

Statistics Types and Summary

D. Uma

Department of Computer Science and Engineering umaprabha@pes.edu



Descriptive & Inferential Statistics

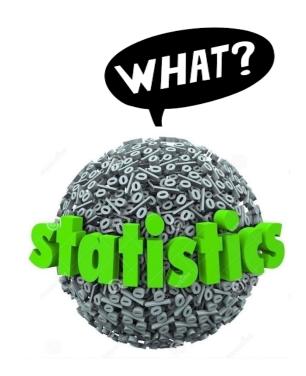
D. Uma

Department of Computer Science and Engineering

Statistics

1. WHAT IS STATISTICS?

2. TYPES OF STATISTICS



3. DESCRIPTIVE STATISTICS



Why Statistics?







To find a way a process behaves the way it does.

Why a process produces defective goods and services?

To check various performance measures of a process.

To prevent problems caused by various causes of variation in process.

To analyze the real world.

Statistics

The word **statistics** convey a **variety of meaning** to people in different walks of life.

The word statistics comes from a **Italian** word **Statista** meaning statement

and

German word statistik meaning political state.

Statistics is a science of data.

It is a **method** of dealing with **quantitative or qualitative information**.





Statistics

"Statistics is the science of collecting, organizing, presenting, analyzing and interpreting numerical data to assist in making more effective decisions."





Statistics is the science of collecting, organizing, presenting, analyzing and interpreting numerical data to assist in making more effective decisions.



Statistics







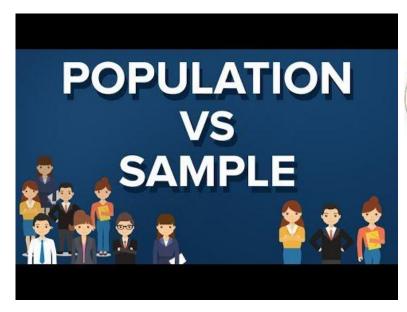
Statistics is the branch of mathematics that transforms data into useful information for decision makers.

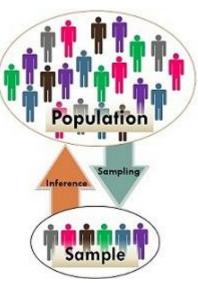
Population vs. Sample



A **population** is the entire collection of all items(or objects) of interest to our study.

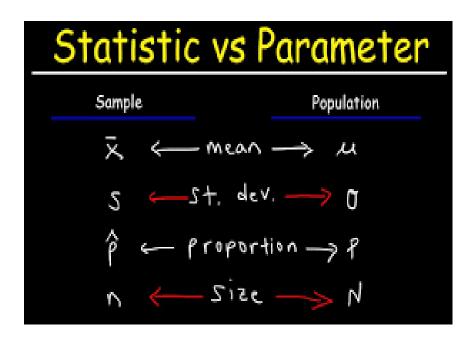
A **sample** is a subset of a population.



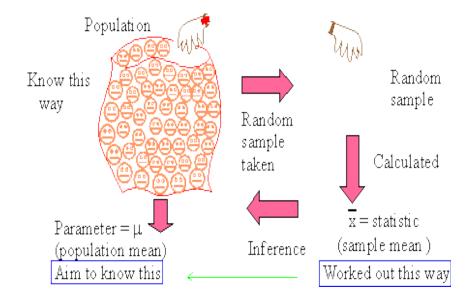


Parameter vs. Statistic

Parameter is a numerical measurement describing some characteristic of a population.



Statistic is a numerical measurement describing some characteristic of a sample.



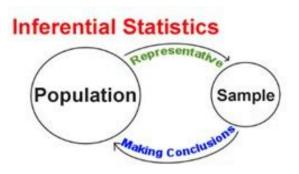


Processes of statistics

Statistics comprises of two processes.

1. Describing set of data

2. Drawing conclusions (making estimates, decisions, predictions, about set of data based on sampling)



- Measures of Central Tendency
- Measures of Dispersion/Spread
- •How it gets accumulates?

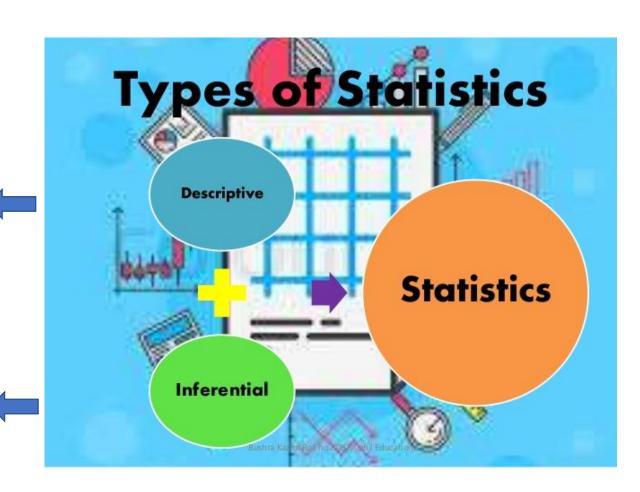


Types of Statistics



Numerical methods to organize, summarize and display data

Draws inferences from a population using sample



Descriptive statistics

- Collect Data
 - e.g. Survey

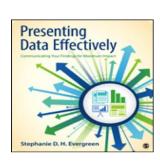


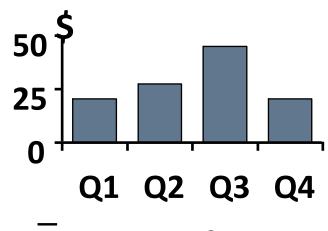
Purpose

Describe Data



- **Present Data**
 - e.g. Tables and graphs





- Characterize Data
 - e.g. Sample mean $\bar{X} = \frac{\sum X}{X}$

$$\bar{X} = \frac{\sum X}{n}$$

Why Descriptive Statistics?

