

The effect of competition in contests: A unifying approach

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 - R&D: Millennium Prize Problems (2000)

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Research question

How should a budget be allocated across prizes to maximize total sales?

A puzzle in the literature

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	Complete Info.	Finite types	Continuum of Types
Linear	(0, v, 100-v)	?	(0, 0, 100)
Concave	(0, 0, 100)	?	(0, 0, 100)
Convex	(0, 50, 50)	?	Depends

References: Barut & Kovenock [EJPE, 1998]; Fang et al. [JPE, 2020]; Moldovanu & Sela [AER, 2001]; Olszewski & Siegel [ECMA, 2016; TE, 2020]; Zhang [TE, 2024]

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- Sisak [JES, 2009]'s conjecture:

The case of asymmetric individuals, where types are private information but drawn from discrete, identical or even different distributions, has not been addressed so far. From the results ..., one could conjecture that multiple prizes might be optimal even with linear costs.

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- Experiment provides qualitative support for these findings
- Novel approach for analyzing symmetric equilibrium in games

Outline

- Model
- Equilibrium characterization
- Effect of competition
 - Linear costs
 - General costs
- Experiment

Model

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A **contest** $v = (v_0, \dots, v_N)$ assigns a prize for each rank:

$$v \in \mathcal{V} = \{v \in \mathbb{R}^{N+1} : v_0 \leq \dots \leq v_N \text{ with } 0 = v_0 < v_N\}$$

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- Symmetric Bayes-Nash equilibrium: (X_1, X_2, \dots, X_K)

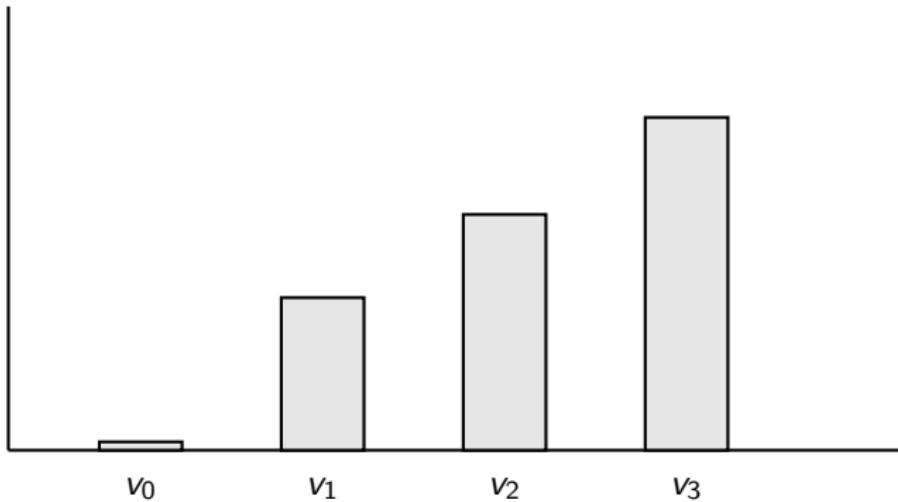
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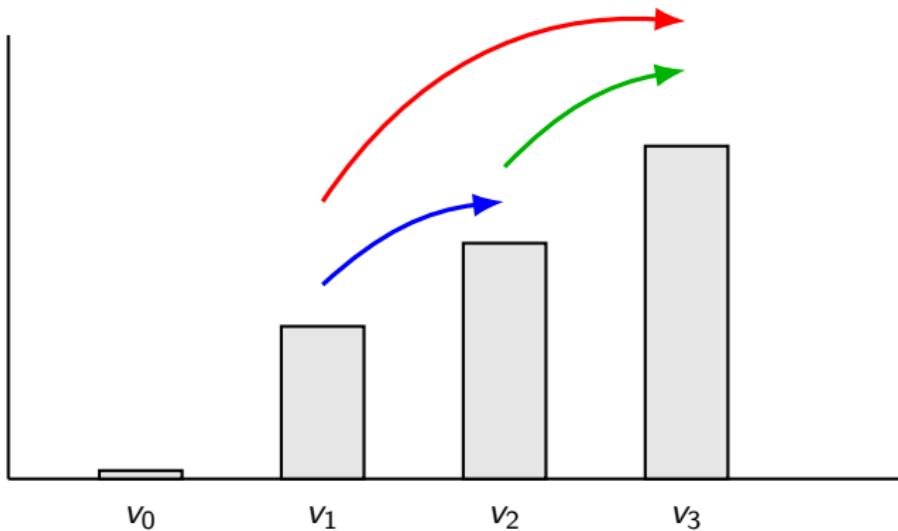
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- Symmetric Bayes-Nash equilibrium: (X_1, X_2, \dots, X_K)
- $\mathbb{E}[X] = \sum_{k=1}^K p_k \mathbb{E}[X_k]$: ex-ante expected effort

