# Introduction to Matrix Algebra-I

Sumit

July 18, 2025

## 1 Definition

A matrix is a rectangular arrangement of numbers. A typical matrix is composed of rows and columns. Let $A$ be a matrix, $m$ be the number of rows, and $n$ be the number of columns. We can write the matrix $A$ as $A_{m \times n}$.

Let's discuss some definitions before we proceed.

- **Order of a matrix**: Any matrix having $m$ rows and $n$ columns has the order $m \times n$.
- **Element of a matrix**: An element (or an entry) within a matrix is denoted by $a_{ij}$ where $i$ is the row number and $j$ the column number.

Some examples will help us understand these two concepts (order and elements).

Consider the information that you collected from *Buhari* about the quantity of *biriyani* and *samsa* sold on a particular day of the week. They sold 2000 plates of *biriyani* and 5000 plates of *samsa* in the month of June 2025. We can write out a matrix (call it $A$). $A = \begin{bmatrix} 2000 & 5000 \end{bmatrix}$

Let's focus on the order of the matrix $A$. The number of rows $(A) = 1$ and the number of columns $(A) = 2$. Therefore, the order (or the dimension) of the matrix is:

$$dim(A) = 1 \times 2$$

What are the elements of $A$? We need to understand the structure here. The data is stored in a single row and two columns.

$a_{11} = 2000$: the first row and the first column

$a_{12} = 5000$: the first row and the second column

Now, you go back to the restaurant and they supply you with one more data point on the quantities of *biriyani* and *samsa*. They sold 2500 plates of *biriyani* and 6000 plates of *samsa* during July 2025. Can we think of a matrix that combines the data that we stored in matrix $A$ and this new information? Let's call it $B$ and we can write:

$$B = \begin{bmatrix} 2000 & 5000 \\ 2500 & 6000 \end{bmatrix}$$

Notice one key feature of this matrix- I have arranged the entries corresponding to a particular item vertically. There are two rows and two columns. Therefore,

$$dim(B) = 2 \times 2$$

What are the elements of $B$?
$a_{11} = 2000$: the first row and the first column
$a_{12} = 5000$: the first row and the second column
$a_{21} = 2500$: the second row and the first column
$a_{22} = 6000$: the second row and the second column

We are all set to write a general matrix.

$$A_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

**Problem**: Construct a $2 \times 2$ matrix whose elements follow the rule: $a_{ij} = \frac{1}{2}|2i - j^2|$.

# 2 Types of Matrices

## 2.1 Row Matrix

A row matrix is a matrix containing a single row. The order of a row matrix is

$$dim(A) = 1 \times n$$

An example:

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}$$

$A$ is a $1 \times 4$ row matrix or a $1 \times 4$ row vector.

## 2.2 Column Matrix

A column matrix is a matrix containing a single column. The order of a column matrix is

$$dim(A) = m \times 1$$

An example:

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}$$

$A$ is a $4 \times 1$ column matrix or a $4 \times 1$ column vector.

## 2.3 Square Matrix

A square matrix is a matrix whose number of rows equals the number of columns. The order of a square matrix is

$$dim(A) = n \times n$$

An example:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$A$ is a $2 \times 2$ matrix.

## 2.4 Diagonal Matrix

Diagonal matrix is a matrix where the entries outside the 'main diagonal' are all zeroes. We must first understand what diagonal means in the context of a matrix. Any element of a matrix where the row number equals the column number is a diagonal element. For example, for a $2 \times 2$ square matrix, $a_{11}$ and $a_{22}$ represent the diagonal elements. Similarly, think of a $3 \times 3$ matrix. What are its diagonal elements? $a_{11}, a_{22}, a_{33}$. Now, we can write the rule:

$$a_{ij} = 0 \text{ if } i \neq j$$

Please note that we do not impose any restrictions on the values that the diagonal elements themselves can take.

An example:

$$A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

$A$ is a $3 \times 3$ diagonal matrix.

## 2.5 Identity Matrix

Identity matrix is a special case of a square diagonal matrix such that

$$a_{ij} = 1 \text{ if } i = j$$
$$a_{ij} = 0 \text{ if } i \neq j$$

An example:

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$A$ is a $2 \times 2$ identity matrix.

## 2.6 Zero Matrix

A zero matrix is a matrix where:

$$a_{ij} = 0 \text{ for all } i, j$$

A couple of examples

$$A = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$A$ is a $1 \times 3$ zero matrix and $B$ is a $3 \times 2$ zero matrix.