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Introduction to Calculus

Learning Objectives

By the end of this lesson, you will be able to:

- Explain the concepts of calculus
- Define differential calculus
- Examine the limits, continuity, and derivatives of a function
- Define integral calculus
- List down the differential and integral formulas

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Basics of Calculus

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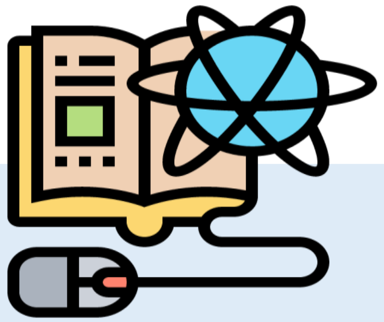
What Is Calculus?

> Calculus is the study of change. It provides a framework for modeling systems in which there is change and ways to make predictions about such models.

What Is Calculus?



- Calculus is one of the key areas of mathematics that deals with continuous change.
- It is based on two key concepts: derivatives and integrals.
- It is also referred to as **infinitesimal calculus** or **the calculus of infinitesimals**.
- The amounts that are virtually equal to nothing but are still not quite zero are known as infinitesimal numbers.
- By and large, old-style calculus is the investigation of continuous change of functions.

Derivatives vs. Integrals

Derivatives

- The derivative of a function is a proportion of the pace of progress of a function, which helps in finding the slope of a curve.
- It explains the function at a specific point.

Integrals

- The integral of a function is a proportion of the area under the curve of the mathematical function f(x) plotted as a function of x.
- It collects the discrete benefits of a function over a range of values.

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Differential Calculus

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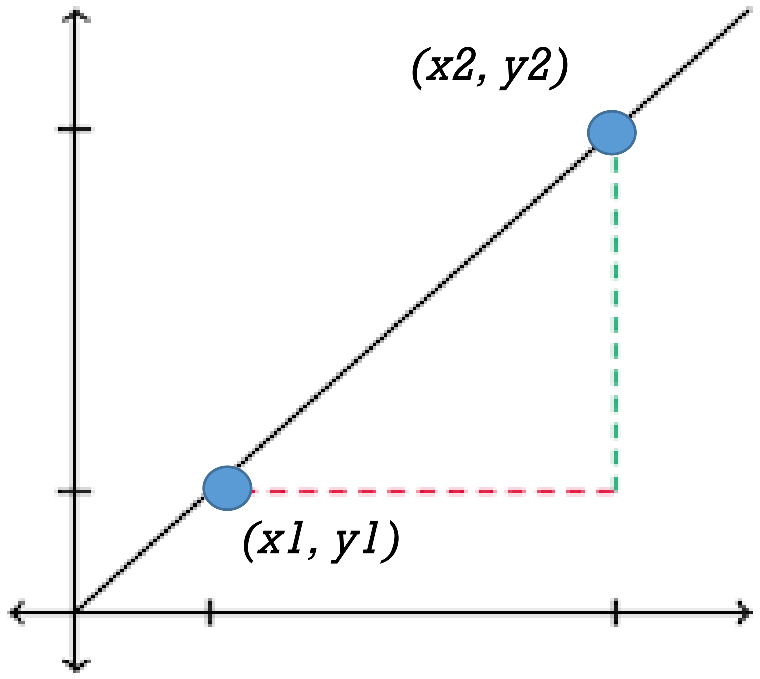
Differential Calculus

Differential calculus is a part of calculus that deals with the study of the rates at which quantities change.



- Consider a scenario where x and y be two real numbers such that y is a function of x, that is, y = f(x).
- If f(x) is the equation of a straight line (linear equation), then the equation is represented as y = mx + b, where m represents the slope.
- The following equation determines the value of m in the slope:

Differential Calculus Equation

$$m = \frac{Change\ in\ y}{Change\ in\ x} = \frac{\Delta y}{\Delta x}$$

Differential Calculus

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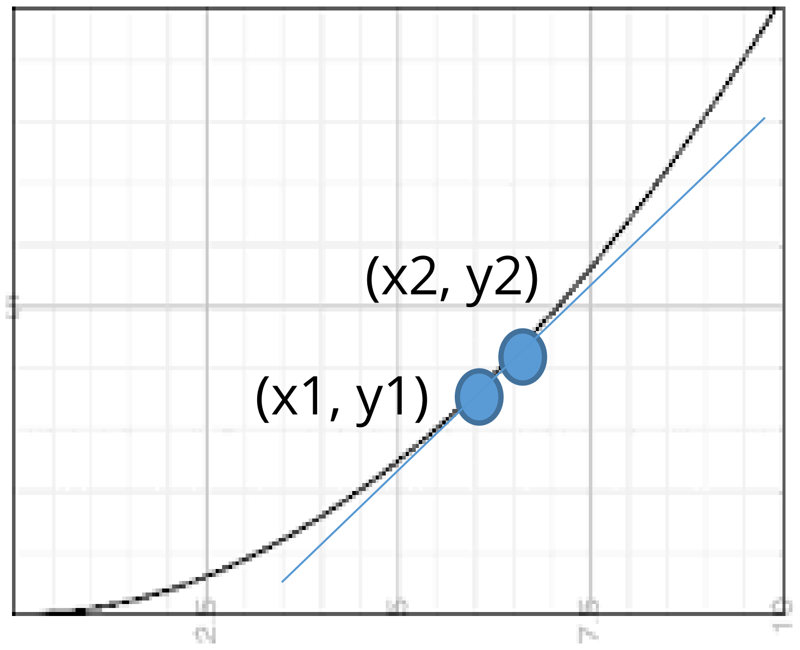

- The **Δy/Δx**, also represented as **dy/dx**, is the derivative of y with respect to x, as well as the rate of change of y per unit change in x.
- The slope of curvature of the graph, which varies at various points, represents the slope of an imaginary straight line drawn through that small graph segment.

Limits

In mathematics, limits are defined as the values that a function approaches as the input approaches a certain value.

The limit equation is expressed as follows:

Limit Equation

$$\lim_{x \to c} f(x) = A$$

This expression is read as **“the limit of f of x as x approaches c equals A”**.

Derivatives

Derivatives are defined as the varying rate of change of a function with respect to an independent variable.

The derivative of a function is expressed as follows:

Derivatives Equation

$$\lim_{x \to h}[f(x + h) - f(x)]/h = A$$

Derivatives are used to measure the sensitivity of one dependent variable with respect to another independent variable.

Continuity

A function f(x) is assumed to be continuous at a specific point where **x = a**, if and only if its three circumstances are fulfilled.

The three continuity circumstances are as follows:

- $f(a)$ is defined.
- $\lim_{x \to a} f(x)$ exists
- $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$

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Differential Formulas

Differential Formulas

Differentiation formulas can be applied to basic algebraic expressions, trigonometric ratios, inverse trigonometry, and exponential terms.

Differential Formula Examples

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}constant = 0$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}a^x = a^x \log a$$

$$\frac{d}{dx}\log x = \frac{1}{x}$$

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\cot x = -\operatorname{cosec}^2 x$$

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\operatorname{cosec} x = -\operatorname{cosec} x \cot x$$

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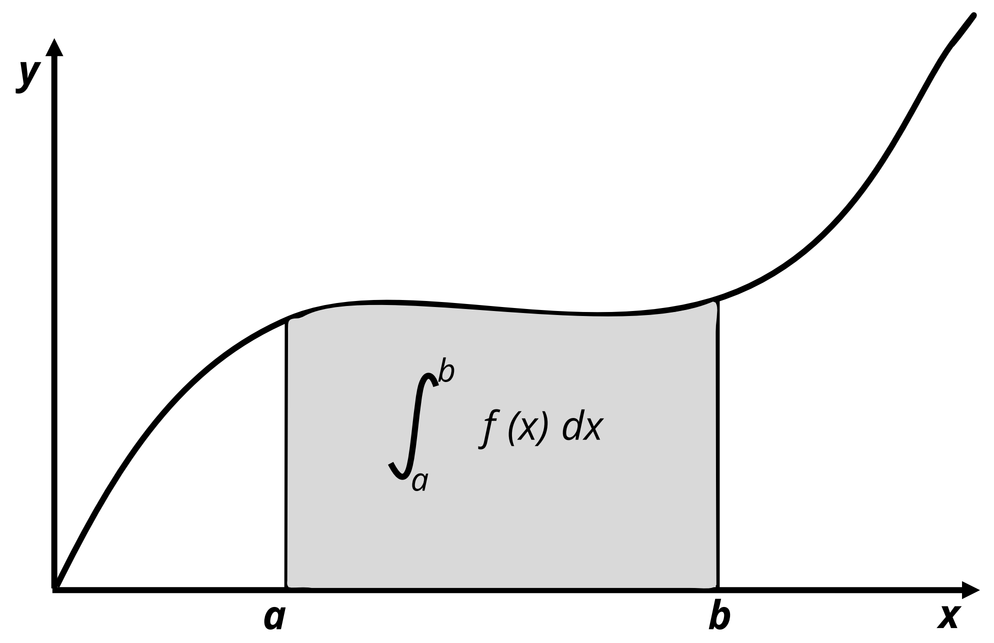
Integral Calculus

