Quiz 04 (Set C: Solution)

SIAS, Krea University (AY 2025-26)
Mathematical Methods for Economics (Course Code: **ECON211**)
12 September 2025

Multiple Choice Questions

1. (1 point) Let $f(x) = \ln(3 + e^x)$. Then, $f''(0)$ is

   A. $\frac{2}{9}$
   B. $\frac{1}{4}$
   C. $\frac{1}{2}$
   D. $\frac{3}{16}$

   Answer: D

   Solution:

$$f'(x) = \frac{e^x}{3 + e^x} \quad \text{(using the chain rule)}$$
$$f''(x) = \frac{3e^x}{(3 + e^x)^2} \quad \text{(using the quotient rule)}$$
$$\implies f''(0) = \frac{3e^0}{(1 + e^0)^2}$$
$$\implies f''(0) = \frac{3}{4^2}$$
$$\implies f''(0) = \frac{3}{16}$$

2. (1 point) Consider the following statements:

   Statement (i):
   $f(x) = e^{x-2}$ is a strictly increasing function.
   Statement (ii):
   $g(x) = 2 - 4x^2$ is a strictly convex function.

   A. Both (i) and (ii) are correct.
   B. Statement (i) is correct but statement (ii) is wrong.
   C. Statement (i) is wrong but statement (ii) is correct.
   D. Both (i) and (ii) are wrong.

   Answer: B

   Solution: A function is strictly increasing if $f'(x) > 0$.

$$f(x) = e^{x-2}$$
$$\implies f'(x) = e^{x-2} \quad \text{(using the chain rule)}$$

We know that e^k is always positive (for any constant k). Therefore, $f'(x) > 0$. Hence, the function is **strictly increasing**.
A function $g(x)$ is strictly convex if $g''(x) > 0$.

$$g(x) = 2 - 4x^2$$
$$\implies g'(x) = -8x$$
$$\implies g''(x) = -8$$

Since the second derivative is negative everywhere in the domain of the function, the function is **strictly concave**.

3. (1 point) Let $f(x) = \sqrt{2x + \sqrt{x}}$. Then $f'(x)$ is

A. $\dfrac{2\sqrt{x}+1}{2\sqrt{x}(\sqrt{2x+\sqrt{x}})}$

B. $\dfrac{2\sqrt{x}+1}{4\sqrt{x}(\sqrt{2x+\sqrt{x}})}$

C. $\dfrac{4\sqrt{x}+1}{4\sqrt{x}(\sqrt{2x+\sqrt{x}})}$

D. $\dfrac{2\sqrt{x}+1}{(\sqrt{2x+\sqrt{x}})}$

Answer: C

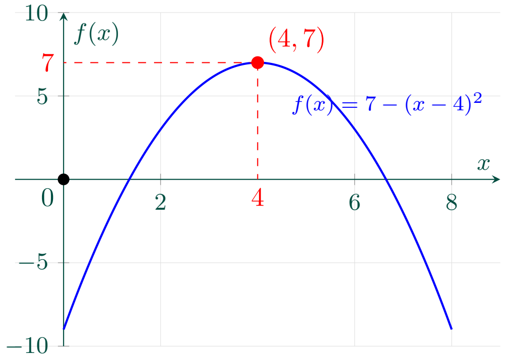
Solution: Let $u = 2x + \sqrt{x}$. Therefore, $f(u) = \sqrt{u}$.

$$\begin{aligned}
\frac{df}{du} &= \frac{1}{2\sqrt{u}} && \text{(using the power rule)}\\
\frac{du}{dx} &= 2 + \frac{1}{2\sqrt{x}} && \text{(using the power rule)}\\
\implies \frac{du}{dx} &= \frac{4\sqrt{x}+1}{2\sqrt{x}}\\
\frac{df}{dx} &= \frac{df}{du}\cdot\frac{du}{dx}\\
&= \frac{1}{2\sqrt{u}}\cdot\frac{4\sqrt{x}+1}{2\sqrt{x}} && \text{(applying the chain rule)}\\
&= \frac{1}{2\sqrt{2x+\sqrt{x}}}\cdot\frac{4\sqrt{x}+1}{2\sqrt{x}}\\
&= \frac{4\sqrt{x}+1}{4\sqrt{x}(\sqrt{2x+\sqrt{x}})}
\end{aligned}$$

Short Answer Questions-I

4. (1 point) Without using calculus, compute the minimum (or the maximum) value of the following function: $f(x) = 7 - (x-4)^2$. (*Hint: Graph the function.*)

Solution: Consider $(x-4)^2$. The minimum value that any square can take is zero. Therefore, $(x-4)^2 \geq 0$. When $x = 4$, $f(x) = 7$. This is the maximum value of the function.



5. (1 point) Let $3xy^2 + x^2y = 5$. Find $\dfrac{dy}{dx}$. Simplify the answer as much as possible.

