# Comparing $C^r$ Values with Network Perturbation Results - Focal Species: Monarch

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# 1 Monarch Notes and Generalized $C^r$

We model a population of Monarch Butterfly building our model based on the formulation in *A general modeling framework for describing spatially structured population dynamics* by Sample et. al.

We track the population at four nodes: node 1 (Nonbreeding Only - winter), node 2 (Breeding Only - April/May/September), node 3 (Breeding Only - May-Sep), node 4 (Breeding Only - June/July/August). The population is modeled through seven times steps each season:

- Winter, all migrants start in the nonbreeding only node. Survival of adults is constant at 0.939.
- April, all migrants are in breeding node 2. Survival of adults is constant at 0.308.
- May, all migrants are in breeding nodes 2 & 3. Survival of adults is constant at 0.308.
- June, July, and August, all migrants are in breeding nodes 3 & 4. Survival of adults is constant at 0.308.
- September, all migrants are in breeding nodes 2 & 3. Survival of adults is constant at 0.308.

We use a generalized network model formulation were we define one time step as

$$\vec{\mathbf{N}}_{t+1} = \mathbf{A}_t \vec{\mathbf{N}}_t \tag{1}$$

Where $\vec{\mathbf{N}}_t$ contains the population at each of the $n$ nodes and $c$ classes at time $t$. The matrix $\mathbf{A}_t$ contains all of the demographic node update and migration data for time $t$. (Christine and Joanna have a draft of a paper with this construction).

Within the monarch code we calculate the $C^r$ values for both the origin and the intermediate nodes during each season. We use a generalized one equation definition of $C^r$ in matrix form.

$$\vec{\mathbf{C}}_t = \left(\prod_{\tau=t}^{t+s-1} \mathbf{A}_\tau^T\right) \vec{\mathbf{1}}_{nc} \tag{2}$$

where $s$ is the total number of seasons in one anual cycle, for the monarchs $s = 7$ and $\vec{\mathbf{C}}_t$ is a column vector that contains the $C^r$ values at each node for each class for focal season, or time, $t$.
We note that because the code is run to equillibrium, the overall growth rate of the nework is equal to one. We can calculate the growth rate using:

$$\lambda_t = \frac{\vec{\mathbf{N}}_t^T}{N_t^{tot}} \vec{\mathbf{C}}_t \tag{3}$$

where $N_t^{tot}$ is the sum of the equilibrium population values across all nodes and all classes in the network and is represented by the following summation

$$N_t^{tot} = \sum_{r=1}^{n} \sum_{x=1}^{c} N_{r,t}^x \tag{4}$$

# 2 Density Dependence in the Monarch Population

For the monarchs, the $f$ function is density dependent and is given by:

$$f_{i,t} = \underbrace{s_i^A \cdot N_{i,t}}_{\text{adults that survive}} + \underbrace{s_i^A \cdot s_i^P \cdot s_i^L \cdot E \cdot N_{i,t}}_{\text{eggs that survive and transition to adults}} \tag{5}$$

$s_i^A$ and $s_i^p$ are constant, but $s_i^L$, which represents larval survival, is dependent on egg density per milkweed stem at node $i$.

# 3 Baseline Simulation

The baseline simulation gives us identical results to *A general modeling framework for describing spatially structured population dynamics* by Sample et. al. We find that the long term carrying capacity for each node during the nonbreeding season, including all classes, is

```
season 1 : k= 104369867 0 0 0
season 2 : k= 0 50678505 0 0
season 3 : k= 0 45341487 20370029 0
season 4 : k= 0 0 53525729 33199558
season 5 : k= 0 0 45933321 88548303
season 6 : k= 0 0 72219163 70020140
season 7 : k= 0 62180832 66308229 0
```

