The effect of competition in contests: A unifying approach

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How should a budget be allocated across prizes to maximize total effort?

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	Complete Info.	Finite types	Continuum of Types
Linear	(0, v, 100-v)	?	(0, 0, 100)
Concave	(0, 0, 100)	?	(0, 0, 100)
Convex	(0, 50, 50)	?	Depends

References: Barut & Kovenock [EJPE, 1998]; Fang et al. [JPE, 2020]; Moldovanu & Sela [AER, 2001]; Olszewski & Siegel [ECMA, 2016; TE, 2020]; Zhang [TE, 2024]

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- Sisak [JES, 2009]'s conjecture:

  The case of asymmetric individuals, where types are private information but drawn from discrete, identical or even different distributions, has not been addressed so far. From the results ..., one could conjecture that multiple prizes might be optimal even with linear costs.

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- Identify conditions under which these effects extend to general costs
- Test theoretical predictions in an online experiment with binary types

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- Experiment provides qualitative support for these findings
- Novel approach for analyzing symmetric equilibrium in games

Outline

- Model
- Equilibrium characterization
- Effect of competition
    - Linear costs
    - General costs
- Experiment

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- $N+1$: number of agents
- $\mathcal{C} = \{c_1, \dots, c_K\}$: finite set of types such that for each k,

$$c_k \in \{c : \mathbb{R}_+ \to \mathbb{R}_+ \mid c(0) = 0, c'(x) > 0, \lim_{x \to \infty} c(x) = \infty\}.$$

  - Ordered: $c_1'(x) > \cdots > c_K'(x)$ for all $x \in \mathbb{R}_+$
  - Parametric: $c_k(x) = \theta_k c(x)$ with $\theta_1 > \cdots > \theta_K$
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A *contest* $v = (v_0, \ldots, v_N)$ assigns a prize for each rank, where

$$v \in \mathcal{V} = \{v \in \mathbb{R}^{N+1} : v_0 \leq \cdots \leq v_N \text{ with } 0 = v_0 < v_N\}.$$

Bayesian game

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- Symmetric Bayes-Nash equilibrium: $(X_1, X_2, \ldots, X_K)$ where $X_k \sim F_k$
- Effect of increasing competition: For prizes $m > m'$, compute

$$\frac{\partial \mathbb{E}[X]}{\partial v_m} - \frac{\partial \mathbb{E}[X]}{\partial v_{m'}},$$

where $\mathbb{E}[X] = \sum_{k=1}^{K} p_k \mathbb{E}[X_k]$ is the ex-ante expected effort.

Equilibrium structure

Lemma 1.

For any $(N+1, \mathcal{C}, p)$ *and* $v \in \mathcal{V}$, *a (symmetric) equilibrium* $(X_1, \ldots, X_K)$ *must be such that there exist boundary points* $0 = b_0 < b_1 < \cdots < b_K$ *so that, for each* k, X_k *is continuously distributed on* $[b_{k-1}, b_k]$.

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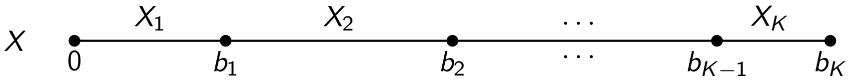

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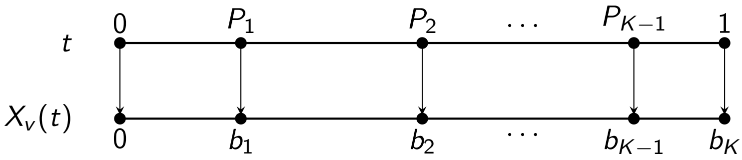

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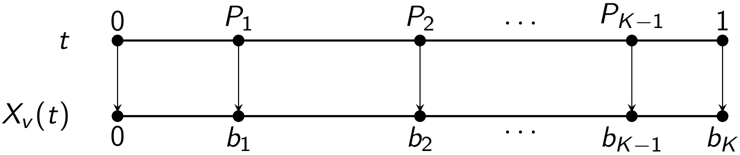

