \text{ from Equation (2) using } u_2) \\
\Rightarrow\ u_3 &= \ldots
\end{aligned}$$

Theorem 1 (Equilibrium Characterization)**.** *For any contest environment* $(N+1, \Theta, p)$ *and contest* $v \in \mathcal{V}$*, there exists a unique symmetric equilibrium* $(X_1, \ldots, X_K)$*. It is such that, for each* $t \in [0, 1]$*,*

$$X_v(t) = \frac{\pi_v(t) - u_{k(t)}}{\theta_{k(t)}}, \tag{3}$$

where $k(t) = \max\{k : P_{k-1} \leq t\}$*, and the equilibrium utilities are given by*

$$u_k = \theta_k \left[\sum_{j=1}^{k-1} \pi_v(P_j) \left(\frac{1}{\theta_{j+1}} - \frac{1}{\theta_j} \right) \right]. \tag{4}$$

Remark 1. *The equilibrium under the finite-type space framework exhibits both the mixed structure characteristic of complete information environments (Barut and Kovenock (1998)) and the monotonic structure observed in environments with a continuum of types (Moldovanu* *and Sela (2001)). The complete information environment is clearly a special case of our model. We also establish an equilibrium convergence result for the continuum type-space environment (Theorem 6 in Appendix D), which implies that the (pure-strategy) equilibrium in any continuum type-space can be well-approximated by the equilibrium of a sufficiently large and appropriately chosen finite type-space. Intuitively, as the finite type-space becomes large, the interval over which an agent of a certain type mixes shrinks, and essentially converges to the effort level prescribed by the pure-strategy equilibrium under the continuum type-space.*

Remark 2. *The equilibrium characterization in Theorem 1 extends to environments in which the type space $\mathcal{C} = \{c_1, \ldots, c_K\}$ consists of cost functions. Assume that for each $k \in [K]$, $c_k(0) = 0$, c_k is strictly increasing and differentiable on $(0, \infty)$, and $\lim_{x\to\infty} c_k(x) = \infty$. Suppose further that types are ordered by marginal costs: for every $x > 0$,*

$$c_1'(x) > c_2'(x) > \cdots > c_K'(x).$$

Then there is a unique symmetric equilibrium in which types mix over disjoint intervals (ordered by type). Moreover, for each $t \in [0, 1]$,

$$X_v(t) = c_{k(t)}^{-1}\big(\pi_v(t) - u_{k(t)}\big),$$

where the equilibrium utilities are determined by the same iterative argument as above (with $b_0 = 0$).

This representation of symmetric equilibrium—formulated in terms of the probability of outperforming an arbitrary agent—provides a unifying framework across environments and is central to our subsequent analysis.

4 Competition

In this section, we examine how increasing competitiveness of a contest, by shifting value to better prizes, influences the expected equilibrium effort.

To begin, the equilibrium characterization in Theorem 1 yields a tractable expression for expected equilibrium effort. Since t denotes the probability of outperforming an arbitrary opponent, we can formally write $t = F(X)$, where $X \sim F$ is the equilibrium effort of an arbitrary agent. By the probability integral transform, $F(X)$ is uniformly distributed on

$[0, 1]$. Hence, t is ex-ante uniformly distributed on $[0, 1]$, and the expected equilibrium effort can be expressed as

$$\mathbb{E}[X] = \int_0^1 X_v(t)dt.$$

This implies that expected effort is linear in prize values. Solving the integral yields the corresponding coefficients.

Lemma 2. *The expected equilibrium effort is*

$$\mathbb{E}[X] = \sum_{m=1}^{N} \alpha_m v_m,$$

where

$$\alpha_m = \frac{1}{N+1}\left[\frac{1}{\theta_K} - \sum_{k=1}^{K-1}\left[H_{\geq m}^{N+1}(P_k) + (N-m)H_m^{N+1}(P_k)\right]\left(\frac{1}{\theta_{k+1}} - \frac{1}{\theta_k}\right)\right]. \tag{5}$$

From Lemma 2, the effect of increasing competition by shifting value from a lower-ranked prize m' to a better-ranked prize m is

$$\frac{\partial \mathbb{E}[X]}{\partial v_m} - \frac{\partial \mathbb{E}[X]}{\partial v_{m'}} = \alpha_m - \alpha_{m'},$$

which can be evaluated explicitly using Equation (5). Hence, the effect depends only on the contest environment $(N + 1, \Theta, p)$ and is independent of the specific contest $v \in \mathcal{V}$. In particular, if $\alpha_m - \alpha_{m'} > 0$, a budget-constrained designer seeking to maximize effort would continually shift value from prize m' to m subject to feasibility constraints. And if $\alpha_m - \alpha_{m'} < 0$, value would be shifted in the opposite direction.

Theorem 2 (Winner-takes-all is optimal)**.** *Consider any contest environment* $(N+1, \Theta, p)$ *with* $|\Theta| > 1$*. For any prize* $m' \in \{1, \ldots, N-1\}$*,*

$$\alpha_N - \alpha_{m'} > 0.$$

Consequently, among all contests $v \in \mathcal{V}$ *satisfying* $\sum_{m=0}^{N} v_m \leq V$*, the winner-takes-all contest* $v = (0, 0, \ldots, 0, V)$ *uniquely maximizes expected equilibrium effort.*

This result resolves the conjecture of Sisak (2009), who—based on results under complete information—suggested that allocating the budget across multiple prizes might be optimal in environments with finitely many types. To see the role of uncertainty, note that when $|\Theta| = K = 1$, we have $\alpha_m - \alpha_{m'} = 0$, so any budget allocation across prizes induces the same expected effort (Barut and Kovenock (1998)). When multiple types are possible, however, the most efficient types are the cheapest to incentivize, and shifting value to the top prize has an encouraging effect on efficient types that more than offsets the discouraging effect on less efficient types. This extends the optimality result of Moldovanu and Sela (2001), established for the continuum case, to environments with finitely many types. Thus, even a small amount of uncertainty (i.e., incomplete information), is sufficient to make the (most competitive) winner-takes-all contest strictly optimal.

Interestingly, however, effort is not necessarily monotonic in the level of competition. Specifically, while transferring value to the best-ranked prize always encourages effort, increasing competition by transferring value to a better-ranked intermediate prize may not.

Proposition 1 (Interior Discouragement Effect)**.** *Consider any contest environment* $(N+1, \Theta, p)$ *with* $|\Theta| = 2$*. For any prize* $m \in \{2, \ldots, N\}$*,*

$$P_1 > \frac{m}{N} \implies \textit{for all } m' < m,\ \alpha_m - \alpha_{m'} < 0.$$

Proposition 1 identifies a sufficient condition under which shifting value to a better-ranked intermediate prize discourages effort. Intuitively, as with shifting value to the best prize, such a transformation discourages inefficient types. However, in contrast to the best prize, the encouraging effect on efficient types is dampened, because the incentive for securing the best prize is also diluted. Consequently, the most efficient types are not as strongly incentivized, and their equilibrium effort may even decline following the transformation (Figure 2). Because this encouraging force is weaker, it may fail to offset the discouragement among inefficient types. In particular, when the inefficient type is sufficiently likely, the overall effect of shifting value to better-ranked intermediate prizes is negative.

In the extreme case where $P_1 \geq \frac{N-1}{N}$, it follows that any shift in value from a lower-ranked prize to a better-ranked prize discourages effort, except when the better prize is the best prize. Consequently, for the design problem, if the value of the best prize is capped, the optimal contest entails allocating the remaining budget evenly across all intermediate prizes.

5 General cost

So far, we have focused on contests with linear effort costs. In this section, we extend the analysis to general cost functions. Our approach is to identify conditions under which the effect of competition under linear costs remains informative under general costs.

Model

Formally, we enrich the model as follows. Fix a contest environment $(N+1, \Theta, p)$ and contest $v \in \mathcal{V}$. If a type-θ_k agent chooses effort x and wins prize v_m, their vNM utility is

$$v_m - \theta_k c(x),$$

where $c : \mathbb{R}_+ \to \mathbb{R}_+$ is a cost function satisfying $c(0) = 0$, $c'(x) > 0$ for $x > 0$, and $\lim_{x\to\infty} c(x) = \infty$.

We denote a symmetric equilibrium by $(X_1^*, X_2^*, \dots, X_K^*)$, and let X^* denote the ex-ante equilibrium effort of an arbitrary agent. As before, we define a mapping

$$X_v^* : [0,1] \to \mathbb{R}_+$$

that assigns to each probability $t \in [0,1]$ of outperforming an arbitrary agent the corresponding equilibrium effort.

