The effect of competition in contests: A unifying approach

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Introduction

- Agents make costly investments to win valuable prizes
  - Examples: sales tournaments, crowdsourcing events, R&D races
- How does contest structure shape investment incentives?
- Rank-order contests:
  - Prizes $v_0 \leq v_1 \leq \cdots \leq v_N$ awarded by rank
  - If agent of type θ exerts effort x and outperforms m agents:

$$\text{Payoff} = v_m - \theta\, c(x).$$

How should a budget be allocated across prizes to maximize total effort?

A puzzle in the literature

	Complete Info.	Finite types	Continuum of Types
Linear	(0, v, 100-v)	?	(0, 0, 100)
Concave	(0, 0, 100)	?	(0, 0, 100)
Convex	(0, 50, 50)	?	Depends

References: Barut & Kovenock [EJPE, 1998]; Fang et al. [JPE, 2020]; Moldovanu & Sela [AER, 2001]; Olszewski & Siegel [ECMA, 2016; TE, 2020]; Zhang [TE, 2024]

- Sisak [JES, 2009]'s conjecture:

  The case of asymmetric individuals, where types are private information but drawn from discrete, identical or even different distributions, has not been addressed so far. From the results ..., one could conjecture that multiple prizes might be optimal even with linear costs.

This paper

- Study rank-order contests with a finite type space
- Characterize unique symmetric equilibrium (novel representation)
- Analyze how competition affects effort under linear costs
- Identify conditions under which these effects extend to general costs
- Test theoretical predictions in an online experiment with binary types

Results

- The (most competitive) winner-takes-all contest is robustly optimal
  - Maximizes total effort of top q agents under linear or concave costs
  - Convex costs?
- Effort is not necessarily monotone increasing in competitiveness
  - Interior discouragement effect: making prizes more unequal may reduce effort if relatively inefficient types are likely
- Experiment provides qualitative support for these findings
- Novel approach for analyzing symmetric equilibrium in games

Outline

- Model
- Equilibrium characterization
- Effect of competition
  - Linear costs
  - General costs
- Experiment

Model

A *contest environment* is defined by $\{N+1, \mathcal{C}, p\}$, where

- $N+1$: number of agents
- $\mathcal{C} = \{c_1, \ldots, c_K\}$: finite set of types such that for each k,

$$c_k \in \{c : \mathbb{R}_+ \to \mathbb{R}_+ \mid c(0) = 0, c'(x) > 0, \lim_{x\to\infty} c(x) = \infty\}.$$

  - Ordered: $c_1'(x) > \cdots > c_K'(x)$ for all $x \in \mathbb{R}_+$
  - Parametric: $c_k(x) = \theta_k c(x)$ with $\theta_1 > \cdots > \theta_K$
- $p = (p_1, \ldots, p_K)$: distribution over $\mathcal{C}$

A *contest* $v = (v_0, \ldots, v_N)$ assigns a prize for each rank, where

$$v \in \mathcal{V} = \{v \in \mathbb{R}^{N+1} : v_0 \leq \cdots \leq v_N \text{ with } 0 = v_0 < v_N\}.$$

Bayesian game

- Given $(N+1, \mathcal{C}, p)$, $v \in \mathcal{V}$, the agents
  1. privately learn their types
  2. choose effort simultaneously
  3. and are awarded corresponding prizes
- Payoffs: if agent of type $c_k \in \mathcal{C}$ exerts effort x_k, and wins prize v_m (outperforms m out of remaining N agents), its vNM utility is

$$v_m - c_k(x_k).$$

- Symmetric Bayes-Nash equilibrium: $(X_1, X_2, \ldots, X_K)$ where $X_k \sim F_k$
- Effect of increasing competition: For prizes $m > m'$, compute

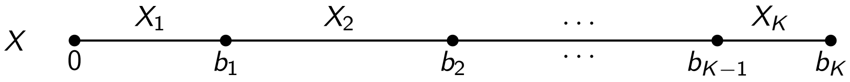
$$\frac{\partial \mathbb{E}[X]}{\partial v_m} - \frac{\partial \mathbb{E}[X]}{\partial v_{m'}},$$

where $\mathbb{E}[X] = \sum_{k=1}^{K} p_k \mathbb{E}[X_k]$ is the ex-ante expected effort.

