



# Automata Formal Languages & Logic

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**Preet Kanwal**

Department of Computer Science & Engineering

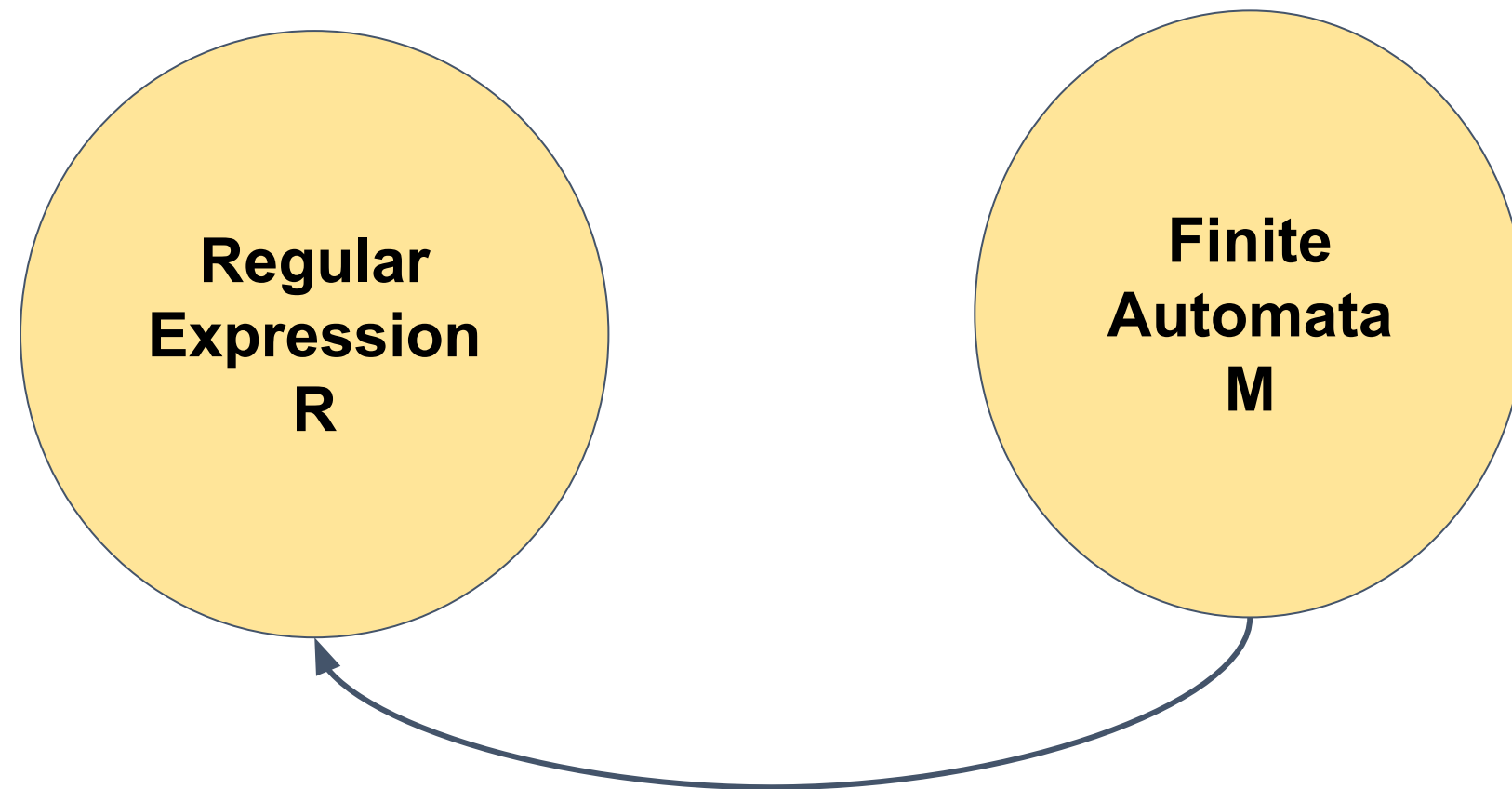
# Automata Formal Languages & Logic

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## Unit 2

**Preet Kanwal**

Department of Mechanical Engineering



**State Elimination Algorithm**

### State Elimination Method

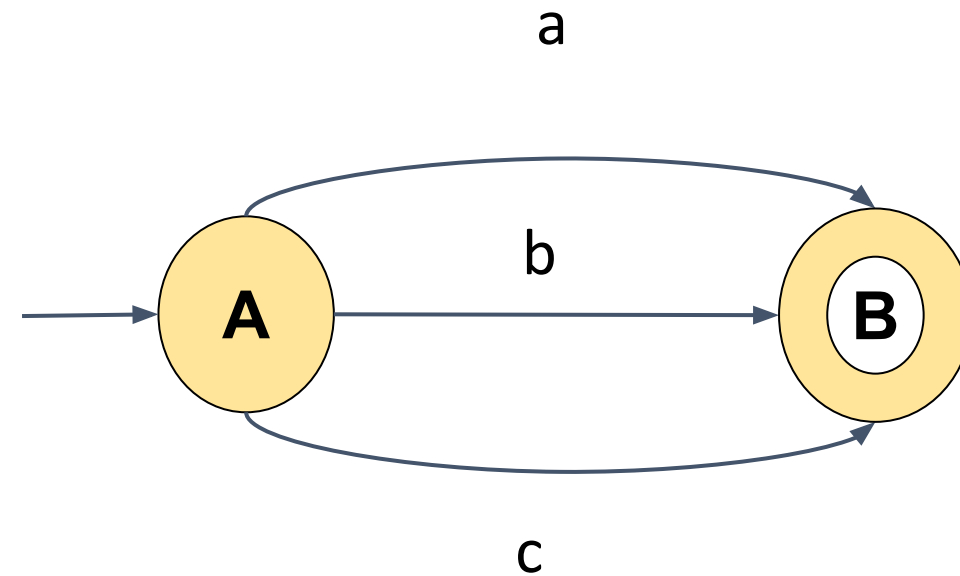
Start with an FA for the language L.

- Add a new start state  $q_s$  and accept state  $q_f$  to the FA.
- Add  $\epsilon$ -transitions from each original accepting state to  $q_f$ , then mark them as not accepting.
  - Repeatedly remove states other than  $q_s$  and  $q_f$  from the FA by “shortcutting” them until only two states remain:  $q_s$  and  $q_f$ .
- The transition from  $q_s$  to  $q_f$  is then a regular expression for the FA.

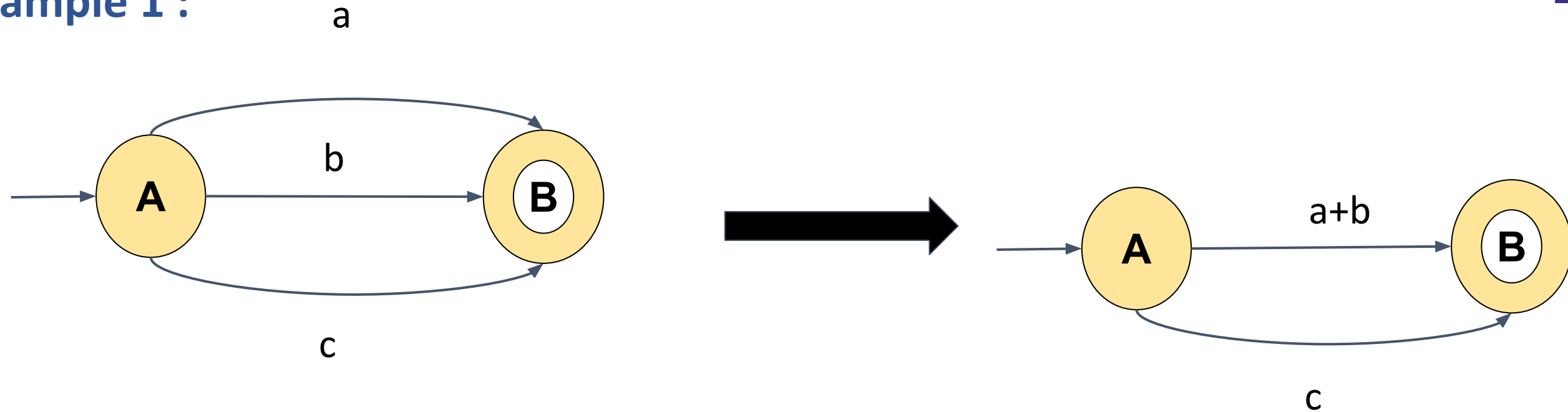
# Automata Formal Languages and Logic

## Unit 2 - Finite Automata to Regular Expression

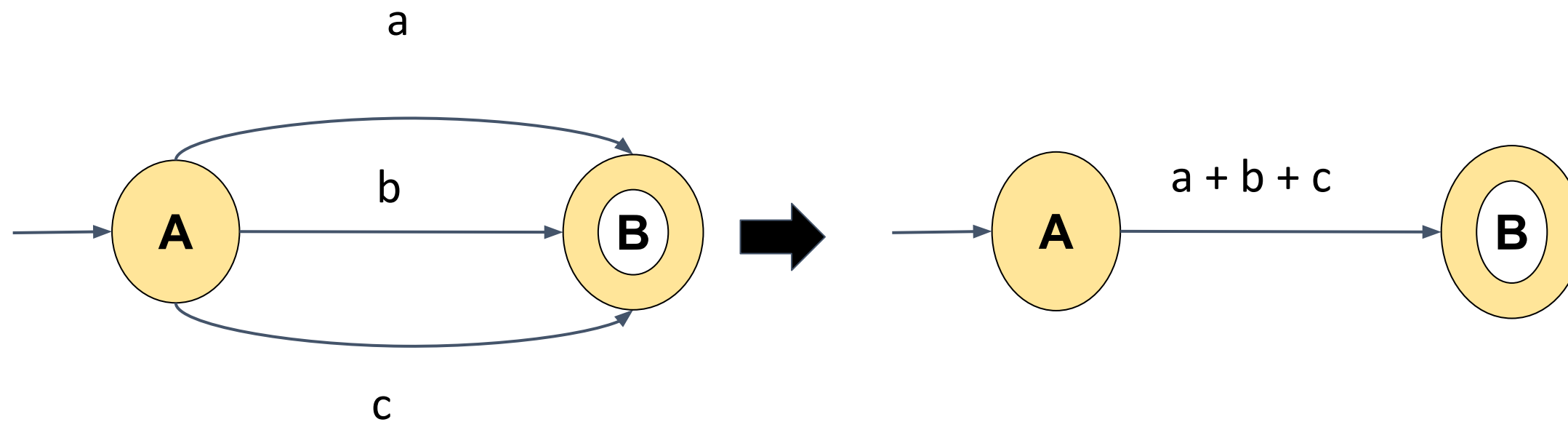
### Example 1 :



Example 1 :



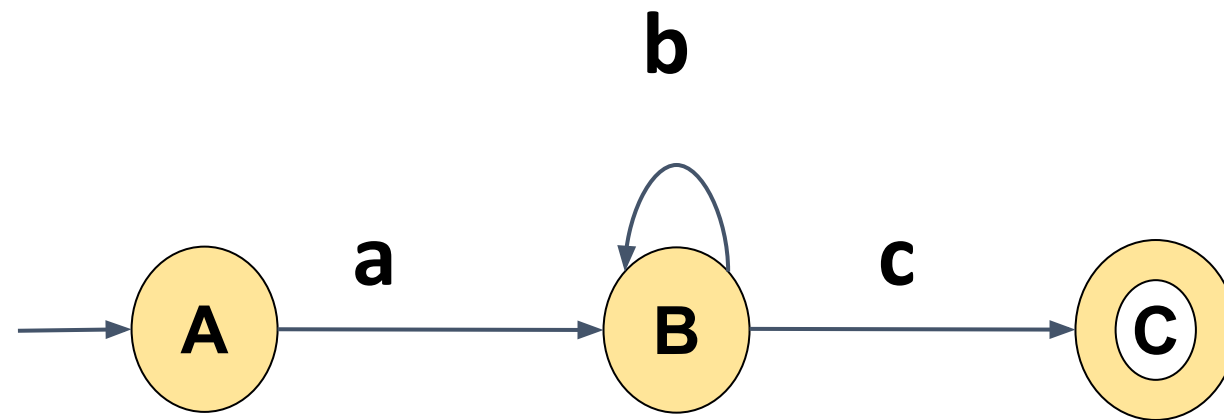
### Example 1 :



# Automata Formal Languages and Logic

## Unit 2 - Finite Automata to Regular Expression

Example 2 :

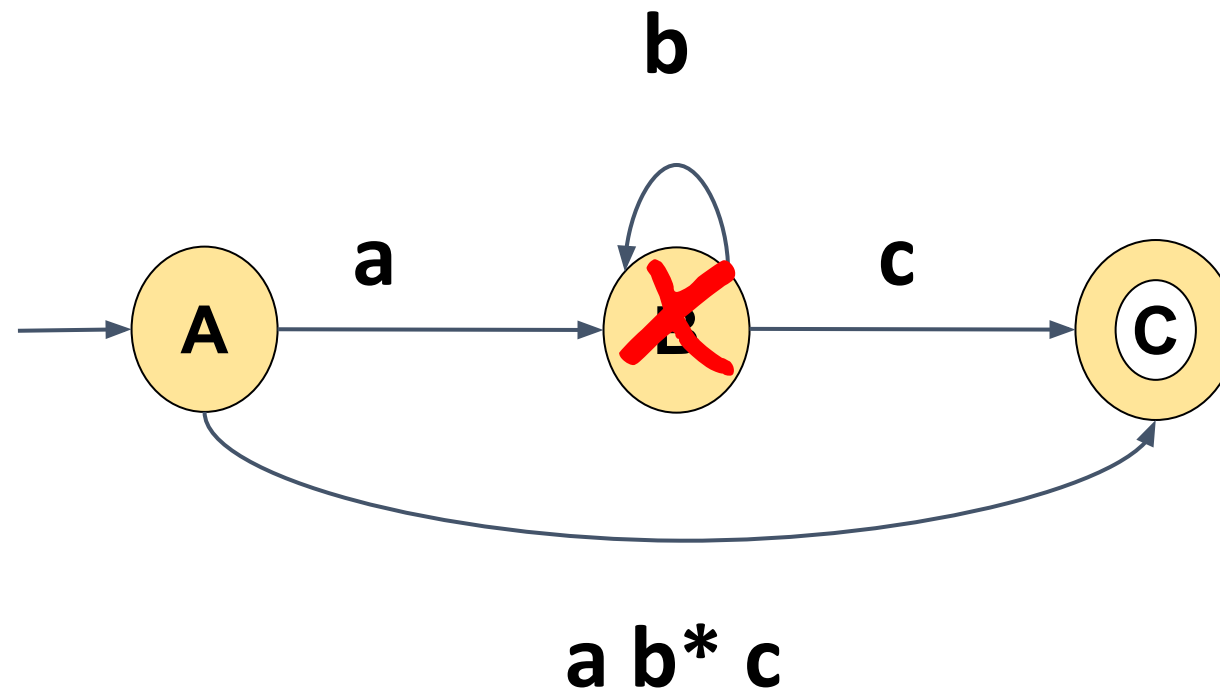




# Automata Formal Languages and Logic

## Unit 2 - Finite Automata to Regular Expression

Example 2 :

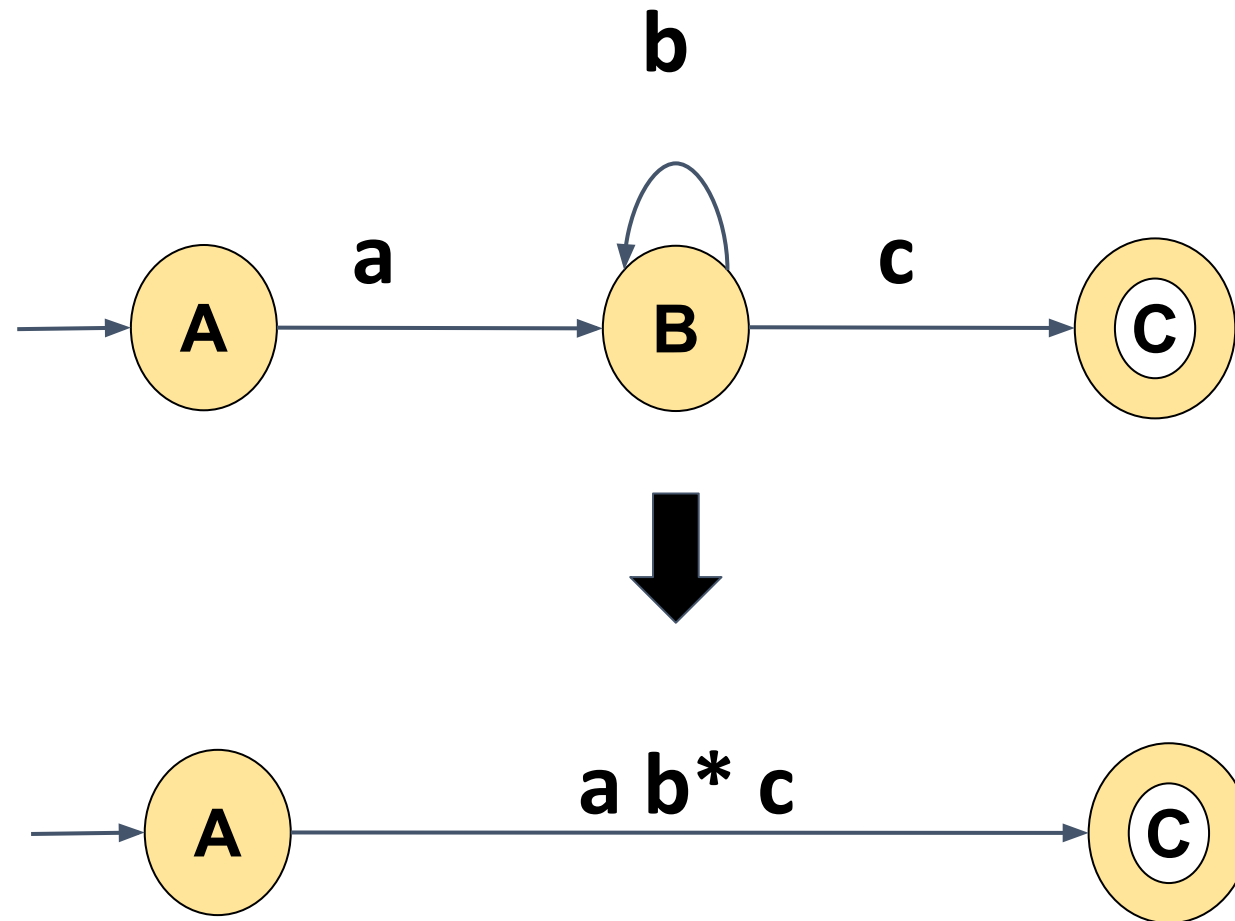


Eliminate B

# Automata Formal Languages and Logic

## Unit 2 - Finite Automata to Regular Expression

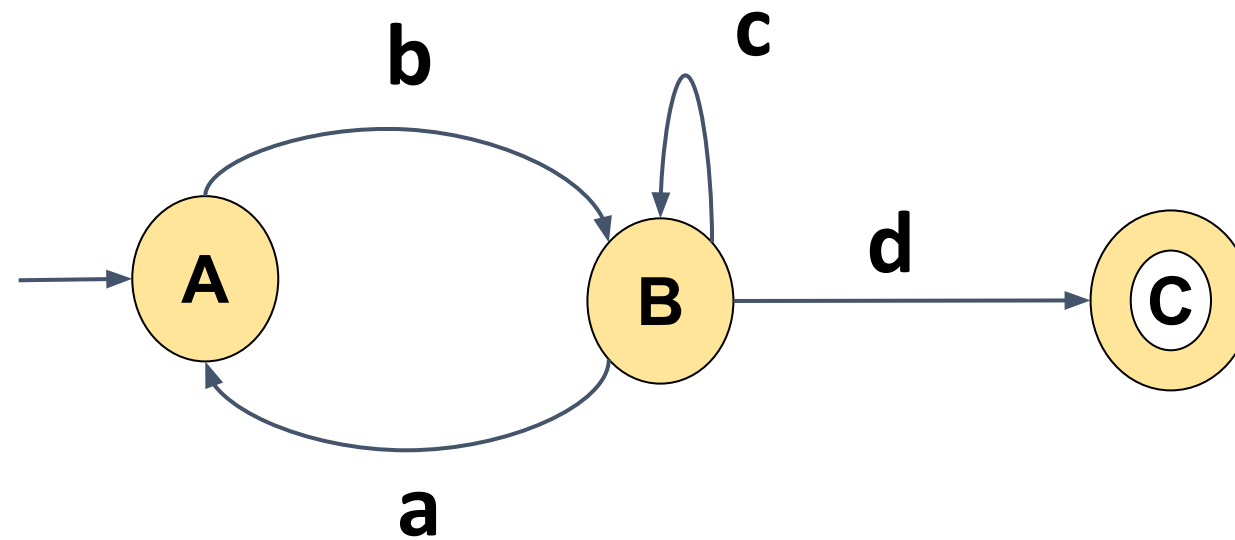
### Example 2 :



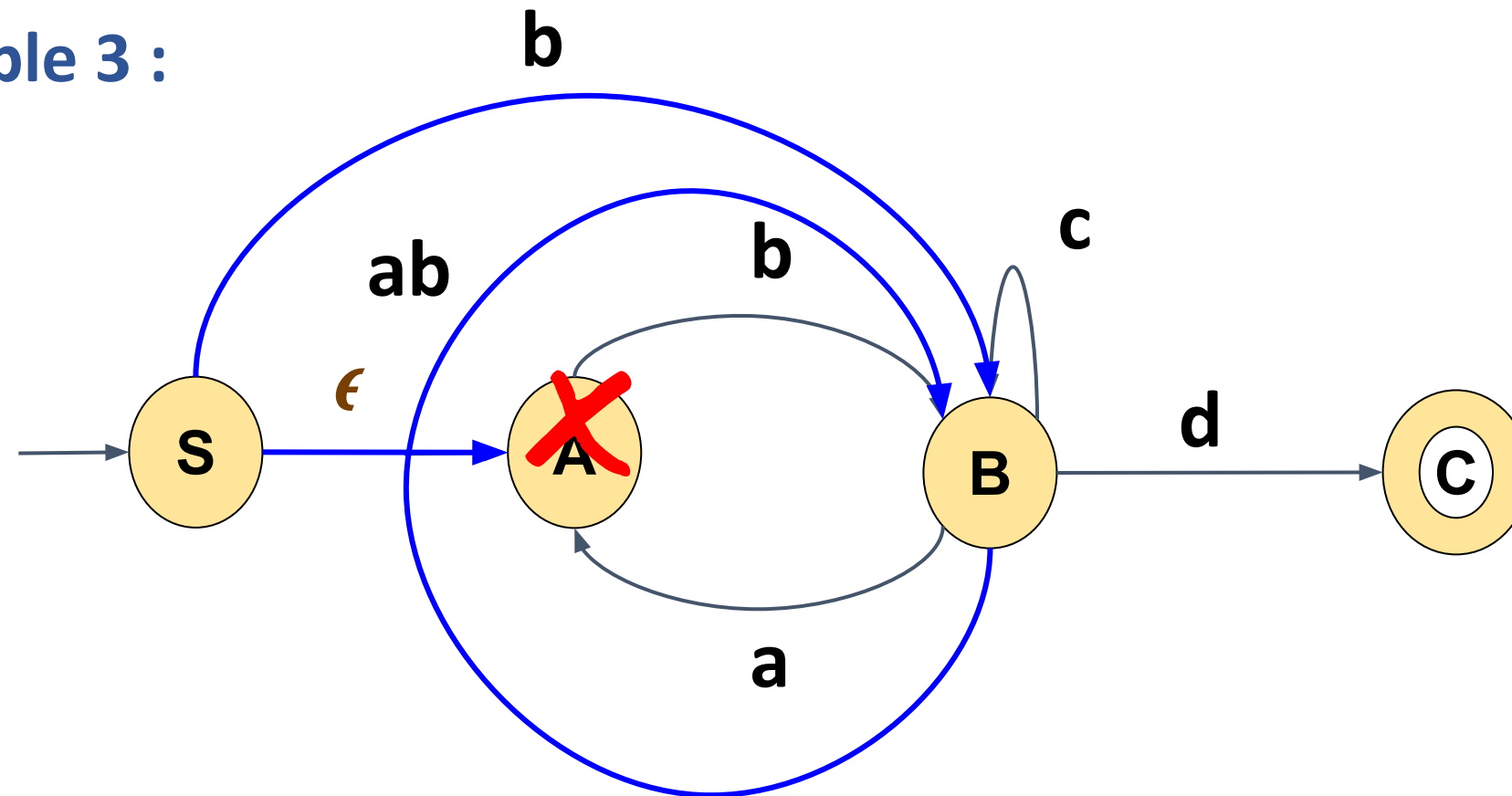
# Automata Formal Languages and Logic

## Unit 2 - Finite Automata to Regular Expression

### Example 3 :

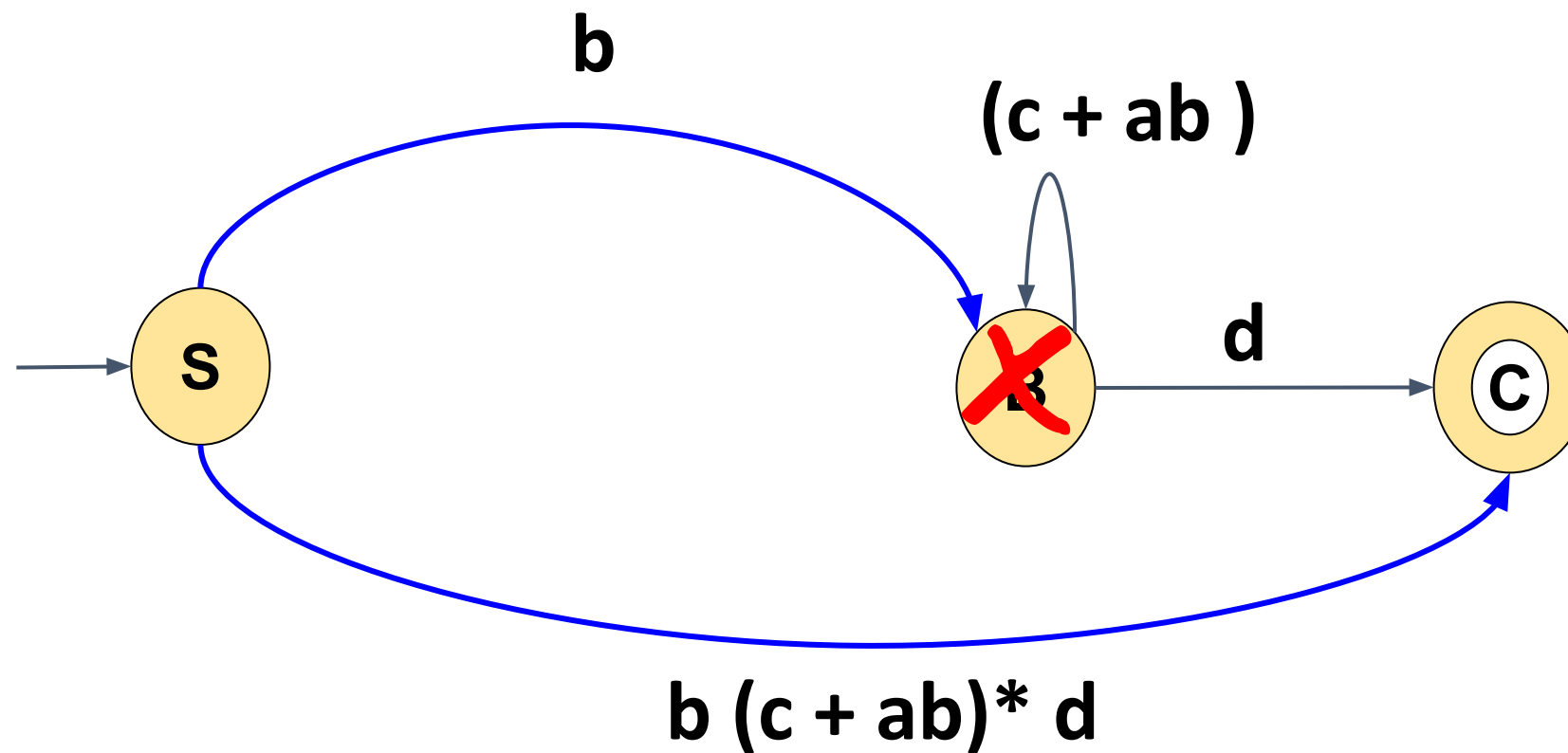


Example 3 :



