

Endterm (Set A)

SIAS, Krea University (AY 2025-26)

Mathematical Methods for Economics (Course Code: ECON211)

25 September 2025

Maximum Points: 30

Duration: 100 minutes

Short Answer Questions-I

1. (1 point) Is $\lim_{x \rightarrow 0} |x - 2| = \lim_{x \rightarrow 0} |x| - 2$? Explain briefly.

Solution: Consider $f(x) = |x - 2|$.

$$f(x) = \begin{cases} 2 - x & x < 2 \\ x - 2 & x > 2 \end{cases}$$

$$\text{LHL: } \lim_{x \rightarrow 0^-} |x - 2| = 2$$

(since $|x - 2| = 2 - x$ when $x < 2$)

$$\text{RHL: } \lim_{x \rightarrow 0^+} |x - 2| = 2$$

(since $|x - 2| = 2 - x$ when $x < 2$)

$$\therefore \lim_{x \rightarrow 0} |x - 2| = 2$$

Let $g(x) = |x| - 2$.

$$g(x) = \begin{cases} -x - 2 & x < 0 \\ x - 2 & x > 0 \end{cases}$$

$$\text{LHL: } \lim_{x \rightarrow 0^-} |x| - 2 = -2$$

(since $|x| = -x$ when $x < 0$)

$$\text{RHL: } \lim_{x \rightarrow 0^+} |x| - 2 = -2$$

(since $|x| = x$ when $x > 0$)

$$\therefore \lim_{x \rightarrow 0} |x| - 2 = -2$$

Answer: $\lim_{x \rightarrow 0} |x - 2| \neq \lim_{x \rightarrow 0} |x| - 2$

2. (1 point) Let $f(x) = \frac{6}{2+x}$. Find $f^{-1}(x)$.

Solution: Let $y = f(x)$.

$$y = \frac{6}{x+2}$$

$$\Rightarrow y(x+2) = 6$$

$$\Rightarrow xy + 2y = 6$$

$$\Rightarrow xy = 6 - 2y$$

$$\Rightarrow x = \frac{6 - 2y}{y}$$

$$\Rightarrow f^{-1}(x) = \frac{6 - 2x}{x}$$

Answer: $f^{-1}(x) = \frac{6 - 2x}{x}$

3. (1 point) Let $f(x) = x^{x-1}$. Find $f'(x)$.

Solution: Let $y = x^{x-1}$.

$$\begin{aligned}\ln y &= (x-1) \ln x && \text{(using the property of } \log a = b^c \implies \ln a = c \ln b) \\ \implies \frac{1}{y} \frac{dy}{dx} &= \frac{d}{dx}((x-1) \ln x) && \text{(differentiating both sides w.r.t. } x) \\ \implies \frac{1}{y} \frac{dy}{dx} &= \ln x \frac{d}{dx}(x-1) + (x-1) \frac{d}{dx}(\ln x) && \text{(applying the product rule to the RHS)} \\ \implies \frac{1}{y} \frac{dy}{dx} &= \ln x + \frac{x-1}{x} \\ \implies \frac{dy}{dx} &= y \left(\ln x + \frac{x-1}{x} \right) \\ \implies \frac{dy}{dx} &= x^{x-1} \left(\ln x + \frac{x-1}{x} \right)\end{aligned}$$

Answer: $\frac{dy}{dx} = x^{x-1} \left(\ln x + \frac{x-1}{x} \right)$

4. (1 point) Determine if the function $f(x) = x^2 - 4x + 3$ is increasing or decreasing in $[1, 3]$.

Solution: First derivative tells us whether (or where) a function is increasing or decreasing.

$$\begin{aligned}f'(x) &= 2x - 4 \\ \text{Condition for increasing function : } 2x - 4 &\geq 0 \\ \therefore \text{ the function is increasing when } x &\geq 2 \\ \text{Condition for decreasing function : } 2x - 4 &\leq 0 \\ \therefore \text{ the function is decreasing when } x &\leq 2\end{aligned}$$

Answer: $f(x)$ is decreasing in $[1, 2]$ and increasing in $[2, 3]$.

