Midterm (Set B): Solution

SIAS, Krea University (AY 2025-26)

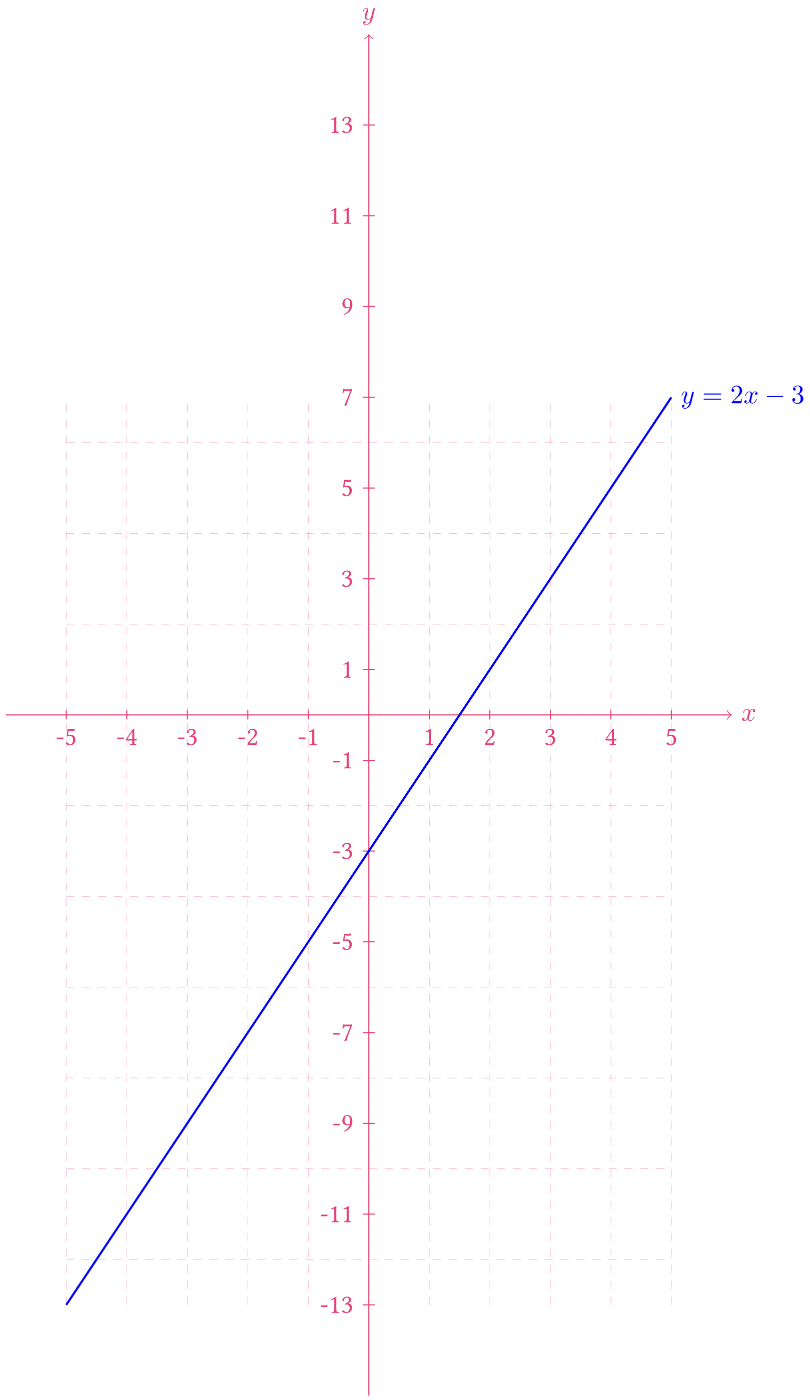
Mathematical Methods for Economics (Course Code: **ECON211**)

18 August 2025

Short Answer Questions-I

1. (2 points) Sketch the graph of the function $y = 2x - 3$ in the following domain: $[-5, 5]$.

Solution:



2. (2 points) Netflix ran a survey on customer preferences on genre. 1,500 customers responded to the survey revealing that 600 preferred romance (R), 450 preferred drama (D), and 350 preferred action (A). The survey also showed that 200 customers preferred both romance and drama, 150 preferred romance and action, and 100 preferred both drama and action movies. How many preferred action but not romance?

Solution:

$$|A| = 350$$
$$|A \cap R| = 150$$

$$|A \setminus R| = |A| - |A \cap R|$$
$$|A \setminus R| = 350 - 150 = 200$$

Answer: The number of customers who preferred action but not romance is 200.

3. (2 points) A sweet shop in Madurai sells *palkova*, *jangiri*, and *badusha*. The sweet shop generously shared some sample data (for three customers) with us.

