深蓝学院
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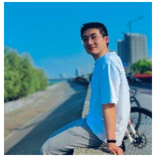
Classic Local Planning Frameworks for Mobile Robots

Lecture 8



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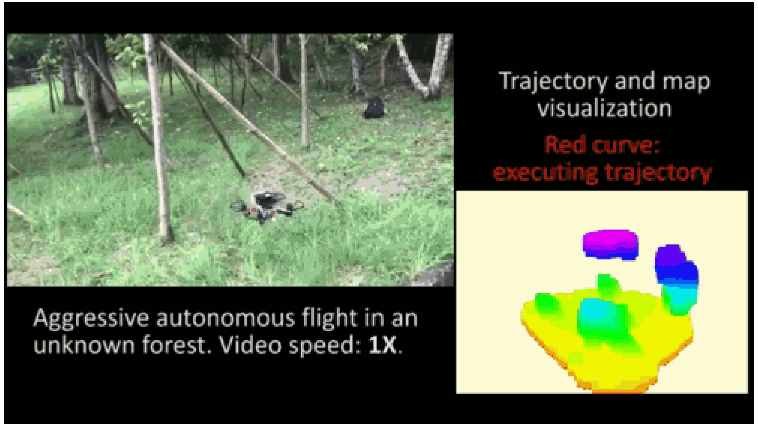
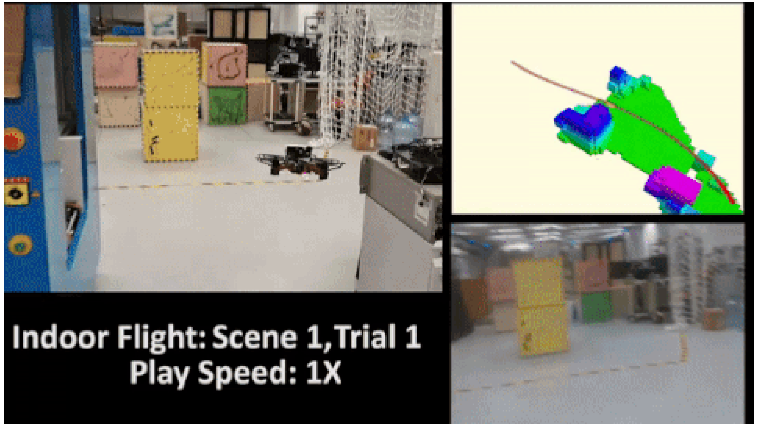
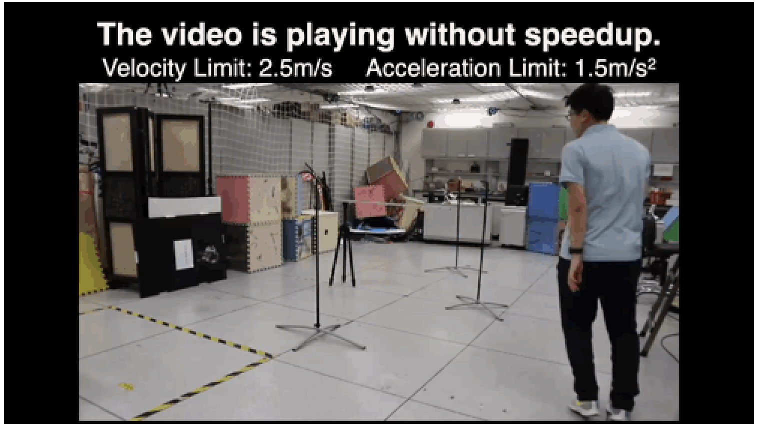
Fast-Planner[1]

[1] Zhou B, Gao F, Wang L, et al. Robust and efficient quadrotor trajectory generation for fast autonomous flight[J]. IEEE RA-L, 2019.

