Chapter 12

Introduction to Simulation Using Crystal Ball

12.0 Introduction

Chapter 1 discussed how the calculations in a spreadsheet can be viewed as a mathematical model that defines a functional relationship between various input variables (or independent variables) and one or more bottom-line performance measures (or dependent variables). The following equation expresses this relationship:

$$Y = f(X_1, X_2, \ldots, X_k)$$

In many spreadsheets, the values of various input cells are determined by the person using the spreadsheet. These input cells correspond to the independent variables $X_1, X_2, \ldots, X_k$ in the previous equation. Various formulas (represented by $f(\bullet)$ above) are entered in other cells of the spreadsheet to transform the values of the input cells into some bottom-line output (denoted by Y above). Simulation is a technique that is helpful in analyzing models in which the value to be assumed by one or more independent variables is uncertain.

This chapter discusses how to perform simulation using a popular commercial spreadsheet add-in called Crystal Ball, created and distributed by Decisioneering, Inc. A limited-life (140-day) version of Crystal Ball and related products can be accessed via the bind-in card accompanying this book. Decisioneering offers students one- and two-year extensions of this software at very affordable prices. You can find additional information about Crystal Ball and these extension options at http://www.decisioneering.com.

12.1 Random Variables and Risk

To compute a value for the bottom-line performance measure of a spreadsheet model, each input cell must be assigned a specific value so that all the related calculations can be performed. However, some uncertainty often exists regarding the value that should be assumed by one or more independent variables (or input cells) in the spreadsheet. This is particularly true in spreadsheet models that represent future conditions. A **random variable** is any variable whose value cannot be predicted or set with certainty. Thus, many input variables in a spreadsheet model represent random variables whose actual values cannot be determined with certainty.

For example, projections of the cost of raw materials, future interest rates, future numbers of employees, and expected product demand are random variables because their true values are unknown and will be determined in the future. If we cannot say with certainty what value one or more input variables in a model will assume, we also cannot say with certainty what value the dependent variable will assume. This uncertainty associated with the value of the dependent variable introduces an element of risk to the decision-making problem. Specifically, if the dependent variable represents some bottom-line performance measure that managers use to make decisions, and its value is uncertain, any decisions made on the basis of this value are based on uncertain (or incomplete) information. When such a decision is made, some chance exists that the decision will not produce the intended results. This chance, or uncertainty, represents an element of **risk** in the decision-making problem.

The term "risk" also implies the *potential* for loss. The fact that a decision's outcome is uncertain does not mean that the decision is particularly risky. For example, whenever we put money into a soft-drink machine, there is a chance that the machine will take our money and not deliver the product. However, most of us would not consider this risk to be particularly great. From past experience, we know that the chance of not receiving the product is small. But even if the machine takes our money and does not deliver the product, most of us would not consider this to be a tremendous loss. Thus, the amount of risk involved in a given decision-making situation is a function of the uncertainty in the outcome of the decision and the magnitude of the potential loss. A proper assessment of the risk present in a decision-making situation should address both of these issues, as the examples in this chapter will demonstrate.

12.2 Why Analyze Risk?

Many spreadsheets built by business people contain *estimated* values for the uncertain input variables in their models. If a manager cannot say with certainty what value a particular cell in a spreadsheet will assume, this cell most likely represents a random variable. Ordinarily, the manager will attempt to make an informed guess about the values such cells will assume. The manager hopes that inserting the expected, or most likely, values for all the uncertain cells in a spreadsheet will provide the most likely value for the cell containing the bottom-line performance measure (Y). The problem with this type of analysis is that it tells the decision maker nothing about the variability of the performance measure.

For example, in analyzing a particular investment opportunity, we might determine that the expected return on a $1,000 investment is $10,000 within two years. But how much variability exists in the possible outcomes? If all the potential outcomes are scattered closely around $10,000 (say from $9,000 to $11,000), then the investment opportunity still might be attractive. If, on the other hand, the potential outcomes are scattered widely around $10,000 (say from −$30,000 up to +$50,000), then the investment opportunity might be unattractive. Although these two scenarios might have the same expected or average value, the risks involved are quite different. Thus, even if we can determine the expected outcome of a decision using a spreadsheet, it is just as important, if not more so, to consider the risk involved in the decision.

12.3 Methods of Risk Analysis

Several techniques are available to help managers analyze risk. Three of the most common are best-case/worst-case analysis, what-if analysis, and simulation. Of these methods, simulation is the most powerful and, therefore, is the technique that we will focus on in this chapter. Although the other techniques might not be completely effective in risk analysis, they probably are used more often than simulation by most managers in business today. This is largely because most managers are unaware of the spreadsheet's ability to perform simulation and of the benefits provided by this technique. So before discussing simulation, let's first look briefly at the other methods of risk analysis to understand their strengths and weaknesses.

12.3.1 BEST-CASE/WORST-CASE ANALYSIS

If we don't know what value a particular cell in a spreadsheet will assume, we could enter a number that we think is the most likely value for the uncertain cell. If we enter such numbers for all the uncertain cells in the spreadsheet, we can easily calculate the most likely value of the bottom-line performance measure. This is also called the **base-case** scenario. However, this scenario gives us no information about how far away the actual outcome might be from this expected, or most likely, value.

One simple solution to this problem is to calculate the value of the bottom-line performance measure using the **best-case**, or most optimistic, and **worst-case**, or most pessimistic, values for the uncertain input cells. These additional scenarios show the range of possible values that might be assumed by the bottom-line performance measure. As indicated in the earlier example about the $1,000 investment, knowing the range of possible outcomes helps us assess the risk involved in different alternatives. However, simply knowing the best-case and worst-case outcomes tells us nothing about the distribution of possible values within this range, nor does it tell us the probability of either scenario occurring.

Figure 12.1 displays several probability distributions that might be associated with the value of a bottom-line performance measure within a given range. Each of these distributions describes variables that have identical ranges and similar average values. But each distribution is very different in terms of the risk it represents to the decision maker. The appeal of best-case/worst-case analysis is that it is easy to do. Its weakness is that it tells us nothing about the shape of the distribution associated with the bottom-line

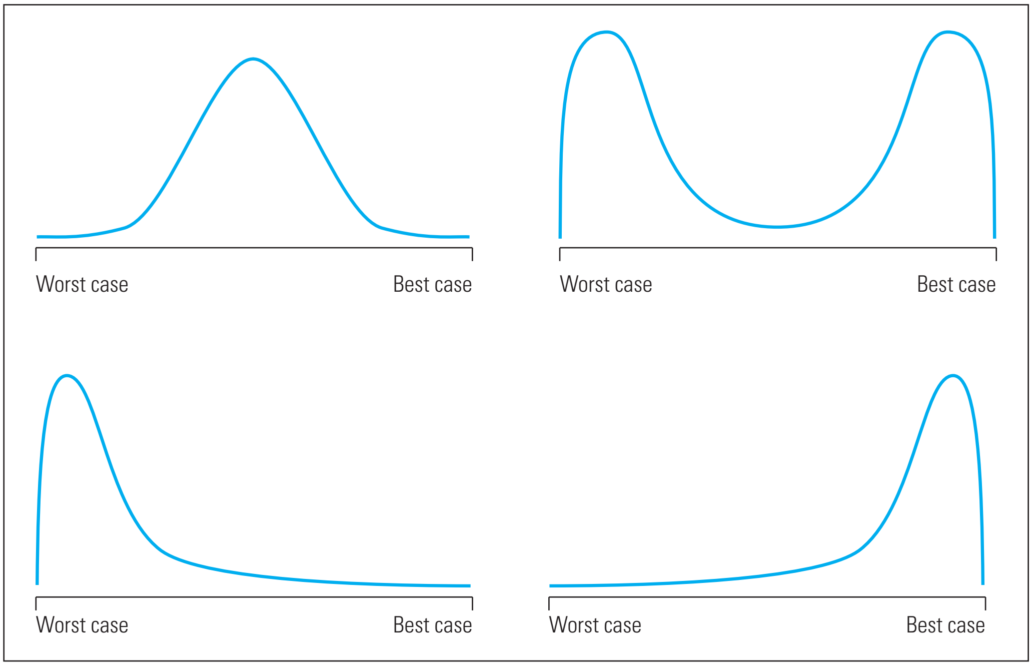


FIGURE 12.1

Possible distributions of performance measure values within a given range performance measure. As we will see later, knowing the shape of the distribution of the bottom-line performance measure can be critically important in helping us answer several managerial questions.

12.3.2 WHAT-IF ANALYSIS

Before the introduction of electronic spreadsheets in the early 1980s, the use of best-case/worst-case analysis often was the only feasible way for a manager to analyze the risk associated with a decision. This process was extremely time-consuming, error prone, and tedious, using only paper, pencil, and calculator to recalculate the performance measure of a model using different values for the uncertain inputs. The arrival of personal computers and electronic spreadsheets made it much easier for a manager to play out a large number of scenarios in addition to the best and worst cases—which is the essence of what-if analysis.

In **what-if analysis**, a manager changes the values of the uncertain input variables to see what happens to the bottom-line performance measure. By making a series of such changes, a manager can gain some insight into how sensitive the performance measure is to changes to the input variables. Although many managers perform this type of manual what-if analysis, it has three major flaws.

First, if the values selected for the independent variables are based only on the manager's judgment, the resulting sample values of the performance measure are likely to be biased. That is, if several uncertain variables can each assume some range of values, it would be difficult to ensure that the manager tests a fair, or representative, sample of all possible combinations of these values. To select values for the uncertain variables that correctly reflect their random variations, the values must be randomly selected from a distribution, or pool, of values that reflects the appropriate range of possible values, and the appropriate relative frequencies of these variables.

Second, hundreds or thousands of what-if scenarios might be required to create a valid representation of the underlying variability in the bottom-line performance measure. No one would want to perform these scenarios manually, nor could anyone make sense of the resulting stream of numbers that would flash on the screen.

The third problem with what-if analysis is that the insight that the manager might gain from playing out various scenarios is of little value when recommending a decision to top management. What-if analysis simply does not supply the manager with the tangible evidence (facts and figures) needed to justify why a given decision was made or recommended. Additionally, what-if analysis does not address the problem identified in our earlier discussion of best-case/worst-case analysis—it does not allow us to estimate the distribution of the performance measure in a formal enough manner. Thus, what-if analysis is a step in the right direction, but it is not quite a large enough step to allow managers to analyze risk effectively in the decisions they face.

12.3.3 SIMULATION

Simulation is a technique that measures and describes various characteristics of the bottom-line performance measure of a model when one or more values for the independent variables are uncertain. If any independent variables in a model are random variables, the dependent variable (Y) also represents a random variable. The objective in simulation is to describe the distribution and characteristics of the possible values of the bottom-line performance measure Y, given the possible values and behavior of the independent variables $X_1, X_2, \ldots, X_k$.

The idea behind simulation is similar to the notion of playing out many what-if scenarios. The difference is that the process of assigning values to the cells in the spreadsheet that represent random variables is automated so that: (1) the values are assigned in a non-biased way, and (2) the spreadsheet user is relieved of the burden of determining these values. With simulation, we repeatedly and randomly generate sample values for each uncertain input variable ($X_1, X_2, \ldots, X_k$) in our model and then compute the resulting value of our bottom-line performance measure (Y). We can then use the sample values of Y to estimate the true distribution and other characteristics of the performance measure Y. For example, we can use the sample observations to construct a frequency distribution of the performance measure, to estimate the range of values over which the performance measure might vary, to estimate its mean and variance, and to estimate the probability that the actual value of the performance measure will be greater than (or less than) a particular value. All these measures provide greater insight into the risk associated with a given decision than a single value calculated based on the expected values for the uncertain independent variables.

On Uncertainty and Decision-Making...

"Uncertainty is the most difficult thing about decision-making. In the face of uncertainty, some people react with paralysis, or they do exhaustive research to avoid making a decision. The best decision-making happens when the mental environment is focused. In a physical environment, you focus on something physical. In tennis, that might be the spinning seams of the ball. In a mental environment, you focus on the facts at hand. That fine-tuned focus doesn't leave room for fears and doubts to enter. Doubts knock at the door of our consciousness, but you don't have to have them in for tea and crumpets."—Timothy Gallwey, author of *The Inner Game of Tennis* and *The Inner Game of Work*.

12.4 A Corporate Health Insurance Example

The following example demonstrates the mechanics of preparing a spreadsheet model for risk analysis using simulation. The example presents a fairly simple model to illustrate the process and give a sense of the amount of effort involved. However, the process for performing simulation is basically the same regardless of the size of the model.

> Lisa Pon has just been hired as an analyst in the corporate planning department of Hungry Dawg Restaurants. Her first assignment is to determine how much money the company needs to accrue in the coming year to pay for its employees' health insurance claims. Hungry Dawg is a large, growing chain of restaurants that specializes in traditional southern foods. The company has become large enough that it no longer buys insurance from a private insurance company. The company is now self-insured, meaning that it pays health insurance claims with its own money (although it contracts with an outside company to handle the administrative details of processing claims and writing checks).

FIGURE 12.2

Original corporate health insurance model with expected values for uncertain variables

Hungry Dawg Restaurants

Initial Conditions				Assumptions		
Number of Covered Employees			18,533	Increasing	2%	per month
Average Claim per Employee			$250	Increasing	1%	per month
Amount Contributed per Employee			$125	Constant		
Month	Number of Employees	Employee Contributions	Avg Claim per Emp.	Total Claims		Company Cost
1	18,904	$2,363,000	$252.50	$4,773,260		$2,410,260
2	19,282	$2,410,250	$255.03	$4,917,488		$2,507,238
3	19,667	$2,458,375	$257.58	$5,065,826		$2,607,451
4	20,061	$2,507,625	$260.15	$5,218,869		$2,711,244
5	20,462	$2,557,750	$262.75	$5,376,391		$2,818,641
6	20,871	$2,608,875	$265.38	$5,538,746		$2,929,871
7	21,289	$2,661,125	$268.03	$5,706,091		$3,044,966
8	21,714	$2,714,250	$270.71	$5,878,197		$3,163,947
9	22,149	$2,768,625	$273.42	$6,055,980		$3,287,355
10	22,592	$2,824,000	$276.16	$6,239,007		$3,415,007
11	23,043	$2,880,375	$278.92	$6,427,154		$3,546,779
12	23,504	$2,938,000	$281.71	$6,621,312		$3,683,312
				Total Company Cost		$36,126,069

The money the company uses to pay claims comes from two sources: employee contributions (or premiums deducted from employees' paychecks), and company funds (the company must pay whatever costs are not covered by employee contributions). Each employee covered by the health plan contributes $125 per month. However, the number of employees covered by the plan changes from month to month as employees are hired and fired, quit, or simply add or drop health insurance coverage. A total of 18,533 employees were covered by the plan last month. The average monthly health claim per covered employee was $250 last month.

An example of how most analysts would model this problem is shown in Figure 12.2 (and in the file Fig12-2.xls on your data disk). The spreadsheet begins with a listing of the initial conditions and assumptions for the problem. For example, cell D5 indicates that 18,533 employees currently are covered by the health plan, and cell D6 indicates that the average monthly claim per covered employee is $250. The average monthly contribution per employee is $125, as shown in cell D7. The values in cells D5 and D6 are unlikely to stay the same for the entire year. Thus, we need to make some assumptions about the rate at which these values are likely to increase during the year. For example, we might assume that the number of covered employees will increase by about 2% per month, and that the average claim per employee will increase at a rate of 1% per month. These assumptions are reflected in cells F5 and F6. The average contribution per employee is assumed to be constant over the coming year.

Using the assumed rate of increase in the number of covered employees (cell F5), we can create formulas for cells B11 through B22 that cause the number of covered employees to increase by the assumed amount each month. (The details of these formulas are covered later.) The expected monthly employee contributions shown in column C are calculated as $125 times the number of employees in each month. We can use the assumed rate of increase in average monthly claims (cell F6) to create formulas for cells D11 through D22 that cause the average claim per employee to increase at the assumed rate.

The total claims for each month (shown in column E) are calculated as the average claim figures in column D times the number of employees for each month in column B. Because the company must pay for any claims that are not covered by the employee contributions, the company cost figures in column G are calculated as the total claims minus the employee contributions (column E minus column C). Finally, cell G23 sums the company cost figures listed in column G, and shows that the company can expect to contribute $36,126,069 of its revenues toward paying the health insurance claims of its employees in the coming year.

12.4.1 A CRITIQUE OF THE BASE CASE MODEL

Now, let's consider the model we just described. The example model assumes that the number of covered employees will increase by *exactly* 2% each month and that the average claim per covered employee will increase by *exactly* 1% each month. Although these values might be reasonable approximations of what might happen, they are unlikely to reflect exactly what will happen. In fact, the number of employees covered by the health plan each month is likely to vary randomly around the average increase per month—that is, the number might decrease in some months and increase by more than 2% in others. Similarly, the average claim per covered employee might be lower than expected in certain months and higher than expected in others.

Both of these figures are likely to exhibit some uncertainty or random behavior, even if they do move in the general upward direction assumed throughout the year. So, we cannot say with certainty that the total cost figure of $36,126,069 is exactly what the company will have to contribute toward health claims in the coming year. It is simply a prediction of what might happen. The actual outcome could be smaller or larger than this estimate. Using the original model, we have no idea how much larger or smaller the actual result could be—nor do we have any idea of how the actual values are distributed around this estimate. We do not know if there is a 10%, 50%, or 90% chance of the actual total costs exceeding this estimate. To determine the variability or risk inherent in the bottom-line performance measure of total company costs, we will apply the technique of simulation to our model.

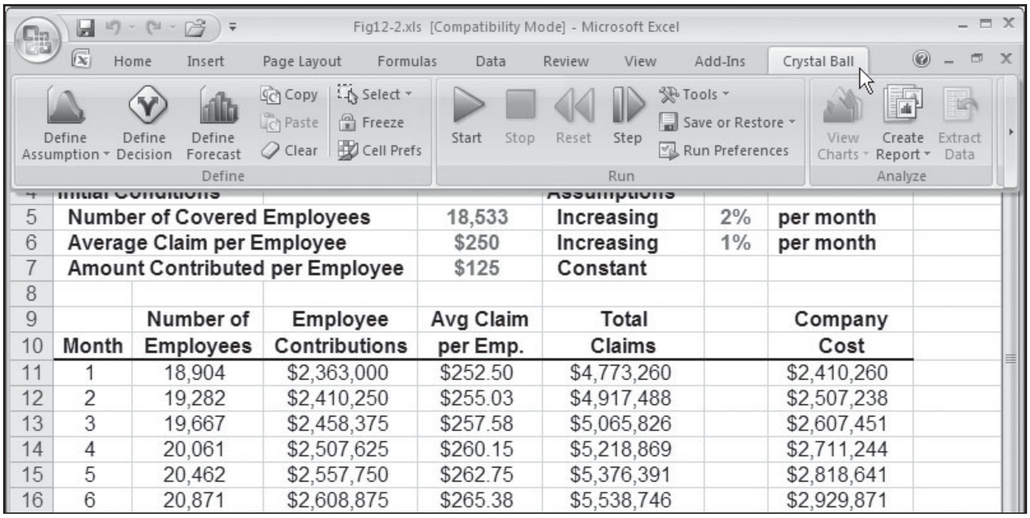
12.5 Spreadsheet Simulation Using Crystal Ball

To perform simulation in a spreadsheet, we must first place a **random number generator** (RNG) in each cell that represents a random, or uncertain, independent variable. Each RNG provides a sample observation from an appropriate distribution that represents the range and frequency of possible values for the variable. After the RNGs are in place, a new value of the bottom-line performance measure is computed every time new sample values are generated for the uncertain input cells. We can sample the spreadsheet *n* times, where *n* is the desired number of replications or scenarios, and the value of the bottom-line performance measure will be stored after each replication. We then analyze these stored observations to gain insights into the behavior and characteristics of the performance measure.

The process of simulation involves a lot of work but, fortunately, the spreadsheet can do most of the work for us fairly easily. In particular, the spreadsheet add-in package Crystal Ball is designed specifically to make spreadsheet simulation a simple process. The Crystal Ball software provides the following capabilities, which are not otherwise available while working in Excel: additional tools that are helpful in generating the

FIGURE 12.3

Crystal Ball Commands in Excel



random numbers needed in simulation; additional commands that are helpful in setting up and running the simulation; and graphical and statistical summaries of the simulation data. As we shall see, these capabilities make simulation a relatively easy technique to apply in spreadsheets.

12.5.1 STARTING CRYSTAL BALL

If you are running Crystal Ball from a local area network (LAN) or in a computer lab, your instructor or LAN coordinator should give you directions on how to access this software. If you have installed Crystal Ball on your own computer, you can load Crystal Ball in Windows as follows:

1. Click Start.
2. Click Programs.
3. Click Crystal Ball 7.
4. Click Crystal Ball.

This loads Crystal Ball in a new instance of Excel and causes the Crystal Ball commands shown in Figure 12.3 to appear. We will refer to the various commands in Figure 12.3 throughout this chapter.

12.6 Random Number Generators

As mentioned earlier, the first step in spreadsheet simulation is to place an RNG in each cell that contains an uncertain value. Each RNG will generate (or return) a number that represents a randomly selected value from a distribution, or pool, of values. The distributions that these samples are taken from should be representative of the underlying pool of values expected to occur in each uncertain cell. Crystal Ball refers to such cells as *assumption cells.* To create the RNGs required for simulating a spreadsheet model,

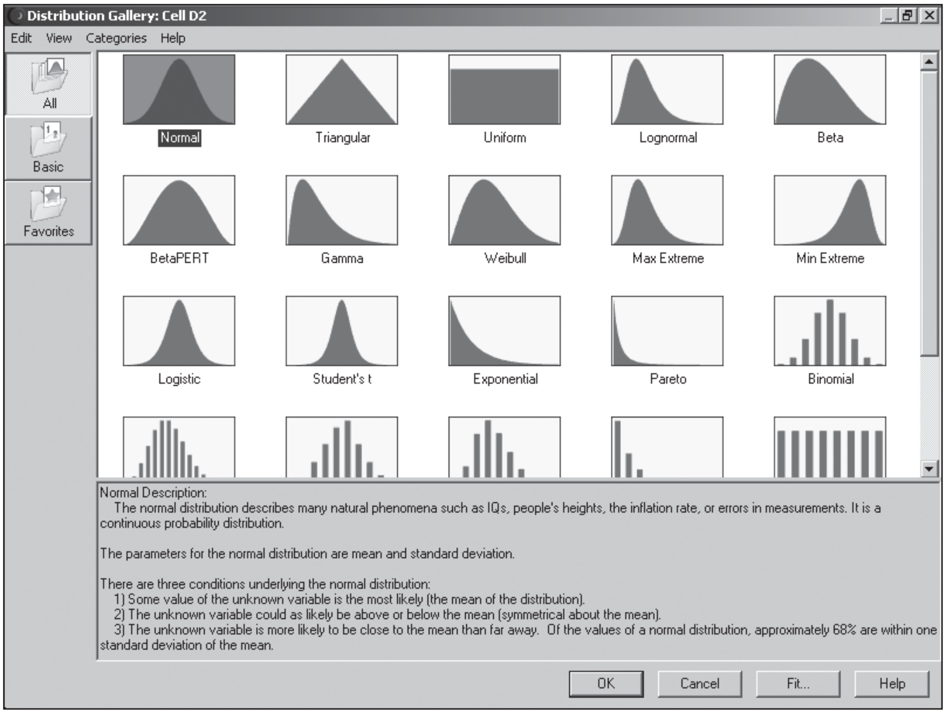


FIGURE 12.4

The Crystal Ball Distribution Gallery

Crystal Ball provides the Distribution Gallery tool shown in Figure 12.4. (The Distribution Gallery is launched by clicking the Define Assumption button on the Crystal Ball tab.)

The distributions shown in Figure 12.4 allow us to generate a variety of random numbers easily. For example, if we think that the behavior of a particular uncertain cell (perhaps representing the average health insurance claim per employee in a given month) could be modeled as a normally distributed random variable with a mean of $250 and standard deviation of $3, we could double-click the Normal distribution button in Figure 12.4. This launches the dialog in Figure 12.5 that allows us to indicate the mean and standard deviation for the assumption (or uncertain) cell in question. (The entries for the mean and standard deviation also could be formulas that refer to other cells in the spreadsheet.) Whenever Crystal Ball creates a new replication of the spreadsheet, the value in this assumption cell would be a randomly selected value from a normal distribution with a mean of 250 and standard deviation of 3.

As a different example, suppose that an assumption cell in a spreadsheet (perhaps representing the number of gallons of white ceiling paint sold on a given day at a paint store) has a 30% chance of assuming the value 10, a 50% chance of assuming the value 15, and a 20% chance of assuming the value 20. We could model the behavior of this random variable using the Custom distribution from Crystal Ball's distribution gallery as shown in Figure 12.6. If we used Crystal Ball to replicate this spreadsheet many times, it would randomly choose the value 10 for this assumption cell approximately 30% of the time, the value 15 approximately 50% of the time, and the value 20 approximately 20% of the time.

Each of the RNGs in Figure 12.4 require different parameters that allow us to generate random numbers from probability distributions with a wide variety of shapes. Figures 12.7 and 12.8 illustrate some example distributions. Additional information about each distribution is available by clicking the Help button on any of the Crystal Ball dialogs.

FIGURE 12.5

Example of creating a normally distributed assumption cell

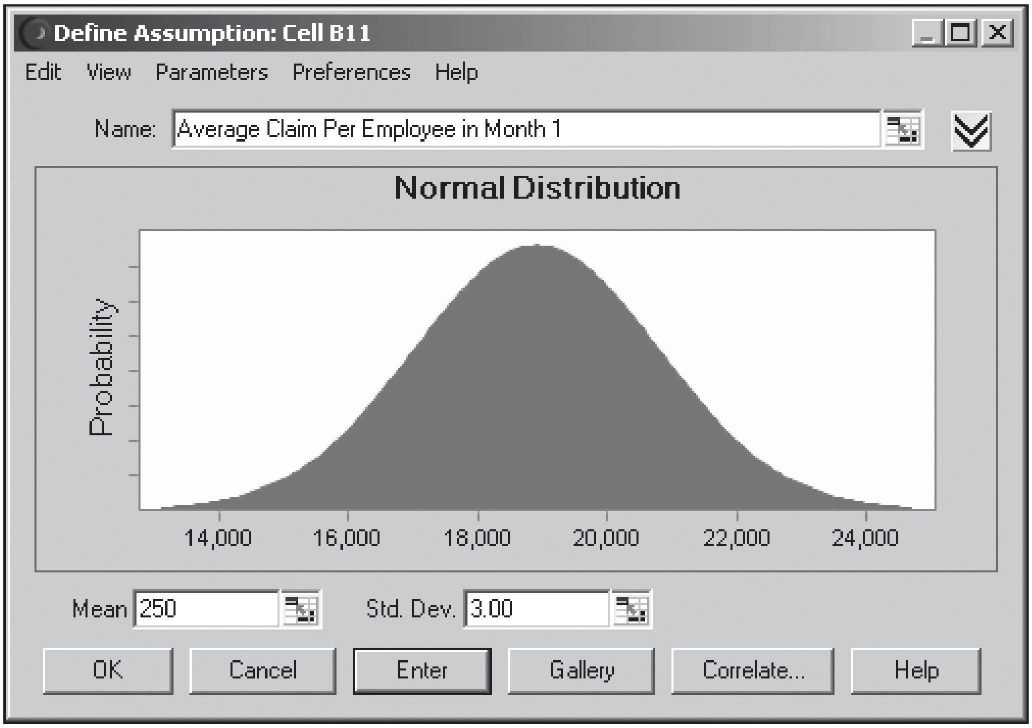
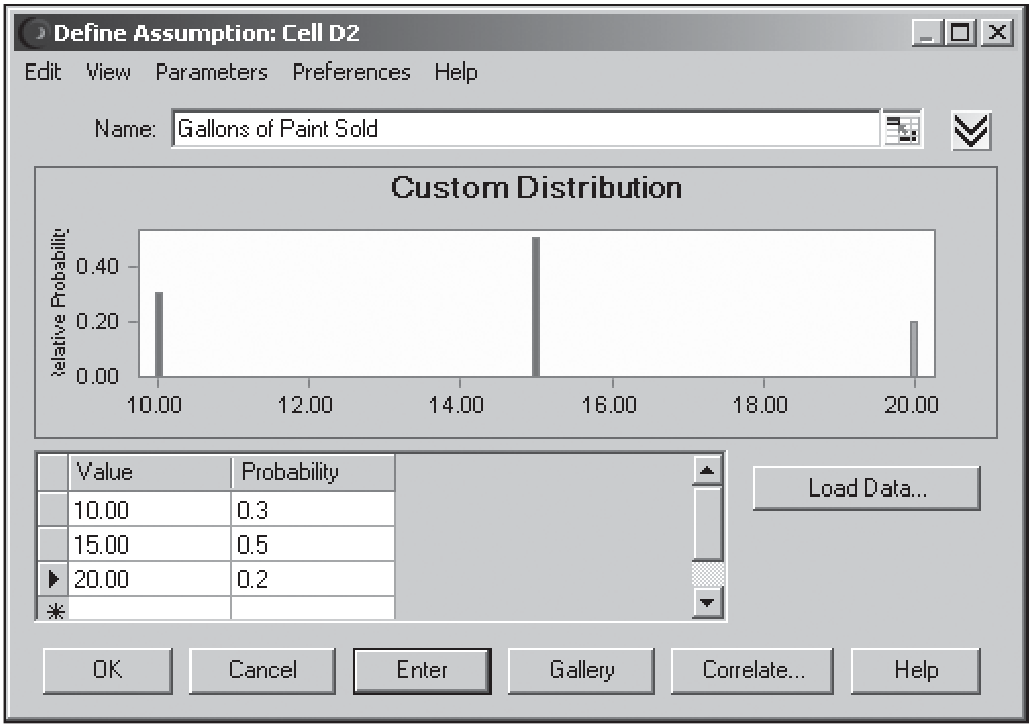


FIGURE 12.6

Example of creating a custom distribution for an assumption cell



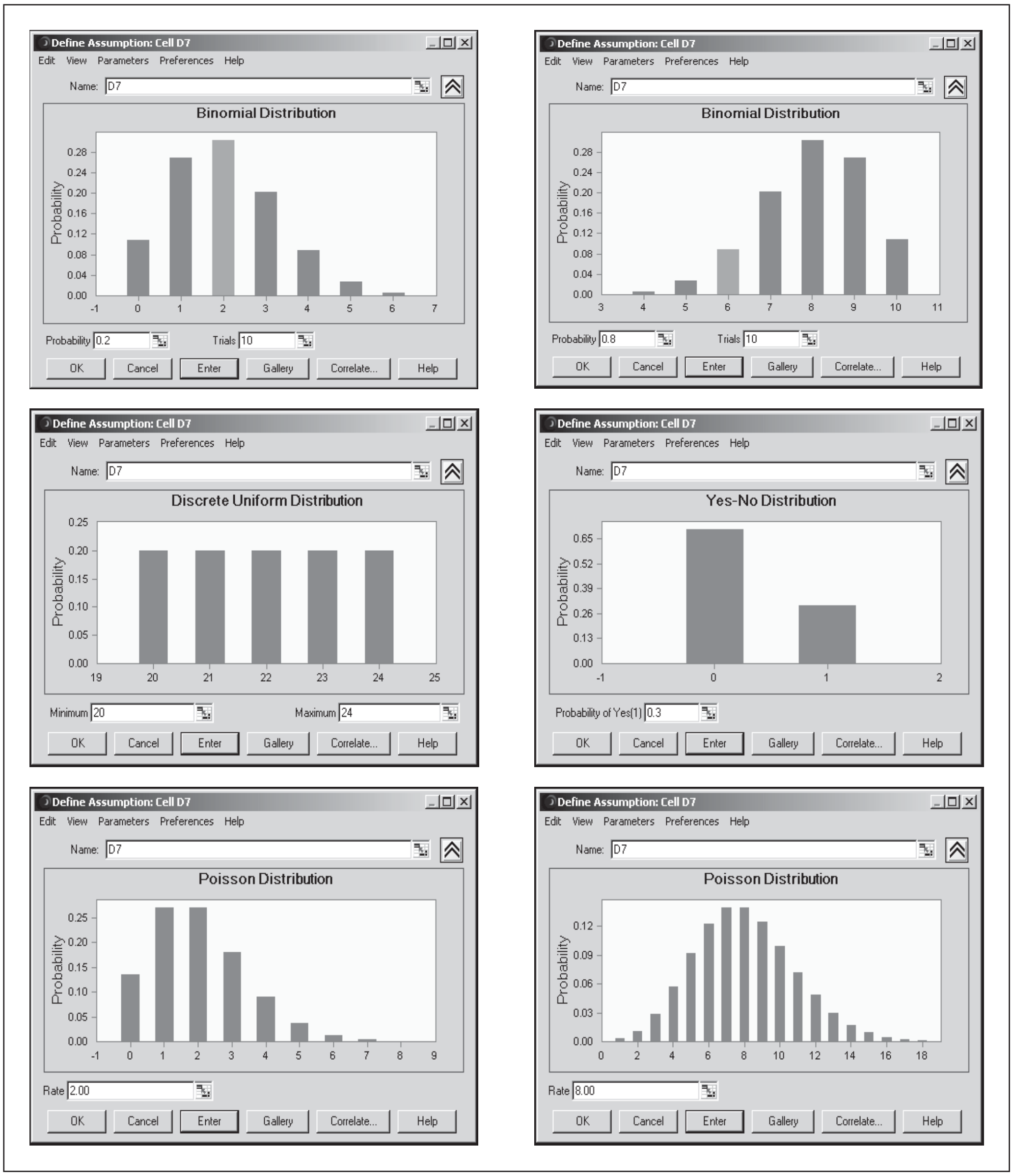


FIGURE 12.7

Examples of selected discrete distributions

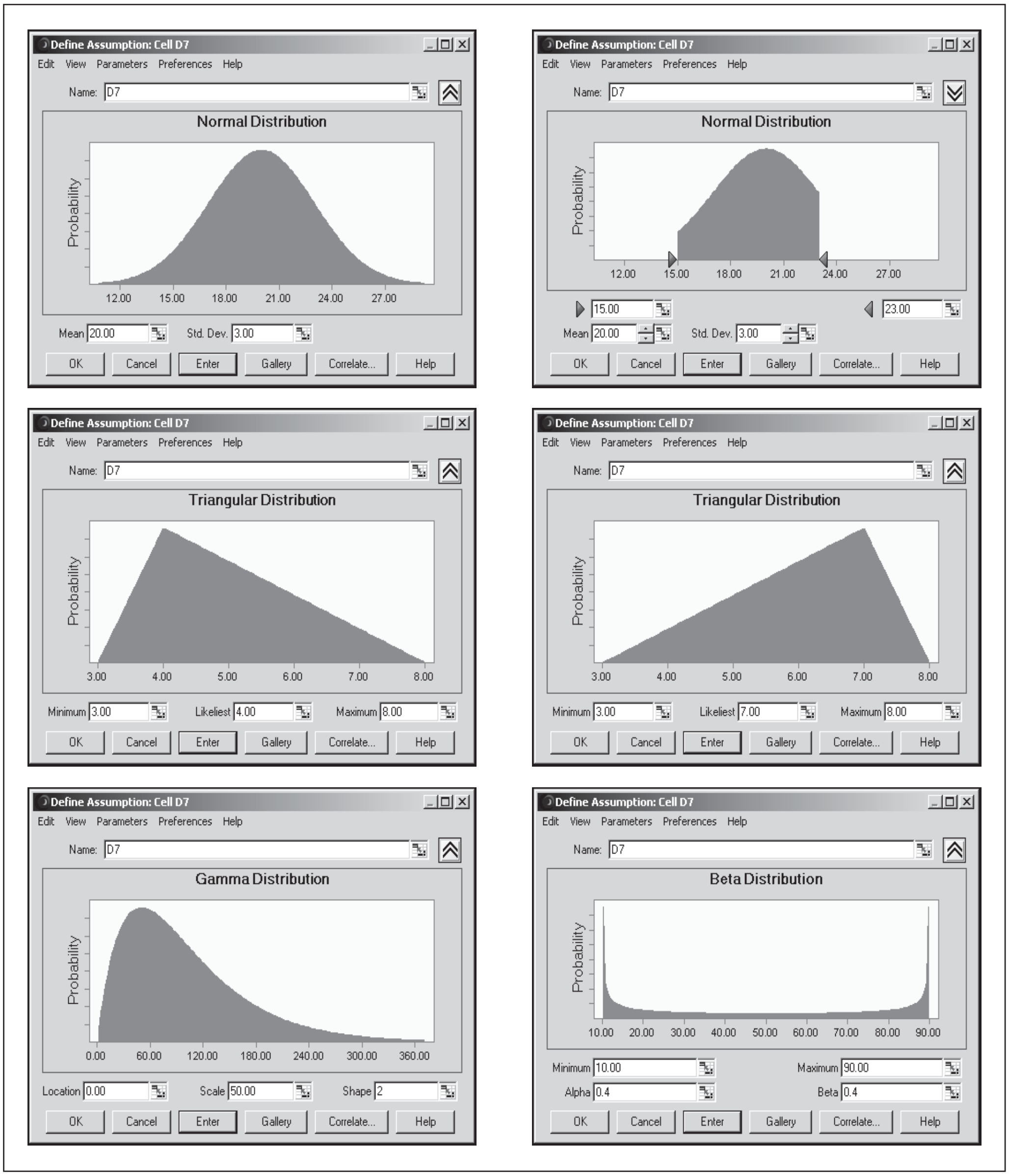
12.6.1 DISCRETE VS. CONTINUOUS RANDOM VARIABLES

An important distinction exists between the graphs in Figures 12.7 and 12.8. In particular, the RNGs depicted in Figure 12.7 generate *discrete* outcomes, whereas those represented in Figure 12.8 generate *continuous* outcomes. That is, some of the RNGs displayed in Figure 12.4 can return only a distinct set of individual values, whereas the other RNGs can return any value from an infinite set of values. The distinction between discrete and continuous random variables is very important.