- $\pi_v(t)$: expected prize function

$$\pi_v(t) = \sum_{m=0}^{N} v_m H_m^N(t), \qquad H_m^N(t) = \binom{N}{m} t^m (1-t)^{N-m}.$$

Example: for $v = (0, \ldots, 0, V)$, $\pi_v(t) = Vt^N$

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- Observe that

$$\begin{aligned} u_1 = 0 &\implies b_1 = c_1^{-1}(\pi_v(P_1)) \\ &\implies u_2 = \pi_v(P_1) - c_2(b_1) \\ &\implies b_2 = c_2^{-1}(\pi_v(P_2) - u_2) \\ &\implies u_3 = \ldots \end{aligned}$$

Equilibrium characterization

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Theorem 2.

For any $(N+1, \mathcal{C}, p)$ *and* $v \in \mathcal{V}$*, the equilibrium* $(X_1, \ldots, X_K)$ *is such that for each* k*, the distribution* $F_k : [b_{k-1}, b_k] \to [0,1]$ *is defined by*

$$\pi_v(P_{k-1} + p_k F_k(x_k)) - c_k(x_k) = u_k \text{ for all } x_k \in [b_{k-1}, b_k],$$

where the boundary points $b = (b_0, \ldots, b_K)$*, with* $b_0 = 0$*, and the equilibrium utilities* $u = (u_1, \ldots, u_K)$*, with* $u_1 = 0$*, satisfy*

$$\pi_v(P_k) - c_k(b_k) = u_k \text{ for all } k \in [K],$$

and

$$\pi_v(P_{k-1}) - c_k(b_{k-1}) = u_k \text{ for all } k \in [K].$$

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Lemma 3.

For any $(N+1, \mathcal{C}, p)$ and $v \in \mathcal{V}$, the expected equilibrium effort is

$$\mathbb{E}[X] = \int_0^1 g_{k(t)}\left(\pi_v(t) - u_{k(t)}\right) dt,$$

where $g_k = c_k^{-1}$ and $k(t) = \max\{k : P_{k-1} \leq t\}$.

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- For any prize $m \in \{1, \ldots, N\}$,

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- For prizes $m > m'$, $\dfrac{\partial \mathbb{E}[X]}{\partial v_m} - \dfrac{\partial \mathbb{E}[X]}{\partial v_{m'}}$ is

$$\int_0^1 g'_{k(t)} \left(\pi_v(t) - u_{k(t)}\right) \left[H_m^N(t) - H_{m'}^N(t) - \left[\frac{\partial u_{k(t)}}{\partial v_m} - \frac{\partial u_{k(t)}}{\partial v_{m'}}\right]\right] dt,$$

which provides a useful general framework in which to study the effect of increasing competition on effort.

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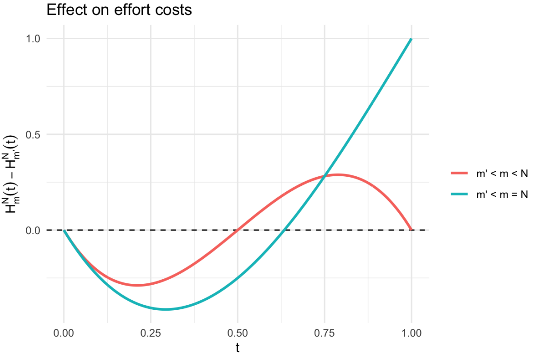

Complete information

Theorem 4.

Suppose $\mathcal{C} = \{c_1\}$. For any m, m' with $m > m'$, the following hold:

1. *If c_1 is concave, then for any $v \in \mathcal{V}$,* $\dfrac{\partial \mathbb{E}[X]}{\partial v_m} - \dfrac{\partial \mathbb{E}[X]}{\partial v_{m'}} \geq 0.$
2. *If c_1 is convex, then for any $v \in \mathcal{V}$,* $\dfrac{\partial \mathbb{E}[X]}{\partial v_m} - \dfrac{\partial \mathbb{E}[X]}{\partial v_{m'}} \leq 0.$

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If c_1 concave, $v^* = (0, \ldots, 0, V)$. If c_1 convex, $v^* = (0, \frac{V}{N}, \ldots, \frac{V}{N})$.