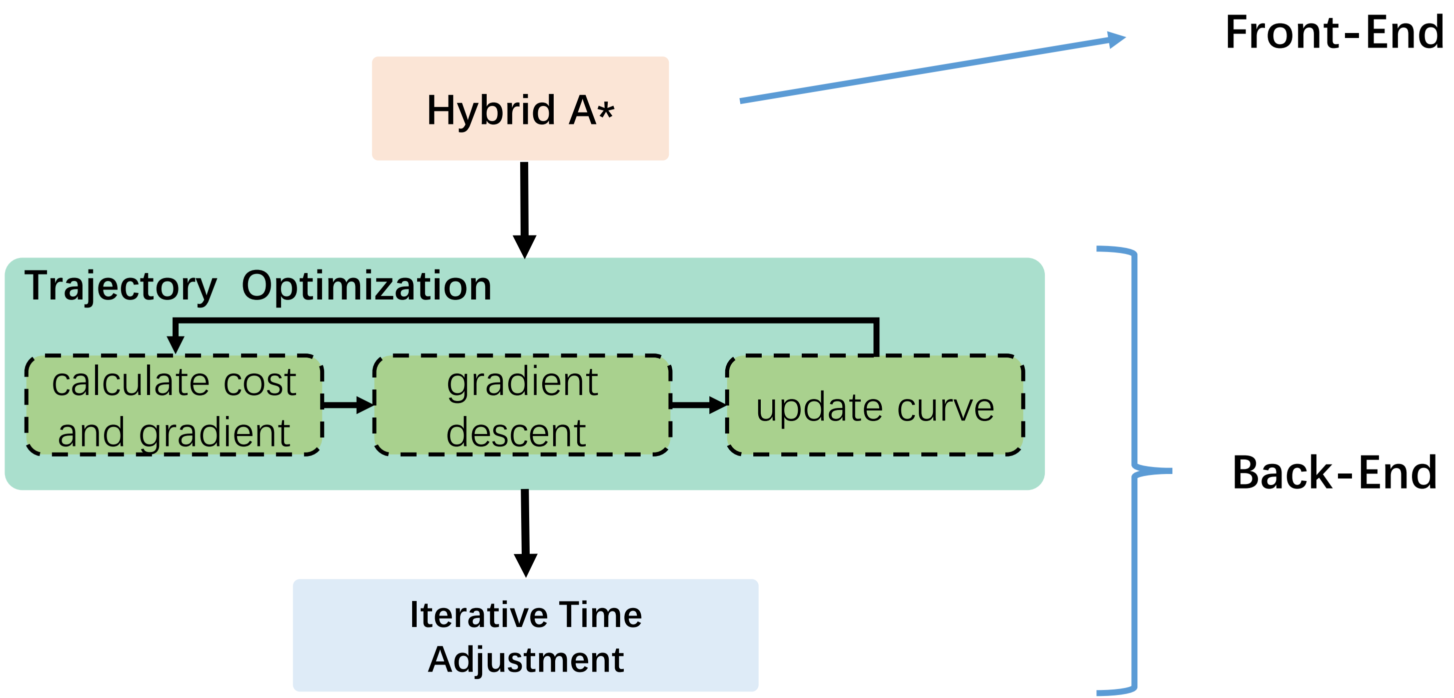
Fast-Planner







Fast-Planner



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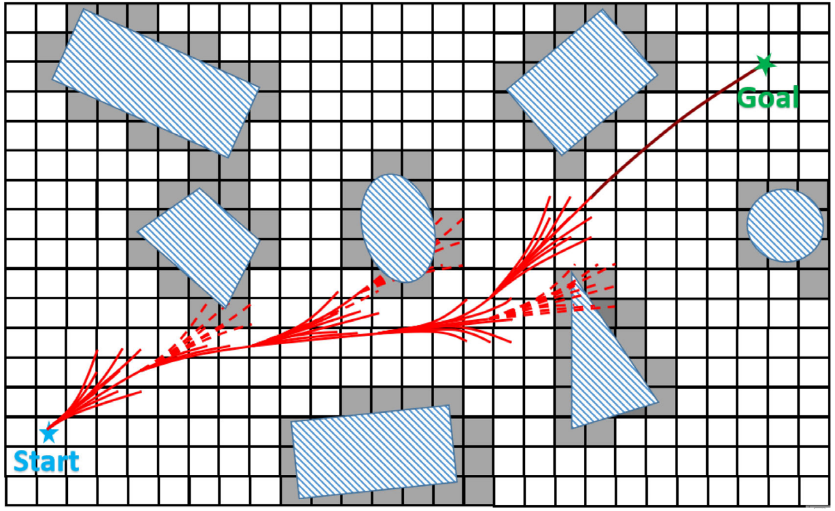
Front-End