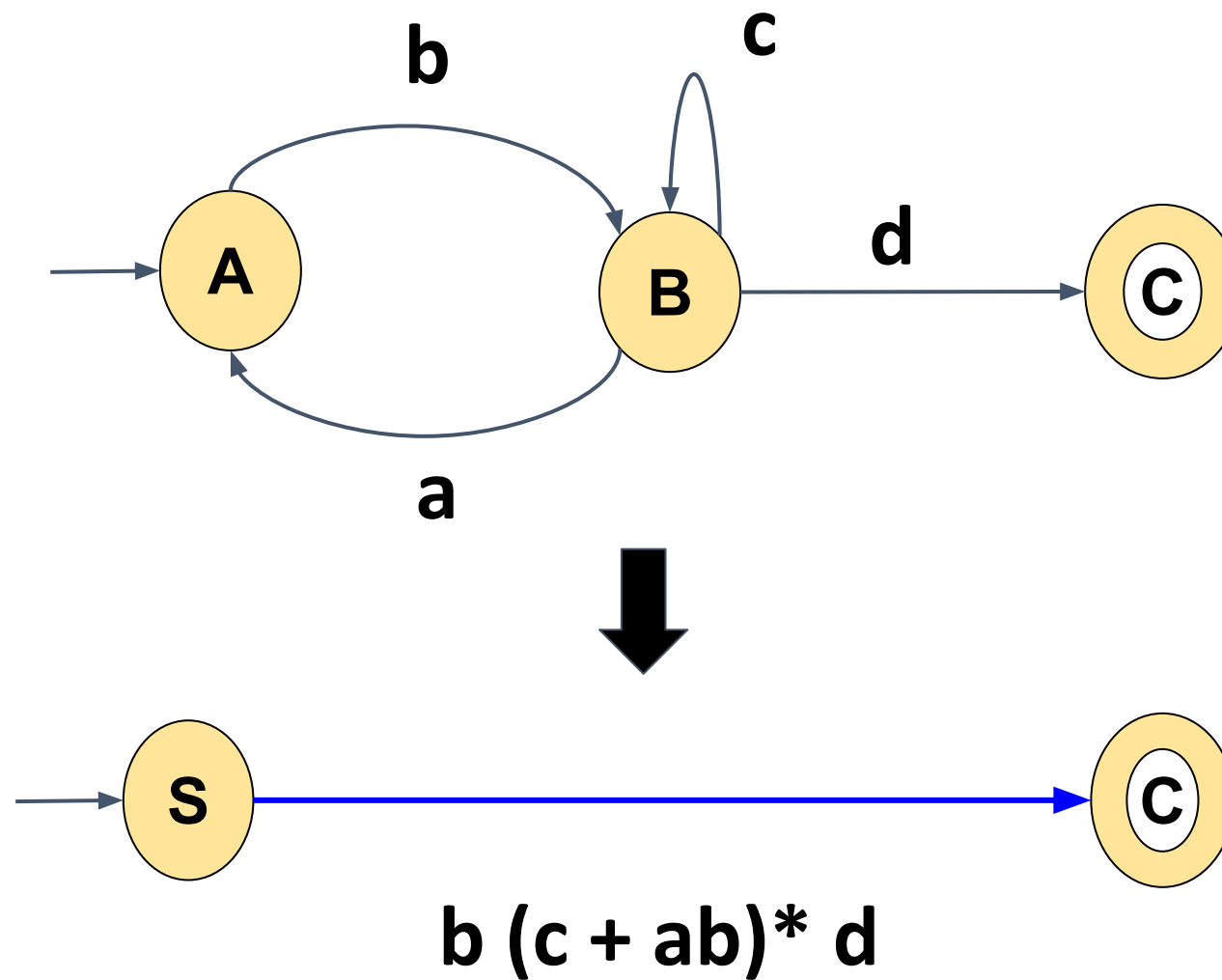
1. Eliminate A

### Example 3 :



1. Eliminate A
2. Eliminate B

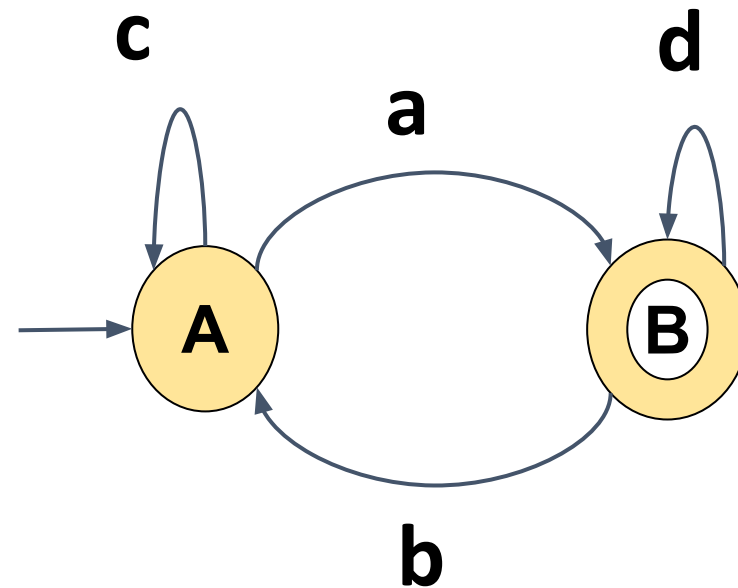
### Example 3 :



# Automata Formal Languages and Logic

## Unit 2 - Finite Automata to Regular Expression

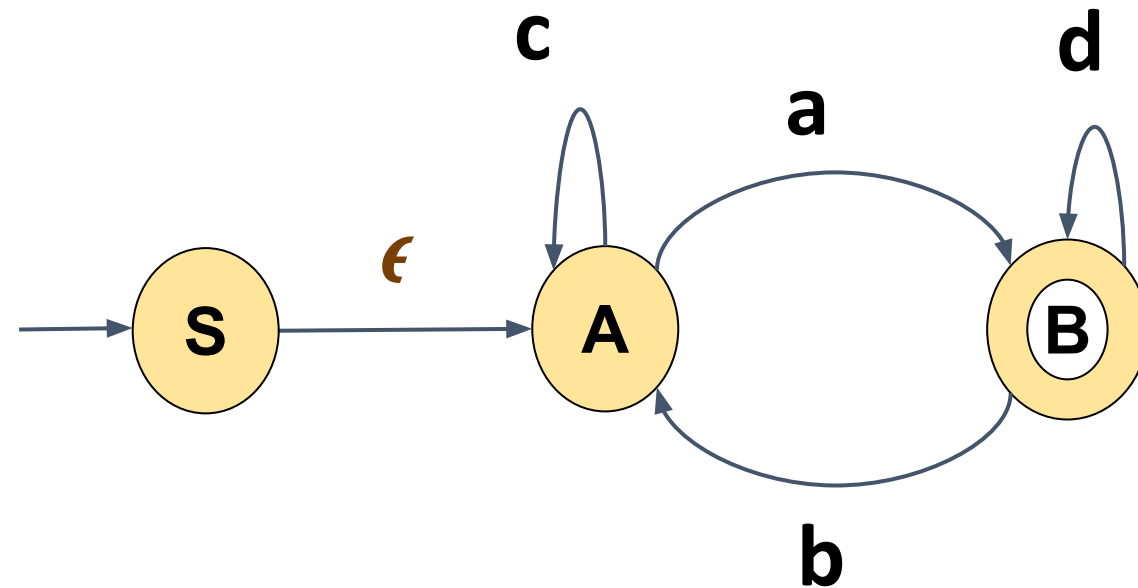
### Example 4 :



# Automata Formal Languages and Logic

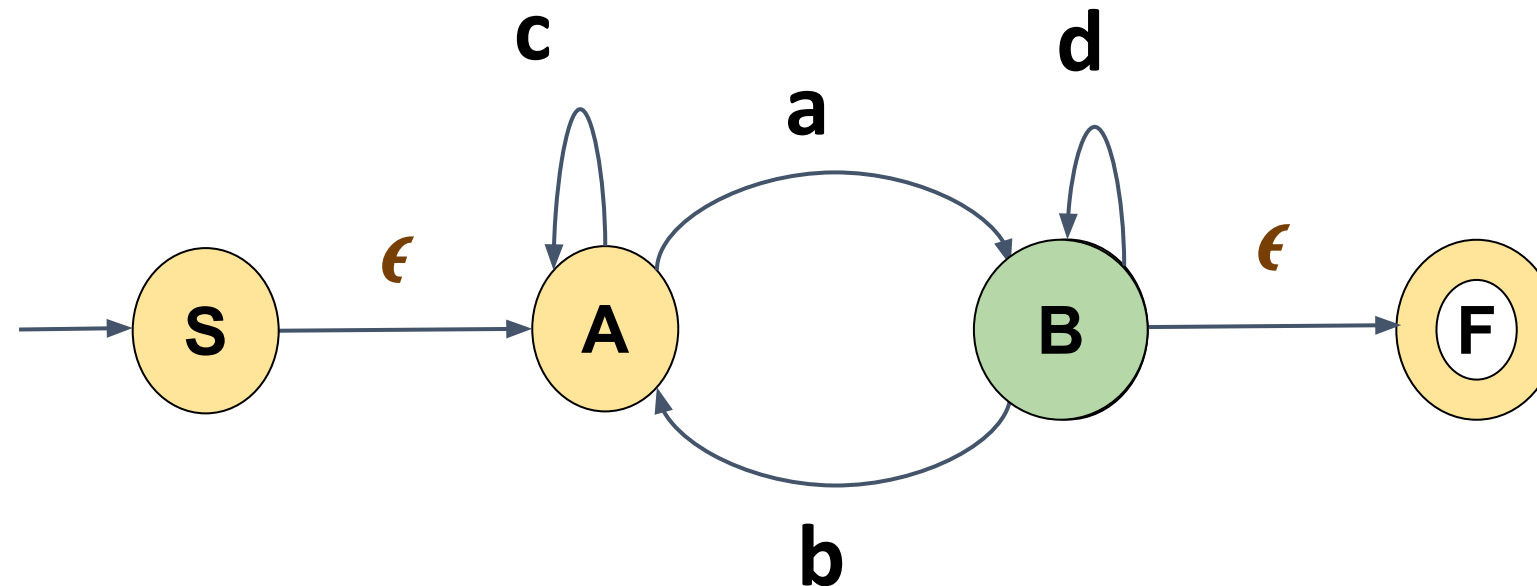
## Unit 2 - Finite Automata to Regular Expression

### Example 4 :



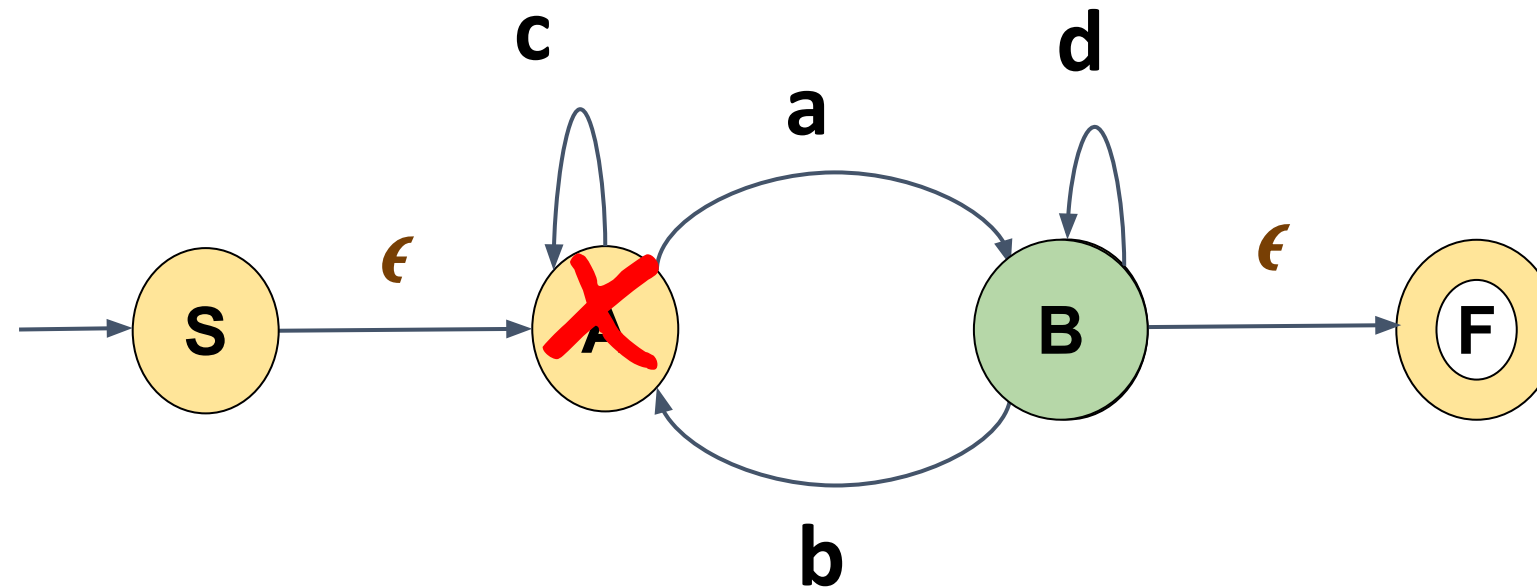


### Example 4 :



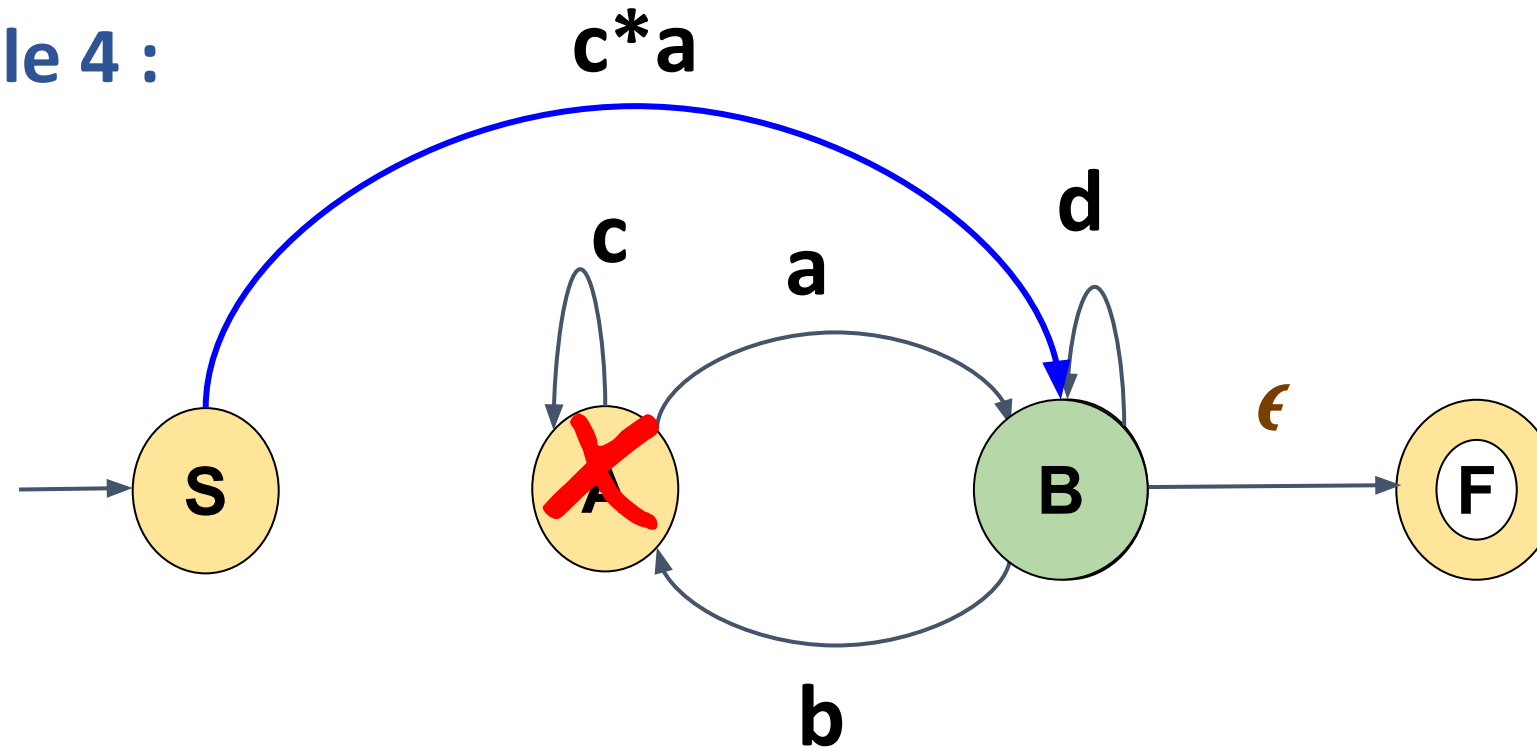
A new Final state F is introduced as there is an Outgoing edge to the existing Final State B.

### Example 4 :



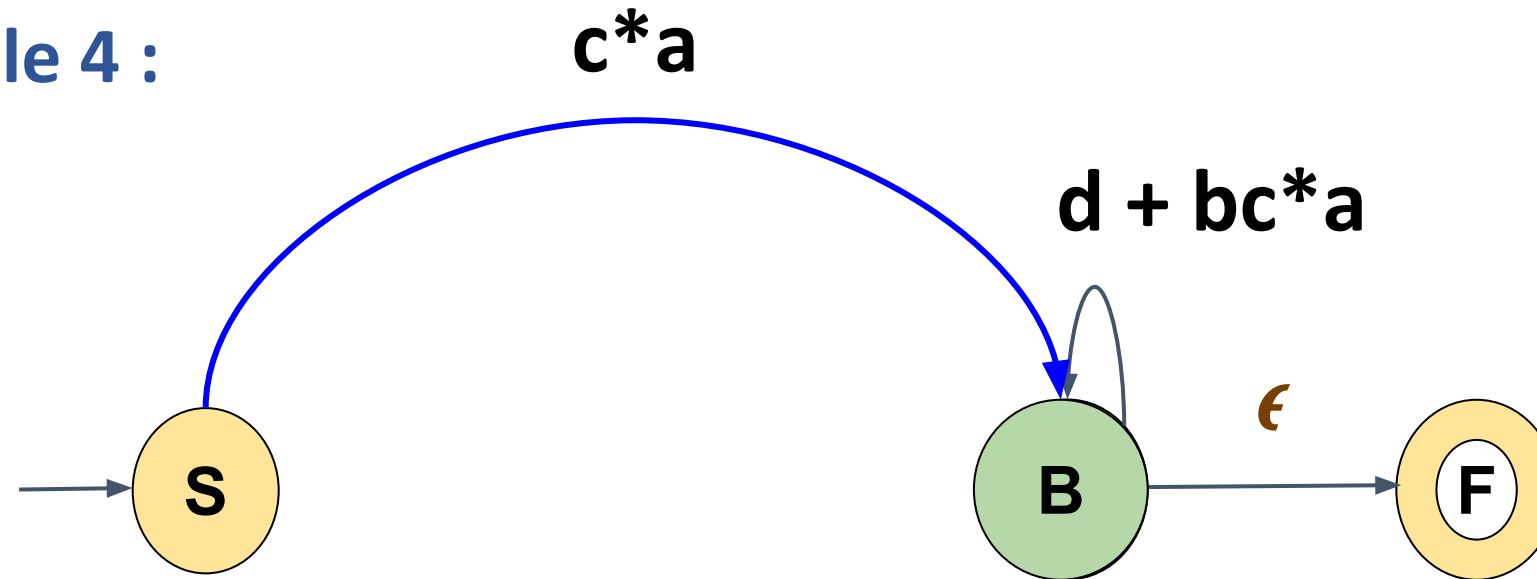
### 1. Eliminate A

Example 4 :



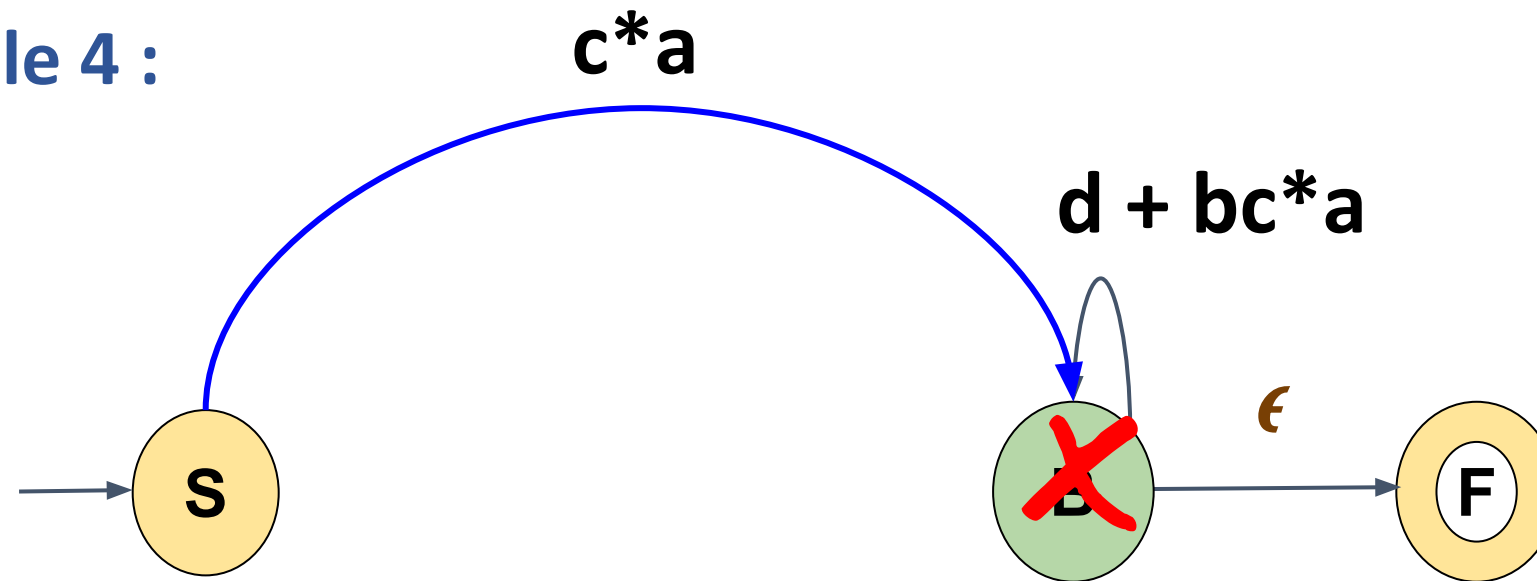
1. Eliminate A

Example 4 :



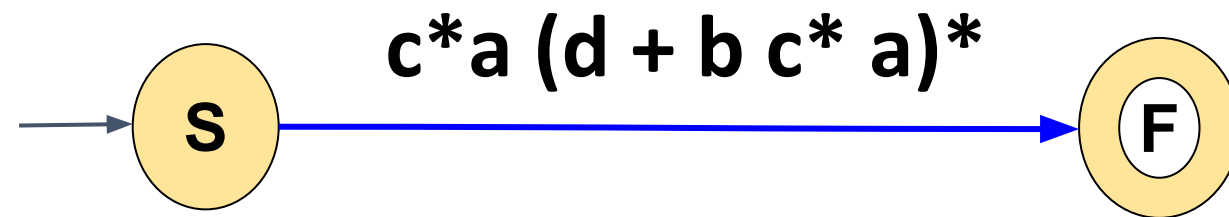
1. Eliminate A

Example 4 :



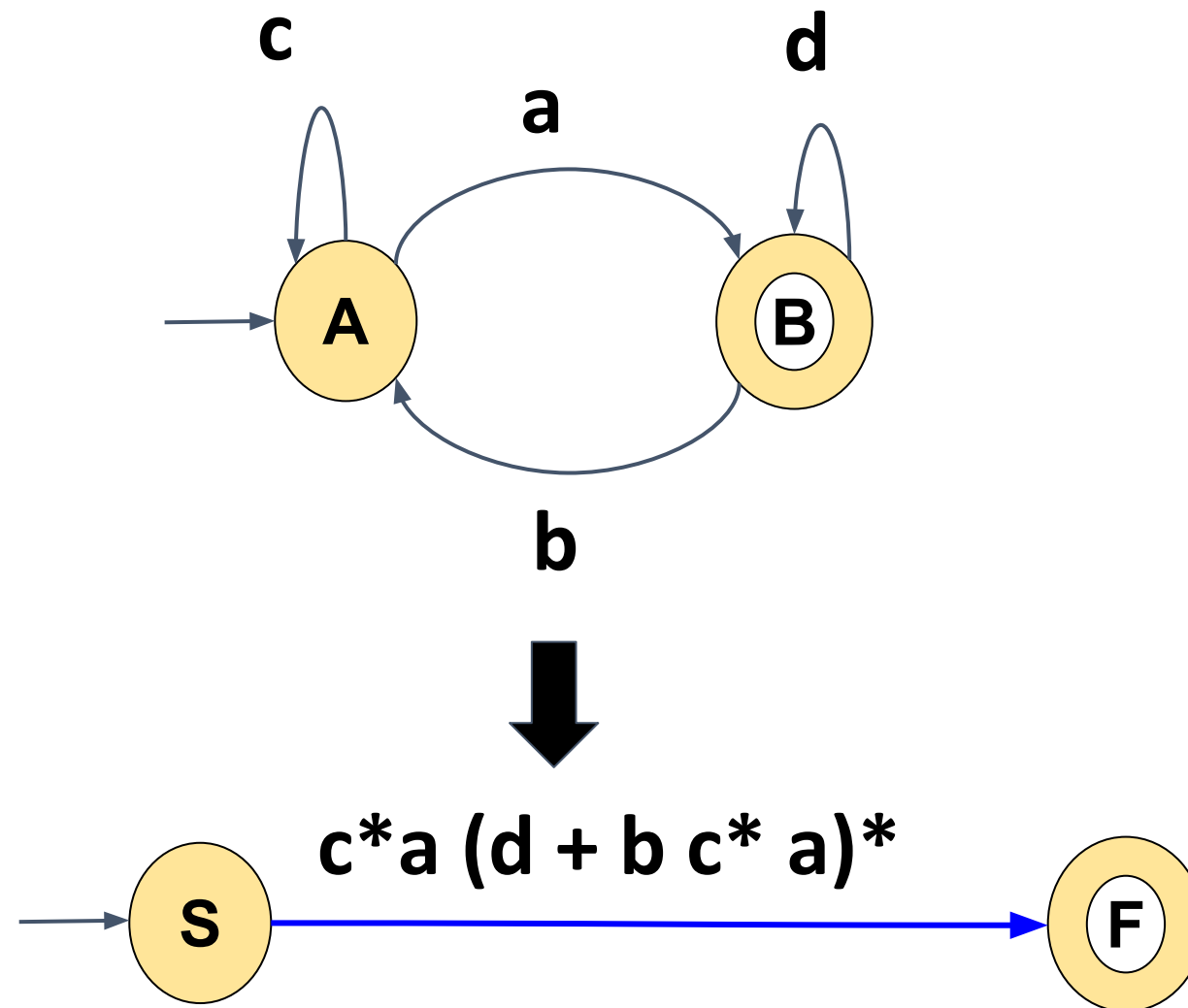
1. Eliminate A
2. Eliminate B

### Example 4 :



1. Eliminate A
2. Eliminate B

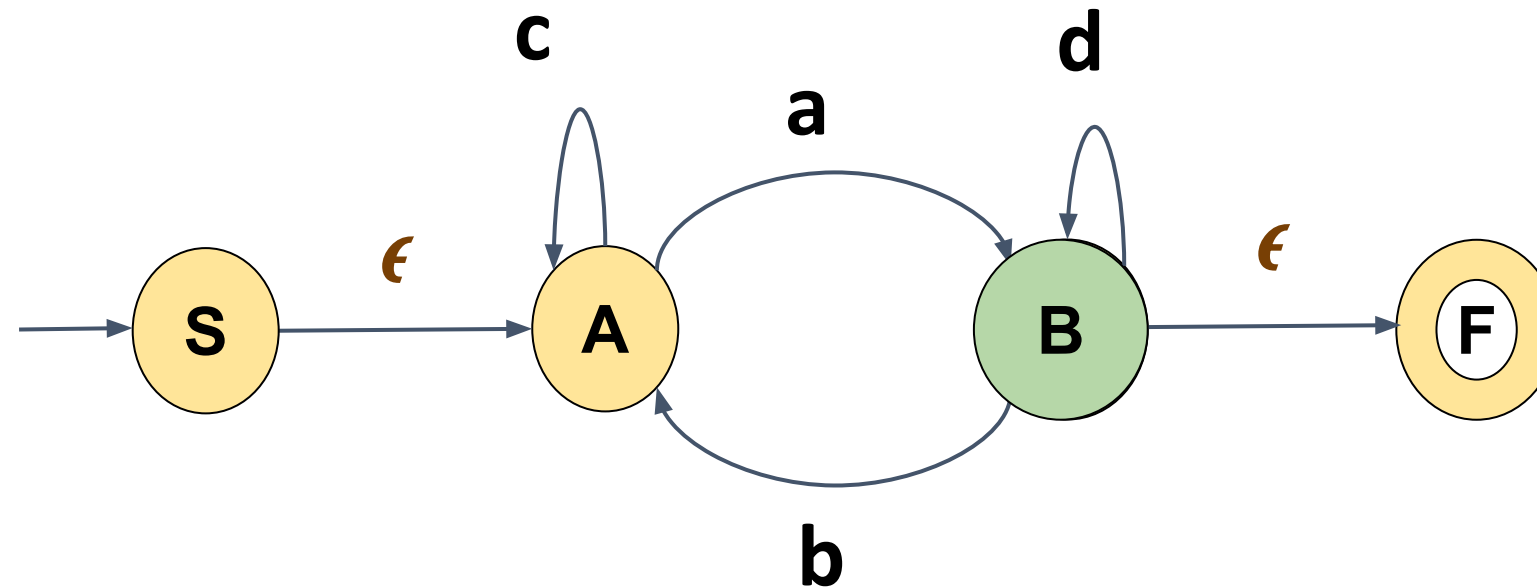
### Example 4 :



# Automata Formal Languages and Logic

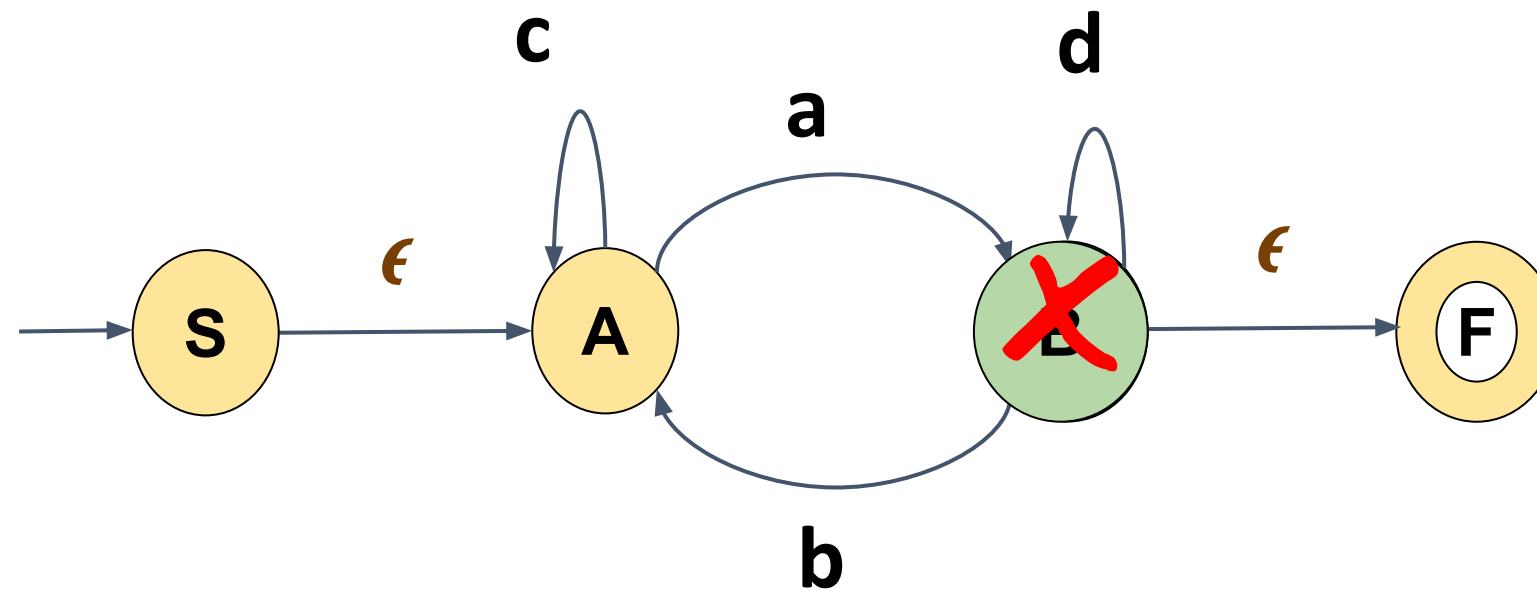
## Unit 2 - Finite Automata to Regular Expression

