Simulating the Effects of a Horizontal Merger

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The full equation of this project is:

$$u_{ijt} = \delta_{jt}(x_j, p_{jt}, \xi_j, \Delta\xi_{jt}; \theta_1) + \mu_{ijt}(x_j, p_{jt}, \nu_j, D_i; \theta_2) + \epsilon_{ijt},$$
$$\delta_{jt} = x_j\beta - \alpha p_{jt} + \xi_j + \Delta\xi_{jt}, \mu = [p_{jt}, x_j]' * (\prod D_i + \sum \nu_i)$$

1 Logit Models

Logit model yields restrictive and unrealistic substitution patterns, and therefore is inadequate for measuring market power, but still a useful tool for getting a feel for the data. These are the results of the Logit models: (i) OLS without brand fixed effects, (ii) OLS with brand fixed effects; (iii) IV without brand fixed effects; (iv) IV with brand fixed effects.

	normal	felm	instrumental variable	felm
	OLS	OLS-FE	IV	IV-FE
	(1)	(2)	(3)	(4)
price	-7.586***	-29.037***	-8.686***	
	(0.860)	(0.989)	(0.867)	
`price(fit)`				-30.186***
				(1.005)
Constant	-2.896***		-2.758***	
	(0.111)		(0.112)	
Observations	2,256	2,256	2,256	2,256
R2		0.433	0.033	0.433
Adjusted R2		0.427	0.032	0.427
Log Likelihood	-3,585.160			
Residual Std. Error		0.912 (df = 2232)	1.186 (df = 2254)	0.913 (df = 2232)
Note:				*p<0.1; **p<0.05; ***p<0.01

Figure 1: model results

2 Markups, Margins and Implied Marginal Costs

For each model of supply, the pricing decision depends on brand-level demand.

$s_j(p)$: market share of brand j;

P: market price;

mc_j: marginal cost of brand j;

M: market size;

C_f: fixed cost of production.

The profits of firm f are:

$$\prod_f = \sum_{j \in F_f} \left(P_j - mc_j\right) Ms_j(p) - C_f \tag{1}$$

Assuming the existence of a pure-strategy Bertrand-Nash equilibrium in prices. The first order condition should be:

$$s_j(p) + \sum_{r \in F_f} (p_r - mc_r) \frac{\partial s_r(p)}{\partial p_r} = 0 \tag{2}$$

The markups can be solved for explicitly by defining $S_{jr} = -\partial s_r / \partial p_j$. The ownership matrix can be defined as:

$$\Omega_{jr} = S_{jr} * \Omega^{\star}_{jr} = S_{jr} * \begin{cases} 1, & if \exists f : \{r, j\} \subset F_f \\ 0, & otherwise \end{cases} \tag{3}$$

Subsidy into the FOC equation, so we can get:

$$p - mc = \Omega^{-1} s(p) \tag{4}$$

From Nevo(2000), equation(5) and equation(6) showed that, $\partial s_r / \partial p_j$ can be calculated from market share:

$$s_{jt} = \frac{exp(x_{jt}\beta - \alpha p_{jt} + \xi_{jt})}{1 + \sum_{k=1}^{J} exp(x_{kt}\beta - \alpha p_{kt} + \xi_{kt})} \tag{5}$$

$$\frac{\partial s_{jt}}{\partial p_{kt}} = \begin{cases} -\alpha p_{jt}(1 - s_{jt}) \frac{s_{jt}}{p_{kt}} & if j = k, \\ \alpha p_{kt} s_{kt} \frac{s_{jt}}{p_{kt}} & otherwise. \end{cases} \tag{6}$$

Now all the variables in the right hand side is known, we can have the markups and margins.

	mean	median	sd
markups	0.037	0.028	0.030
margins	27.63%	22.49%	0.18
mc	0.089	0.092	0.026

3 Merger

Nevo assumed that technology, and all the other factors affecting the marginal cost, to produce the products did not change after merging. Therefore, $MC_{pre} = MC_{post}$, but ownership matrices changed. Ω_{post} will change to: id first digit=3,6 will be same firm, id=2,4 be the same firm. New price can be solved from:

$$p^* - mc = \Omega^{-1}_{post} s(p^*) \tag{7}$$

4 Potential Problems

In this section, I will talk about the two potential problems with the analysis in the previous sections.

