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A simple method to calculate P_{50} from a single blood sample

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Abstract

Hill's equation relating oxygen tension, saturation and P_{50} is used as the basis for a simple method to calculate P_{50} from a single blood sample. The effects of errors of measurement in oxygen tension and saturation are considered using the technique of sensitivity analysis. The method is illustrated using data published by Severinghaus.

Introduction

Estimation of P_{50}, the blood oxygen tension at which blood hemoglobin is 50% saturated, is often of clinical interest, especially in critically ill patients. Several methods to estimate P_{50} have been reported [1–3], but none is simple enough for speedy calculation using a pocket calculator. Here we report on a very simple method for estimating P_{50} from a single arbitrary point on the oxyhemoglobin dissociation curve. The method, based on the Hill equation [4] for the oxyhemoglobin dissociation curve, needs only a single pair of oxygen tension/saturation measurements and a pocket calculator capable of performing power calculations.

Mathematical description of the oxyhemoglobin dissociation curve

Hill [4] has described an equation for the oxyhemoglobin dissociation curve which is appealingly simple:

$$SO_2 = \frac{PO_2^n}{PO_2^n + P_{50}^n}$$

where SO_2 is the fractional oxygen saturation at an oxygen tension PO_2 and n is an empirical constant. Using Severinghaus' data [5] over the clinically relevant oxygen tension range of 15 to 100 mmHg, and choosing P_{50} = 26.6 mmHg from this data, the value of n which minimized the mean square error between the Hill curve and the Severinghaus data was determined (Figure 1). The best fit was obtained using n = 2.711. The quality of the fit is readily apparent by fitting the Hill equation for P_{50} = 26.6 and n = 2.711 through the Severinghaus data (Figure 2).

Table 1. Comparison of this method (Doyle P_{50}) with the method of Canizaro et al. [3] (Canizaro P_{50}) for data obtained from Mountcastle [6]. (The Canizaro data is presented with less precision because it was obtained from a nomogram)

SO_2	PO_2	True P_{50}	Doyle P_{50}	Canizaro P_{50}
P_{50} = 21				
0.9	50	21	21.8	24
0.8	37	21	21.9	22
0.7	30	21	21.8	22
0.6	24	21	20.6	21
P_{50} = 27				
0.9	65	27	28.4	30
0.8	46	27	27.3	28
0.7	37	27	26.9	26
0.6	32	27	27.5	28
P_{50} = 32				
0.9	75	32	32.7	36
0.8	56	32	33.2	34
0.7	45	32	32.7	33
0.6	38	32	32.6	33

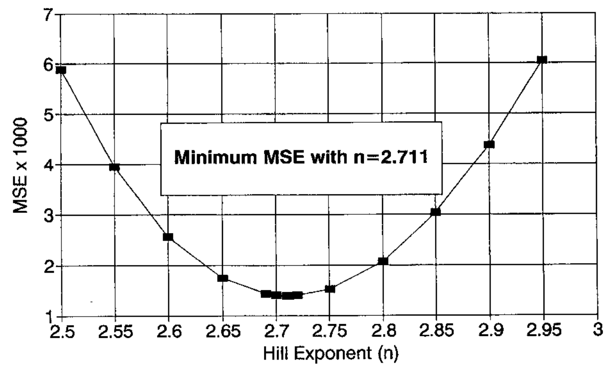

Figure 1. Mean-square error (MSE) between the Hill model and the Severinghaus data in the clinical range of PO_2 of 15 to 100 mmHg for various values of the Hill exponent. A best fit is obtained for n = 2.711.

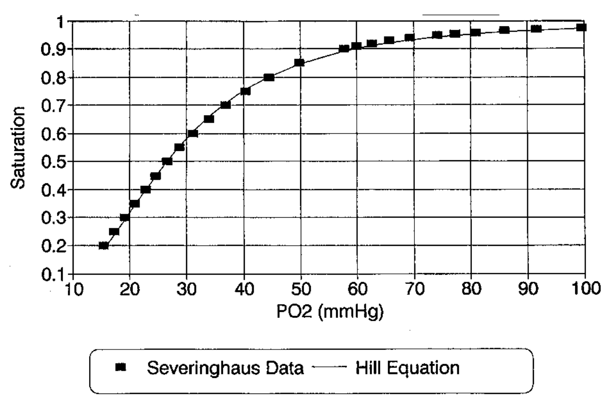

Figure 2. Hill equation with n = 2.711 plotted against the Severinghaus data over the PO_2 range of 15 to 100 mmHg.

Estimation of P_{50}

By a process of straightforward algebraic manipulation of the Hill equation one can solve for P_{50}:

$$P_{50} = PO_2 \cdot \left[\frac{1 - SO_2}{SO_2}\right]^{1/n}$$

where PO_2 and SO_2 are simultaneous measurements of tension and saturation taken from a single blood sample. The method may be illustrated with an example. Suppose we obtain PO_2 = 80 mmHg from a blood gas machine and SO_2 = 0.95 from a co-oximeter. Using n = 2.711 we then obtain P_{50} = 27 mmHg.

Evaluation

The Severinghaus data [5] in the clinical range of 15 to 100 mmHg may be used to carry evaluation of the method. This curve is known to have a P_{50} of 26.6. Each point on the curve allows us to obtain an estimate of P_{50} by the method described. The error in this estimate can be plotted against PO_2 for this data and is shown in Figure 3. The square root of the average square error (root-mean-square error) is 1.27 mmHg. The method was also evaluated using sample data from Mountcastle [6] (Table 1), where the results of our method are compared with the nomogram developed by Canizaro et al. [3]. As can be seen, our method is comparable in accuracy.

Sensitivity analysis

The theoretical sensitivity of the proposed method with respect to errors of measurement in oxygen tension and saturation can be determined using the method of sensitivity analysis [7]. If P_e is the error in the

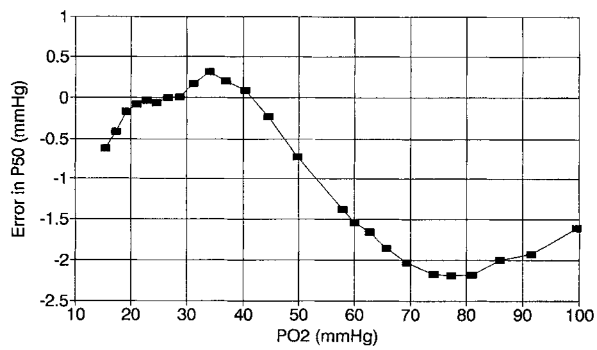

Figure 3. Error in estimating P_{50} using various oxygen tension values for the Severinghaus data over the range of 15 to 100 mmHg.

estimate of the oxygen tension and S_e is the error in the estimation of the fractional oxygen saturation, the error in estimation P_{50} ($P50_e$) is given by:

$$\begin{aligned} P50_e &= \frac{\partial P_{50}}{\partial PO_2} P_e + \frac{\partial P_{50}}{\partial SO_2} S_e \\ &= \left(\frac{1-SO_2}{SO_2}\right)^{1/n} P_e - \frac{PO_2}{nSO_2^2}\left(\frac{1-SO_2}{SO_2}\right)^{\frac{1-n}{n}} S_e \end{aligned}$$

As an example, suppose a blood sample has true values of P_{50} = 27 mmHg, PO_2 = 80 mmHg, and SO_2 = 0.95. If the measured data is obtained as PO_2 = 82 mmHg and SO_2 = 0.96 the error in estimating P_{50} will be approximately $P50_e$ = 0.338 × 2 - 210 × 0.01 = -1.43 mmHg.

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