Customer→	Dilli	Malar	Definite
palkova (kg)	6	2	4
jangiri (kg)	7	1	1
badusha (kg)	9	2	1

The per kg prices of *palkova*, *jangiri*, and *badusha* are ₹500, ₹200, and ₹200 respectively (these prices remain stable over the given period). Create two matrices, one for quantities (call it A) and one for prices (call it B). Compute and interpret AB.

Solution: The only trick here is to set up the matrices correctly.

$$A = \begin{bmatrix} 6 & 7 & 9 \\ 2 & 1 & 2 \\ 4 & 1 & 1 \end{bmatrix}$$
$$B = \begin{bmatrix} 500 \\ 200 \\ 200 \end{bmatrix}$$
$$AB = \begin{bmatrix} 6 \times 500 + 7 \times 200 + 9 \times 200 = 6200 \\ \\ 2 \times 500 + 1 \times 200 + 2 \times 200 = 1600 \\ \\ 4 \times 500 + 1 \times 200 + 1 \times 200 = 2400 \end{bmatrix}$$

Answer: $AB = \begin{bmatrix} 6200 \\ 1600 \\ 2400 \end{bmatrix}$. **The matrix represents the total sales per customer.**

4. (2 points) Compute $\sum_{k=4}^{k=7}(2k^2 - 2k + 1)$.

Solution: There are various possible ways to solve this one, but the easiest solution just involves plugging each k into the expression. The sum, therefore, is:

$$\text{When } k = 4, \sum_{k=4}^{k=7}(2k^2 - 2k + 1) = 2(4)^2 - 2(4) + 1 = 32 - 8 + 1 = 25$$
$$\text{When } k = 5, \sum_{k=4}^{k=7}(2k^2 - 2k + 1) = 2(5)^2 - 2(5) - 1 = 50 - 10 + 1 = 41$$
$$\text{When } k = 6, \sum_{k=4}^{k=7}(2k^2 - 2k + 1) = 2(6)^2 - 2(6) - 1 = 72 - 12 + 1 = 61$$
$$\text{When } k = 7, \sum_{k=4}^{k=7}(2k^2 - 2k + 1) = 2(7)^2 - 2(7) - 1 = 98 - 14 + 1 = 85$$
$$\overline{212}$$

Answer: $\sum_{k=4}^{k=7}(2k^2 - 2k + 1) = 212$

Short Answer Questions-II

5. (3 points) Consider two matrices $A = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Find a matrix B such that $(A - 3I)B = I$.

Solution: It should be clear at the outset that $B = (A - 3I)^{-1}$.

$$
\begin{aligned}
(A-3I) &= \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix} - 3 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
\implies (A-3I) &= \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \\
\implies (A-3I) &= \begin{bmatrix} 3-3 & -1-0 \\ 2-0 & 4-3 \end{bmatrix} \\
\implies (A-3I) &= \begin{bmatrix} 0 & -1 \\ 2 & 1 \end{bmatrix}
\end{aligned}
$$

We can now compute the inverse of this matrix. In order to do so, we need $|A - 3I|$ and $adj(A - 3I)$ as

$$(A-3I)^{-1} = \frac{1}{|A-3I|} adj(A-3I)$$

.

$$
\begin{aligned}
|A-3I| &= (0 \cdot 1 - (-1) \cdot 2) \\
\implies |A-3I| &= 2
\end{aligned}
$$

$$
\begin{aligned}
adj(A-3I) &= \begin{bmatrix} 1 & 1 \\ -2 & 0 \end{bmatrix} \\
\implies (A-3I)^{-1} &= \frac{1}{2}\begin{bmatrix} 1 & 1 \\ -2 & 0 \end{bmatrix} \\
\implies (A-3I)^{-1} &= \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -1 & 0 \end{bmatrix}
\end{aligned}
$$

Answer: $\boxed{B = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -1 & 0 \end{bmatrix}}$

6. (3 points) The value of a machine depreciates continuously at the annual rate of 5%. How many years will it take for the value of the machine to become 25% of its original value?

Solution: Since this is a case of continuous compounding, we can apply the following formula-

$$V_t = V_0 e^{rt}$$

Given: $r = -0.05$ (why?), $V_t = \frac{1}{4}V_0$ (why?)

$$
\begin{aligned}
\frac{1}{4}V_0 &= V_0 e^{-0.05t} && \text{(using the values provided in the question)} \\
e^{-0.05t} &= \frac{1}{4} \\
e^{0.05t} &= 4 \\
0.05t &= \ln(4) && \text{(since } log(exp(x)) = x\text{)} \\
t &= \frac{\ln(4)}{0.05} \\
t &= \frac{1.3863}{0.05} && \text{(using the value provided in the appendix)} \\
t &\approx 28
\end{aligned}
$$

It will take approximately twenty eight years for the machine to become one-fourth of its original value.