A* vs Hybrid A*



A*

Hybrid A*

Front-End

A* vs Hybrid A*

Loop

- If the queue is empty, return FALSE; break;
- Remove the node “n” with the lowest f(n)=g(n)+h(n) from the priority queue
- Mark node “n” as expanded
- If the node “n” is the goal state, return TRUE; break;
- For all unexpanded neighbors “m” of node “n”
  - If g(m) = infinite
    - g(m)= g(n) + Cnm
    - Push node “m” into the queue
  - If g(m) > g(n) + C_{nm}
    - g(m)= g(n) + Cnm
- end

End Loop

A*

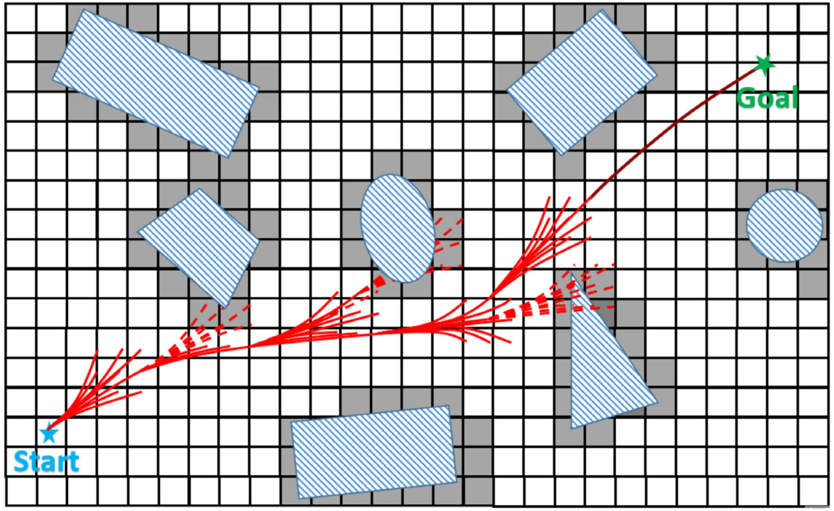
Algorithm 1: Kinodynamic Path Searching.

Initialize();
while $\neg\,\mathcal{P}$.**empty**() **do**
  $n_c \leftarrow \mathcal{P}$.**pop**(), $\mathcal{C}$.**insert**(n_c) ;
  if ReachGoal(n_c) $\vee$ **AnalyticExpand**(n_c) **then**
    return RetrievePath();
  $primitives \leftarrow$ **Expand**(n_c);
  $nodes \leftarrow$ **Prune**($primitives$);
  for n_i ***in*** $nodes$ **do**
    if $\neg\,\mathcal{C}$.**contain**(n_i) $\wedge$ **CheckFeasible**(n_i) **then**
      $g_{temp} \leftarrow n_c.g_c +$ **EdgeCost**(n_i) ;
      if $\neg\,\mathcal{P}$.**contain**(n_i) **then**
        $\mathcal{P}$.**add**(n_i);
      else if $g_{temp} \geq n_i.g_c$ **then**
        continue;
      $n_i.parent \leftarrow n_c$, $n_i.g_c \leftarrow g_{temp}$;
      $n_i.f_c \leftarrow n_i.g_c +$ **Heuristic**(n_i);

Hybrid A*

Front-End

Kinodynamic Path Searching: Hybrid A*



Algorithm 1: Kinodynamic Path Searching.