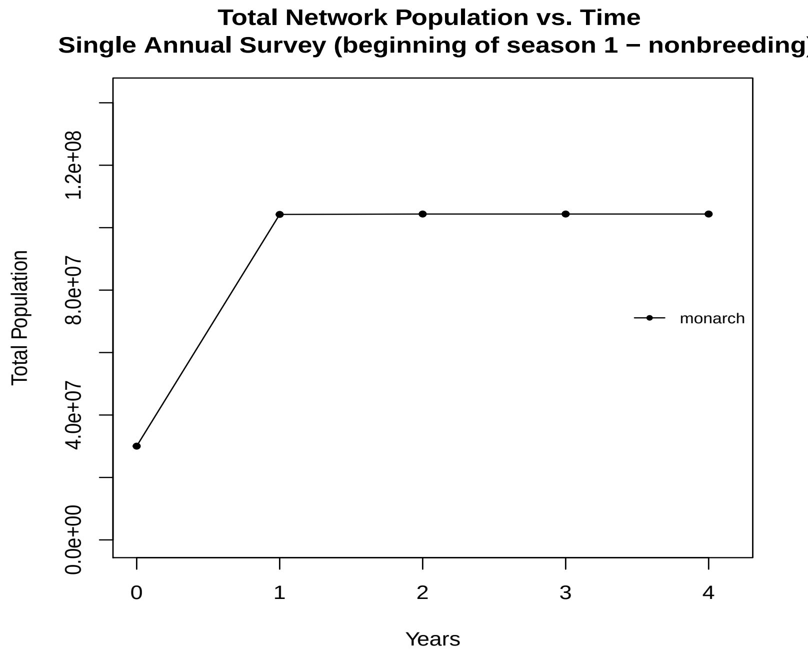


Figure 1: Baseline results: population over time at nonbreeding node.

# 4 Perturbation Experiment - Perturbing Node Survival Rates

To investigate the utility of the metrics as an indicator of the change in carrying capacity $K$ we consider the following perturbations to the survival rate at each node:

$$PERT = .9, .8, .7, .6, .5$$

Some notes about the simulations:

- All simulations are run to equillibrium with an error tollerance of within 1 animal, meaning the network growth rate $\lambda = 1$.
- We only cary out negative perturbations because positive perturbations will cause survival rates to exceed 1.

# 5 Monarch Plots - June 2018 - to Match Pintail Outputs

We will consider metrics for the x-axis of our graph:

- Baseline $C^r$ vs $K$ perturbations - this is the population weighted $C^r$ value averaged across classes and seasons Figure 2
- Baseline $W^r$ vs $K$ perturbations - the population proportions averaged across seasons Figure 3
- Baseline $D_s$ vs $K$ perturbations - the proportional dependence as defined in Bagstad et al, generalizing to account for juveniles: Figure 4
- NEW - Baseline Criticality $KR_i$ vs $K$ perturbations - defined as the network growth rate in the absence of node $i$ using a population weighted average across seasons: Figure 5. Note that because we are at equilibrium the baseline network growth rate is $\lambda = 1$.

## 5.1 Population weighted average $C^r$

In order to be consistent across case studies we decide to calculate $C^r$ using a population weighted average. The general matrix form of $C^r$ results in a $C^r$ values for each node, class, and season. To combine these into a single full year network wide $C^r$ value we first do a population weighted average by class within each node and then do a population weighted average across seasons. This results in a single $C^r$ values for each node.
The Monarch model has seven seasons (Winter, April, May, June, July, August, Sep) and one class (adult females).