7. (3 points) There is a store called *The Revolver Club* in Bandra that sells vinyl records or, simply, vinyls.

(a) (2 points) If the store sells Q vinyls, the price received per vinyl sold is $P = 100 - \frac{1}{5}Q$. The price it has to pay per vinyl is $P = 30 + \frac{2}{3}Q$. In addition, it has to incur transportation cost of ₹5 per vinyl. Express this store's profit π as a function of Q. Find the profit-maximizing quantity.

Solution: We know that profit equals total revenue minus total cost.

$$\pi = TR - TC$$

We have been provided per unit selling price. Therefore, the total revenue is:

$$\begin{aligned} & TR = P \times Q \\ \implies & TR = (100 - \frac{1}{5}Q) \times Q \\ \implies & TR = 100Q - \frac{1}{5}Q^2 \end{aligned}$$

We also the know the per unit cost and the per unit transportation cost. Therefore, the total cost is:

$$\begin{aligned} & TC = P \times Q + t \times Q && (t \text{ is the transportation cost}) \\ \implies & TC = (30 + \frac{2}{3}Q) \times Q + t \times Q \\ \implies & TC = (30Q + \frac{2}{3}Q^2) + 5Q && (\text{since per unit transportation cost is ₹}10) \\ \implies & TC = 35Q + \frac{2}{3}Q^2 \end{aligned}$$

We can now write the profit function.

$$\begin{aligned} & \pi = \underbrace{100Q - \frac{1}{5}Q^2}_{\text{Total Revenue}} - \underbrace{(35Q + \frac{2}{3}Q^2)}_{\text{Total Cost}} \\ \implies & \pi = 65Q - (\frac{1}{5} + \frac{2}{3})Q^2 \\ \implies & \pi = 65Q - \frac{13}{15}Q^2 \end{aligned}$$

We know that, for any quadratic function $f(x) = ax^2 + bx + c$, the maximum value is achieved $x = \frac{-b}{2a}$ if $a < 0$.
In this case $a = -\frac{13}{15}$ and $b = 65$. Therefore, the profit-maximizing quantity is:

$$\begin{aligned} & Q = \frac{-65}{2 \times \frac{-13}{15}} \\ \implies & Q = \frac{65}{2 \times \frac{13}{15}} \\ \implies & Q = \frac{65 \times 15}{2 \times 13} \\ \implies & Q = \frac{75}{2} \\ \implies & Q = 37.5 \end{aligned}$$

Answer: $\boxed{\begin{array}{l} \pi = 65Q - \frac{13}{15}Q^2 \\ Q^* = 37.5 \end{array}}$

(b) (1 point) Suppose the government imposes a tax on the store's product of ₹2 per vinyl. Find the new expression for the store's profit.

Solution: We already have the profit function from the previous part. All we need to do is to subtract the new cost imposed by the government.

$$\pi = 65Q - \frac{13}{15}Q^2 - 2Q$$
$$\implies \pi = 63Q - \frac{13}{15}Q^2$$

Answer: $\boxed{\pi = 63Q - \frac{13}{15}Q^2}$

8. (3 points) Let the universal set $\mathbb{U}$ be the set of all students at a particular university. Moreover, let F denote the set of female students, E the set of all economics students, C the set of students in the university choir, P the set of all psychology students, and T the set of all students who play tennis. Describe the members of the following sets: $\mathbb{U} \setminus P$, $T \cup C$, $E \cap F' \cap T$. No calculation is needed.

Solution:

1. $\mathbb{U} \setminus P$: All university students who are not studying psychology.
2. $T \cup C$: All students who play tennis **or** all students who are in the choir **or** all students who are play tennis and are also in choir.
3. $E \cap F' \cap T$: All male economics students who do play tennis.

Long Answer Questions

9. (5 points) This question tests your knowledge and understanding of present value and interest rates. Assume compounding of the interest rate.

(a) (1 point) A sum of ₹20,000 is invested at 6% annual interest. What will this amount have grown to after 20 years?

Solution:

$$\text{Balance} = P(1+r)^n$$

$P = 20,000$, $r = 0.06$, $n = 20$.

$$\text{Balance} = 20,000(1+0.06)^{20}$$
$$\implies \text{Balance} = 20,000(3.204) \quad \text{(using the value provided in the appendix)}$$
$$\implies \text{Balance} = 64,080$$

Answer: Balance after 20 years will be ₹64,080.

