HW_05_Gupta_S

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1.You hypothesize that the average person is smarter than Sarah Palin. You know her IQ is 100. You give an IQ test to 100 randomly selected people,and get a mean of 104 and standard deviation of 22. Please show your work for each question.

a. What is your null hypothesis?

$H_0 = 100$

b. What is your research hypothesis?

$H_a > 100$

c. What is your test statistic?

$Sample : n = 100, \bar{x} = 104, s = 22, se = \frac{s}{\sqrt{n}} = 2.2$

$TestStatistic = \frac{\bar{x} - \mu_{population}}{se}$

$TestStatistic = \frac{104-100}{2.2} = 1.82$

d. Do you prefer a one-tailed or two-tailed test here, and why?

According to the hypothesis, it is sufficient to do a 1-t test, however, I would prefer a 2 test to get rid of the slightest possibility of getting the sample statistic in the opposite direction of expected. The 2-sided test will allow for both lower extremes and once we reject the Null hypothesis for the two tailed test, we would be assured to get the same result for one-tailed test.

e. I choose alpha=0.05 which is the most common and most conservative value for alpha.

f.

$-1.96 < 1.82 < 1.96$ Here, we are NOT able to reject the null hypothesis with the specifications we have defined .(p-value=0.95 and two_tailed).

f.

Let’s try with the one tailed t test which looks sufficient for this hypothesis:

```
upper <- qt(0.95,99)
upper
```

```
## [1] 1.660391
```

Here, TestStatistic = 1.82 > upper(1.66), so we can reject the null hypothesis which will be a Type I error.

h. $$ CI_{0.95} = [{x} - 1.959964*se, {x} \

- 1.959964*se] $$

$$z_0 = (\bar{x} - \mu_0)/\sigma$$

$$z_0 = 1.82$$

$$-z_{\alpha/2} \leqslant z_0 \leqslant z_{\alpha/2}$$

Therefore,

$$CI_{0.95} = -z_{0.025} \leqslant z_0 \leqslant z_{0.025}$$

```
lower<-qnorm(0.025)
lower
```

```
## [1] -1.959964
```

```
upper<-qnorm(0.975)
upper
```

```
## [1] 1.959964
```

$$CI_{0.95} = -1.96 \leqslant 1.82 \leqslant 1.96$$

i.What is the p-value for your test results?

```
For a 2-sided test, p value will be:
$$P-value = 2[1-\Phi(Z_{0})]$$
$$P-value = 2[1-\Phi(1.82)] = 0.0688$$
```

2. You hypothesize that men and women have different skill levels in playing Tetris. To test this, you have 50 men and 50 women play the game in a controlled setting. The mean score of the men is 1124 with a standard deviation of 200 and the mean score for the women is 1245, also with a standard deviation of 200.

a. Men's sample:

$$n_1 = 50, \bar{x}_1 = 1124, s_1 = 200, se_1 = \frac{s_1}{\sqrt{n_1}} = 28.29$$

Women's sample:

$$n_2 = 50, \bar{x}_2 = 1245, s_2 = 200, se_2 = \frac{s_2}{\sqrt{n_2}} = 28.29$$

$$H_0 = Men\ and\ Women\ have\ same\ skill\ levels\ in\ playing\ Tetris.$$

$$H_a = Men\ and\ Women\ have\ different\ skill\ levels\ in\ playing\ Tetris.$$

Parameters for T-distribution:

$$TestStatistic = \frac{\bar{x}_2 - \bar{x}_1}{se_diff}$$

$$se_diff = sqrt(se_1^2 + se_2^2) = 40$$

so,

$$TestStatistic = \frac{1245 - 1124}{40} = 3.025$$

Since n1=n2 and s1=s2, degree of freedom is calcuated as :

$$df = 2n - 2 = 98$$

Let’s consider alpha=0.05 and calculate the T-statistic values for the rejection region:

```
test_statistic <- qt(c(0.025,0.975),98)
test_statistic
```

```
## [1] -1.984467  1.984467
```

b. Thus, Test Statistic (3.025) doesnot lie in the region [-1.98,+1.98] ie. it lies in the rejected area and hence we reject the Null Hypothesis and accept Alternate hypothesis.

From this experiment we conclude that, Men and Women have different skill level in playing Tetris.

3. You think drinking the night before an exam might help performance on the exam the next morning. To test this, you select 100 of your closest friends, and randomly get 50 of them drunk the night before the exam, which you denote the treatment group. The next day, the treatment group gets a mean of 78 with a standard deviation of 10 and the control group gets a 75 with a standard deviation of 5.

$$H_0 = Drinking\ the\ night\ before\ exam\ helps\ performance\ on\ exam\ next\ morning.$$

$$H_a = Drinking\ the\ night\ before\ exam\ doesnot\ helps\ performance\ on\ exam\ next\ morning.$$

Treatment Sample:

$$n_1 = 50, \bar{x}_1 = 78, s_1 = 10, se_1 = \frac{s_1}{\sqrt{n_1}} = 1.41$$

Control Sample:

$$n_2 = 50, \bar{x}_2 = 75, s_2 = 5, se_2 = \frac{s_2}{\sqrt{n_2}} = 0.71$$

Parameters of T-distribution of the samples:

$$TestStatistic = \frac{\bar{x_2} - \bar{x_1}}{se_{diff}}$$

$$se_{diff} = sqrt(se_1^2 + se_2^2) = 1.58$$

$$TestStatistic = \frac{78 - 75}{1.58} = 1.90$$

Now, degrees of freedom(df):

$$df = \frac{se_{diff}^4}{se_a^4/(n_a - 1) + se_b^4/(n_b - 1)}$$

$$df = \frac{(1.58)^4}{\frac{(1.41)^4}{50-1} + \frac{(0.71)^4}{50-1}} = 73$$

lets assume alpha=0.05 and c alculate the T statitic values for rejection region:

```
T_statistic <- qt(c(0.025,0.975),73)
T_statistic
```

```
## [1] -1.992997  1.992997
```

Thus,

$$-1.99 < TestStatistic(1.90) < 1.99$$

Thus, we cannot reject Null Hypothesis. Hence, we conclude that drinking the night before exam doesnot help performance on the next morning.

