Selling VIX® Futures and Options for Portfolio Return Enhancement

Edward Szado, Ph.D., CFA

Associate Professor of Finance, Providence College, Providence, RI 02918
Director of Research, INGARM (Institute for Global Asset and Risk Management)

Email: eszado@providence.edu Phone: (401) 865-1218

PROVIDENCE
COLLEGE

ABSTRACT

The term structure of VIX® futures is generally upward sloping. The persistent VIX contango may result in abnormally strong performance for VIX futures selling or VIX call writing strategies. However, the high volatility of volatility and significant jump risk may expose short uncovered VIX positions to extreme tail risk.

In this paper we consider the performance of VIX futures and call option selling in a portfolio context in 2008 and 2016 as well as over a 10+ year period beginning in 2006. In addition, we consider alternative strategies including VIX futures spread strategies, and the dynamically de-levering VIX futures selling strategies represented by the VPD^{SM} and VPN^{SM} indexes. The VPD Index had annual returns that were much higher than those of the S&P 500 Index in 2009 and 2016 (years without large spikes in VIX), but the VPD returns were lower than those of the S&P 500 in 2008 and 2018, years with significant upward spikes in the VIX Index.

While the goal of the study is not to make a particular recommendation for a VIX-based investment strategy, meaningful portfolio return enhancements are possible over particular time periods with small allocations to VIX selling. A large allocation to uncovered selling of VIX instruments could have substantial volatility jump risk exposure which can result in economically meaningful and potentially catastrophic losses. Of note, the VPN Index - which sells VIX futures, holds money market instruments and buys VIX calls – had lower volatility and less severe maximum drawdowns than the stock and commodity benchmark indexes studied in the period of analysis.

1.0 Introduction to the Spot VIX Index and Select Literature

The Cboe Volatility Index® (the VIX® Index) was developed in 1993 (see Whaley [1993]) as a forward-looking measure of expected market volatility. The spot VIX index represents the constant maturity 30-day expected volatility of the entire range of out-of-the-money (OTM) S&P 500® (SPX®) put and call options with more than 23 days to expiration and less than 37 days to expiration. In 2014, the VIX index methodology was changed to include weekly SPX options, tightening the range of tenor of the options used. Additional details and historical data are available at www.cboe.com/VIX.

Much of the extant literature on VIX focuses on the characteristics of long VIX exposures. For example, Szado [2009] finds that the strong negative correlation between spot VIX and the S&P 500 results in significant diversification benefits from adding a long spot VIX position to an S&P 500 portfolio. However, a number of researchers have documented the diversification benefits of long VIX exposure may come at a significant potential cost due to the meaningful portfolio drag of long VIX futures that results from frequent contango in the term structure. Research by Mencia and Sentana [2013], Whaley [2013] and Korovilas [2013] imply that inverse VIX futures might provide return enhancing benefits. The historically low levels of VIX in 2017 resulted in an increase in interest in inverse VIX strategies. Furthermore, extant literature suggests that low levels of the VIX index can be predictive of strong negative equity returns (see Giot [2005]). Toikka et al. [2004] of CSFB argue that short volatility should be characterized as an asset class. They demonstrate that long volatility is associated with a negative risk premium, therefore over extended time periods, long volatility positions tend to underperform the market. This is consistent with a number of studies on option writing strategies that find excess risk-adjusted returns for short volatility positions. They also note that volatility tends to be mean reverting, and tends to cluster.

However, number of studies have indicated that the spot VIX index tends to be negatively correlated with the SPX (see Daigler and Rossi [2006]), particularly for large equity market losses. Furthermore, since VIX often experiences upward jumps in such market moves (see Szado [2018b]), return enhancement benefits of uncovered inverse VIX exposure may come with extremely high VIX jump exposure risk. Such an exposure may significantly magnify investors' losses in market corrections or crashes.

Exhibit 1: Spot VIX Index and S&P 500 (SPX) Price Index

1990 to 2018	
Mean Daily VIX	19.3
Median Daily VIX	17.4
Highest VIX Close	80.9
Lowest VIX Close	9.1

Average Daily Close by Year

	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2018
S&P 500	1207.2	1310.5	1477.2	1220.0	948.0	1140.0	1267.6	1379.4	1643.8	1931.4	2061.1	2094.7	2746.2
Spot VIX	12.8	12.8	17.5	32.7	31.5	22.5	24.2	17.8	14.2	14.2	16.7	15.8	16.6
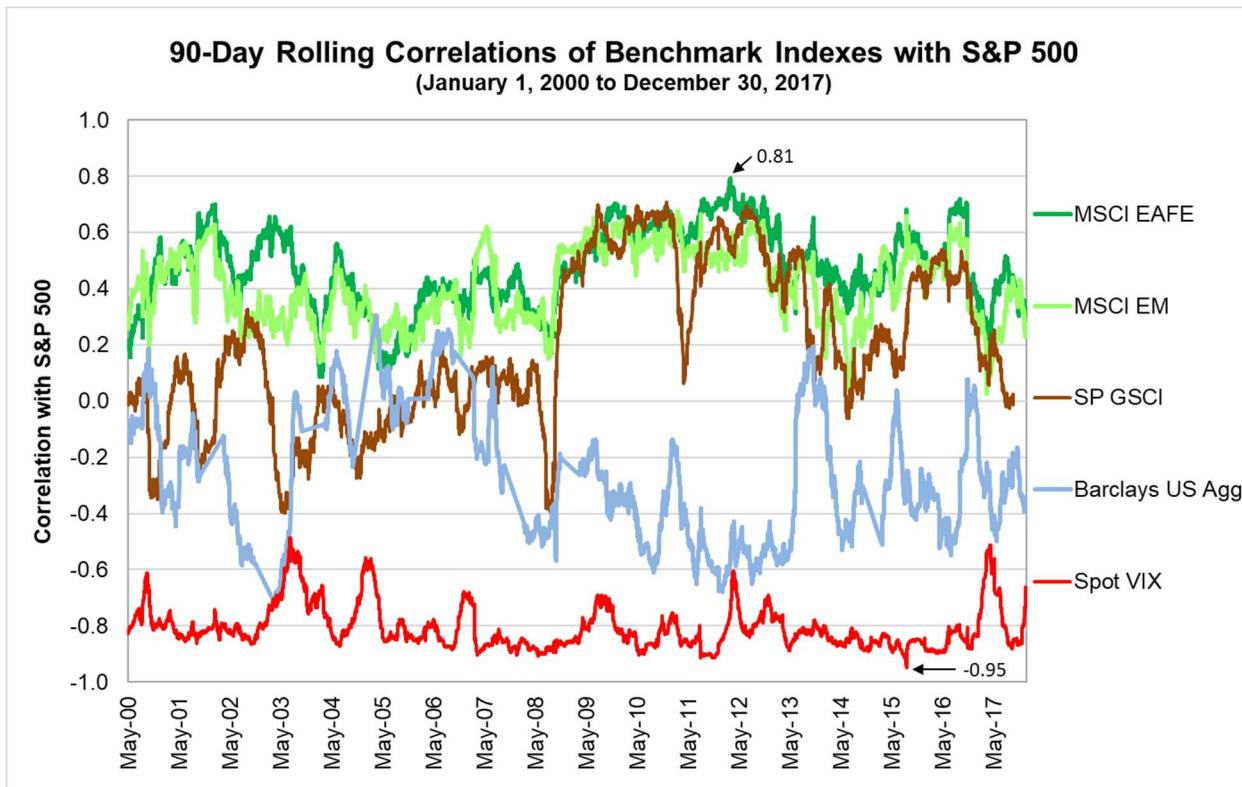
Source: Cboe Exchange Inc., Bloomberg

Exhibit 1: This exhibit illustrates the historical levels of the spot VIX and S&P 500 Price Indexes from 1990 to December 2017. Spikes in the level of the VIX are often concurrent with drawdowns in the S&P 500 index.

90-day rolling correlations between the S&P 500 index and a variety of benchmark indexes are illustrated in Exhibit 2. With the exception of the Barclays U.S. Aggregate bond index and the VIX index, correlations with the S&P 500 increased in the 2008 financial crisis. Asset classes which traditionally experienced low correlations with equity markets provided little diversification benefits in 2008. While bonds were one of the exceptions to this pattern, it could be problematic to assume that they could provide similar diversification benefits in the near future due to the currently historically low yields. It is worth noting that the correlation

between the VIX index and the S&P 500 was consistently negative over the entire period, remaining below -0.65 throughout the financial crisis.

Exhibit 2: 90-Day Rolling Correlations of Benchmark Indexes with the S&P 500

Exhibit 2: *This exhibit provides 90-day rolling correlations of a number of benchmark indexes and spot VIX versus the S&P 500 price index, based on daily data. With the exception of the Barclays U.S. Aggregate bond index and the VIX index, the correlations with the S&P 500 increased significantly in the financial crisis and were quite consistent across the indexes.*

While the rolling correlation between the VIX index and the S&P 500 index was negative for the entire 18-year period, it is well documented that this correlation is particularly strong during equity market distress S&P 500 (see Szado [2009]). Exhibit 3 illustrates the median within-month conditional correlations of benchmark indexes with the S&P 500 categorized by S&P 500 monthly return quintiles. Specifically, months are sorted based on S&P 500 returns and correlations are calculated within each month based on daily returns. Within each quintile, the median of the correlations is calculated and reported in the exhibit. The exhibit clearly indicates a strong negative correlation between the S&P 500 and the VIX index, particularly in the lowest S&P 500 return quintile.

Exhibit 3: Conditional Correlations of Benchmark Indexes with the S&P 500
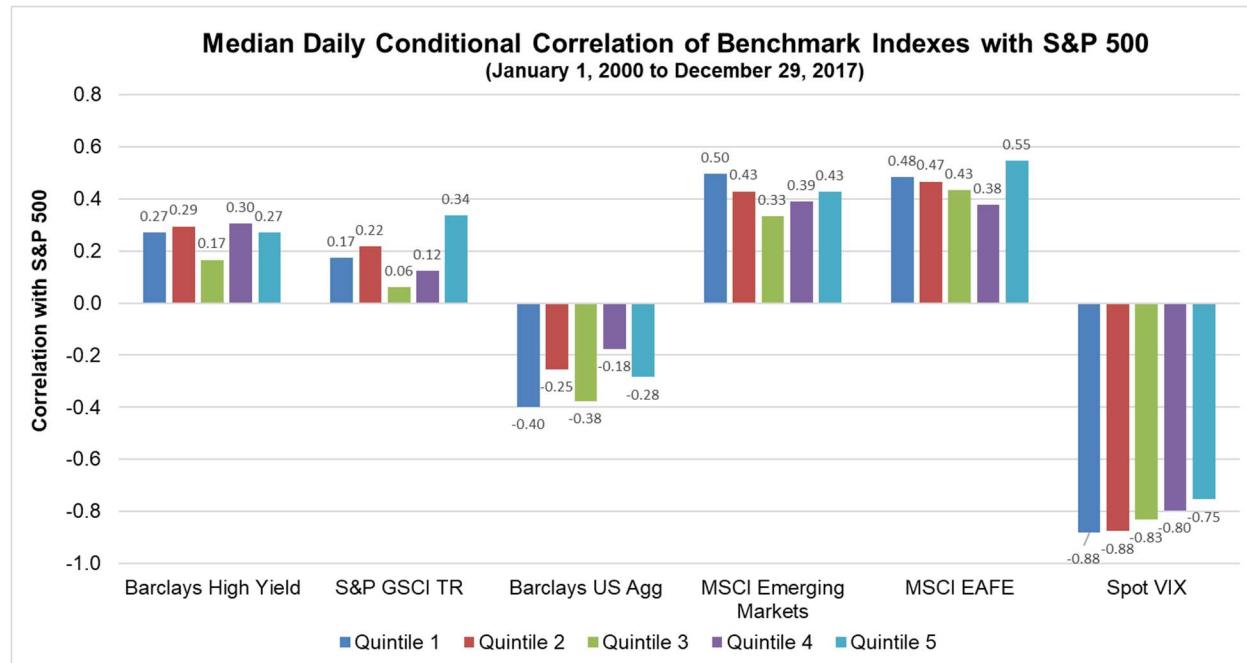
Exhibit 3: *This exhibit provides median monthly conditional correlations between a number of benchmark indexes and the S&P 500 price index based on daily data. The quintiles are based on monthly S&P 500 returns, ranked from low to high. The negative correlation between the VIX index and the S&P 500 tends to be strongest in months with the lowest S&P 500 returns.*

2.0 Introduction to VIX Futures and Options and VIX-Based Benchmark Indexes

While the spot VIX index is not investable, there are a number of available alternatives to gain long or short exposure to the VIX index, including VIX futures, VIX options and a variety of VIX-based exchange traded products and funds based on VIX futures and options.

Exhibit 4 provides some key specifications of VIX futures and options. Many unique characteristics of VIX futures and options are detailed in Szado [2018a]. It is important to note a few distinct attributes of VIX options. VIX options are priced based on the prices of the corresponding VIX futures contract (as opposed to the level of the spot VIX index). VIX futures and VIX options typically expire on the Wednesday which is 30 days prior to the SPX option expiration (which occurs on the third Friday of the month, unless Friday is an exchange holiday). VIX options are European style options (they can only be exercised at expiration). Furthermore, VIX options expire at the day's market open based on a Special Opening Quotation (SOQ).

Exhibit 4: Key Specifications of VIX Futures and Options

	VIX Futures	VIX Options
Exchange	Cboe Futures Exchange (CFE®)	Cboe Options Exchange
Year of Introduction	2004	2006
Ticker	VX and VX01 through VX53	VIX
Multiplier	$1,000	$100
Extended Trading Hours CT (Chicago Time)	5:00 p.m. (previous day) to 8:30 a.m. and 3:30 p.m. to 4:00 p.m.	2:00 a.m. to 8:15 a.m.
Regular Trading Hours CT (Chicago Time)	8:30 a.m. to 3:15 p.m.	
Last Trading Date	Trading hours for expiring VX futures contracts end at 8:00 a.m. Chicago time on the Final Settlement Date.	The last trading day (usually a Tuesday) is the business day prior to the Expiration Date of each contract expiration.
Expiration and Settlement	Usually on a Wednesday morning (30 days before a Friday settlement for SPX options)	
Settlement Value	The exercise-settlement value for VIX futures and options (Ticker: VRO) shall be a Special Opening Quotation (SOQ) of VIX calculated from the sequence of opening prices during regular trading hours for SPX of the options used to calculate the index on the settlement date.	
Number of Contract Expirations	Up to six near-term expiration weeks, nine near-term serial months and five months on the February quarterly cycle (February, May, August, November) for the VX futures contract.	Up to six weekly expirations and up to twelve standard (monthly) expirations in VIX options may be listed.
Options Exercise Style	Not applicable.	European - VIX options generally may be excercised only on the Expiration Date.

Source: Cboe Global Markets. www.Cboe.com/VIX

Exhibit 4: This exhibit provides key specifications of VIX futures and options contracts. The specifications are subject to change.

A variety of VIX-based investment products are based on a series of benchmark indexes that represent the returns to VIX futures and options trading strategies, including those published by Cboe Exchange and S&P Dow Jones Indices. These indexes include:

VPD - Cboe VIX Premium Strategy Index - overlays a sequence of short one-month VIX futures on a money market account; the short VIX futures positions are held until expiration and new VIX futures are then sold (see www.cboe.com/VPD).

VPN - Cboe Capped VIX Premium Strategy Index - tracks the performance of a strategy that systematically sells 1-month VIX futures, capped by the purchase of a VIX call option; the short VIX futures position is capped with long VIX calls struck about 25 points higher than the VIX futures price (see www.cboe.com/VPN).

VSTG - Cboe VIX Strangle Index - a premium capture index that overlays short VIX call and put options with a capped long VIX call option position. The position is collateralized by
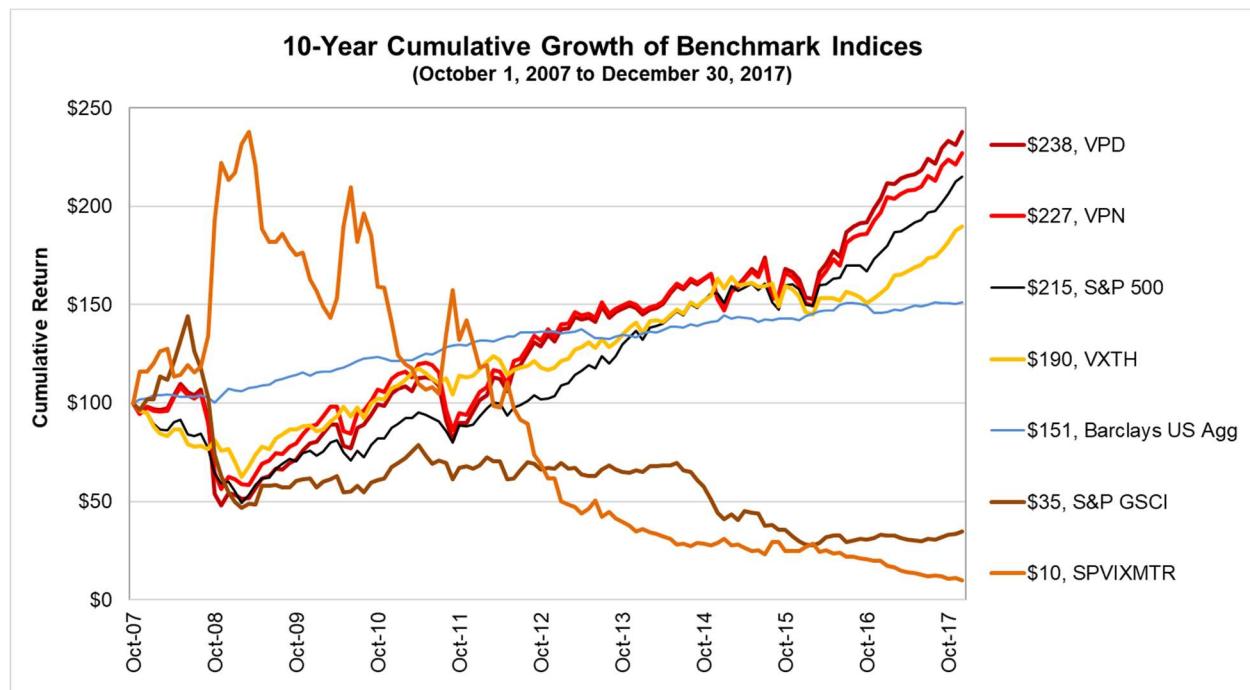
fixing the number of strangles such that 80% of capital is reserved (see www.cboe.com/VSTG).

VXTH - Cboe VIX Tail Hedge Index - buys and holds S&P 500 stocks, and also often buys 30-delta call options on the Cboe Volatility Index (VIX) (see www.cboe.com/VXTH).

SPVIXMTR - S&P 500 VIX Mid-term Futures Index buys a combination fourth, fifth, sixth and seventh month VIX futures in order to reflect the expectations of the VIX Index level in five months. A portion of the VIX futures are rolled daily to maintain a constant average weighted five-month term (see us.spindexes.com/SPVIXMTR).

SPVIXSTR - S&P 500 VIX Short-term Futures Index buys a combination of first and second month VIX futures in order to reflect the expectations of the VIX Index level in one-month. A portion of the VIX futures are rolled daily to maintain a constant average weighted one-month term (see us.spindexes.com/SPVIXSTR).

Exhibit 5: 10-Year Cumulative Growth of Benchmark Indexes

Source: Cboe Exchange Inc., Bloomberg.

Exhibit 5: *This exhibit illustrates the cumulative growth of a variety of benchmark total return indexes (pre-tax, with reinvested dividends) over the past 10+ years. It is not surprising that the long VIX futures-based index (SPVIXMTR) experienced significant drawdowns and volatility, due to the generally pervasive contango in the VIX futures term structure over the period. In contrast, the short (inverse) VIX futures indexes (VPN and VPD) experienced strong performance in the period.*

The historical performance of a selection of VIX-based indexes along with a selection of benchmark indexes is illustrated in Exhibit 5. The S&P 500 VIX Mid-term futures index stands out for its very large drawdown and high volatility. In stark contrast, the VPD and VPN indexes, which sell VIX futures outperformed the S&P 500 on a raw return basis, although they did so with higher volatility. It is important to note that the VPD and VPN indexes may be exposed to the significant tail risk of volatility jumps since they sell VIX futures. However, the VPN index includes a long OTM VIX call option exposure to mitigate this tail risk exposure. While the cost of long call may reduce performance in periods in which VIX does not increase meaningfully, it may provide protection against large volatility jumps. Further details of these indexes can be found in Szado [2018c].

3.0 Inverse VIX Futures and Written Call Options in Two Key Years – 2008 and 2016

The analyses in the following sections of the paper focus on trading VIX futures and options. For this purpose, we construct the returns to a series of VIX futures strategies rather than relying on the published benchmarks. The two main VIX futures trading strategy indexes used in the analyses represent the returns to strategies that take short (inverse) positions in:

1) One-month VIX futures held until the market close on the day prior to the VIX futures morning expiration, at which time they are rolled into one-month VIX futures, and
2) Three-month VIX futures held until the market close on the day prior to the next VIX futures morning expiration, at which time the (now two-month) VIX futures are rolled into three-month VIX futures.

In addition to the two inverse VIX futures trading strategies outlined above, we also similarly create two short VIX option strategies which:

1) Write one-month at-the-money (ATM) and,
2) Write one-month 25% out-of-the-money (OTM) options.

We will begin by considering the inverse VIX futures strategies. The written VIX options strategies will be discussed later in the paper. For a discussion of corresponding long VIX futures and options strategies, see Szado [2018c]. It is important to note that no adjustment has been made for transactions costs throughout this analysis.

As indicated in Szado [2018a], the VIX futures term structure is generally in contango. The often significant cost of holding long futures in contango (when spot price is below futures price) suggests that inverse VIX futures and written VIX call option positions may generate abnormal returns due to contango. On the other hand, the convex relationship and strong negative conditional correlation between the VIX index and the S&P 500 index suggests inverse VIX futures and written call options may expose the investor to catastrophic tail risk. Since inverse

volatility positions are expected to earn positive excess returns in exchange for exposure to potentially significant risk, an active approach rather than a passive short volatility exposure may be appropriate.

