

Multiple Choice Questions

1. (1 point) Find the roots of the following quadratic equation: $4x^2 - 32x - 80 = 0$.

- A. $(-2, -10)$
- B. $(2, 10)$
- C. No real roots exist
- D. $(-2, 10)$

Answer: D

Solution:

$$\begin{aligned} 4x^2 - 32x - 80 &= 0 \\ 4(x^2 - 8x - 20) &= 0 && \text{(taking 4 as a common factor)} \\ x^2 - 8x - 20 &= 0 && \text{(getting rid of a constant is kosher)} \\ x^2 - 10x + 2x - 20 &= 0 && \text{(since } -8x = -10x + 2x) \\ x(x - 10) + 2(x - 10) &= 0 \\ (x - 10)(x + 2) &= 0 \\ x &= 10 \text{ or } x = -2 \end{aligned}$$

Answer: $x = 10, -2$

2. (1 point) Consider the following statements:

Statement (i): The set of equations: $2x + 3y = 7$ and $4x - 6y = 2$ has a unique solution.

Statement (ii): The set of equations: $4x - y = 3$ and $-28x + 7y = 21$ has infinitely many solutions.

- A. Both (i) and (ii) are correct.
- B. Statement (i) is correct but statement (ii) is incorrect.
- C. Statement (ii) is correct but statement (i) is incorrect.
- D. Both (i) and (ii) are incorrect.

Answer: B

Solution: Consider the first equation from Statement (i).

$$\begin{aligned} 2x + 3y &= 7 \\ \Rightarrow 4x + 6y &= 14 && \text{(multiplying the equation by 2)} \\ 4x - 6y &= 2 && \text{(writing the second equation as it is)} \\ \Rightarrow 8x &= 16 && \text{(adding the two equations)} \\ \Rightarrow x &= 2 \\ 2(2) + 3y &= 7 && \text{(plugging the value of } x \text{ in the first equation)} \\ \Rightarrow 3y &= 3 \\ \Rightarrow y &= 1 \end{aligned}$$

There does exist a unique solution. Therefore, the statement is correct.

Consider the second equation from Statement (ii).

$$\begin{aligned} -28x + 7y &= 21 \\ \Rightarrow 4x - y &= -3 && \text{(dividing both sides by -3)} \end{aligned}$$

This is incompatible with $4x - y = -3$. Therefore, this set doesn't have any solution. So, this assertion is incorrect.

3. (1 point) Identify the element a_{35} in the following matrix:

$$A = \begin{bmatrix} 0 & 1 & 3 & 6 \\ 1 & 2 & 3 & 9 \\ 7 & 5 & 2 & 0 \\ 9 & 4 & 1 & 5 \end{bmatrix}$$

- A. 0
- B. 1
- C. Does not exist
- D. 5

Answer: C

Solution: The matrix A is of dimension 4×4 . While the third row is very much there, the fifth column doesn't exist. Therefore, the element a_{35} doesn't exist.

Short Answer Questions-I

4. (1 point) There are two matrices A and B such that:

$$A = \begin{bmatrix} x+5 & 0 & 3 \\ 4 & 0.6y & 8 \end{bmatrix}, \quad B = \begin{bmatrix} 6 & 5 & 0.5 \\ 10 & 2 & -10 \end{bmatrix}, \quad 2A + B = \begin{bmatrix} 14 & 5 & 6.5 \\ 18 & 3.2 & 6 \end{bmatrix}$$

Find x and y .

Solution: Using equality of matrices, we can write:

$$\begin{aligned} 2(x+5) + 6 &= 14 & 2(0.6y) + 2 &= 3.2 \\ \Rightarrow 2(x+5) &= 8 & \Rightarrow 1.2y &= 1.2 \\ \Rightarrow x+5 &= 4 & \Rightarrow y &= 1 \\ \Rightarrow x &= 4-5 \\ \Rightarrow x &= -1 \end{aligned}$$

Answer: $x = -1$, $y = 1$

5. (1 point) Solve for x and y :

$$\begin{aligned} 5x - 6y &= 3 \\ x + 3y &= 2 \end{aligned}$$

Solution:

$$\begin{aligned} 2x + 6y &= 4 & & \text{(multiplying the second equation by 2)} \\ 5x - 6y &= 3 & & \text{(writing the first equation as it is)} \\ \Rightarrow 7x &= 7 & & \text{(adding the two equations)} \\ \Rightarrow x &= 11 + 3y & & \text{(plugging the value of } x \text{ in the second equation)} \\ \Rightarrow 3y &= 1 & & \text{(transferring 1 to the RHS)} \\ \Rightarrow y &= \frac{1}{3} \end{aligned}$$

Answer: $x = 1$, $y = \frac{1}{3}$

6. (1 point) Let $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 6 & 7 \end{bmatrix}$. Compute AB .

