AUTOMATA FORMAL LANGUAGES AND LOGIC

Lecture notes on Converting Regular Expression to Finite Automata

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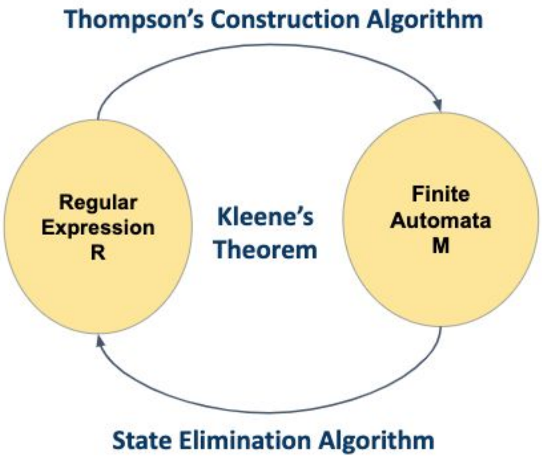
1. Regular Expression to Finite Automata

Regular expressions and finite automata have equivalent expressive power:

Many software tools work by matching regular expressions against text. Text processing utilities use regular expressions to describe advanced search patterns, but NFAs are better suited for execution on a computer.

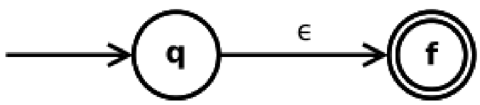
The equivalence of regular expressions and finite automata is known as Kleene's theorem. We use Thompson Construction algorithm to convert a RE to NFA . This algorithm is of practical interest, since it can compile regular expressions into NFAs. We use a State Elimination method in order to convert a finite automata to Regex.



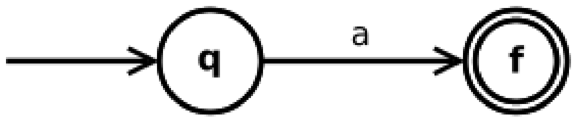
Converting regular expressions to finite automata using Thompson Construction Method:

The algorithm works recursively by splitting an expression into its constituent subexpressions, from which the NFA will be constructed using a set of rules.
Following are the rules :

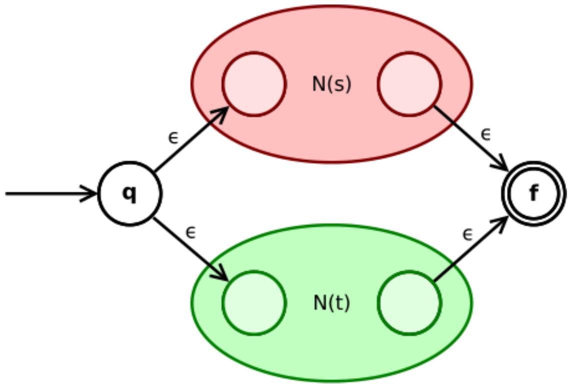
- If the operand is epsilon, then our FA has two states, q (the start state) and F (the final, accepting state), and an epsilon transition from q to F.



- If the operand is a character a, then our FA has two states, q (the start state) and F (the final, accepting state), and a transition from q to F with label a.

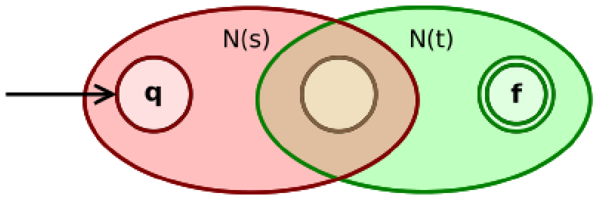


- The **union expression** *s*|*t* is converted to



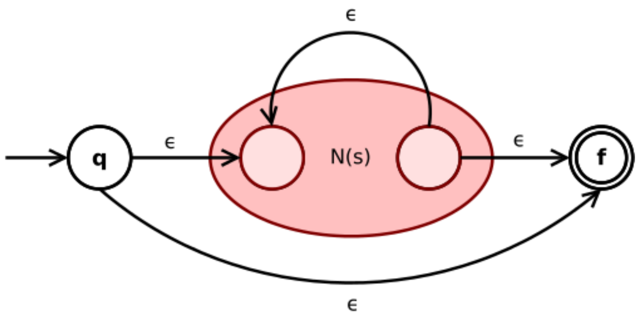
State *q* goes via ε either to the initial state of Automata of s and t . *N*(*s*) or *N*(*t*). Their final states become intermediate states of the whole NFA and merge via two ε-transitions into the final state of the NFA.

