DATA AND
ARTIFICIAL INTELLIGENCE

Coordinate Geometry

Learning Objectives

By the end of this lesson, you will be able to:

- Explain the fundamentals of coordinate geometry and its uses.
- Define the coordinate plane and the coordinates of a point in coordinate geometry.
- Examine the distance and slope formulae used in coordinate geometry.
- List the various coordinate geometry formulas and their components.

DATA AND
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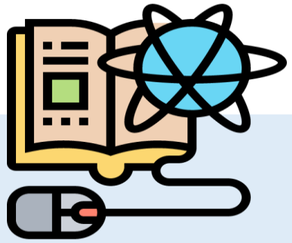
Introduction to Coordinate Geometry

Coordinate Geometry



- Coordinate geometry is the study of geometric figures by plotting them on the coordinate axes.
- It is a branch of mathematics that assists in presenting geometric figures on a two-dimensional plane and to learn the properties of these figures.
- Figures like straight lines, curves, circles, ellipses, hyperbola, and polygons can be easily drawn and presented to scale in the coordinate axes.
- It also aids in algebraic computation and the study of geometric figure attributes using the coordinate system.

The starting points of the coordinate system are zero degrees of **Greenwich Longitude** and zero degrees of **Equator Latitude**.

Coordinate Plane

Coordinate Plane

A cartesian plane divides the plane space into two dimensions and is useful for easily locating the points.

It is also referred to as the coordinate plane.

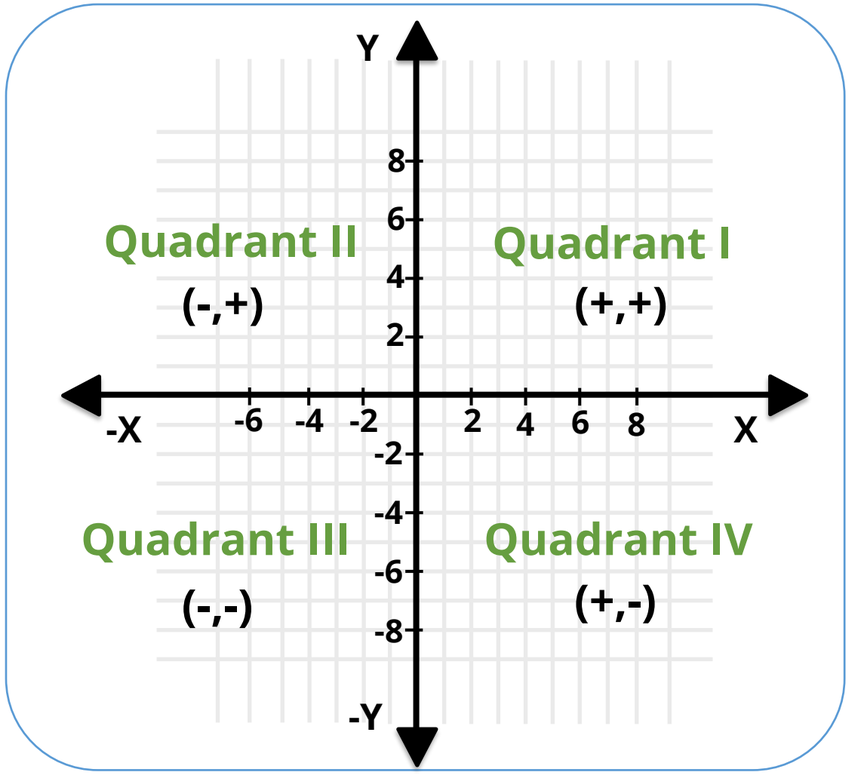
The two axes of the coordinate plane are the horizontal x-axis and the vertical Y-axis.

These coordinate axes divide the plane into four quadrants, and the point of intersection of these axes is the origin (0, 0).

Any point in the coordinate plane is referred to by a point (x, y), where the x and y values are the position of the point with reference to the x-axis and y-axis, respectively.

