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This Slide Deck

- ▶ Permanent Magnet DC Motors
 - ▶ Most small / hobby DC motors
 - ▶ Quadrotors, Mobile Robots, Robot Arms
- ▶ Block Diagram
- ▶ Servo Motor Control

Permanent Magnet Direct Current Motor

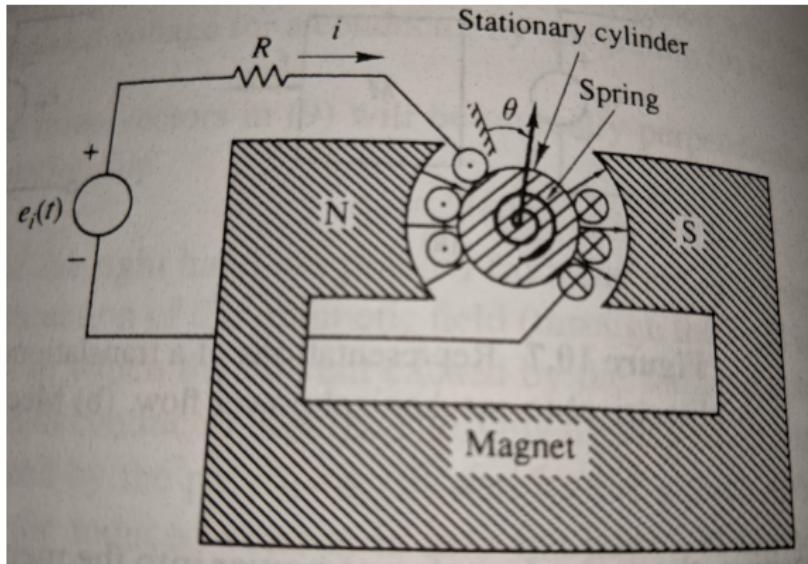


Figure: A PMDC (called a Galvanometer in Text) in Figure 10.8

Permanent Magnet Direct Current Motor

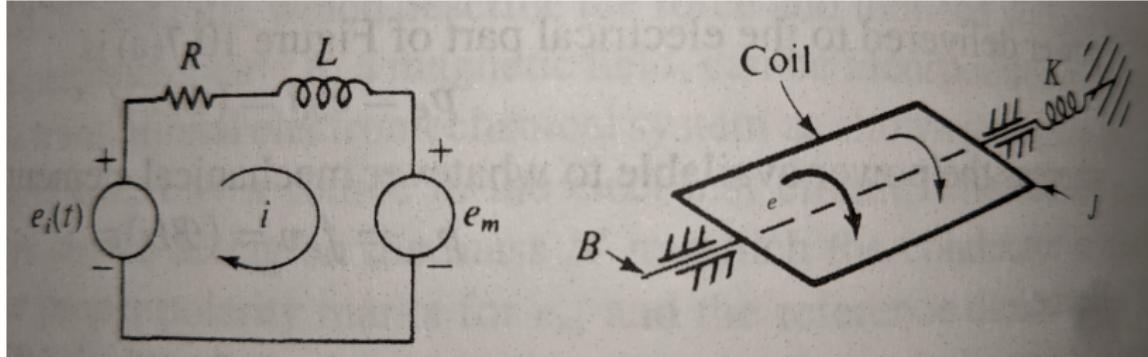


Figure: A lumped eletrical/mechanical model of a PMDC Motor

$$L \frac{di}{dt} + Ri + e_{\text{motor}}(t) = e_i(t) \quad (1)$$

$$J\dot{\omega} + B\omega = \tau_{\text{electric}} + \tau_L(t), \text{ where} \quad (2)$$

$$\omega = \frac{d\theta}{dt}, \quad e_{\text{motor}} = \alpha\omega, \quad \tau_{\text{electric}} = \alpha i(t), \text{ and}$$

Also, instead of a spring torque $K\theta$ on the galvanometer, we use a generic load torque $\tau_L(t)$

PMDC Model

$$L \frac{di}{dt} + Ri + \alpha\omega = e_i(t) \quad (3)$$

$$J\ddot{\omega} + B\dot{\omega} = \alpha i + \tau_L(t) \quad (4)$$

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We can obtain the transfer functions $G_1(s)$ and $G_2(s)$ from the inputs applied voltage $e_i(t)$ and load torque $\tau_L(t)$ to the output ω (rotation speed):

$$\begin{aligned}\hat{\omega}(s) &= G_1(s)\hat{e}_i(s) + G_2(s)\hat{\tau}_L(s) \\ &= \frac{\alpha}{JLs^2 + (JR + BL)s + RB + \alpha^2} \hat{e}_i(s) \\ &\quad - \frac{Ls + R}{JLs^2 + (JR + BL)s + RB + \alpha^2} \hat{\tau}_L(s)\end{aligned} \quad (5)$$

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Note: 2nd order model. Asymptotically stable for most physical values.

