Basic Algebra-II

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1 Inequalities

- For any $a, b \in \mathcal{R}$, either $a < b$, $b < a$, or $a = b$.
- If $a < b$, then $a + c < b + c$ for all $c \in \mathcal{R}$.
- If $a > 0$ and $b > 0$, then $a + b > 0$ and $a \times b > 0$.
- If $a > b$ and $c < 0$, then $ac < bc$.
- If $a > b$ and $c > d$, then $a + c > b + d$.
- If $0 < a < b$, then $0 < \frac{1}{b} < \frac{1}{a}$.

These rules will still hold even if we are dealing with weak inequalities ($\leq$).

2 Absolute Values and Intervals

2.1 Intervals

Let $a, b \in \mathcal{R}$.

- (a, b) represents the open interval from a to b ($a < x < b$).
- $[a, b]$ represents the closed interval from a to b ($a \leq x \leq b$).
- $(a, b]$ represents the half-open interval from a to b ($a < x \leq b$).
- $[a, b)$ represents the half-open interval from a to b ($a \leq x < b$).

2.2 Absolute Value

Let $x \in \mathcal{R}$. The absolute value of x is its distance from zero. We can define:

$$|x| = \begin{cases} x, & \text{if } x > 0 \\ -x, & \text{if } x < 0 \end{cases}$$

Super-optional, but absolute value also tells us that:

1. $|xy| = |x| \times |y|$
2. If $\epsilon > 0$ then $|x| \leq \epsilon$ if and only if $-\epsilon \leq x \leq \epsilon$.
3. $|x + y| \leq |x| + |y|$. This one is known as the *triangle inequality*.

3 Linear Equations

A generic linear equation is of the following form:

$$y = ax + b$$

We will discuss more about this equation when we cover functions, but it will suffice to know that a and b are parameters in this equation. Typically, when you have linear equation with m variables, you will need to have m equations in order to solve for those variables. For instance:

$$\begin{aligned} x + y &= 3 \\ x - y &= 1 \end{aligned}$$

In above equation, we have two variables (x and y) and two equations. Therefore, you can solve for the values of x and y. However, consider the following equation:

$$\begin{aligned} x + y + z &= 6 \\ x - y &= 0 \end{aligned}$$

In this case, we cannot determine the values because the number of equations (2) is less than the number of variables (3).

Where are we going to use linear equations in econ? In micro-economics, you will use it to solve for equilibrium price and quantity of a good. Consider the following demand and supply equations.

$$\begin{aligned} Q^D &= 100 - 2P \\ Q^S &= 3P - 20 \end{aligned}$$

where Q^D is the quantity demanded (of an unknown, unnamed good) and Q^S represents the quantity supplied; P is the price of the good. It seems here that we have two equations but three variables (P, Q^D, Q^S), but as you will learn later (in microecon) that, in equilibrium, quantity demanded equals quantity supplied. Therefore, we also have our third equation which is $Q^D = Q^S = Q$.

$$\begin{aligned} Q &= 100 - 2P \\ Q &= 3P - 20 \end{aligned}$$

You can now easily eliminate Q to first solve for P. It turns out that $P^* = 24$ and $Q^* = 52$. Note here the little asterisk that I have put. This is the formal notation for the equilibrium.

Let's extend this to another case where the government decides to tax the supply by ₹10. So, it seems like the new price will be $P^+ = P + 10$. We will first rewrite the supply equation.

$$\begin{aligned} Q &= 3P - 20 \\ 3P &= Q + 20 \\ P &= \frac{Q + 20}{3} \end{aligned}$$

Therefore, $P^+ = \frac{Q+20}{3} + 10$, or some math torture later, $P^+ = \frac{Q+50}{3}$. This is your new supply equation given taxes. Now, we once again have two equations with two variables.

$$Q = 100 - 2P^+$$
$$P^+ = \frac{Q+50}{3}$$

We should rewrite the demand equation so that we can easily eliminate P^+ to solve for Q^*.

$$P^+ = \frac{100-Q}{2}$$
$$P^+ = \frac{Q+50}{3}$$

A few minutes later, you will discover that $Q^* = 40$ and plugging back this value into one of the two equations will get you $P^* = 30$. This makes a lot of sense to me. Taxation should bump up the prices and lower the demand for the good.

Now, I have a question for you. What happens if the government decides to impose an ad-valorem tax, say 10%, on the supply?

Another use case of linear equation lies in macroeconomics. We have the following set of equations:

$$\begin{aligned}
Y &= C + I + G \\
C &= c_0 + c_1 \times (Y - T) \\
I &= \bar{I} \\
G &= \bar{G}
\end{aligned}$$

where Y happens to be the GDP, C is the consumption, T is total taxes, I is the total investment, and G is the total government spending. c_0 is the consumption people would have even without any income, c_1 is the sensitivity of the consumption to income (and $0 < c_1 < 1$).

The first one is an accounting identity which says that total GDP of a country is the sum of consumption, investment, and government spending. The second one says that the consumption in any economy depends upon the GDP. The third one, for the sake of simplicity, assumes that the total investment in an economy is a fixed amount and so is the government spending.

One big question that bothers economists a lot is the one around the relationship between government spending and the GDP. Let's write an equation.

$$\begin{aligned}
Y &= (c_0 + c_1 \times (Y - T)) + \bar{I} + \bar{G} \\
Y &= c_0 + \bar{I} + \bar{G} - c_1 \times T + c_1 \times Y \\
Y \times (1 - c_1) &= c_0 + \bar{I} + \bar{G} \\
Y &= \frac{c_0 + \bar{I} + \bar{G}}{(1 - c_1)}
\end{aligned}$$

At this point, we will not worry about the meaning of this final equation.

4 Quadratic Equations

The general form of a quadratic equation is

$$ax^2 + bx + c = 0$$

where $a \neq 0$.
If $b^2 - 4ac \geq 0$ and $a \neq 0$, then

$$ax^2 + bx + c = 0 \quad \text{if and only if } x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

If x_1 and x_2 are the solutions (or the roots) of $ax^2 + bx + c = 0$, then

$$ax^2 + bx + c = a(x - x_1)(x - x_2)$$

Let me also discuss a useful (specific) case. If we have a quadratic equation $x^2 + px + q$, then the roots of the equation are:

$$x = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$$

If x_1 and x_2 are the roots of the quadratic equation $x^2 + px + q$, then

$$x_1 + x_2 = -p$$
$$x_1 \times x_2 = q$$

An example will help us understand these rules.

Consider the following quadratic equation, $x^2 - 6x + 8 = 0$. Here, $a = 1$, $b = -6$, and $c = 8$. The first thing that you should always do is to check the square root of the following term: $b^2 - 4ac$. In this case, $b^2 - 4ac = (-6)^2 - 4 \times 1 \times 8 = 36 - 32 = 4$. So, this quadratic equation indeed has real roots.

- Pathway I: Use $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$.
- Pathway II: Use $x = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$

Either of these strategies must yield the solution: $x = \{2, 4\}$.