Chapter-3

Digital Image Processing Operations

1. Compute D_e, D_4, D_8 and D_m distance between two pixels x and y be (0, 0) and (6, 3) for the image below

Solution

Eucledian distance:

$\mathbf{D_e} = \sqrt{(x-s)^2 + (y-t)^2} = \sqrt{(0-6)^2 + (0-3)^2} = \sqrt{45}$

City block distance:

$\mathbf{D_4} = |(x-s)| + |(y-t)| = |0-6| + |0-3| = 9$

Chess board distance:

$\mathbf{D_8} = \max(|(x-s)|, |(y-t)|) = \max(6, 3) = \mathbf{6}$

2. What is the connectivity?

Solution: The 1's are diagonal. Hence 8-connectivity.

3. Median is a non-linear operator as

Hint: median ($x_m + y_m$) is not equal to median (x_m) + median (y_m).

4. Consider the following two images.

$$f_1 = \begin{pmatrix} 10 & 40 & 30 \\ 40 & 100 & 90 \\ 90 & 80 & 70 \end{pmatrix}$$

$$f_2 = \begin{pmatrix} 40 & 140 & 90 \\ 140 & 100 & 90 \\ 90 & 80 & 190 \end{pmatrix}$$

Perform the arithmetic operations. Assume that all the operations are uint8.

<u>Solution</u>

$f_1 + f_2 =$ $$f_2 = \begin{pmatrix} 10+40 & 40+140 & 30+90 \\ 40+140 & 100+100 & 90+90 \\ 90+90 & 80+80 & 70+190 \end{pmatrix} = \begin{pmatrix} 50 & 180 & 120 \\ 180 & 200 & 180 \\ 180 & 160 & 260 \end{pmatrix}$$

If unit 16 is used, the above is the answer. But if unit 8 is used the maximum is 255. So the value 260 is truncated to 255 to yield the matrix

5. What is the difference between image subtraction and image absolute difference?

Solution:

$$f_1 - f_2 = \begin{pmatrix} 10-40 & 40-140 & 30-90 \\ 40-140 & 100-100 & 90-90 \\ 90-90 & 80-80 & 70-190 \end{pmatrix} = \begin{pmatrix} -30 & -100 & -60 \\ -100 & 0 & 0 \\ 0 & 0 & -120 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

If the absolute difference is used, it is then

$$f_1 - f_2 = \begin{pmatrix} 10-40 & 40-140 & 30-90 \\ 40-140 & 100-100 & 90-90 \\ 90-90 & 80-80 & 70-190 \end{pmatrix} = \begin{pmatrix} 30 & 100 & 60 \\ 100 & 0 & 0 \\ 0 & 0 & 120 \end{pmatrix}$$

6. Consider the following two images.

$$f_1 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$f_2 = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Perform the logical operations AND, OR, NOT and difference.

Solution

AND

$$f_1 \ \& \ f_2 = \begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix}$$

OR

$$f_1 \mid f_2 = \begin{pmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} \end{pmatrix}$$

NOT

$$!\, f_1 = \begin{pmatrix} \mathbf{0} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} & \mathbf{0} \end{pmatrix}$$

$$!\, f_2 = \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix}$$

DIFFERENCE:

$$f_1 - f_2 = (f_1)\ \&\ (!f_2) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$f_2 - f_1 = (f_2)\ \&\ (!f_1) = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix}$$

7. Derive the inverse scaling transformation.

Solution:

The inverse scaling transformation is given as $A^{-1} = \dfrac{|Adjo\,\text{int}(A)|^T}{\det|A|}$

The scaling matrix is given as $A = \begin{pmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{pmatrix}$

The adjoint matrix of A is given as $\begin{pmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & s_x s_y \end{pmatrix}$

The determinant of A = $s_x s_y$

Therefore, the inverse transformation is given as = $A^{-1} = \begin{pmatrix} \frac{1}{s_x} & 0 & 0 \\ 0 & \frac{1}{s_y} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

8. Prove that transform and its inverse restore the original object.

Solution:

The original scaling transform is given as

The scaling matrix is given as $A = \begin{pmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{pmatrix}$

The inverse scaling transform is given as

$$A^{-1} = \begin{pmatrix} \frac{1}{s_x} & 0 & 0 \\ 0 & \frac{1}{s_y} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Therefore the multiplication of scaling and its inverse gives a matrix of

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The multiplication of any matrix with unit matrix gives the same matrix. Hence the original object is unchanged.