For example, the number of defective tires on a new car is a discrete random variable because it can assume only one of five distinct values: 0, 1, 2, 3, or 4. On the other hand, the amount of fuel in a new car is a continuous random variable because it can assume any value between 0 and the maximum capacity of the fuel tank. Thus, when selecting

FIGURE 12.8

Examples of selected continuous distributions

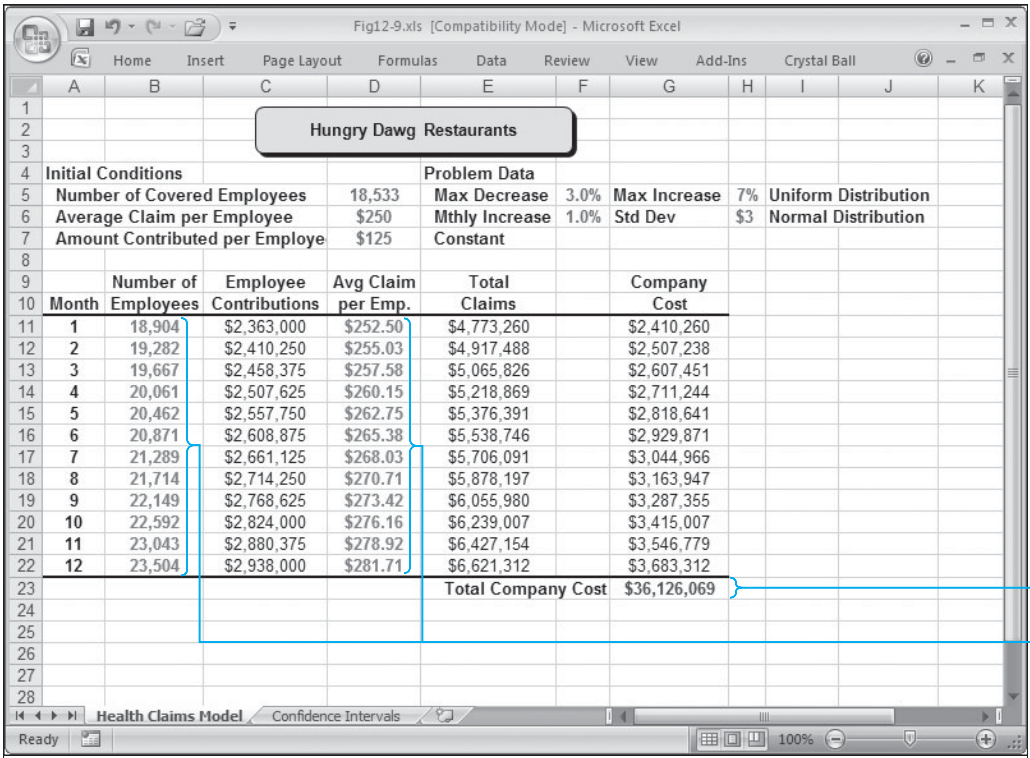


an RNG for an uncertain variable in a model, it is important to consider whether the variable can assume discrete or continuous values.

12.7 Preparing the Model for Simulation

To apply simulation to the model for Hungry Dawg Restaurants described earlier, first we must select appropriate RNGs for the uncertain variables in the model. If available, historical data on the uncertain variables could be analyzed to determine appropriate RNGs for these variables. (Crystal Ball's Batch Fit tool can be used for this purpose.) If past data are not available, or if we have some reason to expect the future behavior of a variable to be significantly different from the past, then we must use judgment in selecting appropriate RNGs to model the random behavior of the uncertain variables.

For our example problem, let's assume that by analyzing historical data, we determined that the change in the number of covered employees from one month to the next is expected to vary uniformly between a 3% decrease and a 7% increase. (Note that this should cause the *average* change in the number of employees to be a 2% increase, because 0.02 is the midpoint between −0.03 and +0.07.) Further, assume that we can model the average monthly claim per covered employee as a normally distributed random variable with the mean increasing by 1% per month and a standard deviation of approximately $3. (Note that this will cause the *average* increase in claims per covered employee from one month to the next to be approximately 1%.) These assumptions are reflected in cells F5 through H6 at the top of Figure 12.9 (and in the file Fig12-9.xls on your data disk).



	A	B	C	D	E	F	G	H	I
2			Hungry Dawg Restaurants						
4	Initial Conditions				Problem Data				
5	Number of Covered Employees			18,533	Max Decrease	3.0%	Max Increase	7%	Uniform Distribution
6	Average Claim per Employee			$250	Mthly Increase	1.0%	Std Dev	$3	Normal Distribution
7	Amount Contributed per Employe			$125	Constant				
9		Number of	Employee	Avg Claim	Total		Company		
10	Month	Employees	Contributions	per Emp.	Claims		Cost		
11	1	18,904	$2,363,000	$252.50	$4,773,260		$2,410,260		
12	2	19,282	$2,410,250	$255.03	$4,917,488		$2,507,238		
13	3	19,667	$2,458,375	$257.58	$5,065,826		$2,607,451		
14	4	20,061	$2,507,625	$260.15	$5,218,869		$2,711,244		
15	5	20,462	$2,557,750	$262.75	$5,376,391		$2,818,641		
16	6	20,871	$2,608,875	$265.38	$5,538,746		$2,929,871		
17	7	21,289	$2,661,125	$268.03	$5,706,091		$3,044,966		
18	8	21,714	$2,714,250	$270.71	$5,878,197		$3,163,947		
19	9	22,149	$2,768,625	$273.42	$6,055,980		$3,287,355		
20	10	22,592	$2,824,000	$276.16	$6,239,007		$3,415,007		
21	11	23,043	$2,880,375	$278.92	$6,427,154		$3,546,779		
22	12	23,504	$2,938,000	$281.71	$6,621,312		$3,683,312		
23					Total Company Cost		$36,126,069		

FIGURE 12.9

Modified corporate health insurance model with RNGs replacing expected values for uncertain variables

Key Cell Formulas & CB Settings

Cell	Formula/Setting	Copied to
B11	CB Distribution: Uniform Minimum: =D5*(1-F5) Maximum: =D5*(1+H5)	--
B12	CB Distribution: Uniform Minimum: =B11*(1-$F$5) Maximum: =B11*(1+$H$5)	B13:B22 (Using CB copy & paste)
C11	=$D$7*B11	C12:C22
D11	CB Distribution: Normal Mean: =$D$6*(1+$F$6)^A11 Standard Deviation: =$H$6	D12:D22 (Using CB copy & paste)
E11	=D11*B11	E12:E22
G11	=E11-C11	G12:G22
G23	=SUM(G11:G22)	--

12.7.1 DEFINING ASSUMPTIONS FOR THE NUMBER OF COVERED EMPLOYEES

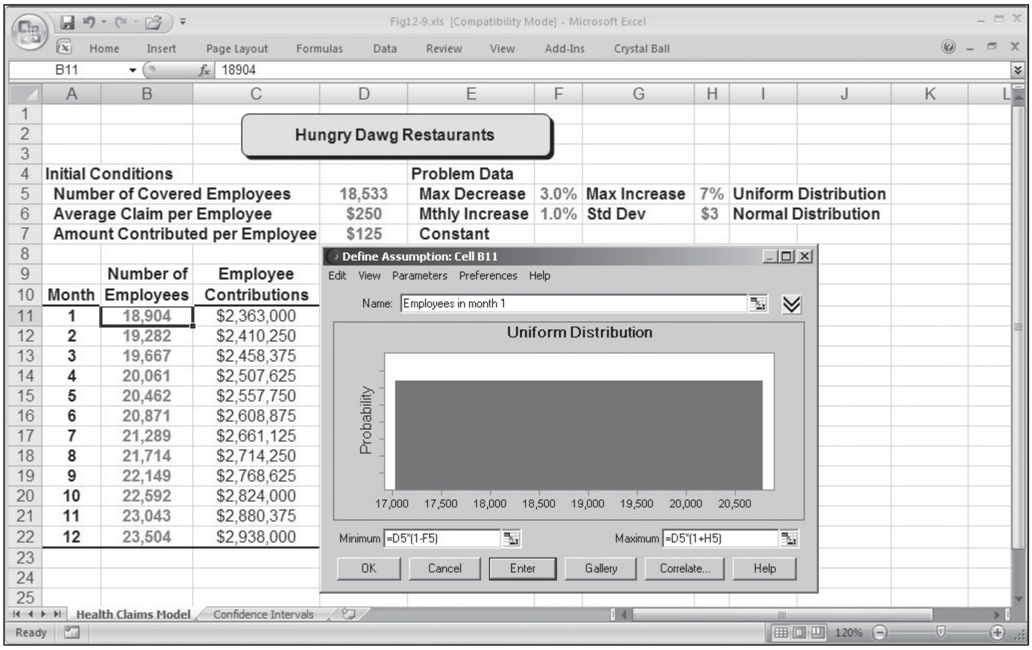
To randomly generate the appropriate number of employees covered by the health plan each month, we will use the Uniform distribution shown in Figure 12.4. Because the change in the number of employees from one month to the next can vary between a 3% decrease and a 7% increase, the number of employees in any month should be between a minimum of 97% and a maximum of 107% of the number of employees in the previous month. Thus, the number of employees in month 1 (represented by cell B11) should vary uniformly between 97% and 107% of the number of employees given in cell D5. Using Crystal Ball, we place this assumption on cell B11 as follows:

1. Select cell B11.
2. Click Crystall Ball, Define Assumption.
3. Select the Uniform distribution.
4. In the Name text box type: Employees in Month 1
5. For the Minimum parameter type the formula: =D5*(1-F5)
6. For the Minimum parameter type the formula: =D5*(1+H5)

The resulting Define Assumption dialog box is shown in Figure 12.10. Notice that we are instructing Crystal Ball to treat cell B11 as an assumption (or uncertain) cell whose value is a uniformly distributed random variable between a minimum of 97% (or 1-F5) of the value in D5 and a maximum of 107% (or 1+H5) of the value in cell D5. So when we run the simulation, Crystal Ball will randomly select values between 17,977 and 19,830 for cell B11.

FIGURE 12.10

Defining the assumptions cell for cell B11



Do You Want to See the Formulas or the Numbers?

Using the Define Assumption dialog as shown in Figure 12.10, you can use the Parameters, Show Cell References command (or Ctrl+ ~) to instruct Crystal Ball to show either the formulas that define the minimum and maximum values or the values produced by the formulas.

When you click OK on the Define Assumption dialog shown in Figure 12.10, Crystal Ball automatically changes the background color of cell B11 to green to indicate that it is now an assumption cell. If you want assumption cells to be indicated by a different color you may indicate your preference using the Define, Cell Preferences command.

Next, we need to place similar assumptions on the number of employees in months 2 through 12 (cells B12 through B22). Again, the number of employees in each month should be between a minimum of 97% and a maximum of 107% of the number of employees in the previous month. We could define the assumptions for each cell individually, just as we did for month 1 (cell B11). However, it is also possible to copy and paste assumption definitions from one cell to other cells when the formulas that describe the distribution follow a common pattern. So if we are careful about how we create the formulas that define the minimum and maximum values for number of employees in month 2 (for cell B12), we can copy the assumptions about that cell to the remaining cells for months 3 through 12 (cells B13 through B22).

Figure 12.11 shows the Define Assumptions dialog box for cell B12. Notice that the formulas for the minimum and maximum number of employees are =B11*(1-$F$5) and

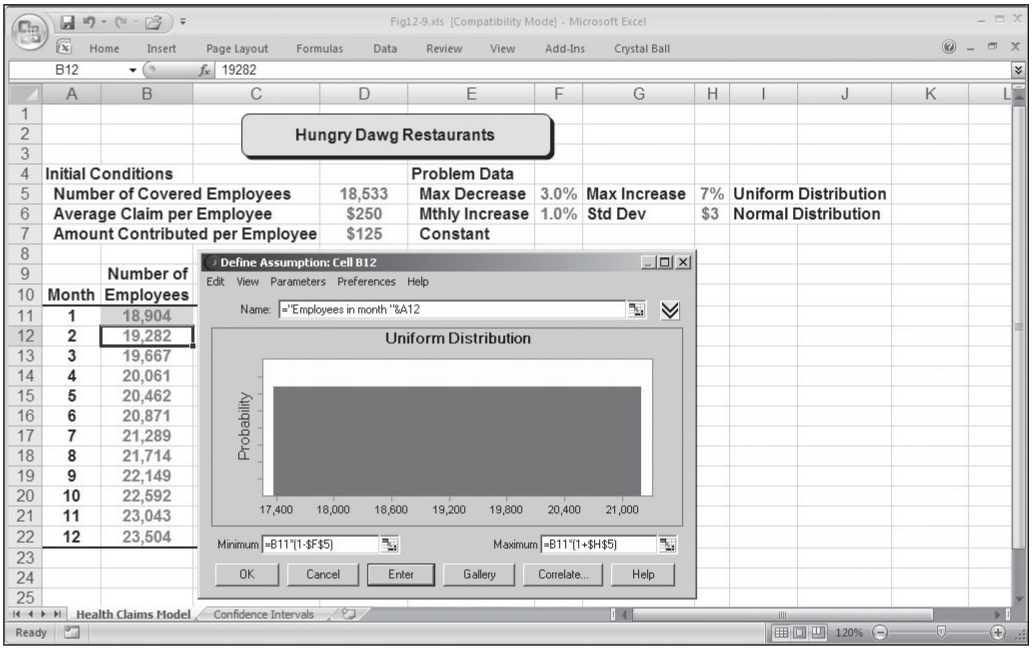


FIGURE 12.11

Defining the assumptions for cell B12

=B11*(1+$H$5), respectively. The dollar signs embedded in the references to cells F5 and H5 indicate that these are absolute cell references that should not change if the definition for this assumption cell is ever copied to another cell. The reference to B11 lacks the dollar signs and will change in the customary relative fashion if the definition for this assumption cell is copied elsewhere. Also notice that the name being defined for cell B12 is given by the formula: ="Employees in month "&A12. (Because the value in cell A12 is two (2), this formula evaluates to the text string "Employees in month 2".) When defining cells whose definitions will be copied, it is good practice to use formulas like this (when possible) for creating names so that these items are identified clearly in subsequent Crystal Ball reports and charts.

Once our assumptions about cell B12 are properly recorded, we can copy its assumption cell definition to cells B13 through B22. However, it is ***critical*** that we use Crystal Ball's copy and paste commands to do this. Excel's copy and paste command ***will not work*** on assumption cell definitions. Thus, to copy the assumption cell definitions from cell B12 to cells B13 through B22:

1. Select cell B12.
2. Click Crystal Ball, Copy.
3. Select cells B13 through B22.
4. Click Crystal Ball, Paste.

Important Software Issue

To copy assumption cell definitions, you must use Crystal Ball's copy data and paste data commands. Excel's copy and paste commands do not work on assumption cell definitions.

12.7.2 DEFINING ASSUMPTIONS FOR THE AVERAGE MONTHLY CLAIM PER EMPLOYEE

To randomly generate the appropriate average claims per covered employee in each month, we will use the Normal distribution shown in Figure 12.4. This distribution requires that we supply the value of the mean and standard deviation of the distribution from which we want to sample. The assumed $3 standard deviation for the average monthly claim, shown in cell H6 in Figure 12.11, is constant from month to month. Thus, we need only to determine the proper mean value for each month.

In this case, the mean for any given month should be 1% larger than the mean in the previous month. For example, the mean for month 1 is:

$$\text{Mean in month } 1 = (\text{original mean}) \times 1.01$$

and the mean for month 2 is:

$$\text{Mean in month } 2 = (\text{mean in month } 1) \times 1.01$$

If we substitute the previous definition of the mean in month 1 into the previous equation, we obtain,

$$\text{Mean in month } 2 = (\text{original mean}) \times (1.01)^2$$

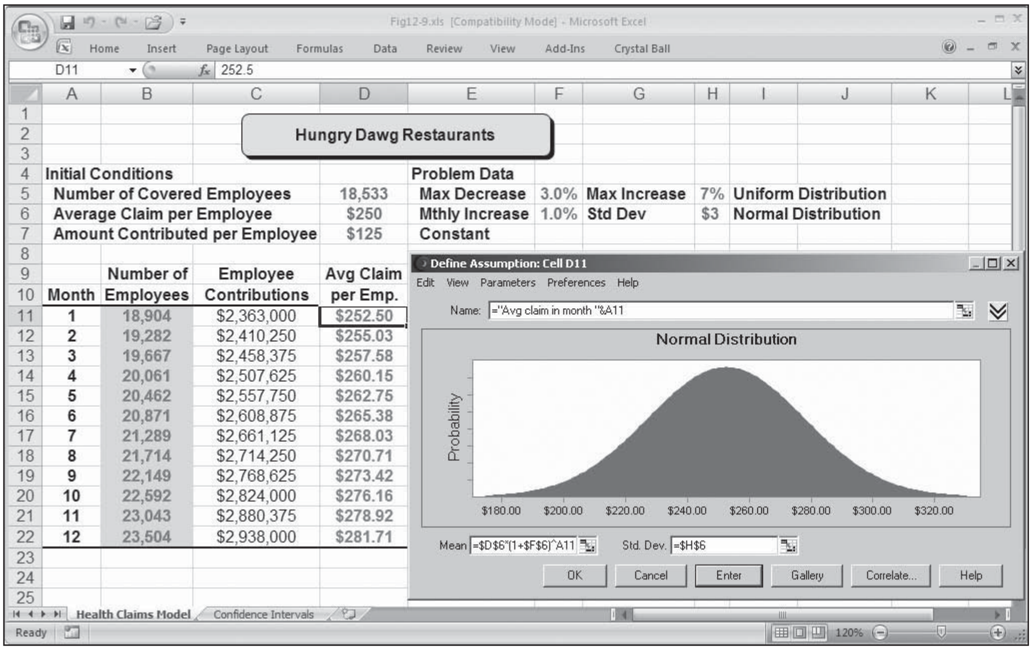


FIGURE 12.12

Defining the assumptions for cell D11

Similarly, the mean in month 3 is:

$$\text{Mean in month 3} = (\text{mean in month 2}) \times 1.01 = (\text{original mean}) \times (1.01)^3$$

So in general, the mean for month n is:

$$\text{Mean in month } n = (\text{original mean}) \times (1.01)^n$$

Thus, to generate the average claim per covered employee for month 1, we use the assumption cell definition for cell D11 shown in Figure 12.12.

Notice that the mean for cell D11 is defined by the formula: =$D$6*(1+$F$6)^A11. This formula implements the general definition of the mean in month n. Similarly, the standard deviation, which is assumed to be constant in all months, is defined by the formula: =$H$6. So after entering our assumptions for cell D11, we can copy this definition to cells D11 through D22 to complete the assumption cell specifications related to the average claim per employee each month. Again, it is ***critical*** that we use Crystal Ball's copy and paste commands to do this because Excel's copy and paste command ***will not work*** on assumption cell definitions.

12.7.3 DEFINING ASSUMPTIONS FOR THE AVERAGE MONTHLY CLAIM PER EMPLOYEE

After entering the appropriate RNGs via our assumption cell definitions, you can use Crystal Ball's Run, Single Step command to automatically select new values for all the cells in the spreadsheet that represent uncertain (or random) values. Similarly, with each replication, a new value for the bottom-line performance measure (total company cost) appears in cell G23. Thus, by executing the Single Step command several times, we can observe representative values of the company's total cost for health claims. This also helps to verify that we implemented the RNGs correctly and that they are generating appropriate values for each uncertain cell.

Why Don't Your Numbers Change?

By default, Crystal Ball displays the mean (or average) value of the distribution for cells containing RNG functions. To make Crystal Ball display new values for your RNG cells whenever you click the Single Step button, click Crystal Ball, Cell Prefs, and deselect the check box labeled "Set cell value to distribution mean."

12.8 Running the Simulation

The next step in performing the simulation involves recalculating the spreadsheet several hundred or several thousand times and recording the resulting values generated for the output cell, or bottom-line performance measure. Fortunately, Crystal Ball can do this for us if we indicate: 1) which cell(s) in the spreadsheet we want to track, and 2) how many times we want it to replicate the model.

12.8.1 SELECTING THE OUTPUT CELLS TO TRACK

We can use the Define Forecast button on the Crystal Ball tab (see Figure 12.3) to indicate the output cell (or cells) that we want Crystal Ball to track during the simulation. In the current example, cell G23 represents the output cell we want Crystal Ball to track. To indicate this:

1. Click cell G23.
2. Click the Define Forecast button on the Crystal Ball tab.
   (This causes the dialog box in Figure 12.13 to appear. If you see a button labeled More, click it.)
3. Make the indicated selections.
4. Click OK.

In some simulations, we might want to analyze several performance measures. In such a case, we can follow the preceding procedure repeatedly to select additional output cells to track.

Software Note...

When you define assumption, decision, or forecast cells, Crystal Ball automatically changes the background color of the cells to distinguish these cells from others on the spreadsheet. You can change the default colors Crystal Ball uses (or suppress this behavior entirely) by clicking Crystal Ball, Cell Prefs.

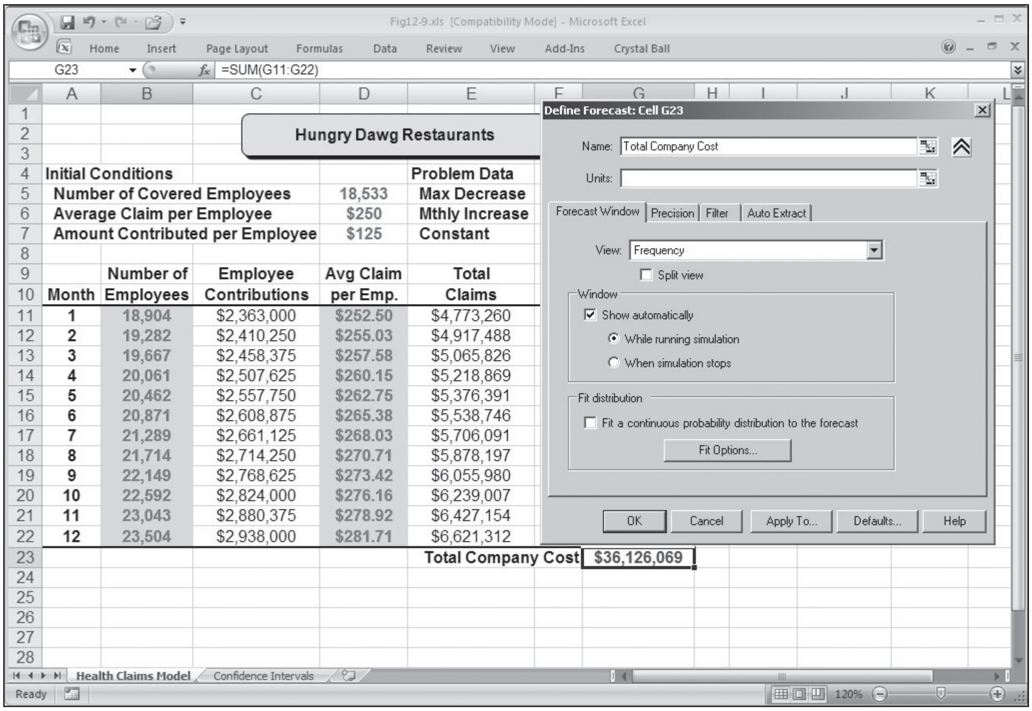


FIGURE 12.13

Defining the forecast cell

12.8.2 SELECTING THE NUMBER OF ITERATIONS

Figure 12.14 shows the Trials card in the Run Preferences dialog box for Crystal Ball. The "Maximum Number of Trials" option in this dialog box allows us to specify the number of iterations (or replications) to include in our simulation. Thus, to indicate that we want to perform 5000 replications of our model:

1. Click the Run Preferences button on the Crystal Ball tab.
2. Type 5000 in the box labeled "Number of trials to run."
3. Click OK.

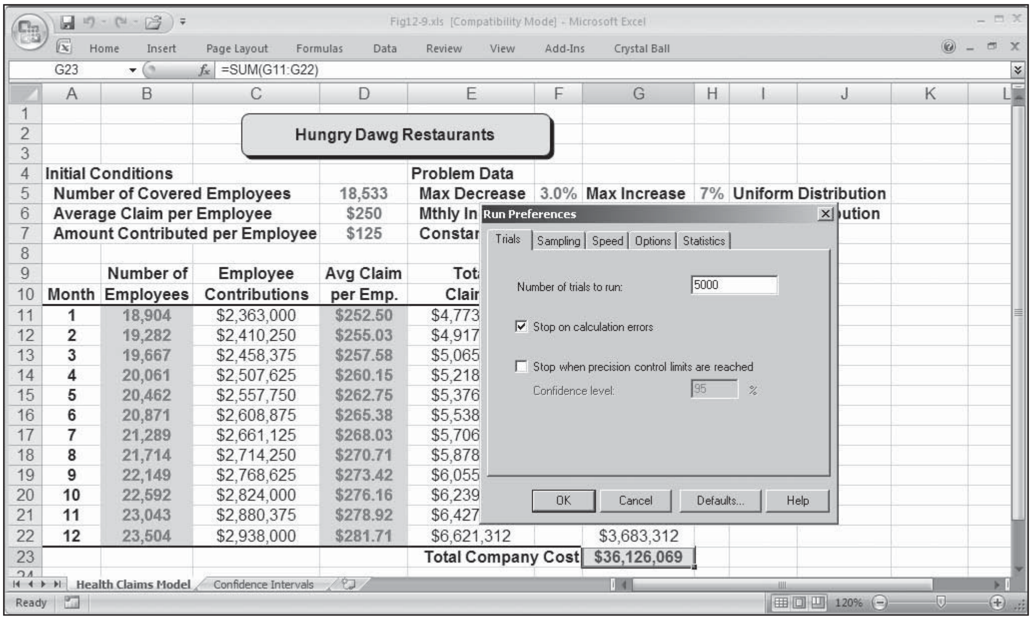
12.8.3 DETERMINING THE SAMPLE SIZE

You might wonder why we selected 5000 replications. Why not 500, or 8000? Unfortunately, there is no easy answer to this question. Remember that the goal in simulation is to estimate various characteristics about the bottom-line performance measure(s) under consideration. For example, we might want to estimate the mean value of the performance measure and the shape of its probability distribution. However, a different value of the bottom-line performance measure occurs each time we manually recalculate the model in Figure 12.8. Thus, there is an infinite number of possibilities—or an **infinite population**—of total company cost values associated with this model.

We cannot analyze all of these infinite possibilities. But by taking a large enough sample from this infinite population, we can make reasonably accurate estimates about the characteristics of the underlying infinite population of values. The larger the sample we

FIGURE 12.14

Trials options in the Run Preferences dialog box



take (that is, the more replications we do), the more accurate our final results will be. But, performing many replications takes time, so we must make a trade-off in terms of estimation accuracy versus convenience. Thus, there is no simple answer to the question of how many replications to perform, but, as a bare minimum you should always perform at least 1000 replications, and more as time permits or accuracy demands.

12.8.4 RUNNING THE SIMULATION

Having identified the output cells to track and the number of replications to perform, we now need to instruct Crystal Ball to perform the simulation by clicking the Start Simulation button on the Crystal Ball tab in Figure 12.3. Crystal Ball then begins to perform the specified number of replications. Depending on the number of iterations selected, the size of the model, and the speed of your computer, it could take anywhere from several seconds to several minutes for these computations to be carried out.

For our example problem, Crystal Ball performs 5000 recalculations of the model, keeping track of the value in cell G23 for each replication. By default, Crystal Ball updates the computer screen after each replication, which can be very time-consuming. If you select the “When simulation stops” option shown in Figure 12.13, Crystal Ball will not update the screen after each replication and will perform the replications more quickly.

12.9 Data Analysis

As mentioned earlier, the objective of performing simulation is to estimate various characteristics of the performance measure resulting from uncertainty in some or all of the input variables. After performing the replications, Crystal Ball summarizes the output data, as shown in Figure 12.15. (Click Crystal Ball, View Charts, Forecast Charts, Total Company Cost, Open.) The summary statistics and frequency chart are shown simultaneously by clicking View, Split View.

