Single Variable Optimization-I

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August 29, 2025

1 Introduction

Suppose that a function f is differentiable in an interval I and c is an interior point of I. Then, for $x = c$ to be a stationary point for f, the necessary condition is that

$$f'(x) = 0$$

The sufficient condition for the extreme value will be

$$\text{Maximum if: } f''(c) < 0$$
$$\text{Minimum if: } f''(c) > 0$$

Example 1: Compute the stationary points for the following function: $f(x) = x^2 - 4x + 4$.

$$\begin{aligned} f'(x) &= 2x - 4 \\ 2x - 4 &= 0 && \text{(applying the necessary condition)} \\ 2x &= 4 \\ x &= 2 \end{aligned}$$

It seems like this function has an extreme point $x = 2$. Is this a maximum or minimum?

$$\begin{aligned} f''(x) &= 2 \\ \implies f''(x) &> 0 \end{aligned}$$

The second order condition seems to suggest that at $x = 2$, the function $f(x)$ is being minimized.
Example 2: Compute the stationary points for the following function: $f(x) = 4 - 2x - x^2$.

$$\begin{aligned} f'(x) &= -2 - 2x \\ -2 - 2x &= 0 && \text{(applying the necessary condition)} \\ \implies x &= -1 \end{aligned}$$

We have determined that $x = 1$ is an extreme point. Let's check for the second order condition.

$$\begin{aligned} f''(x) &= -2 \\ f''(x) &< 0 \end{aligned}$$

The second order condition suggests that $x = -1$ maximizes the function.