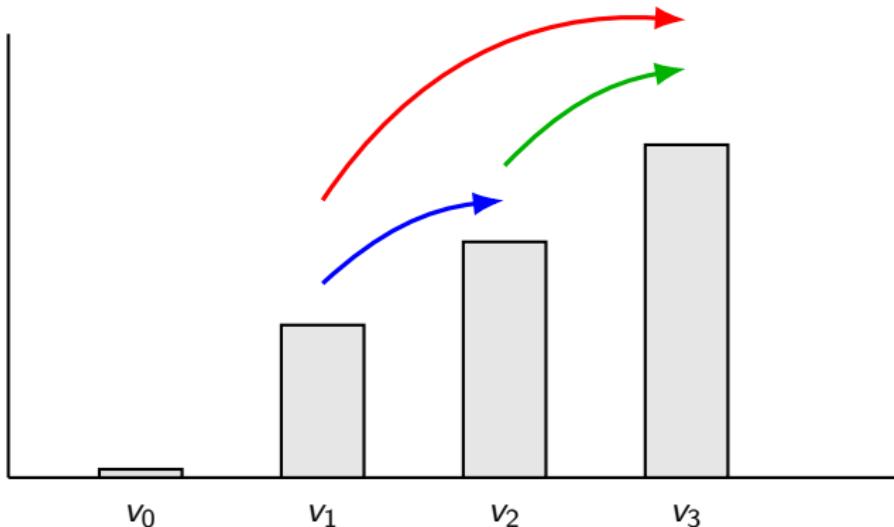
Increasing competition



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Research question

For prizes $m > m'$:

$$\frac{\partial \mathbb{E}[X]}{\partial v_m} - \frac{\partial \mathbb{E}[X]}{\partial v_{m'}}?$$

Equilibrium

Equilibrium structure

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Lemma.

Consider any $(N + 1, \Theta, p)$ and $v \in \mathcal{V}$. If (X_1, \dots, X_K) is an equilibrium, there exist boundary points $b_1 < \dots < b_K$ so that X_k is continuously distributed on $[b_{k-1}, b_k]$.

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④ Monotonicity:

If more efficient type has x, y with $x < y$ in its support, then less efficient type gets a higher payoff from x than y

Equilibrium representation

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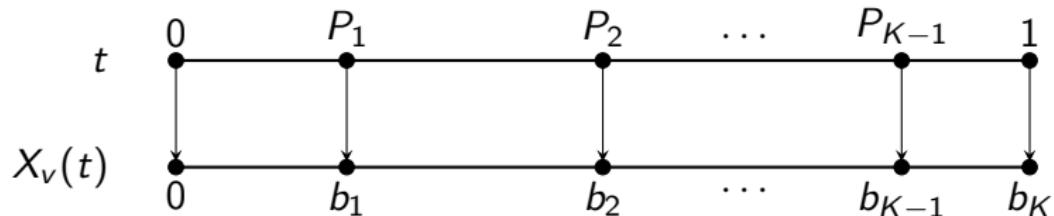
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Equilibrium representation

