

Quiz 04 (Set C: Solution)

SIAS, Krea University (AY 2025-26)

Mathematical Methods for Economics (Course Code: ECON211)

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Multiple Choice Questions

1. (1 point) Let $f(x) = \ln(3 + e^x)$. Then, $f''(0)$ is

- A. $\frac{2}{9}$
- B. $\frac{1}{4}$
- C. $\frac{1}{2}$
- D. $\frac{3}{16}$

Answer: D

Solution:

$$f'(x) = \frac{e^x}{3 + e^x} \quad (\text{using the chain rule})$$

$$f''(x) = \frac{3e^x}{(3 + e^x)^2} \quad (\text{using the quotient rule})$$

$$\Rightarrow f''(0) = \frac{3e^0}{(1 + e^0)^2}$$

$$\Rightarrow f''(0) = \frac{3}{4^2}$$

$$\Rightarrow f''(0) = \frac{3}{16}$$

2. (1 point) Consider the following statements:

Statement (i):

$f(x) = e^{x-2}$ is a strictly increasing function.

Statement (ii):

$g(x) = 2 - 4x^2$ is a strictly convex function.

- A. Both (i) and (ii) are correct.
- B. Statement (i) is correct but statement (ii) is wrong.
- C. Statement (i) is wrong but statement (ii) is correct.
- D. Both (i) and (ii) are wrong.

Answer: B

Solution: A function is strictly increasing if $f'(x) > 0$.

$$\begin{aligned} f(x) &= e^{x-2} \\ \Rightarrow f'(x) &= e^{x-2} \end{aligned} \quad (\text{using the chain rule})$$

We know that e^k is always positive (for any constant k). Therefore, $f'(x) > 0$. Hence, the function is **strictly increasing**.

A function $g(x)$ is strictly convex if $g''(x) > 0$.

$$\begin{aligned} g(x) &= 2 - 4x^2 \\ \Rightarrow g'(x) &= -8x \\ \Rightarrow g''(x) &= -8 \end{aligned}$$

Since the second derivative is negative everywhere in the domain of the function, the function is **strictly concave**.

3. (1 point) Let $f(x) = \sqrt{2x + \sqrt{x}}$. Then $f'(x)$ is

- A. $\frac{2\sqrt{x} + 1}{2\sqrt{x}(\sqrt{2x} + \sqrt{x})}$
- B. $\frac{2\sqrt{x} + 1}{4\sqrt{x}(\sqrt{2x} + \sqrt{x})}$
- C. $\frac{4\sqrt{x} + 1}{4\sqrt{x}(\sqrt{2x} + \sqrt{x})}$
- D. $\frac{2\sqrt{x} + 1}{(\sqrt{2x} + \sqrt{x})}$

Answer: C

Solution: Let $u = 2x + \sqrt{x}$. Therefore, $f(u) = \sqrt{u}$.

$$\frac{df}{du} = \frac{1}{2\sqrt{u}} \quad \text{(using the power rule)}$$

$$\frac{du}{dx} = 2 + \frac{1}{2\sqrt{x}} \quad \text{(using the power rule)}$$

$$\Rightarrow \frac{du}{dx} = \frac{4\sqrt{x} + 1}{2\sqrt{x}}$$

$$\begin{aligned} \frac{df}{dx} &= \frac{df}{du} \cdot \frac{du}{dx} \\ &= \frac{1}{2\sqrt{u}} \cdot \frac{4\sqrt{x} + 1}{2\sqrt{x}} \quad \text{(applying the chain rule)} \end{aligned}$$

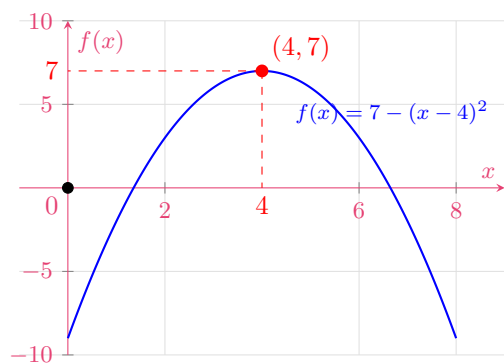
$$= \frac{1}{2\sqrt{2x + \sqrt{x}}} \cdot \frac{4\sqrt{x} + 1}{2\sqrt{x}}$$

$$= \frac{4\sqrt{x} + 1}{4\sqrt{x}(\sqrt{2x} + \sqrt{x})}$$

Short Answer Questions-I

4. (1 point) Without using calculus, compute the minimum (or the maximum) value of the following function: $f(x) = 7 - (x - 4)^2$. (Hint: Graph the function.)

Solution: Consider $(x - 4)^2$. The minimum value that any square can take is zero. Therefore, $(x - 4)^2 \geq 0$. When $x = 4$, $f(x) = 7$. This is the maximum value of the function.



5. (1 point) Let $3xy^2 + x^2y = 5$. Find $\frac{dy}{dx}$. Simplify the answer as much as possible.