Equilibrium structure

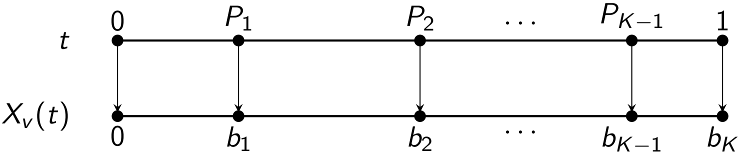
Lemma 1.

For any $(N+1, \mathcal{C}, p)$ *and* $v \in \mathcal{V}$*, a (symmetric) equilibrium* $(X_1, \ldots, X_K)$ *must be such that there exist boundary points* $0 = b_0 < b_1 < \cdots < b_K$ *so that, for each* k*,* X_k *is continuously distributed on* $[b_{k-1}, b_k]$*.*



Equilibrium representation

- $X_v(t)$: equilibrium effort in terms of **probability of outperforming an arbitrary agent**



- $\pi_v(t)$: expected prize function

$$\pi_v(t) = \sum_{m=0}^{N} v_m H_m^N(t), \qquad H_m^N(t) = \binom{N}{m} t^m (1-t)^{N-m}.$$

Example: for $v = (0, \ldots, 0, V)$, $\pi_v(t) = Vt^N$

Indifference conditions

- Let $u_1, u_2, \ldots, u_K$ denote equilibrium utilities.
- For $t \in [P_{k-1}, P_k]$,

$$\pi_v(t) - c_k(X_v(t)) = u_k \implies X_v(t) = c_k^{-1}(\pi_v(t) - u_k).$$

- Observe that

$$\begin{aligned} u_1 = 0 &\implies b_1 = c_1^{-1}(\pi_v(P_1)) \\ &\implies u_2 = \pi_v(P_1) - c_2(b_1) \\ &\implies b_2 = c_2^{-1}(\pi_v(P_2) - u_2) \\ &\implies u_3 = \ldots \end{aligned}$$

Equilibrium characterization

Theorem 2.

For any $(N+1, \mathcal{C}, p)$ *and* $v \in \mathcal{V}$, *the equilibrium* $(X_1, \ldots, X_K)$ *is such that for each* k, *the distribution* $F_k : [b_{k-1}, b_k] \to [0,1]$ *is defined by*

$$\pi_v(P_{k-1} + p_k F_k(x_k)) - c_k(x_k) = u_k \text{ for all } x_k \in [b_{k-1}, b_k],$$

where the boundary points $b = (b_0, \ldots, b_K)$, *with* $b_0 = 0$, *and the equilibrium utilities* $u = (u_1, \ldots, u_K)$, *with* $u_1 = 0$, *satisfy*

$$\pi_v(P_k) - c_k(b_k) = u_k \text{ for all } k \in [K],$$

and

$$\pi_v(P_{k-1}) - c_k(b_{k-1}) = u_k \text{ for all } k \in [K].$$

Expected effort

Since $t = F(X)$ is always $U[0,1]$,

$$\begin{aligned}\mathbb{E}[X] &= \int_0^1 X_v(t)dt \\ &= \int_0^{P_1} c_1^{-1}(\pi_v(t) - u_1)dt + \int_{P_1}^{P_2} c_2^{-1}(\pi_v(t) - u_2)dt + \dots\end{aligned}$$

Lemma 3.

For any $(N+1, \mathcal{C}, p)$ and $v \in \mathcal{V}$, the expected equilibrium effort is

$$\mathbb{E}[X] = \int_0^1 g_{k(t)} \left(\pi_v(t) - u_{k(t)}\right) dt,$$

where $g_k = c_k^{-1}$ and $k(t) = \max\{k : P_{k-1} \le t\}$.

