# The effect of competition in contests: A unifying approach*

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**Abstract**

We study all-pay contests with a general finite type-space and show that the most competitive winner-takes-all contest maximizes both total and maximum effort under linear or concave costs, resolving a long-standing open question in contest design. Nonetheless, the effect of competition is nuanced, as we uncover an *interior discouragement effect*: making prizes more unequal may reduce effort if relatively inefficient types are sufficiently likely. An experiment provides qualitative support for these findings. Our analysis develops a novel methodology based on characterizing equilibrium effort through the probability of outperforming an arbitrary opponent, offering a broadly applicable tool particularly useful in settings where mixed equilibria hinder analysis. Moreover, it provides a unifying framework that reconciles contrasting results under complete information and continuum type-space environments, for which we establish an equilibrium convergence result.

# 1 Introduction

A central question in contest theory is how prize structures shape agents' incentives to exert effort, and in particular, which structures maximize effort. Under complete information, optimal contests typically feature multiple prizes, allocated in minimally competitive ways (Barut and Kovenock (1998); Fang, Noe, and Strack (2020)). By contrast, in incomplete

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information settings with a continuum of types, the most competitive winner-takes-all structure is frequently optimal (Moldovanu and Sela (2001); Zhang (2024)). Yet, what drives this divergence remains unclear, and it is an open question which (if any) of these findings extend to the intermediate and fundamental case of a finite type-space. This gap was also noted in a survey article by Sisak (2009), who conjectured that multiple prizes might be optimal:

> *"The case of asymmetric individuals, where types are private information but drawn from discrete, identical or maybe even different distributions, has not been addressed so far. From the results ... on asymmetric types with full information, one could conjecture that multiple prizes might be optimal even with linear costs."*

In this paper, we address this question by analyzing all-pay contests where ex-ante symmetric agents have private abilities drawn from a finite type-space. The finite type-space framework embeds the complete information as a special case and can approximate any continuum type-space. Thus, our analysis not only bridges a gap in the literature, but provides a unifying approach offering insights into the contrasting results in these extreme environments. Beyond its theoretical appeal, this framework is practically relevant, can accommodate richer non-parametric type-spaces, and enables experimental investigation.

We begin by characterizing the unique symmetric equilibrium of the Bayesian game, showing that it is mixed and monotonic: different types randomize over disjoint but contiguous intervals, with more efficient types always outperforming less efficient ones. To overcome the complexity of analyzing this mixed equilibrium, we introduce a novel representation in terms of the (ex-ante) probability of outperforming an arbitrary opponent, which is always uniformly distributed. Using these ideas, we examine how increasing competition affects effort under linear costs and identify conditions under which they persist under general costs.

Our results show that the winner-takes-all contest maximizes total effort under linear or concave costs. This resolves Sisak (2009)'s conjecture in the negative and, together with Moldovanu and Sela (2001), establishes winner-takes-all as being robustly optimal whenever there is any uncertainty. More broadly, our analysis actually suggests that this optimality persists as long as the cost function is not too convex, and we conjecture that the degree of convexity required to overturn the result shrinks as the environment converges to complete information—where, in contrast, the minimally competitive contest that allocates the budget equally among all but the worst-performing agent is strictly optimal.

Although the most competitive contest is optimal under linear costs, we uncover an *interior discouragement effect*: shifting value from lower to higher ranked (but not the top) prizes can reduce effort when relatively inefficient types are sufficiently likely. Intuitively, because equilibrium is monotonic, such transformations reduce the expected prize for inefficient types while raising it for efficient ones. After accounting for the resulting changes in equilibrium utilities (or information rents, which are always zero for the least efficient type), the residual effect on effort is typically negative for inefficient types, so aggregate effort declines when these types are prevalent. As a consequence, for design problems where the top prize is capped, the remaining budget may be best split equally across all but the lowest-ranked prize.

For general costs, we show that, provided the utility of the most efficient type does not increase, the effect on effort under linear cost extends: non-negative effects carry over to equilibrium under concave costs, and non-positive effects to that under convex costs. This generalizes the complete information analysis of Fang, Noe, and Strack (2020), where increasing competition leaves both equilibrium utilities and expected effort unchanged under linear costs. While our formal result focuses on expected effort, analogous arguments apply to other objectives, and in particular, we show that the winner-takes-all contest also maximizes maximum effort under linear or concave costs.

Our finite type-space framework enables direct experimental testing of the model's predictions. Since implementing a continuum of types is infeasible, prior experiments have relied on large but finite type-spaces, assuming equilibrium properties extend from the continuum setting (Müller and Schotter (2010)). While our convergence result justifies this assumption, our experiment employs a simple binary type-space with linear costs where the inefficient type is relatively likely. We vary the prize structure by gradually increasing competitiveness across four contests, including winner-takes-all. The results reveal an over-provision of effort, particularly by the inefficient type, but aggregate patterns broadly align with our comparative statics: winner-takes-all remains optimal, and the interior discouragement effect receives partial support. Specifically, although efforts do not decline as theory predicts, we observe no significant increase in effort when competition intensifies in the interior.

**Contest theory related literature.** The existing game-theoretic literature in contests has predominantly focused on the design problem in environments where the type-space is either a continuum, or a singleton (the complete information case), and the results highlight how the structure of the optimal contest can vary significantly depending on the environment. For the continuum type-space, the most competitive winner-takes-all contest has been shown to be optimal under linear or concave costs (Moldovanu and Sela (2001)), in some cases under convex costs (Zhang (2024)), with negative prizes (Liu, Lu, Wang, and Zhang (2018)), and with general architectures (Moldovanu and Sela (2006); Liu and Lu (2014)). In comparison, in the complete information environments, the minimally competitive budget distribution (all agents but one receive an equal positive prize) has been shown to be a feature of the optimal contest quite generally (Barut and Kovenock (1998); Letina, Liu, and Netzer (2023, 2020); Xiao (2018)). In a general framework with many agents, Olszewski and Siegel (2016, 2020) show that awarding multiple prizes of descending sizes is optimal under convex costs. Other related work has examined the effect of competition in complete information setting (Fang, Noe, and Strack (2020)), and continuum type-space setting (Goel

(2025); Krishna, Lychagin, Olszewski, Siegel, and Tergiman (2025)).[1]

There is a related literature on contests with a finite type-space, much of which assumes binary type-spaces or a small number of agents and focuses on characterizing equilibrium properties under correlated or asymmetric types. Siegel (2014) establishes the existence of a unique equilibrium under general distributional assumptions. With correlated types, Liu and Chen (2016) show that the symmetric equilibrium may be non-monotonic when the degree of absolute correlation is high, Rentschler and Turocy (2016) highlight the possibility of allocative inefficiency in equilibrium, while Tang, Fu, and Wu (2023) and Kuang, Zhao, and Zheng (2024) explore the impact of reservation prices and information disclosure policies, respectively. With asymmetric type distributions, Szech (2011) shows that agents may benefit from revealing partial information about their private types, while Chen (2021) characterizes equilibrium outcomes for varying levels of signal informativeness.[2]

**Experimental investigations of contests related literature.** There is a long tradition of studying contests through incentivized laboratory experiments (see Dechenaux, Kovenock, and Sheremeta (2015) for a survey). Perhaps the most closely related work is Müller and Schotter (2010), who provide evidence broadly consistent with the predictions of Moldovanu and Sela (2001): winner-takes-all is optimal under linear costs, whereas splitting prizes is favored under convex costs. Two other studies, Barut, Kovenock, and Noussair (2002) and Noussair and Silver (2006), examine all-pay auctions with private valuations, a setting strategically equivalent to all-pay contests with private costs. All three experiments employ large finite type-spaces to approximate continuum-type equilibria, an assumption that our convergence result formally justifies. Like us, these studies observe substantial overbidding relative to theoretical benchmarks. A key difference, however, is that low-valuation (or less efficient) agents in their settings tend to underbid, whereas in our experiment, inefficient types significantly overbid, driving much of the aggregate over-provision of effort.[3]

The paper proceeds as follows. In Section 2, we present the model. In Section 3, we characterize the symmetric Bayes-Nash equilibrium. In Section 4, we study the effect of increasing competition on effort under different contest environments. In Section 5, we present the experimental design and results. Section 6 concludes.

[1] In early work, Glazer and Hassin (1988) highlight the distinction between the two environments by solving the problem in some special cases. Other related studies include Schweinzer and Segev (2012); Drugov and Ryvkin (2020) who examine the budget allocation problem under different contest success functions. For general surveys of the literature in contest theory, see Corchón (2007); Sisak (2009); Konrad (2009); Vojnović (2015); Fu and Wu (2019); Chowdhury, Esteve-González, and Mukherjee (2023); Beviá and Corchón (2024).

[2] Other related work has studied imperfectly discriminating contests (Ewerhart and Quartieri (2020)), contests with altruistic or envious types (Konrad (2004)), and common value all-pay auctions with private asymmetric information (Einy, Goswami, Haimanko, Orzach, and Sela (2017)). There is also some work in mechanism design and auction design with finite type-spaces (Maskin and Riley (1985); Jeong and Pycia (2023); Vohra (2012); Lovejoy (2006); Doni and Menicucci (2013); Elkind (2007)).

[3] In other related work, Brookins and Ryvkin (2014) compares behavior under complete and incomplete information in Tullock contests.

# 2 Model

## Contest environment

There is a set of $N+1$ risk-neutral agents. Each agent has a privately known type, which is their effort cost function. We assume that there are $K$ possible types, each of which is such that agents incur zero cost from zero effort, higher cost from higher effort, and arbitrarily large costs from arbitrarily large effort. Formally, each agent's type is drawn from a finite type-space

$$\mathcal{C} = \{c_k \in \mathcal{F} : k \in [K]\},$$

where the set $\mathcal{F}$ is defined as

$$\mathcal{F} = \{c : \mathbb{R}_+ \to \mathbb{R}_+ \mid c(0) = 0, c'(x) > 0 \text{ for all } x > 0, \text{ and } \lim_{x\to\infty} c(x) = \infty\}.$$

We further assume that the $K$ possible types can be ordered by efficiency, and without loss of generality, let types associated with higher indices be more efficient than those with lower indices. Formally, the type-space $\mathcal{C}$ is an *ordered type-space*, defined as follows:

**Definition 1** (Ordered type-space)**.** A type-space $\mathcal{C} = \{c_k \in \mathcal{F} : k \in [K]\}$ is an *ordered type-space* if, for all $x > 0$,

$$c_1'(x) > \cdots > c_K'(x).$$

A particularly relevant subclass of ordered type-spaces, commonly studied in the literature on contests (with a continuum of types), consists of type-spaces where the types are simply scaled versions of a single base function.

**Definition 2** (Parametric type-space)**.** A type-space $\mathcal{C} = \{c_k \in \mathcal{F} : k \in [K]\}$ is a *parametric type-space* if there exists a (base) cost function $c \in \mathcal{F}$ and parameters $\theta_1, \ldots, \theta_K \in \mathbb{R}_+$, with $\theta_1 > \cdots > \theta_K$, such that for each $k \in [K]$,

$$c_k(x) = \theta_k c(x) \text{ for all } x \in \mathbb{R}_+.$$

Each agent's type is drawn independently from the type-space $\mathcal{C}$ according to distribution $p = (p_1, \ldots, p_K)$, where $p_k > 0$ for all $k$ and $\sum_{k=1}^K p_k = 1$. For each $k$, we let $P_k = \sum_{j=1}^k p_j$. We refer to the tuple $(N+1, \mathcal{C}, p)$ as the *contest environment* and assume it is common knowledge.[4]

## Contest

A *contest* $v = (v_0, \ldots, v_N)$ assigns a prize value to each rank, with $v_0 \leq \cdots \leq v_N$ and $v_0 < v_N$. Given a contest $v$ and their private types, all $N+1$ agents simultaneously choose

[4] Under a parametric type-space, where $c_k(x) = \theta_k c(x)$, the contest environment is strategically equivalent to an all-pay auction environment in which agents have private (marginal) valuations for prizes, given by $\frac{1}{\theta_k}$, and face a common bidding cost function $c(x)$.

their effort. The agents are ranked according to their efforts, with ties broken uniformly at random, and awarded the corresponding prizes. Specifically, the agent who exerts the highest effort (outperforming all other $N$ agents) is awarded the prize $v_N$, and more generally, the agent who outperforms exactly $m \in \{0, \ldots, N\}$ out of the $N$ other agents is awarded the prize $v_m$. If an agent of type $c_k \in \mathcal{C}$ wins prize $v_m$ after exerting effort $x_k \geq 0$, their payoff is equal to the value of the prize minus the cost of exerting the effort:

$$v_m - c_k(x_k).$$

Given a contest environment $(N+1, \mathcal{C}, p)$, a contest $v$ defines a Bayesian game between the $N+1$ agents. Since the game induced by $v$ is strategically equivalent to the game induced by the contest $w$ where $w_m = v_m - v_0$ for all $m \in \{0, \ldots, N\}$, we assume without loss of generality that $v_0 = 0$. Formally, we will restrict our attention to contests in the set

$$\mathcal{V} = \{v \in \mathbb{R}^{N+1} : v_0 \leq v_1 \leq \cdots \leq v_N \text{ where } 0 = v_0 < v_N\}.$$

## Equilibrium

For any contest environment $(N+1, \mathcal{C}, p)$ and contest $v \in \mathcal{V}$, we will focus on the symmetric Bayes-Nash equilibrium of the induced Bayesian game. This is a strategy profile where all agents use the same (potentially mixed) strategy, mapping types to a distribution over non-negative effort levels, such that if an agent has type $c_k$, choosing any effort level in the support of the distribution for $c_k$ yields an expected payoff at least as high as any other effort level, given that all other agents use the same strategy. We denote this symmetric Bayes-Nash equilibrium by $(X_1, X_2, \ldots, X_K)$, where $X_k \sim F_k$ represents the random level of effort exerted by an agent of type $c_k$. We further denote by $X \sim F$ the ex-ante random level of effort exerted in equilibrium by an arbitrary agent, so that for any $x \in \mathbb{R}$,

$$F(x) = \sum_{k=1}^{K} p_k F_k(x),$$

and the expected effort of an arbitrary agent is

$$\mathbb{E}[X] = \sum_{k=1}^{K} p_k \mathbb{E}[X_k].$$

## Competition

We are interested in examining how increasing competitiveness of a contest influences the expected equilibrium effort. As is standard in the literature (Fang, Noe, and Strack (2020); Goel (2025)), we define a contest as being more competitive than another if the prizes are more unequal, measured using the Lorenz order.