The following section considers the impact of a 5% allocation of inverse VIX futures and a 1% allocation to written VIX options to a traditional stock and bond portfolio and a hypothetical endowment portfolio. The size of the VIX futures allocation was chosen not as typical allocation, but rather as an oversized allocation for the purposes of illustrating hypothetical impacts of a VIX futures allocation. The base portfolios were chosen to represent a traditional 60/40 stock/bond portfolio (60% S&P 500 index and 40% Barclays U.S. Aggregate index) and a hypothetical endowment portfolio which is based on the published annual allocations of a well-known endowment applied to a series of indexes which proxy for the asset classes represented in the allocations (S&P 500 index, Barclays U.S. Aggregate index, Barclays High Yield U.S. Corporate index, HFR Global Hedge Fund index, SG CTA index (formerly Newedge CTA index), S&P GSCI TR index, S&P Listed Private Equity index, Dow Jones Equity REIT index, MSCI EAFE® index, MSCI EEM index, and the 3-Month Treasury Bill Secondary Market Rate).

While we consider long-term inverse VIX futures and written options allocations later in the paper, in the following sections we consider two years in which we expect an inverse VIX futures or written options allocation to have meaningful impacts on portfolio returns and risk. One year (2008) was chosen to represent a period unfavorable to positions in inverse VIX futures and written call options, in which we expected to see the impact of significant volatility risk exposure during major volatility events. In addition, 2016 was chosen as a year that was generally favorable to inverse positions in VIX futures and written call options, in which we expected to see return enhancement benefits.

The choice of 2008 as a potentially unfavorable year for inverse VIX futures and written call option exposure is quite obvious due to the high market volatility and large drawdowns of the base portfolios experienced in the last quarter of the year. The choice of a favorable year for inverse VIX exposure is less clear. A number of factors were considered in the choice (see Exhibit 6) including the number of days in which VIX futures were in contango, the average return to VIX futures, the maximum drawdown of a VIX futures strategy and the slope of the VIX term structure (the futures basis). Overall, 2016 appears to be a relatively favorable year for an inverse VIX futures or written call option exposure. It should be noted that 2016 is not the most favorable year based on any single measure considered but rather exhibits a variety of favorable characteristics. Furthermore, 2016 has the advantage of being more recent than some other alternative years (and thus may possibly be more reflective of the current market microstructure).

Exhibit 6: Select Annual Summary Statistics for Spot VIX, SPX and VIX Futures

	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Average Spot VIX Index	17.5	32.7	31.5	22.5	24.2	17.8	14.2	14.2	16.7	15.8	11.1
% in Contango vs Front Month Only	61%	54%	76%	79%	71%	90%	87%	84%	80%	86%	94%
Average VIX Basis (VIX points per day)	-0.003	0.026	-0.006	-0.033	-0.013	-0.042	-0.025	-0.019	-0.014	-0.024	-0.032
Average S&P 500 Realized 30-Day Volatility Risk Premium	1.7%	-2.7%	7.5%	6.0%	3.2%	5.2%	3.5%	3.1%	1.7%	4.5%	4.4%
Spot VIX Average Daily % Change	0.04%	0.07%	-0.07%	-0.02%	0.02%	-0.02%	-0.02%	0.02%	0.00%	-0.02%	-0.01%
Spot VIX Minimum Daily % Change	-6.99%	-17.36%	-10.23%	-12.11%	-12.94%	-4.70%	-3.95%	-4.13%	-5.70%	-5.10%	-3.79%
Spot VIX Maximum Daily % Change	7.16%	16.54%	10.54%	10.47%	16.00%	3.25%	5.21%	4.37%	12.71%	8.51%	4.94%
Spot VIX Annualized Return Volatility	139%	127%	91%	119%	140%	98%	112%	122%	144%	127%	114%
3 Month VIX Futures Average Daily Return	0.23%	0.35%	-0.12%	-0.17%	0.01%	-0.39%	-0.27%	-0.09%	-0.05%	-0.16%	-0.40%
3 Month VIX Futures Minimum Daily Return	-8%	-7%	-7%	-10%	-13%	-11%	-8%	-9%	-9%	-13%	-11%
3 Month VIX Futures Maximum Daily Return	12%	11%	10%	11%	15%	9%	10%	10%	16%	20%	11%
3 Month VIX Futures Maximum Drawdown	-17%	-22%	-47%	-50%	-36%	-69%	-53%	-43%	-37%	-57%	-65%
Average VIX Futures Open Interest	56,019	48,570	31,645	93,591	181,028	320,111	392,584	388,832	332,028	414,813	572,744
Average VIX Futures Volume	4,169	4,301	4,543	17,432	47,744	95,143	158,508	200,521	204,986	238,773	297,694

Source: Cboe Exchange Inc., Bloomberg

Exhibit 6: *This exhibit provides a wide variety of annual summary statistics from 2007 through 2017 for the spot VIX index, the S&P 500 index and VIX futures. The year 2008 was chosen as an unfavorable year for inverse VIX futures and written call options allocations due to the large drawdowns in the S&P 500 index and jump in volatility in 2008. The choice of a favorable year to explore the potential return enhancement benefits of inverse VIX futures and written call options allocations is somewhat less obvious. The year 2016 was chosen based on a number of measures including number of days in contango, average return, maximum drawdown and the slope of the VIX term structure (futures basis).*

As discussed above, inverse positions in VIX futures may generate abnormal returns due to persistent contango. However, while periods of strong contango may generate high returns to inverse VIX futures strategies if a volatility event does not occur, such strategies may generate catastrophic losses in the case of a volatility event such as a major equity market correction. Furthermore, such losses may occur concurrently with losses in many other asset classes. Thus, one might argue that inverse VIX futures may not be well suited to long term buy-and-hold investment since they are susceptible to overwhelming losses, but rather they may be best suited to strategic/selective uses rather than buy-and-hold investment.
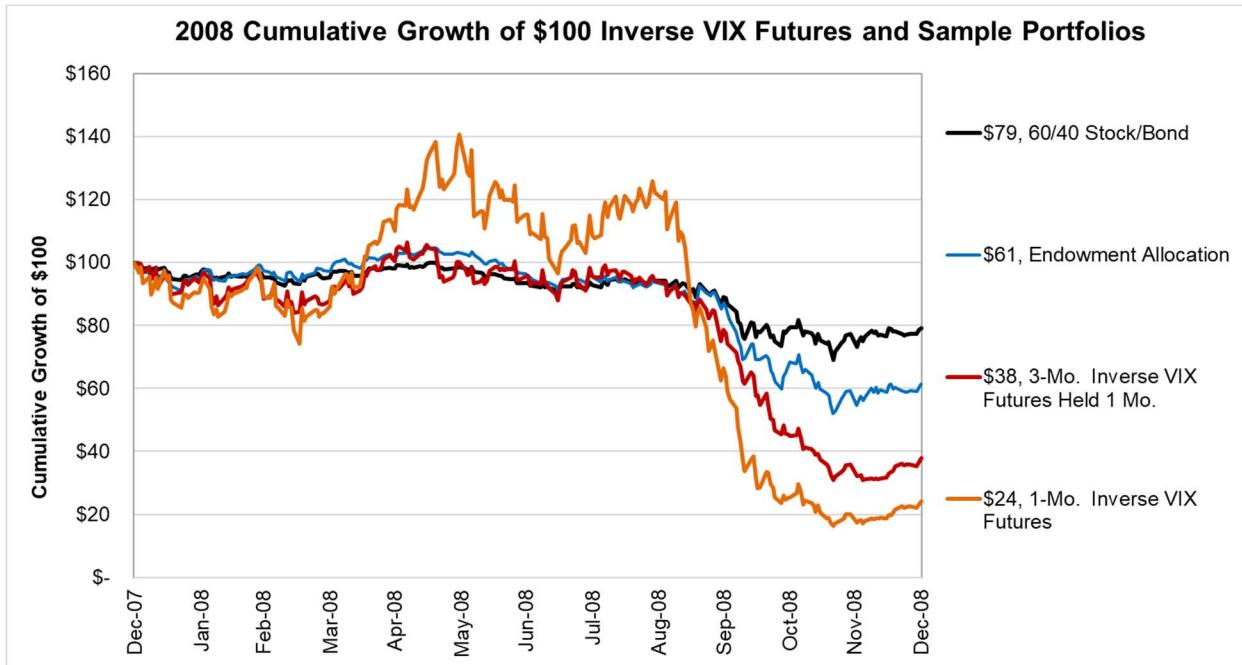
The following section considers the impact of a 5% allocation to inverse one-month or three-month VIX futures or a 1% allocation to written one-month ATM or 25% OTM VIX call options to a traditional stock and bond portfolio and a hypothetical endowment portfolios. Consistent with the analyses above, two years are considered to represent periods generally unfavorable for inverse VIX positions (2008) and generally favorable for inverse VIX positions (2016). One would expect that the historic volatility event of 2008 would result in very poor performance for inverse VIX futures and written call option strategies, while the strong and very persistent contango (and the absence of a volatility spike) would result in strong performance in 2016.

3.1 Inverse VIX Futures and VIX Call Writing-Based Strategies in 2008

Exhibit 7 illustrates the 2008 cumulative performance of the two inverse VIX futures portfolios along with the two base portfolios. The high volatility of the inverse VIX futures portfolios is clearly evident in the exhibit. While the inverse one-month VIX futures portfolio experienced significant returns in the first three quarters of the year, it experienced a dramatic drawdown in the last quarter. The magnitude of the losses of the VIX futures strategies in the third quarter significantly eclipses the losses of the base portfolios during one of the worst equity drawdowns in history. This is consistent with the conditional correlations and high volatility of volatility discussed previously in the paper.

Exhibit 8 provides an illustration of the 2008 cumulative growth and summary statistics of the 60/40 stock/bond portfolio with and without a 5% allocation to inverse VIX futures (one-month or three-month VIX futures) and a 1% allocation to written VIX calls (ATM or 25% OTM one-month VIX calls). The base 60/40 portfolio lost 21% over the period at a 24% annual standard deviation and a -31% maximum drawdown. The portfolios with a 5% allocation to inverse one- and three-month VIX futures performed similarly to each other, increasing the loss of the base portfolio from 21% to 24% and 23%, respectively. In addition, the one- and three-month inverse VIX futures allocations increased standard deviation from 24% to 26% and 25%, respectively and maximum drawdown from -31% to -35% and -33%, respectively. The ex-post skewness was also hurt by adding the inverse VIX futures allocation to the base portfolio. Thus the inverse VIX futures allocation increased risk and reduced returns in 2008, albeit moderately.

**Exhibit 7: 100% Allocations to Inverse VIX Futures-Based Strategies in 2008**
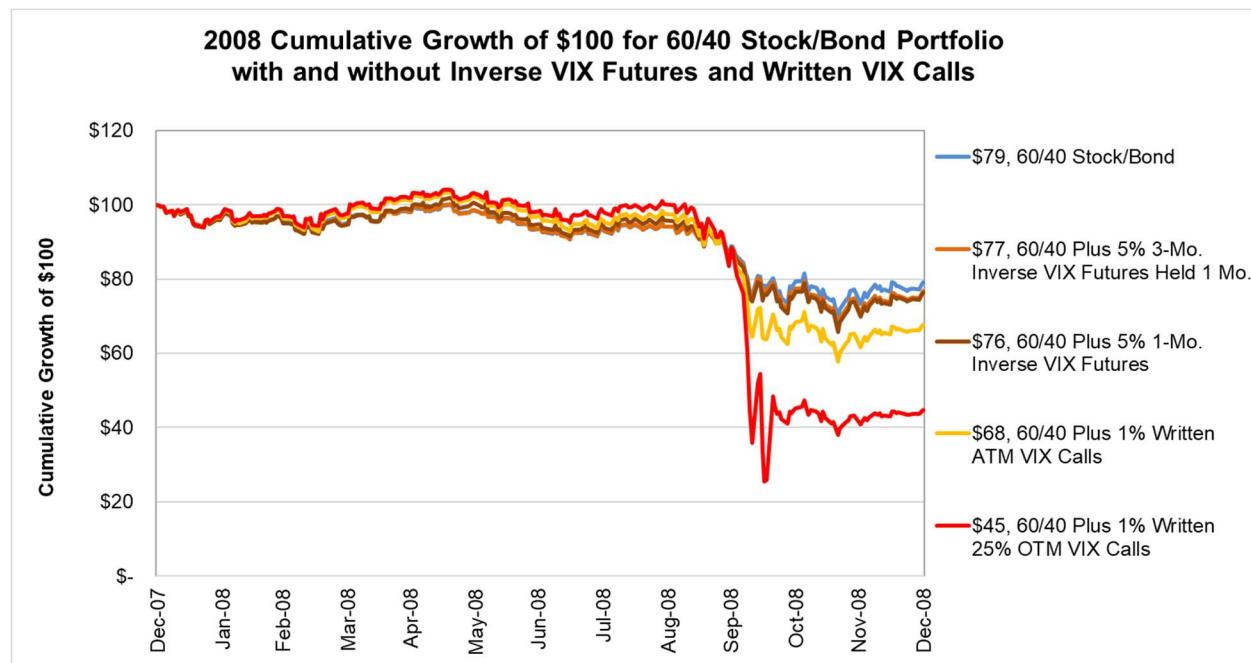
2008 Cumulative Growth of $100 Inverse VIX Futures and Sample Portfolios

- $79, 60/40 Stock/Bond
- $61, Endowment Allocation
- $38, 3-Mo. Inverse VIX Futures Held 1 Mo.
- $24, 1-Mo. Inverse VIX Futures

Summary Statistics for the Year 2008	60/40 Stock/Bond Portfolio	Endowment	100% Allocation to 1-Mo. Inverse VIX Futures	100% Allocation to 3-Mo. Inverse VIX Futures Held 1 Mo.
Annualized Return	-20.9%	-38.6%	-75.7%	-62.0%
Annualized Standard Deviation	24.1%	31.0%	83.2%	48.2%
Sharpe Ratio	-0.87	-1.25	-0.91	-1.29
Maximum Drawdown	-30.9%	-50.4%	-88.4%	-71.0%
Skewness	0.16	-0.13	-0.52	-0.61
Kurtosis	4.14	3.21	1.30	1.03
Correlation w SPX	1.00	0.92	0.82	0.83
Beta	0.58	0.69	1.66	0.97
Alpha	1.53%	-17.63%	-42.64%	-47.98%
Alpha P-Value	0.501	0.158	0.374	0.080

Source: Cboe Exchange Inc., Bloomberg

Exhibit 7: This exhibit illustrates the performance of two traditional portfolios as well as two inverse VIX futures-based portfolios in 2008. The VIX portfolios are both fully collateralized and rebalanced daily. The one-month portfolio takes short positions in one-month VIX futures and rolls out on the close the night before expiration. The Three-month VIX futures portfolio shorts three-month VIX futures and rolls into a new short three-month VIX futures contract after one month at the close before the front month VIX futures contract expires. The traditional portfolios consist of a 60/40 Stock/Bond portfolio and a hypothetical endowment portfolio which invests in a variety of indexes based on the average yearly asset allocation of a representative endowment. It is clear from the exhibit that 2008 was an unfavorable year for inverse VIX futures exposure.

The results for adding a written VIX call option allocation are far more dramatic. The 1% written one-month ATM VIX call allocation reduced returns from -21% to -32%, and increased the standard deviation from 24% to 35%, and maximum drawdown from -31% to -44%, thus doubling losses while increasing risk by half. The 1% allocation to inverse one-month 25% OTM VIX calls was even more extreme, further reducing returns to a 55% loss (from a 21% loss for the base portfolio), and did so at a far higher standard deviation (118% versus 24% for the base portfolio), and it more than doubled maximum drawdown (-76% versus -31% for the base portfolio).

Exhibit 8: 60/40 Portfolio 2008 Performance with Inverse VIX Futures and Written VIX Calls

Summary Statistics for the Year 2008	60/40 Stock/Bond Portfolio	60/40 Plus 5% 1-Mo. Inverse VIX Futures	60/40 Plus 5% 3-Mo. Inverse VIX Futures Held 1 Mo.	60/40 Plus 1% Written 25%-OTM VIX Calls	60/40 Plus 1% Written ATM VIX Calls
Annualized Return	-20.9%	-23.6%	-23.3%	-55.3%	-32.3%
Annualized Standard Deviation	24.1%	26.0%	24.8%	118.2%	34.7%
Sharpe Ratio	-0.87	-0.91	-0.94	-0.47	-0.93
Maximum Drawdown	-30.9%	-35.4%	-33.3%	-75.6%	-44.1%
Skewness	0.16	0.01	0.05	6.45	0.23
Kurtosis	4.14	3.33	3.74	84.28	9.21
Correlation w SPX	1.00	0.99	1.00	0.52	0.94
Beta	0.58	0.63	0.60	1.51	0.80
Alpha	1.53%	0.29%	-0.58%	34.51%	-2.81%
Alpha P-Value	0.501	0.923	0.812	0.733	0.814

Source: Cboe Exchange Inc., Bloomberg

Exhibit 8: *This exhibit illustrates the 2008 performance of a traditional 60/40 portfolio with and without a small allocation to inverse VIX futures or written VIX call options. In this period of crisis, even a small allocation to VIX call*

writing can significantly reduce returns and increase volatility. A 5% allocation to inverse VIX futures has a smaller impact but also reduces returns while slightly increasing volatility and drawdowns.
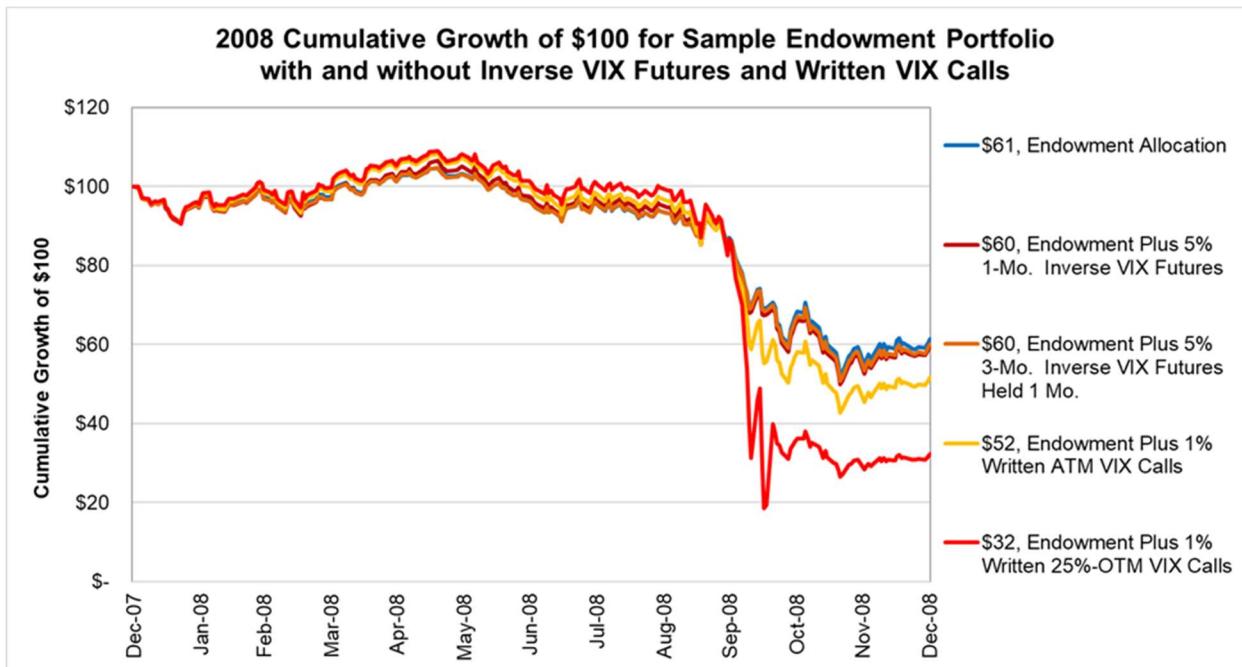
The results above suggest that an inverse VIX futures allocation would have moderately increased losses and risk of a 60/40 portfolio in 2008, while a written call option allocation would have significantly increased losses and risk.

Exhibit 9 presents results which reflect the impact of an addition of inverse VIX futures and written call options to a well-diversified hypothetical endowment portfolio in 2008. The impact of the inverse VIX allocations were similar for both base portfolios. Exhibit 9 provides an illustration of the 2008 cumulative growth and summary statistics of the base hypothetical endowment portfolio with and without a 5% allocation to inverse VIX futures (one-month or three-month VIX futures) and a 1% allocation to written VIX calls (ATM or 25% OTM one-month VIX calls). The base endowment portfolio lost 39% over the period at a 31% annual standard deviation and a -50% maximum drawdown. The portfolios with a 5% allocation to inverse one- and three-month VIX futures performed similarly to each other, slightly increasing the loss of the base portfolio from 39% to 40% and 40%, respectively. The one- and three-month VIX futures allocations also slightly increased standard deviation from 31% to 33% and 31.3%, respectively and maximum drawdown from -50% to -53% and -52%, respectively. The ex-post skewness was also worsened by adding the inverse VIX futures allocation to the base endowment portfolio. Thus the inverse VIX futures allocation had little impact on the endowment portfolio, slightly reducing risk and returns in 2008.

As with the 60/40 portfolio, the results for adding a written VIX call option allocation are far more extreme than those from adding inverse VIX futures. The 1% written one-month ATM VIX call allocation significantly reduced returns from -39% to -48% and increased standard deviation from 31% to 41%, and maximum drawdown from -50% to -61%, thus providing significant degradation in risk and return (in contrast to the inverse VIX futures allocations). The 1% allocation to inverse one-month 25% OTM VIX calls further reduced returns to a 68% loss (from a 39% loss for the base portfolio) and more than quadrupled standard deviation (33% versus 140% for the base portfolio), and significantly increased the maximum drawdown (-83% versus -50% for the base endowment portfolio).

Thus, for both base portfolios, an inverse VIX futures allocation had a small negative impact on risk and return while a written call allocation significantly degraded returns and risk, particularly for the OTM calls. At this point, it is worth recalling the results in Exhibit 7. While the results in this section suggest that a 5% allocation to inverse VIX futures had a small negative impact on the base portfolios in 2008, an investment in the inverse VIX strategies outside of a diversified portfolios would have resulted in far larger losses. It is the inclusion in the diversified portfolios and the small size of the allocation that significantly mitigates the negative impact.

