

Multiple Choice Questions

1. (1 point) Identify the element a_{34} in the following matrix A

$$A = \begin{bmatrix} 0 & 1 & 3 & 6 \\ 1 & 2 & 3 & 9 \\ 7 & 5 & 2 & 0 \\ 9 & 4 & 1 & 5 \end{bmatrix}$$

- A. 1
- B. 0
- C. 3
- D. 2

Answer: B

Solution: $a_{34} = 0$.

$$A = \begin{bmatrix} a_{11} = 0 & a_{12} = 1 & a_{13} = 3 & a_{14} = 6 \\ a_{21} = 1 & a_{22} = 2 & a_{23} = 3 & a_{24} = 9 \\ a_{31} = 7 & a_{32} = 5 & a_{33} = 2 & a_{34} = 0 \\ a_{41} = 9 & a_{42} = 4 & a_{43} = 1 & a_{44} = 5 \end{bmatrix}$$

2. (1 point) Find the roots of the following quadratic equation: $-2x^2 + 40x - 600 = 0$.

- A. $(10, -40)$
- B. $(10 \pm \sqrt{5})$
- C. No real roots exist
- D. $(-10, 30)$

Answer: C

Solution: We will first compute the discriminant to check if the equation has any (real) roots. $a = -2$, $b = 40$, and $c = -600$.

$$\begin{aligned} b^2 - 4ac &= (40)^2 - (4 \times (-2)(-600)) && \text{(using given values)} \\ \Rightarrow b^2 - 4ac &= 1600 - 4 \times (2 \times 600) && \text{(since } (-a) \times (-b) = ab) \\ \Rightarrow b^2 - 4ac &= 1600 - 4 \times 1200 \\ \Rightarrow b^2 - 4ac &= 1600 - 4800 \\ \Rightarrow b^2 - 4ac &< 0 \end{aligned}$$

The equation, therefore, has no real roots.

3. (1 point) Consider the following statements:

Statement (i): The set of equations: $2x + 3y = 5$ and $4x + 6y = 7$ does not have any solution.

Statement (ii): The set of equations: $4x - y = 3$ and $-28x + 7y = 21$ does not have any solution.

- A. Both (i) and (ii) are correct.
- B. Statement (i) is correct but statement (ii) is incorrect.
- C. Statement (ii) is correct but statement (i) is incorrect.
- D. Both statements are incorrect.

Answer: A

Solution: Consider the first equation from Statement (i).

$$\begin{aligned}2x + 3y &= 5 \\ \Rightarrow 2 \times (2x + 3y) &= 2 \times 5 && \text{(multiplying both sides by 2.)} \\ \Rightarrow 4x + 6y &= 10\end{aligned}$$

This is incompatible with $4x + 6y = 7$. Therefore, this set doesn't have a solution. So, the assertion is indeed right.

Consider the second equation from Statement (ii).

$$\begin{aligned}-28x + 7y &= 21 \\ \Rightarrow 4x - y &= -3 && \text{(dividing both sides by -3)}\end{aligned}$$

This is incompatible with $4x - y = -3$. Therefore, this set, too, doesn't have any solution. So, this assertion is also correct.

Short Answer Questions-I

4. (1 point) Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 6 & 7 \end{bmatrix}$. Compute AB .

Solution: Let

$$C = AB = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$

$$\begin{aligned}c_{11} &= (1 \times 0) + (2 \times 6) = 12 \\ c_{12} &= (1 \times -1) + (2 \times 7) = 13 \\ c_{21} &= (3 \times 0) + (4 \times 6) = 24 \\ c_{22} &= (3 \times -1) + (4 \times 7) = 25\end{aligned}$$

Answer:

$$AB = \begin{bmatrix} 12 & 13 \\ 24 & 25 \end{bmatrix}$$

5. (1 point) There are two matrices A and B such that:

$$A = \begin{bmatrix} x+5 & 0 & 3 \\ 4 & 0.75y & 8 \end{bmatrix}, \quad B = \begin{bmatrix} 6 & 5 & 0.5 \\ 10 & 2 & -10 \end{bmatrix}, \quad 2A + B = \begin{bmatrix} 24 & 5 & 6.5 \\ 18 & 5 & 6 \end{bmatrix}$$

Compute x and y .

