filter — freq. selecting circuit

Active filter | Passive filter

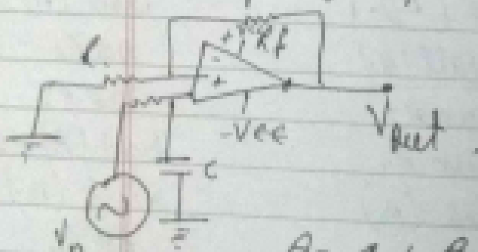
Active components (Ex: op-amp, transistor)

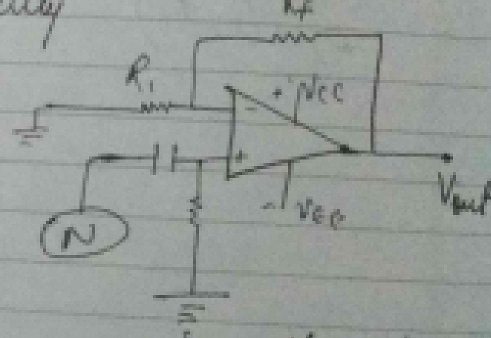
Passive components Ex (L, C, R) — RC, RL, RLC

$V_{out} > V_{in}$

$V_{out} < V_{in}$ gain less.

low pass filter | high pass filter



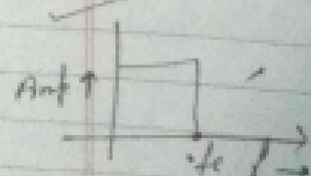


$A_v = 1 + \frac{R_f}{R_1}$ v. gain

$f_c = \frac{1}{2\pi RC}$

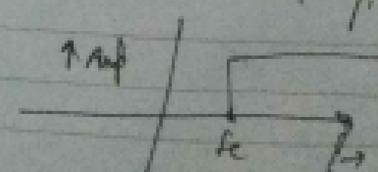
1st order low pass active filter

0 — fc



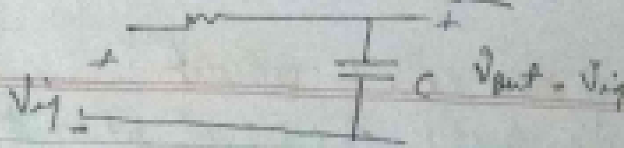
RC low

1st order high pass active filter



RC high

RC low pass filter.



$$V_{out} = \frac{X_c}{X_c + R} \times V_{in}$$

$$|X_c| = \frac{1}{\omega C} \qquad f_c = \frac{1}{2\pi RC}$$

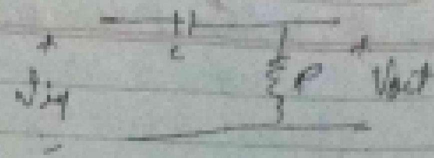
$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{|X_c|}{\sqrt{R^2 + X_c^2}}$$

$$\frac{1}{\sqrt{2}} = \frac{\frac{1}{\omega C}}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$$

$$\frac{1}{2} = \frac{\left(1/\omega C\right)^2}{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$= \omega = \frac{1}{RC}$$

RC high pass.

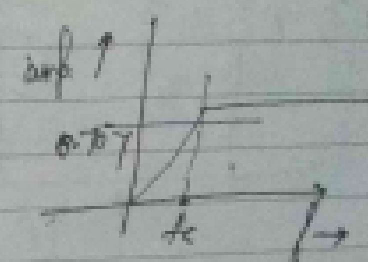


$$|X_c| = \frac{1}{\omega C}$$

$$V_{out} = \frac{R}{R + X_c} \times V_{in}$$

$\omega = 0 \Rightarrow V_{out} = 0$

$\omega = \infty \Rightarrow V_{out} = V_{in}$



$$f_c = \frac{1}{2\pi RC}$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{R}{\sqrt{R^2 + X_c^2}}$$

$$\frac{1}{2} = \frac{R^2}{R^2 + X_c^2}$$

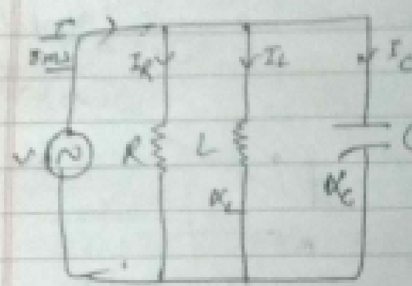
$$R^2 = X_c^2 = \frac{1}{\omega^2 C^2}$$

$$\omega^2 = \frac{1}{R^2 C^2} \Rightarrow \omega = \frac{1}{RC}$$

$$f_c = \frac{1}{2\pi RC}$$

Condition of resonance in ~~series~~ R-L-C circuit

resonance occurs in a parallel RLC circuit when the total circuit current is "in-phase" with the supply voltage as the two reactive components cancel each other. At resonance the admittance of the circuit is at its minimum and is equal to the conductance of the circuit.



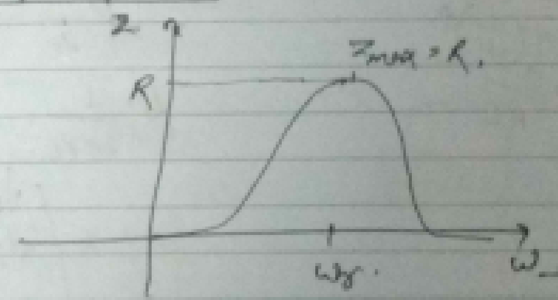
$X_L = X_C$

Inductive reactance = Capacitor reactance

voltage and current are in phase.

