IME611A :
FINANCIAL ENGINEERING

Project Report

on

Equity Portfolio Management

Group Members
Kanika Kukreja (190414)
Rahul Verma (190671)
Sparsh Singhal (190861)
Sumit Sahu (190874)

Portfolio optimization

Some important parameters used in portfolio optimization using Markowitz model and their significance:

Covariance - It is a statistical measure of how two assets move in relation to each other.A positive covariance indicates that two assets move in tandem. A negative covariance indicates that two assets move in opposite directions.

$$\operatorname{cov}(X, Y) = \sum_{i=1}^{N} \frac{(x_i - \bar{x})(y_i - \bar{y})}{N}.$$

While covariance can show the direction between two assets, it cannot be used to calculate the strength of the relationship between the prices.

Portfolio optimization steps:

We form a portfolio using n assets and give weights wi i.e. $w_1, w_2\ldots\ldots w_n$

return on portfolio: $r = w_1 r_1 + w_2 r_2 + \cdots + w_n r_n$

Expected returns: $E(r) = w_1 E(r_1) + w_2 E(r_2) + \cdots + w_n E(r_n)$

Where,

$$w_i = \frac{\textit{Initial value of asset}_i}{\sum_{j=1}^{n} \textit{Initial value of asset}_j}$$

$$r_i = \frac{\textit{Final value of asset}_i - \textit{Initial value of asset}_i}{\textit{Initial value of asset}_i}$$

r_i is called rate of return and R_i $(1 + r_i)$ is called total return.

Suppose,

σ^2 denote the portfolio variance,

σi^2 denote the variance of *ith* stock, and

σij denote the covariance of return on asset *i* and asset *j*.

$$\sigma^2 = E[(r - \bar{r})^2]$$

$$\sigma^2 = E\left[\left(\sum_{i=1}^{n} w_i r_i - \sum_{i=1}^{n} w_i \bar{r}_i\right)^2\right]$$

$$\sigma^2 = E\left[\left(\sum_{i=1}^{n} w_i (r_i - \bar{r}_i)\right)\left(\sum_{j=1}^{n} w_j (r_j - \bar{r}_j)\right)\right]$$

$$\sigma^2 = E\left[\left(\sum_{i,j=1}^{n} w_i w_j (r_i - \bar{r}_i)(r_j - \bar{r}_j)\right)\right]$$

$$\sigma^2 = \sum_{i,j=1}^{n} w_i w_j \sigma_{ij}$$

Returns are calculated using following calculations :

There is no linear compensation in return while using mean return.

There is a mismatch between average return and effective return:

Ex- $(1+0.5)*(1-0.5)=1.75$

Average return $=(0.5-0.5)/2$

So we use geometric mean return

Geometric mean return = $((1+r_1)*(1+r_2)*(1+r_3)*(1+r_4)\ldots\ldots\ldots(1+r_n))^{1/n} - 1$

Annualized Data

Annualize daily performance (we have 247 trading days in the year we are doing calculations)

Arithmetic mean = daily mean * 247

Geometric mean return = $((1+r_1)*(1+r_2)*(1+r_3)*(1+r_4)\ldots\ldots\ldots(1+r_n))^{247/n} - 1$

Volatility = daily volatility * sqrt(247)

Covariance = daily covariance * 247

Here n is the number of data points in our daily stock price data.

Markowitz model optimization-

Assuming short selling is prohibited

$$\text{Minimize} \quad \frac{1}{2}\sum_{i,j=1}^{n} w_i w_j \, \sigma_{ij}$$

$$\text{subject to} \quad \sum_{i=1}^{n} w_i \bar{r}_i = \bar{r}$$

$$\text{and} \quad \sum_{i=1}^{n} w_i = 1$$

$$\text{and} \quad w_i \geq 0 \text{ for } i = 1, 2, \ldots, n$$

Sharpe Ratio - The formula of sharpe ratio measures the risk-adjusted return of a particular portfolio by dividing the excessive returns by the standard deviation of the portfolio returns.

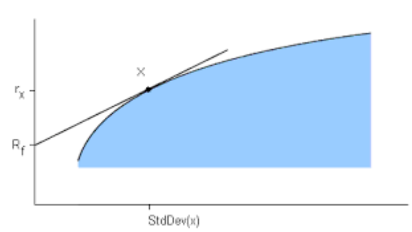
$S=(r_p-r_f)/\sigma_p$

S- Sharpe ratio

r_p- Expected portfolio return

r_f- Risk free rate

σ- Standard deviation of portfolio(risk)



The Sharpe ratio measures the increase in expected return per unit of additional standard deviation.Higher sharpe ratio is desirable.