An Illustration : Which Group is Smarter?					
Class AIQs of 13 Students		Class BIQs of 13 Students			
102	115	127	162		
128	109	131	103		
131	89	96	111		
98	106	80	109		
140	119	93	87		
93	97	120	105		
110		109			



Descriptive statistics

An Illustration : Which Group is Smarter?					
Class AIQs of 13 Students		Class BIQs of 13 Students			
102	115	127	162		
128	109	131	103		
131	89	96	111		
98	106	80	109		
140	119	93	87		
93	97	120	105		
110		109			

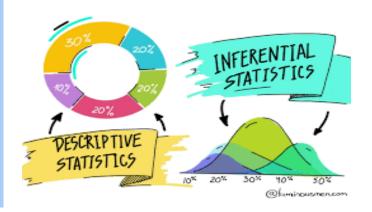


Figure speaks it all !!!

Which group is smarter now?

Class A--Average IQ

Class B--Average IQ

110.54

110.23

They're roughly the same!

With a summary descriptive statistic, it is much easier to answer our question.



Example-Descriptive Statistics



In a recent study, volunteers who had less than 6 hours of sleep were four times more likely to answer incorrectly on a science test than were participants who had at least 8 hours of sleep. Decide which part is the descriptive statistic and what conclusion might be drawn using inferential statistics.

The statement "four times more likely to answer incorrectly" is a descriptive statistic. An inference drawn from the sample is that all individuals sleeping less than 6 hours are more likely to answer science question incorrectly than individuals who sleep at least 8 hours.

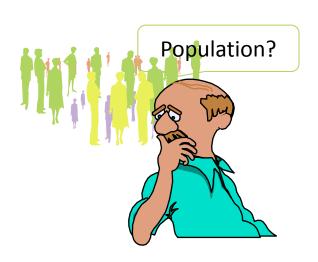
Inferential statistics

- Involves Estimation
 - e.g. Population Parameters
- Hypothesis Testing

Inferential Statistics: Making decisions and drawing conclusions about populations.

Purpose

 Make decision about population characteristics.



Inferential statistics utilizes sample data to make estimates, decisions, predictions or other generalizations about a larger set of data.



Why inferential statistics?



Suppose you want to know the mean income of the subscribers of Netflix

Mean (μ) — a parameter of a population.

You draw a random sample of 100 subscribers and determine that their mean income is \$27,500.

Mean(
$$\bar{x}$$
) = \$27,500 (a statistic).

Conclusion : You conclude that the population mean income μ is likely to be close to \$27,500 as well.

This example is one of statistical inference.

Descriptive vs. Inferential statistics



Descriptive Statistics

- Organize
- Summarize
- Simplify
- Presentation of data

Describing data

Inferential Statistics

- Generalize from samples to population
- Hypothesis testing
- Relationships among variables

Make predictions

Measures of Central Tendency



Something to know about !!!!

When we gather data, we want to uncover the "information" in it. One easy way to do that is to think of: "Shape —Position-Spread"

Shape – What is the shape of the histogram?

Position – What is the mean or median?

Spread – What is the range or standard deviation?

Types of Descriptive Statistics

- Organize Data
 - Tables
 - •Graphs



- Organize Data
 - Tables
 - Frequency Distributions
 - Relative Frequency Distributions
 - Graphs
 - Bar Chart or Histogram
 - . Stem and Leaf Plot
 - Frequency Polygon

Summarizing Data:

- Central Tendency (or Groups' "Middle Values")
 - = Mean
 - Median
 - = Mode
- Variation (or Summary of Differences Within Groups)
 - = Range
 - Interquartile Range
 - Variance
 - Standard Deviation



Variation

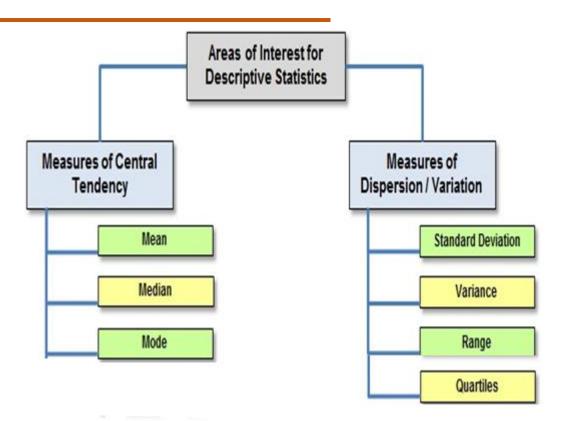
■Summarize Data

Central Tendency

Descriptive statistics

 Descriptive Statistics is a method of organizing, summarizing, and presenting data in a convenient and informative way.

 The actual method used depends on what information we would like to extract.



Descriptive statistics

INDICATORS OF CENTRAL TENDENCY

10

- Mode
 - Most Frequently Occurring Score
- Median
 - •Middle Score
- Mean
 - Arithmetic Average, etc.

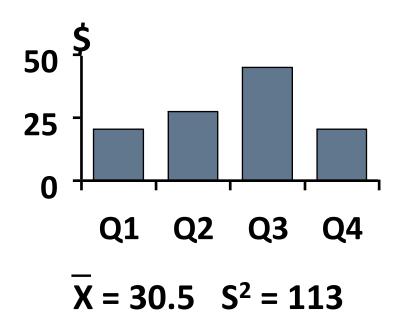
13/07/2018 Descriptive Statistics



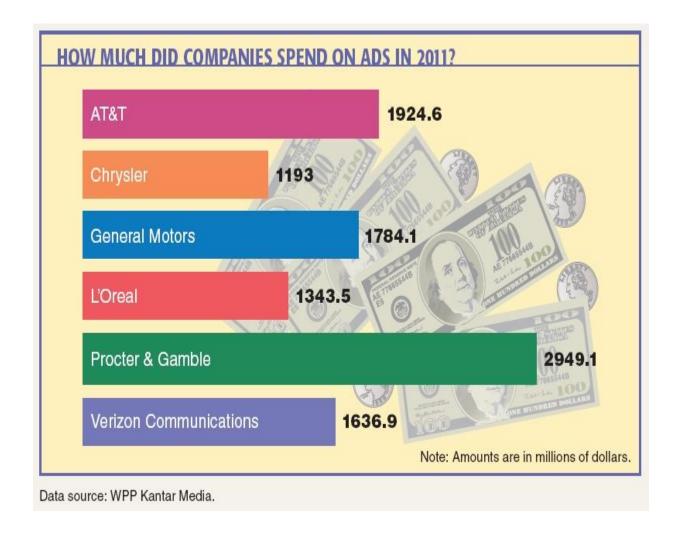
Descriptive statistics

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- Descriptive statistics are methods for organizing and summarizing data.
- For example, tables or graphs are used to organize data, and descriptive values such as the average score are used to summarize data.