5. (1 point) Let $x^2y^3 + x^3y^2 = 5$. Find $\frac{dy}{dx}$.

Solution: Let $u = x^2y^3$, $v = x^3y^2$, and $c = 5$.

Applying the product rule, we get:

$$u' = 2xy^3 + 3x^2y^2 \frac{dy}{dx}$$

$$v' = 2x^3y \frac{dy}{dx} + 3x^2y^2$$

$$c' = 0$$

$$u' + v' = c'$$

$$\therefore \left[2xy^3 + 3x^2y^2 \frac{dy}{dx} \right] + \left[2x^3y \frac{dy}{dx} + 3x^2y^2 \right] = 0$$

$$\Rightarrow (2xy^3 + 3x^2y^2) + (3x^2y^2 + 2x^3y) \frac{dy}{dx} = 0$$

$$\Rightarrow (3x^2y^2 + 2x^3y) \frac{dy}{dx} = -(2xy^3 + 3x^2y^2)$$

$$\Rightarrow \frac{dy}{dx} = -\frac{(2xy^3 + 3x^2y^2)}{(3x^2y^2 + 2x^3y)}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{y(3x + 2y)}{x(2x + 3y)}$$

Answer: $\frac{dy}{dx} = -\frac{y(3x + 2y)}{x(2x + 3y)}$

6. (1 point) Let $f(x) = \sqrt{x} + 3$ and $g(x) = f^{-1}(x)$. Find $g'(5)$.

Solution: We know that: $g'(a) = \frac{1}{f'(g(a))}$

$$\therefore g'(5) = \frac{1}{f'(g(5))}$$

$$\text{Let } g(5) = k \Rightarrow f(k) = 5.$$

$$\sqrt{k} + 3 = 5$$

$$\Rightarrow \sqrt{k} = 5 - 3$$

$$\Rightarrow \sqrt{k} = 2$$

$$\Rightarrow k = 4$$

$$f'(x) = \frac{1}{2\sqrt{x}}$$

$$\Rightarrow f'(4) = \frac{1}{2\sqrt{2}}$$

$$\Rightarrow f'(4) = \frac{1}{4}$$

$$\Rightarrow g'(5) = 4$$

Answer: $g'(5) = 4$

7. (1 point) Suppose that f and g are continuous on $[0, 4]$ and that $\int_0^4 (f(x) - g(x))dx = 2$ and $\int_0^4 (3f(x) - 4g(x))dx = 4$. Find $\int_0^4 (f(x) + g(x))dx$.

Solution: Let $\int_0^4 f(x)dx = a$ and $\int_0^4 g(x)dx = b$. Then,

$$a - b = 2$$

$$3a - 4b = 4$$

$$3a - 3b = 6$$

$$\Rightarrow b = 2$$

$$\Rightarrow a = 4$$

(multiplying the first equation by 3)

(differencing the previous two equations to eliminate a)

$$\begin{aligned}\int_0^4 (f(x) + g(x))dx &= \int_0^4 f(x)dx + \int_0^4 g(x)dx && \text{(applying the sum rule)} \\ \Rightarrow \int_0^4 (f(x) + g(x))dx &= 6\end{aligned}$$

Answer : $\int_0^4 (f(x) + g(x))dx = 6$

8. (1 point) Compute: $\int (3x^2 + \frac{2}{x} + e^{3x})dx$

Solution:

$$\begin{aligned}\int (3x^2 + \frac{2}{x} + e^{3x})dx &= \int 3x^2dx + \int \frac{2}{x}dx + \int e^{3x}dx && \text{(applying the sum rule)} \\ \Rightarrow \int (3x^2 + \frac{2}{x} + e^{3x})dx &= x^3 + 2 \ln |x| + \frac{e^{3x}}{3} + C\end{aligned}$$

Answer : $\int (3x^2 + \frac{2}{x} + e^{3x})dx = x^3 + 2 \ln |x| + \frac{e^{3x}}{3} + C$

Short Answer Questions-II

9. (3 points) Given the demand function for comedy shows on *Ruinmyshow*: $p = \frac{16}{q+3} - 3$,

(a) ($\frac{1}{2}$ points) Compute the total revenue.

