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Linear Algebra

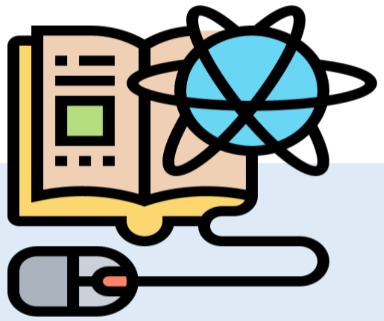
Learning Objectives

By the end of this lesson, you will be able to:

- Explain the concepts of linear algebra
- Solve a linear system of equations
- Describe matrix, forms of matrix, and matrix operations
- Define vectors and list down its properties

Introduction to Linear Algebra

Linear Algebra



- Linear algebra refers to the study of linear combinations.
- For the linear transformations to be carried out, a study of vector spaces, lines, and planes, as well as some mappings, is necessary.
- It contains linear functions, vectors, and matrices.
- It is an examination of the characteristics of linear set transformations.

Linear Equations

Linear algebra's major goal is to establish systematic techniques for solving systems of linear equations.

A linear equation with n variables has the following form:

$$a_1 x_1 + a_2 x_2 + \dots\dots\dots\dots + a_n x_n = b$$

- $x_1 + x_2 + \dots\dots\dots\dots + x_n$ represent the unknown quantities to be found.
- $a_1 + a_2 + \dots\dots\dots\dots + a_n$ are the coefficients.
- b is the constant term.

Linear Equations

A linear equation with two variables x and y is a linear connection between x and y.

The value of one of the variables, often y, is determined by the value of the other, usually x.

The independent variable in this situation is x.

The term "dependent variable" refers to y since it depends on x.

Linear Equations: Example

Here are some instances of linear equations of various types:

Linear equation in one variable	Linear equation in two variables	Linear equation in three variables
$3x+5=0$ $(3/2)x + 7 = 0$ $98x = 49$	$y+7x=3$ $3a+2b = 5$ $6x+9y-12=0$	$x + y + z = 0$ $a - 3b = c$ $3x + 12y = \frac{1}{2} z$

Identifying Linear and Non-linear Equations

Equations	Linear or non-linear
$y = 8x - 9$	Linear
$y = x^2 - 7$	Non-linear, the power of the variable x is 2
$\sqrt{y} + x = 6$	Non-linear, the power of the variable y is 1/2
$y + 3x - 1 = 0$	Linear
$y^2 - x = 9$	Non-linear, the power of the variable y is 2

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Forms of Linear Equation

Linear Equation Forms

The three forms of linear equations are:

- Standard Form
- Slope Intercept Form
- Point Slope Form

Linear Equation in Standard Form

The formula for one-variable single-line calculations is as follows:

The formula for two-variable single-line calculations is as follows:

Equation

$$Ax + B = 0$$

Where:

- A and B are real integers
- x is the variable

Equation

$$Ax + By = C$$

Where:

- A, B, and C are real integers
- x, and y are the variables

Linear Equation in Standard Form

Linear equations take the following form:

Linear Equation

Coefficient of x   Coefficient of y

$$Ax + By = C$$

Variables   Constant

Linear Equation in Slope Intercept Form

A linear equation's slope can be calculated to see how one variable varies in response to a unit change in another variable.

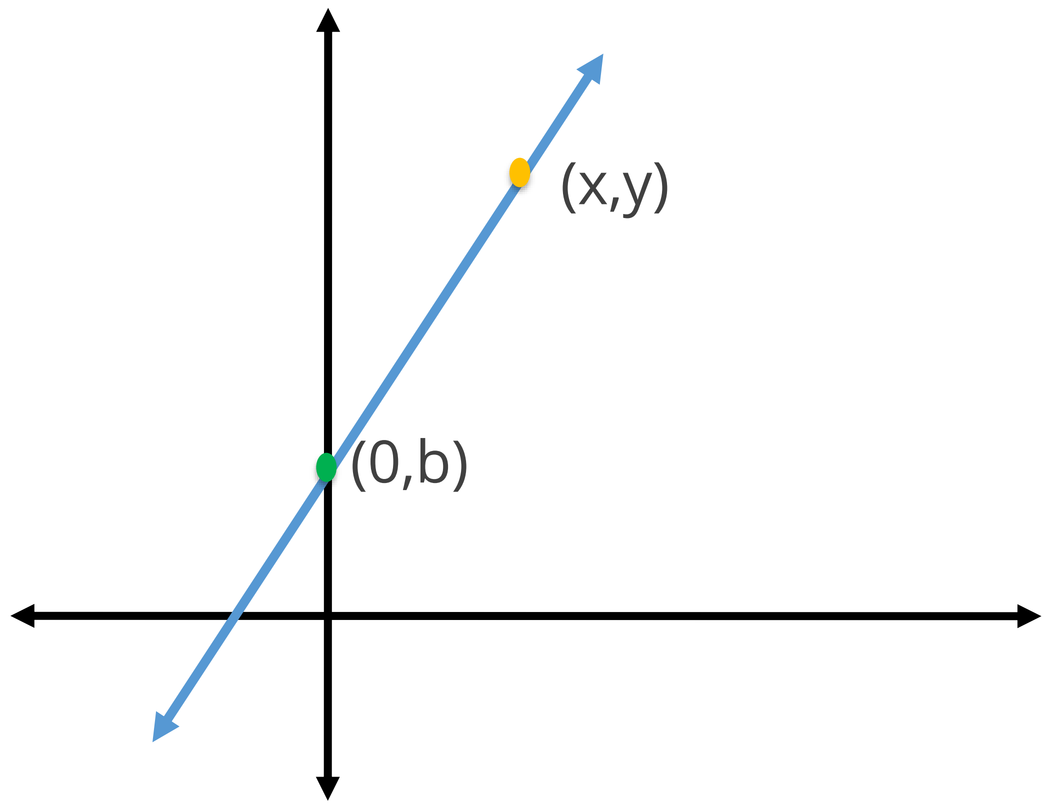
Slope Equation

$$y = mx + b$$

Where:

- m is the slope.
- b is the intercept.
- x and y stand for the line's distance from the x- and y-axes, respectively.

Linear Equation in Slope Intercept Form



The line's (x,y) point represents the distance from the x and y axes, respectively.

The line intercept on the x axes is at (0,b).

Linear Equation: Slope

Slope

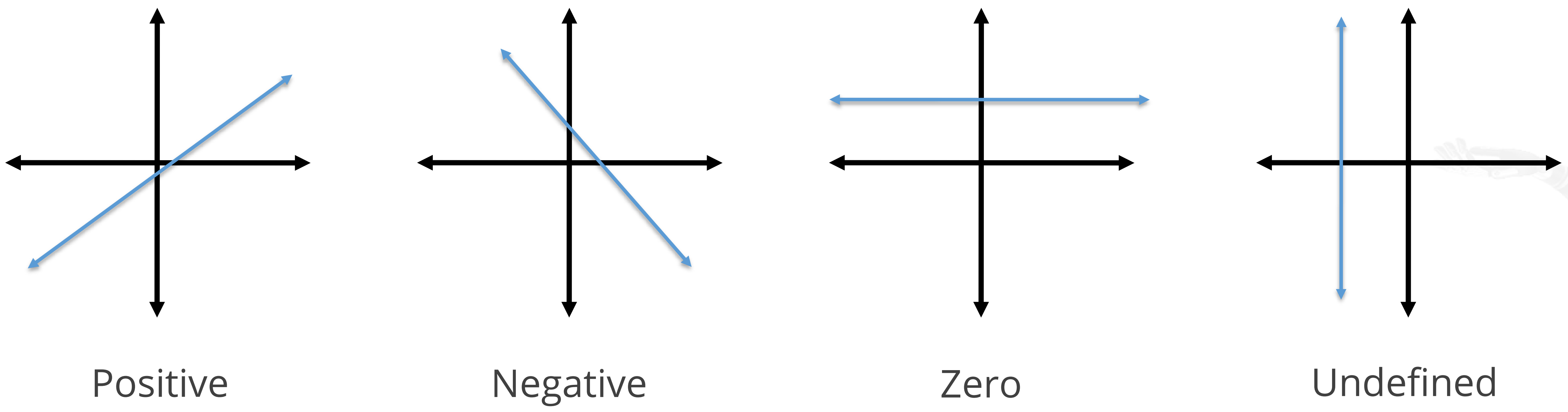
The slope indicates how steep a line is with respect to the y-axis.

When seen from left to right, it indicates whether the line goes up or down.

Slope describes how the independent variable has been changed while the dependent variable is changing.

Types of Slope

There are four types of slopes based on the relationship between the two variables x and y. These are:



Linear Equation in Point Slope Form

A straight line is represented in point slope form by its slope and a point on the line.

Equation

$$y – y1 = m(x – x1)$$

Where:

(x_1, y_1) are the coordinates of the point.

Linear Equations Forms: Example

Consider solving the following linear equation:
(2x – 10)/2 = 3(x – 1)

Step 1:
Clear the fraction

Step 2:
Simplify both sides equation

Step 3:
Clear the fraction

x-5 = 3(x-1)

x-5 = 3x-3
x = 3x+2

x-3x = 2
-2x = 2
x = -1

System of Linear Equations

A system of linear equations is a finite collection of linear equations.

Equation

$$
\begin{aligned}
&a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\
&a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\
&\dots\dots\dots\dots\dots\dots\dots\dots \\
&a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m
\end{aligned}
$$

System of Linear Equations

A linear system can have an unlimited number of solutions, one solution, or no solutions at all in the case of a single linear equation.

A consistent linear system has a solution.

An inconsistent linear system has no solution.

Solving a Linear Equation

Solving Linear Systems of Equations

Following are a few different methods of solving systems of linear equations:

1. Graphic method
2. Substitution method
3. Linear combination method or elimination method
4. Matrix method

Solving Systems of Linear Equations Using Graphing

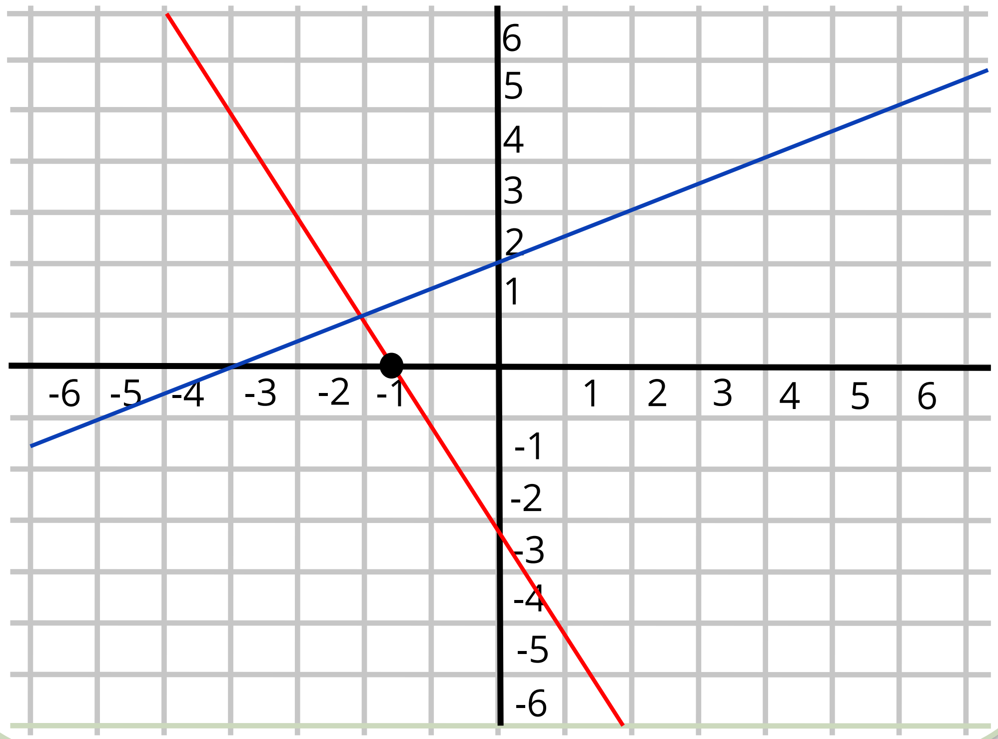
Solve the following system of linear equation by graphing:
$y=0.5x+2$, $y=-2x-3$

- The two equations are in slope-intercept form.
- The first line has a slope of 0.5 and a y-intercept of 2.
- The second line has a slope of -2 and a y-intercept of -3.

Solving Systems of Linear Equations Using Graphing

The following graph depicts the intersection of below two lines:
y = 0.5x + 2 and y =−2 x −3

Graph of the given lines



The intersection of the two lines is at (-2,1).
As a result, the solution is (-2,1). x = 2 and y = 1.

Solving Systems of Linear Equations Using Substitution: Steps

The steps for solving systems of linear equations using the substitution method are as follows:

01 Put one of the equations in the form "variable =........"

02 Substitute that variable in the other equation in its place

03 Take the other equation to solve

04 If required, repeat steps 1 through 3

Solving Systems of Linear Equations Using Substitution

Consider solving the following linear equation:
$3x + 2y = 19, x + y = 8$

- Begin with any equation and variable
- Look at the second equation with the variable "y"

$$3x+2y = 19 \longrightarrow (1)$$

$$x+y = 8 \longrightarrow (2)$$

Solving Systems of Linear Equations Using Substitution

The steps for solving the linear equations $3x + 2y = 19$ and $x + y = 8$ using the substitution method are as follows:

Step 1:
Subtract x from both sides of $x + y = 8$

Step 2:
Replace "y" with "8 − x" in the other equation

Step 3:
Solve using the usual algebra methods

$$3x + 2y = 19$$
$$y = 8 - x$$

$$3x + 2(8 - x) = 19$$
$$y = 8 - x$$

$$3x + 16 - 2x = 19$$
$$y = 8 - x$$

Solving Systems of Linear Equations Using Substitution

The steps for solving the linear equations $3x + 2y = 19$ and $x + y = 8$ using the substitution method are as follows:

Step 4:
Solve $3x – 2x$

$x + 16 = 19$
$y = 8- x$

Step 5:
Solve $19 - 16$

$x = 3$
$y = 8 - x$

Step 6:
Put $x = 3$ in equation $y = 8 - x$

$x = 3$
$y = 8 – 3$
$y = 5$

Answer:
$x = 3, y = 5$

Solving Systems of Linear Equations Using Elimination: Steps

The steps for solving systems of linear equations using the elimination method are as follows:

01 Multiply an equation by a constant (except zero)

02 Add (or subtract) an equation onto another equation

Example:
$3x + 2y = 19$
$x + y = 8$

Solving Systems of Linear Equations Using Elimination

The steps for solving the linear equations 3x + 2y = 19 and x + y = 8 using the elimination method are as follows:

Step 1:
Multiply the second equation by 2

$3x+2y=19$
$2x + 2y = 16$

Step 2:
Subtract the second equation from the first equation

$x = 3$
$2x + 2y =16$

Step 3:
Multiply the second equation by ½

$X =3$
$X + Y = 8$

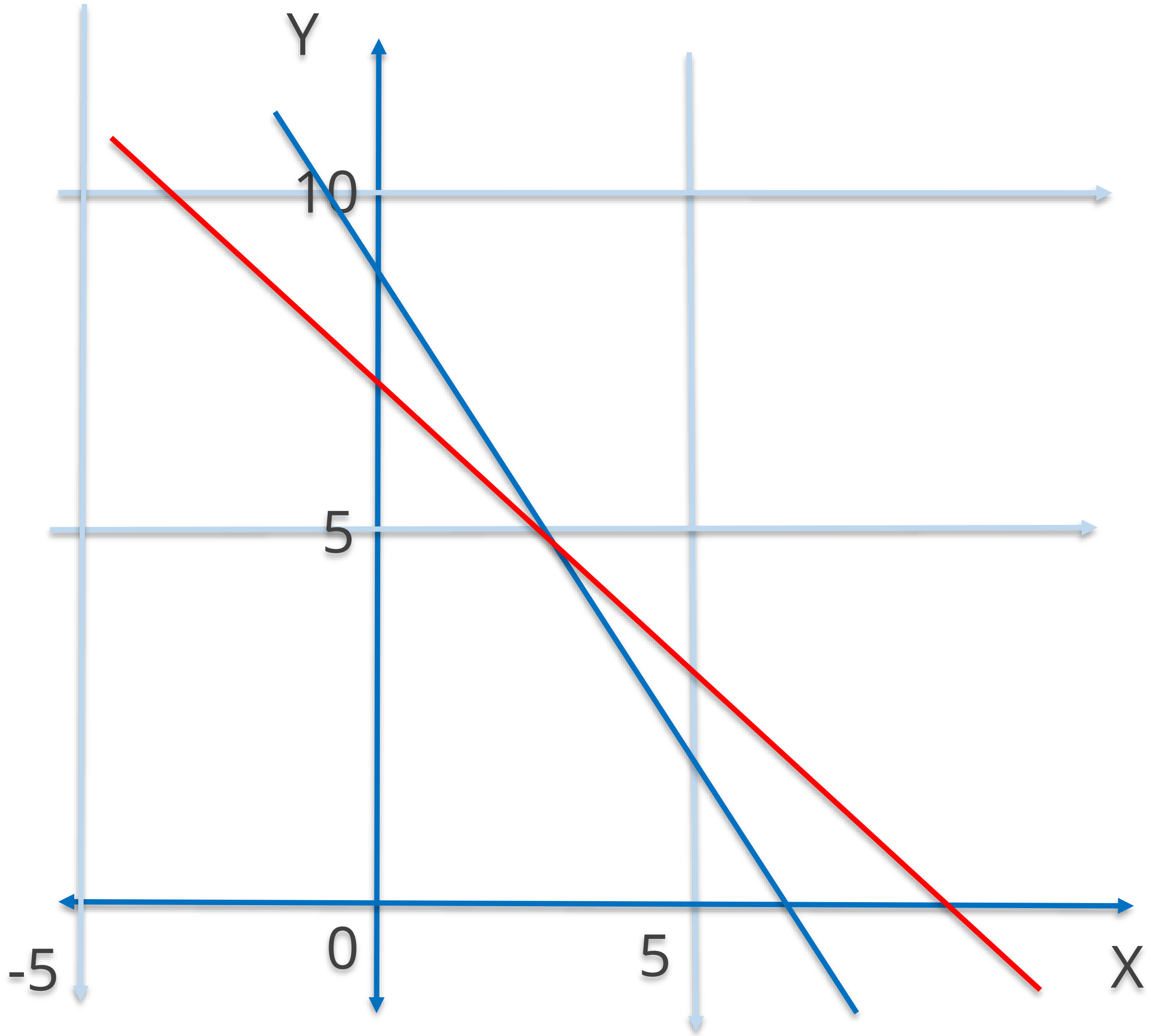
Step 4:
Subtract the first equation from the second equation

$y = 5$

Answer: X=3, Y=5

Solving Systems of Linear Equations Using Elimination

The following graph shows the intersection of the below lines:
3x + 2y=19 and x + y=8



The blue line represents the point at which 3x + 2y = 19 is true.

The red line represents the point at which x + y = 8 is true.

The solution is found at x = 3 and y = 5, where both lines intersect.

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Introduction to Matrices

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Matrix

A matrix is a rectangular array or table with rows and columns of numbers, symbols, or expressions used to represent a mathematical object or an attribute.

Example

$$A = \begin{pmatrix} 2 & 1 & 3 \\ -1 & 2 & 4 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$$

Matrix Size

The matrix's size is expressed as **m X n**
Where:
m is the number of rows
n is the number of columns

A matrix with two rows and three columns is referred to as a **two by three matrix**, a **2X3-matrix**, or a **matrix of dimension 2X3**.

Notation of Matrix

The following is a representation of a matrix with **m** rows and **n** columns:

$$\begin{array}{cc} & \overbrace{\begin{array}{cccc} 1 & 2 & \cdots\cdots & n \end{array}}^{\text{Columns}} \\ \text{Rows} \left\{ \begin{array}{c} 1 \\ 2 \\ 3 \\ \vdots \\ m \end{array} \right. & \begin{bmatrix} a_{11} & a_{12} & \cdots\cdots & a_{1n} \\ a_{21} & a_{22} & \cdots\cdots & a_{2n} \\ a_{31} & a_{32} & \cdots\cdots & a_{3n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots\cdots & a_{mn} \end{bmatrix} \end{array} = \text{Am x n}$$

Forms of Matrix

An element's entry in the matrix of form aii is located on the diagonal.

The matrix is termed a square matrix if n = m, then the number of columns and rows is equal.

