Chapter-7

Image Segmentation

QUESTION 1

Consider a one-dimensional image $f(x)$ = [10 10 10 10 40 40 40 40 20 20] What is the first and second derivative? Locate the position of edge.

F (x,y)	10		10		10		10		40		40		40		40		20		20
First derivative		0		0		0		30		0		0		0		-20		0	
Second derivative			0		0		30		-30		0		0		-20		20		

The location of the edge is shaded.

In the first derivative, the magnitude of change indicates the presence of an edge

In the second derivative, change of sign indicates the presence of an edge. The second derivative gives an edge that is two pixels wide.

QUESTION 2

Consider an image

$$F = \begin{pmatrix} 1 & 2 & 5 \\ 5 & 5 & 5 \\ 5 & 3 & 2 \end{pmatrix}$$

Show the output of any edge detection algorithm.

Let us consider the application of the Prewitt's edge detection filters

The edge detection filters are as below:

gx

-1	-1	-1
0	0	0
1	1	1

gy

-1	0	1
-1	0	1
-1	0	1

Applying gx filter, we get gx = -1-2-5+5+3+2 = 2

Applying gy filter, we get gy = -1-5-5+5+5+2 = 1

The gradient, denoted by $\Delta f = grad(f) = (2\ 1)^T$

Magnitude (length of vector) = $mag(\Delta f) = (gx^2 + gy^2)^{1/2}$

= sqrt (4 + 1) = **2.236**

Direction of the edge = $\tan^{-1}(gy/gx) = \tan^{-1}(1/2)$ = **26.5 degrees**

Thus, we see that the edge is located at an angle of **26.5 degrees wrt x-axis**

QUESTION 3

An black object in a white background is known to occupy 20%,40% and 80% of the histogram. What is the threshold value? If the colours of the object and background are reversed, what is the threshold value?

Reference:

http://www.ece.uvic.ca/~aalbu/computer%20vision%202009/Lecture%209.%20Segmentation-Thresholding.pdf

a priori information: object is brighter/darker than background and occupies a certain known percentile 1/p from the total image area (example: printed text sheet). We set the threshold by finding the intensity level such that 1/p image pixels are below this value.

We set a threshold T, such that the cumulative histogram C(g) = ∑ h(g) , t = 0 to g, and h(k) = n_k / n

C(T) = 1 – 1/p for a bright foreground i.e. bright image on dark background

C(T) = 1/p, for a dark foreground ie. Dark image on bright background

Below are the results of the calculation of the threshold values

Dark Object in a white background

Problem Number	1/P	In decimal	Threshold C(T) 1/P	Threshold value = C(T) * max intensity i.e. 255
(a)	20%	0.200	0.200	51
(b)	40%	0.400	0.400	102
(c)	80%	0.800	0.800	204

Bright object in a dark background

Problem number	1/P	in decimal	Threshold C(T) 1-1/P	Threshold value = C(T) * max intensity i.e. 255
(a)	20%	0.200	0.800	204
(b)	40%	0.400	0.600	153
(c)	80%	0.800	0.200	51

Observation: as the % of darker objects goes down (or brighter object goes up), the threshold intensity is lowered, while the reverse happens when the % of darker objects goes up (or brighter object goes down).

QUESTION 4

Consider the following image

8	4	5
8	4	4
7	5	1

Show the crack edges.

The crack edges along with the magnitude and direction are show below

8	< 4	**4**	>1	**5**
				^ 1
8	< 4	**4**		**4**
^ 1		1 V		^ 3
7	< 2	**5**	< 4	**1**

QUESTION 5

For the given image, apply graph theoretic algorithm and show the flow of the edges.

9	7	9
2	4	3
7	5	1

Step 1:

Calculate the edge weights in the form of a graph

		↓	↑		↓	↑	
	9	7	11	**7**	11	7	**9**
→	16			12			15
←	2			6			3
	2	11	7	**4**	8	10	**3**
→	4			8			11
←	14			10			7
	7	7	11	**5**	5	13	**1**

The weights for each edge are calculated as (MaxIntensity) – (Left pixel intensity – Right pixel intensity)

Assume that the edge only starts from the top row and ends at the bottom row.