Equilibrium

For this Bayesian game, we can equivalently describe agents' choices in terms of the cost of effort $c(x)$ rather than effort itself. In other words, we can reinterpret the game as one in which agents directly choose effort costs, are ranked based on their effort costs (which coincides with their ranking based on effort), and are awarded the corresponding prizes. From this change-of-variable interpretation, it follows that the equilibrium effort under linear costs (Theorem 1) more generally characterizes equilibrium effort costs in the present environment. This leads immediately to the following characterization of equilibrium effort in this more general environment.

Theorem 3 (Equilibrium Characterization)**.** *For any contest environment $(N+1, \Theta, p)$ with cost function $c(\cdot)$ and contest $v \in \mathcal{V}$, there exists a unique symmetric equilibrium $(X_1^*, \dots, X_K^*)$. It is such that, for each $t \in [0,1]$,*

$$X_v^*(t) = c^{-1}(X_v(t)),$$

where $X_v(t)$ denotes the equilibrium effort with linear costs $c(x) = x$ (Theorem 1).

Competition

From the equilibrium characterization, the expected equilibrium effort can be expressed as

$$\mathbb{E}[X^*] = \int_0^1 g(X_v(t))\,dt, \quad \text{where } g = c^{-1}.$$

This representation again follows from the probability integral transform: the probability $t = F(X^*)$ of outperforming an arbitrary agent is ex-ante uniformly distributed on $[0,1]$.

The effect of shifting value from a lower-ranked prize m' to a better-ranked prize m is then

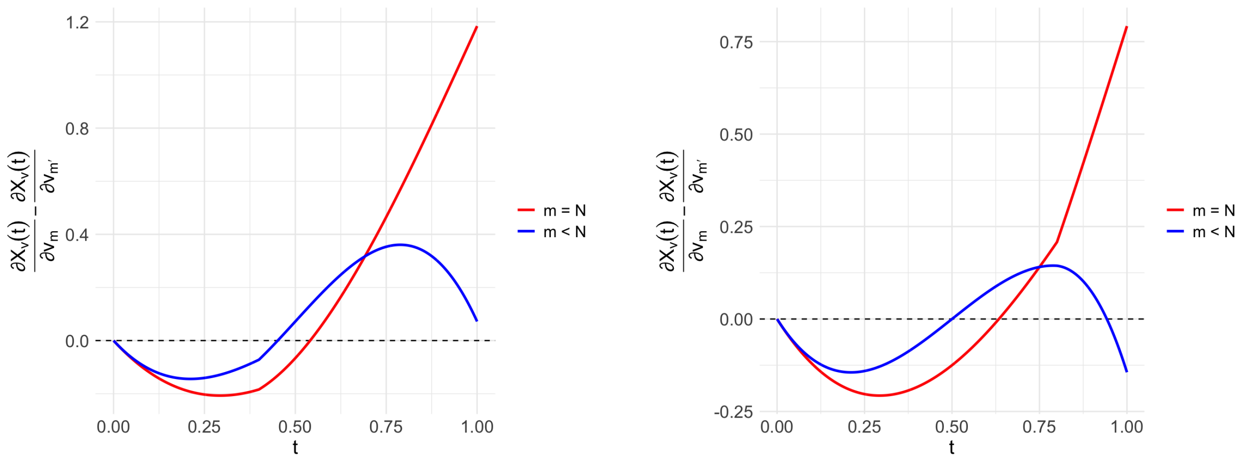
$$\frac{\partial \mathbb{E}[X^*]}{\partial v_m} - \frac{\partial \mathbb{E}[X^*]}{\partial v_{m'}} = \int_0^1 g'(X_v(t)) \underbrace{\left(\frac{\partial X_v(t)}{\partial v_m} - \frac{\partial X_v(t)}{\partial v_{m'}}\right)}_{\substack{\text{i) effect on effort if } c(x)=x \\ \text{ii) effect on effort cost } c(X^*)}} dt, \tag{6}$$

where, from Equations (3) and (4), we obtain that

$$\frac{\partial X_v(t)}{\partial v_m} = \frac{H_m^N(t)}{\theta_{k(t)}} - \left[\sum_{j=1}^{k(t)-1} H_m^N(P_j)\left(\frac{1}{\theta_{j+1}} - \frac{1}{\theta_j}\right)\right]. \tag{7}$$

Equation (6) provides an interpretable and analytically convenient representation for evaluating the effect of shifting value from prize m' to prize m on expected effort. Specifically, $\frac{\partial X_v(t)}{\partial v_m} - \frac{\partial X_v(t)}{\partial v_{m'}}$ represents the effect of the transformation on the effort of a (pseudo)-type t agent under linear cost. From Equation (7), it depends only on the contest environment $(N+1, \Theta, p)$ and the prizes m, m'. As discussed earlier, shifting value to better prizes discourages effort among inefficient types and encourages effort among efficient types. However, this encouraging effect is dampened when the better-ranked prize is interior ($m < N$), and may actually even discourage effort among the most efficient types, as illustrated in Figure 2. The term $g'(X_v(t))$ captures any influence that the cost function $c(\cdot)$ or the contest $v \in \mathcal{V}$ may have on how the transformation affects effort, and will be interpreted as simply assigning different weights to different pseudo-types.

We use the representation in Equation (6) to identify conditions under which the effect of the transformation on effort can be inferred from its effect under linear costs $(\alpha_m - \alpha_{m'})$. Specifically, we show that if, under linear costs, the effect on effort across types is single-crossing, then positive effects persist under concave costs, while negative effects persist under



(a) Single-crossing ($p = (0.4, 0.6)$)

(b) Double-crossing for $m < N$ ($p = (0.8, 0.2)$)

Figure 2: Effect of increasing competition on effort under linear cost ($N = 3, \Theta = \{2, 1\}$ and $m' = 1$)

convex costs. To illustrate the idea, suppose that under linear costs the effect on effort across types is single-crossing and that the aggregate effect is non-negative ($\alpha_m - \alpha_{m'} \geq 0$). If the cost function $c(\cdot)$ is concave, then $g(\cdot)$ is convex, so that $g'(X_v(t))$ is increasing in t. Consequently, this term assigns relatively lower weights to inefficient pseudo-types and larger weights to efficient pseudo-types. Since the aggregate effect on effort under linear costs is assumed to be non-negative, it follows that $\frac{\partial \mathbb{E}[X^*]}{\partial v_m} - \frac{\partial \mathbb{E}[X^*]}{\partial v_{m'}} \geq 0$.

We are now ready to state our main result. The theorem identifies necessary and sufficient conditions for the effect on effort across types under linear costs to be single-crossing, and shows that, under these conditions, positive effects under linear costs persist under concave costs, while negative effects persist under convex costs.

Theorem 4 (Linear to General)**.** *Consider any contest environment* $(N+1, \Theta, p)$. *Let* m, m' *with* $m > m'$ *be such that either*

- $m = N$ *or*
- $\sum_{j=1}^{K-1} \left[H^N_{m'}(P_j) - H^N_m(P_j)\right] \left(\frac{1}{\theta_{j+1}} - \frac{1}{\theta_j}\right) \geq 0.$

Then, the following hold:

1. If $\alpha_m - \alpha_{m'} \geq 0$ and c is concave, then for any $v \in \mathcal{V}$, $\dfrac{\partial \mathbb{E}[X^*]}{\partial v_m} - \dfrac{\partial \mathbb{E}[X^*]}{\partial v_{m'}} \geq 0$.

2. If $\alpha_m - \alpha_{m'} \leq 0$ and c is convex, then for any $v \in \mathcal{V}$, $\dfrac{\partial \mathbb{E}[X^*]}{\partial v_m} - \dfrac{\partial \mathbb{E}[X^*]}{\partial v_{m'}} \leq 0$.

Remark 3. *Theorem 4 nests and extends the result of Fang, Noe, and Strack (2020) from complete information to general contest environments with finite types. To see this, consider the special case of a complete information environment ($|\Theta| = K = 1$). In this setting, for any m, m' with $m > m'$, the conditions of Theorem 4 are satisfied. Moreover, $\alpha_m - \alpha_{m'} = 0$. It therefore follows that increasing competition always encourages effort under concave costs and always discourages effort under convex costs, recovering the complete information result of Fang, Noe, and Strack (2020).*

Other objectives

While we have focused on expected effort, Theorem 4 can be readily generalized to other quantities of interest. To illustrate, consider the expected maximum effort, another commonly studied objective in contest design (Archak and Sundararajan (2009); Wasser and Zhang (2023)). From the equilibrium characterization in Theorem 3, the expected maximum effort can be expressed as

$$\mathbb{E}[X^*_{max}] = (N+1)\int_0^1 g(X_v(t))t^N dt.$$

Thus, the equilibrium effort of a pseudo-type t agent is weighted by the probability that their effort is the maximum, which is t^N.

