Project selection with partially verifiable information*

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Abstract

We study a principal-agent project selection problem with asymmetric information. The principal must choose exactly one of N projects, each defined by the utility it provides to the principal and to the agent. The agent knows all the utilities, and the principal can commit to a mechanism (without transfers) that maps the agent's report about the utilities to a chosen project. Unlike the typical literature, which assumes the agent can lie arbitrarily, we examine the principal's problem under partial verifiability constraints. We characterize the class of truthful mechanisms under a family of partial verifiability constraints and study the principal's problem for the specific cases of no-overselling and no-underselling. Our results suggest significant benefits for the principal from identifying or inducing such partial verifiability constraints, while also highlighting the simple mechanisms that perform well.

1 Introduction

Suppose a principal has to choose exactly one out of a set of available projects, but does not know how profitable these projects are. There is an agent who is fully informed about the profitability of the projects, but also has its own private preference over which project is chosen. Assuming the principal has commitment power, we study the question of whether and how the principal can use the private information held by the agent to choose a profitable project. If transfers are feasible, the principal can use a classical sell-the-firm type mechanism to extract the entire surplus. However, in many contexts, as when a decision maker within a firm delegates information-gathering responsibilities to a subordinate, such transfers are infeasible or impractical. We focus on such environments and study the principal's

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problem of finding a mechanism, mapping the agent's report about its private information to a choice of project, so as to maximize expected profit.

In this setting, if the agent is unconstrained in its reporting, as is typically assumed in the mechanism design literature, the principal cannot really exploit the agent's private information about the profitability of the projects. To see this, consider a naive mechanism in which the principal commits to simply choosing the most profitable project. Clearly, the agent has an incentive to lie about its private information and report its most preferred project to be the most profitable project, thus implementing the mechanism in which the agent's most preferred project is always chosen. More generally, it is straightforward to show that a mechanism is implementable (truthful) if and only if it always selects the agent's most preferred project among those in the range of the mechanism. It follows that if the agent's preference over the projects does not reveal anything about their profitability (independence), the principal is essentially stripped of the power to exploit the private information held by the agent about the profitability of the projects to get higher profits.

However, the principal can potentially, depending upon the environment, identify or induce some partial verifiability constraints that limit the extent to which the agent can misreport, for example by requiring the agent to furnish some evidence in support of its claims. For instance, if a tech firm wants to hire a programmer and the hiring committee is biased towards applicants with better social skills, the firm can require the committee to justify its assessment of applicants' programming skills with accompanying certificates. While the committee can still perhaps undersell certain applicants by not revealing all their credentials, it might be prohibitively costly for the committee to fabricate fake certificates and oversell applicants. Therefore, in designing a mechanism, the firm only needs to guard against potential underselling by the committee and can safely assume that overselling will not occur.

In this paper, we investigate the potential value for the principal from identifying or inducing such partial verifiability constraints. There are different notions of partial verifiability that might be reasonable depending on the environment. For instance, apart from no-overselling, one could have environments where the agent cannot undersell, or when the agent cannot distort the true state significantly. In this work, we focus on partial verifiability constraints that satisfy the assumptions of Nested Range Condition (NRC), Separability, and Agency Freedom. While NRC ensures that the revelation principle applies and we can restrict attention to truthful mechanisms (Green and Laffont [10]), Agency Freedom allows the agent to always be able to claim any preference over the projects, and Separability then ensures that the agent's set of potential misreports about the principal's profits in any state depends only on the principal's true profits and not on the agent's true preference over the projects.

Our first result characterizes the class of truthful mechanisms for the family of partial verifiability constraints satisfying NRC, Separability, and Agency Freedom. In particular, we show that for any such partial verifiability constraint, the class of truthful mechanisms is always a subclass of table mechanisms. A table mechanism can be interpreted as being defined by a table function that determines the set of projects on the table based on the agent's report about the principal's profits, so that the mechanism simply chooses the agent's favorite project from those on the table. The subclass of table mechanisms that characterize truthful mechanisms is then determined by a simple restriction on the set of table functions: the set of projects on the table must be increasing in the set of misreports available.

Next, we study the design problem for the specific partial verifiability constraints of no-overselling and no-underselling, and our results suggest significant benefits for the principal from identifying such partial verifiability constraints, while also illustrating both the simplicity of the optimal mechanisms and the distinction between them. We first use our characterization result to show that truthful mechanisms are characterized by table functions that are increasing in reported profits in the no-overselling case, and decreasing in reported profits in the no-underselling case. We then turn to the specific case where there are two projects. We observe that these characterizations imply the existence of a default project that is always on the table, and so the optimal mechanism identifies conditions under which the alternative project is also on the table. In the no overselling case, the condition is that the reported profit from the alternative project should be high enough, whereas in the no underselling case, it is that the reported profit from the default project should be low enough. Finally, we note that this simple cutoff structure allows for natural comparative statics, and we use this to investigate the well-known ally principle. In particular, we analyze the design problem for the no-overselling case under bivariate normal payoffs and our results provide evidence in support of the principle.

Related literature

There is a vast literature studying mechanism design with evidence or partially verifiable information, and an important theme has been on characterizing implementable mechanisms in different environments. For our principal-agent setting without transfers, Green and Laffont [10] showed that if the message correspondence satisfies the NRC, the revelation principle applies and we can restrict attention to truthful mechanisms. In recent work, Schweighofer-Kodritsch and Strausz [17] present a unifying approach to mechanism design with and without evidence, allowing for a direct application of Myerson's revelation principle (Myerson [15]). Deneckere and Severinov [8] and Bull and Watson [4] study multi-agent environments, identifying conditions under which static mechanisms suffice for implementation.

Another related stream of literature studies the value of principal's commitment power in persuasion games, where the agent tries to persuade the principal to take a certain action and can present some evidence about its type. In such games with binary actions, Glazer and Rubinstein [9] find that neither randomization nor commitment power provides any value to the principal. Sher [18] and Hart et al. [12] show that this result applies under more general conditions, while Ben-Porath et al. [3] obtain similar findings for multi-agent environments. In recent work, Silva [19] demonstrates that this finding is not robust to making the evidence imperfect. The conditions noted above apply in some special cases of our model, suggesting that randomized mechanisms or even commitment power do not always provide additional benefit to the principal in our project-selection framework.

There is other related literature focusing on design problems with some specific forms of partial verifiability, beginning with Townsend [20]. In particular, in models with a single dimensional state, Celik [5], Moore [14], and Krähmer and Strausz [13] also focus on uni-directional incentive constraints of no overstating or no understating. Celik [5] and Moore [14] study a contracting problem and find sufficient conditions under which the uni-directional constraint of the agent not being able to overstate its productivity provides no additional benefit to the principal. In recent work, Krähmer and Strausz [13] study implementable allocation rules with transfers in a general setting and present several examples where uni-directional incentive constraints might be reasonable. This literature on uni-directional incentive constraint, though studying problems different from the project-selection problem considered in this paper, suggests the relevance of such constraints in important design environments.