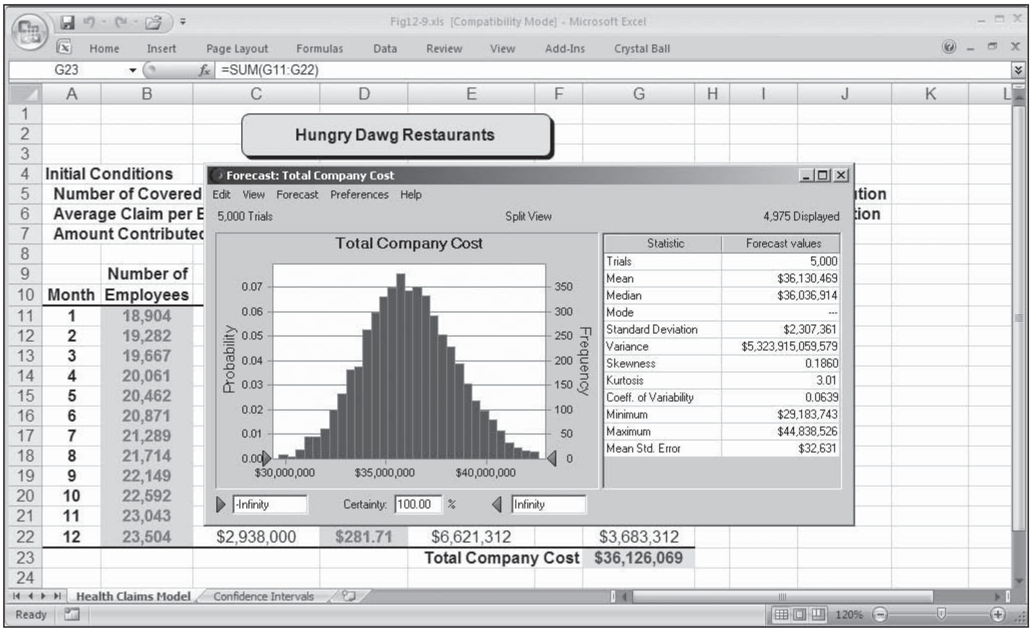


FIGURE 12.15

Summary statistics and histogram for simulation results

12.9.1 THE BEST CASE AND THE WORST CASE

As shown in Figure 12.15, the average (or mean) value for cell G23 is approximately \$36.1 million. (If you are working through this example on a computer, the graphs and statistics you generate might be somewhat different from the results shown here because you might be working with a different sample of 5000 observations.) However, decision makers usually want to know the best-case and worst-case scenarios to get an idea of the range of possible outcomes they might face. This information is available from the simulation results, as shown by the Minimum and Maximum values listed in Figure 12.15.

Although the average total cost value observed in the 5000 replications is \$36.1 million, in one case the total cost is approximately \$29.1 million (representing the best case) and in another case the total cost is approximately \$44.8 million (representing the worst case). These figures should give the decision maker a good idea about the range of possible cost values that might occur. Note that these values might be difficult to determine manually in a complex model with many uncertain independent variables.

12.9.2 THE DISTRIBUTION OF THE OUTPUT CELL

The best- and worst-case scenarios are the most extreme outcomes, and might not be likely to occur. To determine the likelihood of these outcomes requires that we know something about the shape of the distribution of our bottom-line performance measure. A histogram of the simulation data for cell G23 also appears in Figure 12.15. This graph provides a visual summary of the approximate shape of the probability distribution associated with the output cell tracked by Crystal Ball during the simulation.

As shown in Figure 12.15, the shape of the distribution associated with the total cost variable is fairly bell-shaped, with a maximum value around \$44 million and a minimum value around $29 million. Thus, we now have a clear idea of the shape of the distribution associated with our bottom-line performance measure—one of the goals in simulation.

If we tracked more than one output cell during the simulation, we could display histograms of the values occurring in these other cells in a similar fashion.

12.9.3 VIEWING THE CUMULATIVE DISTRIBUTION OF THE OUTPUT CELLS

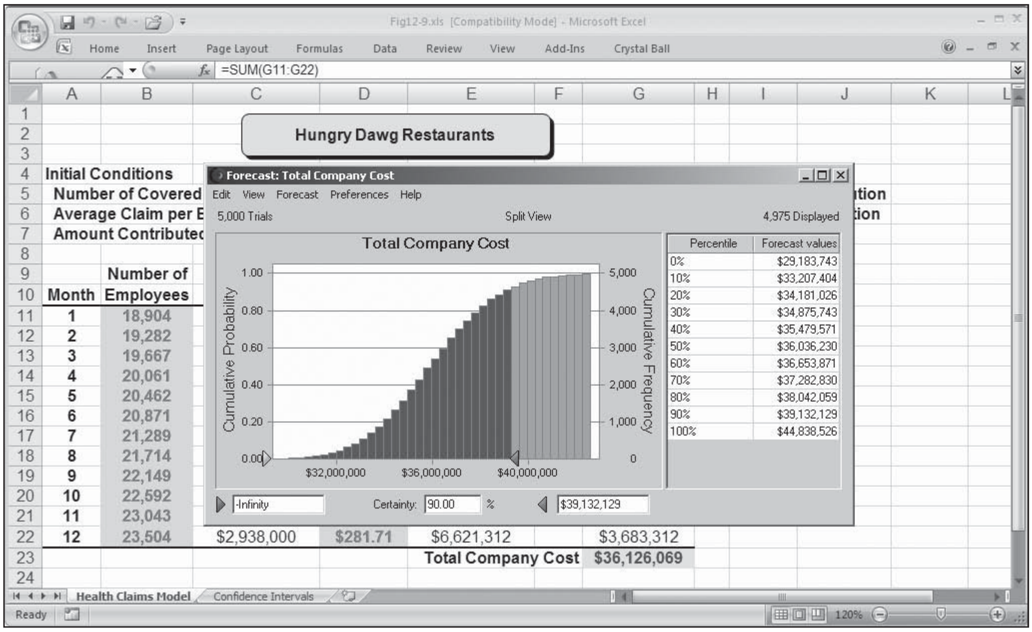
At times, we might want to view a graph of the cumulative probability distribution associated with one of the output cells tracked during a simulation. For example, suppose that the chief financial officer (CFO) for Hungry Dawg would prefer to accrue an excess amount of money to pay health claims rather than not accrue enough money. The CFO might want to know what amount the company should accrue so that there is only a 10% chance of coming up short of funds at the end of the year. So, how much money would you recommend be accrued?

Figure 12.16 shows a graph of the cumulative probability distribution of the values that occurred in cell G23 during the simulation. This graph could help us answer the preceding question. To change the graph shown in Figure 12.15 to the graph and statistics shown in Figure 12.16:

1. Click View.
2. De-select Frequency.
3. Select Cumulative Frequency.
4. De-select Statistics.
5. Select Percentiles.
6. Click and drag the right-hand arrowhead on the X-axis until the certainty level is 90%.

FIGURE 12.16

Cumulative frequency distribution and percentiles of possible total costs



This graph displays the probability of the selected output cell taking on a value smaller than each value on the X-axis. For example, this graph indicates that approximately a 25% chance exists of the output cell (Total Company Cost) assuming a value smaller than approximately $34.5 million. Similarly, this graph indicates that roughly a 75% chance exists of total costs being less than approximately $37.6 million (or a 25% chance of total costs exceeding approximately $37.6 million). Thus, from this graph, we would estimate that roughly a 10% chance exists of the company's costs exceeding approximately $39.1 million.

12.9.4 OBTAINING OTHER CUMULATIVE PROBABILITIES

We also can answer the CFO's question from information in the Percentiles window shown in Figure 12.16. This window reveals several percentile values for the output cell G23. For example, the 10th percentile of the values generated for the output cell is approximately $33.2 million—or 10% of the 5000 values generated for cell G23 are less than or equal to this value. Similarly, the 90th percentile of the distribution of values is approximately $39.1 million. Thus, based on these results, if the company accrues $39.1 million, we would expect that only a 10% chance exists of the actual company costs exceeding this amount.

The ability to perform this type of analysis demonstrates the power and value of simulation and Crystal Ball. For example, how could we have answered the CFO's question about how much money to accrue using best-case/worst-case analysis or what-if analysis? The fact is, we could not have answered the question with any degree of accuracy without using simulation.

12.10 Incorporating Graphs and Statistics into a Spreadsheet

At times, you will want to save some of the graphs or statistics created by Crystal Ball. You can do so by selecting the Create Report option found on the Crystal Ball tab. This option displays the dialog box shown in Figure 12.17. This dialog box provides options for saving different results to Excel. Crystal Ball automatically saves all the selected graphs and statistics in a worksheet in a new workbook.

Similarly, you might want to save the individual trial or replication data generated during the simulation to a separate file for further analysis. You can do so by selecting the Extract Data option from the Crystal Ball tab. This option displays the dialog box shown in Figure 12.18. Here again, Crystal Ball automatically saves all the selected data items in a worksheet in a new workbook.

12.11 The Uncertainty of Sampling

To this point, we have used simulation to generate 5000 observations on our bottom-line performance measure and then calculated various statistics to describe the characteristics and behavior of the performance measure. For example, Figure 12.15 indicates that the mean company cost value in our sample is $36,130,469, and Figure 12.16 shows that a 90% chance exists of this performance measure assuming a value less than $39,132,129. But what if we repeat this process and generate another 5000 observations? Would the

FIGURE 12.17

Saving charts and statistics from the simulation

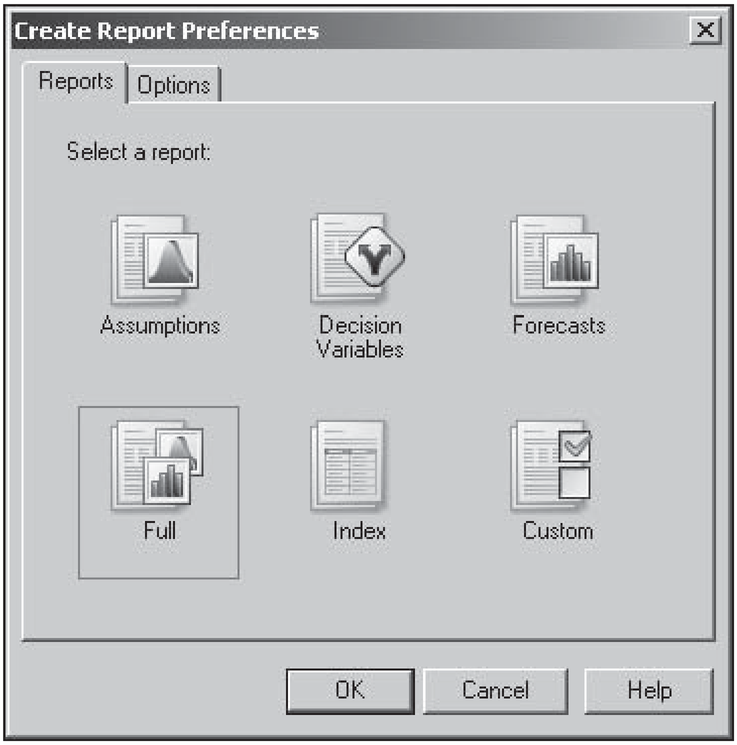
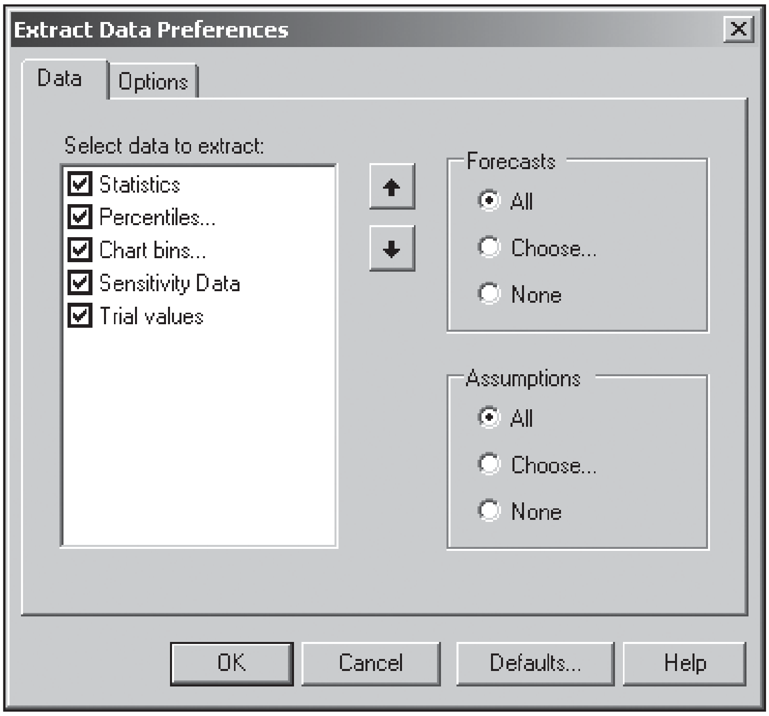


FIGURE 12.18

Saving data from the simulation



sample mean for the new 5000 observations also be exactly \$36,130,469? Or would exactly 90% of the observations in the new sample be less than \$39,132,129?

The answer to both these questions is “probably not.” The sample of 5000 observations used in our analysis was taken from a population of values that theoretically is infinite in size. That is, if we had enough time and our computer had enough memory, we could generate an infinite number of values for our bottom-line performance measure. Theoretically, we then could analyze this infinite population of values to determine its true mean value, its true standard deviation, and the true probability of the performance measure being less than \$39,132,129. Unfortunately, we do not have the time or computer resources to determine these true characteristics (or parameters) of the population. The best we can do is take a sample from this population and, based on our sample, make estimates about the true characteristics of the underlying population. Our estimates will differ depending on the sample we choose and the size of the sample.

So, the mean of the sample we take probably is not equal to the true mean we would observe if we could analyze the entire population of values for our performance measure. The sample mean we calculate is just an estimate of the true population mean. In our example problem, we estimated that a 90% chance exists for our output variable to assume a value less than \$39,132,129. However, this most likely is not equal to the true probability we would calculate if we could analyze the entire population. Thus, there is some element of uncertainty surrounding the statistical estimates resulting from simulation because we are using a sample to make inferences about the population. Fortunately, there are ways of measuring and describing the amount of uncertainty present in some of the estimates we make about the population under study. Typically this is done by constructing confidence intervals for the population parameters being estimated.

12.11.1 CONSTRUCTING A CONFIDENCE INTERVAL FOR THE TRUE POPULATION MEAN

Constructing a confidence interval for the true population mean is a simple process. If $\overline{y}$ and s represent, respectively, the mean and standard deviation of a sample of size n from any population, then assuming n is sufficiently large ($n \geq 30$), the Central Limit Theorem tells us that the lower and upper limits of a 95% confidence interval for the true mean of the population are represented by:

$$\text{95\% Lower Confidence Limit} = \overline{y} - 1.96 \times \frac{s}{\sqrt{n}}$$

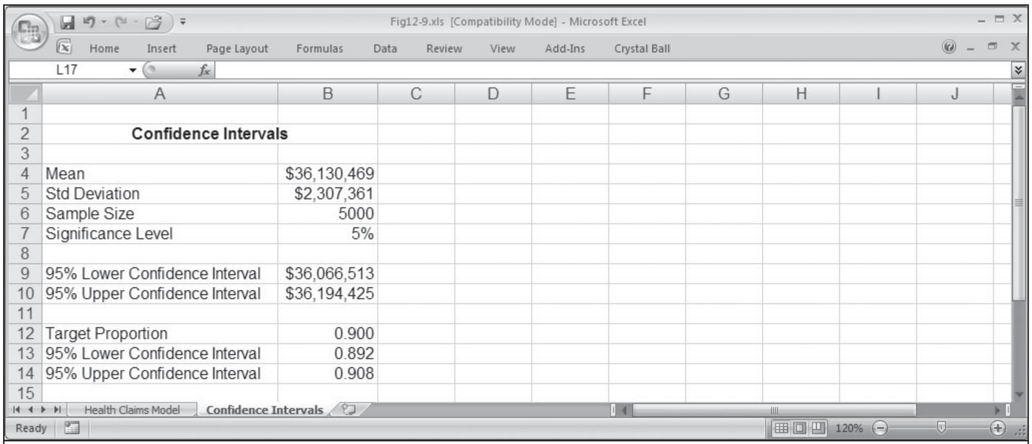
$$\text{95\% Upper Confidence Limit} = \overline{y} + 1.96 \times \frac{s}{\sqrt{n}}$$

Although we can be fairly certain that the sample mean we calculate from our sample data is not equal to the true population mean, we can be 95% confident that the true mean of the population falls somewhere between the lower and upper limits given previously. If we want a 90% or 99% confidence interval, we must change the value 1.96 in the previous equation to 1.645 or 2.575, respectively. The values 1.645, 1.96, and 2.575 represent the 95, 97.5, and 99.5 percentiles of the standard normal distribution. Any percentile of the standard normal distribution can be obtained easily using Excel’s NORMSINV( ) function.

For our example, the lower and upper limits of a 95% confidence interval for the true mean of the population of total company cost values can be calculated easily, as shown

FIGURE 12.19

Confidence intervals for the population mean and population proportion



	A	B
1		
2	**Confidence Intervals**	
3		
4	Mean	$36,130,469
5	Std Deviation	$2,307,361
6	Sample Size	5000
7	Significance Level	5%
8		
9	95% Lower Confidence Interval	$36,066,513
10	95% Upper Confidence Interval	$36,194,425
11		
12	Target Proportion	0.900
13	95% Lower Confidence Interval	0.892
14	95% Upper Confidence Interval	0.908
15		

Key Cell Formulas

Cell	Formula	Copied to
B9	=B4–NORMSINV(1–B7/2)*B5/SQRT(B6)	--
B10	=B4+NORMSINV(1–B7/2)*B5/SQRT(B6)	--
B13	=B12–NORMSINV(1–B7/2)*SQRT(B12*(1–B12)/B6)	--
B14	=B12+NORMSINV(1–B7/2)*SQRT(B12*(1–B12)/B6)	--

in cells B9 and B10 in Figure 12.19. (The sample mean and standard deviation shown in Figure 12.19 were obtained from Figure 12.15.) The formulas for these cells are:

Formula for cell B9: =B4−NORMSINV(1−B7/2)*B5/SQRT(B6)

Formula for cell B10: =B4+NORMSINV(1−B7/2)*B5/SQRT(B6)

Thus, we can be 95% confident that the true mean of the population of total company cost values falls somewhere in the interval from $36,066,513 to $36,194,425.

12.11.2 CONSTRUCTING A CONFIDENCE INTERVAL FOR A POPULATION PROPORTION

In our example, we estimated that 90% of the population of total company cost values fall below $39,132,129 based on our sample of 5000 observations. However, if we could evaluate the entire population of total cost values, we might find that only 80% of these values fall below $39,132,129. Or, we might find that 99% of the entire population falls below this mark. It would be helpful to determine how accurate the 90% value is. So, at times, we might want to construct a confidence interval for the true proportion of a population that falls below (or above) some value, for example Y_p.

To see how this is done, let $\overline{p}$ denote the proportion of observations in a sample of size n that falls below some value Y_p. Assuming that n is sufficiently large ($n \geq 30$), the Central Limit Theorem tells us that the lower and upper limits of a 95% confidence interval for the true proportion of the population falling below Y_p are represented by:

$$95\% \text{ Lower Confidence Limit} = \bar{p} - 1.96 \times \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$95\% \text{ Upper Confidence Limit} = \bar{p} + 1.96 \times \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

Although we can be fairly certain that the proportion of observations falling below Y_p in our sample is not equal to the true proportion of the population falling below Y_p, we can be 95% confident that the true proportion of the population falling below Y_p is contained within the lower and upper limits given previously. Again, if we want a 90% or 99% confidence interval, we must change the value 1.96 in the previous equations to 1.645 or 2.575, respectively.

Using these formulas, we can calculate the lower and upper limits of a 95% confidence interval for the true proportion of the population falling below $39,132,129. From our simulation results, we know that 90% of the observations in our sample are less than $39,132,129. Thus, our estimated value of $\bar{p}$ is 0.90. This value was entered into cell B12 in Figure 12.19. The upper and lower limits of a 95% confidence interval for the true proportion of the population falling below $39,132,129 are calculated in cells B13 and B14 of Figure 12.19 using the following formulas:

Formula for cell B13: =B12−NORMSINV(1−B7/2)*SQRT(B12*(1−B12)/B6)

Formula for cell B14: =B12+NORMSINV(1−B7/2)*SQRT(B12*(1−B12)/B6)

We can be 95% confident that the true proportion of the population of total cost values falling below $39,132,129 is between 0.892 and 0.908. Because this interval is fairly tight around the value 0.90, we can be reasonably certain that the $39.1 million figure quoted to the CFO has approximately a 10% chance of being exceeded.

12.11.3 SAMPLE SIZES AND CONFIDENCE INTERVAL WIDTHS

The formulas for the confidence intervals in the previous section depend directly on the number of replications (n) in the simulation. As the number of replications (n) increases, the width of the confidence interval decreases (or becomes more precise). Thus, for a given level of confidence (for example, 95%), the only way to make the upper and lower limits of the interval closer together (or tighter) is to make n larger—that is, use a larger sample size. A larger sample should provide more information about the population and, therefore, allow us to be more accurate in estimating the true parameters of the population.

12.12 The Benefits of Simulation

What have we accomplished through simulation? Are we really better off than if we had just used the results of the original model proposed in Figure 12.2? The estimated value for the expected total cost to the company in Figure 12.2 is comparable to that obtained through simulation (although this might not always be the case). But remember that the goal of modeling is to give us greater insight into a problem to help us make more informed decisions.

The results of our simulation analysis do give us greater insight into the example problem. In particular, we now have some idea of the best- and worst-case total cost outcomes for the company. We have a better idea of the distribution and variability of the possible outcomes, and a more precise idea about where the mean of the distribution is located. We also now have a way of determining how likely it is for the actual outcome to fall above or below some value. Thus, in addition to our greater insight and understanding of the problem, we also have solid empirical evidence (the facts and figures) to support our recommendations.

Applying Simulation in Personal Financial Planning

A recent article in the *Wall Street Journal* highlighted the importance of simulation in evaluating the risk in personal financial investments. "Many people are taking a lot more risk than they realize. They are walking around with a false sense of security," said Christopher Cordaro, an investment adviser at Bugen Stuart Korn & Cordaro of Chatham, N.J. Using one online retirement calculator, after entering information about your finances and assumptions about investment gains, the screen shows a green or red light indicating whether you have saved enough to retire. What is doesn't tell you is whether there is a 95% chance or just a 60% chance that your plan will succeed. "How certain are you about the green light—is it just about to turn yellow and you're just sneaking by?" Mr. Cordaro asked.

Although any planning is better than nothing, traditional planning models spit out answers that create "the illusion that the number is a certainty, when it isn't," said Ross Levin, a Minneapolis investment adviser who uses simulation. Mutual-fund firm T. Rowe Price applied simulation to its recently launched Retirement Income Manager, a personalized consultation service that helps retirees understand how much income they can afford without outliving their assets or depleting funds they want to leave to heirs. A number of independent financial planning companies are now also using simulation software (see the Advisor Tools section of http://www.financeware.com).

In the face of widely divergent possibilities, the number crunching involved in simulation can bring some peace of mind. Steven Haas of Randolph, N.J., was not sure whether he had saved enough to accept an early retirement package from AT&T at age 53. After his financial advisor used simulation to determine he had a 95% probability that his money would last until he reached age 110, Mr. Haas took the package and retired. Mr. Haas reported that by using simulation, "I found that even under significantly negative scenarios we could live comfortably. It relieved a lot of anxiety."

Adapted from "Monte Carlo Financial Simulator May Be A Good Bet For Planning," *Wall Street Journal*, Section C1, April 27, 2000 by Karen Hube.

12.13 Additional Uses of Simulation

Earlier, we indicated that simulation is a technique that *describes* the behavior or characteristics of a bottom-line performance measure. The next several examples show how describing the behavior of a performance measure gives a manager a useful tool in determining the optimal value for one or more controllable parameters in a decision problem. These examples reinforce the mechanics of using simulation, and also demonstrate some additional capabilities of Crystal Ball.

12.14 A Reservation Management Example

Businesses that allow customers to make reservations for services (such as airlines, hotels, and car rental companies) know that some percentage of the reservations made will not be used for one reason or another, leaving these companies with a difficult decision problem. If they accept reservations for only the number of customers that actually can be served, then a portion of the company's assets will be underutilized when some customers with reservations fail to arrive. On the other hand, if they overbook (or accept more reservations than can be handled), then at times, more customers will arrive than can be served. This typically results in additional financial costs to the company and often generates ill-will among those customers who cannot be served. The following example illustrates how simulation might be used to help a company determine the optimal number of reservations to accept.

Marty Ford is an operations analyst for Piedmont Commuter Airlines (PCA). Recently, Marty was asked to make a recommendation on how many reservations PCA should book on Flight 343—a flight from a small regional airport in New England to a major hub at Boston's Logan airport. The plane used on Flight 343 is a small twin-engine turbo-prop with 19 passenger seats available. PCA sells nonrefundable tickets for Flight 343 for $150 per seat.

Industry statistics show that for every ticket sold for a commuter flight, a 0.10 probability exists that the ticket holder will not be on the flight. Thus, if PCA sells 19 tickets for this flight, there is a fairly good chance that one or more seats on the plane will be empty. Of course, empty seats represent lost potential revenue to the company. On the other hand, if PCA overbooks this flight and more than 19 passengers show up, some of them will have to be bumped to a later flight.

To compensate for the inconvenience of being bumped, PCA gives these passengers vouchers for a free meal, a free flight at a later date, and sometimes also pays for them to stay overnight in a hotel near the airport. PCA pays an average of $325 (including the cost of lost goodwill) for each passenger that gets bumped. Marty wants to determine if PCA can increase profits by overbooking this flight and, if so, how many reservations should be accepted to produce the maximum average profit. To assist in the analysis, Marty analyzed market research data for this flight that reveals the following probability distribution of demand for this flight:

Seats Demanded	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
Probability	0.03	0.05	0.07	0.09	0.11	0.15	0.18	0.14	0.08	0.05	0.03	0.02

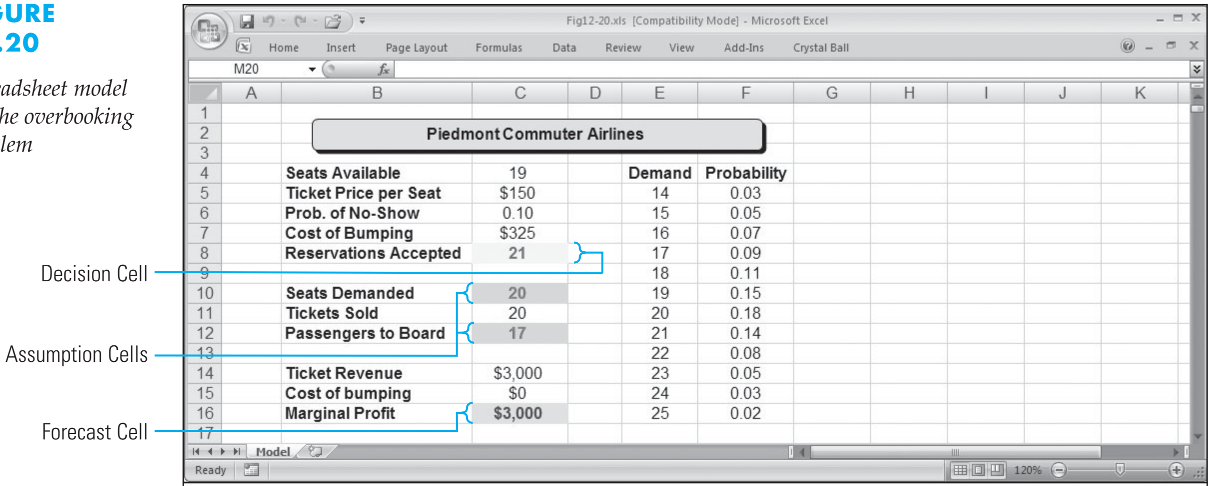
12.14.1 IMPLEMENTING THE MODEL

A spreadsheet model for this problem is shown in Figure 12.20 (and in the file Fig12-20.xls on your data disk). The spreadsheet begins by listing the relevant data from the problem, including the number of seats available on the plane, the price PCA charges for each seat, the probability of a no-show (a ticketed passenger not arriving in time for the flight), the cost of bumping passengers, and the number of reservations that will be accepted.

The distribution of demand for seats on the flight is summarized in columns E and F. With this data, cell C10 is defined as an assumption cell representing the random number of seats demanded for a particular flight using Crystal Ball's Custom distribution with the settings shown in Figure 21.21.

FIGURE 12.20

Spreadsheet model for the overbooking problem



	A	B	C	D	E	F
1						
2		Piedmont Commuter Airlines				
3						
4		Seats Available	19		Demand	Probability
5		Ticket Price per Seat	$150		14	0.03
6		Prob. of No-Show	0.10		15	0.05
7		Cost of Bumping	$325		16	0.07
8		Reservations Accepted	21		17	0.09
9					18	0.11
10		Seats Demanded	20		19	0.15
11		Tickets Sold	20		20	0.18
12		Passengers to Board	17		21	0.14
13					22	0.08
14		Ticket Revenue	$3,000		23	0.05
15		Cost of bumping	$0		24	0.03
16		Marginal Profit	$3,000		25	0.02

Key Cell Formulas & CB Settings

Cell	Formula/Setting	Copied to
C10	CB Distribution: Custom Linked to: =E5:F16	--
C11	=MIN(C10,C8)	--
C12	CB Distribution: Binomial Probability: =1-C6 Trials: =C11	--
C14	=C11*C5	--
C15	=MAX(C12–C4,0)*C7	--
C16	=C14-C15	--

The number of tickets actually sold for a flight cannot exceed the number of reservations the company is willing to accept. Thus, the number of tickets sold is calculated in cell C11 as follows:

Formula for cell C11: =MIN(C10,C8)

Because each ticketed passenger has a 0.10 probability of being a no-show, a 0.9 probability exists that each ticketed passenger will arrive in time to board the flight. So the number of passengers present to board the flight can be modeled as a Binomial random variable. The Binomial RNG returns the number of "successes" in a sample of size n where each trial has a probability p of "success." In this case, the sample size is the number of tickets sold (cell C11) and the probability of "success" is 0.9 – the probability that each ticketed passenger will arrive in time to board the flight (or 1-C6). Thus, Crystal Ball's Binomial distribution is used to define cell C12 as an assumption cell that models the number of ticketed passengers that actually arrive for a flight as shown in Figure 12.22.

Cell C14 represents the ticket revenue that PCA earns based on the number of tickets it sells for each flight. The formula for this cell is:

Formula for cell C14: =C11*C5

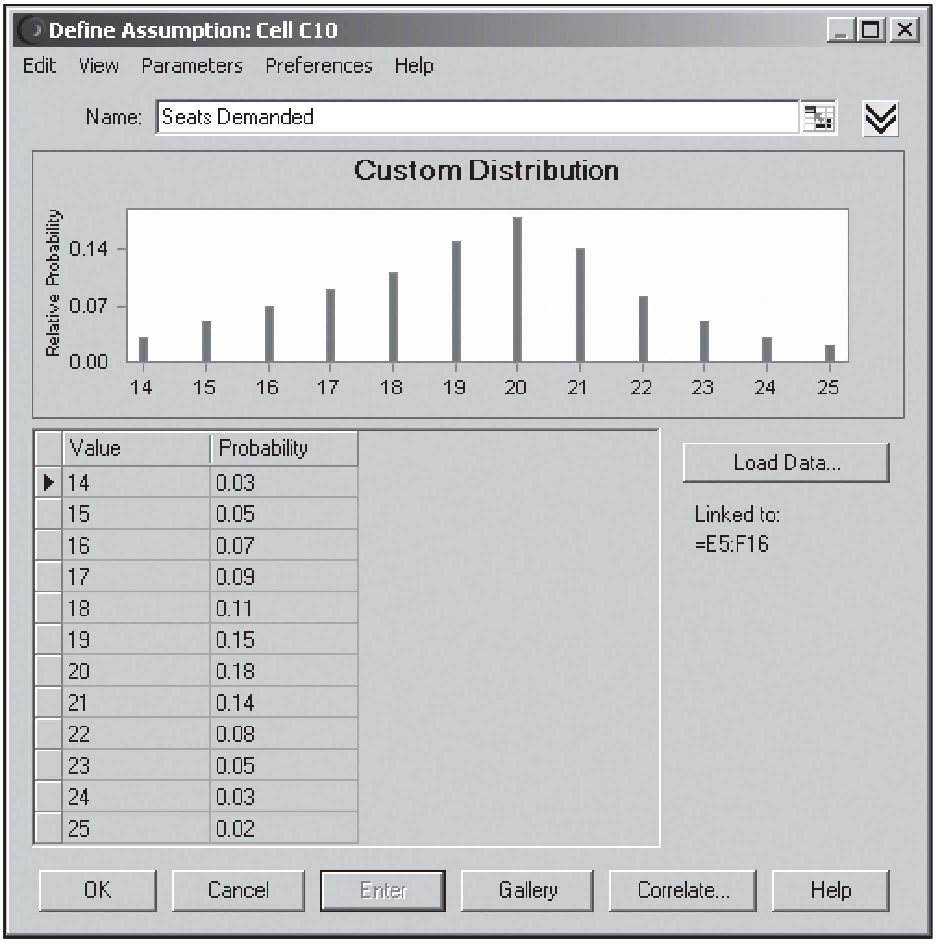


FIGURE 12.21

Defining the assumptions for cell C10

Cell C15 computes the costs PCA incurs when passengers must be bumped (i.e., when the number of passengers wanting to board exceeds the number of available seats).

Formula for cell C15: =MAX(C12−C4,0)*C7

Finally, cell C16 computes the marginal profit PCA earns on each flight.

