

Continuous Random Variable and Poisson Distribution

Full marks: 32

1. The random variable X is a continuous random variable with probability density function

$$f(x) = \begin{cases} kx(2-x) & \text{if } 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

- i) find the value of k. (2)
 - ii) Sketch the probability density function.(3)
 - iii) find $P(1 < X \leq 1.5)$. (3)
 - iv) Find the expectance and variance of X.(3)
 - v) Find the median of X. (2)
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2. Jess is watching a shower of meteors(shooting stars). During the shower, she sees meteors at an average rate of 1.3 per minute.

- i) State conditions required for a Poisson distribution to be a suitable model for the number of meteors which Jess sees during a randomly selected minute.(2)

You may assume that these conditions are satisfied. ii) Find the probability that, during one minute, Jess sees:(4)

a) exactly one meteor b) at least 4 meteors.

- iii) Find the probability that, in a period of 10 minutes, Jess sees 10 meteors.(3)
- iv) Use suitable approximating distribution to find the probability that Jess sees a total of at least 100 meteors during a period of one hour.(5)
- v) Jess watches the shower for t minutes. She wishes to be at least 99% certain that she will see one or more meteors. Find the smallest possible integer value of t. (5)