Derivatives-III

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1 Elasticity and Revenue

We discussed elasticity of demand in the last lecture. In this lecture, we would like to establish an important relationship between the elasticity of demand and total revenue. Let demand be $q = f(p)$. Then, the total revenue will be $TR = p \times q$. Therefore,

$$TR = p \times f(p)$$

Compute the marginal revenue.

$$\begin{aligned}
MR &= \frac{d}{dp}(p \times f(p)) \\
MR &= p\frac{d}{dp}(f(p)) + f(p)\frac{d}{dp}(p) && \text{(applying the product rule)} \\
MR &= pf'(p) + f(p) \\
MR &= f(p)\left[p\frac{f'(p)}{f(p)} + 1\right] \\
MR &= f(p)[1 - \epsilon_p] && \text{(since } e_p = -\frac{p}{q} \times \frac{dq}{dp}\text{)}
\end{aligned}$$

Case-I: The demand is inelastic, that is, $\epsilon_p < 1 \implies 1 - \epsilon_p > 0$. Therefore,

$$MR = f(p)(1 - \epsilon_p) > 0$$

Since $MR > 0$, a small increase (decrease) in price will lead to a rise (fall) in the total revenue.
Case-II: The demand is unitary elastic, that is, $\epsilon_p = 1 \implies 1 - \epsilon_p = 0$. Therefore,

$$MR = 0$$

Since $MR = 0$, any small increase or decrease in the unit price will not affect a change in total revenue.
Case-III The demand is elastic, that is, $\epsilon_p > 1 \implies 1 - \epsilon_p < 0$. Therefore,

$$MR = f(p)(1 - \epsilon_p) < 0$$

Since $MR < 0$, any small increase (decrease) in the price will lead to a fall (rise) in the revenue.

An example will help us understand this relationship. Let $q = 10 - p \quad (0 \le p \le 10)$. Consider three different prices $p = 4$, $p = 5$, and $p = 6$.

We know that $\frac{dq}{dp} = -1$.

When $p = 4$, $q = 6$, and $\epsilon_p = |\frac{4}{6}\frac{dq}{dp}| = \frac{2}{3}$.

What happens when the firm tries to lower its price? Let's say price falls from 4 to $p = 3$. Let's also check what happens when price goes up from $p = 4$ to $p = 4.5$.

$$\begin{aligned} TR(p = 4) &= 4 \times 6 = 24 \\ TR(p = 3) &= 3 \times 7 = 21 \\ TR(p = 4.5) &= 4.5 \times 5.5 = 24.75 \end{aligned}$$

We can see that, when the demand is inelastic, lowering price will be a bad news for the firm's revenue.

When $p = 5$, $q = 5$, and $\epsilon_p = |\frac{5}{5}\frac{dq}{dp}| = 1$.

What happens when the firm tries to, let's say increase the price from 5 to $p = 5.5$, and lower the price to 4.5?

$$\begin{aligned} TR(p = 5) &= 5 \times 5 = 25 \\ TR(p = 4.5) &= 4.5 \times 5.5 = 24.75 \\ TR(p = 5.5) &= 5.5 \times 4.5 = 24.75 \end{aligned}$$

We can conclude that, when the demand is unitary elastic, lowering or raising unit price won't impact firm's revenue.

When $p = 6$, $q = 4$, and $\epsilon_p = \frac{6}{4} = \frac{3}{2}$.

What happens when the firm tries to change its price from this price? Let's say the unit price moves from $p = 6$ to $p = 7$, and then it falls from $p = 6$ to $p = 5$.

$$\begin{aligned} TR(p = 6) &= 6 \times 4 = 24 \\ TR(p = 7) &= 7 \times 3 = 21 \\ TR(p = 5) &= 5 \times 5 = 25 \end{aligned}$$

We can infer from the calculation that, when the demand is elastic, lowering price will raise firm revenue and increasing the unit price will lead to a fall in the total revenue.