Lastly, the paper also contributes to the literature studying principal-agent project selection problems. In closely related work, Che et al. [6] study a model where the agent, who has the same utilities over projects as the principal but a different outside option, can send a cheap-talk message (recommendation) following which the principal chooses the project. They find that in equilibrium, the agent might sometimes recommend Pareto inferior projects. Armstrong and Vickers [1] study a model where the set of available projects is random, and the agent can propose any project whose payoffs can then be verified. They identify properties of the optimal set of projects that the principal should commit to approving, showing in particular that it exhibits a simple cutoff structure. In a similar model, Guo and Shmaya [11] illustrate how the principal, who cares about minimizing regret, can benefit from allowing the agent to propose multiple projects despite selecting only one. In other related work, Ben-Porath et al. [2] and Mylovanov and Zapechelnyuk [16] study the problem of a principal choosing an agent among a fixed set of agents, each of whom wish to get selected and provide some private value to the principal. Overall, in this literature on project selection or agent selection problems, the principal is empowered by ex-post verifiability of the reported values and the ability to use a prohibitively high punishment to deter the agent from telling *any* lie, whereas in our setup, the agent is constrained in its ability to lie by the partial verifiability constraints, but the principal does not have the power to directly deter the agent from lying when it is feasible for the agent to do so. As a result, the resulting incentive considerations in these papers are quite different from ours.

The paper proceeds as follows. In Section 2, we present the model. Section 3 characterizes truthful mechanisms. In Section 4, we study the design problem for the no-overselling case.

In Section 5, we discuss the ally principle and the no-underselling case. Section 6 concludes.

2 Model

There are two parties: a principal and an agent. There is a set of available projects $[N] = \{1, 2, \ldots, N\}$ and the principal must choose exactly one of these projects. If project $i \in [N]$ is chosen, it leads to payoffs $(p_i, a_i) \in \mathbb{R}^2$, where p_i denotes the profit for the principal and a_i denotes the utility to the agent. The payoffs (p_i, a_i) are distributed according to a bi-variate distribution $F_i : \mathbb{R}^2 \to [0, 1]$. We assume that payoffs across projects are independent (not necessarily identical), but F_i might be such that p_i and a_i are correlated. The principal knows the distributions $(F_1, F_2, \ldots, F_N)$, and the agent knows the true payoffs from all the projects $(p, a) \in \mathbb{R}^N \times \mathbb{R}^N = \Theta_p \times \Theta_a = \Theta$.

The principal can commit to a (direct) mechanism $d : \Theta \to [N]$ so that if the agent reports payoffs (π, α) when the true state is (p, a), the project $d(\pi, \alpha)$ is chosen leading to final payoffs $p_{d(\pi,\alpha)}, a_{d(\pi,\alpha)}$ for the principal and agent respectively.

To model partial verifiability, we assume that there is a message correspondence $M : \Theta \rightrightarrows \Theta$, so that $M(\theta) \subseteq \Theta$ denotes the set of admissible/feasible messages when the true state is $\theta \in \Theta$. We will assume that truthful reporting is always feasible so that $\theta \in M(\theta)$ for all $\theta \in \Theta$.

Definition 1. *Given $M : \Theta \rightrightarrows \Theta$, a mechanism $d : \Theta \to [N]$ is truthful under M if for any $(p, a) \in \Theta_p \times \Theta_a$ and $(\pi, \alpha) \in M(p, a)$,*

$$a_{d(p,a)} \geq a_{d(\pi,\alpha)}.$$

Before introducing partial verifiability, we illustrate how the principal cannot exploit the agent's information if there is no partial verifiability[1].

2.1 No partial verifiability

Suppose $M(\theta) = \Theta$ for all $\theta \in \Theta$ so that the agent can always misreport arbitrarily. Notice that the mechanism in which the principal commits to choosing the agent's favorite project is a truthful mechanism. More generally, consider the mechanism in which the principal predetermines a set $S \subset [N]$ of projects and commits to choosing the agent's favorite project among S. It is easy to see that each of these mechanisms, defined by $S \subseteq [N]$, is truthful. It turns out that, when there is no partial verifiability, these are the only truthful mechanisms.

[1]Note that we are assuming transfers are not feasible in this setting. If the principal could use transfers to incentivize the agent, then a classical sell-the-firm mechanism would allow the principal to extract the entire surplus (without any partial verifiability constraints).

Lemma 1. *Suppose $M(\theta) = \Theta$ for all $\theta \in \Theta$. A mechanism $d : \Theta \to [N]$ is truthful if and only if there exists $S \subset [N]$ such that for any $(p, a) \in \Theta$,*

$$d(p, a) \in \arg\max_i \{a_i : i \in S\}.$$

Proof. Consider any mechanism $d : \Theta \to [N]$ and let S denote the range of this mechanism. For any $(p, a) \in \Theta$, the agent will report a $(\pi, \alpha) \in \Theta$ at which the mechanism chooses the agent's favorite project in S. The result follows. □

In particular, it follows from Lemma 1 that under any truthful mechanism, the outcome of the mechanism cannot depend on p. Thus, the best principal can do in this case is to pre-determine a set S of projects (based on $F_1, F_2, \ldots, F_N$) and commit to choosing the agent's favorite project in S. In the extreme cases, if each F_i is such that there is perfect positive correlation ($p_i = a_i$), then the principal should choose $S = [N]$, and if there is perfect negative correlation ($p_i = -a_i$), then the principal should just always pick the project with the highest ex-ante expected profit. Importantly, without partial verifiability, the principal cannot exploit the private information held by the agent about p to choose potentially better projects.

2.2 Partial verifiability

Depending on the environment, there are different notions of partial verifiability that might be reasonable. Some natural examples in the context of our project selection framework include:

1. $M_O(p, a) = \{(\pi, \alpha) \in \Theta : \pi_i \leq p_i \ \forall i \in [N]\}$: no overselling
2. $M_U(p, a) = \{(\pi, \alpha) \in \Theta : \pi_i \geq p_i \ \forall i \in [N]\}$: no underselling
3. $M_P(p, a) = \{(\pi, \alpha) \in \Theta : \pi = p\}$: no lying about principal preferences
4. $M_A(p, a) = \{(\pi, \alpha) \in \Theta : \alpha = a\}$: no lying about agent preferences
5. $M_D(p, a) = \{(\pi, \alpha) \in \Theta : |(p, a) - (\pi, \alpha)| < \varepsilon\}$: no big distortions/deviations

While one can form many more natural examples of such correspondences, even just by taking intersections or unions of the above sets, we will focus in this paper on correspondences that satisfy the following three assumptions.

Assumption 1. *(Nested Range Condition) For any $(p, a) \in \Theta_p \times \Theta_a$,*

$$(\pi, \alpha) \in M(p, a) \implies M(\pi, \alpha) \subset M(p, a).$$

Assumption 2. *(Separable) There exist $M_p : \Theta_p \rightrightarrows \Theta_p$ and $M_a : \Theta_a \rightrightarrows \Theta_a$ so that for any $(p, a) \in \Theta_p \times \Theta_a$,*

$$M(p, a) = M_p(p) \times M_a(a).$$

Assumption 3. *(Agency Freedom) For any* $(p, a) \in \Theta_p \times \Theta_a$,

$$(\pi, \alpha) \in M(p, a) \implies \pi \times \Theta_a \subset M(p, a).$$

The first assumption ensures that the revelation principle applies and we can restrict attention to truthful mechanisms (Green and Laffont [10]). The second assumption says that feasible reports about the principal's profits are determined solely by the principal's actual profits, and those about the agent's utilities are determined solely by the agent's actual utilities, and the set of feasible reports at any (p, a) is any combination of these feasible reports. The third assumption says that the message space is rich enough so that the agent has the freedom to report any α, regardless of (p, a). Informally, the idea is that the agent need not justify its preference to the principal. Note that if $M : \Theta \rightrightarrows \Theta$ satisfies Assumptions 2 and 3, then there exist $M_p : \Theta_p \rightrightarrows \Theta_p$ such that for any $(p, a) \in \Theta$, $M(p, a) = M_p(p) \times \Theta_a$. In such a case, we will just use $M(p)$ to refer to the set $M_p(p) \times \Theta_a$. From the examples of message correspondences given above, one can verify that the first four examples satisfy the Nested Range Condition and Separability, while the first three examples satisfy Agency Freedom. In particular, the first three examples satisfy all three assumptions. In general, a correspondence M satisfies all three assumptions if and only if there is a partial order $\succeq$ on Θ_p such that