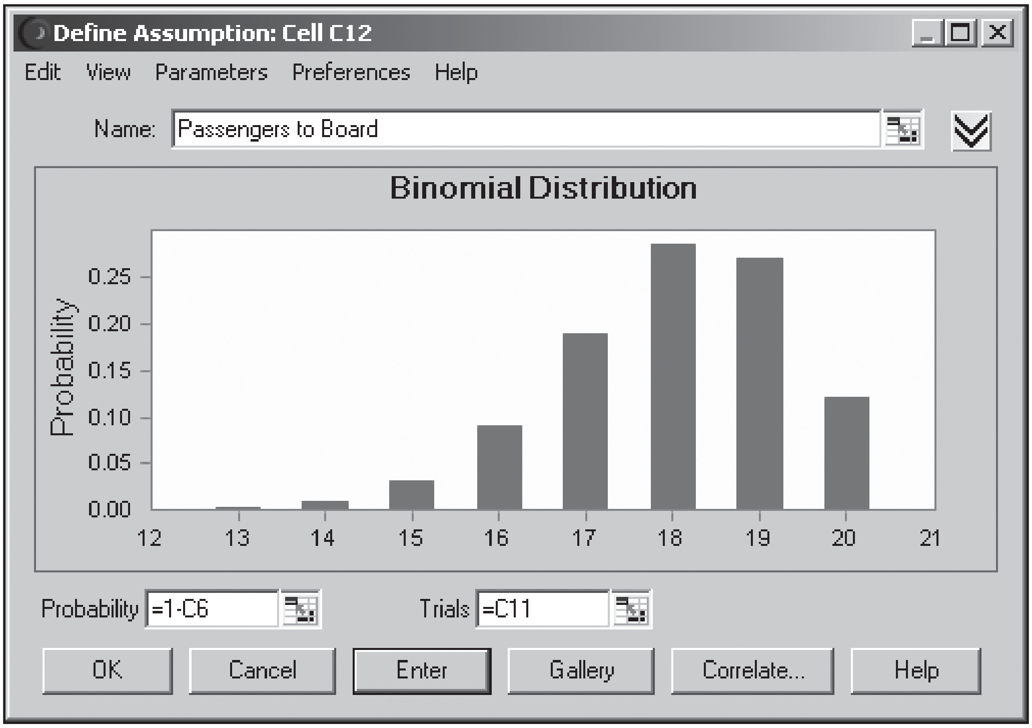
Formula for cell C16: =C14-C15

12.14.2 USING THE DECISION TABLE TOOL

Marty wants to determine the number of reservations to accept that, on average, will result in the highest marginal profit. To do so, he needs to simulate what would happen to the marginal profit cell (C16) if 19, 20, 21, . . . , 25 reservations were accepted. Fortunately, Crystal Ball includes a tool called Decision Table that simplifies this type of

FIGURE 12.22

Defining the assumptions for cell C12



analysis. To use Decision Table, first indicate the marginal profit cell (C16) to be a forecast cell in the usual way:

1. Select C16.
2. Click the Define Forecast button on the Crystal Ball tab.

Next, define cell C8 (number of reservations accepted) to be a decision variable that may assume integer values from 19 to 25. To do this:

1. Select cell C8.
2. Click Crystal Ball, Define Decision.
3. Fill in the values shown in Figure 12.23.
4. Click OK.

When comparing different values for one or more decision variables (*i.e.*, different solutions), it is best if each possible solution is evaluated using exactly the same series of random numbers. In this way, any difference in the performance of two possible solutions can be attributed to the decision variables' values and not the result of a more favorable set of random numbers for one of the solutions. You can ensure that Crystal Ball uses the same set of random numbers for each simulation by selecting the "Use Same Sequence of Random Numbers" option and supplying an "Initial Seed Value" under the Sampling option on Crystal Ball's Run Preferences dialog box shown in Figure 12.24.

The Sampling Method options shown in Figure 12.21 also affect the accuracy of the results of a simulation run. Using the Monte Carlo option, Crystal Ball is free to select any value for a particular RNG during each replication of the model. For example,

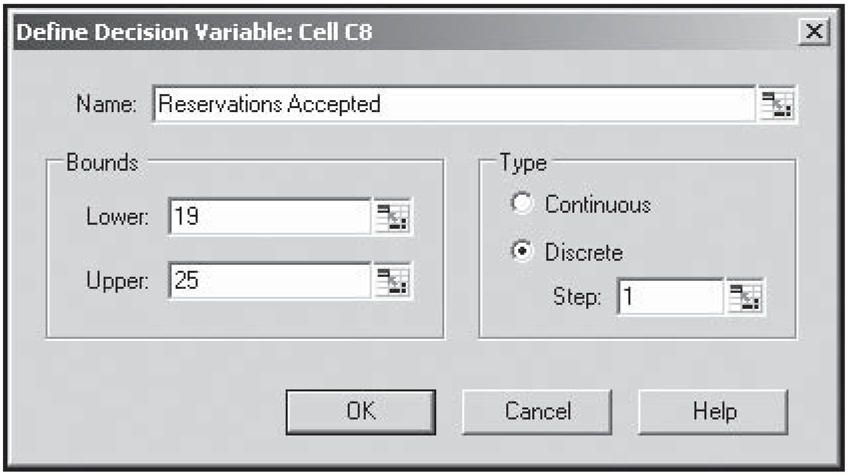


FIGURE 12.23

Defining reservations accepted as a decision variable

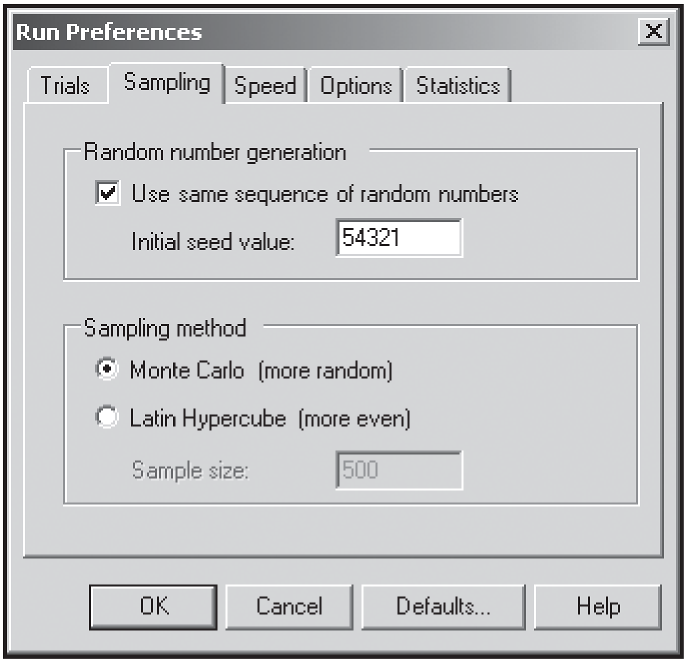


FIGURE 12.24

Crystal Ball's Run Preferences dialog

Crystal Ball might repeatedly generate several very extreme (and rare!) values from the upper tail of a normal distribution. The Latin Hypercube option guards against this by ensuring that a fair representation of values is generated from the entire distribution for each RNG. As you might imagine, the Latin Hypercube sampling option requires a bit more work during each replication of the model, but it tends to generate more accurate simulation results in a fewer number of trials.

After setting a random number seed as shown in Figure 12.24, we are ready to use the Decision Table tool. To invoke the Decision Table tool:

1. Click Crystal Ball, Tools.
2. Click Decision Table.

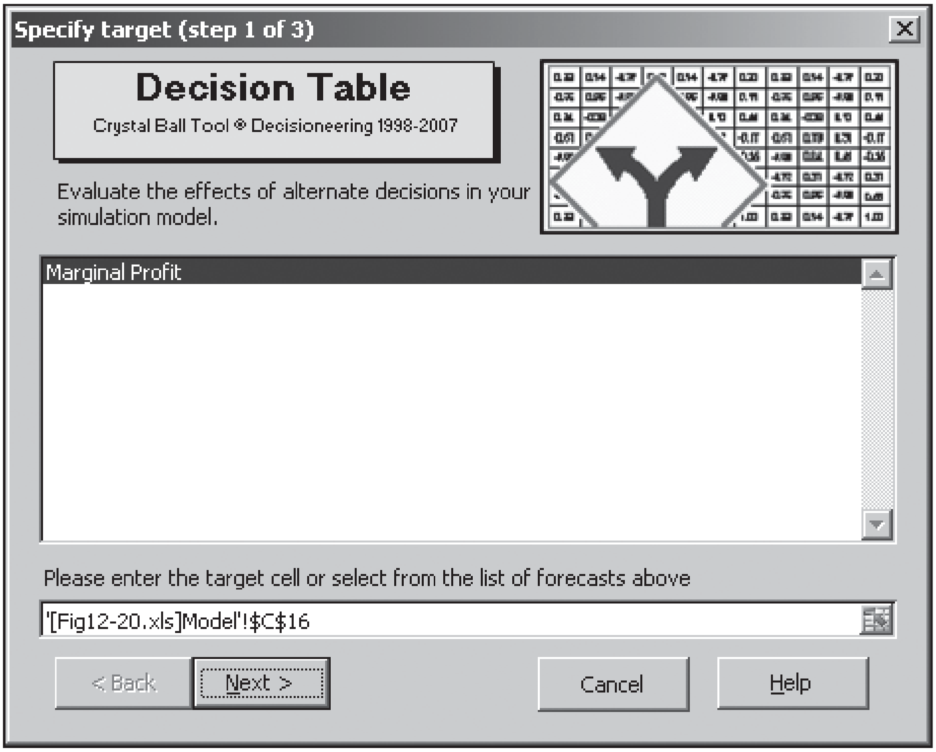


FIGURE 12.25

Specifying the target cell for the Decision Table tool

The dialog box shown in Figure 12.25 appears prompting us to select the forecast (or target) cell we want to track. Here, we select the "Marginal Profit" option and click the Next button.

In the next dialog box, shown in Figure 12.26, we select the decision variable we want to evaluate. In this case there is only one variable (Reservations Accepted). However, the Decision Table tool allows you to select up to two decision variables to analyze simultaneously.

The next dialog box, shown in Figure 12.27, allows us to specify the number of values to test for each decision variable and the number of iterations (trials) to use for each resulting simulation run. In our case, there are seven possible discrete values for Reservations Accepted (i.e., 19, 20, 21, . . . , 25) and we want to test them all. However, for continuous decision variables that can assume any value within some interval, it is impossible to test all the possible values, and we must tell the Decision Table tool the number of possible values to try.

When you click the Start button in Figure 12.27, the Decision Table tool runs a separate simulation for each of the seven possible values we indicated for the decision variable. When it is finished, the Decision Table tool creates a new workbook like the one shown in Figure 12.28 summarizing the average value of the forecast cell (Marginal Profit) for each of the seven simulations. These values indicate that the average marginal profit reaches a maximum value of $2,834 when PCA accepts 22 reservations. We can easily create a bar chart from this data to graphically represent the differences in the average profits under each scenario.

You can also select one or more of the average profit values on row 2 in Figure 12.28 and display a Trend Chart, Overlay Chart, or Forecast Chart for the selected values.

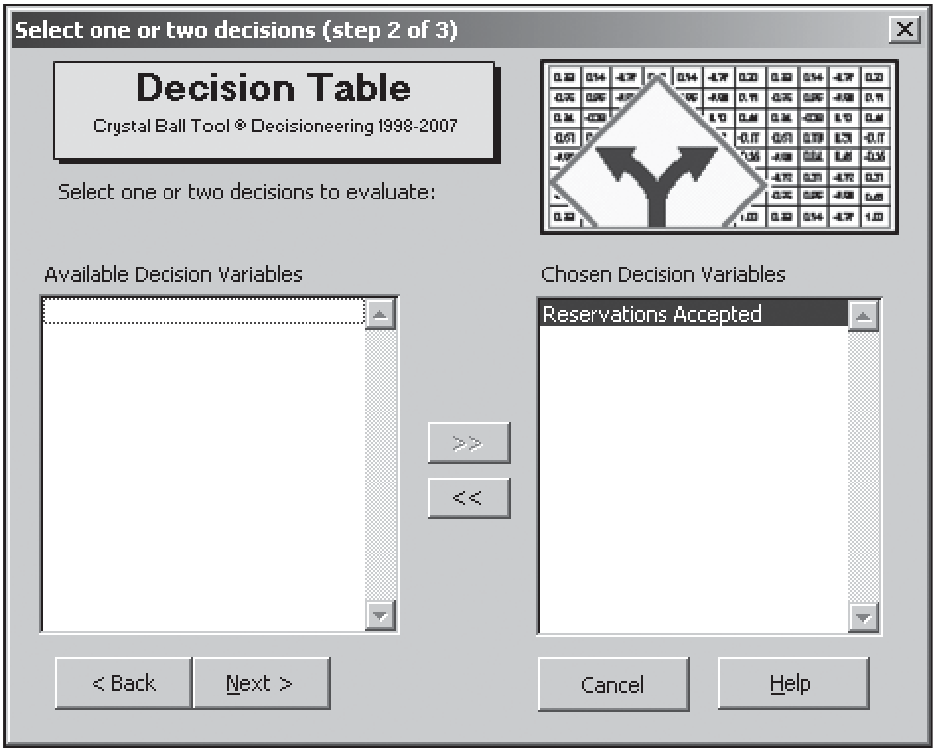


FIGURE 12.26

Choosing the decision variable with the Decision Table tool

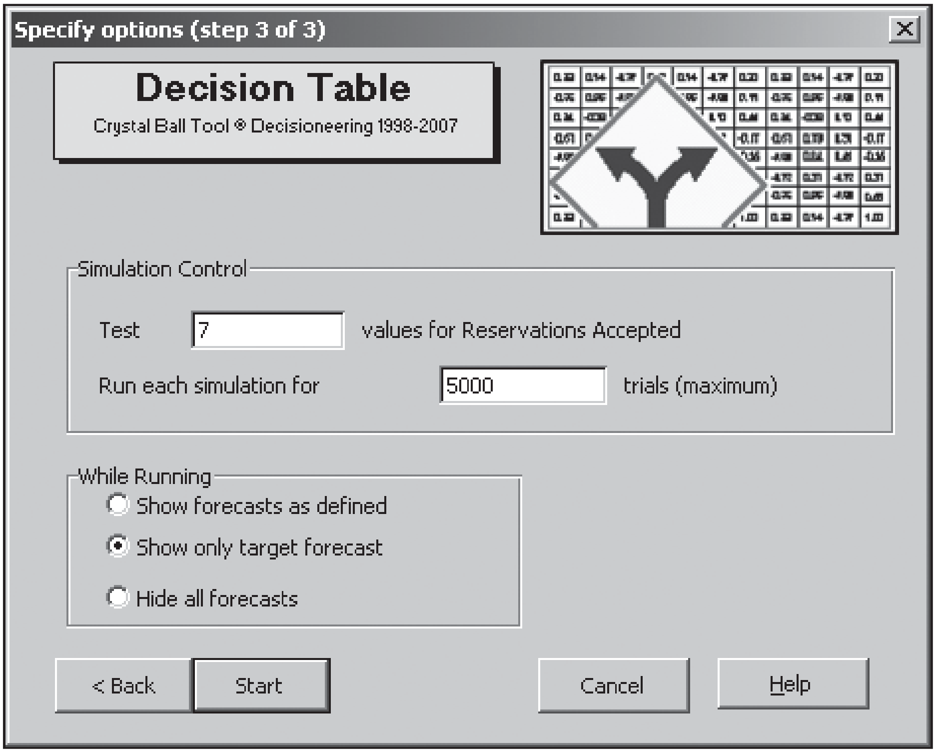


FIGURE 12.27

Setting options with the Decision Table tool

FIGURE 12.28

Results of the Decision Table tool

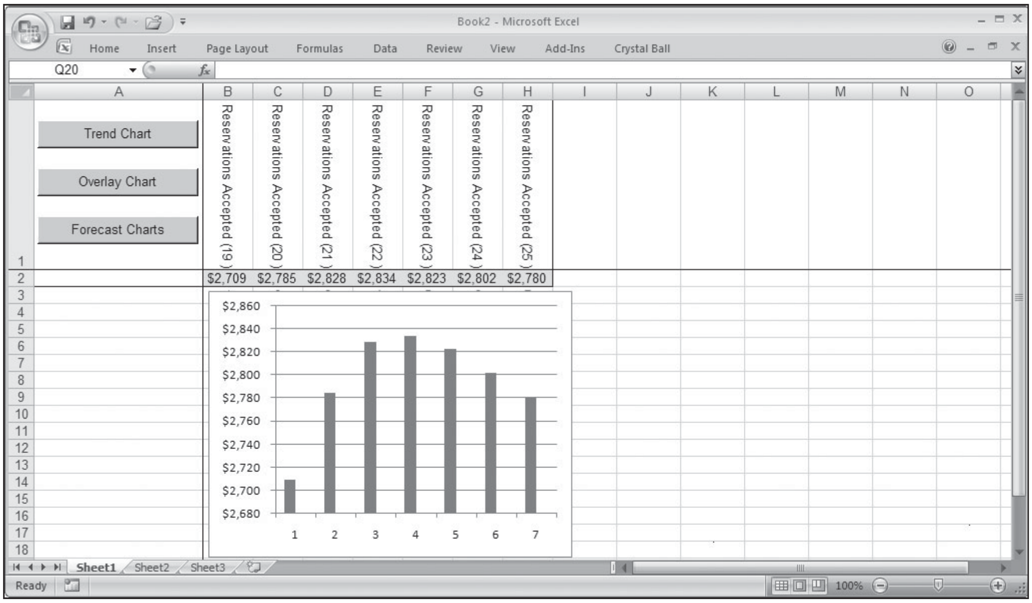


FIGURE 12.29

Overlay chart comparing outcome distributions for 21 and 25 reservations accepted

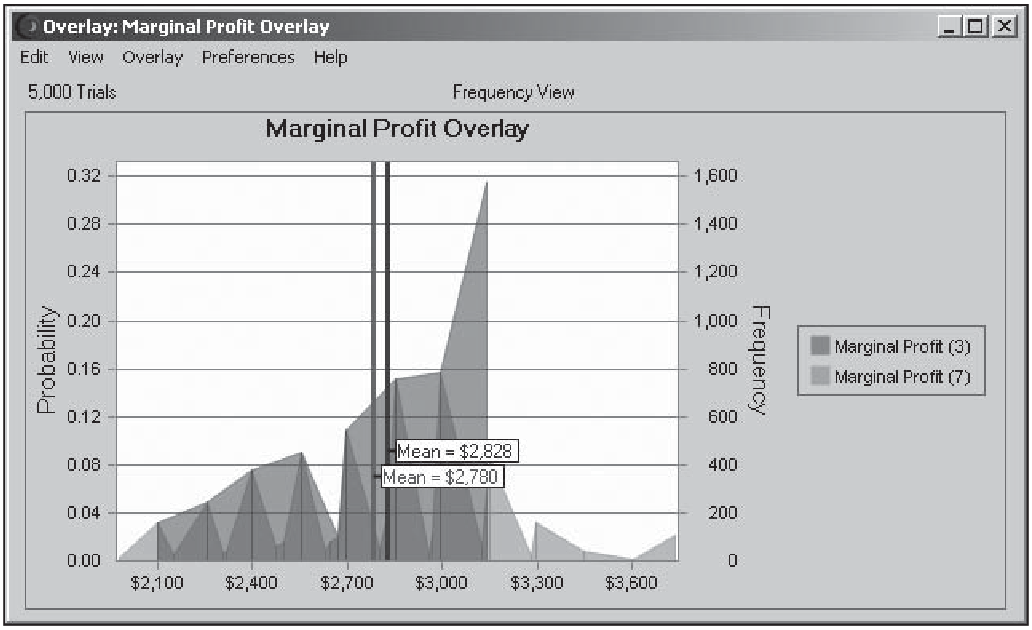


Figure 12.29 shows an Overlay Chart comparing the distributions of values generated in simulation 3 (where up to 21 reservations were accepted) and simulation 7 (where up to 25 reservations were accepted). This type of chart is often helpful in comparing the riskiness of different decision alternatives.

12.15 An Inventory Control Example

According to the *Wall Street Journal*, U.S. businesses recently had a combined inventory worth $884.77 billion. Because so much money is tied up in inventories, businesses face many important decisions regarding the management of these assets. Frequently asked questions regarding inventory include:

- What's the best level of inventory for a business to maintain?
- When should goods be reordered (or manufactured)?
- How much safety stock should be held in inventory?

The study of inventory control principles is split into two distinct areas—one assumes that demand is known (or deterministic), and the other assumes that demand is random (or stochastic). If demand is known, various formulas can be derived that provide answers to the previous questions (an example of one such formula is given in the discussion of the EOQ model in Chapter 8.) However, when demand for a product is uncertain or random, answers to the previous questions cannot be expressed in terms of a simple formula. In these situations, the technique of simulation proves to be a useful tool, as illustrated in the following example.

Laura Tanner is the owner of Millennium Computer Corporation (MCC), a retail computer store in Austin, Texas. Competition in retail computer sales is fierce—both in terms of price and service. Laura is concerned about the number of stockouts occurring on a popular type of computer monitor. Stockouts are very costly to the business because when customers cannot buy this item at MCC, they simply buy it from a competing store and MCC loses the sale (there are no back orders). Laura measures the effects of stockouts on her business in terms of service level, or the percentage of total demand that can be satisfied from inventory.

Laura has been following the policy of ordering 50 monitors whenever her daily ending inventory position (defined as ending inventory on hand plus outstanding orders) falls below her reorder point of 28 units. Laura places the order at the beginning of the next day. Orders are delivered at the beginning of the day and, therefore, can be used to satisfy demand on that day. For example, if the ending inventory position on day 2 is less than 28, Laura places the order at the beginning of day 3. If the actual time between order and delivery, or lead time, turns out to be four days, then the order arrives at the start of day 7.

The current level of on-hand inventory is 50 units and no orders are pending. MCC sells an average of six monitors per day. However, the actual number sold on any given day can vary. By reviewing her sales records for the past several months, Laura determined that the actual daily demand for this monitor is a random variable that can be described by the following probability distribution:

Units Demanded	0	1	2	3	4	5	6	7	8	9	10
Probability	0.01	0.02	0.04	0.06	0.09	0.14	0.18	0.22	0.16	0.06	0.02

The manufacturer of this computer monitor is located in California. Although it takes an average of four days for MCC to receive an order from this company, Laura has determined that the lead time of a shipment of monitors is also a random variable that can be described by the following probability distribution:

Lead Time (days)	3	4	5
Probability	0.2	0.6	0.2

One way to guard against stockouts and improve the service level is to increase the reorder point for the item so that more inventory is on hand to meet the demand occurring during the lead time. However, there are holding costs associated with keeping more inventory on hand. Laura wants to evaluate her current ordering policy for this item and determine if it might be possible to improve the service level without increasing the average amount of inventory on hand.

12.15.1 IMPLEMENTING THE MODEL

To solve this problem, we need to build a model to represent the inventory of computer monitors during an average month of 30 days. This model must account for the random daily demands that can occur and the random lead times encountered when orders are placed. To facilitate this, we first entered the data for these variables as shown in Figure 12.30 (and in the file Fig12-30.xls on your data disk). We will use this data in conjunction with Crystal Ball's Custom distribution to create assumption cells representing the random order lead times and daily demand values needed for this problem.

Figure 12.31 shows a model representing 30 days of inventory activity. Notice that cells M5 and M6 have been reserved to represent, respectively, the reorder point and order quantity for the model.

The inventory on hand at the beginning of each day is calculated in column B in Figure 12.31. The beginning inventory for each day is simply the ending inventory from the previous day. The formulas in column B are:

Formula for cell B6: =50

Formula for cell B7: =F6

(Copy to B8 through B35.)

Column C represents the number of units scheduled to be received each day. We will discuss the formulas in column C after we discuss columns H, I, and J, which relate to ordering and order lead times.

FIGURE 12.30

RNG data for MCC's inventory problem

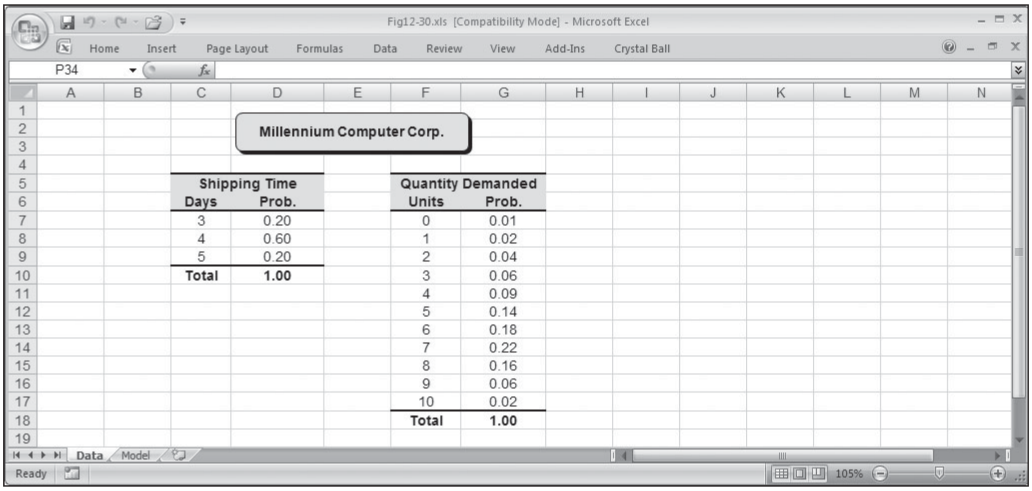
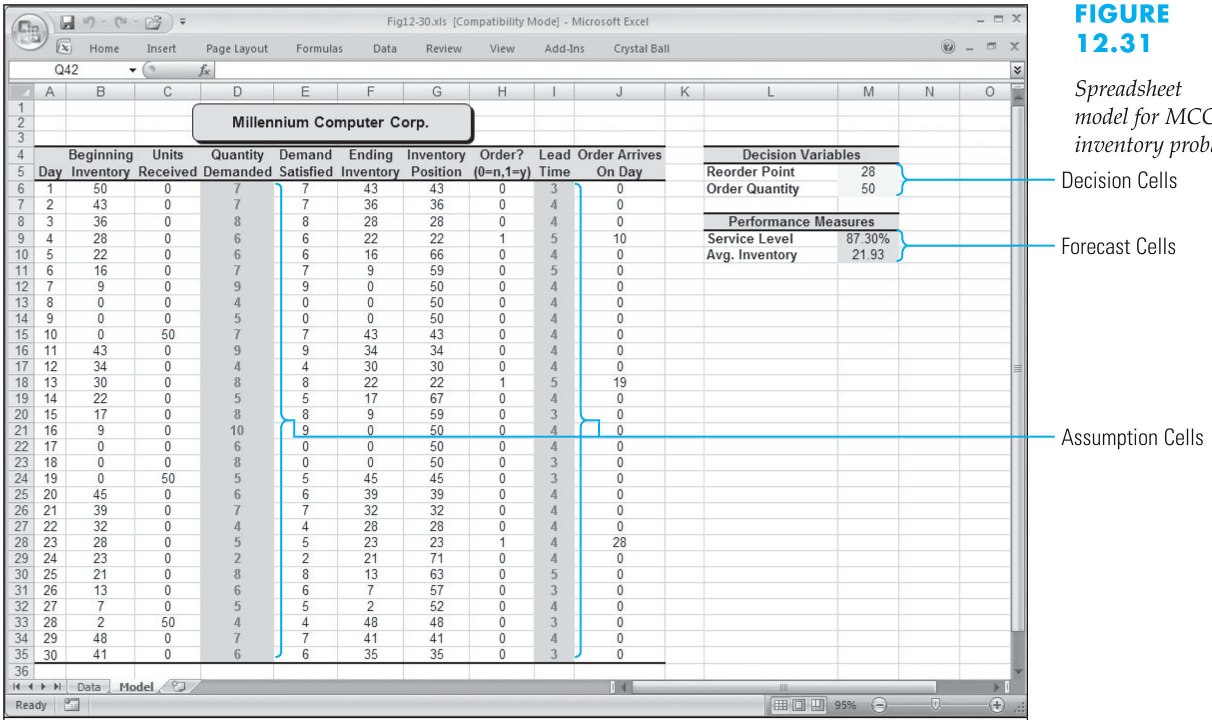


FIGURE 12.31

Spreadsheet model for MCC's inventory problem



Day	Beginning Inventory	Units Received	Quantity Demanded	Demand Satisfied	Ending Inventory	Inventory Position	Order? (0=n,1=y)	Lead Time	Order Arrives On Day
1	50	0	7	7	43	43	0	3	0
2	43	0	7	7	36	36	0	4	0
3	36	0	8	8	28	28	0	4	0
4	28	0	6	6	22	22	1	5	10
5	22	0	6	6	16	66	0	4	0
6	16	0	7	7	9	59	0	5	0
7	9	0	9	9	0	50	0	4	0
8	0	0	4	0	0	50	0	4	0
9	0	0	5	0	0	50	0	4	0
10	0	50	7	7	43	43	0	4	0
11	43	0	9	9	34	34	0	4	0
12	34	0	4	4	30	30	0	4	0
13	30	0	8	8	22	22	1	5	19
14	22	0	5	5	17	67	0	4	0
15	17	0	8	8	9	59	0	3	0
16	9	0	10	9	0	50	0	4	0
17	0	0	6	0	0	50	0	4	0
18	0	0	8	0	0	50	0	3	0
19	0	50	5	5	45	45	0	3	0
20	45	0	6	6	39	39	0	4	0
21	39	0	7	7	32	32	0	4	0
22	32	0	4	4	28	28	0	4	0
23	28	0	5	5	23	23	1	4	28
24	23	0	2	2	21	71	0	4	0
25	21	0	8	8	13	63	0	5	0
26	13	0	6	6	7	57	0	3	0
27	7	0	5	5	2	52	0	4	0
28	2	50	4	4	48	48	0	3	0
29	48	0	7	7	41	41	0	4	0
30	41	0	6	6	35	35	0	3	0

Key Cell Formulas & CB Settings

Cell	Formula/Setting	Copied to
B6	=50	--
B7	=F6	B8:B35
C7	=COUNTIF($J$6:J6,A7)*$M$6	C8:C35
D6	CB Distribution: Custom Linked to: =Data!$F$7:$G$17	D7:D35 (Using CB copy and paste)
E6	=MIN(D6,B6+C6)	E7:E35
F6	=B6+C6-E6	F7:F35
G6	=F6	--
G7	=G6–E7+IF(H6=1,$M$6,0)	G8:G35
H6	=IF(G6<$M$5,1,0)	H7:H35
I6	CB Distribution: Custom Linked to: =Data!$C$7:$D$9	I7:I35 (Using CB copy and paste)
J6	=IF(H6=0,0,A6+1+I6)	J7:J35
M9	=SUM(E6:E35)/SUM(D6:D35)	--
M10	=AVERAGE(B6:B35)	--

In column D, we use Crystal Ball's Custom distribution to define our assumptions about the random behavior of daily demand for monitors. Figure 12.32 shows the Define Assumption dialog box for cell D6. This definition was also copied to cells D7 through D35 (using Crystal Ball's copy data and paste data commands).

Because it is possible for demand to exceed the available supply, column E indicates how much of the daily demand can be met. If the beginning inventory (in column B)

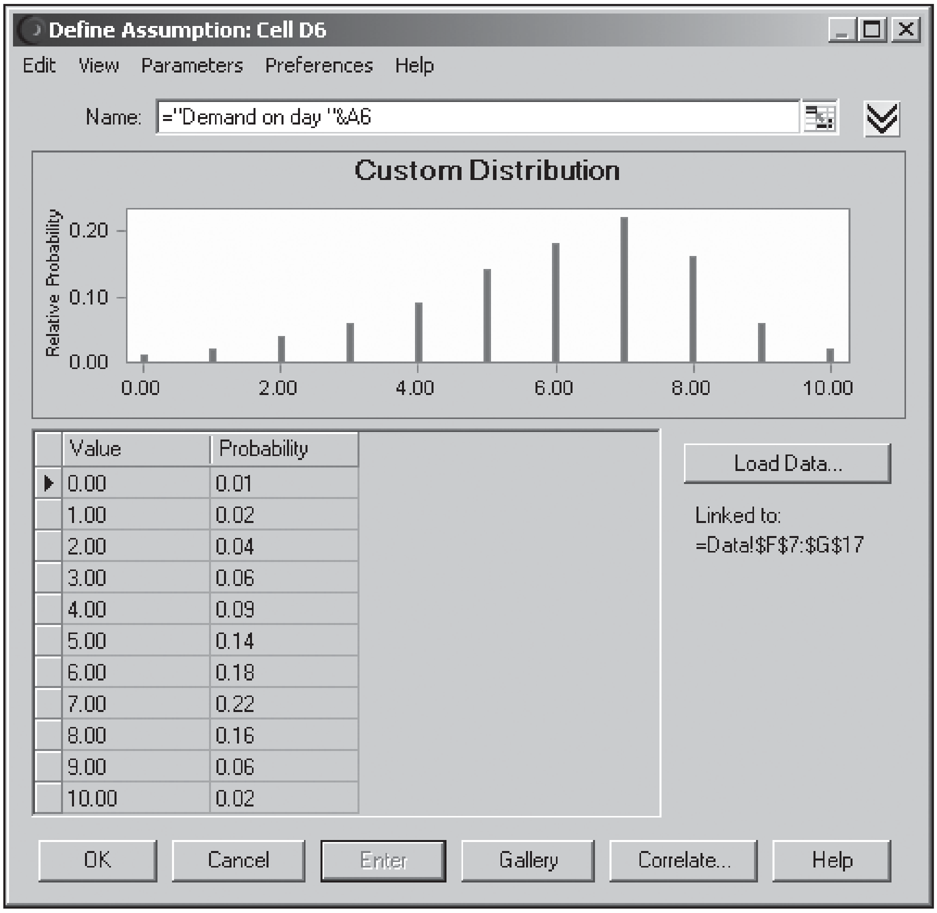


FIGURE 12.32

Defining the assumptions for cell D6

plus the ordered units received (in column C) is greater than or equal to the actual demand, then all the demand can be satisfied; otherwise, MCC can sell only as many units as are available. This condition is modeled as:

Formula for cell E6: =MIN(D6,B6+C6)
(Copy to E7 through E35.)

