EMPIRICAL MODELS OF DEMAND FOR DIFFERENTIATED PRODUCTS: TEACHING NOTES

Aviv Nevo

University of California, Berkeley

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1. INTRODUCTION

A key to the methods of measuring market power we have seen up to now is the consistent estimation of demand. As we use our models for additional applications we will rely even more on the demand parameters. This note explains the problems faced by this estimation and surveys some of the recent methods used to solve them. The emphasis is on the modeling methods and not the econometric details.

2. THE PROBLEM

Probably the most straight-forward approach to estimating a demand system is to specify a system of demand equations

$$q = D(p\,;r)$$

where q is a J-dimensional vector of quantities demanded from the J commodities, p is a J-dimensional vector of prices of the commodities, and r is a vector of exogenous variables that shift demand. The main concern of previous work was to specify $D(\cdot)$ in a way that was both flexible and consistent with economic theory. Such methods include: the Linear Expenditure model (Stone, 1954), the Rotterdam model (Theil, 1965; and Barten 1966), the Translog model (Christensen, Jorgenson, and Lau, 1975), and the Almost Ideal Demand System (Deaton and Muellbauer, 1980a).

The problem in applying any of these methods to estimate demand for differentiated products is the dimensionality problem. Due to the large number of products, even if we were to assume a very simple and restrictive functional form for the demand function, $D(\cdot)$, the number of parameters will be too large to estimate. For example, a linear demand system, $D(p) = Ap$, where A is $J \times J$ matrix of constants, implies J^2 parameters. The number of parameters can be reduced by imposing symmetry of the Slutsky matrix and adding up restrictions. However, the basic problem still remains: the number of parameters to be estimated increases with square the number of products. This problem is augmented if we attempt to use a flexible functional form.

An additional problem with the straight forward approach outlined above is the need to deal with the multicollinearity of prices and the need for an instrumental variable for each of them. In most differentiated products industries prices of the various goods will be highly collinear. This problem is augmented since we require an IV for each price. It is usually very hard to find IV that are both exogenous and will not generate moment conditions that are nearly collinear.

The final problem with the above approach is that it ignores heterogeneity among consumers. Besides the fact that a representative consumer is not always guaranteed to exist, for some applications we would like to explicitly model and estimate the distribution of heterogeneity.

3. SOLUTIONS

Solutions to the problem, discussed above, include: (1) avoiding the problem, (2) symmetric representative consumer models, (3) multi-stage budgeting and (4) discrete choice/address models. This section presents these methods and shows how they solve the dimensionality problem.

3.1 *Avoiding the Problem*

One way to get around the problem discussed in the previous section is to assume it away. This can be done by either (1) focusing on an aggregate (for example Porter, 1983, and Ellison, 1994, aggregate all the eastbound shipments rather than differentiating across cities); or (2) by focusing on a narrowly defined product (for example Borenstien and Shepard, 1996, focus on a self service 87 octane); or (3) by focusing on sub-markets (for example, Baker and Bresnahan, 1985, examine a particular segment in the beer industry). All these examples did not solve the problems defined in the previous section rather they avoided the need to estimate the full demand system. In some cases this is the best solution. You should always ask yourself if you really need to estimate the full demand system. If the answer is positive you have several options detailed below.

3.2 *Symmetric Representative Consumer Models*

A widely used specification in theoretical models of product differentiation is the constant elasticity of substitution (CES) utility function used by Dixit and Stiglitz (1977) and Spence (1976). The CES utility function takes the form

$$U(q_1, ..., q_J) = \left(\sum_{i=1}^{J} q_i^{\rho} \right)^{1/\rho},$$

where ρ is a constant parameter that measures substitution across products. The demand of the representative consumer obtained from this utility function is

$$q_k = \frac{p_k^{-1/(1-\rho)}}{\sum_{i=1}^{J} p_i^{-\rho/(1-\rho)}} I, \qquad k = 1, ..., J,$$

where I is the income of the representative consumer. The dimensionality problem is solved by imposing symmetry between the different products; thus, estimation involves a single parameter, regardless of the number of products, and can be achieved using non-linear estimation methods. However, the symmetry condition is restrictive and indeed for this model implies

$$\frac{\partial q_i}{\partial p_j} \frac{p_j}{q_i} = \frac{\partial q_k}{\partial p_j} \frac{p_j}{q_k}, \text{ for all } i, k, j.$$

The cross-price elasticities are restricted to be equal, regardless of how "close" the products are in some attribute space. This restriction can have important implications for the simulation of mergers and in many cases would lead to the "wrong" conclusions.

An alternative to the CES utility function is

$$U(q_1, ..., q_J) = \sum_{j=1}^{J} \delta_j q_j - \sum_{j=1}^{J} q_j \ln q_j,$$

which as shown by Anderson, de Palma, and Thisse (1992) yields the Logit demand. Estimation of this model involves J parameters and allows for somewhat richer substitution patterns. However, as discussed below the substitution patterns in the Logit model are solely a function of market shares (which here are equivalent to the quantities consumed by the aggregate consumer), and are not related to the characteristics of the products. Or in other words, due to the aggregate IIA property if the price of commodity i increases the representative consumer will keep the same ratio q_j/q_k, for all $j,k \neq i$, instead of consuming relatively more of products that are similar to product *i*.

