Midterm (Set A): Solution

SIAS, Krea University (AY 2025-26)

Mathematical Methods for Economics (Course Code: **ECON211**)

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Short Answer Questions-I

1. (2 points) Netflix ran a survey on customer preferences on genre. 1,500 customers responded to the survey revealing that 600 preferred romance (R), 450 preferred drama (D), and 350 preferred action (A). The survey also showed that 200 customers preferred both romance and drama, 150 preferred romance and action, and 100 preferred both drama and action movies. How many preferred romance but not action?

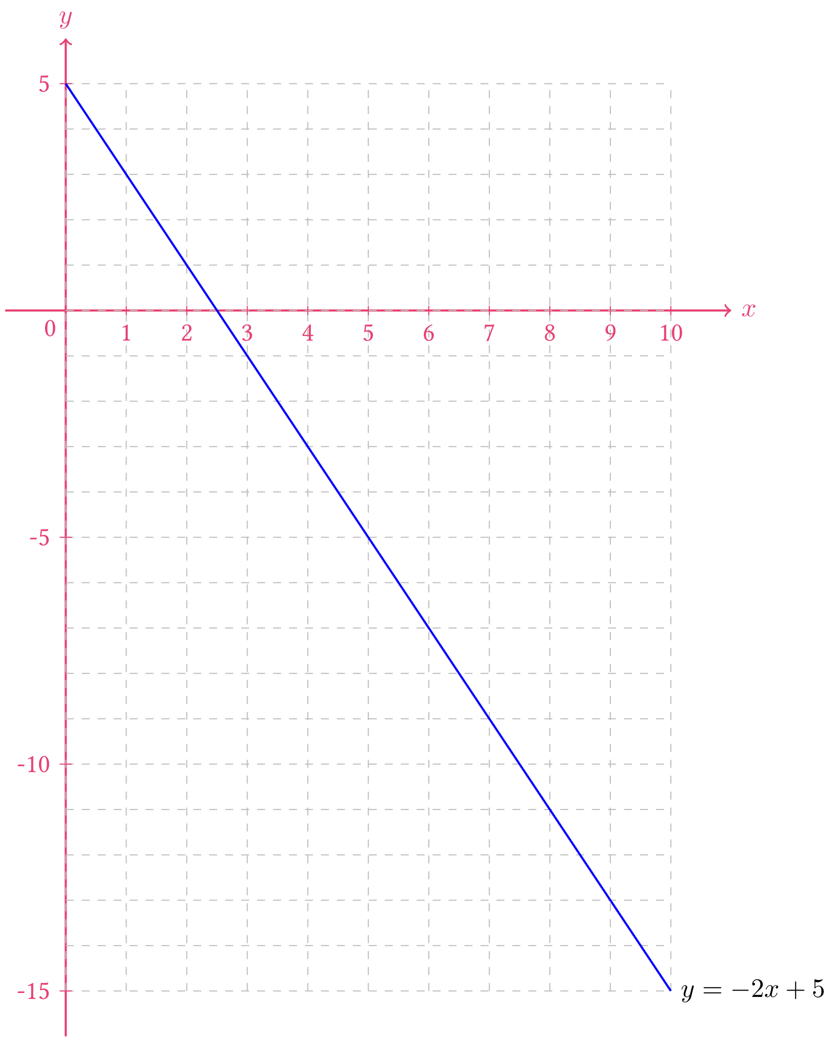
Solution:

$$|R| = 600$$
$$|R \cap A| = 150$$

$$|R \setminus A| = |R| - |R \cap A|$$
$$|R \setminus A| = 600 - 150 = 450$$

Answer: $\boxed{\text{The number of customers who preferred romance but not action is 450.}}$

2. (2 points) Sketch the graph of the function $y = -2x + 5$ in the following domain: $x \in [0, 10]$.



Solution:

3. (2 points) Compute $\sum_{k=3}^{k=6}(2k^2 - 2k - 1)$.

Solution: There are various possible ways to solve this one, but the easiest solution just involves plugging each k into the expression. The sum, therefore, is:

$$\text{When } k=3, \sum_{k=3}^{k=6}(2k^2-2k-1) = 2(3)^2-2(3)-1 = 18-6-1 = 11$$
$$\text{When } k=4, \sum_{k=3}^{k=6}(2k^2-2k-1) = 2(4)^2-2(4)-1 = 32-8-1 = 23$$
$$\text{When } k=5, \sum_{k=3}^{k=6}(2k^2-2k-1) = 2(5)^2-2(5)-1 = 50-10-1 = 39$$
$$\text{When } k=6, \sum_{k=3}^{k=6}(2k^2-2k-1) = 2(6)^2-2(6)-1 = 72-12-1 = 59$$
$$132$$

Answer: $\boxed{\sum_{k=3}^{k=6}(2k^2-2k-1) = 132}$

4. (2 points) A bakery in Indiranagar specializes in baking pound cakes. Each loaf of pound cake requires butter, sugar, and flour. The bakery generously shared some sample data with us.