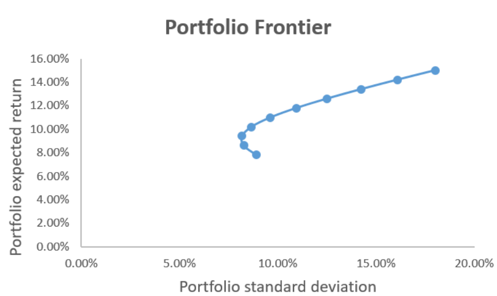
Capital allocation line and efficient frontier - The CAL is a line that graphically depicts the risk and reward profile of assets.

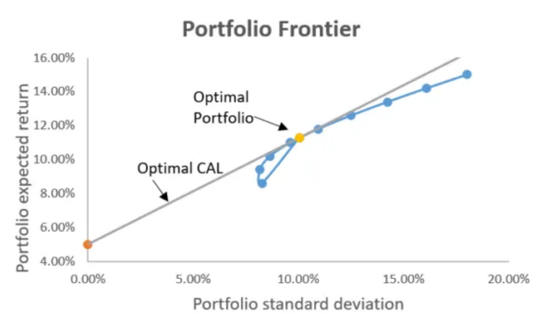
$$E(R_p) = R_f + \frac{(E(R_i) - R_f)}{\sigma_i}\sigma_p$$

Rf - risk free rate
Slope of CAL - Sharpe ratio

A portfolio frontier is a graph that maps out all possible portfolios with different asset weight combinations, with levels of portfolio standard deviation graphed on the x-axis and portfolio expected return on the y-axis.

<u>Example:</u>





The optimal portfolio consists of a risk-free asset and an optimal risky asset portfolio. The optimal risky asset portfolio is at the point where the CAL is tangent to the efficient frontier. This portfolio is optimal because the slope of CAL is the highest, which means we achieve the highest returns per additional unit of risk.

Evaluation:

a) We have chosen 10 stocks from NIFTY 50 :
ONGC, Asianpaints, BhartiAirtel, HDFC Bank, Infosys, ITC, L&T, Reliance, SunPharma, M&M
Stocks are chosen from different sectors so as to diversify the portfolio.
Data for each of these stocks has been taken from **Yahoo Finance.**
The data for 1 year is taken i.e **247 trading days (excluding holidays)**(13th October 2021 to 12th October 2022).

Total Amount to be invested = ₹5,00,000
Risk-free annual rate= 6%

All the relevant parameters like expected return, standard deviation, correlation, etc have been calculated and shown in the excel sheet.

DESCRIPTION TABLE						
	μ	σ	Sharpe ratio	Risk free rate	Random Weights	Random Array
ONGC	-11.93%	0.3955	-45.33%	6.00%	1%	0.026667178
ASIANPAINT	-1.59%	0.2806	-27.04%		7%	0.343152994
BHARTIARTL	12.91%	0.2500	27.65%		16%	0.726539815
HDFCBANK	-13.01%	0.2535	-74.96%		5%	0.220803712
INFY	-14.78%	0.2757	-75.37%		20%	0.916188613
ITC	38.70%	0.2435	134.33%		2%	0.100542465
L&T	10.48%	0.2621	17.11%		14%	0.630988048
RELIANCE	-11.60%	0.2817	-62.46%		19%	0.896445017
SUN PHARMA	14.68%	0.2385	36.40%		3%	0.147769148
M&M	34.78%	0.2933	98.13%		14%	0.640793256
					100%	4.649890246

CORELLATION MATRIX										
	ONGC	ASIANPAINT	BHARTIARTL	HDFCBANK	INFY	ITC	L&T	RELIANCE	SUN PHARMA	M&M
ONGC	1.00	-0.13	0.23	0.00	0.10	0.21	0.04	0.32	0.11	0.10
ASIANPAINT	-0.13	1.00	0.27	0.30	0.35	0.21	0.41	0.24	0.29	0.35
BHARTIARTL	0.23	0.27	1.00	0.36	0.32	0.27	0.36	0.38	0.26	0.36
HDFCBANK	0.00	0.30	0.36	1.00	0.38	0.30	0.44	0.39	0.29	0.38
INFY	0.10	0.35	0.32	0.38	1.00	0.31	0.54	0.36	0.34	0.27
ITC	0.21	0.21	0.27	0.30	0.31	1.00	0.35	0.34	0.35	0.29
L&T	0.04	0.41	0.36	0.44	0.54	0.35	1.00	0.45	0.42	0.48
RELIANCE	0.32	0.24	0.38	0.39	0.36	0.34	0.45	1.00	0.33	0.46
SUN PHARMA	0.11	0.29	0.26	0.29	0.34	0.35	0.42	0.33	1.00	0.27
M&M	0.10	0.35	0.36	0.38	0.27	0.29	0.48	0.46	0.27	1.00

We have assumed that short selling is prohibited thus the weights of every stock needs to be greater than or equal to 0 and carried out further analysis in our project.

b) All the purchases are made on 13 Oct 2022.

