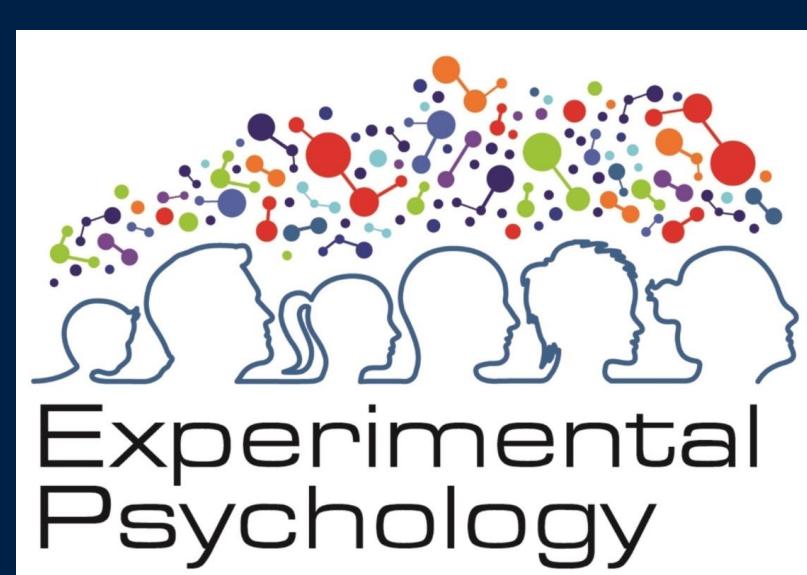




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# Compositional Learning of a Numerical Reasoning Task in Artificial Neural Networks

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## Introduction

- Composition:** the ability to recompose elements of prior knowledge to generate new mental operations or concepts.
- Humans excel at composition, usually after experiencing structured training (curricula), while current artificial neural networks struggle to match human performance.

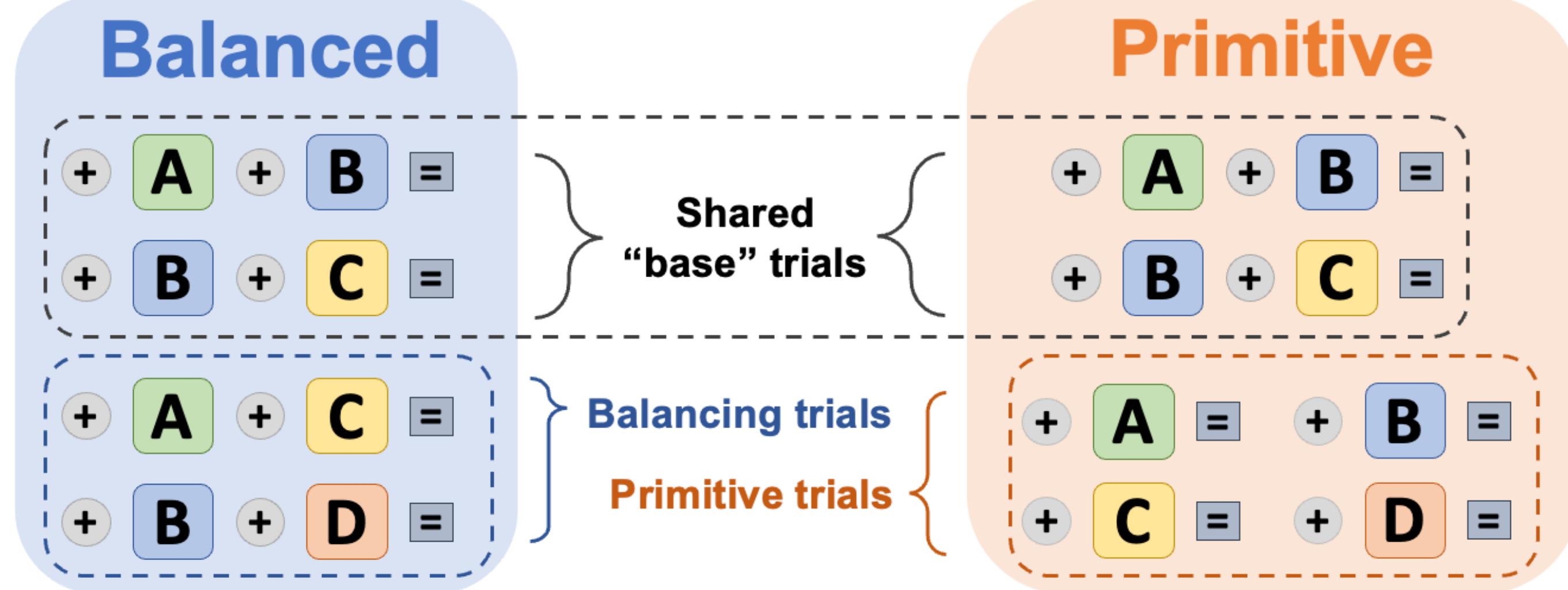
**Can training curricula encourage compositional learning in artificial neural networks?**