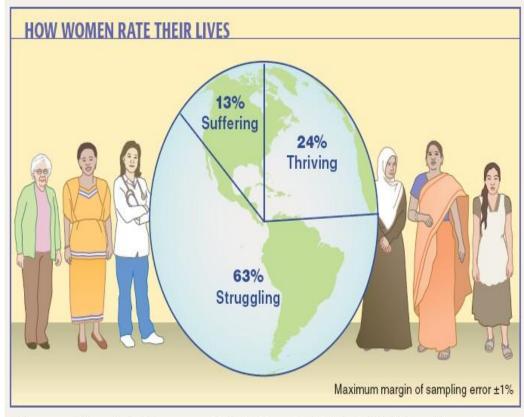


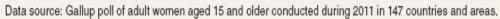
Descriptive statistics





Descriptive statistics







Descriptive statistics

Problem:

Calculate the average number of truck shipments from the United States to five Canadian cities for the following data given in thousands of bags:

Montreal, 64.0; Ottawa, 15.0; Toronto, 285.0; Vancouver, 228.0; Winnipeg, 45.0



Measures of Central Tendency



There are three different types of 'average'. These are the *mean*, the *median* and the *mode*.

They are used by statisticians as a way of summarizing where the 'centre' of the data is.

Mean = sum of all values
total number of values

Median = middle value (when the
data are arranged
in order)

Mode = most common value

Measures of Central Tendency: Mean



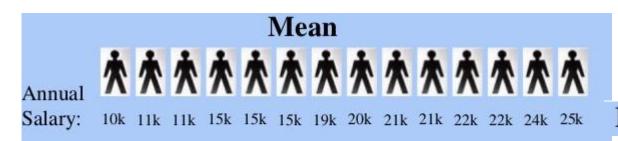
- Mean is the arithmetic average computed by summing all the values in the dataset and dividing the sum by the number of data values.
- The population mean is represented by Greek letter μ.
- For a finite set of dataset with measurement values X1, X2,, Xn (a set of n numbers), it is defined by the formula:

$$\mu_x = \sum_{i=1}^{N} \frac{x_i}{N} = \frac{x_1 + x_2 + \dots + x_N}{N}$$

$$\mu_x = \frac{\sum X}{N}$$
 mean of a population $\overline{X} = \frac{\sum X}{n}$ mean of a sample

Measures of Central Tendency: Mean





$$\bar{X} = \frac{\sum X}{n}$$
 $\bar{X} = \frac{251}{14}$

Mean = 17.9 k.y^{-1}

Disadvantages

- Very sensitive measure
- Can only be used on interval or ratio data

Advantages

- Very sensitive measure
- •Takes into account all the available information
- •Can be combined with means of other groups to give the overall mean

Measures of Central Tendency: Mean



- 1. Add all the values to get the sum.
- 2. To find the mean, divide the sum by the number of data values (i.e. n).

Consider the data given below:

5, 9, 12, 4, 5, 14, 19, 16, 3, 5, 7

Find the Mean:

2. mean = sum/ no. of values= 99/11= 9.

Sometimes the mean will not appear in the original list. It might even be a decimal value.

Measures of Central Tendency: Mean



Advantages:

Takes into account every number in the data set. That means all numbers are included in calculating the mean.

Easy and quick way to represent the entire data values by a single or unique number due to its straightforward method of calculation.

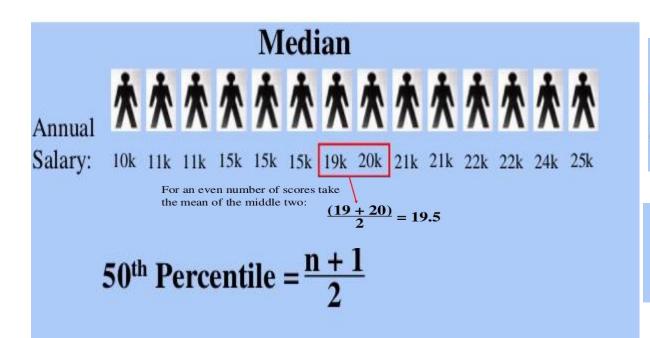
Each set has a unique mean value.

Disadvantages:

Its value is easily affected by extreme values known as the outliers.

Measures of Central Tendency: Median





Advantages

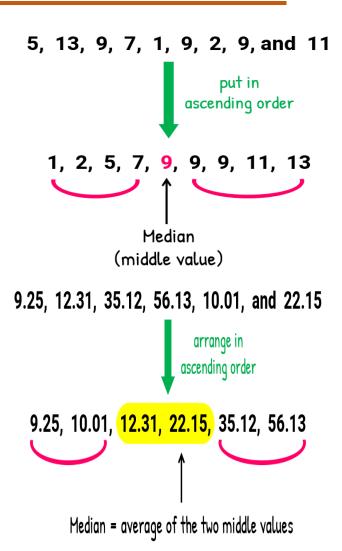
- •Unaffected by extreme scores
- Can be used at all levels above nominal.

Disadvantages

•Only considers order- value ignored.

Measures of Central Tendency: Median

- Arrange all the values in ascending order.
- 2. Find the middle position.
- 3. The element corresponding to middle position is considered as median(if odd number of elements are present).
- 4. If there are even number of elements present then the average of the elements present in the middle positions is considered as median.





Measures of Central Tendency: Median- Example



Consider the data given below:

$$(n=11)$$

The Median

To calculate the median, we need to put the numbers in order and find the middle value.