Solution:

$$\begin{aligned}
\frac{d(3xy^2 + x^2y)}{dx} &= \frac{d(5)}{dx} \\
\frac{d(3xy^2)}{dx} + \frac{d(x^2y)}{dx} &= 0 && \text{(applying the sum rule to the LHS and the constant rule to the RHS)} \\
3x\frac{d(y^2)}{dx} + 3y^2\frac{d(x)}{dx} + x^2\frac{d(y)}{dx} + y\frac{d(x^2)}{dx} &= 0 && \text{(applying the product rule)} \\
6xy\frac{dy}{dx} + 3y^2 + x^2\frac{dy}{dx} + 2xy &= 0 \\
(6xy + x^2)\frac{dy}{dx} + (3y^2 + 2xy) &= 0 \\
(6xy + x^2)\frac{dy}{dx} &= -(3y^2 + 2xy) \\
\frac{dy}{dx} &= -\frac{(3y^2 + 2xy)}{(6xy + x^2)} \\
\frac{dy}{dx} &= -\frac{y}{x}\left(\frac{2x + 3y}{x + 6y}\right)
\end{aligned}$$

6. (1 point) Let $f(x) = \ln(2 + e^{x-1})$ and let $g(x) = f^{-1}(x)$. Find $g'(x)$.

Solution: Let $y = f(x)$.

$$\begin{aligned}
y &= \ln(2 + e^{x-1}) \\
\implies e^y &= 2 + e^{x-1} && \text{(since } e^{\ln(a)} = a\text{)} \\
\implies e^{x-1} &= e^y - 2 \\
\implies x - 1 &= \ln(e^y - 2) && \text{(taking log on both sides)} \\
\implies x &= 1 + \ln(e^y - 2) \\
\implies f^{-1}(x) &= 1 + \ln(e^x - 2) \\
\implies g(x) &= 1 + \ln(e^x - 2) \\
\implies g'(x) &= \frac{1}{e^x - 2}\frac{d}{dx}(e^x - 2) && \text{(applying the chain rule)} \\
\implies g'(x) &= \frac{e^x}{e^x - 2}
\end{aligned}$$

Short Answer Questions-II

7. (2 points) Find and classify all the stationary/inflection points for the following function: $f(x) = x^3 - 3x + 10$.

Solution:

$$\begin{aligned}
f'(x) &= 3x^2 - 3 \\
f''(x) &= 6x
\end{aligned}$$

Set the first derivative to zero to find the stationary points.

$$\begin{aligned}
3x^2 - 3 &= 0 \\
x^2 - 1 &= 0 \\
x^2 &= 1 \\
x &= \pm 1
\end{aligned}$$

When $x = 1$, the second derivative is positive indicating a local minimum. When $x = -1$, the second derivative is negative suggesting a local maximum.

Set the second derivative to zero to look for inflection point(s).

$$6x = 0$$
$$x = 0$$

Take any two points in the vicinity of $x = 0$. Luckily, we already have those points from our previous calculation. Since the sign of the second derivative changes as x moves around $x = 0$, $x = 0$ is an inflection point.

$x = 1$	**Local maximum**
$x = -1$	**Local minimum**
$x = 0$	**Inflection point**

8. (2 points) You work for an online retailer and you have been tasked with estimating the elasticity of demand for their product. The demand function is $q = \frac{2}{3}\sqrt{144 - p^2}$.

   (a) (1 point) Compute the elasticity of demand when $p = 8\sqrt{2}$.

Solution: We know that:

$$\epsilon_p = \left|\frac{p}{q} \cdot \frac{dq}{dp}\right|$$

When $p = 8\sqrt{2}$,

$$\begin{aligned} q &= \frac{2}{3}\sqrt{144 - 128} \\ &= \frac{2}{3}\sqrt{16} \\ &= \frac{8}{3} \end{aligned}$$

Let's compute $\frac{dq}{dp}$.

$$\frac{dq}{dp} = \frac{1}{3}(144 - p^2)^{-\frac{1}{2}}\left(\frac{d(144 - p^2)}{dp}\right) \qquad \text{(applying the chain rule)}$$
$$\frac{dq}{dp} = \frac{-2p}{3(144 - p^2)^{\frac{1}{2}}}$$

When $p = 8\sqrt{2}$,

$$\begin{aligned} \frac{dq}{dp} &= \frac{-16\sqrt{2}}{3 \cdot 4} \\ &= \frac{-4\sqrt{2}}{3} \end{aligned}$$

The elasticity of demand,

$$\epsilon_p = \left|\frac{p}{q} \cdot \frac{dq}{dp}\right|$$
$$\epsilon_p = \left|\frac{8\sqrt{2}}{8/3} \cdot \frac{-4\sqrt{2}}{3}\right|$$
$$\epsilon_p = 8$$

   (b) (1 point) Based on your previous answer, what should be the firm's pricing strategy (increase or decrease the price?) that will boost revenue? Explain briefly.

Solution: The demand, our previous computation suggests, is elastic. Therefore, the retailer can decrease the price a bit to boost the revenue.