## Experimental Design

### Symbolic arithmetic task

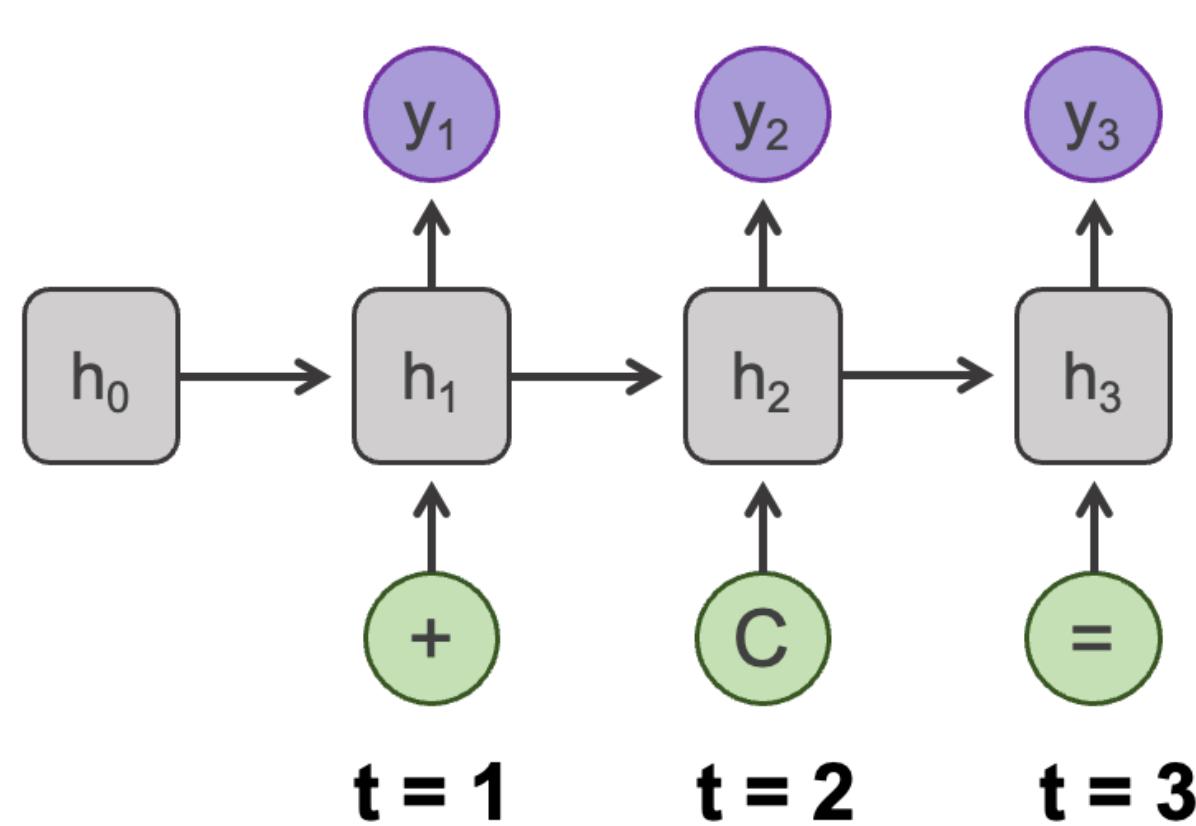
Primitive	Simple Addition	Complex Addition
$+ A = 1$	$+ B + C = 5$	$+ D + A + B = 8$

### Curricula



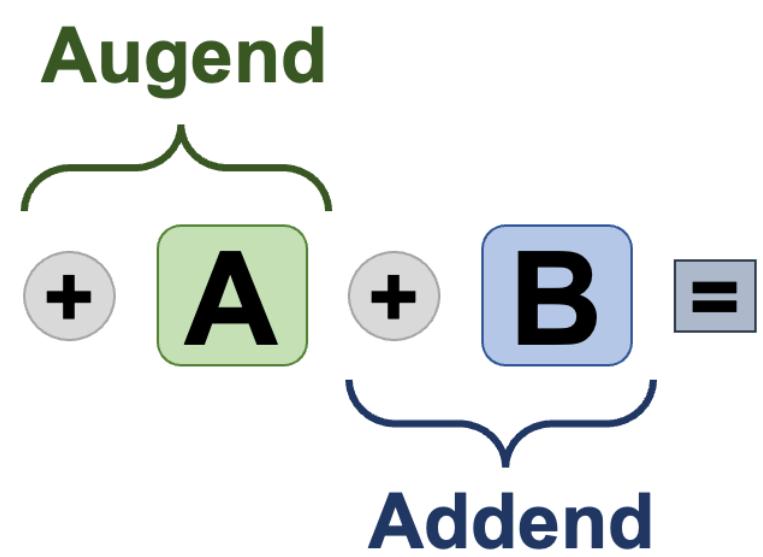
### Models

- Recurrent neural networks (RNNs)
- Sequences of one-hot vectors represented sums
- One set trained on Balanced and one on Primitive curriculum

