Quiz 01 (Set A)

SIAS, Krea University (AY 2025-26)

Mathematical Methods for Economics (Course Code: **ECON211**)

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Multiple Choice Questions

1. (1 point) Consider the following statements:

   Statement (i):

   $(a^r)(a^s) = a^{rs}$

   Statement (ii):

   $(a^r)^s = a^{r+s}$

   A. Both (i) and (ii) are correct.

   B. Statement (i) is correct but statement (ii) is wrong.

   C. Statement (i) is wrong but statement (ii) is correct.

   D. Both (i) and (ii) are wrong.

   Answer: D

   Solution:

   $$a^r \times a^s = a^{r+s}$$
   $$(a^r)^s = a^{rs}$$

   Compare these with the statements provided in the problem.

2. (1 point) If $x^{-2}y^3 = 5$, compute $x^2y^{-3} + 2x^{-10}y^{15}$.

   A. 1250.2

   B. 3125.2

   C. 6250

   D. 6250.2

   Answer: D

   Solution:

   $$\underbrace{x^2y^{-3}}_{A} + \underbrace{2x^{-10}y^{15}}_{B}$$

   $$A = (x^{-2}y^3)^{-1} = \frac{1}{5} = 0.2$$
   $$B = (2(x^{-2}y^5)^3) = 2 \times (5)^5 = 2 \times 3125 = 6250$$
   $$A + B = 6250.2$$

3. (1 point) There are two sets A and B.

   $$A = \{x : x \text{ is a prime number}\}$$
   $$B = \{x : x \text{ is an even number}\}$$

   The universal set is $\mathbb{U} = \{x : 0 \leq x \leq 20\}$.

   What is $A \cap B^c$?

   A. $\{3, 5, 7, 9, 11\}$

   B. $\{2, 3, 5, 6, 9, 11\}$

   C. $\emptyset$

   D. $\{3, 5, 7, 11, 13, 17, 19\}$

   Answer: D

Solution: Write the two sets and their complements.

$$\begin{aligned} A &= \{2, 3, 5, 7, 11, 13, 17, 19\} \\ B &= \{0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20\} \\ A^c &= \{0, 1, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20\} \\ B^c &= \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\} \end{aligned}$$

Therefore, $A \cap B^c = \{3, 5, 7, 11, 13, 17, 19\}$.

Short Answer Questions-I

4. (1 point) The shortest side of a triangle is given by x cm. The longest side and the third side are given by $3x$ cm and $3x-2$ cm respectively. What is the minimum value of x to have the perimeter greater than or equal to 61 cm?

Solution: The perimeter of a triangle with sides (a, b, and c) is given by $P = a + b + c$. All that we need to do is plug the given values. Let $a = x$, $b = 3x$, and $c = 3x - 2$.

$$\begin{aligned} P &= x + 3x + 3x - 2 \\ P &\geq 61 \quad \text{(given)} \\ x + 3x + 3x - 2 &\geq 61 \\ 7x - 2 &\geq 61 \\ 7x &\geq 63 \\ x &\geq 9 \end{aligned}$$

5. (1 point) Simplify the following expression: $p^2 - q^2 + (p - q)$

Solution:

$$\begin{aligned} p^2 - q^2 + (p - q) &= [(p + q)(p - q)] + (p - q) && (\text{ since } a^2 - b^2 = (a + b)(a - b)) \\ p^2 - q^2 + (p - q) &= (p - q)(p + q + 1) && (\text{taking } (p - q) \text{ as common}) \end{aligned}$$

6. (1 point) Solve for x: $|5 - 3x| \leq 4$.

Solution: If $\epsilon > 0$ then $|x| \leq \epsilon$ if and only if $-\epsilon \leq x \leq \epsilon$. We will use this rule.

$$\begin{aligned} &|5 - 3x| \leq 4 \\ &\implies -4 \leq 5 - 3x \leq 4 \\ &\implies -4 - 5 \leq 5 + (-5) - 3x \leq 4 - 5 && \text{(adding -5 to all sides)} \\ &\implies -9 \leq -3x \leq -1 \\ &\implies \frac{1}{3} \leq x \leq 3 && \text{(since we are dividing by -3, we must switch signs.)} \end{aligned}$$

Short Answer Questions-II

7. (2 points) Solve for x.

$$\frac{(x-2)+3(x+1)}{x+3} \leq 0$$

Solution: Looking at the LHS, we already know what the value of x is not going to be.

$$x \neq -3$$

$$\begin{aligned} \frac{(x-2)+3(x+1)}{x+3} &\leq 0 \\ \frac{4x+1}{x+3} &\leq 0 \end{aligned}$$

There are only two scenarios under which the inequality will hold.

Case-I: the numerator has to be negative (or zero) and the denominator has to be positive (can't be zero).

$$\begin{aligned} 4x+1 &\leq 0 \\ x+3 &> 0 \end{aligned}$$

We have: $x \leq \frac{-1}{4}$ and $x > -3$. Therefore,

$$x \in (-3, \frac{1}{4}]$$

Case-II: the numerator has to be positive (or zero) and the denominator has to be negative.

$$\begin{aligned} 4x+1 &\geq 0 \\ x+3 &< 0 \end{aligned}$$

We have: $x \geq \frac{-1}{4}$ and $x < -3$. Unfortunately, there is no such number that can be simultaneously greater than $\frac{-1}{4}$ and also less than -3.

The final answer:

$$x \in (-3, \frac{-1}{4}]$$

8. (2 points) In a survey of 25 students, it was found that 15 had taken Mathematics, 12 had taken Physics and 11 had taken Chemistry, 5 had taken Mathematics and Chemistry, 9 had taken Mathematics and Physics, 4 had taken Physics and Chemistry and 3 had taken all the three subjects. Find the number of students that had none of the subjects.

Solution: Let M be the set that contains students who took Maths, P who took Physics, and C who took Chem. Therefore, $n(M) = 15$, $n(P) = 12$, and $n(C) = 11$. We also know that:

$$
\begin{aligned}
n(M \cap C) &= 5 \\
n(M \cap P) &= 9 \\
n(P \cap C) &= 4 \\
n(M \cap P \cap C) &= 3 \\
n(\mathbb{U}) &= 25
\end{aligned}
$$

We are supposed to compute $n(M^\mathsf{c} \cap P^\mathsf{c} \cap C^\mathsf{c})$.

We know that, for any three sets A, B, C, the following is true:

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

Given the values, we will apply this formula.

$$
\begin{aligned}
& n(M \cup P \cup C) = n(M) + n(P) + n(C) - n(M \cap P) - n(M \cap C) - n(P \cap C) + n(M \cap P \cap C) \\
\implies & n(M \cup P \cup C) = 15 + 12 + 11 - 9 - 5 - 4 + 3 \\
\implies & n(M \cup P \cup C) = 38 - 18 + 3 \\
\implies & n(M \cup P \cup C) = 20 + 3 \\
\implies & n(M \cup P \cup C) = 23
\end{aligned}
$$

A complement of a set is just all the elements in the universal set excluding the ones within the set. Hence,

$$
\begin{aligned}
& n(M^\mathsf{c} \cap P^\mathsf{c} \cap C^\mathsf{c}) = n(\mathbb{U}) - n(M \cup P \cup C) \\
\implies & n(M^\mathsf{c} \cap P^\mathsf{c} \cap C^\mathsf{c}) = 25 - 23 \\
\implies & n(M^\mathsf{c} \cap P^\mathsf{c} \cap C^\mathsf{c}) = 2
\end{aligned}
$$