02 TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2075 Baishakh

Exam.	Back		
Level	BE	Full Marks	80
Programme	ALL (Except B. Arch.)	Pass Marks	32
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II *(SH451)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. State Euler's Theorem for homogeneous function of two variables. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x+y}\right)$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$. [1+4]

2. Obtain the maximum value of xyz such that $x+y+z=24$. [5]

3. Evaluate: $\int_0^a \int_{\sqrt{ax}}^a \frac{y^2}{\sqrt{y^4 - a^2x^2}}\,dy\,dx$ by changing order of integration. [5]

4. Evaluate: $\iiint_R (2x+y)\,dx\,dy\,dz$ where R is closed region sounded by cylinder $z = 4 - x^2$ and planes $x=0,\ y=0,\ y=2,\ z=0$. [5]

5. Show that $\frac{x+4}{3} = \frac{y+6}{5} = \frac{z-1}{-2}$ and $3x-2y+z+5=0=2x+3y+4z-4$ are coplanar lines and find the point of intersection. [5]

6. Show that the shortest distance between the lines $x+a=2y=-12z$ and $x=y+2a=6z-6a$ is 2a. [5]

7. Obtain the equation of tangent plane to sphere $x^2+y^2+z^2+6x-2z+1=0$ which passes through the line $3(16-x)=3z=2y+30$ [5]

8. Find the equation of cone with vertex at $(3,1,2)$ and base $2x^2+3y^2=1,\ z=1$ [5]

   OR

   Find the equation of the right circular cylinder whose guiding curve is the circle: $x^2+y^2+z^2-x-y-z=0,\ x+y+z=1$

9. Solve the initial value problem: $y''-4y'+3y=10e^{-2x},\ y(0)=1,\ y'(0)=3$ [5]

10. Solve the differential equation by power series method: $y''-y=0$ [5]

11. Solve in series, the Legendre's equation $(1-x^2)y''-2xy'+n(n+1)y=0$ [5]

OR

Prove the Bessel's function $J_{\frac{3}{2}}(x)=\sqrt{\frac{2}{\Pi x}}\left(\frac{\sin x}{x}-\cos x\right)$

12. Prove that $\left[\vec{a}\times\vec{b}\ \ \vec{c}\times\vec{d}\ \ \vec{e}\times\vec{f}\right]=\left[\vec{a}\,\vec{b}\,\vec{d}\right]\left[\vec{c}\,\vec{e}\,\vec{f}\right]-\left[\vec{a}\,\vec{b}\,\vec{c}\right]\left[\vec{d}\,\vec{e}\,\vec{f}\right]$ [5]

13. Prove that the necessary and sufficient conditions for the vector function $\vec{a}$ of scalar variable t to have constant direction is $\vec{a}\times\frac{d\vec{a}}{dt}=0$ [5]

14. Find the angle between the normal to the surfaces given by: $x\log z=y^2-1$ and $x^2y+z=2$ at the point $(1,1,1)$ [5]

15. Test the convergence of the series: [5]

$$x+\frac{3}{5}x^2+\frac{8}{10}x^3+\frac{15}{17}x^4+\ldots\ldots+\frac{n^2-1}{n^2+1}x^n+\ldots\ldots,x>0.$$

16. Find the interval and radius of convergence of power series: [5]

$$\frac{1}{1.2}(x-2)+\frac{1}{2.3}(x-2)^2+\frac{1}{3.4}(x-2)^3+\ldots\ldots\ldots+\frac{1}{n(n+1)}(x-2)^n+\ldots\ldots\ldots\ldots$$

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02 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2074 Bhadra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	ALL (Except B. Arch.)	Pass Marks	32
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II *(SH451)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ ***All** questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. If $u = \log\dfrac{x^2+y^2}{x+y}$, then show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 1$

2. Find the minimum value of $x^2+y^2+z^2$ when $x+y+z=3a$.

3. Evaluate $\iint xy(x+y)\,dxdy$ over the area between $y=x^2$ and $y=x$

4. Evaluate $\int_0^a\int_y^a \dfrac{x\,dx\,dy}{x^2+y^2}$ by changing order of integration.

OR

Evaluate $\iiint_v x^2 dx\,dy\,dz$ over the region v boundary by the planes x = 0, y =0, z = 0 and x+y+z=a

5. Obtain the equation of the plane passing through the line of intersection of two planes through the line of intersection of two planes $7x-4y+7z+16=0$ and $4x-3y-2z+13=0$ and perpendicular to plane $x-y-2z+5=0$

6. Find the length and equation of the shortest distance between the lines $\dfrac{x-3}{3}=\dfrac{y-8}{-1}=\dfrac{z-3}{1}$, $2x-3y+27=0;\ 2y-z+20=0$

7. Find the equation of the sphere having the circle $x^2+y^2+z^2+7y-2z+2=0$, $2x+3y+4z-8=0$ as a great circle.

8. Find the equation of right circular cone whose vertex at origin and axis is the line $\dfrac{x}{1}=\dfrac{y}{2}=\dfrac{z}{3}$ with vertical angle 30°

OR

Find the equation of the right circular cylinder of radius 2 whose axis is the line $\dfrac{x-1}{2}=\dfrac{y-2}{1}=\dfrac{z-3}{2}$

9. Solve by power series method the differential equation $y''+xy'+y=0$

10. Express the following in terms of legendre's Polynomials $f(x)=5x^3+x$

11. Prove the Bessel's function $J_{-\frac{5}{2}}(x) = \sqrt{\frac{2}{\pi x}}\left[\frac{3}{x}\sin x + \frac{3-x^2}{x^2}\cos x\right]$

12. Find the set of reciprocal system to the set of vectors $2\vec{i}+3\vec{j}-\vec{k}, -\vec{i}+2\vec{j}-3\vec{k}$ and $3\vec{i}-4\vec{j}+2\vec{k}$

13. Prove that the necessary and sufficient condition for the vector function of scalar variable 't' have constant magnitude is $\vec{a}.\frac{d\vec{a}}{dt} = 0$

14. Find the directional derivative of $\phi(x, y, z) = xy^2 + yz^3$ at the point (2, -1, 1) in the direction of vector $\vec{i}+2\vec{j}+2\vec{k}$