Analysis

$$\hat{\omega}(s) = \textcolor{red}{G_1(s)}\hat{e}_i(s) + \textcolor{blue}{G_2(s)}\hat{\tau}_L(s) \quad (6)$$

If we apply a constant load torque T_L and voltage E_i (step inputs), what would happen?

Analysis

$$\hat{\omega}(s) = \textcolor{red}{G_1(s)}\hat{e}_i(s) + \textcolor{blue}{G_2(s)}\hat{\tau}_L(s) \quad (6)$$

If we apply a constant load torque T_L and voltage E_i (step inputs), what would happen? FVT!

$$\hat{\omega}_{\text{step}}(\infty) = \lim_{s \rightarrow 0} \left(s \left(\textcolor{red}{G_1(s)} \frac{E_i}{s} + \textcolor{blue}{G_2(s)} \frac{T_L}{s} \right) \right) \quad (7)$$

$$= \textcolor{red}{G_1(0)} E_i + \textcolor{blue}{G_2(0)} T_L \quad (8)$$

$$= \frac{\alpha E_i - R T_L}{R B + \alpha^2} \quad (9)$$

$$= c_1 - c_2 T_L \quad (\text{when } e_i(t), \tau_L(t) \text{ are constant}) \quad (10)$$

Steady-State Torque-Speed Curve

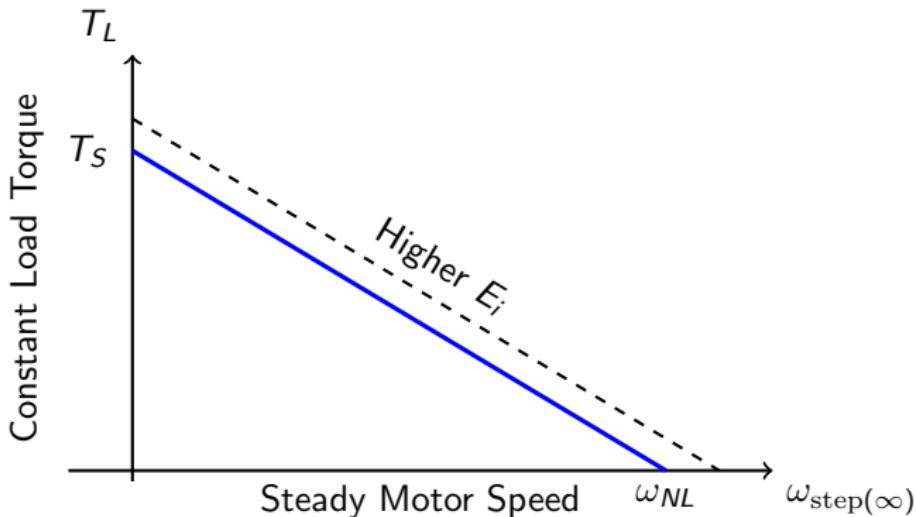


Figure: T_S : Stall Torque. ω_{NL} : No-Load Speed

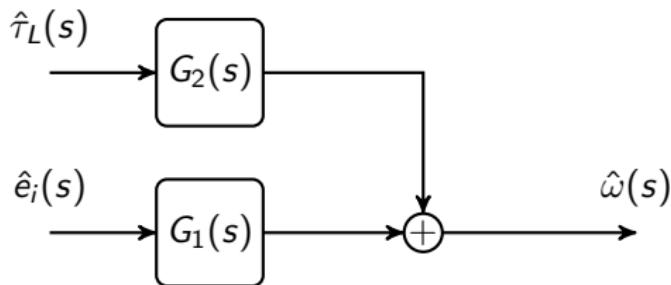
$$T_S = \frac{\alpha E_i}{R}, \quad \omega_{NL} = \frac{\alpha E_i}{RB + \alpha^2}$$

Recap

- ▶ Derived transfer functions (TF) from inputs τ_L and e_i to output $\omega = \dot{\theta}$
- ▶ Used TFs to predict steady-state speed under constant voltage and load torque.
- ▶ Used this prediction to draw a steady-state torque-speed curve.
- ▶ Looked at stall torque and no-load speed.