Solution: Total revenue, $TR = p \cdot q$.

$$\begin{aligned}p &= \frac{16}{q+3} - 3 \\ \Rightarrow TR &= \frac{16q}{q+3} - 3q\end{aligned}$$

(b) ($\frac{1}{2}$ points) Compute the marginal revenue.

Solution: We know that $MR = \frac{d}{dq}(TR)$.

$$TR = \frac{16q}{q+3} - 3q$$

$$\text{Let } u = 16q, \quad v = q + 3$$

$$\Rightarrow u' = 16, \quad v' = 1$$

$$MR = \frac{vu' - uv'}{v^2} - 3$$

$$\Rightarrow MR = \frac{16(q+3) - 16q}{(q+3)^2} - 3$$

$$\Rightarrow MR = \frac{48}{(q+3)^2} - 3$$

$$\Rightarrow MR = \frac{48 - 3(q+3)^2}{(q+3)^2}$$

Answer: $MR = \frac{48 - 3(q+3)^2}{(q+3)^2}$

(c) (2 points) Compute the revenue-maximizing price and quantity.

Solution: We know that the revenue is maximized when $MR = 0$.

$$48 - 3(q+3)^2 = 0 \quad (\text{note that the denominator can't be zero.})$$

$$\Rightarrow 3(q+3)^2 = 48$$

$$\Rightarrow (q+3)^2 = 16$$

$$\Rightarrow (q+3) = \pm 4$$

$$\Rightarrow q+3 = 4$$

(discarding the negative value.)

$$\Rightarrow q = 1$$

Plugging the value into the demand equation, we get $p = \frac{16}{1+3} - 3$

$$\Rightarrow p = \frac{16}{4} - 3$$

$$\Rightarrow p = 4 - 3$$

$$\Rightarrow p = 1$$

Answer: The revenue-maximizing price is $p = 1$ and the quantity is $q = 1$.
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10. (2+1 points) The total cost of producing *Phantom cigarettes* is $C(q) = 2q^2 + 10q + 32$. Find the value of q which minimizes the average cost. Show that the marginal cost is equal to the average cost at this point (where the average cost is being minimized).

Solution: We know that the average cost is:

$$AC(q) = \frac{C(q)}{q}$$

The average cost of producing *Phantom cigarettes* is:

$$AC(q) = 2q + 10 + \frac{32}{q}$$

We need to find the first derivative and set it to zero.

$$\begin{aligned}\frac{d(AC(q))}{dq} &= \frac{d}{dq} \left(2q + 10 + \frac{32}{q} \right) \\ &= 2 - \frac{32}{q^2} \\ \text{FOC: } \frac{d(AC(q))}{dq} &= 0 \\ \Rightarrow 2 - \frac{32}{q^2} &= 0 \\ \Rightarrow 2q^2 &= 32 \\ \Rightarrow q^2 &= 16 \\ \Rightarrow q^* &= 4 \\ \text{SOC: } \frac{d^2(AC(q))}{dq^2} &> 0 \quad (\text{for minimum}) \\ \frac{d^2(AC(q))}{dq^2} &= \frac{64}{q^3} \\ \frac{64}{q^3} &> 0 \text{ when } q = 4\end{aligned}$$

We also need to compute the marginal cost.

$$MC(q) = 4q + 10$$

When $q = 4$,

$$\begin{aligned}\text{Average cost: } AC(q = 4) &= 2q + 10 + \frac{32}{4} \\ &= 8 + 10 + 8 \\ &= 26 \\ \text{Marginal cost: } MC(q = 4) &= 4(4) + 10 \\ &= 26\end{aligned}$$

Answer: The quantity that minimizes the average cost is $q^* = 4$. When $q = 4$, $AC = MC = 26$.

11. (3 points) $f(x, y) = 6x^{1/3}y^{2/3}$

(a) (1 point) Determine the degree of homogeneity.

Solution: The degree of homogeneity can be calculated using:

$$\begin{aligned}f(tx, ty) &= t^k f(x, y) \\ f(tx, ty) &= 6(tx)^{1/3}(ty)^{2/3} \\ &= 6t^{1/3}x^{1/3}t^{2/3}y^{2/3} \\ &= (t^{1/3+2/3})6(x^{1/3}y^{2/3}) \\ &= t^1 f(x, y) \\ \Rightarrow k &= 1\end{aligned}$$

Answer: $f(x, y)$ is homogeneous of degree 1.