Input (in kg)	August 14	August 15	August 16
Butter	6	7	9
Sugar	2	3	4
Flour	4	2	4

The per kg prices of butter, sugar, and flour are ₹1000, ₹200, and ₹100 respectively. Create two matrices, one for quantities (call it A) and one for prices (call it B). Compute and interpret AB.

Solution: The only trick here is to set up the matrices correctly.

$$A = \begin{bmatrix} 6 & 2 & 4 \\ 7 & 3 & 2 \\ 9 & 4 & 4 \end{bmatrix}$$
$$B = \begin{bmatrix} 1000 \\ 200 \\ 100 \end{bmatrix}$$
$$A \times B = \begin{bmatrix} 6 \times 1000 + 2 \times 200 + 4 \times 100 = 6800 \\ 7 \times 1000 + 3 \times 200 + 2 \times 100 = 7800 \\ 9 \times 1000 + 4 \times 200 + 4 \times 100 = 10200 \end{bmatrix}$$

Answer: $AB = \begin{bmatrix} 6,800 \\ 7,800 \\ 10,200 \end{bmatrix}$ This matrix represents the daily cost of producing poundcakes.

Short Answer Questions-II

5. (3 points) Let the universal set $\mathbb{U}$ be the set of all students at a particular university. Moreover, let F denote the set of female students, E the set of all economics students, C the set of students in the university choir, P the set of all psychology students, and T the set of all students who play tennis. Describe the members of the following sets: $\mathbb{U} \setminus E$, $E \cup C$, $P \cap F \cap T'$. No calculation is needed.

Solution:

1. $\mathbb{U} \setminus E$: All university students who are not studying economics.
2. $E \cup C$: All economics students or all students in choir or both studying economics as well as in choir.
3. $P \cap F \cap T'$: All female psychology students who do not play tennis.

6. (3 points) Consider two matrices $A = \begin{bmatrix} 2 & 2 \\ 1 & 5 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Find a matrix C such that $(A - 2I)C = I$.

Solution: It should be clear at the outset that $C = (A - 2I)^{-1}$.

$$\begin{aligned}
(A-2I) &= \begin{bmatrix} 2 & 2 \\ 1 & 5 \end{bmatrix} - 2 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
\implies (A-2I) &= \begin{bmatrix} 2 & 2 \\ 1 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \\
\implies (A-2I) &= \begin{bmatrix} 2-2 & 2-0 \\ 1-0 & 5-2 \end{bmatrix} \\
\implies (A-2I) &= \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix}
\end{aligned}$$

We can now compute the inverse of this matrix. In order to do so, we need $|A - 2I|$ and $adj(A - 2I)$ as

$$(A-2I)^{-1} = \frac{1}{|A-2I|} adj(A-2I)$$

.

$$\begin{aligned}
&|A-2I| = (0 \cdot 3 - 1 \cdot 2) \\
\implies &|A-2I| = -2
\end{aligned}$$

$$\begin{aligned}
adj(A-2I) &= \begin{bmatrix} 3 & -2 \\ -1 & 0 \end{bmatrix} \\
\implies (A-2I)^{-1} &= \frac{1}{-2} \begin{bmatrix} 3 & -2 \\ -1 & 0 \end{bmatrix} \\
\implies (A-2I)^{-1} &= \begin{bmatrix} \frac{-3}{2} & 1 \\ \frac{1}{2} & 0 \end{bmatrix}
\end{aligned}$$

Answer: $\boxed{C = \begin{bmatrix} \frac{-3}{2} & 1 \\ \frac{1}{2} & 0 \end{bmatrix}}$

7. (3 points) The value of a machine depreciates continuously at the annual rate of 10%. How many years will it take for the value of the machine to halve?