### Example 4 :



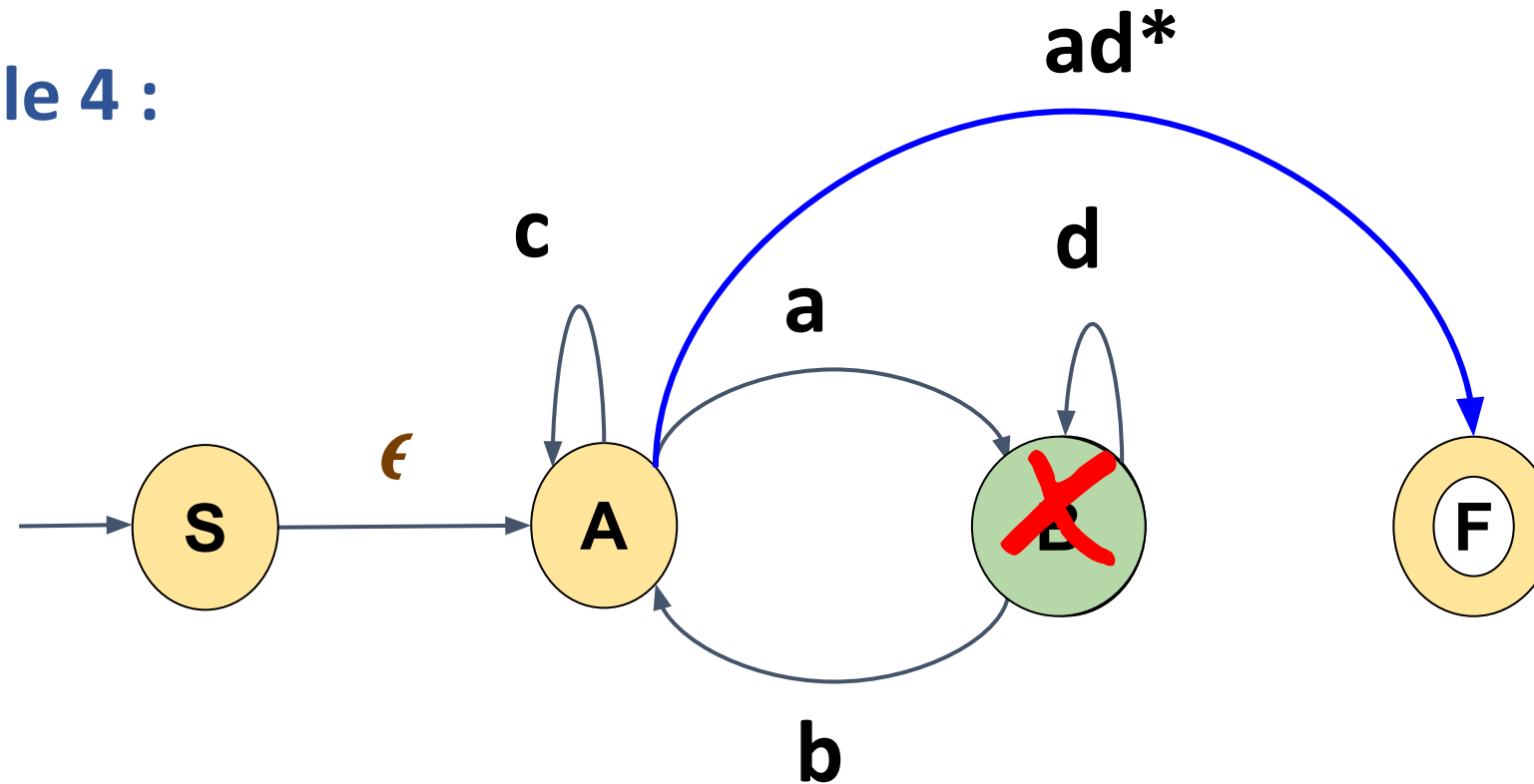


### Example 4 :



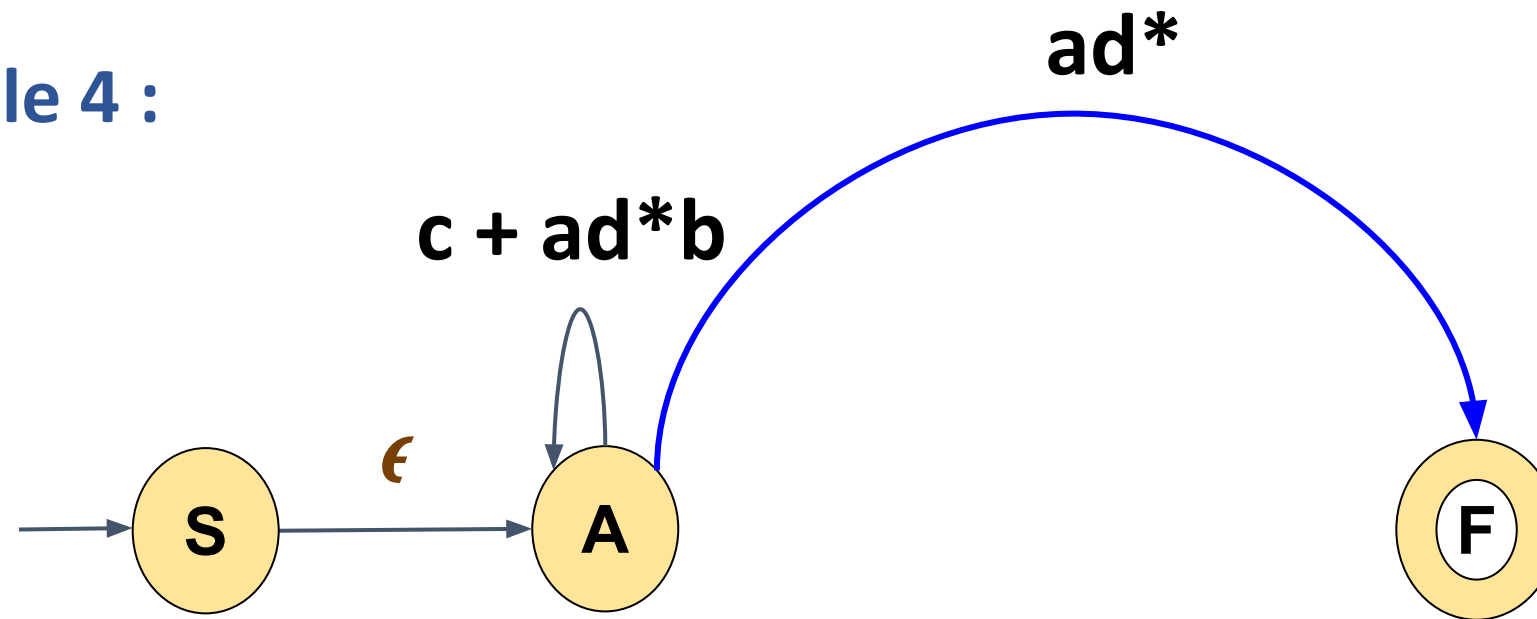
### 1. Eliminate B

Example 4 :



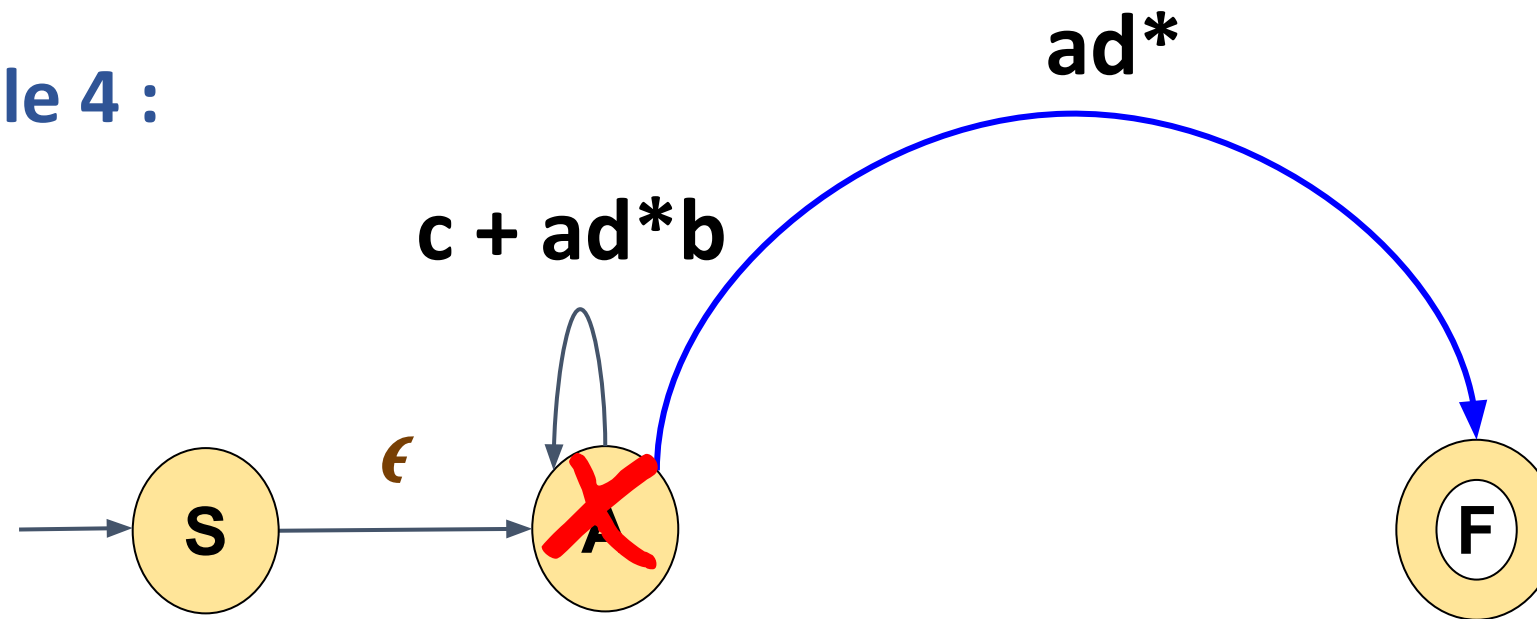
1. Eliminate B

Example 4 :



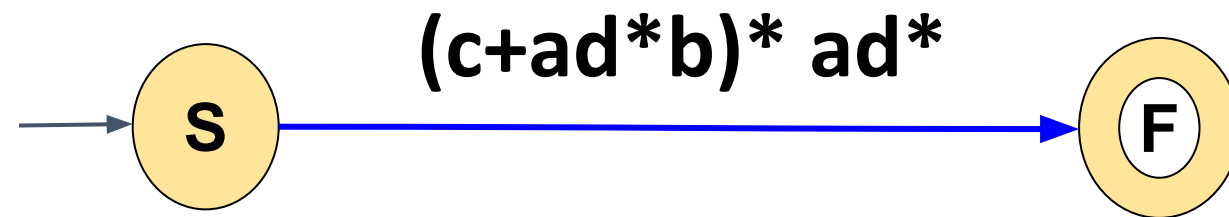
1. Eliminate B

Example 4 :



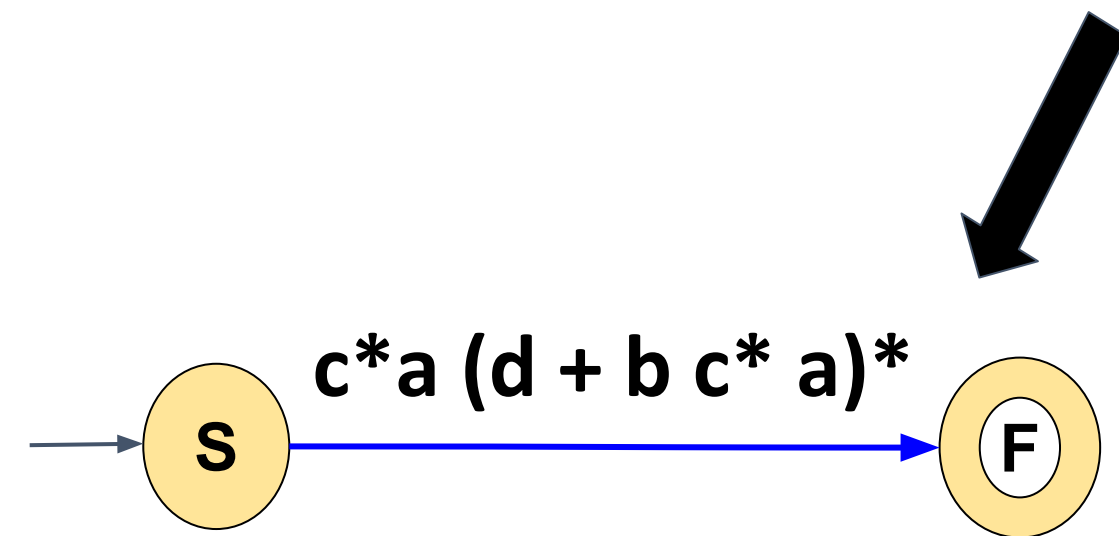
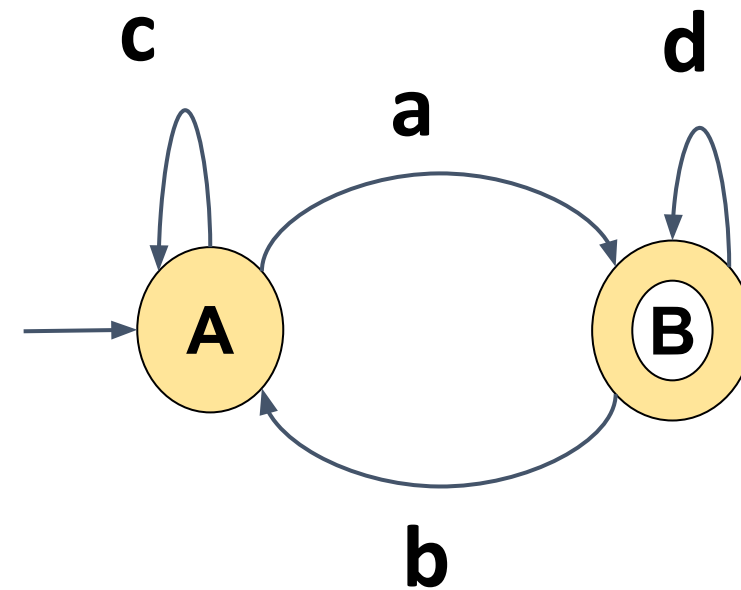
1. Eliminate B
2. Eliminate A

### Example 4 :

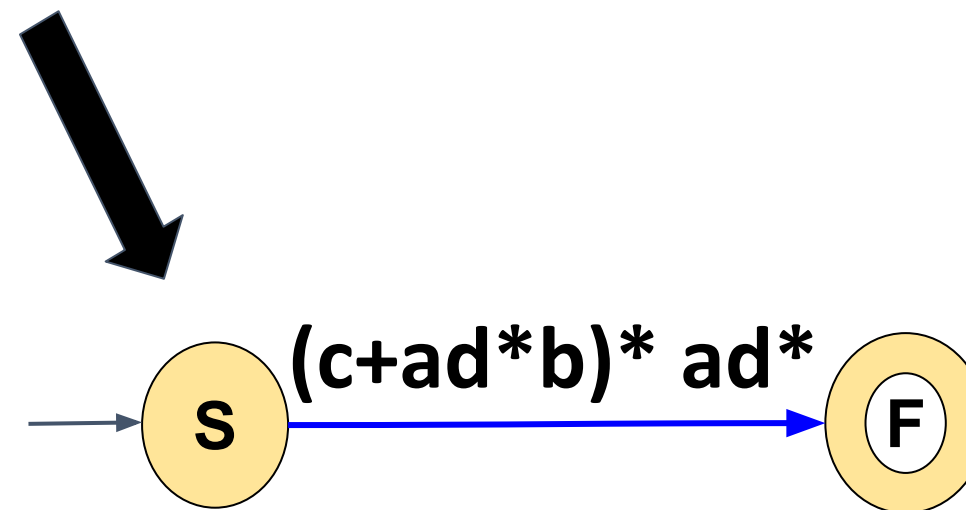


1. Eliminate B
2. Eliminate A

### Example 4 :



1. Eliminate A
2. Eliminate B

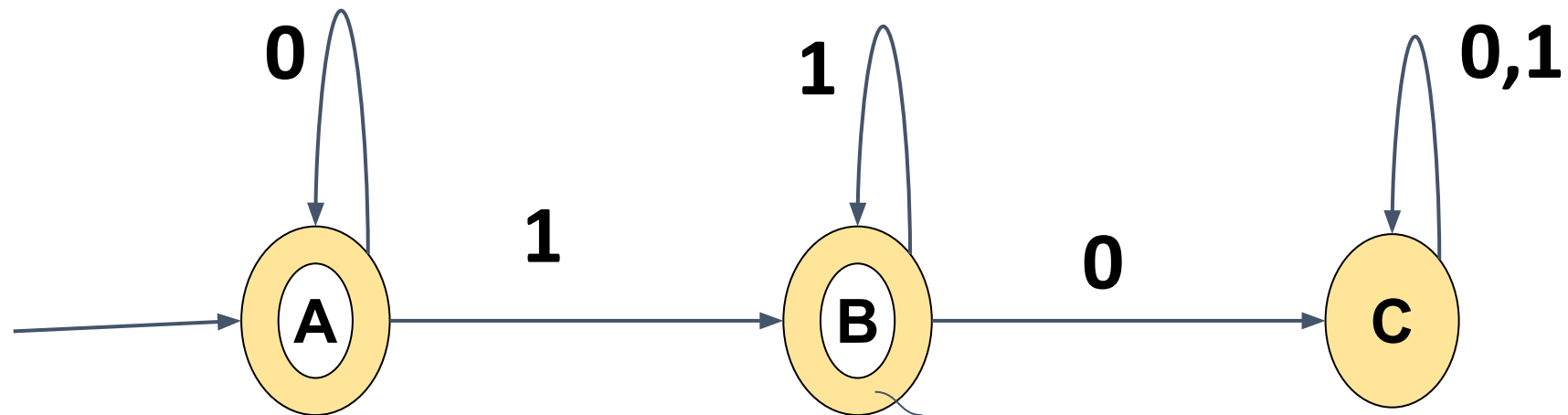


1. Eliminate B
2. Eliminate A

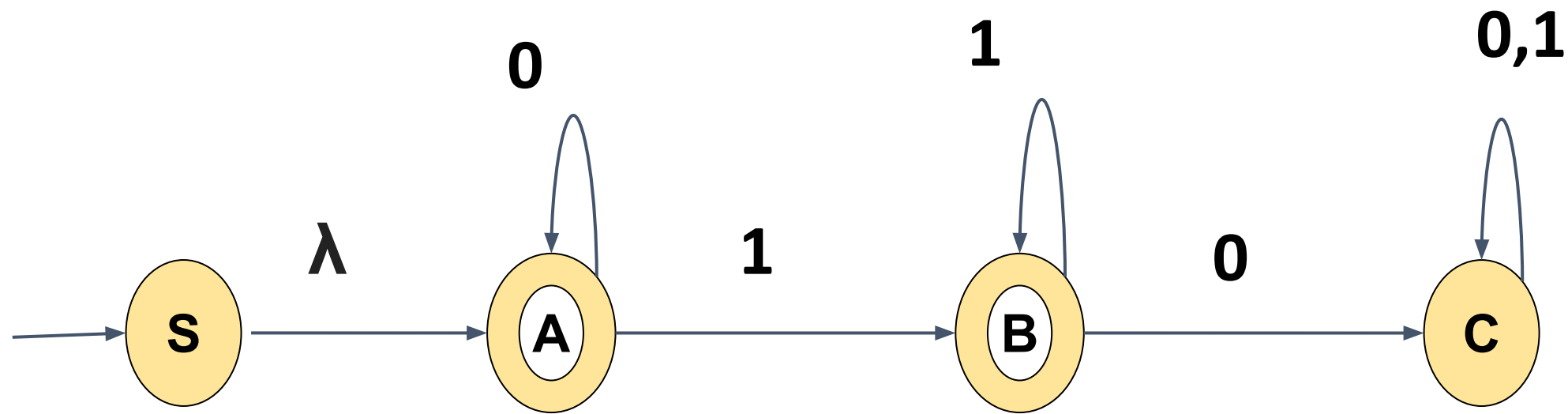
# Automata Formal Languages and Logic

## Unit 2 - Finite Automata to Regular Expression

### Example 5 :



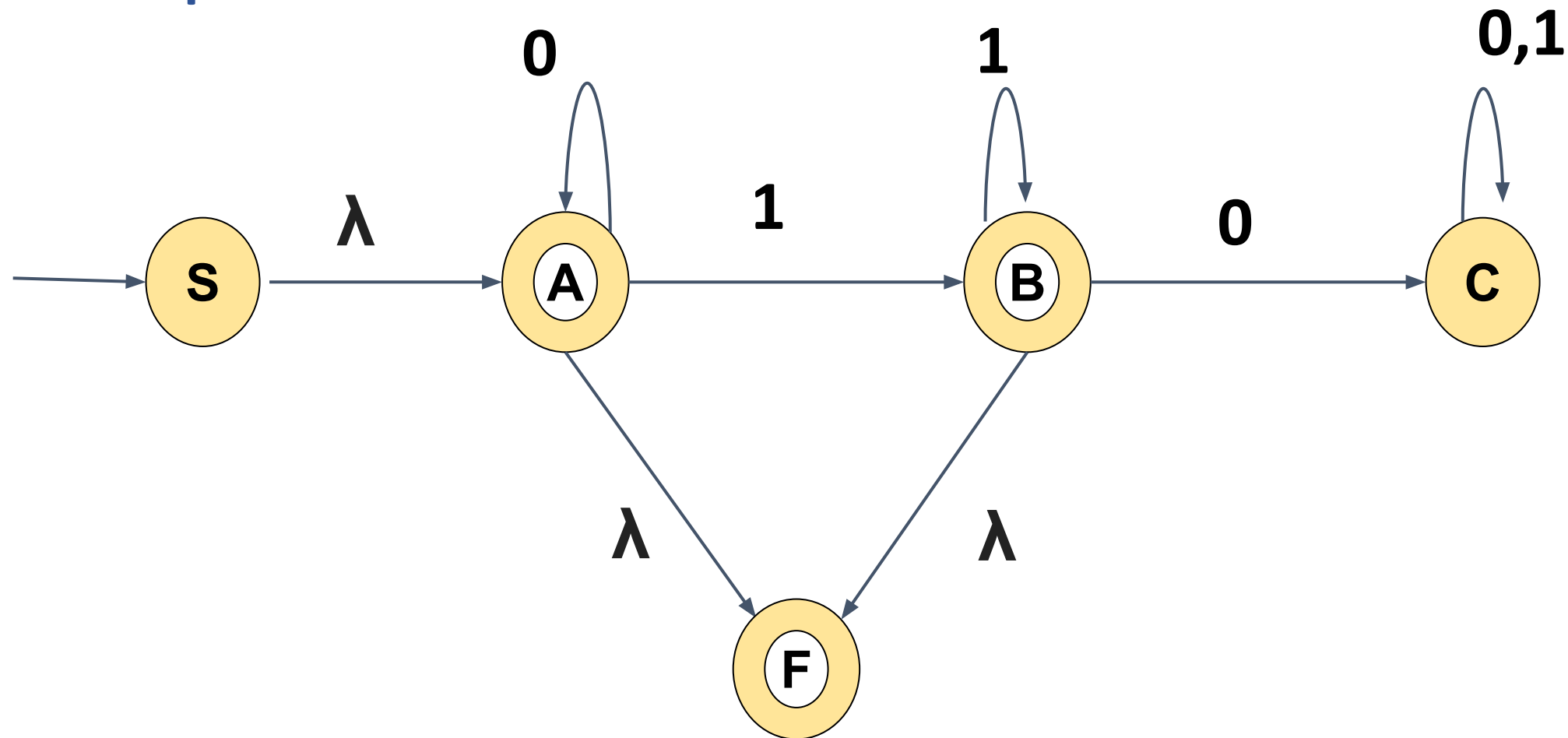
### Example 5 :



A new start state (S) is introduced as there is an incoming edge to the existing start state

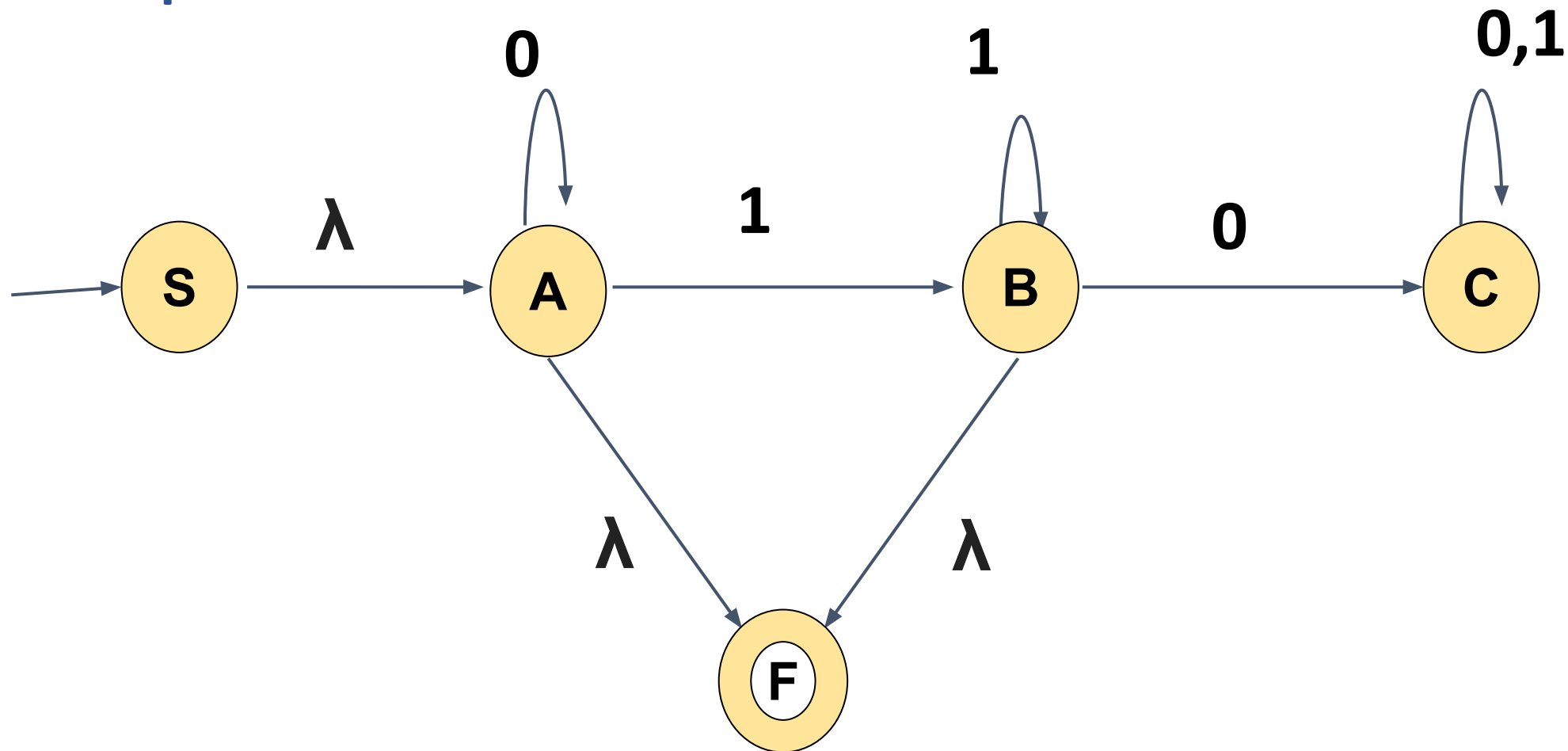


### Example 5 :



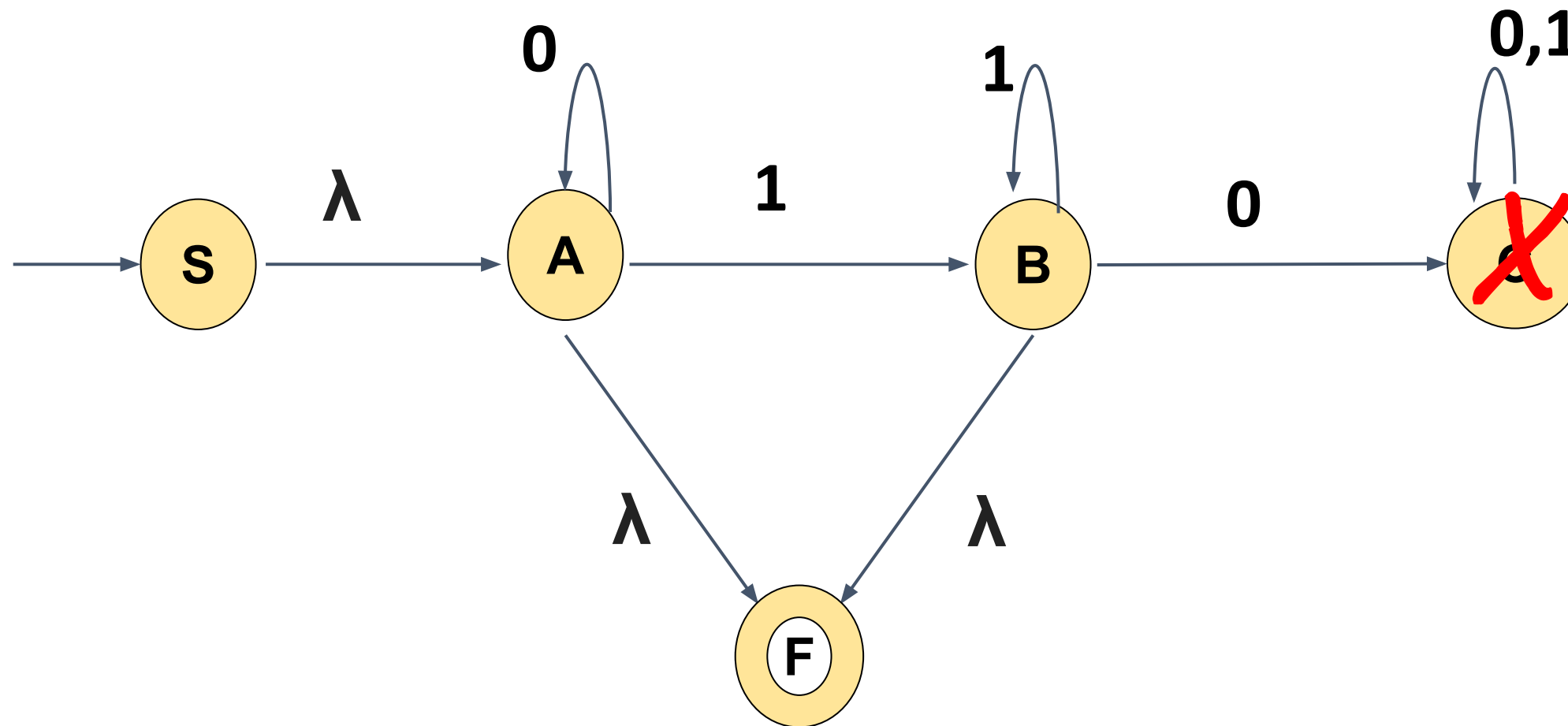
A new final state (F) is introduced as there is an outgoing edge from the existing final state and we must have a single accepting state