Coordinate Plane: Properties

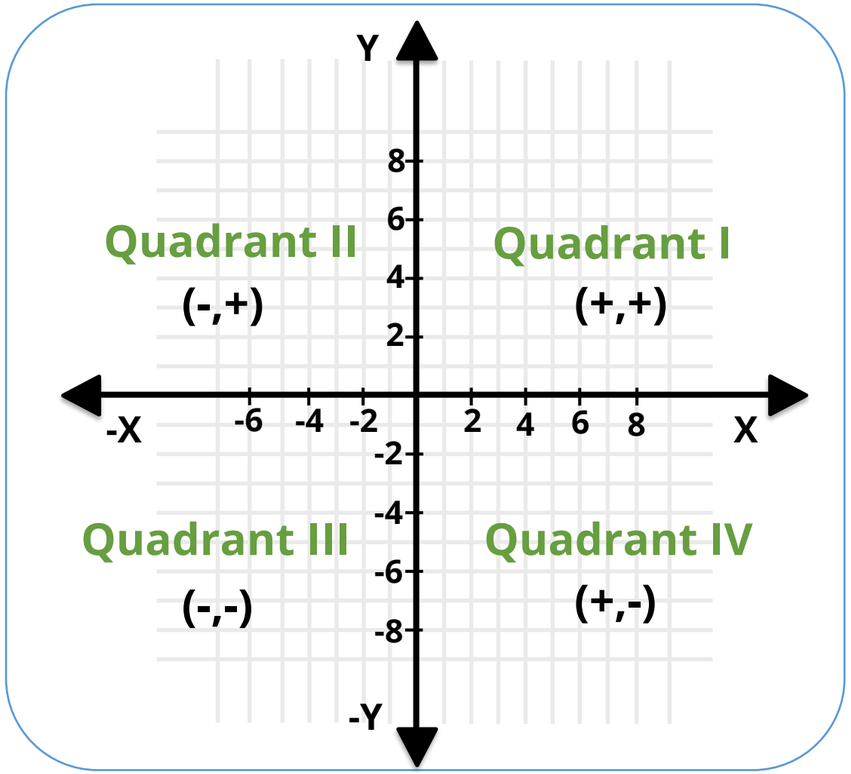
The properties of the point represented in the four quadrants of the coordinate plane are as follows:



- The origin O is the point of intersection of the x-axis and the Y-axis and has the coordinates (0, 0).
- The X-axis to the right and left of the origin O are the positive and negative X-axis, respectively.
- The Y-axis above and below the origin O are the positive and negative Y-axis, respectively.
- The point represented in the first quadrant (X, Y) has both positive values and is plotted with reference to the positive X-axis and the positive Y-axis.

Coordinate Plane: Properties

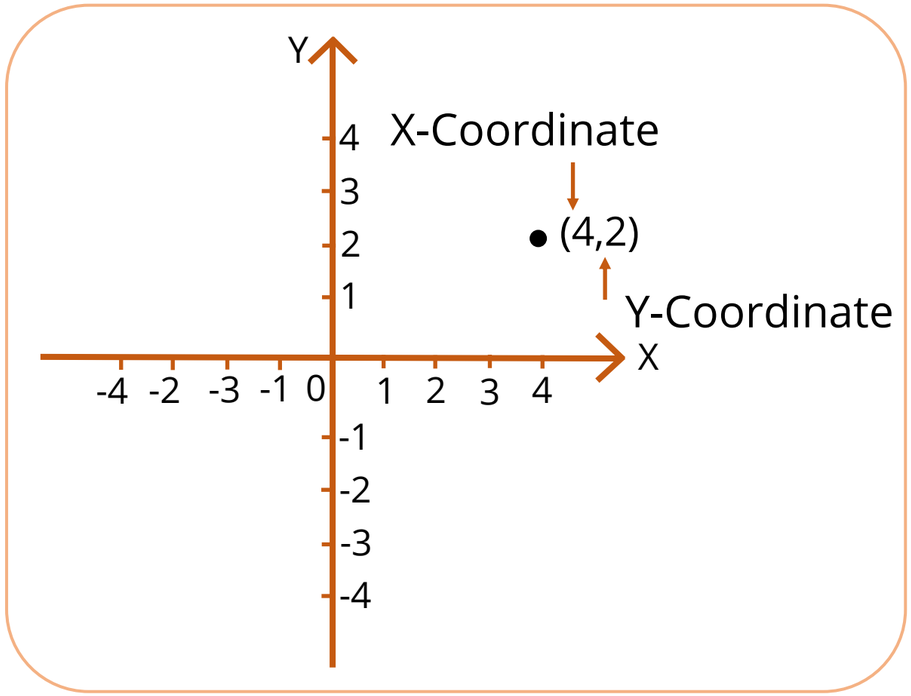
The properties of the point represented in the four quadrants of the coordinate plane are as follows:



- The point represented in the second quadrant is (-X, Y) and is plotted with reference to the negative X-axis and positive Y-axis.
- The point represented in the third quadrant (-X, -Y) is plotted with reference to the negative X-axis and negative Y-axis.
- The point represented in the fourth quadrant (X, -Y) is plotted with reference to the positive X-axis and negative Y-axis.

Coordinates of a Point

A coordinate is an address, which helps to locate a point in space.