Effect of competition

- For any prize $m \in \{1, \ldots, N\}$,

$$\frac{\partial \mathbb{E}[X]}{\partial v_m} = \int_0^1 g'_{k(t)} \left(\pi_v(t) - u_{k(t)}\right) \left[H_m^N(t) - \frac{\partial u_{k(t)}}{\partial v_m}\right] dt.$$

- For prizes $m > m'$, $\dfrac{\partial \mathbb{E}[X]}{\partial v_m} - \dfrac{\partial \mathbb{E}[X]}{\partial v_{m'}}$ is

$$\int_0^1 g'_{k(t)} \left(\pi_v(t) - u_{k(t)}\right) \left[H_m^N(t) - H_{m'}^N(t) - \left[\frac{\partial u_{k(t)}}{\partial v_m} - \frac{\partial u_{k(t)}}{\partial v_{m'}}\right]\right] dt,$$

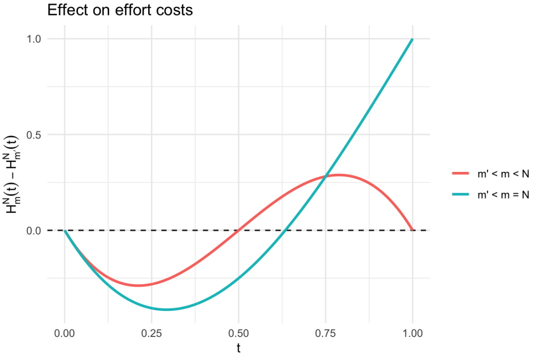
which provides a useful general framework in which to study the effect of increasing competition on effort.

1. Complete information (Fang, Noe, and Strack [2020])

- Suppose $\mathcal{C} = \{c_1\}$.
- For any $v \in \mathcal{V}$, $u_1 = 0$ and thus,

$$\frac{\partial \mathbb{E}[X]}{\partial v_m} - \frac{\partial \mathbb{E}[X]}{\partial v_{m'}} = \int_0^1 g_1'\left(\pi_v(t)\right)\left[H_m^N(t) - H_{m'}^N(t)\right] dt.$$

- For $c_1(x) = x$, this equals $\int_0^1 \left[H_m^N(t) - H_{m'}^N(t)\right] dt = 0$.



Complete information

Theorem 4.

Suppose $\mathcal{C} = \{c_1\}$. For any m, m' with $m > m'$, the following hold:

1. *If c_1 is concave, then for any $v \in \mathcal{V}$, $\dfrac{\partial \mathbb{E}[X]}{\partial v_m} - \dfrac{\partial \mathbb{E}[X]}{\partial v_{m'}} \geq 0$.*
2. *If c_1 is convex, then for any $v \in \mathcal{V}$, $\dfrac{\partial \mathbb{E}[X]}{\partial v_m} - \dfrac{\partial \mathbb{E}[X]}{\partial v_{m'}} \leq 0$.*

If c_1 concave, $v^* = (0, \ldots, 0, V)$. If c_1 convex, $v^* = (0, \frac{V}{N}, \ldots, \frac{V}{N})$.

- In this case, we can compute equilibrium utilities and expected effort.

Lemma 5.

Suppose $(N+1, \mathcal{C}, p)$ is such that $c_k(x) = \theta_k \cdot x$ with $\theta_1 > \cdots > \theta_K$. For any $v \in \mathcal{V}$, the equilibrium utilities are

$$u_k = \theta_k \left[\sum_{j=1}^{k-1} \pi_v(P_j) \left(\frac{1}{\theta_{j+1}} - \frac{1}{\theta_j} \right) \right],$$

and the expected effort is $\mathbb{E}[X] = \sum_{m=1}^{N} \alpha_m v_m$, where

$$\alpha_m = \frac{1}{N+1} \left[\frac{1}{\theta_K} - \sum_{k=1}^{K-1} \left[H_{\geq m}^{N+1}(P_k) + (N-m) H_m^{N+1}(P_k) \right] \left(\frac{1}{\theta_{k+1}} - \frac{1}{\theta_k} \right) \right].$$

Competition under linear costs

- Design problem: $v^* = (0, \ldots, 0, V)$ is uniquely optimal (resolves Sisak [2009]'s conjecture in the negative).
  For any prize $m \in \{1, \ldots, N-1\}$,

$$\alpha_N - \alpha_m > 0.$$

- *Interior discouragement effect*: Suppose $K = 2$. For any interior prize $m \in \{1, \ldots, N-1\}$,

$$p_1 \geq \frac{m+1}{N} \implies \alpha_{m+1} - \alpha_m \leq 0.$$

  In the extreme case, when $p_1 > \frac{N-1}{N}$, transferring value to a better-ranked prize encourages effort iff the better prize is top-ranked.

