Contest design with a finite type-space: A unifying apprach

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- Goal: understand how prize structure affects incentives
- Vast literature: complete or incomplete information (continuum type-space)
- This paper: contests with a finite type-space

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- Sisak [2009]: "The case of asymmetric individuals, where **types are private information but drawn from discrete, identical** or maybe even different distributions, has not been addressed so far. From the results discussed above, especially on asymmetric types with full information, one could **conjecture that multiple prizes might be optimal even with linear costs**."

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- **Finite types**: Xiao [2018], Liu and Chen [2016], Szech [2011], Konrad [2004], Chen [2021]

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- Design problem: given a budget, find $v = (v_1, \dots, v_N)$ to maximize

$$\mathbb{E}[X] = \sum_{k=1}^{K} p_k \mathbb{E}[X(\theta_k)].$$

Equilibrium structure

Lemma 1.

For any contest $v = (v_1, \ldots, v_{N-1}, 0)$, there is a unique symmetric Bayes-Nash equilibrium and it is such that there exist boundary points $b_1 < b_2 < \cdots < b_K$ so that for any $\theta_k \in \Theta$, an agent of type θ_k mixes between $[b_{k-1}, b_k]$ with $b_0 = 0$.

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- More efficient agents (those with lower θ) exert higher effort
- Exhibits both the mixed structure from complete information, and monotonic structure from continuum type-space environments

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6. if $b_k = a_j$, then $\theta_k \geq \theta_j$

Together, the properties imply the structure in the equilibrium.

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- By the indifference condition, for all $x_k \in [b_{k-1}, b_k]$,

$$\pi_v(P_{k-1} + p_k F_k(x_k)) - \theta_k c(x_k) = u_k,$$

where $\pi_v(t) = \sum_{m=1}^{N} v_m H_{N-m}^{N-1}(t)$, and $H_{N-m}^{N-1}(t)$ is the probability $Y \sim Bin(N-1, t)$ takes the value $N - m$.

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- Solve for u_k and b_k by using $u_1 = 0$, $F_k(b_k) = 1$ and $F_{k+1}(b_k) = 0$.

Equilibrium

Theorem 2.

For any v, the unique symmetric Bayes-Nash equilibrium is such that for any $\theta_k \in \Theta$, the distribution function $F_k : [b_{k-1}, b_k] \to [0,1]$ is defined by

$$\pi_v(P_{k-1} + p_k F_k(x_k)) - \theta_k c(x_k) = u_k \text{ for all } x_k \in [b_{k-1}, b_k],$$

where the points $b = (b_1, \ldots, b_K)$ and utilities $u = (u_1, \ldots, u_K)$ are

$$c(b_k) = \sum_{j=1}^{k} \frac{\pi_v(P_j) - \pi_v(P_{j-1})}{\theta_j} \text{ for any } k \in \{1, 2, \ldots, K\},$$

and

$$u_k = \theta_k \left[\sum_{j=1}^{k-1} \pi_v(P_j) \left(\frac{1}{\theta_{j+1}} - \frac{1}{\theta_j} \right) \right] \text{ for any } k \in \{1, 2, \ldots, K\}.$$

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  which does not depend on the cost function c, or even the contest v.
- For expected effort, we can write

$$\pi_v(P_{k-1} + p_k F_k(X_k)) - \theta_k c(X_k) = u_k.$$

  This gives $\mathbb{E}[X_k]$, and since $\mathbb{E}[X] = \sum_{k=1}^{K} p_k \mathbb{E}[X_k]$, we get

$$\mathbb{E}[X] = \sum_{k=1}^{K} \int_{P_{k-1}}^{P_k} g\left(\frac{\pi_v(t) - u_k}{\theta_k} \right) dt.$$

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Suppose $c(x) = x$. *For any contest* $v = (v_1, \ldots, v_{N-1}, 0)$, *the expected equilibrium effort of an arbitrary agent is* $\mathbb{E}[X] = \sum_{m=1}^{N-1} \alpha_m v_m$ *where*

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- Effect of competition: compute $\alpha_b - \alpha_w$ (maybe < 0 when $b \neq 1$)

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1. *If* $\alpha_b - \alpha_w \geq 0$ *and* c *is concave, then for any* v*,* $\frac{\partial \mathbb{E}[X]}{\partial v_b} - \frac{\partial \mathbb{E}[X]}{\partial v_w} > 0$.

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- For some (N, Θ, p), the effect of increasing competition under general costs can be informed by those under linear costs.
- If $K = 1$, increasing competition encourages effort if costs are concave, and discourages effort if costs are convex.
- For $K > 1$, since $\alpha_1 - \alpha_w > 0$, the winner-takes-all contest is strictly optimal under concave costs.

Convergence to continuum

Theorem 5.

Suppose there are N agents and consider a fixed contest $v = (v_1, \ldots, v_{N-1}, 0)$. Let $G : [\underline{\theta}, \overline{\theta}] \to [0, 1]$ be a differentiable CDF and let $G^1, G^2, \ldots,$ be any sequence of CDF's, each with a finite support, such that for all $\theta \in [\underline{\theta}, \overline{\theta}]$,

$$\lim_{n\to\infty} G^n(\theta) = G(\theta).$$

Let $F^n : \mathbb{R} \to [0, 1]$ denote CDF of the equilibrium effort under the finite type-space distribution G^n, and let $F : \mathbb{R} \to [0, 1]$ denote CDF of the equilibrium under continuum type-space distribution G. Then, the sequence of CDF's $F^1, F^2, \ldots,$ converges to the CDF F, i.e., for all $x \in \mathbb{R}$,

$$\lim_{n\to\infty} F^n(x) = F(x).$$

- Intuitively, as K increases, $[b_{k-1}, b_k]$ shrinks, and converges to the pure-strategy equilibrium effort under the continuum type-space.

Summary

- Study effect of increasing competition in contests with a finite type-space
- Provide a unifying approach to studying contests simultaneously in complete and incomplete information environments
- Identify effects under linear costs, and find conditions under which they extend to general costs, which pertain to how competition affects the equilibrium utility of the most efficient agent
- Generate insights into what drives some of the differences in the complete and incomplete information environments
- Solve the design problem under linear and concave costs, showing that the winner-takes-all contest is optimal with any limited uncertainty

Thank you!