The values in column F represent the on-hand inventory at the end of each day, and are calculated as:

Formula for cell F6: =B6+C6−E6
(Copy to F7 through F35.)

To determine whether to place an order, we first must calculate the inventory position, which was defined earlier as the ending inventory plus any outstanding orders. This is implemented in column G as:

Formula for cell G6: =F6
Formula for cell G7: =G6−E7+IF(H6=1,$M$6,0)
(Copy to G8 through G35.)

Column H indicates if an order should be placed based on inventory position and the reorder point, as:

Formula for cell H6: =IF(G6<$M$5,1,0)
(Copy to H7 through H35.)

In column I, we use Crystal Ball's Custom distribution to define our assumptions about the random behavior of order lead times. Figure 12.33 shows the Define Assumption dialog box for cell I6. This definition was also copied to cells I7 through I35 (using Crystal Ball's copy data and paste data commands). Note that column I generates an order lead time for each day, irrespective of if an order is actually placed on a given day. If no order is placed on a given day, the associated lead time value in Column I is ignored.

If an order is placed (as indicated by column H), column J indicates the day on which the order will be received based on its random lead time in column I. This is done as:

Formula for cell J6: =IF(H6=0,0,A6+1+I6)
(Copy to J7 through J35.)

The values in column C are coordinated with those in column J. The nonzero values in column J indicate the days on which orders will be received. For example, cell J9 indicates that an order will be received on day 10. The actual receipt of this order is reflected by the value of 50 in cell C15, which represents the receipt of an order at the beginning of day 10. The formula in cell C15 that achieves this is:

Formula for cell C15: =COUNTIF($J$6:J14,A15)*$M$6

This formula counts how many times the value in cell A15 (representing day 10) appears as a scheduled receipt day between days 1 through 9 in column J. This represents

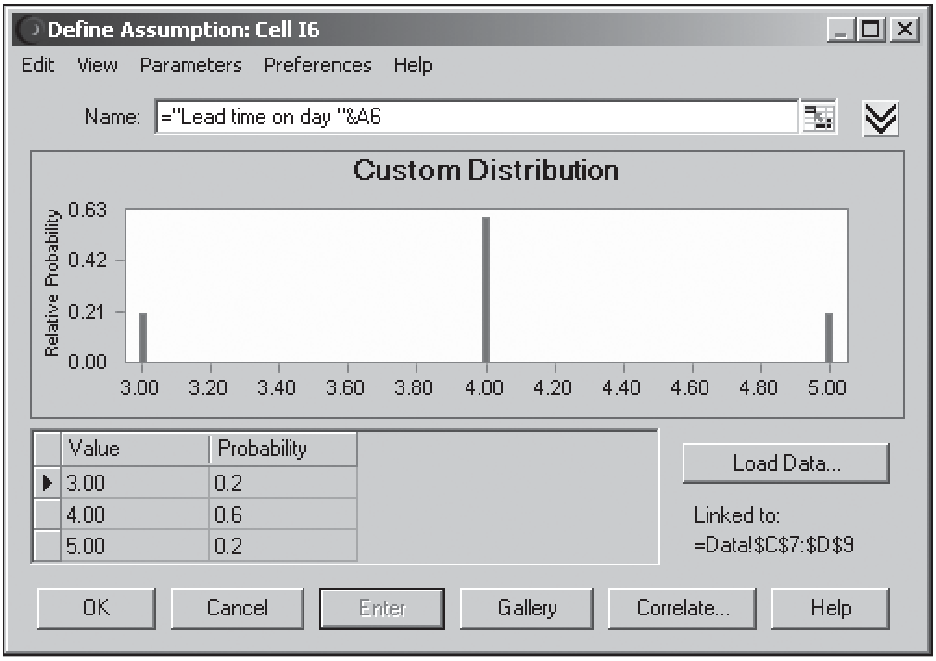


FIGURE 12.33

Defining the assumptions for cell I6

the number of orders scheduled to be received on day 10. We then multiply this by the order quantity (50), given in cell M6 to determine the total units to be received on day 10. Thus, the values in column C are generated as:

Formula for cell C6: =0
Formula for cell C7: =COUNTIF($J$6:J6,A7)*$M$6
(Copy to C8 through C35.)

The service level for the model is calculated in cell M9 using the values in columns D and E as:

Formula for cell M9: =SUM(E6:E35)/SUM(D6:D35)

Again, the service level represents the proportion of total demand that can be satisfied from inventory. The value in cell M9 indicates that in the scenario shown, 87.30% of the total demand is satisfied.

The average inventory level is calculated in cell M10 by averaging the values in column B. This is accomplished as follows:

Formula for cell M10: =AVERAGE(B6:B35)

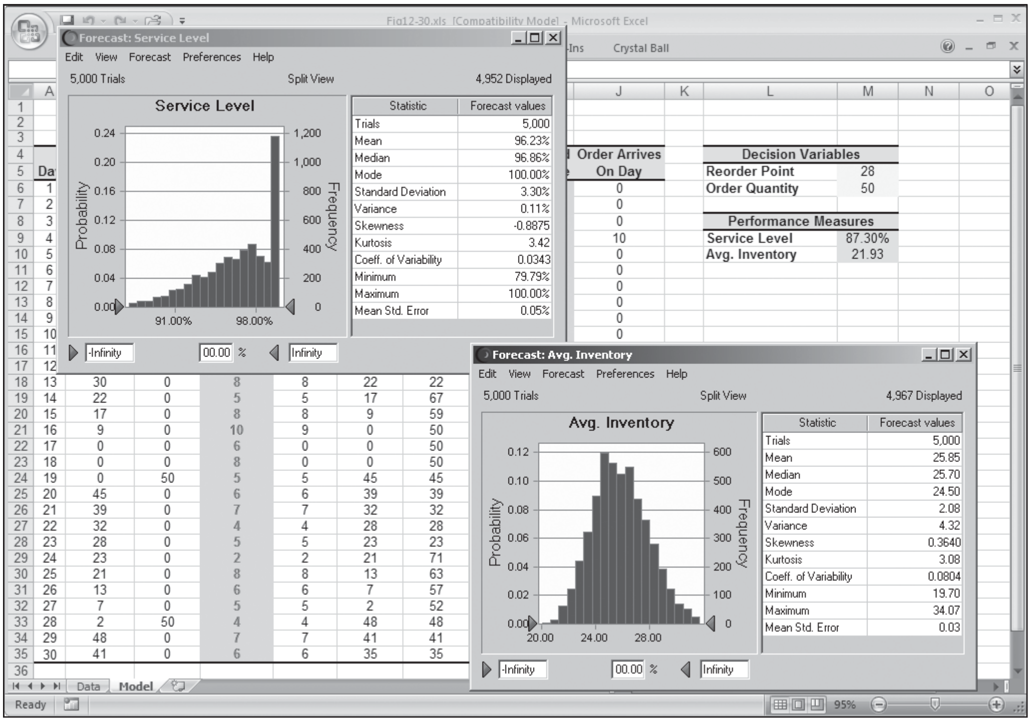
12.15.2 REPLICATING THE MODEL

The model in Figure 12.31 indicates one possible scenario that could occur if Laura uses a reorder point of 28 units for the computer monitor. Figure 12.34 shows the results of using Crystal Ball to replicate this model 5000 times, tracking the values in cells M9 (service level) and M10 (average inventory) as forecast cells.

Figure 12.34 indicates that MCC's current reorder point (28 units) and order quantity (50 units) results in an average service level of approximately 96.2% (with a minimum

FIGURE 12.34

Results of 5000 replications of the MCC model



value of 79.8% and a maximum value of 100%) and an average inventory level of almost 26 monitors (with a minimum value of 19.7 and a maximum value of 34.07).

12.15.3 OPTIMIZING THE MODEL

Suppose Laura wants to determine a reorder point and order quantity that provides an average service level of 98% while keeping the average inventory level as low as possible. One way to do this is to run additional simulations at various reorder point and inventory level combinations trying to find the combination of settings that produce the desired behavior. However, as you might imagine, this could be very time-consuming.

Fortunately, Crystal Ball Professional includes a product called OptQuest that is designed to solve this type of problem. To use OptQuest, first we must use Crystal Ball to define cells M5 (reorder point) and M6 (order quantity) to be decision variable cells. We will assume that Laura wants to consider discrete reorder points in the range from 20 and 50 and discrete order quantity values in the range from 20 to 70. To do this:

1. Select cells M5 and M6.
2. Click the Define Decision button on the Crystal Ball tab.
3. Enter lower and upper bounds for each cell and a discrete step size of 1.
4. Click OK.

Note that cells M9 (service level) and M10 (average inventory level) should already be selected as forecast cells from our earlier simulation run.

Again, when comparing different values for one or more decision variables, it is best if each possible solution is evaluated using exactly the same series of random numbers. We can ensure that Crystal Ball uses the same set of random numbers for each simulation by selecting the "Use same sequence of random numbers" option and supplying an "Initial seed value" under the Sampling option on Crystal Ball's Run Preferences dialog box (shown earlier in Figure 12.24).

After the decision variable cells, forecast cells, random number seed, and an Assumption cell are specified using Crystal Ball, you may start an OptQuest session for the model as follows:

1. Click Crystal Ball, Tools, OptQuest.
2. In OptQuest, click File, New.

The resulting OptQuest screen is shown in Figure 12.35. This screen shows the decision variables for the current problem along with the bounds and restrictions we have placed on these variables. Click the OK button on this screen to accept these selections.

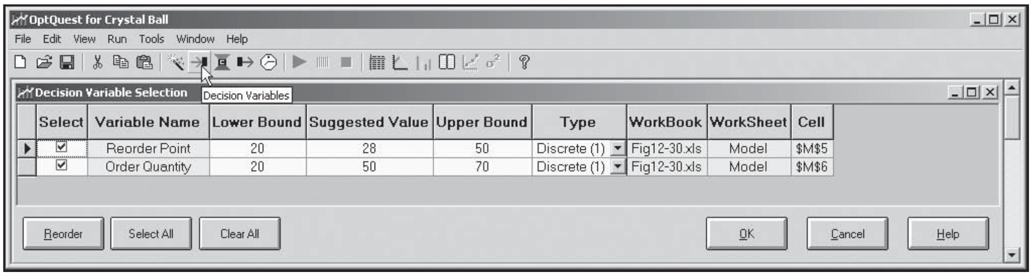


FIGURE 12.35

OptQuest's Decision Variable Selection screen

FIGURE 12.36

OptQuest's Forecast Selection screen

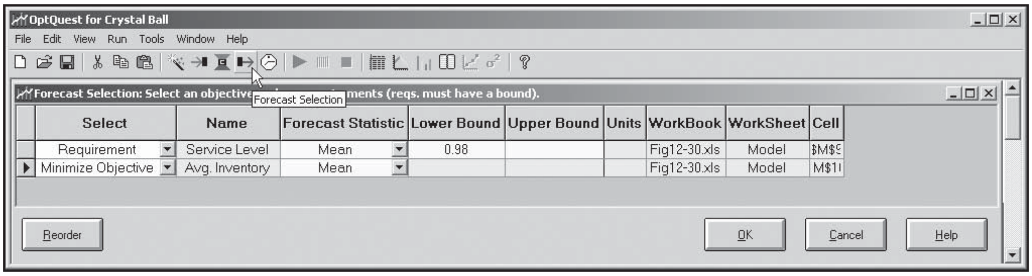
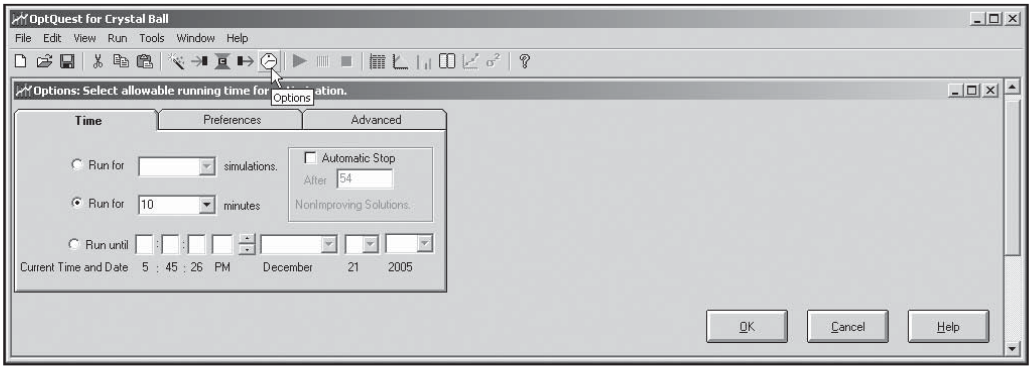


FIGURE 12.37

OptQuest's Options screen



The next step is to tell OptQuest about the objectives we want it to pursue. This is done using the Forecast Selection window shown in Figure 12.36.

In the window shown in Figure 12.36, we indicated that we want OptQuest to minimize the value of the "Avg. Inventory" cell. We also placed a lower bound of 0.98 on the "Service Level" cell and indicated that this is a requirement that must be enforced. (Note that you can make changes to the various cells in the Forecast Selection window by clicking on the appropriate cell.) Click the OK button on this screen to accept these selections.

Next, we use the Options window shown in Figure 12.37 to indicate the amount of time we want OptQuest to search for solutions to the problem. Remember that a separate simulation must be run for each combination of the decision variables that OptQuest chooses. OptQuest uses several heuristics to search intelligently for the best combination of decision variables. However, this is still inherently a very computationally intensive and time-consuming process and complicated models may take hours (or days) of solution time. Note that we are giving OptQuest ten minutes to search for a solution to the MCC problem.

With all the appropriate options set, click the Start button (or click Run, Start). OptQuest then runs multiple simulations using different combinations of values for the decision variables. The Performance Graph and Status and Solutions windows shown in Figure 12.38 provide a summary of OptQuest's efforts on the problem. Ultimately, OptQuest found that a reorder point of 34 and order quantity of 20 resulted in an average service level of 98.4% while requiring an average inventory of approximately 18.3 units per month. Thus, MCC can simultaneously increase its service level and reduce its average inventory level by using the new ordering policy identified by OptQuest.

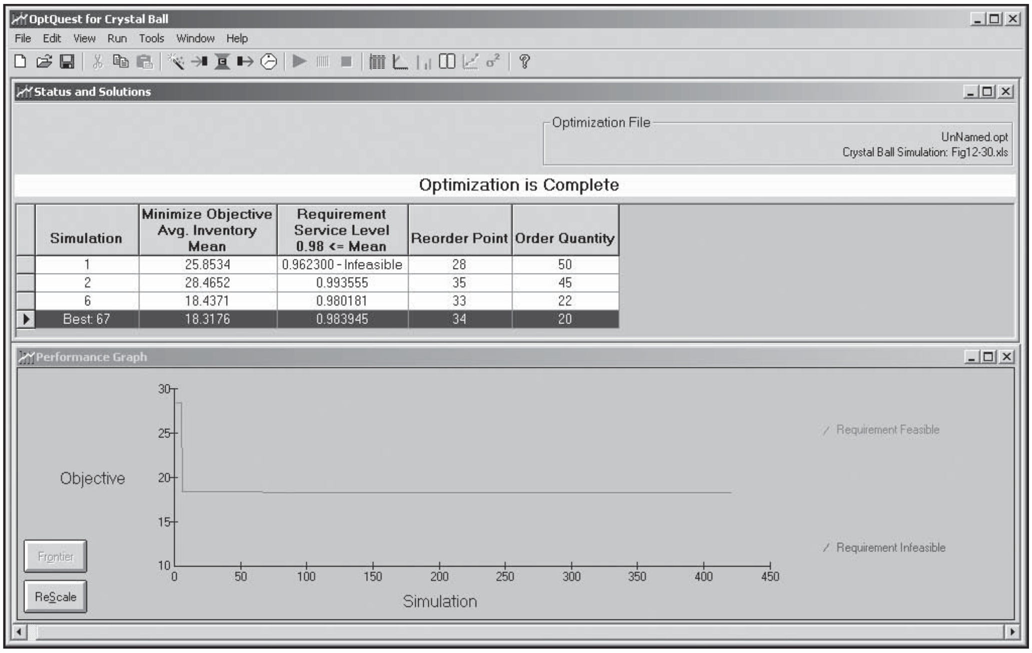


FIGURE 12.38

OptQuest results

A Tip on Using OptQuest...

When using OptQuest to solve a problem, it is often a good idea to reduce the number of trials that Crystal Ball performs during each simulation. This allows OptQuest to explore more possible combinations of decision variable values (solutions) in the allotted amount of time. Then after OptQuest identifies a solution for the problem, you can implement that solution in your model (click Edit, Copy to Excel in OptQuest) and rerun Crystal Ball with a higher number of trials to get a better idea of how well the suggested solution actually performs.

12.15.4 COMPARING THE ORIGINAL AND OPTIMAL ORDERING POLICIES

MCC's manager should be pleased with the new ordering policy OptQuest identified, but she may have other questions about how the reorder point and order quantity will affect the physical storage requirements for this inventory item. Figure 12.39 shows two separate charts illustrating the variability in daily ending inventory under the original and optimal ordering policies. Clearly, in addition to its other benefits, the new ordering policy also requires a more consistent and smaller amount of storage space for this inventory item.

The charts in Figure 12.39 were created using Crystal Ball's Analyze, Trend Charts command. To collect the data for these charts, each of the ending inventory cells shown in column F of Figure 12.31 were defined to be Crystal Ball forecast cells. Two separate simulations then were run: one with the original reorder point and order

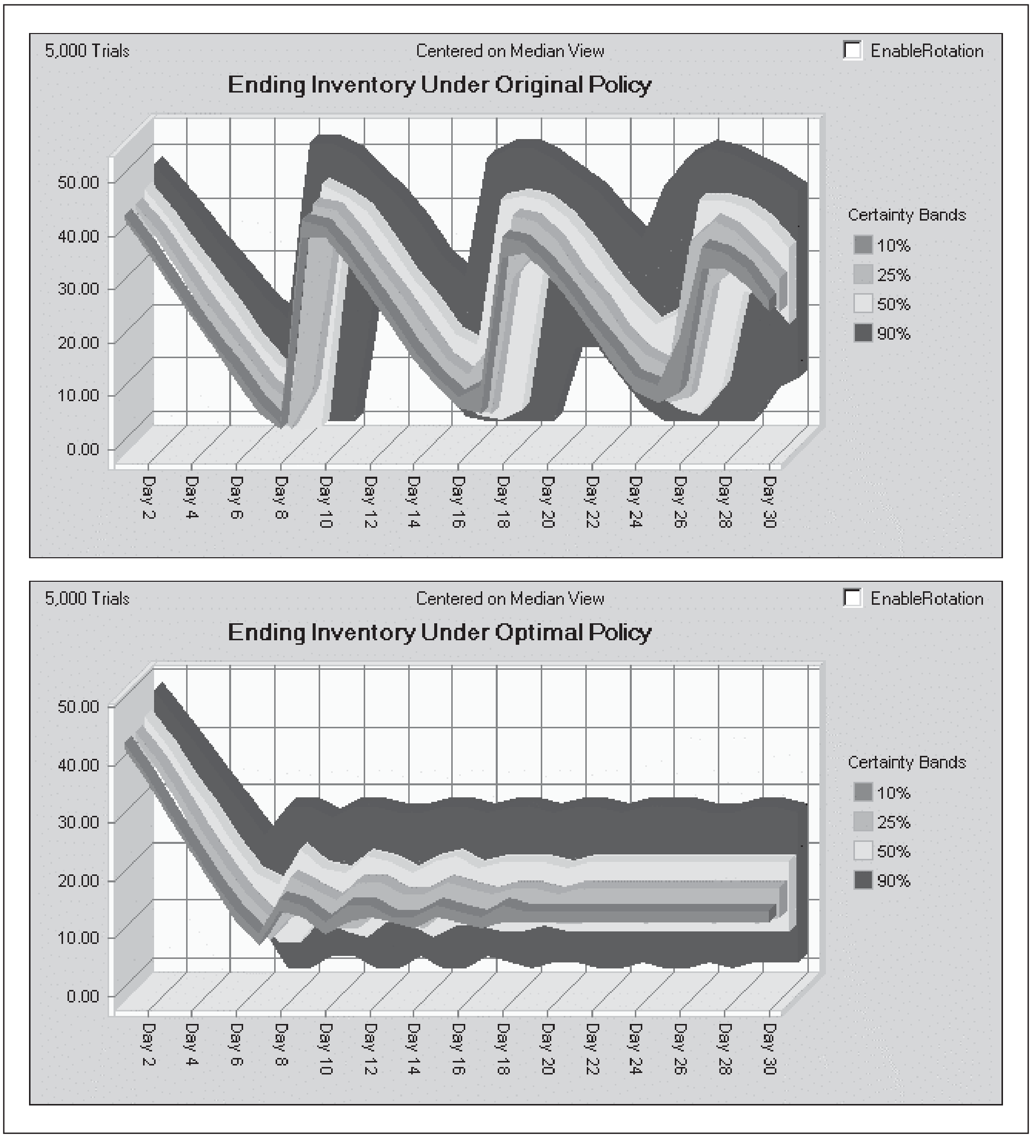


FIGURE 12.39

Comparison of original and optimal inventory policies

quantity values (in cells M5 and M6, respectively), and the other with the optimal reorder point and order quantity values identified by OptQuest. After each simulation, a trend chart was created from the forecast cells representing the daily ending inventory values.

12.16 A Project Selection Example

In Chapter 6, we saw how Solver can be used in project selection problems in which the payoff for each project is assumed to be known with certainty. In many cases, a great deal of uncertainty exists with respect to the ultimate payoff that will be received if a particular project is undertaken. In these situations, Crystal Ball and OptQuest offer an alternate means for deciding which project(s) to undertake. Consider the following example.

TRC Technologies has $2 million to invest in new R&D projects. The following table summarizes the initial cost, probability of success, and revenue potential for each of the projects.

Project	Initial Cost ($1000s)	Probability of Success	Revenue Potential ($1000s) Min.	Most Likely	Max.
1	$250.0	90%	$ 600	$ 750	$ 900
2	$650.0	70%	$1,250	$1,500	$1,600
3	$250.0	60%	$ 500	$ 600	$ 750
4	$500.0	40%	$1,600	$1,800	$1,900
5	$700.0	80%	$1,150	$1,200	$1,400
6	$ 30.0	60%	$ 150	$ 180	$ 250
7	$350.0	70%	$ 750	$ 900	$1,000
8	$ 70.0	90%	$ 220	$ 250	$ 320

TRC's management wants to determine the set of projects that will maximize the firm's expected profit.

12.16.1 A SPREADSHEET MODEL

A spreadsheet model for this problem is shown in Figure 12.40 (and the file Fig12-40.xls on your data disk). Cells C6 through C13 in this spreadsheet indicate which projects will be selected. Using Crystal Ball, we can define these cells to be decision variables that must take on discrete values between zero and one—or operate as binary variables. The values shown in cells C6 though C13 were assigned arbitrarily. We will use OptQuest to determine the optimal values for these variables.

In cell D14, we compute the total initial investment required by the selected projects as follows:

Formula for cell D14: =SUMPRODUCT(D6:D13,C6:C13)

In cell D16, we calculate the amount of unused or surplus investment funds. Using Crystal Ball, we will define this to be a forecast cell. Using OptQuest, we can place a lower bound requirement of zero on the value of this cell to ensure that the projects selected do not require more than $2 million in initial investment funds.

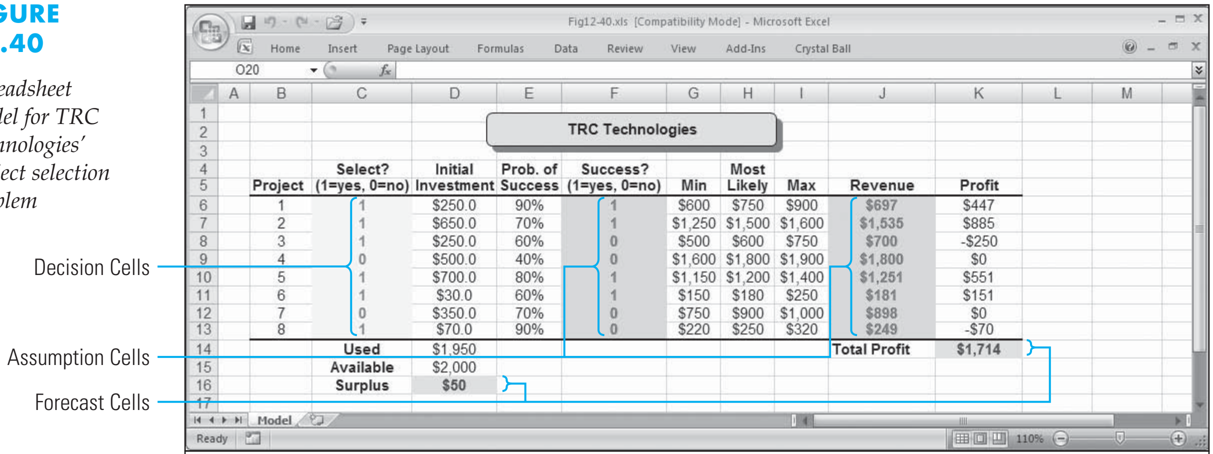
Formula for cell D16: =D15−D14

The potential success or failure of each project may be modeled using Crystal Ball's Yes-No distribution. This distribution randomly returns a value of either 1 (representing "Yes" or success) or 0 (representing "No" or failure) according to a specified probability of success. The probability of success for each project is given in column E. Thus, we model the potential success of project 1 in cell F6 using the Crystal Ball Define Assumption settings shown in Figure 12.41. This definition also was copied to cells F7 through F13 (using Crystal Ball's copy data and paste data commands).

There is also uncertainty about the revenue that each project might generate. Because we have estimates of the minimum, most likely, and maximum possible revenue for

FIGURE 12.40

Spreadsheet model for TRC Technologies' project selection problem



TRC Technologies

Project	Select? (1=yes, 0=no)	Initial Investment	Prob. of Success	Success? (1=yes, 0=no)	Min	Most Likely	Max	Revenue	Profit
1	1	$250.0	90%	1	$600	$750	$900	$697	$447
2	1	$650.0	70%	1	$1,250	$1,500	$1,600	$1,535	$885
3	1	$250.0	60%	0	$500	$600	$750	$700	-$250
4	0	$500.0	40%	0	$1,600	$1,800	$1,900	$1,800	$0
5	1	$700.0	80%	1	$1,150	$1,200	$1,400	$1,251	$551
6	1	$30.0	60%	1	$150	$180	$250	$181	$151
7	0	$350.0	70%	0	$750	$900	$1,000	$898	$0
8	1	$70.0	90%	0	$220	$250	$320	$249	-$70
	Used	$1,950						Total Profit	$1,714
	Available	$2,000							
	Surplus	$50							

Key Cell Formulas & CB Settings

Cell	Formula/Settings	Copied to
D14	=SUMPRODUCT(D6:D13,C6:C13)	--
D16	=D15–D14	--
F6	CB Distribution: Yes-No Probability of Yes(1): =E6	F7:F13 (Using CB copy & paste)
J6	CB Distribution: Triangular Minimum: =G6 Likeliest: =H6 Maximum: =I6	J7:J13 (Using CB copy & paste)
K6	=IF(C6=1,J6*F6-D6,0)	K7:K13
K14	=SUM(K6:K13)	--

each project, we will model the revenues for selected, successful projects using a triangular distribution. We accomplish this for project 1 in cell J6 as shown in Figure 12.42. This definition also was copied to cells J7 through J13 (using Crystal Ball's copy data and paste data commands).

Note that each project's potential success or failure must be generated independently of whether or not the project is selected (even though only selected projects have the potential to be successful). Similarly, each project's potential revenue must be generated independently of whether or the not the project is successful (even though only successful projects generate revenue). Thus, in column K we must compute the profit for each project carefully based on whether the project is selected and, if selected, whether it is also successful. This is accomplished in column K as follows:

Formula for cell K6: =IF(C6=1,J6*F6-D6,0)

(Copy to cells K7 through K13.)

Finally, cell K14 computes the total profit for each replication of the model. We will define this as a forecast cell using Crystal Ball and attempt to maximize its value using OptQuest.

Formula for cell K14: =SUM(K6:K13)

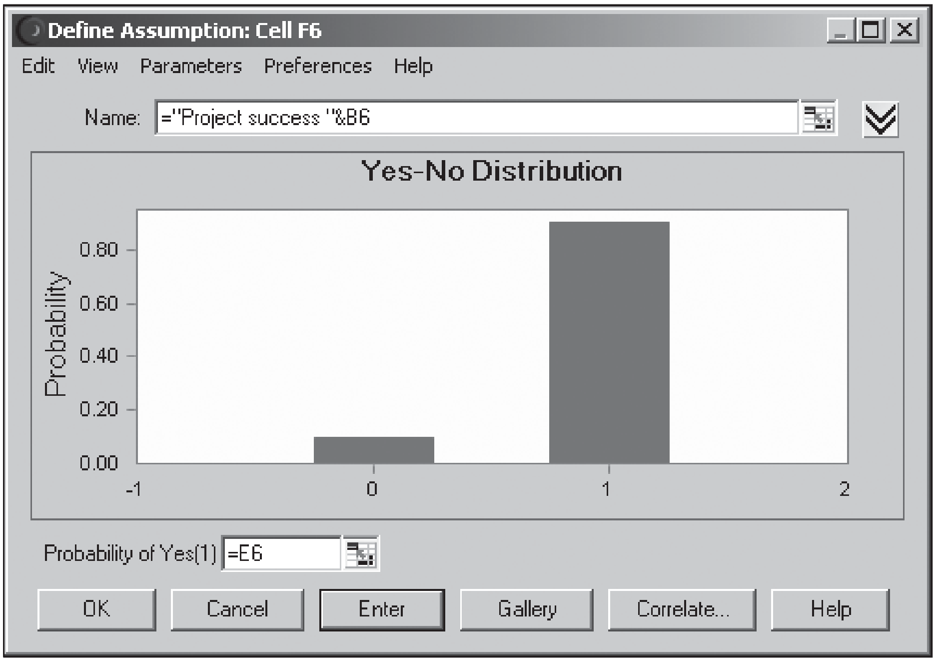


FIGURE 12.41

Defining the assumptions for cell F6

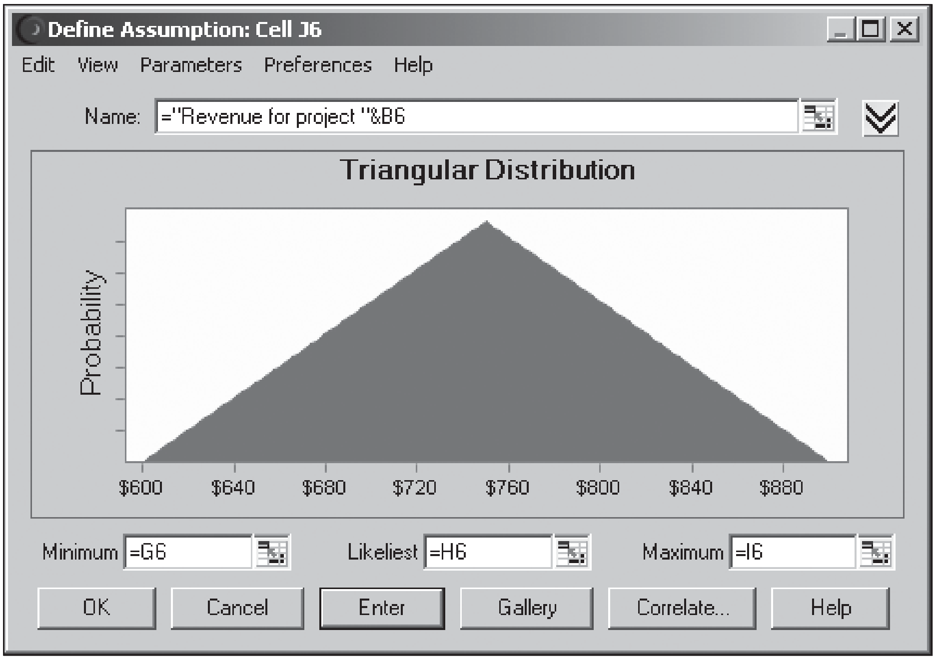


FIGURE 12.42

Defining the assumptions for cell J6

12.16.2 SOLVING THE PROBLEM WITH OptQUEST

The OptQuest settings used to solve this problem are shown in Figure 12.43 and the optimal solution is shown in Figure 12.44. This solution involves selecting projects 1, 2, 4, 6, 7, and 8, requiring an initial investment of $1.85 million and resulting in an expected profit of approximately $1.5 million.