9.

Solution:

The resultant transform is multiplication of three transforms

1. First translate with $-x_r, -y_r$
2. Apply scaling transform
3. Apply translation now with $+x_r, +y_r$

This results in composite transform.

10. Consider the image $F = \begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix}$. Apply replication and interpolation techniques and show the result.

<u>Given</u>

$F = \begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix}$

<u>Solution</u>

<u>Zero-hold interpolation:</u>

$$F' = \begin{pmatrix} 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Linear interpolation:

After Row wise interpolation,

$$F'' = \begin{pmatrix} 2 & 1.5 & 1 & 0.5 \\ 0 & 0 & 0 & 0 \\ 3 & 2 & 1 & 0.5 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

After Column wise interpolation,

$$F'' = \begin{pmatrix} 2 & 1.5 & 1 & 0.5 \\ 2.5 & 1.75 & 1 & 0.5 \\ 3 & 2 & 1 & 0.5 \\ 1.5 & 1 & 0.5 & 0.25 \end{pmatrix}$$

11. Consider the image $F = \begin{pmatrix} 2 & 2 \\ 3 & 2 \end{pmatrix}$. Calculate the mean and entropy of the image.

Given

$$F = \begin{pmatrix} 2 & 2 \\ 3 & 2 \end{pmatrix}$$

Symbols	2	3
p_k	0.75	0.25

Solution

Mean = $\frac{2+2+3+2}{4} = \frac{9}{4} =$ **2.25**

Entropy, $H = -\sum_{k=1}^{n} p_k \log_2 p_k$

$$H = -\sum_{k=1}^{2} p_k \log_2 p_k$$

$$= -[0.75 \log_2 0.75 + 0.25 \log_2 0.25]$$

$$= 0.311 + 0.5$$

$$\boldsymbol{H = 0.811}$$

12. Find the convolution and correlation of the following streams of data:

a) {1 7 9 6} and {1 3 5}

b) {1 2 3 4} and {1 1}

Solution

a) Correlation:

f w

1 7 9 6 1 3 5

After zero padding,

i) 0 0 1 7 9 6 $y_0 = 5$

1 3 5

ii) 0 0 1 7 9 6 $y_1 = 38$

1 3 5

iii) 0 0 1 7 9 6 $y_2 = 67$

1 3 5

iv) 0 0 1 7 9 6 $y_3 = 64$

1 3 5

v) 0 0 1 7 9 6 $y_4 = 27$

1 3 5

v) 0 0 1 7 9 6 $y_5 = 6$

1 3 5

Correlation output = { 5 38 67 64 27 6 }

Convolution:

f w rotated 180^0

1 7 9 6 5 3 1

After zero padding,

i) 0 0 1 7 9 6 $y_0 = 1$

5 3 1

ii) 0 0 1 7 9 6 $y_1 = 10$

5 3 1

iii) 0 0 1 7 9 6 $y_2 = 35$

5 3 1

iv) 0 0 1 7 9 6 $y_3 = 68$

5 3 1

v) 0 0 1 7 9 6 $y_4 = 63$

5 3 1

v) 0 0 1 7 9 6 $y_5 = 30$

5 3 1

Convolution output = { 1 10 35 68 63 30 }

b) Correlation:

f	w
1 2 3 4	1 1

After zero padding,

i) 0 1 2 3 4 $y_0 = 1$

1 1

ii) 0 1 2 3 4 $y_1 = 3$

  1 1

iii) 0 1 2 3 4 $y_2 = 5$

    1 1

iv) 0 1 2 3 4 $y_3 = 7$

      1 1

v) 0 1 2 3 4 $y_4 = 4$

        1 1

Correlation output = { 1 3 5 7 4 }

Convolution:

f	w rotated 180^0
1 2 3 4	1 1

After zero padding,

i) 0 1 2 3 4 $\qquad y_0 = 1$

1 $\underset{\uparrow}{1}$

ii) 0 1 2 3 4 $\qquad y_1 = 3$

1 $\underset{\uparrow}{1}$

iii) 0 1 2 3 4 $\qquad y_2 = 5$

1 $\underset{\uparrow}{1}$

iv) 0 1 2 3 4 $\qquad y_3 = 7$

1 $\underset{\uparrow}{1}$

v) 0 1 2 3 4 $\qquad y_4 = 4$

1 $\underset{\uparrow}{1}$

Convolution output = { 1 3 5 7 4 }

Since the filter is symmetric, correlation and convolution produce the same result.

13. Find the convolution and correlation of the following streams of data:

a) $F = \begin{pmatrix} 7 & 3 & 3 \\ 2 & 2 & 2 \\ 2 & 2 & 1 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

b) $F = \begin{pmatrix} 7 & 2 & 3 \\ 2 & 2 & 1 \\ 2 & 1 & 1 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

Solution

a) Image dimension is 3 X 3.

Template dimension is 2 X 2.

The output dimension will be (3+2-1) X (3+2-1) i.e., 4 X 4.

Correlation:

F is padded with 1 row of 0's on all sides.

After padding,

$$F = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 3 & 3 & 0 \\ 0 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

1) $$\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 3 & 3 & 0 \\ 0 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

2) $$\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 3 & 3 & 0 \\ 0 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

3) $$\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 3 & 3 & 0 \\ 0 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

4) $$\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 3 & 3 & 0 \\ 0 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

5) $$\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 3 & 3 & 0 \\ 0 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

6) $$\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 3 & 3 & 0 \\ 0 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

7) $\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 3 & 3 & 0 \\ 0 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$

8) $\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 3 & 3 & 0 \\ 0 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$

9) $\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 3 & 3 & 0 \\ 0 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$

10) $\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 3 & 3 & 0 \\ 0 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$

11) $\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 3 & 3 & 0 \\ 0 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$

12) $\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 3 & 3 & 0 \\ 0 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$

13) $\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 3 & 3 & 0 \\ 0 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$

14) $\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 3 & 3 & 0 \\ 0 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$

15) $\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 3 & 3 & 0 \\ 0 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$

16) $\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 3 & 3 & 0 \\ 0 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$

Output of the correlation is

$$\begin{pmatrix} -7 & 4 & 0 & 3 \\ 5 & 10 & 6 & 5 \\ 0 & 5 & 5 & 3 \\ 2 & 4 & 3 & 1 \end{pmatrix}$$

Convolution:

F is padded with 1 row of 0's on all sides.

After padding,

$$F = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 3 & 3 & 0 \\ 0 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

filter rotated 180^0

1) $$\begin{array}{ccccc} {}^{-1}0 & {}^{1}0 & 0 & 0 & 0 \\ {}^{1}0 & {}^{1}7 & 3 & 3 & 0 \\ 0 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}$$

2) $$\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ {}^{-1}0 & {}^{1}7 & 3 & 3 & 0 \\ {}^{1}0 & {}^{1}2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}$$

3) $$\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 3 & 3 & 0 \\ {}^{-1}0 & {}^{1}2 & 2 & 2 & 0 \\ {}^{1}0 & {}^{1}2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}$$

4) $$\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 3 & 3 & 0 \\ 0 & 2 & 2 & 2 & 0 \\ {}^{-1}0 & {}^{1}2 & 2 & 1 & 0 \\ {}^{1}0 & {}^{1}0 & 0 & 0 & 0 \end{array}$$

5) $$\begin{array}{ccccc} 0 & {}^{-1}0 & {}^{1}0 & 0 & 0 \\ 0 & {}^{1}7 & {}^{1}3 & 3 & 0 \\ 0 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}$$