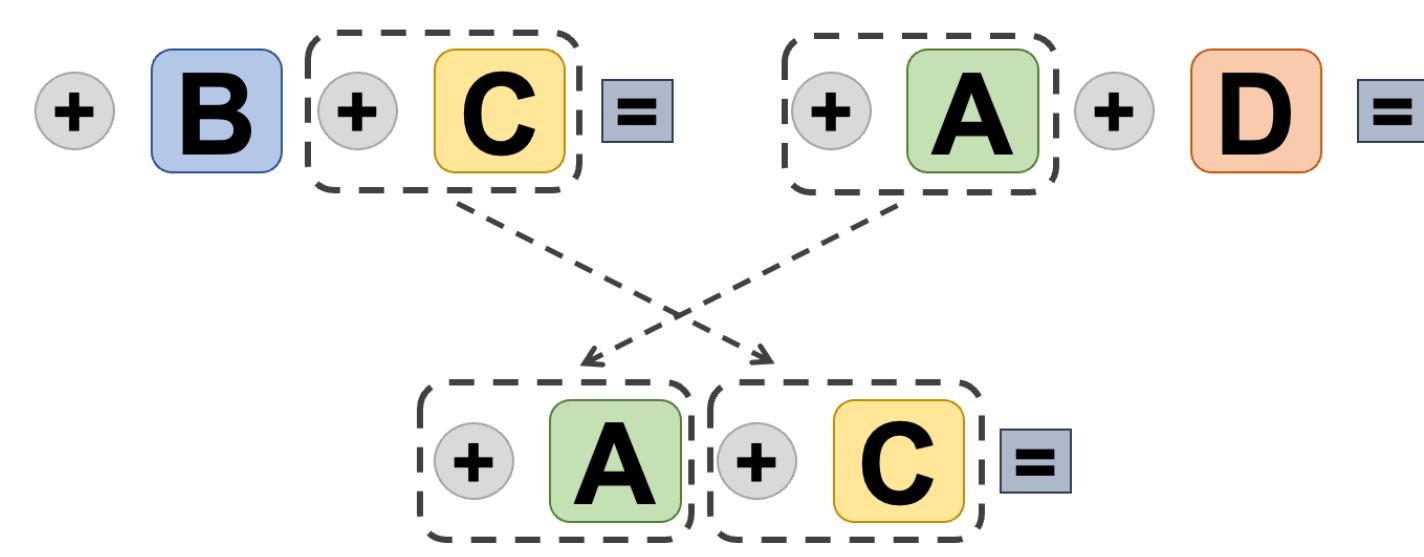


## Experiment 1: Testing systematicity

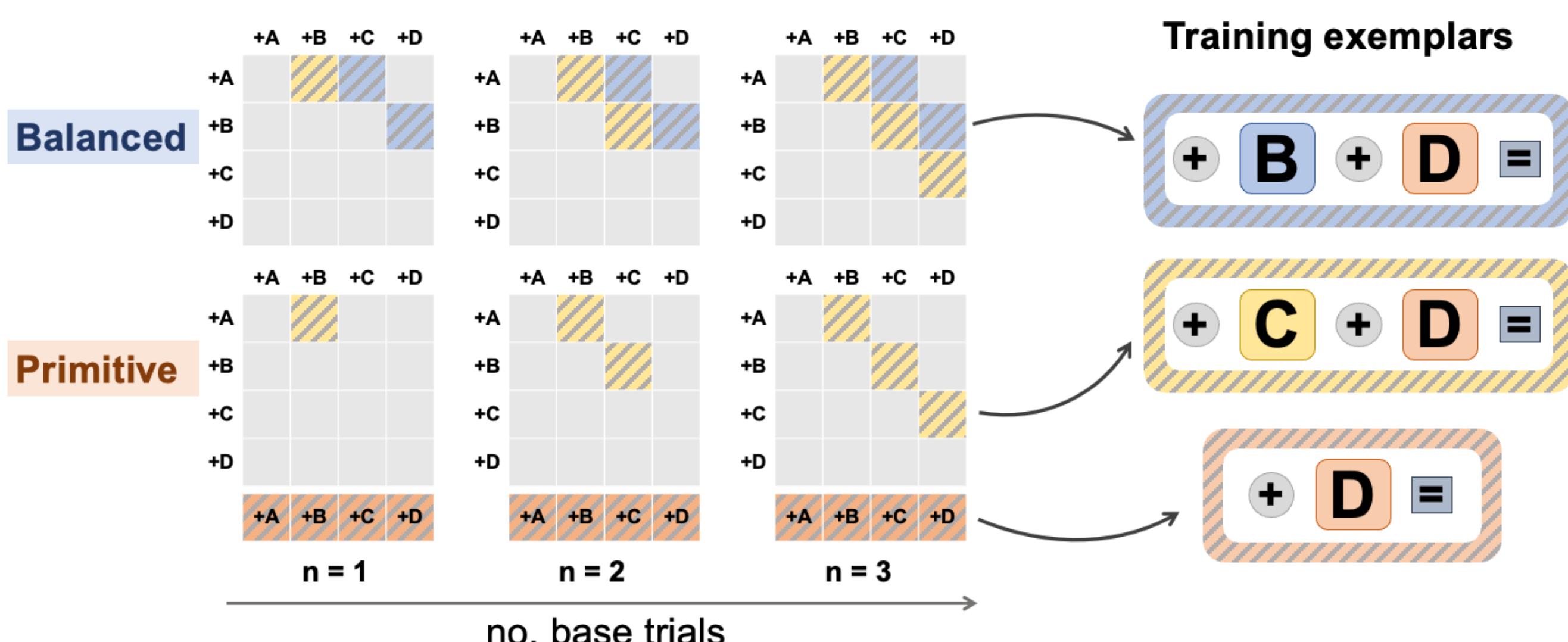
### Simple addition



### Systematicity

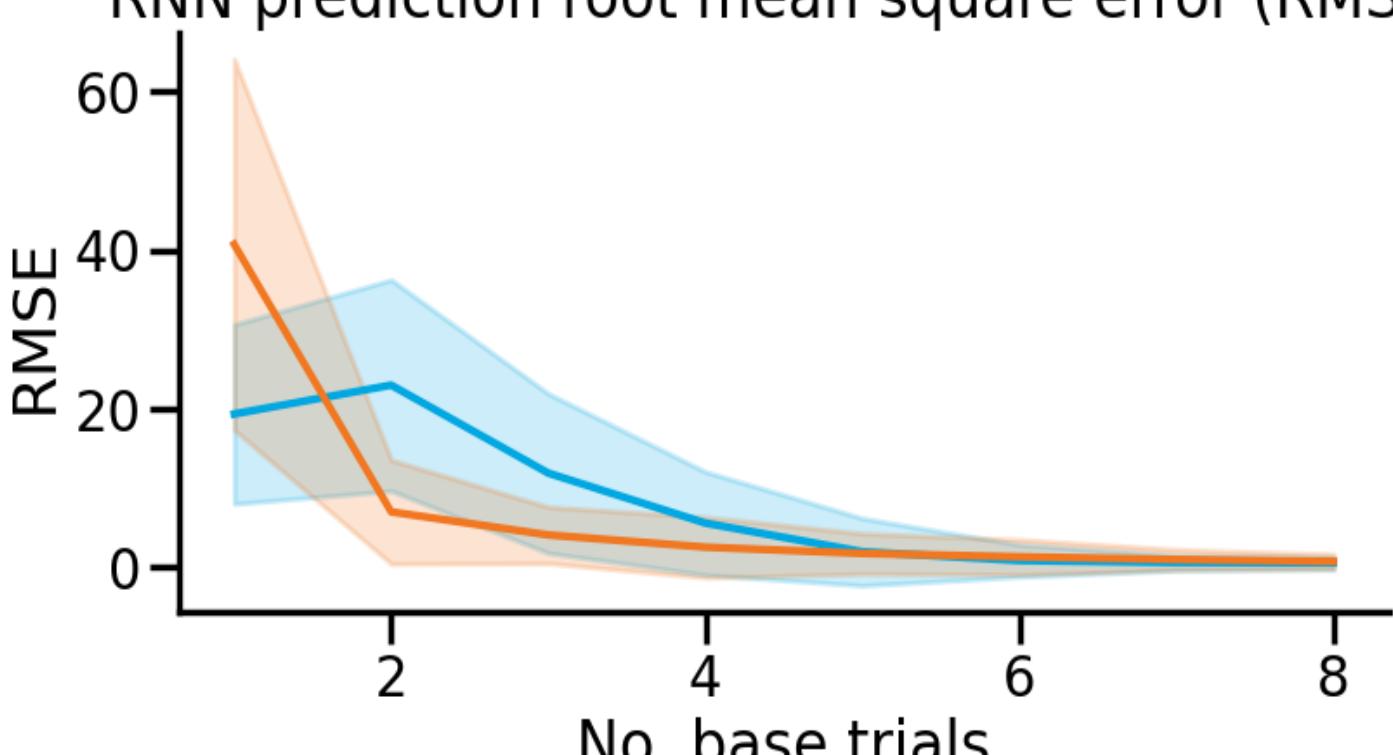


