Solution 1: Using Lists of Trees

```
class MyTreeIter[A](val lst: MyList[MyTree[A]]) extends Iter[A] {
  val getValue = lst match {
    case MyCons(Node(v,_,_), _) => Some(v)
    case _ => None
  }
  def getNext = {
    val remainingTrees : MyList[MyTree[A]] = lst match {
      case MyNil() => throw new Exception("...")
      case MyCons(hd,tl) => hd match {
        case Empty() => throw new Exception("...")
        case Node(_,Empty(),Empty()) => tl
        case Node(_,lt,Empty()) => MyCons(lt,tl)
        case Node(_,Empty(),rt) => MyCons(rt,tl)
        case Node(_,lt,rt) => MyCons(lt,MyCons(rt,tl))
      }
    }
    new MyTreeIter(remainingTrees)
  } }
```

Lazy Iteration using Lists of Trees

```
sealed abstract class MyTree[A] extends Iterable[A]
case class Empty[A]() extends MyTree[A] {
  val iter = new MyTreeIter(MyNil())
}
case class Node[A](value: A,
                   left: MyTree[A],
                   right: MyTree[A]) extends MyTree[A]
{
  val iter = new MyTreeIter(MyCons(this,MyNil()))
}

{ val t: MyTree[Int] = generateTree(200000)
  time (sumN((x:Int) => x)(100, t))
  time (sumN((x:Int) => x)(100000, t))
}
```

Solution 2: Lazy List

```
sealed abstract class LazyList[A] extends Iter[A] {
  def append(lst: LazyList[A]) : LazyList[A]
}

case class LNil[A]() extends LazyList[A] {
  def getValue = None
  def getNext = throw new Exception("")
  def append(lst: LazyList[A]) = lst
}

class LCons[A](hd: A, _tl: =>LazyList[A]) extends LazyList[A] {
  lazy val tl = _tl
  def getValue = Some(hd)
  def getNext = tl
  def append(lst: LazyList[A]) = LCons(hd, tl.append(lst)) }
object LCons {
  def apply[A](hd: A, tl: =>LazyList[A]) = new LCons(hd, tl)
}
```

Note: “append” is not recursive!!!

Lazy Iteration using LazyList

```
sealed abstract class MyTree[A] extends Iterable[A] {
  def iter : LazyList[A]
}
case class Empty[A]() extends MyTree[A] {
  val iter = LNil()
}
case class Node[A](value: A,
                   left: MyTree[A],
                   right: MyTree[A]) extends MyTree[A] {
  lazy val iter = LCons(value, left.iter.append(right.iter))
  // lazy val iter = left.iter.append(LCons(value,right.iter))
  // lazy val iter = left.iter.append(right.iter.append(
  //                    LCons(value,LNil())))
}
{ val t: MyTree[Int] = generateTree(200000)
  time (sumN((x:Int) => x)(100, t))
  time (sumN((x:Int) => x)(100000, t))
}
```

Note: "iter" is not recursive!!!

Wrapper for Inheritance

Using a Wrapper Class

```
abstract class Iter[A] {
  def getValue: Option[A]
  def getNext: Iter[A]
}

class ListIter[A](val list: List[A]) extends Iter[A] {
  def getValue = list.headOption
  def getNext = new ListIter(list.tail)
}

sumElements((x:Int)=>x)(new ListIter(List(1,2,3,4)))
```

MyTree Using ListIter

```
abstract class Iterable[A] {
  def iter : Iter[A]
}
sealed abstract class MyTree[A] extends Iterable[A] {
  def iter : ListIter[A]
}
case class Empty[A]() extends MyTree[A] {
  val iter : ListIter[A] = new ListIter(Nil)
}
case class Node[A](value: A,
                   left: MyTree[A],
                   right: MyTree[A])
  extends MyTree[A] {
  val iter : ListIter[A] = new ListIter(
    value::(left.iter.list ++ right.iter.list))
}
```

Test

```
val t : MyTree[Int] =
  Node(3, Node(4,Node(2,Empty(),Empty()),
    Node(3,Empty(),Empty())),
    Node(5,Empty(),Empty()))

sumElementsGen((x:Int)=>x)(t)
```

Abstract Class With Associate Types

Using an Associate Type

```
abstract class Iterable[A] {
  type iter_t
  def iter: iter_t
  def getValue(i: iter_t) : Option[A]
  def getNext(i: iter_t) : iter_t
}

def sumElements[A](f:A=>Int)(xs: Iterable[A]) : Int = {
  def sumElementsIter(i: xs.iter_t) : Int =
    xs.getValue(i) match {
      case None => 0
      case Some(n) => f(n) + sumElementsIter(xs.getNext(i))
    }
  sumElementsIter(xs.iter)
}
```

