



PES UNIVERSITY, Bangalore
(Established under Karnataka Act No. 16 of 2013)
Department of Computer Science & Engineering

Automata Formal Languages & Logic

Homework

1. Construct a PDA $L = \{a^i b^j \mid 0 \leq i \leq j\}$
2. Construct a PDA $L = \{a^i b^j c^i \mid i, j \geq 0\}$
3. Construct a PDA $\{a^i b^j c^k \mid i + k = j\}$
4. Construct a PDA $\{a^i b^j \mid i \geq 0\} \cup a^* \cup b^*$
5. Construct a PDA $L = \{a^i b^j \mid 2i = 3j + 1\}$
6. Construct a PDA $\{a^i b^j \mid i \neq j\}$
7. Construct a PDA $L = \{a^n b^m \mid m \leq n \leq 2m\}$
8. Construct a PDA $L = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } (i \neq j \text{ or } j \neq k)\}$
9. Construct a PDA for $L = \{a^n b^m \mid n, m \geq 0 \text{ and } m \text{ is even}\}$
10. Construct a PDA for $L = \{b_i \# b_{i+1}^R \mid b_i \text{ is the binary representation of some integer } i, i \geq 0, \text{ without leading zeros}\}$.
11. Construct a PDA for $L = \{a^m b^n c^p d^q \mid m, n, p, q \geq 0 \text{ and } m + n = p + q\}$
12. Construct a PDA for arithmetic, let $\Sigma = \{\text{int}, +, -, (,)\}$ and language $\text{ARITH} = \{w \in \Sigma^* \mid w \text{ is a legal arithmetic expression}\}$.
13. Construct a PDA for the language $L = \{a^n w w^R b^n \mid w \text{ is any string in } (a+b)^*\}$.
14. Let L be the language $\{w \in \{a, b\}^* \mid w \text{ has a prefix containing more } b\text{'s than } a\text{'s}\}$. For example $\text{baa}, \text{abba}, \text{abbaaa} \in L$, but $\text{aab}, \text{aabbab} \notin L$. Construct a PDA that accepts the language.
15. $a^i b^j c^k$ where either $i = j$ (and k is any number) or k is the difference between i and j . Show an accepting sequence of configurations for the input aabbcc . Show how the PDA rejects both aabbcc and aaaabbcc .
16. Construct a PDA for simplified XML tags strings. For example $\langle a \rangle \langle b \rangle \langle /b \rangle \langle /a \rangle \langle a \rangle \langle a \rangle \langle c \rangle \langle /c \rangle \langle /a \rangle \langle b \rangle \langle /b \rangle \langle /a \rangle$ where a, b and c are the only tag names.
17. Construct a PDA for the language of table tags in HTML $\{\text{TABLE}, \text{TH}, \text{TR}, \text{TD}\}$.
18. The language of addition with positive or negative numbers, that is, strings over the alphabet $\{a, b, c, -\}$ of the form:
 - (a) $a^n b^m c^k$ where $k = n + m$ and both numbers are positive; for example, aabbccccc .
 - (b) $-a^n b^m c^k$ where $k = m - n$ if $m \geq n$ and the first number is negative; for example, $-\text{aabbccccc}$.



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(c) $a^n b^m c^k$ where $k = n - m$ if $n > m$ and the first number is negative; for example, -aaaabb-cc.

(d) $a^n b^m c^k$ where $k = n - m$ if $n < m$ and the second number is negative; for example, aaaa-bbcc.

(e) $a^n b^m c^k$ where $k = m - n$ if $m > n$ and the second number is negative; for example, aa-bbbb-cc.

(f) $a^n b^m c^k$ where $k = n + m$ and both numbers are negative; for example, -aaa-bbb-cccccc.

Show an accepting sequence of configurations for each of the example strings shown above.

19. PDA $P = (\{q_0, q_1, q_2, q_3, f\}, \{a, b\}, \{Z, A, B\}, \delta, q_0, Z, \{f\})$ has the following rules defining δ :

$\delta(q_0, a, Z) = (q_1, AAZ)$	$\delta(q_0, b, Z) = (q_1, BZ)$	$\delta(q_0, \lambda, Z) = (f, \lambda)$
$\delta(q_1, a, A) = (q_1, AAA)$	$\delta(q_1, b, A) = (q_1, \lambda)$	$\delta(q_1, \lambda, Z) = (q_0, Z)$
$\delta(q_2, a, B) = (q_3, \lambda)$	$\delta(q_2, b, B) = (q_2, BB)$	$\delta(q_2, \lambda, Z) = (q_0, Z)$
$\delta(q_3, \lambda, B) = (q_2, \lambda)$	$\delta(q_3, \lambda, Z) = (q_1, AZ)$	

- Give an sequence showing that string bab is in $L(P)$
- Give a sequence showing that string abb is in $L(P)$.
- Give the contents of the stack after P has read $b^7 a^4$ from its input.
- Informally describe $L(P)$.

20. Let M be the PDA defined by

$$Q = \{q_0, q_1, q_2\}$$

$$\Sigma = \{a, b\}$$

$$\Gamma = \{a\}$$

$$F = \{q_1, q_2\}$$

$$\delta(q_0, a, \lambda) = \{q_0, a\}$$



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$$\delta(q_0, \lambda, \lambda) = \{q_1, \lambda\}$$

$$\delta(q_0, b, a) = \{q_2, \lambda\}$$

$$\delta(q_1, \lambda, a) = \{q_0, \lambda\}$$

$$\delta(q_2, b, a) = \{q_2, \lambda\}$$

$$\delta(q_2, \lambda, a) = \{q_2, \lambda\}$$

- a. Describe the language accepted by M.
- b. Give the transition diagram of M.
- c. Trace all computations of the strings aab, abb, aba in M.
- d. Show that aabb, aaab $\in L(M)$