### RNNs trained on increasing number of exemplars

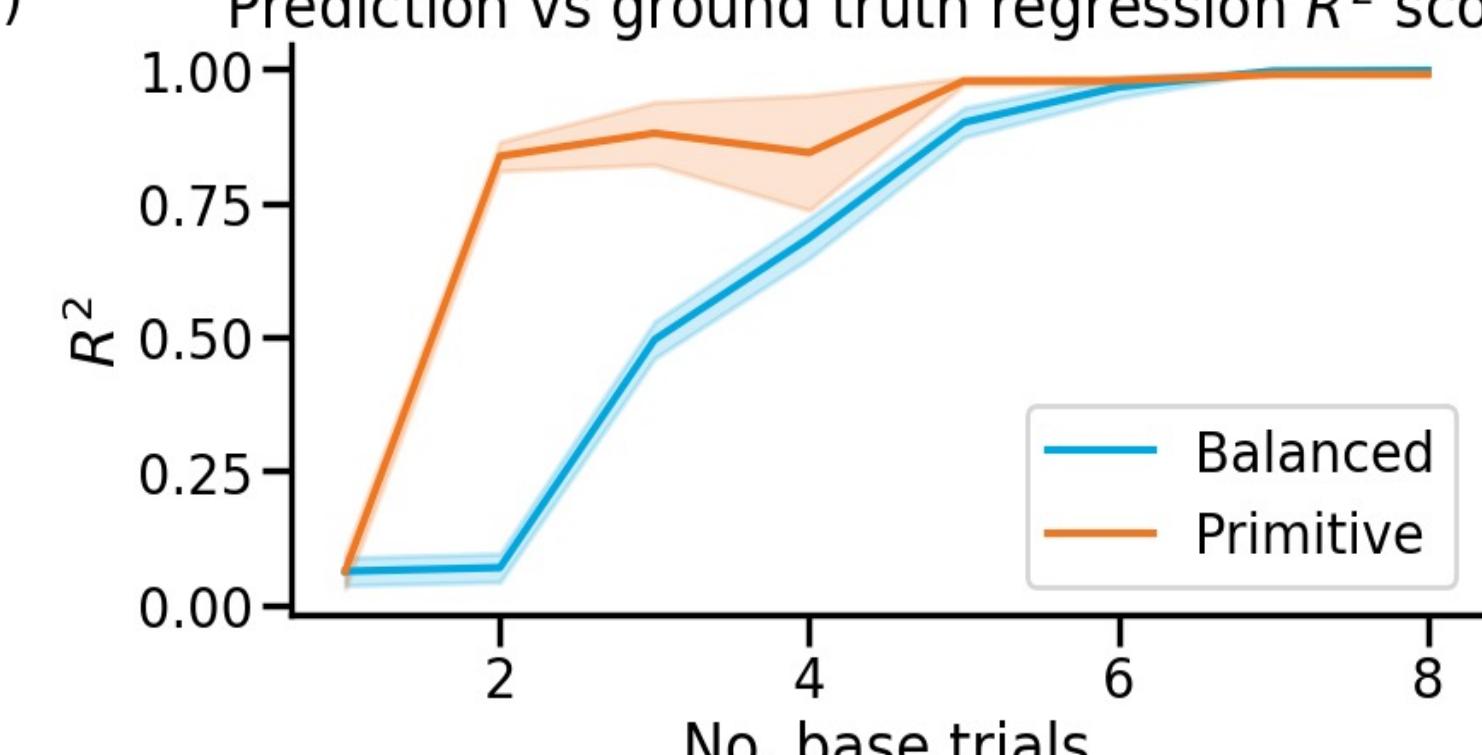


- With few training exemplars, Primitive vs Balanced group generalised better to unseen trials, indicating greater sample efficiency

RNN prediction root mean square error (RMSE)



