CSCI 570 - HW 7

Solutions

Rubric: This homework will have 50 points in total. Each question 25 points. It is more important that you give feedback, rather than the points. Please make sure to explain why you deduct points.

1. In the network below, the demand values are shown on vertices (supply value if negative). Lower bounds on flow and edge capacities are shown as (lower bound, capacity) for each edge. Determine if there is a feasible circulation in this graph. You need to show all your steps.

b:5

c:-4

a:7

(3,4)

(1,4)

(2,3)

(2,5)

(2,6)

(2,5)

(2,4)

d:-11

e:3

1. Turn the circulation with lower bounds problem into a circulation problem without lower bounds.

Solution: Push a flow on edge equal to the lower bound. Compute L(v) = fin(v) – fout(v).

L(a) = 4, L(b)=-2, L(c)=0, L(d)=-5, L(e)=3

Recompute the demands: d’(v) = d(v) – L(v)

d’(a) =3, d’(b) =7, d’(c) =-4, d’(d) =-6, d’(e) =0

(5 pts)

1. Turn the circulation with demands problem into the maximum flow problem.

Solution: Create a source vertex s and connect it with c and d, with weights 4 and 6 respectively. Create a sink vertex t. Connect a and b with t, weights 3 and 7 respectively.

(5 pts)

1. Does a feasible circulation exist? Explain your answer.

Solution: Yes. (5pts) Compute the maximum flow of the new flow graph. Verify that for each node the flow constraint is satisfied (in=out) and the maximum flow equals 10. (3+7/ 4+6) (!0pts)

1. There is a precious diamond that is on display in a museum at *m* disjoint time intervals. There are *n* security guards who can be deployed to protect the precious diamond. Each guard has a list of intervals for which he/she is available to be deployed. Each guard can be deployed to at most *M* time slots and has to be deployed to at least *L* time slots. Design an algorithm that decides if there is a deployment of guards to intervals such that each interval has either one or two guards deployed.

Solution: We create a circulation network as follows. For the *i*-th guard, introduce a vertex *gi* and for the *j*-th time interval, introduce a vertex *tj*. If the i-th guard is available for the jth interval, then introduce an edge from gi to tj of capacity 1. Add a source s and a sink t. To every guard vertex add an edge from s of capacity M and lower bound L. From every interval vertex add an edge to t of capacity 2 and lower bound 1. Add an edge from t to s of infinite capacity.

(15 pts for formalization as network flow. 5 pts for describing the network nodes. 10 pts for describing the flow on edges.)

(The last step “add an edge..” is in fact not necessary)

(This problem is almost the same as the survey design problem in lecture 08)

Next we reduce this to a problem with no lower bounds. We will get the following demands: d(s) = n L, d(*gi*) = -L, d(*tj*) = 1, d(t) = -m.

Then we reduce it to a flow problem, by creating a super source S and super sink T.

We connect S to t and *gi*. We connect s and *tj*  to T. We also reduce capacities.

Claim: there exists a valid deployment if and only if the above network has a max flow equal to Max（m, nL）.

(10 pts for converting the problem)