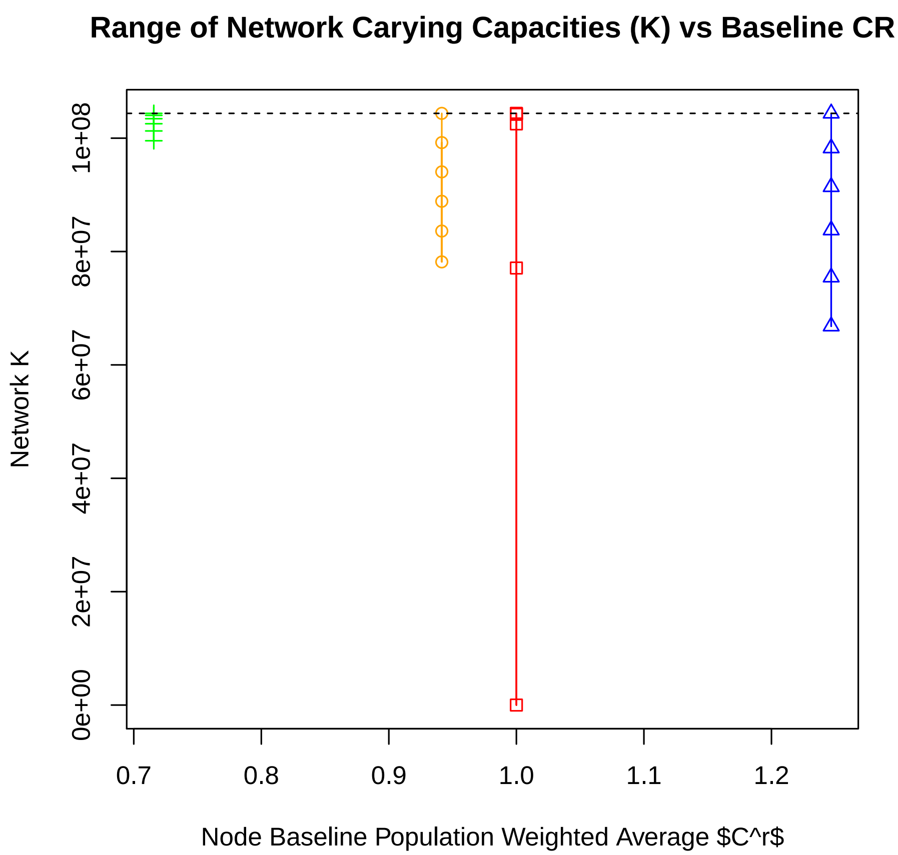


Figure 2: Perturbation results: Range of K values after perturbations to each node, X-axis represents **baseline population weighted $C^r$ values** for each node

| | | |
|---|---|---|
| Mexico | $\square$ | winter |
| South | $\bigcirc$ | April/May/Sep |
| Central | $\triangle$ | May/June/July/August/Sep |
| North | $+$ | June/July/August |

### 5.2 Population Distribution $W^r$

$W^r$ is calculated as the percent of the total population residing at a node. This calculation results in $W^r$ values for each node during each seaons. To get a consistent value for the network, we average (not weighted) across the seasons. The final numbers should sum to one.