### Example 5 :



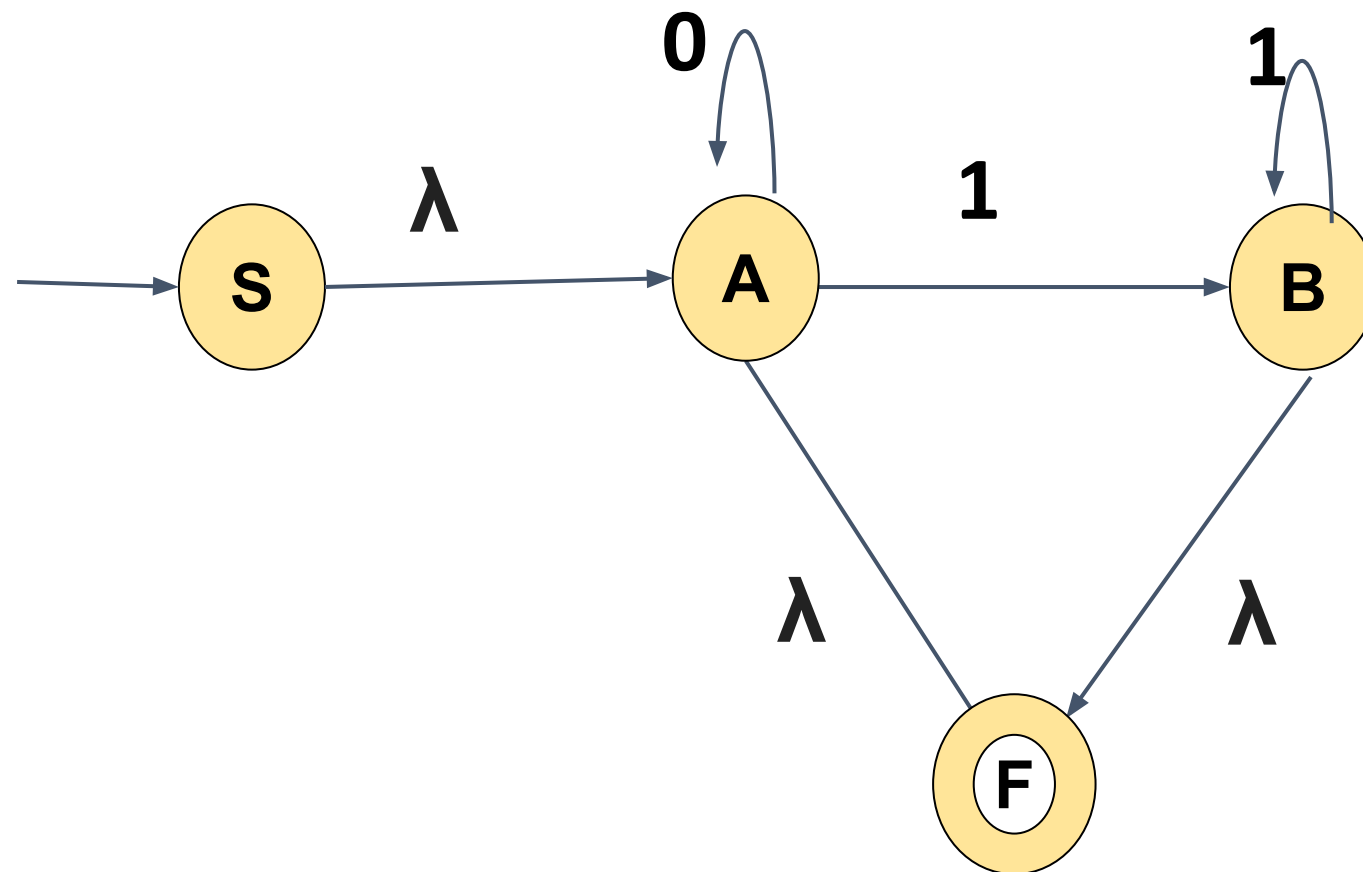
**Previous final state is made as non final state**

### Example 5 :



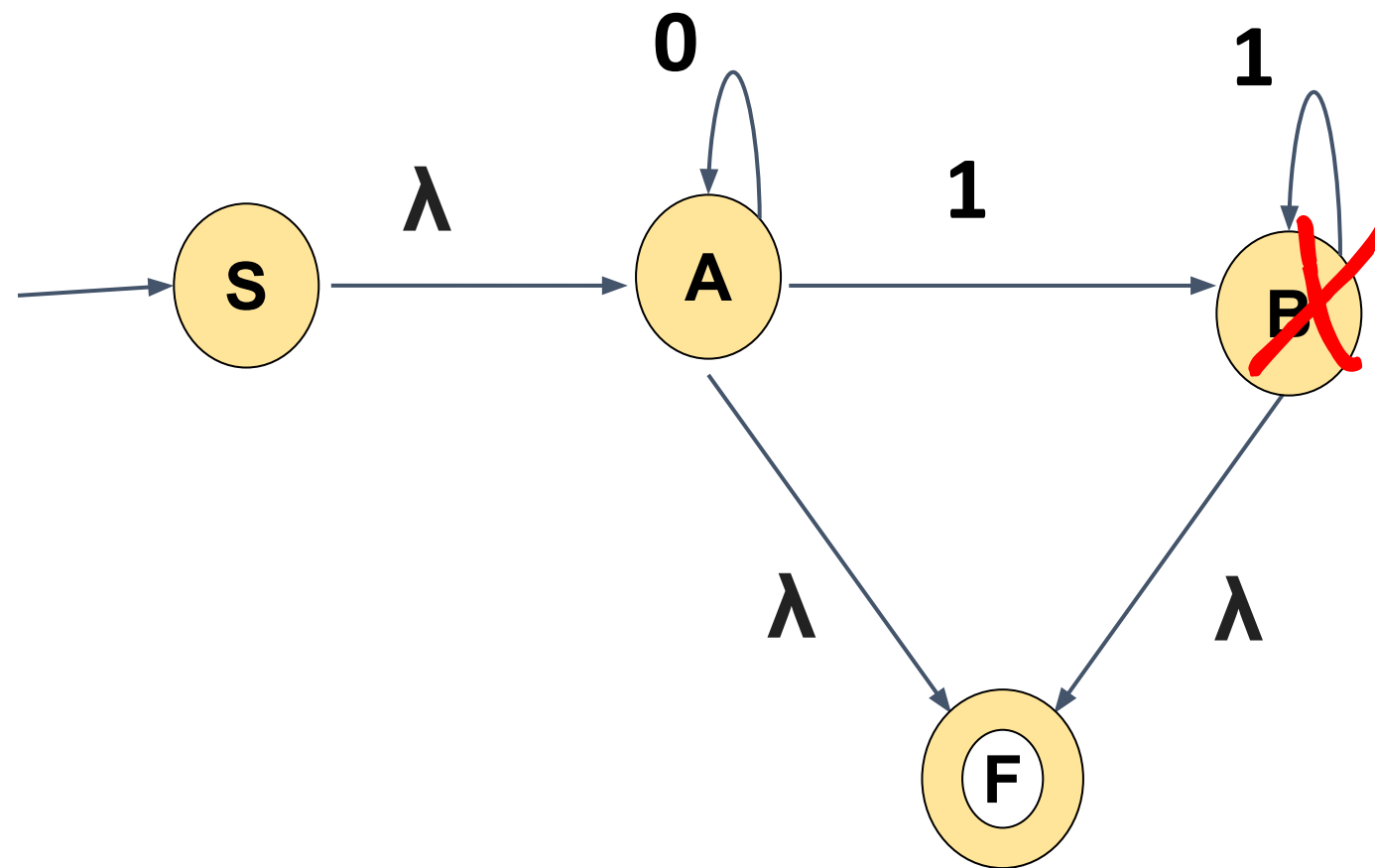
### 1. Eliminate C

### Example 5 :



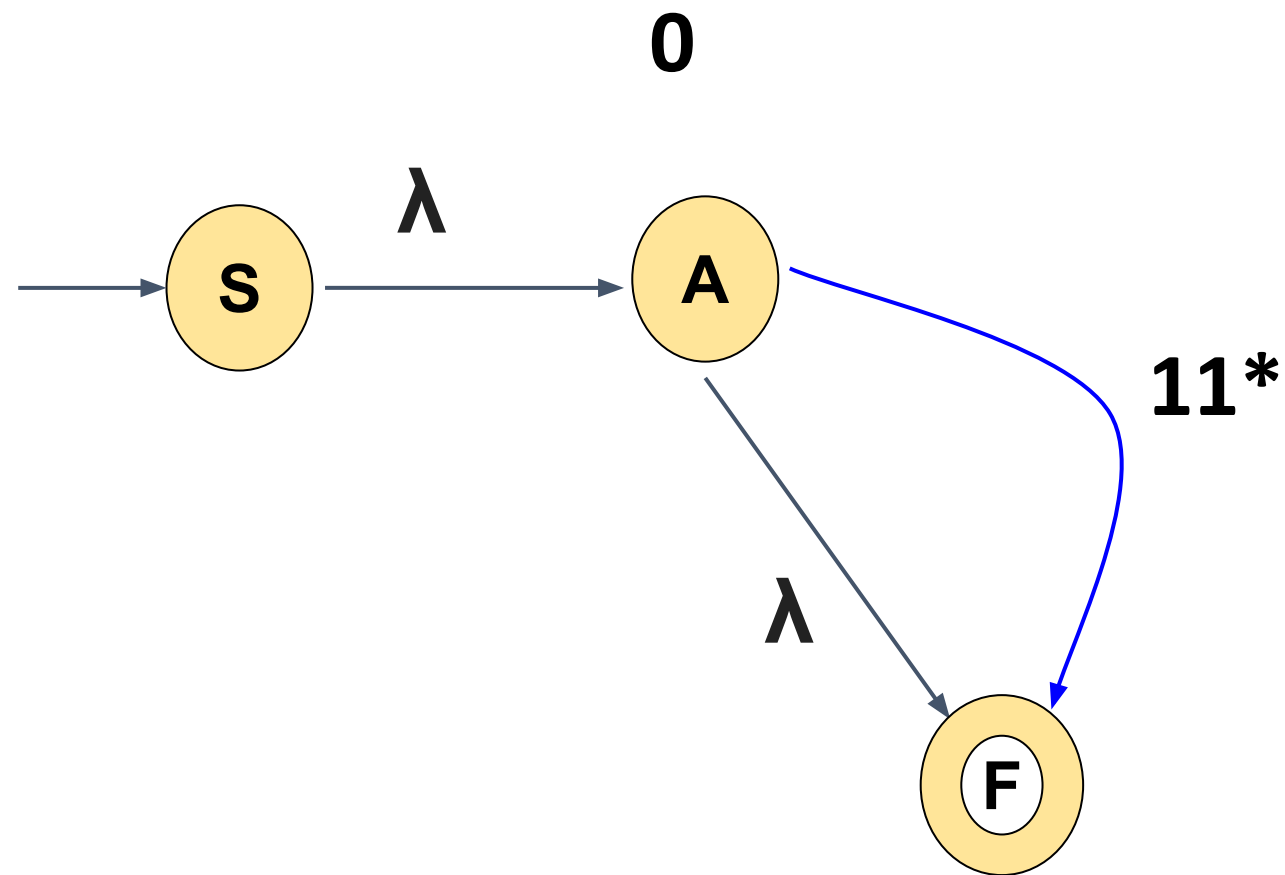
### 1. Eliminate C

### Example 5 :



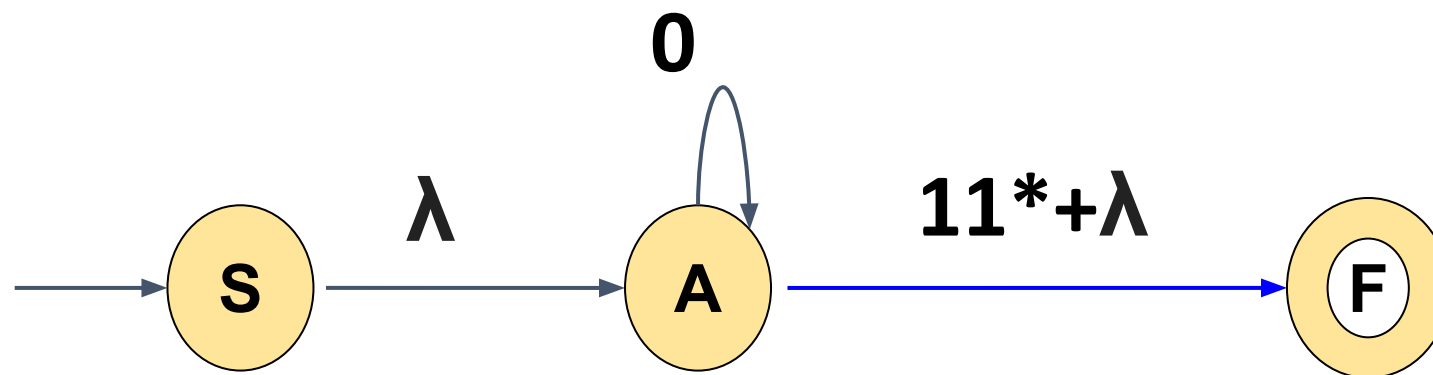
1. Eliminate C
2. Eliminate B

### Example 5 :



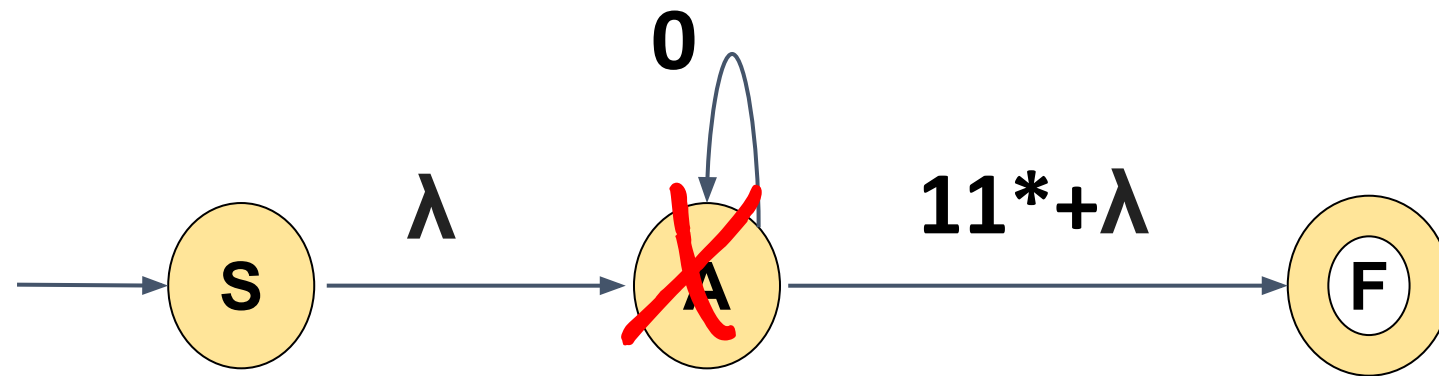
1. Eliminate C
2. Eliminate B

### Example 5 :



1. Eliminate C
2. Eliminate B

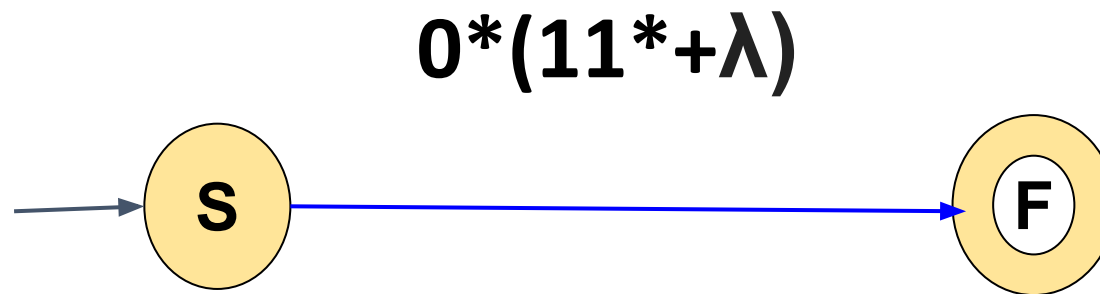
### Example 5 :



