

Linear Functions of Random Variables

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Topics to be covered...



- Different Transformations
- Means of Linear Combinations of Random Variables
- Variance of Linear Combinations of Random Variables
- Independent Random Variables

Linear Functions of Random variables

- We often construct new random variables by performing arithmetic operations on other random variables.
- For example, we might add a constant to a random variable, multiply a random variable by a constant, or add two or more random variables together.



Linear Functions of Random variables – Different Transformations



Addition – Adding a constant to each value of X.

Subtraction – Subtracting a constant from each value of X.

Multiplication – Multiplying each value of X by a constant.

Division – Dividing each value of X by a constant.

where, X represents a Random Variable.

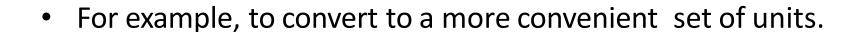
Adding a Constant

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 When a constant is added to a random variable, the mean is increased by the value of the constant, but the variance and standard deviation are unchanged.

Multiplying by a Constant

Often we need to multiply a random variable by a constant.



 Multiplication by a constant affects the mean, variance, and standard deviation of a random variable.



Multiplying by a Constant



• In general, when a random variable is multiplied by a constant, its mean is multiplied by the same constant.

• In general, when a random variable is multiplied by a constant, its variance is multiplied by the square of the constant.

Multiplying by a Constant



• If a random variable is multiplied by a constant and then added to another constant, the effect on the mean and variance.

Applying Transformations to a random variable X



Tranformatio E	Effect on mean	Effect on Variance	Effect on shape of probability histogram	
+ or - a Constant	√ Changes	X Doesn't change	X Doesn't change	
* or / by a Constant	√ Changes	√ Changes	X Doesn't change	

Applying Transformations to a random variable X

Tranformation	Effect on mean	Effect on Variance	Effect on SD	Effect on shape of probability histogram
Add Ex: Y = X + 2	E(Y) = E(X) + 2	Var(Y) = Var(X)	SD(Y) = SD(X)	Doesn't change
Subtract Ex: Y = X - 2	E(Y) = E(X) - 2	Var(Y) = Var(X)	SD(Y) = SD(X)	Doesn't change
Multiplying by a constant Ex: Y = X * 2	E(Y) = E(X) * 2	Var(Y) = Var(X) * 2 ²	SD(Y) = SD(X) * 2	Doesn't change
Dividing by a constant Ex: Y = X/2	E(Y) = E(X) / 2	Var(Y) = Var(X) /2 ²	SD(Y) = SD(X) / 2	Doesn't change



Combining Random Variables

Combining Variables

Many interesting statistics problems require us to examine two or more random variables.

Ex.: Casino games

- Roulette games
- Deal of shuffled cards
- Roll of Dice



Means of Linear Combinations of Random Variables

Consider the case of adding two random variables.



Means of Linear Combinations of Random Variables

To find the mean of a linear combination of random variables.



Variance for Linear Combinations of IRV



Variance for Linear Combinations of IRV



Example

SRS travels offers a half-day trip in a tourist area. There must be at least 2 passengers for the visit to run. The vehicle provided by SRS travels can hold up to 6 passengers.

SRS travels charges Rs. 150 per passenger. The amount spent on petrol and permit by SRS travels per trip is Rs. 100. Number of passengers that turn up on a randomly selected day(X) and the corresponding probabilities are given below.

X	2	3	4	5	6
p(x)	0.15	0.25	0.35	0.20	0.05



Example

Define new random variables for the following:

- 1. The amount SRS travels collects on a randomly selected day and write the probability distribution function (Let the r.v. be Y).
- 2. Profit made by SRS travels on a randomly selected day. (Let the r.v. be Z).



Example

Probability Distribution	X	p(x)	Y (= 150 * X)	p(y)	Z (= Y – 100)	p(z)
	2	0.15	300	0.15	200	0.15
	3	0.25	450	0.25	350	0.25
	4	0.35	600	0.35	500	0.35
	5	0.20	750	0.20	650	0.20
	6	0.05	900	0.05	800	0.05
Mean	E(X) = 3.75		E(Y) = 150 * E(X)		E(Z) = E(Y) - 100	
			= 562.5		= 462.5	
SD	SD(X) = 1.09		SD(Y) = 150 * SD(X)		SD(Z) = SD(Y) = 163.5	
			= 163.5			



Independent Random Variables

- The notion of independence for random variables is very much like the notion of independence for events.
- Two random variables are independent if knowledge concerning one of them does not affect the probabilities of the other.
- When two events are independent, the probability that both occur is found by multiplying the probabilities for each event.

i.e.
$$P(X=a) * P(Y=b)$$

Note: If they are **not independent** then a **covariance term** has to be introduced.

Ex. : Flips of a coin
Rolls of a dice



Independent and Identically Distributed Random Variables

IID Variables

If X₁, X₂, X₃,....,X_n are independent random variables all with same

Distribution then, they are called independent and identically distributed (i. i. d.)

Independent: outcome of one observation does not affect the outcome of other observation.

Identically Distributed: They have same mean and variance.

