

Multiple Choice Questions

1. (1 point) Consider the following statements:

Statement (i): The set of equations: $2x + 3y = 5$ and $4x - 6y = -2$ has a unique solution.

Statement (ii): The set of equations: $4x - y = 3$ and $-28x + 7y = 21$ does not have any solution.

- A. Both (i) and (ii) are correct.
- B. Statement (i) is correct but statement (ii) is incorrect.
- C. Statement (ii) is correct but statement (i) is incorrect.
- D. Both statements are incorrect.

Answer: A

Solution: Consider the first set of equations.

$$\begin{aligned} 2x + 3y &= 5 \\ \Rightarrow 4x + 6y &= 10 && \text{(multiplying the equation by 2)} \\ 4x - 6y &= -2 && \text{(equation two as it is)} \\ \hline 8x &= 8 && \text{(adding the previous two equations)} \\ x &= 1 \\ y &= 1 && \text{(using equation one)} \end{aligned}$$

The set of equations, indeed, has a solution and, therefore, the assertion is correct.

Consider the second equation from Statement (ii).

$$\begin{aligned} -28x + 7y &= 21 \\ \Rightarrow 4x - y &= -3 && \text{(dividing both sides by -3)} \end{aligned}$$

This is incompatible with $4x - y = -3$. Therefore, this set doesn't have any solution. So, this assertion is also correct.

2. (1 point) Identify the element a_{43} in the following matrix A

$$A = \begin{bmatrix} 0 & 1 & 3 & 6 \\ 1 & 2 & 3 & 9 \\ 7 & 5 & 2 & 0 \\ 9 & 4 & 1 & 5 \end{bmatrix}$$

- A. 0
- B. 1
- C. 2
- D. 3

Answer: B

Solution: $a_{43} = 1$.

$$A = \begin{bmatrix} a_{11} = 0 & a_{12} = 1 & a_{13} = 3 & a_{14} = 6 \\ a_{21} = 1 & a_{22} = 2 & a_{23} = 3 & a_{24} = 9 \\ a_{31} = 7 & a_{32} = 5 & a_{33} = 2 & a_{34} = 0 \\ a_{41} = 9 & a_{42} = 4 & a_{43} = 1 & a_{44} = 5 \end{bmatrix}$$

3. (1 point) Find the roots of the following quadratic equation: $2x^2 + 32x + 128 = 0$.

- A. $(-16, 4)$
- B. $(-8, 8)$

C. (-8)

D. $(4, -16)$

Answer: C

Solution:

$$\begin{aligned}2x^2 + 32x + 128 &= 0 \\ \Rightarrow 2(x^2 + 16x + 64) &= 0 && \text{(taking 2 as a common factor)} \\ \Rightarrow x^2 + 16x + 64 &= 0 && \text{(getting rid of a constant is kosher)} \\ \Rightarrow x^2 + 2(8x) + (8)^2 &= 0 \\ \Rightarrow (x + 8)^2 &= 0 && \text{(since } (a^2 + 2ab + b^2) = (a + b)^2\text{)} \\ \Rightarrow (x + 8) &= 0 \\ \Rightarrow x &= -8\end{aligned}$$

Answer : $x = -8$.

Short Answer Questions-I

4. (1 point) Solve for x and y :

$$3x - 4y = -2$$

$$6x + 2y = 6$$

Solution:

$$\begin{aligned}12x + 4y &= 12 && \text{(multiplying the second equation by 2)} \\ 3x - 4y &= -2 && \text{(writing the first equation as it is)} \\ \Rightarrow 15x &= 10 && \text{(adding the two equations we just wrote)} \\ \Rightarrow x &= \frac{2}{3} \\ 3 \times \frac{2}{3} - 4y &= -2 && \text{(plugging the value of } x \text{ back into the first equation)} \\ \Rightarrow 2 - 4y &= -2 \\ \Rightarrow -4y &= -4 \\ \Rightarrow y &= 1\end{aligned}$$

Answer: $x = \frac{2}{3}$, $y = 1$.

5. (1 point) Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 6 & 7 \end{bmatrix}$. Compute BA .

Solution: Let

$$C = BA = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$

$$c_{11} = (0 \times 1) + (-1 \times 3) = -3$$

$$c_{12} = (0 \times 2) + (-1 \times 4) = -4$$

$$c_{21} = (6 \times 1) + (7 \times 3) = 27$$

$$c_{22} = (6 \times 2) + (7 \times 4) = 40$$

Answer:

$$BA = \begin{bmatrix} -3 & -4 \\ 27 & 40 \end{bmatrix}$$

6. (1 point) There are two matrices A and B such that:

$$A = \begin{bmatrix} x+5 & 0 & 3 \\ 4 & 0.6y & 8 \end{bmatrix}, \quad B = \begin{bmatrix} 6 & 5 & 0.5 \\ 10 & 2 & -10 \end{bmatrix}, \quad 2A + B = \begin{bmatrix} 16 & 5 & 6.5 \\ 18 & 5 & 6 \end{bmatrix}$$

Compute x and y .

