

The effect of competition in contests: A unifying approach

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Sales contest

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Research question

How should a budget be allocated across prizes to maximize total sales?

A puzzle in the literature

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	Complete Info.	Finite types	Continuum of Types
Linear	$(0, v, 100-v)$?	$(0, 0, 100)$
Concave	$(0, 0, 100)$?	$(0, 0, 100)$
Convex	$(0, 50, 50)$?	Depends

References: Barut & Kovenock [EJPE, 1998]; Fang et al. [JPE, 2020]; Moldovanu & Sela [AER, 2001]; Olszewski & Siegel [ECMA, 2016; TE, 2020]; Zhang [TE, 2024]

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- Sisak [JES, 2009]'s conjecture:

*The case of asymmetric individuals, where types are private information but **drawn from discrete, identical** or even different distributions, has not been addressed so far. From the results ..., one could conjecture that **multiple prizes might be optimal even with linear costs.***

Contribution

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- Experiment provides qualitative support for these findings
- Novel approach for analyzing symmetric equilibrium in games

- Model
- Equilibrium characterization
- Effect of competition
 - Linear costs
 - General costs
- Experiment

Model

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A **contest** $v = (v_0, \dots, v_N)$ assigns a prize for each rank:

$$v \in \mathcal{V} = \{v \in \mathbb{R}^{N+1} : v_0 \leq \dots \leq v_N \text{ with } 0 = v_0 < v_N\}$$

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- Symmetric Bayes-Nash equilibrium: (X_1, X_2, \dots, X_K)

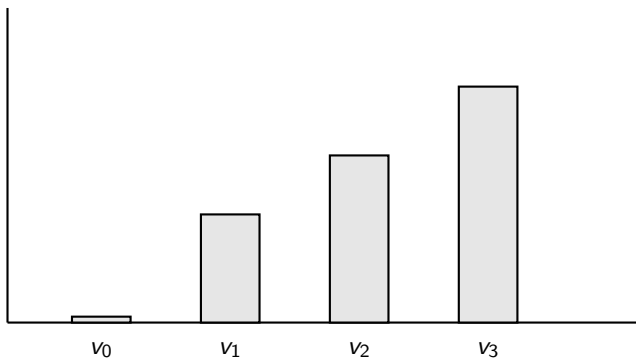
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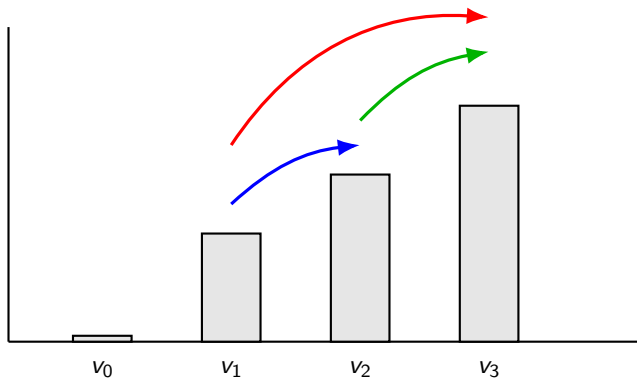
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- Symmetric Bayes-Nash equilibrium: (X_1, X_2, \dots, X_K)
- $\mathbb{E}[X] = \sum_{k=1}^K p_k \mathbb{E}[X_k]$: ex-ante expected effort

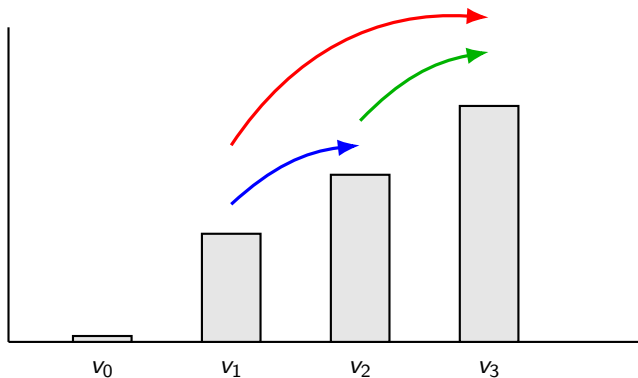
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Research question

For prizes $m > m'$:

$$\frac{\partial \mathbb{E}[X]}{\partial v_m} - \frac{\partial \mathbb{E}[X]}{\partial v_{m'}}?$$

Equilibrium

Equilibrium structure

Lemma.

