Quiz 03 (Set B (Solution))

SIAS, Krea University (AY 2025-26)

Mathematical Methods for Economics (Course Code: **ECON211**)

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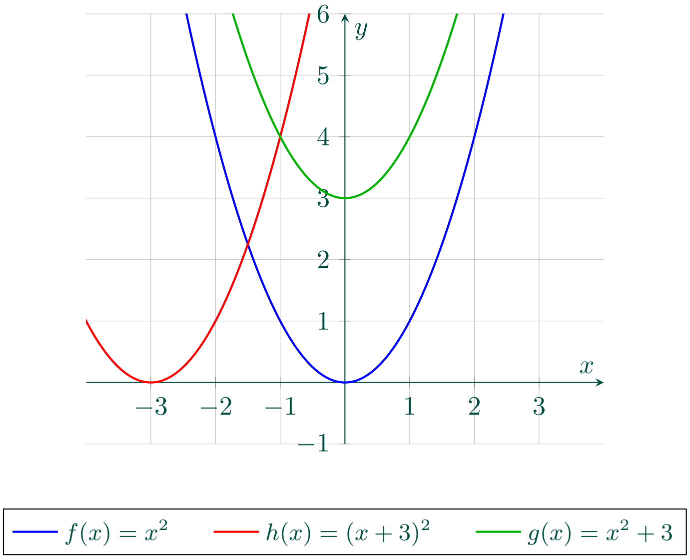
Multiple Choice Questions

1. (1 point) If $f(x) = x^2$, $g(x) = x^2 + 3$ and $h(x) = (x+3)^2$, then
   A. the graph of $g(x)$ can be obtained by shifting $f(x)$ downwards by 3 units.
   B. the graph of $h(x)$ can be obtained by shifting $f(x)$ to the right by 1 unit.
   C. the graph of $h(x)$ can be obtained by shifting $f(x)$ to the left by 1 unit.
   D. the graph of $g(x)$ can be obtained by shifting $f(x)$ upwards by 3 units.

   Answer: D

Solution: This is very straightforward. $g(x)$ is $f(x)$ shifted up three units and $h(x)$ is $f(x)$ being shifted to the left by three units.



2. (1 point) Let $f(x) = 100$. Then,
   A. $f^{-1}(x)$ does not exist.
   B. $f^{-1}(x) = 100$
   C. $f^{-1}(x) = \dfrac{1}{100}$
   D. $f^{-1}(x) = \dfrac{1}{100x}$

   Answer: A

Solution: Consider two points in the domain of the function: $x = 1$ and $x = 2$.

$f(1) = 100$ and $f(2) = 100$.

What happens when you 'invert' this function? You get:

$f^{-1}(100) = 1$ and $f^{-1}(100) = 2$.

This cannot be a valid function as it is not one-to-one. Therefore, the inverse does not exist.

3. (1 point) Consider the following statements:

   Statement (i):

   $\lim_{x\to 2} |x-2|$ does not exist.

   Statement (ii):

   $f(x) = |x-2|$ is not differentiable at $x = 2$.

A. Both (i) and (ii) are correct.
B. Statement (i) is correct but statement (ii) is wrong.
C. Statement (i) is wrong but statement (ii) is correct.
D. Both (i) and (ii) are wrong.

Answer: C

Solution:

$$\text{LHL: } \lim_{x\to 2^-} |x-2| = 0$$
$$\text{RHL: } \lim_{x\to 2^+} |x-2| = 0$$

LHL = RHL. Therefore, the limit does exist.

$$\text{LHD: } \lim_{x\to 2^-} -1 = -1 \qquad (\text{since } |x-2| = 2-x \quad \forall x < 2)$$
$$\text{RHD: } \lim_{x\to 2^+} 1 = 1 \qquad (\text{since } |x-2| = x-2 \quad \forall x > 2)$$

LHD $\neq$ RHD. Therefore, $f(x)$ is not differentiable at $x = 0$.

Short Answer Questions-I

4. (1 point) Compute $\frac{dy}{dx}$ if $y = 4x + \frac{2}{\sqrt{x}}$.

Solution:

$$y = 4x + \frac{2}{\sqrt{x}}$$
$$\frac{dy}{dx} = 4 + \frac{d(\frac{2}{\sqrt{x}})}{dx}$$
$$\frac{dy}{dx} = 4 + 2\frac{d(x^{-1/2})}{dx}$$
$$\frac{dy}{dx} = 4 - 2\left(\frac{1}{2}x^{-3/2}\right)$$

Answer: $\boxed{\frac{dy}{dx} = 4 - x^{-3/2}}$

5. (1 point) Compute the inverse of the following function: $f(x) = \frac{3x-1}{3x+1}$.