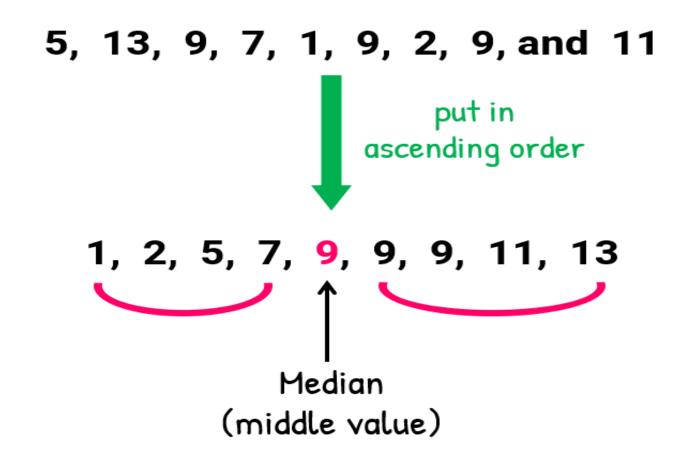
3 4 5 5 5 **7** 9 12 14 16 19

Here the *median* is 7 because this is the middle value.

Half of the other values in the list are below 7 and half are above 7.

Measures of Central Tendency: Median- Example





Measures of Central Tendency: Median- Example



Consider the data given below:

$$(n=6)$$

When there are an even number of values, there is no clear middle value.

In this case, there are two middle values.

6 **7 8** 11 15

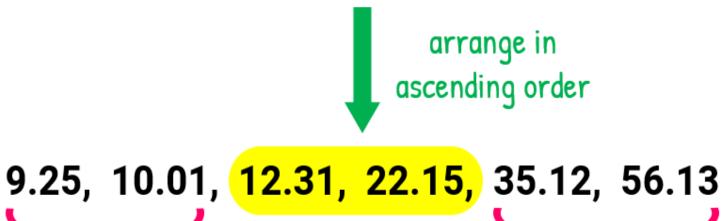
The median is the *mean* of these two middle numbers.7 + 8 / 2 = 7.5So the median for this set of values is **7.5**.

Like the mean, the median value does not always appear in the original list of values.

Measures of Central Tendency: Median- Example



9.25, 12.31, 35.12, 56.13, 10.01, and 22.15



Median = average of the two middle values

Measures of Central Tendency: Median



Advantages:

Not affected by the outliers in the data set.

An outlier is a data point that is radically "distant" or "away" from common trends of values in a given set.

It does not represent a typical number in the set.

The concept of the median is intuitive thus can easily be explained as the center value.

Each set has a unique median value.

Disadvantages:

Its value is perceived as it is. It cannot be utilized for further algebraic treatment.

Measures of Central Tendency: Mode

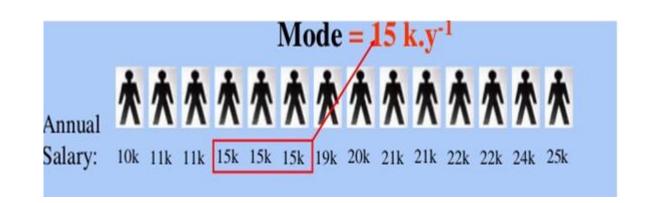
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Mode: Most often value in the data set.

To calculate the mode, we need to look at which value appears the most often.

Disadvantages

- Terminal Statistic
- A given sub-group could make this measure unrepresentative.



Advantages

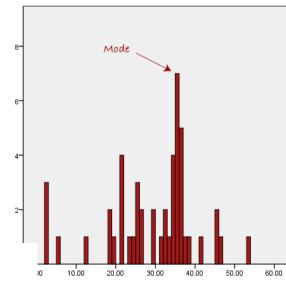
- •Quick and easy to compute
- Unaffected by extreme scores
- •Can be used at any level of measurement.

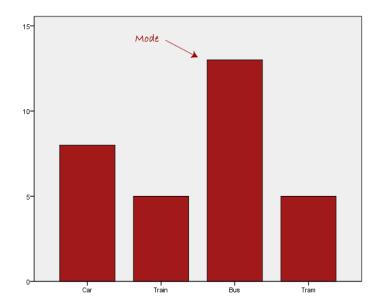
Measures of Central Tendency: Mode

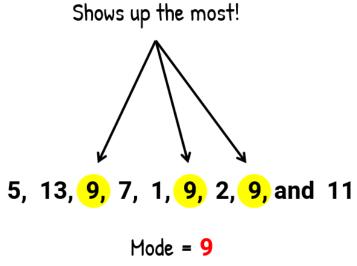


Mode: Most often value in the data set.

To calculate the mode, we need to look at which value appears the most often.





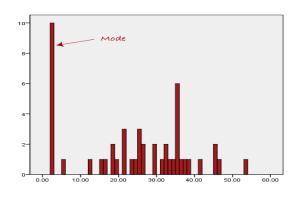


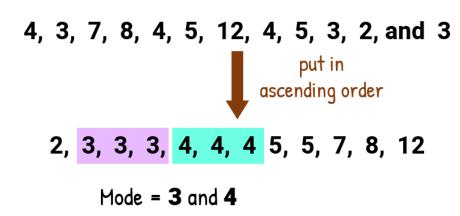
Measures of Central Tendency: Mode - Example

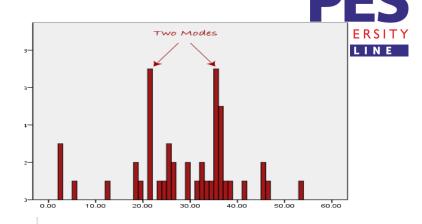
Consider the data given below:

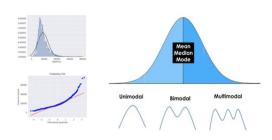
5, 9, 12, 4, 5, 14, 19, 16, 3, 5, 7 3 4 **5 5 7** 9 12 14 16 19 In this list the *mode* is 5, because it appears most often.

Sometimes there will be more than one mode, because two or more values appear the same number of times.









Measures of Central Tendency: Mode



Advantages:

Just like the median, the mode is not affected by outliers.

Useful to find the most "popular" or common item. This includes data sets that do not involve numbers.

Disadvantages:

If the set contains no repeating values, the mode is irrelevant.

In contrast, if there are many values that have the same count, then mode can be meaningless.

Source : Chilimath

Measures of Central Tendency



The most appropriate measure of location depends on ...

the shape of the data's distribution.

Depends on whether or not data are"symmetric" or "skewed".