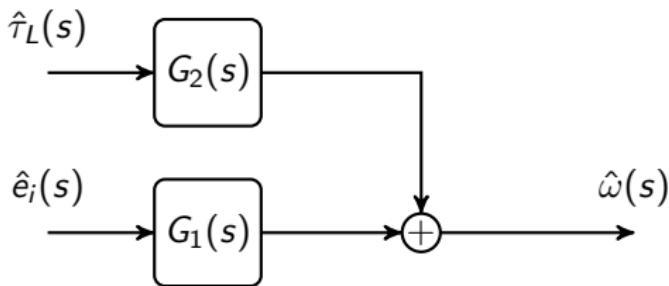
Speed Regulation

We can represent the derived transfer functions using a block diagram.



Speed Regulation

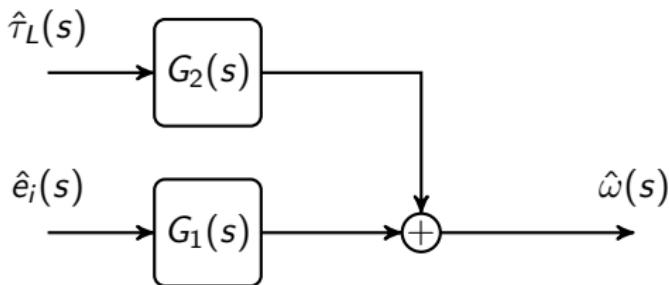
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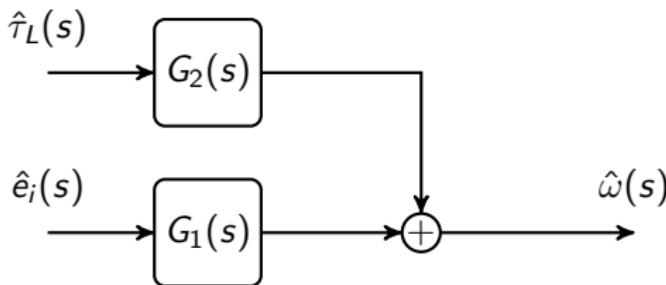


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Challenge: How do we ensure that $y(t) = \omega(t) \rightarrow \omega^*$, where ω^* is a desired steady speed?

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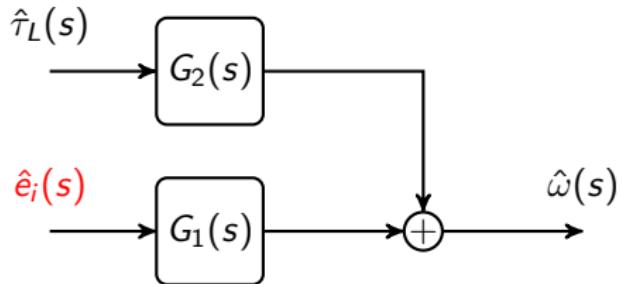


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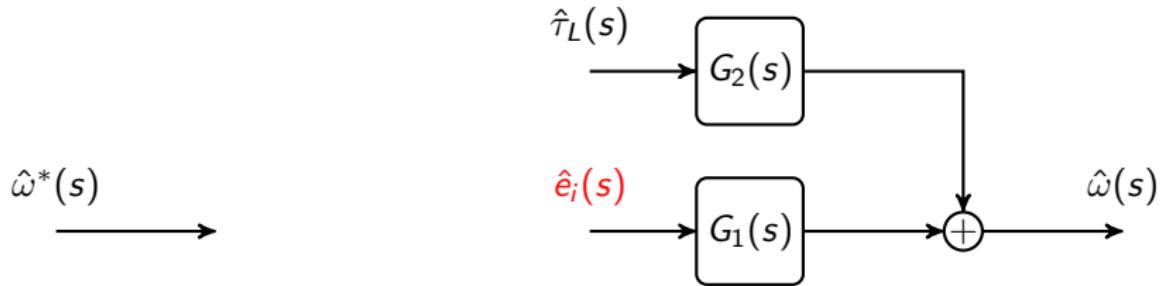
Applications: Electrical power tools, CNC Machines, quadrotor/drones, radio-controlled planes/cars