$$(\pi, \alpha) \in M(p) \iff p \succeq \pi.$$

Going forward, we will first obtain a characterization of truthful mechanisms for an arbitrary message correspondence satisfying Assumptions 1, 2, and 3. Then, we will study the principal's problem of finding a truthful mechanism that maximizes the principal's expected profit under some of the specific partial verifiability constraints defined above. Formally, given $M : \Theta \rightrightarrows \Theta$, we study the problem

$$\max_{\text{d:d is truthful under } M} \mathbb{E}\left[p_{d(p,a)}\right].$$

3 Truthful mechanisms

In this section, we provide a characterization of truthful mechanisms under an arbitrary partial verifiability constraint satisfying the Nested Range Condition, Separability, and Agency Freedom. We begin by defining a special class of mechanisms which we call table mechanisms.

Definition 2. *A mechanism* $d : \Theta_p \times \Theta_a \to [N]$ *is a table mechanism if there exists a (table) function* $f : \Theta_p \to 2^N \setminus \{\emptyset\}$ *such that for any* $(p, a) \in \Theta_p \times \Theta_a$,

$$d(p, a) \in \arg\max_i \{a_i : i \in f(p)\}.$$

Informally, a table function f defines the set of projects on the table as a function of the principal's profits p, and the corresponding mechanism chooses the agent's favorite project among those that are on the table. Observe that for any $(p, a) \in \Theta$, only the ordinal content of the agent's preference is needed, along with the principal's profits p, to determine the outcome of a table mechanism. We let $\mathcal{T}$ denote the set of all table functions $f : \Theta_p \to 2^N \setminus \{\emptyset\}$.

Notice that the mechanisms identified in Lemma 1 characterizing truthful mechanisms under no partial verifiability constraints are also just table mechanisms corresponding to table functions $f \in \mathcal{T}$ that are constant. The following result characterizes truthful mechanisms for a more general class of message correspondences as a subclass of table mechanisms, defined by a simple restriction on table functions $f \in \mathcal{T}$ which depends on M.

Theorem 1. *Suppose $M : \Theta \rightrightarrows \Theta$ satisfies Assumptions 1, 2, and 3. A mechanism $d : \Theta_p \times \Theta_a \to [N]$ is truthful under M if and only if it is a table mechanism with a table function $f \in \mathcal{T}$ that satisfies*

$$M(\pi) \subset M(p) \Longrightarrow f(\pi) \subset f(p). \tag{1}$$

Proof. Suppose d is a table mechanism with a table function f satisfying Condition 1. We want to show that d is truthful under M.

Consider any profile $(p, a) \in \Theta$. If the agent reports some $(\pi, \alpha) \in M(p)$, we know that $M(\pi) \subset M(p)$ since M satisfies the Nested Range Condition (Assumption 1). It follows from Condition 1 then that $f(\pi) \subset f(p)$. Since the agent gets his most preferred project among those available and reporting truthfully maximizes his set of available projects, the agent cannot gain by misreporting. Therefore, d is truthful.

Now suppose that d is a truthful mechanism under M. Define the function $f : \Theta_p \to 2^N$ so that for any $p \in \Theta_p$,

$$i \in f(p) \iff \exists (\pi, \alpha) \in M(p) \text{ such that } d(\pi, \alpha) = i.$$

We will now show that the f is a table function that satisfies Condition 1 and the corresponding table mechanism coincides with d.

For Condition 1, take any $p, p' \in \Theta_p$ such that $M(p) \subseteq M(p')$. If $i \in f(p)$, there exists $(\pi, \alpha) \in M(p)$ so that $d(\pi, \alpha) = i$. But then, $(\pi, \alpha) \in M(p')$ as well and so $i \in f(p')$. Thus, we get that $f(p) \subset f(p')$. It follows that f satisfies Condition 1.

Now to show that d coincides with the table mechanism defined by f, we need to show that for any (p, a),

$$d(p, a) \in \arg\max_i \{a_i : i \in f(p)\}.$$

Suppose towards a contradiction that $d(p, a)$ is not in this set. By definition, $d(p, a) \in f(p)$. Let $j \in \arg\max_i \{a_i : i \in f(p)\}$. Then $a_j > a_{d(p,a)}$ and $j \in f(p)$. But the fact that $j \in f(p)$ implies that there exists $(\pi, \alpha) \in M(p)$ such that $d(\pi, \alpha) = j$. But then, the agent can misreport at state (p, a) to (π, α) and gain from this manipulation. This contradicts the fact that d is truthful and so it must be that $d(p, a) \in \arg\max_i \{a_i : i \in f(p)\}$. It follows then that d is a table mechanism with a table function f satisfying Condition 1. $\square$

We note here that the characterization in Theorem 1 does not depend on the assumption that the payoffs $(p_1, a_1), (p_2, a_2), \ldots, (p_N, a_N)$ are independent across projects. We only make this distributional assumption to solve for an optimal mechanism, which we can now look for in the relevant subclass of table mechanisms for different partial verifiability constraints. To begin, consider the message correspondence M_P in which the agent cannot lie about the profit vector p. Since for any $\pi, p \in \Theta_p$, $M_P(\pi) \subset M_P(p)$ if and only if $\pi = p$, it follows from Theorem 1 that any table function $f \in \mathcal{T}$ defines a truthful mechanism under M_P. It follows then that the optimal mechanism under M_P is for the principal to choose a table function $f \in \mathcal{T}$ such that for any $p \in \Theta_p$, only the principal's favorite project is on the table. In the following sections, we will study the principal's design problem for the more interesting partial verifiability constraints of no-overselling and no-underselling.

4 Optimal mechanism - No overselling

In this section, we will study the principal's problem of finding the optimal mechanism under the specific partial verifiability constraint of no-overselling. Such a constraint would perhaps be reasonable for environments in which the profitability of a project could be illustrated with some positive evidence, which can be hidden from the principal but not fabricated. Thus, the agent may be able to hide some of the positive evidence associated with a project and undersell it, but it might be prohibitively expensive for the agent to fabricate positive evidence and report a project to be more profitable than it actually is.

To study the principal's design problem, we first characterize truthful mechanisms under the no-overselling constraint. Recall that this constraint is defined by the message correspondence

$$M_O(p) = \{(\pi, \alpha) \in \Theta : \pi_i \leq p_i \ \forall i \in [N]\}.$$

Since $M_O(\pi) \subset M_O(p) \iff \pi \leq p$, Theorem 1 leads to the following characterization.

Corollary 1. *A mechanism $d : \Theta_p \times \Theta_a \to [N]$ is truthful under the no-overselling constraint M_O if and only if it is a table mechanism with a table function $f \in \mathcal{T}$ that satisfies*

$$\pi \leq p \Longrightarrow f(\pi) \subset f(p). \tag{2}$$

In words, a mechanism is truthful under no overselling if and only if it is a table mechanism in which the set of projects on the table is increasing in the principal profit values p. We will denote by $\mathcal{T}_O$ the set of table functions in $\mathcal{T}$ that satisfy (2). Notice that these table functions have the property that there is some (default) project that must always be on the table. Formally, for all $f \in \mathcal{T}_O$, there is some $i \in [N]$ such that $i \in f(p)$ for all $p \in \Theta_p$. Before finding the optimal mechanism, we define a subclass of table mechanisms that take a simple cutoff form.