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    return **RetrievePath**();
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  $nodes \leftarrow$ **Prune**($primitives$);
  for n_i ***in*** $nodes$ **do**
    if $\neg\ \mathcal{C}$.**contain**(n_i) $\wedge$ **CheckFeasible**(n_i) **then**
      $g_{temp} \leftarrow n_c.g_c + \boxed{\textbf{EdgeCost}(n_i)}$;
      if $\neg\ \mathcal{P}$.**contain**(n_i) **then**
        $\mathcal{P}$.**add**(n_i);
      else if $g_{temp} \geq n_i.g_c$ **then**
        continue;
      $n_i.parent \leftarrow n_c$, $n_i.g_c \leftarrow g_{temp}$;
      $n_i.f_c \leftarrow n_i.g_c + \boxed{\textbf{Heuristic}(n_i);}$

Front-End

Hybrid A*: Primitives Generation

- represent the trajectory by three independent 1-D time-parameterized polynomial functions 使用三个独立的一维多项式表示轨迹

$$\mathbf{p}(t) \coloneqq \left[p_x(t), p_y(t), p_z(t)\right]^{\mathrm{T}}$$

$$p_\mu(t) = \sum_{k=0}^{K} a_k t^k \,, \mu \in \{x, y, z\}$$

For example, K=2, $\mu = x$

$$p_x(t) = a_0 + a_1 t + a_2 t^2$$

- From the view of quadrotor systems, it corresponds to a linear time-invariant (LTI) system.

state vector: $\mathbf{x}(t) \coloneqq \boxed{\left[\mathbf{p}(t)^{\mathrm{T}}, \dot{\mathbf{p}(t)}^{\mathrm{T}}, \dots, \mathbf{p}^{(n-1)}(t)^{\mathrm{T}}\right]^{\mathrm{T}}} \subset \mathbb{R}^{3n}$

control input: $\mathbf{u}(t) \coloneqq \mathbf{p}^{(n)}(t) \in \mathcal{U} \coloneqq [-u_{max}, u_{max}]^3 \subset \mathbb{R}^3$

Front-End

Hybrid A*: Primitives Generation

state vector: $\mathbf{x}(t) := \left[\mathbf{p}(t)^{\mathrm{T}}, \dot{\mathbf{p}}(t)^{\mathrm{T}}, \ldots, \mathbf{p}^{(n-1)}(t)^{\mathrm{T}}\right]^{\mathrm{T}} \subset \mathbb{R}^{3n}$ 状态空间方程

control input: $\mathbf{u}(t) := \mathbf{p}^{(n)}(t) \in \mathcal{U} := [-u_{max}, u_{max}]^3 \subset \mathbb{R}^3$

- The state space model can be defined as:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I}_3 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_3 & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \cdots & \mathbf{0} & \mathbf{I}_3 \\ \mathbf{0} & \cdots & \cdots & \mathbf{0} & \mathbf{0} \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \\ \mathbf{I}_3 \end{bmatrix}$$

- The state equation： $\mathbf{x}(t) = e^{\mathbf{A}t}\boxed{\mathbf{x}(0)} + \int_0^t e^{\mathbf{A}(t-\tau)}\mathbf{B}\boxed{\mathbf{u}(\tau)}\, d\tau$ 计算整条轨迹