Step 2

Now, use the shortest path routing algorithm to find a path from the top to the bottom using the weighted edges as paths.

		↓	↑		↓	↑	
	9	7	11	**7**	11	7	**9**
→	16			12			15

←	2			6	↓		3
	2	11	7	**4**	8	10	**3**
→	4			8			11
←	14			10	↓		7
	7	7	11	**5**	5	13	**1**

The downward edge is marked in the figure above.

6.Explain in detail the graph theoretic algorithm, with respect to the following image:

4	5	6
11	9	8
3	11	13

Step 1:

Calculate the edge weights in the form of a graph

The weights for each edge are calculated as (MaxIntensity) – (Left pixel intensity – Right pixel intensity)

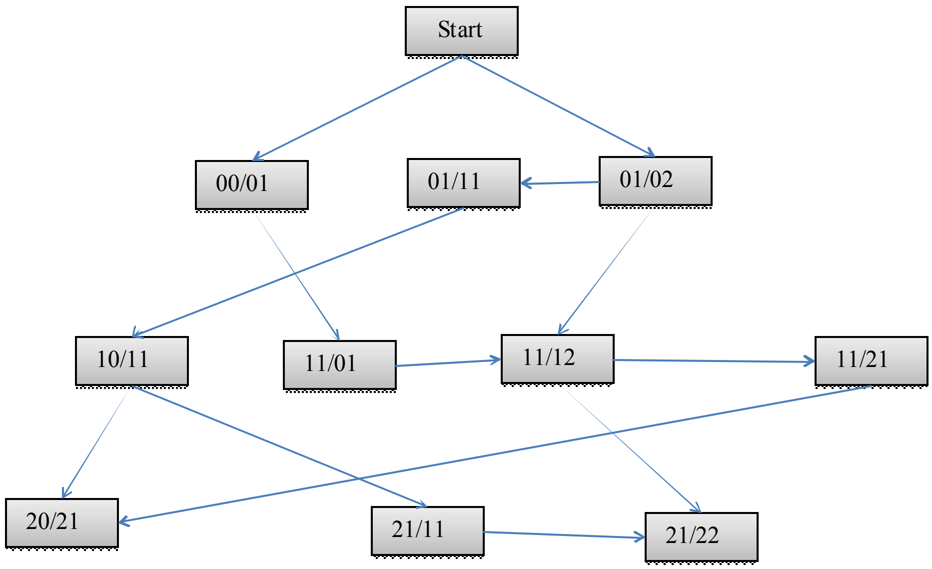
		↓	↑		↓	↑	
	4	14	12	**5**	14	12	**6**
→	6			9			11
←	20			17			15
	11	11	15	**9**	12	14	**8**

→	21			11			8
←	5			15			18
	3	21	5	**11**	15	11	**13**

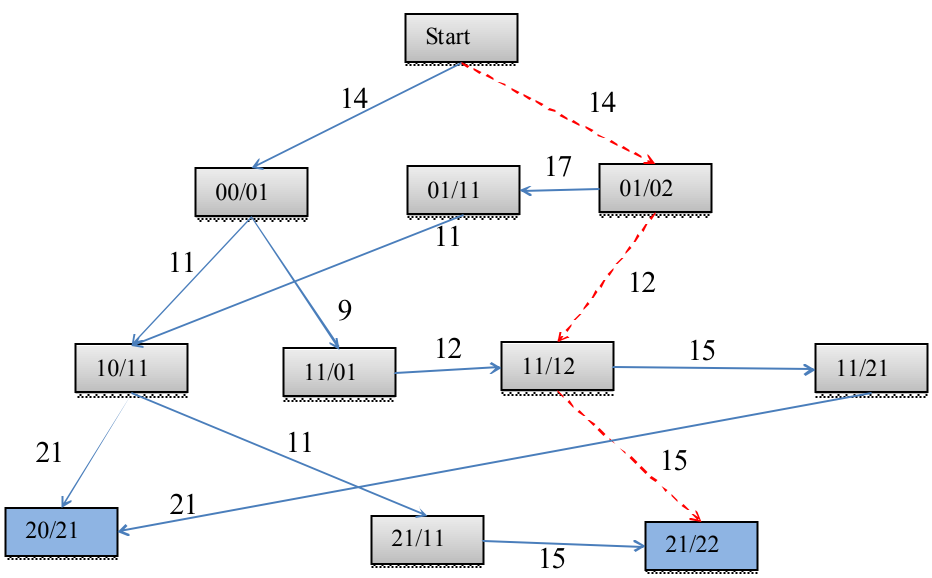
Assume that the edge only starts from the top row and ends at the bottom row. This means that we can remove some of the edges from consideration.