Exhibit 9: Endowment Portfolio in 2008 with Inverse VIX Futures and Written VIX Calls

Summary Statistics for the Year 2008	Endowment	Endowment Plus 5% 1-Mo. Inverse VIX Futures	Endowment Plus 5% 3-Mo. Inverse VIX Futures Held 1 Mo.	Endowment Plus 1% Written 25%-OTM VIX Calls	Endowment Plus 1% Written ATM VIX Calls
Annualized Return	-38.6%	-40.2%	-39.8%	-67.8%	-48.3%
Annualized Standard Deviation	31.0%	32.5%	31.3%	139.6%	41.0%
Sharpe Ratio	-1.25	-1.24	-1.27	-0.49	-1.18
Maximum Drawdown	-50.4%	-53.1%	-51.5%	-83.0%	-60.5%
Skewness	-0.13	-0.21	-0.19	6.93	-0.20
Kurtosis	3.21	2.79	3.07	92.48	5.50
Correlation w SPX	0.92	0.93	0.92	0.50	0.91
Beta	0.69	0.73	0.71	1.70	0.91
Alpha	-17.63%	-18.32%	-19.05%	29.66%	-22.96%
Alpha P-Value	0.158	0.138	0.113	0.807	0.185

Source: Cboe Exchange Inc., Bloomberg
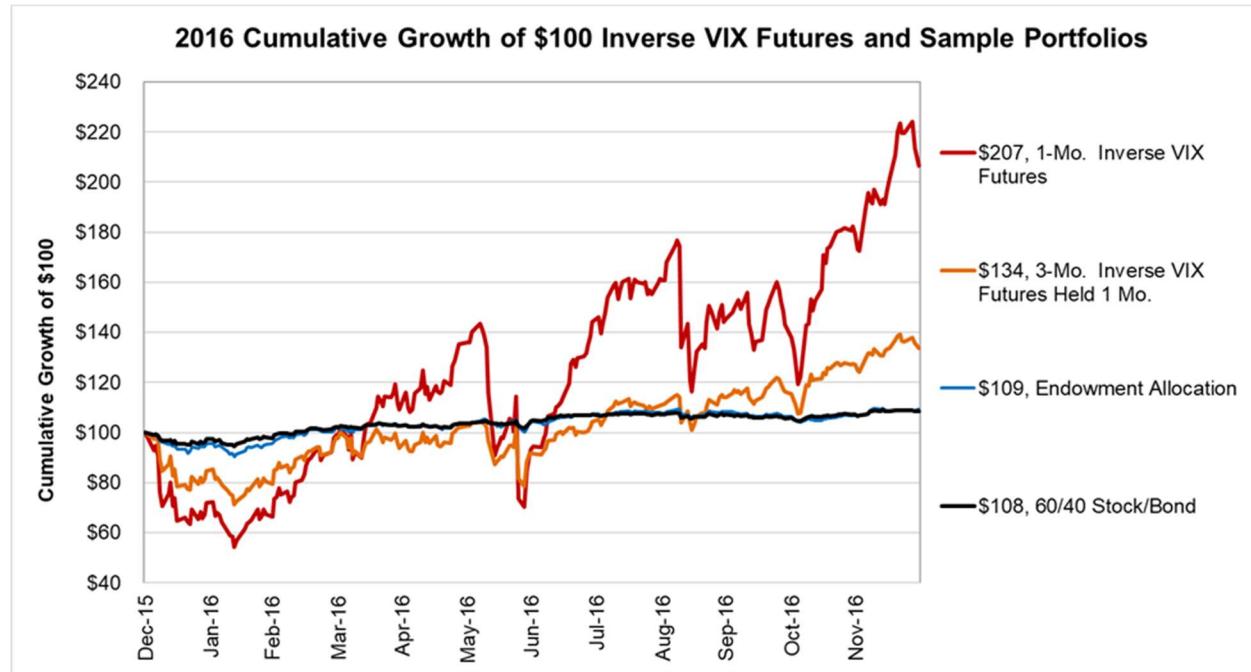
Exhibit 9: This exhibit illustrates the 2008 performance of a hypothetical endowment portfolio with and without a small allocation to inverse VIX futures or written VIX call options. In this period of crisis, even a small allocation to VIX call writing can significantly reduce returns and increase volatility. A 5% allocation to inverse VIX futures has a smaller impact but also reduces returns while slightly increasing volatility and drawdowns.

3.2 Inverse VIX Futures and VIX Call Writing-Based Strategies in 2016

As discussed earlier, 2016 was chosen as a sample year which one would expect would be attractive for an inverse VIX futures or written call option allocation. The following section

considers the impact of a 5% allocation to inverse VIX futures or 1% allocation to written VIX call options on the two base portfolios (the 60/40 portfolio and the hypothetical endowment portfolio).

Exhibit 10: Inverse VIX Futures-Based Strategies in 2016

Summary Statistics for the Year 2016	60/40 Stock/Bond Portfolio	Endowment	100% Allocation to 1-Mo. Inverse VIX Futures	100% Allocation to 3-Mo. Inverse VIX Futures Held 1 Mo.
Annualized Return	8.4%	9.2%	106.5%	33.7%
Annualized Standard Deviation	7.6%	10.0%	88.2%	47.4%
Sharpe Ratio	1.11	0.92	1.21	0.71
Maximum Drawdown	-5.4%	-9.6%	-51.0%	-28.6%
Skewness	-0.29	-0.48	-1.63	-1.12
Kurtosis	2.23	2.56	9.46	9.17
Correlation w SPX	0.99	0.82	0.83	0.84
Beta	0.57	0.62	5.58	3.05
Alpha	1.42%	1.74%	46.51%	3.47%
Alpha P-Value	0.240	0.765	0.348	0.893

Source: Cboe Exchange Inc., Bloomberg

Exhibit 10: This exhibit illustrates the performance of two traditional portfolios as well as two inverse VIX futures-based portfolios in 2016. The VIX portfolios are both fully collateralized and rebalanced daily. The one-month portfolio takes short positions in one-month VIX futures and rolls out on the close the night before expiration. The Three-month VIX futures portfolio shorts three-month VIX futures and rolls into a new short three-month VIX futures contract after one month at the close before the front month VIX futures contract expires. The traditional portfolios consist of a 60/40 Stock/Bond portfolio and a hypothetical endowment portfolio which invests in a variety of indexes based on the average yearly asset allocation of a representative endowment. It is clear from the exhibit that 2016 was a generally favorable year for inverse VIX futures exposure.

Exhibit 10 provides a graphical representation and summary statistics of the 2016 cumulative performance of the one-month and three-month inverse VIX futures strategies along with the two base portfolios. It is evident that 2016 was an overall favorable year for an inverse VIX futures exposure from a total return perspective, albeit a very high volatility. While the futures strategies performed very poorly at the start of the year, both futures strategies ended the year strongly while experiencing many very large drawdowns over the course of the year. In fact, the one-month inverse VIX futures strategy experienced a standard deviation of 88% over the year, with a maximum drawdown of -51% (versus -8% and -5% for the 60/40 portfolio).
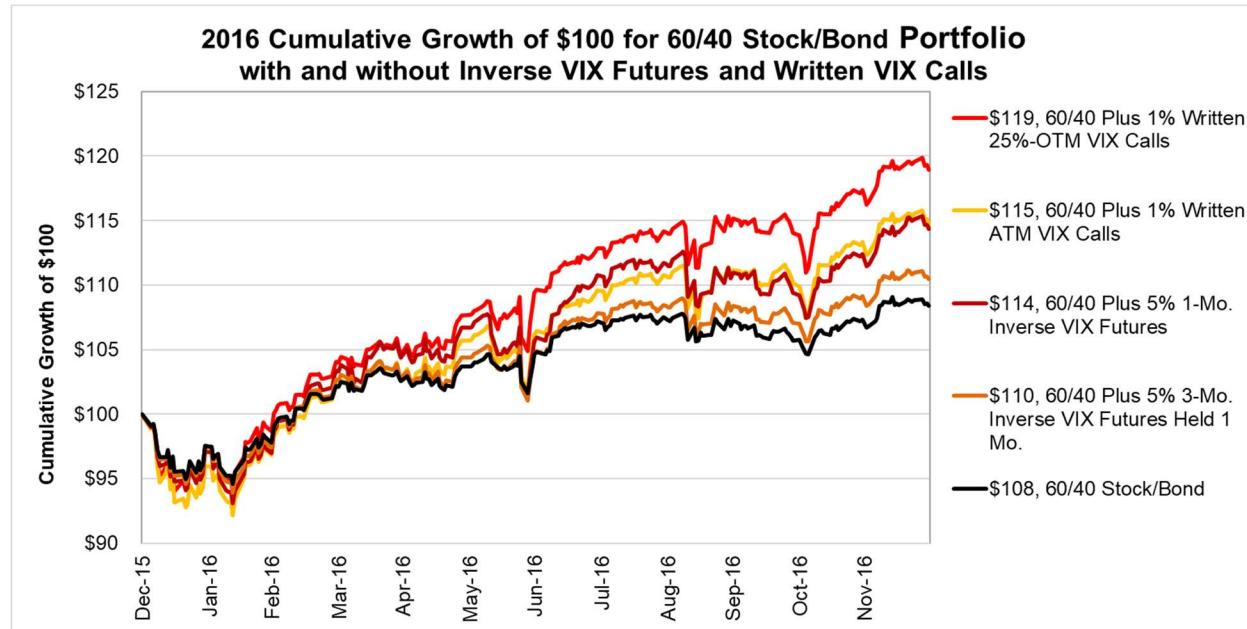
The following section provides an analysis of the impact of an addition of an inverse VIX futures or written call option allocation to the two base portfolios in 2016. We begin by considering a 5% allocation to inverse VIX futures and 1% allocation to written VIX call options in 2016. As mentioned earlier, 2016 was chosen as an example of a favorable year for an inverse VIX allocation as we expect that an inverse VIX futures or written call exposure would benefit from the relatively strong and consistent contango in VIX futures and relative lack of major volatility spikes (see Exhibit 7).

Exhibit 11 provides an illustration of the 2016 cumulative growth and summary statistics of the 60/40 stock/bond portfolio with and without a 5% allocation to inverse VIX futures (one-month or three-month VIX futures) and a 1% allocation to written VIX calls (ATM or 25% OTM one-month VIX calls). The base 60/40 portfolio gained 8% over the period at an 8% annual standard deviation and a -5% maximum drawdown. The portfolios with a 5% allocation to inverse one- and three-month VIX futures outperformed the base portfolio, increasing the returns of the base portfolio from 8% to 14% and 10%, respectively. However, the one- and three-month VIX futures allocations increased standard deviation from 8% to 11% and 9%, respectively and maximum drawdown from -5% to -7% and -6%, respectively. The ex-post skewness was also reduced by adding the inverse VIX futures allocation to the base portfolio. Thus, while the inverse VIX futures allocations improved returns in 2016, they did so at the expense of a small increase in risk. The return to risk tradeoff (as measured by the Sharpe ratio) of the base portfolio was improved by the addition of inverse VIX futures (from 1.11 to 1.30 and 1.14, respectively). Similar improvements are seen in alpha (although the alphas are not statistically significant at 10%).

The results for adding a written VIX call option allocation are similar, yet somewhat more extreme, particularly for the OTM calls. The 1% written one-month ATM VIX call allocation increased returns from 8% to 15% and standard deviation from 8% to 11%, and maximum drawdown from -5% to -8%, thus providing improvements in returns at the cost of increased risk. The 1% written OTM VIX call allocation increased returns from 8% to 19% and standard deviation from 8% to 11%, and maximum drawdown from -5% to -6%. The return to risk tradeoff (as measured by the Sharpe ratio) of the base portfolio was greatly improved by the addition of written VIX call options (from 1.11 to 1.30 and 1.72, respectively).

Thus, in 2016 the addition of inverse VIX futures or written call options to the 60/40 base portfolio resulted in meaningful increases in absolute returns and in risk-adjusted returns based on the Sharpe ratio and alpha (although most are statistically insignificant at 10%, with the exception of the 60/40+OTM call portfolio).

Exhibit 11: 60/40 Portfolio in 2016 with Inverse VIX Futures and Written VIX Calls
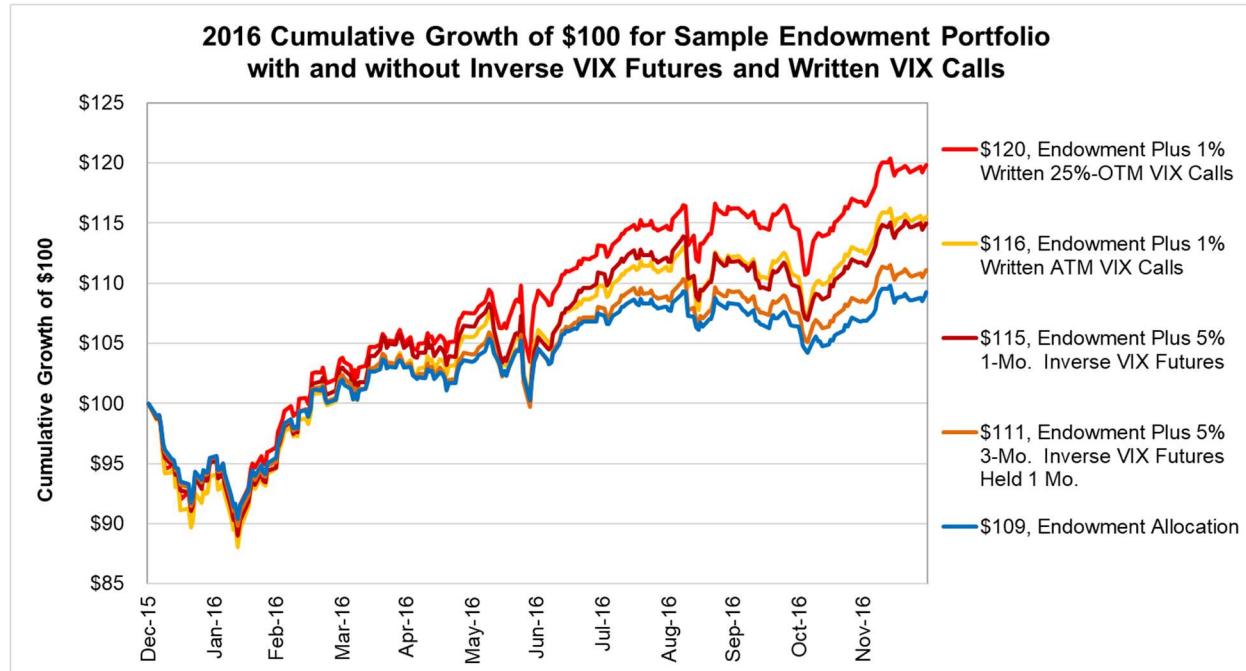

Summary Statistics for the Year 2016	60/40 Stock/Bond Portfolio	60/40 Plus 5% 1-Mo. Inverse VIX Futures	60/40 Plus 5% 3-Mo. Inverse VIX Futures Held 1 Mo.	60/40 Plus 1% Written 25%-OTM VIX Calls	60/40 Plus 1% Written ATM VIX Calls
Annualized Return	8.4%	14.3%	10.4%	18.9%	14.7%
Annualized Standard Deviation	7.6%	11.0%	9.1%	11.0%	11.3%
Sharpe Ratio	1.11	1.30	1.14	1.72	1.30
Maximum Drawdown	-5.4%	-6.9%	-6.1%	-6.0%	-7.8%
Skewness	-0.29	-0.96	-0.51	-0.59	-0.70
Kurtosis	2.23	5.22	3.52	3.89	3.71
Correlation w SPX	0.99	0.96	0.98	0.95	0.95
Beta	0.57	0.81	0.68	0.79	0.82
Alpha	1.42%	4.16%	2.03%	8.30%	4.39%
Alpha P-Value	0.240	0.168	0.257	0.022	0.206

Source: Cboe Exchange Inc., Bloomberg

Exhibit 11: This exhibit illustrates the 2016 performance of a traditional 60/40 portfolio with and without a small allocation to inverse VIX futures or written VIX call options. In this period of low volatility and generally positive market returns, a small allocation to VIX call writing can significantly increase returns with a moderate increase in volatility. A 5% allocation to inverse VIX futures has a smaller impact but also provides a meaningful increase in return with a small increase in volatility and drawdowns.

Exhibit 12: Endowment Portfolio in 2016 with Inverse VIX Futures and Written VIX Calls

Source: Cboe Exchange Inc., Bloomberg

Summary Statistics for the Year 2016	Endowment	Endowment Plus 5% 1-Mo. Inverse VIX Futures	Endowment Plus 5% 3-Mo. Inverse VIX Futures Held 1 Mo.	Endowment Plus 1% Written 25%-OTM VIX Calls	Endowment Plus 1% Written ATM VIX Calls
Annualized Return	9.2%	15.0%	18.9%	19.8%	15.5%
Annualized Standard Deviation	10.0%	13.0%	10.0%	12.7%	13.1%
Sharpe Ratio	0.92	1.15	1.89	1.56	1.19
Maximum Drawdown	-9.6%	-11.0%	-4.4%	-10.0%	-12.0%
Skewness	-0.48	-1.00	0.82	-0.71	-0.74
Kurtosis	2.56	5.32	6.15	3.68	3.33
Correlation w SPX	0.82	0.87	0.31	0.87	0.88
Beta	0.62	0.86	0.24	0.85	0.87
Alpha	1.74%	4.34%	14.92%	8.58%	4.65%
Alpha P-Value	0.765	0.504	0.119	0.165	0.465

Exhibit 12: This exhibit illustrates the 2016 performance of a hypothetical endowment portfolio with and without a small allocation to inverse VIX futures or written VIX call options. In this period of relative calm and positive performance, small allocations to inverse VIX futures or VIX call writing increase returns significantly with a moderate increase in volatility. A 5% allocation to inverse three-month VIX futures doubles returns with no increase in volatility and a reduction in drawdown.

We now consider the addition of inverse VIX futures and written call options to a well-diversified portfolio that contains other asset classes beyond equity and fixed income. Exhibit 12 presents an analysis of an addition of inverse VIX futures and written call options to a well-diversified hypothetical endowment portfolio in 2016. As in 2008, in 2016 the impact of the inverse VIX allocations were similar for both base portfolios. Exhibit 12 provides an illustration of the 2016 cumulative growth and summary statistics of the base hypothetical endowment

portfolio with and without a 5% allocation to inverse VIX futures (one-month or three-month VIX futures) and a 1% allocation to written VIX calls (ATM or 25% OTM one-month VIX calls). The base endowment portfolio gained 9% over the period at a 10% annual standard deviation and a -10% maximum drawdown. The portfolios with a 5% allocation to inverse one- and three-month VIX futures both significantly outperformed the base portfolio, increasing the gains of the base portfolio from 9% to 15% and 19%, respectively. While the one-month inverse VIX futures allocation increased standard deviation from 10% to 13% and maximum drawdown from -10% to -13%, the three-month inverse VIX futures allocation increased returns without a change in standard deviation and actually reduced maximum drawdown from -10% to -4%. Similarly, the ex-post skewness was reduced for the one-month inverse VIX futures allocation but improved by adding the three-month inverse VIX futures allocation to the base endowment portfolio. From a risk-adjusted return basis, the addition of inverse VIX futures provided a large improvement in the performance (as measured by the Sharpe ratio) of the base portfolio (from 0.92 to 1.15 and 1.89, respectively).
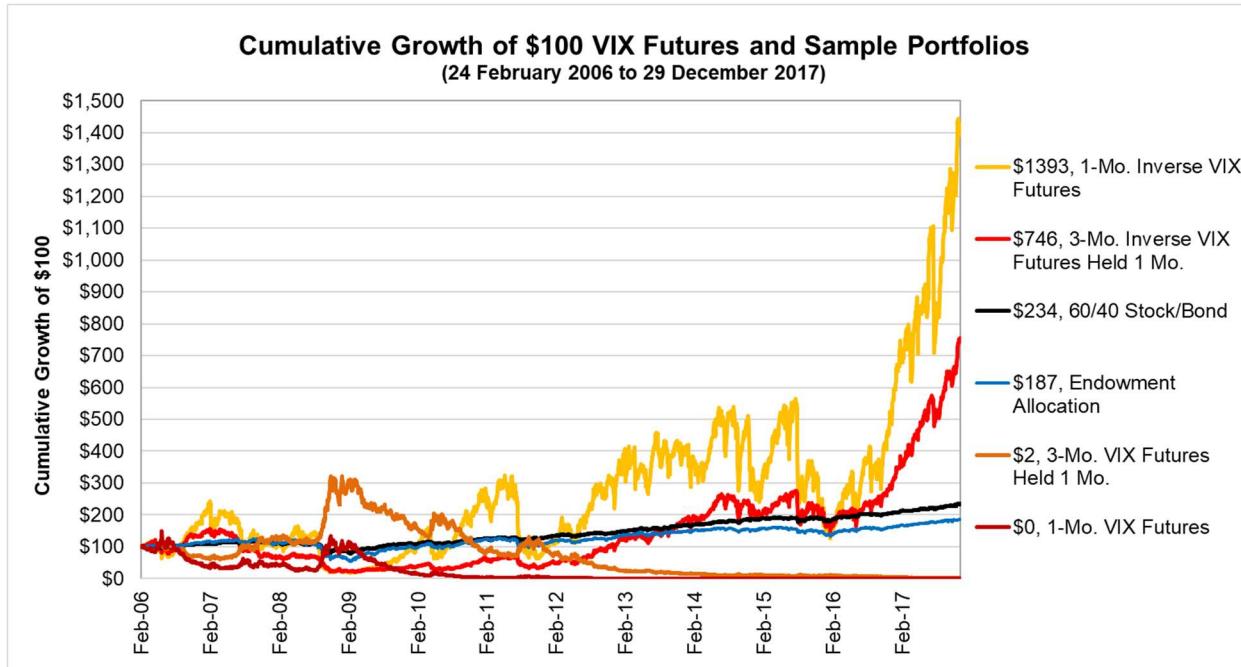
As with the 60/40 portfolio, the results for adding a written VIX call option allocation are similar to those from adding inverse VIX futures. The 1% written one-month ATM VIX call allocation improved returns from 9% to 16% while increasing standard deviation from 10% to 13%, and maximum drawdown from -10% to -12%, thus providing improvements in return with an increase in risk very similar to the one-month inverse VIX futures allocation. The 1% allocation to inverse one-month 25% OTM VIX calls further increased returns to 20% (from 9% for the base portfolio), but did so at a higher standard deviation (13% versus 10% for the base portfolio), and at an essentially identical maximum drawdown (-10.0% versus -9.6% for the base endowment portfolio).

Thus, for both base portfolios, an inverse VIX futures or call allocation resulted in an economically significant increase in returns and a smaller increase in risk, resulting in much higher Sharpe ratios. Similar results are found when considering alpha (although all measured alphas are statistically insignificant at 10%). Unlike the previous analyses, in which the VIX call allocations had more extreme impacts than the VIX futures allocations, the 1% written VIX call allocations in 2016 were very similar to the 5% inverse VIX futures allocations in 2016.

4.0 Performance of Long Term Buy-and-hold Inverse VIX-Based Strategies from 2006 to 2017

As mentioned above, the relatively persistent contango in VIX futures and the high volatility of volatility would likely make a long term buy-and-hold standalone long or inverse VIX futures position a questionable proposition for most investors. For this reason, the previous analysis focused on VIX futures or call option investments in a portfolio context over single year time periods. For the sake of completeness, this section expands the analysis by considering the performance of these strategies over the 10+ years since the introduction of VIX options.