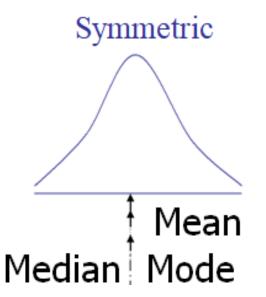
Depends on whether or not data have one ("unimodal") or more ("multimodal") modes.

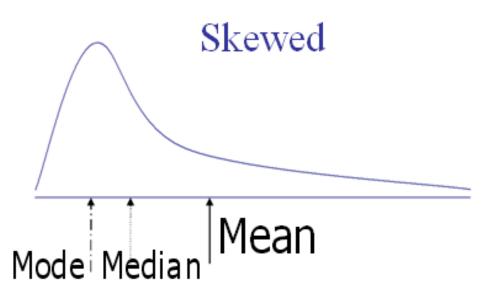
Measures of Central Tendency: Median



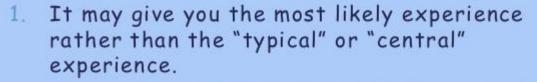
In symmetric distributions, the mean, median, and mode are the same.

In skewed data, the mean and median lie further toward the skew than the mode.



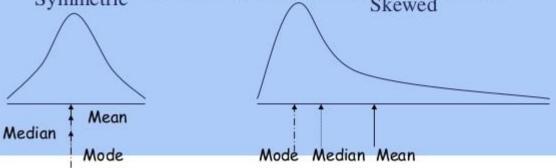


Measures of Central Tendency: Mode



2. In symmetric distributions, the mean, median, and mode are the same.

3. In skewed data, the mean and median lie Symmetric toward the skew than the mode.



When the median and the mean are different, the distribution is skewed. The greater the difference, the greater the skew.



If the skewness is extreme, the researcher should either transform the data to make them better resemble a normal curve or else use a different set of statistics—nonparametric statistics—to carry out the analysis

Problem - Mean, Median and Mode



Alex did a survey of how many games each of his 20 friends owned, and got this:

9, 15, 11, 12, 3, 5, 10, 20, 14, 6, 8, 8, 12, 12, 18, 15, 6, 9, 18, 11

Find the mean, median and mode

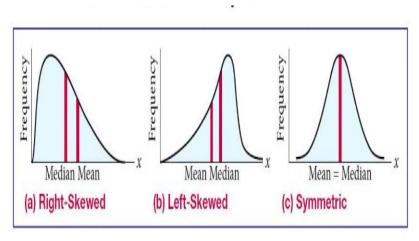
Measures of Central Tendency

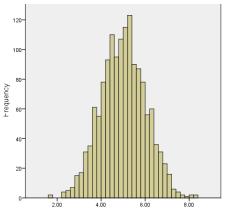


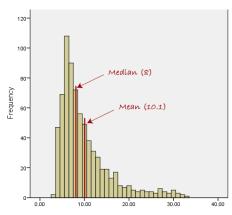
Symmetric and Skewed Distributions:

Symmetric Data: Data sets whose values are evenly spread around the center.

Skewed Data: Data sets that are not symmetric.







Measures of Central Tendency

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Shape: The "shape" of the data is called its "distribution".

If mean = median = mode, the shape of the distribution is symmetric.

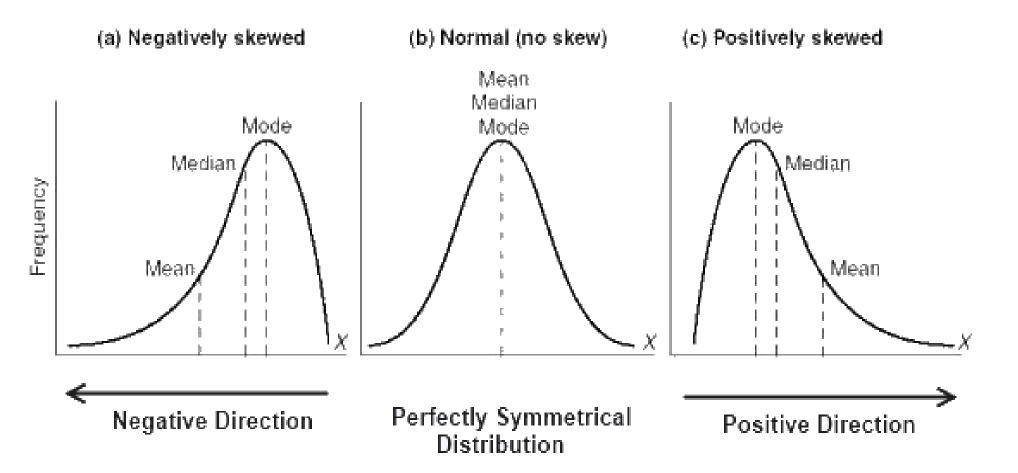
•If mode < median < mean, the shape of the distribution trails to the right, is positively skewed.



- •If mean < median < mode, the shape of the distribution trails to the left, is negatively skewed.
- Distributions of various "shapes" have different properties and names such as the "normal" distribution, which is also known as the "bell curve" (among mathematicians it is called the Gaussian)

Symmetrical vs Skewed data





Measures of Central Tendency

Quantitative data:

- Mode the most frequently occurring observation
- Median the middle value in the data
- Mean arithmetic average

Qualitative data:

Mode – always appropriate

Ex: Maximum Type of Color

Mean – never appropriate

Ex : Average value of Yellow color



When to use Mean, Median and Mode?