Impedance is max =

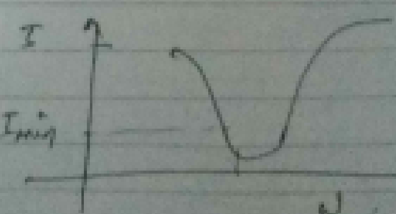
Current is minimum,



$\omega = 0 \Rightarrow X_L = 0$

$\omega = \infty \Rightarrow X_C = \frac{1}{\omega C} = 0$

current & freq:



$I = I_R + I_L + I_C$

$= \frac{V}{R} + \frac{V}{X_L} + \frac{V}{X_C}$

$= V\left[\frac{1}{R} + \frac{1}{j\omega L} + j\omega C\right]$

$= V\left(\frac{1}{R} + j\left[\omega C - \frac{1}{\omega L}\right]\right)$

$=$ Y = admittance. (Inverse of impedance)

$\boxed{I = V \cdot Y}$ rms.

for get the minimum value of Y in we have ..

$$X_L = \frac{1}{\omega C} \Rightarrow Y = \frac{1}{R}$$

$$\Rightarrow I = \frac{V}{R}$$

$\boxed{Z = R}$ Impedance is max^m.

$$\omega^2 = \frac{1}{LC}$$

$$\omega = \frac{1}{\sqrt{LC}}$$

current lags volt. (I) III

$$\cdot f_r = \frac{1}{2\pi\sqrt{LC}}$$ Inductance $X_C > X_L$

for low freq, the circuit will behave as a short circuit and the current will flow through the inductance at $\omega = 0$ $\Rightarrow X_L = 0$

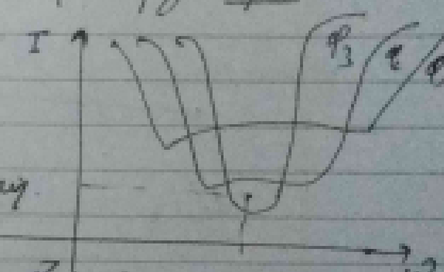
and at $\omega = \infty \Rightarrow X_C = 0$,

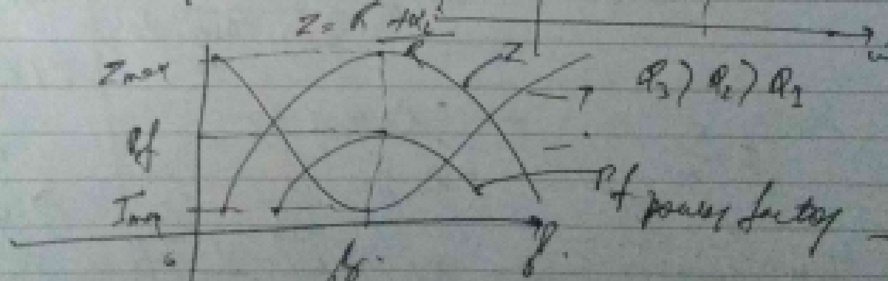
for high freq.

current leads volt.

power factor ($\cos\phi$) = 1
resistive

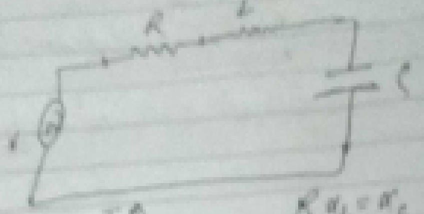
quality factor:





freq response curve of a parallel resonance circuit shows that the magnitude of the current is a function of freq. and plotting this onto a graph, show us that the response starts at its max^m value,

Series LRC circuit



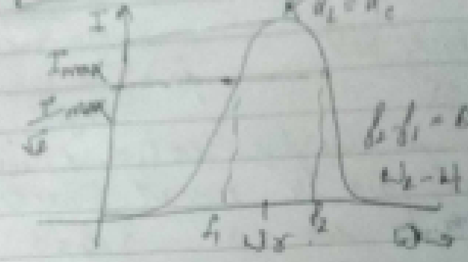
$V = V_0 \sin \omega t$

$X_L = \omega L$

$X_C = \frac{1}{\omega C}$

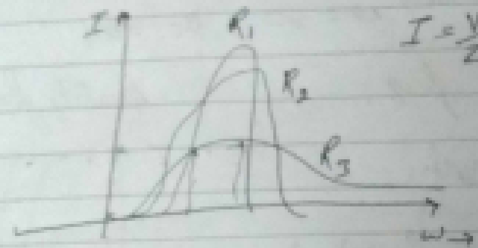
at $\omega = 0$ $X_C = \infty$

$\omega = \infty$ $X_L = \infty$



$f_2 - f_1 = B.W$

$\omega_2 - \omega_1 = B.W$



$I = \frac{V}{Z}$ $R_3 > R_2 > R_1$

Q/ ⇒ $Q = 2\pi \times \frac{\text{energy stored}}{\text{energy dissipated}}$

$Z = R + X_L + X_C$

$= R + j\omega L - \frac{j}{\omega C}$

$Z = R + j\left[\omega L - \frac{1}{\omega C}\right]$

$I = \frac{V}{Z}$

$I_{max} = \frac{V}{Z_{min}}$

$\omega L - \frac{1}{\omega C} = 0 \Rightarrow Z_{min} = R$

$X_L = X_C$

$\Rightarrow \omega L = \frac{1}{\omega C} \Rightarrow Z_{min}$

$I_{max} = \frac{V}{R} \Rightarrow f = \frac{1}{2\pi\sqrt{LC}}$

$\omega L = \frac{1}{\omega C}$

$\omega^2 = \frac{1}{LC}$

$\Rightarrow \omega = \frac{1}{\sqrt{LC}}$

$Q = \frac{I^2 X_L}{I^2 R} = \frac{X_L}{R} = \frac{X_C}{R} = \frac{\omega L}{R} = \frac{1}{\omega C R}$

$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$ $\boxed{Q = \frac{\omega_r}{B.W}}$