MyTree Using List

```
sealed abstract class MyTree[A] extends Iterable[A] {
  type iter_t = List[A]
  def getValue(i: List[A]): Option[A] = i.headOption
  def getNext(i: List[A]): List[A] = i.tail
}
case class Empty[A]() extends MyTree[A] {
  val iter : List[A] = Nil
}
case class Node[A](value: A,
                   left: MyTree[A], right: MyTree[A])
  extends MyTree[A] {
  val iter = value :: (left.iter ++ right.iter) //Pre-order
//val iter = left.iter ++ (value :: right.iter) // In-order
//val iter = left.iter ++ (right.iter ++ List(value))
                                                //Post-order
}
```

Test

```
val t : MyTree[Int] =
  Node(3, Node(4,Node(2,Empty(),Empty()),
    Node(3,Empty(),Empty())),
    Node(5,Empty(),Empty()))

sumElements((x:Int)=>x)(t)
```

Abstract Class with Arguments

Abstract Class with Arguments

```
abstract class IterableH[A] extends Iterable[A] {
  def hasElement(a: A) : Boolean
}
abstract class IterableHE[A](eq: (A,A) => Boolean)
  extends IterableH[A]
{
  def hasElement(a: A) : Boolean = {
    def hasElementIter(i: iter_t) : Boolean =
      getValue(i) match {
        case None => false
        case Some(n) =>
          if (eq(a,n)) true
          else hasElementIter(getNext(i))
      }
    hasElementIter(iter)
  }
```

MyTree

```
sealed abstract class MyTree[A](eq:(A,A)=>Boolean)
  extends IterableHE[A](eq) {
  type iter_t = List[A]
  def getValue(i : List[A]) : Option[A] = i.headOption
  def getNext(i: List[A]) : List[A] = i.tail
}
case class Empty[A](eq: (A,A)=>Boolean)
  extends MyTree[A](eq) {
  val iter : List[A] = Nil
}
case class Node[A](eq: (A,A)=>Boolean,
                   value: A, left: MyTree[A], right: MyTree[A])
  extends MyTree[A](eq) {
  val iter : List[A] = value :: (left.iter ++ right.iter)
}
```

Test

```
val leq = (x:Int,y:Int) => x == y
val lEmpty = Empty(leq)
def lNode(n: Int, t1: MyTree[Int], t2: MyTree[Int]) =
  Node(leq,n,t1,t2)

val t : MyTree[Int] =
  lNode(3, lNode(4,lNode(2,lEmpty,lEmpty),
                   lNode(3,lEmpty,lEmpty)),
           lNode(5,lEmpty,lEmpty))

sumElements((x:Int)=>x)(t)
t.hasElement(5)
t.hasElement(10)
```

Alternatively, Argument Elimination

```
abstract class IterableHE[A]
  extends Iterable[A]
{
  def eq(a:A, b:A) : Boolean
  def hasElement(a: A) : Boolean = {
    def hasElementIter(i: iter_t) : Boolean =
      getValue(i) match {
        case None => false
        case Some(n) =>
          if (eq(a,n)) true
          else hasElementIter(getNext(i))
      }
    hasElementIter(iter)
  }
}
```

MyTree

```
sealed abstract class MyTree[A] extends IterableHE[A] {
  type iter_t = List[A]
  def getValue(i : List[A]) : Option[A] = i.headOption
  def getNext(i: List[A]) : List[A] = i.tail
}
case class Empty[A](_eq:(A,A)=>Boolean) extends MyTree[A] {
  def eq(a:A, b:A) = _eq(a,b)
  val iter : List[A] = Nil
}
case class Node[A](_eq: (A,A)=>Boolean,
                   value: A, left: MyTree[A], right: MyTree[A])
  extends MyTree[A] {
  def eq(a:A, b:A) = _eq(a,b)
  val iter : List[A] = value :: (left.iter ++ right.iter)
}
```

Test

```
val leq = (x:Int,y:Int) => x == y
val lEmpty = Empty(leq)
def lNode(n: Int, t1: MyTree[Int], t2: MyTree[Int]) =
  Node(leq,n,t1,t2)

val t : MyTree[Int] =
  lNode(3, lNode(4,lNode(2,lEmpty,lEmpty),
                   lNode(3,lEmpty,lEmpty)),
           lNode(5,lEmpty,lEmpty))

sumElements((x:Int)=>x)(t)
t.hasElement(5)
t.hasElement(10)
```