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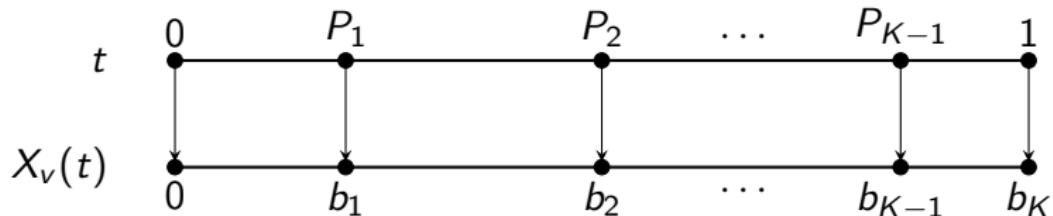
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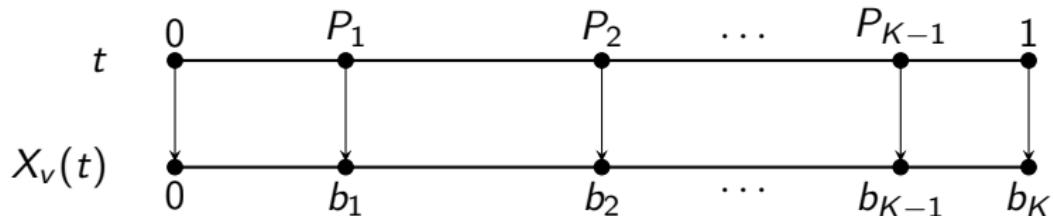
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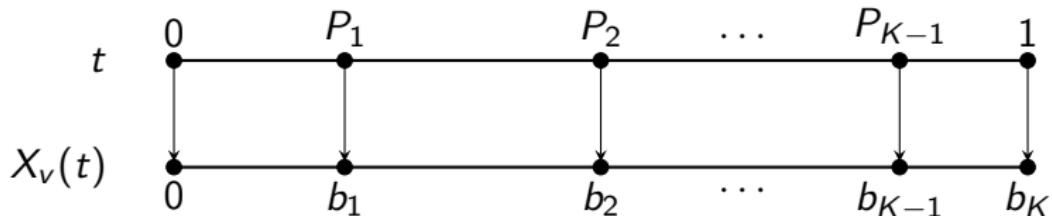


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$$\pi_v(t) = \sum_{m=0}^N v_m H_m^N(t) \quad H_m^N(t) = \binom{N}{m} t^m (1-t)^{N-m}$$

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Example: for $v = (0, \dots, 0, V)$, $\pi_v(t) = Vt^N$

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Equilibrium characterization

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Theorem 1.

For any $(N + 1, \Theta, p)$ and $v \in \mathcal{V}$, there is a unique symmetric equilibrium (X_1, \dots, X_K) . It is such that for each $t \in [0, 1]$,

$$X_v(t) = \frac{\pi_v(t) - u_{k(t)}}{\theta_{k(t)}},$$

where $k(t) = \max\{k : P_{k-1} \leq t\}$ and the equilibrium utilities are

$$u_k = \theta_k \left[\sum_{j=1}^{k-1} \pi_v(P_j) \left(\frac{1}{\theta_{j+1}} - \frac{1}{\theta_j} \right) \right].$$

Competition: Linear Cost

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$$\alpha_m = \frac{1}{N+1} \left[\frac{1}{\theta_K} - \sum_{k=1}^{K-1} \left[H_{\geq m}^{N+1}(P_k) + (N-m)H_m^{N+1}(P_k) \right] \left(\frac{1}{\theta_{k+1}} - \frac{1}{\theta_k} \right) \right].$$

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- For any $v \in \mathcal{V}$ and $m > m'$:

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- Suppose $\Theta = \{\theta_1\}$:

$$\alpha_1 = \alpha_2 = \cdots = \alpha_N = \frac{1}{(N+1)\theta_1}$$

Complete information (Barut & Kovenock [1998])

For any $m > m'$, $\alpha_m - \alpha_{m'} = 0$.