Solution:

$$\begin{aligned} y &= f(x) \\ \implies y &= \frac{3x-1}{3x+1} \\ \implies y(3x+1) &= 3x-1 \\ \implies 3xy + y &= 3x-1 \\ \implies 3xy - 3x &= -1-y \\ \implies x(3y-3) &= -1-y \\ \implies x &= \frac{-1-y}{3y-3} \\ \implies x &= \frac{1+y}{3-3y} \\ \implies x &= \frac{1}{3}\left(\frac{1+y}{1-y}\right) \end{aligned}$$

Answer: $\boxed{f^{-1}(x) = \frac{1}{3}\left(\frac{1+x}{1-x}\right)}$.

6. (1 point) Calculate: $\lim_{x\to\infty} \dfrac{4x^3 - 28x^2 + 20}{5x^3 - 22x^2 + 1009}$.

Solution: Divide the whole expression by x^3.

$$\begin{aligned} &\lim_{x\to\infty} \frac{4 - \frac{28}{x} + \frac{20}{x^3}}{5 - \frac{22}{x} + \frac{1009}{x^3}} \\ &= \frac{4 - \lim_{x\to\infty} \frac{28}{x} + \lim_{x\to\infty} \frac{20}{x^3}}{5 - \lim_{x\to\infty} \frac{22}{x} + \lim_{x\to\infty} \frac{1009}{x^3}} \\ &= \frac{4}{5} \end{aligned}$$

Answer: $\boxed{\lim_{x\to\infty} \frac{4x^3 - 28x^2 + 20}{5x^3 - 22x^2 + 1009} = \frac{4}{5}}$

Short Answer Questions-II

7. (2 points) The demand function for *Ruinmyshow* tickets is given by

$$p = -0.04q + 800$$

(a) (1 point) Compute the marginal revenue.

Solution:

$$\begin{aligned} TR &= (800 - 0.04q)\cdot q \\ TR &= (800 - 0.04q)\cdot q \\ TR &= 800q - 0.04q^2 \\ \implies MR &= 800 - 0.08q && \text{(applying the power rule)} \end{aligned}$$

Answer: $\boxed{\text{Marginal revenue} = 800 - 0.08q}$

(b) (1 point) Calculate the approximate revenue from selling the 5001st ticket.

Solution: We know that $MR(x)$ will approximate $TR(x+1)$. Therefore, we need to compute $MR(5000)$.

$$\begin{aligned} & MR(5000) = 800 - 0.08q \\ \implies & MR(5000) = 800 - 0.08(5000) \\ \implies & MR(5000) = 800 - 400 \\ \implies & MR(5000) = 400 \end{aligned}$$

Answer: $\boxed{\text{The approximate revenue from selling the 5001st ticket is 400.}}$

8. (2 points) There are two parts in this question.

(a) (1 point) Calculate a such that the following function is continuous for all x. $f(x) = \begin{cases} ax - 2 & \text{if } x \leq 1 \\ 2x^2 + 1 & \text{if } x > 1 \end{cases}$

Solution: Condition for continuity at $x = a$: $LHL = RHL = f(a)$.

$$\begin{aligned} \text{LHL: } \lim_{x \to 1^-} f(x) &= \lim_{x \to 1^-} ax - 2 \\ &= a - 2 \\ \text{RHL: } \lim_{x \to 1^+} f(x) &= \lim_{x \to 1^+} 2x^2 + 1 \\ &= 2 + 1 \\ &= 3 \\ f(1) &= a - 2 \\ \implies a - 2 &= 3 \\ \implies a &= 5 \end{aligned}$$

Answer: $\boxed{a = 5}$.

(b) (1 point) Compute $\dfrac{dy}{dx}$ if $f(x) = \dfrac{2 - x^2}{2 + x^2}$.

Solution: Let $u = 2 - x^2$ and $v = 2 + x^2$.

$$\begin{aligned} u' &= -2x \\ v' &= 2x \end{aligned}$$

We know that, if $f(x) = \dfrac{u}{v}$, $f'(x) = \dfrac{vu' - uv'}{v^2}$.
Applying the quotient rule, we get:

$$\begin{aligned} & \frac{dy}{dx} = \frac{(2 + x^2)(-2x) - (2 - x^2)(2x)}{(2 + x^2)^2} \\ \implies & \frac{dy}{dx} = \frac{-8x}{(2 + x^2)^2} \end{aligned}$$

Answer: $\boxed{\dfrac{dy}{dx} = \dfrac{-8x}{(2 + x^2)^2}}$