

Problem Set on Matching Model, Wages, and Fluctuations

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Problem 1

Consider a matching model with a labor force of size H . The matching function is Cobb-Douglas:

$$m(U, V) = \omega \cdot U^\eta \cdot V^{1-\eta},$$

where U is the number of unemployed workers, V is the number of vacant jobs, and $\eta \in (0, 1)$ is the matching elasticity. All workers are paid at a minimum wage $w > 0$. Firms have a production function

$$y(N) = a \cdot N^\alpha,$$

where a governs labor productivity, N denotes the number of producers in the firm, and $\alpha \in (0, 1)$ indicates diminishing marginal returns to labor. Firms incur a recruiting cost of $r > 0$ recruiters per vacancy and face a job-destruction rate $s > 0$. The labor market tightness is $\theta = V/U$.

- A) Compute the job-finding rate $f(\theta)$ and vacancy-filling rate $q(\theta)$. Assuming that labor-market flows are balanced, compute the recruiter-producer ratio $\tau(\theta)$.
- B) Assuming that labor-market flows are balanced, compute labor supply $L^s(\theta, H)$.
- C) Firms choose employment to maximize flow profits:

$$y(N) - [1 + \tau(\theta)] \cdot w \cdot N.$$

Compute the labor demand $L^d(\theta, a, w)$ by solving this maximization problem.

- D) Give the equation that determines tightness in the model. Explain the origin of this equation.
- E) Using the equation that you have just obtained, compute the elasticity of tightness with respect to the minimum wage, ϵ_w^θ . Is the elasticity positive or negative? Discuss your finding in light of the empirical literature on the minimum wage.
- F) Now assume that productivity depends on the wage: $a(w) = \mu \cdot w^\beta$ with $\mu > 0$ and $\beta > 0$. Give possible reasons why $\beta > 0$. Recompute the elasticity of tightness with respect to the minimum wage under this new assumption. How does your answer compare with the answer you gave above?

Problem 2

Consider a large firm with $L(t)$ workers. There are two types of workers: $N(t)$ producers and $R(t)$ recruiters. All workers are paid a wage $w > 0$. The firm's production function is

$$y(t) = a \cdot N(t)^\alpha,$$

where a governs labor productivity and $\alpha \in (0, 1)$ indicates diminishing marginal returns to labor. The firm face a job-separation rate $s > 0$, so it must post $V(t)$ vacancies to replace the workers who are leaving. Each vacancy requires the attention of $r > 0$ recruiters and is filled at a rate $q > 0$. The parameters satisfy $s \times r < q$. The firm discounts future profits at rate $\delta > 0$. The firm maximizes the discounted sum of future profits, taking the initial number of workers $L(0)$ as given.

- A) Formulate the firm's problem using the number of producers $N(t)$ as control variable and the total number of employees $L(t)$ as state variable. Write down the associated current-value Hamiltonian.
- B) Write down the optimality conditions for the firm's problem, and use them to derive the differential equation governing the optimal number of producers. The equation should be a first-order nonlinear differential equation involving $\dot{N}(t)$, $N(t)$, and parameters.
- C) Compute the critical point of the dynamical system composed of the differential equations governing the evolution of $L(t)$ and $N(t)$ over time. How does the critical point relate to the labor demand computed in lecture? Under which conditions do they overlap?
- D) Plot the phase diagram describing the evolution of employment in the firm. (The phase diagram should have $L(t)$ on the x-axis and $N(t)$ on the y-axis.) Does the phase diagram represent a sink, a source, or a saddle? Display the trajectory of employment for a given initial condition.