Solution: Let

$$C = AB = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$

$$c_{11} = (2 \times -1) + (1 \times 6) = 4$$

$$c_{12} = (2 \times 0) + (1 \times 7) = 7$$

$$c_{21} = (3 \times -1) + (4 \times 6) = 21$$

$$c_{22} = (3 \times 0) + (4 \times 7) = 28$$

Answer:

$$AB = \begin{bmatrix} 4 & 7 \\ 21 & 28 \end{bmatrix}$$

Short Answer Questions-II

7. (2 points) Given the following supply and demand equations:

$$P = 2Q_S^2 + 9Q_S + 10$$

$$P = -Q_D^2 - 3Q_D + 73$$

Calculate the equilibrium price and quantity.

Solution: We know that, at equilibrium, the following is true:

$$\text{Supply} = \text{Demand}$$

$$Q_S = Q_D = Q$$

Therefore,

$$\begin{aligned} 2Q^2 + 9Q + 10 &= -Q^2 - 3Q + 73 \\ \Rightarrow 3Q^2 + 12Q - 63 &= 0 && \text{(rearranging terms)} \\ \Rightarrow Q^2 + 4Q - 21 &= 0 && \text{(dividing both sides by 3)} \\ \Rightarrow Q^2 + 7Q - 3Q - 14 &= 0 && \text{(since } 4Q = 7Q - 3Q) \\ \Rightarrow Q(Q + 7) - 3(Q + 7) &= 0 \\ \Rightarrow (Q + 7)(Q - 3) &= 0 && \text{(since } a(b + k) - c(b + k) = (a - c)(b + k)) \\ \Rightarrow Q^* &= 3 && \text{(since quantities cannot be negative)} \\ P^* &= 2(Q^*)^2 + 9Q^* + 10 && \text{(using the supply equation)} \\ \Rightarrow P^* &= 2(3)^2 + 9 \times 3 + 10 && \text{(since } Q^* = 3) \\ \Rightarrow P^* &= 55 \end{aligned}$$

Answer: $Q^* = 3$, $P^* = 55$

8. (2 points) Use Cramer's rule OR matrix inverse method to solve the following set of equations:

$$7x_1 + 5x_2 = 12$$

$$3x_1 - 9x_2 = -6$$

Solution: Cramer's Rule

First, write all the matrices you need.

$$A = \begin{bmatrix} 7 & 5 \\ 3 & -9 \end{bmatrix}, \quad B = \begin{bmatrix} 12 \\ -6 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

We also need D_{x_1} and D_{x_2} .

$$\begin{aligned} D_{x_1} &= \begin{vmatrix} 12 & 5 \\ -6 & -9 \end{vmatrix} \\ D_{x_2} &= \begin{vmatrix} 7 & 12 \\ 3 & -6 \end{vmatrix} \\ |A| &= \begin{vmatrix} 7 & 5 \\ 3 & -9 \end{vmatrix} = -78 \end{aligned}$$

Therefore,

$$\begin{aligned} x_1 &= \frac{D_{x_1}}{|A|} \\ x_2 &= \frac{D_{x_2}}{|A|} \\ D_{x_1} &= -78 \\ D_{x_2} &= -78 \\ \Rightarrow x_1 &= 1, \quad x_2 = 1 \end{aligned}$$

Matrix Inverse Method

We know that $X = A^{-1}B$ and $A^{-1} = \frac{adj(A)}{|A|}$.

$$\begin{aligned} A^{-1} &= \frac{1}{-78} \begin{bmatrix} -9 & -5 \\ -3 & 7 \end{bmatrix} \Rightarrow A^{-1}B = \frac{1}{78} \begin{bmatrix} -9 & -5 \\ -3 & 7 \end{bmatrix} \times \begin{bmatrix} 12 \\ -6 \end{bmatrix} \\ \Rightarrow A^{-1}B &= \frac{1}{-78} \begin{bmatrix} (-9 \times 12) + (-5 \times -6) \\ (-3 \times 12) + (7 \times -6) \end{bmatrix} \\ \Rightarrow A^{-1}B &= \frac{1}{-78} \begin{bmatrix} -78 \\ -78 \end{bmatrix} \\ \Rightarrow A^{-1}B &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ \Rightarrow X &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} \end{aligned}$$

Answer: $x_1 = 1$, $x_2 = 1$.