OR

If $\vec{a}$ is a constant vector and $\vec{r}$ be the position vector then prove that $\left(\vec{a}\times\nabla\right)\times\vec{r} = -2\vec{a}$

15. Test convergent or divergent of the series $1+\frac{x}{2} = \frac{2!}{3^2}x^2 + \frac{3!}{4^3}x^3 + \ldots\ldots\infty$

16. Find the internal and radius of convergence of the power series $\sum_{n=1}^{\infty}\frac{2^n(x-3)^n}{n+3}$

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02 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2073 Bhadra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II *(SH451)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ ***All** questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. State Euler's Theorem for a homogeneous function of two independent variables and verify it for the function $u = x^n \tan^{-1}\left(\frac{y}{x}\right)$
2. Find the extreme value of $x^2+y^2+z^2$ connected by the relation $ax+by+cz = p$.
3. Evaluate $\iint xy(x+y)dxdy$ over the area between $y = x^2$ and $y = x$.
4. Evaluate the integral by changing to polar coordinates $\int_0^a \int_0^{\sqrt{a^2-x^2}} y^2 \sqrt{x^2+y^2}\, dy.dx$

OR

Find by triple integration the volume of the sphere $x^2+y^2+z^2 = a^2$.

5. Find the equation of the plane through the line $2x+3y-5z = 4$ and $3x-4y+5z = 6$ and parallel to the coordinate axes.
6. Find the length and equation of shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $2x-3y+27 = 0$, $2y-z+20 = 0$.
7. Obtain the centre and radius of the circle $x^2+y^2+z^2+x+y+z = 4$, $x+y+z = 0$.
8. The plane through OX and OY includes an angle α, prove that their line of intersection lies on the cone $z^2(x^2+y^2+z^2) = x^2y^2\tan^2\alpha$

OR

Find the equation of the right circular cylinder of radius 2 whose axis is the line $\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{2}$.

9. Solve by power series method the differential equation $y'' - 4xy' + (4x^2 - 2)y = 0$.
10. Express $f(x) = x^3 - 5x^2 + x + 2$ in terms of Legendre's polynomial.
11. Show that $J_{\left(\frac{5}{2}\right)}(x) = \sqrt{\frac{2}{\pi x}}\left(\frac{3}{x}\sin x + \frac{3-x^2}{x^2}\cos x\right)$.

12. Prove that $\left[\vec{b}\times\vec{c}\;\;\vec{c}\times\vec{a}\;\;\vec{a}\times\vec{b}\right]=\left[\vec{a}\;\vec{b}\;\vec{c}\right]^2$

13. A partical moves along the curve x = a cost, y = a Sint and z = bt. Find the velocity and acceleration at t = 0 and t = π/2.

14. Find the directional derivative of $\phi(x, y, z) = xy^2+yz^3$ at the point (2, -1, 1) in the direction of vector $\vec{i}+2\vec{j}+2\vec{k}$.

OR

If $\vec{a}$ is a constant vector and $\vec{r}$ be the position vector then prove that $(\vec{a}\times\nabla)\times\vec{r}=-2\vec{a}$.

15. Test the convergence of the series

$$\frac{x}{1.2}+\frac{x^2}{2.3}+\frac{x^3}{3.4}+\ldots\ldots\ldots\ldots,\ x>0$$

16. Find the interval and radius of convergence of the power series $\sum_{n=1}^{\infty}\frac{(-3)^n x^n}{\sqrt{n+1}}$.

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02 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2073 Magh

Exam.	New Back (2066 & Later Batch)		
Level	BE	Full Marks	80
Programme	All (Except B. Arch)	Pass Marks	32
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II *(SH451)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ ***All** questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. If $u = \log\left(\frac{x^2+y^2}{x+y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$.
2. Obtain the maximum value of xyz such that $x+y+z = 24$.
3. Evaluate: $\iint xy(x+y)dxdy$ over the area between $y = x^2$ and $y = x$.
4. Evaluate the integral by changing to polar co-ordinates: $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{x^2+y^2}\, dydx$

   OR

   Find by triple integration the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.
5. Show that the lines $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ and $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$ are coplanar. Find their common point.
6. Find the S.D between the lines $\frac{x-6}{3} = \frac{y-7}{-1} = \frac{z-4}{1}$ and $\frac{x}{-3} = \frac{y+9}{2} = \frac{z-2}{4}$. Find also the equation of shortest distance.
7. Find the equation of spheres passing through the circle $x^2+y^2+z^2-6x-2z+5 = 0$, $y = 0$ and touching the plane $3y+4z+5 = 0$.
8. Find the equation of the cone whose vertex is the origin and base the circle $y^2+z^2 = b^2$, and $x = a$.

   OR

   Find the equation to the right circle cylinder of radius 2 and whose is the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}$.
9. Solve by Power series method $y'' - y = x$.
10. Express in terms of Legendre's polynomials $f(x) = x^3-5x^2+6x+1$.
11. Prove the Bessel's Function

$$J_{3/2}(x) = \sqrt{\frac{2}{\Pi x}}\left(\frac{\sin x}{x} - \cos x\right)$$

12. Find the set of reciprocal system to the set of vectors $2\vec{i}+3\vec{j}-\vec{k}$, $-\vec{i}+2\vec{j}-3\vec{k}$, and $3\vec{i}-4\vec{j}+2\vec{k}$.

13. Prove that $\left[\vec{b}\times\vec{c}\quad \vec{c}\times\vec{a}\quad \vec{a}\times\vec{b}\right]=\left[\vec{a}\ \vec{b}\ \vec{c}\right]^2$

14. If $\vec{r}$ be the position vector and $\vec{a}$ is constant vector then prove that $\nabla\left(\frac{\vec{a}\bullet\vec{r}}{r^n}\right)=\frac{\vec{a}}{r^n}-\frac{n}{r^{n+2}}\left(\vec{a}\bullet\vec{r}\right)\vec{r}$

OR

Find the value of n so that $r^n\,\vec{r}$ is solenoidal.