**Definition 3.** A contest $v \in \mathcal{V}$ is *more competitive* than $w \in \mathcal{V}$ if $v$ is more unequal than $w$ in the Lorenz order, i.e.,

$$\sum_{i=0}^{m} v_i \leq \sum_{i=0}^{m} w_i \text{ for all } m \in \{0, 1, \dots, N\},$$

with equality for $m = N$.

Observe that, given a fixed budget $V \in \mathbb{R}_+$, the contest that awards the entire budget to only the best-performing agent, $v = (0, 0, \dots, 0, V)$, is more competitive than any other contest $w \in \mathcal{V}$ that distributes the entire budget. At the other extreme, the contest that distributes the budget equally among all but the worst-performing agent, $v = (0, \frac{V}{N}, \dots, \frac{V}{N})$, is less competitive than any other contest $w \in \mathcal{V}$ that distributes the entire budget.

Importantly, if $v \in \mathcal{V}$ is more competitive than $w \in \mathcal{V}$, $v$ can be obtained from $w$ through a sequence of transfers from lower-ranked prizes to higher-ranked prizes. Given a contest environment $(N + 1, \mathcal{C}, p)$ and contest $v \in \mathcal{V}$, the marginal effect of such a transfer from a lower-ranked prize $m'$ to a higher-ranked prize $m$ $(m > m')$ on expected equilibrium effort is captured by

$$\frac{\partial \mathbb{E}[X]}{\partial v_m} - \frac{\partial \mathbb{E}[X]}{\partial v_{m'}}.$$

Our objective is to understand how this effect depends on the specific pair of prizes, the underlying contest, and the contest environment, with the aim of identifying conditions under which increasing the competitiveness of a contest systematically leads to more—or less—effort from the agents. We will further explore implications of these findings for the classical design problem of allocating a fixed budget across different prizes to maximize expected equilibrium effort.

## Notation

We now introduce some notation used throughout the rest of the paper. We let

$$H_m^N(t) = \binom{N}{m} t^m (1-t)^{N-m}$$

denote the probability that a binomial random variable $Y \sim Bin(N, t)$ takes the value $m$. We also let

$$H_{\leq m}^N(t) = \sum_{i=0}^{m} H_i^N(t) \text{ and } H_{\geq m}^N(t) = \sum_{i=m}^{N} H_i^N(t),$$

denote the probabilities that $Y$ takes a value at most $m$ and at least $m$, respectively.

Given a contest $v \in \mathcal{V}$, if an agent outperforms each of the $N$ other agents independently with probability $t \in [0, 1]$, notice that $H_m^N(t)$ represents the probability that the agent outperforms exactly $m$ out of these $N$ agents, in which case they are awarded the prize $v_m$. We define

$$\pi_v(t) = \sum_{m=0}^{N} v_m H_m^N(t),$$

as the expected value of the prize an agent is awarded under contest $v \in \mathcal{V}$ if it outperforms each of the $N$ other agents independently with probability $t \in [0, 1]$.

# 3 Equilibrium

In this section, we characterize the symmetric Bayes-Nash equilibrium of the Bayesian game. Before presenting the complete characterization, we establish a robust structural property: different agent types mix over contiguous intervals, with more efficient types choosing higher effort than less efficient ones.

**Lemma 1.** *Consider any contest environment $(N+1, \mathcal{C}, p)$ where $\mathcal{C}$ is an ordered type-space. For any contest $v \in \mathcal{V}$, any symmetric Bayes-Nash equilibrium $(X_1, \dots, X_K)$ must be such that there exist boundary points $b_0 < b_1 < \cdots < b_K$, with $b_0 = 0$, so that for each $k \in [K]$, $X_k$ is continuously distributed on $[b_{k-1}, b_k]$.*

*Proof sketch.* We show that a symmetric equilibrium must satisfy the following:

1. The equilibrium must be in mixed strategies, and cannot have any atoms. This is because if an agent of type $c_k$ chose $x_k$ with positive probability, there is a positive probability that all agents are tied at $x_k$, and an agent of type $c_k$ would obtain a strictly higher payoff by choosing $x_k + \epsilon$ than that from choosing $x_k$ for $\epsilon > 0$ and small enough.

2. The support of the effort distribution across types should be essentially disjoint, with at most one effort level in the intersection of support of any two different types. This is because going from one effort level to another, the change in expected prize is the same irrespective of type, but the change in cost depends on the type. It follows that two different agent-types cannot both be indifferent between two different effort levels.

3. The supports of the different types must be connected, i.e, there shouldn't be any gaps. This is because if there is any gap $(d_1, d_2)$ in the support, an agent-type that has $d_2$ in the support would obtain a strictly higher payoff by choosing $d_1$. In doing so, the expected prize awarded to the agent remains the same, while the effort cost is lower.

4. Finally, the effort must be monotonic in types. This is because if the distribution of type $c_k$ contains $x$ and $y$ in its support with $x < y$, then the indifference condition of type $c_k$, together with the ordered structure of $\mathcal{C}$, implies that for any less efficient type $c_j$ with $j < k$, choosing $x$ would lead to a strictly higher payoff than choosing $y$.

Together, these properties imply the result. The full proof is in the appendix. □

In words, for any environment $(N+1, \mathcal{C}, p)$ and contest $v \in \mathcal{V}$, the equilibrium takes the following form: agents of the least-efficient type $c_1$ mix over the interval $[0, b_1]$, agents of type $c_2$ mix over $[b_1, b_2]$, and so forth, up to agents of the most-efficient type $c_K$, who mix over $[b_{K-1}, b_K]$. Consequently, more efficient agents always exert greater effort than less efficient agents, and mixing arises solely because agents may compete against others of the same type.

Now for the complete characterization, it remains to determine the exact distributions, which we derive using the indifference condition. Specifically, if an agent of type $c_k \in \mathcal{C}$ mixes on the interval $[b_{k-1}, b_k]$, they must be indifferent across all effort levels in $[b_{k-1}, b_k]$. This indifference condition uniquely pins down the equilibrium effort distribution $F_k$ on $[b_{k-1}, b_k]$ for each $k \in [K]$. The following result fully characterizes the unique symmetric Bayes-Nash equilibrium of the Bayesian game.[5]

**Theorem 1.** *Consider any contest environment $(N+1, \mathcal{C}, p)$ where $\mathcal{C}$ is an ordered type-space. For any contest $v \in \mathcal{V}$, the symmetric Bayes-Nash equilibrium $(X_1, \ldots, X_K)$ is such that for each $k \in [K]$, the distribution $F_k : [b_{k-1}, b_k] \to [0, 1]$ is defined by*

$$\pi_v(P_{k-1} + p_k F_k(x_k)) - c_k(x_k) = u_k \text{ for all } x_k \in [b_{k-1}, b_k], \tag{1}$$

*where the boundary points $b = (b_0, \ldots, b_K)$, with $b_0 = 0$, and the equilibrium utilities $u = (u_1, \ldots, u_K)$, with $u_1 = 0$, satisfy*

$$\pi_v(P_k) - c_k(b_k) = u_k \text{ for all } k \in [K], \tag{2}$$

*and*

$$\pi_v(P_{k-1}) - c_k(b_{k-1}) = u_k \text{ for all } k \in [K]. \tag{3}$$

*Proof.* Suppose $(X_1, X_2, \ldots, X_K)$ is a symmetric Bayes-Nash equilibrium. From Lemma 1, there exist boundary points $b_0 < b_1 < b_2 < \cdots < b_K$, with $b_0 = 0$, so that $X_k$ is continuously distributed on $[b_{k-1}, b_k]$. It follows that an agent of type $c_k$ must be indifferent between all effort levels in this interval. Suppose $(F_1, F_2, \ldots, F_K)$ is an equilibrium distribution. Notice that if an agent of type $c_k \in \mathcal{C}$ chooses $x_k \in [b_{k-1}, b_k]$, it outperforms any arbitrary agent with probability $P_{k-1} + p_k F_k(x_k)$, and thus, the expected value of the prize that this agent is awarded is $\pi_v(P_{k-1} + p_k F_k(x_k))$. Moreover, the cost of choosing $x_k$ is $c_k(x_k)$. Thus, by the indifference condition, the equilibrium distribution function $F_k$ must satisfy Equation (1).

[5] The equilibrium under our finite-type space framework exhibits both the mixed structure characteristic of complete information environments (Barut and Kovenock (1998)) and the monotonic structure observed in environments with a continuum of types (Moldovanu and Sela (2001)). The complete information environment is clearly a special case of our model. We also establish an equilibrium convergence result for the continuum type-space environment (Theorem 4 in Appendix C), which implies that the (pure-strategy) equilibrium in any continuum type-space can be well-approximated by the equilibrium of a sufficiently large and appropriately chosen finite type-space. Intuitively, as the finite type-space becomes large, the interval over which an agent of a certain type mixes shrinks, and essentially converges to the effort level prescribed by the pure-strategy equilibrium under the continuum type-space.

It remains to solve for the boundary points and equilibrium utilities. Plugging in $x_k = b_k$ in Equation (1) leads to Equation (2), and plugging in $x_k = b_{k-1}$ in Equation (1) leads to Equation (3). Starting from $b_0 = 0$, Equation (3) gives $u_1 = 0$, and then, Equation (2) gives $b_1 = c_1^{-1}(\pi_v(P_1))$. In general, once we have $b_{k-1}$, Equation (3) gives $u_k$, and then, Equation (2) gives $b_k$. Proceeding iteratively in this way, we can recover all the equilibrium boundary points and utilities from Equations (2) and (3). Together, these three equations fully characterize the unique symmetric Bayes-Nash equilibrium of the Bayesian game. $\square$

We now provide our reinterpretation of the equilibrium in Theorem 1, which allows us to circumvent the analytical challenges associated with directly analyzing the equilibrium distributions $F_k$. Specifically, we embed the equilibrium in an alternative space–by associating with each probability $t \in [0, 1]$ of outperforming an arbitrary agent the corresponding equilibrium effort level. For instance, the probability $t = 0$ corresponds to zero effort, $t = 1$ corresponds to maximum effort $b_K$, and $t = P_k$ corresponds to effort $b_k$. In general, from Equation (1), the equilibrium effort associated with probability $t \in (P_{k-1}, P_k)$ is

$$c_k^{-1} \left(\pi_v(t) - u_k\right).$$

This interpretation–assigning to each probability $t \in [0, 1]$ of outperforming an arbitrary agent the corresponding equilibrium effort–provides a unified framework for analyzing symmetric equilibrium across environments, and is central to our subsequent analysis.

To begin, the interpretation allows us to obtain a tractable representation for the expected equilibrium effort. Since the probability $t$ of outperforming an arbitrary agent is ex-ante uniformly distributed on $[0, 1]$, we immediately obtain the following representation.[6]

**Lemma 2.** *Consider any contest environment $(N+1, \mathcal{C}, p)$ where $\mathcal{C}$ is an ordered type-space. For any contest $v \in \mathcal{V}$, the expected equilibrium effort of an arbitrary agent is*

$$\mathbb{E}[X] = \int_0^1 g_{k(t)} \left(\pi_v(t) - u_{k(t)}\right) dt,$$

*where $g_k = c_k^{-1}$ and $k(t) = \max\{k : P_{k-1} \leq t\}$.*

More generally, even for other quantities of interest—such as maximum effort and minimum effort—we can capture the randomness in the effort space by translating it into the $t$-space, and use the above interpretation to obtain tractable representations. For instance, we can show that the expected maximum effort takes the form

$$\mathbb{E}[X_{max}] = (N+1) \int_0^1 g_{k(t)} \left(\pi_v(t) - u_{k(t)}\right) t^N dt,$$

while the expected minimum effort is

$$\mathbb{E}[X_{min}] = (N+1) \int_0^1 g_{k(t)} \left(\pi_v(t) - u_{k(t)}\right) (1-t)^N dt.$$

[6] Formally, the probability $t$ can be interpreted as $F(X)$, where $X \sim F$ represents the equilibrium effort of an arbitrary agent. By the probability integral transform, $F(X)$ is uniformly distributed on $[0, 1]$.

# 4 Effect of competition on expected effort

In this section, we examine how increasing competitiveness of a contest influences the expected equilibrium effort, and also solve the designer's problem of allocating a fixed budget across prizes so as to maximize expected equilibrium effort. The same approach would yield corresponding results for maximum and minimum effort, and we briefly discuss the maximum effort case at the end of the section.