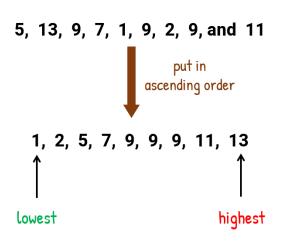
TYPE OF VARIABLE	BEST MEASURE OF CENTRAL TENDENCY
Nominal	Mode
Ordinal	Median
Interval / Ratio (not skewed)	Mean
Interval / Ratio (skewed)	Median

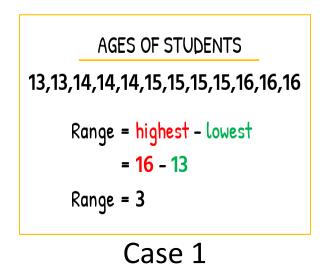


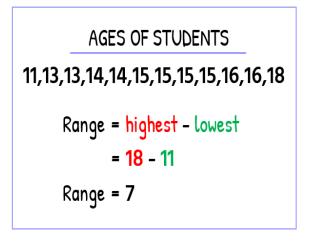
Measures of spread: Range



Range = Maximum Value – Minimum Value







Case 2

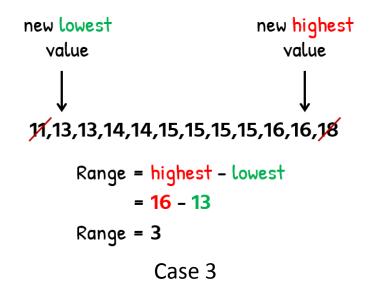
Observations:

Since the range of Class A is **smaller** than in Class B, can we claim that the age distribution in Class A is more clustered (closely related) than in Class B? In other words, are the ages listed in Class A more uniform than in Class B?

Measures of spread: Range

Limitations:

- 1. Using the range to describe the spread of data within a set.
- 2. It can drastically be affected by outliers (values that are not typical as compared to the rest of the elements in the set).





Measures of spread: Range



Range (in statistics) is the difference between the maximum and minimum value of the set. What the range provides is a quick and rough estimate of the spread of data values within a set.

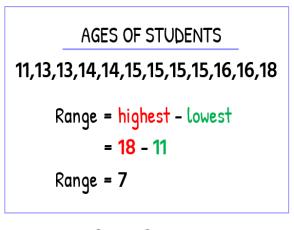
Here we have two classes taking Data Science and the ages of the students in each class.

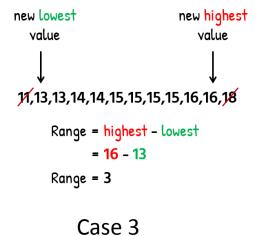
AGES OF STUDENTS

13,13,14,14,14,15,15,15,15,16,16,16

Range = highest - lowest
= 16 - 13

Range = 3





Case 1

Case 2

Measures of spread: Range



Advantages:

Just like the median, the mode is not affected by outliers.

Useful to find the most "popular" or common item. This includes data sets that do not involve numbers.

Disadvantages:

If the set contains no repeating values, the mode is irrelevant.

In contrast, if there are many values that have the same count, then mode can be meaningless.

Measures of Dispersion: Range - Example



AGES OF STUDENTS

13,13,14,14,14,15,15,15,15,16,16,16

Range
$$= 3$$

Measures of Dispersion: Range - Example



```
new lowest
                           new highest
                              value
  value
11,13,13,14,14,15,15,15,15,16,16,18
      Range = highest - lowest
            = 16 - 13
      Range = 3
```

Measures of Dispersion: Range



The Range Can Be Misleading

The range can sometimes be misleading when there are extremely high or low values.

Example: In **{8, 11, 5, 9, 7, 6, 3616}**:

The lowest value is 5,

and the highest is 3616,

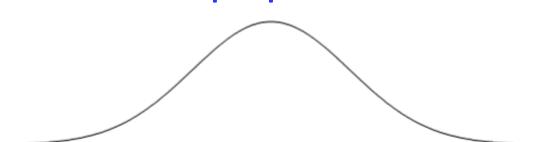
So the range is 3616-5 = 3611.

The single value of 3616 makes the range large, but most values are around 10.

Measures of Dispersion: Quartiles

As the name 'quartile' suggests, we want to divide

the data into four equal parts



The first quartile is the 25th percentile

The median is the 50th percentile

The third quartile is the 75th percenile

Figure :: Graph representing heights of adult males.

In the above graph, we want to divide the area under our curve into four equal areas.

First put the list of numbers in order

Then cut the list into four equal parts

The Quartiles are at the "cuts"



Measures of Dispersion: Quartiles

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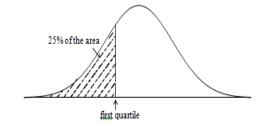
The first quartile is the 25th percentile

The median is the 50th percentile

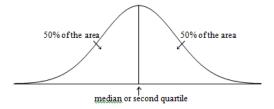
The third quartile is the 75th percenile

Measures of Dispersion: Quartiles

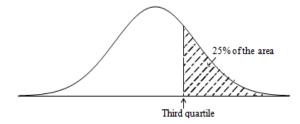
The first quartile is the 25th percentile



The median is the 50th percentile



The third quartile is the 75th percentile





Measures of Dispersion: Quartiles

The first quartile

The first quartile is the point which gives us 25% of the area to the left of it and 75% to the right of it.

This means that 25% of the observations are less than or equal to the first quartile and 75% of the observations greater than or equal to the first quartile.

25% of the area

The first quartile is also called the 25th percentile.



Measures of Dispersion: Quartiles



To find the first quartile, compute the value 0.25(n + 1).

If this is an integer, then the sample value in that position is the first quartile.

If not, then take the average of the sample values on either side of this value.

Measures of Dispersion: Quartiles

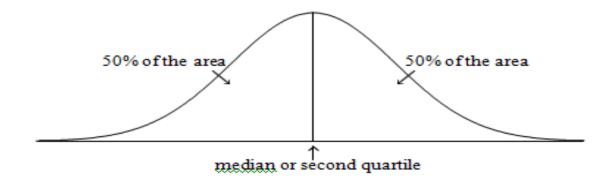
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The second quartile or median

It is easy to see how to divide the area in Figure 9 into two equal parts, since the graph is symmetric.

The point which gives us 50% of the area to the left of it and 50% to the right of it is called the second quartile or median

Second quartile is calculated using the value 0.5(n+1)



Measures of Dispersion: Quartiles



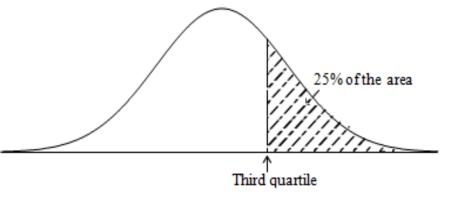
The third quartile

The third quartile is the point which gives us 75% of the area to the left of it and 25% of the area to the right of it.