$I = \frac{V}{R}$

Laplace transform of unit step function

unit step funⁿ:

$$f(t) \text{ or } u(t) = \begin{cases} 0, & t<0 \\ 1, & t\geq 0 \end{cases}$$

$$L\{f(t)\} = f(s) = \int_0^\infty e^{-st} f(t)\,dt.$$

$$\Rightarrow L\{u(t)\} = u(s) = \int_0^\infty e^{-st} u(t)\,dt$$

$$= \int_0^\infty e^{-st} \cdot 1\,dt.$$

$$\Rightarrow u(s) \; \frac{e^{-st}}{-s}\Big|_0^\infty = \frac{-1}{s}[0-1] = 1/s$$

unit impulse function:

$$u(t-a) \text{ or } u(t-a) = \begin{cases} 0, & t<a. \\ 1, & t\geq a. \end{cases}$$

$$u(t) = \begin{cases} 1, & a<t<b \\ 0, & \text{otherwise} \end{cases} \Big\} - \text{impulse function}$$

$$L\{u(t-a)\} = \int_0^\infty e^{-st} u(t-a)\,dt$$

$$= \int_0^a 0 \cdot e^{-st} u(t-a)\,dt + \int_a^\infty e^{-st} u(t-a)\,dt.$$

$$= \int_a^\infty e^{-st} \cdot 1\,dt.$$

$$= \frac{-1}{s}(e^{-as})$$

$$L\{u(t)\} = \int_0^a e^{-st} u(t)\,dt + \int_a^b e^{-st} u(t)\,dt + \int_b^\infty e^{-st} u(t)\,dt$$

$$\Rightarrow \int_a^b e^{-st} u(t)\,dt = \frac{e^{-st}}{-s}\Big|_a^b = e^{-sb} - e^{-as} \Big/ \frac{-1}{s}$$

Laplace transformation of any function $f(t)$ in time domain is -

$$L[f(t)] = \int_0^\infty f(t) e^{-st} dt$$

$= F(s)$ where $s = \sigma + j\omega$

The function is said to be laplace transform when $\int_0^\infty f(t) e^{-st} dt < \infty$

This transformation is in time domain $0 < t < \infty$ to freq. domain for analysis.

inverse of this is.

$$L^{-1}[F(s)] = f(t) = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} F(s) e^{st} ds.$$

$$L[e^{-at} f(t)] = \int_0^\infty e^{-at} f(t) e^{-st} dt$$

$$= \int_0^\infty f(t) e^{-(s+a)t} dt$$

$$= F(s+a)$$

Convolution theorem: This theorem state that the convolution of two real function is equal to the multiplication of their respective function.

if $L[f_1(t)] = F_1(s)$ & $L[f_2(t)] = F_2(s)$

then by convolution it can be written as:

$$L[f_1(t) * f_2(t)] = F_1(s) \cdot F_2(s)$$

let $F(s) = L[f_1(t) * f_2(t)]$

$$= \int_0^\infty \left[\int_0^t f_1(t-\tau) f_2(\tau) d\tau\right] e^{-st} dt$$

$$= \int_0^\infty \left[\int_0^{t=\infty} f_1(t-\tau) f_2(\tau) u(t-\tau) e^{-st} d\tau\right] dt.$$

since $u(t-\tau) = 0$ for all $t < \tau$, the inverse integral is multiplied by $u(t-\tau)$ & upper limit of inner integral becomes $t \to \infty$

$$F(s) = \int_0^\infty f_2(\tau)\left[\int_0^\infty f(t-\tau)\,u(t-\tau)\,e^{-st}\,dt\right]d\tau$$

$$= \int_0^\infty f_2(\tau)\,e^{-s\tau}\,F_1(s)\,d\tau$$

$$= F_1(s)\int_0^\infty f_2(\tau)\,e^{-s\tau}\,d\tau = F_1(s)\cdot F_2(s).$$

$$L\left(f_1(t)e^{-st}dt\right) = F_1(s)$$

$$L\left(f_2(t)e^{-st}dt\right) = F_2(s)$$

$$L(\sin\omega t) = L\left[\frac{e^{j\omega t} - e^{-j\omega t}}{2j}\right]$$

$$= \frac{1}{2j}\left[\int_0^\infty e^{j\omega t}e^{-st}\,dt - \int_0^\infty e^{-j\omega t}e^{-st}\,dt\right]$$

$$= \frac{1}{2j}\left[\int_0^\infty e^{-(s-j\omega)t}\,dt - \int_0^\infty e^{-(s+j\omega)t}\,dt\right]$$

$$= \frac{1}{2j}\left[\frac{1}{s-j\omega} - \frac{1}{s+j\omega}\right] = \frac{\omega}{s^2+\omega^2}$$

$$L(\cancel{\sin\omega t}\ \cos\omega t) = L\left[\frac{e^{j\omega t} + e^{-j\omega t}}{2}\right]$$

$$= \int_0^\infty \left(\frac{e^{j\omega t} + e^{-j\omega t}}{2}\right)e^{-st}\,dt$$

$$= \frac{1}{2}\left[\int_0^\infty e^{j\omega t}e^{-st}\,dt + \int_0^\infty e^{-j\omega t}e^{-st}\,dt\right]$$

$$= \frac{1}{2}\left[\int_0^\infty e^{-(s-j\omega)t}\,dt + \int_0^\infty e^{-(s+j\omega)t}\,dt\right]$$

$$= \frac{1}{2}\left[\frac{1}{s-j\omega} + \frac{1}{s+j\omega}\right] = \frac{1}{2}\,\frac{2s}{(s^2+\omega^2)} = \frac{s}{s^2+\omega^2}$$

Ramp function: $f(t) = t$ (A unit ramp.)

