

# **AUTOMATA FORMAL LANGUAGES AND LOGIC**



## **Lecture Notes on Non-deterministic Finite Acceptors (NFA/ $\lambda$ -NFA)**

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# 1.Non-deterministic Finite Accepters/Automata(NFA)

An NFA can be represented by a 5-tuple  $(Q, \Sigma, \delta, q_0, F)$  where –

- $Q$  is a finite set of states.
- $\Sigma$  is a finite set of symbols called the alphabets.
- $\delta$  is the transition function where  $\delta: Q \times \Sigma \rightarrow 2^Q$   
(Here the power set of  $Q$  ( $2^Q$ ) has been taken because in case of NFA, from a state, transition can occur to any combination of  $Q$  states)
- $q_0$  is the initial state from where any input is processed ( $q_0 \in Q$ ).
- $F$  is a set of final state/states of  $Q$  ( $F \subseteq Q$ ).

In an NFA, a (state, symbol) combination may lead to several states simultaneously.

If a transition is labeled with the empty string as its input symbol, the NFA may change states without consuming input.

An NFA may have undefined transitions.

In DFA we have to be determined about the transition of every input symbol on every state.

IN NFA we do not have this restriction.

## Example 1:

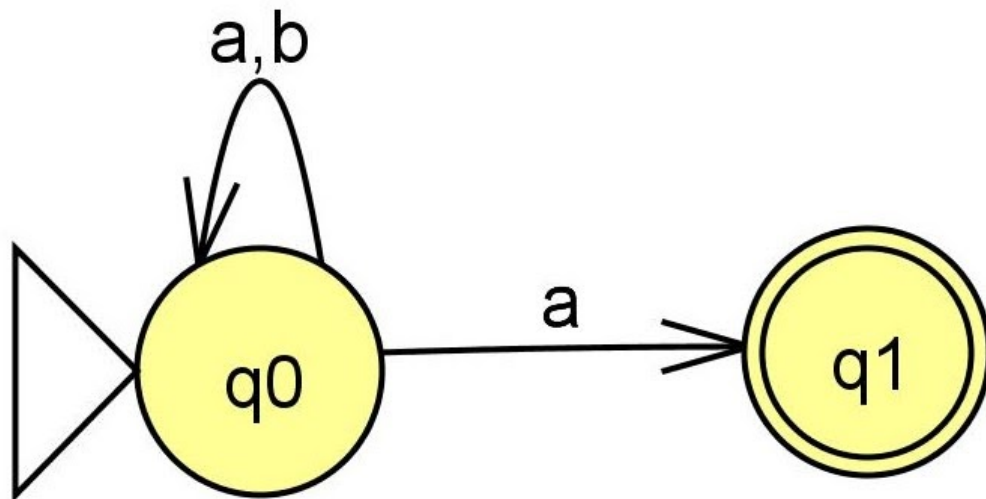
Assume  $\Sigma = \{a, b\}$

Consider language  $L1 = \{\text{set of string that end in 'a'}\}$

Minimum string : a

$L1 = \{a, (a's \text{ or } b's) a\}$

We will first construct the NFA for the set of strings ending in a.



- In NFA we can have multiple transitions on the same symbol from a given state or there can be no transition on a symbol from a given state.
- We have two transitions on 'a' in  $q_0$ , one remains in the same state and the other moves to state  $q_1$ .
- In  $q_1$ , there is no transition on a and b. This is allowed in NFA but not in DFA.
- It is easier to construct a NFA, since we don't have to worry about the transition of every symbol in every state.
- We can convert NFA to DFA using an algorithm.

- We will try out simultaneously all the possible paths the string can take ,even if one of the paths leads us to the final state then we will accept the string.

The diagram illustrates the execution of a finite automaton for the input string "abab". It shows four snapshots of the automaton's state at different points in time:

- Snapshot 1 (Top):** The automaton is in state  $q_0$  (the start state). It has two outgoing transitions: one labeled  $a$  to another  $q_0$  state, and one labeled  $b$  to a  $q_1$  state. The  $q_1$  state is an accepting state, indicated by a yellow box labeled  $x$  below it.
- Snapshot 2 (Middle):** After reading the first 'a', the automaton is in state  $q_0$ . It has two outgoing transitions: one labeled  $a$  to another  $q_0$  state, and one labeled  $b$  to a  $q_1$  state. The  $q_1$  state is an accepting state, indicated by a yellow box labeled  $x$  below it.
- Snapshot 3 (Bottom):** After reading the first 'ab', the automaton is in state  $q_1$ . It has two outgoing transitions: one labeled  $a$  to a  $q_0$  state, and one labeled  $b$  to another  $q_1$  state. The  $q_1$  state is an accepting state, indicated by a yellow box labeled  $x$  below it.
- Snapshot 4 (Bottom):** After reading the full input "abab", the automaton is in state  $q_0$ . It has two outgoing transitions: one labeled  $a$  to another  $q_0$  state, and one labeled  $b$  to a  $q_1$  state. The  $q_1$  state is an accepting state, indicated by a yellow box labeled  $x$  below it.

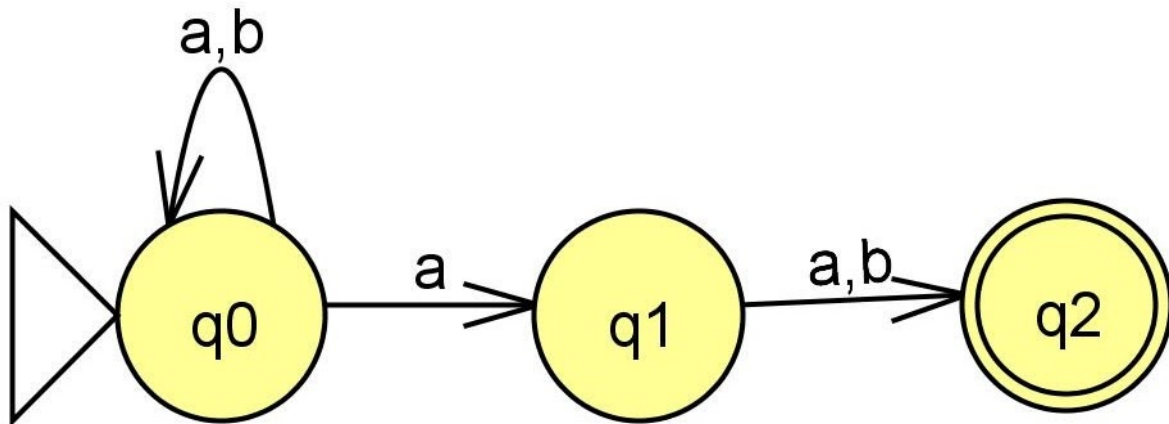
- On  $q_0$ , if we see 'a' there are two possibilities, one it remains in  $q_0$ , other it goes to state  $q_1$ . We will check both the possibilities and see which path takes us to the final state.

- On  $q_0$ , we can go to state  $q_1$  after seeing 'a' but in  $q_1$  we do not have any transition, hence we do not choose this path (we haven't reached the end of the string).
- We will pick the other path where on  $q_0$ , after seeing 'a' we remain in  $q_0$ .
- Now, we are in  $q_0$ , the next symbol from the string "abaa" is '**b**'.
- In  $q_0$ , on 'b' we remain in  $q_0$ .
- Now, we are still in  $q_0$ , the next symbol from the string "abaa" is '**a**'.
- On  $q_0$ , we can go to state  $q_1$  after seeing 'a' but in  $q_1$  we do not have any transition, hence we do not choose this path.
- We will pick the other path where on  $q_0$ , after seeing 'a' we remain in  $q_0$ .

## Example 2:

Assume  $\Sigma = \{a, b\}$

Consider language  $L2 = \{\text{Strings where the second symbol from RHS is 'a'}\}$   
(a's or b's) a (a or b)



# $\lambda$ -Non-deterministic Finite Acceptors/Automata( $\lambda$ -NFA)

A nondeterministic finite automata with  $\lambda$ -transitions is a 5-tuple  $(Q, \Sigma, q_0, A, \delta)$ , where  $Q$  and  $\Sigma$  are finite sets,  $q_0 \in Q$ ,  $A \subseteq Q$ , and  $\delta : Q \times (\Sigma \cup \{\lambda\}) \rightarrow 2^Q$ .

When we have a  $\lambda$ -transition we jump to states without consuming any input.

## Example 1:

Assume  $\Sigma = \{a, b\}$

Consider language  $L1 = \{\text{Strings that start and end with the same symbol}\}$

$L1 = \{awa \text{ or } bw b \mid w \in \{a, b\}^*\}$

We construct two NFA one each for  $awa$  and  $bwb$  and combine the NFA with a single new start state.