Exhibit 13: Inverse VIX Futures-Based Strategies Buy-and-Hold Performance for over 10 years from 2006 to 2017

Cumulative Growth of $100 VIX Futures and Sample Portfolios
(24 February 2006 to 29 December 2017)

- $1393, 1-Mo. Inverse VIX Futures
- $746, 3-Mo. Inverse VIX Futures Held 1 Mo.
- $234, 60/40 Stock/Bond
- $187, Endowment Allocation
- $2, 3-Mo. VIX Futures Held 1 Mo.
- $0, 1-Mo. VIX Futures

24 Feb 2006 to 29 Dec 2017	60/40 Stock/Bond Portfolio	Endowment	1-Mo. VIX Futures	3-Mo. VIX Futures Held 1-Mo.	1-Mo. Inverse VIX Futures	3-Mo. Inverse VIX Futures Held 1 Mo.
Annualized Return	7.5%	5.4%	-55.8%	-28.8%	25.0%	18.5%
Annualized Standard Deviation	11.4%	16.5%	78.0%	43.6%	78.0%	43.6%
Sharpe Ratio	0.65	0.33	-0.72	-0.66	0.32	0.42
Maximum Drawdown	-34.8%	-57.4%	-100.0%	-99.4%	-92.3%	-86.5%
Skewness	-0.10	-0.24	1.04	0.64	-1.04	-0.64
Kurtosis	12.38	10.18	5.44	4.00	5.44	4.00
Correlation w SPX	0.99	0.89	-0.74	-0.74	0.74	0.74
Beta	0.58	0.75	-2.94	-1.66	2.94	1.66
Alpha	1.95%	-1.00%	-21.83%	-7.54%	24.02%	9.69%
Alpha P-Value	0.000	0.648	0.156	0.374	0.118	0.254

Source: Cboe Exchange Inc., Bloomberg

Exhibit 13: *This exhibit illustrates the performance of two traditional portfolios as well as two inverse VIX futures-based portfolios and two long VIX futures-based portfolios over the entire, more than 10 year period of study. The volatility and drawdowns of the four VIX futures-based portfolios suggest that VIX futures may be best suited for shorter term strategic portfolio risk management or return enhancement rather than long term buy-and-hold strategies.*
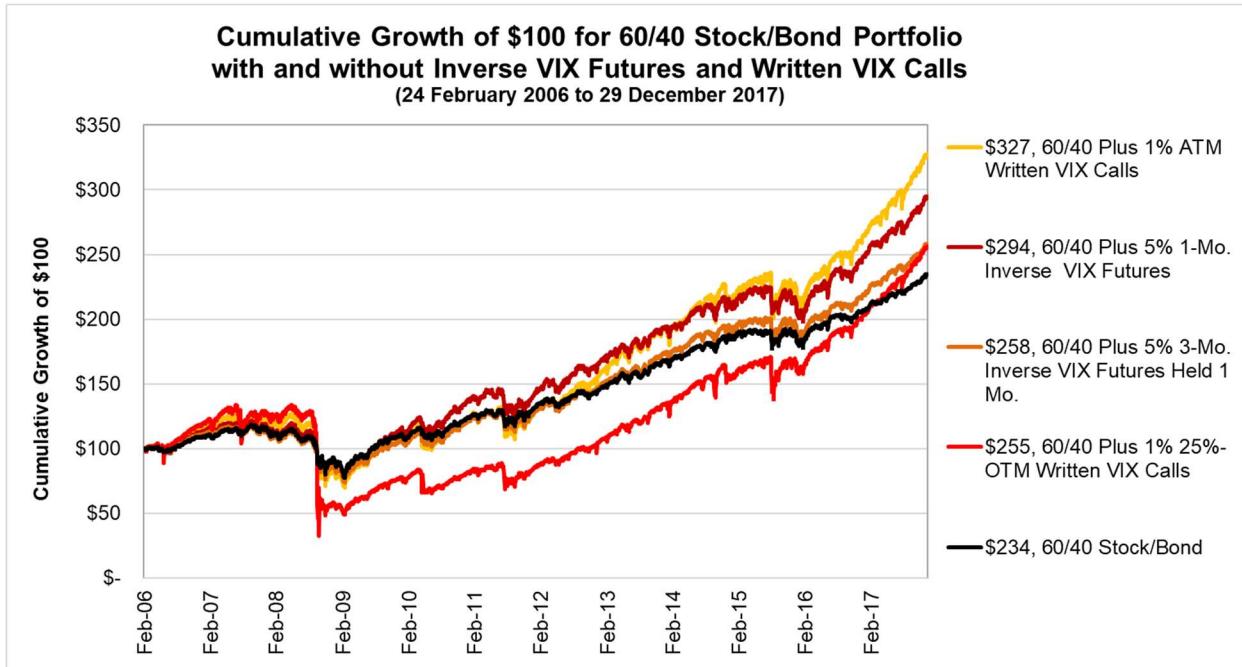
Exhibit 13 provides a graphical representation and summary statistics of the cumulative performance of the one-month and three-month long and inverse VIX futures strategies along with the two base portfolios since the inception of VIX options. It is evident that the overall

period was favorable for an inverse VIX futures exposure from a total return perspective and unfavorable for a long VIX futures exposure. However, the volatility of the long and inverse VIX exposures were exceedingly high (ranging from 32% to 78%) as were the maximum drawdowns (ranging from -90% to -100%). The maximum drawdowns for the one-month and three-month long VIX futures strategies were catastrophically high at -100% and -99%, respectively. The following sections will consider the full period performance of long and inverse VIX futures and call options in a portfolio context.

4.1 Inverse VIX Futures and Written Options-Based Strategies Portfolio Allocations from 2006 to 2017

This section considers the performance of inverse VIX futures and VIX call writing strategies within 60/40 and hypothetical endowment portfolios over the 10+ years since the introduction of VIX options. Exhibit 14 illustrates the 10+ year cumulative growth and provides summary statistics of the 60/40 stock/bond portfolio with and without a 5% allocation to inverse VIX futures (one-month or three-month VIX futures) and a 1% allocation to written VIX calls (ATM or 25% OTM one-month VIX calls). The base 60/40 portfolio generated a 7.5% annualized return over the period at an 11% annualized standard deviation with a -35% maximum drawdown. Both of the 5% inverse VIX futures allocations moderately increased the annualized return (from 7.5% to 9.5% and 8.3%, respectively) and standard deviation (from 11% to 14% and 12%, respectively) of the base portfolio. The maximum drawdown also increased from -35% to -39% and -38%, respectively. The ex-post skewness was also degraded by adding the inverse VIX futures allocations to the base portfolio. Thus over the full period, the inverse VIX futures allocations improved absolute returns and risk. Based on the Sharpe ratio and alpha, the inverse VIX futures allocation improved risk-adjusted returns. The Sharpe ratio increased slightly from 0.65 to 0.69 and 0.67, respectively and alpha increased from 2.8% to 4.4% and 3.4%, respectively.

Exhibit 14: 60/40 Portfolio Performance with Inverse VIX Futures and Written VIX Calls from 2006 to 2017

Cumulative Growth of $100 for 60/40 Stock/Bond Portfolio with and without Inverse VIX Futures and Written VIX Calls
(24 February 2006 to 29 December 2017)

Legend:
- $327, 60/40 Plus 1% ATM Written VIX Calls
- $294, 60/40 Plus 5% 1-Mo. Inverse VIX Futures
- $258, 60/40 Plus 5% 3-Mo. Inverse VIX Futures Held 1 Mo.
- $255, 60/40 Plus 1% 25%-OTM Written VIX Calls
- $234, 60/40 Stock/Bond

24 Feb 2006 to 29 Dec 2017	60/40 Stock/Bond Portfolio	60/40 Plus 5% 1-Mo. Inverse VIX Futures	60/40 Plus 5% 3-Mo. Inverse VIX Futures Held 1 Mo.	60/40 Plus 1% Written 25% OTM VIX Calls	60/40 Plus 1% Written ATM VIX Calls
Annualized Return	7.5%	9.5%	8.3%	8.3%	10.5%
Annualized Standard Deviation	11.4%	13.7%	12.4%	38.6%	17.4%
Sharpe Ratio	0.65	0.69	0.67	0.21	0.60
Maximum Drawdown	-34.8%	-38.5%	-37.7%	-75.6%	-44.8%
Skewness	-0.10	-0.41	-0.27	15.45	-0.26
Kurtosis	12.38	7.43	9.43	629.17	19.36
Correlation w SPX	0.99	0.98	0.99	0.58	0.93
Beta	0.58	0.69	0.63	1.14	0.83
Alpha	1.95%	3.04%	2.36%	2.77%	3.11%
Alpha P-Value	0.000	0.000	0.000	0.763	0.105

Source: Bloomberg, CFE, Cboe Exchange Inc., Optionmetrics

Exhibit 14: *This exhibit illustrates the performance of a 60/40 traditional portfolio with and without a small allocation to inverse VIX futures or written VIX call options. The returns of the 60/40 portfolio are improved with the addition of VIX call writing or inverse VIX futures, although the VIX call writing strategies significantly increase both volatility and drawdowns.*

The results for adding a written ATM VIX call option allocation are generally similar to the results for the inverse VIX futures allocations. The 1% one-month written ATM VIX call allocation increased returns from 7.5% to 10.5% and standard deviation from 11% to 17%, and maximum drawdown from -35% to -45%. In contrast, while the 1% allocation to written one-month 25% OTM VIX calls had a similar return increase to the other inverse VIX allocations, it increased standard deviation far more than the written VIX call or inverse VIX futures allocations. Returns increased from 7.5% to 8.3% while standard deviation tripled from 11% to 39%, and maximum

drawdown more than doubled from -35% to -76%. From a risk-adjusted return perspective, the results are mixed. The written ATM and OTM VIX call allocations reduced the Sharpe ratio of the base portfolio 0.65 to 0.60 and 0.21, respectively, while alpha increased from 2.8% to 4.5% and 4.1%, respectively, (although the alpha for the written OTM call portfolios is statistically insignificant at 10%).
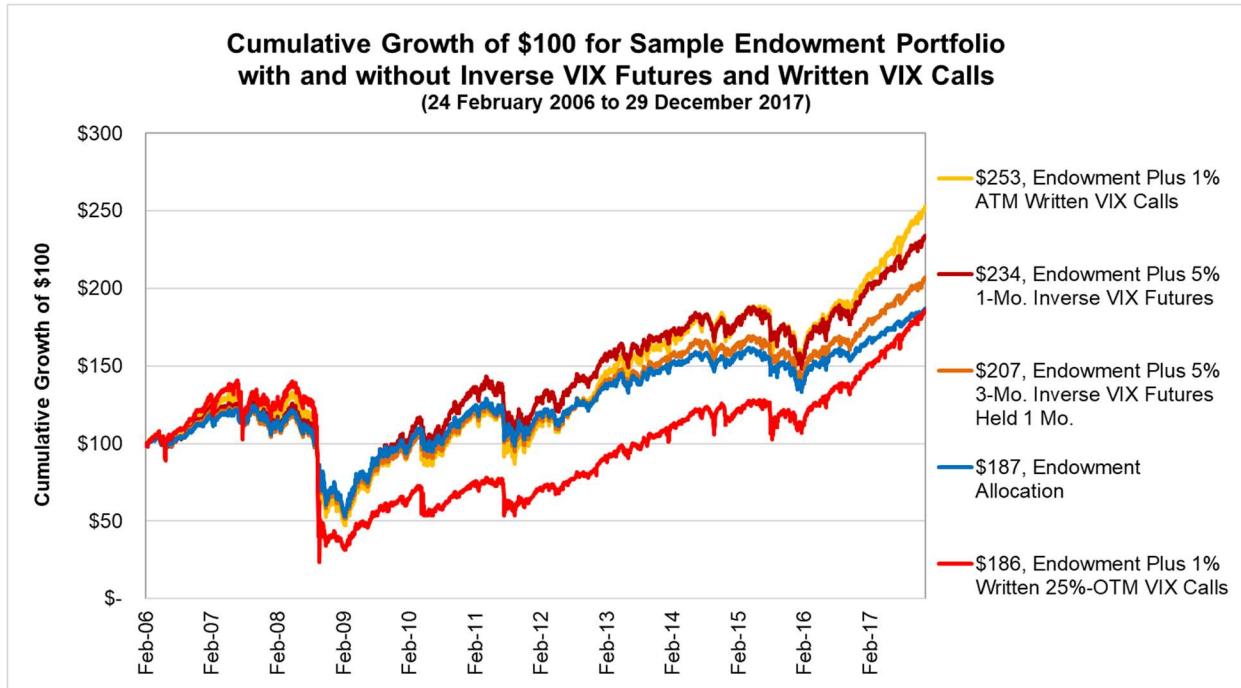
The results above suggest that an inverse VIX futures or written call allocation would have improved performance from a raw return perspective at the expense of increased risk. However, the results are mixed from a risk-adjusted return perspective. This suggests that permanent buy-and-hold allocations to inverse VIX futures or written calls may provide small improvement in returns but at the expense of increased risk over the long term.

This section assesses the impact of an added allocation of inverse VIX futures or call options to a well-diversified hypothetical endowment portfolio over the full 10+ year period. Exhibit 15 provides an illustration of the full period cumulative growth and summary statistics of the base hypothetical endowment portfolio with and without a 5% allocation to inverse VIX futures (one-month or three-month VIX futures) or a 1% allocation to written VIX calls (ATM or 25% OTM one-month VIX calls). The base endowment portfolio experienced a 5% annualized return over the period at a 17% annualized standard deviation and a -57% maximum drawdown. The portfolios with a 5% allocation to inverse one- and three-month VIX futures outperformed the base portfolio, experiencing moderately higher returns (8% and 6% versus 5% for the base portfolio) although standard deviations were also somewhat higher (18.3% and 17.1% versus 16.5% for the base portfolio). The maximum drawdown also slightly increased from -57% to -59% and -58%, respectively. The ex-post skewness was also worsened by adding the inverse VIX futures allocations to the base endowment portfolio. From a risk-adjusted return perspective, the inverse VIX futures allocations improved performance of the endowment portfolio with the Sharpe ratio increasing from 0.33 to 0.41 and 0.37 and alpha increasing from -1.4% to +0.2% and -0.7%, respectively.

The results for adding a written VIX call option allocation are quite different from the inverse VIX futures allocations. The 1% written one-month ATM or OTM VIX call allocation reduced returns from 5% to 3% and 2%, respectively, and increased standard deviation from 17% to 16% and 14%, respectively, and maximum drawdown from -57% to -43% and -50%, respectively. From a risk-adjusted return perspective, the inverse VIX call allocations reduced performance of the endowment portfolio based on the Sharpe ratio (dropping from 0.33 to 0.16 for both written call strategies). However the results for alpha are mixed, with alpha decreasing from -1.4% to -2.8% and increasing from -1.4% to +0.2%, respectively (although the alphas are not statistically significant).

Thus, while the inverse VIX futures allocations improved absolute and risk-adjusted returns for both base portfolios, the call allocation results are mixed for both absolute returns and risk-adjusted returns.

Exhibit 15: Endowment with Inverse VIX Futures and Written VIX Calls from 2006 to 2017

Cumulative Growth of $100 for Sample Endowment Portfolio with and without Inverse VIX Futures and Written VIX Calls
(24 February 2006 to 29 December 2017)

- $253, Endowment Plus 1% ATM Written VIX Calls
- $234, Endowment Plus 5% 1-Mo. Inverse VIX Futures
- $207, Endowment Plus 5% 3-Mo. Inverse VIX Futures Held 1 Mo.
- $187, Endowment Allocation
- $186, Endowment Plus 1% Written 25%-OTM VIX Calls

24 Feb 2006 to 29 Dec 2017	Endowment	Endowment Plus 5% 1-Mo. Inverse VIX Futures	Endowment Plus 5% 3-Mo. Inverse VIX Futures Held 1 Mo.	Endowment Plus 1% Written 25% OTM VIX Calls	Endowment Plus 1% Written ATM VIX Calls
Annualized Return	5.4%	7.5%	6.3%	2.2%	2.6%
Annualized Standard Deviation	16.5%	18.3%	17.1%	13.7%	16.2%
Sharpe Ratio	0.33	0.41	0.37	0.16	0.16
Maximum Drawdown	-57.4%	-59.0%	-58.3%	-50.2%	-42.9%
Skewness	-0.24	-0.44	-0.34	0.04	-0.40
Kurtosis	10.18	7.66	8.90	11.36	28.10
Correlation w SPX	0.89	0.91	0.91	0.71	0.44
Beta	0.75	0.85	0.79	0.50	0.37
Alpha	-1.00%	0.17%	-0.48%	-1.96%	0.11%
Alpha P-Value	0.648	0.939	0.820	0.482	0.978

Source: Bloomberg, CFE, Cboe Exchange Inc., Optionmetrics

Exhibit 15: This exhibit illustrates the performance of a hypothetical endowment portfolio with and without a small allocation to inverse VIX futures or written VIX call options. The returns of the endowment portfolio are improved with the addition of inverse VIX futures with a small increase in volatility. In contrast, VIX call writing reduces returns with a small impact on volatility.

5.0 VIX Spread Strategies

Spread trading is very popular in futures trading. VIX futures are no exception. The amount of VIX futures open interest held in the form of futures spreads is a significant portion of total open interest and has grown significantly since 2012 (see Szado [2018a]). While there are a wide variety of VIX futures spread strategies, the following analysis focuses on a simple term
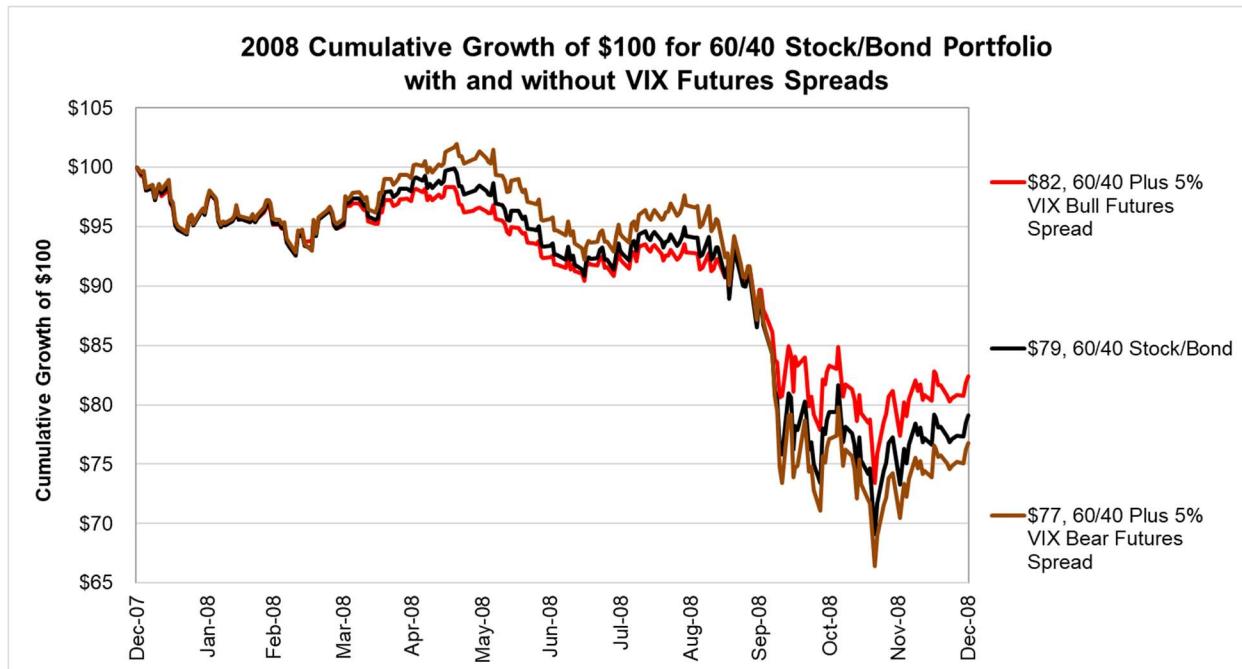
structure spread strategy which takes a position in the one-month VIX futures contract versus an opposite position in the equal number of three-month VIX futures, with both contracts held for one-month before rolling. The following sections consider the addition of a bull or bear VIX Futures Spread to the two base portfolios in 2008, in 2016 and in the overall 10+ year period. The bull VIX Futures Spread consists of a long one-month VIX futures position and a short three-month VIX futures position, while a bear VIX Futures Spread is short one-month VIX futures and long three-month VIX futures. It is worth noting that the terms "bull" and "bear" do not refer to bull or bear equity exposure, but rather to the exposure to volatility through VIX futures. A bull spread in VIX futures might be considered in some way comparable to a bear spread in S&P 500 futures since one would expect VIX futures to rise as S&P 500 futures fall. The spread strategies can be implemented to take advantage of a wide variety of characteristics of VIX futures, including the relative betas, volatility, or cost of carry of futures contracts of different tenor or expectations of changes in the term structure. Spread strategies may also be implemented as a less "expensive" means of accessing VIX exposure since the volatility of volatility exposure is expected to be less in a term spread than in a direct inverse futures exposure, or the cost of carry drag caused by steep contango may be less in a term spread than in a direct long futures position.

5.1 VIX Spread Strategies in 2008

Exhibit 16 provides results pertaining to the 2008 performance of the base 60/40 stock/bond portfolio with and without a 5% allocation to VIX bull or bear futures spreads (as defined above). Previously discussed summary statistics for the 60/40 portfolio with a 5% long or inverse VIX futures allocation are also provided for reference.

The base 60/40 portfolio lost 21% over the period at a 24% annual standard deviation and a -31% maximum drawdown. The portfolios with a 5% allocation to bull VIX futures spreads performed similarly to long one-month VIX futures, although the bull VIX futures portfolio underperforms the one-month futures portfolio. The spread portfolio experienced a -18% return at a 20% standard deviation versus a -13% return and 18% standard deviation for the portfolio with the one-month long VIX futures allocation. Risk-adjusted return for the bull spread allocation portfolio is also lower with an alpha of 1.7% versus 5.5%, although both are not statistically significant. In contrast, the performance of the portfolios with allocations to bear VIX futures spreads and to inverse one-month VIX futures performed almost identically by most of the calculated measures.

