Quiz 03 (Set C (Solution))

SIAS, Krea University (AY 2025-26)

Mathematical Methods for Economics (Course Code: **ECON211**)

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Multiple Choice Questions

1. (1 point) Let $f(x) = 200$. Then,

   A. $f^{-1}(x) = 200$

   B. $f^{-1}(x)$ does not exist.

   C. $f^{-1}(x) = \frac{1}{200}$

   D. $f^{-1}(x) = \frac{1}{200x}$

   Answer: B

   Solution: Consider two points in the domain of the function: $x = 1$ and $x = 2$.
   $f(1) = 200$ and $f(2) = 200$.
   What happens when you 'invert' this function? You get:
   $f^{-1}(200) = 1$ and $f^{-1}(200) = 2$.
   This cannot be a valid function as it is not one-to-one. Therefore, the inverse does not exist.

2. (1 point) Consider the following statements:

   Statement (i):
   $\lim_{x\to-3} |x + 3| = 0$.

   Statement (ii):
   $f(x) = |x + 3|$ is differentiable at $x = -3$.

   A. Both (i) and (ii) are correct.

   B. Statement (i) is correct but statement (ii) is wrong.

   C. Statement (i) is wrong but statement (ii) is correct.

   D. Both (i) and (ii) are wrong.

   Answer: B

   Solution:

   $$\text{LHL: } \lim_{x\to-3^-} |x + 3| = 0$$
   $$\text{RHL: } \lim_{x\to-3^+} |x + 3| = 0$$

   LHL = RHL. Therefore, the limit does exist and is equal to zero.

   $$\text{LHD: } \lim_{x\to-3^-} -1 = -1 \qquad (\text{since } |x + 3| = -x - 3 \quad \forall x < 0)$$
   $$\text{RHD: } \lim_{x\to-3^+} 1 = 1 \qquad (\text{since } |x + 3| = x + 3 \quad \forall x > 0)$$

   LHD $\neq$ RHD. Therefore, $f(x)$ is not differentiable at $x = 0$.

3. (1 point) If $f(x) = x^2$, $g(x) = x^2 + 2$ and $h(x) = (x + 2)^2$, then

   A. the graph of $g(x)$ can be obtained by shifting $f(x)$ downwards by 2 units.

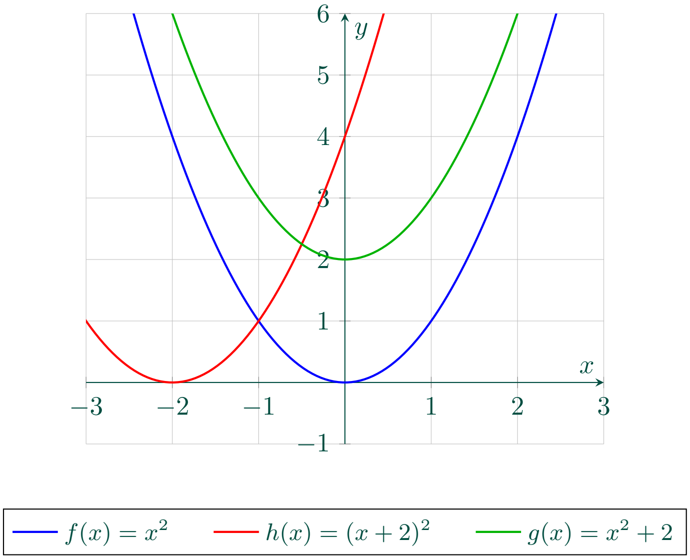
   B. the graph of $h(x)$ can be obtained by shifting $f(x)$ upwards by 1 unit.

C. the graph of $h(x)$ can be obtained by shifting $f(x)$ to the left by 2 units.

D. the graph of $g(x)$ can be obtained by shifting $f(x)$ to the right by 2 units.

Answer: C

Solution: This is very straightforward. $g(x)$ is $f(x)$ shifted up two units and $h(x)$ is $f(x)$ being shifted to the left by two units.



Short Answer Questions-I

4. (1 point) Compute the inverse of the following function: $f(x) = \dfrac{5x-1}{5x+1}$.

Solution:

$$\begin{aligned} y &= f(x) \\ \implies y &= \frac{5x-1}{5x+1} \\ \implies y(5x+1) &= 5x-1 \\ \implies 5xy + y &= 5x-1 \\ \implies 5xy - 5x &= -1 - y \\ \implies x(5y-5) &= -1-y \\ \implies x &= \frac{-1-y}{5y-5} \\ \implies x &= \frac{1+y}{5-5y} \\ \implies x &= \frac{1}{5}\left(\frac{1+y}{1-y}\right) \end{aligned}$$

Answer: $\boxed{f^{-1}(x) = \frac{1}{5}\left(\frac{1+x}{1-x}\right)}$.