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Lemma 5.

Suppose $(N+1, \mathcal{C}, p)$ is such that $c_k(x) = \theta_k \cdot x$ with $\theta_1 > \cdots > \theta_K$. For any $v \in \mathcal{V}$, the equilibrium utilities are

$$u_k = \theta_k \left[\sum_{j=1}^{k-1} \pi_v(P_j) \left(\frac{1}{\theta_{j+1}} - \frac{1}{\theta_j} \right) \right],$$

and the expected effort is $\mathbb{E}[X] = \sum_{m=1}^{N} \alpha_m v_m$, where

$$\alpha_m = \frac{1}{N+1} \left[\frac{1}{\theta_K} - \sum_{k=1}^{K-1} \left[H_{\geq m}^{N+1}(P_k) + (N-m) H_m^{N+1}(P_k) \right] \left(\frac{1}{\theta_{k+1}} - \frac{1}{\theta_k} \right) \right].$$

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- *Interior discouragement effect*: Suppose $K = 2$. For any interior prize $m \in \{1, \ldots, N-1\}$,

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  In the extreme case, when $p_1 > \frac{N-1}{N}$, transferring value to a better-ranked prize encourages effort iff the better prize is top-ranked.

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- For any $v \in \mathcal{V}$, u_k is as in Lemma 5 and independent of c.
- Further,

$$\frac{\partial \mathbb{E}[X]}{\partial v_m} - \frac{\partial \mathbb{E}[X]}{\partial v_{m'}} = \int_0^1 g'\left(\frac{\pi_v(t) - u_{k(t)}}{\theta_{k(t)}}\right)(\lambda_m(t) - \lambda_{m'}(t))\,dt,$$

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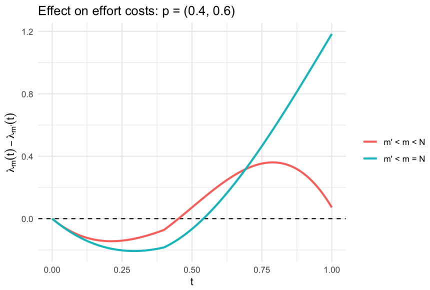
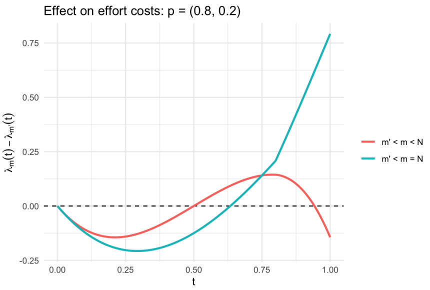
- If $\lambda_m(t) - \lambda_{m'}(t)$ is single-crossing (i.e. $\lambda_m(1) - \lambda_{m'}(1) \geq 0$), then $\int_0^1 (\lambda_m(t) - \lambda_{m'}(t))\, dt = \alpha_m - \alpha_{m'}$ may be informative.

Illustration (Binary types)





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Suppose $(N+1, \mathcal{C}, p)$ *is such that* $c_k(x) = \theta_k \cdot c(x)$ *with* $\theta_1 > \cdots > \theta_K$*. Let* $m, m' \in \{1, \ldots, N\}$ *with* $m > m'$ *be such that, either* $m = N$ *or* $\left(\frac{\partial u_K}{\partial v_m} - \frac{\partial u_K}{\partial v_{m'}}\right) \leq 0$*. Then, the following hold:*

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1. *If* $\alpha_m - \alpha_{m'} \geq 0$ *and* c *is concave, then for any* $v \in \mathcal{V}$, $\frac{\partial \mathbb{E}[X]}{\partial v_m} - \frac{\partial \mathbb{E}[X]}{\partial v_{m'}} \geq 0$.
2. *If* $\alpha_m - \alpha_{m'} \leq 0$ *and* c *is convex, then for any* $v \in \mathcal{V}$, $\frac{\partial \mathbb{E}[X]}{\partial v_m} - \frac{\partial \mathbb{E}[X]}{\partial v_{m'}} \leq 0$.