Exhibit 16: 60/40 Portfolio with and without VIX Futures Spreads in 2008

2008 Cumulative Growth of $100 for 60/40 Stock/Bond Portfolio with and without VIX Futures Spreads

| 2008	60/40 Stock/Bond Portfolio	60/40 Plus 5% "Bull" Futures Spread	60/40 Plus 5% "Bear" Futures Spread	60/40 Plus 5% 1 Mo. Futures	60/40 Plus 5% 1 Mo. Inverse VIX Futures
Annual Return	-20.9%	-17.6%	-23.2%	-13.4%	-23.6%
Annualized Standard Deviation	24.1%	20.4%	26.0%	18.4%	26.0%
Sharpe Ratio	-0.87	-0.86	-0.89	-0.73	-0.91
Maximum Drawdown	-30.9%	-26.6%	-34.8%	-22.1%	-35.4%
Skewness	0.16	0.22	0.17	0.36	0.01
Kurtosis	4.14	3.69	4.91	4.08	3.33
Correlation w SPX	1.00	0.98	0.98	0.97	0.99
Beta	0.58	0.49	0.62	0.44	0.63
Alpha	2.22%	1.73%	0.78%	5.52%	0.42%
Alpha P-Value	0.501	0.745	0.914	0.357	0.923
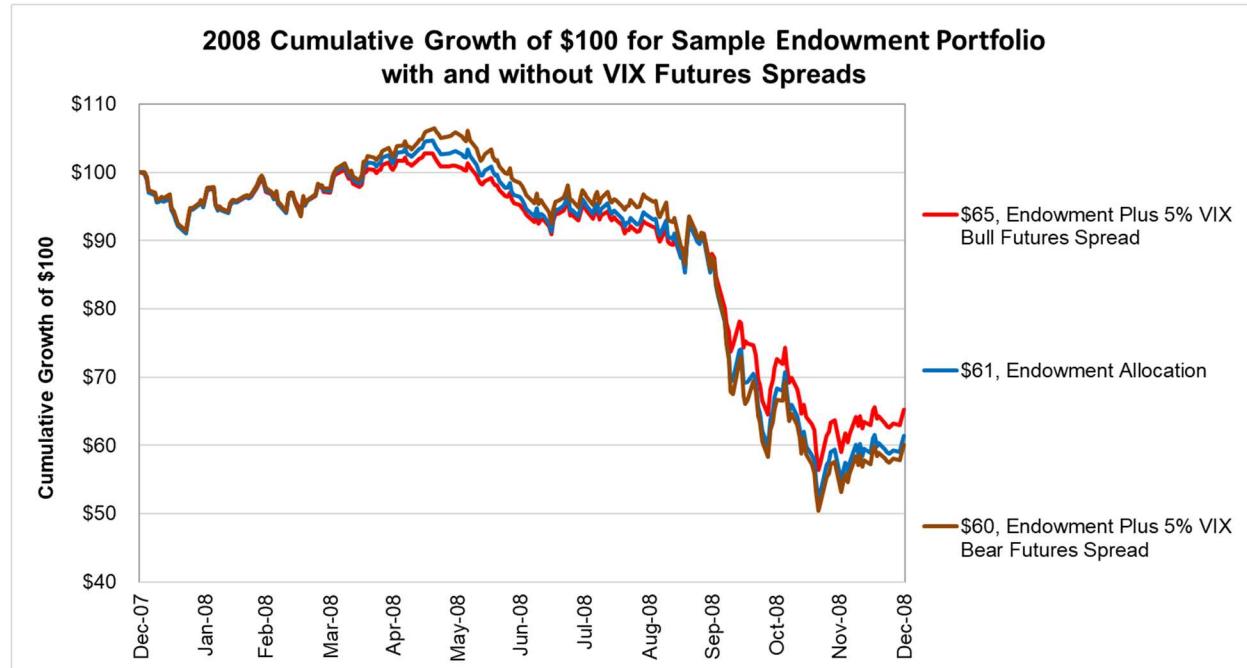
Source: Bloomberg, CFE, Cboe Exchange Inc., Optionmetrics

Exhibit 16: This exhibit illustrates the 2008 performance of a 60/40 portfolio with and without the addition of bear VIX futures spreads and bull VIX futures spreads. The strategy of a 60-40 portfolio plus a 5% allocation to a VIX futures bull spread had slight improvements in return, standard deviation, and drawdown when compared to the 60/40 portfolio.

Similar results are found in Exhibit 17 for the endowment-based portfolios. Once again, the portfolio with the bull spread allocation underperformed the long one-month VIX futures allocation portfolio with a lower return (-35% versus -31%) and higher standard deviation (27% versus 25%), while the bear spread allocation portfolio performed virtually identically to the long one-month VIX futures allocation portfolio.

Thus, in 2008 both the bull VIX futures spread allocations and the long one-month VIX futures allocation improved performance of the base 60/40 and endowment portfolios in 2008, however the bull spread allocation was less effective than the long one-month VIX futures allocation. In contrast, the bear VIX futures spread allocations and the inverse one-month VIX futures allocation both slightly reduced performance of the base 60/40 and endowment portfolios in 2008, and did so virtually identically to one another.

Exhibit 17: Endowment Portfolio with and without VIX Futures Spreads in 2008

2008	Endowment	Endowment Plus 5% "Bull" Futures Spread	Endowment Plus 5% "Bear" Futures Spread	Endowment Plus 5% 1 Mo. Futures	Endowment Plus 5% 1 Mo. Inverse VIX Futures
Annual Return	-38.6%	-34.7%	-39.9%	-31.1%	-40.2%
Annualized Standard Deviation	31.0%	27.0%	32.5%	25.2%	32.5%
Sharpe Ratio	-1.25	-1.29	-1.23	-1.23	-1.24
Maximum Drawdown	-50.4%	-45.2%	-52.7%	-41.5%	-53.1%
Skewness	-0.13	-0.08	-0.17	-0.02	-0.21
Kurtosis	3.21	3.38	3.13	3.61	2.79
Correlation w SPX	0.92	0.89	0.92	0.87	0.93
Beta	0.69	0.59	0.73	0.54	0.73
Alpha	-25.54%	-24.21%	-26.16%	-19.88%	-26.53%
Alpha P-Value	0.158	0.170	0.166	0.269	0.138

Source: Bloomberg, CFE, Cboe Exchange Inc., Optionmetrics

Exhibit 17: *This exhibit illustrates the 2008 performance of a theoretical endowment portfolio with and without the addition of bear VIX futures spreads and bull VIX futures spreads. The endowment portfolio plus a 5% allocation to a VIX futures bull spread had slight improvements in return and standard deviation when compared to the endowment portfolio.*

5.2 VIX Spread Strategies in 2016

Exhibit 18 provides a graphical representation and summary statistics of the 2016 performance of the base 60/40 stock/bond portfolio with and without a 5% allocation to VIX bull or bear futures spreads. Previously discussed 2016 summary statistics for the 60/40 portfolio with a 5% long or inverse VIX futures allocation are also provided for reference. The base 60/40 portfolio gained 8% over the period at an 8% annual standard deviation and a -5% maximum drawdown. The portfolio with a 5% allocation to bear VIX futures spreads outperformed the base portfolio but slightly underperformed the one-month inverse VIX futures allocation portfolio from an absolute return perspective, increasing the returns of the base portfolio from 8.4% to 13.6% (versus 14.3% for the inverse futures allocation portfolio). However, the bear VIX futures spread allocation portfolio increased standard deviation by less than the one-month inverse VIX futures allocation (from 8% to 9% versus 11%), while maximum drawdown was increased by approximately the same amount for both VIX allocations (from -5% to -7% and -6%, respectively). From a risk-adjusted return perspective, both the bear VIX futures spread and inverse VIX futures allocations improved the 60/40 portfolio's Sharpe ratio and alpha, although the bear spread allocation provided a greater improvement. Sharpe ratio was increased from 1.11 to 1.49 and 1.30, respectively and alpha was increased from 2.1% to 7.6% and 6.0%, respectively (although only the bear spread allocation alpha is not statistically significant at 10%).
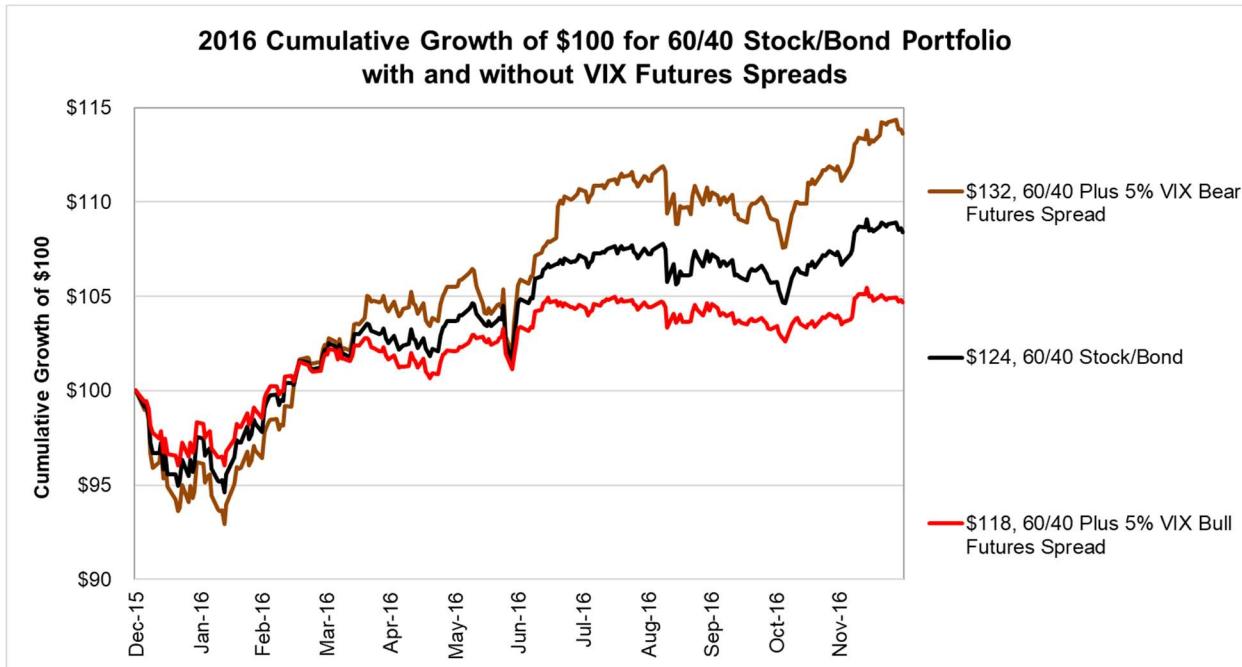
As expected, the bull VIX futures allocation and the long one-month VIX futures allocation both reduced the performance of the 60/40 portfolio in 2016. The bull spread allocation reduced returns by less than the long one-month futures allocation (from 8% to 5% versus 2%, respectively), although it also reduced standard deviation by less (from 8% to 6% versus 4%, respectively). From a risk-adjusted return perspective, both allocations reduced performance, yet the bull spread allocation portfolio reduced performance by less than the long one-month futures allocation. Sharpe ratio was reduced from 1.11 to 0.79 and 0.52, respectively and alpha was decreased from 2.1% to -0.7% and -1.4%, respectively (although none of the alphas are statistically significant at 10%).

Similar results are found in Exhibit 19 for the endowment based portfolios. Once again, the portfolio with the bull spread allocation underperformed the long one-month VIX futures allocation portfolio with a lower return (6% versus 3%) at a higher standard deviation (9% versus 7%), while both had higher returns and lower standard deviations than the base endowment portfolio. Both portfolios also experienced lower Sharpe ratios and alphas than the base portfolio although, once again, the bull spread allocation outperformed the long one-month VIX futures allocation portfolio.

The bear spread allocation and inverse one-month VIX futures allocation also had similar impacts on the endowment portfolio as they did on the 60/40 portfolio. Both allocations increased absolute and risk-adjusted return, while slightly increasing standard deviation and maximum drawdown. Once again, the inverse one-month VIX futures allocation increased

absolute return by more than the bear spread allocation, although the bear spread allocation provided a greater increase in risk-adjusted return (Sharpe ratio and alpha).

Exhibit 18: 60/40 Portfolio with and without VIX Futures Spreads in 2016

| 2016	60/40 Stock/Bond Portfolio	60/40 Plus 5% "Bull" Futures Spread	60/40 Plus 5% "Bear" Futures Spread	60/40 Plus 5% 1 Mo. Futures	60/40 Plus 5% 1 Mo. Inverse VIX Futures
Annual Return	8.4%	4.7%	13.6%	2.3%	14.3%
Annualized Standard Deviation	7.6%	5.9%	9.1%	4.4%	11.0%
Sharpe Ratio	1.11	0.79	1.49	0.52	1.30
Maximum Drawdown	-5.4%	-4.0%	-7.1%	-2.9%	-6.9%
Skewness	-0.29	0.05	-0.39	0.57	-0.96
Kurtosis	2.23	1.58	2.29	1.77	5.22
Correlation w SPX	0.99	0.96	0.94	0.81	0.96
Beta	0.57	0.43	0.65	0.27	0.81
Alpha	2.06%	-0.73%	7.57%	-1.36%	6.02%
Alpha P-Value	0.240	0.762	0.093	0.714	0.168
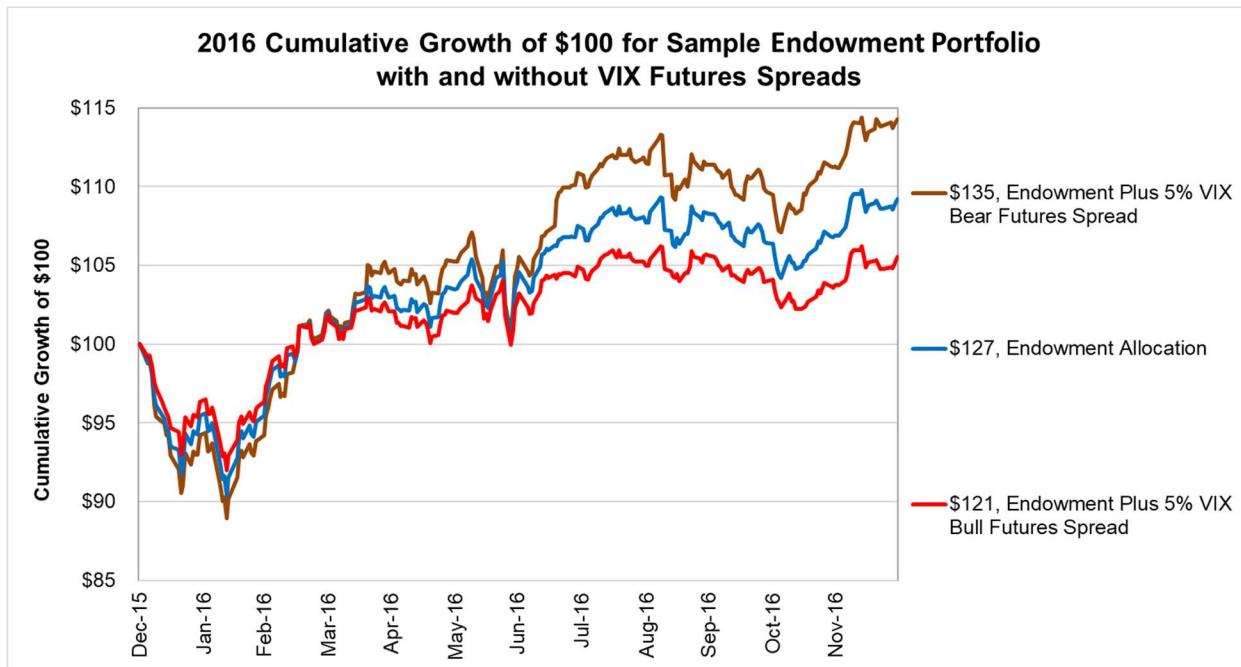
Source: Bloomberg, CFE, Cboe Exchange Inc., Optionmetrics

Exhibit 18: *This exhibit illustrates the 2016 performance of a 60/40 portfolio with and without the addition of bear VIX futures spreads and bull VIX futures spreads. The strategy of a 60-40 portfolio plus a 5% allocation to a VIX futures bear spread had the highest Sharpe Ratio among all five strategies in the table above.*

Thus, in 2016 both the VIX futures spread allocations generated higher risk-adjusted returns than their corresponding one-month VIX futures allocations (for both base portfolios). In contrast, in 2008 the bear VIX futures spread allocations provided essentially identical impacts

on absolute and risk-adjusted returns, while the bull VIX futures spread allocations provided somewhat lower improvement in absolute and risk-adjusted returns when compared to one-month VIX futures allocations.

Exhibit 19: Endowment Portfolio with and without VIX Futures Spreads in 2016

2016 Cumulative Growth of $100 for Sample Endowment Portfolio with and without VIX Futures Spreads

2016	Endowment	Endowment Plus 5% "Bull" Futures Spread	Endowment Plus 5% "Bear" Futures Spread	Endowment Plus 5% 1 Mo. Futures	Endowment Plus 5% 1 Mo. Inverse VIX Futures
Annual Return	9.2%	5.5%	14.3%	3.2%	15.0%
Annualized Standard Deviation	10.0%	8.5%	11.2%	7.3%	13.0%
Sharpe Ratio	0.92	0.65	1.28	0.44	1.15
Maximum Drawdown	-9.6%	-8.0%	-11.1%	-6.4%	-11.0%
Skewness	-0.48	-0.36	-0.53	-0.27	-1.00
Kurtosis	2.56	2.17	2.58	1.79	5.32
Correlation w SPX	0.82	0.75	0.83	0.58	0.87
Beta	0.62	0.48	0.71	0.32	0.86
Alpha	2.52%	-0.22%	7.85%	-0.77%	6.29%
Alpha P-Value	0.765	0.979	0.390	0.928	0.504

Source: Bloomberg, CFE, Cboe Exchange Inc., Optionmetrics

Exhibit 19: This exhibit illustrates the 2016 performance of a theoretical endowment portfolio with and without the addition of bear VIX futures spreads and bull VIX futures spreads. The strategy of the endowment portfolio plus a 5% allocation to a VIX futures bear spread had the highest Sharpe Ratio among all five strategies in the table above.
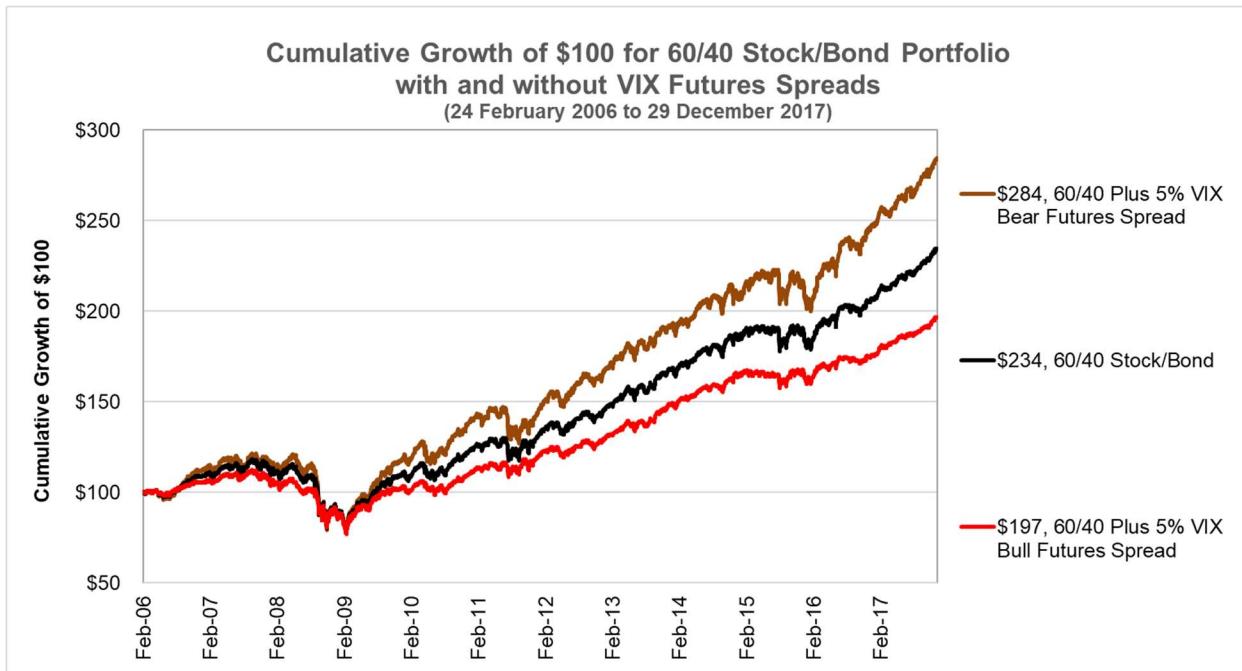
5.3 VIX Spread Strategies from 2006 to 2017

This section considers the performance of the bull and bear VIX futures spread strategies over a 10+ year period since the introduction of VIX options (24 February 2006 to 31 December 2017). Exhibit 20 provides a graphical representation and summary statistics of the cumulative performance of the base 60/40 stock/bond portfolio with and without a 5% allocation to VIX bull or bear futures spreads while Exhibit 21 provides the corresponding results for the cumulative performance of the base endowment portfolio with and without an allocation to VIX futures spreads. The exhibits also include summary statistics for the base portfolios with a 5% long or inverse VIX futures allocation.

Consistent with the previously presented results, the bull VIX spread allocation and the long one-month VIX futures allocation both reduce the performance of the 60/40 base portfolio (see Exhibit 20). This is true both from an absolute return perspective and a risk-adjusted return perspective. However, the bear VIX futures spread allocation portfolio outperforms the long VIX futures allocation portfolio. The performance of the bear VIX futures spread allocation portfolio lies close to mid-way between the performance of the 60/40 portfolio and the performance of the long VIX futures allocation portfolio by most measures. For example, the returns of the base, bull spread allocation and long futures allocation portfolios are 8%, 6% and 4%, respectively, the Sharpe ratios are 0.65, 0.62 and 0.54, respectively and alphas are 2.8%, 2.0% and 1.1%, respectively. The risk of the portfolios follow a similar pattern with standard deviations of 11%, 10% and 8%, respectively and maximum drawdowns of -35%, -31% and -27%. This is consistent with the 2008 results shown above in which the long one-month futures allocation portfolio provided greater protection than the bull spread allocation portfolio.

The bear spread and inverse VIX futures allocations both improved absolute returns and risk-adjusted returns over the 10+ years since the inception of VIX option trading, while also moderately increasing risk. While the bear spread allocation increased return by less than the inverse VIX futures allocation (from 7.5% to 9.2% and 9.5%, respectively), the bear spread allocation increased risk-adjusted return by more (Sharpe ratio increased from 0.65 to 0.73 and 0.69 while the alpha increased from 2.8% to 4.7% and 4.4%, respectively). The risk of the bear VIX futures spread allocation portfolio was close to mid-way between the risk of the 60/40 portfolio and the long VIX futures allocation portfolio as measured by the standard deviation and the maximum drawdown.