Serial No.	Instrument	Name	Action	Current Price	Buying Price	Quantity	Change	Transaction Date	Current Invested	Value Invested	Portfolio(%)
1	Equity	ONGC	Buy	140.45	129.85	-	10.6	13/10/22	₹ -	₹ -	0%
2	Equity	ASIANPAINT	Buy	3055.4	3204.38	-	-148.98	13/10/22	₹ -	₹ -	0%
3	Equity	BHARTIARTL	Buy	826.9	768.9	-	58	13/10/22	₹ -	₹ -	0%
4	Equity	HDFCBANK	Buy	1611.15	1393.6	-	217.55	13/10/22	₹ -	₹ -	0%
5	Equity	INFY	Buy	1570.1	1404.55	-	165.55	13/10/22	₹ -	₹ -	0%
6	Equity	ITC	Buy	356.35	328.65	1,036	27.7	13/10/22	₹ 369,312.38	₹ 340,604.78	68%
7	Equity	L&T	Buy	2011.1	1876.15	-	134.95	13/10/22	₹ -	₹ -	0%
8	Equity	RELIANCE	Buy	2631.8	2382.8	-	249	13/10/22	₹ -	₹ -	0%
9	Equity	SUN PHARMA	Buy	1013.55	968.4	-	45.15	13/10/22	₹ -	₹ -	0%
10	Equity	M&M	Buy	1287.1	1248.2	128	38.9	13/10/22	₹ 164,362.75	₹ 159,395.22	32%

c)

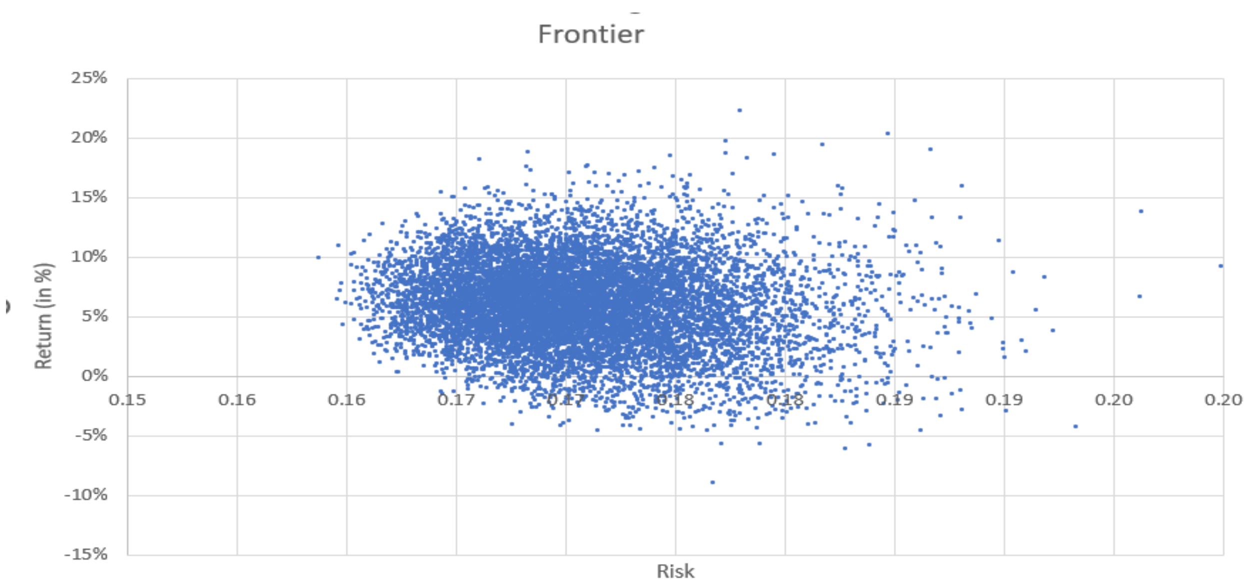
Portfolio Return			
Equal weights(0.1)	Max μ	Min σ	Max Sharpe ratio
10%	0%	12%	0%
10%	0%	16%	0%
10%	0%	13%	0%
10%	0%	15%	0%
10%	0%	5%	0%
10%	100%	16%	68%
10%	0%	0%	0%
10%	0%	0%	0%
10%	0%	19%	0%
10%	0%	4%	32%
100%	100%	100%	100%
5.867%	38.704%	7.760%	37.453%
0.16543	0.24296	0.15792	0.21234
-0.80%	134.61%	11.15%	148.13%

Optimization of portfolio:

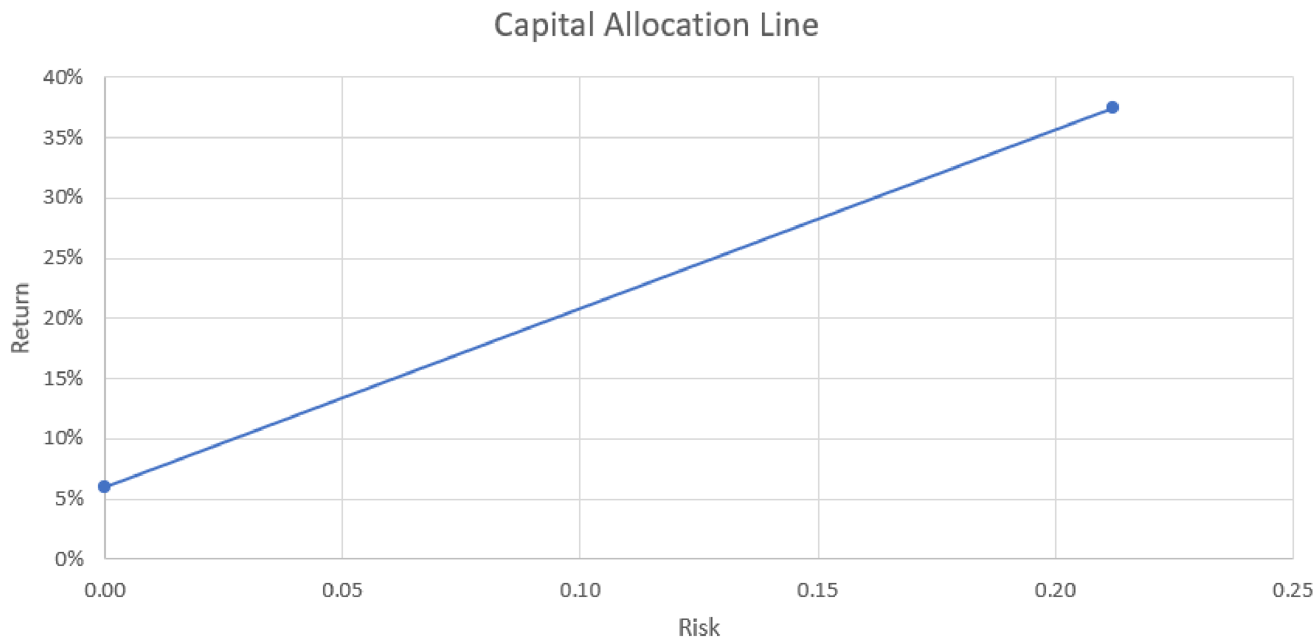
1. Initially the weights are set to be equal (i.e., 10%)
2. The weights have been re-evaluated using the solve function of excel to optimize the return (to maximize it).
3. The acquired weights have been re-evaluated using the solve function of excel to optimize the standard deviation (to minimize it).
4. The newly acquired weights have been re-evaluated using the solve function of excel to optimize the sharpe ratio (to maximize it).

Sharpe ratio = **148.13**

d)



This is a scatter plot of the portfolio frontier that we obtained from stock return-risk data by assigning a series of random weights to each stock using the RANDARRAY() function in Excel. An outer boundary can be drawn on the left portion of the plot to obtain the efficient frontier.



This plot consists of a Capital Allocation Line, drawn using a risk free rate of 6%. The optimal risky asset portfolio is at the point where the CAL is tangent to the efficient frontier and thus slope of CAL becomes equal to sharpe ratio of Portfolio. From our calculations

Slope (m) = Sharpe Ratio = 148.13

Intercept (c) = risk free asset = 6%

Thus equation of line becomes :-

$$\boxed{r = 148.13 * \sigma + 6}$$

e) Considerations to make for the COVID-induced situation to correlation matrix

Some studies have investigated the impact of COVID-19 on stock markets.To analyze the risk correlation between stock markets during the COVID-19 pandemic (Wu J, Zhang C, Chen Y) studied the correlation among stocks in terms of correlation network (they used 0.6 as threshold for having an edge between two stocks).

They concluded that :

The outbreak of COVID-19 has changed the topology of the correlation network. The impact of COVID-19 has made the risk correlation of the stock markets closer. In addition, the density of the network has become tighter. COVID-19 has made it easier for system risks to spread around the world, i.e. there seems to be a rise in correlation between various stocks, values in correlation matrix tends to be higher during covid period than the normal market days. Thus in such scenarios portfolios may need revision or need further dilution to decrease the overall risk.

f) We had invested a sum of ₹ 5,00,000 in our risky portfolio. We bought 1036 shares of ITC (68% of the portfolio) and 128 shares of M&M (32% of the portfolio). The final value of the portfolio as of 11/11/2022 stands at ₹ 5,33,675.13 .