The utility function for the Logit representative consumer has two terms. The first suggests that the representative consumer will consume only the product with the highest δ_j. The second term is an entropy term and expresses a variety-seeking behavior.[1] Through this second term we get consumption of more than one product, but its functional form illuminates the similarity between the aggregate IIA property and the symmetry condition embodied in the CES utility: all products enter this entropy term in a symmetric way.

In summary, models of this class solve the dimensionality problem by imposing symmetry conditions which implicitly suggest an extreme form of non-localized competition. Although for some industries this model of differentiation is adequate, for most markets this is not the case.

3.3 *Separability and Multi-Stage Budgeting*

A different approach to solving the dimensionality problem is to divide the products into smaller groups and allow for a flexible functional form within each group. The justification of such a procedure relies on two closely related ideas: the separability of preferences and multi-stage budgeting.

The first notion is that of (weak) separability of preferences. If this holds commodities can be partitioned into groups so that preferences within each group are independent of the quantities in other groups. For example, the utility function can be written as

$$U(q_1, q_2, ..., q_J) \;=\; f[\,v_1(q_1, q_2), v_2(q_3, q_4), ..., v_G(q_{J'}, ..., q_J)\,],$$

where $f(\cdot)$ is some increasing function and $v_1, ..., v_G$ are the sub-utility functions associated with the separate groups. The groups could be broad categories such as food, shelter and entertainment, and each group can possibly be divided into one or more sub-grouping.

A slightly different notion is that of multi-stage budgeting. This occurs when the consumer can allocate total expenditure in stages; at the highest stage expenditure is allocated to

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[1] In the theoretical literature (for example, Anderson, de Palma, and Thisse, 1992) this term will be multiplied by a positive parameter that captures the relative importance of the variety-seeking behavior. As this parameter goes to zero variety is not valued and one product is purchased by the representative consumer (i.e., there is no heterogeneity in the population); while as this parameter goes to infinity consumption is divided equally among all the products. I follow here the empirical literature that, for identification reasons, normalizes this parameter to one.

broad groups, while at lower stages group expenditure is allocated to sub-groups, until expenditures are allocated to individual products. At each stage the allocation decision is a function of only that group total expenditure and prices of commodities in that group (or price indexes for the sub-groupings). All these allocations must equal those that would occur if the maximization was done in one complete information step.

The two notions, of weak separability and multi-stage budgeting, are closely related; however, they are not identical, nor does one imply the other. Weak separability is necessary and sufficient for the last stage of the multi-stage budgeting; if a subset of products appears only in a separable sub-utility function, then the quantities demanded of these products can always be written as only a function of group expenditures and prices of other products within the group. The higher stages, the allocation of expenditures between groups, are more problematic and have to rely on the composite commodity theorem (Hicks, 1936; or Leontief, 1936), on various restrictions on preferences, or on stronger notions of separability (see Gorman, 1959; or Deaton and Muellbauer, 1980b chapter 5). From an empirical point of view the most useful of these theorems is the requirement (1) that the indirect utility functions for each segment are of the Generalized Gorman Polar Form, and (2) that the overall utility is separable additive in the sub-utilities.

Originally, these methods were developed for the estimation of fairly broad categories of products. Hausman, Leonard, and Zona (1994) and Hausman (1996) use the idea of multi-stage budgeting to construct a multi-level demand system for differentiated products. The actual application involves a three stage system: the top level corresponds to overall demand for the product (beer or ready-to-eat cereal, in their applications); the middle level involves demand for different market segments (for example, family, kids and adults cereal); and the bottom level involves a flexible brand demand system corresponding to the competition between the different brands within each segment.

For each of these stages a flexible parametric functional form is assumed. The choice of functional form is driven by the need for flexibility, but also requires that the conditions for multi-stage budgeting are met. A typical application has the AIDS model (see Deaton and Muellbauer, 1980a) at the lowest level: the demand for brand i within segment g in city c at quarter t is

$$s_{jct} = \alpha_{jc} + \beta_j \log(y_{gct}/P_{gct}) + \sum_{k=1}^{J} \gamma_{jk} \log p_{kct} + \varepsilon_{ict}, \tag{1}$$

$$j = 1, \ldots, J, \quad c = 1, \ldots, C, \quad t = 1, \ldots, T,$$

where s_{jct} is the dollar sales share of total segment expenditure, y_{gct} is overall per capita segment expenditure, P_{gct} is the price index and p_{kct} is the price of the kth brand in city c at quarter t. This system defines a flexible functional form that can allow for a wide variety of substitution patterns within the segment. It has two additional advantages over other flexible demand systems (like the Rotterdam system or the Translog model): (1) it aggregates well over individuals; and (2) it is easy to impose (or test) theoretical restrictions, like adding-up, homogeneity of degree zero and symmetry (for details see Deaton and Muellbauer, 1980a).