More on Classes

Motivating Example

```
class Primes(val prime: Int, val primes: List[Int]) {
  def getNext: Primes = {
    val p = computeNextPrime(prime + 2)
    new Primes(p, primes ++ (p :: Nil))
  }
  def computeNextPrime(n: Int) : Int =
    if (primes.forall((p:Int) => n%p != 0)) n
    else computeNextPrime(n+2)
}

def nthPrime(n: Int): Int = {
  def go(primes: Primes, k: Int): Int =
    if (k <= 1) primes.prime
    else go(primes.getNext, k - 1)
  if (n <= 0) 2 else go(new Primes(3, List(3)), n)
}
nthPrime(10000)
```

Multiple Constructors

```
class Primes(val prime: Int, val primes: List[Int]) {
  def this() = this(3, List(3))
  def getNext: Primes = {
    val p = computeNextPrime(prime + 2)
    new Primes(p, primes ++ (p :: Nil))
  }
  def computeNextPrime(n: Int) : Int =
    if (primes.forall((p:Int) => n%p != 0)) n
    else computeNextPrime(n+2)
}

def nthPrime(n: Int): Int = {
  def go(primes: Primes, k: Int): Int =
    if (k <= 1) primes.prime
    else go(primes.getNext, k - 1)
  if (n == 0) 2 else go(new Primes, n)
}
nthPrime(10000)
```

Access Modifiers

- Access Modifiers
  - Private: Only the class can access the member.
  - Protected: Only the class and its sub classes can access the member.

Using Access Modifiers

```
class Primes private (val prime:Int, protected val primes:List[Int])
{ def this() = this(3, List(3))
  def getNext: Primes = {
    val p = computeNextPrime(prime + 2)
    new Primes(p, primes ++ (p :: Nil))
  }
  private def computeNextPrime(n: Int) : Int =
    if (primes.forall((p:Int) => n%p != 0)) n
    else computeNextPrime(n+2)
}

def nthPrime(n: Int): Int = {
  def go(primes: Primes, k: Int): Int =
    if (k <= 1) primes.prime
    else go(primes.getNext, k - 1)
  if (n == 0) 2 else go(new Primes, n)
}
nthPrime(10000)
```

Traits for Multiple Inheritance

Multiple Inheritance Problem

- Multiple Inheritance
  - The famous "diamond problem"

```
class A(val a: Int)
class B extends A(10)
class C extends A(20)
class D extends B, C.
```

Problem 1: What is the value of (new D).a ?

Problem 2: The constructor of A must be executed once because A may contain side effects such as sending messages over the network.

Scala's Solution: Trait

- Traits
  - A trait can implement any of its methods,
    but should have only one constructor with no arguments.
  - An [abstract] class (resp. trait) X can
    "extends" one trait or [abstract] class with any (resp. no) arguments
    "with" multiple traits T_1 , … , T_n
    such that, for each i,
    the least superclass of T_i, if exists, should be a superclass of X
    where
    C is a superclass of T if C is an (abstract) class and T transitively "extends" C.
  - No cyclic inheritance is allowed.
- Property
  - For any ancestor class in the inheritance tree of a class:
    - Its constructor with arguments can appear at most once
    - Its constructor with no argument can appear multiple times

Example

```
class A(val a : Int) {
  def this () = this(0)
}
trait B {
  def f(x: Int): Int = x
}
trait C extends A with B {
  def g(x: Int): Int = x + a
}
trait D extends B {
  def h(x: Int): Int = f(x + 50)
}
class E extends A(10) with C with D {
  override def f(x: Int) = x * a
}

val e = new E
```

Algorithm for Multiple Inheritance

➢ Algorithm

- Give a linear order among all ancestors by "post-order" traversing without revisiting the same node.
- Invoke the constructors once in that order.

  Note. Post-order traversal of a class C means
  - Recursively post-order traverse C's first parent; …;
  - Recursively post-order traverse C's last parent; and
  - Visit C.

  By post-order traversing from "E" in the previous example, we have the order: `A(10) -> B -> C -> D -> E`

```
val e = new E
e.a // 10                    e.f(100) // 100*10
e.g(100) // 100 + 10         e.h(100) // (100 + 50) * 10
```

- A constructor with arguments is always visited before the same constructor with no arguments.
- Compile error if the same field is implemented by multiple classes

A Simple Example With Traits

Motivation

```
abstract class Iter[A] {
  def getValue: Option[A]
  def getNext: Iter[A]
}

class ListIter[A](val list: List[A]) extends Iter[A] {
  def getValue = list.headOption
  def getNext = new ListIter(list.tail)
}

abstract class Dict[K,V] {
  def add(k: K, v: V): Dict[K,V]
  def find(k: K): Option[V]
}
```

Q: How can we extend ListIter and implement Dict?