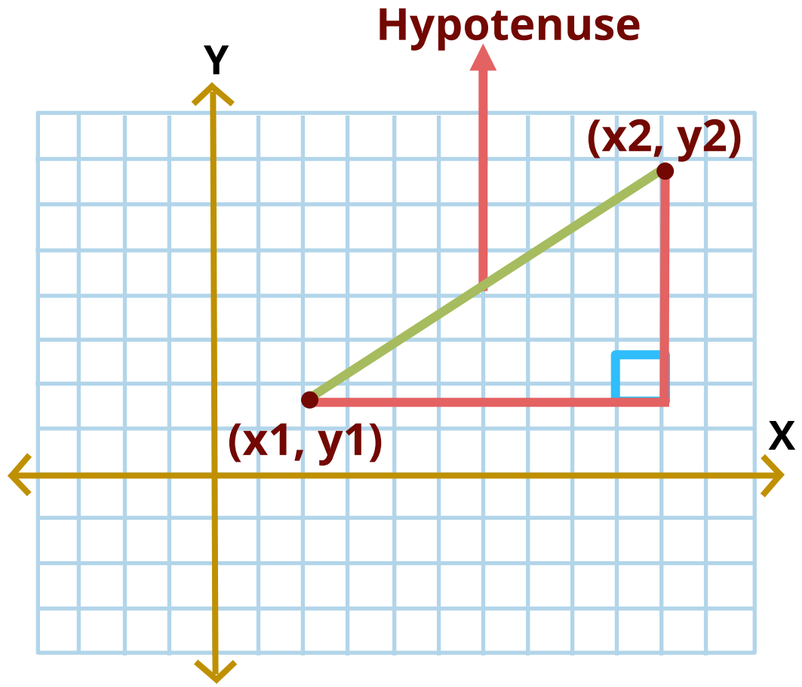


- For a two-dimensional space, the coordinates of a point are (X, Y).
- **Abscissa:** It is the X value in the point (X, Y), and is the distance of this point along the X-axis, from the origin.
- **Ordinate:** It is the Y value in the point (X, Y), and is the perpendicular distance of the point from the X-axis, which is parallel to the Y-axis.
- The coordinates of a point are useful to perform numerous operations of finding distance, midpoint, the slope of a line, and equation of a line.

Coordinate Geometry Formulas

Distance Formula

The distance formula is used to calculate the distance between two points, which is the length of the line segment connecting them.



- Consider two points A and B having coordinates as (x1,y1) and (x2,y2) respectively.
- The distance between these two points is calculated as follows:

Distance Formula

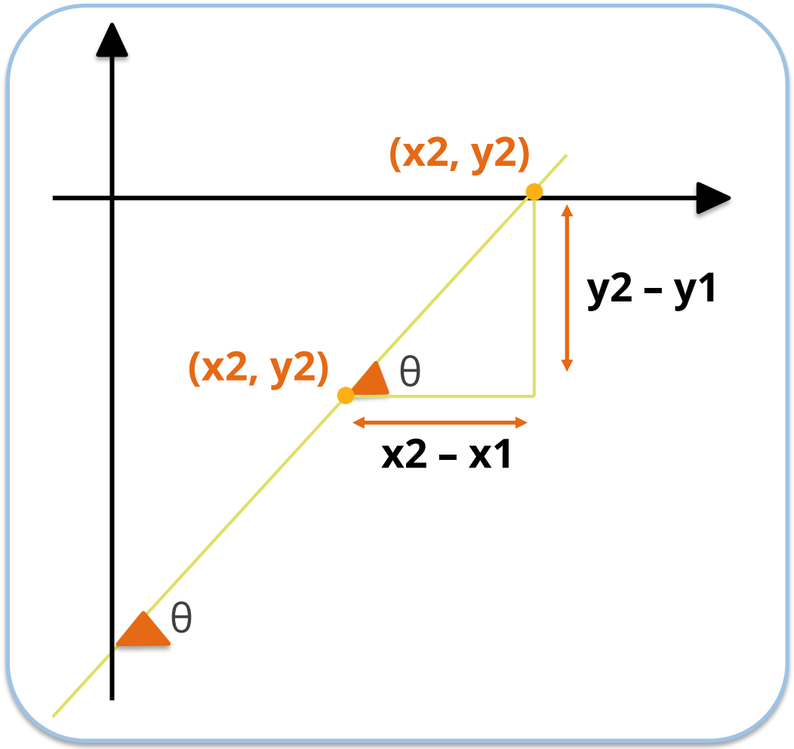
$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

where, $(x_2 - x_1) = Change\ in\ x$

$(y_2 - y_1) = Change\ in\ y$

Slope Formula

The slope of a line, often known as its gradient, is a numerical representation of the steepness and direction of the line.