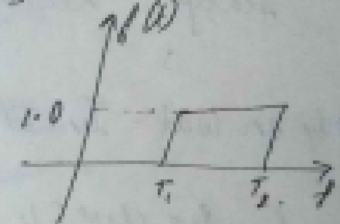
$$\mathcal{L}[f(t)] = \int_0^\infty t \cdot e^{-st}\,dt$$

Integrating by parts, $= uv - \int v\,du$.   where, $u = t$, $dv = \frac{e^{-st}}{-s}$, $du = dt$, $v = \frac{e^{-st}}{-s}$

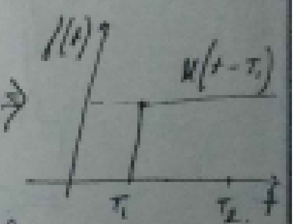
$$= t \cdot \frac{e^{-st}}{-s}\Big|_0^\infty - \int_0^\infty \frac{e^{-st}}{-s} \cdot 1 \cdot dt$$

$$= \frac{1}{s^2}.$$

gate function:



$\Rightarrow$



Applying the laplace transform we have

$$\mathcal{L}(f(t)) = \int_0^\infty f(t)\,e^{-st}\,dt.$$

Also, we have;

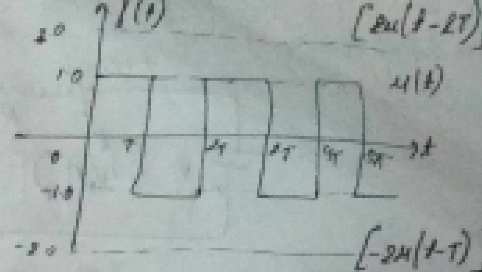
$$\mathcal{L}[u(t-T_1) - u(t-T_2)]$$

$$= \mathcal{L}[u(t-T_1)] - \mathcal{L}[u(t-T_2)]$$

$$= \int_0^\infty u(t-T_1)e^{-st}\,dt - \int_0^\infty u(t-T_2)e^{-st}\,dt$$

$$= \frac{e^{-sT_1}}{s} - \frac{e^{-sT_2}}{s}$$

2



according to laplace we will have,

$$f(t) = u(t) - 2u(t-T) + 2u(t-2T) + \cdots$$

$$\Rightarrow L(f(t)) = \int_0^\infty u(t)e^{-st}dt - 2u(t-T) + 2u(t-2T) + \cdots$$

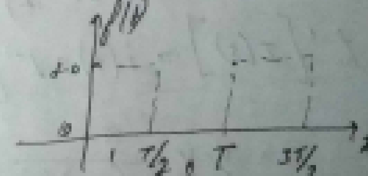
$$= \int_0^\infty e^{-st}dt - 2u(t-T) + \cdots$$

$$= \frac{1}{s}\left|e^{-st}\right|_0^\infty$$

$$= \frac{1}{s} - 2\frac{e^{-Ts}}{s} + 2\frac{e^{-2Ts}}{s} + \cdots$$

$$= \frac{1}{s}\left[1 - 2e^{-Ts}(1 - e^{-Ts} + e^{-2Ts} - e^{-3Ts} + \cdots)\right]$$

3



$$f(t) = 1; \quad 0 \le t \le T/2$$
$$= 0; \quad T/2 \le t \le T$$

$$L(f(t)) = \left(\frac{1}{1-e^{-sT}}\right)\left[\int_0^{T/2} 1 \cdot e^{-st}dt + \int_{T/2}^{T} 0 \times e^{-st}dt\right]$$

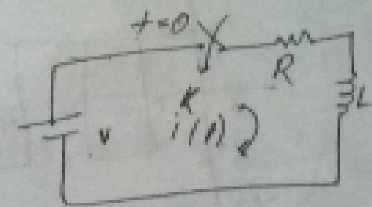
$$= \frac{1}{s}\frac{(1-e^{-sT/2})}{(1-e^{-sT})}$$

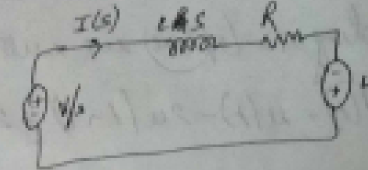
$V = \frac{V}{R}$

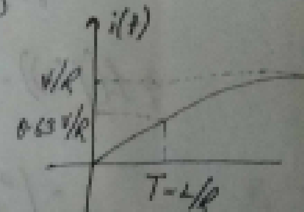
$I \cdot \quad I =$

$V = IR$

$I = \frac{V}{R} = \frac{5}{3}$







KVL:

$$L\frac{di(t)}{d(t)} + Ri(t) = V$$

Taking laplace transform:

$$L[sI(s) - i(0+)] + RI(s) = V/s$$

Now, $i(0+) = 0$ as the initial current through inductor L is zero.

so, $$I(s) = \frac{V}{Ls(s+R/L)}$$

$$L^{-1}[I(s)] = i(t) = L^{-1}\left[\frac{A}{s} + \frac{B}{s+R/L}\right]$$

where,

$$A = \frac{V}{L} \times \frac{s}{s(s+R/L)}\Bigg|_{s=0} = V/R$$

$$B = \frac{V}{L}\frac{(s+R/L)}{s(s+R/L)}\Bigg|_{s=-R/L} = -V/R$$

$$i(t) = \frac{V}{R}L^{-1}\left[\frac{1}{s} - \frac{1}{s+R/L}\right] = V/R\left[1 - e^{-t/(L/R)}\right]$$

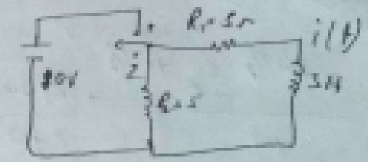
for $t \geq 0$.

at $t=0$; $i(t) = 0$

$t = \infty$; $i(t) = V/R$

$t = L/R$; $i(t) = V/R(1-e^{-1}) = 0.63\ V/R$.