		↓	↑		↓	↑	
	4	14		**5**	14		**6**
→				9			
←				17			
	11	11	15	**9**	12	14	**8**
→				11			
←				15			
	3	21		**11**	15		**13**

These edges can now be represented as a graph as below

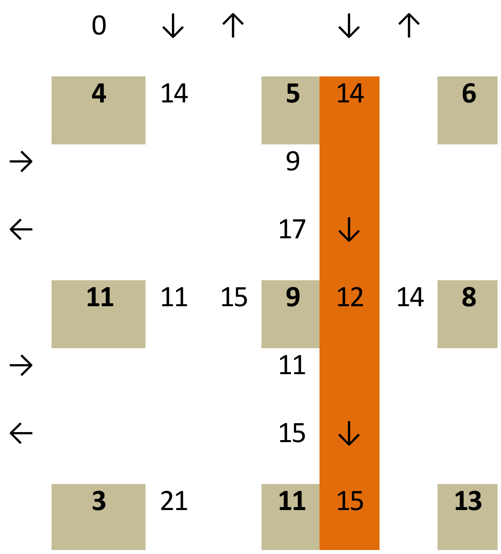


The weights as shown in the table above are put on the edges on the graph, and the shortest path algorithm like Djikistra is used to find the path with minimum cost.



The minimum cost in this case is:

So the edge is present along the path as shown in the graph below:



QUESTION 7

Find an optimal threshold for the image where the probability distribution is given as follows:

$$P_1(z) = \{0 \text{ if z<1; } \frac{1}{2}z + 1 \text{ if } 1 \le z \le 4; 0 \text{ when z>3}$$

$$P_2(z) = \{0 \text{ if z<1; } -\frac{1}{2}z - 1 \text{ if } 1 \le z \le 3; 0 \text{ when z>2}$$

We can find the threshold by solving the equation $P_1(z) = P_2(z)$.

(1/2) z + 1 = - (1/2) z – 1

Z = - 2

A value of z = -2 will be rounded off to a pixel intensity of 0

So the threshold value for the given probability distribution is 0

QUESTION 8

For the given image,

1	1	9	8	7
0	1	8	**8**	8
0	0	7	9	8
0	**1**	8	8	9
1	2	8	8	9

What is the result in the threshold is 3. If the threshold condition is changed as (pixel – seed) < 0.1 x (maximum – minimum) of 8-neighborhood, what is the resultant image?

The following figure illustrates how the regions grow if the threshold value i.e. the difference between the 8 neighbour and the center pixel intensity value > 3

1	1	9	8	7
0	1	8	**8**	8
0	0	7	9	8

1	1	9	8	7
0	1	8	**8**	8
0	0	7	9	8

0	**1**	8	8	9
1	2	8	8	9

0	**1**	8	8	9
1	2	8	8	9

1	1	9	8	7
0	1	8	**8**	8
0	0	7	9	8
0	**1**	8	8	9
1	2	8	8	9

1	1	9	8	7
0	1	8	**8**	8
0	0	7	9	8
0	**1**	8	8	9
1	2	8	8	9

The following illustrates the growth with the modified threshold condition.