1. Eliminate C
2. Eliminate B
3. Eliminate A



### Example 5 :

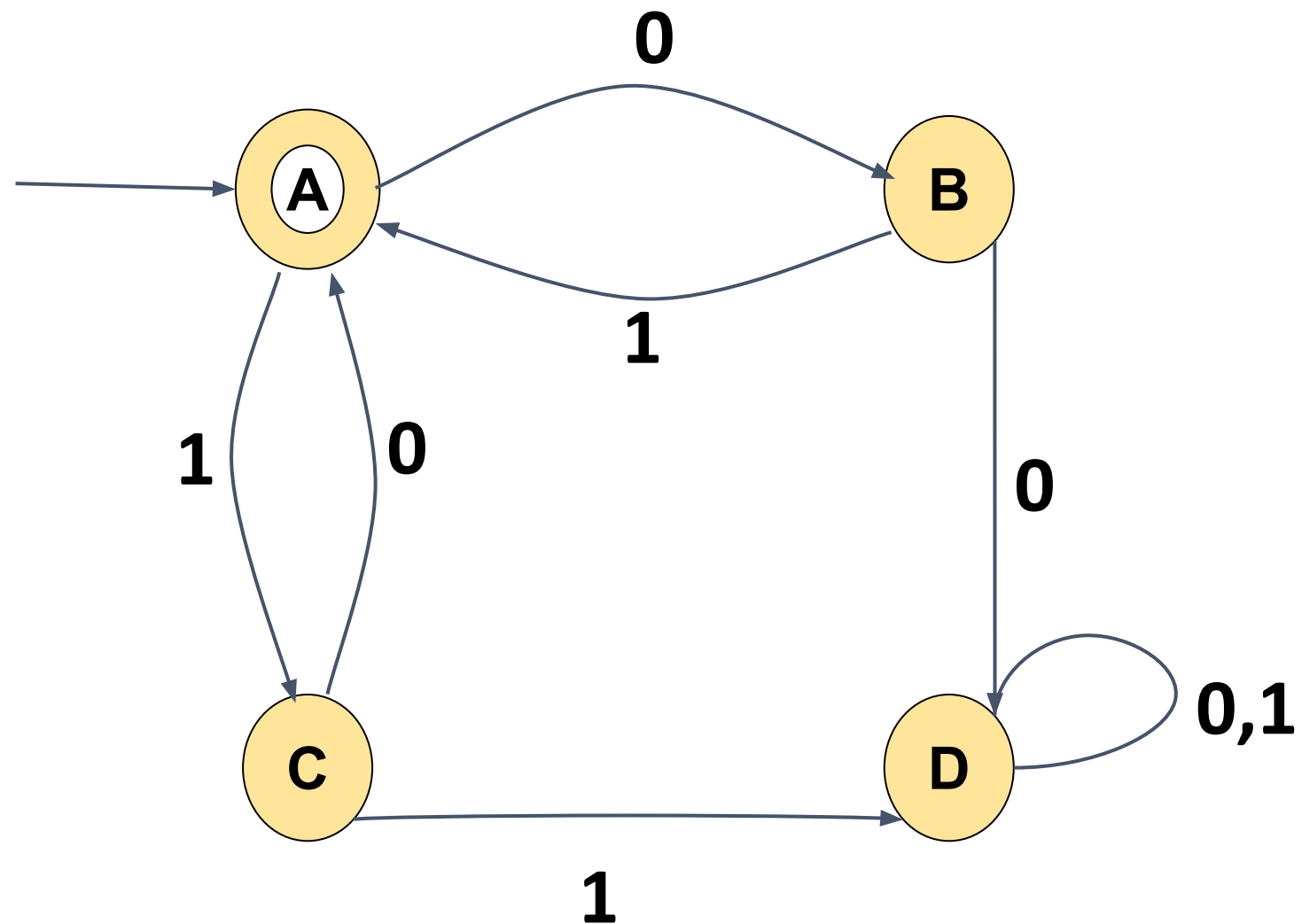


$$= 0(11^*+\lambda)$$

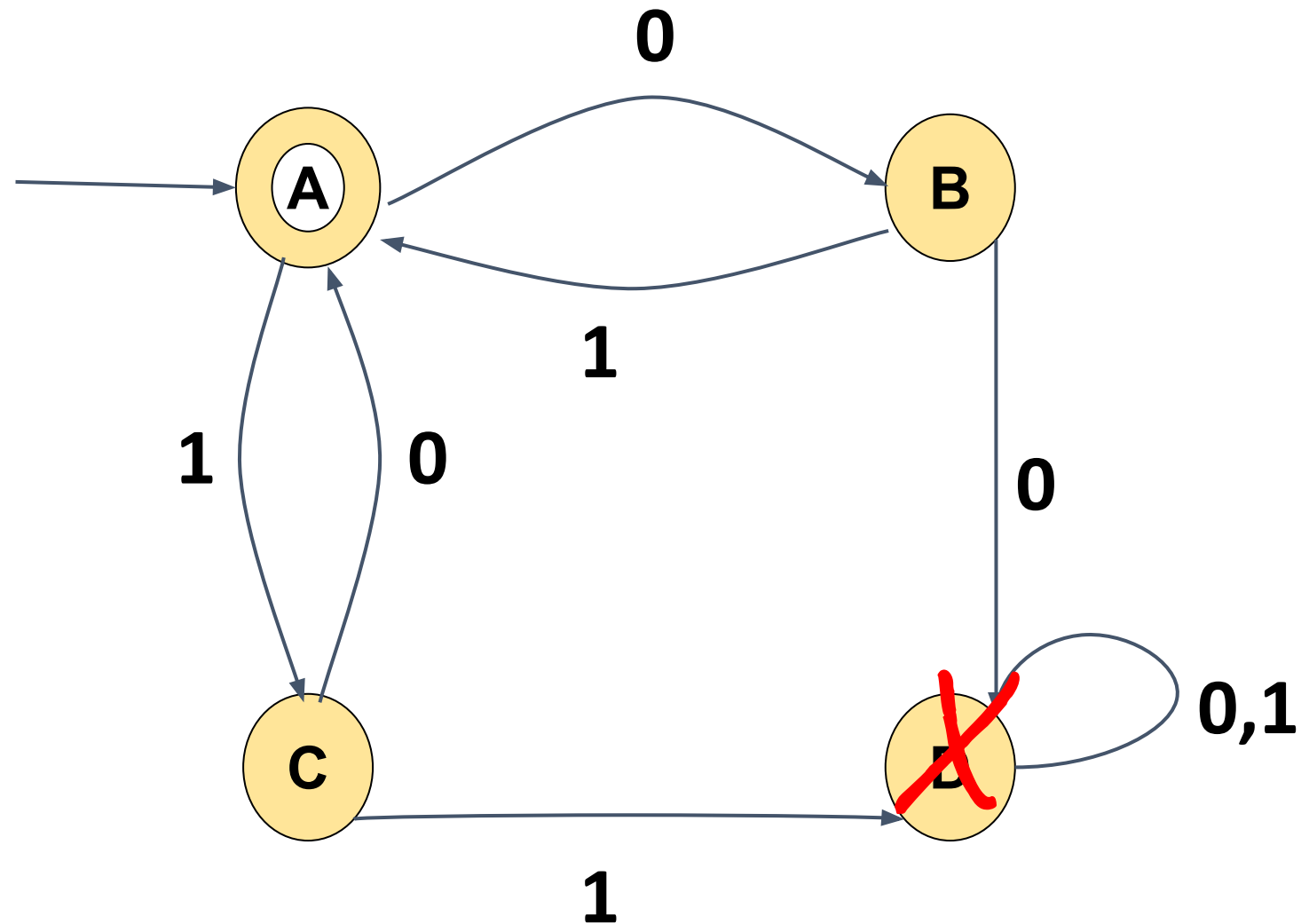
$$= 0^*(1^++\lambda)$$

$$\text{RE} = 0^*1^*$$

### Example 6 :



### Example 6 :

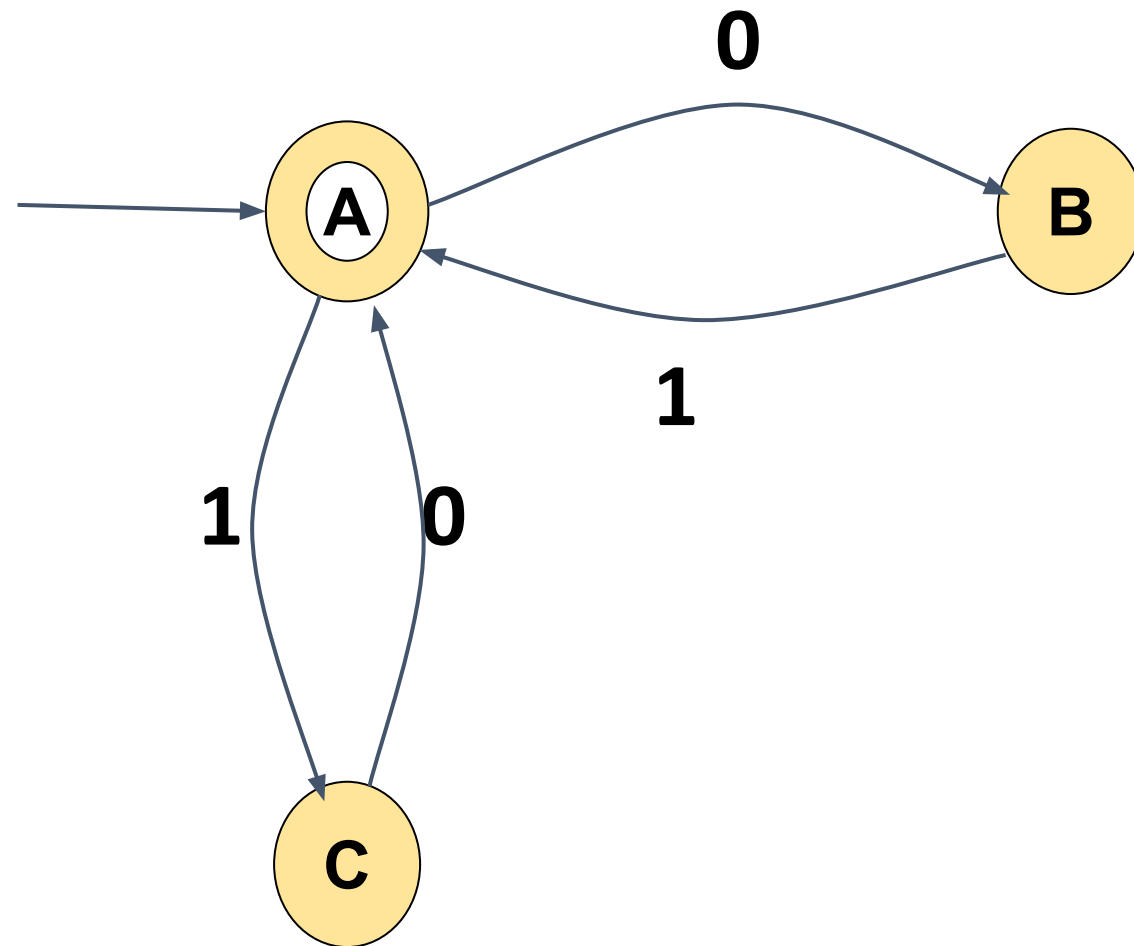


1. Eliminate D  
(Dead state)

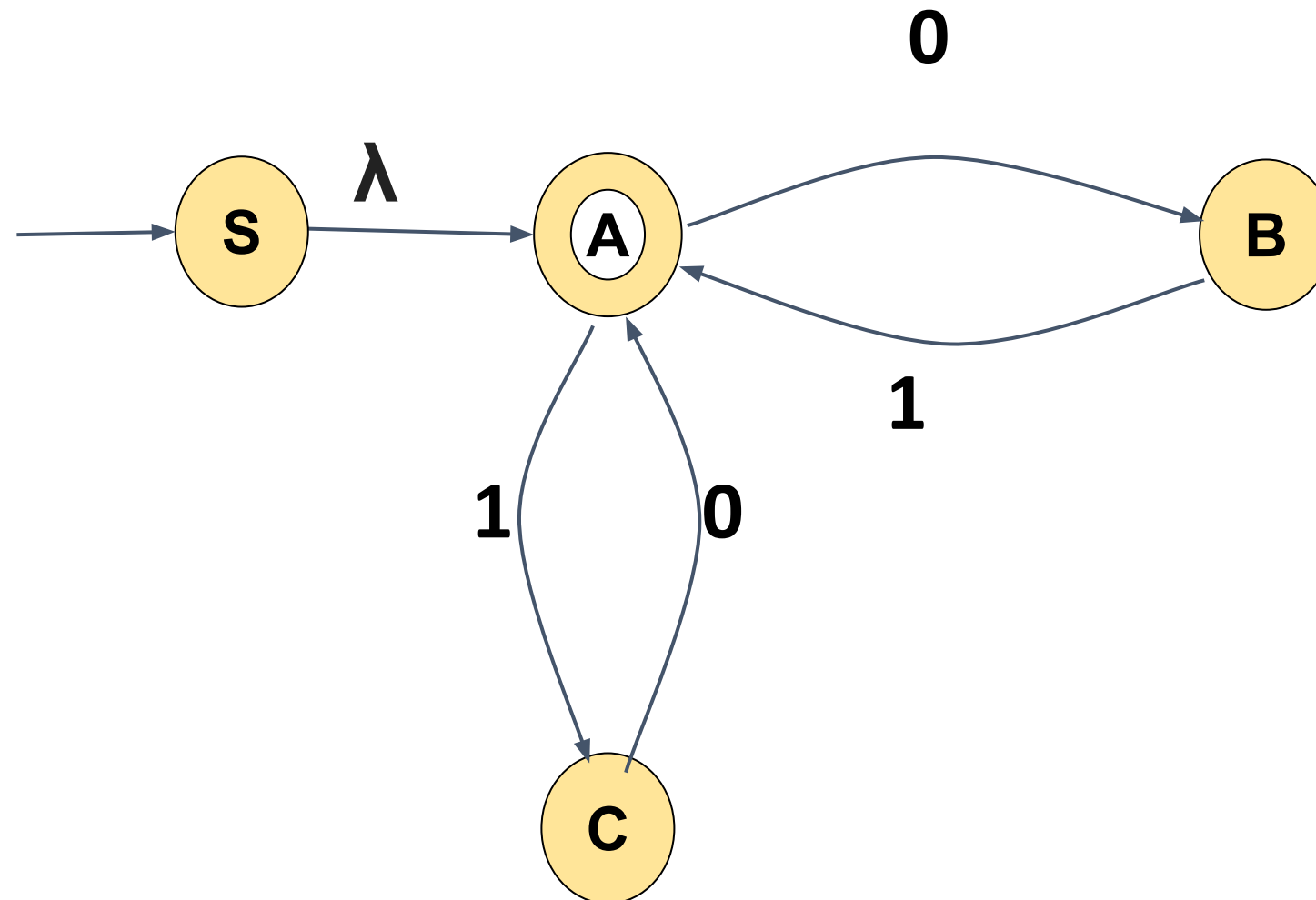
# Automata Formal Languages and Logic

## Unit 2 - Finite Automata to Regular Expression

### Example 6 :

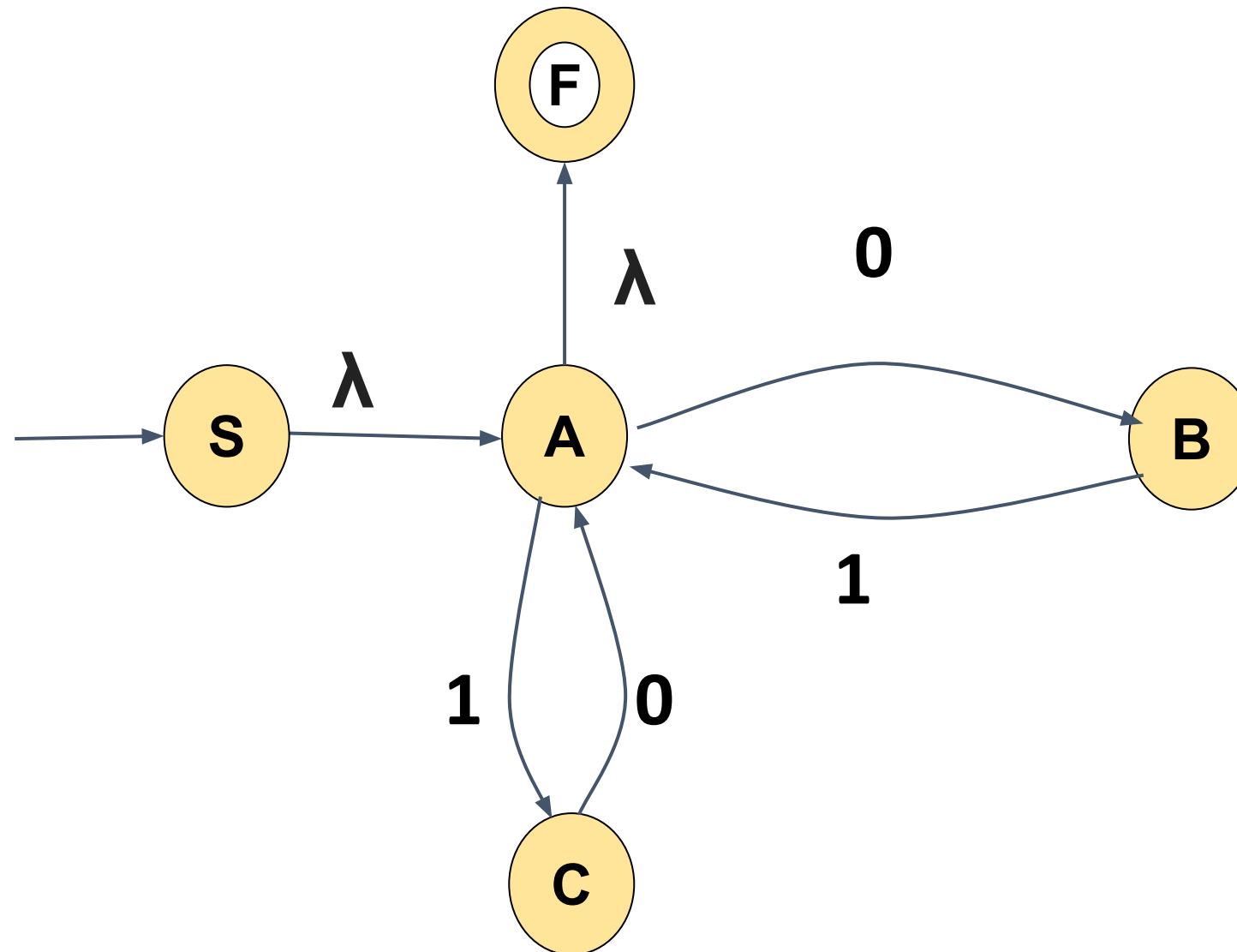


### Example 6 :



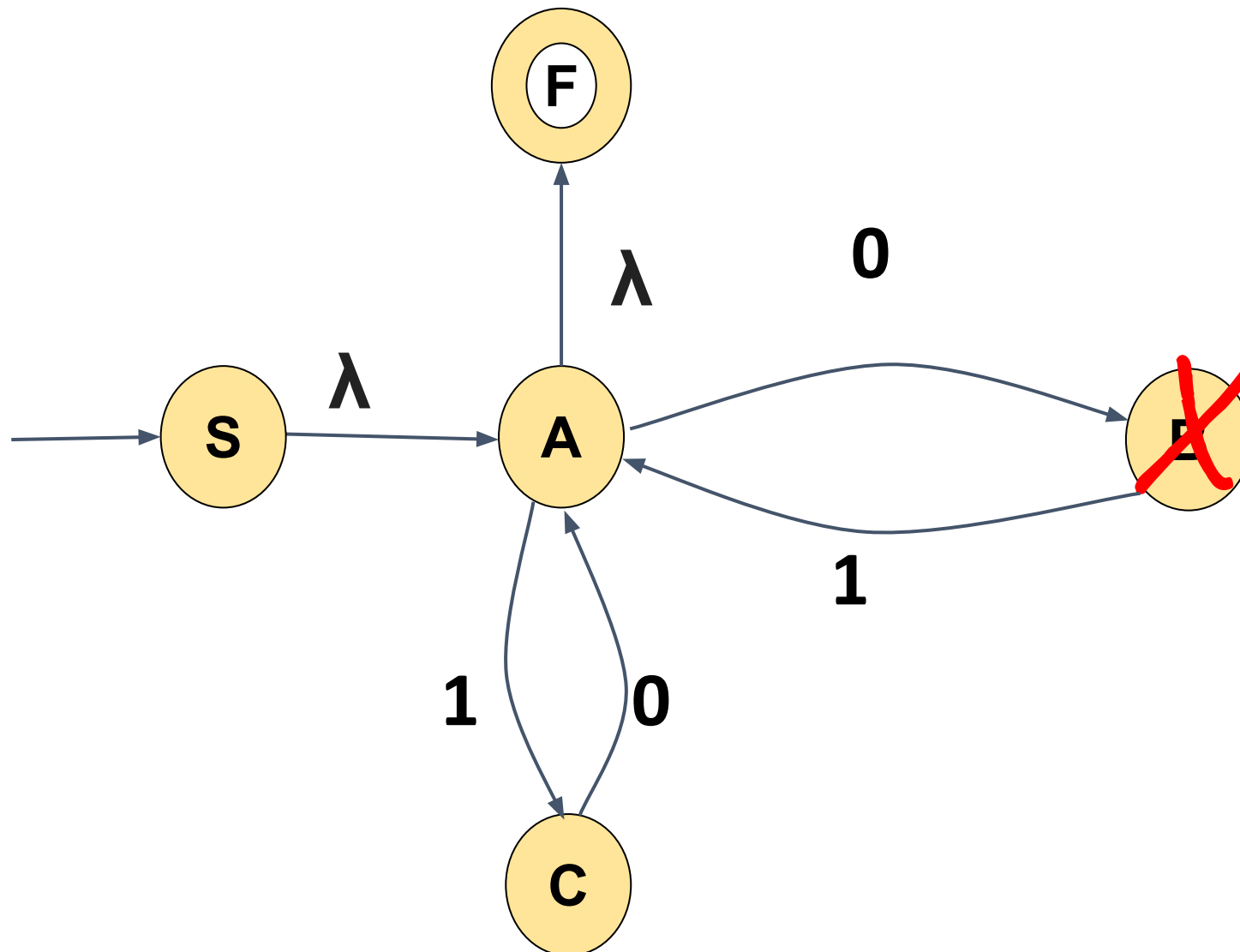
A new start state (S) is introduced as there is an incoming edge to the existing start state

### Example 6 :



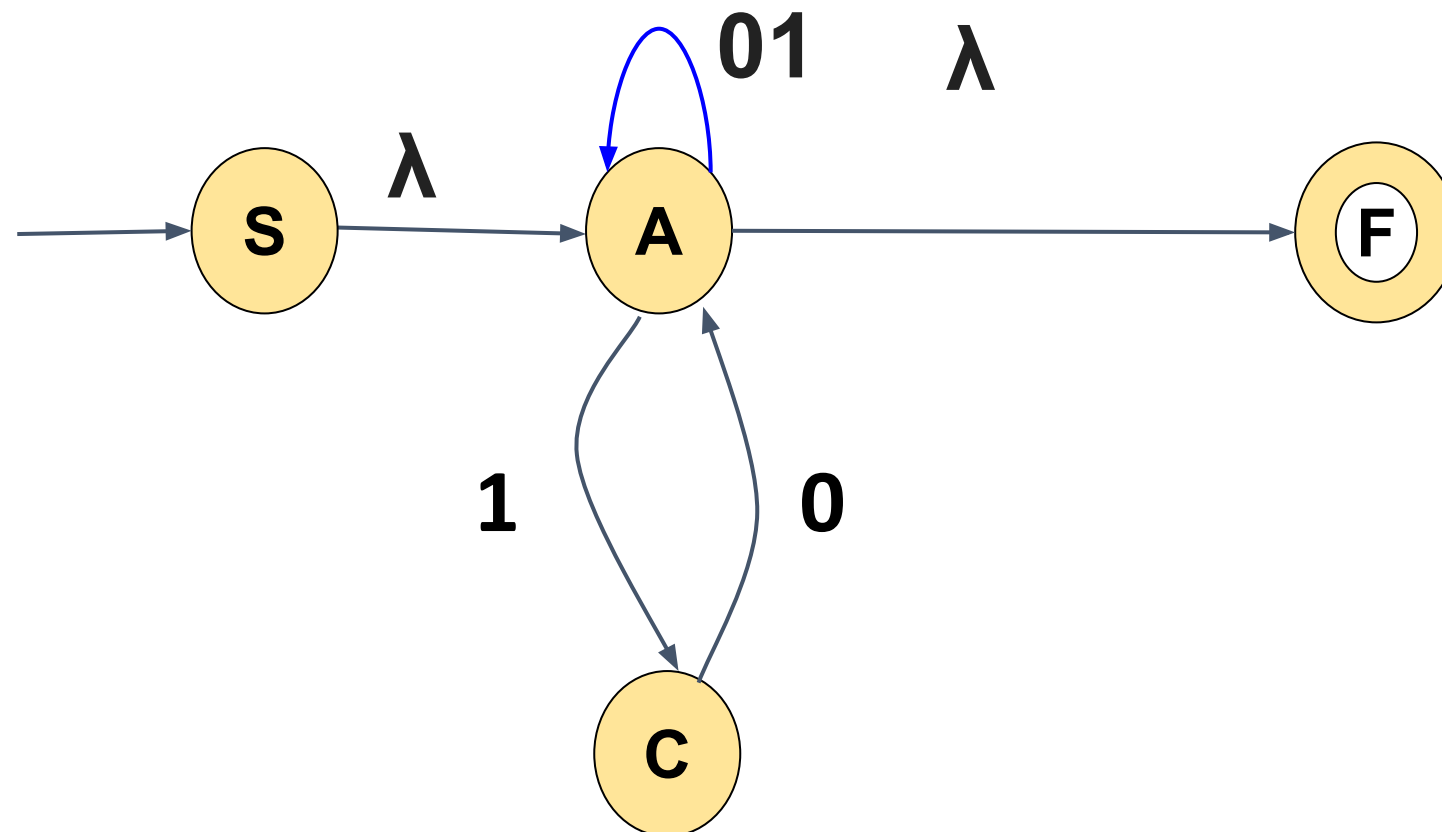
A new final state (F) is introduced as there is an outgoing edge from the existing final state

### Example 6 :



1. Eliminate D
2. Eliminate B

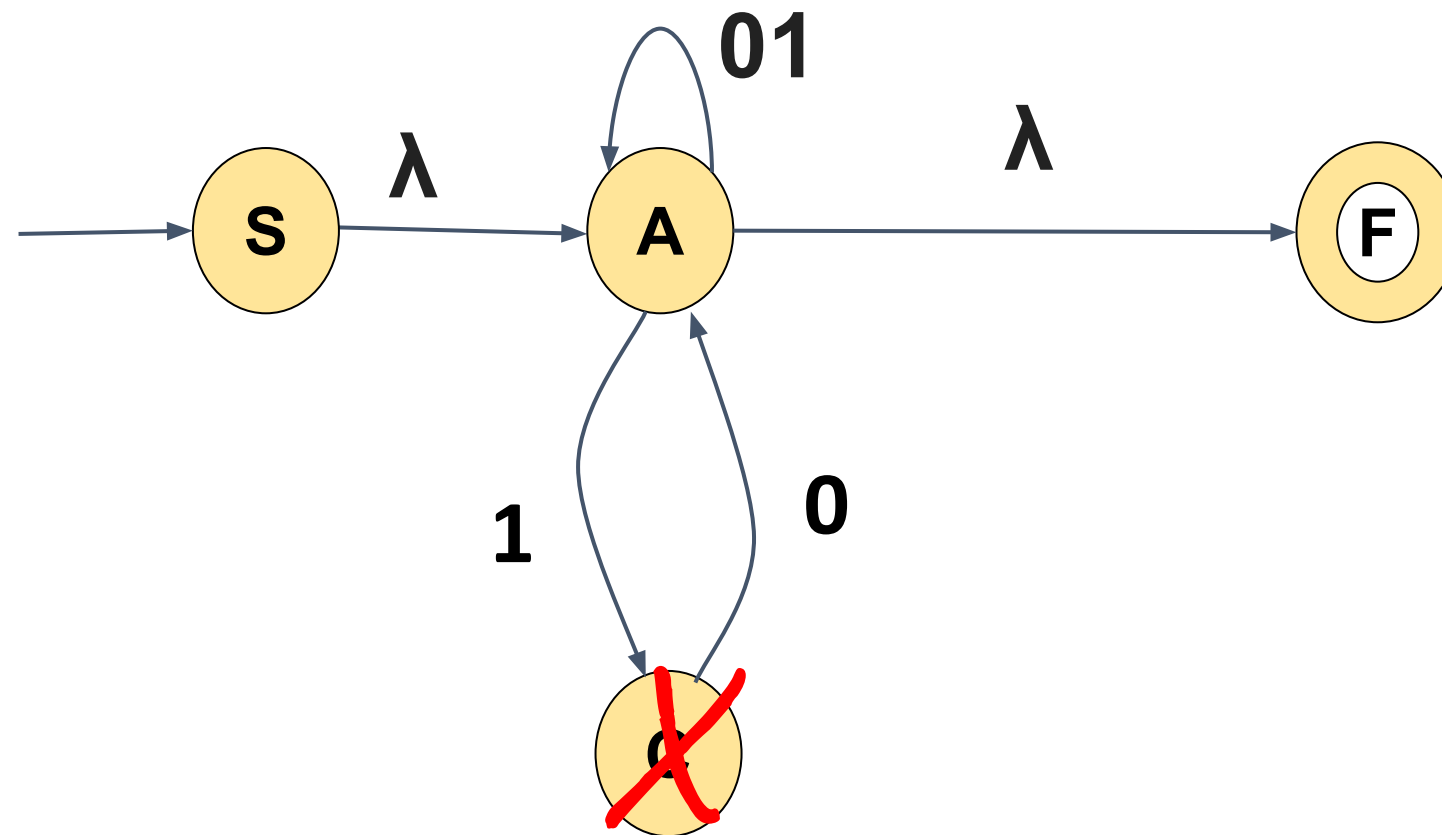
### Example 6 :



1. Eliminate D
2. Eliminate B

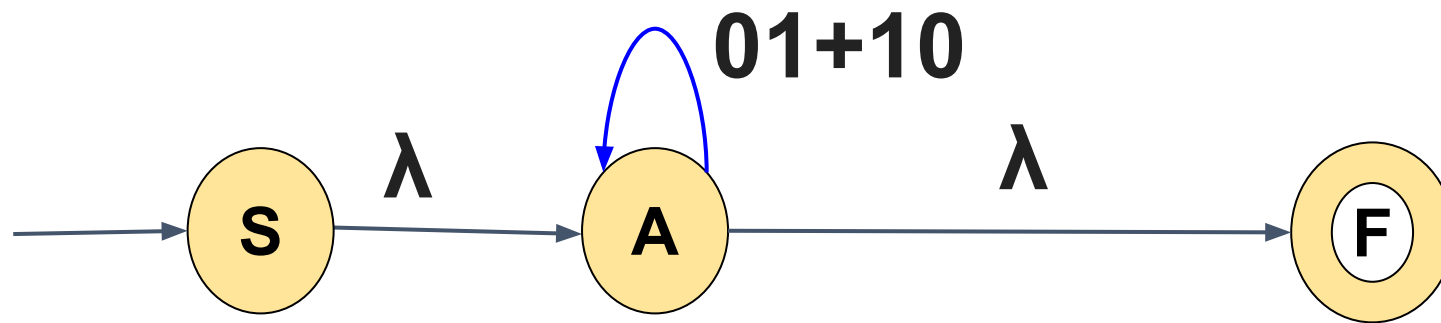


### Example 6 :



1. Eliminate D
2. Eliminate B
3. Eliminate C

### Example 6 :

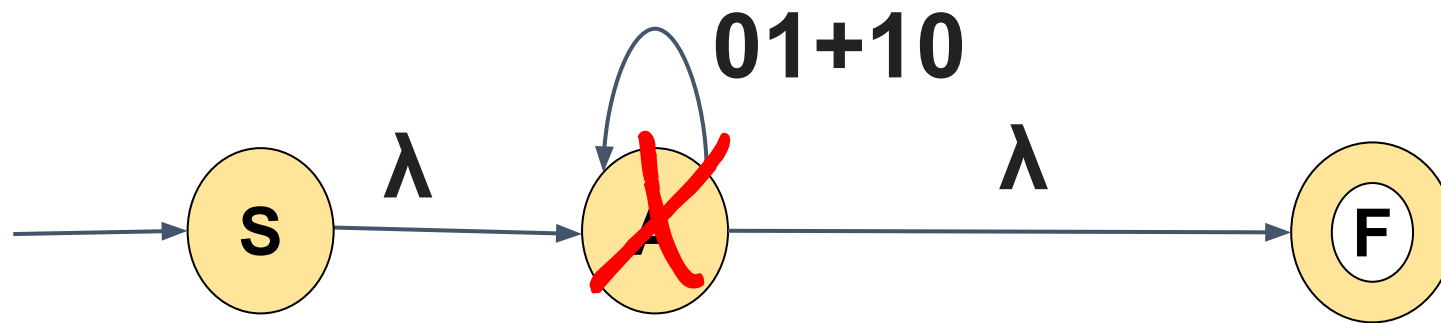


1. Eliminate D
2. Eliminate B
3. Eliminate C

# Automata Formal Languages and Logic

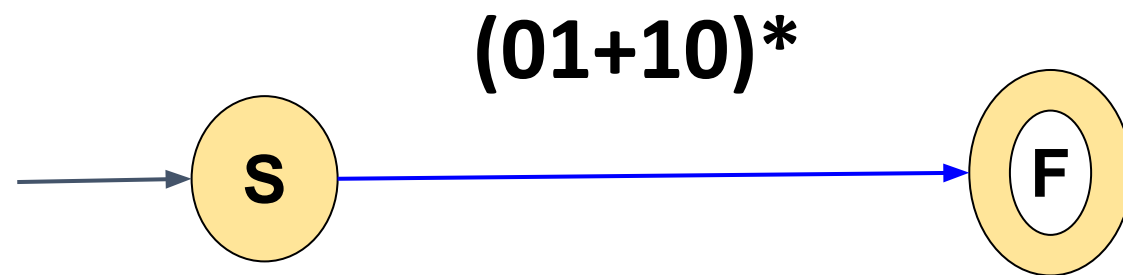
## Unit 2 - Finite Automata to Regular Expression

### Example 6 :



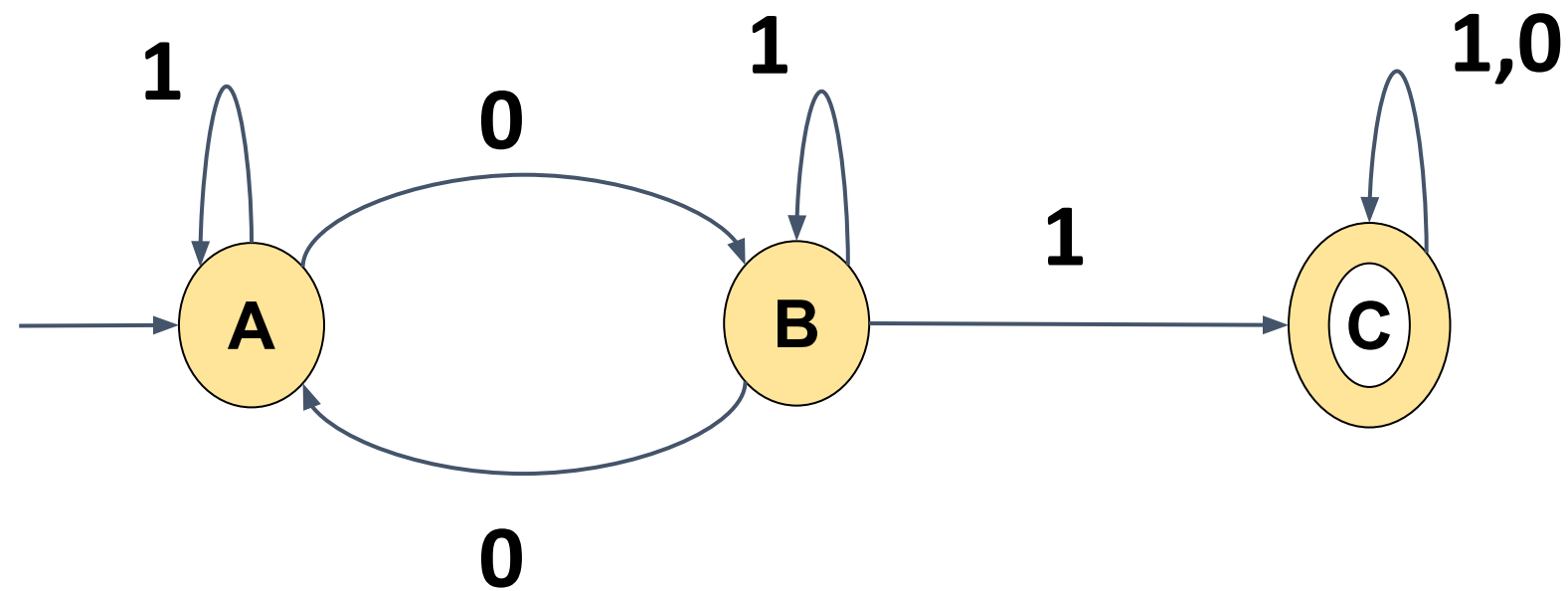
1. Eliminate D
2. Eliminate B
3. Eliminate C
4. Eliminate A

### Example 6 :

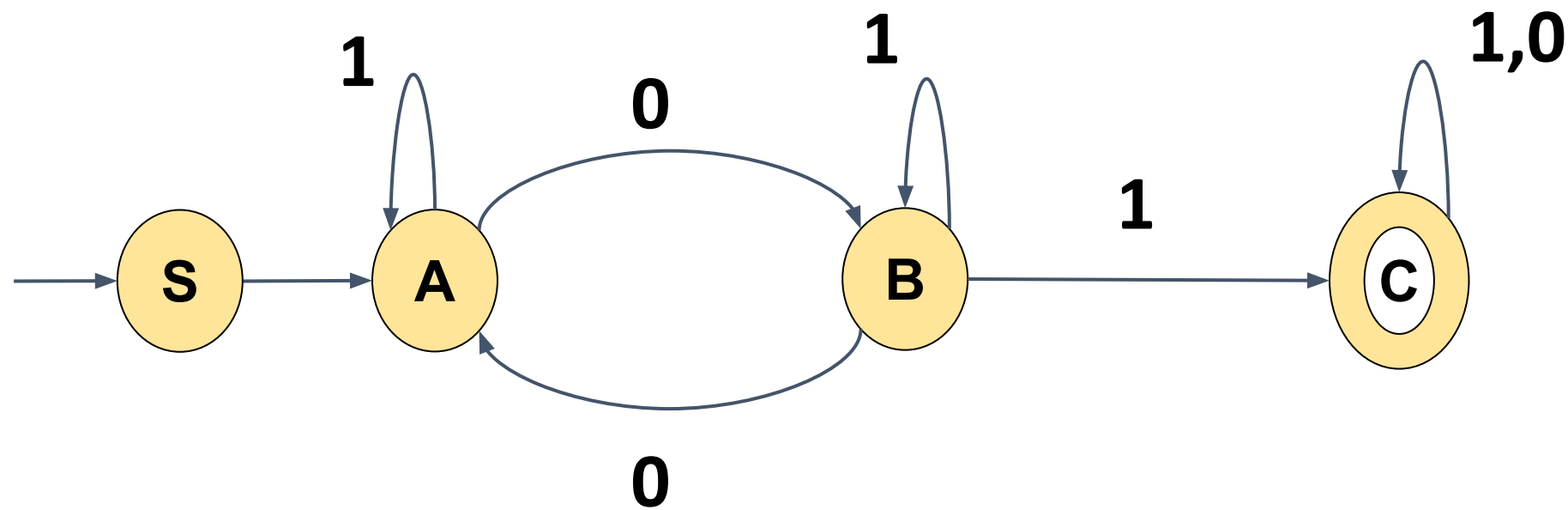


**$RE=(01+10)^*$**

### Example 7 :

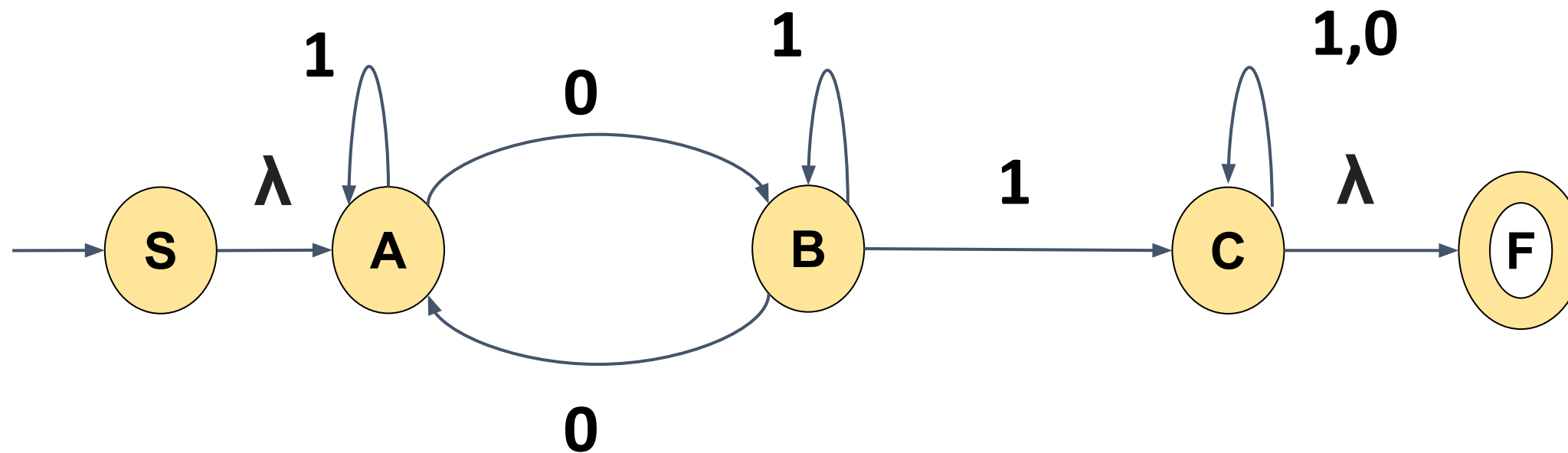


### Example 7 :



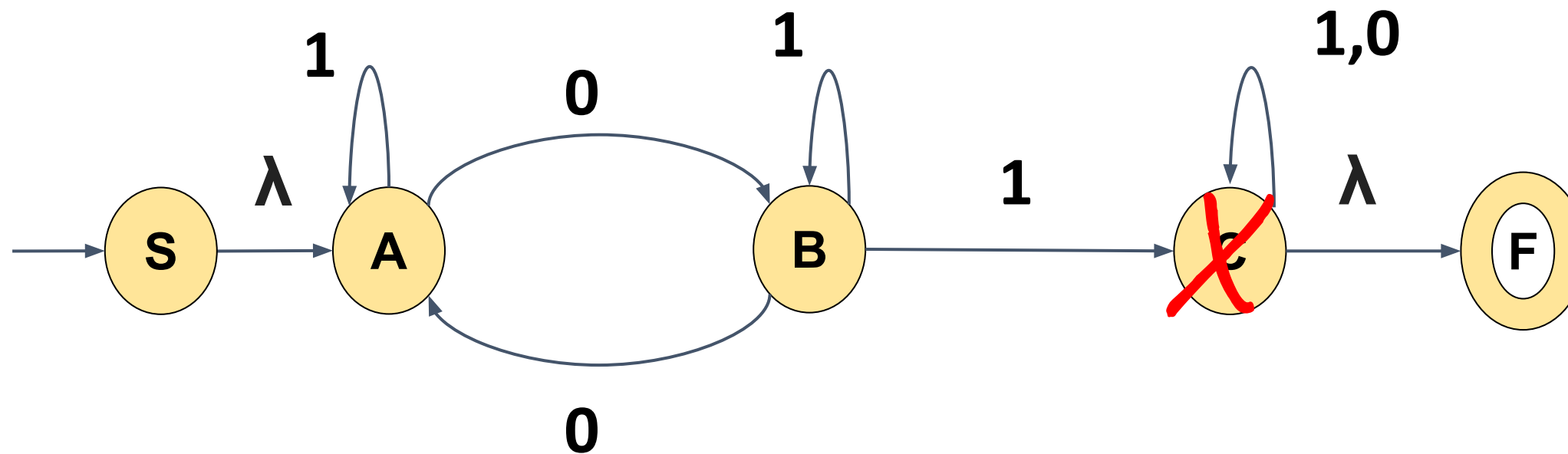
A new start state (S) is introduced as there is an incoming edge to the existing start state