Consider any $(N + 1, \Theta, p)$ and $v \in \mathcal{V}$. If (X_1, \dots, X_K) is an equilibrium, there exist boundary points $b_1 < \dots < b_K$ so that X_k is continuously distributed on $[b_{k-1}, b_k]$.

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④ Monotonicity:

If more efficient type has x, y with $x < y$ in its support, then less efficient type gets a higher payoff from x than y

Equilibrium representation

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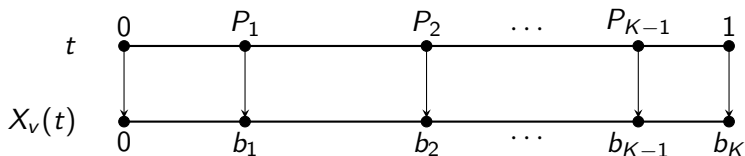
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Equilibrium representation

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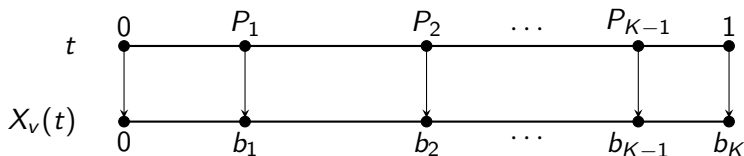
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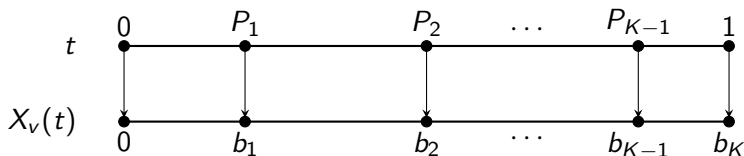
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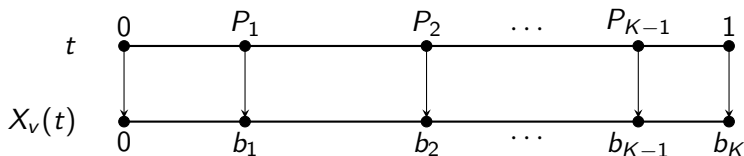


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$$\pi_v(t) = \sum_{m=0}^N v_m H_m^N(t) \quad H_m^N(t) = \binom{N}{m} t^m (1-t)^{N-m}$$

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Example: for $v = (0, \dots, 0, V)$, $\pi_v(t) = Vt^N$

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Equilibrium characterization

Theorem 1.

For any $(N + 1, \Theta, p)$ and $v \in \mathcal{V}$, there is a unique symmetric equilibrium (X_1, \dots, X_K) . It is such that for each $t \in [0, 1]$,

$$X_v(t) = \frac{\pi_v(t) - u_{k(t)}}{\theta_{k(t)}},$$

where $k(t) = \max\{k : P_{k-1} \leq t\}$ and the equilibrium utilities are

$$u_k = \theta_k \left[\sum_{j=1}^{k-1} \pi_v(P_j) \left(\frac{1}{\theta_{j+1}} - \frac{1}{\theta_j} \right) \right].$$

Competition: Linear Cost

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$$\alpha_m = \frac{1}{N+1} \left[\frac{1}{\theta_K} - \sum_{k=1}^{K-1} \left[H_{\geq m}^{N+1}(P_k) + (N-m)H_m^{N+1}(P_k) \right] \left(\frac{1}{\theta_{k+1}} - \frac{1}{\theta_k} \right) \right].$$

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- For any $v \in \mathcal{V}$ and $m > m'$:

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- Suppose $\Theta = \{\theta_1\}$:

$$\alpha_1 = \alpha_2 = \dots = \alpha_N = \frac{1}{(N+1)\theta_1}$$

Complete information (Barut & Kovenock [1998])

For any $m > m'$, $\alpha_m - \alpha_{m'} = 0$.

Incomplete information

Theorem 2.

