Automata Formal Languages & Logic

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Unit 3

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A context-free grammar is in **Greibach normal form** (GNF) if the right-hand sides of all production rules start with a terminal symbol, optionally followed by some variables.

All productions have form:

$$A \to \underbrace{a}_{\text{symbol}} \underbrace{V_1 V_2 \cdots V_k}_{\text{variables}} \qquad k \geq 0$$

https://en.wikipedia.org/wiki/Greibach_normal_form
https://www.cs.colostate.edu/~massey/Teaching/cs301/RestrictedAccess/Slides/301lecture10.pdf

Examples:

$$S \to cAB$$
$$A \to aA \mid bB \mid b$$
$$B \to b$$

Greibach Normal Form

$$S \to abSb$$
$$S \to aa$$

Not Greibach Normal Form

Steps to convert a CFG to Greibach normal form (GNF) :

- Eliminate lambda, unit and useless productions (in the sequence mentioned).
- We attempt to show the conversion using basic examples in a brief manner (direct conversion).
- Conversion is a little tedious if we go by the algorithm (**skipped**)

Note : Conversion of a CFG to Greibach normal form (GNF) :

If **$G = (V, T, P, S)$** is a CFG then,

we can construct another CFG **$G_1 = (V_1, T, P_1, S)$ in GNF** such that,

$$L(G_1) = L(G) - \{\lambda\}$$

For every context-free grammar G where **$\lambda \notin L(G)$**, there exists an equivalent grammar in GNF.

However, a non-strict forms makes an exception to this format restriction for allowing the empty word (λ) to be a member of the described language ($S \rightarrow \lambda$).

http://www.iitg.ac.in/gkd/ma513/oct/oct18/note.pdf
https://en.wikipedia.org/wiki/Greibach_normal_form

Conversion to Greibach Normal Form:

$$S \to abSb$$
$$S \to aa$$

Not Greibach Normal Form

$$S \to aBSB$$
$$B \to b$$
$$S \to aA$$
$$A \to a$$

Greibach Normal Form

Example 1:

$G : S \rightarrow aSa \mid bSb \mid SS \mid \lambda$

Solution :

We get $L(G_1) = L(G) - \{\lambda\}$ in a stricter Form

The grammar G_1 in GNF is

1) Eliminate Lambda Production:

$G_1 : S \rightarrow aSa \mid bSb \mid S \mid SS \mid aa \mid bb$

2) Eliminate Unit Production ($S \rightarrow S$)

$G_1 : S \rightarrow aSa \mid bSb \mid SS \mid aa \mid bb$

3) No useless production

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Example 1 (continued):

G : S $\rightarrow$ aSa | bSb | SS | λ

Solution :

We get $L(G_1) = L(G) - \{\lambda\}$ in a stricter Form

4) Convert to GNF

Step 1 (Ensure RHS of each productions starts with a Terminal):

S $\rightarrow$ aSa | bSb | aSaS | bSbS | aaS | bbS | aa | bb λ

Step 2 (conversion to GNF):

S $\rightarrow$ aSA | bSB | aSAS | bSBS | aAS | bBS | aA | bB | λ

A $\rightarrow$ a

B $\rightarrow$ b

Example 2:

$S \rightarrow XY \mid Xn \mid p$

$X \rightarrow mX \mid m$

$Y \rightarrow Wn \mid o$

Solution :

- There are no Lambda and Unit productions.
- Variable Y is useless as it doesn’t derive a terminal, hence the grammar becomes :

$S \rightarrow Xn \mid p$

$X \rightarrow mX \mid m$

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Example 2:

S → XY | Xn | p

X → mX | m

Y → Wn| o

Solution :

The grammar in GNF is

Step 1:(Ensure RHS of each productions starts with a Terminal)

S → mXn | mn | p

X → mX | m

Step 2 :(conversion to GNF):

S → mXN | mN | p

N → n

X → mX | m

Example 3:

S->aA|bB

B->bB|λ

A->aA|λ

Solution :

Eliminate lambda Productions

S->aA|bB | a | b

B->bB|b

A->aA|a

No Unit and Useless Productions

Above grammar is already in GNF.

- Convert given CFG to GNF

- From the Start state(let's say q0) without consuming any input symbol, push the Start Symbol to Stack and transit to next state (let's say q1).

$$\delta(q0, \lambda, Z_0) = (q1, SZ_0)$$

- For every production of the form

$$A \longrightarrow a\alpha$$

  add the transition :

$$\delta(q1, a, A) = (q1, \alpha)$$

- Finally add the following transition :

$$\delta(q1, \lambda, Z_0) = (qf, Z_0)$$

  where, qf is the final state.

❖ PDA constructed using above method will always have only 3 states.
❖ Stack contents are the Variables in the sentential form of corresponding string derivation by the grammar.

