Quiz 03 (Set A (Solution))

SIAS, Krea University (AY 2025-26)
Mathematical Methods for Economics (Course Code: **ECON211**)
05 September 2025

Multiple Choice Questions

1. (1 point) Consider the following statements:

   Statement (i):

   $\lim_{x \to 0} |x|$ does not exist.

   Statement (ii):

   $f(x) = |x|$ is differentiable at $x = 0$.

   A. Both (i) and (ii) are correct.
   B. Statement (i) is correct but statement (ii) is wrong.
   C. Statement (i) is wrong but statement (ii) is correct.
   D. Both (i) and (ii) are wrong.

   Answer: D

Solution:

$$\text{LHL: } \lim_{x \to 0^-} |x| = 0$$
$$\text{RHL: } \lim_{x \to 0^+} |x| = 0$$

LHL = RHL. Therefore, the limit does exist.

$$\text{LHD: } \lim_{x \to 0^-} -1 = -1 \qquad (\text{since } |x| = -x \quad \forall x < 0)$$
$$\text{RHD: } \lim_{x \to 0^+} 1 = 1 \qquad (\text{since } |x| = x \quad \forall x > 0)$$

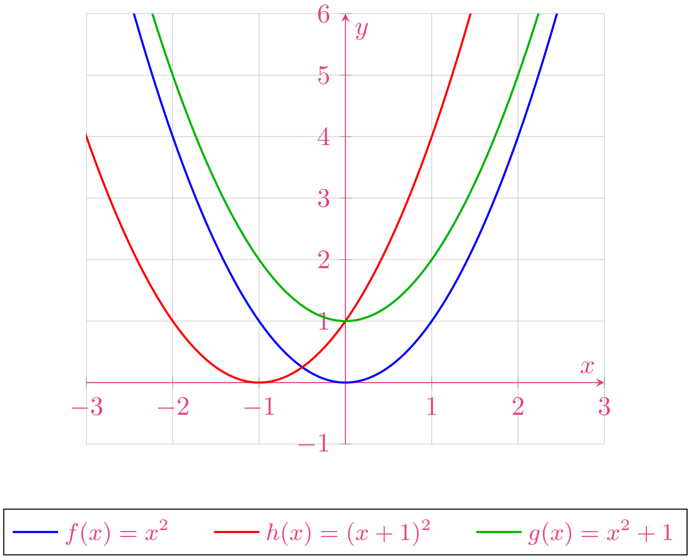
LHD $\neq$ RHD. Therefore, $f(x)$ is not differentiable at $x = 0$.

2. (1 point) If $f(x) = x^2$, $g(x) = x^2 + 1$ and $h(x) = (x + 1)^2$, then

   A. the graph of $g(x)$ can be obtained by shifting $f(x)$ downwards by 1 unit.
   B. the graph of $h(x)$ can be obtained by shifting $f(x)$ upwards by 1 unit.
   C. the graph of $h(x)$ can be obtained by shifting $f(x)$ to the left by 1 unit.
   D. the graph of $g(x)$ can be obtained by shifting $f(x)$ to the right by 1 unit.

   Answer: C

Solution: This is very straightforward. $g(x)$ is $f(x)$ shifted up one unit and $h(x)$ is $f(x)$ being shifted to the left by 1 unit.



3. (1 point) Let $f(x) = 2$. Then,
   A. $f^{-1}(x) = 2$
   B. $f^{-1}(x) = \frac{1}{2}$
   C. $f^{-1}(x) = \frac{1}{2x}$
   D. $f^{-1}(x)$ does not exist.

   Answer: D

Solution: Consider two points in the domain of the function: $x = 1$ and $x = 2$.
$f(1) = 2$ and $f(2) = 2$.
What happens when you 'invert' this function? You get:
$f^{-1}(2) = 1$ and $f^{-1}(2) = 2$.
This cannot be a valid function as it is not one-to-one. Therefore, the inverse does not exist.

Short Answer Questions-I

4. (1 point) Calculate: $\lim_{x\to\infty} \dfrac{x^3 - 68x^2 + 20}{4x^3 - 2x^2 + 1009}$.

Solution: Divide the whole expression by x^3.

$$\begin{aligned}&\lim_{x\to\infty} \frac{1 - \frac{68}{x} + \frac{20}{x^3}}{4 - \frac{2}{x} + \frac{1009}{x^3}}\\ &= \frac{1 - \lim_{x\to\infty} \frac{68}{x} + \lim_{x\to\infty} \frac{20}{x^3}}{4 - \lim_{x\to\infty} \frac{2}{x} + \lim_{x\to\infty} \frac{1009}{x^3}}\\ &= \frac{1}{4}\end{aligned}$$

Answer: $\boxed{\lim_{x\to\infty} \frac{x^3 - 68x^2 + 20}{4x^3 - 2x^2 + 1009} = \frac{1}{4}}$

5. (1 point) Compute $\dfrac{dy}{dx}$ if $y = 2x + \dfrac{1}{\sqrt{x}}$.