initial state control input

Front-End

Hybrid A*: Primitives Generation

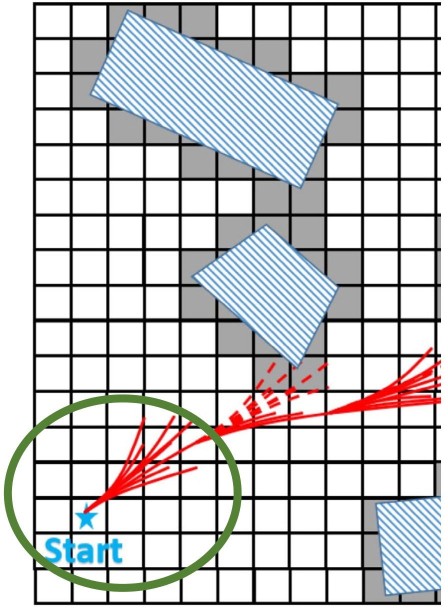
- The state equation： $\mathbf{x}(t) = e^{\mathbf{A}t}\boxed{\mathbf{x}(0)} + \int_0^t e^{\mathbf{A}(t-\tau)}\boxed{\mathbf{Bu}(\tau)}\,d\tau$

initial state   control input

- Each axis $[-u_{max}, u_{max}]$ is discretized uniformly as

$$\left\{-u_{max}, -\frac{r-1}{r}u_{max}, -\frac{r-2}{r}u_{max}, \dots, \frac{r-2}{r}u_{max}, \frac{r-1}{r}u_{max}, u_{max}\right\}$$

which results in $(2r+1)^3$primitives.



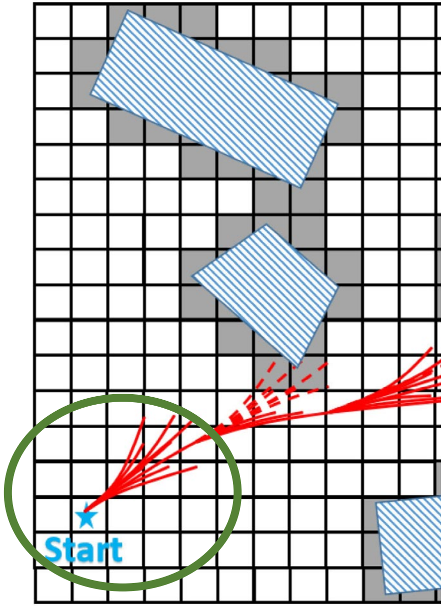
Front-End

Hybrid A*: Primitives Generation

- The state equation： $\mathbf{x}(t) = e^{\mathbf{A}t}\mathbf{x}(0) + \int_0^t e^{\mathbf{A}(t-\tau)}\mathbf{B}\mathbf{u}(\tau)\, d\tau$

  state vector: $\mathbf{x}(t) := \left[\mathbf{p}(t)^{\mathrm{T}}, \dot{\mathbf{p}(t)}^{\mathrm{T}}, \ldots, \mathbf{p}^{(n-1)}(t)^{\mathrm{T}}\right]^{\mathrm{T}} \subset \mathbb{R}^{3n}$

  control input: $\mathbf{u}(t) := \mathbf{p}^{(n)}(t) \in \mathcal{U} := [-u_{max}, u_{max}]^3 \subset \mathbb{R}^3$



- In Fast-Planner

  state vector:

$$\mathbf{x}(t) := \left[\mathbf{p}(t)^{\mathrm{T}}, \dot{\mathbf{p}(t)}^{\mathrm{T}}\right]^{\mathrm{T}} = [p_x(t), p_y(t), p_z(t), v_x(t), v_y(t), v_z(t)]^{\mathrm{T}} \subset \mathbb{R}^6$$

