



**PES UNIVERSITY, Bangalore**  
(Established under Karnataka Act No. 16 of 2013)  
**Department of Computer Science & Engineering**

**Automata Formal Languages & Logic**

**Q&A-Context Free Grammar**

1. Describe the language generated by  $G = (\{S, A\}, \{a, b\}, P, S)$ . The set of productions  $P$  is given as:

$S \rightarrow aA \mid bA$

$A \rightarrow aAa \mid bAb \mid aAb \mid bAa \mid \lambda$

**Solution:**

This language rejects the empty string, but can start with either a or b. The next state the system has, A, is independent of whether we started with an a or b. Next, also observe that after this, the string can take the form of all possible strings of even length (since all 4 combinations: aa, ab, ba or bb are allowed, along with the empty string). Hence, this language accepts all strings which have an odd length.  $w \mid w$  is a string over  $\{a, b\}$  with an odd length

$L = \{ w \mid w \text{ is a string over } \{a, b\}^* \text{ with an odd length} \}$ .

2. Construct the CFG for the language  $L = \{a^n b^m c^k \mid n \neq m \text{ or } m \neq k\}$ .

**Solution:**

There are four 'cases' to consider:

1. More a's than b's (w/ any number of c's).
2. More b's than a's (w/ any number of c's).
3. More b's than c's (w/ any number of a's).
4. More c's than b's (w/ any number of a's).

Production Rules:  $S \rightarrow S_1 S_3 \mid S_2 S_3 \mid S_4 S_5 \mid S_4 S_6$

Each of the four 'cases' are accounted for (from left to right in the above production).

$S_1 \rightarrow aS_1b \mid aS_1 \mid a$

$S_2 \rightarrow aS_2b \mid S_2b \mid b$

$S_3 \rightarrow S_3c \mid \lambda$

$S_4 \rightarrow aS_4 \mid \lambda$

$S_5 \rightarrow bS_5c \mid bS_5 \mid b$

$S_6 \rightarrow bS_6c \mid S_6c \mid c$



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3. For the regular expression  $(011+1)^*(01)^*$  obtain a context free grammar.

**Solution:**

The regular expression is  $(011+1)^*(01)^*$  of the form  $A^*B^*$  where A can be 011 or 1 and B is 01. The regular expression  $A^*B^*$  means that any number of a's (possibly none) are followed by any number of b's (possibly none). Any number of a's can be generated using the productions.

$G=(V,T,P,S)$  where  $V= \{S,A,B\}$ ,  $T= \{0,1\}$ , S is the start symbol.

$P= \{$   
 $S \rightarrow AB$   
 $A \rightarrow 011A \mid 1A \mid \lambda$   
 $B \rightarrow 01B \mid \lambda$   
 $\}$

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4. Is the following grammar ambiguous?

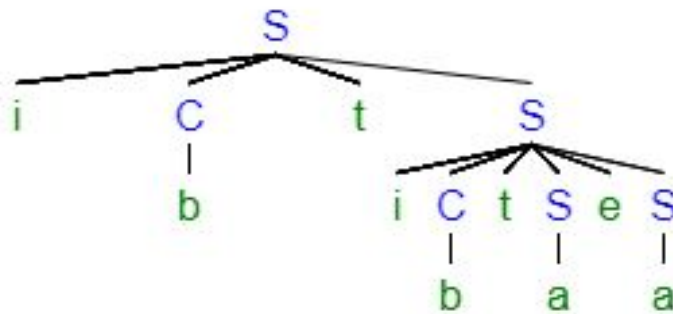
$S \rightarrow iCtS \mid iCtSeS \mid a$

$C \rightarrow b$

**Solution:**

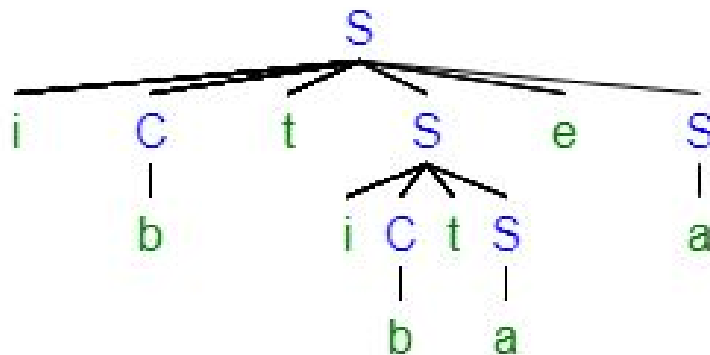
Leftmost derivation1

$S \Rightarrow iCtS \Rightarrow ibtS \Rightarrow ibtiCtSeS \Rightarrow ibtibtSeS \Rightarrow ibtibtaeS \Rightarrow ibtibtaea$



