



PES UNIVERSITY, Bangalore
(Established under Karnataka Act No. 16 of 2013)
Department of Computer Science & Engineering

Automata Formal Languages & Logic

Homework

1. Let $\Sigma = \{a,b\}$ and let $D = \{w \mid w \text{ contains an equal number of occurrences of the substring } 01 \text{ and } 10\}$. Thus $101 \in D$ because 101 contains a single 01 and a single 10 , but $1010 \notin D$ because 1010 contains two 10 s and only one 01 . Show that D is a regular language.
2. Determine whether each of the following languages is regular.
 - $\{a^n a^n a^n \mid n > 0\}$
 - $\{www \mid w \in \{x,y,z\}^*, |w| < 10^{100}\}$
 - $\{vw \mid v, w \in \{a,b\}^*\}$
 - $\{ww \mid w \in \{a\}^*\}$
3. Let $L = \{0^n 1^n \mid n \geq 0\}$. Is the complement of L a regular language?
4. Consider the following statement: "If A is a non regular language and B is a language such that $B \subseteq A$, then B must be non regular." If the statement is true, give a proof. If it is not true, give a counterexample showing that the statement doesn't always hold.
5. Prove that if we add a finite set of strings to a regular language, the result is regular language.
6. Prove that if we remove a finite set of strings from a regular language, the result is a regular language.
7. Consider the following statement: "If A is a non regular language and B is a language such that $B \subseteq A$, then B must be non regular." If the statement is true, give a proof. If it is not true, give a counterexample showing that the statement doesn't always hold.
8. Show how to perform the product construction on NFA's (without lambda transition)
9. Show how to perform the product construction on lambda NFA's .
10. Show how to modify the product construction so the resulting DFA accepts the difference of the languages of the two given DFA's
11. Show how to modify the product construction so the resulting DFA accepts the union of the languages of the two given DFA's.
12. Give an algorithm to tell whether a regular language L contains at least 1000 strings.
13. Give an algorithm to tell whether two regular languages L_1 and L_2 have at least one string in common.
14. Give an algorithm to tell, for two regular languages L_1 and L_2 over the same alphabet Σ , whether there is any string Σ^* that is neither L_1 nor L_2 .
15. The language of regular expression $(0+10)^*$ is the set of all strings of 0's and 1's such that every 1 is immediately followed by a 0. Describe the complement of this language over the same alphabet.
16. For each claim below, state whether it is true or false, and prove your answer.
 - (a) A regular expression denotes an infinite language if and only if it contains the $*$



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operation.

(b) For all languages A and B , $(A^* \cap B^*)^* = (A \cap B)^*$.

(c) For all languages A and B , $(A^* \cup B^*)^* = (A \cup B)^*$.

17. For each statement, indicate whether it is true or false. Prove that your answer is correct.

(a) If languages A and B are non-regular, then their intersection $A \cap B$ is non-regular.

(b) If each of the languages L_0, L_1, L_2, \dots is regular, then their union $\bigcup_{i \in \mathbb{N}} L_i$ is Regular

18. Define the following two languages:

$L_a = \{w \{a, b\}^* : \text{in each prefix } x \text{ of } w, \#a(x) = \#b(x)\}$.

$L_b = \{w \{a, b\}^* : \text{in each prefix } x \text{ of } w, \#b(x) = \#a(x)\}$.

a) Let $L_1 = L_a \cap L_b$. Is L_1 regular? Prove your answer.

b) $L_2 = L_a \cup L_b$. Is L_2 regular? Prove your answer.

19. If L is regular language, prove that $L_1 = \{uv : u \in L, |v| = 3\}$ is also regular.

20. Show that the question, Given a FA M over Σ , does M accept a string of length < 2 ? is decidable

21. Show that the question: Does $L = \Sigma^*$? for regular language L is decidable.

22. Suppose h is the homomorphism from $\{0,1,2\}$ to $\{a,b\}$ defined by $h(0)=a, h(1)=ab$ and $h(2) = ba$

a. What is $h(21120)$?

b. If $L = 01^*2$, what is $h(L)$?

23. This problem is to illustrate proofs of (the many) closure properties of regular languages.

(a) For a language L let $\text{FUNKY}(L) = \{w \mid w \in L \text{ but no proper prefix of } w \text{ is in } L\}$.

Prove that if L is regular then $\text{FUNKY}(L)$ is also regular using the following technique. Let $M = (Q, \Sigma, \delta, s, A)$ be a DFA accepting L . Describe a NFA N in terms of M that accepts $\text{FUNKY}(L)$. Explain the construction of your NFA.

(b) In Lab 3 we saw that $\text{insert}_1(L)$ is regular whenever L is regular. Here we consider a different proof technique. Let r be a regular expression. We would like to show that there is another regular expression r_0 such that $L(r_0) = \text{insert}_1(L(r))$.

i. For each of the base cases of regular expressions ϵ , Σ and $\{a\}$, $a \in \Sigma$ describe a regular expression for $\text{insert}_1(L(r))$.



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li. Suppose r_1 and r_2 are regular expressions, and r_{01} and r_{02} are regular expressions for the languages $\text{insert1}(L(r_1))$ and $\text{insert1}(L(r_2))$ respectively. Describe a regular expression for the language $\text{insert1}(L(r_1 + r_2))$ using r_1, r_2, r_{01}, r_{02} .

iii. Same as the previous part but now consider $L(r_1 r_2)$.

iv. Same as the previous part but now consider $L((r_1)^*)$.

24. We know that if M is a DFA that recognizes language A , swapping the accepting and non accepting states yields a new DFA recognizing the complement of A . We then concluded that the class of regular languages is closed under complement.

a. Show, by giving an example, that if N is an NFA that recognizes language B , swapping the accepting and non accepting states in N does not necessarily yield a new NFA that recognizes the complement of B .

b. Is the class of languages recognized by an NFA closed under complement? Explain your answer

25. Let L_1 and L_2 be regular languages. Show that the following languages are also regular.

a) The difference $L_1 - L_2 = \{w \in L_1 : w \notin L_2\}$.

b) The symmetric difference $L_1 \oplus L_2 = (L_1 - L_2) \cup (L_2 - L_1)$.

c) The reversal $L^R = \{w^R : w \in L\}$.

Hint: You can use the closure properties presented in class for union, intersection, \cdot , concatenation, or the fact that every regular language has a DFA/NFA. Try to find the shortest answer

26.

21.

23.