First, Both markups and margins, mc have used $\partial s_{jt}/_{kt}$. However, since in most cases the market shares are small, the factor $\alpha(1 - s_{jt})$ is nearly constant; hence, the own-price elasticities are proportional to own price.

$$\frac{\partial s_{jt}}{\partial p_{kt}} = \begin{cases} -\alpha p_{jt}(1 - s_{jt})\frac{s_{jt}}{p_{kt}} & if\, j = k, \\ \alpha p_{kt} s_{kt}\frac{s_{jt}}{p_{kt}} & otherwise. \end{cases} \tag{8}$$

Therefore, the lower the price, the lower the elasticity (in absolute value), which implies that a standard pricing model predicts a higher markup for the lower-priced brands. This is possible only if the marginal cost of a cheaper brand is lower (not just in absolute value, but as a percentage of price) than that of a more expensive product. For some products this will not be true.

The second problem is with the cross-price elasticities. For example, in the context of RTE cereals the cross-price elasticities imply that if Quaker CapN Crunch (a childern's cereal) and Post Grape Nuts (a wholesome simple nutrition cereal) have similar market shares, then the substitution from General Mills Lucky Charms (a children's cereal) toward either of them will be the same. Intuitively, if the price of one children's cereal goes up, we would expect more consumers to substitute to another children's cereal than to a nutrition cereal. Yet, the Logit model restricts consumers to substitute towards other brands in proportion to market shares, regardless of characteristics.

5 Model Estimation

I have tried Matlab and R to replicate the project. The whole model estimation steps are followed:

1.Draw u_{ns} $N(0, I)$, for ns=1,...,NS

2.Compute initial δ from homogeneous model.

3.Outer loop iteration l=1,2,...,given the value of θ_2^l

3.1 Inversion: given a starting δ^0. First, use Matlab file *mktsh.m, ind_sh.m* to compute the shares $s_{jt}(\delta_t^h; \theta_2^l)$ via simulation. Then use *meanval.m* to iterate on $\delta_t^{h+1} = \delta_t^h + logS_t - logs_{jt}(\delta_t^h; \theta_2^l)$ until convergence.

3.2 use *gmmobjg.m* to calculate $\theta_1 = (\bar{\beta}, \bar{\alpha})$, get residuals $\xi_{jt} = \xi_{jt}(\theta_2) - (\bar{\beta}' x_{jt} - \bar{\alpha} p_{jt})$ and then evaluate objective function $Q(\theta_2)$

4.Iterate minimization of $Q(\theta_2)$ until convergence by using *rc_dc.m*.

The R code was replicated to the ownership part. So far, I can get similar results from these two method. The estimation methods/steps are identical, but since the time limitation of this project, I will try to work on the rest part later on.

```
|    mean       sigma      income   income^2    age    child

constant
   -2.2751    0.2581    3.5819          0    0.2847         0

   (1,1)       0.2541
   (1,2)       0.1209
   (1,3)       1.0544
   (1,5)       0.4230

price
  -31.6864    2.4317   21.6313   -1.0233          0    4.5870

   (1,1)       8.5752
   (1,2)       0.8719
   (1,3)     181.5619
   (1,4)       9.3905
   (1,6)       3.7320

sugar
    0.1327    0.0075   -0.2216          0    0.0536         0

   (1,1)       0.2541
   (1,2)       0.0102
   (1,3)       0.0389
   (1,5)       0.0250

mushy
   -0.1400    0.1238    1.4420          0   -0.9902         0

   (1,1)       0.0133
   (1,2)       0.1597
   (1,3)       0.5325
   (1,5)       0.5936


**************************************************************************

GMM objective:  19.3717
MD R-squared:  0.50691
MD weighted R-squared:  0.072066
run time (minutes):  0.15767
```

6 Markup Change

This is the markup after iterations. The mean slightly increased and standard deviation significantly increased. Median value decreased, and changed to a negative number. This problem might be caused from the samples.

	mean	median	sd
new_markups	0.043	-0.003	1.215
old_markups	0.037	0.028	0.030