From Lemma 2, it follows that for any environment $(N+1, \mathcal{C}, p)$ and contest $v \in \mathcal{V}$, the marginal effect of increasing prize $m \in [N]$ on expected effort is

$$\frac{\partial \mathbb{E}[X]}{\partial v_m} = \int_0^1 g'_{k(t)}\left(\pi_v(t) - u_{k(t)}\right)\left[H_m^N(t) - \frac{\partial u_{k(t)}}{\partial v_m}\right] dt.$$

And thus, for any pair of prizes $m, m' \in [N]$ with $m > m'$, the marginal effect of increasing competition by transferring value from worse-ranked prize $m'$ to better-ranked prize $m$ is

$$\frac{\partial \mathbb{E}[X]}{\partial v_m} - \frac{\partial \mathbb{E}[X]}{\partial v_{m'}} = \int_0^1 g'_{k(t)}\left(\pi_v(t) - u_{k(t)}\right)\left[H_m^N(t) - H_{m'}^N(t) - \left[\frac{\partial u_{k(t)}}{\partial v_m} - \frac{\partial u_{k(t)}}{\partial v_{m'}}\right]\right] dt. \quad (4)$$

To interpret Equation (4), consider again an agent who outperforms an arbitrary agent with probability $t$. Transferring value from $m'$ to $m$ results in a marginal increase in this agent's expected prize of $H_m^N(t) - H_{m'}^N(t)$. By subtracting the subsequent marginal increase in utility $\left[\frac{\partial u_{k(t)}}{\partial v_m} - \frac{\partial u_{k(t)}}{\partial v_{m'}}\right]$, we isolate the marginal increase in effort costs, which is then translated into the marginal effect on effort. Finally, taking a uniform expectation over $t \in [0,1]$ gives the overall impact of the transformation on expected effort. Equation (4) provides a general and useful framework in which to think about the effect of competition on effort. We will now use this framework to analyze the effect of competition under some important contest environments.

## 4.1 Complete information

We begin with the complete information environment, captured by a type-space containing only a single type. This complete information case was the focus of Fang, Noe, and Strack (2020), who showed that increasing competition encourages effort when the cost function is concave, and discourages effort when it is convex. We now recover this result in our framework, introducing and illustrating some key ideas that will be useful later.

Consider a complete information environment with type-space $\mathcal{C} = \{c_1\}$ where $c_1 \in \mathcal{F}$. From Theorem 1, we know that for any contest $v \in \mathcal{V}$, the equilibrium utility $u_1 = 0$. Consequently, the effect of increasing competition, as captured by Equation (4), simplifies to

$$\frac{\partial \mathbb{E}[X]}{\partial v_m} - \frac{\partial \mathbb{E}[X]}{\partial v_{m'}} = \int_0^1 g'_1\left(\pi_v(t)\right)\left[H_m^N(t) - H_{m'}^N(t)\right] dt.$$

Observe that $\left[H_m^N(t) - H_{m'}^N(t)\right]$, which captures the marginal effect on effort costs, is negative for small $t$-values and positive for large $t$-values. Moreover, the aggregate effect on effort cost is

$$\int_0^1 \left[H_m^N(t) - H_{m'}^N(t)\right] dt = 0.$$

Thus, increasing competition essentially shifts equilibrium effort costs from low $t$-values to high $t$-values. Now for the effect on effort, the term $g_1'(\pi_v(t))$ can be interpreted as assigning different weights to the effect on effort costs across different $t$-values. If these weights are monotonic in $t$ (which they are when $c_1$ is concave or convex), we can recover the effect on effort from the effect on effort costs. We formalize this idea in the following lemma.

**Lemma 3.** *Suppose $a_2 : [0,1] \to \mathbb{R}$ is such that there exists $t^* \in [0,1]$ so that $a_2(t) \leq 0$ for $t \leq t^*$ and $a_2(t) \geq 0$ for $t \geq t^*$. Then, for any increasing function $a_1 : [0,1] \to \mathbb{R}$,*

$$\int_0^1 a_1(t)a_2(t)dt \geq a_1(t^*) \int_0^1 a_2(t)dt.$$

From here, a straightforward application of Lemma 3 with $a_2(t) = \left[H_m^N(t) - H_{m'}^N(t)\right]$ leads to the following result about the effect of increasing competition on expected effort in complete information environments (Fang, Noe, and Strack (2020)).

**Theorem 2.** *Consider any contest environment $(N+1, \mathcal{C}, p)$ where $\mathcal{C} = \{c_1\}$ and $c_1 \in \mathcal{F}$. For any pair $m, m' \in [N]$ with $m > m'$, the following hold:*

1. *If $c_1$ is concave, then for any contest $v \in \mathcal{V}$, $\dfrac{\partial \mathbb{E}[X]}{\partial v_m} - \dfrac{\partial \mathbb{E}[X]}{\partial v_{m'}} \geq 0$.*

2. *If $c_1$ is convex, then for any contest $v \in \mathcal{V}$, $\dfrac{\partial \mathbb{E}[X]}{\partial v_m} - \dfrac{\partial \mathbb{E}[X]}{\partial v_{m'}} \leq 0$.*

Thus, in a complete information environment, the effect of increasing competition on expected effort is determined solely by the structure of the cost function. It encourages effort if the cost is concave, and discourages effort if the cost is convex. If the cost is linear, so that it is both concave and convex, increasing competition has no effect on expected effort (Barut and Kovenock (1998)).

For the design problem of allocating a budget, the solution follows directly from Theorem 2, and we note it in the following corollary.

**Corollary 1.** *Consider any contest environment $(N+1, \mathcal{C}, p)$ where $\mathcal{C} = \{c_1\}$ and $c_1 \in \mathcal{F}$. Suppose any contest $v \in \mathcal{V}$ such that $\sum_{m=0}^N v_m \leq V$ is feasible.*

1. *If $c_1$ is strictly concave, the contest $v = (0, 0, \ldots, 0, V)$ uniquely maximizes $\mathbb{E}[X]$.*

2. *If $c_1$ is linear, any contest $v \in \mathcal{V}$ such that $\sum_{m=1}^N v_m = V$ maximizes $\mathbb{E}[X]$.*

3. *If $c_1$ is strictly convex, the contest $v = \left(0, \frac{V}{N}, \ldots, \frac{V}{N}\right)$ uniquely maximizes $\mathbb{E}[X]$.*

## 4.2 Incomplete information: Linear cost

We now turn to the incomplete information environment. Compared to the complete information case, the analysis here is more nuanced, as increasing competition not only effects the equilibrium effort, but also the equilibrium utilities of the different agent-types. Moreover, these effects on utilities may depend in an intricate way on the specific structure of the type-space $\mathcal{C}$. To begin, we focus on the special case where all cost functions are linear.

Consider a contest environment $(N+1, \mathcal{C}, p)$ where $\mathcal{C}$ is such that $c_k(x) = \theta_k \cdot x$, with $\theta_1 > \cdots > \theta_K > 0$. It turns out that in this case, we can explicitly solve for the expected equilibrium effort. From Lemma 2, we can express the expected effort under contest $v \in \mathcal{V}$ as

$$\mathbb{E}[X] = \sum_{k=1}^{K} p_k \cdot \frac{1}{\theta_k} \cdot \left[ \int_{P_{k-1}}^{P_k} \frac{\pi_v(t)}{p_k} dt - u_k \right],$$

In this representation, observe that $\int_{P_{k-1}}^{P_k} \frac{\pi_v(t)}{p_k} dt$ is simply the expected prize awarded to an agent of type $c_k \in \mathcal{C}$, and is linear in $v_m$ for $m \in [N]$. Further, using Equations (2) and (3), we can solve for the equilibrium utilities and show that:

$$u_k = \theta_k \left[ \sum_{j=1}^{k-1} \pi_v(P_j) \left( \frac{1}{\theta_{j+1}} - \frac{1}{\theta_j} \right) \right] \text{ for } k \in [K], \tag{5}$$

which is also linear in $v_m$ for $m \in [N]$. Substituting these expressions, we derive the following representation for the expected effort.

**Lemma 4.** *Consider any contest environment $(N+1, \mathcal{C}, p)$ where $\mathcal{C}$ is such that $c_k(x) = \theta_k \cdot x$, with $\theta_1 > \cdots > \theta_K > 0$. For any contest $v \in \mathcal{V}$, the expected equilibrium effort is*

$$\mathbb{E}[X] = \sum_{m=1}^{N} \alpha_m v_m,$$

*where*

$$\alpha_m = \frac{1}{N+1} \left[ \frac{1}{\theta_K} - \sum_{k=1}^{K-1} \left[ H_{\geq m}^{N+1}(P_k) + (N-m) H_m^{N+1}(P_k) \right] \left( \frac{1}{\theta_{k+1}} - \frac{1}{\theta_k} \right) \right]. \tag{6}$$

Thus, in the incomplete information environment with linear types, the expected equilibrium effort is linear in the values of the different prizes, with coefficients that depend on the specifics of the environment. It follows then that the effect of increasing competition in this environment, as captured by Equation (4), simplifies to

$$\frac{\partial \mathbb{E}[X]}{\partial v_m} - \frac{\partial \mathbb{E}[X]}{\partial v_{m'}} = \alpha_m - \alpha_{m'},$$

which can now be explicitly evaluated using Equation (6).

In particular, we first note that increasing competition by transferring value to the best-ranked prize always encourages effort. To see why, notice from Equation (6) that for any prize $m' \in \{1, \ldots, N-1\}$,

$$\alpha_N - \alpha_{m'} = \frac{1}{N+1}\left[\sum_{k=1}^{K-1}\left[H_{\geq m'}^{N+1}(P_k) - H_{\geq N}^{N+1}(P_k) + (N-m')H_{m'}^{N+1}(P_k)\right]\left(\frac{1}{\theta_{k+1}} - \frac{1}{\theta_k}\right)\right].$$

With $m' < N$ and $K \geq 2$, it is straightforward to verify that $\alpha_N - \alpha_{m'} > 0$. It follows that for any contest $v \in \mathcal{V}$, transferring value from any lower-ranked prize $m'$ to the top-prize $N$ leads to an increase in expected effort.

Consequently, for the design problem, allocating the entire budget to the best-performing agent is strictly optimal.

**Corollary 2.** *Consider any contest environment $(N+1, \mathcal{C}, p)$ where $\mathcal{C}$ is such that $c_k(x) = \theta_k \cdot x$, with $\theta_1 > \cdots > \theta_K > 0$ and $K \geq 2$. Among all contests $v \in \mathcal{V}$ such that $\sum_{m=0}^{N} v_m \leq V$, the contest $v = (0, 0, \ldots, 0, V)$ uniquely maximizes $\mathbb{E}[X]$.*

This result resolves the conjecture of Sisak (2009), who, based on results in complete information settings, suggested that allocating the budget across multiple prizes might be optimal in environments with finitely many types. Instead, we show that the winner-takes-all contest is strictly optimal under arbitrary finite type-space environments, thereby extending the result of Moldovanu and Sela (2001) for the continuum case. Thus, as soon as there is even a small amount of uncertainty (i.e., incomplete information), the (most competitive) winner-takes-all contest is robustly optimal under linear costs.

Interestingly, however, effort is not necessarily monotonic in the level of competition. Specifically, while transferring value to the best-ranked prize always encourages effort, increasing competition by transferring value to better-ranked intermediate prizes may not. To see this, consider a contest environment with just two types. In this case, with $K = 2$, we can show that for any $m \in \{1, \ldots, N-1\}$, the marginal effect of transferring value from prize $m$ to prize $m+1$ on expected effort is

$$\alpha_{m+1} - \alpha_m \geq 0 \iff P_1 \leq \frac{m+1}{N}.$$

It follows that the effect of the transformation depends on the relative likelihood of efficient and inefficient types. In particular, if the inefficient type is relatively rare ($P_1 < \frac{2}{N}$), increasing competition by transferring value to better-ranked prizes always encourages effort. However, if the inefficient type is highly likely ($P_1 > \frac{N-1}{N}$), increasing competition actually generally discourages effort, except when transferring value to the best-ranked prize. Intuitively, increasing competition encourages effort from the efficient types, while discouraging effort from the inefficient types. Thus, if the population is more likely to be efficient, the overall effect is positive, but if it is more likely to be inefficient, increasing competition by transferring value across intermediate prizes may actually discourage effort, which we refer to as the interior discouragement effect.

## 4.3 Incomplete information: General cost

In this subsection, we continue our analysis of the incomplete information environment, allowing for more general cost functions. Unlike the linear case, where expected effort depends linearly on the value of prizes, the relationship between expected effort and prize values under general costs can be significantly more complex. For tractability, we focus on parametric type-spaces. Our approach builds on the ideas and techniques developed in the complete information setting, and we identify conditions under which the effect of increased competition on equilibrium effort can be inferred from its impact under linear costs.