Interface using Traits

```
// abstract class Dict[K,V] {
//   def add(k: K, v: V): Dict[K,V]
//   def find(k: K): Option[V] }

trait Dict[K,V] {
  def add(k: K, v: V): Dict[K,V]
  def find(k: K): Option[V]
}
```

Implementing Traits

```
class ListIterDict[K,V]
      (eq: (K,K)=>Boolean, list: List[(K,V)])
      extends ListIter[(K,V)](list)
         with Dict[K,V]
{
  def add(k:K,v:V): ListIterDict[K,V] =
    new ListIterDict(eq,(k,v)::list)
  def find(k: K) : Option[V] = {
    def go(l: List[(K, V)]): Option[V] = l match {
        case Nil => None
        case (k1, v1) :: tl =>
          if (eq(k, k1)) Some(v1) else go(tl) }
    go(list) }
}
```

Test

```
def sumElements[A](f: A=>Int)(xs: Iter[A]) : Int =
  xs.getValue match {
    case None => 0
    case Some(n) => f(n) + sumElements(f)(xs.getNext)
  }

def find3(d: Dict[Int,String]) = {
  d.find(3)
}

val d0 = new ListIterDict[Int,String]((x,y)=>x==y,Nil)
val d = d0.add(4,"four").add(3,"three")

sumElements[(Int,String)](x=>x._1)(d)
find3(d)
```

Mixin with Traits

Motivation: Mixin Functionality

```
abstract class Iter[A] {
  def getValue: Option[A]
  def getNext: Iter[A]
}

class ListIter[A](val list: List[A]) extends Iter[A]
{
  def getValue = list.headOption
  def getNext: ListIter[A] = new ListIter(list.tail)
}

trait MRIter[A] extends Iter[A] {
  def mapReduce[B,C](combine: (B,C)=>C, ival: C, f: A=>B): C = ???
}
```

Mixin Composition

```
trait MRIter[A] extends Iter[A] {
  override def getNext: MRIter[A]
  def mapReduce[B,C](combine: (B,C)=>C, ival: C, f: A=>B): C =
    getValue match {
      case None => ival
      case Some(v) =>
        combine(f(v), getNext.mapReduce(combine, ival, f))
    }
}

class MRListIter[A](list: List[A])
  extends ListIter (list) with MRIter[A]
{
 override def getNext = new MRListIter(super.getNext.list)
                    // new MRListIter(list.tail)
}
val mr = new MRListIter[Int](List(3,4,5))
mr.mapReduce[Int,Int]((b,c)=>b+c,0,(a)=>a*a)
```

Mixin Composition: A Better Way

```
trait MRIter[A] extends Iter[A] {
  def mapReduce[B,C](combine: (B,C)=>C, ival: C, f: A=>B): C = {
    def loop(c: Iter[A]): C = c.getValue match {
      case None => ival
      case Some(v) => combine(f(v), loop(c.getNext))
    }
    loop(this)
  }
}

class MRListIter[A](list: List[A])
  extends ListIter (list) with MRIter[A]

val mr = new MRListIter[Int](List(3,4,5))

// or, val mr = new ListIter(List(3,4,5)) with MRIter[Int]

mr.mapReduce[Int,Int]((b,c)=>b+c,0,(a)=>a*a)
```

Syntactic Sugar: new A with B with C { … }

```
new A(...) with B1 ⋯ with Bm {
  code
}
```

is equivalent to

```
{
  class _tmp_(args) extends A(args) with B1 ⋯ with Bm {
    code
  }
  new _tmp_(...)
}
```

Intersection Types

Intersection Types

➢ Typing Rule

$$\frac{t : T1 \qquad t : T2}{t : T1 \text{ with } T2}$$

➢ Example

```
trait A { val a: Int = 0 }
trait B { val b: Int = 0 }
class C extends A with B {
  override val a = 10
  override val b = 20
  val c = 30
}
val x = new C
val y: A with B = x
y.a // 10
y.b // 20
y.c // type error
```

Subtype Relation for “with”

The subtype relation for “with” is structural.