Solution: Since this is a case of continuous compounding, we can apply the following formula-

$$V_t = V_0 e^{rt}$$

Given: $r = -0.1$ (why?), $V_t = \frac{1}{2} V_0$ (why?)

$$\begin{aligned}
\frac{1}{2} V_0 &= V_0 e^{-0.1t} && \text{(using the values provided in the question)} \\
e^{-0.1t} &= \frac{1}{2} \\
e^{0.1t} &= 2 \\
0.1t &= \ln(2) && \text{(since } log(exp(x)) = x\text{)} \\
t &= \frac{\ln(2)}{0.1} \\
t &= \frac{0.693}{0.1} && \text{(using the value provided in the appendix)} \\
t &\approx 7
\end{aligned}$$

Answer: It will take approximately seven years for the machine to become half of its original value.

8. (3 points) There is an old store in Moore Market that sells audio cassettes.

(a) (2 points) If the store sells Q audio cassettes, the price received per cassette sold is $P = 100 - \frac{1}{3}Q$. The price it has to pay per cassette is $P = 80 + \frac{1}{5}Q$. In addition, it has to incur transportation cost of ₹10 per cassette. Express this store's profit π as a function of Q. Find the profit-maximizing quantity.

Solution: We know that profit equals total revenue minus total cost.

$$\pi = TR - TC$$

We have been provided per unit selling price. Therefore, the total revenue is:

$$\begin{aligned} & TR = P \times Q \\ \implies & TR = (100 - \frac{1}{3}Q) \times Q \\ \implies & TR = 100Q - \frac{1}{3}Q^2 \end{aligned}$$

We also the know the per unit cost and the per unit transportation cost. Therefore, the total cost is:

$$\begin{aligned} & TC = P \times Q + t \times Q && (t \text{ is the transportation cost}) \\ \implies & TC = (80 + \frac{1}{5}Q) \times Q + t \times Q \\ \implies & TC = (80Q + \frac{1}{5}Q^2) + 10Q && (\text{since per unit transportation cost is ₹10}) \\ \implies & TC = 90Q + \frac{1}{5}Q^2 \end{aligned}$$

We can now write the profit function.

$$\begin{aligned} & \pi = \underbrace{100Q - \frac{1}{3}Q^2}_{\text{Total Revenue}} - \underbrace{(90Q + \frac{1}{5}Q^2)}_{\text{Total Cost}} \\ \implies & \pi = 10Q - (\frac{1}{3} + \frac{1}{5})Q^2 \\ \implies & \pi = 10Q - \frac{8}{15}Q^2 \end{aligned}$$

We know that, for any quadratic function $f(x) = ax^2 + bx + c$, the maximum value is achieved $x = \frac{-b}{2a}$ if $a < 0$. In this case $a = -\frac{8}{15}$ and $b = 10$. Therefore, the profit-maximizing quantity is:

$$\begin{aligned} & Q = \frac{-10}{2 \times \frac{-8}{15}} \\ \implies & Q = \frac{10}{\frac{16}{15}} \\ \implies & Q = \frac{10 \times 15}{16} \\ \implies & Q = \frac{75}{8} \\ \implies & Q = 9.375 \end{aligned}$$

Answer: $\boxed{\begin{array}{l} \pi = 10Q - \frac{8}{15}Q^2 \\ Q = 9.375 \end{array}}$

(b) (1 point) Suppose the government imposes a tax on the store's product of ₹2 per cassette. Find the new expression for the store's profit.

Solution: We already have the profit function from the previous part. All we need to do is to subtract the new cost imposed by the government.

$$\pi = 10Q - \frac{8}{15}Q^2 - 2Q$$
$$\implies \pi = 8Q - \frac{8}{15}Q^2$$

Answer: $\boxed{\pi = 8Q - \frac{8}{15}Q^2}$

Long Answer Questions

9. (5 points) Calculate the domain and the range of the following functions:

   (a) (2 points)

$$f(x) = \frac{3x+2}{x-6}$$

Solution: The domain of a function is defined as all possible values of the input for which the function is defined. In this case, the function $f(x)$ is not defined at $x = 6$. Therefore, the domain of $f(x)$ is

$$x \in (-\infty, 6) \cup (6, \infty)$$

In order to compute the range, we will write out the input in terms of the output.

$$y = \frac{3x+2}{x-6}$$
$$\implies y(x-6) = 3x+2$$
$$\implies xy - 6y = 3x+2$$
$$\implies xy - 3x = 6y+2$$
$$\implies x(y-3) = 6y+2$$
$$\implies x = \frac{6y+2}{y-3}$$

Since the range is the set of all valid values of the output, it doesn't seem like y can be ever equal to three. Therefore, the range of the function $f(x)$ is:

$$f(x) \in (-\infty, 3) \cup (3, \infty)$$

Answer:

Domain: $x \in (-\infty, 6) \cup (6, \infty)$

Range: $f(x) \in (-\infty, 3) \cup (3, \infty)$

   (b) (3 points)

$$g(x) = \frac{1}{\sqrt{x^2 - 1}}$$

Solution: Observing the denominator of the function, we can say that

$$x^2 - 1 > 0$$

Why? Because, this expression sits inside a square root. Moreover, while the square root of 0 is indeed a valid value, the function won't be defined when $x^2 = 1$. Therefore,

$$x > 1 \text{ or } x < -1$$

The domain of the function is:

$$x \in (-\infty, -1) \cup (1, \infty)$$

As $x \to \pm\infty$, $f(x) \to 0$ and when $x \to \pm 1$, $f(x) \to \infty$, but we will show this more formally.

$$\begin{aligned}
y &= \frac{1}{\sqrt{x^2-1}} \\
y^2 &= \frac{1}{x^2-1} && \text{(squaring both sides)} \\
y^2(x^2-1) &= 1 \\
x^2y^2 - y^2 &= 1 \\
x^2y^2 &= y^2+1 \\
x^2 &= \frac{y^2+1}{y^2} \\
x &= \sqrt{\frac{y^2+1}{y^2}} \\
x &= \sqrt{1+\frac{1}{y^2}}
\end{aligned}$$

We can now say, with some degree of confidence, the values that won't be possible for the function to take. $y \neq 0$. At this stage, it is tempting to write the range as $f(x) \in \mathbb{R} \setminus 0$, but look at the function itself. $f(x) \not< 0$. Therefore, the range of the function is:

$$f(x) \in (0, \infty)$$

Answer: **Domain:** $x \in (-\infty, -1) \cup (1, \infty)$

Range: $f(x) \in (0, \infty)$

10. (5 points) This question tests your knowledge and understanding of present value and interest rates. Assume compounding of the interest rate.

(a) (1 point) A sum of ₹10,000 is invested at 5% annual interest. What will this amount have grown to after 10 years?

Solution:

$$\text{Balance} = P(1+r)^n$$

$P = 10,000$, $r = 0.05$, $n = 10$.

$$\begin{aligned}
& \text{Balance} = 10,000(1+0.05)^{10} \\
\implies & \text{Balance} = 10,000(1.629) && \text{(using the value provided in the appendix)} \\
\implies & \text{Balance} = 16,290
\end{aligned}$$

Answer: Balance after ten years will be ₹16,290.

(b) (2 points) Which terms are preferable for a borrower: (i) an annual interest rate of 21.5%, with interest paid yearly; or (ii) an annual interest rate of 20%, with interest paid quarterly?

Solution: We need to compute the EAR for (ii). We know that:

$$EAR = \left(1+\frac{r}{m}\right)^m - 1$$

$r = 0.2$, $m = 4$. Therefore,

$$\begin{aligned}
EAR_{ii} &= \left(1+\frac{0.2}{4}\right)^4 - 1 \\
EAR_{ii} &= (1+0.05)^4 - 1 \\
EAR_{ii} &= (1.05)^4 - 1
\end{aligned}$$

Using the value from the appendix, we get:

$$\begin{aligned}
EAR_{ii} &= 1.216 - 1 \\
EAR_{ii} &= 0.216 \\
EAR_{ii} &= 21.6\%
\end{aligned}$$

$EAR_i < EAR_{ii}$.

Answer: the borrower will prefer term (i).

(c) (2 points) An account has been dormant for many years earning interest at the constant rate of 4% per year, with interest being compounded every six months. The current balance is ₹750,000. How much was in the account 10 years ago?

Solution: We know that-

$$PV = \frac{\text{Balance}}{\left(1 + \frac{r}{m}\right)^p}$$

Given: Balance $= 7,50,000$, $r = 4\%$, $m = 2$, $p = 10 \times 2 = 20$.

$$\begin{aligned}
PV &= \frac{750000}{\left(1 + \frac{0.04}{2}\right)^{20}} \\
\implies PV &= \frac{750000}{(1+0.02)^{20}} \\
\implies PV &= \frac{750000}{(1.02)^{20}} \\
\implies PV &\approx \frac{750000}{1.5} && \text{(using value provided in the appendix)} \\
\implies PV &\approx 5,00,000
\end{aligned}$$

Answer: the balance in the account ten years ago was ₹5,00,000.