15. Test the series for convergence or divergence

$$2x+\frac{3x^2}{8}+\frac{4x^3}{27}+\ldots\ldots+\frac{(x+1)x^n}{n^3}+\ldots\ldots\quad (x>0)$$

16. Find the interval of convergence and the radius of convergence of the power series $\sum_{n=1}^{\infty}\frac{2^n(x-3)^n}{n+3}$

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05 TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2072 Ashwin

Exam.	Regular		
Level	BE	Full Marks	80
Programme	All (Except B. Arch)	Pass Marks	32
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II *(SH451)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ ***All** questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. If $u=\log(x^3+y^3+z^3-3xyz)$, show that $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u=-\frac{9}{(x+y+z)^2}$

2. Find the minimum value of the function $x^2+xy+y^2+3z^2$ under the condition $x+2y+4z=60$.

3. Evaluate: $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}\,dy\,dx$ by changing to polar coordinates.

4. Evaluate: $\iiint_V x\,dv$ where V is bounded by the coordinate planes and the plane x+y+z=1

OR

Evaluate: $\iint_R xy\,dx\,dy$ where R is the region bounded by the x-axis, the ordinate x = 2a and the curve $x^2=4ay$

5. Find the distance of the point (1, -2, 3) from the plane $x-y+z=5$ measured parallel to the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$

6. Show that the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $4x-3y+1=0=5x-3z+2$ are coplanar and find their point of intersection.

7. Obtain the equation of the tangent planes to the sphere $x^2+y^2+z^2+6x-2z+1=0$ which passes through the line $x+z-16=0, 2y-3z+30=0$

8. Prove that the equation $ax^2+by^2+cz^2+2ux+2vy+2wz+d=0$ represents a cone if $\frac{u^2}{a}+\frac{v^2}{b}+\frac{w^2}{c}=d$

9. Solve by the power series method the differential equation $y''-4xy'+(4x^2-2)y=0$

10. Prove that the Legendre's function $x^5=\frac{8}{63}\left[P_5(x)+\frac{7}{2}P_3(x)+\frac{27}{8}P_1(x)\right]$

11. Prove that $J_{\frac{5}{2}}(x)=\sqrt{\frac{2}{\pi x}}\left\{\frac{3}{x}\sin x+\frac{3-x^2}{x^2}\cos x\right\}$

12. Find the set of reciprocal system to the set of vectors: $2\hat{i}-3\hat{j}+\hat{k}$; $\hat{i}+2\hat{j}-\hat{k}$ and $3\hat{i}-\hat{j}+2\hat{k}$

13. Prove that the necessary and sufficient condition for the vector functions $\vec{a}$ of scalar variable 't' to have a constant direction is $\vec{a}\times\frac{d\vec{a}}{dt}=0$

14. Find the directional derivative of $\phi = x^2yz+4xz^2$ at the point (1, -2, -1) in the derection of vector $2\vec{i}-\vec{j}-2\vec{k}$

OR

If $\vec{a}$ is a constant vector and $\vec{r}$ be the position vector, then prove that $\nabla\times(\vec{a}\times\vec{r})=2\vec{a}$

15. Determine whether the following series is convergent or divergent:

$$1+\frac{1^2}{2^2}+\frac{2^2}{3^3}+\frac{3^3}{4^4}+\frac{4^4}{5^5}+\text{.........}$$

16. Find the radius and interval of convergence of the power series $\sum_{n=0}^{\infty}\frac{(-1)^n(x-3)^n}{n+1}$

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05 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2072 Magh

Exam.	New Back (2066 & Later Batch)		
Level	BE	Full Marks	80
Programme	All (Except B.Arch)	Pass Marks	32
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II *(SH451)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ ***All** questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. If $u = \frac{1}{\sqrt{x^2+y^2+z^2}}$ prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$

2. Find the extreme value of $x^2+y^2+z^2$ connected by the relation $x+y+z=3a$

3. Evaluate: $\iint r \sin\theta \, dr.d\theta$ over the area of the cardioid $r = a\,(1+\cos\theta)$ above the initial line.

4. Evaluate: $\int_0^a \int_0^{\sqrt{(a^2-x^2)}} y^2.\sqrt{(x^2+y^2)}\; dy.dx$ by changing polar coordinates.

OR

Evaluate: $\iiint x^{l-1}.y^{m-1}.z^{n-1} dx.dy.dz$

Evaluate: x, y, z are all positive but $\left(\frac{x}{a}\right)^p + \left(\frac{y}{b}\right)^q + \left(\frac{z}{c}\right)^r \le 1$

5. Find the length of perpendicular from the point (3, -1, 11) to the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$.
Also obtain the equation of the perpendicular.

6. Find the magnitude of the line of the shortest distance between the lines $\frac{X}{4} = \frac{Y+1}{3} = \frac{Z-2}{2}$, $5x-2y-3z+6=0,\ x-3y+2z-3=0$

7. Find the centre and radius of the circle in which the sphere $x^2+y^2+z^2-8x+4y+8z-45=0$ is cut by the phane $x-2y+2z=3$

8. The plane through OX and OY include an angle α, show that their line of intersection lies on the cone $z^2(x^2+y^2+z^2) = x^2y^2\tan^2\alpha$

9. Solve by power series method of the differential equation y''-y=0

10. Express $f(x) = x^3-5x^2+x+2$ in terms of Legendre's polynomials.

11. Prove the Bessel's function: $J_{3/2}(x) = \sqrt{\frac{2}{\pi x}}\left(\frac{\sin x}{x} - \cos x\right)$

12. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{a}', \vec{b}', \vec{c}'$ are the reciprocal system of vectors then prove that

$$\vec{a}' \times \vec{b}' + \vec{b}' \times \vec{c}' + \vec{c}' \times \vec{a}' = \frac{\vec{a}+\vec{b}+\vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]}, \left[\vec{a}\,\vec{b}\,\vec{c}\right] \neq 0$$

13. If $\vec{r} = a\cos t\,\vec{i} + a\sin t\,\vec{j} + at\tan\alpha\,\vec{k}$, find $\left[\vec{r}\;\frac{d\vec{r}}{dt}\;\frac{d^2\vec{r}}{dt^2}\right]$

14. Find the directional derivative of $\phi(x,y,z) = xy^2 + yz^3$ at the point (2, -1, 1) in the direction of victor $\vec{i} + 2\vec{j} + 2\vec{k}$

OR

If $\vec{r}$ be the position vector and $\vec{a}$ is constant vector then prove that $\nabla.\left(\frac{\vec{a} \times \vec{r}}{r}\right) = 0$

15. Determine whether the series $\sum \frac{n}{1+n\sqrt{n+1}}$ is convergent or divergent.

16. Find the radius and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n.2^n}$

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52 TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2071 Bhadra

Exam.	Regular / Back		
Level	BE	Full Marks	80
Programme	B.Arch.	Pass Marks	32
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II *(SH454)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ ***All** questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. If $u = \log \frac{x^2+y^2}{x+y}$, then prove that $x\frac{\delta u}{\delta x} + y\frac{\delta u}{\delta y} = 1$.