A is called a diagonal matrix if aij = 0, where i ≠ j

Matrix Operations

Matrix Operations: Addition

Consider the following two matrices:

Example

$$A = \begin{pmatrix} 22 & 32 \\ 11 & 16 \end{pmatrix} \qquad B = \begin{pmatrix} 13 & 8 \\ 13 & 16 \end{pmatrix}$$

$$A + B = \begin{pmatrix} 22 + 13 & 32 + 8 \\ 11 + 13 & 16 + 16 \end{pmatrix}$$

$$A + B = \begin{pmatrix} 35 & 40 \\ 24 & 32 \end{pmatrix}$$

Matrix Operations: Addition Rules

Only matrices with the same number of rows and columns can be added.

Matrix addition follows the Commutative Property.
A + B = B + A

Matrix Operations: Subtraction

Consider the following matrices:

Example

$$A = \begin{pmatrix} 22 & 32 \\ 11 & 16 \end{pmatrix} \qquad B = \begin{pmatrix} 13 & 8 \\ 13 & 16 \end{pmatrix}$$

$$A - B = \begin{pmatrix} 22 - 13 & 32 - 8 \\ 11 - 13 & 16 - 16 \end{pmatrix}$$

$$A - B = \begin{pmatrix} 9 & 24 \\ -2 & 0 \end{pmatrix}$$

Matrix Operations: Subtraction Rules

Only matrices with the same number of rows and columns can be subtracted.

Matrix subtraction does not follow the Commutative Property.

$$A - B \neq B - A$$

Matrix Operations: Multiplication

Consider the following matrices:

Example

$$A = \begin{pmatrix} 22 & 32 \\ 11 & 16 \end{pmatrix} \qquad B = \begin{pmatrix} 13 & 8 \\ 13 & 16 \end{pmatrix}$$

$$A.B = \begin{pmatrix} (22 \times 13) + (32 \times 13) & (22 \times 8) + (32 \times 16) \\ (11 \times 13) + (16 \times 13) & (11 \times 8) + (16 \times 16) \end{pmatrix}$$

$$C = \begin{pmatrix} 702 & 688 \\ 351 & 344 \end{pmatrix}$$

The 1^{st} and 2^{nd} rows of A are multiplied with the 1^{st} and 2^{nd} columns of B and added.

Matrix Operations: Multiplication Rules

Let AB = C. Use the formula $C_{ik} = \Sigma_j A_{ij}B_{jk}$ to calculate the value of each member in the 2 x 2 matrix C.

The matrix product AB is defined only when the number of columns in A equals the number of rows in B.

The matrix product BA is defined only when the number of columns in B equals the number of rows in A.

AB is not always equal to BA.

Matrix Operations: Transpose

A transpose is a matrix formed by turning all the rows of a given matrix into columns and vice versa. The transpose of matrix A is denoted as AT.

Example

$$A = \begin{pmatrix} 22 & 32 \\ 11 & 16 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 22 & 11 \\ 32 & 16 \end{pmatrix}$$

Matrix Operations: Inverse

If A is a non-singular square matrix, there exists a n x n matrix A^{-1} , known as A's inverse matrix, that satisfies the following property:

Formula

$$AA^{-1} = A^{-1}A = I$$

where I is the Identity matrix

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

AB = BA = In
Where:
In denotes the nXn identity matrix.

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Special Matrix Types

Symmetric Matrix

A matrix A is said to be symmetric if A = AT

Diagonal Matrix

A matrix D is diagonal only if Dij = 0 for all i ≠ j

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Identity Matrix

The identity matrix is denoted as In, where:
A X In = A

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Special Matrix Types: Tensors

- Tensors are arrays with more than two axes.
- A tensor can have N dimensions.
- $A_{i,j,k}$ is the value at the coordinates i, j, k.