3. Incomplete information, general cost

- Suppose $\mathcal{C}$ is such that $c_k(x) = \theta_k \cdot c(x)$ with $\theta_1 > \cdots > \theta_K$.
- Reinterpret the game as one in which agents choose $c(x)$ instead of x: If $X \sim F$ when $c(x) = x$, then $c(X) \sim F$.
- For any $v \in \mathcal{V}$, u_k is as in Lemma 5 and independent of c.
- Further,

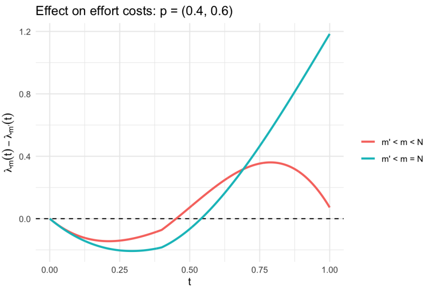
$$\frac{\partial \mathbb{E}[X]}{\partial v_m} - \frac{\partial \mathbb{E}[X]}{\partial v_{m'}} = \int_0^1 g' \left(\frac{\pi_v(t) - u_{k(t)}}{\theta_{k(t)}} \right) (\lambda_m(t) - \lambda_{m'}(t))\, dt,$$

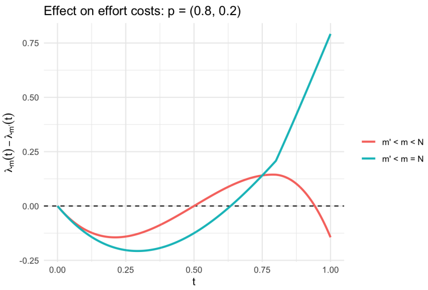
where

$$\lambda_m(t) = \left(\frac{H_m^N(t)}{\theta_{k(t)}} - \frac{1}{\theta_{k(t)}} \frac{\partial u_{k(t)}}{\partial v_m} \right).$$

- If $\lambda_m(t) - \lambda_{m'}(t)$ is single-crossing (i.e. $\lambda_m(1) - \lambda_{m'}(1) \geq 0$), then $\int_0^1 (\lambda_m(t) - \lambda_{m'}(t))\, dt = \alpha_m - \alpha_{m'}$ may be informative.

Illustration (Binary types)





Main result: linear to general

Theorem 6.

Suppose $(N+1, \mathcal{C}, p)$ *is such that* $c_k(x) = \theta_k \cdot c(x)$ *with* $\theta_1 > \cdots > \theta_K$*. Let* $m, m' \in \{1, \ldots, N\}$ *with* $m > m'$ *be such that, either* $m = N$ *or* $\left(\frac{\partial u_K}{\partial v_m} - \frac{\partial u_K}{\partial v_{m'}}\right) \leq 0$*. Then, the following hold:*

1. *If* $\alpha_m - \alpha_{m'} \geq 0$ *and* c *is concave, then for any* $v \in \mathcal{V}$, $\frac{\partial \mathbb{E}[X]}{\partial v_m} - \frac{\partial \mathbb{E}[X]}{\partial v_{m'}} \geq 0$.
2. *If* $\alpha_m - \alpha_{m'} \leq 0$ *and* c *is convex, then for any* $v \in \mathcal{V}$, $\frac{\partial \mathbb{E}[X]}{\partial v_m} - \frac{\partial \mathbb{E}[X]}{\partial v_{m'}} \leq 0$.

- Design problem: if c is concave, $v^* = (0, \ldots, 0, V)$.

Other objectives

- Theorem 6 can be easily generalized with implications for other quantities of interest.
  For example, $\mathbb{E}[X_{max}] = (N+1)\int_0^1 g_{k(t)}\left(\pi_v(t) - u_{k(t)}\right) t^N dt$.
- Replace $g'(\cdot)$ with the corresponding weighting function and derive analogous results.
  For $\mathbb{E}[X_{max}]$, it will be $g'\left(\frac{\pi_v(t)-u_{k(t)}}{\theta_{k(t)}}\right) t^N$.