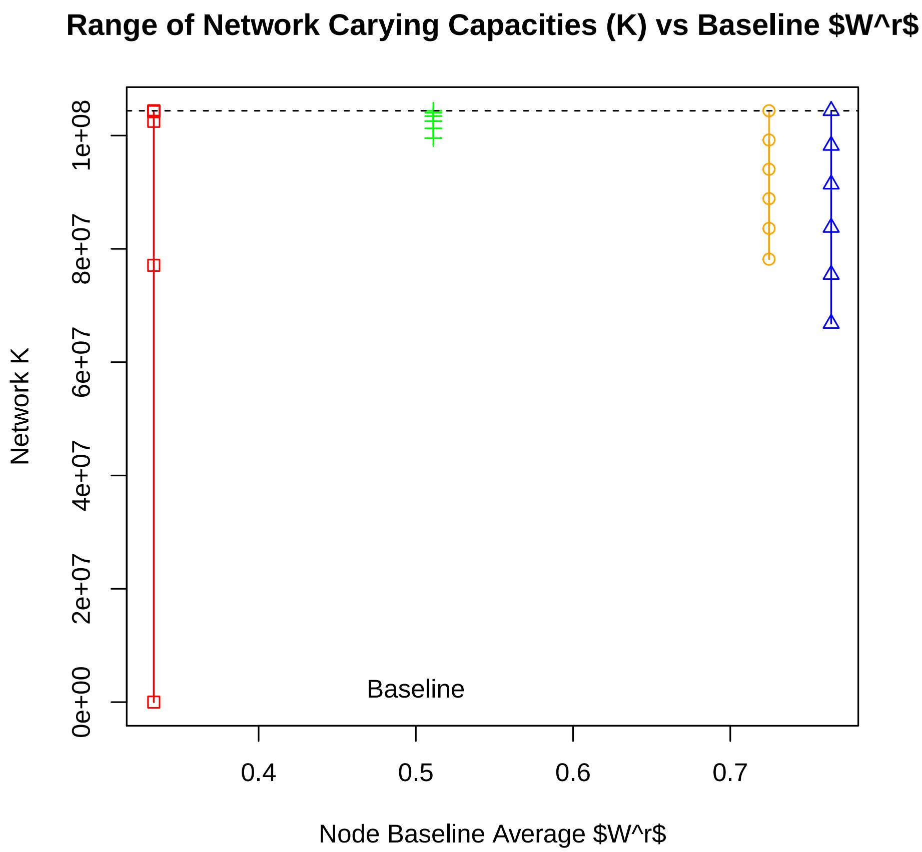


Figure 3: Perturbation results: Range of K values after perturbations to each node, X-axis represents baseline average $W^r$ value for each node

| | | |
|---|---|---|
| Mexico | $\square$ | winter |
| South | $\bigcirc$ | April/May/Sep |
| Central | $\triangle$ | May/June/July/August/Sep |
| North | $+$ | June/July/August |

### 5.3 Proportional Dependence $D_s$

$D_s$ is calculated following Bagstad et al. We first find the population weighted $C^r$ values for each of the seasons and then average across the anual cycle.

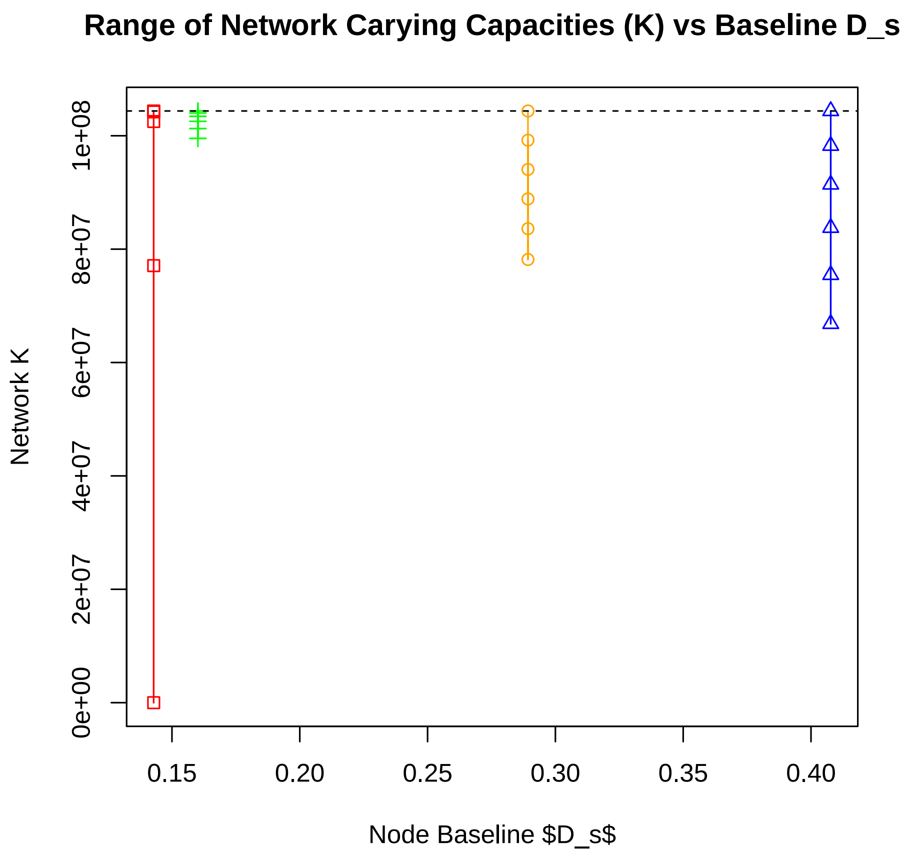


Figure 4: Perturbation results: Range of K values after perturbations to each node, X-axis represents baseline $D_s$ value for each node

| | | |
|---|---|---|
| Mexico | $\square$ | winter |
| South | $\bigcirc$ | April/May/Sep |
| Central | $\triangle$ | May/June/July/August/Sep |
| North | $+$ | June/July/August |