## Exhibit 20: 60/40 Portfolio with and without VIX Futures Spreads
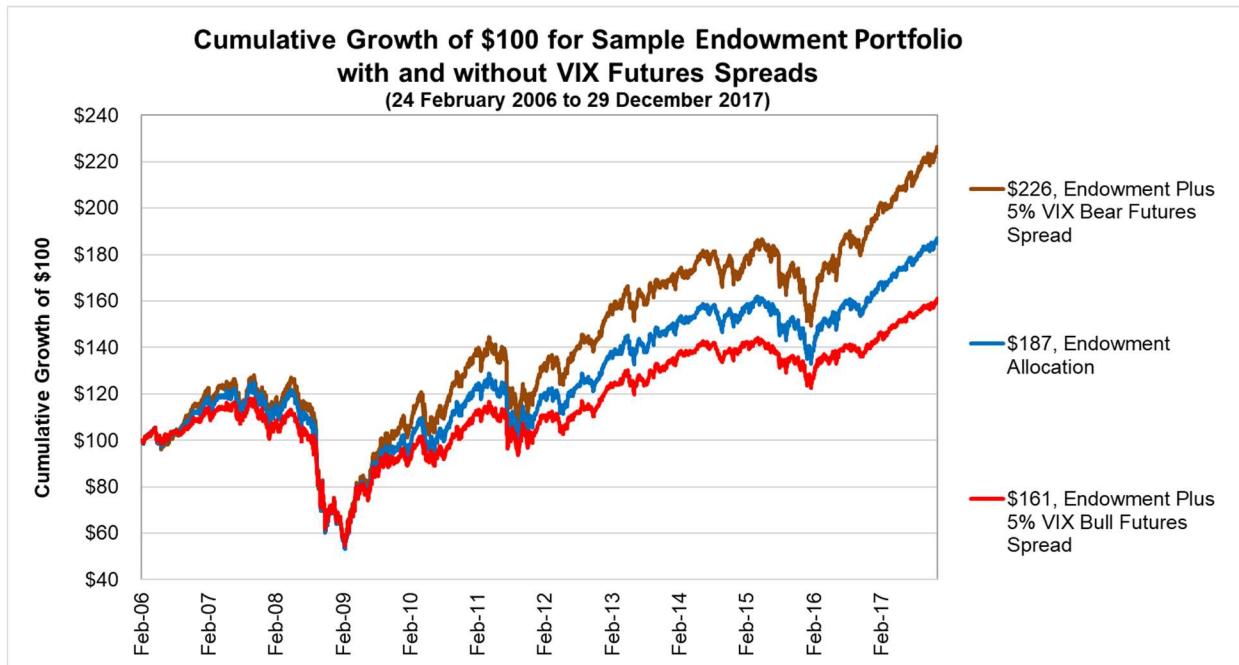
Cumulative Growth of $100 for 60/40 Stock/Bond Portfolio with and without VIX Futures Spreads
(24 February 2006 to 29 December 2017)

$284, 60/40 Plus 5% VIX Bear Futures Spread

$234, 60/40 Stock/Bond

$197, 60/40 Plus 5% VIX Bull Futures Spread

24 Feb 2006 to 29 Dec 2017	60/40 Stock/Bond Portfolio	60/40 Plus 5% "Bull" Futures Spread	60/40 Plus 5% "Bear" Futures Spread	60/40 Plus 5% 1 Mo. Futures	60/40 Plus 5% 1 Mo. Inverse VIX Futures
Annual Return	7.5%	5.9%	9.2%	4.4%	9.5%
Annualized Standard Deviation	11.4%	9.5%	12.6%	8.1%	13.7%
Sharpe Ratio	0.65	0.62	0.73	0.54	0.69
Maximum Drawdown	-34.8%	-31.4%	-35.9%	-26.5%	-38.5%
Skewness	-0.10	0.08	-0.18	0.35	-0.41
Kurtosis	12.38	12.32	12.37	16.25	7.43
Correlation w SPX	0.99	0.97	0.97	0.92	0.98
Beta	0.58	0.47	0.63	0.38	0.69
Alpha	2.82%	1.96%	4.66%	1.12%	4.40%
Alpha P-Value	0.000	0.037	0.000	0.408	0.000

Source: Bloomberg, CFE, Cboe Exchange Inc., Optionmetrics

Exhibit 20: This exhibit illustrates the performance of a 60/40 portfolio with and without the addition of bear VIX futures spreads and bull VIX futures spreads over the full period of the study. The strategy of a 60-40 portfolio plus a 5% allocation to a VIX futures bear spread had the highest Sharpe Ratio among all five strategies in the table above.

Exhibit 21: Endowment Portfolio with and without VIX Futures Spreads

Cumulative Growth of $100 for Sample Endowment Portfolio with and without VIX Futures Spreads
(24 February 2006 to 29 December 2017)

24 Feb 2006 to 29 Dec 2017	Endowment	Endowment Plus 5% "Bull" Futures Spread	Endowment Plus 5% "Bear" Futures Spread	Endowment Plus 5% 1 Mo. Futures	Endowment Plus 5% 1 Mo. Inverse VIX Futures
Annual Return	5.4%	4.1%	7.1%	2.8%	7.5%
Annualized Standard Deviation	16.5%	14.4%	17.2%	13.1%	18.3%
Sharpe Ratio	0.33	0.28	0.41	0.21	0.41
Maximum Drawdown	-57.4%	-53.8%	-57.5%	-50.4%	-59.0%
Skewness	-0.24	-0.10	-0.35	0.02	-0.44
Kurtosis	10.18	10.78	9.45	12.41	7.66
Correlation w SPX	0.89	0.86	0.90	0.80	0.91
Beta	0.75	0.63	0.79	0.54	0.85
Alpha	-1.44%	-2.02%	0.47%	-2.78%	0.24%
Alpha P-Value	0.648	0.516	0.885	0.397	0.939

Source: Bloomberg, CFE, Cboe Exchange Inc., Optionmetrics

Exhibit 21: *This exhibit illustrates the performance of a theoretical endowment portfolio with and without the addition of bear VIX futures spreads and bull VIX futures spreads over the full period of the study. The strategy of the endowment portfolio plus a 5% allocation to a VIX futures bear spread had the highest Sharpe ratio among all five strategies in the table above.*

Thus, over the full period of study, the bull and bear spread allocation portfolios both provided greater risk-adjusted returns than their corresponding one-month futures allocation portfolios. Furthermore, over the long term, long VIX futures and bull spreads historically reduced risk-adjusted performance while inverse VIX futures and bear spreads improved risk-adjusted performance, providing an economically significant increase in ex post alpha.

Similar results are found in Exhibit 21 for the endowment based portfolios. Once again, the portfolio with the bull spread allocation outperformed the long one-month VIX futures

allocation portfolio with a higher return (6% versus 4%), but at a higher standard deviation (10% versus 8%), while both had lower returns and standard deviations than the base endowment portfolio. Both portfolios also provided lower Sharpe ratios and alphas than the base portfolio and, once again, the bull spread allocation outperformed the long one-month VIX futures allocation portfolio.

The bear spread allocation and inverse one-month VIX futures allocation also had similar impacts on the endowment portfolio as they did on the 60/40 portfolio. Both allocations increased absolute and risk-adjusted return, while slightly increasing standard deviation and maximum drawdown. The inverse one-month VIX futures allocation increased absolute return by more than the bear spread allocation, although the bear spread allocation provided a greater increase in risk-adjusted return based on alpha (both provided the same Sharpe ratio).

6.0 Inverse VIX Futures with Dynamic De-Levering - the VPD and VPN Indexes

This section considers the performance of two benchmark indexes that provide exposure to inverse VIX futures with a dynamically varying degree of leverage, with or without additional risk mitigation from the addition of long VIX calls. The Cboe VIX Premium Strategy Index (VPD) sells a varying number of one-month VIX futures contracts versus a money market account investment representing the full value of a VIX futures contract. The number of futures contracts sold is determined on the roll date such that 75 percent of the capital will be preserved in the face of a 25-point increase in the VIX futures price (see www.cboe.com/VPD). Specifically, the number of one-month VIX futures contracts sold on the roll date is determined by the following formula:

$N_{NEW} = M_t * (1 + R_t - 0.75) / (1000 * 25)$

where M_t is the amount in the money market account at the roll and Rt is the effective three-month Treasury bill rate to the next roll.

Since the notional value of the short VIX futures position is determined by the level of the futures price and the number of contracts sold, the level of VIX futures will have a significant impact on the degree of leverage. The futures position is typically de-levered since it is generally over-collateralized. For example, if the VIX futures are at 25.0 at the roll, approximately 0.25 futures contracts will be sold versus a money market position representing the full value of one futures contract (de-levered) and if the VIX futures are at 100.0 at the roll, the short futures position will be approximately fully collateralized (unlevered). At VIX futures levels significantly above 100.0, the short VIX futures will be partially collateralized (levered). Thus, the short VIX futures position is expected to be de-levered unless VIX futures are at extremely high levels.

The Cboe Capped VIX Premium Strategy Index (VPN) is a risk-mitigated version of the VPD index in which the short VIX futures positions are protected by purchasing OTM one-month VIX calls

(see www.cboe.com/VPN). The number of futures contracts sold is determined by the same methodology as used for the VPD index. The risk mitigation is accomplished by purchasing OTM VIX calls at a strike price 25 points above the VIX futures price (or nearest available strike price below the calculated level).
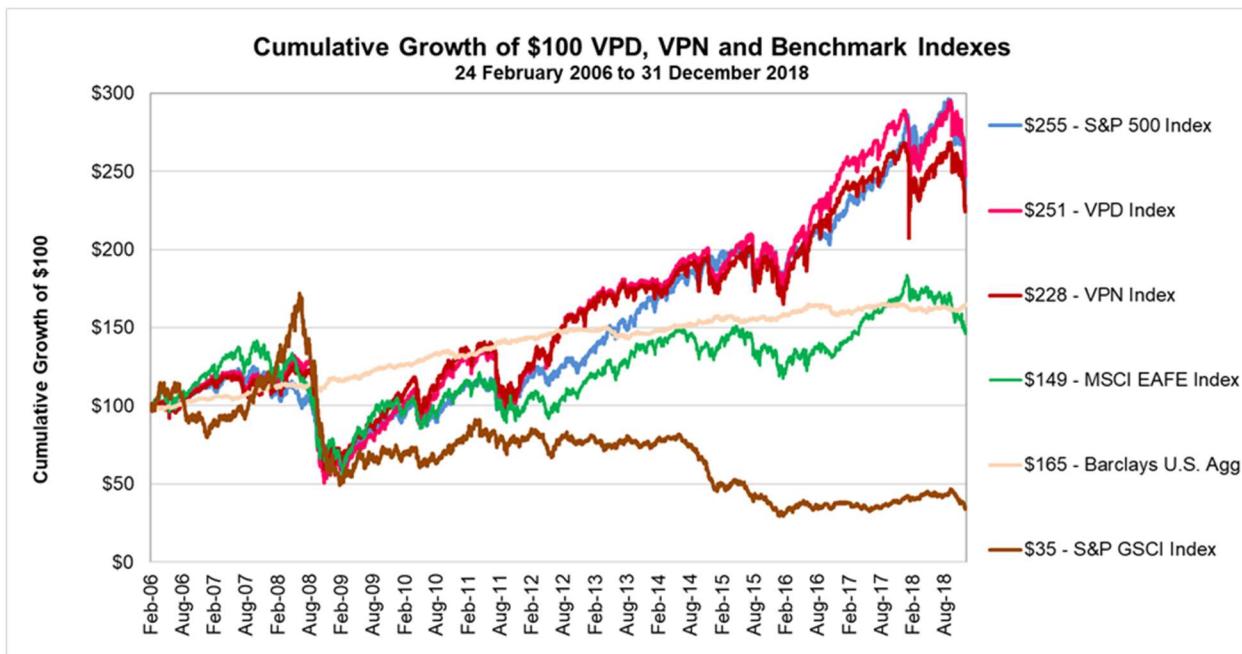
In contrast, two indexes which sell VIX futures (VPD and VPN) outperformed the S&P 500 on a raw return basis, albeit with increased volatility. The VPD index sells hypothetical fully collateralized VIX futures and is therefore exposed to significant tail risk in the case of a major volatility event. To mitigate this tail risk exposure, the VPN index includes a long OTM VIX call option exposure. While the long call results in a drag on performance in periods in which VIX does not increase significantly, it does provide some protection against catastrophic volatility events.

6.1 Performance of the VPD and VPN Indexes from 2006 to 2018

This section considers the 12+ year performance of the VPD and VPN Indexes over the period of study from the inception of VIX options 24 February 2006 to the end of 2018 relative to a variety of benchmark indexes. In addition, the performance is compared to a 60/40 portfolio with a 5% allocation to inverse one-month or three-month (held for one month) VIX futures for the period from 24 February 2006 to the end of 2017.

Exhibit 22 provides an illustration of the 12+ year cumulative growth and summary statistics of the VPD and VPN indexes and four traditional benchmark total return indexes (S&P 500, Barclays U.S. Aggregate bond index, MSCI EAFE equity index, and S&P GSCI commodity index). The VPD and VPN Indexes generated a 7.4% and 6.5% annualized return, at a 22% and 19% annualized standard deviation, and a -62% and -53% maximum drawdown, respectively. While the VPD index generated a higher return than the VPN index, the VPN index experienced lower volatility and a smaller maximum drawdown and slightly higher risk-adjusted returns in terms of the Sharpe ratio (0.34 versus 0.33), although VPD had a higher alpha (1.51% versus 1.31%). This suggests that the protective long call positions may be effective at reducing risk in an efficient manner. Only the S&P 500 index generated higher returns than the VPD and VPN Indexes, albeit only slightly higher. With the exception of the Barclays U.S. Aggregate bond index, the traditional indexes exhibited generally similar standard deviations to those of the VPD and VPN indexes. From a risk-adjusted return perspective, the VPD and VPN indexes had similar Sharpe ratios to the S&P 500, but far lower than the Barclays U.S. Aggregate bond index, although the alphas of the VPD and VPN indexes were only surpassed by that of the Barclays U.S. Aggregate bond index (although only the Barclays Aggregate Index alpha is statistically significant).

**Exhibit 22: VPD and VPN Indexes and Benchmark Indexes from 2006 to 2018**
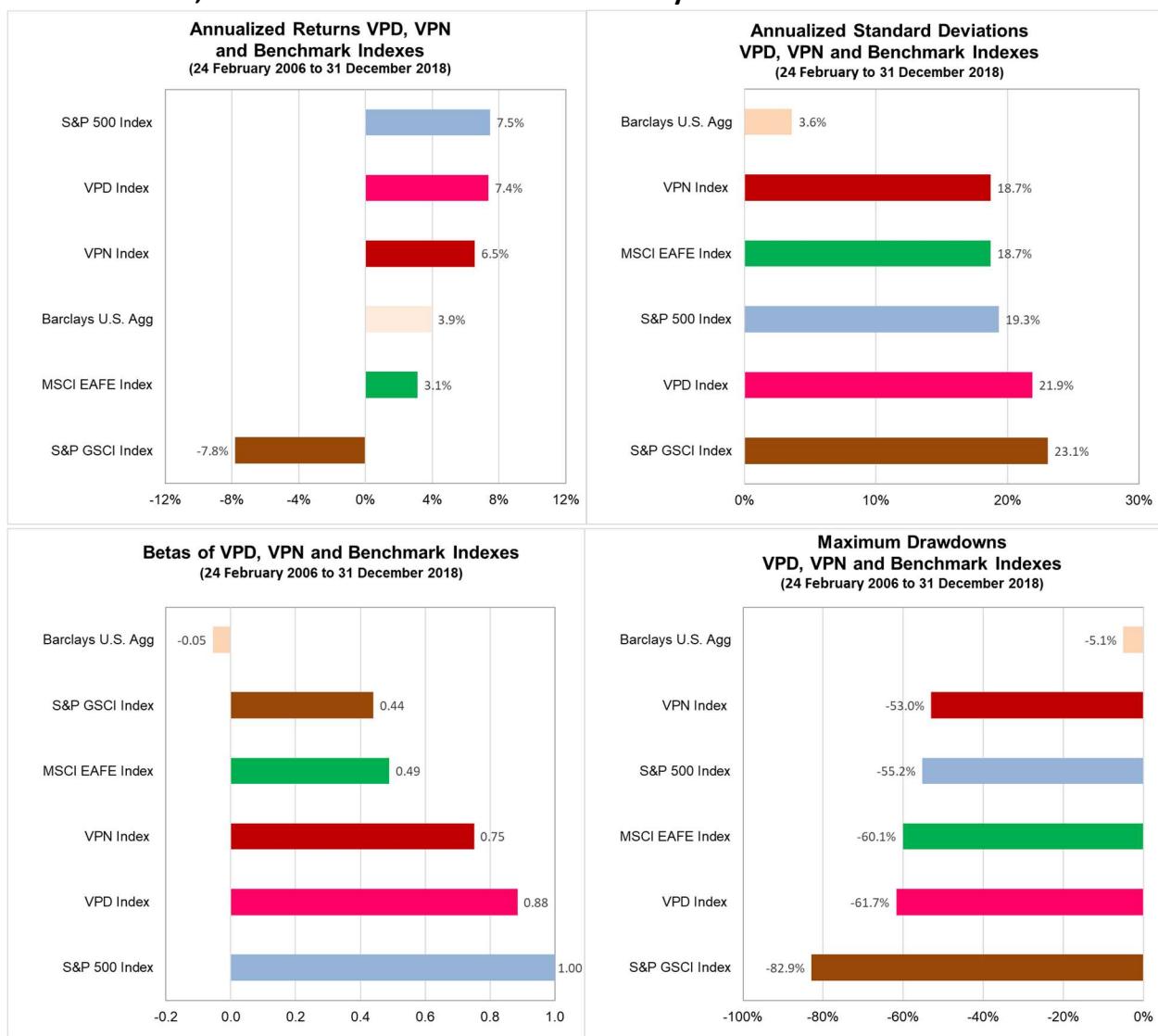
24 Feb 2006 to 31 Dec 2018	VPD Index	VPN Index	S&P 500 Index	Barclays U.S. Agg	S&P GSCI Index	MSCI EAFE Index
Annualized Return	7.4%	6.5%	7.5%	3.9%	-7.8%	3.1%
Annualized Standard Deviation	21.9%	18.7%	19.3%	3.6%	23.1%	18.7%
Sharpe Ratio	0.33	0.34	0.38	1.06	-0.34	0.16
Maximum Drawdown	-61.7%	-53.0%	-55.2%	-5.1%	-82.9%	-60.1%
Skewness	-1.26	-1.29	-0.12	-0.09	-0.18	-0.27
Kurtosis	36.22	26.83	11.38	1.96	3.11	7.72
Correlation w SPX	0.78	0.78	1.00	-0.29	0.37	0.51
Beta	0.88	0.75	1.00	-0.05	0.44	0.49
Alpha	1.51%	1.31%		4.47%	-9.61%	0.39%
Alpha P-Value	0.692	0.692		0.000	0.109	0.932

Source: Cboe Exchange Inc., Bloomberg

Exhibit 22: *This exhibit illustrates the performance of the VPD and VPN Indexes as well as a variety of traditional benchmark indexes from the inception of the VPD and VPN Indexes to the end of 2018. The VPD and VPN Indexes exhibited similar volatility to that of the traditional indexes (with the exception of the Barclays U.S. Aggregate Bond Index) and higher returns than all indexes except for the S&P 500 index. Only the Barclays U.S. Aggregate Bond Index exhibited a significantly higher Sharpe ratio and alpha than the VPN Index.*

Exhibit 23 provides select summary statistics for the VPD and VPN indexes and traditional benchmark indexes from the inception of the VPD and VPN indexes through 2018. The VPD and VPN indexes generated slightly lower returns than the S&P 500 index at similar levels of risk and similar Sharpe ratios while also generating positive alphas relative to the S&P 500 index.
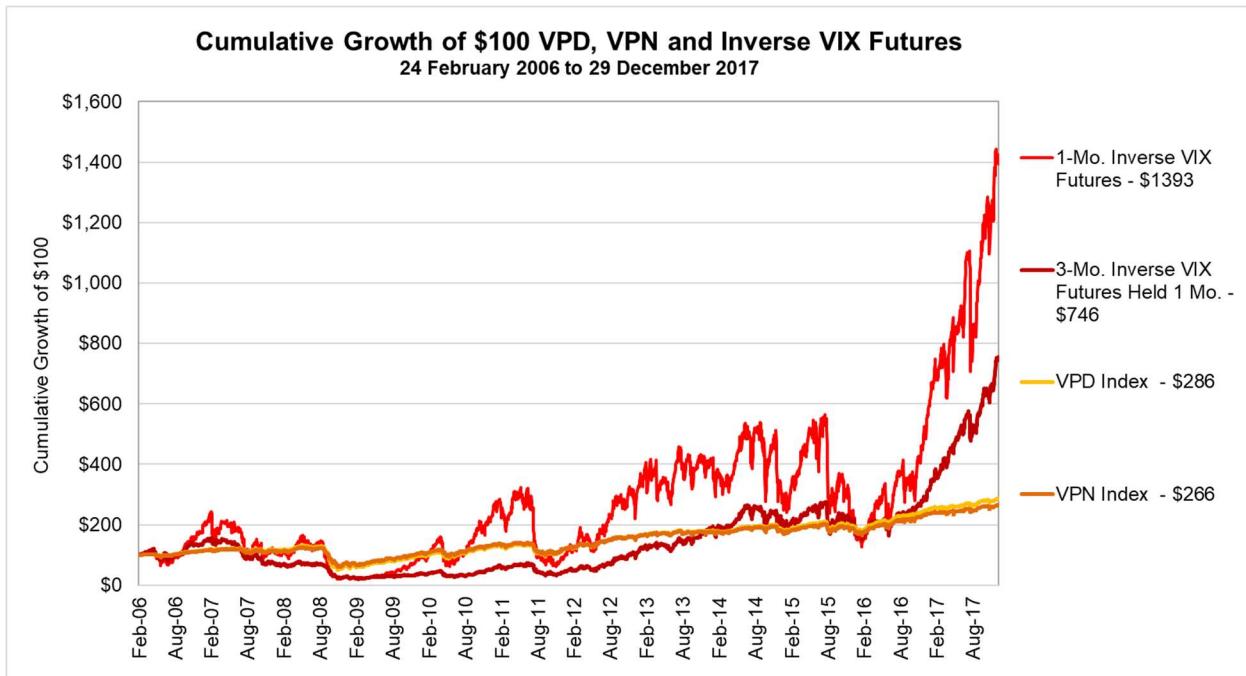
Exhibit 23: VPD, VPN and Benchmark Indexes Summary Statistics from 2006 to 2018

Annualized Returns VPD, VPN and Benchmark Indexes
(24 February 2006 to 31 December 2018)

- S&P 500 Index: 7.5%
- VPD Index: 7.4%
- VPN Index: 6.5%
- Barclays U.S. Agg: 3.9%
- MSCI EAFE Index: 3.1%
- S&P GSCI Index: -7.8%

Annualized Standard Deviations VPD, VPN and Benchmark Indexes
(24 February to 31 December 2018)

- Barclays U.S. Agg: 3.6%
- VPN Index: 18.7%
- MSCI EAFE Index: 18.7%
- S&P 500 Index: 19.3%
- VPD Index: 21.9%
- S&P GSCI Index: 23.1%

Betas of VPD, VPN and Benchmark Indexes
(24 February 2006 to 31 December 2018)

- Barclays U.S. Agg: -0.05
- S&P GSCI Index: 0.44
- MSCI EAFE Index: 0.49
- VPN Index: 0.75
- VPD Index: 0.88
- S&P 500 Index: 1.00

Maximum Drawdowns VPD, VPN and Benchmark Indexes
(24 February 2006 to 31 December 2018)

- Barclays U.S. Agg: -5.1%
- VPN Index: -53.0%
- S&P 500 Index: -55.2%
- MSCI EAFE Index: -60.1%
- VPD Index: -61.7%
- S&P GSCI Index: -82.9%

Source: Bloomberg, CFE, Cboe Exchange Inc.