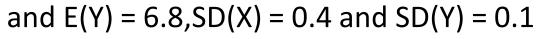
Ex.: Casino games

- Roulette games
- Deal of shuffled cards
- Roll of Dice



Example

If X and ,Y are independent random variables such that E(X) = 9.5



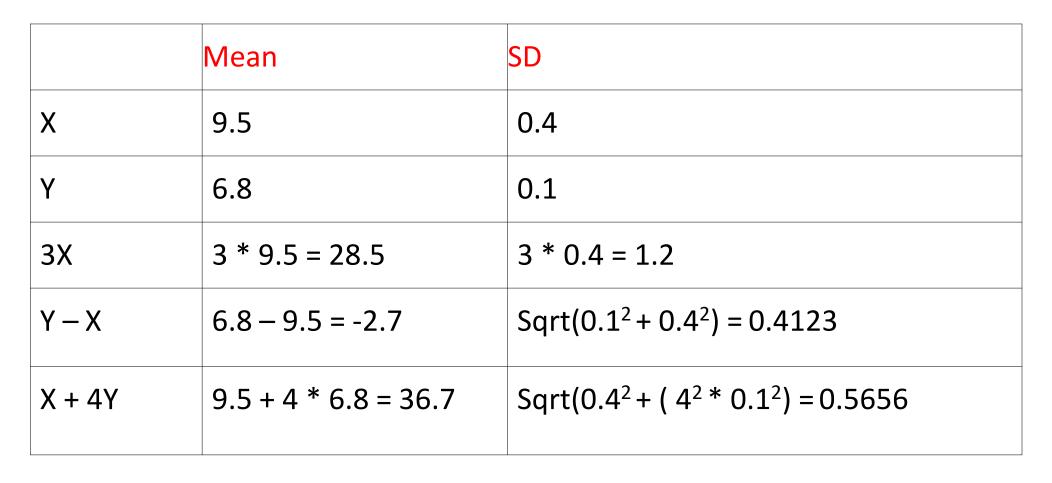
Find Means and SD of the following:

- 1) 3X
- 2) Y-X
- 3) X + 4Y



Example

Solution:





Combinations of Two Random Variables

- •Means combine very easily with addition or subtraction.
- •We can't add standard deviations.
- Variances can be added,
- •Add variances even if subtracting the random variable.

Note: Attention!!!

X + X not same as 2X



Example

Rectangular plastic covers for a compact disc (CD) tray have specifications regarding length and width. Let X be the length and Y be the width, each measured to the nearest millimeter, of a randomly sampled cover. The probability mass function of X is given by P(X = 129) = 0.2, P(X = 130) = 0.7, and P(X = 131) = 0.1. The probability mass function of Y is given by P(Y = 15) = 0.6 and P(Y = 16) = 0.4. The area of a cover is given by A = XY. Assume X and Y are independent. Find the probability that the area is 1935 mm².



Example

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Solution

The area will be equal to 1935 if X = 129 and Y = 15. Therefore

$$P(A = 1935) = P(X = 129 \text{ and } Y = 15)$$

= $P(X = 129)P(Y = 15)$ since X and Y are independent
= $(0.2)(0.6)$
= 0.12

Example

A piston is placed inside a cylinder. The clearance is the distance between the edge of the piston and the wall of the cylinder and is equal to one-half the difference between the cylinder diameter and the piston diameter. Assume the piston diameter has a mean of 80.85 cm with a standard deviation of 0.02 cm. Assume the cylinder diameter has a mean of 80.95 cm with a standard deviation of 0.03 cm. Find the mean clearance. Assuming that the piston and cylinder are chosen independently, find the standard deviation of the clearance.



Example

Solution

Let X_1 represent the diameter of the cylinder and let X_2 the diameter of the piston. The clearance is given by $C = 0.5X_1 - 0.5X_2$. Using Equation (2.49), the mean clearance is

$$\mu_C = \mu_{0.5X_1 - 0.5X_2}$$

$$= 0.5\mu_{X_1} - 0.5\mu_{X_2}$$

$$= 0.5(80.95) - 0.5(80.85)$$

$$= 0.050$$

Since X_1 and X_2 are independent, we can use Equation (2.53) to find the standard deviation σ_C :

$$\sigma_C = \sqrt{\sigma_{0.5X_1 - 0.5X_2}^2}$$

$$= \sqrt{(0.5)^2 \sigma_{X_1}^2 + (-0.5)^2 \sigma_{X_2}^2}$$

$$= \sqrt{0.25(0.02)^2 + 0.25(0.03)^2}$$

$$= 0.018$$



Example

Do It Yourself!!!

- X represents the number of passengers on a randomly selected trip with SRS travels.
- Y represents the number of passengers on a randomly selected trip with VRL Logistics.

The probability distributions of X and Y are given below:

		5 0.20	
	4 0.2		

Define a new random variable T that represents number of passengers that SRS and VRL can expect on a randomly selected day.



Example

Do It Yourself!!!



- 1. Define a new random variable T that represents number of passengers that SRS and VRL can expect on a randomly selected day.
- 2. Find mean and variance of T.



THANK YOU

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