### Example 7 :



A new final state (F) is introduced as there is an outgoing edge from the existing final state

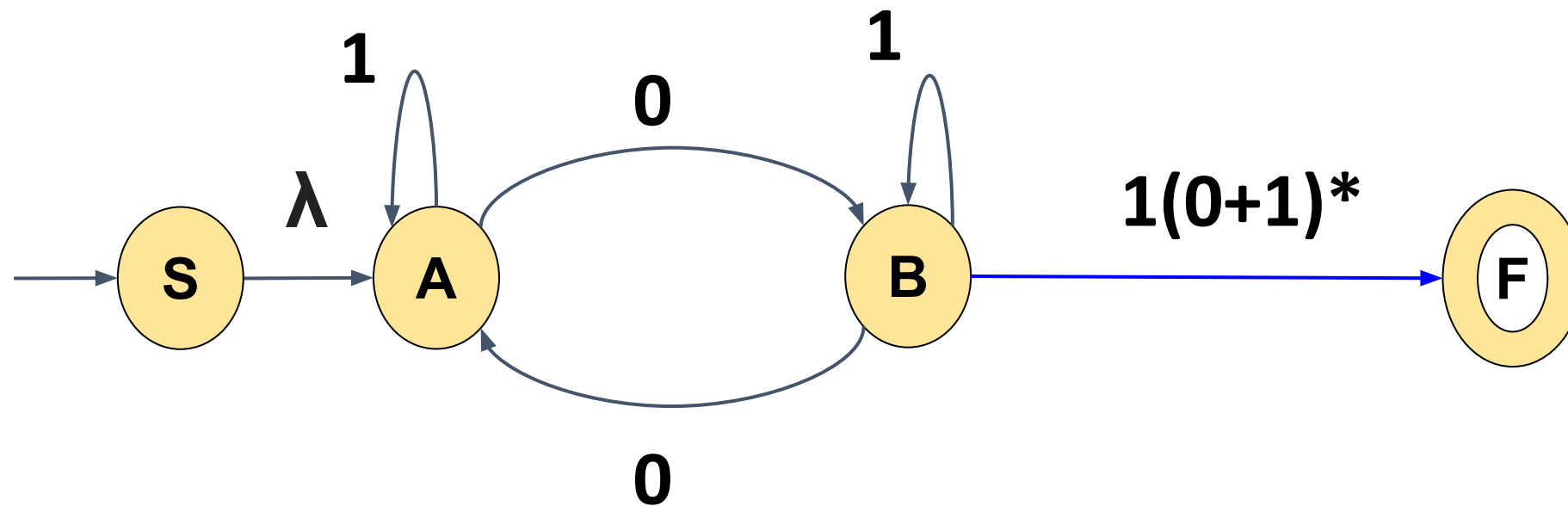
### Example 7 :



### 1. Eliminate C

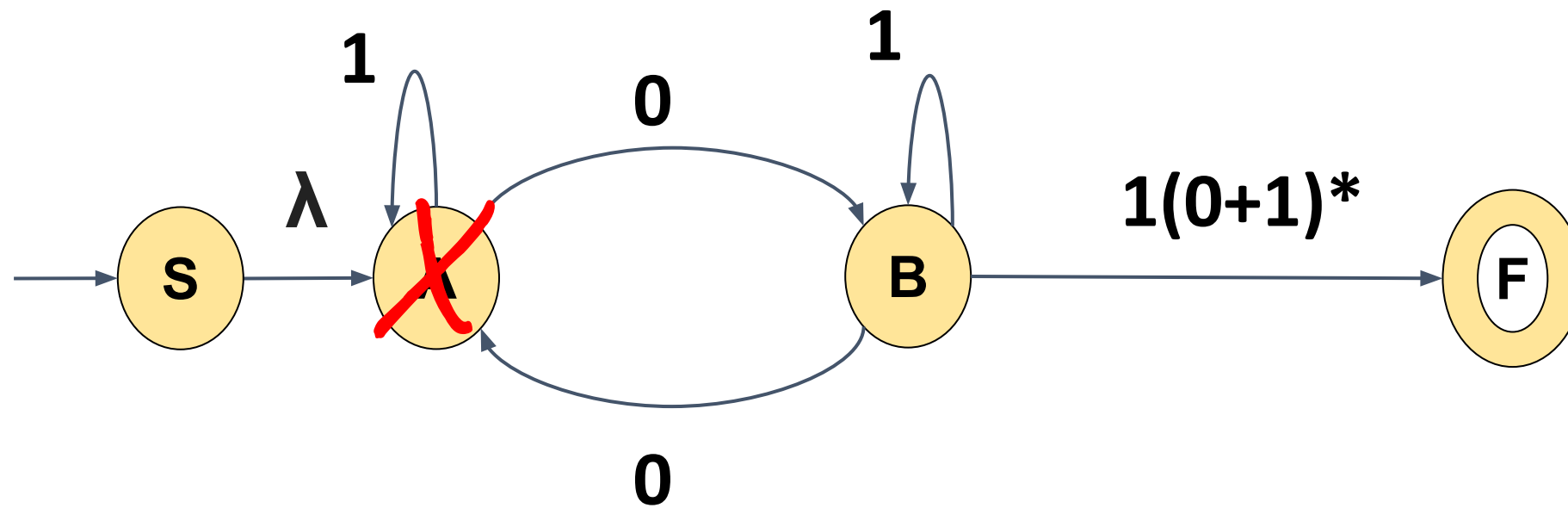


### Example 7 :



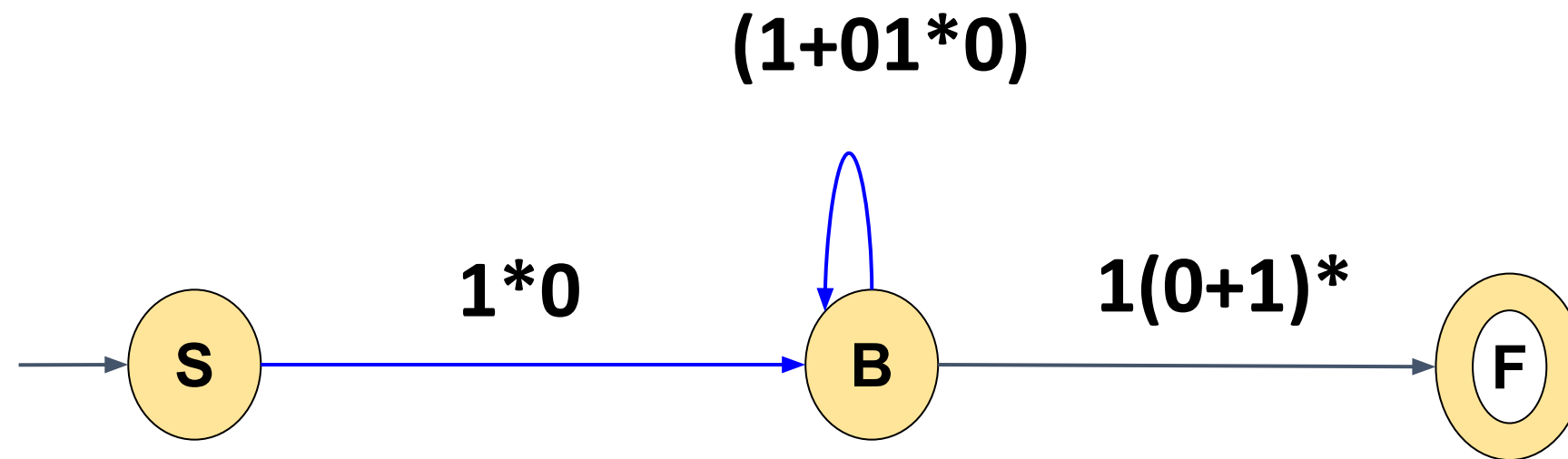
### 1. Eliminate C

### Example 7 :



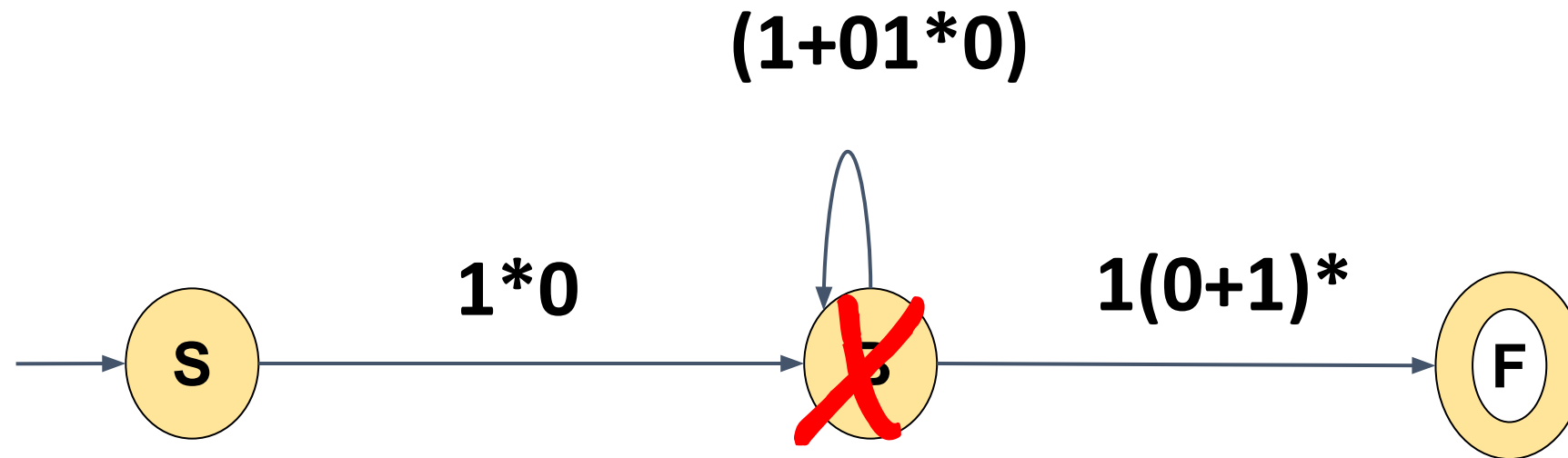
1. Eliminate C
2. Eliminate A

### Example 7 :



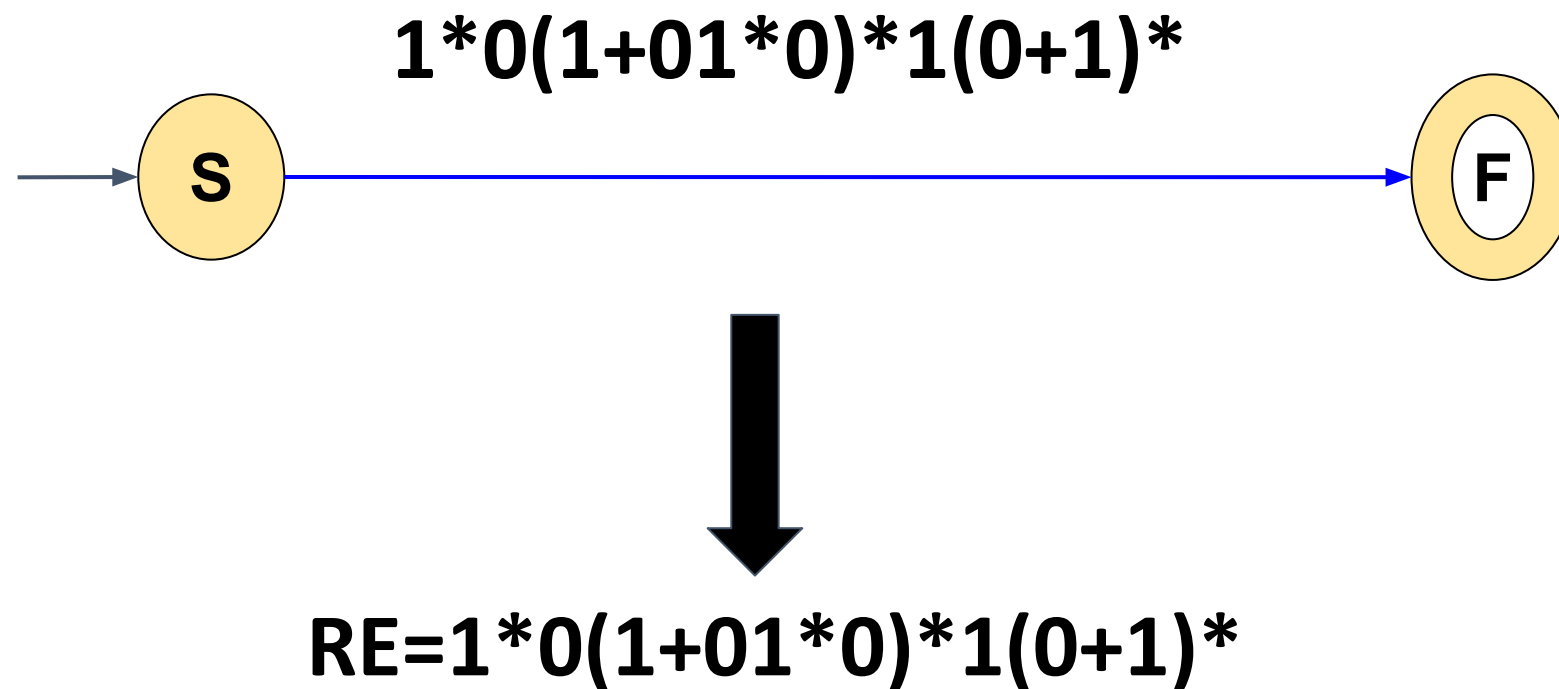
1. Eliminate C
2. Eliminate A

### Example 7 :

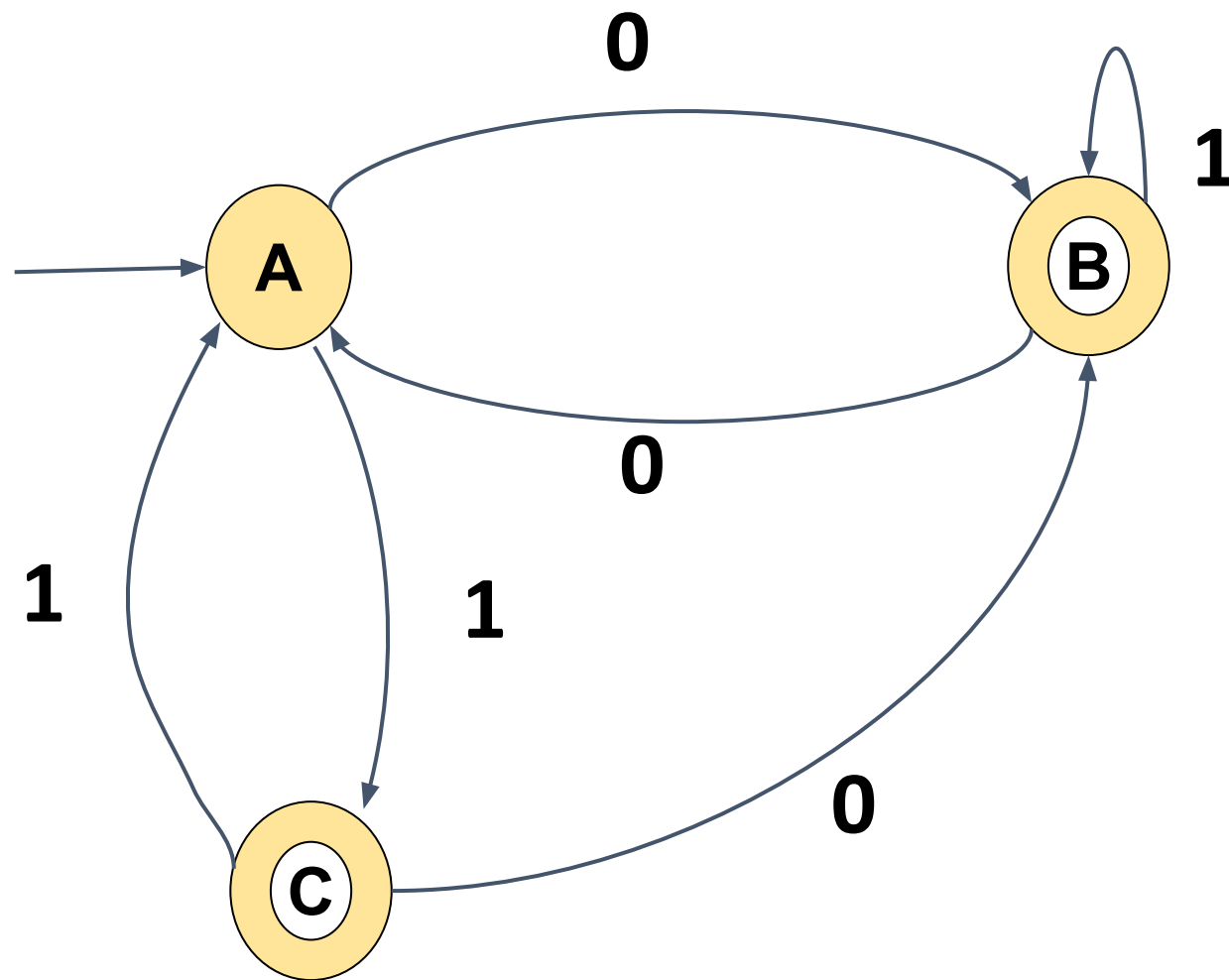


1. Eliminate C
2. Eliminate A
3. Eliminate B

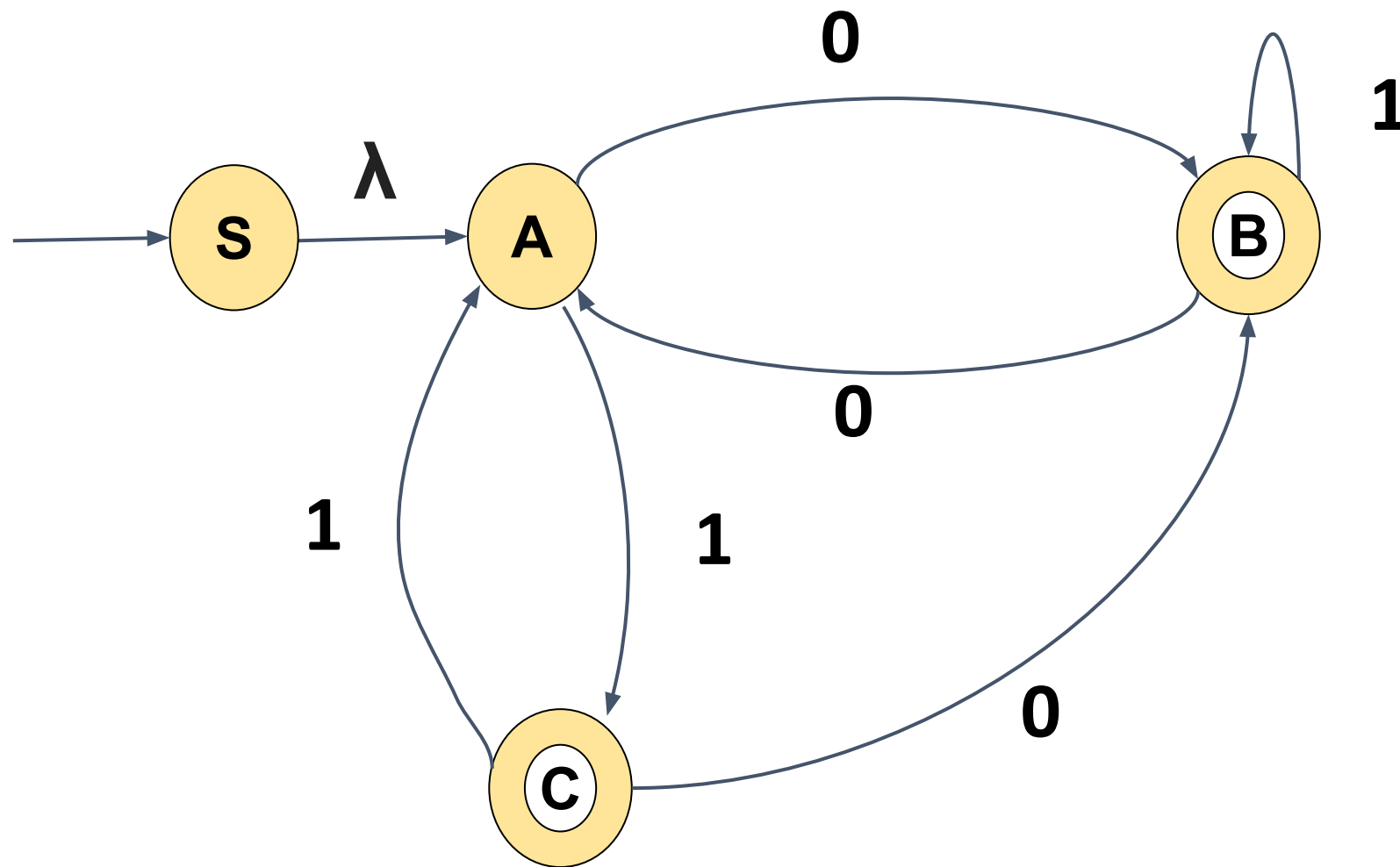
### Example 7 :



### Example 8 :

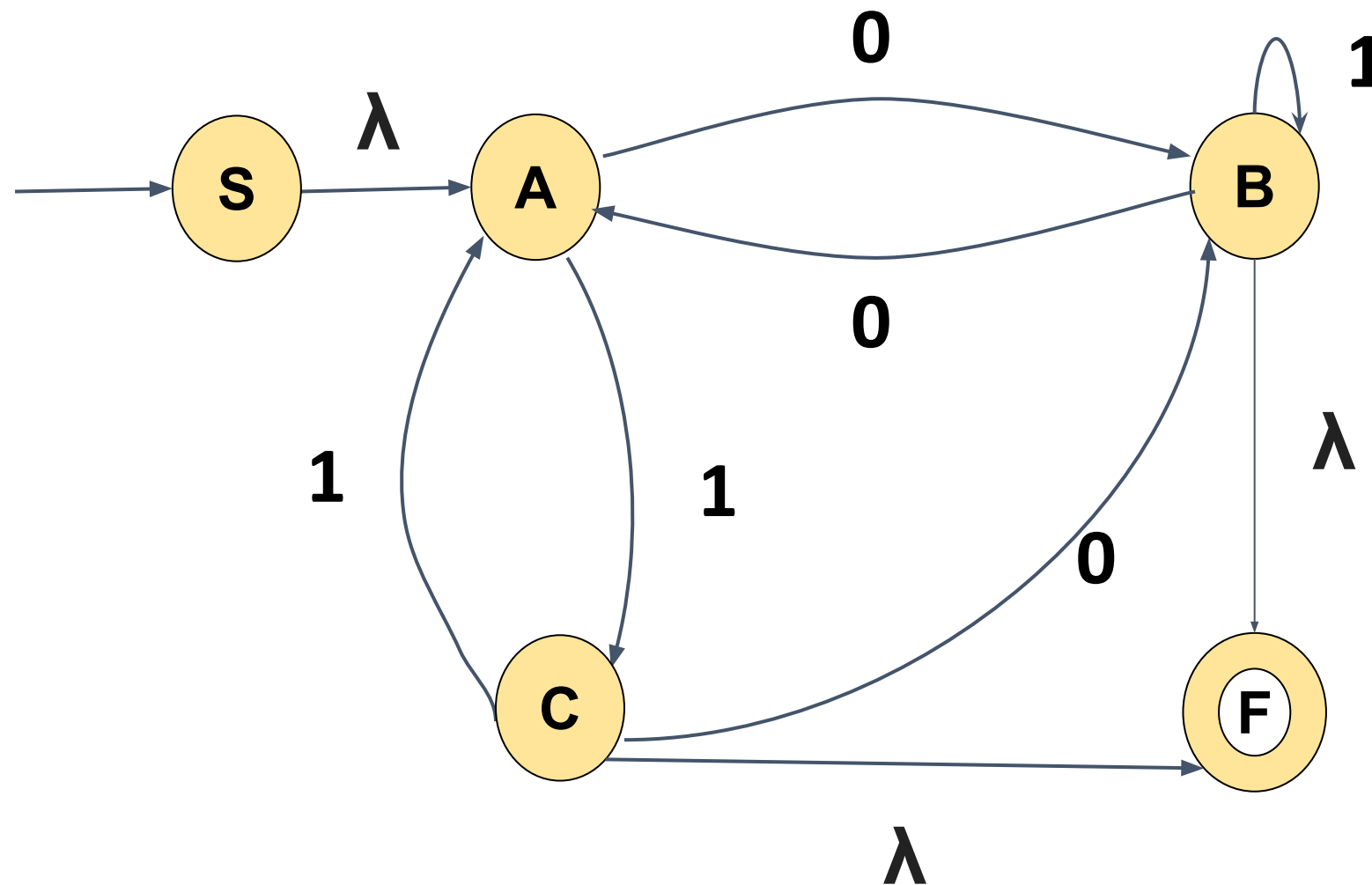


### Example 8 :



A new start state (S) is introduced as there is an incoming edge to the existing start state

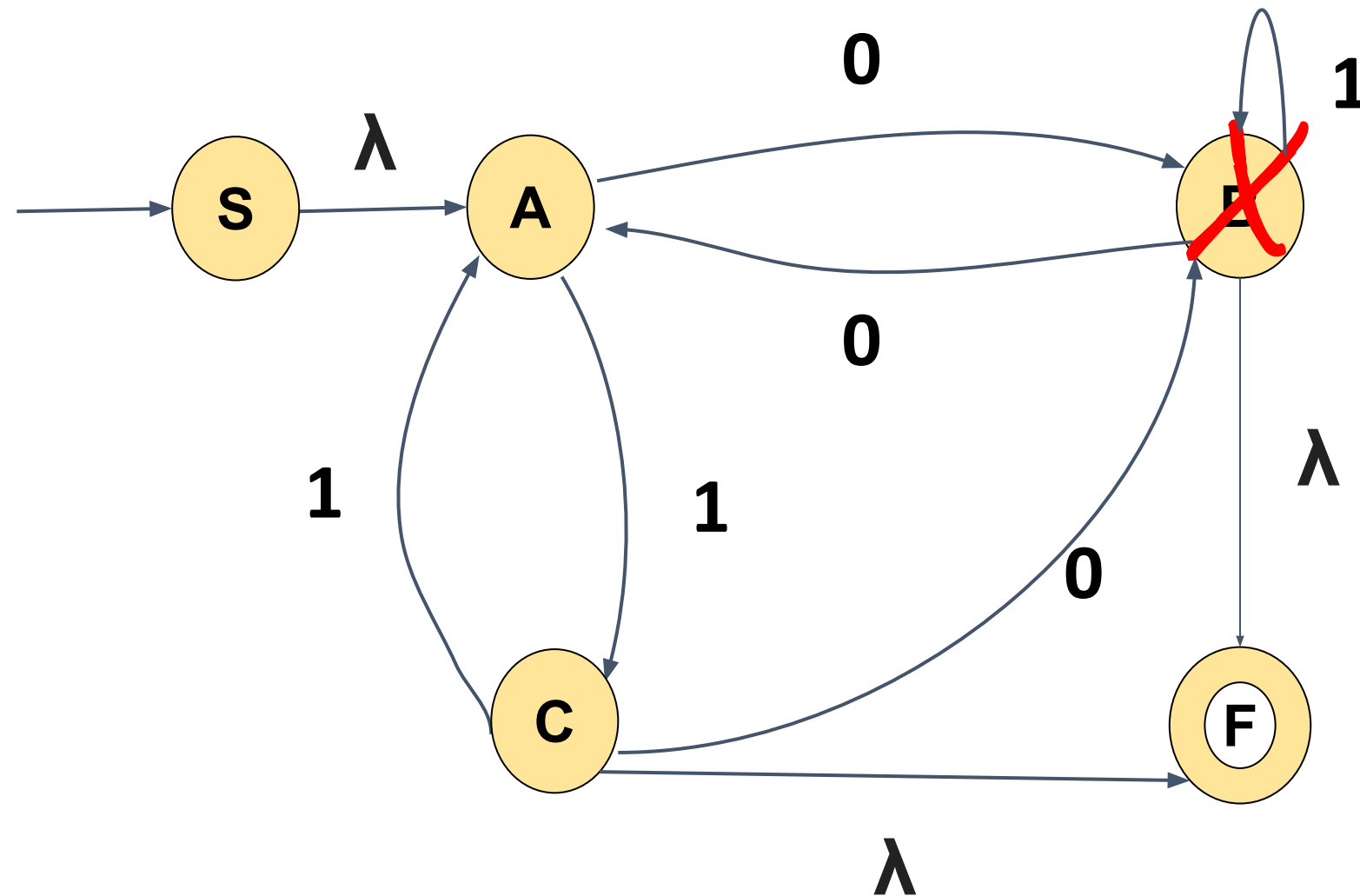
### Example 8 :