6) $$\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ 0 & {}^{-1}7 & {}^{1}3 & 3 & 0 \\ 0 & {}^{1}2 & {}^{1}2 & 2 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}$$

7) $$\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 3 & 3 & 0 \\ 0 & {}^{-1}2 & {}^{1}2 & 2 & 0 \\ 0 & {}^{1}2 & {}^{1}2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}$$

8) $$\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 3 & 3 & 0 \\ 0 & 2 & 2 & 2 & 0 \\ 0 & {}^{-1}2 & {}^{1}2 & 1 & 0 \\ 0 & {}^{1}0 & {}^{1}0 & 0 & 0 \end{array}$$

9) $$\begin{array}{ccccc} 0 & 0 & {}^{-1}0 & {}^{1}0 & 0 \\ 0 & 7 & {}^{1}3 & {}^{1}3 & 0 \\ 0 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}$$

10) $$\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & {}^{-1}3 & {}^{1}3 & 0 \\ 0 & 2 & {}^{1}2 & {}^{1}2 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}$$

11) $$\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 3 & 3 & 0 \\ 0 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

12) $$\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 3 & 3 & 0 \\ 0 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

13) $$\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 3 & 3 & 0 \\ 0 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

14) $$\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 3 & 3 & 0 \\ 0 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

15) $$\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 3 & 3 & 0 \\ 0 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

16) $$\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 3 & 3 & 0 \\ 0 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

Output of the convolution is

$$\begin{pmatrix} 7 & 10 & 6 & 3 \\ 9 & 0 & 4 & -1 \\ 4 & 4 & 3 & -1 \\ 2 & 0 & -1 & -1 \end{pmatrix}$$

b) Image dimension is 3 X 3.

Template dimension is 2 X 2.

The output dimension will be (3+2-1) X (3+2-1) i.e., 4 X 4.

<u>Correlation:</u>

F is padded with 1 row of 0's on all sides.

After padding,

$$F = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 2 & 3 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 2 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Initial filter position is

$$\begin{array}{cccccc} 1 & 0^{1} & 0 & 0 & 0 & 0 \\ 1 & 0^{1} & 7 & 3 & 3 & 0 \\ & 0 & 2 & 2 & 2 & 0 \\ & 0 & 2 & 2 & 1 & 0 \\ & 0 & 0 & 0 & 0 & 0 \end{array}$$

Then the mask is moved through all the elements to obtain the final output as

$$\begin{pmatrix} \mathbf{7} & \mathbf{9} & \mathbf{5} & \mathbf{3} \\ \mathbf{9} & \mathbf{13} & \mathbf{8} & \mathbf{4} \\ \mathbf{4} & \mathbf{7} & \mathbf{5} & \mathbf{2} \\ \mathbf{2} & \mathbf{3} & \mathbf{2} & \mathbf{1} \end{pmatrix}$$

<u>Convolution:</u>

F is padded with 1 row of 0's on all sides.

After padding,

$$F = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 2 & 3 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 2 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Initial filter position is

$$\begin{array}{cccccc} 1 & 0^{1} & 0 & 0 & 0 & 0 \\ 1 & 0^{1} & 7 & 3 & 3 & 0 \\ & 0 & 2 & 2 & 2 & 0 \\ & 0 & 2 & 2 & 1 & 0 \\ & 0 & 0 & 0 & 0 & 0 \end{array}$$

Then the mask is moved through all the elements to obtain the final output as

$$\begin{pmatrix} 7 & 9 & 5 & 3 \\ 9 & 13 & 8 & 4 \\ 4 & 7 & 5 & 2 \\ 2 & 3 & 2 & 1 \end{pmatrix}$$

Since the filter is symmetric, both correlation and convolution produce same output.

14. A triangle is marked by the points A(1, 1),B(5, 5) and C(10, 10).

a) Apply translation of $\delta x = 3$ and $\delta y = 4$

b) Apply rotation with degrees 45, 90, 180 and 270

c) Consider the image perform zooming and mirroring along x, y and z axes

<u>Given</u>

A triangle with A(1, 1),B(5, 5) and C(10, 10).