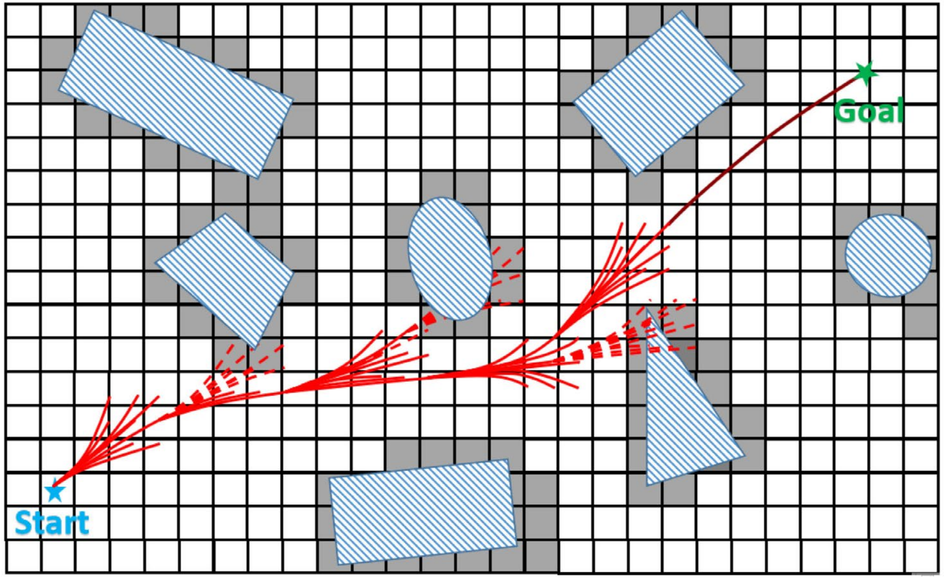
  control input:

$$\mathbf{u}(t) := \mathbf{p}^{(2)}(t) = [a_x(t), a_y(t), a_z(t)]^{\mathrm{T}} \subset \mathbb{R}^3$$

Front-End

The state equation： $\mathbf{x}(t) = e^{\mathbf{A}t}\mathbf{x}(0) + \int_0^t e^{\mathbf{A}(t-\tau)}\mathbf{B}\mathbf{u}(\tau)\,d\tau$

Hybrid A*: Actual Cost



actual cost of one primitive : $e_c = (\|\mathbf{u}_d\|^2 + \rho)\tau$

actual cost of one trajectory consists of J primitives:

$$g_c = \sum_{j=1}^{J}\left(\|(\mathbf{u}_d)_j\|^2 + \rho\right)\tau$$

Algorithm 1: Kinodynamic Path Searching.

```
Initialize();
while ¬ P.empty() do
    n_c ← P.pop(), C.insert(n_c) ;
    if ReachGoal(n_c) ∨ AnalyticExpand(n_c) then
        return RetrievePath();
    primitives ← Expand(n_c);
    nodes ← Prune(primitives);
    for n_i in nodes do
        if ¬ C.contain(n_i) ∧ CheckFeasible(n_i) then
            g_temp ← n_c.g_c + EdgeCost(n_i) ;
            if ¬ P.contain(n_i) then
                P.add(n_i);
            else if g_temp ≥ n_i.g_c then
                continue;
            n_i.parent ← n_c; n_i.g_c ← g_temp;
            n_i.f_c ← n_i.g_c + Heuristic(n_i);
```

actual cost of an optimal trajectory from the start state to the current state

Front-End

Hybrid A*: Heurisitic Cost

$$p(t) = a_3 t^3 + a_2 t^2 + a_1 t + a_0$$

$$p(0) = p_{\mu c}, \qquad \dot{p}(0) = v_{\mu c}$$
$$p(T) = p_{\mu g}, \qquad \dot{p}(T) = v_{\mu g}$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ T^3 & T^2 & T & 1 \\ 3T^2 & 2T & 1 & 0 \end{bmatrix} \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} p_{\mu c} \\ v_{\mu c} \\ p_{\mu g} \\ v_{\mu g} \end{bmatrix}$$

- We compute a closed form trajectory that minimizes $\mathcal{J}(T)$ from $\mathbf{x}_{\mu c}$ to the goal state $\mathbf{x}_{\mu g}$ by applying the **Pontryagins minimum principle**:

$$\boxed{p_\mu^*(t) = \frac{1}{6}\alpha_\mu t^3 + \frac{1}{2}\beta_\mu t^2 + v_{\mu c} t + p_{\mu c}}$$

$$\begin{bmatrix} \alpha_\mu \\ \beta_\mu \end{bmatrix} = \frac{1}{T^3} \begin{bmatrix} -12 & 6T \\ 6T & -2T^2 \end{bmatrix} \begin{bmatrix} p_{\mu g} - p_{\mu c} - v_{\mu c} T \\ v_{\mu g} - v_{\mu c} \end{bmatrix}$$