Exhibit 23: *This exhibit illustrates the performance of the VPD and VPN indexes as well as a variety of traditional benchmark indexes from the inception of VIX options on 24 February 2006 to the end of 2018. The VPD and VPN indexes exhibited similar volatility to the traditional indexes (with the exception of the Barclays U.S. Aggregate Bond index) and higher returns than all indexes except for the S&P 500 index. The VPN Index (that is collateralized and buys VIX calls) had less severe maximum drawdowns than the stock and commodity indexes.*

Exhibit 24: VPD and VPN Indexes and Inverse VIX Futures from 2006 to 2017

24 Feb 2006 to 29 Dec 2017	VPD Index	VPN Index	1-Mo. Inverse VIX Futures	3-Mo. Inverse VIX Futures Held 1 Mo.
Annualized Return	9.2%	8.5%	24.6%	18.2%
Annualized Standard Deviation	21.4%	17.8%	78.0%	43.6%
Sharpe Ratio	0.42	0.47	0.31	0.42
Maximum Drawdown	-61.7%	-53.0%	-92.3%	-86.5%
Skewness	-0.84	-0.72	-1.04	-0.64
Kurtosis	29.93	6.99	5.44	4.00
Correlation w SPX	0.80	0.80	0.74	0.74
Beta	0.87	0.73	2.94	1.66
Alpha	2.28%	2.39%	24.02%	9.69%
Alpha P-Value	0.542	0.441	0.118	0.254
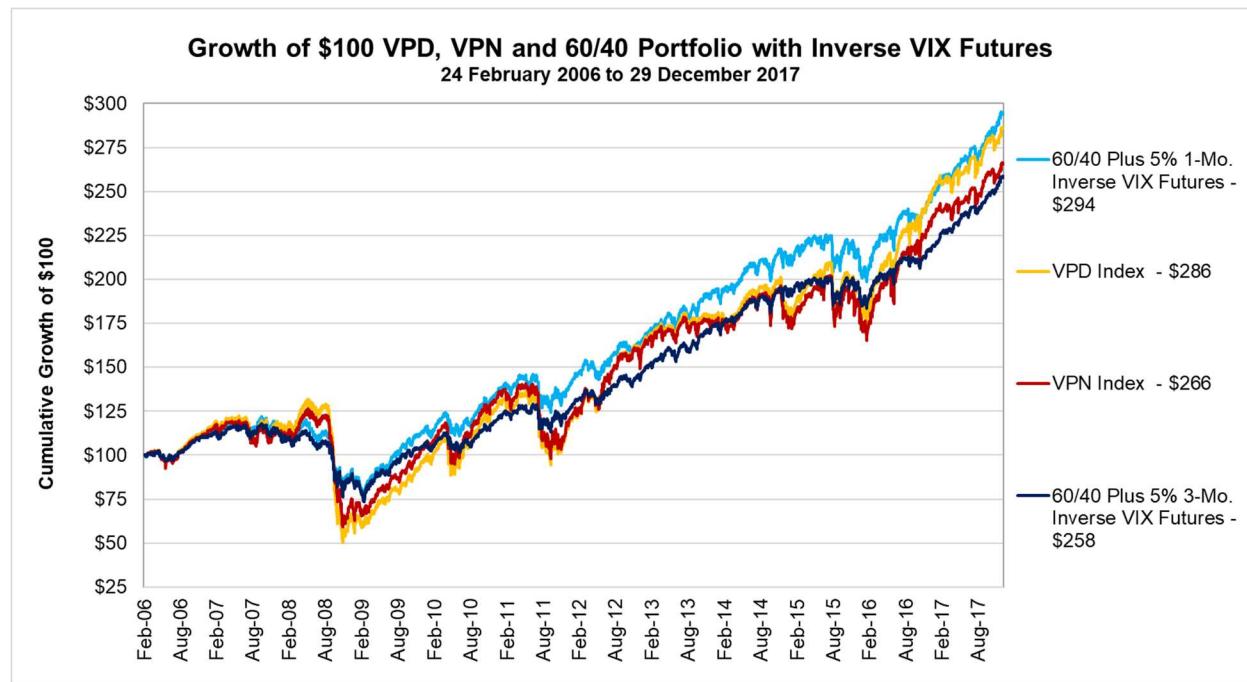
Source: Cboe Exchange Inc., Bloomberg

Exhibit 24: This exhibit illustrates the performance of the VPD and VPN Indexes as well as the one-month and three-month inverse VIX futures strategies. The risk-mitigating impact of the overcollateralization of the VPD and VPN indexes and the long call purchasing of the VPN indexes is clearly evident in the exhibit, as the standard deviations and maximum drawdowns for the VPD and VPN indexes were much lower than for the other two inverse VIX futures strategies.

Exhibit 24 provides details on the performance of the VPD and VPN Indexes relative to inverse one-month and three-month VIX futures strategies from 24 February 2006 to 29 December 2017. The de-levering and resulting reduced risk of the VPD and VPN indexes is clearly evident in the exhibit. The returns of the VPD and VPN Indexes are about half of that of the three-

month inverse VIX futures strategy and less than half of the return of the one-month inverse VIX futures strategy. Similarly, the standard deviations of the VPD and VPN Indexes are less than half of that of the three-month inverse VIX futures strategy and less than half of the return of the one-month inverse VIX futures strategy. Interestingly, the standard deviation of the VPD and VPN Indexes are about one quarter of the standard deviation of the one-month inverse VIX futures strategy (which matches the tenor of the VPD and VPN indexes).

Exhibit 25: VPD and VPN Indexes and 60/40 with Inverse VIX Futures from 2006 to 2017

24 Feb 2006 to 29 Dec 2017	VPD Index	VPN Index	60/40 Plus 5% 1-Mo. Inverse VIX Futures	60/40 Plus 5% 3-Mo. Inverse VIX Futures
Annualized Return	9.2%	8.5%	9.4%	8.2%
Annualized Standard Deviation	21.4%	17.8%	13.7%	12.4%
Sharpe Ratio	0.42	0.47	0.68	0.65
Maximum Drawdown	-61.7%	-53.0%	-38.5%	-37.7%
Skewness	-0.84	-0.72	-0.40	-0.27
Kurtosis	29.93	6.99	7.42	9.42
Correlation w SPX	0.80	0.80	0.98	0.99
Beta	0.87	0.73	0.69	0.63
Alpha	2.28%	2.39%	3.04%	2.36%
Alpha P-Value	0.542	0.441	0.000	0.000

Source: Bloomberg, CFE, Cboe Exchange Inc., Optionmetrics

Exhibit 25: This exhibit illustrates the performance of the VPD and VPN Indexes as well as four 60/40 stock/bond portfolios with allocations to inverse VIX futures. The VPD and VPN Indexes exhibited similar returns to the 60/40 portfolios with inverse VIX futures allocations at much higher volatility and maximum drawdowns.

Exhibit 25 details the performance of the VPD and VPN Indexes relative to a 60/40 portfolio with a 5% allocation to inverse VIX futures. The returns and standard deviations of the VPD and VPN Indexes are much nearer those of the 60/40 portfolios with 5% allocations to inverse VIX futures than to those of the inverse VIX futures strategies presented in Exhibit 24.
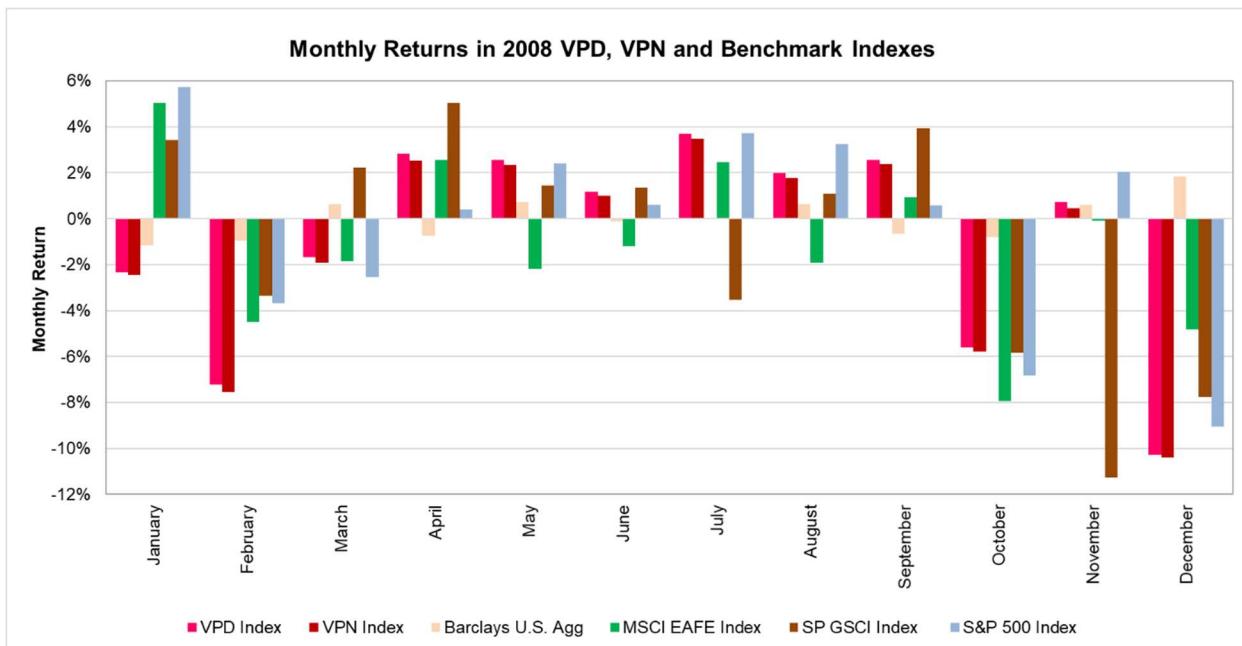
The VPD and VPN indexes generated virtually identical returns to the 60/40 portfolios with one- and three-month inverse allocations at 9.2% and 8.5%, versus 9.4% and 8.2%, respectively. However, they did so at much higher standard deviations of 21% and 18% versus 14% and 12%, respectively. A similar pattern is found in the maximum drawdowns. While the VPD and VPN indexes generated much lower Sharpe ratios than the 60/40-based portfolios, the alphas were generally quite similar. It is worth noting that the 60/40-based portfolios benefit from the diversification of the 40% allocation to bonds. Adding such an allocation to a VPD and VPN index-based portfolio may improve risk-adjusted performance accordingly.

6.2 Performance of the VPD and VPN Indexes in 2018

The VIX index was near record low levels near the end of 2017. In the first half of 2018, the VIX index experienced higher levels and a very sharp spike in the month of February. In the following sections, we consider the performance of the VPD and VPN Indexes in 2018 followed by performance in February 2018.

Exhibit 26 provides monthly returns in 2018 for the VPD and VPN Indexes as well as select benchmark indexes. The VPD and VPN Indexes exhibited significant drawdowns in February and December as the spot VIX index spiked. Despite the large spikes in the spot VIX index in 2018, the volatility of the VPD and VPN Indexes was generally similar to that of the S&P 500 index.

Exhibit 26: VPD and VPN Indexes and Benchmark Indexes Monthly Returns in 2018

2018 Monthly Returns	VPD Index	VPN Index	Barclays U.S. Agg	MSCI EAFE Index	SP GSCI Index	S&P 500 Index
January	-2.33%	-2.47%	-1.15%	5.03%	3.42%	5.72%
February	-7.21%	-7.54%	-0.95%	-4.48%	-3.34%	-3.69%
March	-1.67%	-1.92%	0.64%	-1.84%	2.22%	-2.54%
April	2.83%	2.52%	-0.74%	2.55%	5.04%	0.38%
May	2.57%	2.35%	0.71%	-2.17%	1.45%	2.41%
June	1.19%	0.98%	-0.12%	-1.18%	1.36%	0.62%
July	3.68%	3.48%	0.02%	2.48%	-3.53%	3.72%
August	1.99%	1.77%	0.64%	-1.90%	1.08%	3.26%
September	2.54%	2.37%	-0.64%	0.92%	3.93%	0.57%
October	-5.59%	-5.77%	-0.79%	-7.95%	-5.84%	-6.84%
November	0.74%	0.46%	0.60%	-0.10%	-11.28%	2.04%
December	-10.29%	-10.41%	1.84%	-4.82%	-7.75%	-9.03%

Source: Cboe Exchange Inc., Bloomberg
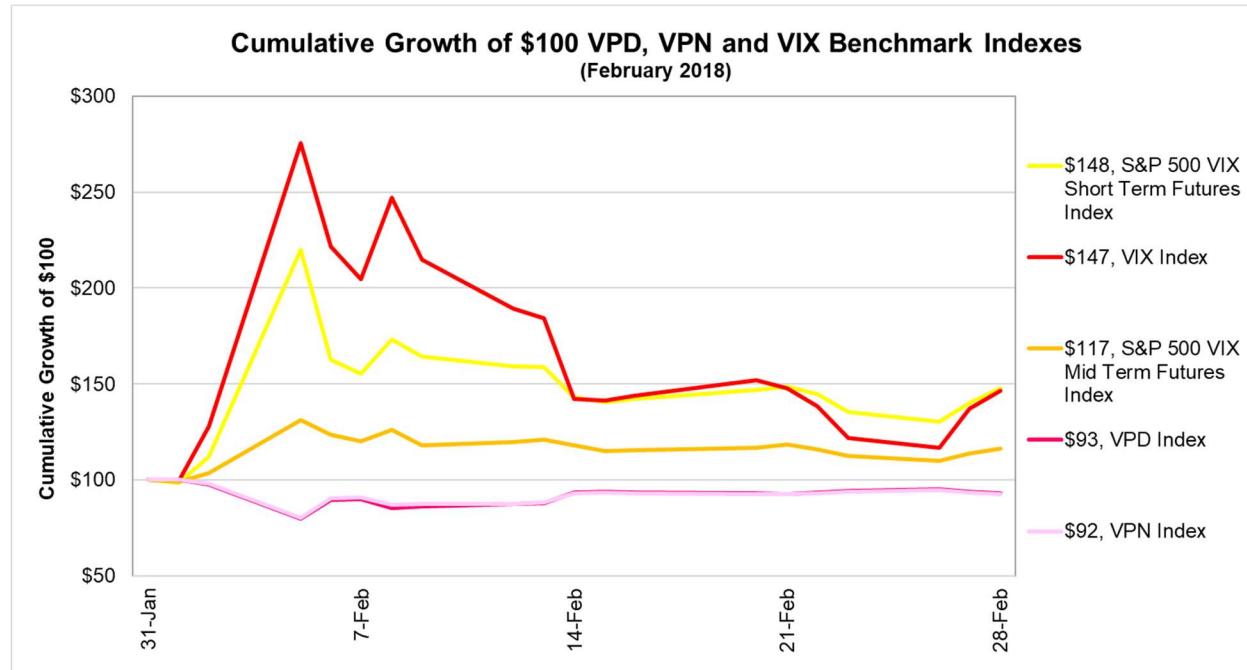
Exhibit 26: This exhibit illustrates the monthly performance of the VPD and VPN Indexes as well as a variety of traditional benchmark indexes in 2018. The VPD and VPN Indexes increased in every month from April through September, but exhibited significant drawdowns in February and December. In 2018, the volatility of the VPD and VPN Indexes was generally similar to that of the S&P 500 index.

6.3 The Volatility Spike of February 2018

Exhibit 27 illustrates the performance of the VPD and VPN Indexes as well the spot VIX index and two VIX futures indexes over the month of February 2018. The February 5, 2018 spike in the VIX index is clearly visible. On that day, the VIX index had its largest ever daily return of over 100%. While the spot VIX index level more than doubled in a single day, the increases in the VIX futures indexes were far less extreme while the VPD and VPN Indexes experienced a meaningful

decrease in value. However, the VPD and VPN Indexes experienced a decrease in value of far smaller magnitude than the increase in front-month VIX futures, largely due to their de-levered and risk-mitigating positions.

Exhibit 27: VPD and VPN Indexes and VIX Futures Index Performance in February 2018
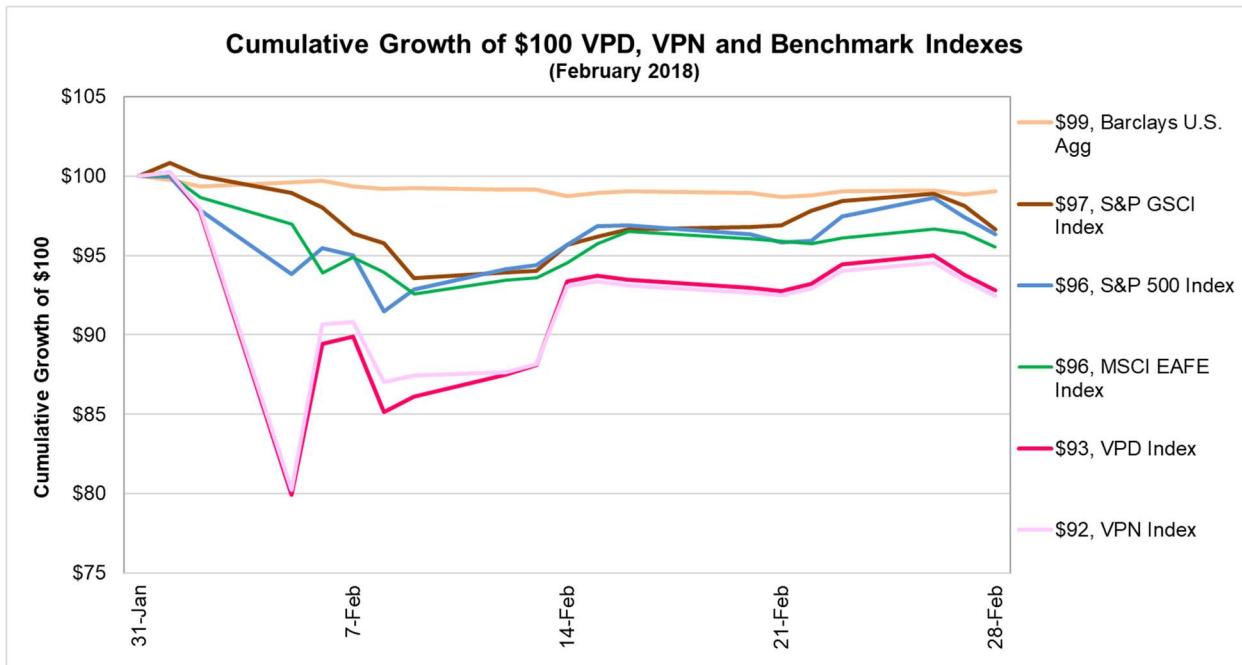

February 2018	VPD Index	VPN Index	S&P 500 VIX Mid Term Futures Index	S&P 500 VIX Short Term Futures Index
Period Return	-7.2%	-7.5%	16.5%	47.6%
Annualized Standard Deviation	88.2%	88.1%	110.1%	379.1%
Sharpe Ratio	-0.08	-0.09	0.15	0.13
Maximum Drawdown	-20.2%	-20.0%	-16.0%	-40.8%
Skewness	-1.44	-1.17	2.94	3.42
Kurtosis	7.06	7.37	10.90	13.85
Correlation w SPX	0.81	0.76	-0.80	-0.75
Beta	2.61	2.47	-3.24	-10.49
Period Alpha	4.55%	3.71%	7.93%	38.40%
Alpha P-Value	0.763	0.821	0.676	0.594

Source: Cboe Exchange Inc., Bloomberg

Exhibit 27: *This exhibit illustrates the performance of the VPD and VPN Indexes as well the spot VIX index and two VIX futures indexes over the month of February 2018. The February 5, 2018 spike in the VIX index is clearly visible in which the VIX index had its largest ever daily change of over 100%. While the spot VIX index level more than doubled in a single day, the increases in the VIX futures indexes were far less extreme. Due to their de-levered and risk-mitigating positions, the VPD and VPN Indexes experienced a decrease in value of far smaller magnitude than the increase in front-month VIX futures.*

Exhibit 28: VPD, VPN and Benchmark Performance in February 2018

February 2018	VPD Index	VPN Index	S&P 500 VIX Mid Term Futures Index	S&P 500 VIX Short Term Futures Index	S&P 500 Index	Barclays U.S. Agg	S&P GSCI Index	MSCI EAFE Index
Period Return	-7.2%	-7.5%	16.5%	47.6%	-3.7%	-0.9%	-3.3%	-4.5%
Annualized Standard Deviation	88.2%	88.1%	110.1%	379.1%	27.3%	3.5%	16.4%	18.4%
Sharpe Ratio	-0.08	-0.09	0.15	0.13	-0.14	-0.31	-0.21	-0.25
Maximum Drawdown	-20.2%	-20.0%	-16.0%	-40.8%	-8.5%	-1.3%	-7.2%	-7.5%
Skewness	-1.44	-1.17	2.94	3.42	-1.02	-0.24	-0.33	-0.88
Kurtosis	7.06	7.37	10.90	13.85	0.56	-1.15	-0.43	0.73
Correlation w SPX	0.81	0.76	-0.80	-0.75	1.00	0.08	0.35	0.32
Beta	2.61	2.47	-3.24	-10.49	1.00	0.01	0.21	0.22
Period Alpha	4.55%	3.71%	7.93%	38.40%		-0.91%	-2.56%	-3.69%
Alpha P-Value	0.763	0.821	0.676	0.594		0.370	0.565	0.466

Source: Cboe Exchange Inc., Bloomberg

Exhibit 28: This exhibit illustrates the performance of the VPD and VPN Indexes as well four traditional benchmark indexes over the month of February 2018. It is clear that the February 5, 2018 spike in the VIX index did not benefit the VPD and VPN Indexes in this case as the VPD and VPN Indexes exactly tracks the S&P 500 index through the first half of the month. This is illustrative of one of the limits of market expectation-based dynamic allocation strategies in which unanticipated events such as the February 5, 2018 spike in the VIX index may not be captured since they are not expected.