Integral Calculus

Integral calculus assigns numbers to functions to describe displacement, area, volume, and other concepts that arise by combining infinitesimal data.



- Consider a function f of a real variable x and an interval [a, b] of the real line.
- The definite integral is defined as the signed area of the region in the xy-plane that is bounded by the graph of f, the x-axis, and the vertical lines x = a and x = b.

An integral is the inverse of a differential and vice versa.

Integral Calculus

The following is an expression for the integral calculus:

Integral Calculus Equation

$$\int_a^b f(x)dx$$

An integral is the inverse of a differential and vice versa.

Definite Integral

- A definite integral has a specific limit or cutoff for the computation of the function.
- The upper and lower cutoff points of the free factor of a function are highlighted.

The following is the numerical representation of a definite integral:

Definite Integral Equation

$$\int a^b \; f(x)\, dx = F(x)$$

Indefinite Integral

- An indefinite integral does not have a particular limit, for example, no upper and lower limit is characterized.
- The integration value is then constantly connected by a constant value (C).

The following is an expression for the indefinite integral:

Indefinite Integral Equation

$$\int f(x)\,dx = F(x) + C$$

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Integration Formulas

Integration Formulas

Integrals Formulas can be derived from differentiation formulas, and are complimentary to differentiation formulas.

Integration Examples

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int 1 dx = x + C$$

$$\int e^x dx = e^x + C$$

$$\int (\frac{1}{x}) dx = \log|x| + C$$

$$\int a^x dx = (\frac{a^x}{\log a}) + C$$

$$\int \cos x\, dx = \sin x + C$$

$$\int \sin x\, dx = -\cos x + C$$

$$\int \sec^2 x\, dx = \tan x + C$$

$$\int \mathrm{cosec}^2 x\, dx = -\cot x + C$$

$$\int \sec x \tan x\, dx = \sec x + C$$

$$\int \mathrm{cosec}\, x \cot x\, dx = -\mathrm{cosec}\, x + C$$

Key Takeaways

- Differential calculus is a part of the calculus that deals with the study of the rates at which quantities change.
- Integral calculus assigns numbers to functions to describe displacement, area, volume, and other concepts that arise by combining infinitesimal data.
- A definite integral has a particular limit or cutoff for the computation of the function.
- Derivatives address the momentary pace of progress of an amount with respect to another.

Knowledge Check

Knowledge Check 1

What is the limit of sin(θ)/θ when θ approaches zero?

A. 1

B. sin(θ)

C. 0

D. None of these

Knowledge Check 1	What is the limit of sin(θ)/θ when θ approaches zero?

A. 1

B. sin(θ)

C. 0

D. None of these

The correct answer is **A**

sin(θ)/θ) = 1 when θ approaches 0.

Knowledge Check 2

What is meant by the differential?

A. A word used a lot on a popular medical television series.

B. A method of directly relating how changes in a dependent variable affect changes in an independent variable.

C. A gearbox on the back end of your car.

D. None of these

Knowledge Check 2

What is meant by the differential?

A. A word used a lot on a popular medical television series.

B. A method of directly relating how changes in a dependent variable affect changes in an independent variable.

C. A gearbox on the back end of your car.

D. None of these

The correct answer is **B**

The differential is a method of directly relating how changes in a dependent variable affect changes in an independent variable.