A new final state (F) is introduced as there is an outgoing edge from the existing final state

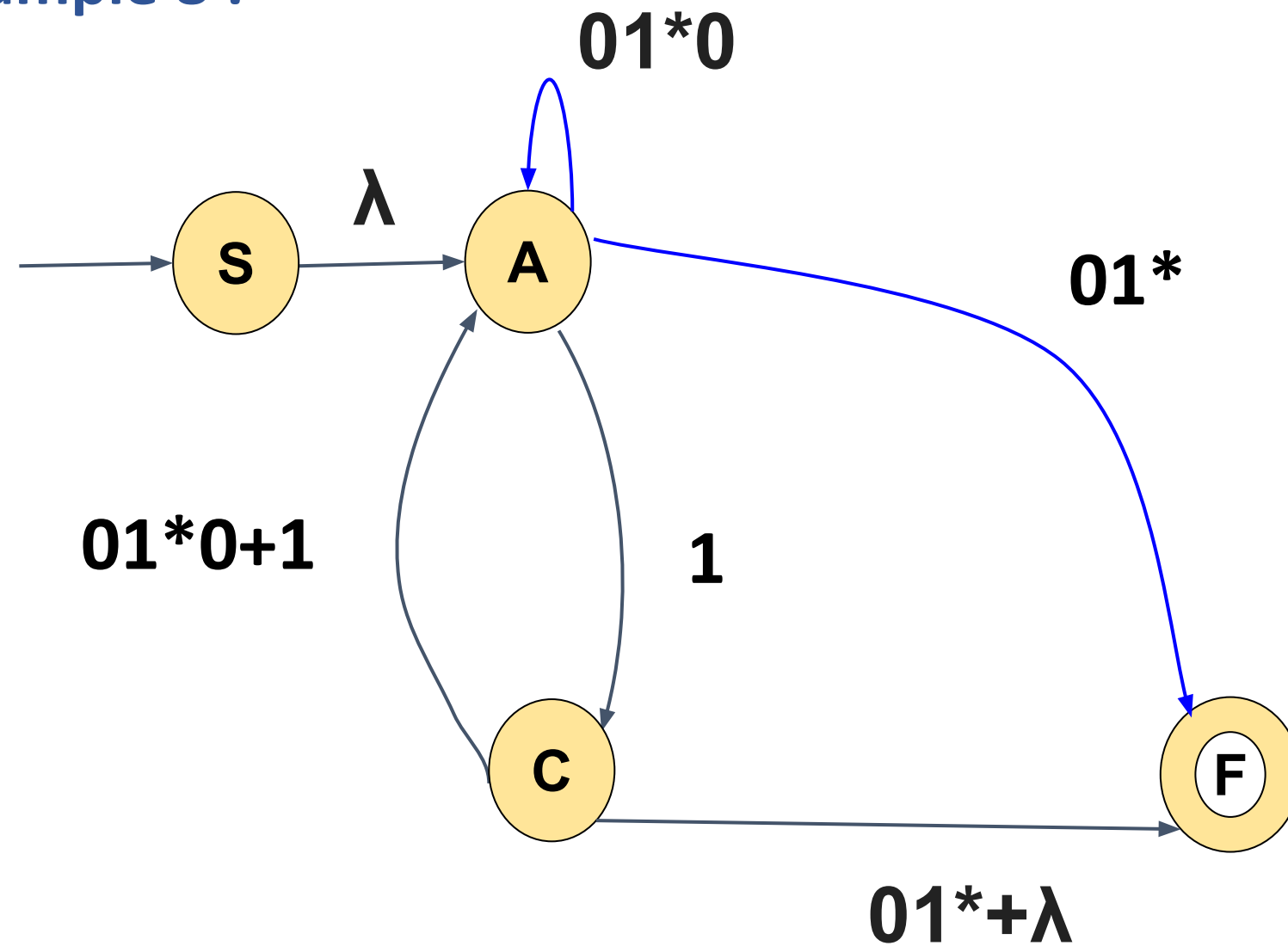


### Example 8 :



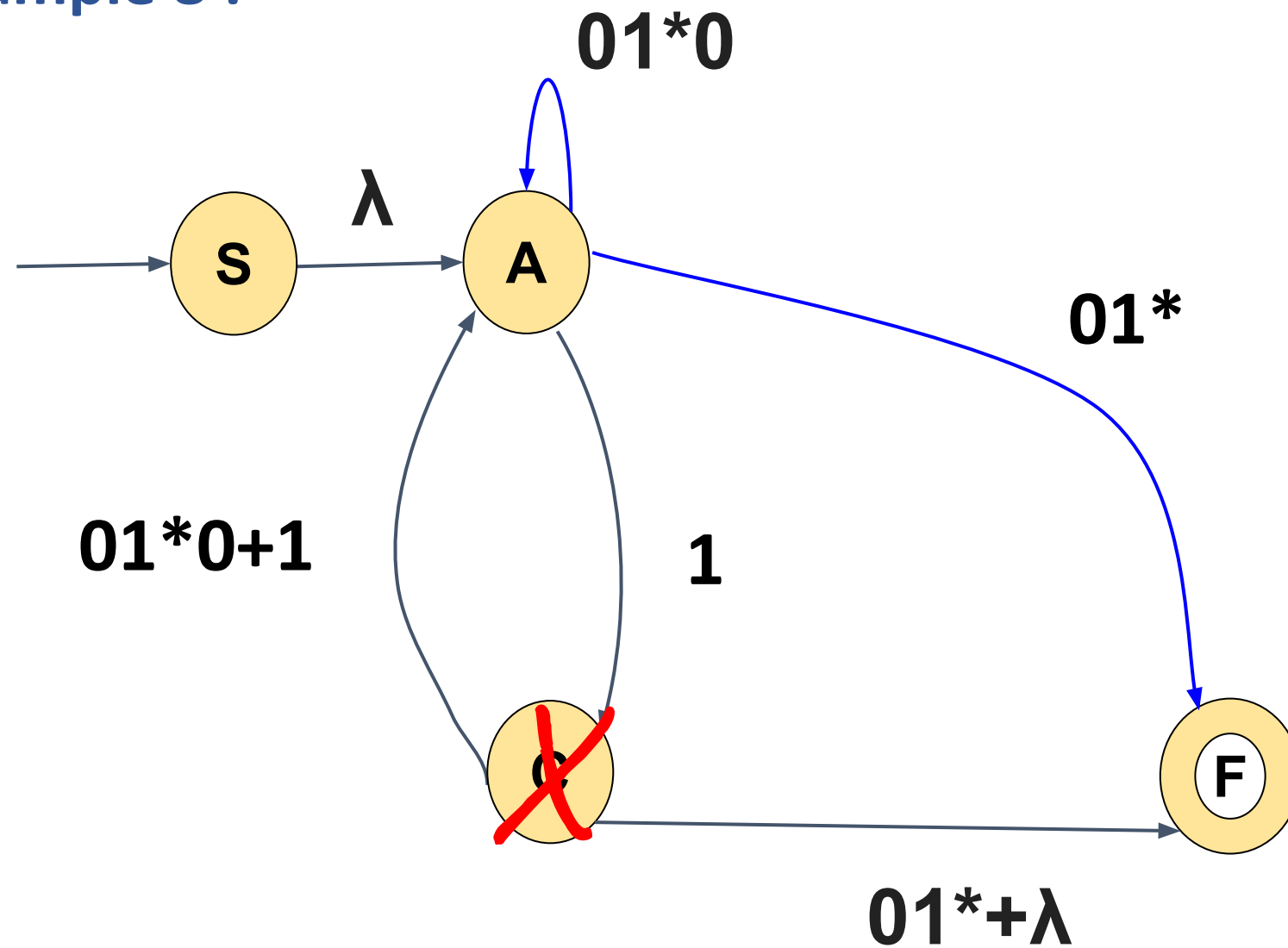
### 1. Eliminate B

### Example 8 :



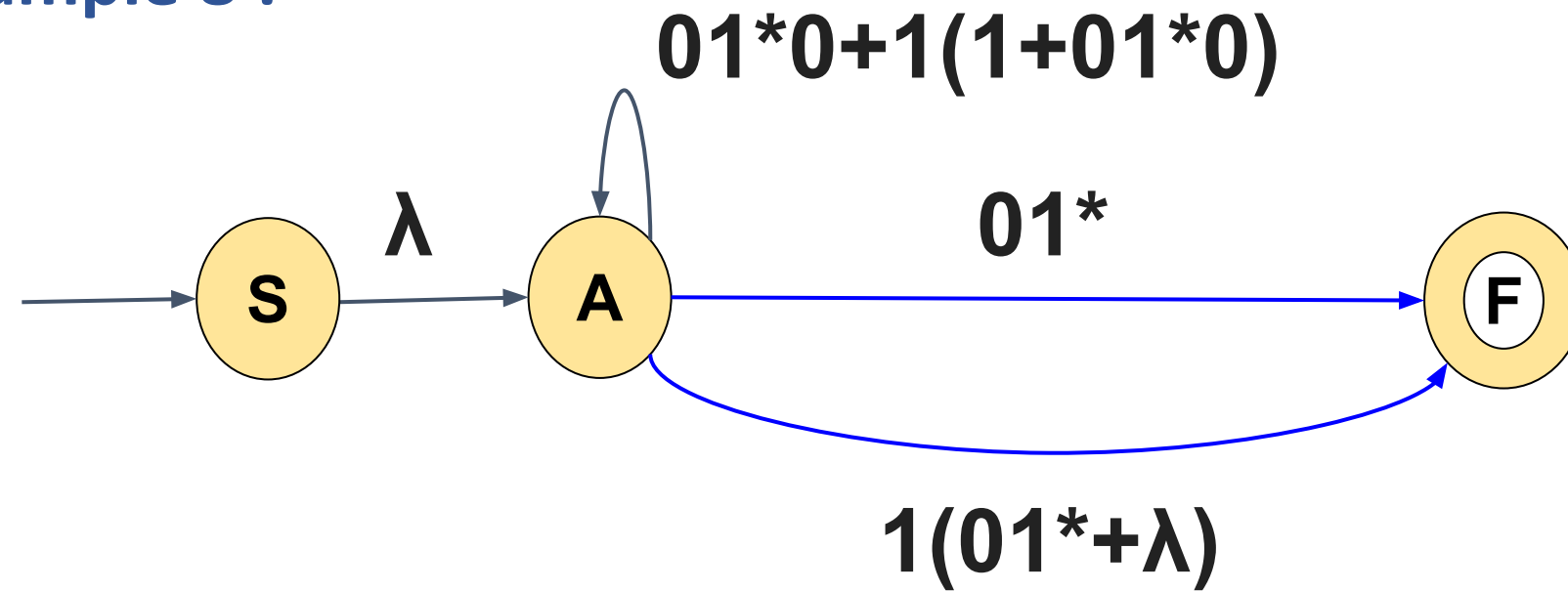
### 1. Eliminate B

### Example 8 :



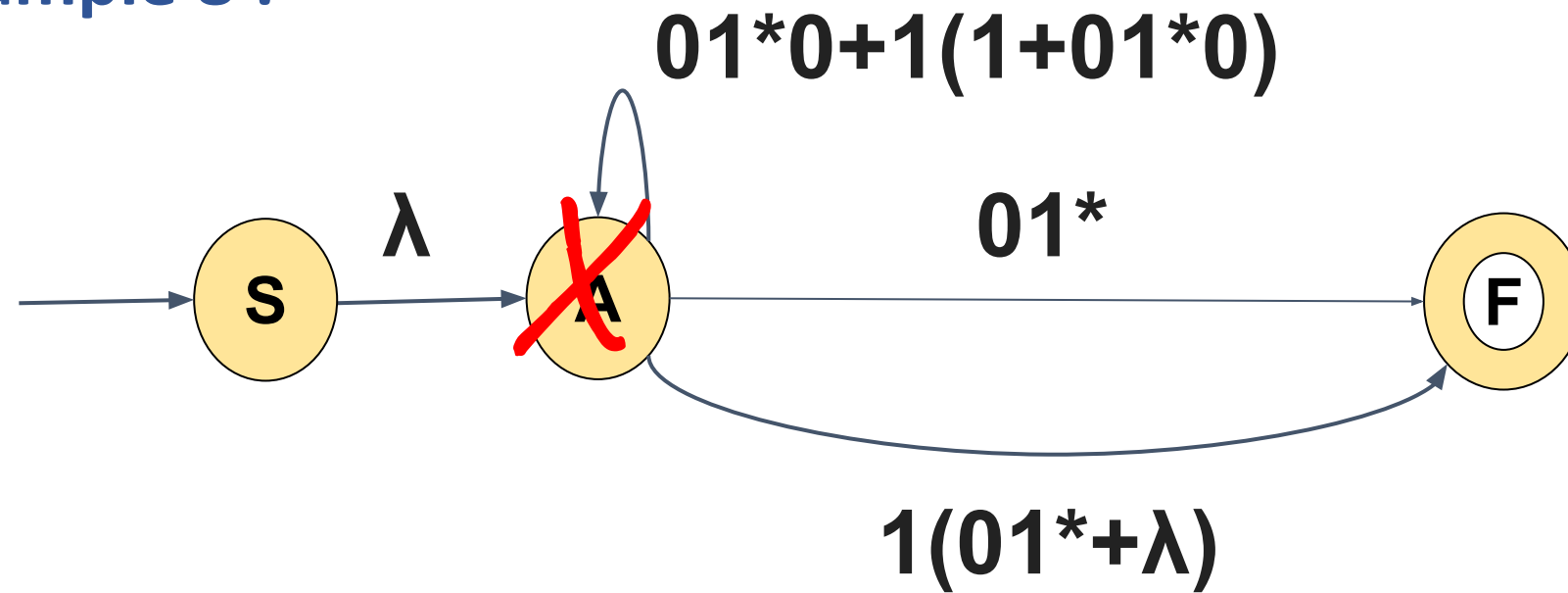
1. Eliminate B
2. Eliminate C

### Example 8 :



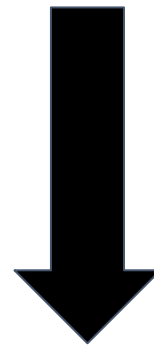
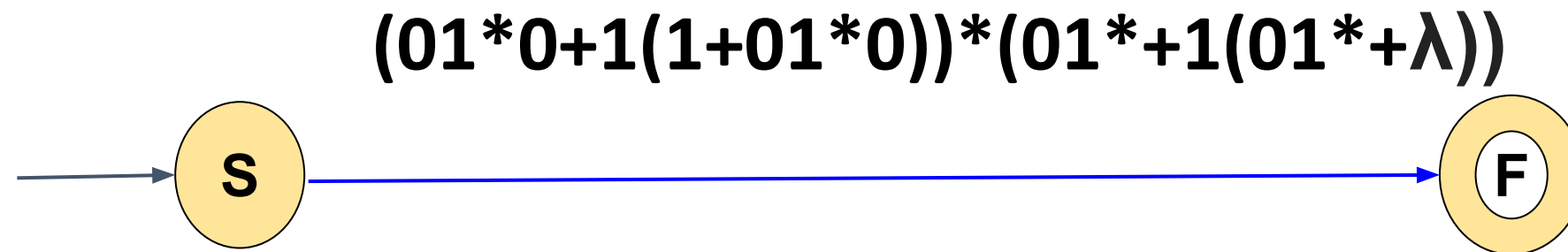
1. Eliminate B
2. Eliminate C

### Example 8 :



1. Eliminate B
2. Eliminate C
3. Eliminate A

### Example 8 :



$RE=(01^*0+1(1+01^*0))^*(01^*+1(01^*+\lambda))$

### Example 8 :

$$RE = (01^*0 + 1(1 + 01^*0))^*(01^* + 1(01^* + \lambda))$$

$$= (01^*0 + 11 + 101^*0)^*(01^* + (101^* + 1))$$

### Example 8 :

$$RE = (01^*0 + 1(1 + 01^*0))^* (01^* + 1(01^* + \lambda))$$

$$= (01^*0 + 11 + 101^*0)^* (01^* + (101^* + 1))$$

$$= (101^*0 + 01^*0 + 11)^* (01^*(\lambda + 1) + 1)$$



### Example 8 :

$$RE = (01^*0 + 1(1 + 01^*0))^*(01^* + 1(01^* + \lambda))$$

$$= (01^*0 + 11 + 101^*0)^*(01^* + (101^* + 1))$$

$$= (101^*0 + 01^*0 + 11)^*(01^*(\lambda + 1) + 1)$$

$$= (01^*0(1 + \lambda) + 11)^*(01^*(\lambda + 1) + 1)$$

### Example 8 :

$$RE = (01^*0 + 1(1 + 01^*0))^* (01^* + 1(01^* + \lambda))$$

$$= (01^*0 + 11 + 101^*0)^* (01^* + (101^* + 1))$$

$$= (101^*0 + 01^*0 + 11)^* (01^*(\lambda + 1) + 1)$$

$$= (01^*0(1 + \lambda) + 11)^* (01^*(\lambda + 1) + 1)$$

$$= (01^*0(1 + \lambda))^* (11)^* (01^*(\lambda + 1) + 1)$$

### Example 8 :

$$RE = (01^*0 + 1(1 + 01^*0))^* (01^* + 1(01^* + \lambda))$$

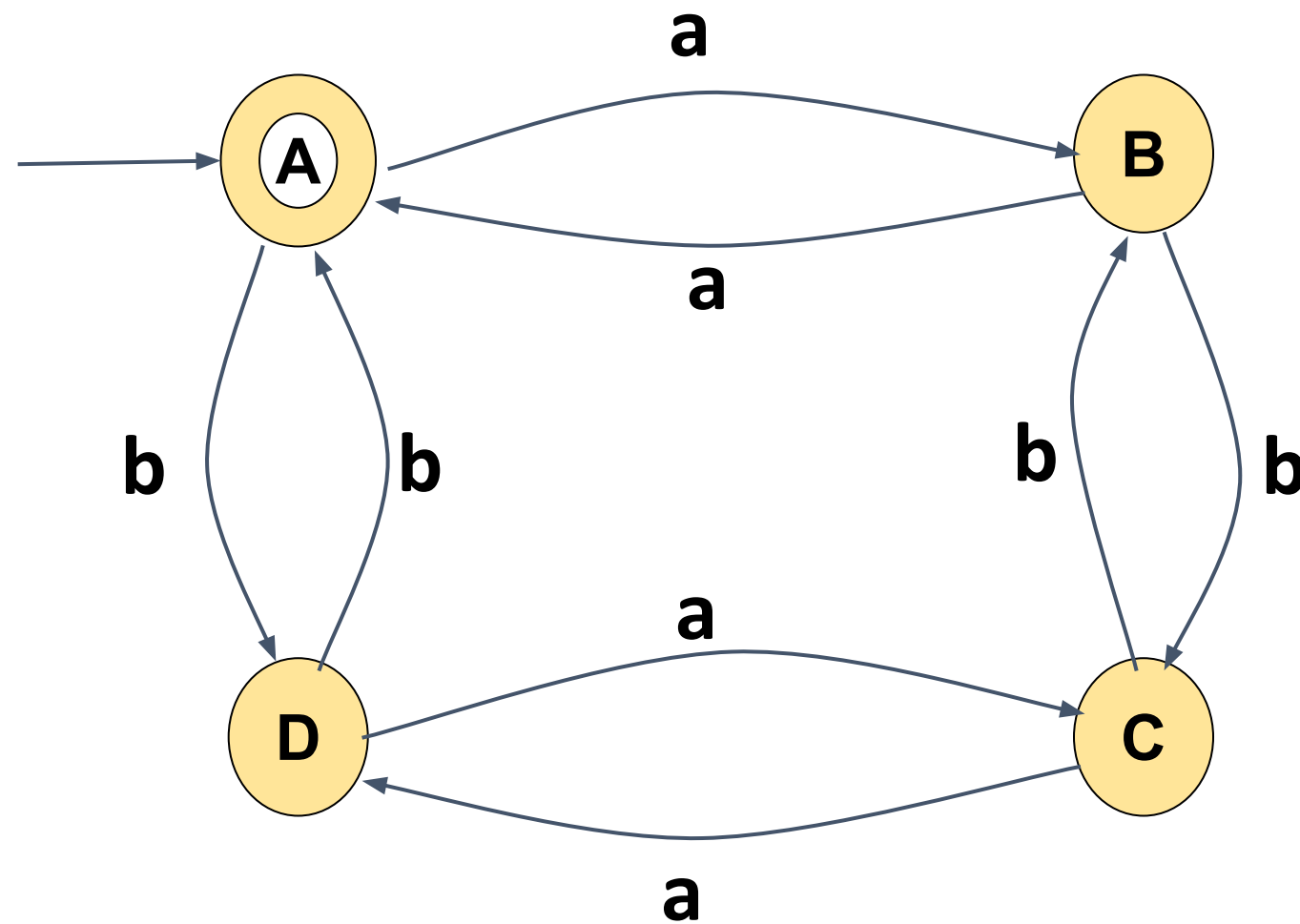
$$= (01^*0 + 11 + 101^*0)^* (01^* + (101^* + 1))$$

$$= (101^*0 + 01^*0 + 11)^* (01^*(\lambda + 1) + 1)$$

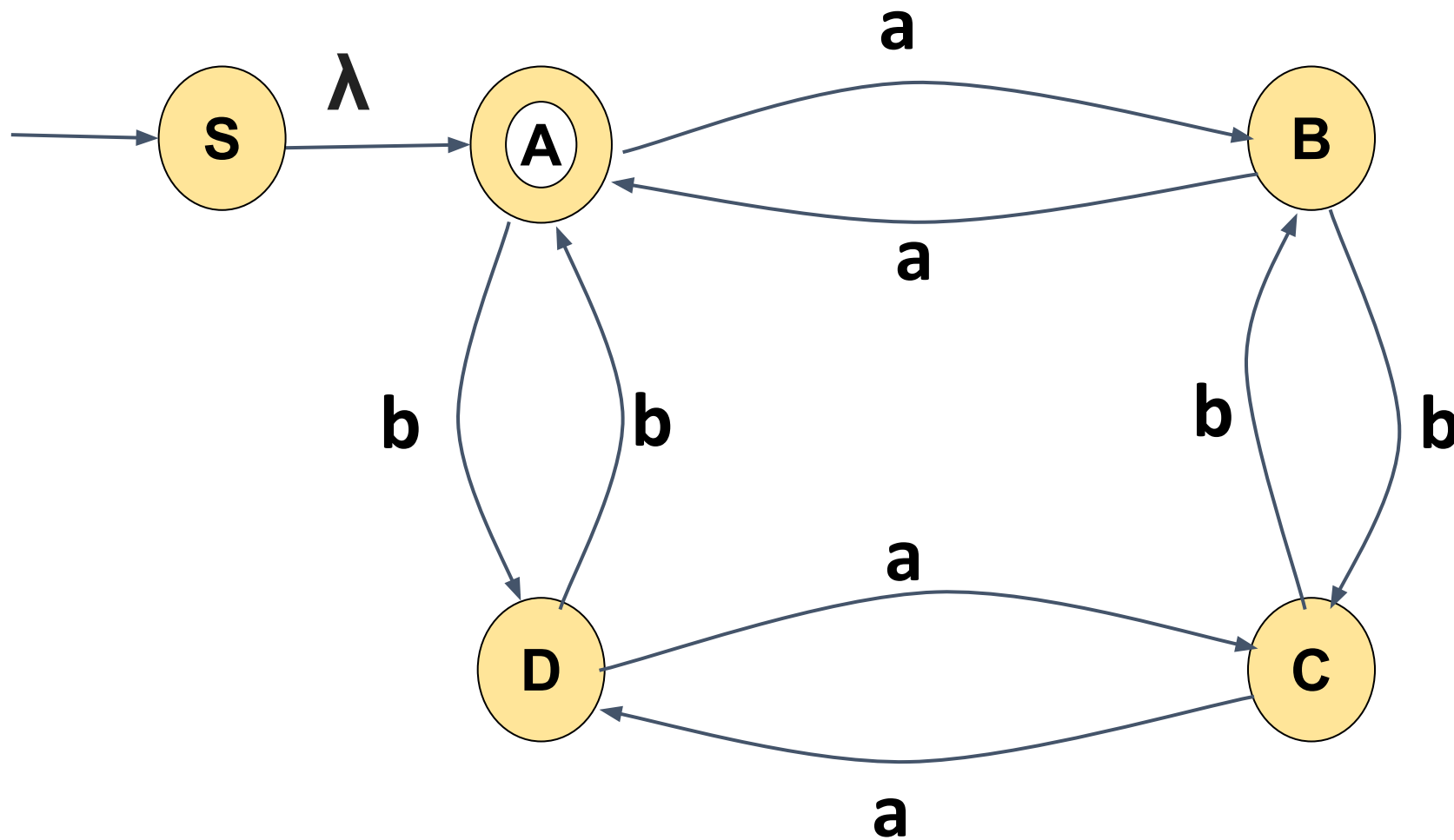
$$= (01^*0(1 + \lambda) + 11)^* (01^*(\lambda + 1) + 1)$$

$$RE = (01^*0(1 + \lambda))^* (11)^* (01^*(\lambda + 1) + 1)$$

### Example 9 :order of elimination( B,D,C,A)

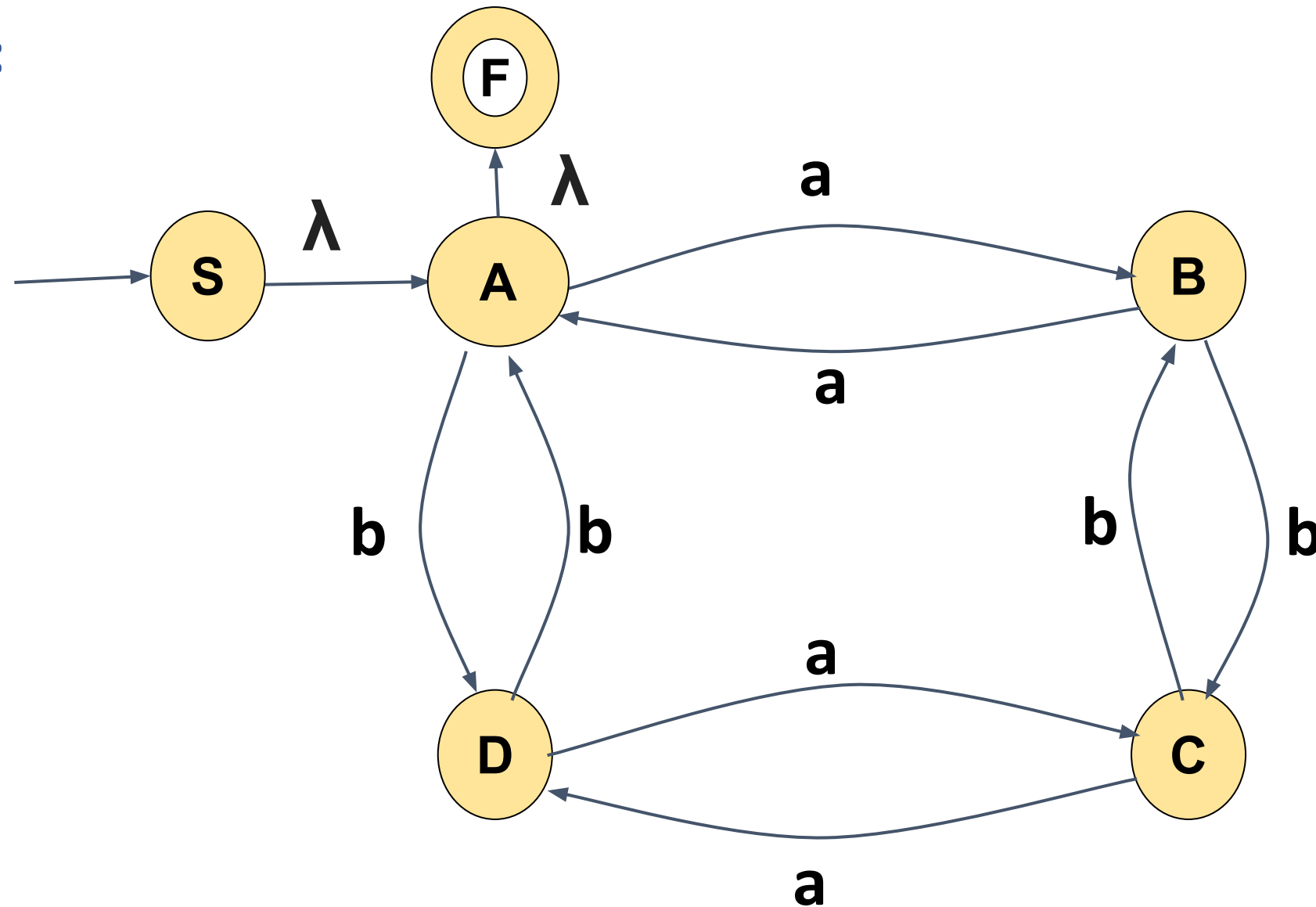


### Example 9 :



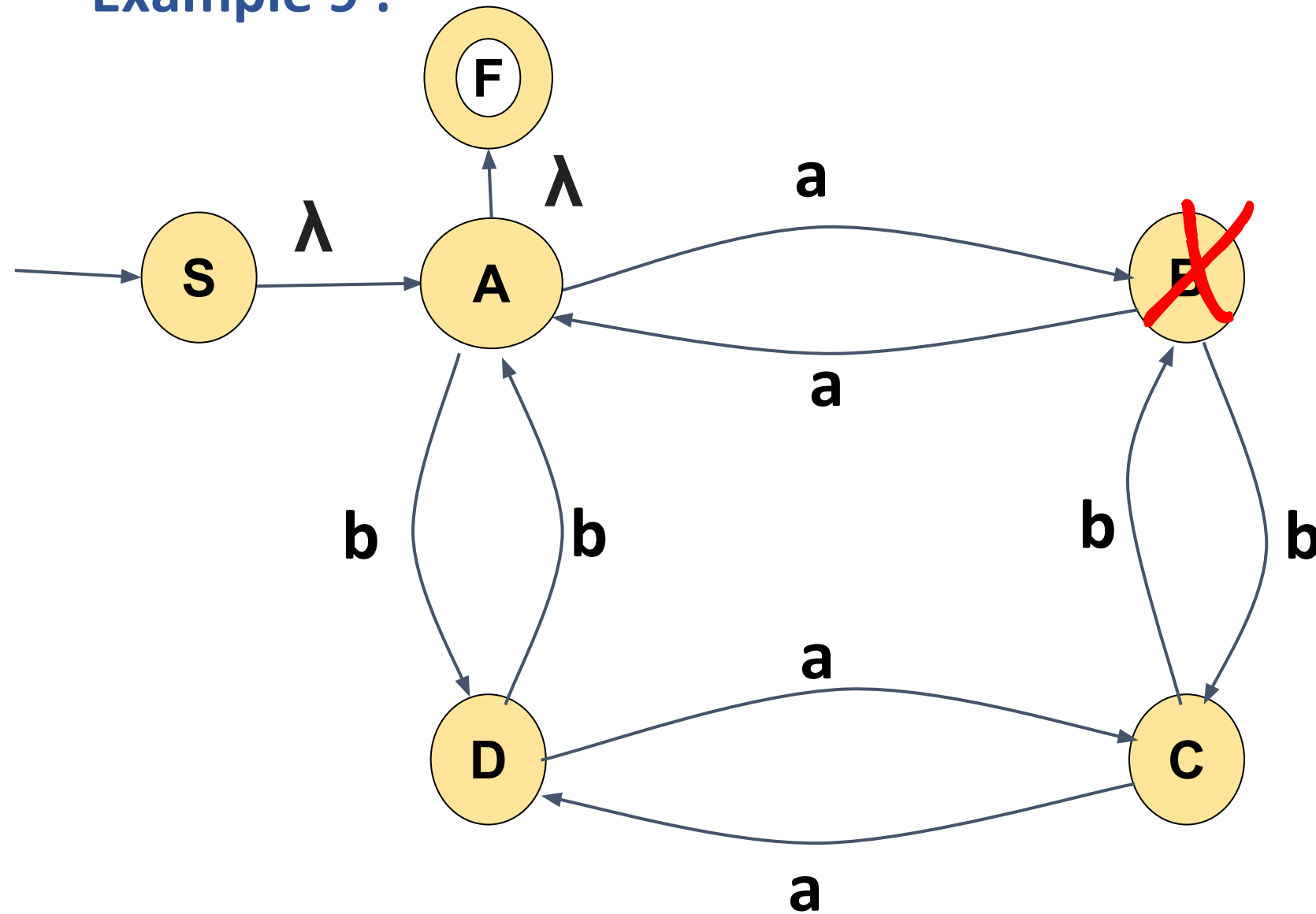
A new start state (S) is introduced as there is an incoming edge to the existing start state