Definition 3. *A mechanism $d : \Theta_p \times \Theta_a \to [N]$ is a cutoff mechanism if it is a table mechanism with a table function $f \in \mathcal{T}_O$, defined by cutoffs $c_1, c_2, \ldots, c_{N-1}, c_N$ with $c_i = -\infty$*

for some $i \in [N]$, *so that for any* $p \in \Theta_p$,

$$i \in f(p) \iff p_i \geq c_i.$$

In a cutoff mechanism, a project is on the table if and only if the principal's profit from the project meets a cutoff. The mechanism then simply chooses the agent's favorite project among those that meet their respective cutoffs.

Next, we'll consider and solve the principal's problem for the case of $N = 2$ projects. We'll then discuss the case of $N > 2$ projects towards the end of this section.

4.1 $N = 2$ projects

For the case of two projects, we find that the optimal mechanism for the principal under the partial verifiability constraint of no-overselling takes the form of a simple cutoff mechanism.

Theorem 2. *Suppose there are* $N = 2$ *projects, each with* $(p_i, a_i) \sim F_i$. *Then, the optimal mechanism under no-overselling constraint* M_O *is a cutoff mechanism. Moreover, the cutoffs are given either by* $c_1 = c_1^*$ *and* $c_2 = -\infty$ *or* $c_1 = -\infty$ *and* $c_2 = c_2^*$, *where* c_i^* *solves*

$$\mathbb{E}\left[(p_i - p_{-i})\mathbb{I}_{a_i \geq a_{-i}} | p_i = c_i^*\right] = 0. \tag{3}$$

Proof. To prove this result, we will first show that for any arbitrary truthful mechanism, there is a cutoff mechanism that is weakly better for the principal. And then, we will solve for the optimal cutoff.

For the first step, suppose d is an arbitrary table mechanism defined by a table function $f \in \mathcal{T}_O$. Assume d is such that $2 \in f(p)$ for all p_1, p_2. The other case is analogous. Let $c = \sup\{p_1 : 1 \notin f(p_1, p_1)\}$ and define the cutoff mechanism d' with a table function $f' \in \mathcal{T}_O$ so that $1 \in f'(p) \iff p_1 \geq c$. We'll show that the expected profit for the principal from d' is at least as high as the expected profit from d. In fact, we will show that this holds conditional on (p_1, p_2), and hence in expectation. Consider the following (exhaustive and mutually exclusive) cases depending on whether $p_i \geq c$ or $p_i < c$:

- $p_1 \in (-\infty, c), p_2 \in (-\infty, c)$: For any such p_1, p_2, we know both $f(p) = f'(p) = \{2\}$ and therefore, the second project is chosen for all such profiles. Thus, the two mechanisms are identical and generate the same profit for the principal in this case.

- $p_1 \in (\infty, c), p_2 \in [c, \infty)$: In this case, $f'(p) = \{2\}$ and thus project 2 is chosen for sure. Note that the principal prefers project 2 over 1 in these profiles and thus, the profit from the cutoff mechanism is weakly higher for any such p_1, p_2.

- $p_1 \in [c, \infty), p_2 \in (-\infty, c)$: Now $f'(p) = \{1, 2\}$ while $f(p)$ can be either $\{2\}$ or $\{1, 2\}$. Observe that the principal strictly prefers project 1 over 2 in all these profiles. Thus, the cutoff mechanism again leads to weakly higher profits for such p_1, p_2.

- $p_1 \in [c, \infty), p_2 \in [c, \infty)$: Here we have $f(p) = f'(p) = \{1, 2\}$. Thus, the two mechanisms are identical in this set and lead to same profits for the principal.

This shows that for any truthful mechanism, there is a cutoff mechanism under which the principal's expected profit is weakly higher. Thus, the optimal truthful mechanism must be a cutoff mechanism. Now we want to find the optimal cutoff c.

Again, assume that $2 \in f(p)$ for all $p \in \Theta_p$. Consider the decision problem of the principal for any given p_1, p_2, a_1, a_2. The principal can either

- not make project 1 available and get p_2
- make project 1 available and get $p_1 \mathbb{I}_{a_1 \geq a_2} + p_2 \mathbb{I}_{a_2 > a_1}$

The constraint imposed by truthfulness and optimality of cutoff mechanisms imply that the principal can only base this decision on the value of p_1. Thus, taking expectation with respect to p_2, a_1, a_2, we get that the two alternatives are:

- not make project 1 available and get $\mathbb{E}[p_2 | p_1]$
- make project 1 available and get $\mathbb{E}\left[p_1 \mathbb{I}_{a_1 \geq a_2} + p_2 \mathbb{I}_{a_2 > a_1} | p_1\right]$

Thus, the principal would want to make project 1 available if and only if

$$\begin{aligned}
&\mathbb{E}\left[p_1 \mathbb{I}_{a_1 \geq a_2} + p_2 \mathbb{I}_{a_2 > a_1} | p_1\right] \geq \mathbb{E}[p_2 | p_1] \\
\iff &\mathbb{E}\left[p_1 \mathbb{I}_{a_1 \geq a_2} | p_1\right] \geq \mathbb{E}\left[p_2 \mathbb{I}_{a_1 \geq a_2} | p_1\right] \\
\iff &\mathbb{E}\left[(p_1 - p_2) \mathbb{I}_{a_1 \geq a_2} | p_1\right] \geq 0.
\end{aligned}$$

At the optimal cutoff, the principal should be indifferent between the two alternatives and so the cutoff c_1^* is defined by the solution to the equation

$$\mathbb{E}\left[(p_1 - p_2) \mathbb{I}_{a_1 \geq a_2} | p_1 = c_1^*\right] = 0.$$

We can repeat the exercise assuming that $1 \in f(p)$ for all $p \in \Theta_p$, and find the optimal cutoff c_2^*. The optimal mechanism is the one that gives a higher profit among these two.

□

Thus, under the no-overselling constraint and with $N = 2$ projects, the optimal mechanism has a simple cutoff structure. In more detail, the principal designates one of the two projects as a default project which is always on the table, and the other project is put on the table if and only if its profit report meets a certain threshold. Among the projects on the table, the agent's favorite project is chosen.

4.2 N projects

In this section, we discuss the principal's problem for $N > 2$ projects and we restrict attention to distributions F_i such that p_i and a_i are independent, and also the agent's utilities a_i are drawn from identical distributions so that the agent is equally likely to have any preference over the N projects. Now for any truthful mechanism d, defined by an increasing table function $f \in \mathcal{T}_O$, and for any $p \in \Theta_p$, independence implies that all of the $|f(p)|$ projects that are on the table are equally likely to be the agent's favorite project. Thus, the principal's expected payoff given p is simply $\left[\frac{\sum_{i\in f(p)} p_i}{|f(p)|}\right]$. It follows then that for the independence case, the principal's design problem can be written as

$$max_{f\in\mathcal{T}_O}\mathbb{E}\left[\frac{\sum_{i\in f(p)} p_i}{|f(p)|}\right]$$

where

$$\mathcal{T}_O = \{f : \Theta_p \to 2^{[N]} \setminus \{\emptyset\} \mid p \leq p' \implies f(p) \subset f(p')\}.$$

Given the optimality of the cutoff mechanism for the $N = 2$ projects case, we believe that a cutoff mechanism is optimal in this case as well but we haven't been able to exactly prove it. However, we can obtain a bound on the performance of cutoff mechanisms using Theorem 1 of Correa et al. [7], who study a very different problem of designing revenue-maximizing posted-price mechanisms for buyers arriving in a random order. First, we note their result in the following Lemma.