(b) (2 points) Compute all first and second order partial derivatives.

f_x	$\begin{aligned}\frac{\partial f}{\partial x} &= \frac{\partial}{\partial x} (6x^{1/3}y^{2/3}) \\ &= 6 \cdot \frac{1}{3}x^{-2/3}y^{2/3} \\ &= 2x^{-2/3}y^{2/3} \\ &= 2\left(\frac{y}{x}\right)^{2/3}\end{aligned}$
f_y	$\begin{aligned}\frac{\partial f}{\partial y} &= \frac{\partial}{\partial y} (6x^{1/3}y^{2/3}) \\ &= 6x^{1/3} \cdot \frac{2}{3}y^{-1/3} \\ &= 4x^{1/3}y^{-1/3} \\ &= 4\left(\frac{x}{y}\right)^{1/3}\end{aligned}$
f_{xx}	$\begin{aligned}\frac{\partial^2 f}{\partial x^2} &= \frac{\partial}{\partial x} (2x^{-2/3}y^{2/3}) \\ &= 2y^{2/3} \cdot \left(-\frac{2}{3}\right)x^{-5/3} \\ &= -\frac{4}{3}x^{-5/3}y^{2/3} \\ &= -\frac{4}{3}\left(\frac{y^2}{x^5}\right)^{1/3}\end{aligned}$
Solution: f_{yy}	$\begin{aligned}\frac{\partial^2 f}{\partial y^2} &= \frac{\partial}{\partial y} (4x^{1/3}y^{-1/3}) \\ &= 4x^{1/3} \cdot \left(-\frac{1}{3}\right)y^{-4/3} \\ &= -\frac{4}{3}x^{1/3}y^{-4/3} \\ &= -\frac{4}{3}\left(\frac{x}{y^4}\right)^{1/3}\end{aligned}$
f_{xy}	$\begin{aligned}\frac{\partial^2 f}{\partial x \partial y} &= \frac{\partial}{\partial y} (2x^{-2/3}y^{2/3}) \\ &= 2x^{-2/3} \cdot \frac{2}{3}y^{-1/3} \\ &= \frac{4}{3}x^{-2/3}y^{-1/3} \\ &= \frac{4}{3}\left(\frac{1}{x^2y}\right)^{1/3}\end{aligned}$
f_{yx}	$\begin{aligned}\frac{\partial^2 f}{\partial y \partial x} &= \frac{\partial}{\partial x} (4x^{1/3}y^{-1/3}) \\ &= 4y^{-1/3} \cdot \frac{1}{3}x^{-2/3} \\ &= \frac{4}{3}x^{-2/3}y^{-1/3} \\ &= \frac{4}{3}\left(\frac{1}{x^2y}\right)^{1/3}\end{aligned}$

12. (1+1+1 points) Let $U = 2\sqrt{x} + y$. Compute the marginal utilities and the marginal rate of substitution.

Solution:

$$\begin{aligned}MU_x &= \frac{1}{\sqrt{x}} \\ MU_y &= 1 \\ MRS_{x,y} &= \frac{MU_x}{MU_y} \\ MRS_{x,y} &= \frac{1}{\sqrt{x}}\end{aligned}$$

Long Answer Questions

13. (5 points) Consider $f(x, y) = \frac{x^3}{3} + \frac{y^3}{3} + x^2 + \frac{y^2}{2} - 3x - 6y + 5$. Find and classify all stationary points.