Consider a contest environment $(N+1, \mathcal{C}, p)$ where $\mathcal{C}$ is a parametric type-space, defined by parameters $\theta_1 > \cdots > \theta_K$ and a (base) cost function $c \in \mathcal{F}$, so that $c_k(x) = \theta_k \cdot c(x)$. In this case, if we let $g = c^{-1}$, notice that we can express $g_k(y) = g\left(\frac{y}{\theta_k}\right)$, so that $g_k'(y) = \frac{1}{\theta_k} g'\left(\frac{y}{\theta_k}\right)$. Thus, for any contest $v \in \mathcal{V}$, the effect of increasing competition on expected effort, as captured by Equation (4), can be expressed as

$$\begin{aligned}\frac{\partial \mathbb{E}[X]}{\partial v_m} - \frac{\partial \mathbb{E}[X]}{\partial v_{m'}} &= \int_0^1 g'\left(\frac{\pi_v(t) - u_{k(t)}}{\theta_{k(t)}}\right)\left[\frac{H_m^N(t) - H_{m'}^N(t)}{\theta_{k(t)}} - \frac{1}{\theta_{k(t)}}\left[\frac{\partial u_{k(t)}}{\partial v_m} - \frac{\partial u_{k(t)}}{\partial v_{m'}}\right]\right] dt \\ &= \int_0^1 g'\left(\frac{\pi_v(t) - u_{k(t)}}{\theta_{k(t)}}\right)\left[\lambda_m(t) - \lambda_{m'}(t)\right] dt, \qquad (7)\end{aligned}$$

where

$$\lambda_m(t) = \left(\frac{H_m^N(t)}{\theta_{k(t)}} - \frac{1}{\theta_{k(t)}}\frac{\partial u_{k(t)}}{\partial v_m}\right).$$

To analyze this, we first discuss how increasing competition affects the equilibrium utilities of the different agent types. Notice that we can reinterpret the Bayesian game as one where agents directly choose their effort cost, $c(x)$, instead of choosing effort $x$. Consequently, the properties of equilibrium effort $x$ under linear costs, derived in Subsection 4.2, actually more generally describe properties of equilibrium effort cost $c(x)$ under cost function $c$.[7] In particular, it follows that the equilibrium utilities are exactly as those described in Equation (5), so that the marginal effect of increasing competition on utility of type $c_k(\cdot) = \theta_k \cdot c(\cdot)$ is

$$\frac{\partial u_k}{\partial v_m} - \frac{\partial u_k}{\partial v_{m'}} = \theta_k \left[\sum_{j=1}^{k-1} H_m^N(P_j) - H_{m'}^N(P_j)\left(\frac{1}{\theta_{j+1}} - \frac{1}{\theta_j}\right)\right].$$

[7]Formally, for any contest $v \in \mathcal{V}$, if $X \sim F$ when $c(x) = x$, then $c(X) \sim F$. In particular, from Lemma 4, we have that $\mathbb{E}[c(X)] = \alpha_m v_m$, where $\alpha_m$ is as described in Equation (6). Moreover, the winner-takes-all contest $v = (0, \ldots, 0, V)$ uniquely maximizes $\mathbb{E}[c(X)]$ among all contests feasible with a budget $V$.

Notice that this effect of competition on equilibrium utilities is independent of the cost function $c$, and also the contest $v$.

We now return to analyzing the effect of competition on equilibrium effort, as described in Equation (7). It follows from above that the cost function $c$ only influences the first term of the integrand, $g'\left(\frac{\pi_v(t)-u_{k(t)}}{\theta_{k(t)}}\right)$, which we will interpret as simply assigning weights across different $t$-values (as in our analysis of the complete information case). The second term, $\lambda_m(t)-\lambda_{m'}(t)$, captures the marginal effect of the transformation on (base) effort costs across different $t$-values. From our analysis of the linear cost case ($c(x) = x$), we know that

$$\int_0^1 \left[\lambda_m(t) - \lambda_{m'}(t)\right] dt = \alpha_m - \alpha_{m'},$$

which we can now interpret more generally as the marginal effect of increasing competition on expected equilibrium (base) effort cost, and compute using Equation (6).

If the function $\lambda_m(t) - \lambda_{m'}(t)$ further exhibits the single-crossing property (as it does under the complete information case), this effect on effort cost may be informative about the effect on effort itself. It is straightforward to verify that $\lambda_m(t) - \lambda_{m'}(t)$ is continuous in $t$, and is almost everywhere differentiable with a derivative that has the same sign as the derivative of $H^N_m(t) - H^N_{m'}(t)$. It follows that, starting from zero at $t = 0$, the function initially decreases, then increases, and eventually decreases again (unless $m = N$). As a result, the condition $\lambda_m(1) - \lambda_{m'}(1) \geq 0$ ensures that $\lambda_m(t) - \lambda_{m'}(t)$ is single-crossing in $t$. In words, while increasing competition reduces the effort cost associated with low $t$-values, this condition ensures that it increases effort costs associated with all higher $t$-values. The condition is clearly satisfied, for instance, when an increase in competition does not increase the equilibrium utility of the most efficient type.

Our main result extends Theorem 2 (Fang, Noe, and Strack (2020)) to the incomplete information setting by showing that, under the above condition (which is always satisfied in the complete information case), the effect of increasing competition on equilibrium effort under general cost functions can be inferred from its effect on effort costs (or, equivalently, from its effect on effort under linear costs).

**Theorem 3.** *Consider any contest environment $(N+1, \mathcal{C}, p)$ where $\mathcal{C}$ is a parametric type-space, defined by parameters $\theta_1 > \cdots > \theta_K$ and (base) function $c \in \mathcal{F}$, so that $c_k(x) = \theta_k \cdot c(x)$. Let $m, m' \in [N]$ with $m > m'$ be such that, either $m = N$ or*

$$\left(\frac{\partial u_K}{\partial v_m} - \frac{\partial u_K}{\partial v_{m'}}\right) \leq 0 \iff \sum_{k=1}^{K-1} \left(H^N_m(P_k) - H^N_{m'}(P_k)\right)\left(\frac{1}{\theta_{k+1}} - \frac{1}{\theta_k}\right) \leq 0.$$

*Then, the following hold:*

1. *If $\alpha_m - \alpha_{m'} \geq 0$ and $c$ is concave, then for any contest $v \in \mathcal{V}$,* $\dfrac{\partial \mathbb{E}[X]}{\partial v_m} - \dfrac{\partial \mathbb{E}[X]}{\partial v_{m'}} \geq 0.$

2. *If $\alpha_m - \alpha_{m'} \leq 0$ and $c$ is convex, then for any contest $v \in \mathcal{V}$,* $\dfrac{\partial \mathbb{E}[X]}{\partial v_m} - \dfrac{\partial \mathbb{E}[X]}{\partial v_{m'}} \leq 0$.

In words, if increasing competition does not lead to an increase in the equilibrium utility of the most-efficient type, or if it involves transferring value to the best-ranked prize, then its effect on effort may be inferred from its effect on effort costs. In such cases, the effect on effort costs (or effort under linear costs) extends to the effect on effort under concave costs if it is positive, and to the effect on effort under convex costs if it is negative. Despite being somewhat limited in its scope, Theorem 3 provides a convenient method to check if increasing competitiveness of a contest would encourage or discourage effort under fairly general environments.

In particular, it allows us to solve the design problem of allocating a budget across prizes for the case of concave costs. To see how, fix any contest environment $(N+1, \mathcal{C}, p)$ where $\mathcal{C}$ is a parametric type-space with a concave cost $c \in \mathcal{F}$. For any contest $v \in \mathcal{V}$, consider the effect of transferring value from an arbitrary prize $m' \in \{1, \ldots, N-1\}$ to the best-ranked prize $m = N$. From Theorem 3, if $\alpha_N - \alpha_{m'} \geq 0$, the transformation will have an encouraging effect on expected equilibrium effort. But we know from our analysis of the linear costs in Subsection 4.2 that $\alpha_N - \alpha_{m'} \geq 0$. It follows that for any concave cost $c \in \mathcal{F}$ and contest $v \in \mathcal{V}$, transferring value to the best-ranked prize encourages expected equilibrium effort. As a result, it is optimal to allocate the entire budget to the top-ranked prize.

**Corollary 3.** *Consider any contest environment $(N+1, \mathcal{C}, p)$ where $\mathcal{C}$ is a parametric type-space, defined by parameters $\theta_1 > \cdots > \theta_K$ and (base) function $c \in \mathcal{F}$, so that $c_k(x) = \theta_k \cdot c(x)$. If $c$ is (weakly) concave, among all contests $v \in \mathcal{V}$ such that $\sum_{m=0}^{N} v_m \leq V$, the contest $v = (0, 0, \ldots, 0, V)$ maximizes $\mathbb{E}[X]$.*

This result extends the optimality of the winner-takes-all contest under concave costs, previously established for a continuum type-space environment by Moldovanu and Sela (2001), to the finite type-space setting. Together with Corollaries 1 and 2, it follows that the winner-takes-all contest is robustly optimal for maximizing expected effort under concave costs, irrespective of whether the environment is a complete or incomplete information environment.

While Theorem 3 focuses on expected effort, it can be readily generalized with implications for other quantities of interest. The key insight is that the concavity or convexity of the cost function $c$ in Theorem 3 ensures only that the weighting term in Equation (7), $g'\left(\frac{\pi_v(t) - u_{k(t)}}{\theta_{k(t)}}\right)$, is monotonic in $t$. As such, we may replace this term with any monotonic weighting function and derive analogous results.

To illustrate, consider expected maximum effort, which is another commonly studied objective in contest design (Wasser and Zhang (2023)). From the representation derived earlier, we can verify that the effect of transferring value from prize $m'$ to $m$ on $\mathbb{E}[X_{max}]$ is

$$\frac{\partial \mathbb{E}[X_{max}]}{\partial v_m} - \frac{\partial \mathbb{E}[X_{max}]}{\partial v_{m'}} = (N+1) \int_0^1 g'\left(\frac{\pi_v(t) - u_{k(t)}}{\theta_{k(t)}}\right) [\lambda_m(t) - \lambda_{m'}(t)]\, t^N dt,$$

which mirrors the structure of Equation (7), now with a different density function. Following the same reasoning as in the proof of Theorem 3, we obtain an analogous result for expected maximum effort, with the original condition on $c$ replaced by the requirement that the new weighting function $g'\left(\frac{\pi_v(t)-u_{k(t)}}{\theta_{k(t)}}\right) t^N$ is monotonic. In particular, when $c$ is concave, this weighting function is strictly increasing. Applying the logic of Corollary 3, we conclude that the winner-takes-all contest is optimal not only for maximizing expected effort, but also for maximizing expected maximum effort under concave costs.

**Corollary 4.** *Consider any contest environment $(N+1, \mathcal{C}, p)$ where $\mathcal{C}$ is a parametric type-space, defined by parameters $\theta_1 > \cdots > \theta_K$ and (base) function $c \in \mathcal{F}$, so that $c_k(x) = \theta_k \cdot c(x)$. If $c$ is (weakly) concave, among all contests $v \in \mathcal{V}$ such that $\sum_{m=0}^{N} v_m \leq V$, the contest $v = (0, 0, \ldots, 0, V)$ maximizes $\mathbb{E}[X_{max}]$.*

# 5 Experiment

In this section, we present findings from an incentivized experiment designed to test the equilibrium predictions of our model regarding the effect of competition on effort. Our primary objective is to test the optimality of the (most competitive) winner-takes-all contest under linear costs, originally established by Moldovanu and Sela (2001) in a continuum type-space setting. Our result for the finite type-space model establishes its robustness under incomplete information and enables experimental investigation. We test this by comparing effort levels in a winner-takes-all structure with those in less competitive alternatives.

Our second goal is to test for the interior discouragement effect of competition, which our model predicts emerges when inefficient types are sufficiently likely. To test this, we consider an environment where this condition holds and gradually increase the competitiveness of the prize structure within an interior range, where the model predicts a decrease in expected equilibrium effort. Below, we present the experimental design, implementation details, and our findings.

## 5.1 Experimental Design

In our experiment, subjects competed in groups of four for monetary prizes. Effort levels were chosen independently and privately, without any possibility of communication. There were two possible types (marginal costs of effort): 1 and 2, assigned with probabilities 20% and 80%, respectively.[8] We implemented the strategy method so that for each contest, sub-

[8] In terms of our notation, the contest environment $(N+1, \mathcal{C}, p)$ is defined by $N+1 = 4$, the type-space $\mathcal{C} = \{c_1(x) = 2x, c_2(x) = x\}$, and type distribution $p = (0.8, 0.2)$.

jects submitted two effort choices (between 0 and 100), one for each possible type. At the end of the experiment, each subject was independently assigned a type according to the type distribution, and the corresponding decisions were used to determine payoffs.

The experimental treatments varied the contest $v = (v_0, v_1, v_2, v_3)$, which awards prizes based on effort rankings: $v_3$ for the highest effort, $v_2$ for the second highest, $v_1$ for the third highest, and $v_0 = 0$ for the lowest. The most competitive treatment, *WTA*, is a winner-takes-all structure with $(0, 0, 0, 100)$. We consider three other treatments with the same total prize but progressively less competitive: *High* with $v = (0, 0, 25, 75)$, *Med* with $v = (0, 0, 50, 50)$, and *Low* with $v = (0, 25, 25, 50)$. Table 1 summarizes the treatments and their equilibrium effort predictions.

The contest order was randomized across subjects. The group composition in each contest was also randomized and the subjects were unable to identify with each other between contests. Feedback on contest outcomes was withheld until the end of the experiment, however on-screen information informed subjects of their potential earnings for each possible prize they could win (i.e. prize minus effort costs) to ensure proper comprehension of the payoffs. Subjects were paid for one randomly selected contest, with tokens converted to U.S. dollars at a rate of 50:1, plus a $2 show up fee. The protocols for subject group matching, feedback, and payment were chosen to minimize reputation-building or repeated-play concerns, thereby inducing the one-shot nature of the game under study.

| Treatment | $(0, v_1, v_2, v_3)$ | Equilibrium Effort ($\mathbb{E}[X]$) | | | Observed Effort | | |
|---|---|---|---|---|---|---|---|
| | | $c_k(x) = x$ | $c_k(x) = 2x$ | Pooled | $c_k(x) = x$ | $c_k(x) = 2x$ | Pooled |
| *WTA* | $(0, 0, 0, 100)$ | 48.2 | 6.4 | 14.76 | 52.8 | 40.5 | 42.96 |
| *High* | $(0, 0, 25, 75)$ | 37.0 | 8 | 13.80 | 46.9 | 37.4 | 39.3 |
| *Med* | $(0, 0, 50, 50)$ | 25.8 | 9.6 | 12.84 | 43.3 | 36.5 | 37.86 |
| *Low* | $(0, 25, 25, 50)$ | 24.6 | 10.2 | 13.08 | 42.9 | 35.9 | 37.3 |

Table 1: Equilibrium and Observed Efforts, by Treatment and Cost Type

We recruited 700 subjects from Prolific, an online labor market, during April 2025. We administered a comprehension quiz to ensure that subjects understood the instructions, the structure of the contests, and how the earnings were determined. Subjects who failed to pass the quiz after a second attempt were not allowed to continue, resulting in a final sample of 445 subjects. The experimental interface and instructions are provided in Appendix E.