Lemma 2. *(Theorem 1 of Correa et al. [7]) Given N independent non-negative random variables $p_1, \dots, p_N$ with $p_i \sim F_i^p$, there exist values $c_1, \dots, c_N$ such that*

$$\mathbb{E}\left[\frac{\sum_{i=1}^n p_i Y_i}{\sum_{i=1}^n Y_i}\right] \geq \left(1 - \frac{1}{e}\right)\mathbb{E}[\max\{p_1, \dots, p_N\}],$$

where Y_i is a Bernoulli random variable that has value 1 if $p_i > c_i$. Here, when evaluating the expectation on the left-hand side, we define $0/0 = 0$.

Lemma 2 can be applied directly to our setting in the case all projects give non-negative payoffs and that one of the projects gives both the agent and the principal utility 0 (i.e. when there is the option to select no project). As the following Corollary shows, Lemma 2 can also be used to get a weaker bound in the general case in which the projects give non-negative payoffs.

Corollary 2. *Suppose there are N projects, and each project $i \in [N]$ leads to non-negative payoffs $(p_i, a_i) \sim F_i$ where p_i and a_i are independent, and a_i's are identically distributed. Then, there exists a cutoff mechanism d such that*

$$\mathbb{E}[p_{d(p,a)}] \geq \frac{1}{2}\frac{N-1}{N}\left(1 - \frac{1}{e}\right)\mathbb{E}[\max\{p_1, \dots, p_N\}].$$

Proof. Let $\pi = \max_i p_i$ and $\pi_{-j} = \max_{i \neq j} p_i$. Observe that $\sum_{j=1}^N \pi_{-j} \geq (N-1)\pi$, so

$$N \max_j (\mathbb{E}[\pi_{-j}]) \geq \sum_{j=1}^N \mathbb{E}[\pi_{-j}] \geq (N-1)\mathbb{E}[\pi].$$

In particular, there is some k such that $\mathbb{E}[\pi_{-k}] \geq \frac{N-1}{N}\mathbb{E}[\pi]$.

Consider a cutoff mechanism where project k is always on the table and each project $i \neq k$ is on the table if $p_i \geq c_i$. Denoting by Y_i the indicator that $p_i \geq c_i$, the principal's expected payoff is given by

$$\mathbb{E}\left[\frac{\sum_{i \neq k} p_i Y_i + p_k}{\sum_{i \neq k} Y_i + 1}\right] \geq \frac{1}{2}\mathbb{E}\left[\frac{\sum_{i \neq k} p_i Y_i}{\sum_{i \neq k} Y_i}\right].$$

By Lemma 2, there exist cutoffs c_i such that

$$\mathbb{E}\left[\frac{\sum_{i \neq k} p_i Y_i}{\sum_{i \neq k} Y_i}\right] \geq \left(1 - \frac{1}{e}\right)\mathbb{E}[\pi_{-k}] \geq \left(1 - \frac{1}{e}\right)\frac{N-1}{N}\mathbb{E}[\pi].$$

$\square$

It follows from Corollary 2 that for N large, and any non-negative random variables p_i, there exists a cutoff mechanism that guarantees the principal at least a $\frac{1}{2}\left(1 - \frac{1}{e}\right) \approx 0.316$ fraction of the maximum possible profit. The proof of Lemma 2 gives explicit cutoffs for achieving the bound in Corollary 2: some project k is always on the table ($c_k = -\infty$), and for all $i \neq k$, either project i is never on the table ($c_i = \infty$), or it is on the table with probability exactly equal to the probability that project i is the best non-default project for the principal ($1 - F_i(c_i) = \Pr[p_i \geq p_j \text{ for all } j \neq k]$).

Restricting attention to the particular case where p_i and a_i are distributed i.i.d. $U([0,1])$ for all $i \in [N]$, we are able to solve for the optimal mechanism within the class of cutoff mechanisms and we conjecture that this mechanism is optimal in the class of all table mechanisms defined by table functions $f \in \mathcal{T}_O$[2].

Lemma 3. *Suppose there are N projects, each with payoffs $(p_i, a_i) \sim F$ where F is uniform on $[0,1]^2$. Then, the optimal cutoff mechanism under the no-overselling constraint M_O has a single cutoff c with $c_i = c$ for $i < N$ and $c_N = -\infty$, where c is the unique solution in $(0,1)$ to the equation*

$$N(1-c)(1-c+c^N) = 1 - c^N.$$

[2]For more general distributions, we can show that the principal's expected profit is quasiconcave in c_i, and we can obtain the first-order conditions whose solution determines the optimal cutoff mechanism. We illustrate these calculations in the proof of Lemma 3, where we use the uniform assumption only towards the end of the argument.

5 Discussion

In this section, we briefly discuss some related questions from our model that provide important additional insights into the structure of optimal mechanisms under partially verifiable information. In particular, we will study how the optimal mechanism changes as the principal and agent payoffs become more aligned, and we will study the design of optimal mechanisms under the no-underselling partial verifiability constraint. Throughout this section, we will focus on the case where there are only $N = 2$ projects.

5.1 Ally principle

In this subsection, we discuss the implications of our model for the well-known ally principle which states that a principal delegates more authority to an agent with more aligned preferences. Towards this goal, we focus on the instance of our model in which the principal-agent payoffs follow a bivariate normal distribution, i.e., $(p_i, a_i) \sim N(0, 0, 1, 1, \rho)$, so that the correlation parameter $\rho \in [-1, 1]$ captures the degree to which the principal and agent preferences are aligned.

As before, we focus on the case with $N = 2$ projects and the no-overselling partial verifiability constraint given by M_O. Note that in this case, we already know from Theorem 2 that a cutoff mechanism is optimal. Given that the projects are ex-ante identical, we let $c_2 = -\infty$ so that project 1 will be on the table if and only if the profit from project 1 is at least more than c_1 which is defined by Equation (3). This simple structure of the optimal mechanism leads to a natural test for the ally principal in our framework, in that we can simply study the relationship between the correlation parameter ρ and the optimal cutoff c_1.

Proposition 1. *Suppose there are $N = 2$ projects with $(p_i, a_i) \sim N(0, 0, 1, 1, \rho)$. Then, an optimal mechanism under no-overselling constraint M_O is a cutoff mechanism with $c_2 = -\infty$ and $c_1 \in \mathbb{R}$ defined by the equation*

$$c_1 \Phi(tc_1) + t\phi(tc_1) = 0,$$

where $t = \frac{\rho}{\sqrt{2-\rho^2}}$, and Φ, ϕ are the CDF and PDF of the standard Normal distribution. The optimal cutoff c_1 is decreasing in ρ.

It follows from Proposition 1 that as ρ increases and the payoffs of the principal and agent become more aligned, the principal grants more freedom to the agent in choosing its favorite project by lowering the cutoff c at which it leaves project 1 on the table. Thus, this finding provides some evidence in favor of the ally principle.

5.2 No underselling

So far, we've focused on the principal's design problem for the specific partial verifiability constraint of no-overselling. In this subsection, we briefly look at a somewhat analogous case of no-underselling, where the agent cannot report a project to be less profitable than it actually is. Such a constraint might hold relevance in environments where projects are associated with some negative evidence, which can be hidden from the principal but not fabricated.