FIGURE 12.43

OptQuest settings for the project selection problem

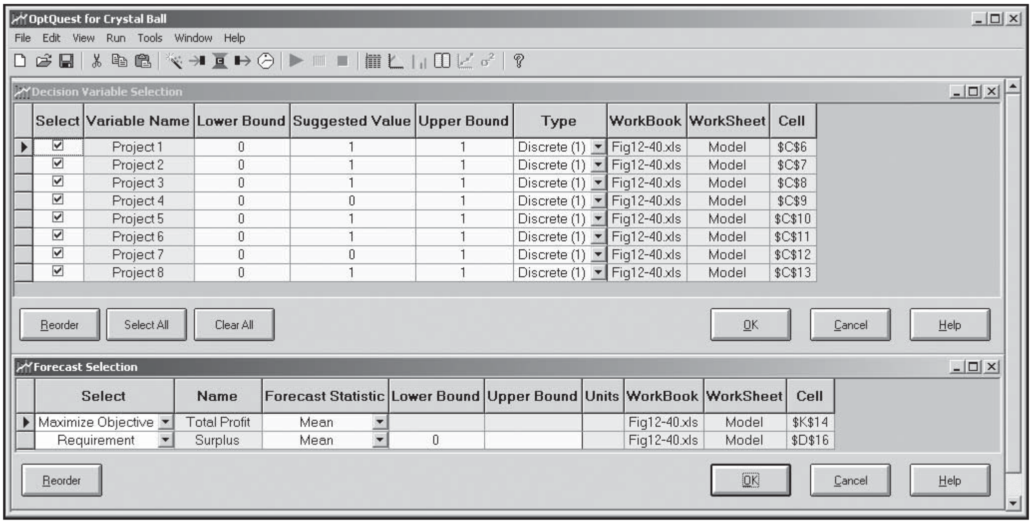
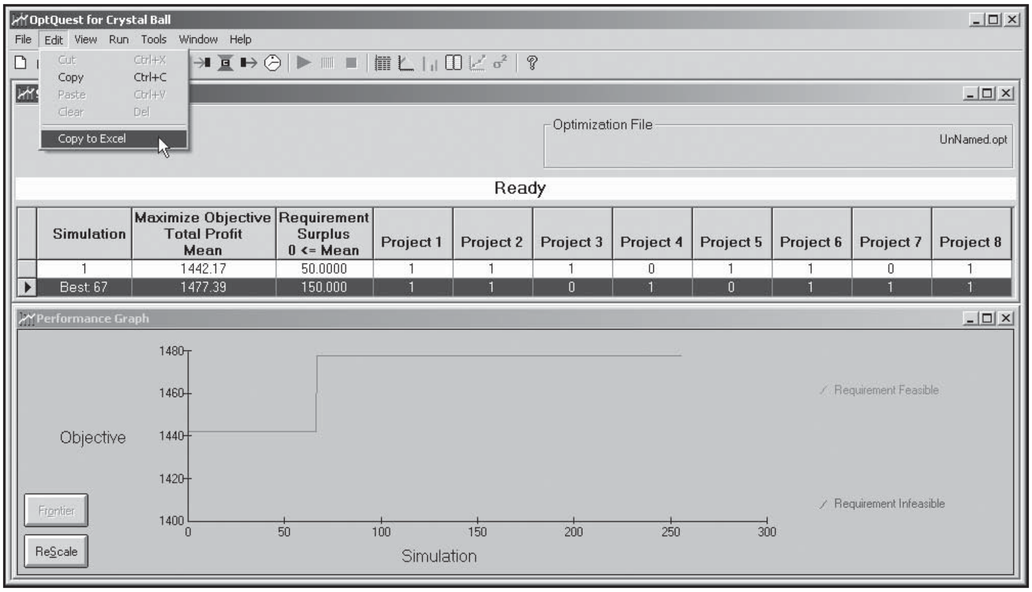


FIGURE 12.44

Optimal solution to the project selection problem



To investigate this solution in a bit more detail, we can copy it back into Crystal Ball (click Edit, Copy to Excel in OptQuest), and view the statistics and frequency distribution associated with the solution. These results are shown in Figures 12.45.

Although the expected (mean) profit associated with this solution is approximately \$1.5 million, the range of the possible outcomes is fairly wide (approximately \$5.5 million). The worst-case outcome observed with this solution resulted in approximately a \$1.6 million loss, whereas the best-case outcome resulted in approximately a \$3.86 million profit. Because each of the projects is a one-time occurrence that either can succeed or fail, the decision makers in this problem do not have the luxury of selecting this set of projects over and over and realizing the average profit level of \$1.5 million over time. Also, we see that there is about a 10% chance of losing money if this solution

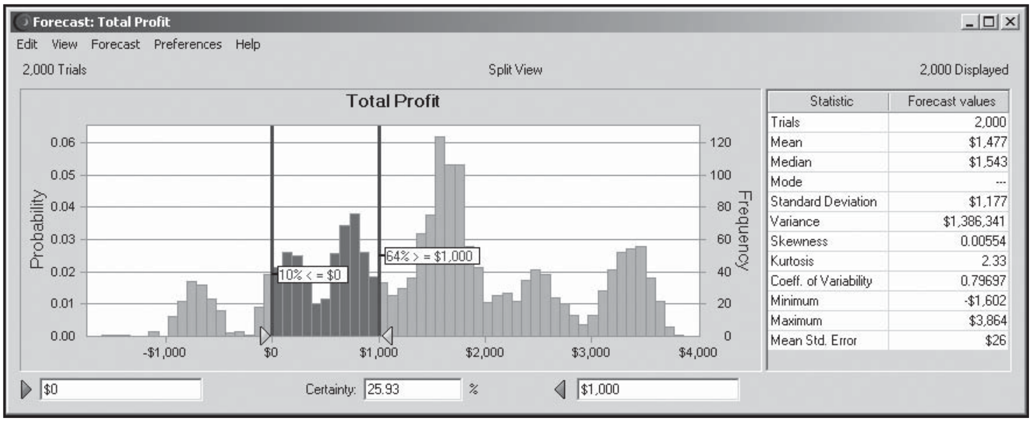


FIGURE 12.45

Results for the project selection problem

is implemented, but about a 64% chance of making at least $1 million. Thus, there is a significant risk associated with this solution that is not apparent if one simply looks at its expected profit level of $1.5 million.

12.16.3 CONSIDERING OTHER SOLUTIONS

The decision makers in this problem might be interested in considering other alternatives. Because each of the projects has the potential to fail and lose money, the only way to completely avoid the possibility of a loss is not to invest in any of the projects. Unfortunately, this also completely avoids the possibility of earning profit! In most situations, greater levels of risk are required to achieve higher levels of return.

After considering the earlier solution and their attitudes toward risk and return, suppose the decision makers at TRC decide that they are comfortable with a 10% chance of losing money. But while assuming this risk, they want to ensure that they maximize the chance of earning a profit of at least $1 million. The OptQuest Forecast Selections settings required to solve this revised problem are shown in Figure 12.46. Notice that now

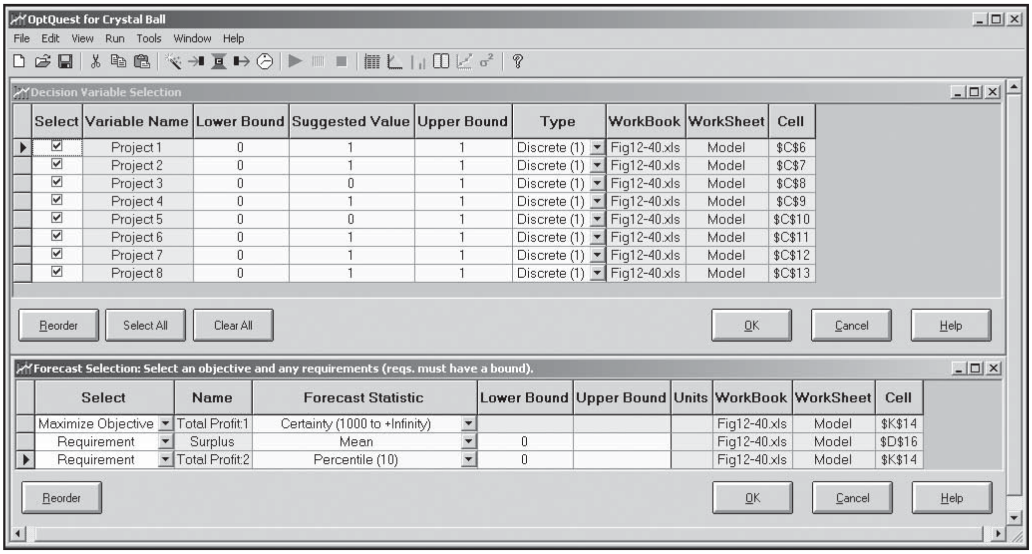


FIGURE 12.46

OptQuest settings for the revised project selection problem we are attempting to maximize the number of times that the forecasted profit value (in cell K14 in Figure 12.40) falls above $1 million (as opposed to maximizing the average profit value). We have implemented a requirement that the 10th percentile of the distribution of forecasted profit values be greater than zero (or have a lower bound of 0). This should ensure that the resulting solution will have at most a 10% chance of losing money. To place more than one requirement on the same forecast cell (as was done for cell K14 in Figure 12.46), in OptQuest's Forecast Selection window,

1. Click anywhere on the row corresponding to the forecast cell you want to duplicate.
2. Click Edit, Duplicate.

Placing Multiple Requirements on the Same Forecast Cell...

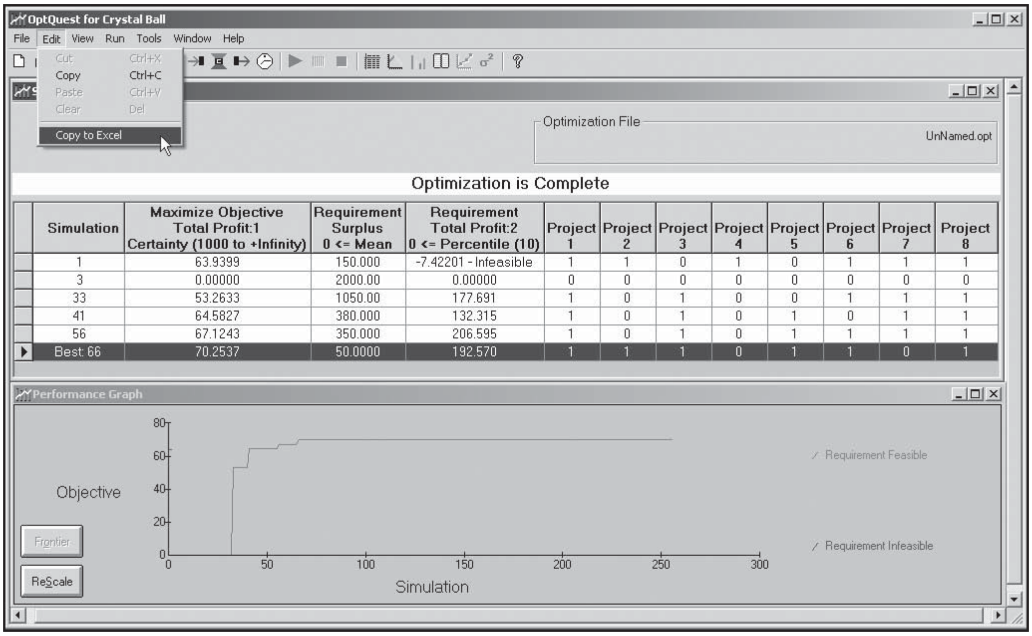
To place more than one requirement on the same forecast cell (as done in Figure 12.46), in OptQuest's Forecast Selection window, click anywhere on the row corresponding to the forecast cell you want to duplicate. Then click Edit, Duplicate.

OptQuest's solution to this problem is shown in Figure 12.47. This solution indicates that TRC should accept projects 1, 2, 3, 5, 6, and 8, consuming $1.95 million in initial capital. To investigate this solution in more detail, we copy this solution back into our spreadsheet (in OptQuest click Edit, Copy to Excel) and generate the solution statistics and frequency distribution shown in Figure 12.48.

In Figure 12.48, notice that the expected (mean) profit for this solution is about $1.44 million, representing a decrease of approximately $35,000 from the earlier solution.

FIGURE 12.47

OptQuest solution for the revised project selection problem



Simulation	Maximize Objective Total Profit:1 Certainty (1000 to +Infinity)	Requirement Surplus 0 <= Mean	Requirement Total Profit:2 0 <= Percentile (10)	Project 1	Project 2	Project 3	Project 4	Project 5	Project 6	Project 7	Project 8
1	63.9399	150.000	-7.42201 - Infeasible	1	1	0	1	0	1	1	1
3	0.00000	2000.00	0.00000	0	0	0	0	0	0	0	0
33	53.2633	1050.00	177.691	1	0	1	0	0	1	1	1
41	64.5827	380.000	132.315	1	0	1	0	1	0	1	1
56	67.1243	350.000	206.595	1	0	1	0	1	1	1	1
Best: 66	70.2537	50.0000	192.570	1	1	1	0	1	1	0	1

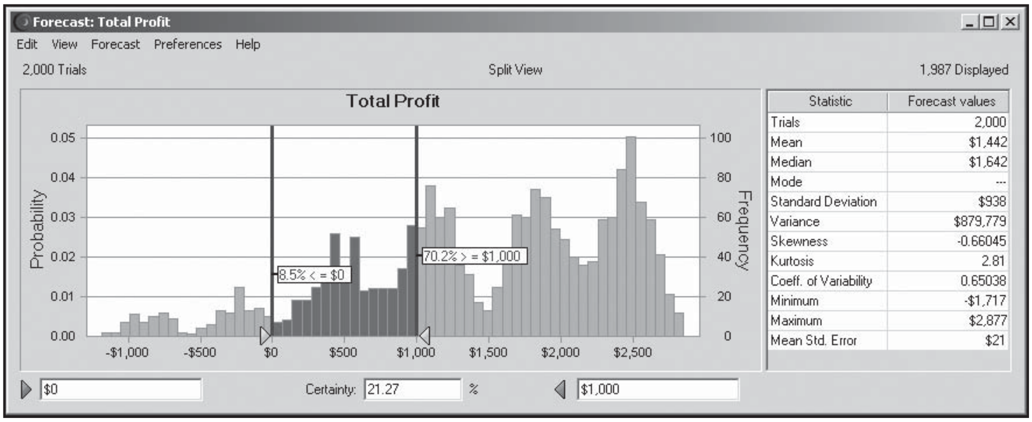


FIGURE 12.48

Statistics for the revised project selection problem

The range of possible outcomes also has decreased to about \$4.6 million, with a worst-case outcome of a \$1.7 million loss, and a best-case outcome of approximately a \$2.9 million profit. This solution *reduces* the chances of realizing a loss to approximately 8.5% and *increases* the chances of making at least \$1 million to approximately 70.2%. Thus, although the best possible outcome realized under this solution (\$2.9 million) is not as large as that of the earlier solution (\$3.86 million), it reduces the downside risk in the problem and makes it more likely for the company to earn at least \$1 million—but it also requires a slightly larger initial investment. It is also interesting to note that the probability of *all* the selected projects being successful under this solution is 0.1633 (i.e., $0.1633 = .9 \times .7 \times .6 \times .8 \times .6 \times .9$), whereas the probability of all selected projects being successful under the first solution is only 0.0953 (i.e., $0.0953 = .9 \times .7 \times .4 \times .6 \times .7 \times .9$).

So, what is the best solution to this problem? It depends on the risk attitudes and preferences of the decision makers at TRC. However, the simulation techniques that we have described clearly provide valuable insights into the risks associated with various solutions.

12.17 A Portfolio Optimization Example

In Chapter 8, we saw how Solver can be used to analyze potential tradeoffs between risk and return for a given set of stocks using the idea of an ***efficient frontier***. The efficient frontier represents the highest level of return a portfolio can achieve for any given level of risk. Whereas portfolio optimization and efficient frontier analysis is most commonly associated with financial instruments such as stocks and bonds, it also can be applied to physical assets. OptQuest contains an efficient frontier calculation process that makes identifying the efficient frontier a relatively simple process. This will be illustrated using the following example.

> In recent years, a fundamental shift occurred in power plant asset ownership. Traditionally, a single regulated utility would own a given power plant. Today, more and more power plants are owned by merchant generators that provide power to a competitive wholesale marketplace. This makes is possible for an investor to buy, for example, 10% of ten different generating assets rather than 100% of a single power plant. As a result, non-traditional power plant owners have emerged in the form of investment groups, private equity funds, and energy hedge funds.

The McDaniel Group is a private investment company in Richmond, VA, that currently has a total of $1 billion that it wants to invest in power generation assets. Five different types of investments are possible: natural gas, oil, coal, nuclear, and wind-powered plants. The following table summarizes the megawatts (MW) of generation capacity that can be purchased per each $1 million investment in the various types of power plants.

	Generation Capacity per Million $ Invested				
Fuel Type	**Gas**	**Coal**	**Oil**	**Nuclear**	**Wind**
MWs	2.0	1.2	3.5	1.0	0.5

The return on each type of investment varies randomly and is determined primarily by fluctuations in fuel prices and the spot price (or current market value) of electricity. Assume that the McDaniel Group analyzed historical data to determine that the return per MW produced by each type of plant can be modeled as normally distributed random variables with the following means and standard deviations.

	Normal Distribution Return Parameters by Fuel Type				
	Gas	**Coal**	**Oil**	**Nuclear**	**Wind**
Mean	16%	12%	10%	9%	8%
Std Dev	12%	6%	4%	3%	1%

Additionally, while analyzing the historical data on operating costs, it was observed that many of the returns are correlated. For example, when the returns from plants fueled by natural gas are high (due to low gas prices), returns from plants fueled by coal and oil tend to be low. So there is a negative correlation between the returns from gas plants and the returns from coal and oil plants. The following table summarizes all the pairwise correlations between the returns from different types of power plants.

	Correlations Between Returns by Fuel Type				
	Gas	**Coal**	**Oil**	**Nuclear**	**Wind**
Gas	1	−0.49	−0.31	0.16	0.12
Coal		1	−0.41	0.11	0.07
Oil			1	0.13	0.09
Nuclear				1	0.04
Wind					1

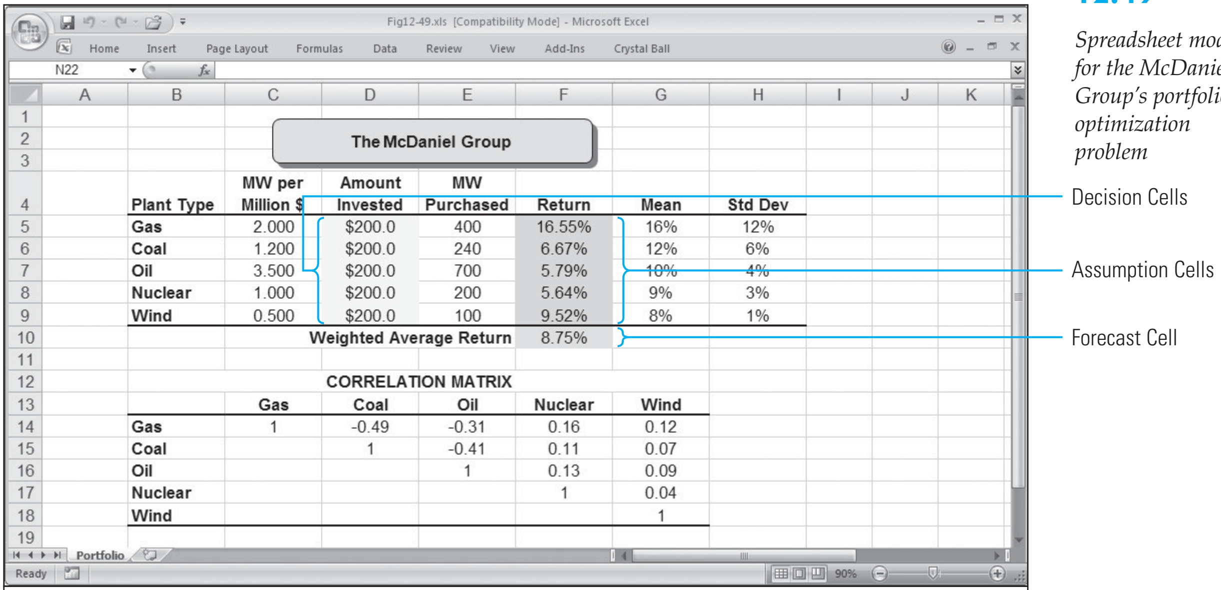
The McDaniel Group would like to determine the efficient frontier for its investment options in power generation assets.

12.17.1 A SPREADSHEET MODEL

A spreadsheet model for this problem is shown in Figure 12.49 (and the file Fig12-49.xls on your data disk). Cells D5 through D9 in this spreadsheet indicate how much money (in millions) will be invested in each type of generation asset. Using Crystal Ball, we can define these cells to be decision variables that must take on values between zero and $1,000. The values shown in cells D5 though D19 were assigned arbitrarily. We will use OptQuest to determine the optimal values for these variables. In OptQuest, we also will create a constraint that requires these values to sum to $1,000 (or $1 billion).

FIGURE 12.49

Spreadsheet model for the McDaniel Group's portfolio optimization problem



Plant Type	MW per Million $	Amount Invested	MW Purchased	Return	Mean	Std Dev
Gas	2.000	$200.0	400	16.55%	16%	12%
Coal	1.200	$200.0	240	6.67%	12%	6%
Oil	3.500	$200.0	700	5.79%	10%	4%
Nuclear	1.000	$200.0	200	5.64%	9%	3%
Wind	0.500	$200.0	100	9.52%	8%	1%
		Weighted Average Return		8.75%		

CORRELATION MATRIX

	Gas	Coal	Oil	Nuclear	Wind
Gas	1	-0.49	-0.31	0.16	0.12
Coal		1	-0.41	0.11	0.07
Oil			1	0.13	0.09
Nuclear				1	0.04
Wind					1

Key Cell Formulas & CB Settings

Cell	Formula/Setting	Copied to
E5	=C5*D5	E6:E9
F5	CB Distribution: Normal Mean: =G5 Standard Deviation: =H5	F6:F9 (Using CB copy & paste) (Correlations entered after copying.)
F10	=SUMPRODUCT(F5:F9,E5:E9)/SUM(E5:E9)	--

In column E we compute the number of MW of generation capacity purchased in each asset category as follows:

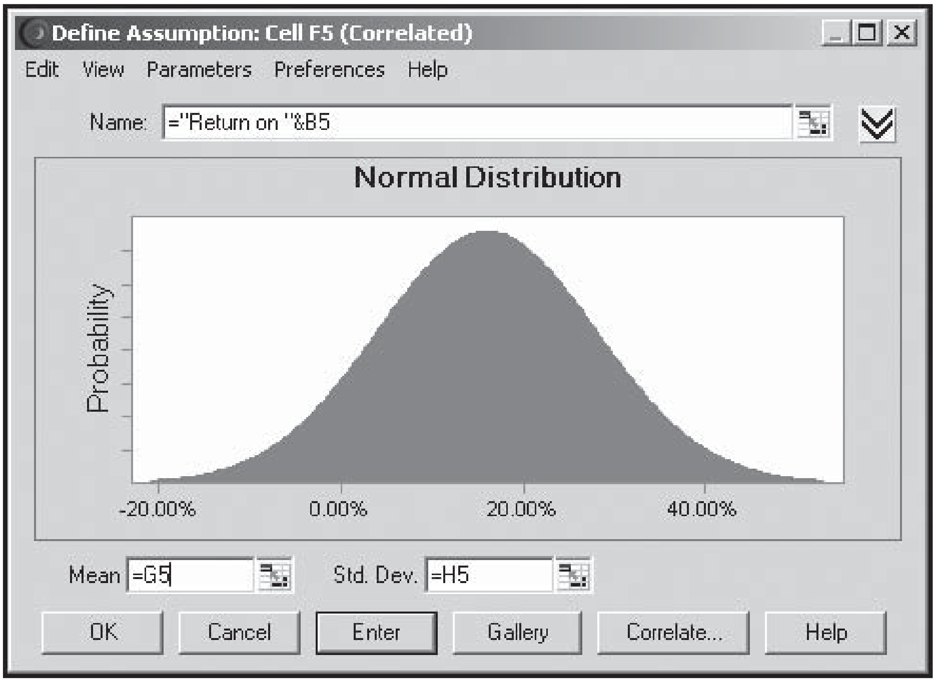
Formula for cell E5: =C5*D5
(Copy to cells E6 through E9.)

The assumption cells representing the random return for each asset category are implemented in column F using Crystal Ball's Normal distribution with the means and standard deviations specified in columns G and H, respectively. We accomplish this for investments in gas-fueled plants in cell F5 as shown in Figure 12.50. This definition also was copied to cells F6 through F9 (using Crystal Ball's copy data and paste data commands).

The "Correlate . . ." button on Crystal Ball's Define Assumption dialog box (shown in Figure 12.50) allows us to define correlations among the random variables (or assumption cells) in our spreadsheet models. However, there is not an accurate way to copy correlations from one assumption cell to another assumption cell. So first we must define all our assumption cells and then go back and individually specify any correlations between them. The Define Correlation dialog box for cell F5 is shown in Figure 12.51. The "Choose . . ." button on this dialog box allows you to select any assumption cells on the spreadsheet and individually specify their correlations to the assumption cell being

FIGURE 12.50

Defining the assumptions for cell F5



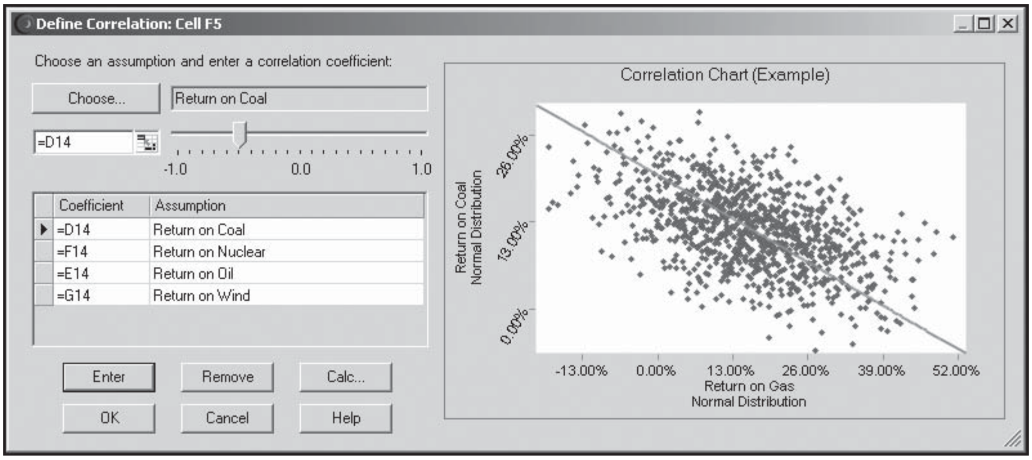
defined. In Figure 12.51 we indicated that the return on gas-fueled plants (represented by cell F5) is correlated with the returns on the other types of plants as indicated by the values in cells D14 through G14 on our spreadsheet model (shown in Figure 12.49). The graph in Figure 12.51 shows an example of the assumed negative correlation between the return of gas assets (assumption cell F5) and the return on coal assets (assumption cell F6). Again, using Crystal Ball's Define Correlation dialog, we have to enter the correlations repeatedly for each individual assumption cell.

Finally, in cell F10, we calculate the weighted average return on investments. Using Crystal Ball, we will define this to be a forecast cell:

Formula for cell F10: =SUMPRODUCT(F5:F9,E5:E9)/SUM(E5:E9)

FIGURE 12.51

Defining correlations for cell F5



Another Way of Specifying Correlations...

Crystal Ball also offers a Correlation Matrix tool (launched via Crystal Ball, Tools, Correlation Matrix) that allows you to upload an entire correlation matrix in a small number of steps. The Correlation Matrix tool is especially useful if you have many correlated assumption cells.

12.17.2 SOLVING THE PROBLEM WITH OptQUEST

The OptQuest settings used to solve this problem are shown in Figure 12.52. The Decision Variable Selection window indicates that we are asking OptQuest to determine the optimal amounts (in millions) to invest in gas, coal, oil, nuclear and wind generation assets, represented by cells D5 through D9, respectively, on our spreadsheet. The Constraints window indicates that the sum of the decision variables must equal 1000 (which is equivalent to $1 billion).

Recall that the McDaniel Group is interested in examining solutions on the efficient frontier of its possible investment options, these power generation assets. This requires determining the portfolios that provide the maximum expected (or average) return at a variety of different risk levels. In this case, we will define risk to be the standard deviation of a portfolio's weighted average return. Thus, in the Forecast Selection window, we indicate that our objective is to maximize the mean value of the weighted average return calculated in cell F10 in our spreadsheet. Finally, we also specify a variable requirement on the allowable upper bound of the standard deviation of the weighted

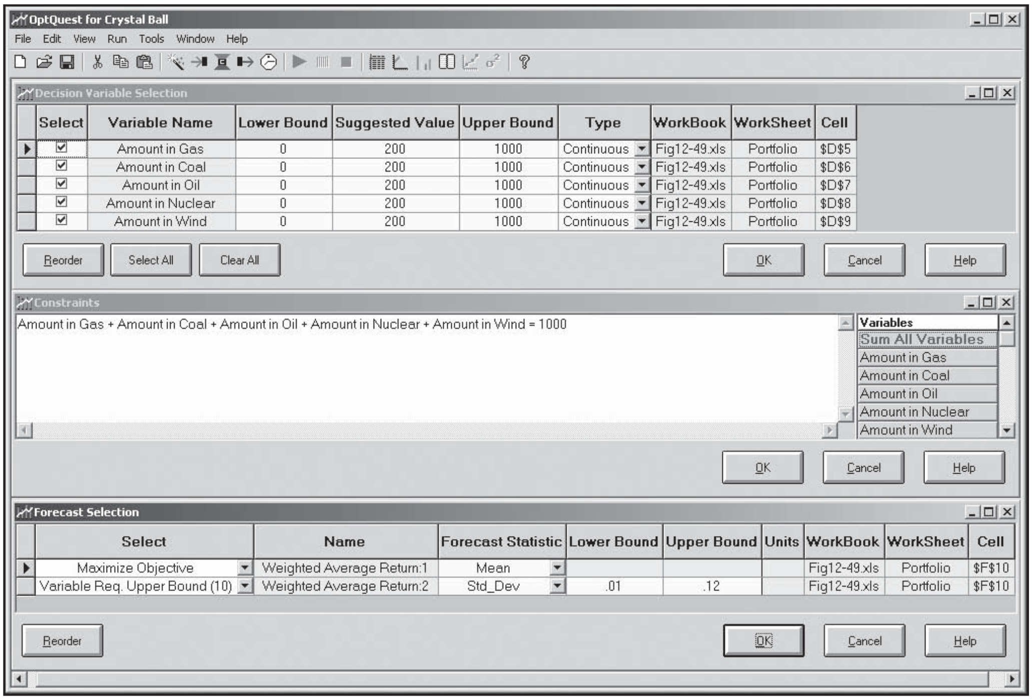
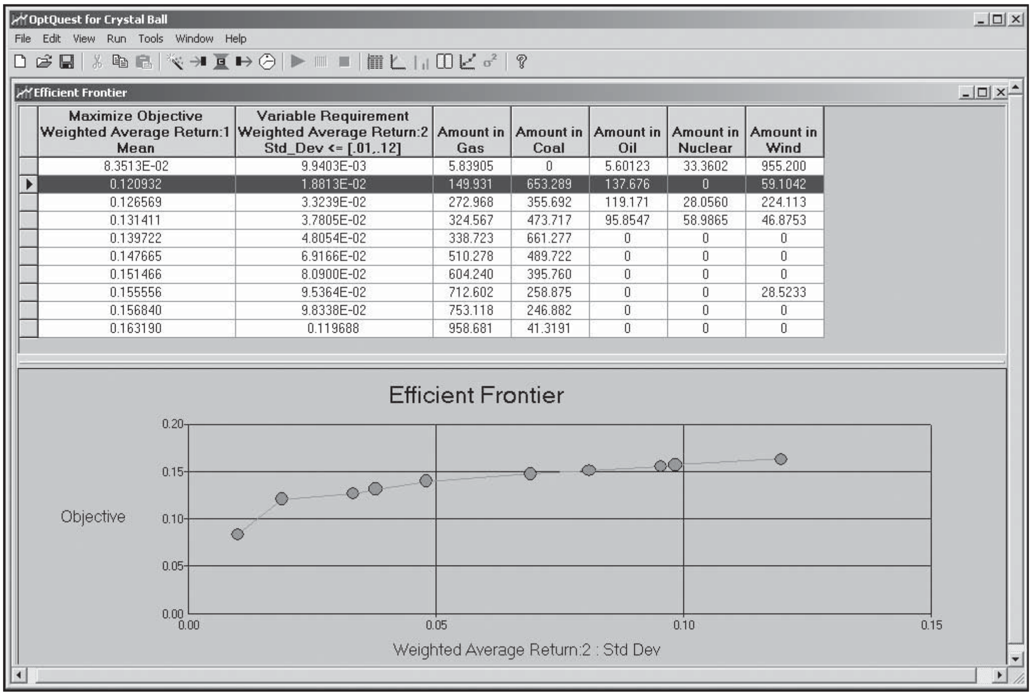


FIGURE 12.52

OptQuest settings for the portfolio optimization problem

FIGURE 12.53

Efficient frontier for the portfolio optimization problem



Maximize Objective Weighted Average Return:1 Mean	Variable Requirement Weighted Average Return:2 Std_Dev <= [.01,.12]	Amount in Gas	Amount in Coal	Amount in Oil	Amount in Nuclear	Amount in Wind
8.3513E-02	9.9403E-03	5.83905	0	5.60123	33.3602	955.200
0.120932	1.8813E-02	149.931	653.289	137.676	0	59.1042
0.126569	3.3239E-02	272.968	355.692	119.171	28.0560	224.113
0.131411	3.7805E-02	324.567	473.717	95.8547	58.9865	46.8753
0.139722	4.8054E-02	338.723	661.277	0	0	0
0.147665	6.9166E-02	510.278	489.722	0	0	0
0.151466	8.0900E-02	604.240	395.760	0	0	0
0.155556	9.5364E-02	712.602	258.875	0	0	28.5233
0.156840	9.8338E-02	753.118	246.882	0	0	0
0.163190	0.119688	958.681	41.3191	0	0	0

average return via the forecast selection labeled "Variable Req. Upper Bound (10)." When you select a variable required upper (or lower) bound on a forecast cell statistic, OptQuest will prompt you to indicate the number of possible upper (or lower) bounds you want to consider. In this case, we indicated we want to investigate 10 different upper bound values for the standard deviation of the forecast cell. The indicated lower and upper bounds of 1% and 12%, respectively, defined the range over which the standard deviation's upper bound will be allowed to vary. (Note that 1% and 12% were selected since they correspond, respectively, to the standard deviations on the returns for the least risky and most risky individual assets available in our portfolio). With these forecast selection settings, OptQuest will automatically identify portfolios that provide the maximum expected return for 10 different levels of risk as measured by portfolio return standard deviations between 1% and 12%.