5. (1 point) Calculate: $\lim_{x\to\infty} \dfrac{2x^3 - 88x^2 + 2000}{5x^3 - 2x^2 + 10}$.

Solution: Divide the whole expression by x^3.

$$\begin{aligned}
&\lim_{x\to\infty} \frac{2 - \dfrac{88}{x} + \dfrac{2000}{x^3}}{5 - \dfrac{2}{x} + \dfrac{10}{x^3}} \\
&= \frac{2 - \lim_{x\to\infty} \dfrac{88}{x} + \lim_{x\to\infty} \dfrac{2000}{x^3}}{5 - \lim_{x\to\infty} \dfrac{2}{x} + \lim_{x\to\infty} \dfrac{10}{x^3}} \\
&= \frac{2}{5}
\end{aligned}$$

Answer: $\boxed{\lim_{x\to\infty} \frac{2x^3 - 88x^2 + 2000}{5x^3 - 2x^2 + 10} = \frac{2}{5}}$

6. (1 point) Compute $\frac{dy}{dx}$ if $y = 3x + \frac{6}{\sqrt{x}}$.

Solution:

$$\begin{aligned}
y &= 3x + \frac{6}{\sqrt{x}} \\
\frac{dy}{dx} &= 3 + \frac{d(\frac{6}{\sqrt{x}})}{dx} \\
\frac{dy}{dx} &= 3 + 6\frac{d(x^{-1/2})}{dx} \\
\frac{dy}{dx} &= 3 - 6\Big(\frac{1}{2}x^{-3/2}\Big)
\end{aligned}$$

Answer: $\boxed{\frac{dy}{dx} = 3 - 3x^{-3/2}}$

Short Answer Questions-II

7. (2 points) There are two parts in this question.

(a) (1 point) Calculate a such the following function is continuous for all x. $f(x) = \begin{cases} 2ax - 1 & \text{if } x \leq 1 \\ 6x^2 + 3 & \text{if } x > 1 \end{cases}$

Solution: Condition for continuity at $x = a$: $LHL = RHL = f(a)$.

$$\begin{aligned}
\text{LHL: } \lim_{x\to 1^-} f(x) &= \lim_{x\to 1^-} 2ax - 1 \\
&= 2a - 1 \\
\text{RHL: } \lim_{x\to 1^+} f(x) &= \lim_{x\to 1^+} 6x^2 + 3 \\
&= 6 + 3 \\
&= 9 \\
f(1) &= 2a - 1 \\
\implies 2a - 1 &= 9 \\
\implies a &= 5
\end{aligned}$$

Answer: $\boxed{a = 5}$.

(b) (1 point) Compute $\frac{dy}{dx}$ if $f(x) = \frac{3 - x^2}{3 + x^2}$.

Solution: Let $u = 3 - x^2$ and $v = 3 + x^2$.

$$u' = -2x$$
$$v' = 2x$$

We know that, if $f(x) = \frac{u}{v}$, $f'(x) = \frac{vu' - uv'}{v^2}$.
Applying the quotient rule, we get:

$$\frac{dy}{dx} = \frac{(3+x^2)(-2x) - (3-x^2)(2x)}{(3+x^2)^2}$$
$$\implies \frac{dy}{dx} = \frac{-12x}{(3+x^2)^2}$$

Answer: $\boxed{\frac{dy}{dx} = \frac{-12x}{(3+x^2)^2}}$

8. (2 points) The demand function for tickets on *Ruinmytrip* is given by

$$p = 400 - 0.02q$$

(a) (1 point) Compute the marginal revenue.

Solution:

$$TR = (400 - 0.02) \cdot q$$
$$TR = (400 - 0.02q) \cdot q$$
$$TR = 400q - 0.02q^2$$
$$\implies MR = 400 - 0.04q \qquad \text{(applying the power rule)}$$

Answer: $\boxed{\text{Marginal revenue} = 400 - 0.04q}$

(b) (1 point) Calculate the approximate revenue for the 2001st ticket.

Solution: We know that $MR(x)$ will approximate $TR(x+1)$. Therefore, we need to compute $MR(2000)$.

$$MR(2000) = 400 - 0.04q$$
$$\implies MR(2000) = 400 - 0.04(2000)$$
$$\implies MR(2000) = 400 - 80$$
$$\implies MR(2000) = 320$$

Answer: $\boxed{\text{The approximate revenue from selling the 2001st ticket is 320.}}$

Rough Work