2. If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then show that $x\frac{\delta u}{\delta x} + y\frac{\delta u}{\delta y} = -\frac{1}{2}\cot u$.

3. Obtain the maximum value of xyz such that $x + y + z = 24$.

4. Express the equation of the line $x + 2y + 3z - 6 = 0$, $3x + 4y + 5z - 2 = 0$ in symmetrical form.

5. Find the length of the perpendicular from the point (3, –1, 11) to the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. Also obtain the equation of perpendicular.

6. Prove that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $4x - 3y + 1 = 0 = 5x - 3z + 2$ are coplanar.

7. Find the equation of the sphere through the points (0, 0, 0), (0, 1, –1), (–1, 2, 0) and (1, 2, 3).

8. Find the equation of the cylinder whose generators are parallel to the line $x = -\frac{y}{2} = \frac{z}{3}$ and whose guiding curve is $x^2 + 2y^2 = 1$, $z = 3$.

   OR

   Obtain the equation of the cone whose vertex is the origin and base the circle $x = a$, $y^2 + z^2 = b^2$.

9. Show that $[\vec{a}+\vec{b} \quad \vec{b}+\vec{c} \quad \vec{c}+\vec{a}] = 2[\vec{a}\,\vec{b}\,\vec{c}]$

10. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{a}', \vec{b}', \vec{c}'$ are reciprocal system of vectors, then show that $\vec{a}\cdot\vec{a}' + \vec{b}\cdot\vec{b}' + \vec{c}\cdot\vec{c}' = 3$.

11. If $\phi = \log(x^2 + y^2 + z^2)$, find div (grad ϕ) and curl (grad ϕ).

12. Test the convergence of the series $\frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4} + \ldots\ldots\ldots$

13. Find the radius of convergence and the interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(-1)^n (x-3)^n}{n+1}$.

14. Evaluate: $\int_0^1 \int_0^{x^2} e^{\frac{x}{y}} dydx$

15. Evaluate, the following integral, by changing to polar coordinates

$\int_0^1 \int_x^{\sqrt{2x-x^2}} (x^2+y^2) dydx$.

16. Evaluate: $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} \frac{dxdydz}{(x+y+z+1)^3}$

OR

Find, by double integration, the area of the region bounded by $y^2 = x^3$ and $y = x$.

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02

TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2071 Magh

Exam.	[redacted]		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II *(SH451)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt All questions.*
- ✓ *All questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. Prove that $\vec{i}\times(\vec{a}\times\vec{i})+\vec{j}\times(\vec{a}\times\vec{j})+\vec{k}\times(\vec{a}\times\vec{k})=2\vec{a}$ where $\vec{i}, \vec{j}, \vec{k}$ are mutually perpendicular unit vectors along the coordinate axes.

2. Find the set of reciprocal system to the set of vectors $2\vec{i}+3\vec{j}-\vec{k}$, $-\vec{i}+2\vec{j}-3\vec{k}$ and $3\vec{i}-4\vec{j}+2\vec{k}$.

3. Prove that the necessary and sufficient condition for the function $\vec{a}$ of scalar variable t to have a constant direction is $\vec{a}\times\frac{d\vec{a}}{dt}=0$.

OR

If $\vec{a}$ is a constant vector and $\vec{r}$ be the position vector, then prove that $\nabla\times(\vec{a}\times\vec{r})=2\vec{a}$.

4. Evaluate: $\iint_R y\,dy.dx$ where R is the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$.

5. Evaluate the integral by changing to polar coordinates $\int_0^a\int_0^{\sqrt{a^2-x^2}}\sqrt{x^2+y^2}\,.dy.dx$.

6. Find by double integration the smaller of the areas bounded by the circle $x^2 + y^2 = 9$ and the line $x + y = 3$.

7. Find the length of perpendicular from the point (3, –1, 11) to the line $\frac{x}{2}=\frac{y-2}{3}=\frac{z-3}{4}$. Also obtain the equation of perpendicular.

8. Find the magnitude and the equation of S.D. between the lines.
$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$

9. Obtain the centre and radius of the circle.
$x^2+y^2+z^2+x+y+z=4$ and $x+y+z=0$

10. Prove that the plane $2x-y+2z-14=0$ touches the sphere $x^2+y^2+z^2-4x+2y-4=0$. Find the point of contact.

11. Find the equation of cone with vertex (α, β, γ) and guiding curve is parabola $y^2 = 4ax$, $z = 0$.

OR

Obtain the equation of right circular cylinder of radius 4 and axis the line $x = 2y = -z$.

12. If $x = \gamma \cos\theta$, $y = \gamma \sin\theta$. Prove that

$$\frac{\partial^2 \gamma}{\partial x^2} + \frac{\partial^2 \gamma}{\partial y^2} = \frac{1}{\gamma}\left[\left(\frac{\partial \gamma}{\partial x}\right)^2 + \left(\frac{\partial \gamma}{\partial y}\right)^2\right]$$

13. If $u = \operatorname{cosec}^{-1}\left(\frac{x^{1/2} + y^{1/2}}{x^{1/3} + y^{1/3}}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -\frac{1}{6}\tan u$.