Figure 12.53 displays OptQuest's Efficient Frontier window summarizing the ten portfolios it found and their relative trade-offs in terms of risk and return. The expected returns on these portfolios vary from 8.3% to 16.3% with standard deviations varying from approximately 1% to 12% with higher expected returns being associated with higher levels of risk. Determining the portfolio that is optimal for the McDaniel Group depends on the firm's preferences for risk versus return. But this analysis should help the firm identify a portfolio that provides the maximum return for the desired level of risk—or the minimum level of risk for the desired level of return.

12.18 Summary

This chapter introduced the concept of risk analysis and simulation. Many of the input cells in a spreadsheet represent random variables whose values cannot be determined with certainty. Any uncertainty in the input cells flows through the spreadsheet model to create a related uncertainty in the value of the output cell(s). Decisions made on the basis of these uncertain values involve some degree of risk.

Various methods of risk analysis are available, including best-case/worst-case analysis, what-if analysis, and simulation. Of these three methods, simulation is the only technique that provides hard evidence (facts and figures) that can be used objectively in making decisions. This chapter introduced the use of the Crystal Ball add-in to perform spreadsheet simulation. To simulate a model, RNGs are used to select representative values for each uncertain independent variable in the model. This process is repeated over and over to generate a sample of representative values for the dependent variable(s) in the model. The variability and distribution of the sample values for the dependent variable(s) can then be analyzed to gain insight into the possible outcomes that might occur. We also illustrated the use of OptQuest in determining the optimal value of controllable parameters or decision variables in simulation models.

12.19 References

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THE WORLD OF MANAGEMENT SCIENCE

The U.S. Postal Service Moves to the Fast Lane

Mail flows into the U.S. Postal Service at the rate of 500 million pieces per day, and it comes in many forms. There are standard-sized letters with 9-digit ZIP codes (with or without imprinted bar codes), 5-digit ZIP codes, typed addresses that can be read by optical character readers, handwritten addresses that are barely decipherable, Christmas cards in red envelopes addressed in red ink, and so on. The enormous task of sorting all these pieces at the sending post office and at the destination has caused postal management to consider and adopt many new forms of technology. These include operator-assisted mechanized sorters, optical character readers (last-line and multiple-line), and bar code sorters. Implementation of new technology brings with it associated policy decisions, such as rate discounts for bar coding by the customer, finer sorting at the origin, and so on.

(Continued)

A simulation model called META (model for evaluating technology alternatives) assists management in evaluating new technologies, configurations, and operating plans. Using distributions based on experience or projections of the effects of new policies, META simulates a random stream of mail of different types; routes the mail through the system configuration being tested; and prints reports detailing total pieces handled, capacity utilization, work hours required, space requirements, and cost.

META has been used on several projects associated with the Postal Service corporate automation plan. These include facilities planning, benefits of alternative sorting plans, justification of efforts to enhance address readability, planning studies for reducing the time that carriers spend sorting vs. delivering, and identification of mail types that offer the greatest potential for cost savings.

According to the Associate Postmaster General, ". . . META became the vehicle to help steer our organization on an entirely new course at a speed we had never before experienced."

Source: Cebry, Michael E., Anura H. deSilva and Fred J. DiLisio. "Management Science in Automating Postal Operations: Facility and Equipment Planning in the United States Postal Service." *Interfaces*, vol. 22, no. 1, January–February 1992, pages 110–130.

Questions and Problems

1. Under what condition(s) is it appropriate to use simulation to analyze a model? That is, what characteristics should a model possess for simulation to be used?
2. The graph of the probability distribution of a normally distributed random variable with a mean of 20 and standard deviation of 3 is shown in Figure 12.8. The Excel function =NORMINV(Rand( ),20,3) also returns randomly generated observations from this distribution.
   a. Use Excel's NORMINV( ) function to generate 100 sample values from this distribution.
   b. Produce a histogram of the 100 sample values that you generated. Does your histogram look like the graph for this distribution in Figure 12.8?
   c. Repeat this experiment, with 1,000 sample values.
   d. Produce a histogram for the 1,000 sample values you generated. Does the histogram now more closely resemble the graph in Figure 12.8 for this distribution?
   e. Why does your second histogram look more "normal" than the first one?
3. Refer to the Hungry Dawg Restaurant example presented in this chapter. Health claim costs actually tend to be seasonal, with higher levels of claims occurring during the summer months (when kids are out of school and more likely to injure themselves) and during December (when people schedule elective procedures before the next year's deductible must be paid). The following table summarizes the seasonal adjustment factors that apply to RNGs for average claims in the Hungry Dawg problem. For instance, the average claim for month 6 should be multiplied by 115% and claims for month 1 should be multiplied by 80%.

Month	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Seasonal Factor	0.80	0.85	0.87	0.92	0.93	1.15	1.20	1.18	1.03	0.95	0.98	1.14

Suppose that the company maintains an account from which it pays health insurance claims. Assume that there is $2.5 million in the account at the beginning of month 1. Each month, employee contributions are deposited into this account and claims are paid from the account.

a. Modify the spreadsheet shown in Figure 12.9 to include the cash flows in this account. If the company deposits $3 million into this account every month, what is the probability that the account will have insufficient funds to pay claims at some point during the year? Use 5000 replications. (*Hint*: You can use the COUNTIF( ) function to count the number of months in a year in which the ending balance in the account is below 0.)
b. If the company wants to deposit an equal amount of money in this account each month, what should this amount be if they want there to only be a 5% chance of having insufficient funds?

4. One of the examples in this chapter dealt with determining the optimal reorder point for a computer monitor sold by Millennium Computer Corp. Suppose that it costs MCC $0.30 per day in holding costs for each monitor in beginning inventory, and it costs $20 to place an order. Each monitor sold generates a profit of $45, and each lost sale results in an opportunity cost of $65 (including the lost profit of $45 and $20 in lost goodwill). Modify the spreadsheet shown in Figure 12.31 to determine the reorder point and order quantity that maximize the average monthly profit associated with this monitor.
5. A debate recently erupted about the optimal strategy for playing a game on the TV show called "Let's Make a Deal." In one of the games on this show, the contestant would be given the choice of prizes behind three closed doors. A valuable prize was behind one door and worthless prizes were behind the other two doors. After the contestant selected a door, the host would open one of the two remaining doors to reveal one of the worthless prizes. Then, before opening the selected door, the host would give the contestant the opportunity to switch the selection to the other door that had not been opened. The question is, should the contestant switch?
   a. Suppose a contestant is allowed to play this game 500 times, always picks door number 1, and never switches when given the option. If the valuable prize is equally likely to be behind each door at the beginning of each play, how many times would the contestant win the valuable prize? Use simulation to answer this question.
   b. Now suppose the contestant is allowed to play this game another 500 times. This time the player always selects door number 1 initially and switches when given the option. Using simulation, how many times would the contestant win the valuable prize?
   c. If you were a contestant on this show, what would you do if given the option of switching doors?
6. Suppose that a product must go through an assembly line that consists of five sequential operations. The time it takes to complete each operation is normally distributed with a mean of 180 seconds and standard deviation of 5 seconds. Let X denote the cycle time for the line, so that after X seconds each operation is supposed to be finished and ready to pass the product to the next operation in the assembly line.
   a. If the cycle time $X = 180$ seconds, what is the probability that all five operations will be completed?
   b. What cycle time will ensure that all operations are finished 98% of the time?
7. Suppose that a product must go through an assembly line that consists of five sequential operations. The time it takes to complete each operation is normally

distributed with a mean of 180 seconds and standard deviation of 5 seconds. Define the flow time to be the total time it takes a product to go through the assembly line from start to finish.

a. What is the mean and standard deviation of the flow time? What is the probability that the total time will be less than 920 seconds?
b. Now assume that the time required to complete each operation has a 0.40 correlation with the operation time immediately preceding it. What is the mean and standard deviation of the flow time? What is the probability that the total time will be less than 920 seconds?
c. Now assume that the time required to complete each operation has a −0.40 correlation with the operation time immediately preceding it. What is the mean and standard deviation of the flow time? What is the probability that the total time will be less than 920 seconds?
d. Explain the effects of positive and negative correlations on the previous results.

8. WVTU is a television station that has 20 thirty-second advertising slots during its regularly scheduled programming each evening. The station is now selling advertising for the first few days in November. It could sell all the slots immediately for $4,500 each, but because November 7 will be Election Day, the station manager knows she might be able to sell slots at the last minute to political candidates in tight races for $8,000 each. The demand for these last minute slots is estimated as follows:

	Demand											
	8	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**
Probability	0.03	0.05	0.10	0.15	0.20	0.15	0.10	0.05	0.05	0.05	0.05	0.02

Slots not sold in advance and not sold to political candidates at the last minute can be sold to local advertisers for $2,000.

a. If the station manager sells all the advertising slots in advance, how much revenue will the station receive?
b. How many advertising slots should be sold in advance if the station manager wants to maximize expected revenue?
c. If the station manager sells in advance the number of slots identified in the previous question, what is the probability that the total revenue received will exceed the amount identified in part a where all slots are sold in advance?

9. The owner of a ski apparel store in Winter Park, CO, must decide in July how many ski jackets to order for the following ski season. Each ski jacket costs $54 each and can be sold during the ski season for $145. Any unsold jackets at the end of the season are sold for $45. The demand for jackets is expected to follow a Poisson distribution with a average rate of 80. The store owner can order jackets in lot sizes of 10 units.

a. How many jackets should the store owner order if she wants to maximize her expected profit?
b. What are the best-case and worst-case outcomes the owner might face for this product if she implements your suggestion?
c. How likely is it that the store owner will make at least $7,000 if she implements your suggestion?
d. How likely is it that the store owner will make between $6,000 to $7,000 if she implements your suggestion?

10. The owner of a golf shop in Myrtle Beach, SC, must decide how many sets of beginner golf clubs to order for the coming tourist season. Demand for golf clubs is random but follows a Poisson distribution with the average demand rates indicated in

the following table for each month. The expected selling price of the clubs also is shown for each month.

	May	June	July	August	September	October
Average Demand	60	90	70	50	30	40
Selling Price	$145	$140	$130	$110	$80	$60

In May, each set of clubs can be ordered at a cost of $75. This price is expected to drop 5% a month during the remainder of the season. Each month, the owner of the shop also gives away a free set of clubs to anyone who makes a hole-in-one from a short practice tee next to the shop. The number of people making a hole-in-one on this tee each month follows a Poisson distribution with a mean of 3. Any sets of clubs left over at the end of October are sold for $45 per set.

a. How many sets of clubs should the shop owner order if he wants to maximize the expected profit on this product?
b. What are the best-case and worst-case outcomes the owner might face on this product if he implements your suggestion?
c. How likely is it that the store owner will make at least $17,000 if he implements your suggestion?
d. How likely is it that the store owner will make between $12,000 to $14,000 if he implements your suggestion?
e. What percentage of the total demand for this product (excluding the free give-aways) will the owner be able to meet if he implements your suggestion?

11. Large Lots is planning a seven-day promotion on a discontinued model of 31" color television sets. At a price of $575 per set, the daily demand for this type of TV has been estimated as follows:

	Units Demanded per Day					
	0	1	2	3	4	5
Probability	0.15	0.20	0.30	0.20	0.10	0.05

Large Lots can order up to 50 of these TVs from a surplus dealer at a cost of $325. This dealer has offered to buy back any unsold sets at the end of the promotion for $250 each.

a. How many TVs should Large Lots order if it wants to maximize the expected profit on this promotion?
b. What is the expected level of profit?
c. Suppose the surplus dealer will buy back a maximum of only four sets at the end of the promotion. Would this change your answer? If so, how?

12. The monthly demand for the latest computer at Newland Computers follows a normal distribution with a mean of 350 and standard deviation of 75. Newland purchases these computers for $1,200 and sells them for $2,300. It costs the company $100 to place an order and $12 for every computer held in inventory at the end of each month. Currently, the company places an order for 1000 computers whenever the inventory at the end of a month falls below 100 units. Assume that the beginning inventory is 400 units, unmet demand in any month is lost to competitors, and orders placed at the end of one month arrive at the beginning of the next month.

a. Create a spreadsheet model to simulate the profit that the company will earn on this product over the next two years. Use 5000 replications. What is the average level of profit the company will earn?

b. Suppose that the company wants to determine the optimum reorder point and order quantity. Which combination of reorder point and order quantity will provide the highest average profit over the next two years?

13. The manager of Moore's Catalog Showroom is trying to predict how much revenue will be generated by each major department in the store during 2006. The manager has estimated the minimum and maximum growth rates possible for revenues in each department. The manager believes that any of the possible growth rates between the minimum and maximum values are equally likely to occur. These estimates are summarized in the following table:

		Growth Rates	
Department	**2005 Revenues**	**Minimum**	**Maximum**
Electronics	\$6,342,213	2%	10%
Garden Supplies	\$1,203,231	−4%	5%
Jewelry	\$4,367,342	−2%	6%
Sporting Goods	\$3,543,532	−1%	8%
Toys	\$4,342,132	4%	15%

Create a spreadsheet to simulate the total revenues that could occur in the coming year. Run 5000 replications of the model and do the following:

a. Construct a 95% confidence interval for the average level of revenues that the manager could expect for 2006.

b. According to your model, what are the chances that total revenues in 2006 will be more than 5% larger than those in 2005?

14. The Harriet Hotel in downtown Boston has 100 rooms that rent for \$150 per night. It costs the hotel \$30 per room in variable costs (cleaning, bathroom items, etc.) each night a room is occupied. For each reservation accepted, there is a 5% chance that the guest will not arrive. If the hotel overbooks, it costs \$200 to compensate guests whose reservations cannot be honored.

a. How many reservations should the hotel accept if it wants to maximize the average daily profit? Use 1000 replications for each reservation level that you consider.

15. Lynn Price recently completed her MBA and accepted a job with an electronics manufacturing company. Although she likes her job, she also is looking forward to retiring one day. To ensure that her retirement is comfortable, Lynn intends to invest \$3,000 of her salary into a tax-sheltered retirement fund at the end of each year. Lynn is not certain what rate of return this investment will earn each year, but she expects that each year's rate of return could be modeled appropriately as a normally distributed random variable with a mean of 12.5% and standard deviation of 2%.

a. If Lynn is 30 years old, how much money should she expect to have in her retirement fund at age 60? (Use 5000 replications.)

b. Construct a 95% confidence interval for the average amount that Lynn will have at age 60.

c. What is the probability that Lynn will have more than \$1 million in her retirement fund when she reaches age 60?

d. How much should Lynn invest each year if she wants there to be a 90% chance of having at least \$1 million in her retirement fund at age 60?

e. Suppose that Lynn contributes \$3,000 annually to her retirement fund for eight years and then terminates these annual contributions. How much of her salary would she have contributed to this retirement plan and how much money could she expect to have accumulated at age 60?

f. Now suppose that Lynn contributes nothing to her retirement fund for eight years and then begins contributing $3,000 annually until age 60. How much of her salary would she have contributed to this retirement plan and how much money could she expect to have accumulated at age 60?
g. What should Lynn (and you) learn from the answers to questions e and f?

16. Employees of Georgia-Atlantic are permitted to contribute a portion of their earnings (in increments of $500) to a flexible spending account from which they can pay medical expenses not covered by the company's health insurance program. Contributions to an employee's "flex" account are not subject to income taxes. However, the employee forfeits any amount contributed to the "flex" account that is not spent during the year. Suppose Greg Davis makes $60,000 per year from Georgia-Atlantic and pays a marginal tax rate of 33%. Greg and his wife estimate that in the coming year their normal medical expenses not covered by the health insurance program could be as small as $500, as large as $5,000, and most likely about $1,300. However, Greg also believes there is a 5% chance that an abnormal medical event could occur which might add $10,000 to the normal expenses paid from their flex account. If their uncovered medical claims exceed their contribution to their "flex" account, they will have to cover these expenses with the after-tax money Greg brings home.
    a. Use simulation to determine the amount of money Greg should contribute to his flexible spending account in the coming year if he wants to maximize his disposable income (after taxes and all medical expenses are paid). Use 5000 replications for each level of "flex" account contribution you consider.

17. Acme Equipment Company is considering the development of a new machine that would be marketed to tire manufacturers. Research and development costs for the project are expected to be about $4 million but could vary between $3 and $6 million. The market life for the product is estimated to be 3 to 8 years with all intervening possibilities being equally likely. The company thinks it will sell 250 units per year, but acknowledges that this figure could be as low as 50 or as high as 350. The company will sell the machine for about $23,000. Finally, the cost of manufacturing the machine is expected to be $14,000 but could be as low as $12,000 or as high as $18,000. The company's cost of capital is 15%.
    a. Use appropriate RNGs to create a spreadsheet to calculate the possible net present values (NPVs) that could result from taking on this project.
    b. Replicate the model 5000 times. What is the expected NPV for this project?
    c. What is the probability of this project generating a positive NPV for the company?

18. Representatives from the American Heart Association are planning to go door-to-door throughout a community, soliciting contributions. From past experience, they know that when someone answers the door, 80% of the time it is a female and 20% of the time it is a male. They also know that 70% of the females who answer the door make a donation, whereas only 40% of the males who answer the door make donations. The amount of money that females contribute follows a normal distribution with a mean of $20 and standard deviation of $3. The amount of money that males contribute follows a normal distribution with a mean of $10 and standard deviation of $2.
    a. Create a spreadsheet model that simulates what might happen whenever a representative of the American Heart Association knocks on a door and someone answers.
    b. Replicate your model 5000 times. What is the average contribution the Heart Association can expect to receive when someone answers the door?

c. Suppose that the Heart Association plans to visit 300 homes on a given Saturday. If no one is home at 25% of the residences, what is the total amount that the Heart Association can expect to receive in donations?

19. Techsburg, Inc. uses a stamping machine to manufacture aluminum bodies for light-weight miniature aircraft used for military reconnaissance. Currently, forms in the stamping machine are changed after every 60 hours of operation or whenever a form breaks, whichever happens first. The lifetime of each form follows a Weibull distribution with location, scale, and shape parameters of 50, 25, and 2, respectively. The machine is operated 5840 hours per year. It costs $800 to replace the stamping forms. If a form breaks before its scheduled replacement time (or in less than 60 hours of use), the shop loses 8 hours of production time. However, if a form lasts until its scheduled replacement after 60 hours of use, the shop only loses 2 hours of production time. The company estimates that each hour of lost production time costs $1200.
    a. On average, how much does Techsburg spend maintaining this stamping machine per year?
    b. Suppose Techsburg wanted to minimize its total maintenance cost for the stamping machine. How often should the company plan on changing the stamping forms and how much money would it save?
    c. Suppose the cost to replace the stamping forms is expected to increase. What impact should this have on the optimal planned replacement time of the forms? Explain.
    d. Suppose the cost of lost production time is increased. What effect should this have on the optimal planned replacement time of the forms? Explain.

20. After spending ten years as an assistant manager for a large restaurant chain, Ray Clark has decided to become his own boss. The owner of a local submarine sandwich store wants to sell the store to Ray for $65,000, to be paid in installments of $13,000 in each of the next five years. According to the current owner, the store brings in revenue of about $110,000 per year and incurs operating costs of about 63% of sales. Thus, once the store is paid for, Ray should make about $35,000–$40,000 per year before taxes. Until the store is paid for, he will make substantially less—but he will be his own boss. Realizing that some uncertainty is involved in this decision, Ray wants to simulate what level of net income he can expect to earn during the next five years as he operates and pays for the store. In particular, he wants to see what could happen if sales are allowed to vary uniformly between $90,000 and $120,000, and if operating costs are allowed to vary uniformly between 60% and 65% of sales. Assume that Ray's payments for the store are not deductible for tax purposes and that he is in the 28% tax bracket.
    a. Create a spreadsheet model to simulate the annual net income Ray would receive during each of the next five years if he decides to buy the store.
    b. Given the money he has in savings, Ray thinks he can get by for the next five years if he can make at least $12,000 from the store each year. Replicate the model 5000 times and track: 1) the minimum amount of money Ray makes over the five-year period represented by each replication, and 2) the total amount Ray makes during the five-year period represented by each replication.
    c. What is the probability that Ray will make at least $12,000 in each of the next five years?
    d. What is the probability that Ray will make at least $60,000 total over the next five years?

21. Road Racer Sports, Inc. is a mail-order business dedicated to the running enthusiast. The company sends out full-color catalogs several times a year to several hundred

thousand people on its mailing list. Production and mailing costs are fairly expensive for direct mail advertising, averaging about $3.25 per catalog. As a result, management does not want to continue sending catalogs to persons who do not buy enough to cover the costs of the catalogs they receive. Currently, the company removes customers from their mailing list if they receive six consecutive catalogs without placing an order. The following table summarizes the probability of a customer placing an order.

Last Order	Prob. of Order
1 catalog ago	0.40
2 catalogs ago	0.34
3 catalogs ago	0.25
4 catalogs ago	0.17
5 catalogs ago	0.09
6 catalogs ago	0.03

According to the first row in this table, if customers receive a catalog and place an order, there is a 40% chance they will place another order when they receive their next catalog. The second row indicates there is a 34% chance that customers will receive a catalog, place an order, and then not order again until they receive two more catalogs. The remaining rows in this table have similar interpretations.

a. How much profit must the company earn on an average order in order to cover the cost of printing and distributing the catalogs?
b. Approximately what percentage of the names on the mailing list will be purged before each catalog mailing?

22. Sammy Slick works for a company that allows him to contribute up to 10% of his earnings into a tax-deferred savings plan. The company matches a portion of the contributions its employees make based on the organization's financial performance. Although the minimum match is 25% of the employee's contributions and the maximum match is 100%, in most years the company match is about 50%. Sammy is currently 30 years old and makes $35,000. He wants to retire at age 60. He expects his salary to increase in any given year to be at least 2% per year, at most 6%, and most likely 3%. The funds contributed by Sammy and his employer are invested in mutual funds. Sammy expects the annual return on his investments to vary according to a normal distribution with a mean of 12.5% and standard deviation of 2%.
    a. If Sammy contributes 10% of his income to this plan, how much money could he expect to have at age 60?
    b. Suppose Sammy makes 10% contributions to this plan for eight years, from age 30 to 37, and then stops contributing. How much of his own money would he have invested and how much money could he expect to have at age 60?
    c. Now suppose Sammy contributes nothing to the plan his first eight years and then contributes 10% for 23 years from age 38 to age 60. How much of his own money would he have invested and how much money could he expect to have at age 60?
    d. What do you learn from Sammy's example?
23. Podcessories manufactures several accessories for a popular digital music player. The company is trying to decide whether to discontinue one of the items in this product line. Discontinuing the item would save the company $600,000 in fixed costs (consisting of leases on building and machinery) during the coming year. However, the company is anticipating that it might receive an order for 60,000 units from a large discount retailer that could be very profitable. Unfortunately, the company is

being forced to decide about renewing the leases required to continue this item before knowing if it will receive the large order from the discount retailer. The variable cost per unit for this item is $6. The regular selling price of the item is $12 per unit. However, the company has offered the discount retailer a price of $10.50 per unit due to the size of its potential order. Podcessories believes there is a 60% chance it will receive the order from the discount retailer. Additionally, it believes general demand for this product (apart from the discount retailer's order) will vary between 45,000 to 115,000 units with a most likely outcome of 75,000 units.

a. Create a spreadsheet model for this problem.
b. How much money might the company lose next year (worst case) if it continues this line?
c. How much money might the company make next year (best case) if it continues this line?
d. If the company loses money, on average how much could it expect to lose? (*Hint*: Use the filter option under the forecast window's preferences command.)
e. If the company makes money, on average how much could it expect to make? (*Hint*: Use the filter option under the forecast window's preferences command.)
f. What other actions might you suggest this company take to improve its chance of making a decision with a good outcome?

24. Bob Davidson owns a newsstand outside the Waterstone office building complex in Atlanta, near Hartsfield International Airport. He buys his papers wholesale at $0.50 per paper and sells them for $0.75. Bob wonders what is the optimal number of papers to order each day. Based on history, he has found that demand (even though it is discrete) can be modeled by a normal distribution with a mean of 50 and standard deviation of 5. When he has more papers than customers, he can recycle all the extra papers the next day and receive $0.05 per paper. On the other hand, if he has more customers than papers, he loses some goodwill in addition to the lost profit on the potential sale of $0.25. Bob estimates the incremental lost goodwill costs five days' worth of business (that is, dissatisfied customers will go to a competitor the next week, but come back to him the week after that).
    a. Create a spreadsheet model to determine the optimal number of papers to order each day. Use 5000 replications and round the demand values generated by the normal RNG to the closest integer value.
    b. Construct a 95% confidence interval for the expected payoff from the optimal decision.
25. Vinton Auto Insurance is trying to decide how much money to keep in liquid assets to cover insurance claims. In the past, the company held some of the premiums it received in interest-bearing checking accounts and put the rest into investments that are not quite as liquid, but tend to generate a higher investment return. The company wants to study cash flows to determine how much money it should keep in liquid assets to pay claims. After reviewing historical data, the company determined that the average repair bill per claim is normally distributed with a mean of $1,700 and standard deviation of $400. It also determined that the number of repair claims filed each week is a random variable that follows the probability distribution shown in the following table:

Number of Claims	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Probability	0.1	0.1	0	0.2	0.3	0.1	0.1	0.1	0.1

In addition to repair claims, the company also receives claims for cars that have been "totaled" and cannot be repaired. A 20% chance of receiving this type of claim exists in any week. These claims for "totaled" cars typically cost anywhere from $2,000 to $35,000, with $13,000 being the most common cost.

a. Create a spreadsheet model of the total claims cost incurred by the company in any week.
b. Replicate the model 5000 times and create a histogram of the distribution of total cost values that were generated.
c. What is the average cost that the company should expect to pay each week?
d. Suppose that the company decides to keep $20,000 cash on hand to pay claims. What is the probability that this amount would not be adequate to cover claims in any week?
e. Create a 95% confidence interval for the true probability of claims exceeding $20,000 in a given week.

26. Executives at Meds-R-Us have decided to build a new production facility for the company's best-selling high-blood-pressure drug. The problem they now face is determining the size of the facility (in terms of production capacity). Last year, the company sold 1,085,000 units of this drug at a price of $13 per unit. They estimate the demand for the drug to be normally distributed with a mean increasing by approximately 59,000 units per year over the next 10 years with a standard deviation of 30,000 units. They expect the price of the drug to increase with inflation at a rate of 3% per year. Variable production costs currently are $9 per unit and are expected to increase in future years at the rate of inflation. Other operating costs are expected to be $1.50 per unit of capacity in the first year of operation, increasing at the rate of inflation in subsequent years. The plant construction cost is expected to be $18 million for 1 million units of annual production capacity. The company can increase the annual production capacity above this level at a cost of $12 per unit of additional capacity. Assume that the company must pay for the plant when it is completed and all other cash flows occur at the end of each year. The company uses a 10% discount rate on cash flows for financial decisions.

a. Create a spreadsheet model to compute the net present value (NPV) for this decision.
b. What is the expected NPV for a plant with a production capacity of 1.2 million units per year?
c. What is the expected NPV for a plant with a production capacity of 1.4 million units per year?
d. How large a plant should the company build if they want to be 90% certain of obtaining a positive NPV for this project?

27. The owner of a local car dealership has just received a call from a regional distributor stating that a $5,000 bonus will be awarded if the owner's dealership sells at least 10 new cars next Saturday. On an average Saturday, this dealership has 75 potential customers look at new cars, but there is no way to determine exactly how many customers will come this particular Saturday. The owner is fairly certain that the number would not be less than 40, but also thinks it would be unrealistic to expect more than 120 (which is the largest number of customers ever to show up in one day). The owner determined that, on average, about one out of ten customers who look at cars at the dealership actually purchases a car—or, a 0.10 probability (or 10% chance) exists that any given customer will buy a new car.

a. Create a spreadsheet model for this problem and generate 5000 random outcomes for the number of cars the dealership might sell next Saturday.
b. What is the probability that the dealership will earn the $5,000 bonus?
c. If you were this dealer, what is the maximum amount of money you would be willing to spend on sales incentives to try to earn this bonus?

28. Dr. Sarah Benson is an ophthalmologist who, in addition to prescribing glasses and contact lenses, performs optical laser surgery to correct nearsightedness. This surgery is fairly easy and inexpensive to perform. Thus, it represents a potential gold mine for her practice. To inform the public about this procedure, Dr. Benson advertises in the local paper and holds information sessions in her office one night a week at which she shows a videotape about the procedure and answers any questions that potential patients might have. The room where these meetings are held can seat ten people, and reservations are required. The number of people attending each session varies from week to week. Dr. Benson cancels the meeting if two or fewer people have made reservations. Using data from the previous year, Dr. Benson determined that the distribution of reservations is as follows:

Number of Reservations	0	1	2	3	4	5	6	7	8	9	10
Probability	0.02	0.05	0.08	0.16	0.26	0.18	0.11	0.07	0.05	0.01	0.01

Using data from the past year, Dr. Benson determined that each person who attends an information session has a 0.25 probability of electing to have the surgery. Of those who do not, most cite the cost of the procedure—$2,000—as their major concern.

a. On average, how much revenue does Dr. Benson's practice in laser surgery generate each week? (Use 5000 replications.)
b. On average, how much revenue would the laser surgery generate each week if Dr. Benson did not cancel sessions with two or fewer reservations?
c. Dr. Benson believes that 40% of the people attending the information sessions would have the surgery if she reduced the price to $1,500. Under this scenario, how much revenue could Dr. Benson expect to realize per week from laser surgery?

29. Calls to the 24-hour customer support line for Richman Financial Services occur randomly following a Poisson distribution with the following average rates during different hours of the day:

Time Period	Avg Calls Per Hour	Time Period	Avg Calls Per Hour
Midnight–1 A.M.	2	Noon–1 P.M.	35
1 A.M.–2 A.M.	2	1 P.M.–2 P.M.	20
2 A.M.–3 A.M.	2	2 P.M.–3 P.M.	20
3 A.M.–4 A.M.	4	3 P.M.–4 P.M.	20
4 A.M.–5 A.M.	4	4 P.M.–5 P.M.	18
5 A.M.–6 A.M.	8	5 P.M.–6 P.M.	18
6 A.M.–7 A.M.	12	6 P.M.–7 P.M.	15
7 A.M.–8 A.M.	18	7 P.M.–8 P.M.	10
8 A.M.–9 A.M.	25	8 P.M.–9 P.M.	6
9 A.M.–10 A.M.	30	9 P.M.–10 P.M.	5
10 A.M.–11 A.M.	25	10 P.M.–11 P.M.	4
11 A.M.–Noon	20	11 P.M.–Midnight	2

The Richman customer service representatives spend approximately seven minutes on each call and are assigned to work eight-hour shifts that begin at the top of each hour. Richman wants to ensure that, on average, they can provide a 98% service level.

a. Determine the customer service schedule that allows Richman to achieve its service level objective using the fewest number of employees.
b. According to your solution, how many customer service representatives should Richman employ and how should they be scheduled?

30. A European call option gives a person the right to buy a particular stock at a given price (the strike price) on a specific date in the future (the expiration date). This type of call option typically is sold at the net present value of the expected value of the option on its expiration date. Suppose you own a call option with a strike price of \$54. If the stock is worth \$59 on the expiration date, you would exercise your option and buy the stock, making a \$5 profit. On the other hand, if the stock is worth \$47 on the expiration date, you would not exercise your option and make \$0 profit. Researchers have suggested the following model for simulating the movement of stock prices:

$$P_{k+1} = P_k\,(1 + (\mu t + z\sigma\sqrt{t})$$

where:

P_k = price of the stock at time period k

$\mu = v + 0.5\sigma^2$

v = the stock's expected annual growth rate

σ = the standard deviation on the stock's annual growth rate

t = time period interval (expressed in years)

z = a random observation from a normal distribution with mean 0 and standard deviation of 1.