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Introduction to Vectors

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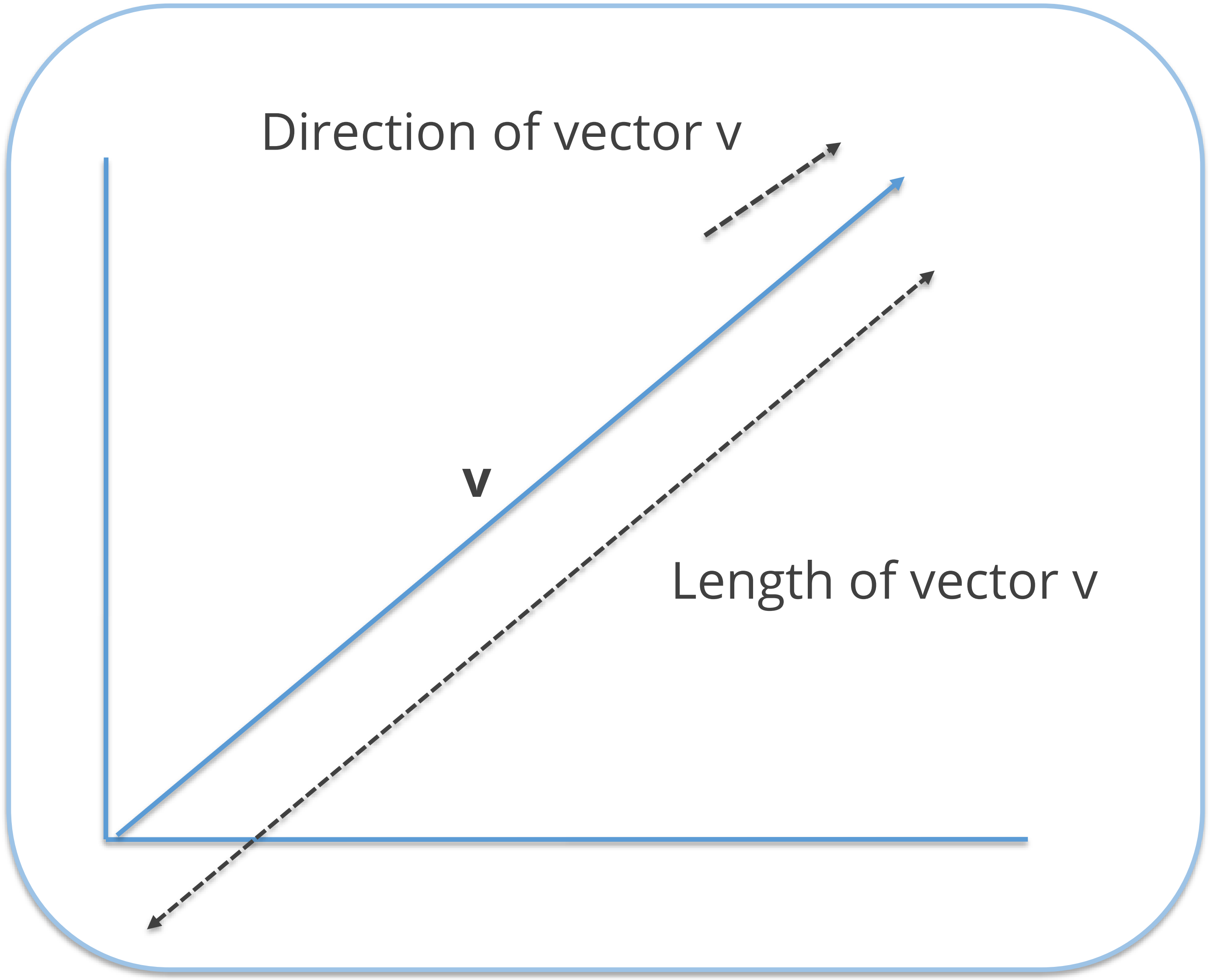
Vector

Objects with both magnitude and direction are called vectors. The magnitude of the vector determines its size.



Vector

- It is represented as a line with an arrow, where the length of the line indicates the vector's magnitude and the arrow points in the desired direction.
- Other names for it include Euclidean vector, Geometric vector, Spatial vector, and simply "vector."
- Its length is indicated by the symbol ||v|| and it begins at the origin (0,0).

Notation of a Vector

The standard form of representation of a vector is:

Formula

$$A = a\hat{i} + b\hat{j} + c\hat{k}$$

Where:
a, b, and c are numeric values.
The unit vectors along the x, y, and z axes are i^, j^, and k^ respectively.

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Types and Properties of Vector

Types of Vectors

The vectors are of following types:

Vector Name	Description
Zero Vector	Vector with zero magnitude
Unit Vector	Vector whose magnitude is one unit
Coinitial Vector	Two or more vectors with same initial point
Collinear Vector	Two or more vectors lying on the same or parallel lines
Equal Vector	Two or more vectors with same magnitude and direction
Negative Vector	Vectors with same magnitude but opposite direction as that of the given vector