The price index, P_{gct}, is computed as either the Stone logarithmic price index

$$P_{gct} = \sum_{k \in g} s_{kct} \log p_{kct}, \tag{2}$$

or the Deaton and Muellbauer exact price index

$$P_{gct} = \alpha_0 + \sum_{k \in g} \alpha_k p_k + \frac{1}{2} \sum_{j \in g} \sum_{k \in g} \gamma_{kj} \log p_k \log p_j. \tag{3}$$

The exact form of the price index does not seem to be very important for the results (see Deaton and Muellbauer, 1980a pg 316-317). If the latter is used the estimation is non-linear, while with the Stone index the estimation can be performed using linear methods.

The middle level of demand captures the allocation between segments and can be modeled using the AIDS model, in which case the demand specified by equation (1) is used with both expenditure shares and prices aggregated to a segment level (the prices are aggregated using either equations (2) or (3)). An alternative is the log-log equation used by Hausman, Leonard, and Zona (1994) and Hausman (1996):

$$\log q_{gct} = \beta_g \log y_{Rct} + \sum_{k=1}^{G} \delta_k \log \pi_{kct} + \alpha_{gc} + \varepsilon_{gct};$$

$$g = 1, \ldots, G, \quad c = 1, \ldots, C, \quad t = 1, \ldots, T,$$

where q_{gct} is the quantity of the gth segment in city c at quarter t, y_{Rct} is total ready-to-eat cereal expenditure, and π_{kct} are the segment price indexes (computed using either equations (2) or (3)).

Since the lower level of the demand system is the AIDS, which satisfies the Generalized

Gorman Polar Form (GGPF), the preferences of the second level should be additively separable (i.e., overall utility from ready-to-eat cereal should be additively separable in the sub-utilities from the various segments), in order to be consistent with exact two-stage budgeting.[2] Neither the second level AIDS, nor the log-log system satisfy this requirement. Also, in order for exact multi-stage budgeting to hold to the next level of aggregation these preferences should be of the GGPF.

Finally, at the top level the demand for the whole ready-to-eat cereal category is specified as

$$\log q_{ct} = \beta_0 + \beta_1 \log y_{ct} + \beta_2 \log \pi_{ct} + Z_{ct}\delta + \varepsilon_{ct}$$

where q_{ct} is the overall consumption of cereal in city c at quarter t, y_{ct} is real income, π_{ct} is the price index for cereal and Z_{ct} are variables that shift demand (demographics and time factors). We note that this does satisfy additive separability, which is required for exact two-stage budgeting.

3.4 *Discrete Choice Models*

The last class of models are discrete choice models, which model products as bundles of characteristics. Preferences are defined over the characteristics space, making the dimension of this space the relevant dimension for empirical work. Heterogeneity is modeled and estimated explicitly. These models have been estimated using either individual data or aggregate market shares. The data used impacts both the econometric methods and the setup of the model. For the case were aggregate data is observed the details of the model are described in Nevo (1998). We will also discuss the case were the household data is observed (we will focus on Goldberg, 1995). The issues there are essentially the same but some of the modeling details and the estimation methods differ.

4. Comparing the Different Methods

In general the symmetric average consumer models are the least adequate for modeling demand for differentiated products. The main weakness of these methods is in estimating the

[2]Instead of using the notion of exact two-stage budgeting one can rely of approximate two-stage budgeting. Deaton and Muellbauer (1980b, pg. 132-133) show that for the Rotterdam model approximate two-stage budgeting implies that the higher stages also have a Rotterdam functional form, but require two price indexes to sum the price in each group. They claim that in practice these indexes are collinear and therefore can be treated as one. I do not know of any such derivation to justify this practice with the AIDS.

"closeness" of the various products, which is the key measure for many applications. Despite this problem the Logit model has been used frequently due to its tractability and ease of use. These models should usually be used only as a first step in the analysis.

Choosing between the multi-stage budgeting approach and the random coefficients discrete choice method is harder. Both have advantages and weakness compared to each other. The multi-level model requires a priori segmentation of the market into relatively small groups, which in some cases might be hard to define. Second, as mentioned above the empirical specification does not always meet the theoretical requirements. Also, the derivation of the AIDS assumes that there no corner solutions, i.e., all consumers consume all products. When dealing with broad categories like food and shelter, as in the original model of Deaton and Muellbuer (1980a), this is a reasonable assumption. For differentiated products, however, it is rather unlikely that all consumers consume all varieties. Finally, from the applied estimation point of view it is harder to find exogenous instrumental variables.

On the plus side this method has two clear advantages. First, it is closer to classical estimation methods and neo-classical theory, and therefore is easier and more intuitive to understand. An additional point, important for practitioners, is that the computation time is lower.

Discrete choice models require characteristics of products, in general are more computational intense, and rely on distributional assumptions and functional forms. All these problems can be treated to some extent, as we will see.

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