As in the no-overselling case, we first characterize truthful mechanisms under the no-underselling constraint. Recall that this constraint is defined by

$$M_U(p) = \{(\pi, \alpha) \in \Theta : \pi_i \geq p_i \ \forall i \in [N]\}.$$

Since $M_U(\pi) \subset M_U(p) \iff \pi \geq p$, Theorem 1 gives the following characterization.

Corollary 3. *A mechanism $d : \Theta_p \times \Theta_a \to [N]$ is truthful under no-underselling constraint M_U if and only if it is a table mechanism with a table function $f \in \mathcal{T}$ that satisfies*

$$\pi \geq p \Longrightarrow f(\pi) \subset f(p). \tag{4}$$

In words, a mechanism is truthful under no underselling if and only if it is a table mechanism in which the set of projects on the table is decreasing in the principal profit values p. We will denote by $\mathcal{T}_U$ the set of table functions in $\mathcal{T}$ that satisfy (4). Notice that these table functions have the property that there is some (default) project that must always be on the table. The following result describes the optimal mechanism under the no-underselling constraint.

Theorem 3. *Suppose there are $N = 2$ projects, each with $(p_i, a_i) \sim F_i$. Then, the optimal mechanism under no-underselling constraint M_U is a table mechanism with a table function $f \in \mathcal{T}_U$ defined by cutoffs c_1, c_2 such that for any $p \in \Theta_p$,*

$$i \in f(p) \iff p_{-i} \leq c_i.$$

Moreover, the cutoffs are given either by $c_1 = c_1^*$ and $c_2 = \infty$ or $c_1 = \infty$ and $c_2 = c_2^*$, where c_i^* solves

$$\mathbb{E}\left[(p_i - p_{-i})\mathbb{I}_{a_i \geq a_{-i}} | p_{-i} = c_i^*\right] = 0 \tag{5}$$

Thus, under the no-underselling constraint, the optimal mechanism again has a simple cutoff structure. The principal designates one of the two projects as a default project which is always on the table, and the other project is put on the table if and only if the profit report from the default project is below a certain threshold. Among the projects on the table, the agent's favorite project is chosen.

For the special case where principal and agent payoffs are independent, the following Corollary describes the optimal mechanisms under the no-underselling and also the no-overselling constraint, highlighting both the simplicity of the optimal mechanisms, and the important distinction between them.

Corollary 4. *Suppose there are $N = 2$ projects, each with payoffs $(p_i, a_i) \sim F$ where F is such that p_i, a_i are independent.*

1. *(No-overselling) An optimal mechanism under M_O is a table mechanism with a table function f such that for any $p \in \Theta_p$, $2 \in f(p)$ and*

$$1 \in f(p) \iff p_1 \geq \mathbb{E}[p_i].$$

2. *(No-underselling) An optimal mechanism under M_U is a table mechanism with a table function f such that for any $p \in \Theta_p$, $2 \in f(p)$ and*

$$1 \in f(p) \iff p_2 \leq \mathbb{E}[p_i].$$

In contrast to the no-overselling case, the decision of whether or not to have a project on the table in the no-underselling case is based on the profit report of the other project. Our results and analysis of the two-project case suggest that in the general scenario with more than two projects, while the optimal mechanism under the no-overselling constraint likely has a simple cutoff structure, so project i is on the table if p_i is high enough, the optimal mechanism under the no-underselling constraint is more complex, with project i being on the table if some monotone function of p_{-i} is small enough.

6 Conclusion

We study a principal-agent project selection problem with asymmetric information. In the absence of transfers, the principal is essentially powerless in this setting if the agent can report arbitrarily. The power to constrain the agent's ability to misreport arises naturally in settings where, for example, the principal can require the agent to furnish evidence to support its claims. In this paper, we study how the principal can leverage this power. In particular, we show that this power allows the principal to restrict the set of projects available for the agent to choose from in a way that depends on the realized value of the projects, then delegate selection to the agent within this set.

To quantify the potential value of this power, we focus on two particular constraints: no-overselling, under which the agent cannot report a project to be more valuable to the principal than it actually is, and no-underselling, under which the agent cannot report a project to be less valuable to the principal than it actually is. These constraints model restrictions that arise naturally in settings in which the agent must furnish evidence and operationalize the idea that it is much easier to hide evidence than it is to fabricate it. Under the no-overselling constraint, a natural mechanism is for the principal to make a project available for the agent to choose as long as its value to the principal is high enough. We show that such cutoff-based mechanisms perform quite well, and that the optimal mechanism takes this form in the case of two projects. Under the no-underselling constraint, we show that in the optimal mechanism, whether one of the projects is available to the agent actually depends on the value of the other project, in sharp contrast to cutoff-based mechanisms in which the availability of a project is determined only by its own value.

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A Proofs for Section 4 (Optimal mechanism - No overselling)

Lemma 3. *Suppose there are N projects, each with payoffs $(p_i, a_i) \sim F$ where F is uniform on $[0,1]^2$. Then, the optimal cutoff mechanism under the no-overselling constraint M_O has a single cutoff c with $c_i = c$ for $i < N$ and $c_N = -\infty$, where c is the unique solution in $(0,1)$ to the equation*

$$N(1-c)(1-c+c^N) = 1-c^N.$$

Proof. We will work with the general case, assuming $p_i \sim F_i^p$, $a_i \sim U[0,1]$, and p_i, a_i independent, and only use the assumption that F_i^p is $U[0,1]$ towards the end of the argument. Also, we refer to F_i^p by F_i to ease notation.

Step 1: First, we show that for any F_i and any cutoff mechanism defined by $c = (c_1, c_2, \ldots, c_N)$, the principal's expected profit is quasiconcave in c_i. It follows that the optimal cutoffs are interior.

Suppose $f \in \mathcal{T}_O$ defines an arbitrary cutoff mechanism d with cutoffs $c = (c_1, c_2, \ldots, c_N)$ where $c_N = -\infty$ so that project N is always on the table. Let $N(c)$ denote the probability that the default project N is chosen at cutoff profile c, and let $\mu_i(c_i) = \mathbb{E}[p_i | p_i \geq c_i]$. Then, the principal's expected profit under this mechanism is simply

$$EU_p(c) = \sum_{i=1}^{N} \mu_i(c_i) \mathbb{P}(d = i).$$

Now observe that

$$\begin{aligned}\mathbb{P}(d = i) &= (1 - F_i(c_i))\mathbb{P}(d = N | c_i = -\infty) \\ &= (1 - F_i(c_i))N(c_i = -\infty, c_{-i})\end{aligned}$$

That is, conditional on $p_i \geq c_i$, the probability that the project chosen is i is the same as the probability that the decision is N when the cutoff $c_i = -\infty$ and the remaining cutoffs are the same. Plugging in, we get

$$EU_p(c) = \sum_{i=1}^{N} N(c_i = -\infty, c_{-i}) \int_{c_i}^{\infty} x f_i(x) dx$$

Now to find the probability that the last project N is chosen as a function of c, we condition on its rank, which we say is k if there are exactly $k-1$ projects with higher a_is. Doing so, we get

$$N(c) = \mathbb{P}(d = N)$$

$$
\begin{aligned}
&= \sum_{k=1}^{N} \mathbb{P}(\text{rank of } N = k)\mathbb{P}(d = N| \text{ rank of } N = k) \\
&= \frac{1}{N}\sum_{k=1}^{N} \mathbb{P}(d = N| \text{ rank of } N = k) \\
&= \frac{1}{N}\sum_{k=1}^{N} \sum_{S\subset[N-1]:|S|=k-1} \frac{\Pi_{i\in S}F_i(c_i)}{\binom{N-1}{k-1}}
\end{aligned}
$$