Solution: Using equality of matrices, we can write:

$$\begin{aligned}2(x+5) + 6 &= 24 && 2(0.75y) + 2 = 5 \\ \Rightarrow 2(x+5) &= 18 && \Rightarrow \frac{6}{4}y = 3 \\ \Rightarrow x+5 &= 9 && \Rightarrow 6y = 12 \\ \Rightarrow x &= 9-5 && \Rightarrow y = 2 \\ \Rightarrow x &= 4 && \end{aligned}$$

Answer: $x = 4$, $y = 2$

6. (1 point) Solve for x and y :

$$\begin{aligned}4x - 3y &= 1 \\ 2x + 9y &= 4\end{aligned}$$

Solution:

$$12x - 9y = 3$$

(multiplying the first equation by 3)

$$2x + 9y = 4$$

$$\Rightarrow 14x = 7$$

(adding the two equations)

$$\Rightarrow x = \frac{1}{2}$$

$$4\left(\frac{1}{2}\right) - 3y = 1$$

(plug the value of x in the first equation)

$$\Rightarrow 2 - 3y = 1$$

$$\Rightarrow 3y = 1$$

(rearranging terms)

$$\Rightarrow y = \frac{1}{3}$$

Answer: $x = \frac{1}{2}$, $y = \frac{1}{3}$.

Short Answer Questions-II

7. (2 points) Given the following supply and demand equations:

$$\text{Supply: } P = 2Q_S^2 + 10Q_S + 10$$

$$\text{Demand: } P = -Q_D^2 - 5Q_D + 52$$

Calculate the equilibrium price and quantity.

Solution: We know that, at equilibrium, the following is true:

$$\text{Supply} = \text{Demand}$$

$$Q_S = Q_D = Q$$

Therefore,

$$2Q^2 + 10Q + 10 = -Q^2 - 5Q + 52$$

$$\Rightarrow 3Q^2 + 15Q - 42 = 0$$

(rearranging terms)

$$\Rightarrow Q^2 + 5Q - 14 = 0$$

(dividing both sides by 3)

$$\Rightarrow Q^2 + 7Q - 2Q - 14 = 0$$

(since $5Q = 7Q - 2Q$)

$$\Rightarrow Q(Q + 7) - 2(Q + 7) = 0$$

$$\Rightarrow (Q + 7)(Q - 2) = 0$$

(since $a(b + k) - c(b + k) = (a - c)(b + k)$)

$$\Rightarrow Q^* = 2$$

(since quantities cannot be negative)

$$P^* = 2(Q^*)^2 + 10Q^* + 10$$

(using the supply equation)

$$\Rightarrow P^* = 2(2)^2 + 10 \times 2 + 10$$

(since $Q^* = 2$)

$$\Rightarrow P^* = 38$$

Answer: $Q^* = 2$, $P^* = 38$

8. (2 points) Use Cramer's rule OR matrix inverse method to solve the following set of equations:

$$5x_1 + 9x_2 = 14$$

$$7x_1 - 3x_2 = 4$$

Solution: Cramer's Rule

First, write all the matrices you need.

$$A = \begin{bmatrix} 5 & 9 \\ 7 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 14 \\ 4 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

We also need D_{x_1} and D_{x_2} .

$$D_{x_1} = \begin{vmatrix} 14 & 9 \\ 4 & -3 \end{vmatrix}$$

$$D_{x_2} = \begin{vmatrix} 5 & 14 \\ 7 & 4 \end{vmatrix}$$

$$|A| = \begin{vmatrix} 5 & 9 \\ 7 & -3 \end{vmatrix} = -78$$

Therefore,

$$x_1 = \frac{D_{x_1}}{|A|}$$

$$x_2 = \frac{D_{x_2}}{|A|}$$

$$D_{x_1} = -78$$

$$D_{x_2} = -78$$

$$\Rightarrow x_1 = 1, \quad x_2 = 1$$

Matrix Inverse Method

We know that $X = A^{-1}B$ and $A^{-1} = \frac{adj(A)}{|A|}$.

$$\begin{aligned} A^{-1} &= \frac{1}{-78} \begin{bmatrix} -3 & -9 \\ -7 & 5 \end{bmatrix} \Rightarrow A^{-1}B = \frac{1}{78} \begin{bmatrix} -3 & -9 \\ -7 & 5 \end{bmatrix} \times \begin{bmatrix} 14 \\ 4 \end{bmatrix} \\ \Rightarrow A^{-1}B &= \frac{1}{-78} \begin{bmatrix} (-3 \times 14) + (-9 \times 4) \\ (-7 \times 14) + (5 \times 4) \end{bmatrix} \\ \Rightarrow A^{-1}B &= \frac{1}{-78} \begin{bmatrix} -78 \\ -78 \end{bmatrix} \\ \Rightarrow A^{-1}B &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ \Rightarrow X &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} \end{aligned}$$

Answer: $x_1 = 1$, $x_2 = 1$.