14. Find the extreme values of $x^2 + y^2 + z^2$ subjected to the condition $x + y + z - 1 = 0$ and $xyz + 1 = 0$.

15. Test the series for convergence or divergence

$$2x + \frac{3x^2}{8} + \frac{4x^3}{27} + \dots\dots + \frac{(n+1)x^n}{n^3} + \dots\dots\dots\dots\dots (x > 0)$$

16. Find the radius and interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{2^n (x-3)^n}{n+3}.$$

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02

TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2070 Bhadra

Exam.		Regular	
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II *(SH451)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt All questions.*
- ✓ *All questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. If $u = \log\frac{x^2+y^2}{x+y}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$.

2. Find the extreme value of $x^2 + y^2 + z^2$ connected by the relation $ax + by + cz = p$.

3. Evaluate $\int_0^a\int_{\sqrt{ax}}^a \frac{y^2\,dy\,dx}{\sqrt{y^4 - a^2x^2}}$ by changing order of integration.

4. Evaluate $\int_0^{\log 2}\int_0^x\int_0^{x+\log y} e^{x+y+z}\,dz\,dy\,dx$.

5. Find the length of the perpendicular from the point (3, −1, 11) to the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. Also obtain the equation of perpendicular.

6. Find the magnitude and the equation of S.D. between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $2x - 3y + 27 = 0,\ 2y - z + 20 = 0$.

7. Find the equation of the sphere through the circle $x^2 + y^2 = 4,\ z = 0$ and is intersected by the plane $x + 2y + 2z = 0$ is a circle of radius 3.

OR

Find the equations of the tangent planes to the sphere $x^2 + y^2 + z^2 + 6x - 2z + 1 = 0$ which passes through the line $x + z - 16 = 0,\ 2y - 3z + 30 = 0$.

8. Find the equation of the right circular cone whose vertex at origin and axis is the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ with vertical angle 30°.

OR

Find the equation of the right circular cylinder of radius 2 whose axis is the line $\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{2}$.

9. Solve the differential equation $y'' - 4xy' + (4x^2 - 2)y = 0$ by power series method.

10. Express $f(x) = x^3 - 5x^2 + x + 2$ interms of Legendre polynomials.

11. Show that $4J_n''(x) = J_{n-2}(x) - 2J_n(x) + J_{n+2}(x)$.

12 Find a set of vectors reciprocal to the following vectors $2\vec{i}+3\vec{j}-\vec{k}$, $\vec{i}-\vec{j}-2\vec{k}$, $-\vec{i}+2\vec{j}+2\vec{k}$.

13. Prove that the necessary and sufficient condition for the vector function of a scalar variable t to have constant magnitude is $\vec{a}\cdot\frac{d\vec{a}}{dt} = 0$.

14. A particle moves along the curve $x = 4\cos t$, $y = t^2$, $z = 2t$. Find velocity and acceleratior at time $t = 0$ and $t = \frac{\pi}{2}$.

15. Test the convergence of the series $1+\frac{x}{2}+\frac{2!}{3^2}x^2+\frac{3!}{4^3}x^3+\ldots$

16. Find the radius and interval of convergence of the power series $\sum_{n=1}^{\infty}\frac{(-1)^n(x-3)^n}{n+1}$.

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02 TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2070 Magh

Exam.	New Back (2066 & Later Batch)		
Level	BE	Full Marks	80
Programme	All (Except B.Arch)	Pass Marks	32
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II *(SH451)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ ***All** questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. Find $\frac{du}{dt}$ if $u = \sin\left(\frac{x}{y}\right)$, $x = e^t$ & $y = t^2$

2. Find the extreme value of $x^2 + y^2 + z^2$ connected by the relation $x+z = 1$ and $2y+z = 2$

3. Evaluate: $\iint_R xy\, dx.dy$ where R is the region over the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in the first quadrant.

4. Evaluate the integral by changing to polar coordinates $\int_0^a\int_0^{\sqrt{a^2-x^2}} y^2.\sqrt{x^2+y^2}\,.dy.dx$

   OR

   Evaluate: $\iiint x^{l-1}.y^{m-1}.z^{n-1}.dx.dy.dz$, where x,y,z are all positive but $\left(\frac{x}{a}\right)^p + \left(\frac{y}{b}\right)^q + \left(\frac{z}{c}\right)^r \le 1$

5. Find the equation of the plane through the line 2x+3y-5z = 4 and 3x-4y+5z = 6 and parallel to the coordinates axes.

6. Show that the lines $\frac{x-5}{4}=\frac{y-7}{4}=\frac{z-3}{-5}$ & $\frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}$ are coplanar. Find their point of intersection and equation of plane in which they lie.

7. Find the centre and radius of the circles $x^2+y^2+z^2-8x+4y+8z-45=0$, x-2y+2z-3=0

8. Find the equation of a right circular cone with vertex (1,1,1) and axis is the line $\frac{x-1}{1}=\frac{y-1}{2}=\frac{z-1}{3}$ and semi vertical angle 30°.

9. Solve by power series method the differential equation $y'' + xy' + y = 0$

10. Find the general solution of the Legendre's differential equation.

11. Prove Bessel's Function $\frac{d[x^{-n}J_n(x)]}{dx} = -x^{-n}J_{n+1}$

12. Prove that: $\left[\vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}\ \ \vec{a}\times\vec{b}\right] = \left[\vec{a}\ \vec{b}\ \vec{c}\right]^2$

13. Find n so that $r^n \vec{r}$ is solonoidal.

14. Prove that the necessary and sufficient condition for a function $\vec{a}$ of scalar variable to have a constant direction is $\vec{a} \times \frac{d\vec{a}}{dt} = 0$

15. Test the series for convergence or divergence

$$x + \frac{3}{5}x^2 + \frac{8}{10}x^3 + \frac{15}{17}x^4 + \ldots\ldots + \frac{n^2-1}{n^2+1}x^n + \ldots..(x > 0)$$

16. Find the radius of convergence and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n.2^n}$

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02 TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2069 Bhadra

Exam.	**Regular (2066 & Later Batch)**		
Level	BE	**Full Marks**	80
Programme	All	**Pass Marks**	32
Year / Part	I / II	**Time**	3 hrs.

