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Experimentally Observing Brownian Motion and Modeling it

Abstract

The Einstein model for Brownian motion uses the collective random movement of particles to gain useful information about a system [1]. In this lab, we tracked the relatively random motion of particles in a suspended medium and represented them in the form of a histogram to compare to the model. Further, we calculated experimental and theoretical values for the diffusion coefficient of our system to also compare to one another. We found that our data confirms the model, and that our calculated values are within enough error to draw conclusions about the system from as the Stokes and Einstein equations dictate.

Introduction

The motion of particles in a suspended medium in thermal equilibrium has been observed to be a somewhat random phenomenon that would appear to be extremely difficult, if not impossible, to model accurately [2]. This motion is named after the scientist Robert Brown who discovered the phenomenon in 1827 while observing the behavior of pollen particles immersed in water. Later, in 1905, Albert Einstein published an explanation for the behavior and explained that the random motion was caused by the collisions between water molecules which surrounded the pollen. In addition, Einstein also proposed that this motion could actually be modelled by plotting the general trend of motion rather than the specific motion of a single moving particle -- This motion would result in a characteristic bell-shaped curve which it's standard deviation could be used to calculate other information about the system. Our experiment was to follow in the footsteps of these scientists and observe/track particles exhibiting Brownian motion and analytically compare our results to Einstein's model.

Methods

To begin with, we turned on our Olympus CX43 imaging microscope to prepare for image taking. Then, using a set of calibration cards and the software program ImageCapture, we took images of similar distances at different magnifications to record the proportion of distance vs pixels (calibration 1: 1div = 0.01 mm, calibration 2: 1 div = 0.1 mm). For the 40x magnification, we found a calibration of $5.3\ px/\mu m$ by using a measurement from between the lines on the slide in pixels and dividing by the distance in micrometers to get a simplified value. Next, using a solution composed of 2-micrometer microparticles suspended in deionized water, we put a few drops on an empty slide and constructed a small well using small pieces of glass to hold the fluid and allow us to view it under a microscope (Figure 1). Then, we used our microscope to take pictures of the particle's motion every frame for 5 seconds at a frame rate of 60 frames per second. Feeding this video into an image processing software, we were able to track each individual particle; This gave us a total of 300 frames of data for about 500 particles per video. We then exported this data into a spreadsheet and began analyzing the trends of what we collected.

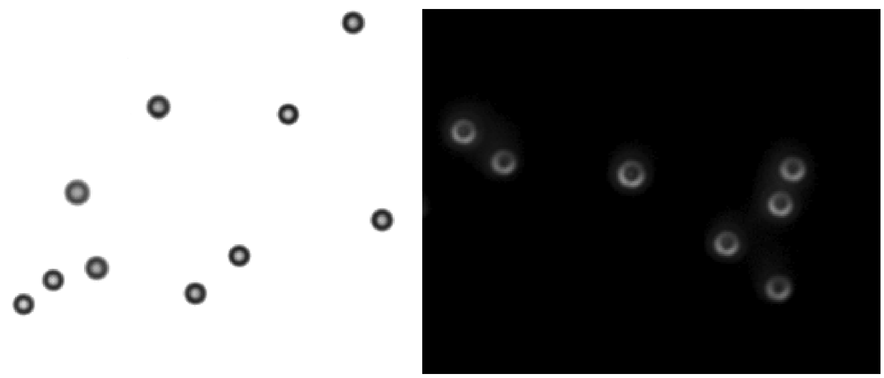

Figure 1. An image of suspended particles held in the slide well being measured by our microscope.

Results

Plotting the pure x and y position of a singular particle alongside time measured in frames shows us the movement of it over our given time, both diverging unpredictably from our initial position (Figure 2). However, plotting the change as a histogram showing the frequency of the particle’s similar position changes, we can see a relatively normal distribution of data both in the x and y directions (Figure 3, Figure 4). Now, for further experimentation, we calculated the Diffusion coefficient using our experimental data and compared it to the one calculated from generally known and accepted values. Doing this, we calculated a theoretical value of

$D = \mu * k_B * T$ where $\mu = 1/(6\pi * d * \eta)$ (η- viscosity of water, d-radius of microparticle).

$$D = \left(\frac{1}{6\pi*(0.965\ \mu m)*(1.0016 +- 0.05 * 10^{-3} Pa*s)}\right)(1.38 * 10^{-23}\ Pa * s)\ (294.38 +- 1\ K)$$

$$D = (1.995 +- 0.007) * 10^{-13}\ m^2/s$$

As well as experimental values of

$D_x = (1.4 +- 0.4) * 10^{-13}\ m^2/s$ and $D_y = (0.865 +- 0.403) * 10^{-13}\ m^2/s$,

using the method outlined by the process in Figure 6 with the values collected for σ_x and σ_y, alongside the error calculated in Figure 5. For the x calculation of the diffusion coefficient, this discrepancy works out to be an 8.7% error overall, while the y calculation has almost double that with a 16.8% overall error (Figure 7).