- Permutation

$$\frac{}{\ldots \text{ with } T1 \text{ with } T2 \ldots <: \ldots \text{ with } T2 \text{ with } T1 \ldots}$$

- Width

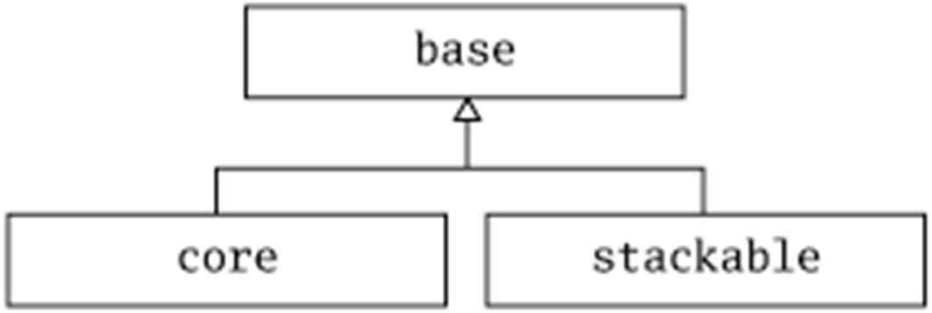
$$\frac{}{\ldots \text{ with } T \ldots <: \ldots \ldots}$$

- Depth

$$\frac{T <: S}{\ldots \text{ with } T \ldots <: \ldots \text{ with } S \ldots}$$

Stacking with Traits

Typical Hierarchy in Scala



- **BASE**
  Interface (trait or abstract class)

- **CORE**
  Functionality (trait or concrete class)

- **CUSTOM**
  Modifications (each in a separate, composable trait)

IntStack: Base

➢**BASE**

```
trait Stack[A] {
  def get(): (A, Stack[A])
  def put(x: A): Stack[A]
}
```

IntStack: Core

➢**CORE**

```
class BasicIntStack protected (xs: List[Int]) extends Stack[Int]
{
  override val toString = "Stack:" + xs.toString
  def this() = this(Nil)

  def get():(Int,Stack[Int]) = (xs.head,new BasicIntStack(xs.tail))
  def put(x:Int): Stack[Int] = new BasicIntStack(x :: xs)
}

val s0 = new BasicIntStack
val s1 = s0.put(3)
val s2 = s1.put(-2)
val s3 = s2.put(4)
val (v1,s4) = s3.get()
val (v2,s5) = s4.get()
```

IntStack: Custom Modifications

➢**CUSOM**

```
trait Doubling extends Stack[Int] {
  abstract override def put(x: Int): Stack[Int] = super.put(2 * x)
}

trait Incrementing extends Stack[Int] {
  abstract override def put(x: Int): Stack[Int] = super.put(x + 1)
}

trait Filtering extends Stack[Int] {
  abstract override def put(x: Int): Stack[Int] =
    if (x >= 0) super.put(x) else this
}
```

IntStack: Stacking

➢**Stacking**

```
class DIFIntStack protected (xs: List[Int])
  extends BasicIntStack(xs)
  with Doubling with Incrementing with Filtering
{
  def this() = this(Nil)
}

val s0 = new DIFIntStack
val s1 = s0.put(3)
val s2 = s1.put(-2)
val s3 = s2.put(4)
val (v1,s4) = s3.get()
val (v2,s5) = s4.get()
val (v2,s6) = s5.get()
```

IntStack: Core (Correct)

➢**CORE**

```
class BasicIntStack protected (xs: List[Int]) extends Stack[Int]
{
  override val toString = "Stack:" + xs.toString
  def this() = this(Nil)
  protected def mkStack(xs: List[Int]): Stack[Int] =
    new BasicIntStack(xs)
  def get(): (Int,Stack[Int]) = (xs.head, mkStack(xs.tail))
  def put(x: Int): Stack[Int] = mkStack(x :: xs)
}

val s0 = new BasicIntStack
val s1 = s0.put(3)
val s2 = s1.put(-2)
val s3 = s2.put(4)
val (v1,s4) = s3.get()
val (v2,s5) = s4.get()
```

IntStack: Stacking (Correct)

➢**Stacking**

```
class DIFIntStack protected (xs: List[Int])
  extends BasicIntStack(xs)
  with Doubling with Incrementing with Filtering
{
  def this() = this(Nil)
  override def mkStack(xs: List[Int]): Stack[Int] =
    new DIFIntStack(xs)
}

val s0 = new DIFIntStack
val s1 = s0.put(3)
val s2 = s1.put(-2)
val s3 = s2.put(4)
val (v1,s4) = s3.get()
val (v2,s5) = s4.get()
```