Solution: Using equality of matrices, we can write:

$$2(x+5) + 6 = 16$$

$$\Rightarrow 2(x+5) = 10$$

$$\Rightarrow x+5 = 5$$

$$\Rightarrow x = 5 - 5$$

$$\Rightarrow x = 0$$

$$2(0.6y) + 2 = 5$$

$$\Rightarrow \frac{6}{5}y = 3$$

$$\Rightarrow 6y = 15$$

$$\Rightarrow y = \frac{5}{2}$$

Answer: $x = 0$, $y = \frac{5}{2}$

Short Answer Questions-II

7. (2 points) Use Cramer's rule OR matrix inverse method to solve the following set of equations:

$$9x_1 + 7x_2 = 16$$

$$4x_1 - 5x_2 = -1$$

Solution: Cramer's Rule

First, write all the matrices you need.

$$A = \begin{bmatrix} 9 & 7 \\ 4 & -5 \end{bmatrix}, \quad B = \begin{bmatrix} 16 \\ -1 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

We also need D_{x_1} and D_{x_2} .

$$D_{x_1} = \begin{vmatrix} 16 & 7 \\ -1 & -5 \end{vmatrix}$$

$$D_{x_2} = \begin{vmatrix} 9 & 16 \\ 4 & -1 \end{vmatrix}$$

$$|A| = \begin{vmatrix} 9 & 7 \\ 4 & -5 \end{vmatrix} = -73$$

Therefore,

$$x_1 = \frac{D_{x_1}}{|A|}$$

$$x_2 = \frac{D_{x_2}}{|A|}$$

$$D_{x_1} = -73$$

$$D_{x_2} = -73$$

$$\Rightarrow x_1 = 1, \quad x_2 = 1$$

Matrix Inverse Method

We know that $X = A^{-1}B$ and $A^{-1} = \frac{\text{adj}(A)}{|A|}$.

$$\begin{aligned} A^{-1} &= \frac{1}{-73} \begin{bmatrix} -5 & -7 \\ -4 & 9 \end{bmatrix} \Rightarrow A^{-1}B = \frac{1}{-73} \begin{bmatrix} -5 & -7 \\ -4 & 9 \end{bmatrix} \times \begin{bmatrix} 16 \\ -1 \end{bmatrix} \\ \Rightarrow A^{-1}B &= \frac{1}{-73} \begin{bmatrix} (-5 \times 16) + (-7 \times -1) \\ (-4 \times 16) + (9 \times -1) \end{bmatrix} \\ \Rightarrow A^{-1}B &= \frac{1}{-73} \begin{bmatrix} -73 \\ -73 \end{bmatrix} \\ \Rightarrow A^{-1}B &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ \Rightarrow X &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} \end{aligned}$$

Answer: $x_1 = 1$, $x_2 = 1$.

8. (2 points) Given the following supply and demand equations:

$$\begin{aligned} \text{Supply: } P &= 2Q_S^2 + 11Q_S + 9 \\ \text{Demand: } P &= -Q_D^2 - 7Q_D + 57 \end{aligned}$$

Calculate the equilibrium price and quantity.

Solution: We know that at equilibrium, the following is true:

$$\begin{aligned} \text{Supply} &= \text{Demand} \\ Q_S &= Q_D = Q \end{aligned}$$

Therefore,

$$\begin{aligned} 2Q^2 + 11Q + 9 &= -Q^2 - 7Q + 57 \\ \Rightarrow 3Q^2 + 18Q - 48 &= 0 && \text{(rearranging terms)} \\ \Rightarrow Q^2 + 6Q - 16 &= 0 && \text{(dividing both sides by 3)} \\ \Rightarrow Q^2 + 8Q - 2Q - 16 &= 0 && \text{(since } 6Q = 8Q - 2Q) \\ \Rightarrow Q(Q + 8) - 2(Q + 8) &= 0 \\ \Rightarrow (Q + 8)(Q - 2) &= 0 && \text{(since } a(b + k) - c(b + k) = (a - c)(b + k)) \\ \Rightarrow Q^* &= 2 && \text{(since quantities cannot be negative)} \\ P^* &= 2(Q^*)^2 + 11Q^* + 10 && \text{(using the supply equation)} \\ \Rightarrow P^* &= 2(2)^2 + 11 \times 2 + 9 && \text{(since } Q^* = 2) \\ \Rightarrow P^* &= 39 \end{aligned}$$

Answer: $Q^* = 2$, $P^* = 39$