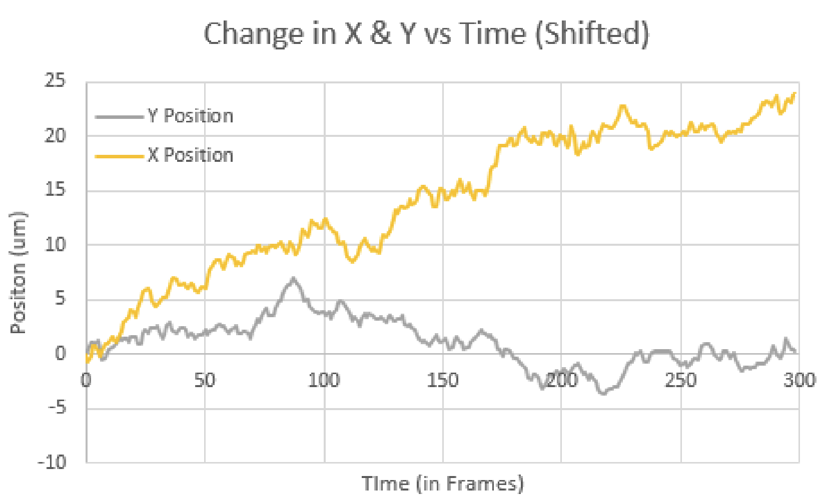

Figure 2. Direct position data of both x and y plotted alongside time measured in frames. Both data sets are shifted to the origin by subtracting the first point from the entire data set.

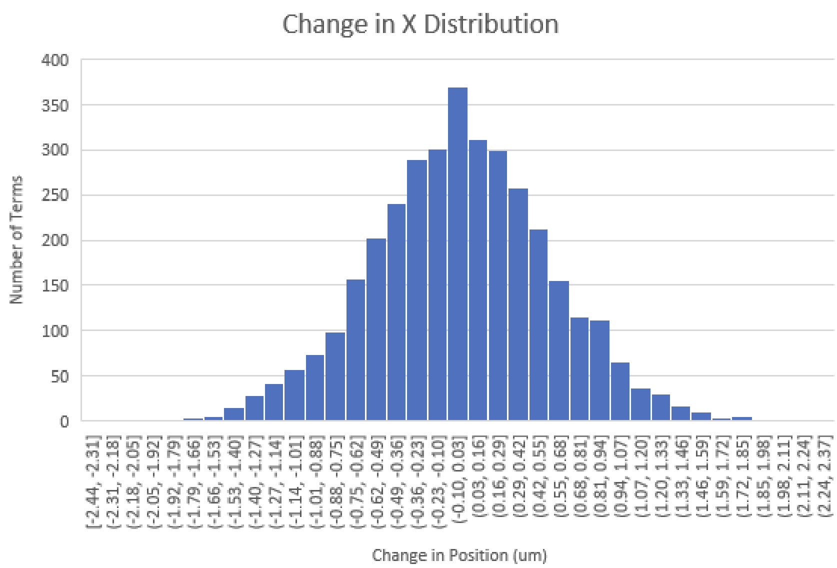

Figure 3. Histogram of the difference between x-direction points, where the x-axis is that value and the y-axis is the number of terms that fall in the categories.

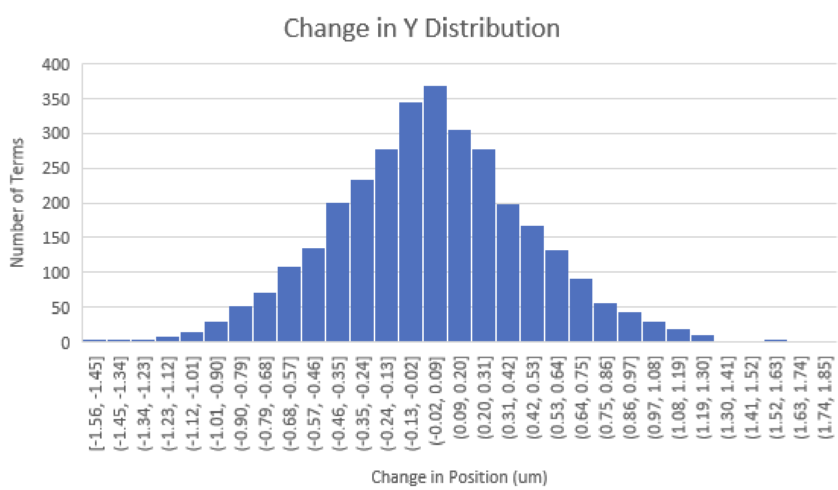

Figure 4. Histogram of the difference between y-direction points, where the x-axis is that value and the y-axis is the number of terms that fall in the categories.