$T = L/R$ is known as time constant of circuit & is defined as the interval after which current rises to 63% of steady state value



let us determine the steady state condition just before the switch is moved to position 2.

i.e; $i_L(0-) = \frac{10}{5} = 2A = i_L(0+)$

when the switch is in position 2 by KVL;

$\Rightarrow L\frac{di}{dt} + (R_1+R_2)i = 0$

now taking laplace transform:-

$\Rightarrow L[sI(s) - i_L(0+)] + (R_1+R_2)I(s) = 0$

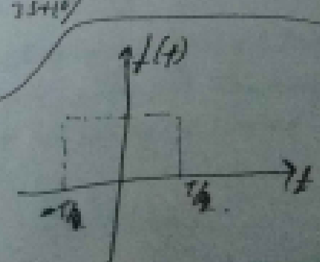
As, $L = 3H$, $R_1 = R_2 = 5\Omega$

$I(s)(3s+10) = Li_L(0+) = 3$.

$I(s) = \frac{3}{3(s+10)} = \frac{3}{3s+10}$

$\Rightarrow i(t) = L^{-1}(I(s)) = L^{-1}\left(\frac{3}{3s+10}\right)$

$= 3e^{-\frac{10}{3}t}$



$f(t) = \begin{cases} 0 & \text{for } t < -T/2 \\ 1 & \text{for } -\frac{T}{2} < t < \frac{T}{2} \\ 0 & \text{for } t > \frac{T}{2} \end{cases}$

$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt$

$= \int_{-\infty}^{-T/2} f(t)e^{-j\omega t}\,dt + \int_{-T/2}^{T/2} f(t)e^{-j\omega t}\,dt + \int_{T/2}^{\infty} f(t)e^{-j\omega t}\,dt.$

$$= \int_{-T/2}^{T/2} (1) e^{-j\omega t}\, dt = \left[\frac{e^{-j\omega t}}{-j\omega} \right]_{-T/2}^{T/2}$$

$$= \frac{e^{j\omega T/2} - e^{j\omega T/2}}{j\omega} = \frac{2j \sin(\omega T/2)}{j\omega}$$

$$= \frac{2}{\omega} \sin(\omega T/2)$$

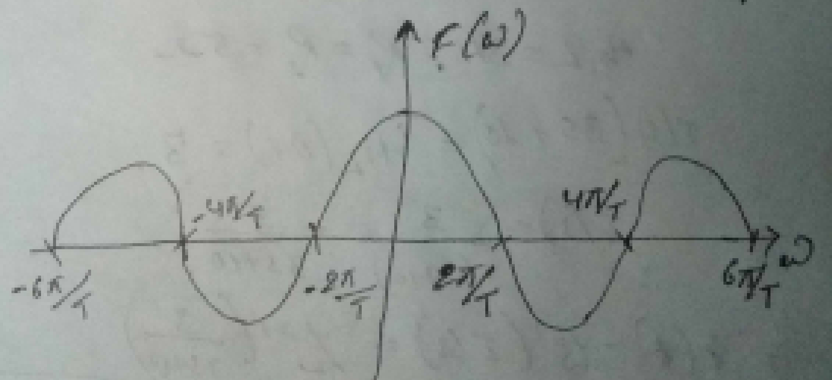
$$F(\omega) = \frac{T \cdot \sin(\omega T/2)}{(\omega T/2)} = \frac{T \sin x}{x} = T\, Sa(x)$$

where, $x = \omega T/2$ & $Sa(x) = \frac{\sin x}{x}$ is called sampling function.

Now, when, $x \to 0$, $Sa(x) \to 1$

& $x = \pm n\pi$, $Sa(x) = 0$

$\therefore \omega T/2 = \pm m\pi$, $\omega = \pm \frac{2\pi}{T} m$.

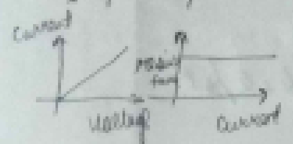


Q) diff between linear and non-linear network?

linear.

Any network whose parameters are constant i.e they donot change with voltage or current passing through that network.

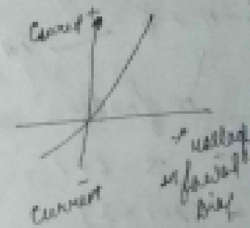
principle of superposition is valid only in linear network.

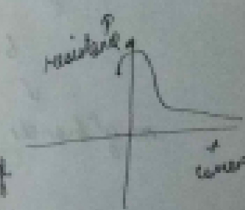


non-linear

It is that network whose parameters are changed with voltage or current passing through it.

Ex network having diode rectifier, any parameter of diode i.e diode resistance is changed with voltage across it.





diff between node & mesh

node

A terminal of any branch of a network common to two or more branches is known as node. Voltage of any node with respect to ground is the node voltage. voltage between any pair of nodes is node pair voltage.

mesh

A set of branches forming a closed path, with the omission of any branch making the path open - but not have any other circuit inside it.

Active

A network containing energy sources (voltage or current sources) together with other circuit elements.

passive

A network containing circuit elements without any energy source (as voltage or current source)

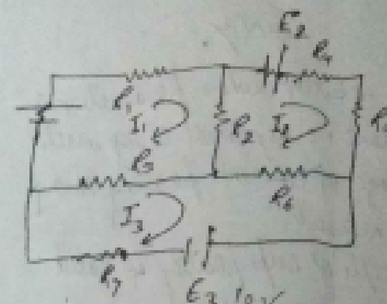
lumped

A network in which physical separate resistors, capacitors & inductors can be represented.

distributed

Network in which resistors, capacitors & inductors cannot be electrically separated & individually isolated as separate elements.

Ex. Transmission network.



no. of loop eq^n to be solved!