The effect of shifting value from a lower-ranked prize m' to a better-ranked prize m is then

$$\frac{\partial \mathbb{E}[X^*_{max}]}{\partial v_m} - \frac{\partial \mathbb{E}[X^*_{max}]}{\partial v_{m'}} = (N+1)\int_0^1 \underbrace{g'(X_v(t))\, t^N}_{\text{weights}} \underbrace{\left(\frac{\partial X_v(t)}{\partial v_m} - \frac{\partial X_v(t)}{\partial v_{m'}}\right)}_{\text{single-crossing?}} dt. \tag{8}$$

As in Theorem 4, under monotonic weights and single-crossing, the sign of $\alpha_m - \alpha_{m'}$ may be informative about how the transformation affects expected maximum effort. Thus, Theorem 4 provides a simple and general recipe for assessing whether shifting value towards better-ranked prizes encourages or discourages expected effort and related objectives.

Design problem

The result has implications for the design problem of allocating a fixed prize budget. Suppose the cost function c is concave. It follows from Theorem 4 that, for any contest $v \in \mathcal{V}$, shifting value from any lower-ranked prize m' to the top prize $m = N$ has a positive effect on expected effort. This is because for $m = N$, the single-crossing property holds, and Theorem 2 establishes that $\alpha_N - \alpha_{m'} \geq 0$. A similar implication holds for expected maximum effort. Under concave costs, the weights in Equation (8) are increasing in t, implying that the same transformation also increases expected maximum effort. More generally, the following result establishes that, under both linear and concave costs, the winner-takes-all contest is optimal not only for maximizing total effort and maximum effort, but also for maximizing the total effort of the top q agents, for any q.

Theorem 5 (Winner-takes-all is optimal)**.** *Consider any contest environment* $(N+1, \Theta, p)$ *with cost function* $c(\cdot)$*. If* c *is (weakly) concave, among all contests* $v \in \mathcal{V}$ *satisfying* $\sum_{m=0}^{N} v_m \leq V$*, the winner-takes-all contest* $v = (0, 0, \ldots, 0, V)$ *maximizes expected total effort of top* q *agents for any* $q \in [N+1]$*.*

Intuitively, as discussed earlier under linear costs, the winner-takes-all contest creates strong incentives for the most efficient pseudo-types—those who are cheapest to incentivize—and this more than compensates for the discouragement it induces among less efficient pseudo-types. Consequently, aggregate effort is maximized (Theorem 2). When the designer instead cares about the total effort of the top q agents, the objective effectively places greater weight on the effort of the more efficient pseudo-types. Since winner-takes-all disproportionately strengthens incentives precisely for these types, its optimality persists under this more selective objective. This conclusion extends to concave cost functions as well. Under concavity, marginal effort costs are decreasing, making it even cheaper to induce higher effort levels from efficient pseudo-types. Thus, shifting value to the best prize amplifies effort where it is most cost-effective, reinforcing the optimality of the winner-takes-all contest.

This intuition, however, does not extend directly to the case of convex costs. When costs are convex, marginal effort costs are increasing, so inducing additional effort at higher effort levels becomes progressively more expensive. As a result, concentrating prize value at the top no longer has an unambiguously dominant effect, since the designer must now weigh stronger incentives for efficient pseudo-types against the rising marginal costs of effort. Nevertheless, we show that the optimality of the winner-takes-all contest persists under moderate convexity. In other words, as long as costs are not too convex, the incentive gains from concentrating prize value at the top continue to outweigh the associated marginal cost increases. The following proposition formalizes this for a parametric class of cost functions.

Proposition 2. *Consider any contest environment* $(N+1, \Theta, p)$ *with* $|\Theta| > 1$ *and cost* $c(x) = x^{\alpha}$. *For any* $q \in [N+1]$, *there exists* $\alpha^* > 1$ *such that for all* $\alpha \in [1, \alpha^*)$, *among all contests* $v \in \mathcal{V}$ *satisfying* $\sum_{m=0}^{N} v_m \leq V$, *the winner-takes-all contest* $v = (0, 0, \ldots, 0, V)$ *maximizes the expected total effort of the top* q *agents.*

Proposition 2 reveals a sharp contrast with the complete-information environment, where the (most competitive) winner-takes-all contest instead minimizes total effort under convex costs (Remark 3). Our analysis suggests a reconciliation of these contrasting findings in that the supremacy of the winner-takes-all contest over alternative prize structures in incomplete-information environments diminishes as the environment approaches complete information. This pattern is illustrated in Figure 3, which shows that in a binary-type environment, the effect of shifting value to the top prize—while always positive—shrinks as the probability of the inefficient type, p_1, approaches 0 or 1. Based on these findings, we conjecture that the degree of convexity required to overturn the optimality of the winner-takes-all contest likewise decreases as the environment approaches complete information.

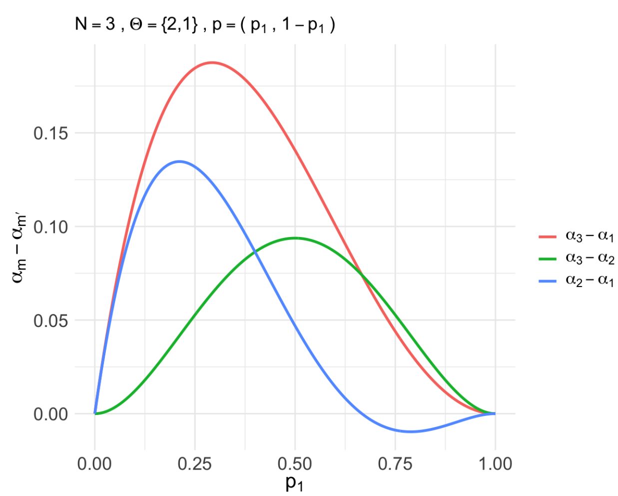


Figure 3: Effect of increasing competition: Complete and Incomplete information.

Remark 4. *Our results (Theorem 5 and Proposition 2) establish that the winner-takes-all contest is robustly optimal in the sense that it maximizes the total effort of the top* q *agents—an objective that places relatively greater weight on the effort of better-ranked* *agents—provided effort costs are not too convex. More generally, we can identify conditions under which equi-split contests—contests that divide the prize budget equally across a subset of prizes—are optimal. Such contests, together with the null contest $v = (0, 0, \ldots, 0)$, constitute the extreme points of the feasible set, allowing us to invoke Bauer's maximum principle to identify conditions when they are optimal. As an application, under linear effort costs and any linear objective of the designer—assigning arbitrary weights to the efforts of differently ranked agents—we can show that there always exists an optimal equi-split contest.*

6 Experiment

In this section, we present findings from an incentivized experiment designed to test the equilibrium predictions of our model regarding the effect of competition on effort. Our primary objective is to test the optimality of the (most competitive) winner-takes-all contest under linear costs, originally established by Moldovanu and Sela (2001) in a continuum type-space setting. Our result for the finite type-space model establishes its robustness under incomplete information and enables experimental investigation. We test this by comparing effort levels in a winner-takes-all structure with those in less competitive alternatives.

Our second goal is to test for the interior discouragement effect of competition, which our model predicts emerges when inefficient types are sufficiently likely. To test this, we consider an environment where this condition holds and gradually increase the competitiveness of the prize structure within an interior range, where the model predicts a decrease in expected equilibrium effort. Below, we present the experimental design, implementation details, and our findings.

6.1 Experimental Design

In our experiment, subjects competed in groups of four for monetary prizes. Effort levels were chosen independently and privately, without any possibility of communication. There were two possible types (marginal costs of effort): 1 and 2, assigned with probabilities 20% and 80%, respectively.[6] We implemented the strategy method so that for each contest, subjects submitted two effort choices (between 0 and 100), one for each possible type. At the

[6] In terms of our notation, the contest environment $(N+1, \Theta, p)$ is defined by $N+1 = 4$, the type-space $\Theta = \{c_1(x) = 2x, c_2(x) = x\}$, and type distribution $p = (0.8, 0.2)$.

end of the experiment, each subject was independently assigned a type according to the type distribution, and the corresponding decisions were used to determine payoffs.