(b) (2 points) Which terms are preferable for a borrower: (i) an annual interest rate of 20.5%, with interest paid yearly; or (ii) an annual interest rate of 18%, with interest paid monthly?

Solution: We need to compute the EAR for (ii). We know that:

$$EAR = \left(1 + \frac{r}{m}\right)^m - 1$$

$r = 0.18$, $m = 12$. Therefore,

$$EAR_{ii} = \left(1 + \frac{0.18}{12}\right)^{12} - 1$$
$$EAR_{ii} = (1 + 0.015)^{12} - 1$$
$$EAR_{ii} = (1.015)^{12} - 1$$

Using the value from the appendix, we get:

$$EAR_{ii} = 1.1956 - 1$$
$$EAR_{ii} = 0.1956$$
$$EAR_{ii} = 19.56\%$$

$EAR_{ii} < EAR_i$.
Answer: the borrower will prefer term (ii).

(c) (2 points) An account has been dormant for many years earning interest at the constant rate of 8% per year, with interest being compounded every quarter. Now the amount is ₹240,000. How much was in the account 6 years ago?

Solution: We know that-

$$PV = \frac{\text{Balance}}{\left(1+\frac{r}{m}\right)^p}$$

Given: Balance $= 2,40,000$, $r = 8\%$, $m = 4$, $p = 6 \times 4 = 24$.

$$\begin{aligned} PV &= \frac{240000}{\left(1+\frac{0.08}{4}\right)^{24}} \\ \implies PV &= \frac{240000}{(1+0.02)^{24}} \\ \implies PV &= \frac{240000}{(1.02)^{24}} \\ \implies PV &\approx \frac{240000}{1.6} && \text{(using value provided in the appendix)} \\ \implies PV &\approx 1,50,000 \end{aligned}$$

Answer: the balance in the account ten years ago was ₹1,50,000.

10. (5 points) Calculate the domain and the range of the following functions:

(a) (2 points)

$$f(x) = \frac{5x+2}{x-4}$$

Solution: The domain of a function is defined as all possible values of the input for which the function is defined. In this case, the function $f(x)$ is not defined at $x = 4$. Therefore, the domain of $f(x)$ is

$$x \in (-\infty, 4) \cup (4, \infty)$$

In order to compute the range, we will write out the input in terms of the output.

$$\begin{aligned} y &= \frac{5x+2}{x-4} \\ \implies y(x-4) &= 5x+2 \\ \implies xy - 4y &= 5x+2 \\ \implies xy - 5x &= 4y+2 \\ \implies x(y-5) &= 4y+2 \\ \implies x &= \frac{4y+2}{y-5} \end{aligned}$$

Since the range is the set of all valid values of the output, it doesn't seem like y can be ever equal to five. Therefore, the range of the function $f(x)$ is:

$$f(x) \in (-\infty, 5) \cup (5, \infty)$$

Answer:

Domain: $x \in (-\infty, 4) \cup (4, \infty)$

Range: $f(x) \in (-\infty, 5) \cup (5, \infty)$

(b) (3 points)

$$g(x) = \frac{4}{\sqrt{x^2-4}}$$

Solution: Observing the denominator of the function, we can say that

$$x^2 - 4 > 0$$

Why? Because, this expression sits inside a square root. Moreover, while the square root of 0 is indeed a valid value, the function won't be defined when $x^2 = 4$. Therefore,

$$x > 2 \text{ or } x < -2$$

The domain of the function is:

$$x \in (-\infty, -2) \cup (2, \infty)$$

As $x \to \pm\infty$, $f(x) \to 0$ and when $x \to \pm 2$, $f(x) \to \infty$, but we will show this more formally.

$$\begin{aligned}
y &= \frac{4}{\sqrt{x^2-4}} \\
y^2 &= \frac{16}{x^2-4} && \text{(squaring both sides)} \\
y^2(x^2-4) &= 16 \\
x^2y^2 - 4y^2 &= 16 \\
x^2y^2 &= 4y^2 + 16 \\
x^2 &= \frac{4y^2+16}{y^2} \\
x &= \sqrt{\frac{4y^2+16}{y^2}} \\
x &= \sqrt{4 + \frac{16}{y^2}}
\end{aligned}$$

We can now say, with some degree of confidence, the values that won't be possible for the function to take. $y \neq 0$. At this stage, it is tempting to write the range as $f(x) \in \mathbb{R} \setminus 0$, but look at the function itself. $f(x) \nless 0$. Therefore, the range of the function is:

$$f(x) \in (0, \infty)$$

Answer: | **Domain:** $x \in (-\infty, -1) \cup (1, \infty)$ | **Range:** $f(x) \in (0, \infty)$