## 5.2 Experimental Results

Table 1 summarize our findings on mean effort. The mean effort is highest under the *WTA* (42.96). Also, efforts are higher for the efficient type ($c_k(x) = x$) and lower for the inefficient type ($c_k(x) = 2x$), as expected. Regression analysis, reported in column 1 of Table 4 in Appendix D, confirms the statistical significance of these findings.

To directly test whether behavior aligns with the theory, we use Lemma 4, which represents the expected equilibrium effort as a linear combination of the contest prizes. For the environment considered in the experiment, we have that for any contest $v \in \mathcal{V}$,

$$\mathbb{E}[X] = 0.119 \cdot v_1 + 0.109 \cdot v_2 + 0.148 \cdot v_3 \, .$$

In this representation, observe that the largest coefficient is on the highest-ranked prize ($v_3$), which implies that transferring value from any lower-ranked prize to the highest-ranked prize increases expected effort, thus resulting in the optimality of the WTA prize structure. The interior discouragement effect is reflected in the fact that the coefficient $v_2$ is smaller than that of $v_1$, so that increasing competition by transferring value from lower-ranked prize $v_1$ to higher-ranked prize $v_2$ decreases expected equilibrium effort.

We can also decompose the expected effort by cost type and obtain that

$$\mathbb{E}[X \mid c_k(x) = x] = -0.014 \cdot v_1 + 0.034 \cdot v_2 + 0.482 \cdot v_3$$

and

$$\mathbb{E}[X \mid c_k(x) = 2x] = 0.152 \cdot v_1 + 0.128 \cdot v_2 + 0.064 \cdot v_3 \, .$$

Observe that the highest-ranked prize ($v_3$) has the largest coefficient for the efficient type and the smallest for the inefficient type. This suggests that the overall optimality of the WTA structure is primarily driven by the strong incentive it creates for the efficient type. For the interior prizes, transferring value from $v_1$ to $v_2$ encourages effort from the efficient type, but discourages effort from the inefficient type. The overall interior discouragement effect arises because the inefficient type is significantly more likely.

Table 2 presents results from a linear regression of effort choices on the three prizes.[9] We find that, in contrast to theoretical predictions, the highest-ranked prize carries the largest coefficient not only for the efficient type, but also for the inefficient type. Moreover, both the intermediate prizes appear to be overweighted in observed effort choices relative to theoretical expectations, for both types. Even so, in the aggregate, the highest-ranked prize has the largest coefficient for mean effort, and therefore, the transfer of value from any lower-ranked prize to the highest-ranked prize is empirically correlated with an increase in effort. This finding speaks to the global optimality of winner-take-all contest, in line with the theory.

[9] Since the expected equilibrium effort is zero when all prizes are zero, we estimate the regression without a constant to align directly with the theoretical specification.

For the interior discouragement effect, we find that the difference in the estimated coefficients is negligible (0.006), and statistically insignificant (the Wald test for the coefficients being equal yields a $p$-value of 0.850). Although we do not find a significant negative effect of transferring value from the $v_1$ to $v_2$, we also do not find a positive effect. We interpret this result as partially in line with equilibrium behavior, as it provides evidence that increasing the competitiveness of the prize structure need not lead to higher effort.

| Prize | Equilibrium Weight | | | Estimated Coefficient | | |
|---|---|---|---|---|---|---|
| | $c_k(x) = x$ | $c_k(x) = 2x$ | Pooled | $c_k(x) = x$ | $c_k(x) = 2x$ | Pooled |
| First place prize ($v_3$) | 0.482 | 0.064 | 0.148 | 0.524 | 0.401 | 0.426 |
| Second place prize ($v_2$) | 0.034 | 0.128 | 0.109 | 0.335 | 0.321 | 0.324 |
| Third place prize ($v_1$) | -0.014 | 0.152 | 0.119 | 0.334 | 0.314 | 0.318 |

Table 2: Linear Decomposition of the Expected Effort: Equilibrium and Regression Results

*Note: Lemma 4 establishes that the equilibrium effort can be expressed as a linear function of the contest prize vector. The columns labeled "Equilibrium Weight" report the theoretical coefficients for each prize under the two cost types and in the aggregate. The columns labeled "Estimated Coefficient" present the corresponding regression estimates. Full regression results are provided in Table 3.*

## 5.3 Discussion of Experimental Results

In the preceding subsection, we focused on testing the theoretical predictions. While we find qualitative support for the main comparative statics of the model, our results also reveal evidence of rent dissipation in the form of excessive effort, especially from the inefficient type. This phenomenon of over-provision of effort has been previously identified in contest experiments.

We briefly discuss the non-strategic determinants of behavior by examining how individual characteristics correlate with effort choices, as is common in the analysis of experimental data. For this purpose, we elicited self-reported measures of risk attitudes (willingness to take risks on a 0–10 Likert scale) and competitiveness (willingness to compete on a 0–10 Likert scale) at the end of the experiment to proxy for the intrinsic *joy of winning* that subjects may derive beyond the pecuniary earnings. We also control for gender. Note that these factors have been shown to influence behavior in prior studies on contests (Dechenaux, Kovenock, and Sheremeta (2015)). Regression results, reported in column 2 of Table 4 in Appendix D, show that subjects who declare a higher willingness to take risks (coefficient 1.528, $p < 0.01$) and claim to be more competitive (coefficient 1.063, $p < 0.05$) choose significantly higher efforts. We find no significant gender differences in effort choices.

# 6 Conclusion

We analyze all-pay contests with finite type-spaces and show that the most-competitive winner-takes-all contest maximizes both total and maximum effort under linear or concave costs, thereby resolving a long-standing open question. At the same time, we uncover an interior discouragement effect: when inefficient types are sufficiently likely, gradually increasing competition by reallocating value to higher-ranked prizes may fail to raise effort. We complement the theory with an experiment which reveals significant over-provision of effort, especially among inefficient types, but aggregate patterns broadly align with the model's comparative statics, including the optimality of winner-takes-all and the interior discouragement effect. Our analysis rests on a novel methodology that represents equilibrium effort in terms of the probability of outperforming an arbitrary opponent, enabling tractable analysis of otherwise complex mixed-strategy equilibria.

Our findings open several promising avenues for future research. First, the question of optimal contest design under convex costs remains unresolved. While we do not present formal results, our analysis suggests that winner-takes-all remains optimal as long as costs are not too convex, with the threshold of convexity shrinking as information approaches completeness. Second, exploring non-parametric type-spaces, particularly unordered environments, may yield new insights into contest design. Third, allowing for more general mechanisms beyond rank-order allocation is an important direction, as emphasized in recent work by Letina, Liu, and Netzer (2023) and Zhang (2024). More broadly, the techniques we develop apply to a wider class of contest and auction environments and may prove especially valuable in settings where mixed-strategy equilibria have previously impeded tractable analysis. Finally, finite type-spaces offer a natural framework for experimental work: with explicit equilibrium characterization and convergence results, our analysis provides a strong foundation for testing the many theoretical predictions in the growing literature on contests under incomplete information.

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# A Proofs for Section 3 (Equilibrium)

**Lemma 1.** *Consider any contest environment $(N+1, \mathcal{C}, p)$ where $\mathcal{C}$ is an ordered type-space. For any contest $v \in \mathcal{V}$, any symmetric Bayes-Nash equilibrium $(X_1, \ldots, X_K)$ must be such that there exist boundary points $b_0 < b_1 < \cdots < b_K$, with $b_0 = 0$, so that for each $k \in [K]$, $X_k$ is continuously distributed on $[b_{k-1}, b_k]$.*

*Proof.* Suppose $(X_1, X_2, \ldots, X_K)$ is a symmetric Bayes-Nash equilibrium, and let $X \sim F$ denote the ex-ante effort of an arbitrary agent. Let $u_k$ denote the payoff of agent of type $c_k \in \mathcal{C}$ under this symmetric strategy profile.

1. **Mixed strategies:** We first show that $X_k$ cannot have any atoms. Suppose instead that $\Pr[X_k = x_k] > 0$. We will argue that an agent of type $c_k$ obtains a strictly higher payoff from choosing $x_k + \epsilon$ as compared to $x_k$ for $\epsilon > 0$ and small enough. Notice that under the given profile, there is a positive probability that all $N+1$ agents are tied at effort level $x_k$, in which case the ties are broken uniformly at random. Thus, choosing $x_k + \epsilon$ results in a discontinuous jump in the expected value of the prize awarded to the agent (since $v_0 < v_N$), even though the additional cost $c_k(x_k + \epsilon) - c_k(x_k)$ can be made arbitrarily small with $\epsilon$ small enough. It follows that for agent of type $c_k \in \mathcal{C}$, the payoff from choosing $x_k + \epsilon$ is strictly higher than that from choosing $x_k$, which is a contradiction. Thus, $X_k$ must be a continuous random variable. Consequently, we assume, without loss of generality, that the support of $X_k$ is closed.

2. **Disjoint support (essentially) across types:** We now show that for any $j \neq k$, the support of $X_j$ and $X_k$ have at most one point of intersection. Suppose instead that both $x, y$ are in the support of both $X_j$ and $X_k$ and $x \neq y$. Since an agent of type $c_k$ must be indifferent between all actions in the support of $X_k$, it must be that

$$u_k = \pi_v(F(x)) - c_k(x) = \pi_v(F(y)) - c_k(y),$$

and similarly for agent of type $c_j$, it must be that

$$u_j = \pi_v(F(x)) - c_j(x) = \pi_v(F(y)) - c_j(y).$$

But this implies that

$$\pi_v(F(x)) - \pi_v(F(y)) = c_k(x) - c_k(y) = c_j(x) - c_j(y),$$

which contradicts the fact that $\mathcal{C}$ is ordered.

3. **No gaps in support:** We now show that there cannot be any gaps in the support of $X$, and that it must take the form $[0, b_K]$. Suppose instead that there is an interval $(d_1, d_2)$ which is not in the support of $X$. Then, an agent with a type that has $d_2$ in its support obtains a strictly higher payoff from choosing $d_1$, as this agent is still awarded the same expected prize, but the cost incurred by this agent is lower. It follows that the support of $X$ must be convex. An analogous argument leads to the property that the lower bound of the support must be 0.

4. **Monotonicity across types:** Lastly, we show that there exist boundary points $b_1 < b_2 < \cdots < b_K$ such that the support of $X_k$ is $[b_{k-1}, b_k]$. Suppose that $x, y$ with $x < y$ is in the support of $X_k$. We will show that for an agent of type $c_j$ where $j < k$, choosing $x$ leads to a strictly higher payoff than choosing $y$. Observe that

$$u_k = \pi_v(F(x)) - c_k(x) = \pi_v(F(y)) - c_k(y).$$

Now the payoff of agent of type $c_j$ from choosing $y$ is

$$\pi_v(F(y)) - c_j(y) = u_k + c_k(y) - c_j(y),$$

and that from choosing $x$ will be

$$\pi_v(F(x)) - c_j(x) = u_k + c_k(x) - c_j(x).$$

Since $\mathcal{C}$ is ordered,

$$c_j(y) - c_j(x) > c_k(y) - c_k(x) \implies c_k(x) - c_j(x) > c_k(y) - c_j(y).$$

It follows that the agent of type $c_j$ obtains a strictly higher payoff from choosing $x$ as compared to $y$.

Together, the properties imply that the equilibrium exhibits the structure in the Lemma. $\square$

**Lemma 2.** *Consider any contest environment $(N+1, \mathcal{C}, p)$ where $\mathcal{C}$ is an ordered type-space. For any contest $v \in \mathcal{V}$, the expected equilibrium effort of an arbitrary agent is*

$$\mathbb{E}[X] = \int_0^1 g_{k(t)}\left(\pi_v(t) - u_{k(t)}\right) dt,$$

*where $g_k = c_k^{-1}$ and $k(t) = \max\{k : P_{k-1} \leq t\}$.*

*Proof.* We first find the expected effort exerted in equilibrium by an agent of type $c_k$. From Theorem 1, we have that the (random) level of effort $X_k$ satisfies

$$\pi_v(P_{k-1} + p_k F_k(X_k)) - c_k(X_k) = u_k.$$

Rearranging and taking expectations on both sides, we obtain

$$\begin{aligned}
\mathbb{E}[X_k] &= \mathbb{E}\left[g_k\left(\pi_v(P_{k-1} + p_k F_k(X_k)) - u_k\right)\right] && \text{(Since } g_k = c_k^{-1}\text{)} \\
&= \int_{b_{k-1}}^{b_k} g_k\left(\pi_v(P_{k-1} + p_k F_k(x_k)) - u_k\right) f_k(x_k) dx_k \\
&= \int_0^1 g_k\left(\pi_v(P_{k-1} + p_k t) - u_k\right) dt && \text{(Substituting } F_k(x_k) = t\text{).}
\end{aligned}$$