$b - n + s$ — no. of separate part

b → no. of branches, n → no. of nodes

$= 8 - 6 + 1$

$= 2 + 1 = 3$

In loop (i)

$$I_1R_1 + (I_1 - I_2)R_2 + (I_1 - I_3)R_3 - E_1 = 0$$

$$\Rightarrow I_1(R_1 + R_2 + R_3) - I_2R_2 - I_3R_3 = E_1 \quad \text{— (i)}$$

In loop (ii)

$$-E_2 + I_2R_4 + I_2R_5 + (I_2 - I_3)R_6 + (I_2 - I_1)R_2 = 0$$

$$\Rightarrow I_2(R_4 + R_5 + R_6 + R_2) - I_1R_2 - I_3R_6 = E_2 \quad \text{— (ii)}$$

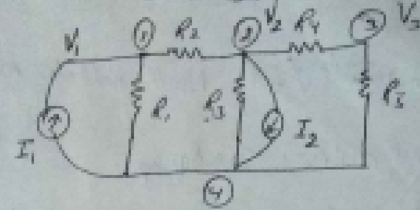
In loop (iii)

$$(I_3 - I_1)R_3 + (I_3 - I_2)R_6 + E_3 \cancel{=0} + I_3R_7 = 0$$

$$\Rightarrow I_3(R_3 + R_6 + R_7) - I_1R_3 - I_2R_6 = -E_3 \quad \text{— (iii)}$$

$$\begin{bmatrix} (R_1+R_2+R_3) & -R_2 & -R_3 \\ -R_2 & (R_4+R_5+R_6+R_2) & -R_6 \\ -R_3 & -R_6 & (R_3+R_6+R_7) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ -E_3 \end{bmatrix}$$

node analysis



for same.

$n = 1$

$n = 4 - 1$

$= 3$ eq $\rightarrow$

$$\begin{bmatrix} \frac{1}{2} - \frac{1}{2j} & +\frac{1}{2j} & 0 \\ \frac{1}{2j} & -\frac{1}{2j} + \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix} \begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ \end{bmatrix}$$

if $(n-1) > (b-n+1)$

$\rightarrow$ loop analysis is preffered.

if $(n-1) < (b-n+1)$

$\rightarrow$ nodal analysis is preffered.

at node - I using (KCL):

$$I_1 - V_1/R_1 - (V_1 - V_2)/R_2 = 0$$

$$V_1\left(\frac{1}{R_1} \cdot 1 + \frac{1}{R_2}\right) - V_2 \cdot \frac{1}{R_2} = I_1 \quad \text{— (i)}$$

At node - II;

$$-I_2 - V_2/R_3 - \frac{(V_2 - V_1)}{R_2} + \frac{V_3 - V_2}{R_4} = 0$$

$$\Rightarrow I_2 = V_2\left(\frac{1}{R_3} + \frac{1}{R_2} + \frac{1}{R_4}\right) - \frac{V_1}{R_2} - \frac{V_3}{R_4} \quad \text{— (ii)}$$

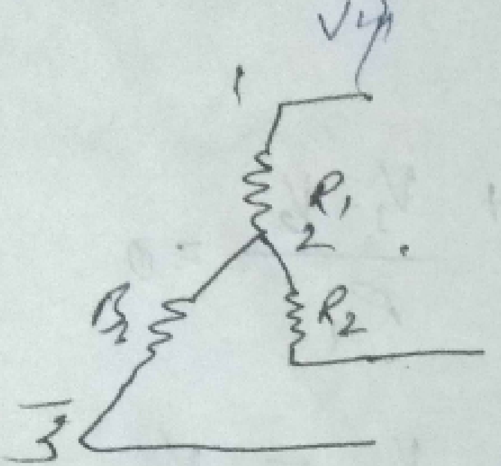
At node - III:

$$-\frac{V_3}{R_5} - 0 + \frac{(V_2 - V_3)}{R_4}$$

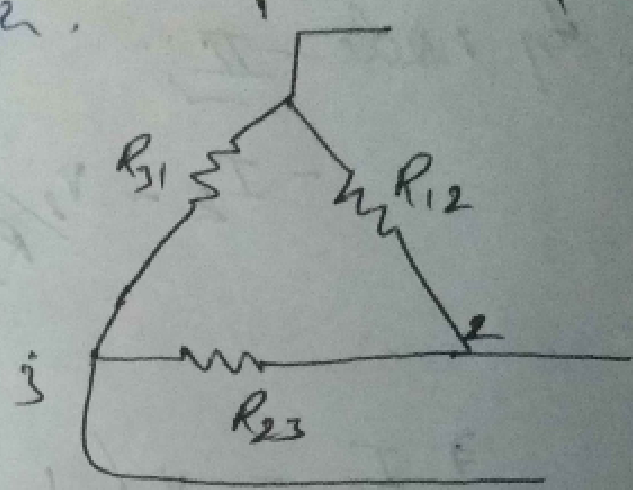
$$V_3\left(\frac{1}{R_5} + \frac{1}{R_4}\right) - \frac{V_2}{R_4} \quad \text{— (iii)}$$

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} & 0 \\ -\frac{1}{R_2} & \frac{1}{R_3} + \frac{1}{R_2} + \frac{1}{R_4} & -\frac{1}{R_4} \\ 0 & -\frac{1}{R_4} & \frac{1}{R_5} + \frac{1}{R_4} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ -I_2 \\ 0 \end{bmatrix}$$

Shyambazar.



star network



eqn delta network.

resi betn (1) & (ii)

$R_1 \| (R_2 + R_3)$

$$\frac{R_1 \times (R_2 + R_3)}{R_1 + R_2 + R_3}$$

$= R_{12} \,\cancel{+}\, \cancel{R_{23}}$ — (i)

(ii) $R_2 \| (R_3 + R_1)$

R_{23} — (ii) $\quad R_{31}$ — (iii)