The experimental treatments varied the contest $v = (v_0, v_1, v_2, v_3)$, which awards prizes based on effort rankings: v_3 for the highest effort, v_2 for the second highest, v_1 for the third highest, and $v_0 = 0$ for the lowest. The most competitive treatment, *WTA*, is a winner-takes-all structure with $(0, 0, 0, 100)$. We consider three other treatments with the same total prize but progressively less competitive: *High* with $v = (0, 0, 25, 75)$, *Med* with $v = (0, 0, 50, 50)$, and *Low* with $v = (0, 25, 25, 50)$. Table 1 summarizes the treatments and their equilibrium effort predictions.

The contest order was randomized across subjects. The group composition in each contest was also randomized and the subjects were unable to identify with each other between contests. Feedback on contest outcomes was withheld until the end of the experiment, however on-screen information informed subjects of their potential earnings for each possible prize they could win (i.e. prize minus effort costs) to ensure proper comprehension of the payoffs. Subjects were paid for one randomly selected contest, with tokens converted to U.S. dollars at a rate of 50:1, plus a \$2 show up fee. The protocols for subject group matching, feedback, and payment were chosen to minimize reputation-building or repeated-play concerns, thereby inducing the one-shot nature of the game under study.

Treatment	$(0, v_1, v_2, v_3)$	Equilibrium Effort ($\mathbb{E}[X]$)			Observed Effort		
		$c_k(x) = x$	$c_k(x) = 2x$	Pooled	$c_k(x) = x$	$c_k(x) = 2x$	Pooled
WTA	$(0, 0, 0, 100)$	48.2	6.4	14.76	52.8	40.5	42.96
High	$(0, 0, 25, 75)$	37.0	8	13.80	46.9	37.4	39.3
Med	$(0, 0, 50, 50)$	25.8	9.6	12.84	43.3	36.5	37.86
Low	$(0, 25, 25, 50)$	24.6	10.2	13.08	42.9	35.9	37.3

Table 1: Equilibrium and Observed Efforts, by Treatment and Cost Type

We recruited 700 subjects from Prolific, an online labor market, during April 2025. We administered a comprehension quiz to ensure that subjects understood the instructions, the structure of the contests, and how the earnings were determined. Subjects who failed to pass the quiz after a second attempt were not allowed to continue, resulting in a final sample of 445 subjects.[7]

6.2 Experimental Results

Table 1 summarize our findings on mean effort. The mean effort is highest under the *WTA*. Also, efforts are higher for the efficient type ($c_k(x) = x$) and lower for the inefficient type ($c_k(x) = 2x$), as expected. Regression analysis, reported in column 1 of Table 4 in Appendix E, confirms the statistical significance of these findings.

To directly test whether behavior aligns with the theory, we use Lemma 2, which represents the expected equilibrium effort as a linear combination of the contest prizes. For the environment considered in the experiment, we have that for any contest $v \in \mathcal{V}$,

$$\mathbb{E}[X] = 0.119 \cdot v_1 + 0.109 \cdot v_2 + 0.148 \cdot v_3 \,.$$

In this representation, observe that the largest coefficient is on the best prize (v_3), which implies that transferring value from any lower-ranked prize to the best prize increases expected effort, thus resulting in the optimality of the WTA prize structure. The interior discouragement effect is reflected in the fact that the coefficient v_2 is smaller than that of v_1, so that increasing competition by transferring value from lower-ranked prize v_1 to higher-ranked prize v_2 decreases expected equilibrium effort.

We can also decompose the expected effort by cost type and obtain that

$$\mathbb{E}[X \mid c_k(x) = x] = -0.014 \cdot v_1 + 0.034 \cdot v_2 + 0.482 \cdot v_3$$

and

$$\mathbb{E}[X \mid c_k(x) = 2x] = 0.152 \cdot v_1 + 0.128 \cdot v_2 + 0.064 \cdot v_3 \,.$$

Observe that the best prize (v_3) has the largest coefficient for the efficient type and the smallest for the inefficient type. This suggests that the overall optimality of the WTA structure is primarily driven by the strong incentive it creates for the efficient type. For the interior prizes, transferring value from v_1 to v_2 encourages effort from the efficient type, but discourages effort from the inefficient type. The overall interior discouragement effect arises

[7]The experimental interface and instructions are provided here.

because the inefficient type is significantly more likely.

Table 2 presents results from a linear regression of effort choices on the three prizes.[8] We find that, in contrast to theoretical predictions, the best prize carries the largest coefficient not only for the efficient type, but also for the inefficient type. Moreover, both the intermediate prizes appear to be overweighted in observed effort choices relative to theoretical expectations, for both types. Even so, in the aggregate, the best prize has the largest coefficient for mean effort, and therefore, the transfer of value from any lower-ranked prize to the best prize is empirically correlated with an increase in effort. This finding speaks to the global optimality of winner-take-all contest, in line with the theory.

For the interior discouragement effect, we find that the difference in the estimated coefficients is negligible (0.006), and statistically insignificant (the Wald test for the coefficients being equal yields a p-value of 0.850). Although we do not find a significant negative effect of transferring value from the v_1 to v_2, we also do not find a positive effect. We interpret this result as partially in line with equilibrium behavior, as it provides evidence that increasing the competitiveness of the prize structure need not lead to higher effort.

Prize	Equilibrium Weight			Estimated Coefficient		
	$c_k(x) = x$	$c_k(x) = 2x$	Pooled	$c_k(x) = x$	$c_k(x) = 2x$	Pooled
First place prize (v_3)	0.482	0.064	0.148	0.524	0.401	0.426
Second place prize (v_2)	0.034	0.128	0.109	0.335	0.321	0.324
Third place prize (v_1)	-0.014	0.152	0.119	0.334	0.314	0.318

Table 2: Linear Decomposition of the Expected Effort: Equilibrium and Regression Results

Note: Lemma 2 establishes that the equilibrium effort can be expressed as a linear function of the contest prize vector. The columns labeled "Equilibrium Weight" report the theoretical coefficients for each prize under the two cost types and in the aggregate. The columns labeled "Estimated Coefficient" present the corresponding regression estimates. Full regression results are provided in Table 3.

[8]Since the expected equilibrium effort is zero when all prizes are zero, we estimate the regression without a constant to align directly with the theoretical specification.

6.3 Discussion of Experimental Results

In the preceding subsection, we focused on testing the theoretical predictions. While we find qualitative support for the main comparative statics of the model, our results also reveal evidence of rent dissipation in the form of excessive effort, especially from the inefficient type. This phenomenon of over-provision of effort has been previously identified in contest experiments.

We briefly discuss the non-strategic determinants of behavior by examining how individual characteristics correlate with effort choices, as is common in the analysis of experimental data. For this purpose, we elicited self-reported measures of risk attitudes (willingness to take risks on a 0–10 Likert scale) and competitiveness (willingness to compete on a 0–10 Likert scale) at the end of the experiment to proxy for the intrinsic *joy of winning* that subjects may derive beyond the pecuniary earnings. We also control for gender. These factors have been shown to influence behavior in prior studies on contests (Dechenaux, Kovenock, and Sheremeta (2015)). Regression results, reported in column 2 of Table 4 in Appendix E, show that subjects who declare a higher willingness to take risks (coefficient 1.528, $p < 0.01$) and claim to be more competitive (coefficient 1.063, $p < 0.05$) choose significantly higher efforts. We find no significant gender differences in effort choices.

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A Proofs for Section 3 (Equilibrium)

Lemma 1. *If $(X_1, \ldots, X_K)$ is an equilibrium, then there exist boundary points*

$$0 = b_0 < b_1 < \cdots < b_K$$

such that, for each k, X_k is continuously distributed on $[b_{k-1}, b_k]$.

Proof. Suppose $(X_1, \ldots, X_K)$ is an equilibrium, and $X \sim F$ is the ex-ante equilibrium effort. We establish the result in three steps.

Step 1: For each k, X_k is a continuous random variable.

Suppose, for contradiction, that $\Pr[X_k = x] > 0$ for some x. Under the given profile, there is a positive probability that all $N + 1$ agents choose effort x, in which case ties are broken uniformly at random. If a type-θ_k agent deviates to $x + \epsilon$, it obtains a discontinuous jump in expected prize (since $v_N > v_0$) while the additional cost $\theta_k(x + \epsilon) - \theta_k x$ can be made arbitrarily small. Hence the deviation is profitable, a contradiction.

Without loss of generality, we assume that the support of X_k is closed.