1	1	9	8	7
0	1	8	**8**	8
0	0	7	9	8
0	**1**	8	8	9
1	2	8	8	9

-2	-2	5.2
-2	**1**	6.2

0.2	-0.8	-1.8
-0.8	**8**	-0.8

=>

1	1	9	8	7
0	1	8	**8**	8
0	0	7	9	8
0	**1**	8	8	9

-1	0.2	6.2

-1.8	0.2	-0.8

1	2	8	8	9

Consider neighbourhood of px 1,1 wrt seed '1'

1	1	9
0	**1**	8
0	0	7

-0.9	-0.9	7.1
-1.9	-0.9	6.1
-1.9	-1.9	5.1

=>

1	1	9	8	7
0	1	8	**<u>8</u>**	8
0	0	7	9	8
0	**<u>1</u>**	8	8	9
1	2	8	8	9

Consider neighbourhood of px 2,4 wrt seed '8'

<u>8</u>	8
9	8
8	9

-0.1	-0.1
0.9	-0.1
-0.1	0.9

=>

1	1	9	8	7
0	1	8	**<u>8</u>**	8
0	0	7	9	8
0	**<u>1</u>**	8	8	9
1	2	8	8	9

Consider neighbourhood of px 3,3 wrt seed '8'

7	9	8

-1.2	0.8	-

1	1	9	8	7
0	1	8	**<u>8</u>**	8
0	0	7	9	8

8	8	9
8	8	9

		0.2
-0.2	-0.2	0.8
-0.2	-0.2	0.8

=>

0	**<u>1</u>**	8	8	9
1	2	8	8	9

QUESTION 9

For the given image, $F = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 9 & 9 & 9 & 9 & 2 & 2 & 2 & 2 \\ 9 & 9 & 9 & 9 & 2 & 2 & 2 & 2 \\ 9 & 9 & 9 & 9 & 2 & 2 & 2 & 2 \\ 9 & 9 & 9 & 9 & 2 & 2 & 2 & 2 \\ 9 & 9 & 9 & 9 & 2 & 2 & 2 & 2 \\ 9 & 9 & 9 & 9 & 2 & 2 & 2 & 2 \\ 9 & 9 & 9 & 9 & 2 & 2 & 2 & 2 \end{pmatrix}$, show the result of split and merge algorithm.

At the start of the algorithm the entire image is considered as a candidate region as below:

1	2	3	4	5	6	7	8
9	9	9	9	2	2	2	2
9	9	9	9	2	2	2	2

9	9	9	9	2	2	2	2
9	9	9	9	2	2	2	2
9	9	9	9	2	2	2	2
9	9	9	9	2	2	2	2
9	9	9	9	2	2	2	2

First split

1	2	3	4	5	6	7	8
9	9	9	9	2	2	2	2
9	9	9	9	2	2	2	2
9	9	9	9	2	2	2	2
9	9	9	9	2	2	2	2
9	9	9	9	2	2	2	2
9	9	9	9	2	2	2	2
9	9	9	9	2	2	2	2

Second split

1	2	3	4	5	6	7	8
9	9	9	9	2	2	2	2
9	9	9	9	2	2	2	2

9	9	9	9	2	2	2	2
9	9	9	9	2	2	2	2
9	9	9	9	2	2	2	2
9	9	9	9	2	2	2	2
9	9	9	9	2	2	2	2

Third split

1	2	3	4	5	6	7	8
9	9	9	9	2	2	2	2
9	9	9	9	2	2	2	2
9	9	9	9	2	2	2	2
9	9	9	9	2	2	2	2
9	9	9	9	2	2	2	2
9	9	9	9	2	2	2	2
9	9	9	9	2	2	2	2

Results of split and region separation

1	2	3	4	5	6	7	8
9	9	9	9	2	2	2	2
9	9	9	9	2	2	2	2

9	9	9	9	2	2	2	2
9	9	9	9	2	2	2	2
9	9	9	9	2	2	2	2
9	9	9	9	2	2	2	2
9	9	9	9	2	2	2	2

QUESTION 10

The contingency table of a segmentation algorithm is given as

Expert Human	Present	Absent	Total
ROI present	8	1	9
absent	1	1	2
Total	9	2	11

What is the efficiency of the segmentation algorithm?

Efficiency = (TP + TN) / (TP + TN + FP + FN)

= (8 + 2) / (8+ 2 + 1 + 1)

= 10/12

= 83 %