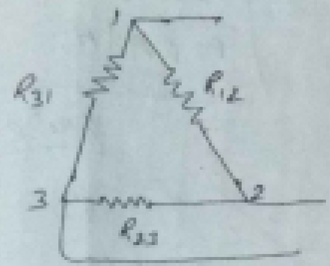
$$R_{12} \alpha R_{23} = \frac{R_1 \times (R_2 + R_3)}{R_1 + R_2 + R_3} \cdot \frac{R_2 (R_3 + R_1)}{R_1 + R_2 + R_3}$$

$$R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

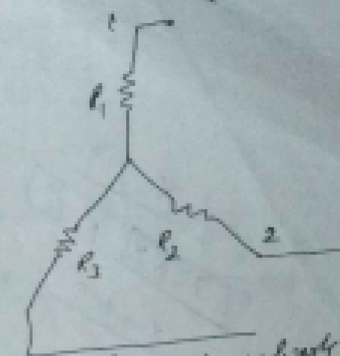
$$R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1}$$

$$R_{31} = R_3 + R_1 + \frac{R_3 R_1}{R_2}$$

delta/star conversion



delta network



Equi star network

resistance between node (1) & (2): $R_{12} \| (R_{23} + R_{31})$

$$= \frac{R_{12} \times (R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}}$$

= equivalent resistance between node 1 & 2 in star network.

$= R_1 + R_2$ --- (i)

resistance between node (2) & (3): $\frac{R_{23} \| (R_{31} + R_{12})}{R_{12} + R_{23} + R_{31}}$

$= R_2 + R_3$ — (ii)

similarly,

$$\frac{R_{31} \times (R_{23} + R_{12})}{R_{12} + R_{23} + R_{31}} = R_3 + R_1 \text{ — (iii)}$$

sub eqn (i) from (ii) and adding result to eqn (iii), we get,

$R_2 + R_3 - R_1 - R_2$ $\quad$ $R_3 - \cancel{R_1} + R_3 + \cancel{R_1}$

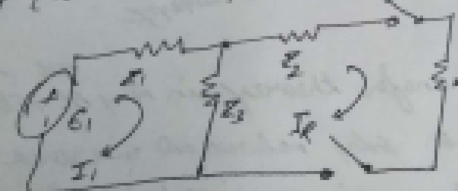
$$2R_3 = \frac{2R_{23}R_{31}}{R_{12} + R_{23} + R_{31}}$$

$$R_2 = \frac{R_{12} \cdot R_{23}}{R_{12} + R_{23} + R_{31}} \qquad R_1 = \frac{R_{12}R_{31}}{R_{12} + R_{23} + R_{31}}$$

* Circuit will resonate when reactive or watt less component of current is zero. & frequency is called resonance freq.

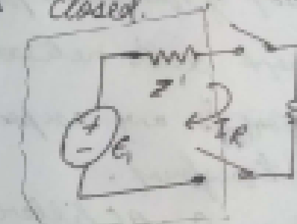
Thevenin's theorem! Norton's theorem!

Any two terminal linear, bilateral, active network containing energy sources (voltage or current sources) & impedency can be replaced with an equivalent circuit consisting of a voltage source E' in series with an impedance Z'. The value of E' is the open circuit voltage between the terminals of the network & Z' is the impedance measured between the terminals of network with all energy sources eliminated (but not their internal impedency if any).



I_R is flown when Thevenin's circuit is closed.

$\Rightarrow$



when Z_R is open:

$$E_1 = I_1(Z_1 + Z_3)$$

$$I_1 = E_1/(Z_1 + Z_3)$$

at open circuit of Z_R terminal no I_R is flowing. so, no voltage drop in Z_2 ($\because Z_2 I_R = 0$)

So, open circuit voltage $= E_1' = Z_3 I_1 = \dfrac{E_1 Z_3}{Z_1 + Z_3}$

Thevenin's Equivalent is

$$Z' (Z_1 \,//\, Z_3) + Z_2 = \frac{Z_1 Z_3}{Z_1 + Z_3} + Z_2$$

$$= \frac{Z_1 + Z_3 + Z_2 Z_1 + Z_2 Z_3}{Z_1 + Z_3}$$

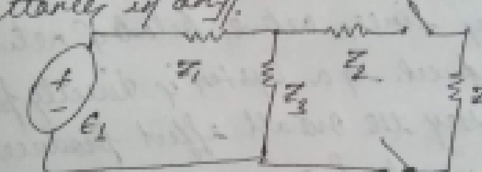
So current passing through

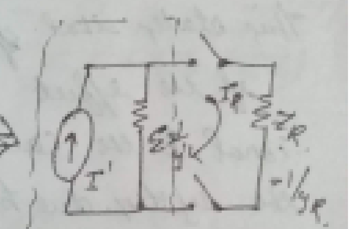
$$Z_R = E_1 \Big/ \left[(Z' + Z_R) - \left(\frac{E_1 Z_3}{Z_1 + Z_3} \right) \right]$$

$$\left\{ \left(\frac{Z_1 Z_3 + Z_1 Z_2 + Z_2 Z_3}{Z_1 + Z_3} \right) + Z_R \right\}$$

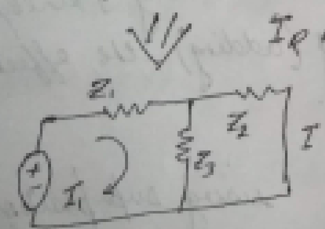
Norton's

Any two terminal linear, active, bilateral network containing energy sources (voltage or current sources) can be replaced by an equivalent circuit consisting of a current source I' in parallel with an admittance Y'. The value of I' is the short circuit current between the terminals of the network or Y' is the admittance measured between the terminals with all energy sources eliminated (but not their internal admittances if any).