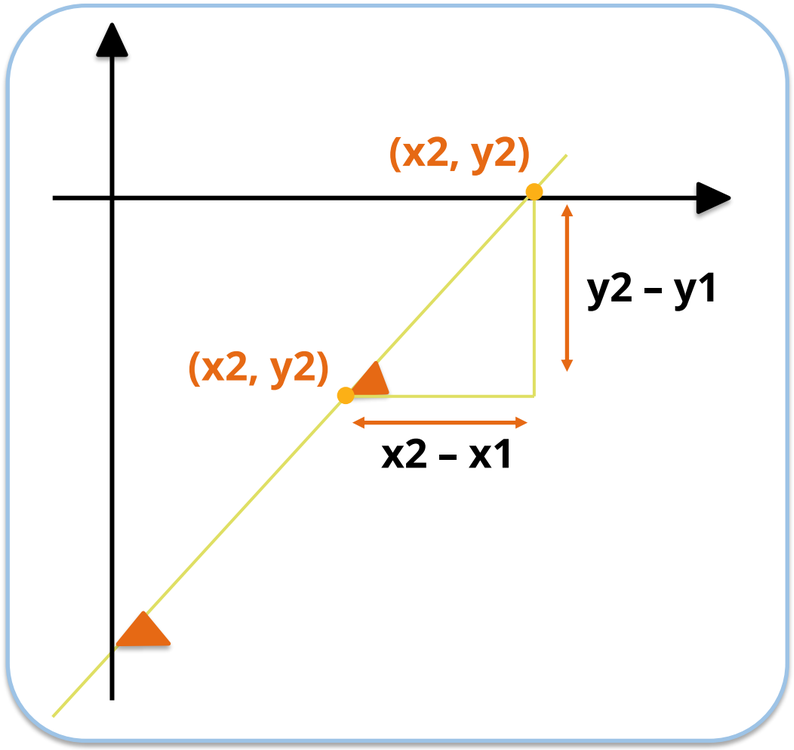


- It is the difference between the change in the x and y coordinates.
- It can be calculated by taking any two consecutive points or by using the angle that the line on the positive x-axis forms.
- The slope of the vertical line at an angle θ with the x-axis vertical is:

$$m = \tan(\theta)$$

Slope Formula

The slope of a line, often known as its gradient, is a numerical representation of the steepness and direction of the line.



- For a line volume made up of two points, the slope at an angle θ can be calculated using the formula below :

Slope Formula

$$\tan(\theta) = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan(\theta)$$

Coordinate Geometry Formulas

Formula Name	Formula
Slope Formula	$Slope, m = \frac{Change\ in\ y}{Change\ in\ x} = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$ • Parallel lines have equal slopes • The slopes of perpendicular lines are opposite reciprocals of each other
General Formula	$Ax + By = C$
Slope Intercept Form	$y = mx + b$ • Where, m is the slope and b is the y-intercept

Coordinate Geometry Formulas

Formula Name	Formula
Point Slope Form	$(y - y_1) = m(x - x_1)$ • Where, m is the Slope
Midpoint Formula	$(x_m, y_m) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right)$
Distance Formula	$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Key Takeaways

- Coordinate geometry is a discipline of mathematics that studies geometric figures using the coordinate system and assists in algebraic computations.
- The coordinate plane splits plane space into two dimensions by having two axes that further divide the plane into four quadrants, with the origin (0,0) being the point of intersection of these axes.
- The distance formula is used to find the distance between two lines in a coordinate plane.

Key Takeaways

- The slope of a line is a numerical representation of its steepness and direction.
- The midpoint formula is used to find a point that is halfway between two points in a coordinate plane.

Knowledge Check

Knowledge Check

1

The distance of the point P(2, 3) from the x-axis is

A. 2

B. 3

C. 1

D. 5

Knowledge Check

1

The distance of the point P(2, 3) from the x-axis is

A. 2

B. 3

C. 1

D. 5

The correct answer is **B**

The distance from x-axis is equal to its ordinate i.e., 3

Knowledge Check

2

The distance of the point (α, β) from the origin is

A. $\alpha + \beta$

B. $\alpha^2 + \beta^2$

C. $|\alpha| + |\beta|$

D. $\sqrt{\alpha^2 + \beta^2}$

Knowledge Check

2

The distance of the point (α, β) from the origin is

A. $\alpha + \beta$

B. $\alpha^2 + \beta^2$

C. $|\alpha| + |\beta|$

D. $\sqrt{\alpha^2 + \beta^2}$

The correct answer is **B**

The distance of (α, β) **from origin** $(0, 0) = \sqrt{(\alpha - 0)^2 + (\beta - 0)^2} = \sqrt{\alpha^2 + \beta^2}$