Solution:

$$\begin{aligned}y &= 2x + \frac{1}{\sqrt{x}}\\ \frac{dy}{dx} &= 2 + \frac{d(\frac{1}{\sqrt{x}})}{dx}\\ \frac{dy}{dx} &= 2 + \frac{d(x^{-1/2})}{dx}\\ \frac{dy}{dx} &= 2 - \frac{1}{2}x^{-3/2}\end{aligned}$$

Answer: $\boxed{\frac{dy}{dx} = 2 - \frac{1}{2}x^{-3/2}}$

6. (1 point) Compute the inverse of the following function: $f(x) = \dfrac{2x - 1}{2x + 1}$.

Solution:

$$
\begin{aligned}
y &= f(x) \\
\implies y &= \frac{2x-1}{2x+1} \\
\implies y(2x+1) &= 2x-1 \\
\implies 2xy + y &= 2x-1 \\
\implies 2xy - 2x &= -1-y \\
\implies x(2y-2) &= -1-y \\
\implies x &= \frac{-1-y}{2y-2} \\
\implies x &= \frac{1+y}{2-2y} \\
\implies x &= \frac{1}{2}\left(\frac{1+y}{1-y}\right)
\end{aligned}
$$

Answer: $\boxed{f^{-1}(x) = \frac{1}{2}\left(\frac{1+x}{1-x}\right)}$.

Short Answer Questions-II

7. (2 points) There are two parts in this question.

(a) (1 point) Calculate a such the following function is continuous for all x. $f(x) = \begin{cases} ax-1 & \text{if } x \leq 1 \\ 3x^2+1 & \text{if } x > 1 \end{cases}$

Solution: Condition for continuity at $x = a$: $LHL = RHL = f(a)$.

$$
\begin{aligned}
\text{LHL: } \lim_{x\to 1^-} f(x) &= \lim_{x\to 1^-} ax - 1 \\
&= a - 1 \\
\text{RHL: } \lim_{x\to 1^+} f(x) &= \lim_{x\to 1^+} 3x^2 + 1 \\
&= 3 + 1 \\
&= 4 \\
f(1) &= a - 1 \\
\implies a - 1 &= 4 \\
\implies a &= 5
\end{aligned}
$$

Answer: $\boxed{a = 5}$.

(b) (1 point) Compute $\frac{dy}{dx}$ if $f(x) = \frac{1-x^2}{1+x^2}$.

Solution: Let $u = 1 - x^2$ and $v = 1 + x^2$.

$$
\begin{aligned}
u' &= -2x \\
v' &= 2x
\end{aligned}
$$

We know that, if $f(x) = \frac{u}{v}$, $f'(x) = \frac{vu' - uv'}{v^2}$.
Applying the quotient rule, we get:

$$
\begin{aligned}
\frac{dy}{dx} &= \frac{(1+x^2)(-2x) - (1-x^2)(2x)}{(1+x^2)^2} \\
\implies \frac{dy}{dx} &= \frac{-4x}{(1+x^2)^2}
\end{aligned}
$$

Answer: $\boxed{\frac{dy}{dx} = \frac{-4x}{(1+x^2)^2}}$

8. (2 points) The demand function for *Lollafalooda* tickets is given by

$$p = 8000 - 100q$$

(a) (1 point) Compute the marginal revenue.

Solution:

$$\begin{aligned} TR &= p \cdot q \\ TR &= (8000 - 100q) \cdot q \\ TR &= 8000q - 100q^2 \\ \implies MR &= 8000 - 200q \qquad \text{(applying the power rule)} \end{aligned}$$

Answer: $\boxed{\text{Marginal revenue} = 8000 - 200q}$

(b) (1 point) Calculate the approximate revenue from selling the 41st ticket.

Solution: We know that $MR(x)$ will approximate $TR(x+1)$. Therefore, we need to compute $MR(40)$.

$$\begin{aligned} MR &= 8000 - 200q \\ \implies MR(40) &= 8000 - 200 \cdot 40 \\ \implies MR(40) &= 8000 - 8000 \\ \implies MR(40) &= 0 \end{aligned}$$

Answer: $\boxed{\text{The approximate revenue from selling the 41st ticket is zero.}}$