Prediction vs ground truth regression  $R^2$  score

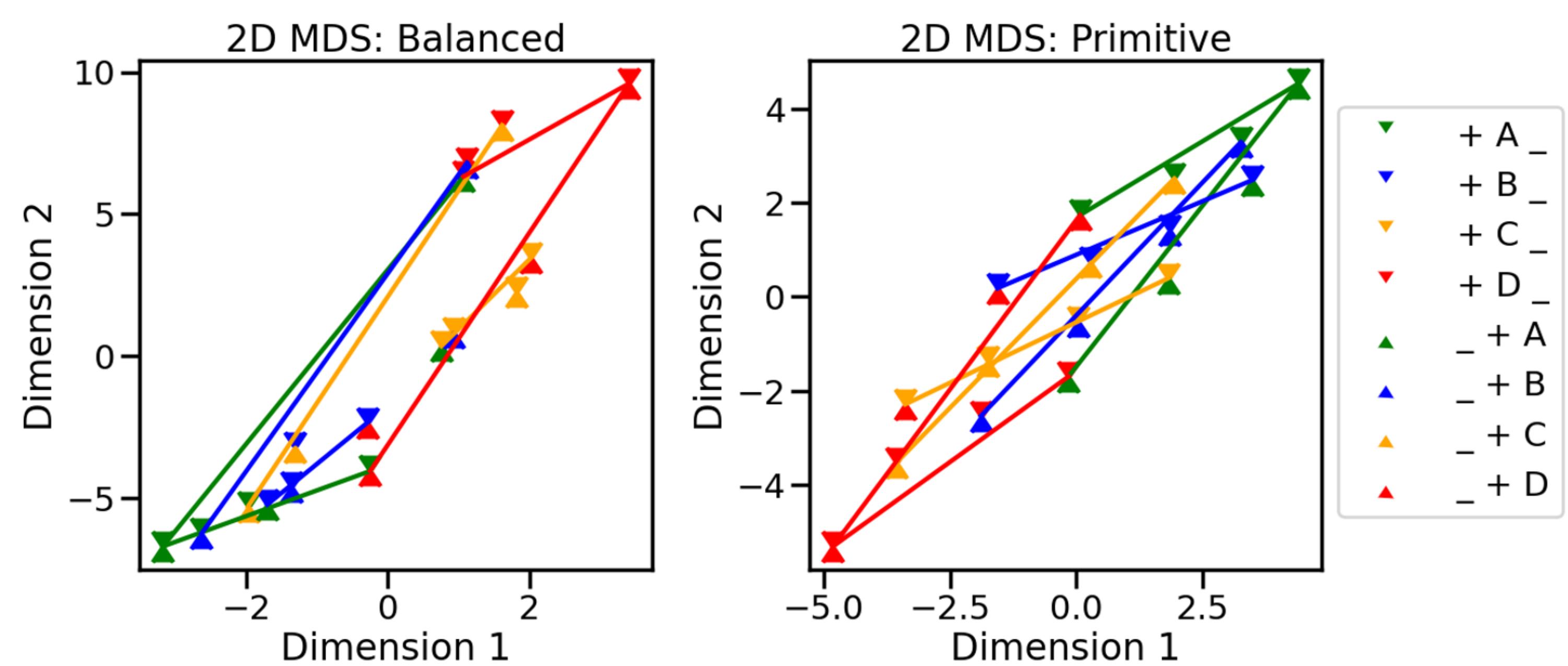


## Representation Similarity Analysis (RSA)

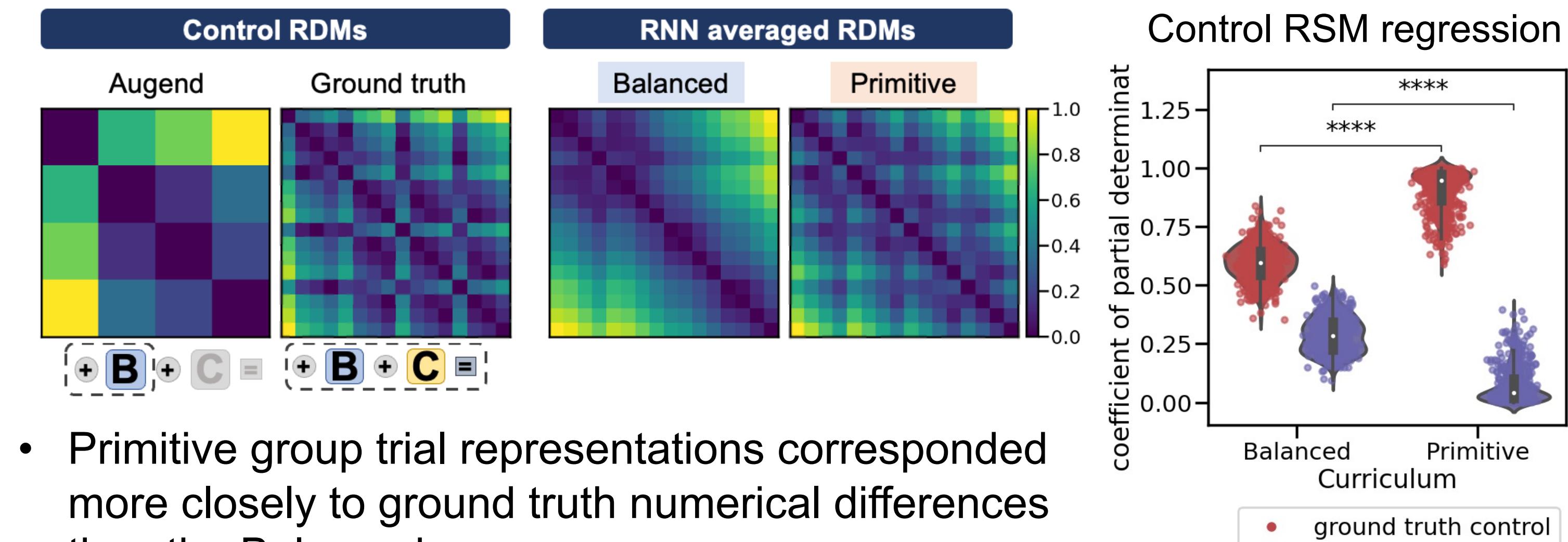
### Multi Dimensional Scaling (MDS)

Hidden layer activations for the full set of 16 unique simple addition sequences