Subject: - Engineering Mathematics II ***(SH451)***

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ ***All** questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. If $\sin u = \dfrac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$, show that $x\dfrac{\delta u}{\delta x}+y\dfrac{\delta x}{\delta y}=0.$

2. Obtain the maximum value of xyz such that $x + y + z = 24$.

3. Evaluate: $\iint xy(x+y)dxdy$ over the area between $y = x^2$ and $y = x$.

4. Evaluate $\iiint_V x^2\, dx\, dy\, dz$ over the region V bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = a$.

5. Find the image of the point (2, −1, 3) in the plane $3x-2y-z-9=0$.

6. Find the S.D. between the line $\dfrac{x-6}{3}=\dfrac{7-y}{1}=\dfrac{z-4}{1}$ and $\dfrac{x}{-3}=\dfrac{y+9}{2}=\dfrac{2-z}{-4}$. Find also equation of S.D.

7. Obtain the equation of the sphere through the circle $x^2 + y^2 + z^2 = 9$, $x - 2y + 2z = 5$ as a great circle.

8. Find the equation of cone with vertex (3, 1, 2) and base $2x^2 + 3y^2 = 1$, $z = 1$.

OR

Find the equation of right circular cylinder whose axis is the line $\dfrac{x-\alpha}{\ell}=\dfrac{y-\beta}{m}=\dfrac{z-r}{n}$ and whose radius 'r'

9. Solve the initial value problem $y'' + 2y' + 5y = 0$, given $y(0) = 1$, $y'(0) = 5$.

10. Define power series. Solve by power series method of differential equation, $y' + 2xy = 0$.

11. Prove the Bessell's function $\dfrac{d}{dx}\left[x^n J_n(x)\right]=x^n J_{n-1}(x)$.

12. Prove if $\vec{\ell}, \vec{m}, \vec{n}$ be three non-coplanar vectors then

$$\left[\vec{\ell}\,\vec{m}\,\vec{n}\right]\left(\vec{a}\times\vec{b}\right)=\begin{vmatrix} \vec{\ell}.\vec{a} & \vec{\ell}.\vec{b} & \vec{\ell} \\ \vec{m}.\vec{a} & \vec{m}.\vec{b} & \vec{m} \\ \vec{n}.\vec{a} & \vec{n}.\vec{b}, & \vec{n} \end{vmatrix}.$$

13. Prove that the necessary and sufficient condition for the vector function of a scalar variable t have a constant magnitude is $\vec{a} \cdot \frac{d\vec{a}}{dt} = 0$.

14. Find the angle between the normal to the surfaces $x \log z = y^2 - 1$ and $x^2 y + z = 2$ at the point (1, 1, 1).

15. Test the convergence of the series $\frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4} + \ldots\ldots\ldots\ldots$

16. Find the interval of cgt, radius of cgt and centre of cgt of power series $\sum \frac{2^n x^n}{n!}$

02 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2069 Poush

Exam.	New Back (2066 & Later Batch)		
Level	BE	Full Marks	80
Programme	All except B.Arch.	Pass Marks	32
Year / Part	I / II	Time	3 hrs

Subject: - Engineering Mathematics II *(SH451)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt All questions.*
- ✓ *All questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. State Euler's theorem on homogeneous functions of two independent variables. And if $\text{Sin } u = \dfrac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$ then prove $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=0$

2. Find the minimum value of the function $F(x,y,z)=x^2+y^2+z^2$ when $\dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}=1$

3. Evaluate: $\iint r^3 dr\, d\theta$ over the area included between the circles $r = 2\sin\theta$ and $r = 4\sin\theta$

4. Evaluate $\int_1^e \int_1^{\log y} \int_1^{e^x} \log z \, dz\, dx\, dy$

OR

Find the volume of sphere $x^2+y^2+z^2 = a^2$ using Diritchlet's integral.

5. Prove that the lines $\dfrac{x+1}{-3}=\dfrac{y-3}{2}=\dfrac{z+2}{1}$ and $x=\dfrac{y-7}{-3}=\dfrac{z+7}{2}$ are coplanar and find the equation of plane in which they lie.

6. Show that the shortest distance between two skew lines $\dfrac{x-1}{2}=\dfrac{y-2}{3}=\dfrac{z-3}{4}$ and $\dfrac{x-2}{3}=\dfrac{y-4}{4}=\dfrac{z-5}{5}$ is $1/\sqrt{6}$

7. A variable plane is parallel to the plane $\dfrac{x}{a}+\dfrac{y}{b}+\dfrac{z}{c}=1$ and meets the axes in A, B, C. Prove that the circle ABC lies on the cone $\left(\dfrac{b}{c}+\dfrac{c}{b}\right)yz+\left(\dfrac{c}{a}+\dfrac{a}{c}\right)zx+\left(\dfrac{a}{b}+\dfrac{b}{a}\right)xy=0$

8. Find the equation of the right circular cylinder of radius 4 and axis the line $x = 2y = -z$

9. Show that the solutions of $x^2y'''-3xy''+3y'=0, (x>0)$ are linearly independent.

OR

Solve the equation $x^2\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}+(x^2-4)y=0$ in series form.

10. Prove that $4J_n''(x)=J_{n-2}(x)-2J_n(x)+J_{n+2}(x)$ where the symbols have their usual meanings.

11. Apply the power series method to the following differential equation $\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}+y=0$

OR

Find the general solution of Legendre's differential equation.

12. Show that $(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})=\left[\vec{a}\,\vec{b}\,\vec{c}\right]\vec{c}$ and deduce $\left[\vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}\ \ \vec{a}\times\vec{b}\right]=\left[\vec{a}\,\vec{b}\,\vec{c}\right]^2$

13. Prove that the necessary and sufficient condition for the function $\vec{a}$ of scalar variable to have a constant direction is $\vec{a}\times\dfrac{d\vec{a}}{dt}=0$

14. Find the angle between the surface $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point (2,-1,2)

15. Test the convergence of the series $\sum\dfrac{(n+1)^n x^n}{n^{n+1}}$

16. Find the radius of convergence and the interval of convergence of the power series $\sum_{n=1}^{\infty}\dfrac{(-1)^n(x-1)^2}{\sqrt{n}}$

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02 TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2068 Bhadra