- $p_{\mu c}, v_{\mu c}, p_{\mu g}, v_{\mu g}$ are the current and goal position and velocity.

$$a_\mu^*(t) = \alpha_\mu t + \beta_\mu$$

$$\mathbf{u}(t) := [a_x(t), a_y(t), a_z(t)]^{\mathrm{T}}$$

$$\mathcal{J}^*(T) = \int_0^T \|\mathbf{u}(t)\|^2 \, dt + \rho T$$
$$= \sum_{\mu \in \{x,y,z\}} \left(\frac{1}{3}{\alpha_\mu}^2 T^3 + \frac{1}{2}\alpha_\mu \beta_\mu T^2 + {\beta_\mu}^2 T \right) + \rho T$$

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                continue;
            n_i.parent ← n_c, n_i.g_c ← g_temp;
            n_i.f_c ← n_i.g_c + Heuristic(n_i);
```

Front-End

Hybrid A*: Heurisitic Cost

$$\begin{bmatrix}\alpha_\mu \\ \beta_\mu\end{bmatrix} = \frac{1}{T^3}\begin{bmatrix}-12 & 6T \\ 6T & -2T^2\end{bmatrix}\begin{bmatrix}p_{\mu g} - p_{\mu c} - v_{\mu c}T \\ v_{\mu g} - v_{\mu c}\end{bmatrix}$$

$$\mathcal{J}^*(T) = \sum_{\mu \in \{x,y,z\}} \left(\frac{1}{3}\alpha_\mu{}^2 T^3 + \frac{1}{2}\alpha_\mu \beta_\mu T^2 + \beta_\mu{}^2 T\right) + \rho T$$

- To find the optimal time T that minimize the cost, we substitute α_μ, β_μ into $\mathcal{J}^*(T)$ and find the roots of

$$\frac{\partial \mathcal{J}^*(T)}{\partial T} = 0$$

- The root making a minimum cost min $\mathcal{J}^*$ and feasible trajectory is selected and denoted as T_h.

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            n_i.f_c ← n_i.g_c + Heuristic(n_i);
```
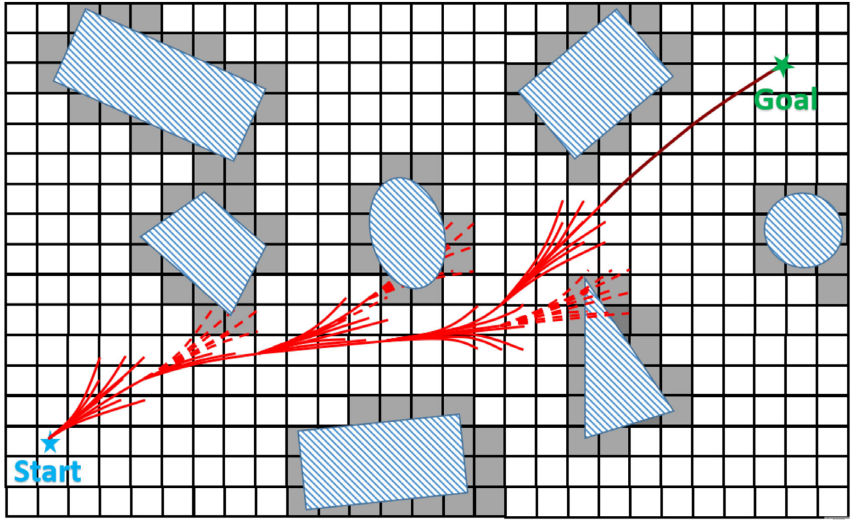

We use $\mathcal{J}^*(T_h)$ as the heuristic.

Front-End

Kinodynamic Path Searching: Hybrid A*



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