Example 1:

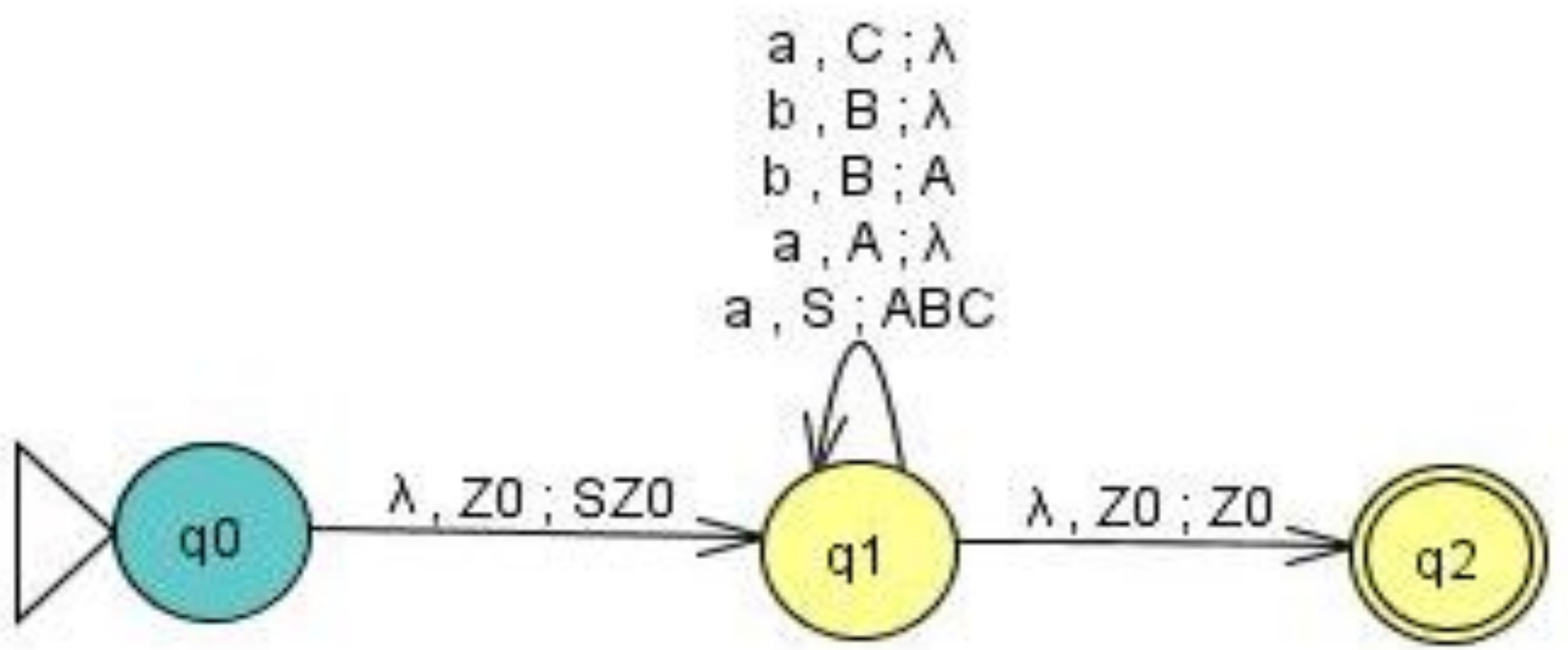
S -> aABC

A -> aB | a

B -> bA | b

C -> a

Solution :



Example 1:

S -> aABC

A -> aB | a

B -> bA | b

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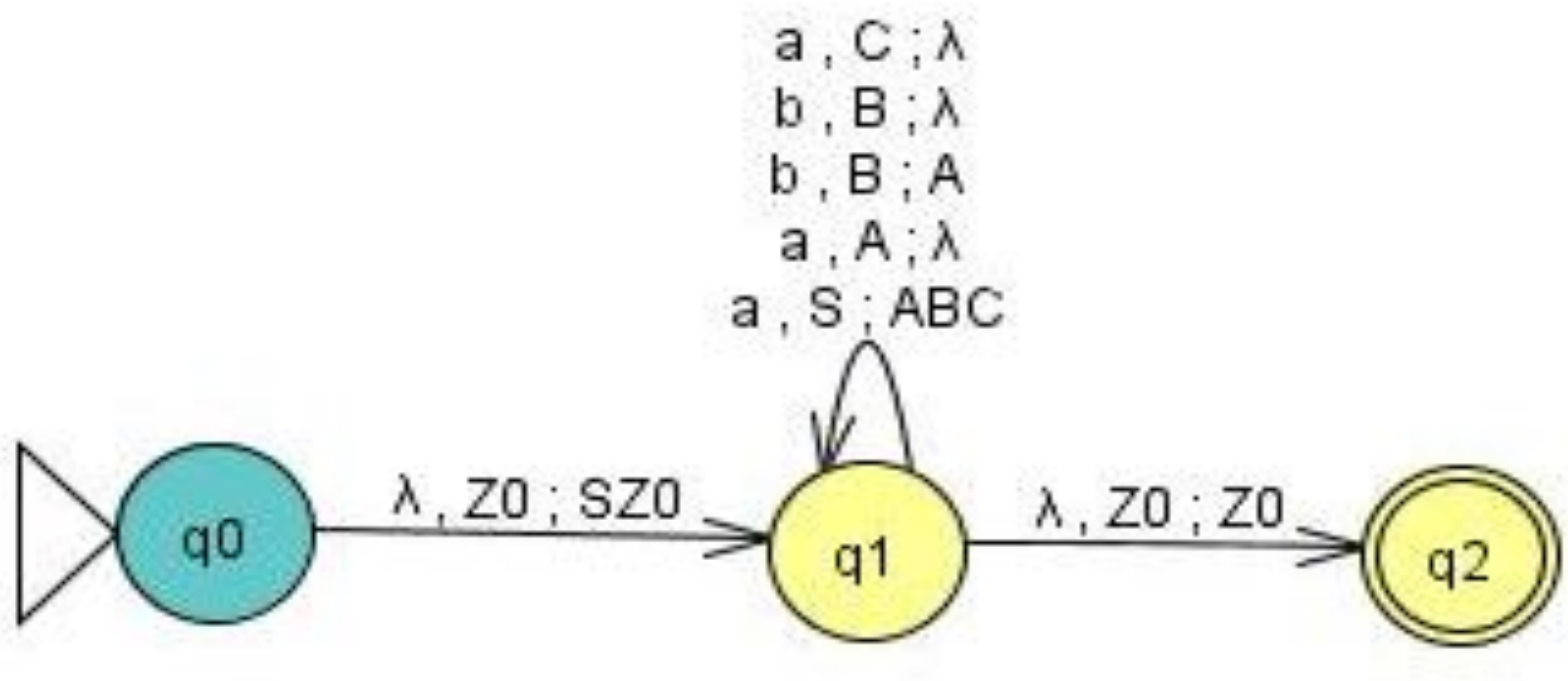


Trace the string "aababa" on the above PDA.

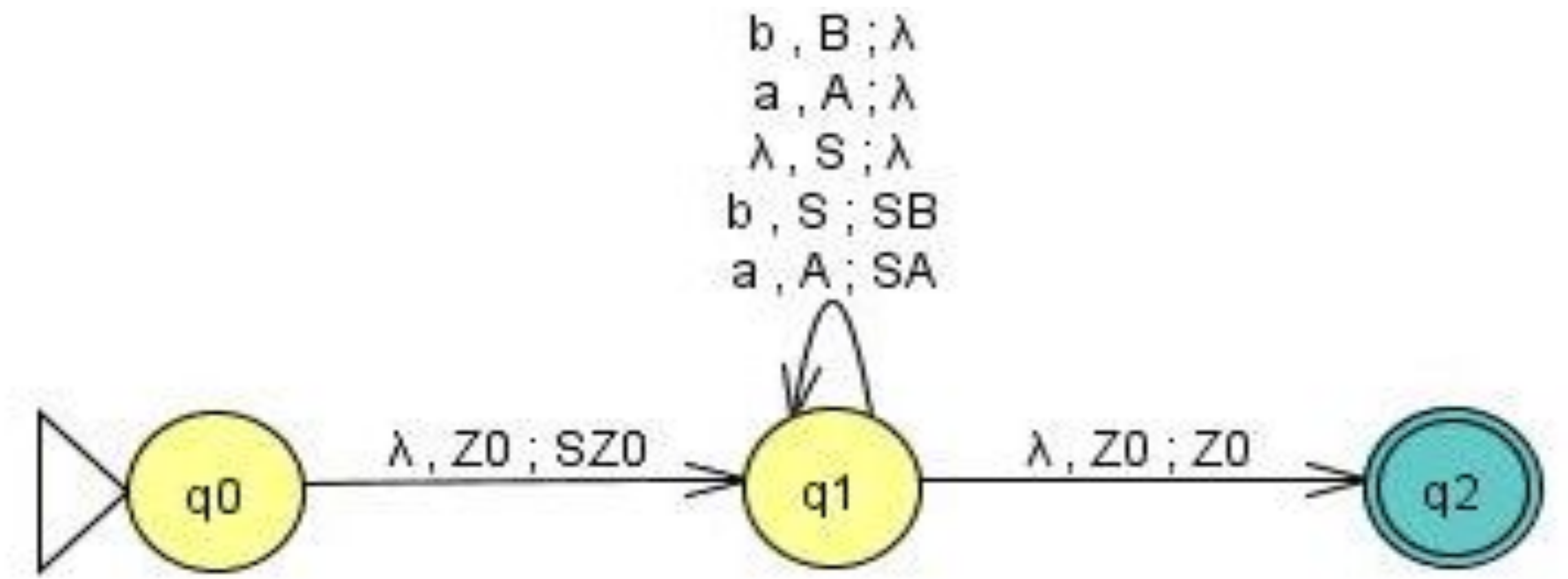
Example 2:

S -> aSA | bSB

A -> a

B -> b

Soluti



THANK YOU

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