Project selection with partially verifiable information

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Introduction

- A principal has to chose between N projects, but does not know how profitable they are
- The agent knows how profitable the projects are, but may also have its own preferences over these projects
- How can the principal use the agent's information to chose the right project?

Model

- Two parties: principal and agent
- $[N] = \{1, 2, \dots, N\}$: set of available projects
- $(p_i, a_i) \sim F$: payoff values for principal and agent from project i
- $\Theta = X^N$ where $X \subset \mathbb{R}^2$ is support of F
- The principal commits to a direct mechanism $d : \Theta \to [N]$
- $(p_{d(\pi,\alpha)}, a_{d(\pi,\alpha)})$: payoffs for principal and agent under mechanism d when true state is $\theta = (p, a)$ and agent reports $\theta' = (\pi, \alpha)$

Optimal mechanism

- By the revelation principle, restrict attention to truthful mechanisms
- A mechanism d is truthful iff there exists $S \subset [N]$ s.t. for all $(p, a) \in \Theta$
$$d(p, a) = \text{argmax}_{i \in S} a_i$$
- If F is such that the principal and agent payoffs are independent, then a constant mechanism that always chooses the same project is optimal.
- Thus, the principal can not gain from agent's private information about the projects.

No overselling

- The agent's ability to manipulate might be limited due to environmental constraints or if it is required to support its claims with some form of evidence.
- We assume the partial verifiability constraint of no-overselling.
- Formally, for any $\theta = (p, a) \in \Theta$, the set of messages available to the agent are

$$M(p, a) = \{(\pi, \alpha) \in \Theta : \pi_i \leq p_i \; \forall i \in [N]\}$$

- We study the principal's problem under this no-overselling constraint on the agent.

Revelation principle

- Does the revelation principle hold?
- Restricted to direct mechanisms only
- Since M satisfies the Nested Range condition (Green and Laffont [1986]), the revelation principal applies and we can restrict attention to truthful mechanisms

$$\theta' \in M(\theta) \implies M(\theta') \subset M(\theta)$$

- Principal's problem:

$$\max_{d:d \text{ is truthful}} \mathbb{E}[p_{d(p,a)}]$$

Outline

- Characterization of truthful mechanisms
- Maximizing expected profit
  - 2 projects
  - Ally principle
  - N projects
- Maximizing probability of choosing best project
  - 2 projects
  - Ally principle

Truthful mechanisms

- A mechanism d is truthful if for any (p, a) and $(\pi, \alpha) \in M(p, a)$,

$$a_{d(p,a)} \geq a_{d(\pi,\alpha)}$$

- A mechanism d is a table mechanism if there exists a function $f : \Theta \to 2^N$ such that
  - $f(p, a) \neq \phi$
  - $f(p, a) = f(p, a') = f(p)$
  - $p \leq p' \implies f(p) \subset f(p')$

  and
  - $d(p, a) = \arg max_i\{a_i : i \in f(p)\}$.

Theorem

d is truthful if and only if it is a table mechanism.

Proof

- Table $\implies$ Truthful
  - Suppose d is a table mechanism. Under (p, a), if agent reports $(\pi, \alpha) \in M(p, a)$, it must be that $\pi \leq p$.
  - Since f is increasing, $f(\pi) \subset f(p)$ and so the agent cannot gain from misreporting.
- Truthful $\implies$ Table
  - Suppose d is truthful.
  - Define $f : \Theta \to 2^N$ so that $i \in f(p, a) \iff$ there exists some $(p', a') \in M(p, a)$ such that $d(p', a') = i$.
  - Verify that f satisfies the properties of table mechanism.

Cutoff mechanisms

- d is a *cutoff mechanism* if it is a table mechanism and there exist cutoffs c_i such that $i \in f(p)$ if and only if $p_i \geq c_i$.
- d is a *single cutoff mechanism* if it is a cutoff mechanism where $c_i = c$ for all $i \in N \setminus \{N\}$.