Exam.		Regular	
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. State Euler's theorem for homogeneous function of two variables. If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -\frac{1}{2}\text{Cot } u$. [1+4]
2. Find the minimum value of $x^2 + xy + y^2 + 3z^2$ under the condition $x + 2y + 4z = 60$. [5]
3. Change the order of integration and hence evaluate the same.
   $$\int_0^a\int_0^x \frac{\cos y\, dydx}{\sqrt{(a-x)(a-y)}}$$ [5]
4. Find by double integration, the volume bounded by the plane $z = 0$, surface $z = x^2 + y^2 + 2$ and the cylinder $x^2 + y^2 = 4$. [5]
5. Prove that the plane through the point (α, β, γ) and the line $x = py + q = rz + s$ is given by:
   $$\begin{vmatrix} x & py+q & rz+s \\ \alpha & p\beta+q & r\gamma+s \\ 1 & 1 & 1 \end{vmatrix} = 0.$$ [5]
6. Find the magnitude and equation of the shortest distance between the lines: [5]
   $$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \text{ and } \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$$
7. Find the equation of the sphere through the circle $x^2 + y^2 + z^2 - 3x + 4y - 2z - 5 = 0$, $5x - 2y + 4z + 7 = 0$ as a great circle. [5]

   OR

   Find the equation which touches the sphere $x^2 + y^2 + z^2 + 2x - 6y + 1 = 0$ at $(1, 2, -2)$ and passes through the point $(1, -1, 0)$. [5]
8. Find the equation of the cone with vertex $(\propto, \beta, \gamma)$ and base $y^2 = 4ax$, $z = 0$ [5]
9. Solve the initial value problem
   $y'' - 4y' + 3y = 10e^{-2x}$, $y(0) = 1$, $y'(0) = 3$. [5]
10. Solve by power series method the differential equation $y'' - 4xy' + (4x^2 - 2)\,y = 0$. [5]

10. Solve by power series method the differential equation

11. Express $f(x) = x^3 - 5x^2 + 6x + 1$ in terms of Legendre's polynomials.

OR

Prove that $\frac{d}{dx}[x^{-n}J_n(x)] = -x^{-n}J_{n+1}(x)$. [5]

12. Find a set of vectors reciprocal to the following vectors: [5]

$-\vec{i} + \vec{j} + \vec{k}, \vec{i} - \vec{j} + \vec{k}, \vec{i} + \vec{j} - \vec{k}$

13. Prove that $\vec{b} \times \vec{c}, \vec{c} \times \vec{a}$ and $\vec{a} \times \vec{b}$ are coplanar or non-coplanar according as $\vec{a}, \vec{b}, \vec{c}$ are coplanar or non-coplanar.

14. Prove that curl $(\vec{a} \times \vec{b}) = \vec{a}\ div\ \vec{b} - (\vec{a}.\nabla)$ ***OR*** [5]

If $u = x+y+z$, $v = x^2 + y^2 + z^2$ and $w = xy + yz + zx$, show that (gradu grad v grad w] = 0

15. Test the convergence of the series: [5]

$$2x + \frac{3x^2}{8} + \frac{4x^3}{27} + \cdots \ldots + \frac{(n+1)}{n^3}x^n + \cdots \ldots$$

16. Find the radius of convergence and the convergence of the power series: [5]

$$\sum_{n=0}^{\infty} \frac{(-1)^n (x-3)^n}{n+1}$$

02 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2068 Magh

Exam.	New Back (2066 & Later Batch)		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ ***All** questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. State Euler's theorem for a homogeneous function of two independent variables. If $\sin v = \dfrac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$ then prove that $x\dfrac{\partial v}{\partial x}+y\dfrac{\partial v}{\partial y}=0$. fmin = 3

2. Find the minimum value of the function $x^2 + y^2 + z^2$ when subjected to the conditions $x + y + z = 1$ and $xyz + 1 = 0$. f(3)

3. Evaluate the integral $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}dydx$ by changing into the polar co-ordinates.

4. Evaluate the following double integrals: $\int_0^1\int_{3y}^3 e^{x^2}dxdy$.

   OR

   Prove that $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sqrt{x^2+y^2+z^2}\; e^{-(x^2+y^2+z^2)}dxdydz = 2\pi$.

   21x − 19y + 22z + 12 = 0
   λ = 9/4

5. Show that the lines $\dfrac{x+5}{3}=\dfrac{y+4}{1}=\dfrac{z-7}{-2}$ and $3x + 2y + z - 2 = 0 = x - 3y + 2z - 13$ are coplanar and find the equation of the plane in which they lie.

   $\frac{1}{\sqrt{6}}$ 11x + 2y − 7z − 6 = 0
   7x + y − 5z + 2 = 0

6. Find the shortest distance between the lines:
   $\dfrac{x-3}{2}=\dfrac{y-4}{3}=\dfrac{z-5}{4}$ and $\dfrac{x-4}{3}=\dfrac{y-5}{4}=\dfrac{z-7}{5}$. Also find the equation of the shortest distance line.

7. Show that the equation of the sphere through the circle $x^2 + y^2 + z^2 - 2x - 3y + 4z + 8 = 0$ and $x^2 + y^2 + z^2 + 4x + 5y - 6z + 2 = 0$ and having its centre on the plane $4x - 5y - z - 3 = 0$ is $x^2 + y^2 + z^2 + 7x + 9y - 11z - 1 = 0$.

8. Prove that $ax^2 + by^2 + cz^2 + 2ux + 2vy + 2\omega z + d = 0$ represents a cone if $\dfrac{u^2}{a}+\dfrac{v^2}{b}+\dfrac{\omega^2}{c}=d$

   OR

   Find the equation of a right circular cylinder whose guiding curve is the circle $x^2 + y^2 + z^2 = 9,\ x - y + z = 3$.

9. Apply the power series method and solve the differential equation $y'' + x^2y = 0$.

10. Prove the following Bessel's function: $J_{n+3}(x) + J_{n+5}(x) = \frac{2}{x}(n+4)J_{n+4}(x)$.

11. Find the general solution of Legendre's differential equation.

12. Show that $(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a}) = [\vec{a}\ \vec{b}\ \vec{c}]\vec{c}$ and deduce $[\vec{b}\times\vec{c}\quad \vec{c}\times\vec{a}\quad \vec{a}\times\vec{b}] = [\vec{a}\ \vec{b}\ \vec{c}]^2$

13. Prove that the necessary and sufficient condition for a function $\vec{a}$ of scalar variable to have a constant direction is $\vec{a}\times\frac{d\vec{a}}{dt} = 0$.