- The **concatenation expression** *st* is converted to



The initial state of *N*(*s*) is the initial state of the whole NFA. The final state of *N*(*s*) becomes the initial state of *N*(*t*). The final state of *N*(*t*) is the final state of the whole NFA.

- The **Kleene star expression** s^* is converted to



An ε-transition connects the initial and final state of the NFA with the sub-NFA .

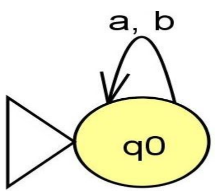
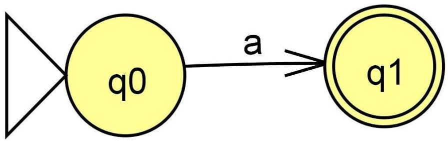
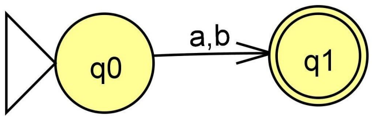
N(*s*) Automata for s in between. Another ε-transition from the inner final to the inner initial state of *N*(*s*) allows for repetition of expression *s* according to the star operator.

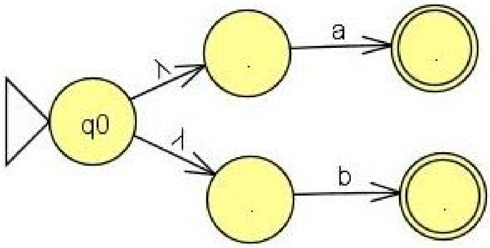
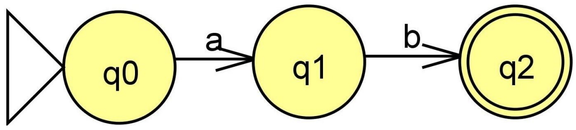
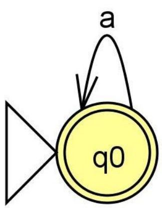
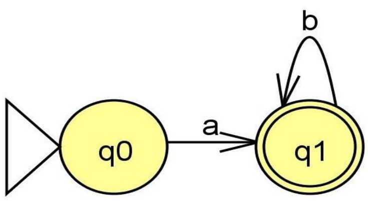
- The **parenthesized expression** (*s*) is converted to *N*(*s*) itself.

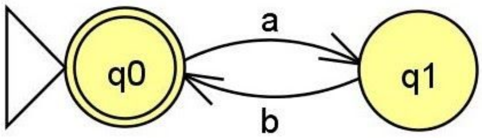
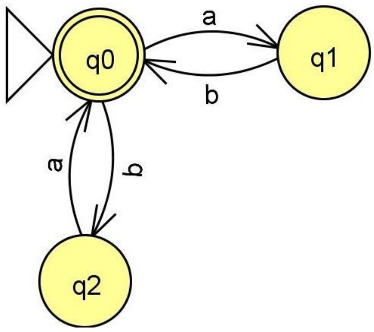
With these rules, using the **empty expression** and **symbol** rules as base cases, it is possible to prove with mathematical induction that any regular expression may be converted into an equivalent NFA.

1.1. Basic Regular Expressions to NFA/ λ -NFA

Let us start with some basic regular expressions.
Assume Σ={a,b}.