Then,

$$\begin{aligned}
\mathbb{E}[X] &= \sum_{k=1}^{K} p_k \mathbb{E}[X_k] \\
&= \sum_{k=1}^{K} p_k \int_0^1 g_k\left(\pi_v(P_{k-1} + p_k t) - u_k\right) dt \\
&= \sum_{k=1}^{K} \int_{P_{k-1}}^{P_k} g_k\left(\pi_v(p) - u_k\right) dp && \text{(Substituting } P_{k-1} + p_k t = p\text{)} \\
&= \int_0^1 g_{k(t)}\left(\pi_v(t) - u_{k(t)}\right) dt && \text{(where } k(t) = \max\{k : P_{k-1} \le t\}\text{)}
\end{aligned}$$

as required. □

# B Proofs for Section 4 (Effect of competition on expected effort)

**Lemma 3.** *Suppose $a_2 : [0,1] \to \mathbb{R}$ is such that there exists $t^* \in [0,1]$ so that $a_2(t) \le 0$ for $t \le t^*$ and $a_2(t) \ge 0$ for $t \ge t^*$. Then, for any increasing function $a_1 : [0,1] \to \mathbb{R}$,*

$$\int_0^1 a_1(t) a_2(t) dt \ge a_1(t^*) \int_0^1 a_2(t) dt.$$

*Proof.* Observe that

$$\begin{aligned}
\int_0^1 a_1(t) a_2(t) dt &= \int_0^{t^*} a_1(t) a_2(t) dt + \int_{t^*}^1 a_1(t) a_2(t) dt \\
&\ge \int_0^{t^*} a_1(t^*) a_2(t) dt + \int_{t^*}^1 a_1(t^*) a_2(t) dt \\
&= a_1(t^*) \int_0^1 a_2(t) dt.
\end{aligned}$$

□

**Theorem 2.** *Consider any contest environment $(N+1, \mathcal{C}, p)$ where $\mathcal{C} = \{c_1\}$ and $c_1 \in \mathcal{F}$. For any pair $m, m' \in [N]$ with $m > m'$, the following hold:*

1. *If $c_1$ is concave, then for any contest $v \in \mathcal{V}$, $\dfrac{\partial \mathbb{E}[X]}{\partial v_m} - \dfrac{\partial \mathbb{E}[X]}{\partial v_{m'}} \ge 0$.*
2. *If $c_1$ is convex, then for any contest $v \in \mathcal{V}$, $\dfrac{\partial \mathbb{E}[X]}{\partial v_m} - \dfrac{\partial \mathbb{E}[X]}{\partial v_{m'}} \le 0$.*

*Proof.* From Theorem 1, we know that $u_1 = 0$, and thus, from Equation (4), we have that

$$\frac{\partial \mathbb{E}[X]}{\partial v_m} - \frac{\partial \mathbb{E}[X]}{\partial v_{m'}} = \int_0^1 g_1'(\pi_v(t)) \left[H_m^N(t) - H_{m'}^N(t)\right] dt.$$

If $c_1$ is concave, $g_1 = c_1^{-1}$ is convex, and thus, $g_1'(\pi_v(t))$ is increasing in $t$. Applying Lemma 3 with $a_1(t) = g_1'(\pi_v(t))$ and $a_2(t) = \left[H_m^N(t) - H_{m'}^N(t)\right]$ gives the result.

If $c_1$ is convex, $g_1 = c_1^{-1}$ is concave, and thus, $g_1'(\pi_v(t))$ is decreasing in $t$. Applying Lemma 3 with $a_1(t) = -g_1'(\pi_v(t))$ and $a_2(t) = \left[H_m^N(t) - H_{m'}^N(t)\right]$ gives the result.

□

**Lemma 4.** *Consider any contest environment $(N+1, \mathcal{C}, p)$ where $\mathcal{C}$ is such that $c_k(x) = \theta_k \cdot x$, with $\theta_1 > \cdots > \theta_K > 0$. For any contest $v \in \mathcal{V}$, the expected equilibrium effort is*

$$\mathbb{E}[X] = \sum_{m=1}^{N} \alpha_m v_m,$$

*where*

$$\alpha_m = \frac{1}{N+1}\left[\frac{1}{\theta_K} - \sum_{k=1}^{K-1}\left[H_{\geq m}^{N+1}(P_k) + (N-m)H_m^{N+1}(P_k)\right]\left(\frac{1}{\theta_{k+1}} - \frac{1}{\theta_k}\right)\right]. \tag{6}$$

*Proof.* Using the representation in Lemma 2, we have that for any contest $v \in \mathcal{V}$,

$$\begin{aligned}
\mathbb{E}[X] &= \int_0^1 g_{k(t)}\left(\pi_v(t) - u_{k(t)}\right) dt \\
&= \int_0^1 \frac{\left(\pi_v(t) - u_{k(t)}\right)}{\theta_{k(t)}} dt && \left(g_k(y) = \frac{y}{\theta_k}\right) \\
&= \sum_{k=1}^{K} p_k \cdot \frac{1}{\theta_k} \cdot \left[\int_{P_{k-1}}^{P_k} \frac{\pi_v(t)}{p_k} dt - u_k\right].
\end{aligned}$$

1. Notice that for any $k \in [K]$, $\displaystyle\int_{P_{k-1}}^{P_k} \frac{\pi_v(t)}{p_k} dt$ is the expected prize awarded to an agent of type $c_k$. To compute this, we instead compute the ex-ante expected total prize awarded to agents of type $c_k$. Notice that for any prize $m \in \{0, \ldots, N\}$, the ex-ante probability that this prize is awarded to an agent of type $c_k$ is simply

$$\left[H_{\geq m+1}^{N+1}(P_k) - H_{\geq m+1}^{N+1}(P_{k-1})\right].$$

Thus, the ex-ante expected total prize awarded to agents of type $c_k$ is

$$\sum_{m=1}^{N} v_m \left[H_{\geq m+1}^{N+1}(P_k) - H_{\geq m+1}^{N+1}(P_{k-1})\right].$$

By an alternative calculation, which entails adding up over the $N+1$ agents, this expectation should equal

$$(N+1)\cdot p_k \cdot \int_{P_{k-1}}^{P_k} \frac{\pi_v(t)}{p_k} dt.$$

Equating these two, we get that

$$\int_{P_{k-1}}^{P_k} \pi_v(t)dt = \frac{\sum_{m=1}^{N} v_m \left[H_{\geq m+1}^{N+1}(P_k) - H_{\geq m+1}^{N+1}(P_{k-1})\right]}{N+1}.$$

Alternatively, we can also directly use the following fact to compute this integral:

$$\frac{\partial H_{\geq m+1}^{N+1}(t)}{\partial t} = (N+1)H_m^N(t)$$

2. For the equilibrium utilities $u_k$, we simply solve the Equations (2) and (3). For the given type-space $\mathcal{C}$ with $c_k(x) = \theta_k \cdot x$, these equations can be rewritten as

$$\pi_v(P_k) - \theta_k b_k = u_k \text{ and } \pi_v(P_{k-1}) - \theta_k b_{k-1} = u_k.$$

Solving this system of equations gives

$$b_k = \sum_{j=1}^{k} \frac{\pi_v(P_j) - \pi_v(P_{j-1})}{\theta_j} \text{ for } k \in [K],$$

and

$$u_k = \theta_k \left[\sum_{j=1}^{k-1} \pi_v(P_j)\left(\frac{1}{\theta_{j+1}} - \frac{1}{\theta_j}\right)\right] \text{ for } k \in [K].$$

Substituting these expressions in the above representation, we get that

$$\mathbb{E}[X] = \sum_{k=1}^{K} \frac{1}{(N+1)\theta_k} \sum_{m=1}^{N} v_m \left[H_{\geq m+1}^{N+1}(P_k) - H_{\geq m+1}^{N+1}(P_{k-1})\right] - \sum_{k=1}^{K} \frac{p_k u_k}{\theta_k}.$$

From here, it follows that we can write

$$\mathbb{E}[X] = \sum_{m=1}^{N} \alpha_m v_m$$

where

$$\alpha_m = \sum_{k=1}^{K} \frac{\left[H_{\geq m+1}^{N+1}(P_k) - H_{\geq m+1}^{N+1}(P_{k-1})\right]}{(N+1)\theta_k} - \sum_{k=1}^{K} p_k \sum_{j=1}^{k-1} H_m^N(P_j)\left(\frac{1}{\theta_{j+1}} - \frac{1}{\theta_j}\right)$$

$$= \sum_{k=1}^{K} \frac{\left[H_{\geq m+1}^{N+1}(P_k) - H_{\geq m+1}^{N+1}(P_{k-1})\right]}{(N+1)\theta_k} - \sum_{k=1}^{K-1}(1-P_k)H_m^N(P_k)\left(\frac{1}{\theta_{k+1}} - \frac{1}{\theta_k}\right)$$

$$= \frac{1}{N+1}\left[\frac{1}{\theta_K} - \sum_{k=1}^{K-1} H_{\geq m+1}^{N+1}(P_k)\left(\frac{1}{\theta_{k+1}} - \frac{1}{\theta_k}\right)\right] - \frac{(N+1-m)}{N+1}\sum_{k=1}^{K-1}\left[H_m^{N+1}(P_k)\left(\frac{1}{\theta_{k+1}} - \frac{1}{\theta_k}\right)\right]$$

$$= \frac{1}{N+1}\left[\frac{1}{\theta_K} - \sum_{k=1}^{K-1}\left[H_{\geq m}^{N+1}(P_k) + (N-m)H_m^{N+1}(P_k)\right]\left(\frac{1}{\theta_{k+1}} - \frac{1}{\theta_k}\right)\right].$$

□

**Theorem 3.** *Consider any contest environment $(N+1, \mathcal{C}, p)$ where $\mathcal{C}$ is a parametric type-space, defined by parameters $\theta_1 > \cdots > \theta_K$ and (base) function $c \in \mathcal{F}$, so that $c_k(x) = \theta_k \cdot c(x)$. Let $m, m' \in [N]$ with $m > m'$ be such that, either $m = N$ or*

$$\left(\frac{\partial u_K}{\partial v_m} - \frac{\partial u_K}{\partial v_{m'}}\right) \leq 0 \iff \sum_{k=1}^{K-1}\left(H_m^N(P_k) - H_{m'}^N(P_k)\right)\left(\frac{1}{\theta_{k+1}} - \frac{1}{\theta_k}\right) \leq 0.$$

*Then, the following hold:*

1. *If $\alpha_m - \alpha_{m'} \geq 0$ and $c$ is concave, then for any contest $v \in \mathcal{V}$, $\dfrac{\partial \mathbb{E}[X]}{\partial v_m} - \dfrac{\partial \mathbb{E}[X]}{\partial v_{m'}} \geq 0$.*
2. *If $\alpha_m - \alpha_{m'} \leq 0$ and $c$ is convex, then for any contest $v \in \mathcal{V}$, $\dfrac{\partial \mathbb{E}[X]}{\partial v_m} - \dfrac{\partial \mathbb{E}[X]}{\partial v_{m'}} \leq 0$.*

*Proof.* For the given parametric type-space, we have from Equation (4) that for any contest $v \in \mathcal{V}$ and any pair of prizes $m, m' \in [N]$ with $m > m'$,

$$\frac{\partial \mathbb{E}[X]}{\partial v_m} - \frac{\partial \mathbb{E}[X]}{\partial v_{m'}} = \int_0^1 g'\left(\frac{\pi_v(t) - u_{k(t)}}{\theta_{k(t)}}\right)\left(\lambda_m(t) - \lambda_{m'}(t)\right) dt,$$

where

$$\lambda_m(t) = \left(\frac{H_m^N(t)}{\theta_{k(t)}} - \frac{1}{\theta_{k(t)}}\frac{\partial u_{k(t)}}{\partial v_m}\right).$$

Further, we know from Theorem 1 that the equilibrium boundary points $b = (b_1, \ldots, b_K)$ and utilities $u = (u_1, \ldots, u_K)$ must satisfy Equations (2) and (3). Solving these equations, we get that the equilibrium utilities are as described in Equation (5), and thus, we get that

$$\frac{\partial u_k}{\partial v_m} = \theta_k\left[\sum_{j=1}^{k-1} H_m^N(P_j)\left(\frac{1}{\theta_{j+1}} - \frac{1}{\theta_j}\right)\right].$$

Plugging in, we get that

$$\lambda_m(t) - \lambda_{m'}(t) = \left(\frac{H_m^N(t) - H_{m'}^N(t)}{\theta_{k(t)}}\right) - \left[\sum_{j=1}^{k(t)-1}\left(H_m^N(P_j) - H_{m'}^N(P_j)\right)\left(\frac{1}{\theta_{j+1}} - \frac{1}{\theta_j}\right)\right].$$

From here, one can verify that

1. $\lambda_m(0) - \lambda_{m'}(0) = 0$

2. $\lambda_m(1) - \lambda_{m'}(1) = \begin{cases} \frac{1}{\theta_K} - \frac{1}{\theta_K}\left(\frac{\partial u_K}{\partial v_m} - \frac{\partial u_K}{\partial v_{m'}}\right) & \text{if } m = N \\ -\frac{1}{\theta_K}\left(\frac{\partial u_K}{\partial v_m} - \frac{\partial u_K}{\partial v_{m'}}\right) & \text{otherwise} \end{cases}$

3. $\lambda_m(t) - \lambda_{m'}(t)$ is continuous in $t$

4. $\lambda_m(t) - \lambda_{m'}(t)$ is differentiable at $t \in [0,1]$ for $t \neq P_k$, and at any such $t$, the derivative has the same sign as the derivative of $H_m^N(t) - H_{m'}^N(t)$ with respect to $t$.

Since $m, m'$ are such that either $m = N$ or $\left(\frac{\partial u_K}{\partial v_m} - \frac{\partial u_K}{\partial v_{m'}}\right) \leq 0$, we get that $\lambda_m(1) - \lambda_{m'}(1) \geq 0$. Together with the above properties, this implies that there is some $t^* \in [0,1]$ such that $\lambda_m(t) - \lambda_{m'}(t) \leq 0$ for $t \in [0, t^*]$, and $\lambda_m(t) - \lambda_{m'}(t) \geq 0$ for $t \in [t^*, 1]$.

