Handout

1 Bernoulli Random Variable

A Bernoulli random variable is the simplest kind of random variable. It can take on two values, 1 and 0. It takes on a 1 if an experiment with probability p resulted in success and a 0 otherwise. Some example uses include a coin flip, a random binary digit, whether a disk drive crashed, and whether someone likes a Netflix movie.

If X is a Bernoulli random variable, denoted by P(X=x) or P(x) then the Probability Mass Distribution Function is given by :

$$P(X = x) or p(x) = p^x(1-p)^{1-x} \tag{1}$$

for x=0,1
Also, The Mean or Expectaion is given by :

$$\mu \quad or \quad E(X) = p \tag{2}$$

The Variance is given by :

$$\sigma^2 \quad or \quad Var(X) = p(1-p) \tag{3}$$

Now lets derive the Mean and Variance for Bernoulli Random Variable.

1.1 Mean

Consider the Mean(μ) $or\ Expectation\ for\ a\ Discrete\ Random\ Variable$,

$\mu \quad or \quad E(X) = \sum_x xp(x)$
substitute the value of Probability Mass Distribution function or p(x) given in Equation(1) to the above equation. Now we have,
$\mu \quad or \quad E(X) = 0.p^0(1-p)^{1-0} + 1.p^1(1-p)^{1-1}$
$\mu \quad or \quad E(X) = 0.(1-p) + 1.p$
$\mu \quad or \quad E(X) = p$
Hence Proved.

1.2 Variance

We know that,

$Var(X) = E[(X - \mu)^2]$,
That is,
$Var(X) = \sum_x (x - \mu)^2.p(x)$

But a very handy relationship for the same by using one of the axiom of Expectation is given by ,

$$Var(X) \ \ or \ \ E[(X - \mu)^2] = E(X^2) - [E(X)]^2 \tag{4}$$

In the above equation we have already derived what is E(X) i.e mean. So lets consider the first part in the above equation i.e,
$E(X^2) = \sum_x x^2.p(x)$
$= 0.p^0(1-p)^{1-0} + 1^2.p^1(1-p)^{1-1}$
$= 0^2.(1-p) + 1^2.p$
$= p$
No lets substitute the above value of E(X$^2)andalsoE(X)intheequation(4), so,$
$Var(X) = E(X^2) - [E(X)]^2$
$= p - p^2$
$= p(1-p)$
Hence proved

2 Binomial Random Variable

A binomial random variable is random variable that represents the number of successes in n successive independent trials of a Bernoulli experiment. Some example uses include the number of heads in n coin flips, the number of disk drives that crashed in a cluster of 1000 computers, and the number of advertisements that are clicked when 40,000 are served. If X is a Binomial random variable, we denote this X as Bin(n,p, where p is the probability of success in a given trial. A binomial random variable has the following properties:

$$P(X = x) or p(x) =^n C_r p^x (1-p)^{1-x} \tag{5}$$

for x=0,1,...,n
Also, The Mean or Expectaion is given by :

$$\mu \quad or \quad E(X) = np \tag{6}$$

The Variance is given by :

$$\sigma^2 \quad or \quad Var(X) = np(1-p) \tag{7}$$

Now lets derive the Mean and Variance for Binomial Random Variable.

From equation(6) and equation(7) we can observe that the mean and variance of Binomial Random Variable is nothing but n times the mean and variance of Bernoulli Random Variable obtained in equation(2) and equation(3). Hence we derive our mean and variance for Binomial Random Variable with the help of mean and variance of Bernoulli Random Variable.

2.1 Mean

Let $\mathrm{U}_1, U_2, U_3, ..., U_n$ $be\ the\ independent\ Bernoulli\ Random\ Variables.$
$Hence\ the\ \mu\ or\ Expectation E(U_i) = p$
and
$\sigma^2\ or\ Var(U_i) = p(1-p).$
$Here, X = U_1 + U_2 + U_3 + ... + U_n$
$So,$
$\mu\ or\ E(U_1 + U_2 + U_3 + ... + U_n)$
$\mu\ or\ E(U_1 + U_2 + U_3 + ... + U_n)$
$\mu\ or\ E(X) = p + p + ... + p$
$\mu\ or\ E(X) = np$
$Hence Proved.$

2.2 Variance

Let $\mathrm{U}_1, U_2, U_3, ..., U_n$ $be\ the\ independent\ Bernoulli\ Random\ variables.$
$Hence\ the\ \mu\ or\ Expectation E(U_i) = p$
and
$\sigma^2\ or\ Var(U_i) = p(1-p).$
$Here, X = U_1 + U_2 + U_3 + ... + U_n$
$So,$
$\sigma^2\ or\ Var(X) = Var(U_1 + U_2 + U_3 + ... + U_n)$
$\sigma^2\ or\ Var(X) = Var(U_1) + Var(U_2) + ... + U_n)$
$\sigma^2\ or\ Var(X) = p(1-p) + p(1-p) + ... + p(1-p)$
$\sigma^2\ or\ Var(X) = np(1-p)$
$Hence Proved.$