- Primitive group formed a sheared grid-like arrangement
- Balanced group clustered representations by the augend



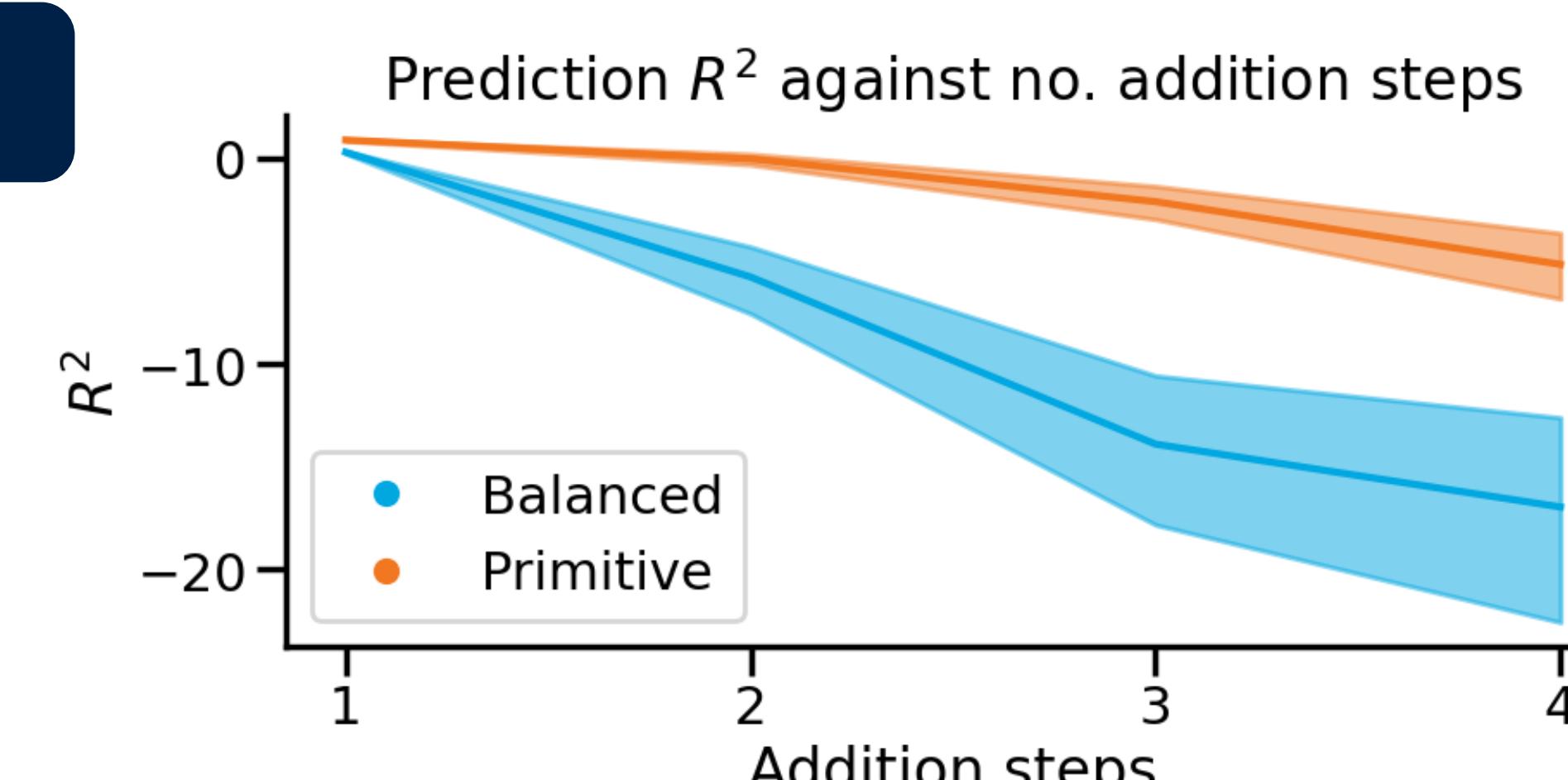
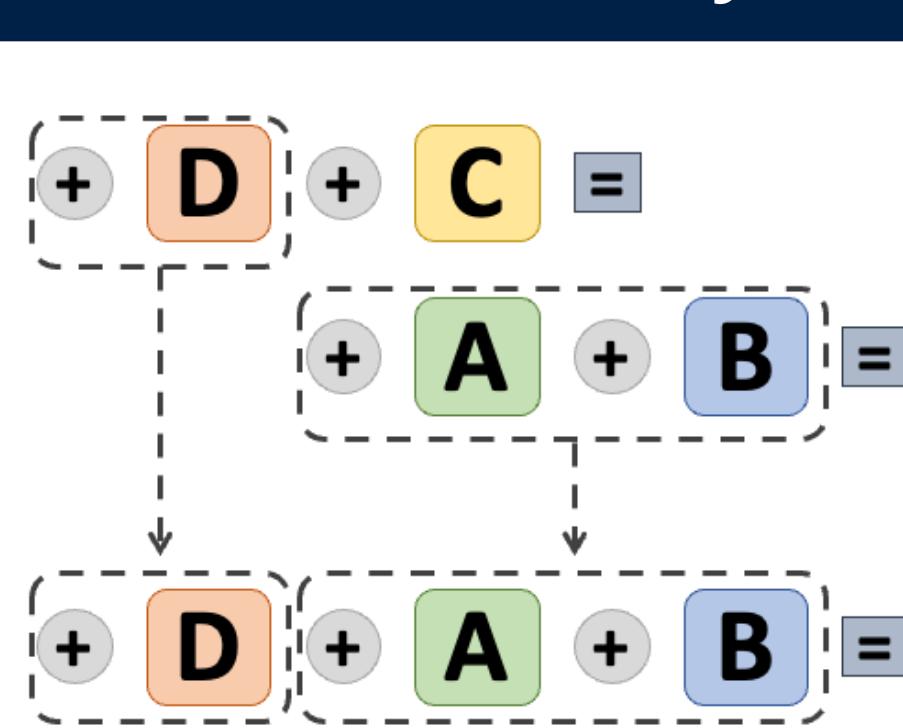
### Representation Dissimilarity Matrix (RDM) analysis