<u>Solution</u>

a) Apply translation of $tx = 3$ and $ty = 4$

$$[\, x' \;\; y' \;\; 1 \,] = [\, x \;\; y \;\; 1 \,] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ tx & ty & 1 \end{bmatrix}$$

<u>For A:</u>

$$[\, x' \;\; y' \;\; 1 \,] = [\, 1 \;\; 1 \;\; 1 \,] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 4 & 1 \end{bmatrix}$$ (where $tx = 3$ and $ty = 4$)

$[\, x' \;\; y' \;\; 1 \,] = [\, 4 \;\; 5 \;\; 1 \,]$ $\therefore$ **$[\, x' \;\; y' \,] = [\, 4 \;\; 5 \,]$**

<u>For B:</u>

[x' y' 1] = [1 5 5] $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 4 & 1 \end{bmatrix}$ (where $tx = 3$ and $ty = 4$)

[x' y' 1] = [8 9 1] ∴ **[x' y'] = [8 9]**

<u>For C:</u>

[x' y' 1] = [1 10 10] $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 4 & 1 \end{bmatrix}$ (where $tx = 3$ and $ty = 4$)

[x' y' 1] = [13 14 1] ∴ **[x' y'] = [13 14]**

So, after translation, the triangle will be marked by the points

A'(4, 5), B'(8, 9) and C'(13, 14).

b) Apply rotation with degrees 45, 90, 180 and 270

[x' y' 1] = [x y 1] $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

<u>$\theta = 45°$</u>

<u>For A:</u>

[x' y' 1] = [1 1 1] $\begin{bmatrix} 0.707 & -0.707 & 0 \\ 0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

[x' y' 1] = [1.414 0 1] ∴ **[x' y'] = [1.414 0]**

<u>For B:</u>

[x' y' 1] = [5 5 1] $\begin{bmatrix} 0.707 & -0.707 & 0 \\ 0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

[x' y' 1] = [7.07 0 1] ∴ **[x' y'] = [7.07 0]**

<u>For C:</u>

[x' y' 1] = [10 10 1] $\begin{bmatrix} 0.707 & -0.707 & 0 \\ 0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

[x' y' 1] = [14.14 0 1] ∴ **[x' y'] = [14.14 0]**

So, after rotation, the triangle will be marked by the points

A'(1.414, 0), B'(7.07, 0) and C'(14.14, 0).

$\underline{\theta = 90^\circ}$

For A:

$$[\,x'\ \ y'\ \ 1\,] = [\,1\ \ 1\ \ 1\,]\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

[x' y' 1] = [1 -1 1] ∴ **[x' y'] = [1 -1]**

For B:

$$[\,x'\ \ y'\ \ 1\,] = [\,5\ \ 5\ \ 1\,]\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

[x' y' 1] = [5 -5 1] ∴ **[x' y'] = [5 -5]**

For C:

$$[\,x'\ \ y'\ \ 1\,] = [\,10\ \ 10\ \ 1\,]\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

[x' y' 1] = [10 -10 1] ∴ **[x' y'] = [10 -10]**

So, after rotation, the triangle will be marked by the points

A'(1, -1), B'(5, -5) and C'(10, -10).

$\underline{\theta = 180^\circ}$

For A:

$$[\,x'\ \ y'\ \ 1\,] = [\,1\ \ 1\ \ 1\,]\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

[x' y' 1] = [-1 -1 1] ∴ **[x' y'] = [-1 -1]**

For B:

$$[\,x'\ \ y'\ \ 1\,] = [\,5\ \ 5\ \ 1\,]\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

[x' y' 1] = [-5 -5 1] ∴ **[x' y'] = [-5 -5]**

For C:

[x' y' 1] = [10 10 1] $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

[x' y' 1] = [-10 -10 1] ∴ **[x' y'] = [-10 -10]**

So, after rotation, the triangle will be marked by the points

A'(-1, -1), B'(-5, -5) and C'(-10, -10).