Step 2: There exists $b_K > 0$ such that X is continuously distributed on $[0, b_K]$.

Suppose there exists an interval (d_1, d_2) that is not contained in the support of X. Consider a type whose support contains d_2. By deviating to d_1, the agent obtains the same expected prize while incurring strictly lower cost, yielding a profitable deviation. Hence the support of X cannot contain gaps. It follows that the support is a convex subset of $\mathbb{R}_+$.

Step 3: There exist $b_1 < b_2 < \cdots < b_K$ such that the support of X_k is $[b_{k-1}, b_k]$.

Let $x < y$ lie in the support of X_k. Since a type-θ_k agent must be indifferent between the two,

$$\pi_v(F(y)) - \theta_k y = \pi_v(F(x)) - \theta_k x.$$

For a type-θ_j agent, the payoff difference between choosing y and x is

$$\pi_v(F(y)) - \theta_j y - \big(\pi_v(F(x)) - \theta_j x\big) = (\theta_k - \theta_j)(y - x).$$

This expression is strictly positive if $\theta_j < \theta_k$ and strictly negative if $\theta_j > \theta_k$. Hence no other type is indifferent between x and y, implying that the supports of X_j and X_k intersect in at most boundary points. The ordering follows from the observation that if $\theta_j < \theta_k$, a type-θ_j agent obtains a higher payoff from y then x. Therefore there exist boundary points $0 = b_0 < b_1 < \cdots < b_K$ such that the support of X_k is $[b_{k-1}, b_k]$. $\square$

Theorem 1 (Equilibrium Characterization)**.** *For any contest environment* $(N+1, \Theta, p)$ *and contest* $v \in \mathcal{V}$*, there exists a unique symmetric equilibrium* $(X_1, \ldots, X_K)$*. It is such that, for each* $t \in [0, 1]$*,*

$$X_v(t) = \frac{\pi_v(t) - u_{k(t)}}{\theta_{k(t)}}, \tag{3}$$

where $k(t) = \max\{k : P_{k-1} \leq t\}$*, and the equilibrium utilities are given by*

$$u_k = \theta_k \left[\sum_{j=1}^{k-1} \pi_v(P_j) \left(\frac{1}{\theta_{j+1}} - \frac{1}{\theta_j} \right) \right]. \tag{4}$$

Proof. It remains only to establish the equilibrium utilities, which we do by induction.

Base case. Since zero effort lies in the support of the type-θ_1 strategy and yields payoff 0, we have $u_1 = 0$.

Induction step. Suppose

$$u_k = \theta_k \left[\sum_{j=1}^{k-1} \pi_v(P_j) \left(\frac{1}{\theta_{j+1}} - \frac{1}{\theta_j} \right) \right].$$

Evaluating the equilibrium effort at $t = P_k$ yields

$$\begin{aligned} X_v(P_k) &= \frac{\pi_v(P_k) - u_k}{\theta_k} \\ &= \frac{\pi_v(P_k)}{\theta_k} - \sum_{j=1}^{k-1} \pi_v(P_j) \left(\frac{1}{\theta_{j+1}} - \frac{1}{\theta_j} \right) \\ &= b_k. \end{aligned}$$

Since b_k lies in the support of the type-θ_{k+1} strategy, their equilibrium utility is

$$\begin{aligned} u_{k+1} &= \pi_v(P_k) - \theta_{k+1} b_k \\ &= \pi_v(P_k) - \theta_{k+1} \left[\frac{\pi_v(P_k)}{\theta_k} - \sum_{j=1}^{k-1} \pi_v(P_j) \left(\frac{1}{\theta_{j+1}} - \frac{1}{\theta_j} \right) \right] \\ &= \theta_{k+1} \left[\sum_{j=1}^{k} \pi_v(P_j) \left(\frac{1}{\theta_{j+1}} - \frac{1}{\theta_j} \right) \right], \end{aligned}$$

which establishes the inductive claim. $\square$

B Proofs for Section 4 (Competition)

Lemma 2. *The expected equilibrium effort is*

$$\mathbb{E}[X] = \sum_{m=1}^{N} \alpha_m v_m,$$

where

$$\alpha_m = \frac{1}{N+1}\left[\frac{1}{\theta_K} - \sum_{k=1}^{K-1}\left[H_{\geq m}^{N+1}(P_k) + (N-m)H_m^{N+1}(P_k)\right]\left(\frac{1}{\theta_{k+1}} - \frac{1}{\theta_k}\right)\right]. \tag{5}$$

Proof. Since $t = F(X)$ is uniformly distributed on $[0,1]$,

$$\begin{aligned}
\mathbb{E}[X] &= \int_0^1 X_v(t)dt \\
&= \int_0^1 \frac{\pi_v(t) - u_{k(t)}}{\theta_{k(t)}}dt && \text{(Theorem 1)} \\
&= \sum_{k=1}^{K} p_k \cdot \frac{1}{\theta_k} \cdot \left[\int_{P_{k-1}}^{P_k} \frac{\pi_v(t)}{p_k}dt - u_k\right].
\end{aligned}$$

Notice that for any $k \in [K]$, $\int_{P_{k-1}}^{P_k} \frac{\pi_v(t)}{p_k}dt$ is the expected prize awarded to a type-θ_k agent. To compute this, we instead compute the ex-ante expected total prize awarded to type-θ_k agents. Notice that for any prize $m \in \{0, \ldots, N\}$, the ex-ante probability that this prize is awarded to a type-θ_k agent is simply

$$\left[H_{\geq m+1}^{N+1}(P_k) - H_{\geq m+1}^{N+1}(P_{k-1})\right].$$

Thus, the ex-ante expected total prize awarded to type-θ_k agents is

$$\sum_{m=1}^{N} v_m \left[H_{\geq m+1}^{N+1}(P_k) - H_{\geq m+1}^{N+1}(P_{k-1})\right].$$

By an alternative calculation, which entails adding up over the $N+1$ agents, this expectation should equal

$$(N+1) \cdot p_k \cdot \int_{P_{k-1}}^{P_k} \frac{\pi_v(t)}{p_k}dt.$$

Equating these two, we get that

$$\int_{P_{k-1}}^{P_k} \pi_v(t)dt = \frac{\sum_{m=1}^{N} v_m \left[H_{\geq m+1}^{N+1}(P_k) - H_{\geq m+1}^{N+1}(P_{k-1})\right]}{N+1}.$$

Alternatively, we can also directly use the following fact to compute this integral:

$$\frac{\partial H^{N+1}_{\geq m+1}(t)}{\partial t} = (N+1) H^N_m(t)$$

Substituting this in the above representation, we get

$$\mathbb{E}[X] = \sum_{k=1}^{K} \frac{1}{(N+1)\theta_k} \sum_{m=1}^{N} v_m \left[H^{N+1}_{\geq m+1}(P_k) - H^{N+1}_{\geq m+1}(P_{k-1})\right] - \sum_{k=1}^{K} \frac{p_k u_k}{\theta_k}.$$

From here, we write

$$\mathbb{E}[X] = \sum_{m=1}^{N} \alpha_m v_m$$

where

$$\begin{aligned}
\alpha_m &= \sum_{k=1}^{K} \frac{\left[H^{N+1}_{\geq m+1}(P_k) - H^{N+1}_{\geq m+1}(P_{k-1})\right]}{(N+1)\theta_k} - \sum_{k=1}^{K} p_k \sum_{j=1}^{k-1} H^N_m(P_j) \left(\frac{1}{\theta_{j+1}} - \frac{1}{\theta_j}\right) \\
&= \sum_{k=1}^{K} \frac{\left[H^{N+1}_{\geq m+1}(P_k) - H^{N+1}_{\geq m+1}(P_{k-1})\right]}{(N+1)\theta_k} - \sum_{k=1}^{K-1} (1-P_k) H^N_m(P_k) \left(\frac{1}{\theta_{k+1}} - \frac{1}{\theta_k}\right) \\
&= \frac{1}{N+1}\left[\frac{1}{\theta_K} - \sum_{k=1}^{K-1} H^{N+1}_{\geq m+1}(P_k) \left(\frac{1}{\theta_{k+1}} - \frac{1}{\theta_k}\right)\right] - \frac{(N+1-m)}{N+1} \sum_{k=1}^{K-1} \left[H^{N+1}_m(P_k) \left(\frac{1}{\theta_{k+1}} - \frac{1}{\theta_k}\right)\right] \\
&= \frac{1}{N+1}\left[\frac{1}{\theta_K} - \sum_{k=1}^{K-1} \left[H^{N+1}_{\geq m}(P_k) + (N-m) H^{N+1}_m(P_k)\right] \left(\frac{1}{\theta_{k+1}} - \frac{1}{\theta_k}\right)\right].
\end{aligned}$$