### 5.4 Criticality - NEW $K^r$

Criticality $K^r$ is a new metric concieved of by Christine and her colleagues. The basic idea is that from $C^r$ we can calculate the network growth rate $\lambda$. In the case of a population in equilibrium, like our simulations, $\lambda = 1$. Then one could also calculate a theoretical $C^r$ value as if on of the nodes was removed. From this theoretical $C^r$ value we could calulcate a new network growth rate $\gamma$. We define criticality as

$$K^r = \lambda - \gamma$$

In other words $K^r$ represents the amount of the network growth rate that flows through the focal node $r$. In our equilibrium case is $K^r = 1$ then all of the network growth rate must flow through $r$ and removal of $r$ reduces the new growth rate $\gamma$ to zero. Alternatively, if $K^r = 0$ then none of the original network growth rate flows through $r$ and removal of $r$ does not change the growth rate, $\lambda = \gamma = 1$.
The $K^r$ calculation results in criticality vales for each node during each season. To get a single yearly value we use a population weighted average.

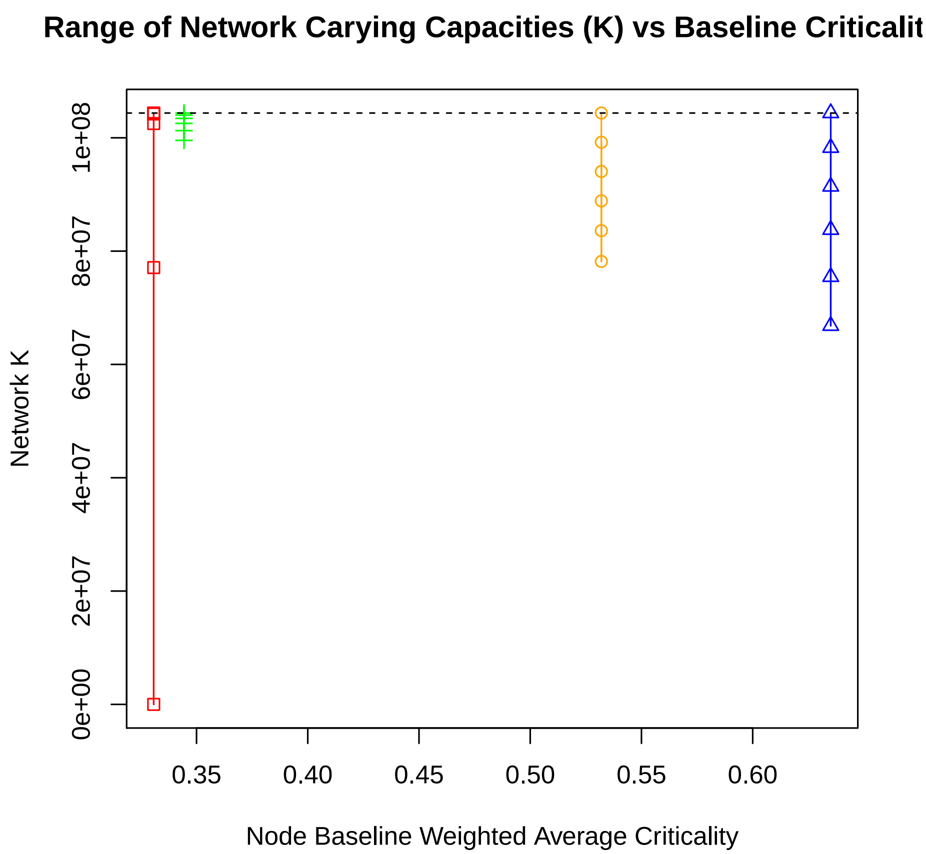


Figure 5: Perturbation results: Range of K values after perturbations to each node, X-axis represents baseline Criticality value for each node

| | | |
|---|---|---|
| Mexico | □ | winter |
| South | ○ | April/May/Sep |
| Central | △ | May/June/July/August/Sep |
| North | + | June/July/August |