14. Prove that $\text{curl}(\vec{a}\times\vec{b}) = \vec{a}\,\text{div}\,\vec{b} - (\vec{a}\cdot\nabla)\vec{b}$.

OR

Find the divergence and curl of $\vec{F}$ where $\vec{F} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$.

15. Test the convergence of the series $1 + \frac{1}{2^2} + \frac{2^2}{3^3} + \frac{3^3}{4^4} + \dots\dots\dots$ $\frac{1}{e} < 1$

16. Find the radius of convergence and the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{2^n(x-3)^n}{n+3}$.

$\frac{1}{2}$, $\frac{5}{2} \le x < \frac{7}{2}$

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02

TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2067 Mangsir

Exam.	Regular / Back		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. State Euler's Theorem for a homogeneous function of two independent variables and verify it for the function: [1+4]

$$u = \frac{x^{1/4} + y^{1/4}}{x^{1/2} + y^{1/5}}.$$

2. Find the extreme value of $\phi = x^2+y^2+z^2$ connected by the relation $ax + by + cz = p$ [5]

3. Evaluate: $\iint_R xy\,dxdy$ where R is the region over the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in the first quadrant. [5]

4. Transform to polar coordinates and complete the integral $\int_0^{2a}\int_0^{\sqrt{2ax-x^2}} (x^2 + y^2)dydx$. [5]

OR

Evaluate: $\iiint x^{\ell-1}.y^{m-1}.z^{n-1}dxdydz$

where x, y, z are all positive but $\left(\frac{x}{a}\right)^p + \left(\frac{y}{b}\right)^q + \left(\frac{z}{c}\right)^r \le 1.$

5. Find the length of perpendicular from the point (3, –1, 11) to the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. Also obtain the equation of the perpendicular. [5]

6. Find the length and equation of the shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$; $2x - 3y + 27 = 0 = 2y - z + 20.$ [5]

7. Find the centre and radius of the circle in which the sphere $x^2 + y^2 + z^2 - 8x + 4y + 8z - 45 = 0$ is cut by the plane $x - 2y + 2z = 3$. [5]

8. Plane through OX and OY include an angle α. Show that their line of intersection lies on the cone $z^2(x^2 + y^2 + z^2) = x^2y^2 \tan^2 \alpha$. [5]

OR

Find the equation of the right circular cylinder whose guiding curve is the circle $x^2 + y^2 + z^2 - x - y - z = 0, x + y + z = 1.$

9. Solve in series: [5]

$$(1+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=0$$

10. Show that: [5]

$$J_{\frac{5}{2}}(x)=\frac{\sqrt{2}}{\pi x}\left(\frac{3-x^2}{x^2}\sin x-\frac{3}{x}\cos x\right)$$

11. Show that: [5]

$$P_n(x)=\frac{1}{2^n n}\frac{d^n}{dx^n}\left(x^2-1\right)^n$$

12. Prove that $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})+(\vec{a}\times\vec{c})\times(\vec{d}\times\vec{b})+(\vec{a}\times\vec{d})\times(\vec{b}\times\vec{c})=-2\times[\vec{b}\ \vec{c}\ \vec{d}\,]\,\vec{a}$ [5]

13. Prove that the necessary and sufficient condition for the vector function $\vec{a}$ of scalar variable λ to have a constant magnitude is $\left(\vec{a}\,\frac{d\vec{a}}{dt}\right)=0$. [5]

14. Apply the power series method to solve following differential equation [5]

$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0$$

15. Test the convergence of the series $\frac{1}{2}+\frac{2}{3}x+\left(\frac{3}{4}\right)^2x^2+\left(\frac{4}{5}\right)^3x^3+\dots\dots\dots$ [5]

16. Show that $J_4(x)=\left(\frac{48}{x^3}-\frac{3}{x}\right)J_1(x)+\left(1-\frac{24}{x^2}\right)J_0(x)$. [5]

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03 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2067 Chaitra

Exam.	New Back (2066 Batch Only)		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

T)

1. State Euler's theorem of homogeneous equation of two variables. If $u = \sin^{-1}\dfrac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$. Show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 0$. [1+4]

2. Find the extreme value of $x^2+y^2+z^2$ subject to the condition $x+y+z=1$. [5]

3. Evaluate $\iint_R xy\,dx\,dy$ where R is the region over the area of the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ in the first quadrant. [5]

4. Evaluate the integral by changing to polar co-ordinates. $\int_0^1\int_x^{\sqrt{2x-x^2}}(x^2+y^2)\,dy\,dx$.

OR

Find by triple integral, the volume common to the cylinders $x^2+y^2=a^2$ and $x^2+z^2=a^2$. [5]

5. Prove that $(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a}) = [\vec{a}\ \vec{b}\ \vec{c}]\,\vec{c}$ and deduce that $[\vec{b}\times\vec{c},\ \vec{c}\times\vec{a},\ \vec{a}\times\vec{b}] = [\vec{a}\ \vec{b}\ \vec{c}]^2$. [5]

6. Prove that the necessary and sufficient condition for the vector function of a scalar variable t have constant magnitude is $\vec{a}\dfrac{d\vec{a}}{dt}=0$. [5]

7. The position vector of a moving particle at any point is given by $\vec{r}=(t^2+1)\,\vec{i}+(4t-3)\,\vec{j}+(2t^2-6)\,\vec{k}$. Find the velocity and acceleration at t = 1. Also obtain the magnitudes. [5]

8. Prove that the lines $x = ay+b$, $z = cy+d$ and $x = a'y+b'$, $z = c'y+d'$ are perpendicular if $aa'+cc'+1=0$. [5]

9. Prove that the lines $\dfrac{x-1}{2}=\dfrac{y+1}{-3}=\dfrac{z+10}{8}$ and $\dfrac{x-4}{1}=\dfrac{y+3}{-4}=\dfrac{z+1}{7}$ intersect. Find also their point of intersection and plane through them. [5]

10. Find the centre and radius of the circle $x^2+y^2+z^2+x+y+z=4$, $x+y+z=0$. [5]