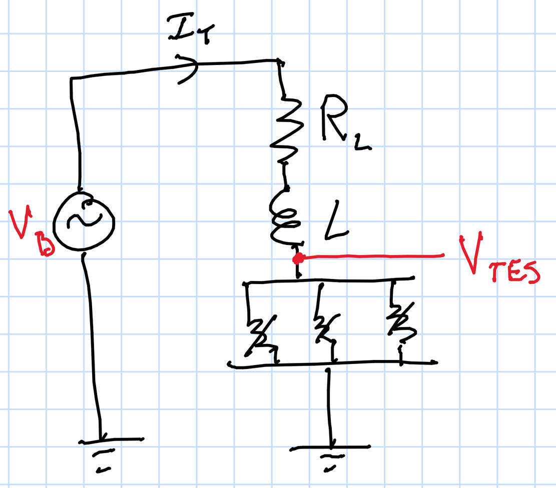

State variable: $\vec{X} = \begin{bmatrix} \vec{I} \\ \vec{T} \end{bmatrix}$

$$L \dot{I}_T = -I_T (R_L + R_{||}) + V_B(0) + \delta V_B \quad \text{eq 1)} \Rightarrow \dot{I}_T = f(\vec{X}) + \delta Q$$

$$C_j \dot{T}_j = I_j^2 R_j (T_j I_j) - P_{cool}(T_j) + \delta Q_j \quad \text{eq 2)} \quad \dot{T}_j = f(\vec{X}) + \delta Q_j$$

$$V_{TES} = I_T R_{||} = I_j R_j$$ $\Rightarrow$ voltage across every TES is the same! $\Rightarrow$ constraints

from $\dot{I}_T$ & $\vec{X}$ find $\dot{I}_j$

$$V_{TES} = I_T R_{||} = I_j R_j$$

$$\dot{V}_{TES} = \underbrace{\dot{I}_T}_{\text{known from eq1}} \underbrace{R_{||}}_{f(\vec{X})} + I_T \underbrace{\dot{R}_{||}}_{f(\vec{X})} = \dot{I}_j R_j + I_j \dot{R}_j$$

$$= \dot{I}_j R_j + I_j \left(\frac{\partial R_j}{\partial T_j} \dot{T}_j + \frac{\partial R}{\partial I_j} \dot{I}_j \right)$$

$$= \dot{I}_j R_j + I_j \left(\alpha_j \frac{R_j}{T_j} \dot{T}_j + \beta_j \frac{R_j}{I_j} \dot{I}_j \right)$$

$$\dot{I}_T R_{||} + I_T \dot{R}_{||} = \dot{I}_j R_j (1+\beta_j) + I_j \frac{\alpha_j R_j}{T_j} \dot{T}_j$$

from page 2

$$\dot{R}_{||} = +R_{||}^2 \left(\sum_k \frac{\alpha_k}{R_k T_k} \dot{T}_k + \frac{\beta_k}{R_k I_k} \dot{I}_k \right)$$

$\therefore$

$$\dot{I}_T R_{||} + I_T R_{||}^2 \left(\sum_k \frac{\alpha_k}{R_k T_k} \dot{T}_k + \frac{\beta_k}{R_k I_k} \dot{I}_k \right) = \dot{I}_j R_j (1+\beta_j) + \frac{I_j \alpha_j R_j \dot{T}_j}{T_j}$$

$\Downarrow$ let's group the $\dot{I}$ all one one side

from eq1

$$\underbrace{\dot{I}_T R_{||} + I_T R_{||}^2 \sum_k \frac{\alpha_k}{R_k T_k} \dot{T}_k - \frac{I_j \alpha_j R_j \dot{T}_j}{T_j}}_{Y_j} = \dot{I}_j R_j (1+\beta_j) - I_T R_{||}^2 \sum_k \frac{\beta_k}{R_k I_k} \dot{I}_k$$

$$= \sum_k \left[\delta_{jk} R_k (1+\beta_k) - \frac{I_T R_{||}^2 \beta_k}{R_k I_k} \right] \dot{I}_k$$

$$Y_j = M_{jk} \dot{I}_k$$

$$\boxed{M^{-1} \vec{Y} = \dot{\vec{I}}}$$

let's check that this makes sense in the n=1 limit

$$\dot{I}_T = \dot{I}_1$$
$$R_{||} = R_1$$

$$\dot{I}_1 R_1 + \cancel{I_1 R_1^2 \frac{\alpha_1 \dot{T}_1}{R_1 T_1}} - \cancel{\frac{I_1 \alpha_1 R_1}{T_1} \dot{T}_1} = \left[R_1 (1+\beta_1) - \frac{\cancel{I_1} R_1^2 \beta_1}{\cancel{R_1} \cancel{I_1}} \right] \dot{I}_1$$

$$\dot{I}_1 R_1 = R_1 \dot{I}_1$$

let's see if we can simplify a bit ... $\sum_j$ on both sides

$$n \left[\dot{I}_T R_{||} + I_T R_{||}^2 \sum_k \frac{\alpha_k}{R_k T_k} \dot{T}_k \right] - \sum_j \frac{I_j \alpha_j R_j}{T_j} \dot{T}_j = \sum_j R_j (1+\beta_j) \dot{I}_j - n \sum_k \frac{I_T R_{||}^2 \beta_k \dot{I}_k}{R_k I_k}$$

$$R_{11} = \frac{1}{\sum_j \frac{1}{R_j}}$$

$$\dot{R}_{11} = \frac{-1}{\left(\sum_j \frac{1}{R_j}\right)^2} \left(\sum_j \frac{d}{dt} \frac{1}{R_j} \right)$$

$$\dot{R}_{11} = -R_{11}^2 \left(\sum_j \frac{-1}{R_j^2} \dot{R}_j \right)$$

$$= -R_{11}^2 \sum_j \frac{-1}{R_j^2} \left(\frac{\partial R_j}{\partial T_j} \dot{T}_j + \frac{\partial R_j}{\partial I_j} \dot{I}_j \right)$$

$$\alpha_j = \frac{\partial R_j}{\partial T_j} \frac{T_j}{R_j} \qquad \beta_j = \frac{\partial R_j}{\partial I_j} \dot{I}_j$$

$$\dot{R}_{11} = -R_{11}^2 \left(\sum_j \frac{-1}{R_j^{\cancel{2}}} \left[\alpha_j \frac{\cancel{R_j}}{T_j} \dot{T}_j + \frac{\beta_j \cancel{R_j}}{I_j} \right] \right)$$

$$\boxed{\dot{R}_{11} = +R_{11}^2 \left(\sum_j \frac{\alpha_j}{R_j T_j} \dot{T}_j + \frac{\beta_j}{R_j I_j} \dot{I}_j \right)}$$