Regular Expression	NFA/ λ -NFA
ϕ	 ϕ represents not accepting anything that is,no accepting/final state . In state q0,any input symbol is rejected.
a	 In state q0 ,we will wait for input symbol ‘a’. Once we see ‘a’ in state q0 we will move to state q1 and accept it.
a+b	 We accept either a or b. In state q0,if we see ‘a’ we move to state q1 and accept it or

	if we see ‘b’ we move to state q1 and accept it. a+b can also be represented by treating a and b as two different regular expressions ,we get the automata as shown below: 
a.b	 a followed by b and accept the string.
a*	 a* represents any number of ‘a’s including λ (empty string). So we make q0 as the final state. When we see * we put a self loop.
ab*	 ab*. ‘a’ is the minimum string. In state q0 we look for the symbol ‘a’ . Once we see the symbol ‘a’ in q0 we move to state q1 and

	accept it . In state q1 we can accept any number of b's. When we see a * we put a loop.
(ab)*	 (ab)*=sequences of ab= λ ,ab,abab,ababab,.......... We look for 'a' followed by a 'b' . In state q0 we look for 'a' and move to state q1. In state q1 we see a 'b' and move back to state q0 and accept it. Since there is a *,we make q0 as the final state.
(ab+ba)*	 Strings accepted by automata=Sequences of "ab" and/or sequences of "ba".

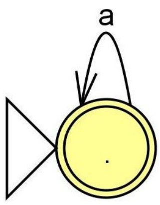
1.2: Higher end Problems on converting Regular Expression to Finite Automata

Problem #1:

Convert the regular expression a*+b*+c* to finite automata.

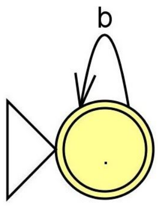
Solution :

Automata accepting a* :



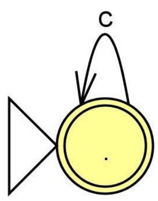
- a* indicates any number of a’s including λ .
- When we see * we put a loop.

Automata accepting b*:



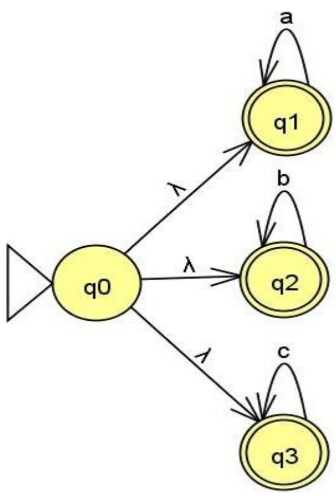
- b* indicates any number of b’s including λ .
- When we see * we put a loop.

Automata accepting c*:



- c* indicates any number of c’s including λ .
- When we see * we put a loop.

We can combine the automata accepting a*,b* and c* by introducing a new start state q0 with λ ,-transitions connecting to the old start states of a*,b* and c*.

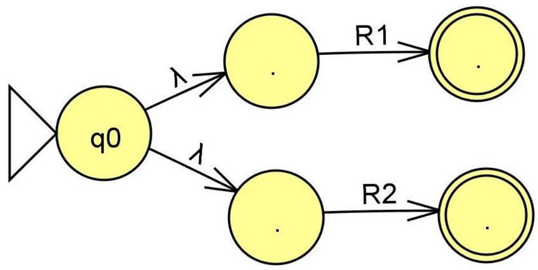


Problem #2:

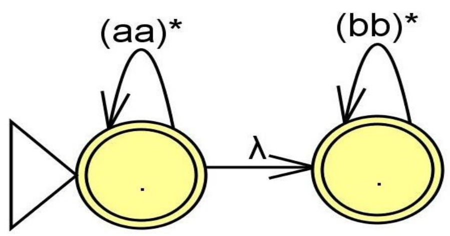
Convert the regular expression (aa)*(bb)*+a(aa)*b(bb)* to finite automata.

Let,

- R1=(aa)*(bb)*
- R2=a(aa)*b(bb)*.
- Construct accepter/automata for R1
- Construct accepter/automata for R2.
- Introducing a new start state on λ we reach the start state of R1 and R2.