Incomplete information

Theorem 2.

Consider any $(N + 1, \Theta, p)$ with $|\Theta| > 1$. For any interior prize $m' \in \{1, \dots, N - 1\}$,

$$\alpha_N - \alpha_{m'} > 0.$$

For the design problem, $v^* = (0, \dots, 0, V)$ is uniquely optimal.

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Resolves Sisak [2009]'s conjecture in the negative

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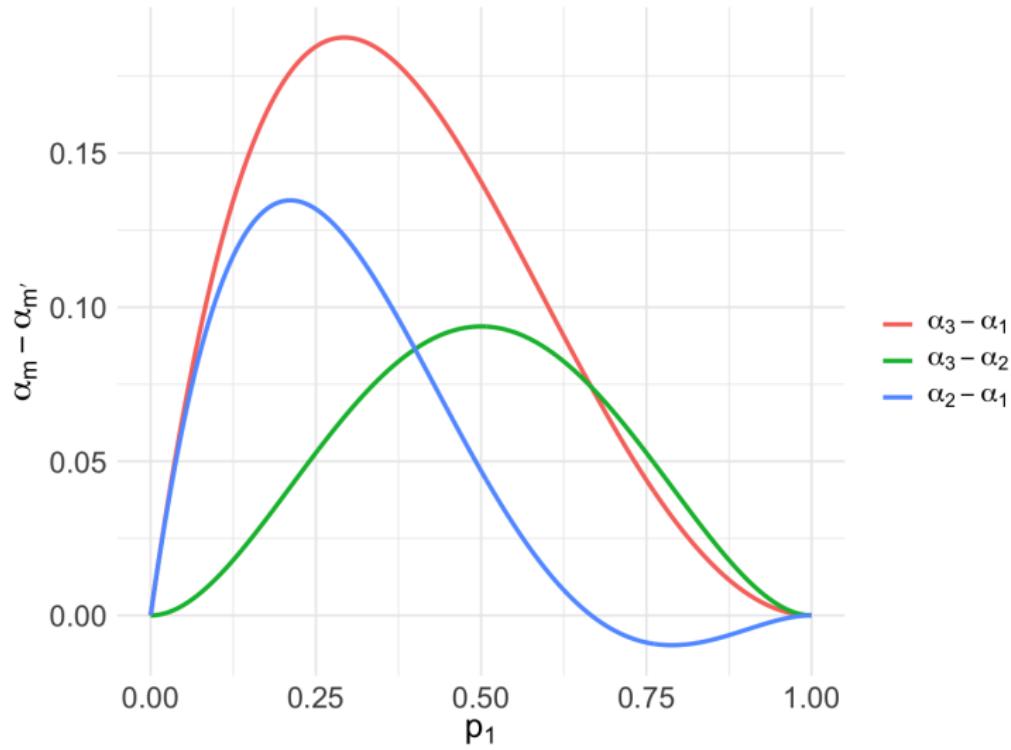
- Extreme case: If $p_1 > \frac{N-1}{N}$, for any $m > m'$,

$$\alpha_m - \alpha_{m'} \geq 0 \iff m = N.$$

Competition (Linear Cost)

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$$N = 3, \Theta = \{2, 1\}, p = (p_1, 1 - p_1)$$



Competition: General Cost

General costs

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- Given $(N+1, \Theta, p)$ and $v \in \mathcal{V}$, payoffs are

$$v_m - \theta_k \cdot c(x),$$

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$$c(X_v^*(t)) = X_v(t)$$

where $X_v(t)$ is the equilibrium effort under $c(x) = x$ (Theorem 1)