Suppose that a stock has an initial price (P_0) of \$80, an expected annual growth rate (v) of 15%, and a standard deviation (σ) of 25%.

a. Create a spreadsheet model to simulate this stock's price behavior for the next 13 weeks (note $t = 1/52$ because the time period is weekly).
b. Suppose you are interested in purchasing a call option with a strike price of \$75 and an expiration date at week 13. On average, how much profit would you earn with this option? (Use 5000 replications.)
c. Assume that a risk-free discount rate is 6%. How much should you be willing to pay for this option today? (*Hint*: Use Excel's NPV function.)
d. If you purchase the option, what is the probability that you will make a profit?

31. Refer to the previous question. Another type of option is the Asian option. Its payoff is not based on the price of the stock on the expiration date but, instead, on the average price of the stock over the lifetime of the option.
Suppose a stock has an initial price (P_0) of \$80, an expected annual growth rate (v) of 15%, and a standard deviation (σ) of 25%.

a. Create a spreadsheet model to simulate this stock's price behavior for the next 13 weeks (note $t = 1/52$ because the time period is weekly).
b. Suppose you are interested in purchasing a call option with a strike price of \$75 and an expiration date at week 13. On average, how much profit would you earn with this option? (Use 5000 replications.)
c. Assume a risk-free discount rate is 6%. How much should you be willing to pay for this option today? (*Hint*: Use Excel's NPV function.)
d. If you purchase the option, what is the probability that you will make a profit?

32. Amanda Green is interested in investing in the following set of mutual funds whose returns are all normally distributed with the indicated means and standard deviations:

	Windsor	**Columbus**	**Vanguard**	**Integrity**	**Nottingham**
Mean	17.0%	14.0%	11.0%	8.0%	5.0%
Std Dev	9.0%	6.5%	5.0%	3.5%	2.0%

The correlations between the mutual funds are as follows:

	Windsor	Columbus	Vanguard	Integrity	Nottingham
Windsor	1	0.1	0.05	0.3	0.6
Columbus		1	0.2	0.15	0.1
Vanguard			1	0.1	0.2
Integrity				1	0.4
Nottingham					1

a. What is the expected return and standard deviation on a portfolio where Amanda invests her money equally in all five mutual funds?
b. Suppose Amanda is willing to assume the risk associated with a 5% standard deviation in returns on her portfolio. What portfolio will give her the greatest expected return for this level of risk?
c. Construct the efficient frontier for this portfolio. How would you explain this graph to Amanda?

33. Martin manufacturing company uses a piece of machinery that has three different bushings that periodically fail in service. The probability distribution of the life of each bushing is identical and is summarized by the following table.

	Bushing Life (operating hours)									
	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900
Probability	0.08	0.13	0.25	0.15	0.12	0.09	0.07	0.05	0.04	0.02

When a bushing fails, the machine stops, a repair person is called, and a new bushing is installed. Each bushing costs $35. Downtime for the machine costs the company an estimated $15 per minute. The direct on-site cost for the repair person is $50 per hour. The amount of time required for the repair person to arrive after a bushing fails is approximately normally distributed with a mean of 10 minutes and standard deviation of 2 minutes. The amount of time required to change a bushing follows a triangular distribution with minimum, most likely, and maximum values of 15, 20, and 30 minutes, respectively.
a. On average, what is the total bushing-related cost that Martin incurs to operate this machine for 20,000 hours?
b. Martin is considering implementing another repair policy: if any of the bushings fail, all three are replaced. What is the total bushing-related cost that Martin incurs to operate this machine for 20,000 hours under this policy?
c. Should Martin implement the new policy?

34. Michael Abrams runs a specialty clothing store that sells collegiate sports apparel. One of his primary business opportunities involves selling custom screenprinted sweatshirts for college football bowl games. He is trying to determine how many sweatshirts to produce for the upcoming Tangerine Bowl game. During the month before the game, Michael plans to sell his sweatshirts for $25 apiece. At this price, he believes the demand for sweatshirts will be triangularly distributed with a minimum demand of 10,000, maximum demand of 30,000 and a most likely demand of 18,000. During the month after the game, Michael plans to sell any remaining sweatshirts for $12 apiece. At this price, he believes the demand for sweatshirts will be triangularly distributed with a minimum demand of 2,000, maximum demand of 7,000, and a most likely demand of 5,000. Two months after the game, Michael plans to sell any remaining sweatshirts to a surplus store that has agreed to buy up to 2,000

sweatshirts for a price of $3 per shirt. Michael can order custom screenprinted sweatshirts for $8 apiece in lot sizes of 3,000.

a. On average, how much profit would Michael earn if he orders 18,000 sweatshirts? Use 5000 replications.
b. How many sweatshirts should he order if he wants to maximize his expected profit? Again use 5000 replications in each simulation you perform.

35. The Major Motors Corporation is trying to decide whether to introduce a new mid-size car. The directors of the company only want to produce the car if it has at least an 80% chance of generating a positive net present value over the next ten years. If the company decides to produce the car, it will have to pay an uncertain initial start-up cost that is estimated to follow a triangular distribution with a minimum value of $2 billion, maximum value of $2.4 billion, and a most likely value of $2.1 billion. In the first year, the company would produce 100,000 units. Demand during the first year is uncertain but expected to be normally distributed with a mean of 95,000 and standard deviation of 7,000. For any year in which the demand exceeds production, production will be increased by 5% in the following year. For any year in which the production exceeds demand, production will be decreased by 5% in the next year, and the excess cars will be sold to a rental car company at a 20% discount. After the first year, the demand in any year will be modeled as a normally distributed random variable with a mean equal to the actual demand in the previous year and standard deviation of 7,000. In the first year, the sales price of the car will be $13,000 and the total variable cost per car is expected to be $9,500. Both the selling price and variable cost is expected to increase each year at the rate of inflation, which is assumed to be uniformly distributed between 2% and 7%. The company uses a discount rate of 9% to discount future cash flows.

    a. Create a spreadsheet model for this problem and replicate it 5000 times. What is the minimum, average, and maximum NPV Major Motors can expect if it decides to produce this car? (*Hint*: Consider using the NPV( ) function to discount the profits Major Motors would earn each year.)
    b. What is the probability of Major Motors earning a positive NPV over the next ten years?
    c. Should Major Motors produce this car?

36. Each year, the Schriber Corporation must determine how much to contribute to the company's pension plan. The company uses a ten-year planning horizon to determine the contribution which, if made annually in each of the next ten years, would allow for only a 10% chance of the fund running short of money. The company then makes that contribution in the current year and repeats this process in each subsequent year to determine the specific amount to contribute each year. (Last year, the company contributed $23 million to the plan.) The pension plan covers two types of employees: hourly and salaried. In the current year, there will be 6,000 former hourly employees and 3,000 former salaried employees receiving benefits from the plan. The change in the number of retired hourly employees from one year to the next is expected to vary according to a normal distribution with a mean of 4% and standard deviation of 1%. The change in the number of retired salaried employees from one year to the next is expected to vary between 1% and 4% according to a truncated normal distribution with a mean of 2% and standard deviation of 1%. Currently, hourly retirees receive an average benefit of $15,000 per year, whereas salaried retirees receive an average annual benefit of $40,000. Both of these averages are expected to increase annually with the rate of inflation, which is assumed to vary between 2% and 7% according to a triangular distribution with a most likely value of 3.5%. The current balance in the company's pension fund is $1.5 billion. Investments in this

fund earn an annual return that is assumed to be normally distributed with a mean of 12% and standard deviation of 2%. Create a spreadsheet model for this problem and use simulation to determine the pension fund contribution that the company should make in the current year. What is your recommendation?

CASE 12.1 Live Well, Die Broke

(Inspired by a presentation given by Dr. John Charnes, University of Kansas, at the 2005 Crystal Ball Users Group meeting.)

For investment advisors, a major consideration in planning for a client in retirement is the determination of a withdrawal amount that will provide the client with the funds necessary to maintain a desired standard of living throughout the client's remaining lifetime. If a client withdraws too much or if investment returns fall below expectations, there is a danger of either running out of funds or reducing the desired standard of living. A sustainable retirement withdrawal is the inflation-adjusted monetary amount a client can withdraw periodically from retirement funds for an assumed planning horizon. This amount cannot be determined with complete certainty because of the random nature of investment returns. Usually, the sustainable retirement withdrawal is determined by limiting the probability of running out of funds to some specified level, such as 5%. The sustainable retirement withdrawal amount typically is expressed as a percentage of the initial value of the assets in the retirement portfolio, but is actually the inflation-adjusted monetary amount that the client would like each year for living expenses.

Assume that an investment advisor, Roy Dodson, is assisting a widowed client in determining a sustainable retirement withdrawal. The client is a 59 year-old woman who will turn 60 in two months. She has $1,000,000 in a tax-deferred retirement account that will be the primary source of her retirement income. Roy has designed a portfolio for his client with returns that he expects to be normally distributed with a mean of 8% and a standard deviation of 2%. Withdrawals will be made at the beginning of each year on the client's birthday.

Roy assumes that the inflation rate will be 3%, based on long-term historic data. So if her withdrawal at the beginning of the first year is $40,000, her inflation-adjusted withdrawal at the beginning of the second year will be $41,200, and third year's withdrawal will be $42,436, etc.

For his initial analysis, Roy wants to assume that his client will live until age 90. In consultation with his client, he also wants to limit the chance that she will run out of money before her death to a maximum of 5%.

a. What is the maximum amount that Roy should advise his client to withdraw on her 60th birthday? If she lives until age 90, how much should the client expect to leave to her heirs?
b. Roy is now concerned about basing his analysis on the assumption that his client will live to age 90. After all, she is healthy and might live to be 110, or she could be in a car accident and die at age 62. To account for this uncertainty in the client's age at death, Roy would like to model the client's remaining life expectancy as a random variable between 0 and 50 years that follows a lognormal distribution with a mean of 20 and standard deviation of 10 (rounded to the nearest integer). Under this assumption, what is the maximum amount that Roy should advise his client to withdraw on her 60th birthday and how much should the client expect to leave to her heirs? *Hint:* Modify your spreadsheet to accommodate ages up to 110 and use a VLOOKUP( ) function to return the client's ending balance in her randomly determined year of death.)

c. Roy is pleased to now be modeling the uncertainty in his client's life expectancy. But he is now curious about limiting to 5% the chance that his client will run out of money before her death. In particular, he is wondering how sensitive the sustainable withdrawal amount is to changes in this 5% assumption. To answer this question, create an efficient frontier showing the maximum sustainable withdrawal amount as the chance of running out of money is varied from 1% to 10%. How should Roy explain the meaning of this chart to his client?
d. Suppose that Roy's client has three children and wants there to be a 95% chance that they will each inherit at least $250,000 when she dies. Under this assumption, what is the maximum amount that Roy should advise his client to withdraw on her 60th birthday and how much should the client expect to leave to her heirs?

Death and Taxes

CASE 12.2

Benjamin Franklin once said, "In this world nothing is certain but death and taxes." Although that might be true, there is often great uncertainty involved in when one will encounter death and how much one must pay in taxes before arriving there. Another Benjamin made a very significant contribution toward assessing the uncertainty associated with both death and taxes. Benjamin Gompertz (1779–1865) was a British mathematician who, by studying Mediterranean fruit flies, theorized that mortality rates increase at an exponential rate as age increases (*i.e.*, as an organism gets older, its chance of dying per unit of time increases exponentially). Gompertz's Law of Mortality has since become a cornerstone of actuarial and financial planning activities.

In a group of people of a given age (for example, 65), some proportion of those people will not live another year. Let q_x represent the proportion of people of age x who will die before reaching age $x + 1$. The value q_x is sometimes referred to as the ***mortality rate*** at age x. The following formula, based on Gompertz's Law, is sometimes used to model mortality rates.

$$q_x = 1 - \text{EXP}\left(\frac{(\text{LN}(1-q_{x-1}))^2}{\text{LN}(1-q_{x-2})}\right)$$

Mortality rates play an important role in numerous financial planning and retirement decisions. For instance, most individuals do not want to retire unless they are reasonably certain that they have enough assets to sustain themselves financially for the rest of their life. The uncertainties associated with this sort of decision create a perfect application for spreadsheet simulation.

The following questions give you the opportunity to explore several issues that actuaries and financial planners face on a daily basis. Assume the mortality rates for males at ages 63 and 64 are $q_{63} = 0.0235$ and $q_{64} = 0.0262$, respectively, and those of females at ages 63 and 64 are $q_{63} = 0.0208$ and $q_{64} = 0.0225$, respectively.

a. On average, to what age should a 65-year-old male expect to live?
b. What is the probability of a 65-year-old male living to at least age 80?
c. What is the probability of a 65-year-old male living to exactly age 80?
d. On average, to what age should a 70-year-old male expect to live?
e. What is the probability of a 70-year-old male living to at least age 80?
f. What is the probability of a 70-year-old male living to exactly age 80?
g. Suppose a 65-year-old male has $1,200,000 in retirement investments earning an 8% interest rate. Assume he intends to withdraw $100,000 in his first year of retirement and 3% more in subsequent years to adjust for inflation. Annual interest earnings are credited on the beginning balance minus one half the amount withdrawn. For

example, in the first year interest earnings would be $0.08 \times (\$1{,}200{,}000 - \$100{,}000/2) = \$92{,}000$. What is the probability that this individual would outlive his retirement assets (assuming he spends all that he withdraws each year)?

h. Refer to the previous question. Suppose the interest rate each year can be modeled as a normally distributed random variable with a mean of 8% and standard deviation of 1.5%. Further suppose that the rate of inflation each year can be described as a random variable following a triangular distribution with minimum, most likely, and maximum values of 2%, 3%, and 5%, respectively. Under these conditions, what is the probability that this individual would outlive his retirement assets (assuming that he spends all he withdraws each year)?

i. Suppose that the person described in the previous question has a 65-year-old wife who is joint owner of the retirement assets described earlier. What is the probability that the retirement assets would be depleted before both spouses die (assuming they spend all they withdraw each year)?

j. Refer to the previous question. How much money should this couple plan on withdrawing in the first year if they want there to be a maximum of a 5% chance of depleting their retirement assets before they both die?

CASE 12.3 The Sound's Alive Company

(Contributed by Dr. Jack Yurkiewicz, Lubin School of Business, Pace University.)

Marissa Jones is the president and CEO of Sound's Alive, a company that manufactures and sells a line of speakers, CD players, receivers, high-definition televisions, and other items geared for the home entertainment market. Respected throughout the industry for bringing many high-quality, innovative products to market, Marissa is considering adding a speaker system to her product line.

The speaker market has changed dramatically during the last several years. Originally, high-fidelity aficionados knew that to reproduce sound covering the fullest range of frequencies—from the lowest kettle drum to the highest violin—a speaker system had to be large and heavy. The speaker had various drivers: a woofer to reproduce the low notes, a tweeter for the high notes, and a mid-range driver for the broad spectrum of frequencies in between. Many speaker systems had a minimum of three drivers, but some had even more. The trouble was that such a system was too large for anything but the biggest rooms, and consumers were reluctant to spend thousands of dollars and give up valuable wall space to get the excellent sound these speakers could reproduce.

The trend has changed during the past several years. Consumers still want good sound, but they want it from smaller boxes. Therefore, the satellite system became popular. Consisting of two small boxes that house either one driver (to cover the mid-range and high frequencies) or two (a mid-range and tweeter), a satellite system can be mounted easily on walls or shelves. To reproduce the low notes, a separate subwoofer that is approximately the size of a cube 18 inches on a side also is needed. This subwoofer can be placed anywhere in the room. Taking up less space than a typical large speaker system and sounding almost as good, yet costing hundreds of dollars less, these satellite systems are hot items in the high-fidelity market.

Recently, the separate wings of home entertainment—high fidelity (receivers, speakers, CD players, CDs, cassettes, and so on), television (large screen monitors, video cassette recorders, laser players), and computers (games with sounds, virtual reality software, and so on)—have merged into the home theater concept. To simulate the movie environment, a home theater system requires the traditional stereo speaker system plus additional speakers placed in the rear of the room so that viewers are literally surrounded with sound. Although the rear speakers do not have to match the high quality of the front speakers and, therefore, can be less expensive, most consumers choose a system in which the front and rear speakers are of equal quality, reproducing the full range of frequencies with equal fidelity.

This is the speaker market that Marissa wants to enter. She is considering having Sound's Alive manufacture and sell a home theater system that consists of seven speakers. Three small speakers—each with one dome tweeter that could reproduce the frequency range of 200 Hertz to 20,000 Hertz (upper-low frequencies to the highest frequencies)—would be placed in front, and three similar speakers would be placed strategically around the sides and back of the room. To reproduce the lowest frequencies (from 35 Hertz to 200 Hertz), a single subwoofer also would be part of the system. This subwoofer is revolutionary because it is smaller than the ordinary subwoofer, only 10 inches per side, and it has a built-in amplifier to power it. Consumers and critics are thrilled with the music from early prototype systems, claiming that these speakers have the best balance of sound and size. Marissa is extremely encouraged by these early reviews, and although her company has never produced a product with its house label on it (having always sold systems from established high-fidelity companies), she believes that Sound's Alive should enter the home theater market with this product.

Phase One: Projecting Profits

Marissa decides to create a spreadsheet that will project profits over the next several years. After consulting with economists, market analysts, employees in her own company, and employees from other companies that sell house brand components, Marissa is confident that the gross revenues for these speakers in 2007 would be around $6 million. She also must figure that a small percentage of speakers will be damaged in transit, or some will be returned by dissatisfied customers shortly after the sales. These returns and allowances (R&As) usually are calculated as 2% of the gross revenues. Hence, the net revenues are simply the gross revenues minus the R&As. Marissa believes that the 2007 labor costs for these speakers will be $995,100. The cost of materials (including boxes to ship the speakers) should be $915,350 for 2007. Finally, her overhead costs (rent, lighting, heating in winter, air conditioning in summer, security, and so on) for 2007 should be $1,536,120. Thus, the cost of goods sold is the sum of labor, material, and overhead costs. Marissa figures the gross profit as the difference between the net revenues and the cost of goods sold. In addition, she must consider the selling, general, and administrative (SG&A) expenses. These expenses are more difficult to estimate, but the standard industry practice is to use 18% of the net revenues as the nominal percentage value for these expenses. Therefore, Marissa's profit *before taxes* is the gross profit minus the SG&A value. To calculate taxes, Marissa multiplies her profits before taxes times the tax rate, currently 30%. If her company is operating at a loss, however, no taxes would have to be paid. Finally, Marissa's net (or after tax) profit is simply the difference between the profit before taxes and the actual taxes paid.

To determine the numbers for 2008 through 2010, Marissa assumes that gross revenues, labor costs, material costs, and overhead costs will increase over the years. Although the rates of increase for these items are difficult to estimate, Marissa figures that gross revenues will increase by 9% per year, labor costs will increase by 4% per year, material costs will increase by 6% per year, and overhead costs will increase by 3% per year. She figures that the tax rate will not change from the 30% mark, and she assumes that the SG&A value will remain at 18%.

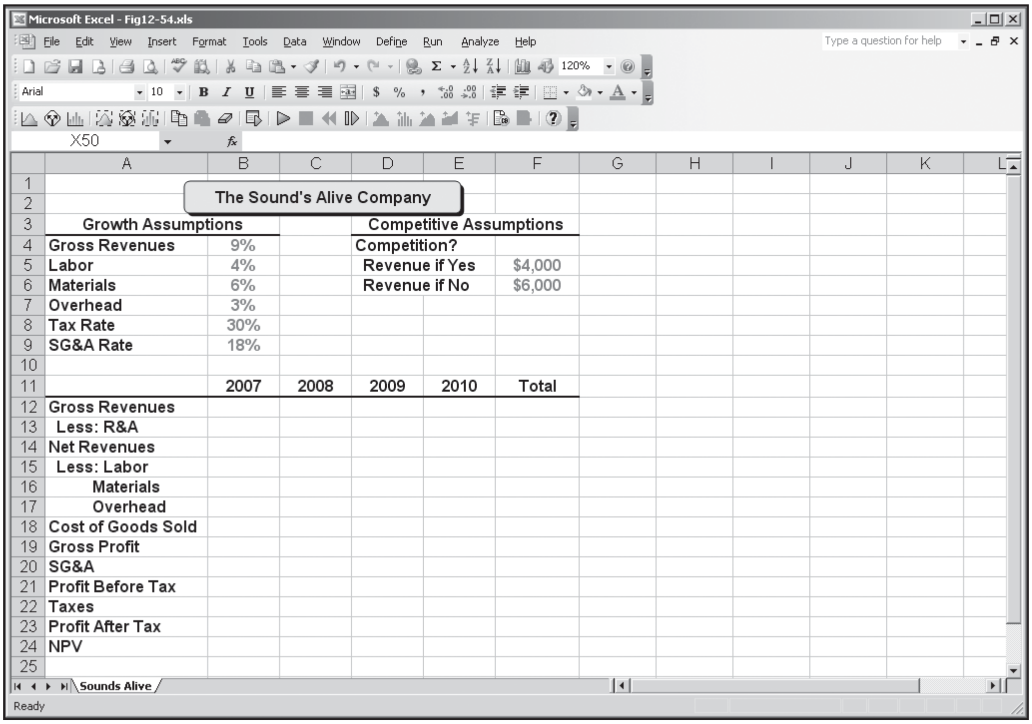


FIGURE 12.54

Spreadsheet template for the Sound's Alive case

The basic layout of the spreadsheet that Marissa creates is shown in Figure 12-54 (and in the file Fig12-54.xls on your data disk). (Ignore the Competitive Assumptions section for now; we will consider it later.) Construct the spreadsheet, determine the values for the years 2007 through 2010, and then determine the totals for the four years.

Marissa not only wants to determine her net profits for 2007 through 2010, she also must justify her decisions to the company's Board of Trustees. Should she even consider entering this market, from a financial point of view? One way to answer this question is to find the net present value (NPV) of the net profits for 2007 through 2010. Use Excel's NPV capability to find the NPV, at the current interest rate of 5%, of the profit values for 2007 through 2010.

To avoid large values in the spreadsheet, enter all dollar calculations in thousands. For example, enter labor costs as 995.10 and overhead costs as 1536.12.

Phase Two: Bringing Competition into the Model

With her spreadsheet complete, Marissa is confident that entering the home theater speaker market would be lucrative for Sound's Alive. However, she has not considered one factor in her calculations—competition. The current market leader and company she is most concerned about is the Bose Corporation. Bose pioneered the concept of a satellite speaker system, and its AMT series is very successful. Marissa is concerned that Bose will enter the home market, cutting into her gross revenues. If Bose does enter the market, Marissa believes that Sound's Alive still would make money; however, she would have to revise her gross revenues estimate from $6 million to $4 million for 2007.

To account for the competition factor, Marissa revises her spreadsheet by adding a Competitive Assumptions section. Cell F4 will contain either a 0 (no competition) or a 1 (if Bose enters the market). Cells F5 and F6 provide the gross revenue estimates (in thousands of dollars) for the two possibilities. Modify your spreadsheet to take these options into account. Use the IF( ) function for the gross revenues for 2007 (cell B12). If Bose does enter the market, not only would Marissa's gross revenues be lower, but the labor, materials, and overhead costs also would be lower because Sound's Alive would be making and selling fewer speakers. Marissa thinks that if Bose enters the market, her 2007 labor costs would be $859,170; 2007 material costs would be $702,950; and 2007 overhead costs would be $1,288,750. She believes that her growth rate assumptions would stay the same whether or not Bose enters the market. Add these possible values to your spreadsheet using the IF( ) function in the appropriate cells.

Look at the net profits for 2007 through 2010. In particular, examine the NPV for the two scenarios: Bose does or does not enter the home theater speaker market.

Phase Three: Bringing Uncertainty into the Model

Jim Allison, the chief of operations at Sound's Alive and a quantitative methods specialist, plays a key role in providing Marissa with estimates for the various revenues and costs. He is uneasy about the basic estimates for the growth rates. For example, although market research indicates that a 9% gross revenue increase per year is reasonable, Jim knows that if this value is 7%, for example, the profit values and the NPV would be quite different. Even more troublesome is a potential tax increase, which would hit Sound's Alive hard. Jim believes that the tax rate could vary around the expected 30% figure. Finally, Jim is uncomfortable with the industry's standard estimate of 18% for the SG&A rate. Jim thinks that this value could be higher or even lower.

The Sound's Alive problem is too complicated for solving with what-if analysis because seven assumed values could change: the growth rates for gross revenues, labor, materials, overhead costs, tax rate, SG&A percent, and whether or not Bose enters the market. Jim believes that a Monte Carlo simulation would be a better approach. Jim thinks that the behavior of these variables can be modeled as follows:

Gross Revenues (%): normally distributed, mean = 9.9, std dev = 1.4

Labor Growth (%): normally distributed, mean = 3.45, std dev = 1.0

Materials (%)	Probability
4	0.10
5	0.15
6	0.15
7	0.25
8	0.25
9	0.10

Overhead (%)	Probability
2	0.20
3	0.35
4	0.25
5	0.20

Tax Rate (%)	Probability
30	0.15
32	0.30
34	0.30
36	0.25

SG&A (%)	Probability
15	0.05
16	0.10
17	0.20
18	0.25
19	0.20
20	0.20

Finally, Jim and Marissa agree that there is a 50/50 chance that Bose will enter the market.

a. Use simulation to analyze the Sound's Alive problem. Based on your results, what is the expected net profit for the years 2007 through 2010, and what is the expected NPV for this business venture?
b. The Board of Trustees told Marissa that the stockholders would feel comfortable with this business venture if its NPV is at least $5 million. What are the chances that Sound's Alive home theater venture will result in an NPV of $5 million or more?

CASE 12.4 The Foxridge Investment Group

(Inspired by a case written by MBA students Fred Hirsch and Ray Rogers for Professor Larry Weatherford at the University of Wyoming.)

The Foxridge Investment Group buys and sells rental income properties in Southwest Virginia. Bill Hunter, president of Foxridge, has asked for your assistance in analyzing a small apartment building the group is interested in purchasing.

The property in question is a small two-story structure with three rental units on each floor. The purchase price of the property is $170,000 representing $30,000 in land value and $140,000 in buildings and improvements. Foxridge will depreciate the buildings and improvements value on a straight-line basis over 27.5 years. The Foxridge Group will make a down payment of $40,000 to acquire the property and finance the remainder of the purchase price over 20 years with an 11% fixed-rate loan with payments due annually. Figure 12.55 (and the file Fig12-55.xls on your data disk) summarizes this and other pertinent information.

FIGURE 12.55

Assumptions for the Foxridge Investment Group

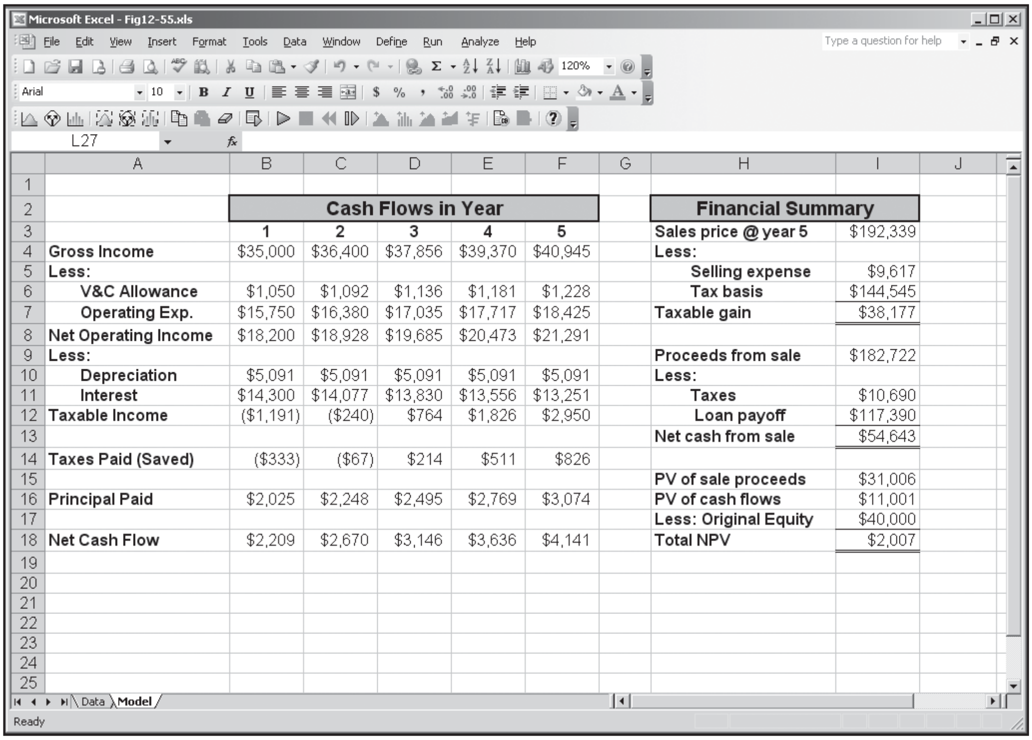
	B	C
2	**Foxridge Investment Group**	
4	**Acquisition Data**	
5	Land Value	$30,000
6	Buildings/Improvements	$140,000
7	Purchase Price	$170,000
9	**Financing Data**	
10	Down Payment	$40,000
11	Amount Financed	$130,000
12	APR	11.0%
13	Term	20
14	Annual Payment	$16,325
16	**Economic Assumptions**	
17	Annual Gross Rental Income	$35,000
18	Rental Income Growth Rate	4.0%
19	V&C Allowance	3.0%
20	Operating Expenses	45.0%
21	Tax Rate	28.0%
22	Property Value Growth Rate	2.5%
23	Sales Commission	5.0%
24	Discount Rate	12.0%

If all units are fully occupied, Mr. Hunter expects the property to generate rental income of $35,000 in the first year and expects to increase the rent at the rate of inflation (currently 4%). Because vacancies occur and some residents might not always be able to pay their rent, Mr. Hunter factors in a 3% vacancy & collection (V&C) allowance against rental income. Operating expenses are expected to be approximately 45% of rental income. The group's marginal tax rate is 28%.

If the group decides to purchase this property, their plan is to hold it for five years and then sell it to another investor. Presently, property values in this area are increasing at a rate of approximately 2.5% per year. The group will have to pay a sales commission of 5% of the gross selling price when they sell the property.

Figure 12.56 shows a spreadsheet model that Mr. Hunter developed to analyze this problem. This model first uses the data and assumptions given in Figure 12.55 to generate the expected net cash flows in each of the next five years. It then provides a final summary of the proceeds expected from selling the property at the end of five years. The total net present value (NPV) of the project is then calculated in cell I18 using the discount rate of 12% in cell C24 of Figure 12.55. Thus, after discounting all the future cash flows associated with this investment by 12% per year, the investment still generates an NPV of $2,007.

Although the group has been using this type of analysis for many years to make investment decisions, one of Mr. Hunter's investment partners recently read an article in the *Wall Street Journal* about risk analysis and simulation using spreadsheets. As a result, the partner realizes that there is quite a bit of uncertainty associated with many of the economic assumptions shown in Figure 12.55. After explaining the potential problem to Mr. Hunter, the two have decided to apply simulation to this model before making a decision. Because neither of them know how to do simulation, they have asked for your assistance.



	Cash Flows in Year				
	1	2	3	4	5
Gross Income	$35,000	$36,400	$37,856	$39,370	$40,945
Less:					
V&C Allowance	$1,050	$1,092	$1,136	$1,181	$1,228
Operating Exp.	$15,750	$16,380	$17,035	$17,717	$18,425
Net Operating Income	$18,200	$18,928	$19,685	$20,473	$21,291
Less:					
Depreciation	$5,091	$5,091	$5,091	$5,091	$5,091
Interest	$14,300	$14,077	$13,830	$13,556	$13,251
Taxable Income	($1,191)	($240)	$764	$1,826	$2,950
Taxes Paid (Saved)	($333)	($67)	$214	$511	$826
Principal Paid	$2,025	$2,248	$2,495	$2,769	$3,074
Net Cash Flow	$2,209	$2,670	$3,146	$3,636	$4,141

Financial Summary	
Sales price @ year 5	$192,339
Less:	
Selling expense	$9,617
Tax basis	$144,545
Taxable gain	$38,177
Proceeds from sale	$182,722
Less:	
Taxes	$10,690
Loan payoff	$117,390
Net cash from sale	$54,643
PV of sale proceeds	$31,006
PV of cash flows	$11,001
Less: Original Equity	$40,000
Total NPV	$2,007

FIGURE 12.56

Cash flow and financial summary for the Foxridge Investment Group

To model the uncertainty in this decision problem, Mr. Hunter and his partner have decided that the growth in rental income from one year to the next could vary uniformly from 2% to 6% in years 2 through 5. Similarly, they believe that the V&C allowance in any year could be as low as 1% and as high as 5%, with 3% being the most likely outcome. They think that the operating expenses in each year should be normally distributed with a mean of 45% and standard deviation of 2% but never should be less than 40% and never greater than 50% of gross income. Finally, they believe that the property value growth rate could be as small as 1% or as large as 5%, with 2.5% being the most likely outcome.

a. Revise the spreadsheets shown in Figures 12.55 and 12.56 to reflect the uncertainties outlined.
b. Construct a 95% confidence interval for the average total NPV that the Foxridge Investment Group can expect if they undertake this project. (Use 500 replications.) Interpret this confidence interval.
c. Based on your analysis, what is the probability of this project generating a positive total NPV if the group uses a 12% discount rate?
d. Suppose the investors are willing to buy the property if the expected total NPV is greater than zero. Based on your analysis, should they buy this property?
e. Assume that the investors decide to increase the discount rate to 14% and repeat questions 2, 3, and 4.
f. What discount rate results in a 90% chance of the project generating a positive total NPV?