Note that

$$
\frac{\partial N(c)}{\partial c_i} = \lambda(c_{-i})f_i(c_i)
$$

where $\lambda(c_{-i})$ is increasing in c_j. Then, we have that for all $i \in [N-1]$,

$$
\begin{aligned}
\frac{\partial EU_p(c)}{\partial c_i} &= -N(c_i = -\infty, c_{-i})c_i f_i(c_i) + \sum_{j\neq i} \frac{\partial N(c_j = -\infty, c_{-j})}{\partial c_i} \int_{c_j}^{\infty} x f_j(x)dx \\
&= \left[\sum_{j\neq i} \lambda(c_j = -\infty, c_{-i,j}) \int_{c_j}^{\infty} x f_j(x)dx - c_i N(c_i = -\infty, c_{-i})\right] f_i(c_i).
\end{aligned}
$$

Notice that the marginal profit with respect to c_i goes from being positive to negative as c_i increases, and thus, it follows that $EU_p(c)$ is quasiconcave in c_i for all $i \in [N-1]$. Therefore, the cutoffs should be interior, and satisfy the first-order condition.

Step 2: For the case where F_i is identically distributed so that $F_i = F$ for all $i \in [N]$, we derive the optimal single cutoff mechanism, defined by $c \in \mathbb{R}$ so that $c_i = c$ for all $i \in [N-1]$.

Suppose $f \in \mathcal{T}_O$ defines a single-cutoff mechanism with cutoff $c \in \mathbb{R}$ so that for any $p \in \Theta_p$,

$$
i \in f(p) \iff [p_i \geq c \text{ or } i = N].
$$

Let X be the random variable denoting $|f(p)| - 1$. It follows that $X \sim Bin(N-1, 1-F(c))$ and we can write

$$
\begin{aligned}
N(c) &= \mathbb{P}[d = N] \\
&= \mathbb{E}\left[\frac{1}{X+1}\right] \\
&= \left[\frac{1 - F(c)^N}{N(1 - F(c))}\right]
\end{aligned}
$$

Here, we use the fact that if $Y \sim Bin(n, p)$, then $\mathbb{E}\left[\frac{1}{Y+1}\right] = \frac{1-(1-p)^{n+1}}{(n+1)p}$. It follows by symmetry that

$$EU_p(c) = \mu(c)(1 - N(c)) + \mu N(c).$$

Note that

$$\mu'(c) = \frac{\mu(c) - c}{1 - F(c)} f(c) \text{ and } N'(c) = \frac{f(c)}{N(1 - F(c))^2} \left[1 - F(c)^N - NF(c)^{N-1}(1 - F(c))\right].$$

Now taking the derivative, we get

$$\begin{aligned}
EU_p'(c) &= \mu'(c)(1 - N(c)) - \mu(c)N'(c) + \mu N'(c) \\
&= \frac{\mu(c) - c}{1 - F(c)} f(c) \left(1 - \frac{1 - F(c)^N}{N(1 - F(c))}\right) \\
&- (\mu(c) - \mu)\frac{f(c)}{N(1 - F(c))^2} \left[1 - F(c)^N - NF(c)^{N-1}(1 - F(c))\right] \\
&= \frac{f(c)}{N(1 - F(c))^2} \\
&\times \left[(\mu(c) - c)(N - 1 - NF(c) + F(c)^N) - (\mu(c) - \mu)F(c)^N \left(N - 1 - \frac{N}{F(c)} + \frac{1}{F(c)^N}\right)\right]
\end{aligned}$$

Taking the first-order condition, we get that the optimal cutoff must be a solution to the equation

$$\frac{\mu(c) - c}{\mu(c) - \mu} \left(\frac{N - 1 - NF(c) + F(c)^N}{N - 1 - \frac{N}{F(c)} + \frac{1}{F(c)^N}}\right) = F(c)^N.$$

For the uniform, the equation reduces to

$$N(1 - c)(1 - c + c^N) = 1 - c^N.$$

Observe that

$$N(1 - c)(1 - c + c^N) - (1 - c^N) = (1 - c)^2 \cdot (N - \phi(c)),$$

where $\phi(c) = \sum_{k=0}^{N-1}(k + 1)c^k$. Observe that $\phi(0) = 1 < N$ and $\phi(1) = \binom{N+1}{2} > N$, so $N = \phi(c)$ has a solution in $(0, 1)$. Moreover, since ϕ is increasing, this solution is unique.

Step 3: To complete the proof, we now show that when F_i is $U[0, 1]$, the optimal cutoff mechanism cannot have different cutoffs.

Suppose that the mechanism d is such that there exist i, j with $c_i > c_j$. Define t so that $\bar{c} + t = c_i$ and $\bar{c} - t = c_j$. From the above calculations, we know that if we write $EU_p(c)$ in expanded form and plug in $c_i = \bar{c} + t$ and $c_j = \bar{c} - t$, we get a polynomial that is at most cubic in t. This is because we get a term that is at most quadratic in t for $\mathbb{P}(d = k)$ for any

$k \in [n]$ and in the expected profit calculation, we multiply that with $\mu(c_k) = \frac{1+c_k}{2}$. Note that by the symmetry of the projects, the principal should get the same expected profit if we changed t to $-t$. Therefore, the polynomial should be of the form $at^2 + b$. Now, if a is > 0 or < 0, the principal gains from increasing or decreasing t which is possible since we know that the solution is interior and $c_i > c_j$. Therefore, d cannot be optimal in either case. When $a = 0$, the principal is indifferent to increasing t till one of the cutoffs reaches an extreme of 1 or 0 which we know cannot be optimal. Therefore, a cutoff mechanism with different cutoffs cannot be optimal. □

B Proofs for Section 5 (Discussion)

Proposition 1. *Suppose there are $N = 2$ projects with $(p_i, a_i) \sim N(0, 0, 1, 1, \rho)$. Then, an optimal mechanism under no-overselling constraint M_O is a cutoff mechanism with $c_2 = -\infty$ and $c_1 \in \mathbb{R}$ defined by the equation*

$$c_1 \Phi(tc_1) + t\phi(tc_1) = 0,$$

where $t = \frac{\rho}{\sqrt{2-\rho^2}}$, and Φ, ϕ are the CDF and PDF of the standard Normal distribution. The optimal cutoff c_1 is decreasing in ρ.

Proof. We know that the optimal cutoff $c(\rho)$ is the solution to the following equation

$$\mathbb{E}\left[(p_1 - p_2)\mathbb{P}[a_1 \geq a_2 | p_2] | p_1 = c(\rho)\right] = 0$$

Let us simplify the above expression. First, we want to find $\mathbb{P}[a_1 \geq a_2 | p_1, p_2]$. We know that if $X, Y \sim N(\mu_x, \mu_y, \sigma_x^2, \sigma_y^2, \rho)$, then the conditional distribution

$$X \mid Y \sim N\left(\mu_x + \rho\frac{\sigma_x}{\sigma_y}\left(y - \mu_y\right), \sigma_x^2(1-\rho^2)\right)$$

and so in our case, $a_i | p_i \sim N(\rho p_i, 1 - \rho^2)$. Also, since the payoffs are independent across projects, we get that

$$a_1 - a_2 | p_1, p_2 \sim N(\rho(p_1 - p_2), 2(1 - \rho^2))$$

Using this, we get that

$$\mathbb{P}[a_1 - a_2 \geq 0 | p_1, p_2] = \Phi\left(\frac{\rho(p_1 - p_2)}{\sqrt{2(1-\rho^2)}}\right)$$

where Φ is the standard normal cdf.