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- Equilibrium: For each $t \in [0, 1]$,

$$X_v^*(t) = g(X_v(t))$$

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Approach: Linear to General

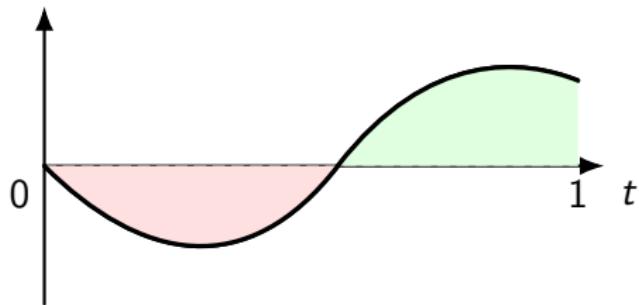
Approach: Linear to General

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Approach: Linear to General

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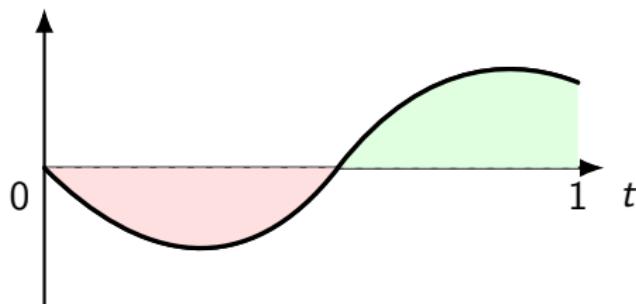
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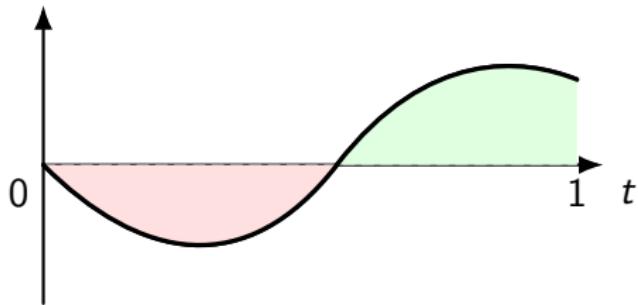


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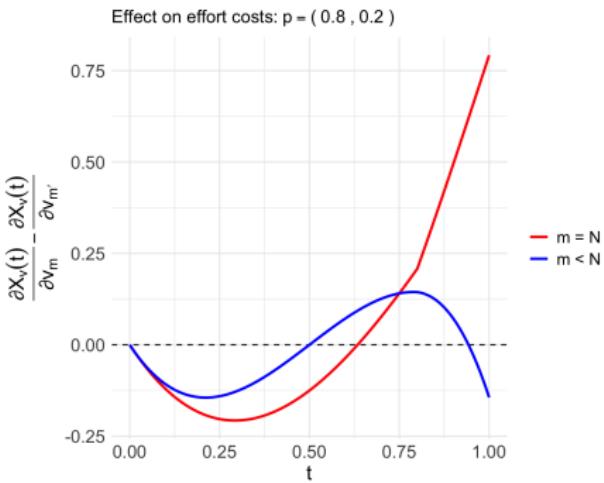
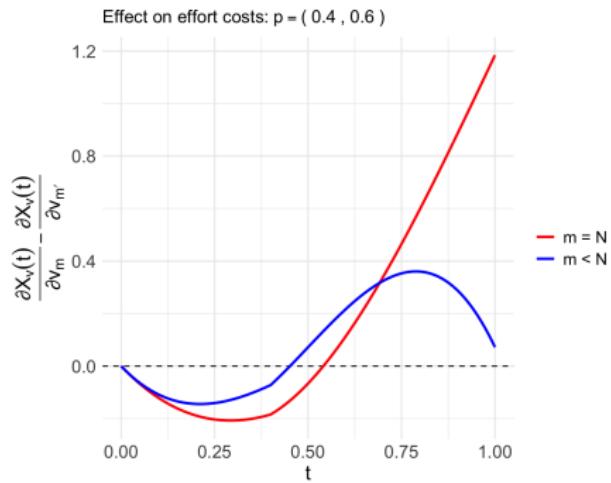
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Single-crossing?