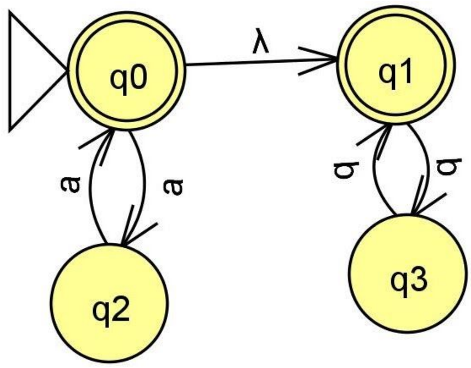


Brief diagram of automata for R1=(aa)*(bb)*



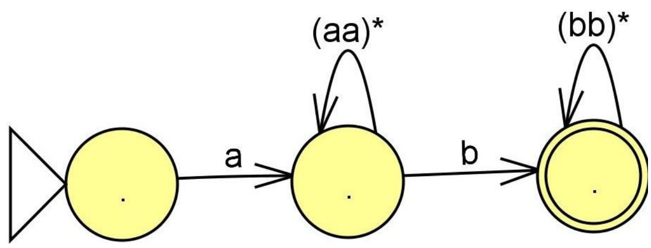
We look for sequences of any number of "aa" 's followed by sequences of any number of "bb"'s.

Expanded form of automata for R1=(aa)*(bb)*



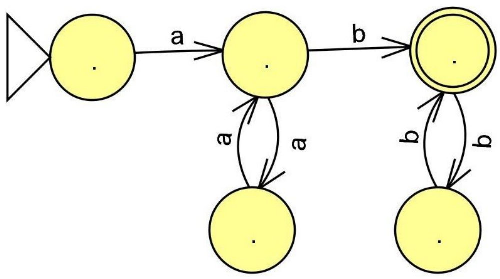
In state q0 we look for ‘a’ followed by symbol ‘a’ or ‘b’ followed by symbol ‘b’.

Brief diagram of automata for R2=a(aa)*b(bb)*

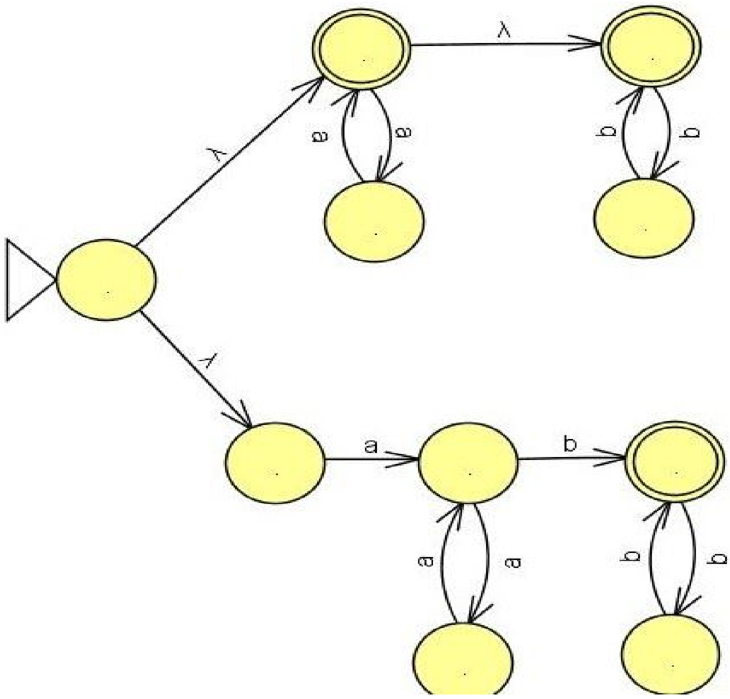


We look for “a “ followed by any number of sequences of “aa” ,followed by a ”b” and sequences of any number of “bb”. When we see a star we put a loop.

Expanded form of automata for R2=a(aa)*b(bb)*



Automata for the regular expression R1+R2= (aa)*(bb)*+a(aa)*b(bb)*.
We will combine the automata for R1 and automata for R2 by introducing a new start state on λ we reach the start state of R1 and R2.