This means that 75% of the observations are less than or equal to the third quartile and 25% of the observation are greater than or equal to the third quartile.

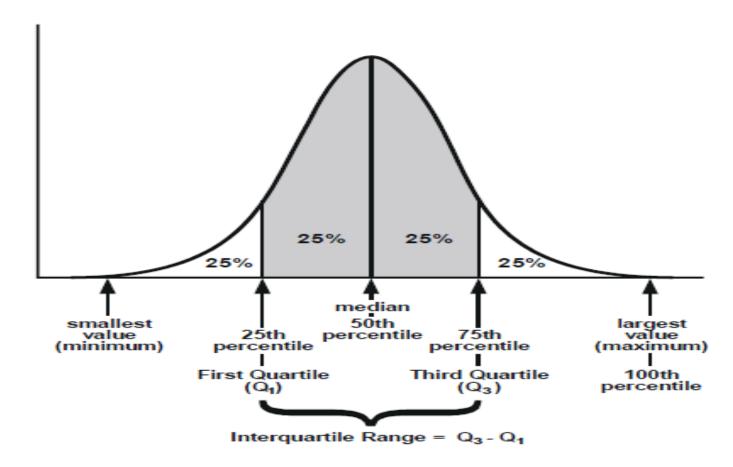
The third quartile is also called the 75th percen

The third quartile is computed in the same way the value 0.75(n+1) is used.



Measures of Dispersion: Quartiles Summary

The first (Q_1) , second (Q_2) and third (Q_3) quartiles divide the distribution into four equal parts.

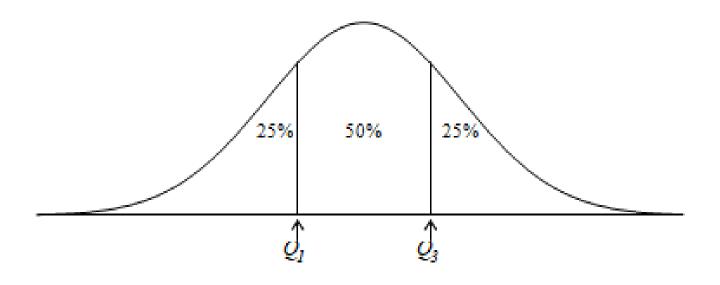




Measures of Spread: InterQuartile Range

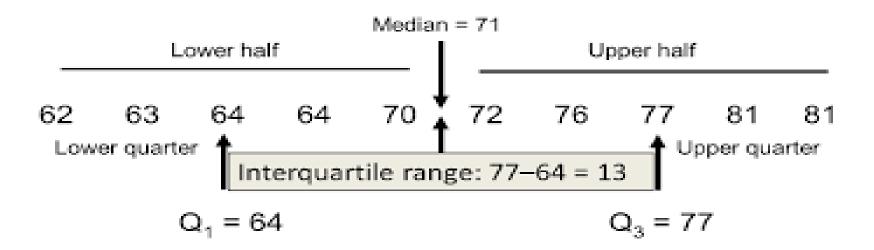
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The interquartile range quantifies the difference between the third and first quartiles.



Measures of Spread: InterQuartile Range

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Measures of Spread: InterQuartile Range



Interquartile Range = Upper Quartile(Q3) – Lower Quartile(Q1)

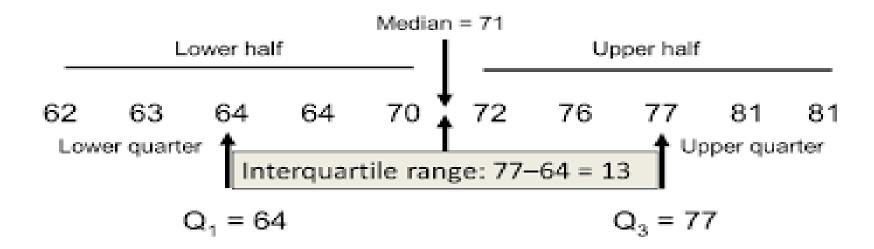
IQR = Q3 –Q1

Steps to find IQR:

- 1. Arrange the data scores in ascending order.
- 2. Find the median of the data set(the number in the middle).
- 3. Find the median of the lower half of the scores (Q1).
- 4. Find the median of the upper half of the scores (Q3).

Note: If the number of scores is even, the median is the average of the two middle scores.

Measures of Spread: IQR-Example





Measures of Spread: IQR-Example



For the following data sets, calculate the quartiles and find the interquartile range.

The following numbers represent the time in minutes that twelve employees took to get to work on a particular day.

18 34 68 22 10 92 46 52 38 29 45 37

Measures of spread: Variance

Average of the distance that each score is from the mean (Squared deviation from the mean).

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- 1. Find the mean value of the given data values.
- 2. Subtract mean from each data value.
- 3. Square each value that is obtained from step2.
- 4. Find the sum of all values that is obtained from step 3.
- 5. Divide the result that is obtained from step4 by N(for population) and n-1(for sample).

Variance

$$s^{2} = \frac{\sum (x - \overline{x})^{2}}{n - 1}$$
 Sample Variance

$$\sigma^2 = \frac{\sum (x - \mu)^2}{N}$$
 Population Variance

54

Measures of spread: Variance And Standard Deviation

Consider these two list of numbers:-28,29,30,31,32 and 10,20,30,40,50. Find their means.



Both the list have same mean i.e. 30

But,

Clearly list differs which is not captured by mean

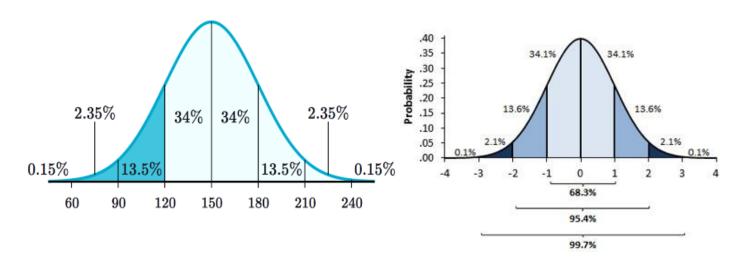


Measures of spread: Standard Deviation

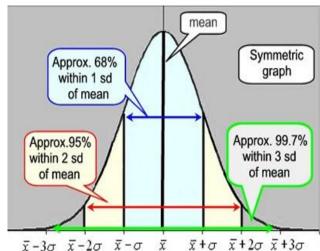
Standard deviation signifies the deviation of the terms from the mean value

of the distribution.