Main result: linear to general

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- Design problem: if c is concave, $v^* = (0, \ldots, 0, V)$.

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Corollary 7.

Suppose $(N+1, \mathcal{C}, p)$ is such that $c_k(x) = \theta_k \cdot c(x)$ with $\theta_1 > \cdots > \theta_K$. If c is (weakly) concave, the WTA contest maximizes expected total effort of the top q agents for any $q \in \{1, \ldots, N+1\}$.

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- Can use continuity arguments to obtain formal results
- Sharp contrast to the complete information environment
- Conjecture: The degree of convexity required to overturn the optimality of WTA shrinks as the contest environment approaches complete information.

Convergence

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Theorem 8.

Fix any contest $v \in \mathcal{V}$ *and cost function* $c : \mathbb{R}_+ \to \mathbb{R}_+$. *Let* $G : [\underline{\theta}, \overline{\theta}] \to [0, 1]$ *be a differentiable CDF and let* $G^1, G^2, \ldots,$ *be any sequence of CDF's, each with a finite support, such that for all* $\theta \in [\underline{\theta}, \overline{\theta}]$,

$$\lim_{n \to \infty} G^n(\theta) = G(\theta).$$

Then, the corresponding sequence of finite-type space equilibrium CDF's, $F^1, F^2, \ldots,$ *converges to the continuum type-space equilibrium CDF* F, *i.e., for all* $x \in \mathbb{R}$,

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- Intuitively, as K increases, $[b_{k-1}, b_k]$ shrinks, and converges to the pure-strategy equilibrium effort under the continuum type-space.

Experiment

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- Prolific : 445 participants
- Contest environment: 4 agents, $\mathcal{C} = \{2x, x\}$, $p = (0.8, 0.2)$
- Four contests which are progressively less competitive:
  WTA $= (0, 0, 0, 100)$
  High $= (0, 0, 25, 75)$
  Med $= (0, 0, 50, 50)$
  Low $= (0, 25, 25, 50)$
- Strategy method, contest order randomized

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Expected effort

Treatment	$(0, v_1, v_2, v_3)$	Equilibrium Effort ($\mathbb{E}[X]$)			Observed Effort		
		$c_k(x) = x$	$c_k(x) = 2x$	Pooled	$c_k(x) = x$	$c_k(x) = 2x$	Pooled
WTA	$(0, 0, 0, 100)$	48.2	6.4	14.76	52.8	40.5	42.96
High	$(0, 0, 25, 75)$	37.0	8.0	13.80	46.9	37.4	39.30
Med	$(0, 0, 50, 50)$	25.8	9.6	12.84	43.3	36.5	37.86
Low	$(0, 25, 25, 50)$	24.6	10.2	13.08	42.9	35.9	37.30

Table: Equilibrium and observed efforts by treatment and cost type.

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Table: Equilibrium and observed efforts by treatment and cost type.

- Efficient type exerts higher effort than the less efficient type
- Significant over-provision compared to equilibrium across treatments
- WTA remains optimal
- Going from Low to Med does not lead to a significant change in effort

Regression estimates

From Lemma 5 : $X = \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 + \epsilon$.

Prize	Equilibrium Weight			Estimated Coefficient		
	$c_k(x) = x$	$c_k(x) = 2x$	Pooled	$c_k(x) = x$	$c_k(x) = 2x$	Pooled
α_3	0.482	0.064	0.148	0.524	0.401	0.426
α_2	0.034	0.128	0.109	0.335	0.321	0.324
α_1	-0.014	0.152	0.119	0.334	0.314	0.318

Table: Expected Effort: Equilibrium and Regression Results

Summary

- The (most competitive) winner-takes-all is robustly optimal for maximizing effort under linear or concave costs.
- Despite this, increasing competition in the interior may discourage effort if inefficient types are relatively likely.
- An experiment provides qualitative support to these findings.
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Thank you!