Now if $c$ is concave, $g = c^{-1}$ is convex, and thus, $g'\left(\frac{\pi_v(t) - u_{k(t)}}{\theta_{k(t)}}\right)$ is increasing in $t$. Applying Lemma 3 with $a_1(t) = g'\left(\frac{\pi_v(t) - u_{k(t)}}{\theta_{k(t)}}\right)$ and $a_2(t) = \lambda_m(t) - \lambda_{m'}(t)$ gives

$$\begin{aligned} \frac{\partial \mathbb{E}[X]}{\partial v_m} - \frac{\partial \mathbb{E}[X]}{\partial v_{m'}} &\geq g'\left(\frac{\pi_v(t^*) - u_{k(t^*)}}{\theta_{k(t^*)}}\right) \int_0^1 (\lambda_m(t) - \lambda_{m'}(t))dt \\ &= g'\left(\frac{\pi_v(t^*) - u_{k(t^*)}}{\theta_{k(t^*)}}\right)(\alpha_m - \alpha_{m'}) \end{aligned}$$

and the result follows. An analogous argument applies for the case where $c$ is convex.

□

# C Convergence to continuum type-space equilibrium

In this section, we establish an equilibrium convergence result for the continuum type-space. Specifically, we show that if a sequence of (parametric) finite type-space distributions converges to a differentiable distribution over a continuum type-space, then the corresponding sequence of mixed-strategy equilibria converges to the pure-strategy equilibrium in the continuum model. Intuitively, as the finite type-space becomes large, the interval over which a given type mixes shrinks, and essentially converges to the effort level prescribed by the pure-strategy equilibrium under the continuum type-space. Thus, the equilibrium in an appropriate and sufficiently large finite-type space provides a reasonable approximation to the equilibrium strategy under the continuum type-space, and vice versa.

We begin by recalling the symmetric equilibrium under a (parametric) continuum type-space (Moldovanu and Sela (2001)). For this section, we focus on the linear cost case ($c(x) = x$), which is without loss of generality due to the equivalence between convergence in effort cost and in effort.

**Lemma 5.** *Suppose there are $N+1$ agents, each with a private type (marginal cost of effort) drawn from $\Theta = [\underline{\theta}, \overline{\theta}]$ according to a differentiable CDF $G : [\underline{\theta}, \overline{\theta}] \to [0, 1]$. For any contest $v \in \mathcal{V}$, there is a unique symmetric Bayes-Nash equilibrium and it is such that for any $\theta \in \Theta$,*

$$X(\theta) = \int_{\theta}^{\overline{\theta}} \frac{\pi_v'(1 - G(t))g(t)}{t} dt.$$

*Proof.* Suppose $N$ agents are playing a strategy $X : [\underline{\theta}, \overline{\theta}] \to \mathbb{R}_+$ so that if an agent's type is $\theta$, it exerts effort $X(\theta)$. Further suppose that $X(\theta)$ is decreasing in $\theta$. Now we want to find the remaining agent's best response to this strategy of the other agents. If the agent's type is $\theta$ and it pretends to be an agent of type $t \in [\underline{\theta}, \overline{\theta}]$, its payoff is

$$\pi_v(1 - G(t)) - \theta X(t).$$

Taking the first order condition, we get

$$\pi_v'(1 - G(t))(-g(t)) - \theta X'(t) = 0.$$

Now we can plug in $t = \theta$ to get the condition for $X(\theta)$ to be a symmetric Bayes-Nash equilibrium. Doing so, we get

$$\pi_v'(1 - G(\theta))(-g(\theta)) - \theta X'(\theta) = 0$$

so that

$$X(\theta) = \int_{\theta}^{\overline{\theta}} \frac{\pi_v'(1 - G(t))g(t)}{t} dt.$$

□

We now state and prove the convergence result.

**Theorem 4.** *Suppose there are $N+1$ agents and fix any contest $v \in \mathcal{V}$. Let $G : [\underline{\theta}, \overline{\theta}] \to [0, 1]$ be a differentiable CDF and let $G^1, G^2, \ldots,$ be any sequence of CDF's, each with a finite support, such that for all $\theta \in [\underline{\theta}, \overline{\theta}]$,*

$$\lim_{n \to \infty} G^n(\theta) = G(\theta).$$

*Let $F^n : \mathbb{R} \to [0, 1]$ denote CDF of the equilibrium effort under $G^n$, and let $F : \mathbb{R} \to [0, 1]$ denote CDF of the equilibrium effort under $G$. Then, the sequence of CDF's $F^1, F^2, \ldots,$ converges to the CDF $F$, i.e., for all $x \in \mathbb{R}$,*

$$\lim_{n \to \infty} F^n(x) = F(x).$$

*Proof.* For the finite support CDF $G^n$, let $\Theta^n = (\theta_1^n, \theta_2^n, \ldots, \theta_{K(n)}^n)$ denote the support and $p^n = (p_1^n, p_2^n, \ldots, p_{K(n)}^n)$ denote the probability mass function. From Theorem 1, let

$b^n = (b_1^n, b_2^n, \ldots, b_{K(n)}^n)$ denote the boundary points, $u^n = (u_1^n, u_2^n, \ldots, u_{K(n)}^n)$ denote the equilibrium utilities, and $F_k^n$ denote the equilibrium CDF of agent of type $\theta_k^n$ on support $[b_{k-1}^n, b_k^n]$. Then, the CDF of the equilibrium effort, $F^n : \mathbb{R} \to [0,1]$, is such that for any $x \in \mathbb{R}$,

$$F^n(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ P_{k-1}^n + p_k^n F_k^n(x) & \text{if } x \in [b_{k-1}^n, b_k^n] \\ 1 & \text{if } x \geq b_{K(n)}^n \end{cases}. \tag{8}$$

For the continuum CDF $G : [\underline{\theta}, \overline{\theta}] \to [0,1]$, the CDF of the equilibrium effort, $F : \mathbb{R} \to [0,1]$, is such that for any $x \in \mathbb{R}$,

$$F(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ 1 - G(\theta(x)) & \text{if } x \in [0, B] \\ 1 & \text{if } x \geq B \end{cases}. \tag{9}$$

where $\theta(x)$ is the inverse of $X(\theta)$ (from Lemma 5) and $B = X(\underline{\theta})$. The following Lemma will be the key to showing that $F^n(x)$ converges to $F(x)$ for all $x \in \mathbb{R}$.

**Lemma 6.** *Consider any $\theta \in (\underline{\theta}, \overline{\theta})$ and for any $n \in \mathbb{N}$, let $k(n) \in \{0, 1, 2, \ldots, K(n)\}$ be such that $\theta_{k(n)}^n > \theta \geq \theta_{k(n)+1}^n$ (where $\theta_0^n = \infty$ and $\theta_{K(n)+1}^n = 0$). Then,*

$$\lim_{n\to\infty} b_{k(n)}^n = X(\theta) \text{ and } \lim_{n\to\infty} F^n(b_{k(n)}^n) = 1 - G(\theta).$$

*Proof.* From Lemma 5 and Theorem 1, we have

$$X(\theta) = \int_\theta^{\overline{\theta}} \frac{\pi_v'(1 - G(t))g(t)}{t} dt \text{ and } b_{k(n)}^n = \sum_{j=1}^{k(n)} \frac{\pi_v(P_j^n) - \pi_v(P_{j-1}^n)}{\theta_j^n}.$$

Observe that

$$\begin{aligned}
b_{k(n)}^n &= \left[ \frac{\pi_v(P_{k(n)}^n)}{\theta_{k(n)}^n} - \sum_{j=1}^{k(n)-1} \pi_v(P_j^n) \left[ \frac{1}{\theta_{j+1}^n} - \frac{1}{\theta_j^n} \right] \right] \\
&= \int_0^{1/\theta_{k(n)}^n} \left[ \pi_v(P_{k(n)}^n) - \pi_v(1 - G^n(1/x)) \right] dx \\
&\xrightarrow{n\to\infty} \int_0^{\frac{1}{\theta}} \left[ \pi_v(1 - G(\theta)) - \pi_v(1 - G(1/x)) \right] dx \quad \text{(dominated convergence)} \\
&= \underbrace{\left[ x(\pi_v(1 - G(\theta)) - \pi_v(1 - G(1/x))) \right]_0^{\frac{1}{\theta}}}_{\text{this is 0}} + \int_0^{\frac{1}{\theta}} \frac{\pi_v'(1 - G(1/x))g(1/x)}{x} dx \\
&= \int_\theta^\infty \frac{\pi_v'(1 - G(t))g(t)}{t} \, \mathrm{d}t \quad \text{(substitute } t = 1/x\text{)} \\
&= X(\theta)
\end{aligned}$$

By definition, we have

$$\begin{aligned}\lim_{n\to\infty} F^n(b^n_{k(n)}) &= \lim_{n\to\infty} P^n_{k(n)}\\ &= \lim_{n\to\infty}[1 - G^n(\theta)]\\ &= 1 - G(\theta)\end{aligned}$$

□

Returning to the proof of Theorem 4, fix any $x \in (0, B)$ and let $\theta \in (\underline{\theta}, \overline{\theta})$ be such that $X(\theta) = x$. Then, we know that

$$F(x) = 1 - G(\theta).$$

We want to show that

$$\lim_{n\to\infty} F^n(x) = 1 - G(\theta).$$

Take $\epsilon > 0$. Find $\theta' < \theta$ and $\theta'' > \theta$ such that

$$0 < G(\theta) - G(\theta') = G(\theta'') - G(\theta) < \frac{\epsilon}{4}.$$

Let $x' = X(\theta')$, $x'' = X(\theta'')$, so that $x' > x > x''$. Let $\delta = \min\{x' - x, x - x''\}$. From Lemma 6, let $N(\epsilon)$ be such that for all $n > N(\epsilon)$,

$$\max\{|b^n_{k(n)} - x|, |b^n_{k'(n)} - x'|, |b^n_{k''(n)} - x''|\} < \frac{\delta}{2}$$

and

$$\max\{|F^n(b^n_{k'(n)}) - (1 - G(\theta'))|, |F^n(b^n_{k''(n)}) - (1 - G(\theta''))|\} < \frac{\epsilon}{4},$$

where $k(n), k'(n), k''(n)$ are sequences as defined in Lemma 6 for $\theta$, $\theta'$ and $\theta''$ respectively. Then, for all $n > N(\epsilon)$,

$$\begin{aligned}F^n(x) &> F^n(b^n_{k''(n)})\\ &> 1 - G(\theta'') - \frac{\epsilon}{4}\\ &> 1 - G(\theta) - \frac{\epsilon}{2}\end{aligned}$$

and

$$\begin{aligned}F^n(x) &< F^n(b^n_{k'(n)})\\ &< 1 - G(\theta') + \frac{\epsilon}{4}\\ &< 1 - G(\theta) + \frac{\epsilon}{2}\end{aligned}$$

so that $|F^n(x) - (1 - G(\theta))| < \epsilon$. Thus, $\lim_{n\to\infty} F^n(x) = 1 - G(\theta) = F(x)$ for all $x \in \mathbb{R}$. □

# D Regression tables

Table 3: OLS for Effort Choice as Function of Prizes

| | $c_k(x) = x$ | $c_k(x) = 2x$ | Pooled |
|---|---|---|---|
| First place prize ($v_3$) | 0.524*** | 0.401*** | 0.426*** |
| | (0.012) | (0.010) | (0.009) |
| Second place prize ($v_2$) | 0.335*** | 0.321*** | 0.324*** |
| | (0.015) | (0.016) | (0.015) |
| Third place prize ($v_1$) | 0.334*** | 0.314*** | 0.318*** |
| | (0.026) | (0.030) | (0.026) |
| $N$ | 1780 | 1780 | 1780 |
| $R^2$ | 0.806 | 0.734 | 0.786 |

Standard errors in parentheses clustered at the subject level.

Constant omitted from estimation. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 4: OLS for Effort

| | Model 1 | Model 2 |
|---|---|---|
| Constant | 43.866*** | 26.087*** |
| | (1.004) | (3.252) |
| Treatment *Med* | 0.472 | 0.472 |
| | (0.664) | (0.665) |
| Treatment *High* | 2.715*** | 2.715*** |
| | (0.687) | (0.687) |
| Treatment *WTA* | 7.203*** | 7.203*** |
| | (0.735) | (0.736) |
| Inefficient type ($c_k(x) = 2x$) | -8.900*** | -8.900*** |
| | (0.939) | (0.939) |
| Male (1 =yes) | | -2.285 |
| | | (1.542) |
| Willingness to take Risks (0-10) | | 1.528*** |
| | | (0.499) |
| Willingness to Compete (0-10) | | 1.063** |
| | | (0.503) |
| $N$ | 3560 | 3560 |
| $R^2$ | 0.051 | 0.106 |

Standard errors in parentheses clustered at the subject level.

Treatment *Low* is the omitted category. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

# E Experimental Instructions

**Welcome to the experiment**
This experiment is conducted by a research team at New York University Abu Dhabi. Our objective is to investigate the bidding behavior of participants in competitive situations under conditions of uncertainty regarding the characteristics of their opponents.

**Voluntary Participation**
Your participation in this experiment is entirely voluntary. If you choose to withdraw, you will forfeit any payment. Please review the information below carefully before deciding whether to participate.

**About Your Participation**

1. In this experiment, you will take part in various contests, bidding against other participants for monetary rewards. Your performance relative to others will determine your earnings.
2. Your personal information will remain confidential. We may share anonymized data from this study publicly, ensuring that no personal identifiers are included.
3. The experiment is expected to last approximately 15 minutes.
4. You can expect to earn around 5 US dollars. This includes a participation fee of 2 US dollars and a bonus component ranging from 0 to 6 US dollars, depending on your bids and the bids of your opponents.
5. The bonus portion of your payment will be distributed via Prolific within one week after the experiment concludes.
6. To be eligible for any payment, you must pass a comprehension quiz. If you do not pass, you will not receive any payment.
7. There are no known risks associated with this experiment.