Constant	Value	Error	Source
k_b	$1.3806 * 10^{-23} m^2 kg\, s^{-2} K^{-1}$	N/A	[3]
T	294.35 K	+- 1 K	Arbitrary reading of thermometer in room.
d	$0.965\ \mu m$	+- $0.05\ \mu m$	Polysciences Inc
η	$1.0016 * 10^{-3} Pa * s$	+- $5.008 * 10^{-5} Pa * s$ (based on T error)	[4]
σ_x	$0.1069\ \mu m$	+- $0.174\ \mu m$ (⅓ of a pixel)	Figure 6
σ_y	$0.0849\ \mu m$	+- $0.174\ \mu m$ (⅓ of a pixel)	Figure 6

Figure 5. Table of uncertainty values calculated and found from various sources [3],[4].

First, we can start by finding out what one pixel is in micrometers using our calibration value for 40x magnification.

$1\, px * \mu m = 5.73\, px * (x\, \mu m) \rightarrow x = 0.174\, \mu m$ (1 pixel in micrometers)

Our estimated error is ⅓ of a pixel, so we can say

$$(0.174) * (1/3) = 0.058\ \mu m.$$

Then, we can use the formula

$$\sigma = 2Dt \text{ where } t = 5/60$$

We can say

$$(5.8 * 10^{-8} m)^2 * (60\ frames/5\ s) = 0.403 * 10^{-13}\ m^2/s$$

$$\sigma_{x,y}\ Uncertainty = 0.403 * 10^{-13}\ m^2/s$$

Figure 6. Calculation of the uncertainty of σ_x and σ_y using our time and Calibration values.

$$\%\ \text{Error in X} = \frac{(1.995*10^{-13}\ m^2/s)-(1.37*10^{-13})}{(1.995*10^{-13}\ m^2/s)}(100) = 8.7\%$$

$$\%\ \text{Error in Y} = \frac{(1.995*10^{-13}\ m^2/s)-(8.65*10^{-14})}{(1.995*10^{-13}\ m^2/s)}(100) = 16.8\%$$

Figure 7. Percent error of D_x and D_y experimental error vs theoretical.

Discussion

Generally, our data seemed to follow pretty closely to the Einstein model for Brownian motion. Graphically, however, numerically some of our analytical extensions seemed to differ slightly from what was expected. Our Histograms, over a 5 second interval with around 500 particles, showed a very clear normal distribution in both the x and y axis despite the motion of each individual particle being relatively random. This part follows cleanly with what we would expect from data like this -- we did, however, seem to have a slight tendency for particles moving minutely, leading to larger than normal bars around the very center of our histogram. Though, from here we tried to use the standard deviation of our histogram to further analyze our results by calculating a value for the diffusion coefficient and comparing that value with one calculated from known values. Carrying this out gave us an 8.7% error in the diffusion coefficient from our x-values and a 16.8% error in the diffusion coefficient from our y-values. This means that we have almost double the amount of error in our y-values than we do in our x-values: this may be because of particles stuck to the coverslip or multiple particles that were grouped together restricting movement. Moreover, these numbers are slightly mitigated when the uncertainty for all the component values is considered, which brings the experimental values closer to the predicted values. Overall, our numbers are close enough to what we should expect that we can conclude that our data follows the model in a way that confirms what we sought out to model.

Conclusion

Though the chaotic movement of particles may appear impossible to model, looking at the general trend can provide key insight into the motion of the individual particles. In our experiment, we saw a clear and discernible normal distribution when we compiled the x and y positions of our particles over a period of 5 seconds, and was able to calculate a value for the diffusion coefficient within 10-20% of the expected value. These results fall close enough in line with the model for this phenomenon, as proposed by Einstein and Stokes, that we can say that our particles exhibited Brownian motion as presented in the theoretical model.

References

[1] "Brownian Motion." *Brownian Motion - an Overview | ScienceDirect Topics*, www.sciencedirect.com/topics/chemistry/brownian-motion.

[2] *Random Walks*, www.mit.edu/~kardar/teaching/projects/chemotaxis(AndreaSchmidt)/random.htm.

[3] "Boltzmann Constant." *Encyclopædia Britannica*, Encyclopædia Britannica, Inc., www.britannica.com/science/Boltzmann-constant.

[4] "Viscosity of Water – Viscosity Table and Viscosity Chart: Anton Paar Wiki." *Anton Paar*, wiki.anton-paar.com/en/water/.