Plugging this into the equation, we get that the optimal cutoff satisfies

$$\mathbb{E}\left[(p_1 - p_2)\Phi\left(\frac{\rho(p_1 - p_2)}{\sqrt{2(1-\rho^2)}}\right)\middle| p_1 = c(\rho)\right] = 0.$$

We can find that the expectation is equal to

$$p_1 \Phi\left(\frac{\rho p_1}{\sqrt{2-\rho^2}}\right) + \frac{\rho}{\sqrt{2-\rho^2}} \phi\left(\frac{\rho p_1}{\sqrt{2-\rho^2}}\right)$$

and letting $t = \frac{\rho}{\sqrt{2-\rho^2}}$, we have that the optimal cutoff c is implicitly defined by

$$c\Phi(tc) + t\phi(tc) = 0.$$

Observe that $t \in [-1, 1]$ and is increasing in ρ. Now letting $F(c, t) = c\Phi(tc) + t\phi(tc)$, we can use the implicit function theorem to get that

$$\begin{aligned} c'(t) &= -\frac{F_t}{F_c} \\ &= -\frac{c^2\phi(tc) + \phi(tc) - t^2c^2\phi(tc)}{\Phi(tc) + tc\phi(tc) - t^3c\phi(tc)} \\ &= -\frac{\phi(tc)\left(1 + c^2(1-t^2)\right)}{\Phi(tc) + tc\phi(tc)(1-t^2)} \\ &= -\frac{\phi(tc)\left(1 + c^2(1-t^2)\right)}{\Phi(tc)\left(1 - c^2(1-t^2)\right)}. \end{aligned}$$

Note that $c(0) = 0$ and so $c'(0) = -2\phi(0) < 0$. Also observe from the above expression that $c'(t)$ is never 0 and so it follows from the smoothness of c that $c'(t) < 0$ for all $t \in (-1, 1)$. Thus, we have that the optimal cutoff is decreasing in ρ. □

Theorem 3. *Suppose there are $N = 2$ projects, each with $(p_i, a_i) \sim F_i$. Then, the optimal mechanism under no-underselling constraint M_U is a table mechanism with a table function $f \in \mathcal{T}_U$ defined by cutoffs c_1, c_2 such that for any $p \in \Theta_p$,*

$$i \in f(p) \iff p_{-i} \leq c_i.$$

Moreover, the cutoffs are given either by $c_1 = c_1^*$ and $c_2 = \infty$ or $c_1 = \infty$ and $c_2 = c_2^*$, where c_i^* solves

$$\mathbb{E}\left[(p_i - p_{-i})\mathbb{I}_{a_i \geq a_{-i}} | p_{-i} = c_i^*\right] = 0 \tag{5}$$

Proof. To prove this result, we will first show that for any arbitrary truthful mechanism $f \in \mathcal{T}_U$, there is a cutoff mechanism that is weakly better for the principal. And then, we will solve for the optimal cutoff.

For the first step, suppose d is an arbitrary table mechanism defined by a table function $f \in \mathcal{T}_U$. Assume d is such that $2 \in f(p)$ for all p_1, p_2. The other case is analogous. Let $c = \sup\{p_1 : 1 \in f(p_1, p_1)\}$ and define the cutoff mechanism d' with a table function $f' \in \mathcal{T}_U$ so that $1 \in f'(p) \iff p_2 \leq c$. We'll show that the expected profit for the principal from d' is at least as high as the expected profit from d. In fact, we will show that this holds conditional on (p_1, p_2), and hence in expectation. Consider the following (exhaustive and mutually exclusive) cases depending on whether $p_i \geq c$ or $p_i < c$:

- $p_1 \in (-\infty, c], p_2 \in (-\infty, c]$: For any such p_1, p_2, we know both $f(p) = f'(p) = \{1, 2\}$. Thus, the two mechanisms are identical in this set and lead to same profits for the principal.

- $p_1 \in (\infty, c], p_2 \in (c, \infty)$: In this case, $f'(p) = \{2\}$ and thus project 2 is chosen for sure. Note that the principal prefers project 2 over 1 in these profiles and thus, the profit from the cutoff mechanism is weakly higher for any such p_1, p_2.

- $p_1 \in (c, \infty), p_2 \in (-\infty, c]$: Now $f'(p) = \{1, 2\}$ while $f(p)$ can be either $\{2\}$ or $\{1, 2\}$. Observe that the principal strictly prefers project 1 over 2 in all these profiles. Thus, the cutoff mechanism again leads to weakly higher profits for such p_1, p_2.

- $p_1 \in (c, \infty), p_2 \in (c, \infty)$: Here we have $f(p) = f'(p) = \{2\}$. Therefore, the second project is chosen for all such profiles. Thus, the two mechanisms are identical and generate the same profit for the principal in this case.

This shows that for any truthful mechanism, there is a cutoff mechanism under which the principal's expected profit is weakly higher. Thus, the optimal truthful mechanism must be a cutoff mechanism. Now we want to find the optimal cutoff c.

Again, assume that $2 \in f(p)$ for all $p \in \Theta_p$. Consider the decision problem of the principal for any given p_1, p_2, a_1, a_2. It can either

- not make project 1 available and get p_2

- make project 1 available and get $p_1 \mathbb{I}_{a_1 \geq a_2} + p_2 \mathbb{I}_{a_2 > a_1}$

The constraint imposed by truthfulness and optimality of cutoff mechanisms imply that the principal can only base this decision on the value of p_2. Thus, taking expectation with respect to p_1, a_1, a_2, we get that the two alternatives are:

- not make project 1 available and get $\mathbb{E}[p_2|p_2]$

- make project 1 available and get $\mathbb{E}\left[p_1 \mathbb{I}_{a_1 \geq a_2} + p_2 \mathbb{I}_{a_2 > a_1} | p_2\right]$

Thus, the principal would want to make project 1 available if and only if

$$\begin{aligned}
&\mathbb{E}\left[p_1 \mathbb{I}_{a_1 \geq a_2} + p_2 \mathbb{I}_{a_2 > a_1} | p_2\right] \geq \mathbb{E}[p_2|p_2] \\
\iff &\mathbb{E}\left[p_1 \mathbb{I}_{a_1 \geq a_2} | p_2\right] \geq \mathbb{E}\left[p_2 \mathbb{I}_{a_1 \geq a_2} | p_2\right] \\
\iff &\mathbb{E}\left[(p_1 - p_2) \mathbb{I}_{a_1 \geq a_2} | p_2\right] \geq 0 \\
\iff &\mathbb{E}\left[(p_1 - p_2) \mathbb{P}[a_1 \geq a_2 | p_1] | p_2\right] \geq 0
\end{aligned}$$

At the optimal cutoff, the principal should be indifferent between the two alternatives and so the cutoff c_1^* is defined by the solution to the equation

$$\mathbb{E}\left[(p_1 - p_2) \mathbb{I}_{a_1 \geq a_2} | p_2 = c_1^*\right] = 0.$$

We can repeat the exercise assuming that $1 \in f(p)$ for all $p \in \Theta_p$, and find the optimal cutoff c_2^*. The optimal mechanism is the one that gives a higher profit among these two. $\square$