$\theta = 270^\circ$

For A:

[x' y' 1] = [1 1 1] $\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

[x' y' 1] = [-1 1 1] ∴ **[x' y'] = [-1 1]**

For B:

[x' y' 1] = [5 5 1] $\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

[x' y' 1] = [-5 5 1] ∴ **[x' y'] = [-5 5]**

For C:

[x' y' 1] = [10 10 1] $\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

[x' y' 1] = [-10 10 1] ∴ **[x' y'] = [-10 10]**

So, after rotation, the triangle will be marked by the points

A'(-1, 1), B'(-5, 5) and C'(-10, 10).

c) Consider the image perform mirroring along x, y and z axes

Along x-axis:

$$[\,x'\ \ y'\ \ 1\,] = [\,x\ \ y\ \ 1\,]\begin{bmatrix}1 & 0 & 0\\0 & -1 & 0\\0 & 0 & 1\end{bmatrix}$$

For A:

$$[\,x'\ \ y'\ \ 1\,] = [\,1\ \ 1\ \ 1\,]\begin{bmatrix}1 & 0 & 0\\0 & -1 & 0\\0 & 0 & 1\end{bmatrix}$$

$[\,x'\ \ y'\ \ 1\,] = [\,1\ \ -1\ \ 1\,]$ $\quad \therefore$ **$[\,x'\ \ y'\,] = [\,1\ \ -1\,]$**

For B:

$$[\,x'\ \ y'\ \ 1\,] = [\,5\ \ 5\ \ 1\,]\begin{bmatrix}1 & 0 & 0\\0 & -1 & 0\\0 & 0 & 1\end{bmatrix}$$

$[\,x'\ \ y'\ \ 1\,] = [\,5\ \ -5\ \ 1\,]$ $\quad \therefore$ **$[\,x'\ \ y'\,] = [\,5\ \ -5\,]$**

For C:

$$[\,x'\ \ y'\ \ 1\,] = [\,10\ \ 10\ \ 1\,]\begin{bmatrix}1 & 0 & 0\\0 & -1 & 0\\0 & 0 & 1\end{bmatrix}$$

$[\,x'\ \ y'\ \ 1\,] = [\,10\ \ -10\ \ 1\,]$ $\quad \therefore$ **$[\,x'\ \ y'\,] = [\,10\ \ -10\,]$**

So, after rotation, the triangle will be marked by the points

A'(1, -1), B'(5, -5) and C'(10, -10).

Along y-axis:

$$[\,x'\ \ y'\ \ 1\,] = [\,x\ \ y\ \ 1\,]\begin{bmatrix}-1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

For A:

$$[\,x' \;\; y' \;\; 1\,] = [\,1 \;\; 1 \;\; 1\,]\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$[\,x' \;\; y' \;\; 1\,] = [\,-1 \;\; 1 \;\; 1\,]$ $\quad$ **∴ [x' y'] = [-1 1]**

For B:

$$[\,x' \;\; y' \;\; 1\,] = [\,5 \;\; 5 \;\; 1\,]\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$[\,x' \;\; y' \;\; 1\,] = [\,-5 \;\; 5 \;\; 1\,]$ $\quad$ **∴ [x' y'] = [-5 5]**

For C:

$$[\,x' \;\; y' \;\; 1\,] = [\,10 \;\; 10 \;\; 1\,]\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$[\,x' \;\; y' \;\; 1\,] = [\,-10 \;\; 10 \;\; 1\,]$ **∴ [x' y'] = [-10 10]**

So, after rotation, the triangle will be marked by the points

A'(-1, 1), B'(-5, 5) and C'(-10, 10).

Since this is 2D triangle, mirroring along z-axis gives the same points.

15.

Solution:

$$mean = \frac{548}{9} = 60.89$$

mode = 10

median = 67

Sum of difference = 15342.888

Variance = 1704.7654

Standard Deviation = 41.2888

16.

Solution:

$= 0.4 \log(0.4) + 0.3 \log(0.3) + 0.2 \log(0.2) + 0.1 \log(0.1)$

$= -[-0.5288-0.5211-0.4644-0.3322]$

$= 1.8465$