It quantifies the amount of variation of a set of data values.







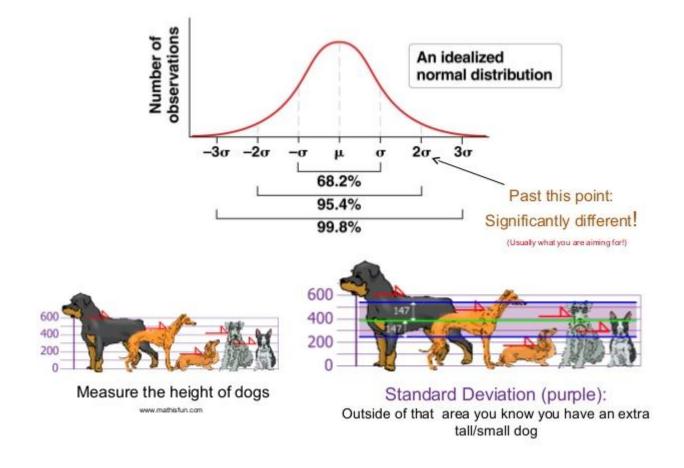
Measures of spread: Standard Deviation



Standard deviation signifies the deviation of the terms from the mean value of the distribution.

It quantifies the amount of variation of a set of data values.

Measures of spread: Standard Deviation





Measures of spread: Variance and Standard Deviation



Standard Deviation = Square root of Variance

Example

Find the standard deviation and variance

The variance

$$s^2 = \frac{\sum (x - \overline{x})^2}{n - 1} = 32 / 2 = 16$$

The standard deviation

$$s^2 = \frac{\sum (x - \overline{x})^2}{n - 1} = 32 / 2 = 16$$
 $s = \sqrt{16} = 4$

Measures of spread: Standard Deviation



Problem:

The heights of the players (in centimeters) from a basketball team are represented by the table:

Height	[170, 175)	[175, 180)	[180, 185)	[185, 190)	[190, 195)	[195, 2.00)
No. of players	1	3	4	8	5	2

Calculate standard deviation.

Measures of spread : Variance - Example



$$\mu_{x} = 59.11 \frac{46.64.54.77.67.68.62.56.38}{\sigma^{2}} = \frac{\sum (x - \mu_{x})^{2}}{N} = \frac{1146.88}{9} = 127.43$$
Random Sample 38.62.67.62 $\bar{x} = 57.25$
 $n = 4$

$$s_{x}^{2} = \frac{\sum (x - \bar{x})^{2}}{n - 1} = \frac{510.75}{3} = 170.25$$

Trimmed Mean

The trimmed mean is computed by arranging the sample values in order, "trimming" an equal number of them from each end, and computing the mean of those remaining.

If p% of the data are trimmed from each end, the resulting trimmed mean is called the "p% trimmed mean."

There are no hard-and-fast rules on how many values to trim.

The most commonly used trimmed means are the 5%, 10%, and 20% trimmed means.

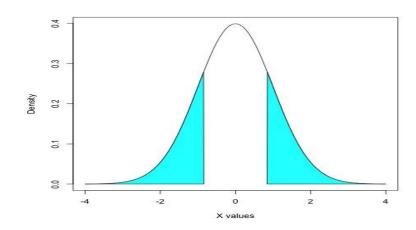


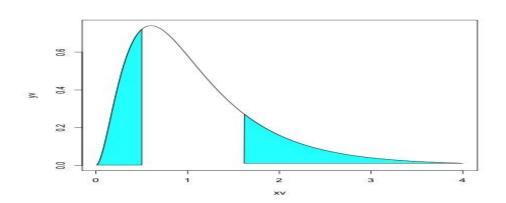
Trimmed Mean

If the sample size is denoted by n, and a p% trimmed mean is desired, the number of data points to be trimmed is np/100

It is used to reduce the effects of outliers on the calculated average.

This method is best suited for data with large, erratic deviations or extremely skewed distributions







Trimmed Mean



Questions

For the following data

30 75 79 80 80 105 126 138 149 179 179 191

223 232 232 236 240 242 245 247 254 274 384 470

Compute the mean, median, and the 5%, 10%, and 20% trimmed means.

Solution:-

•••••



Trimmed Mean

The mean is found by averaging together all 24 numbers, which produces a value of 195.42.

The median is the average of the 12th and 13th numbers, which is (191 + 223)/2 = 207.00.

To compute the 5% trimmed mean, we must drop 5% of the data from each end. This comes to (0.05)(24) = 1.2 observations.

We round 1.2 to 1, and trim one observation off each end.





Trimmed Mean



The 5% trimmed mean is the average of the remaining 22 numbers: $75 + 79 + \cdots + 274 + 384/22 = 190.45$



To compute the 10% trimmed mean, round off (0.1)(24) = 2.4 to 2.

Drop 2 observations from each end, and then average the remaining 20:

To compute the 20% trimmed mean, round off (0.2)(24) = 4.8 to 5. Drop 5 observations from each end, and then average the remaining 14:

$$105 + 126 + \cdots + 242 + 245/14 = 194.07$$

Percentile





The pth percentile of a sample, for a number p between 0 and 100, divides the sample such that

- 1. p% of the sample values are less than the pth percentile
- 2. And (100-p%) are greater.

Percentile

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Steps:-

Order the n samples values from smallest to largest

Compute the quantity (p/100)(n+1), where n is the sample size.

If this quantity is an integer, the sample value in this position is the percentile.

Otherwise, average the two sample values pn either side.

Tertile and Decile



Steps:-

Order the n samples values from smallest to largest

Compute the quantity (p/100)(n+1), where n is the sample size.

If this quantity is an integer, the sample value in this position is the percentile.

Otherwise, average the two sample values pn either side.



THANK YOU

D. Uma

Department of Computer Science and Engineering umaprabha@pes.edu

+91 99 7251 5335