□

Theorem 2 (Winner-takes-all is optimal)**.** *Consider any contest environment* $(N+1, \Theta, p)$ *with* $|\Theta| > 1$*. For any prize* $m' \in \{1, \ldots, N-1\}$*,*

$$\alpha_N - \alpha_{m'} > 0.$$

Consequently, among all contests $v \in \mathcal{V}$ *satisfying* $\sum_{m=0}^{N} v_m \leq V$*, the winner-takes-all contest* $v = (0, 0, \ldots, 0, V)$ *uniquely maximizes expected equilibrium effort.*

Proof. From Equation (5), we have that for any prize $m' \in \{1, \ldots, N-1\}$,

$$\alpha_N - \alpha_{m'} = \frac{1}{N+1}\left[\sum_{k=1}^{K-1} \left[H^{N+1}_{\geq m'}(P_k) - H^{N+1}_{\geq N}(P_k) + (N-m') H^{N+1}_{m'}(P_k)\right] \left(\frac{1}{\theta_{k+1}} - \frac{1}{\theta_k}\right)\right].$$

With $|\Theta| = K > 1$, it is straightforward to verify that $\alpha_N - \alpha_{m'} > 0$. It follows that for any contest $v \in \mathcal{V}$, transferring value from any lower-ranked prize m' to the top-prize N leads to an increase in expected effort. □

Proposition 1 (Interior Discouragement Effect)**.** *Consider any contest environment* $(N+1, \Theta, p)$ *with* $|\Theta| = 2$*. For any prize* $m \in \{2, \ldots, N\}$*,*

$$P_1 > \frac{m}{N} \implies \textit{for all } m' < m,\ \alpha_m - \alpha_{m'} < 0.$$

Proof. From Equation (5), we have that for any prize $m \in \{2, \ldots, N\}$,

$$\begin{aligned}
\alpha_m - \alpha_{m-1} &= \frac{1}{N+1}\left[\left[H_{m-1}^{N+1}(P_1) + (N-(m-1))H_{m-1}^{N+1}(P_1) - (N-m)H_m^{N+1}(P_1)\right]\left(\frac{1}{\theta_2} - \frac{1}{\theta_1}\right)\right] \\
&= \frac{H_{m-1}^{N+1}(P_1)}{N+1}\left(\frac{1}{\theta_2} - \frac{1}{\theta_1}\right)\left[N - m + 2 - (N-m)\frac{N-m+2}{m}\frac{P_1}{1-P_1}\right] \\
&= \frac{(N-m+2)H_{m-1}^{N+1}(P_1)}{N+1}\left(\frac{1}{\theta_2} - \frac{1}{\theta_1}\right)\left[1 - \frac{(N-m)}{m}\frac{P_1}{1-P_1}\right].
\end{aligned}$$

It is straightforward to verify that if $P_1 > \frac{m}{N}$, then $\alpha_m - \alpha_{m-1} < 0$.

Now for any $m' < m$,

$$\alpha_m - \alpha_{m'} = (\alpha_m - \alpha_{m-1}) + (\alpha_{m-1} - \alpha_{m-2}) + \cdots + (\alpha_{m'+1} - \alpha_{m'}),$$

and the result follows.

□

C Proofs for Section 5 (General cost)

Lemma 3. *Suppose* $a_2 : [0,1] \to \mathbb{R}$ *is such that there exists* $t^* \in [0,1]$ *so that* $a_2(t) \leq 0$ *for* $t \leq t^*$ *and* $a_2(t) \geq 0$ *for* $t \geq t^*$*. Then, for any increasing function* $a_1 : [0,1] \to \mathbb{R}$*,*

$$\int_0^1 a_1(t)a_2(t)dt \geq a_1(t^*)\int_0^1 a_2(t)dt.$$

Proof. Observe that

$$\begin{aligned}
\int_0^1 a_1(t)a_2(t)dt &= \int_0^{t^*} a_1(t)a_2(t)dt + \int_{t^*}^1 a_1(t)a_2(t)dt \\
&\geq \int_0^{t^*} a_1(t^*)a_2(t)dt + \int_{t^*}^1 a_1(t^*)a_2(t)dt \\
&= a_1(t^*)\int_0^1 a_2(t)dt.
\end{aligned}$$

□

Theorem 4 (Linear to General)**.** *Consider any contest environment $(N+1, \Theta, p)$. Let m, m' with $m > m'$ be such that either*

- $m = N$ *or*
- $\sum_{j=1}^{K-1} \left[H_{m'}^N(P_j) - H_m^N(P_j)\right] \left(\frac{1}{\theta_{j+1}} - \frac{1}{\theta_j}\right) \geq 0.$

Then, the following hold:

1. *If $\alpha_m - \alpha_{m'} \geq 0$ and c is concave, then for any $v \in \mathcal{V}$, $\dfrac{\partial \mathbb{E}[X^*]}{\partial v_m} - \dfrac{\partial \mathbb{E}[X^*]}{\partial v_{m'}} \geq 0.$*
2. *If $\alpha_m - \alpha_{m'} \leq 0$ and c is convex, then for any $v \in \mathcal{V}$, $\dfrac{\partial \mathbb{E}[X^*]}{\partial v_m} - \dfrac{\partial \mathbb{E}[X^*]}{\partial v_{m'}} \leq 0.$*

Proof. From Equation (6), we have that for any contest $v \in \mathcal{V}$ and prizes m, m' with $m > m'$,

$$\frac{\partial \mathbb{E}[X^*]}{\partial v_m} - \frac{\partial \mathbb{E}[X^*]}{\partial v_{m'}} = \int_0^1 g'\left(X_v(t)\right) \left(\frac{\partial X_v(t)}{\partial v_m} - \frac{\partial X_v(t)}{\partial v_{m'}}\right) dt$$

where, from Equation (7), we obtain that

$$\frac{\partial X_v(t)}{\partial v_m} - \frac{\partial X_v(t)}{\partial v_{m'}} = \left(\frac{H_m^N(t) - H_{m'}^N(t)}{\theta_{k(t)}}\right) - \left[\sum_{j=1}^{k(t)-1} \left(H_m^N(P_j) - H_{m'}^N(P_j)\right) \left(\frac{1}{\theta_{j+1}} - \frac{1}{\theta_j}\right)\right].$$

From here, one can verify that

1. $\left.\frac{\partial X_v(t)}{\partial v_m} - \frac{\partial X_v(t)}{\partial v_{m'}}\right|_{t=0} = 0$
2. $\left.\frac{\partial X_v(t)}{\partial v_m} - \frac{\partial X_v(t)}{\partial v_{m'}}\right|_{t=1} = \begin{cases} \frac{1}{\theta_K} - \left[\sum_{j=1}^{K-1} \left(H_m^N(P_j) - H_{m'}^N(P_j)\right)\left(\frac{1}{\theta_{j+1}} - \frac{1}{\theta_j}\right)\right] & \text{if } m = N \\ \left[\sum_{j=1}^{K-1} \left(H_{m'}^N(P_j) - H_m^N(P_j)\right)\left(\frac{1}{\theta_{j+1}} - \frac{1}{\theta_j}\right)\right] & \text{otherwise} \end{cases}$
3. $\frac{\partial X_v(t)}{\partial v_m} - \frac{\partial X_v(t)}{\partial v_{m'}}$ is continuous in t
4. $\frac{\partial X_v(t)}{\partial v_m} - \frac{\partial X_v(t)}{\partial v_{m'}}$ is differentiable at all t, except when $t = P_k$. At any $t \in (0,1)$ such that $t \neq P_k$, the derivative has the same sign as the derivative of $H_m^N(t) - H_{m'}^N(t)$ with respect to t.

The conditions on m and m' ensure that $\left.\frac{\partial X_v(t)}{\partial v_m} - \frac{\partial X_v(t)}{\partial v_{m'}}\right|_{t=1} \geq 0$. Together with the above properties, this implies that there is some $t^* \in [0,1]$ such that $\frac{\partial X_v(t)}{\partial v_m} - \frac{\partial X_v(t)}{\partial v_{m'}} \leq 0$ for $t \in [0, t^*]$, and $\frac{\partial X_v(t)}{\partial v_m} - \frac{\partial X_v(t)}{\partial v_{m'}} \geq 0$ for $t \in [t^*, 1]$.