Maximizing profit for $N = 2$ projects

Theorem

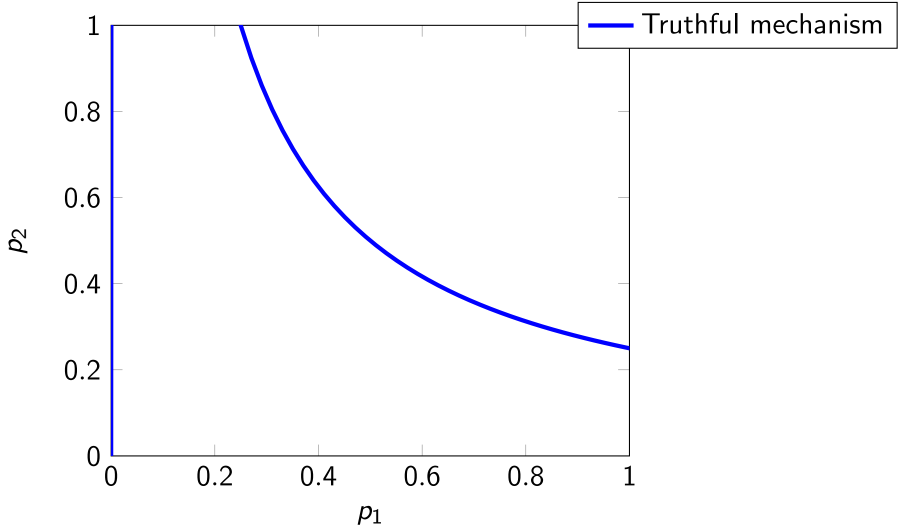
For two projects with $(p_i, a_i) \sim F$, the optimal mechanism is a cutoff mechanism. The optimal cutoff c_1 is defined by

$$\mathbb{E}\left[(p_1 - p_2)\Pr(a_1 > a_2 | p_2) | p_1 = c_1\right] = 0$$

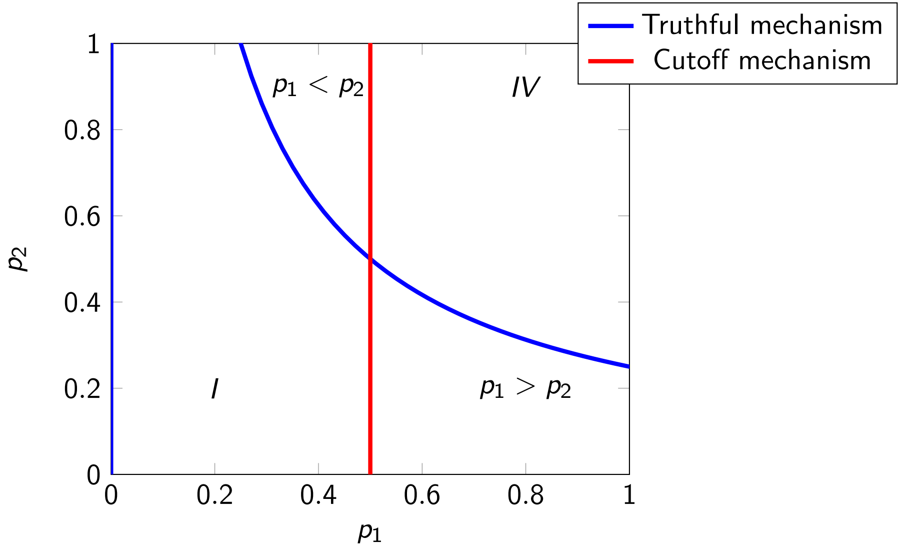
Corollary

Suppose F is such that the principal and agent payoffs are independent. Then the optimal cutoff is $c_1 = \mathbb{E}[p_1]$.

A truthful (table) mechanism



Cutoff is optimal



The ally principle

- Principal delegates more authority to an agent who shares their preferences.
- To discuss the principle in our model, we consider the case where $(p_i, a_i) \sim N(0, 0, 1, 1, \rho)$ and ask if the optimal cutoff $c(\rho)$ is decreasing in ρ.