Speed Regulation



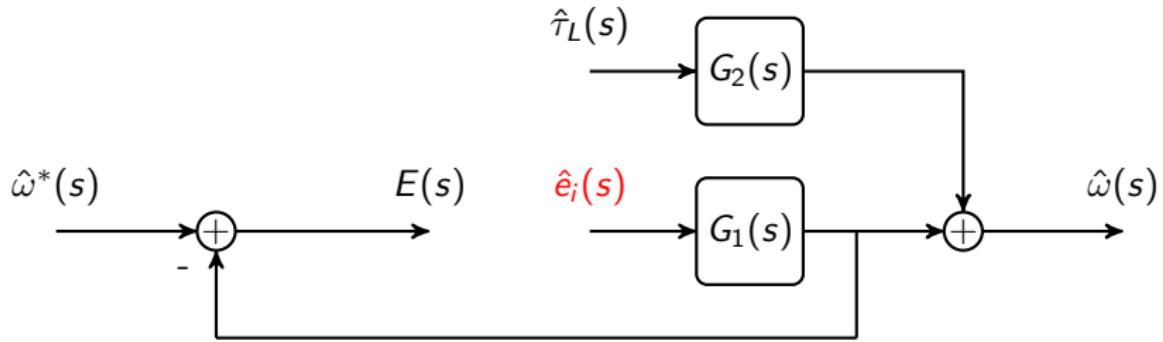
In hand-held power drills, we visually observe $\omega(t)$ and press a button to increase or decrease $\hat{e}_i(t)$

Speed Regulation



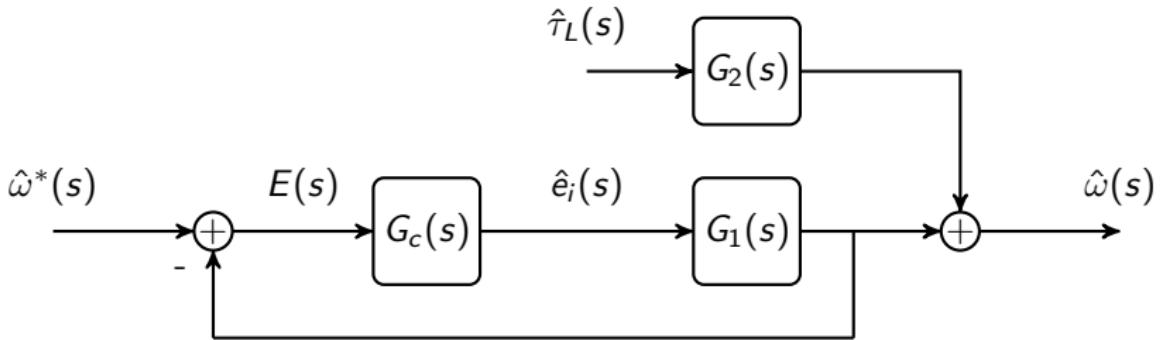
Feedback control is the automated version of this approach to regulating $\omega(t)$.

Speed Regulation



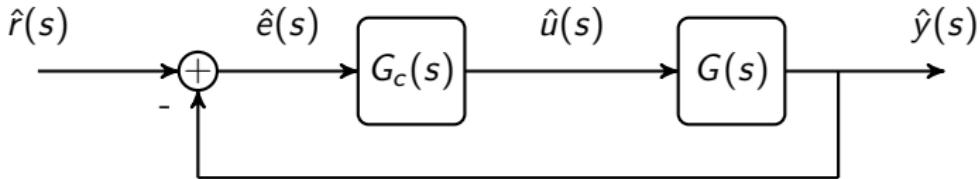
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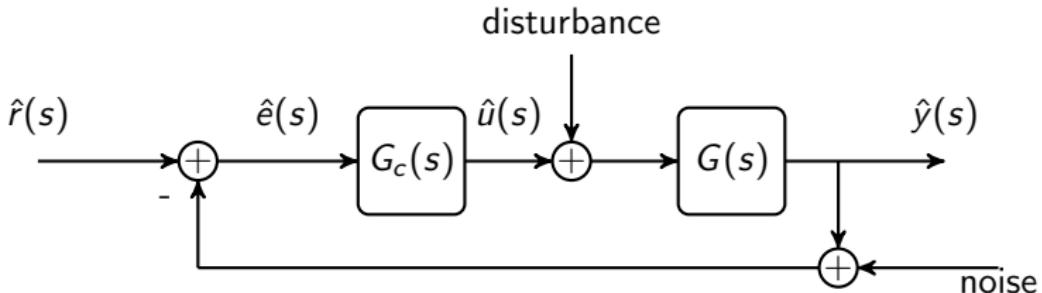
Feedback control is the automated version of this approach to regulating $\omega(t)$. $G_c(s)$ represents a **feedback control**.

Servo Loop



- ▶ $\hat{r}(s)$ is a reference signal
- ▶ $\hat{e}(s)$ is the error signal
- ▶ $\hat{u}(s)$ is the input or control signal
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