- Primitive group trial representations corresponded more closely to ground truth numerical differences than the Balanced

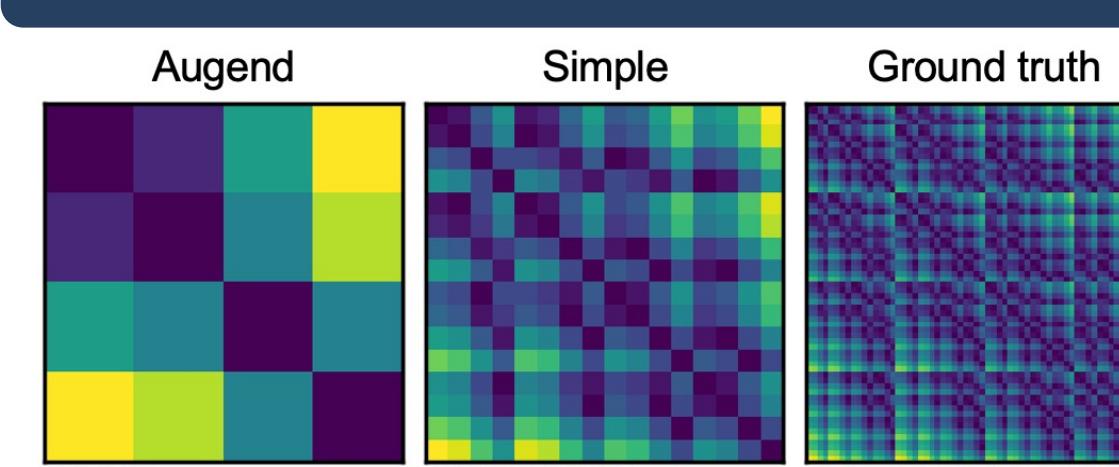
## Experiment 2: Testing productivity

### Productivity

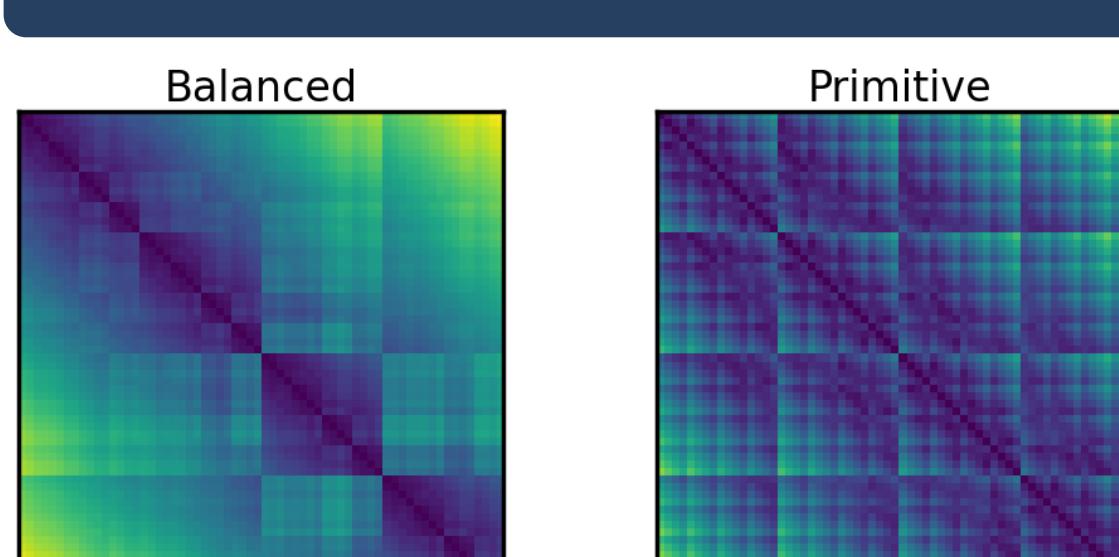


- RNNs trained on simple addition trials (2 base trials) were evaluated on 2, 3, and 4 step complex addition trials
- Performance degraded with increasing addition steps for both groups, but drop was more pronounced for the Balanced.

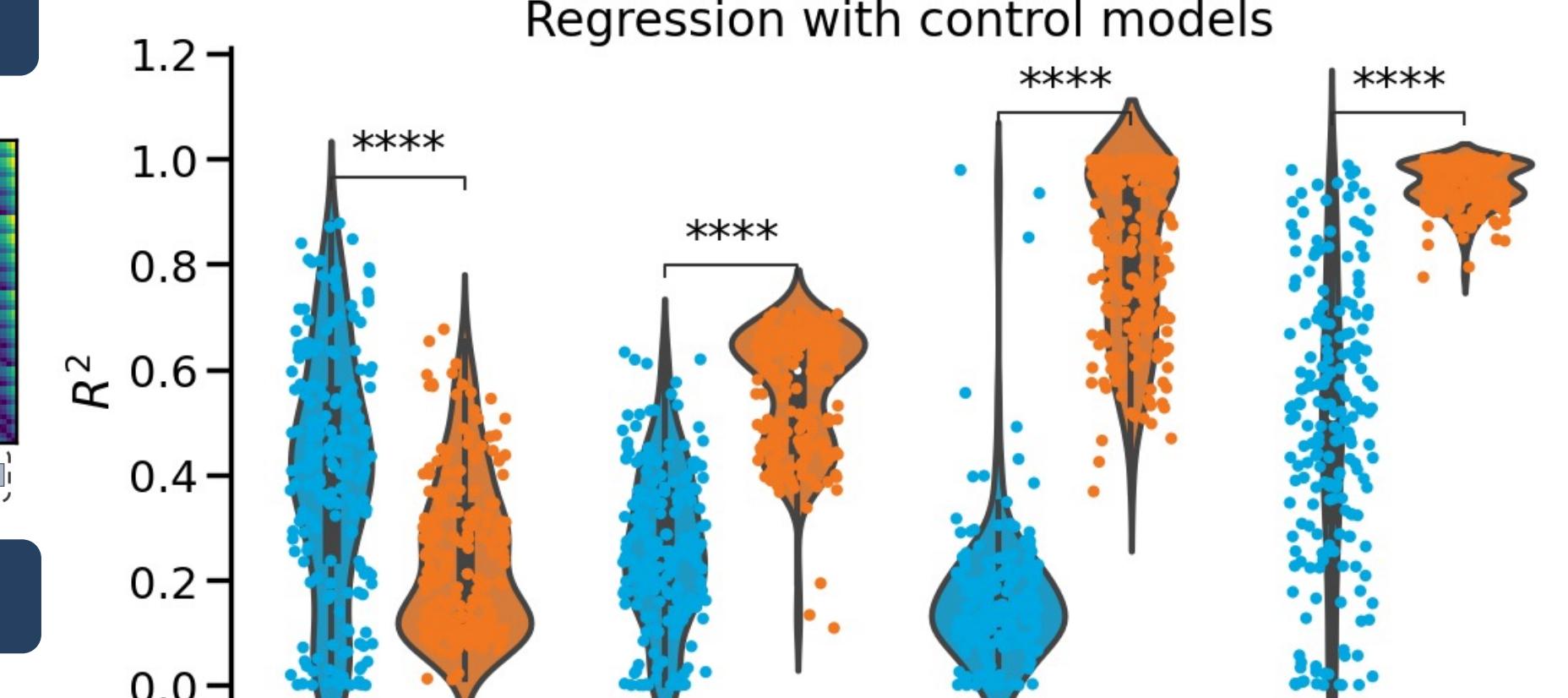
### Control RDMs



### Control RDMs



### Two Step Complex Addition RDM Regression with control models



- Primitive group RNNs generalised better to longer addition sequences

## Conclusions

- Primitive training facilitated compositional generalization in neural networks in an arithmetic task.
  - improving sample efficiency, systematicity, and productivity.
- RNNs with primitive training vs without had distinct representational geometries that corresponded to the ground truth numerical differences more closely.