Exhibit 28 illustrates the performance of the VPD and VPN Indexes as well four traditional benchmark indexes over the month of February 2018. It is clear that the 5 February 2018 spike in the VIX index resulted in a meaningful drawdown (-20% maximum drawdown from the start of February to February 5th) in the VPD and VPN indexes, although the drawdown was of far
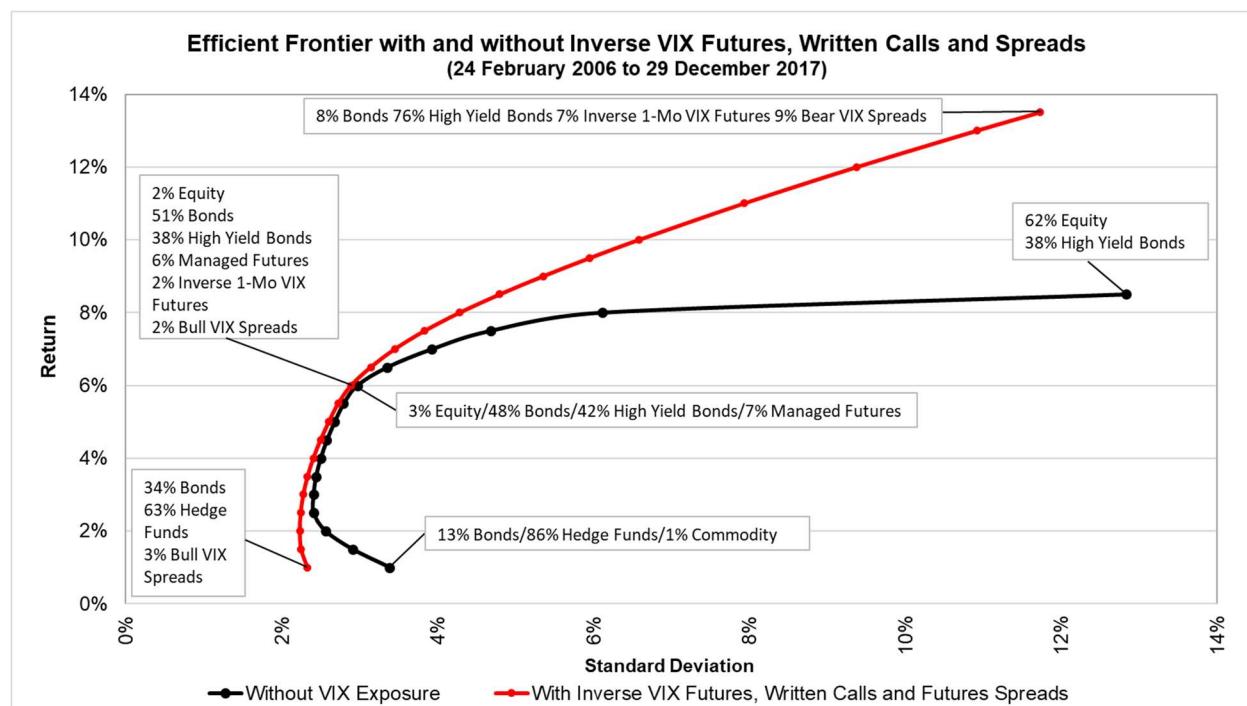
smaller magnitude than the increase in the VIX index. The -20% drawdown over the course of 5 calendar days was certainly economically meaningful, however, it was only 1/5 the size of the one-day 5th of February 2018 increase in the spot VIX index. This difference clearly illustrates the impact of the reduced leverage of the VPD and VPN Indexes. The size of the one-day loss of the VPD and VPN indexes (-18%) may exclude them from consideration as standalone investments for most investors and suggest that they may be more appropriate as return enhancers or diversifiers in well diversified portfolios.

7.0 Return Enhancement and Risk Reduction Benefits – Expansion of Efficient Frontier

Much of the discussion in the previous sections focused on the return enhancement benefits of inverse VIX futures and written VIX calls. This section approaches the analysis from a slightly different perspective by considering impacts on efficient frontiers over the entire 10+ year period since the inception of VIX options. Exhibit 29 provides efficient frontiers with and without the inclusion of inverse VIX exposure. The efficient frontiers are constructed by considering long-only exposure to the following ten total return indexes representing a wide range of asset classes: SP500 (Equity), Barclays U.S. Aggregate (Bonds), Barclays High Yield U.S. Corporate (High Yield Bonds), HFR Global Hedge Fund Index (Hedge Funds), SG CTA Index, (formerly Newedge CTA Index) (Managed Futures), S&P GSCI (Commodity), S&P Listed Private Equity Index (Private Equity), Dow Jones Equity REIT Index (Real Estate), MSCI EAFE Index USD (EAFE), and MSCI EEM Index USD (Emerging Markets), with or without the inclusion of inverse VIX futures, written VIX call options and bull or bear VIX futures spreads. The exhibit presents two efficient frontiers; one calculated by considering inclusion of the ten indexes listed above and the second calculated by considering inclusion of the ten indexes plus the indexes representing the VIX-based trading strategies discussed in the previous sections of the paper (inverse one-month and long three-month VIX futures, one-month VIX ATM and 25% OTM call options and bull or bear VIX one-month/three-month futures spreads).

It is clear that the addition of inverse VIX futures, written VIX call option and VIX futures spread exposure expanded the efficient frontier across the full range of returns and standard deviations, illustrating possible return enhancement abilities of inverse VIX exposures. Labels are provided on the efficient frontiers to indicate the approximate asset allocations at three points on each frontier. For the sake of clarity, any allocations below 1% are not noted in the labels but are included in the calculations. It is worth noting that the addition of bull and bear VIX futures spreads to the efficient frontier calculations effectively allows the inclusion of long VIX futures exposures through the spreads thus potentially expanding the efficient frontier beyond a frontier that only allows inverse VIX exposure.

Exhibit 29: Efficient Frontier with and without Inverse VIX Futures, Written Calls and Spreads

Efficient Frontier with and without Inverse VIX Futures, Written Calls and Spreads
(24 February 2006 to 29 December 2017)

Labels within chart:
- 8% Bonds 76% High Yield Bonds 7% Inverse 1-Mo VIX Futures 9% Bear VIX Spreads
- 2% Equity / 51% Bonds / 38% High Yield Bonds / 6% Managed Futures / 2% Inverse 1-Mo VIX Futures / 2% Bull VIX Spreads
- 62% Equity / 38% High Yield Bonds
- 3% Equity/48% Bonds/42% High Yield Bonds/7% Managed Futures
- 34% Bonds / 63% Hedge Funds / 3% Bull VIX Spreads
- 13% Bonds/86% Hedge Funds/1% Commodity

Axis: Return (vertical), Standard Deviation (horizontal)

Legend: Without VIX Exposure — With Inverse VIX Futures, Written Calls and Futures Spreads

Source: Bloomberg, CFE, Cboe Exchange Inc.

Exhibit 29: *This exhibit illustrates the diversification benefits of including inverse VIX exposure to a sample universe of diversified investments in the construction of an efficient frontier. The black efficient frontier illustrates an efficient frontier optimized across ten asset classes (Equity, Bonds, High Yield Bonds, Hedge Funds, Managed Futures, Commodities, Private Equity, Real Estate, EAFE, Emerging Markets) without considering the inclusion of VIX exposure and without allowing short positions. The red efficient frontiers optimizes the portfolio allocations across the ten asset classes as above plus 1-month and 3-month inverse VIX futures, written VIX ATM and OTM calls, and bull and bear 1-month/3-month VIX futures spreads. The efficient frontier depicts the ex post optimum portfolios that are constructed to maximize returns for each level of standard deviation and minimize standard deviation for each level of returns. The addition of inverse VIX exposure expands the upper end and lower end of the efficient frontier. The approximate allocations are provided in the labels in the exhibit. Any allocations below 1% are not reported in the labels, but are included in the calculations. For a target portfolio return of 1%, the addition of inverse VIX exposure reduced standard deviation by about one percentage point. Similarly, at a target portfolio return of 8%, the addition of inverse VIX exposure reduced standard deviation by about two percentage points. It is worth noting that the efficient frontier depicts optimal portfolios that are constructed with perfect hindsight considering only return and standard deviation, with no consideration of the appropriateness of these portfolios. For example, a 3% Equity, 48% Bonds, 42% High Yield Bonds, 7% Managed Futures portfolio generated a 6% return at a standard deviation of 2.97%. Of all possible portfolios that could have been constructed using the 10 indexes considered in the black efficient frontier, no portfolio could have generated a 6% return at a standard deviation below 2.97%. It is worth noting that it would be unrealistic to assume that any individual would have the foresight to have chosen one of the portfolios on the frontier ex ante.*

It should be emphasized that the efficient frontier depicts optimal portfolios that are constructed with perfect hindsight considering only return and standard deviation, with no consideration of the appropriateness of these portfolios. For example, a 3% Equity, 48% Bonds, 42% High Yield Bonds, 7% Managed Futures portfolio generated a 6% return at a standard deviation of 2.97%. Of all possible portfolios that could have been constructed using the 10 indexes considered in the black efficient frontier, no portfolio could have generated a 6% return at a standard deviation below 2.97%. It would be unrealistic to assume that any individual would have the foresight to have chosen one of the portfolios on the frontier ex ante.

Exhibit 30: Efficient Frontier Asset Allocation without Inverse VIX Exposure
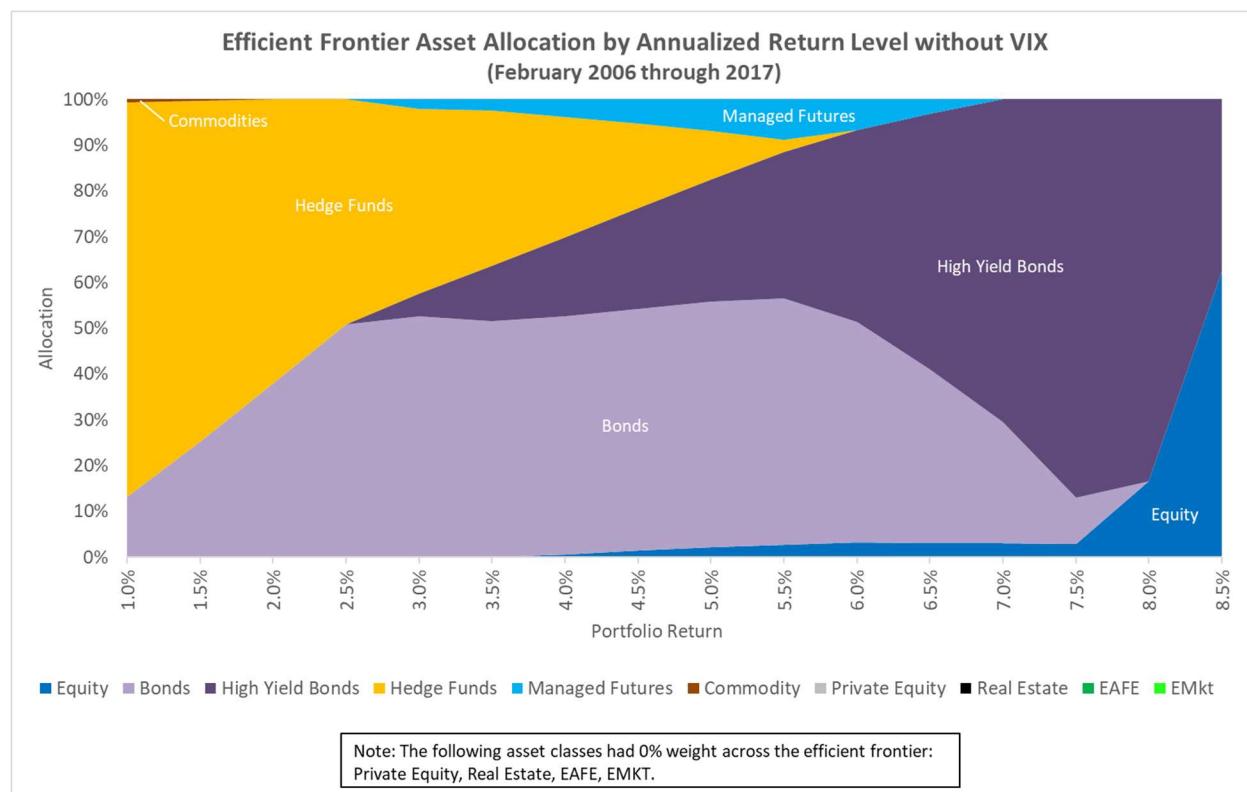
Exhibit 30: *This exhibit illustrates the asset allocations at each target return level represented in the black efficient frontier in Exhibit 29 (without VIX exposure). An efficient frontier depicts the ex post optimum portfolios that are constructed to maximize returns for each level of standard deviation and minimize standard deviation for each level of returns. For example, of all possible portfolios, the portfolio with a 6% return that had the lowest standard deviation was approximately 3% Equity, 48% Bonds, 42% High Yield Bonds, and 7% Managed Futures. All ten indexes which comprise the sample theoretical endowment portfolio were considered in the optimization and short positions were not allowed. It is worth noting that the following asset classes had zero weights across the entire frontier: Private Equity, Real Estate, EAFE, and Emerging Markets.*

Exhibit 31: Efficient Frontier Asset Allocation with Inverse VIX Futures, Written VIX Calls, and Bull and Bear VIX Futures Spreads
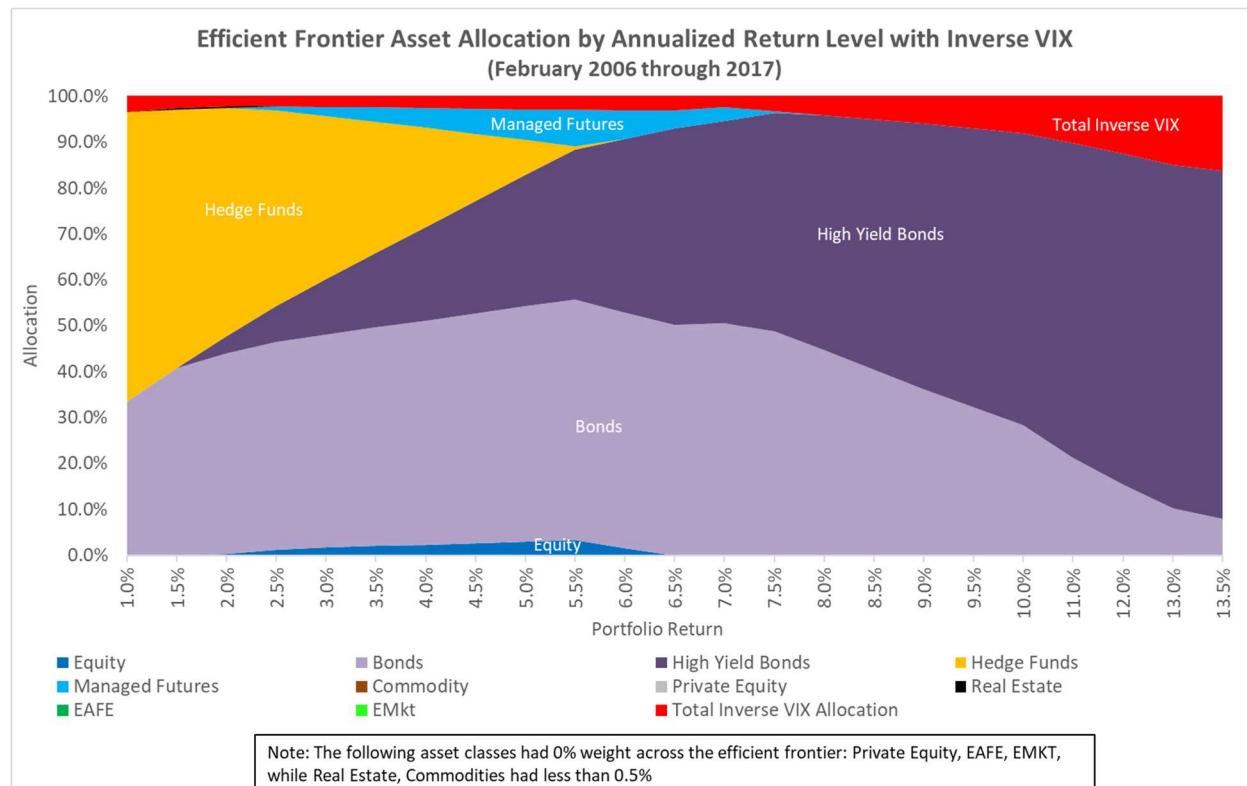
Exhibit 31: This exhibit illustrates the asset allocations at each target return level represented in the red efficient frontier in Exhibit 29 (with inverse VIX exposure). An efficient frontier depicts the ex post optimum portfolios that are constructed to maximize returns for each level of standard deviation and minimize standard deviation for each level of returns. For example, of all possible portfolios, the portfolio with a 6% return that had the lowest standard deviation was approximately 2% Equity, 51% Bonds, 38% High Yield Bonds, 6% Managed Futures, 2% Inverse 1-Mo VIX Futures, and 2% Bull VIX Spreads. All ten indexes which comprise the sample theoretical endowment portfolio were considered in the optimization as well as the inverse VIX future, written VIX call and VIX futures spread strategies used previously in the paper, and short index positions were not permitted. It is worth noting that the following asset classes had zero weights across the entire frontier: Private Equity, EAFE, and Emerging Markets, while Real Estate, and Commodities had allocations below 0.5% across the frontier.

Exhibits 30 and 31 detail the asset allocations underlying the efficient frontiers illustrated in Exhibit 29 at each target portfolio return level (noted on the x-axis). The results displayed in the exhibits reinforce the idea that an ex post efficient frontier is, by its very nature, more of an academic exercise than a realistic representation of the performance that could have been expected of any particular portfolio manager. As previously mentioned, efficient frontiers are created with perfect hindsight, utilizing optimization techniques, and thus represent the best possible return/standard deviation combinations that were possible over the time period and based on the investments included in the optimization procedure. For example, at a 1% target return, the optimal portfolio consisted of 34% bonds, 63% hedge funds and 3% bull VIX futures

spreads. It questionable whether a typical portfolio manager would have expected this portfolio to be an optimum portfolio (at that return level), or that they would have considered this an appropriate portfolio for their clients.

8.0 Conclusions

The goal of this study is not to make a particular recommendation for a VIX-based investment strategy but rather to detail some important of characteristics of VIX futures and options investments and investigate the potential effectiveness of inverse VIX futures or written VIX calls as return enhancers over short periods of rising equity markets. While the results indicate that such strategies may be effective over particular time periods, a large allocation to uncovered selling of VIX instruments may result in potentially substantial volatility jump risk exposure which can result in meaningful and potentially catastrophic losses. Two benchmark indexes that sell VIX futures and hold money market instruments, the VPD and VPN indexes, have achieved strong returns in the past with mitigated downside risk.

Select References

Alexander, C. and Korovilas, D., 2013. Volatility exchange-traded notes: Curse or cure? The Journal of Alternative Investments, 16(2), p.52.

Arditti, F.D. "Risk and the Required Return on Equity." Journal of Finance, Vol. 22, No. 1 (1967), pp. 19–36.

Bakshi, G., and N. Kapadia. "Delta-Hedged Gains and the Negative Market Volatility Risk Premium." Review of Financial Studies, Vol. 16, No. 2 (2003), pp. 527–566.

Black, K. "Improving Hedge Fund Risk Exposures by Hedging Equity Market Volatility, or How the VIX Ate My Kurtosis." Journal of Trading (Spring 2006), pp. 6–15.

Bollen, N.P., O'Neill, M.J. and Whaley, R.E., 2017. Tail wags dog: Intraday price discovery in VIX markets. Journal of Futures Markets, 37(5), pp.431-451.

Cboe Exchange, Inc.. ".Cboe VIX White Paper – Cboe Volatility Index" 2019. (Available at www.cboe.com/VIX.)

Daigler, R.T., and L. Rossi. "A Portfolio of Stocks and Volatility." Journal of Investing (Summer 2006), pp. 99–106.

Dash, S., and M.T. Moran. "VIX as a Companion for Hedge Fund Portfolios." Journal of Alternative Investments (Winter 2005), pp. 75–80.

Giot, P. "Relationships Between Implied Volatility Indexes and Stock Index Returns." The Journal of Portfolio Management (Spring 2005), 31(3), pp. 92-100.

Grant, M., K. Gregory, and J. Lui. "Considering All Options." Goldman Sachs Global Investment Research, August 14, 2007a.

——. "Volatility as an Asset.", Goldman Sachs Global Investment Research, November 15, 2007b.

Mencia, J. and Sentana, E., 2013. Valuation of VIX derivatives. Journal of Financial Economics, 108(2), pp.367-391.

Samuelson, Paul A. "Proof that properly anticipated prices fluctuate randomly," Industrial Management Review, 6(2):41–49, Spring 1965.

Spurgin, R. "How to Game Your Sharpe Ratio." Journal of Alternative Investments (Winter 2001), pp. 38–46.

Szado, Edward. "The Distinctive Characteristics of VIX Futures and Options," Working Paper, 2018a

——. "The Portfolio Diversification Potential of Long VIX Futures and Options," Working Paper, 2018c

——. "VIX Futures and Options – A Case Study of Portfolio Diversification During the 2008 Financial Crisis," Journal of Alternative Investments, Fall 2009, Vol. 12, No. 2, pp. 68-85

Toikka, M., E.K. Tom, S. Chadwick, and M. Bolt-Christmas. "Volatility as an Asset?" CSFB Equity Derivatives Strategy, February 26, 2004.

Ungar, J., and M.T. Moran, "The Cash-Secured Put-Write Strategy and Performance of Related Benchmark Indexes." Journal of Alternative Investments (Spring 2009), pp. 43–56.

Whaley, R.E. "Derivatives on Market Volatility: Hedging Tools Long Overdue." Journal of Derivatives, 1 (1993), pp. 71–84.

——. "The investor fear gauge." The Journal of Portfolio Management, (2000), 26(3), pp. 12-17.

——. "Understanding the VIX." Journal of Portfolio Management, 3 (2009), pp. 98–105.

——. "Trading volatility: At what cost?" Journal of Portfolio Management, (2013), 40(1), p.95.

Important Disclosures