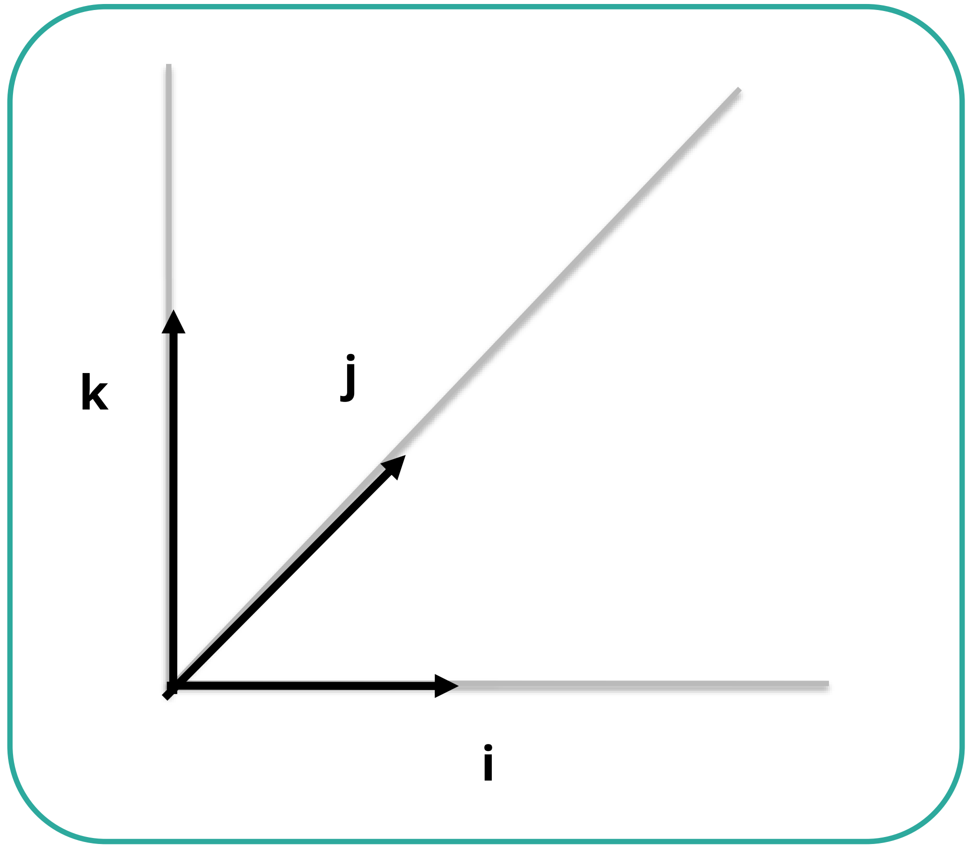
Properties of Vectors

1. A vector having a unit norm or unit length is referred to as a unit vector.

2. If xTy = 0, then a vector x and a vector y are orthogonal to one another. Additionally, this indicates that both vectors are 90 degrees apart from one another.

3. An orthonormal vector is an orthogonal vector with a unit norm.

Properties of Vectors



- In this case, vectors i and k are orthogonal.
- If they are taken to be of the unit norm, they may be orthonormal.
- k and j are not orthogonal vectors.
- j and i are not orthogonal vectors.

Properties of Vectors

A square matrix that has columns and rows that are mutually orthonormal is an orthogonal matrix.

$A^TA = AA^T = I$ in this instance

$A^{-1} = A^T$ if the matrix is orthogonal.

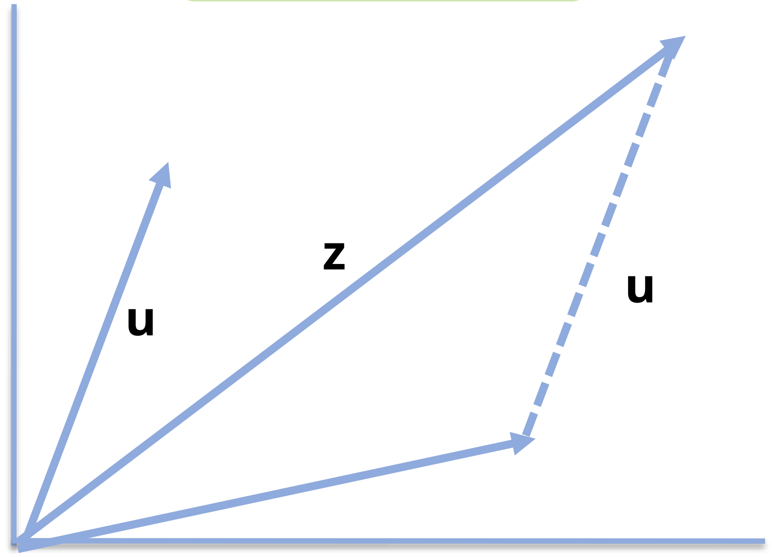
Vector Operations

Vector Operations: Addition

Vector addition is the process of adding two or more vectors to create a vector sum.

The vector sum for the two vectors **u** and **v** is:

u + v = z

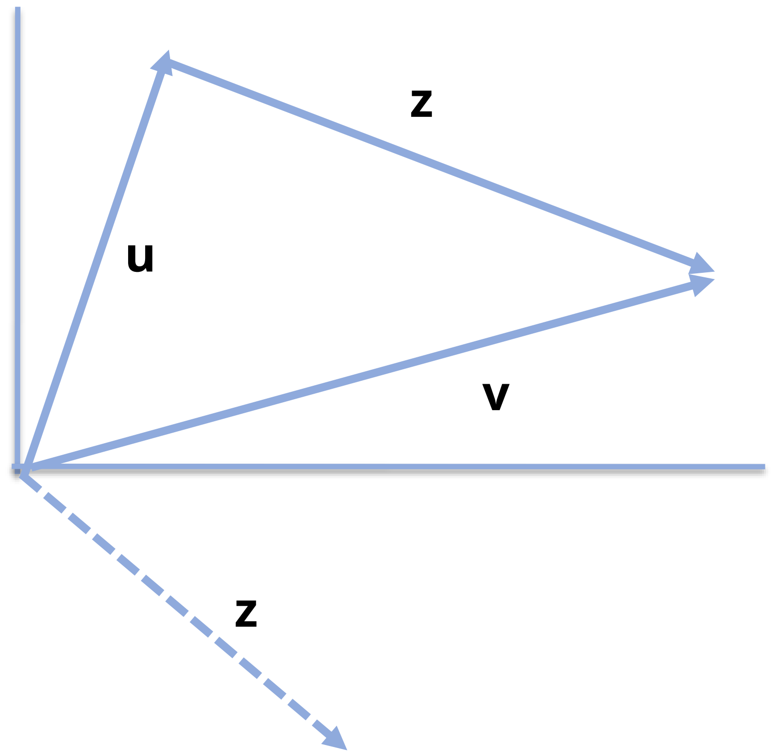


Vector Operations: Subtraction

Vector subtraction is the process of subtracting two or more vectors to get a vector difference.