If c is concave, $g = c^{-1}$ is convex, and thus, $g'(X_v(t))$ is increasing in t. Applying Lemma 3 with $a_1(t) = g'(X_v(t))$ and $a_2(t) = \frac{\partial X_v(t)}{\partial v_m} - \frac{\partial X_v(t)}{\partial v_{m'}}$ gives

$$\begin{aligned}\frac{\partial \mathbb{E}[X^*]}{\partial v_m} - \frac{\partial \mathbb{E}[X^*]}{\partial v_{m'}} &\geq g'(X_v(t^*)) \int_0^1 \left(\frac{\partial X_v(t)}{\partial v_m} - \frac{\partial X_v(t)}{\partial v_{m'}} \right) dt \\ &= g'(X_v(t^*)) (\alpha_m - \alpha_{m'})\end{aligned}$$

and the result follows. An analogous argument applies for the case where c is convex.

□

Theorem 5 (Winner-takes-all is optimal)**.** *Consider any contest environment $(N+1, \Theta, p)$ with cost function $c(\cdot)$. If c is (weakly) concave, among all contests $v \in \mathcal{V}$ satisfying $\sum_{m=0}^{N} v_m \leq V$, the winner-takes-all contest $v = (0, 0, \ldots, 0, V)$ maximizes expected total effort of top q agents for any $q \in [N+1]$.*

Proof. We first establish the result for $q = 1$. For any contest $v \in \mathcal{V}$, the effect of shifting value from any prize $m' \in [N-1]$ to the best prize $m = N$ on expected maximum effort is

$$\frac{\partial \mathbb{E}[X^*_{max}]}{\partial v_N} - \frac{\partial \mathbb{E}[X^*_{max}]}{\partial v_{m'}} = (N+1) \int_0^1 \underbrace{g'(X_v(t))\, t^N}_{\text{increasing weights}} \underbrace{\left(\frac{\partial X_v(t)}{\partial v_N} - \frac{\partial X_v(t)}{\partial v_{m'}} \right)}_{\text{single-crossing}} dt.$$

Since $\alpha_N - \alpha_{m'} > 0$ (Theorem 2), applying Lemma 3 with $a_1(t) = g'(X_v(t))\, t^N$ and $a_2(t) = \frac{\partial X_v(t)}{\partial v_N} - \frac{\partial X_v(t)}{\partial v_{m'}}$ yields

$$\frac{\partial \mathbb{E}[X^*_{max}]}{\partial v_N} - \frac{\partial \mathbb{E}[X^*_{max}]}{\partial v_{m'}} \geq 0.$$

Thus, shifting value toward the best prize increases expected maximum effort, implying that the winner-takes-all contest is optimal for $q = 1$.

We now extend the argument to arbitrary $q \in [N+1]$. The same structure applies. The key observation is that the probability that a pseudo-type t agent ranks among the top q agents is increasing in t. Consequently, under concave costs, the induced weights remain monotone increasing in t. The single-crossing property holds as before, and Lemma 3 delivers the same sign result. Hence, shifting value toward the best prize increases the expected total effort of the top q agents, implying that the winner-takes-all contest is optimal for any q. □

Proposition 2. *Consider any contest environment $(N+1, \Theta, p)$ with $|\Theta| > 1$ and cost $c(x) = x^\alpha$. For any $q \in [N+1]$, there exists $\alpha^* > 1$ such that for all $\alpha \in [1, \alpha^*)$, among* *all contests $v \in \mathcal{V}$ satisfying $\sum_{m=0}^{N} v_m \leq V$, the winner-takes-all contest $v = (0, 0, \ldots, 0, V)$ maximizes the expected total effort of the top q agents.*

Proof. We first establish the result for $q = N + 1$, i.e., for total expected effort. For any contest $v \in \mathcal{V}$, the effect of shifting value from any prize $m' \in [N-1]$ to the top prize $m = N$ on expected effort is

$$\frac{\partial \mathbb{E}[X^*]}{\partial v_N} - \frac{\partial \mathbb{E}[X^*]}{\partial v_{m'}} = \int_0^1 g'(X_v(t)) \left(\frac{\partial X_v(t)}{\partial v_N} - \frac{\partial X_v(t)}{\partial v_{m'}} \right) dt,$$

where

$$g'(X_v(t)) = \frac{1}{\alpha} X_v(t)^{\frac{1}{\alpha}-1}.$$

From Theorem 2, we know that at $\alpha = 1$ this expression is strictly positive. By continuity of the expression in both prize values v and the curvature parameter α, there exist $\epsilon_v > 0$ and $\delta_v > 0$ such that for all contests v' in an ϵ_v-neighborhood of v, and all $\alpha < 1 + \delta_v$, the effect of the transformation remains strictly positive.

The collection of such neighborhoods of contests forms an open cover of the feasible set of contests, which is compact. We may therefore extract a finite subcover. Let $\delta_{m'}$ denote the minimum of the corresponding δ_v across this finite subcover. It follows that for all feasible contests v and all $\alpha < 1+\delta_{m'}$, shifting value from prize m' to the top prize increases expected effort.

Repeating this argument for each interior prize $m' \in [N-1]$ and taking the minimum across these interior prizes yields $\alpha^* > 1$ such that, for all $\alpha \in [1, \alpha^*)$, starting from any feasible contest $v \in \mathcal{V}$, shifting value from any lower-ranked prize to the best prize increases expected effort. This establishes the result for $q = N + 1$.

The argument for general $q \in [N+1]$ is identical. The effect of shifting value to the best prize is strictly positive under linear costs (Theorem 5), continuous in both v and α, and the feasible set remains compact. The same continuity and covering argument therefore delivers the result. □

D Convergence

In this section, we establish an equilibrium convergence result for the continuum type-space. Specifically, we show that if a sequence of (parametric) finite type-space distributions converges to a differentiable distribution over a continuum type-space, then the corresponding sequence of mixed-strategy equilibria converges to the pure-strategy equilibrium in the continuum model. Intuitively, as the finite type-space becomes large, the interval over which a given type mixes shrinks, and essentially converges to the effort level prescribed by the pure-strategy equilibrium under the continuum type-space. Thus, the equilibrium in an appropriate and sufficiently large finite-type space provides a reasonable approximation to the equilibrium strategy under the continuum type-space, and vice versa.

We begin by recalling the symmetric equilibrium under a (parametric) continuum type-space (Moldovanu and Sela (2001)). For this section, we focus on the linear cost case ($c(x) = x$), which is without loss of generality due to the equivalence between convergence in effort cost and in effort.

Lemma 4. *Suppose there are $N+1$ agents, each with a private type (marginal cost of effort) drawn from $\Theta = [\underline{\theta}, \overline{\theta}]$ according to a differentiable CDF $G : [\underline{\theta}, \overline{\theta}] \to [0, 1]$. For any contest $v \in \mathcal{V}$, there is a unique symmetric Bayes-Nash equilibrium and it is such that for any $\theta \in \Theta$,*

$$X(\theta) = \int_{\theta}^{\overline{\theta}} \frac{\pi_v'(1 - G(t))g(t)}{t} dt.$$

Proof. Suppose N agents are playing a strategy $X : [\underline{\theta}, \overline{\theta}] \to \mathbb{R}_+$ so that if an agent's type is θ, it exerts effort $X(\theta)$. Further suppose that $X(\theta)$ is decreasing in θ. Now we want to find the remaining agent's best response to this strategy of the other agents. If the agent's type is θ and it pretends to be an agent of type $t \in [\underline{\theta}, \overline{\theta}]$, its payoff is

$$\pi_v(1 - G(t)) - \theta X(t).$$

Taking the first order condition, we get

$$\pi_v'(1 - G(t))(-g(t)) - \theta X'(t) = 0.$$

Now we can plug in $t = \theta$ to get the condition for $X(\theta)$ to be a symmetric Bayes-Nash equilibrium. Doing so, we get

$$\pi_v'(1 - G(\theta))(-g(\theta)) - \theta X'(\theta) = 0$$

so that

$$X(\theta) = \int_{\theta}^{\overline{\theta}} \frac{\pi_v'(1 - G(t))g(t)}{t} dt.$$

□

We now state and prove the convergence result.