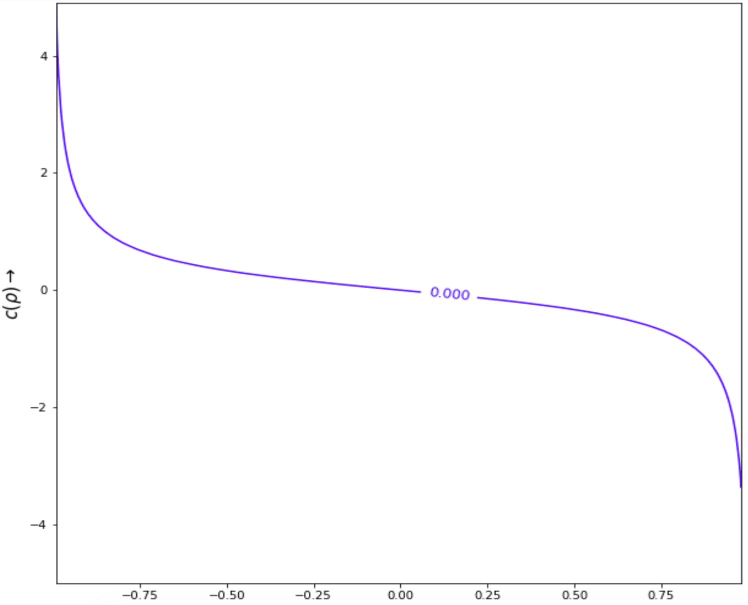
Theorem

For $N = 2$ projects with $(p_i, a_i) \sim N(0, 0, 1, 1, \rho)$, the optimal cutoff is defined by the equation

$$c\Phi(tc) + t\phi(tc) = 0$$

where $t = \frac{\rho}{\sqrt{2-\rho^2}}$ and Φ, ϕ represent standard normal cdf and pdf.

$c(\rho)$



N projects

- Under the independence assumption, the principal's problem is:

$$max_f \mathbb{E}\left[\frac{\sum_{i\in f(p)} p_i}{|f(p)|}\right]$$

where $f : \mathbb{R}^n \to 2^{[n]}$ must be such that
  - it is never empty ($f(p) \neq \phi$ for any p)
  - and it is increasing ($p \leq p' \implies f(p) \subset f(p')$).
- For the uniform case, we show that
  1. the principal's expected utility from the optimal cutoff mechanism is

$$V_N = 1 - \frac{1}{\sqrt{N}} + o\left(\frac{1}{\sqrt{N}}\right)$$

  2. there is a function $\gamma(N) \in o(1)$ such that

$$V(f) \leq 1 - \frac{1 + \gamma(N)}{8\sqrt{N}}.$$

Maximizing probability of choosing best project

- The principal wants to choose a table mechanism d to maximize $\mathbb{P}[p_{d(p,a)} \geq p_j \text{ for all } j]$

Theorem

For two projects with $(p_i, a_i) \sim F$, the optimal mechanism is a cutoff mechanism. The optimal cutoff c_1 is defined by

$$\mathbb{P}[p_1 \geq p_2 | p_1 = c_1, a_1 \geq a_2] = \frac{1}{2}$$

Corollary

Suppose F is such that the principal and agent payoffs (p_i, a_i) are independent. Then the optimal cutoff is given by $c_1 = Median[p_1]$.

The ally principle

Theorem

For $N = 2$ projects with $(p_i, a_i) \sim N(0,0,1,1,\rho)$, the optimal cutoff $c(\rho)$ is given by the equation

$$\frac{BVN(c, tc)}{\Phi(tc)} = \frac{1}{2}$$

where $t = \frac{\rho}{\sqrt{2-\rho^2}}$, Φ is standard normal cdf, BVN is cdf of standard normal random variables with correlation t.

- We believe that the optimal mechanism for the payoff and probability objectives actually coincide for the case $(p_i, a_i) \sim N(0,0,1,1,\rho)$.

Lit review

- **Implementation with partially verifiable information**: Green and Laffont [1986], Singh and Wittman [2001], Kartik [2009], Caragiannis et al. [2012], Ben-Porath and Lipman [2012], Fotakis et al. [2017]
- **Optimal mechanisms under partial verifiability**: Moore [1984], Deneckere and Severinov [2001], Celik [2006], Munro et al. [2014]
- **Project selection**: Armstrong and Vickers [2010], Ben-Porath et al. [2014], Mylovanov and Zapechelnyuk [2017], Guo and Shmaya [2021]

Conclusion

- We illustrate how identifying or inducing partial verifiability constraints like no-overselling can significantly improve the principal's payoffs in asymmetric information environments.
- We characterize the class of truthful mechanisms under the no-overselling constraint and find the optimal mechanism for two natural objectives- maximizing expected payoff and maximizing probability of choosing the best project.
- In the special case when payoffs are bivariate normal, we find that the optimal cutoff decreases as the correlation between principal and agent payoffs increases, and thus, our model lends support to the well-known ally principle.

Thank you!