Consider any $(N + 1, \Theta, p)$ with $|\Theta| > 1$. For any interior prize $m' \in \{1, \dots, N - 1\}$,

$$\alpha_N - \alpha_{m'} > 0.$$

For the design problem, $v^ = (0, \dots, 0, V)$ is uniquely optimal.*

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Resolves Sisak [2009]'s conjecture in the negative

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$$p_1 \geq \frac{m' + 1}{N} \implies \alpha_{m'+1} - \alpha_{m'} \leq 0.$$

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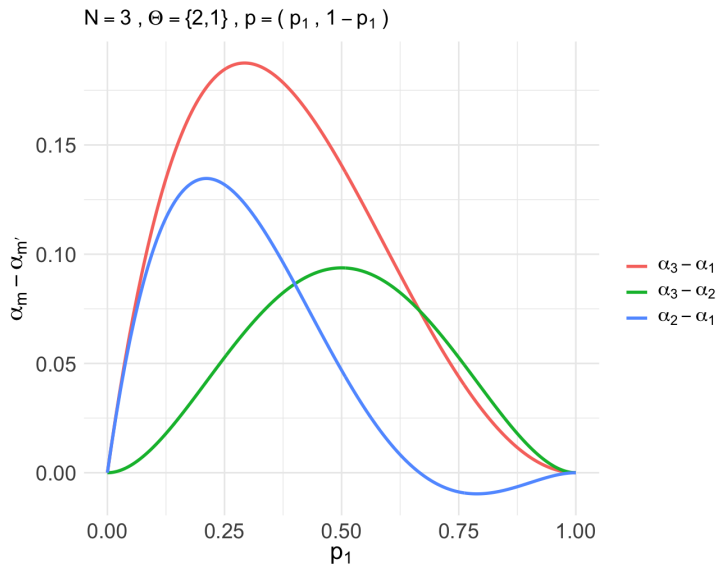
$$p_1 \geq \frac{m' + 1}{N} \implies \alpha_{m'+1} - \alpha_{m'} \leq 0.$$

- Extreme case: If $p_1 > \frac{N-1}{N}$, for any $m > m'$,

$$\alpha_m - \alpha_{m'} \geq 0 \iff m = N.$$

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Competition: General Cost

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$$c(X_v^*(t)) = X_v(t)$$

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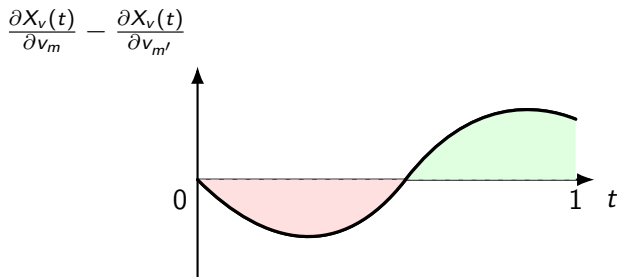
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$$\frac{\partial \mathbb{E}[X^*]}{\partial v_m} - \frac{\partial \mathbb{E}[X^*]}{\partial v_{m'}} = \int_0^1 \underbrace{g'(X_v(t))}_{\text{weights}} \underbrace{\left(\frac{\partial X_v(t)}{\partial v_m} - \frac{\partial X_v(t)}{\partial v_{m'}} \right)}_{\text{single-crossing?}} dt$$

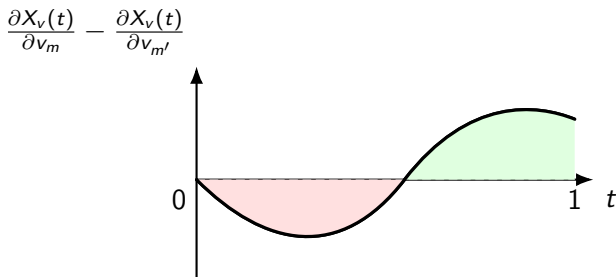
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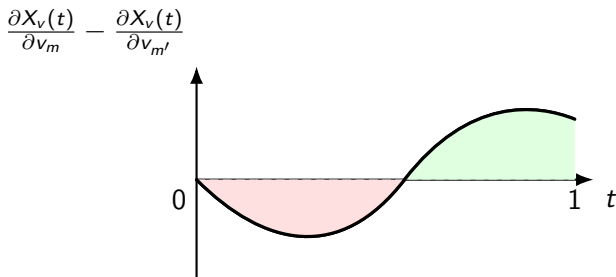
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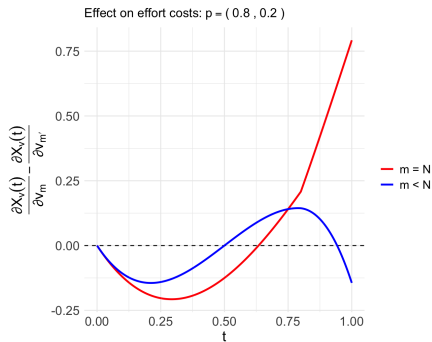
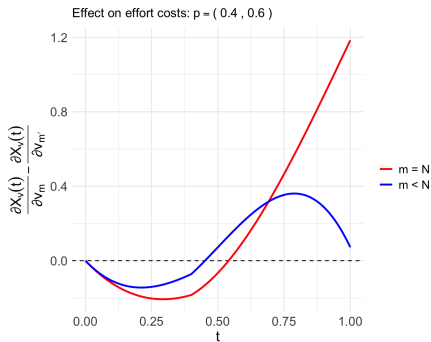
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- $m = N$: ✓
- $m < N$: ✓ or ✗



Main Result

Theorem 3.