Theorem 6. *Suppose there are $N+1$ agents and fix any contest $v \in \mathcal{V}$. Let $G : [\underline{\theta}, \overline{\theta}] \to [0,1]$ be a differentiable CDF and let $G^1, G^2, \ldots,$ be any sequence of CDF's, each with a finite support, such that for all $\theta \in [\underline{\theta}, \overline{\theta}]$,*

$$\lim_{n\to\infty} G^n(\theta) = G(\theta).$$

Let $F^n : \mathbb{R} \to [0,1]$ denote CDF of the equilibrium effort under G^n, and let $F : \mathbb{R} \to [0,1]$ denote CDF of the equilibrium effort under G. Then, the sequence of CDF's $F^1, F^2, \ldots,$ converges to the CDF F, i.e., for all $x \in \mathbb{R}$,

$$\lim_{n\to\infty} F^n(x) = F(x).$$

Proof. For the finite support CDF G^n, let $\Theta^n = (\theta_1^n, \theta_2^n, \ldots, \theta_{K(n)}^n)$ denote the support and $p^n = (p_1^n, p_2^n, \ldots, p_{K(n)}^n)$ denote the probability mass function. From Theorem 1, let $b^n = (b_1^n, b_2^n, \ldots, b_{K(n)}^n)$ denote the boundary points, $u^n = (u_1^n, u_2^n, \ldots, u_{K(n)}^n)$ denote the equilibrium utilities, and F_k^n denote the equilibrium CDF of agent of type θ_k^n on support $[b_{k-1}^n, b_k^n]$. Then, the CDF of the equilibrium effort, $F^n : \mathbb{R} \to [0,1]$, is such that for any $x \in \mathbb{R}$,

$$F^n(x) = \begin{cases} 0 & \text{if } x \le 0 \\ P_{k-1}^n + p_k^n F_k^n(x) & \text{if } x \in [b_{k-1}^n, b_k^n] \\ 1 & \text{if } x \ge b_{K(n)}^n \end{cases}. \tag{9}$$

For the continuum CDF $G : [\underline{\theta}, \overline{\theta}] \to [0,1]$, the CDF of the equilibrium effort, $F : \mathbb{R} \to [0,1]$, is such that for any $x \in \mathbb{R}$,

$$F(x) = \begin{cases} 0 & \text{if } x \le 0 \\ 1 - G(\theta(x)) & \text{if } x \in [0, B] \\ 1 & \text{if } x \ge B \end{cases}. \tag{10}$$

where $\theta(x)$ is the inverse of $X(\theta)$ (from Lemma 4) and $B = X(\underline{\theta})$. The following Lemma will be the key to showing that $F^n(x)$ converges to $F(x)$ for all $x \in \mathbb{R}$.

Lemma 5. *Consider any $\theta \in (\underline{\theta}, \overline{\theta})$ and for any $n \in \mathbb{N}$, let $k(n) \in \{0, 1, 2, \ldots, K(n)\}$ be such that $\theta_{k(n)}^n > \theta \ge \theta_{k(n)+1}^n$ (where $\theta_0^n = \infty$ and $\theta_{K(n)+1}^n = 0$). Then,*

$$\lim_{n\to\infty} b_{k(n)}^n = X(\theta) \text{ and } \lim_{n\to\infty} F^n(b_{k(n)}^n) = 1 - G(\theta).$$

Proof. From Lemma 4 and Theorem 1, we have

$$X(\theta) = \int_{\theta}^{\bar{\theta}} \frac{\pi_v'(1-G(t))g(t)}{t} dt \text{ and } b^n_{k(n)} = \sum_{j=1}^{k(n)} \frac{\pi_v(P^n_j) - \pi_v(P^n_{j-1})}{\theta^n_j}.$$

Observe that

$$\begin{aligned}
b^n_{k(n)} &= \left[\frac{\pi_v(P^n_{k(n)})}{\theta^n_{k(n)}} - \sum_{j=1}^{k(n)-1} \pi_v(P^n_j) \left[\frac{1}{\theta^n_{j+1}} - \frac{1}{\theta^n_j} \right] \right] \\
&= \int_0^{1/\theta^n_{k(n)}} \left[\pi_v(P^n_{k(n)}) - \pi_v(1 - G^n(1/x)) \right] dx \\
&\xrightarrow{n\to\infty} \int_0^{\frac{1}{\theta}} \left[\pi_v(1-G(\theta)) - \pi_v(1-G(1/x)) \right] dx \qquad \text{(dominated convergence)} \\
&= \underbrace{\left[x(\pi_v(1-G(\theta)) - \pi_v(1-G(1/x))) \right]_0^{\frac{1}{\theta}}}_{\text{this is 0}} + \int_0^{\frac{1}{\theta}} \frac{\pi_v'(1-G(1/x))g(1/x)}{x} dx \\
&= \int_{\theta}^{\infty} \frac{\pi_v'(1-G(t))g(t)}{t} \,\mathrm{d}t \qquad \text{(substitute } t = 1/x\text{)} \\
&= X(\theta)
\end{aligned}$$

By definition, we have

$$\begin{aligned}
\lim_{n\to\infty} F^n(b^n_{k(n)}) &= \lim_{n\to\infty} P^n_{k(n)} \\
&= \lim_{n\to\infty} [1 - G^n(\theta)] \\
&= 1 - G(\theta)
\end{aligned}$$

□

Returning to the proof of Theorem 6, fix any $x \in (0, B)$ and let $\theta \in (\underline{\theta}, \bar{\theta})$ be such that $X(\theta) = x$. Then, we know that

$$F(x) = 1 - G(\theta).$$

We want to show that

$$\lim_{n\to\infty} F^n(x) = 1 - G(\theta).$$

Take $\epsilon > 0$. Find $\theta' < \theta$ and $\theta'' > \theta$ such that

$$0 < G(\theta) - G(\theta') = G(\theta'') - G(\theta) < \frac{\epsilon}{4}.$$

Let $x' = X(\theta')$, $x'' = X(\theta'')$, so that $x' > x > x''$. Let $\delta = \min\{x' - x, x - x''\}$. From Lemma 5, let $N(\epsilon)$ be such that for all $n > N(\epsilon)$,

$$\max\{|b^n_{k(n)} - x|, |b^n_{k'(n)} - x'|, |b^n_{k''(n)} - x''|\} < \frac{\delta}{2}$$

and

$$\max\{|F^n(b^n_{k'(n)}) - (1 - G(\theta'))|, |F^n(b^n_{k''(n)}) - (1 - G(\theta''))|\} < \frac{\epsilon}{4},$$

where $k(n), k'(n), k''(n)$ are sequences as defined in Lemma 5 for θ, θ' and θ'' respectively. Then, for all $n > N(\epsilon)$,

$$\begin{aligned} F^n(x) &> F^n(b^n_{k''(n)}) \\ &> 1 - G(\theta'') - \frac{\epsilon}{4} \\ &> 1 - G(\theta) - \frac{\epsilon}{2} \end{aligned}$$

and

$$\begin{aligned} F^n(x) &< F^n(b^n_{k'(n)}) \\ &< 1 - G(\theta') + \frac{\epsilon}{4} \\ &< 1 - G(\theta) + \frac{\epsilon}{2} \end{aligned}$$

so that $|F^n(x) - (1 - G(\theta))| < \epsilon$. Thus, $\lim_{n\to\infty} F^n(x) = 1 - G(\theta) = F(x)$ for all $x \in \mathbb{R}$. $\square$

E Regression tables

Table 3: OLS for Effort Choice as Function of Prizes

	$c_k(x) = x$	$c_k(x) = 2x$	Pooled
First place prize (v_3)	0.524***	0.401***	0.426***
	(0.012)	(0.010)	(0.009)
Second place prize (v_2)	0.335***	0.321***	0.324***
	(0.015)	(0.016)	(0.015)
Third place prize (v_1)	0.334***	0.314***	0.318***
	(0.026)	(0.030)	(0.026)
N	1780	1780	1780
R^2	0.806	0.734	0.786

Standard errors in parentheses clustered at the subject level.

Constant omitted from estimation. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 4: OLS for Effort

	Model 1	Model 2
Constant	43.866***	26.087***
	(1.004)	(3.252)
Treatment *Med*	0.472	0.472
	(0.664)	(0.665)
Treatment *High*	2.715***	2.715***
	(0.687)	(0.687)
Treatment *WTA*	7.203***	7.203***
	(0.735)	(0.736)
Inefficient type ($c_k(x) = 2x$)	-8.900***	-8.900***
	(0.939)	(0.939)
Male (1 =yes)		-2.285
		(1.542)
Willingness to take Risks (0-10)		1.528***
		(0.499)
Willingness to Compete (0-10)		1.063**
		(0.503)
N	3560	3560
R^2	0.051	0.106

Standard errors in parentheses clustered at the subject level.

Treatment *Low* is the omitted category. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.