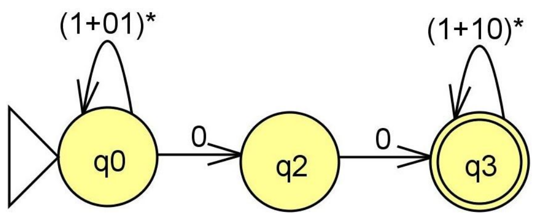
Problem #3:

Convert the regular expression (1+01)*00(1+10)* to finite automata.

Solution :

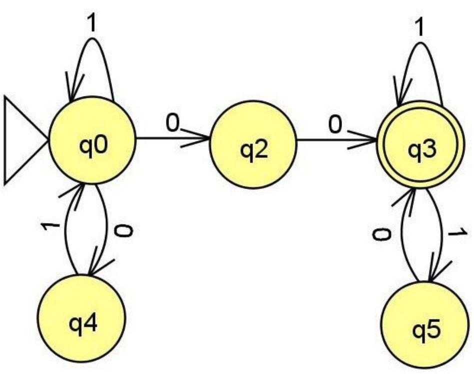
(1+01)*00(1+10)* is the concatenation of (1+01)* , 00 and (1+10)* .

Brief diagram of automata for (1+01)*00(1+10)*



- When we see a * we put a loop.

Expanded form of automata for (1+01)*00(1+10)*



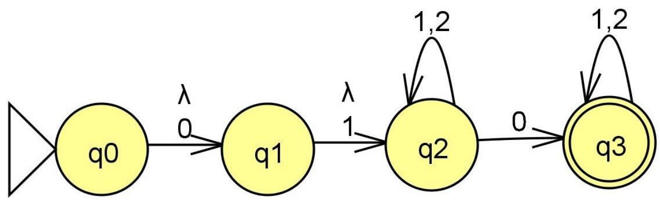
Problem #4:

Convert the regular expression (0+ λ)(1+ λ)(1+2)*0(2+1)* to finite automata.

Solution :

(0+ λ)(1+ λ)(1+2)*0(2+1)* is the concatenation of (0+ λ) , (1+ λ) , (1+2)*, 0 and (2+1)*.

Automata for (0+ λ)(1+ λ)(1+2)*0(2+1)*



When we see * we put a self loop.

Problem #5:

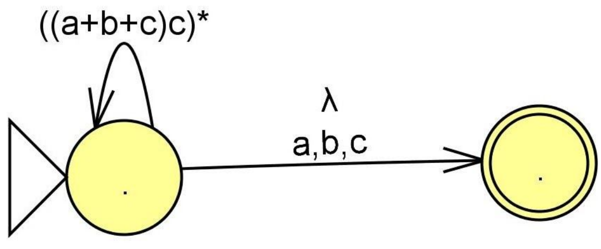
Convert the regular expression ((a+b+c)c)*(a+b+c+ λ) to finite automata.

Solution :

((a+b+c)c) indicates a or b or c followed by a c ,these are strings of length 2.

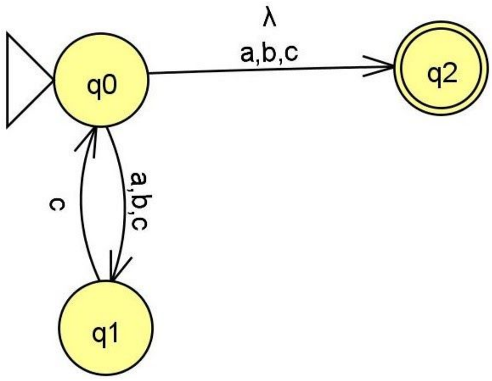
((a+b+c)c)* strings of length 2 repeated any number of times.

((a+b+c)c)*(a+b+c+ λ) strings of length 2 repeated any number of times. Followed by ‘a’ or ’b’ or ‘c’ or nothing (λ).



When we see “* “ we put a self loop.

Expanded form of automata for ((a+b+c)c)*(a+b+c+ λ)



- In q0,when we see ‘ a’ or ‘b’ or ‘c’ it has to be followed by “‘c“ . That is strings of length 2 (‘a’ or ‘b’ or ‘c’ followed by ‘c’)repeated any number of times.
- Followed by ‘a’ or ’b’ or ‘c’ or nothing (λ).