Solution: To find the stationary points, we first compute the first-order partial derivatives and set them equal to zero.

$$\begin{aligned}f_x &= \frac{\partial f}{\partial x} = (x^2 + 2x - 3) = 0 \\ f_y &= \frac{\partial f}{\partial y} = (y^2 + y - 6) = 0\end{aligned}$$

We have two quadratic equations to be solved.

$$\begin{aligned}(x + 3)(x - 1) &= 0 \\ (y + 3)(y - 2) &= 0 \\ x &= 1, -3 \\ y &= 2, -3\end{aligned}$$

The stationary points are the combinations of these x and y values:

$$\begin{array}{cc}(-3, -3) & (-3, 2) \\ (1, -3) & (1, 2)\end{array}$$

To classify the stationary points, we use the second derivative test, which requires the second-order partial derivatives.

$$\begin{aligned}f_{xx} &= \frac{\partial^2 f}{\partial x^2} = 2x + 2 \\ f_{yy} &= \frac{\partial^2 f}{\partial y^2} = 2y + 1 \\ f_{xy} &= \frac{\partial^2 f}{\partial x \partial y} = 0\end{aligned}$$

The determinant of the Hessian matrix is $D = f_{xx}f_{yy} - (f_{xy})^2 = (2x + 2)(2y + 1)$.

We evaluate the Hessian determinant $D = f_{xx}f_{yy} - (f_{xy})^2$ at each stationary point:

- If $D > 0$ and $f_{xx} > 0$: **Local Minimum**
- If $D > 0$ and $f_{xx} < 0$: **Local Maximum**
- If $D < 0$: **Saddle Point**
- If $D = 0$: **Test Inconclusive**

Stationary Point	f_{xx}	f_{yy}	D	Classification
$(-3, -3)$	-4	-5	20 (> 0)	Local Maximum
$(-3, 2)$	-4	5	-20 (< 0)	Saddle Point
$(1, -3)$	4	-5	-20 (< 0)	Saddle Point
$(1, 2)$	4	5	20 (> 0)	Local Minimum

14. (5 points) The demand for robots in *Tatooine* is given by $p = 20 - 2q$ and the supply of robots is given by $p = 4 + 2q$.

(a) (1 point) Compute the equilibrium price and quantity.

Solution: The equilibrium can be found out by setting demand = supply.

$$20 - 2q = 4 + 2q$$

$$\Rightarrow 4q = 16$$

$$\Rightarrow q^* = 4$$

$$\Rightarrow p^* = 12$$

Answer: The equilibrium quantity is $q^* = 4$ and the equilibrium price is $p^* = 12$.

(b) (1+1 points) Compute the consumer surplus and producer surplus.

Solution: Given inverse demand ($D(q)$) and inverse supply ($S(q)$), we know that:

$$CS = \int_{q=0}^{q=q^*} D(q) dq - p^* q^*$$

$$PS = p^* q^* - \int_{q=0}^{q=q^*} S(q) dq$$

We also know from the previous calculation that $p^* = 12$ and $q^* = 4$.

$$\begin{aligned} CS &= \int_0^4 (20 - 2q) dq - 48 \\ &= \left| 20q - q^2 \right|_0^4 - 48 \\ &= 80 - 16 - 48 \\ &= 16 \end{aligned}$$

$$\begin{aligned} PS &= 48 - \int_0^4 (4 + 2q) dq \\ &= 48 - \left| 4q + q^2 \right|_0^4 \\ &= 48 - (16 + 16) \\ &= 16 \end{aligned}$$

Answer: $CS = 16$, $PS = 16$

(c) (1+1 points) Now, suppose that the Damiyo (the ruler of Tatooine), sensing that the robots are valuable, announces a price floor of 14. Compute the new consumer surplus and producer surplus.

Solution: When $p = 14$, we should compute the quantity demanded and the quantity supplied.

$$20 - 2q = 14$$

$$2q = 6$$

$$\text{Demand: } q = 3$$

$$4 + 2q = 14$$

$$2q = 10$$

$$\text{Supply: } q = 5$$

At this price, three robots will be sold in *Tatooine*.

$$\begin{aligned} CS &= \int_0^3 (20 - 2q) - 14 \times 3 \\ &= \left| 20q - q^2 \right|_0^3 - 42 \\ &= (60 - 9) - 42 \\ &= 9 \end{aligned}$$

$$\begin{aligned} PS &= 42 - \int_0^3 (4 + 2q) dq \\ &= 42 - \left| 4q + q^2 \right|_0^3 \\ &= 42 - (12 + 9) \\ &= 21 \end{aligned}$$

Answer: $CS = 9$, $PS = 21$