4. Using data of your choosing (or using simulated data), use R to conduct the following tests, and explain the results you get:

a. A standard one-sample hypothesis test.

```
datasets::cars
```

```
##    speed dist
## 1      4    2
## 2      4   10
## 3      7    4
## 4      7   22
## 5      8   16
## 6      9   10
## 7     10   18
## 8     10   26
## 9     10   34
## 10    11   17
## 11    11   28
## 12    12   14
## 13    12   20
## 14    12   24
## 15    12   28
## 16    13   26
## 17    13   34
## 18    13   34
## 19    13   46
## 20    14   26
## 21    14   36
## 22    14   60
## 23    14   80
## 24    15   20
## 25    15   26
## 26    15   54
## 27    16   32
## 28    16   40
## 29    17   32
## 30    17   40
## 31    17   50
## 32    18   42
## 33    18   56
## 34    18   76
## 35    18   84
## 36    19   36
## 37    19   46
## 38    19   68
## 39    20   32
## 40    20   48
## 41    20   52
## 42    20   56
## 43    20   64
## 44    22   66
## 45    23   54
## 46    24   70
## 47    24   92
## 48    24   93
## 49    24  120
## 50    25   85
```

```
t.test(cars$dist,alternative="two.sided",mu=60)
```

```
##
##  One Sample t-test
##
## data:  cars$dist
## t = -4.6703, df = 49, p-value = 2.372e-05
## alternative hypothesis: true mean is not equal to 60
## 95 percent confidence interval:
##  35.65642 50.30358
## sample estimates:
## mean of x
##     42.98
```

Here, in the cars dataset, we have, Null Hypothesis

$$H_0 : \mu = 60$$

Since, p value < alpha (0.05), we reject the null hypothesis. We can see that the T statistic is (35.65, 50.30) and a good estimate for $\mu = 42.98$

b. A difference-in-means test with independent samples.

```
Non_Drinkers <- c(18,22,21,17,20,17,23,20,22,21)
Drinkers <- c(16,20,14,21,20,18,13,15,17,21)
t.test(Non_Drinkers,Drinkers,mu=0,conf=0.95,alternative="two.sided")
```

```
##
##  Welch Two Sample t-test
##
## data:  Non_Drinkers and Drinkers
## t = 2.2573, df = 16.376, p-value = 0.03798
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  0.1628205 5.0371795
## sample estimates:
## mean of x mean of y
##      20.1      17.5
```

Here the numbers are related to expriment scores for people who are drinkers and non-drinkers. We want to see if being a drinker would affect the cases of obesity.

Here, the Null hypothesis, $H_0 : \mu = 0$ However, since the T statistic has the rejection area P(0.162<mu<5.03) so, which doesnot lie in it and hence, Null hypothesis can be rejected. A good estimate for $\mu = 20.1 - 17.5 = 2.6$

c. A difference-in-means test with dependent samples (ie., a paired t-test)

Here, we'll use an example data set, which contains the weight of 10 mice before and after the treatment.

```
# Weight of the mice before treatment
before <-c(200.1, 190.9, 192.7, 213, 241.4, 196.9, 172.2, 185.5, 205.2, 193.7)
# Weight of the mice after treatment
after <-c(392.9, 393.2, 345.1, 393, 434, 427.9, 422, 383.9, 392.3, 352.2)

# Compute t-test
t.test(before, after, paired = TRUE)
```

```
##
##  Paired t-test
##
## data:  before and after
## t = -20.883, df = 9, p-value = 6.2e-09
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -215.5581 -173.4219
## sample estimates:
## mean of the differences
##                 -194.49
```

As we can see that the p value is significantly less than alpha, hence we reject Null hypothesis.

d. Manually verify the results in (a) using the mean and sd as calculated by R (ie, you don’t have to manually calculate the mean or sd by hand!).

```
x_bar <- mean(cars$dist)
x_bar
```

```
## [1] 42.98
```

```
std_dev <- sd(cars$dist)
std_dev
```

```
## [1] 25.76938
```

```
n <- length(cars$dist)
n
```

```
## [1] 50
```

```
se <- std_dev/(sqrt(n))
se
```

```
## [1] 3.64434
```

```
mu <- 60
z <- (x_bar - mu)/se
z
```

```
## [1] -4.670255
```

```
low_int <- qt(0.025,n-1)
low_int
```

```
## [1] -2.009575
```

```
high_int <-qt(0.975,n-1)
high_int
```

```
## [1] 2.009575
```

```
low_CI <- ((low_int)*se)+x_bar
low_CI
```

```
## [1] 35.65642
```

```
high_CI <- ((high_int)*se)+x_bar
high_CI
```

```
## [1] 50.30358
```

This verifies the results from part a and Null hypothesis can be rejected since mu lies in the rejected area.