Norton's



I_R is flown when closed circuit

At loop-I

$$E_1 - I_1 Z_1 + (I_1 - I') Z_3 = 0$$

$$I_1 (Z_1 + Z_3) - I' Z_3 = E_1 \quad \text{--- (i)}$$

at loop-II

$$Z_2 I' + (I' - I_1) Z_3 = 0$$

$$I_1 Z_3 = I' (Z_2 + Z_3)$$

$$I_1 = I' (Z_2 + Z_3) / Z_3 \quad \text{--- (ii)}$$

putting (ii) in (i) at the value of I_1, we get

$$I' = E_1 \Big/ \left\{ \left(1 + \frac{Z_2}{Z_3} \right) \left(1 + Z_1 / Z_3 \right) - Z_3 \right\}$$

Norton's equivalent admittance $Y' =$

where $Z' = (Z_1 // Z_3) + Z_2$

$$= \frac{Z_1 Z_3}{Z_1 + Z_3} + Z_2$$

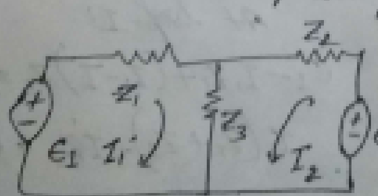
$$= \left(\frac{Z_1 Z_3 + Z_1 Z_2 + Z_3 Z_2}{Z_1 + Z_3}\right).$$

So,

$$I_R = \left(\frac{Z'}{Z' + Z_R}\right) \cdot I'$$

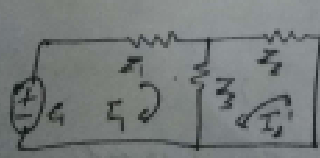
Superposition theorem:

This states that in any linear, active, bilateral network, if the effect produced in a system is directly proportional to the cause, then the overall effect produced in the system due to a number of cause acting jointly, can be determined by superposing (adding) the effects of each source across separately.



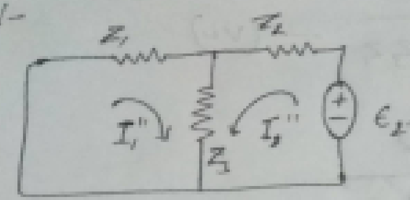
(a)

using superposition, consider E_1 source only. but E_2 is replaced with its internal impedance if any,



(b)

similarly, using that theorey considering only E_2 source but E_1 is replaced by its internal impedence of any:-



(c)

from fig (a)

$$E_1 = I_1(Z_1+Z_3) + I_2 Z_3 \quad -(i)$$

$$E_2 = I_1 Z_3 + I_2(Z_2+Z_3) - (ii)$$

solving!

$$I_1 = \frac{(Z_2+Z_3)E_1}{(Z_1Z_2+Z_2Z_3+Z_3Z_1)} - \frac{Z_3 \cdot E_2}{(Z_1Z_2+Z_2Z_3+Z_3Z_1)} \quad -(iii)$$

$$I_2 = \frac{-Z_3 E_1}{Z_1Z_2+Z_2Z_3+Z_3Z_1} + \frac{(Z_1+Z_2)E_2}{Z_1Z_2+Z_2Z_3+Z_3Z_1} - (iv)$$

from fig(b)

$$E_1 = I'(Z_1+Z_3) + I_2' Z_3$$

$$0 = I_1' Z_3 + I_2'(Z_2+Z_3)$$

solving,

$$I_1' = \left[\frac{Z_2+Z_3}{Z_1Z_2+Z_2Z_3+Z_3Z_1}\right]E_1 - (v)$$

$$I_2' = \frac{-Z_3 \cdot E_1}{Z_1Z_2+Z_2Z_3+Z_3Z_1} \quad -(vi)$$

from fig (c):

$$0 = I_1''(Z_1+Z_3) + I_2'' Z_3$$

$$E_2 = I_1'' Z_3 + I_2''(Z_3+Z_2)$$

solving!

$$I_1'' = \frac{-Z_3 E_2}{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1} \quad \text{— (vi)}$$

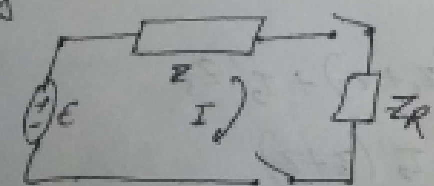
$$I_2'' = \frac{(Z_1 + Z_3) E_2}{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}$$

Now, $I_1 = I_1' + I_1''$ as in eq (vi)

$I_2 = I_2' + I_2''$ as in eq (i)

Maximum power transfer!

Maximum power will be delivered by a network to an impedence Z_R if the impedence of Z_R is the complex conjugate of the impedence of Z of the network measured looking back into the terminals of the network.



I is flow when circuit is closed.

Say, $Z = R + jX$, $Z_R = R_R + jX_R$.

$V = IR$

$$I = \frac{E}{Z + Z_R} = \frac{E}{(R + R_R) + j(X + X_R)}$$

power delivers to load:

$$P = I^2 R_R = \frac{E^2 R_R}{(R + R_R)^2 + (X + X_R)^2}$$

for maximum power delivery:-

$$\boxed{\frac{dp}{dx_R}=0} = \frac{-2E^2 R_R(x_R+x)}{\left[(R_R+R)^2+(x_R+x)^2\right]^2}$$

or, $x_R + x = 0$

$$\boxed{x_R = -x} \qquad \boxed{P = I^2 R_R}$$

putting $x_R = -x$ in power delivery eqn we have,

$$P = \frac{E^2 R_R}{(R_R+R)^2}$$

for max power:-

$$\frac{dP}{dR_R} = 0 = \frac{E^2(R_R+R)^2 - 2E^2 R_R(R+R_R)}{(R_R+R)^4}$$

or $E^2(R_R+R) - 2E^2 R_R = 0$ $\cancel{\left[E^2(R_R+R)^2 - R_R+R)R_R\right.}$

$= \underline{R_R = R}$

So, conclusion is maximum power is delivered to any load if load impedence is complex conjugate of internal network impedance

Maximum power transferred to any load is

$$P_{max} = \frac{E^2 R_R}{(R_R+R)^2} = \frac{E^2 R_R}{(2R_R)^2} = \frac{E^2 R_R}{4R_R^2} = \frac{E^2}{4R_R}$$

Ans