- $m = N$: ✓
- $m < N$: ✓ or ✗



Main Result

Theorem 3.

Consider any $(N + 1, \Theta, p)$ with cost $c(\cdot)$. Let m, m' with $m > m'$ be such that either $m = N$ or $\left(\frac{\partial u_K}{\partial v_m} - \frac{\partial u_K}{\partial v_{m'}} \right) \leq 0$. Then, the following hold:

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Complete information (Fang, Noe, and Strack [2020])

Suppose $|\Theta| = 1$.

- ① If c is concave, competition encourages effort.
- ② If c is convex, competition discourages effort.

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Theorem 4.

Consider any $(N + 1, \Theta, p)$ with cost $c(\cdot)$. If c is (weakly) concave, the winner-takes-all contest maximizes expected total effort of top q agents for any $q \in \{1, \dots, N + 1\}$.

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Conjecture

The degree of convexity required to overturn the optimality of WTA shrinks as the contest environment approaches complete information.

Convergence

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Theorem 6.

Fix any contest $v \in \mathcal{V}$ and cost function $c : \mathbb{R}_+ \rightarrow \mathbb{R}_+$. Let $G : [\underline{\theta}, \bar{\theta}] \rightarrow [0, 1]$ be a differentiable CDF and let G^1, G^2, \dots , be any sequence of CDF's, each with a finite support, such that for all $\theta \in [\underline{\theta}, \bar{\theta}]$,

$$\lim_{n \rightarrow \infty} G^n(\theta) = G(\theta).$$

Then, the corresponding sequence of finite-type space equilibrium CDF's, F^1, F^2, \dots , converges to the continuum type-space equilibrium CDF F , i.e., for all $x \in \mathbb{R}$,

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- Experimental literature: Large but finite type-spaces

Experiment

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- Prolific : 445 participants
- Contest environment: 4 agents, $\Theta = \{2x, x\}$, $p = (0.8, 0.2)$
- Four contests which are progressively less competitive:
 - WTA = (0, 0, 0, 100)
 - High = (0, 0, 25, 75)
 - Med = (0, 0, 50, 50)
 - Low = (0, 25, 25, 50)
- Strategy method, contest order randomized

Expected effort

Treatment	$(0, v_1, v_2, v_3)$	Equilibrium Effort ($\mathbb{E}[X]$)			Observed Effort		
		$c_k(x) = x$	$c_k(x) = 2x$	Pooled	$c_k(x) = x$	$c_k(x) = 2x$	Pooled
WTA	(0, 0, 0, 100)	48.2	6.4	14.76	52.8	40.5	42.96
High	(0, 0, 25, 75)	37.0	8.0	13.80	46.9	37.4	39.30
Med	(0, 0, 50, 50)	25.8	9.6	12.84	43.3	36.5	37.86
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Table: Equilibrium and observed efforts by treatment and cost type.

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- Going from Low to Med does not lead to a significant change in effort

Regression estimates

$$X = \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 + \epsilon.$$

Prize	Equilibrium Weight			Estimated Coefficient		
	$c_k(x) = x$	$c_k(x) = 2x$	Pooled	$c_k(x) = x$	$c_k(x) = 2x$	Pooled
α_3	0.482	0.064	0.148	0.524	0.401	0.426
α_2	0.034	0.128	0.109	0.335	0.321	0.324
α_1	-0.014	0.152	0.119	0.334	0.314	0.318

Table: Expected Effort: Equilibrium and Regression Results

Summary

- The (most competitive) winner-takes-all is robustly optimal for maximizing effort under linear or concave costs.
- Despite this, increasing competition in the interior may discourage effort if inefficient types are relatively likely.
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