The vector difference for the two vectors **u** and **v** is:

u - v = z



Vector Operations: Multiplication

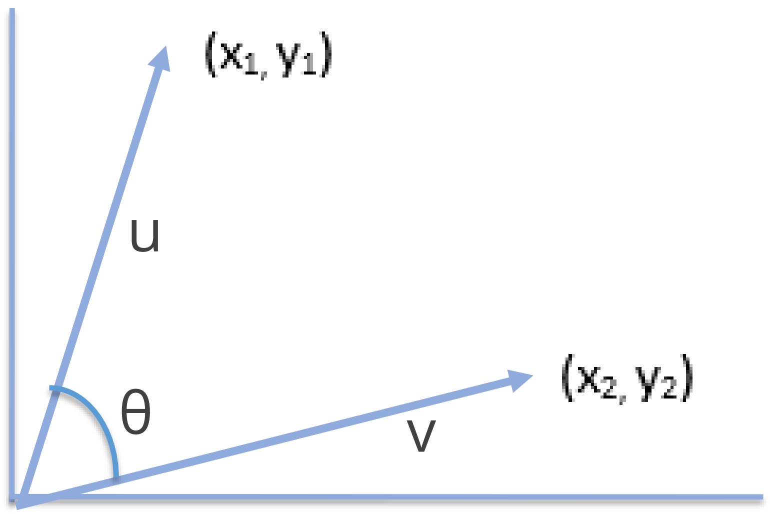
The term **vector multiplication** describes a method for multiplying two or more vectors by themselves.

u * v = z

$$\mathbf{u*v = x_1x_2 + y_1y_2 = \sum(x_iy_i)}$$

This is equivalent to:

$$\mathbf{u*v = \|x\| \|y\| \cos\theta}$$



Vector Operations: Norm

A vector's size is referred to as a norm in machine learning. The distance from the origin to a given point is expressed as the vector's norm.

Consider a vector with a Lp norm where p ≥ 1.

Example

L^p norm

$$||x||_p = \left(\sum_i |x_i|^p \right)^{\frac{1}{p}}$$

Vector Operations: Norm Features

The L2 norm, often known as the Euclidean norm, has p = 2 and is the most widely used norm.

It is the Euclidean distance between the origin and point x.

The 2 in the L2 norm is often omitted, thus $||x||_2$ is simply written as $||x||$

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Vector Operations: Norm Features

The vector size is frequently measured as squared L2 norm and equals xTx. This is preferable since its differential is dependent on x.

Equation

$$L1norm, p = 1 :$$

$$||x||_1 = \sum_i |x_i|$$

Vector Operations: Norm Features

- The L1 norm increases by every time an element of x moves away from 0 by ϵ.
- Infinite maximum norm can be equated as follows:

Equation

$$p : ||x||_\infty = max_i |x_i|$$

Key Takeaways

- Linear algebra finds systematic methods for solving systems of linear equations.
- The three forms of linear equations are standard form, slope-intercept form, and point-slope form.
- Different methods of solving linear equations are the graphing method, substitution method, elimination method, and matrix method.
- A matrix is an m × n array of scalars from a given field. The individual values in the matrix are called entries.
- Vectors are objects which have both magnitude and direction.

Knowledge Check

Knowledge Check 1

Which of the following is the necessary condition for the existence of a solution to this system?

A. ‘A’ must be invertible

B. ‘b’ must be linearly depended on the columns of A

C. ‘b’ must be linearly independent of the columns of A

D. None of these

Knowledge Check 1

Which of the following is the necessary condition for the existence of a solution to this system?

A. 'A' must be invertible

B. 'b' must be linearly depended on the columns of A

C. 'b' must be linearly independent of the columns of A

D. None of these

The correct answer is **A**

For a set of linear equations, Ax = b, the inverse of matrix A exists (|A| ≠ 0). This is a prerequisite for the existence of a solution for this system.

Knowledge Check 2	Suppose that price of 2 balls and 1 bat is 100 units, then what will be the representation of problems in linear algebra in the form of x and y?

A. $2x + y = 100$

B. $2x + 2y = 100$

C. $2x + y = 200$

D. $x + y = 100$

Knowledge Check 2

Suppose that price of 2 balls and 1 bat is 100 units, then what will be the representation of problems in linear algebra in the form of x and y?

A. $2x + y = 100$

B. $2x + 2y = 100$

C. $2x + y = 200$

D. $x + y = 100$

The correct answer is **A**

Suppose the price of a bat is ₹ 'x' and the price of a ball is ₹ 'y'. Values of 'x' and 'y' can be anything depending on the situation. 'x' and 'y' are variables. Therefore, 2x + y = 100 is the answer.

Knowledge Check 3	What does a linear equation in three variables represent?

A. Flat objects

B. Line

C. Planes

D. Both A and C

Knowledge Check 3

What does a linear equation in three variables represent?

A. Flat objects

B. Line

C. Planes

D. Both A and C

The correct answer is **C**

A linear equation in three variables represents the set of all points whose coordinates satisfy the equations. A linear equation in three variables represents a plane.

Knowledge Check 4

Which of the following is NOT a type of matrix?

A. Square matrix

B. Scalar matrix

C. Trace matrix

D. Term matrix

Knowledge Check 4

Which of the following is NOT a type of matrix?

A. Square matrix

B. Scalar matrix

C. Trace matrix

D. Term matrix

The correct answer is **D**

Term matrix is not a type of matrix in linear algebra.