Solution:

$$\begin{aligned}\frac{d(3xy^2 + x^2y)}{dx} &= \frac{d(5)}{dx} \\ \frac{d(3xy^2)}{dx} + \frac{d(x^2y)}{dx} &= 0 && \text{(applying the sum rule to the LHS and the constant rule to the RHS)} \\ 3x \frac{d(y^2)}{dx} + 3y^2 \frac{d(x)}{dx} + x^2 \frac{d(y)}{dx} + y \frac{d(x^2)}{dx} &= 0 && \text{(applying the product rule)} \\ 6xy \frac{dy}{dx} + 3y^2 + x^2 \frac{dy}{dx} + 2xy &= 0 \\ (6xy + x^2) \frac{dy}{dx} + (3y^2 + 2xy) &= 0 \\ (6xy + x^2) \frac{dy}{dx} &= -(3y^2 + 2xy) \\ \frac{dy}{dx} &= -\frac{(3y^2 + 2xy)}{(6xy + x^2)} \\ \frac{dy}{dx} &= -\frac{y}{x} \left(\frac{2x + 3y}{x + 6y} \right)\end{aligned}$$

6. (1 point) Let $f(x) = \ln(2 + e^{x-1})$ and let $g(x) = f^{-1}(x)$. Find $g'(x)$.

Solution: Let $y = f(x)$.

$$\begin{aligned}y &= \ln(2 + e^{x-1}) \\ \Rightarrow e^y &= 2 + e^{x-1} && \text{(since } e^{\ln(a)} = a) \\ \Rightarrow e^{x-1} &= e^y - 2 \\ \Rightarrow x - 1 &= \ln(e^y - 2) && \text{(taking log on both sides)} \\ \Rightarrow x &= 1 + \ln(e^y - 2) \\ \Rightarrow f^{-1}(x) &= 1 + \ln(e^x - 2) \\ \Rightarrow g(x) &= 1 + \ln(e^x - 2) \\ \Rightarrow g'(x) &= \frac{1}{e^x - 2} \frac{d}{dx}(e^x - 2) && \text{(applying the chain rule)} \\ \Rightarrow g'(x) &= \frac{e^x}{e^x - 2}\end{aligned}$$

Short Answer Questions-II

7. (2 points) Find and classify all the stationary/inflection points for the following function: $f(x) = x^3 - 3x + 10$.

Solution:

$$\begin{aligned}f'(x) &= 3x^2 - 3 \\ f''(x) &= 6x\end{aligned}$$

Set the first derivative to zero to find the stationary points.

$$\begin{aligned}3x^2 - 3 &= 0 \\ x^2 - 1 &= 0 \\ x^2 &= 1 \\ x &= \pm 1\end{aligned}$$

When $x = 1$, the second derivative is positive indicating a local minimum. When $x = -1$, the second derivative is negative suggesting a local maximum.

Set the second derivative to zero to look for inflection point(s).

$$\begin{aligned}6x &= 0 \\ x &= 0\end{aligned}$$

Take any two points in the vicinity of $x = 0$. Luckily, we already have those points from our previous calculation. Since the sign of

the second derivative changes as x moves around $x = 0$, $x = 0$ is an inflection point.

$x = 1$	Local maximum
$x = -1$	Local minimum
$x = 0$	Inflection point

8. (2 points) You work for an online retailer and you have been tasked with estimating the elasticity of demand for their product. The demand function is $q = \frac{2}{3}\sqrt{144 - p^2}$.

(a) (1 point) Compute the elasticity of demand when $p = 8\sqrt{2}$.

Solution: We know that:

$$\epsilon_p = \left| \frac{p}{q} \cdot \frac{dq}{dp} \right|$$

When $p = 8\sqrt{2}$,

$$\begin{aligned} q &= \frac{2}{3}\sqrt{144 - 128} \\ &= \frac{2}{3}\sqrt{16} \\ &= \frac{8}{3} \end{aligned}$$

Let's compute $\frac{dq}{dp}$.

$$\begin{aligned} \frac{dq}{dp} &= \frac{1}{3}(144 - p^2)^{-\frac{1}{2}} \left(\frac{d(144 - p^2)}{dp} \right) && \text{(applying the chain rule)} \\ \frac{dq}{dp} &= \frac{-2p}{3(144 - p^2)^{\frac{1}{2}}} \end{aligned}$$

When $p = 8\sqrt{2}$,

$$\begin{aligned} \frac{dq}{dp} &= \frac{-16\sqrt{2}}{3 \cdot 4} \\ &= \frac{-4\sqrt{2}}{3} \end{aligned}$$

The elasticity of demand,

$$\begin{aligned} \epsilon_p &= \left| \frac{p}{q} \cdot \frac{dq}{dp} \right| \\ \epsilon_p &= \left| \frac{8\sqrt{2}}{8/3} \cdot \frac{-4\sqrt{2}}{3} \right| \\ \epsilon_p &= 8 \end{aligned}$$

- (b) (1 point) Based on your previous answer, what should be the firm's pricing strategy (increase or decrease the price?) that will boost revenue? Explain briefly.

Solution: The demand, our previous computation suggests, is elastic. Therefore, the retailer can decrease the price a bit to boost the revenue.