Corollary 7.

Suppose $(N+1, \mathcal{C}, p)$ is such that $c_k(x) = \theta_k \cdot c(x)$ with $\theta_1 > \cdots > \theta_K$. If c is (weakly) concave, the WTA contest maximizes expected total effort of the top q agents for any $q \in \{1, \ldots, N+1\}$.

Convex costs?

- Analysis suggests optimality of the WTA persists as long as cost is not too convex
- Can use continuity arguments to obtain formal results
- Sharp contrast to the complete information environment
- Conjecture: The degree of convexity required to overturn the optimality of WTA shrinks as the contest environment approaches complete information.

Convergence

Theorem 8.

Fix any contest $v \in \mathcal{V}$ *and cost function* $c : \mathbb{R}_+ \to \mathbb{R}_+$*. Let* $G : [\underline{\theta}, \overline{\theta}] \to [0, 1]$ *be a differentiable CDF and let* $G^1, G^2, \ldots,$ *be any sequence of CDF's, each with a finite support, such that for all* $\theta \in [\underline{\theta}, \overline{\theta}]$,

$$\lim_{n \to \infty} G^n(\theta) = G(\theta).$$

Then, the corresponding sequence of finite-type space equilibrium CDF's, $F^1, F^2, \ldots,$ *converges to the continuum type-space equilibrium CDF* F*, i.e., for all* $x \in \mathbb{R}$,

$$\lim_{n \to \infty} F^n(x) = F(x).$$

- Intuitively, as K increases, $[b_{k-1}, b_k]$ shrinks, and converges to the pure-strategy equilibrium effort under the continuum type-space.

Experiment

- Prolific : 445 participants
- Contest environment: 4 agents, $\mathcal{C} = \{2x, x\}$, $p = (0.8, 0.2)$
- Four contests which are progressively less competitive:
  WTA $= (0, 0, 0, 100)$
  High $= (0, 0, 25, 75)$
  Med $= (0, 0, 50, 50)$
  Low $= (0, 25, 25, 50)$
- Strategy method, contest order randomized

Expected effort

Treatment	$(0, v_1, v_2, v_3)$	**Equilibrium Effort** ($\mathbb{E}[X]$)			**Observed Effort**		
		$c_k(x) = x$	$c_k(x) = 2x$	Pooled	$c_k(x) = x$	$c_k(x) = 2x$	Pooled
WTA	$(0, 0, 0, 100)$	48.2	6.4	14.76	52.8	40.5	42.96
High	$(0, 0, 25, 75)$	37.0	8.0	13.80	46.9	37.4	39.30
Med	$(0, 0, 50, 50)$	25.8	9.6	12.84	43.3	36.5	37.86
Low	$(0, 25, 25, 50)$	24.6	10.2	13.08	42.9	35.9	37.30

Table: Equilibrium and observed efforts by treatment and cost type.

- Efficient type exerts higher effort than the less efficient type
- Significant over-provision compared to equilibrium across treatments
- WTA remains optimal
- Going from Low to Med does not lead to a significant change in effort

Regression estimates

From Lemma 5 : $X = \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 + \epsilon$.

Prize	Equilibrium Weight			Estimated Coefficient		
	$c_k(x) = x$	$c_k(x) = 2x$	Pooled	$c_k(x) = x$	$c_k(x) = 2x$	Pooled
α_3	0.482	0.064	0.148	0.524	0.401	0.426
α_2	0.034	0.128	0.109	0.335	0.321	0.324
α_1	-0.014	0.152	0.119	0.334	0.314	0.318

Table: Expected Effort: Equilibrium and Regression Results

Summary

- The (most competitive) winner-takes-all is robustly optimal for maximizing effort under linear or concave costs.
- Despite this, increasing competition in the interior may discourage effort if inefficient types are relatively likely.
- An experiment provides qualitative support to these findings.
- The techniques we develop are broadly applicable, and may be valuable where mixed equilibria impede analysis.
- The convex cost and noisy output environments provide interesting avenues for future work.

Thank you!