If you consent to participate, please click **'I agree'** below.

I agree

I do not agree

→

Restart Survey | Place Bookmark | Tools | Share Preview

What is your Prolific ID?
*Please note that this response should auto-fill with the correct ID.*

→

Powered by Qualtrics

**About the experiment**

In this experiment, you and other participants will compete in a contest to win monetary prizes by submitting costly bids.

Each participant's **final payout** will be:

**Endowment - Bidding cost + Prize won.**

The endowment, bidding costs, and prizes will be denominated in tokens. You will be paid a bonus of **1 US dollar for every 50 tokens** in your final payout. So, if your final payout is 225 tokens, your bonus payment will be 4.5 US dollars.

We will now describe each of the three components that make up the final payout.

**1) Endowment**
Each participant's endowment is 200 tokens.

**2) Bidding cost**
Each participant will be assigned a **cost-per-bid**. This will be either 1 token with a 20% chance, or 2 tokens with an 80% chance.

Participants will not be informed about the cost-per-bid assigned to other participants.

If a participant is assigned a cost-per-bid of 1 token and bids 45, the bidding cost will be 45 tokens.

If a participant is assigned a cost-per-bid of 2 tokens and bids 45, the bidding cost will be 90 tokens.

**3) Prize won**
Each participant’s **prize won** (in tokens) will be determined by their bid and the bid(s) of their opponent(s) in the contest. The exact rules will be explained later.

Participants will now be required to pass a comprehension quiz to continue with the experiment.

→

**Comprehension Quiz: Attempt #1**

There are four questions in this quiz.
You must answer all questions correctly to pass.
You have two attempts to pass the quiz.
If you pass, you will proceed with the experiment.
If you do not pass in two attempts, your participation will be discontinued and no compensation will be provided.

Please answer the following questions carefully.

Suppose you are participating in the following contest:

**No. of participants:** 2 (including you)
**1st prize:** 100 tokens
**2nd prize:** 0 tokens

**Rules:** Participants will be ranked based on their bids and awarded the corresponding prizes. Ties will be broken uniformly at random.

**Reminder:**

- You pay a bidding cost equal to *cost-per-bid* × *your bid* tokens, regardless of the prize you win.
- Each participant's cost-per-bid will be either 1 token with a 20% chance, or 2 tokens with an 80% chance.

Suppose your cost-per-bid is 2 tokens.
What is the probability that your opponent has a cost-per-bid of 2 tokens?

0%

20%

80%

100%

Suppose your cost-per-bid is 2 tokens, and you submit a bid of 30.
What will be your bidding cost?

15 tokens

30 tokens

60 tokens

None of the above

Suppose your cost-per-bid is 2 tokens, and you bid 30. Further suppose your opponent bids 45. Recall that the 1st prize is 100 tokens, and the 2nd prize is 0 tokens.
What will be your prize won?

0 tokens

50 tokens

100 tokens

Depends on my opponent's cost-per-bid

Recall that your endowment is 200 tokens and that your final payout is:

endowment - bidding cost + prize won

What will be your final payout (in tokens) in the situation described above?

70 tokens

140 tokens

240 tokens

270 tokens

Please check your answers carefully.
You can also go back to read the instructions again.
If all your answers are correct, you will proceed further in the experiment.

← →

**Attempt #1: Pass**

You have answered all questions correctly and passed the comprehension quiz. Please click “Next” to proceed with the experiment.

→

**Four Contests**

Participants will now be asked to submit bids for four different contests. In each contest, you will be competing against 3 other opponents. The contests will differ in the values of the prizes.

Participants will be required to submit bids for each of the four contests under both possible cost-per-bid assignments: 1 token or 2 tokens.

After the experiment, each participant’s final payout will be determined as follows:

1. One of the four contests will be randomly selected.
2. The participant's opponents for that contest will be randomly chosen.
3. The participant and their opponents will each be independently assigned a cost-per-bid, which will be either 1 token with a 20% chance, or 2 tokens with an 80% chance.
4. Given the selected contest (Step 1), the assigned opponents (Step 2), and the cost-per-bid assignments (Step 3), we will use participants’ bids to determine the final payout.

Participants will now see the details of the four contests and will be required to submit their bids for each, under both possible cost-per-bid assignments.

→

**Contest #1**

Here are the details of this contest:

**No. of participants:** 4 (including you)
**1st prize:** 50 tokens
**2nd prize:** 50 tokens
**3rd prize:** 0 tokens
**4th prize:** 0 tokens

**Rules:** Participants will be ranked in order of their bids and awarded the corresponding prizes. Ties will be broken uniformly at random.

**Reminder:**

- You pay your bidding cost, calculated as *cost-per-bid* × *your bid*, regardless of the prize you receive.
- Each participant's cost-per-bid will be 1 token with a 20% chance, or 2 tokens with an 80% chance.

Please choose your bid for this contest using the slider below.

0 10 20 30 40 50 60 70 80 90 100

Your bid (if your cost-per-bid=1 token)

Your bid (if your cost-per-bid=2 tokens)

→

**Bid Check**

If contest #1 is selected and you are assigned a cost-per-bid of 1 tokens, your final payout will be:

$$200 - 1 \times 59 + \text{Your prize}$$
$$= 141 + \text{Your prize}$$

Your final payout will depend on your opponents' bids, as shown below:

| Your rank | Your prize | Your final payout |
|---|---|---|
| 1 | 50 tokens | 191 tokens |
| 2 | 50 tokens | 191 tokens |
| 3 | 0 tokens | 141 tokens |
| 4 | 0 tokens | 141 tokens |

---

If contest #1 is selected and you are assigned a cost-per-bid of 2 tokens, your final payout will be:

$$200 - 2 \times 35.5 + \text{Your prize}$$
$$= 129 + \text{Your prize}$$

Your final payout will depend on your opponents' bids, as shown below:

| Your rank | Your prize | Your final payout |
|---|---|---|
| 1 | 50 tokens | 179 tokens |
| 2 | 50 tokens | 179 tokens |
| 3 | 0 tokens | 129 tokens |
| 4 | 0 tokens | 129 tokens |

If you wish to revise your bids, please click the **Previous** button and choose your bids again.
Otherwise, please click the **Next** button to continue.

← →

**Contest #2**

Here are the details of this contest:

**No. of participants:** 4 (including you)
**1st prize:** 75 tokens
**2nd prize:** 25 tokens
**3rd prize:** 0 tokens
**4th prize:** 0 tokens

**Rules:** Participants will be ranked in order of their bids and awarded the corresponding prizes. Ties will be broken uniformly at random.

**Reminder:**

- You pay your bidding cost, calculated as *cost-per-bid* × *your bid*, regardless of the prize you receive.
- Each participant's cost-per-bid will be 1 token with a 20% chance, or 2 tokens with an 80% chance.

Please choose your bid for this contest using the slider below.

0 10 20 30 40 50 60 70 80 90 100

Your bid (if your cost-per-bid=1 token)

Your bid (if your cost-per-bid=2 tokens)

→

**Bid Check**

If contest #2 is selected and you are assigned a cost-per-bid of 1 tokens, your final payout will be:

$$200 - 1 \times 50.8 + \text{Your prize}$$
$$= 149.2 + \text{Your prize}$$

Your final payout will depend on your opponents' bids, as shown below:

| Your rank | Your prize | Your final payout |
|---|---|---|
| 1 | 75 tokens | 224.2 tokens |
| 2 | 25 tokens | 174.2 tokens |
| 3 | 0 tokens | 149.2 tokens |
| 4 | 0 tokens | 149.2 tokens |

---

If contest #2 is selected and you are assigned a cost-per-bid of 2 tokens, your final payout will be:

$$200 - 2 \times 30.4 + \text{Your prize}$$
$$= 139.2 + \text{Your prize}$$

Your final payout will depend on your opponents' bids, as shown below:

| Your rank | Your prize | Your final payout |
|---|---|---|
| 1 | 75 tokens | 214.2 tokens |
| 2 | 25 tokens | 164.2 tokens |
| 3 | 0 tokens | 139.2 tokens |
| 4 | 0 tokens | 139.2 tokens |

If you wish to revise your bids, please click the **Previous** button and choose your bids again.
Otherwise, please click the **Next** button to continue.

← →

**Contest #3**

Here are the details of this contest:

**No. of participants:** 4 (including you)
**1st prize:** 50 tokens
**2nd prize:** 25 tokens
**3rd prize:** 25 tokens
**4th prize:** 0 tokens

**Rules:** Participants will be ranked in order of their bids and awarded the corresponding prizes. Ties will be broken uniformly at random.

**Reminder:**

- You pay your bidding cost, calculated as *cost-per-bid* × *your bid*, regardless of the prize you receive.
- Each participant's cost-per-bid will be 1 token with a 20% chance, or 2 tokens with an 80% chance.

Please choose your bid for this contest using the slider below.

0 10 20 30 40 50 60 70 80 90 100

Your bid (if your cost-per-bid=1 token)

Your bid (if your cost-per-bid=2 tokens)

→

**Bid Check**

If contest #3 is selected and you are assigned a cost-per-bid of 1 tokens, your final payout will be:

$$200 - 1 \times 37.7 + \text{Your prize}$$
$$= 162.3 + \text{Your prize}$$

Your final payout will depend on your opponents' bids, as shown below:

| Your rank | Your prize | Your final payout |
|---|---|---|
| 1 | 50 tokens | 212.3 tokens |
| 2 | 25 tokens | 187.3 tokens |
| 3 | 25 tokens | 187.3 tokens |
| 4 | 0 tokens | 162.3 tokens |

---

If contest #3 is selected and you are assigned a cost-per-bid of 2 tokens, your final payout will be:

$$200 - 2 \times 25.7 + \text{Your prize}$$
$$= 148.6 + \text{Your prize}$$

Your final payout will depend on your opponents' bids, as shown below:

| Your rank | Your prize | Your final payout |
|---|---|---|
| 1 | 50 tokens | 198.6 tokens |
| 2 | 25 tokens | 173.6 tokens |
| 3 | 25 tokens | 173.6 tokens |
| 4 | 0 tokens | 148.6 tokens |

If you wish to revise your bids, please click the **Previous** button and choose your bids again.
Otherwise, please click the **Next** button to continue.

← →

**Contest #4**

Here are the details of this contest:

**No. of participants:** 4 (including you)
**1st prize:** 100 tokens
**2nd prize:** 0 tokens
**3rd prize:** 0 tokens
**4th prize:** 0 tokens

**Rules:** Participants will be ranked in order of their bids and awarded the corresponding prizes. Ties will be broken uniformly at random.

**Reminder:**

- You pay your bidding cost, calculated as *cost-per-bid* × *your bid*, regardless of the prize you receive.
- Each participant's cost-per-bid will be 1 token with a 20% chance, or 2 tokens with an 80% chance.

Please choose your bid for this contest using the slider below.

0 10 20 30 40 50 60 70 80 90 100

Your bid (if your cost-per-bid=1 token)

Your bid (if your cost-per-bid=2 tokens)

→

**Bid Check**

If contest #4 is selected and you are assigned a cost-per-bid of 1 tokens, your final payout will be:

$$200 - 1 \times 51.7 + \text{Your prize}$$
$$= 148.3 + \text{Your prize}$$

Your final payout will depend on your opponents' bids, as shown below:

| Your rank | Your prize | Your final payout |
|---|---|---|
| 1 | 100 tokens | 248.3 tokens |
| 2 | 0 tokens | 148.3 tokens |
| 3 | 0 tokens | 148.3 tokens |
| 4 | 0 tokens | 148.3 tokens |

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If contest #4 is selected and you are assigned a cost-per-bid of 2 tokens, your final payout will be:

$$200 - 2 \times 36.7 + \text{Your prize}$$
$$= 126.6 + \text{Your prize}$$

Your final payout will depend on your opponents' bids, as shown below:

| Your rank | Your prize | Your final payout |
|---|---|---|
| 1 | 100 tokens | 226.6 tokens |
| 2 | 0 tokens | 126.6 tokens |
| 3 | 0 tokens | 126.6 tokens |
| 4 | 0 tokens | 126.6 tokens |

If you wish to revise your bids, please click the **Previous** button and choose your bids again.
Otherwise, please click the **Next** button to continue.

← →

**Final Questions**

You have completed the main experiment. In this final part, we will ask you some additional questions. Your answers will not affect your payment.

What is your gender?

Male

Female

Other

Do not wish to disclose

How do you see yourself? Are you generally someone who is fully prepared to take risks, or do you try to avoid taking risks? Please tick a box on the scale below, where 0 means "not at all willing to take risks" and 10 means "fully prepared to take risks."

Not at all willing to take risks — Fully prepared to take risks

0 1 2 3 4 5 6 7 8 9 10

How do you see yourself? Are you generally a person who is competitive, or do you try to avoid competitive environments? Please tick a box on the scale below, where 0 means "not at all competitive" and 10 means "extremely competitive."

Not at all competitve — Extremely competitive

0 1 2 3 4 5 6 7 8 9 10

A pillow and a blanket cost $110 in total. The blanket costs $100 more than the pillow. How much does the pillow cost?

$5

$10

$100

$105

If it takes 5 machines 5 minutes to make 5 pens, how long would it take 100 machines to make 100 pens?

5 minutes

100 minutes

500 minutes

None of the above

In a lake, there is a patch of lily pads. Every day, the patch doubles in size. If it takes 50 days for the patch to cover the entire lake, how long would it take for the patch to cover half of the lake?

1 day

25 days

45 days

49 days

When choosing how much to bid in the different contests, can you explain your reasoning process?

→