Leftmost derivation2

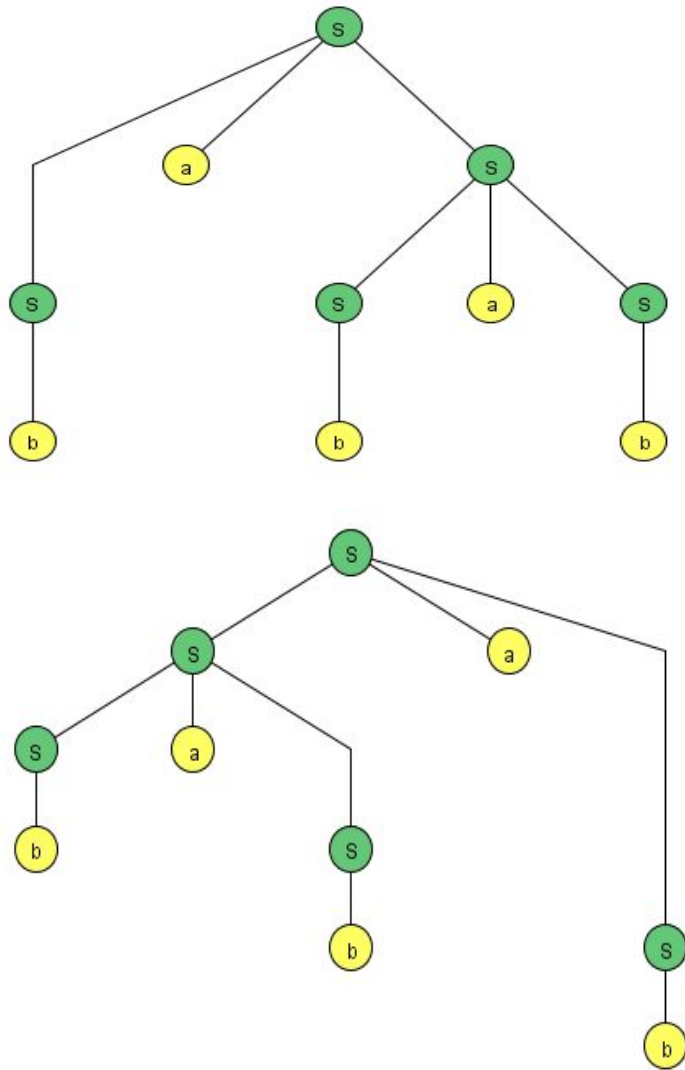
$S \Rightarrow iCtSeS \Rightarrow ibtSeS \Rightarrow ibtiCtSeS \Rightarrow ibtibtSeS \Rightarrow ibtibtaeS \Rightarrow ibtibtaea$



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5. Show that  $S \rightarrow SaS \mid b$  is ambiguous. Construct an unambiguous equivalent of the grammar.

**Solution:** Consider the string *babab*. There are two different leftmost derivations for this string as shown in the two parse trees below. Hence it is ambiguous.



An unambiguous equivalent is the regular grammar:  $S \rightarrow baS \mid b$

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6. Using the grammar  $G=(V,T,P,S)$ , with  $V=\{S\}$   $P=\{S \rightarrow S \cup S \mid SS \mid S^* \mid (S) \mid 0 \mid 1 \mid \lambda\}$ , give the left derivation and the corresponding parse tree for the string  $(0 \cup (10)^*1)^*$ .

**Solution:**

A derivation for  $(0 \cup (10)^*1)^*$  is

$S \Rightarrow S^* \Rightarrow (S)^* \Rightarrow (S \cup S)^* \Rightarrow (0 \cup S)^* \Rightarrow (0 \cup SS)^* \Rightarrow (0 \cup S^*S)^* \Rightarrow (0 \cup (S)^*S)^* \Rightarrow (0 \cup (SS)^*S)^* \Rightarrow (0 \cup (1S)^*S)^* \Rightarrow (0 \cup (10)^*S)^* \Rightarrow (0 \cup (10)^*1)^*$

The corresponding parse tree is,