Consider any $(N + 1, \Theta, p)$ with cost $c(\cdot)$. Let m, m' with $m > m'$ be such that either $m = N$ or $\left(\frac{\partial u_K}{\partial v_m} - \frac{\partial u_K}{\partial v_{m'}} \right) \leq 0$. Then, the following hold:

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Complete information (Fang, Noe, and Strack [2020])

Suppose $|\Theta| = 1$.

- ① If c is concave, competition encourages effort.
- ② If c is convex, competition discourages effort.

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Theorem 4.

Consider any $(N + 1, \Theta, p)$ with cost $c(\cdot)$. If c is (weakly) concave, the winner-takes-all contest maximizes expected total effort of top q agents for any $q \in \{1, \dots, N + 1\}$.

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Consider any $(N + 1, \Theta, p)$ with $|\Theta| > 1$ and cost $c(x) = x^\alpha$. For any q , there exists $\alpha^ > 1$ such that for all $\alpha \in [1, \alpha^*)$, the WTA maximizes expected total effort of top q agents.*

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Conjecture

The degree of convexity required to overturn the optimality of WTA shrinks as the contest environment approaches complete information.

Convergence

Theorem 6.

Fix any contest $v \in \mathcal{V}$ and cost function $c : \mathbb{R}_+ \rightarrow \mathbb{R}_+$. Let $G : [\underline{\theta}, \bar{\theta}] \rightarrow [0, 1]$ be a differentiable CDF and let G^1, G^2, \dots , be any sequence of CDF's, each with a finite support, such that for all $\theta \in [\underline{\theta}, \bar{\theta}]$,

$$\lim_{n \rightarrow \infty} G^n(\theta) = G(\theta).$$

Then, the corresponding sequence of finite-type space equilibrium CDF's, F^1, F^2, \dots , converges to the continuum type-space equilibrium CDF F , i.e., for all $x \in \mathbb{R}$,

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- Experimental literature: Large but finite type-spaces

Experiment

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- Prolific : 445 participants
- Contest environment: 4 agents, $\Theta = \{2x, x\}$, $p = (0.8, 0.2)$
- Four contests which are progressively less competitive:
 WTA = (0, 0, 0, 100)
 High = (0, 0, 25, 75)
 Med = (0, 0, 50, 50)
 Low = (0, 25, 25, 50)
- Strategy method, contest order randomized

Expected effort

Treatment	$(0, v_1, v_2, v_3)$	Equilibrium Effort ($\mathbb{E}[X]$)			Observed Effort		
		$c_k(x) = x$	$c_k(x) = 2x$	Pooled	$c_k(x) = x$	$c_k(x) = 2x$	Pooled
<i>WTA</i>	$(0, 0, 0, 100)$	48.2	6.4	14.76	52.8	40.5	42.96
<i>High</i>	$(0, 0, 25, 75)$	37.0	8.0	13.80	46.9	37.4	39.30
<i>Med</i>	$(0, 0, 50, 50)$	25.8	9.6	12.84	43.3	36.5	37.86
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Table: Equilibrium and observed efforts by treatment and cost type.

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- WTA remains optimal
- Going from Low to Med does not lead to a significant change in effort

Regression estimates

$$X = \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 + \epsilon.$$

Prize	Equilibrium Weight			Estimated Coefficient		
	$c_k(x) = x$	$c_k(x) = 2x$	Pooled	$c_k(x) = x$	$c_k(x) = 2x$	Pooled
α_3	0.482	0.064	0.148	0.524	0.401	0.426
α_2	0.034	0.128	0.109	0.335	0.321	0.324
α_1	-0.014	0.152	0.119	0.334	0.314	0.318

Table: Expected Effort: Equilibrium and Regression Results

Summary

- The (most competitive) winner-takes-all is robustly optimal for maximizing effort under linear or concave costs.
- Despite this, increasing competition in the interior may discourage effort if inefficient types are relatively likely.
- An experiment provides qualitative support to these findings.
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