### Example 9 :



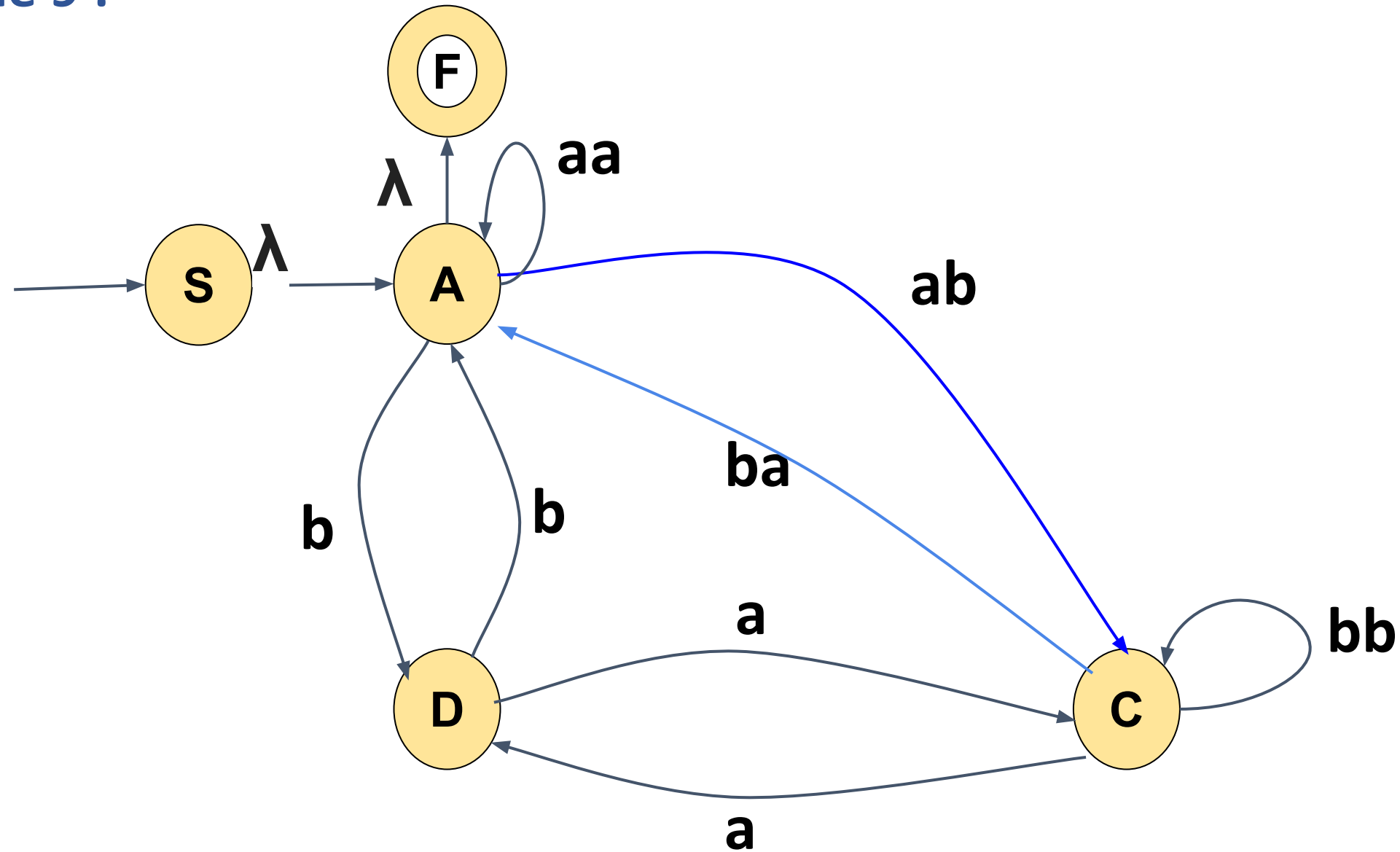
A new final state (F) is introduced as there is an outgoing edge from the existing final state

### Example 9 :



### 1. Eliminate B

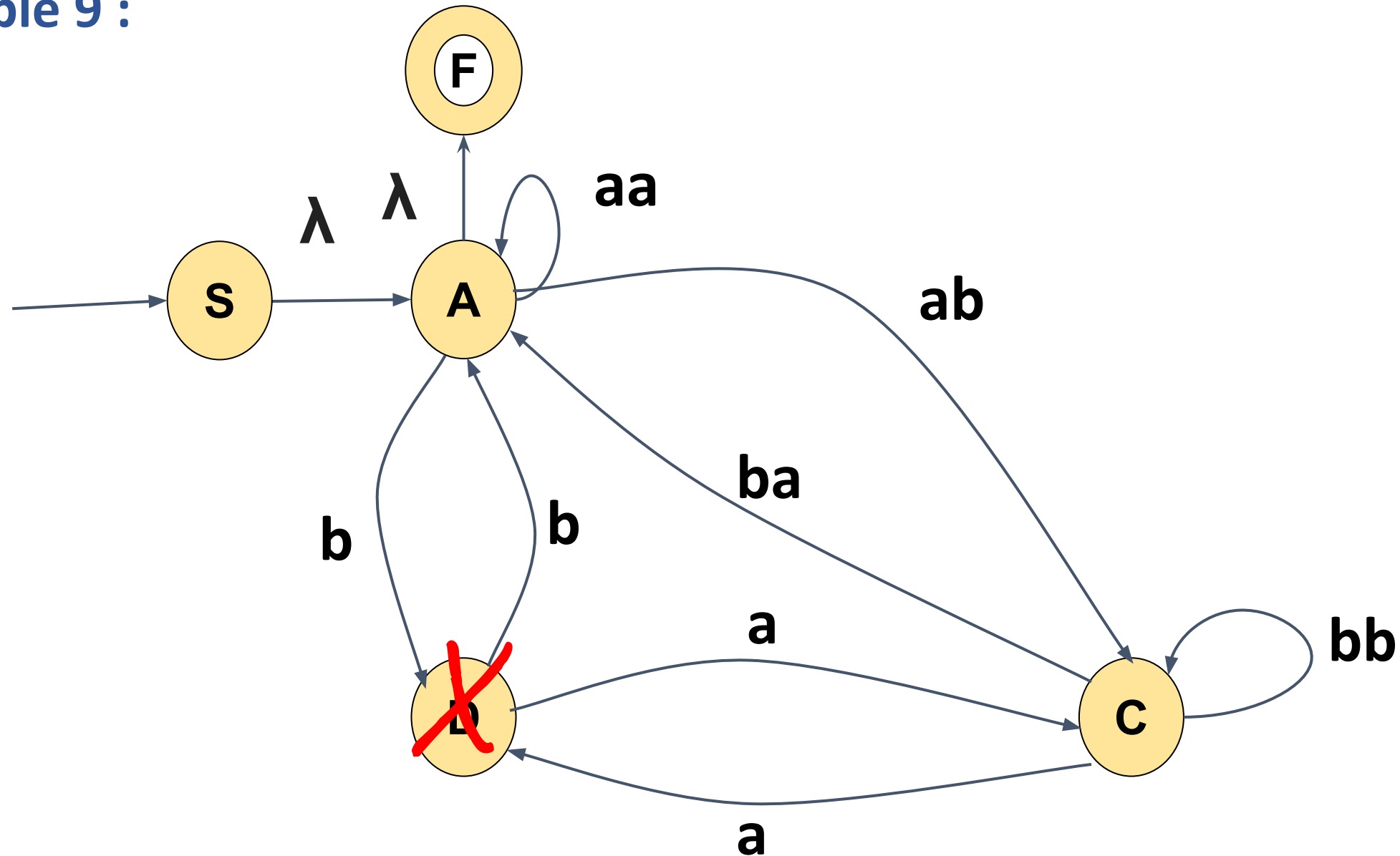
### Example 9 :



### 1. Eliminate B

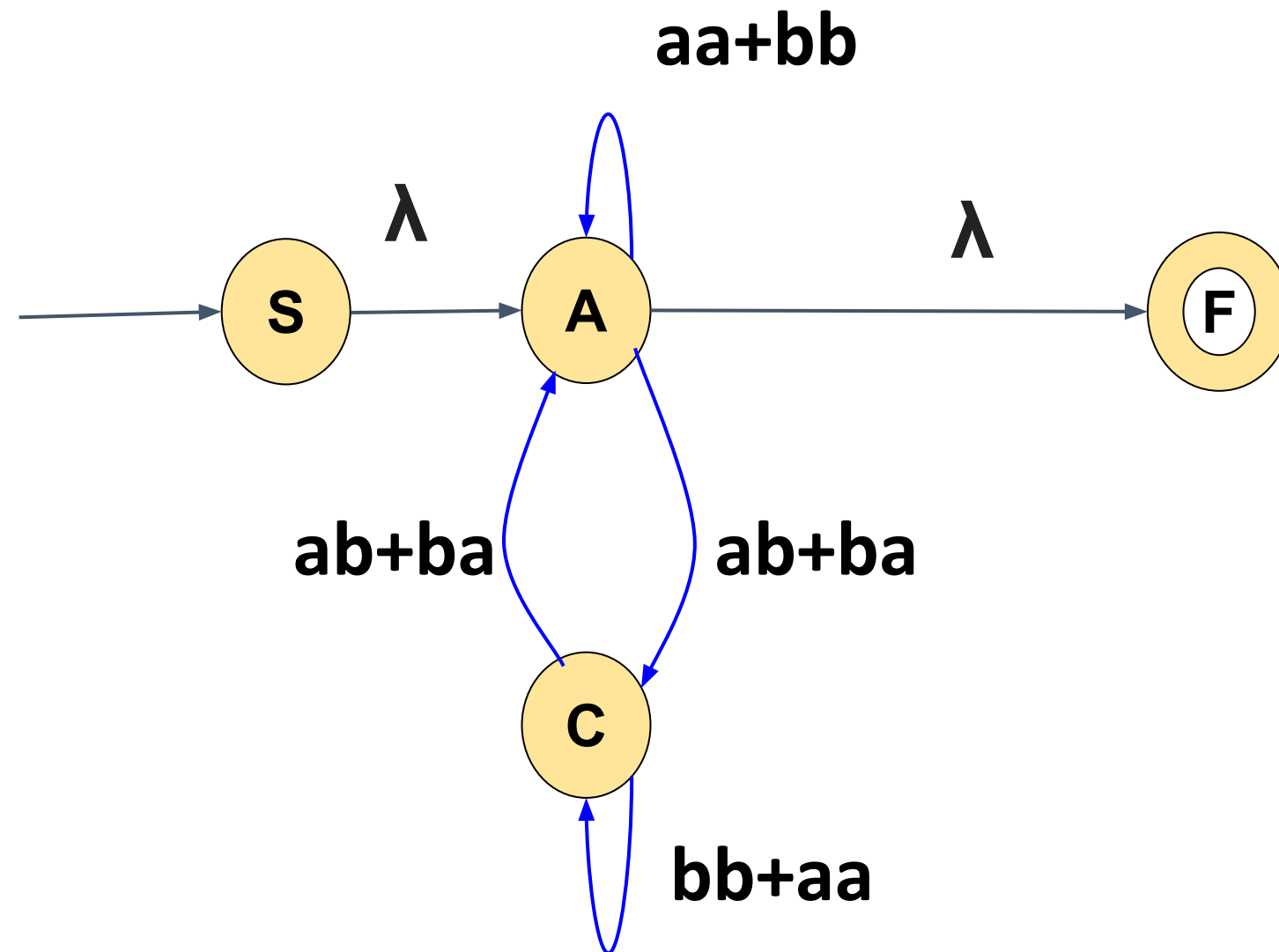


### Example 9 :



1. Eliminate B
2. Eliminate D

### Example 9 :

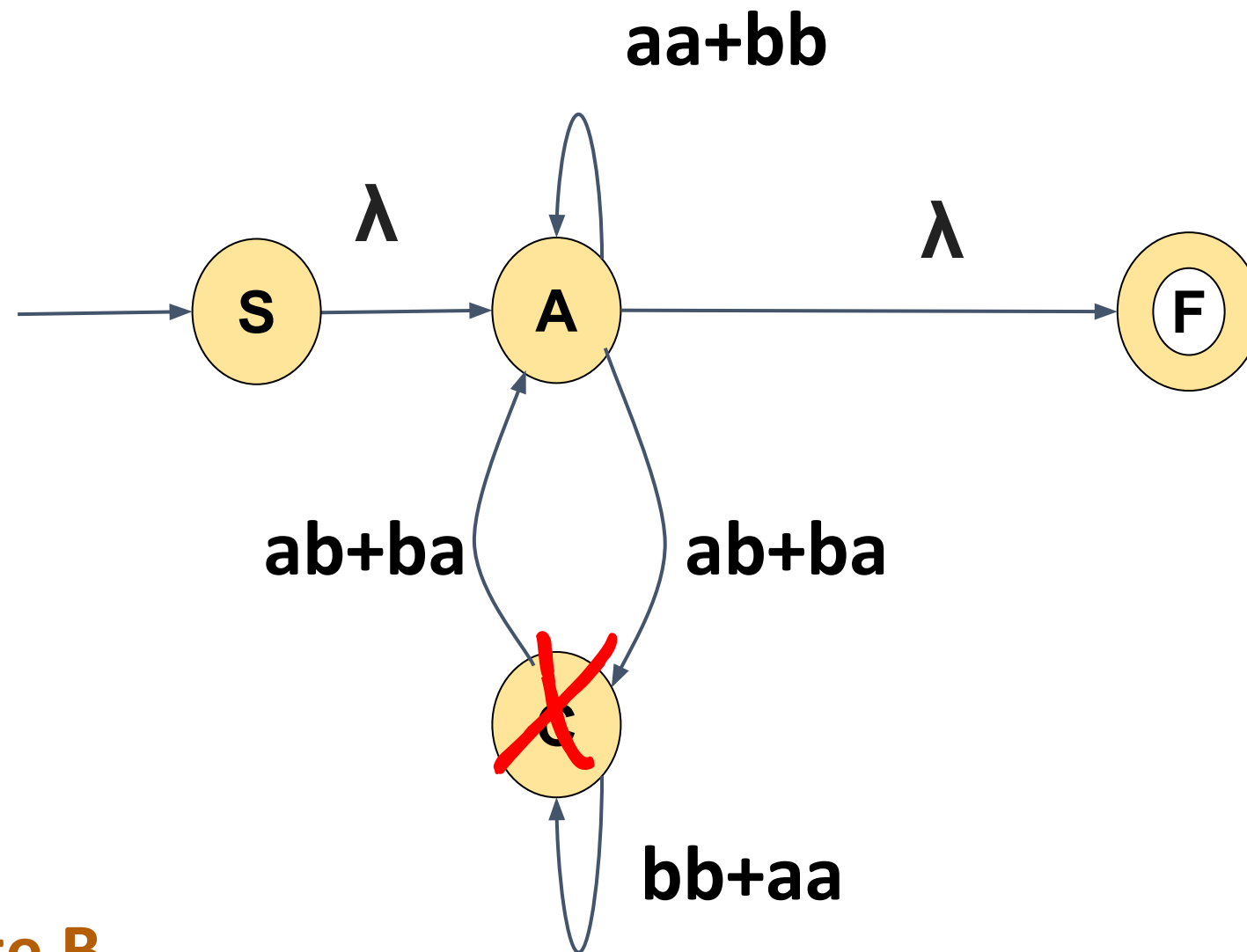


1. Eliminate B
2. Eliminate D

# Automata Formal Languages and Logic

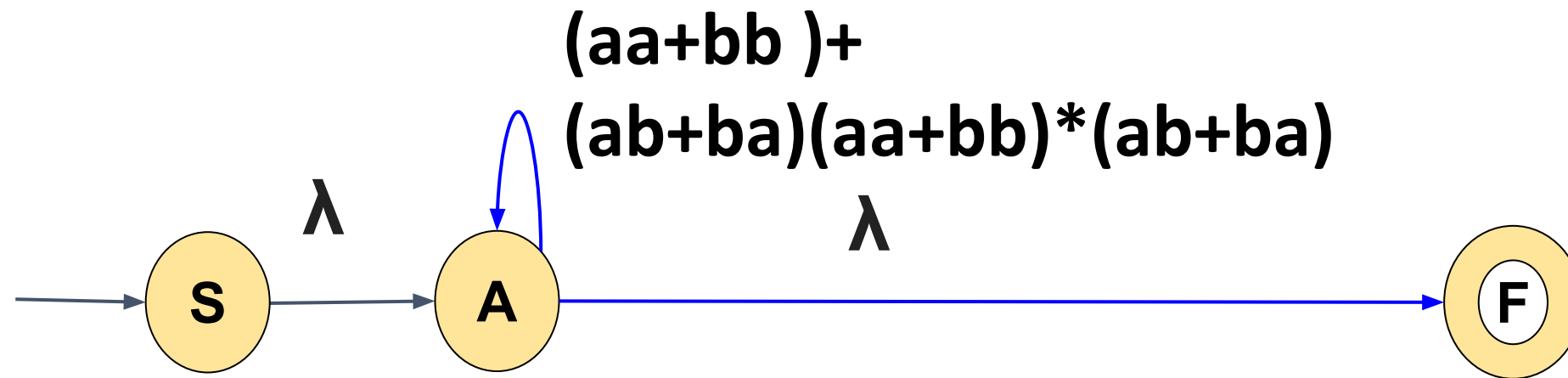
## Unit 2 - Finite Automata to Regular Expression

Example 9 :



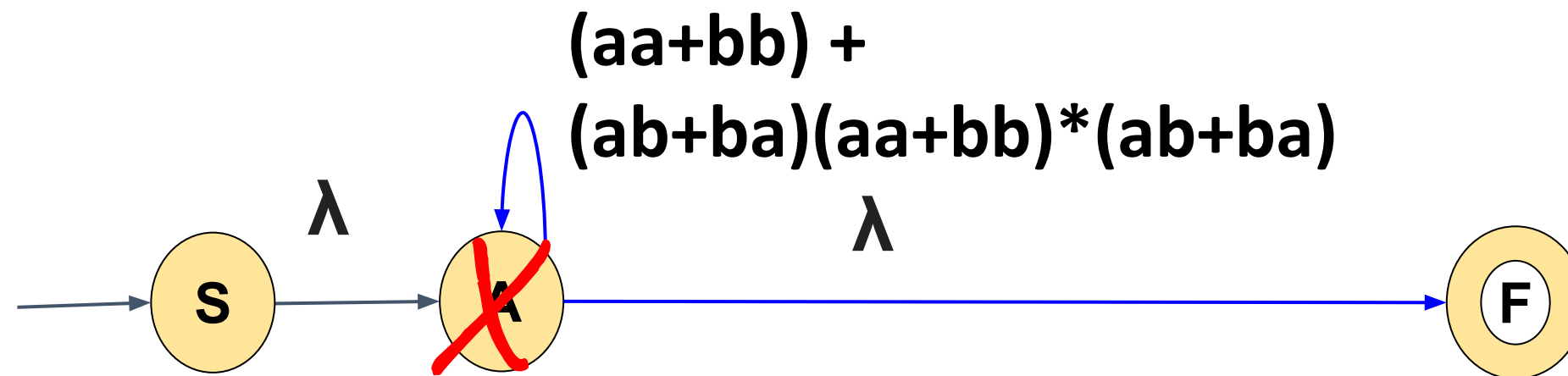
1. Eliminate B
2. Eliminate D
3. Eliminate C

### Example 9 :



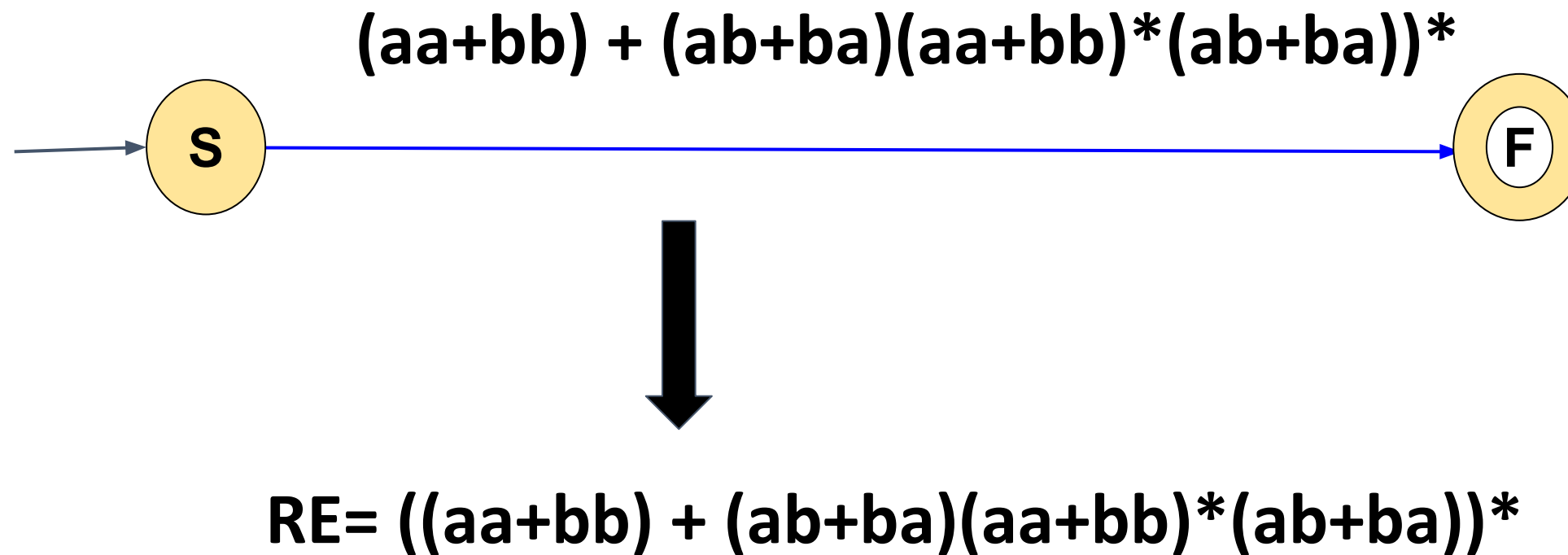
1. Eliminate B
2. Eliminate D
3. Eliminate C

### Example 9 :



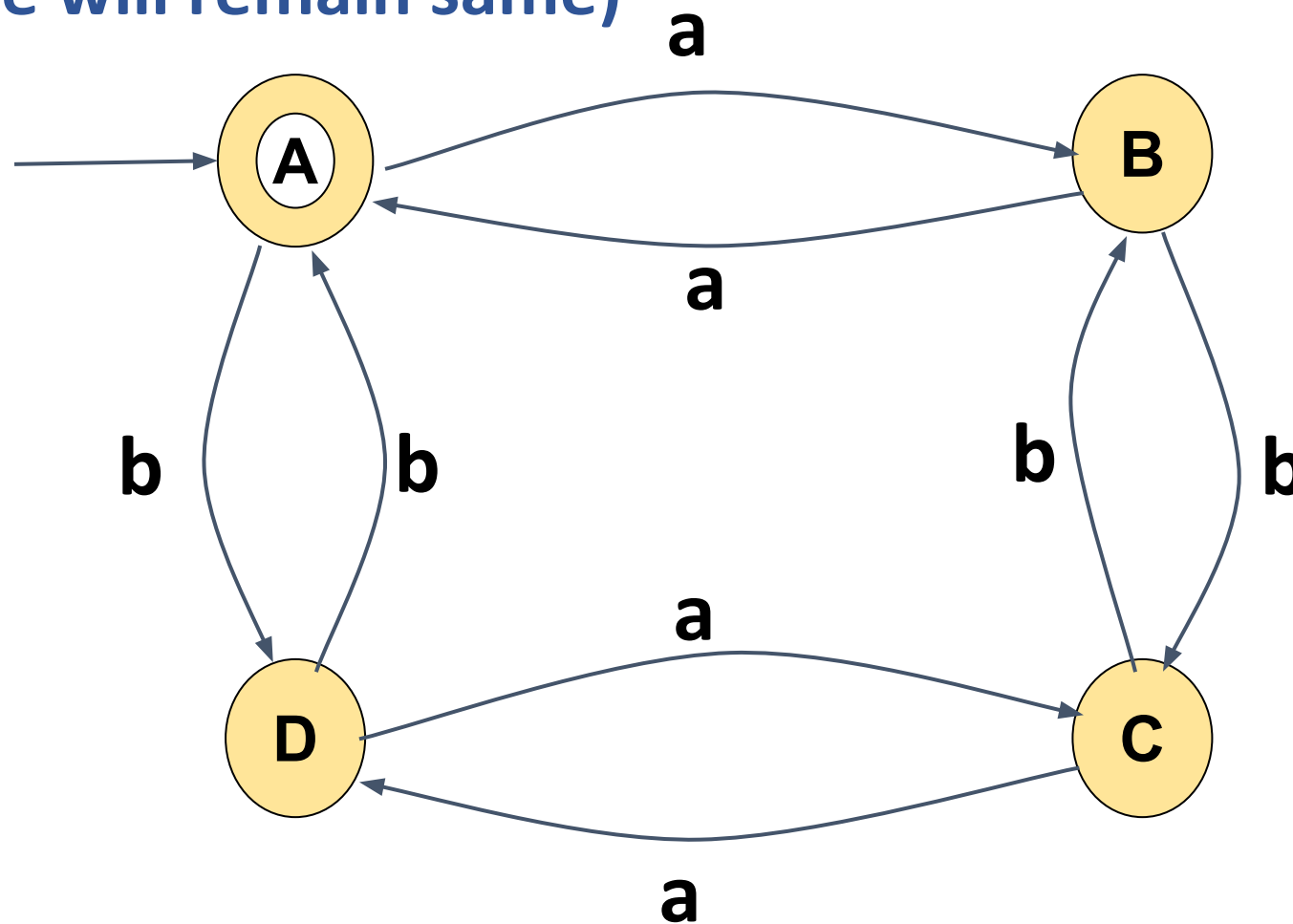
1. Eliminate B
2. Eliminate D
3. Eliminate C
4. Eliminate A

Example 9 :

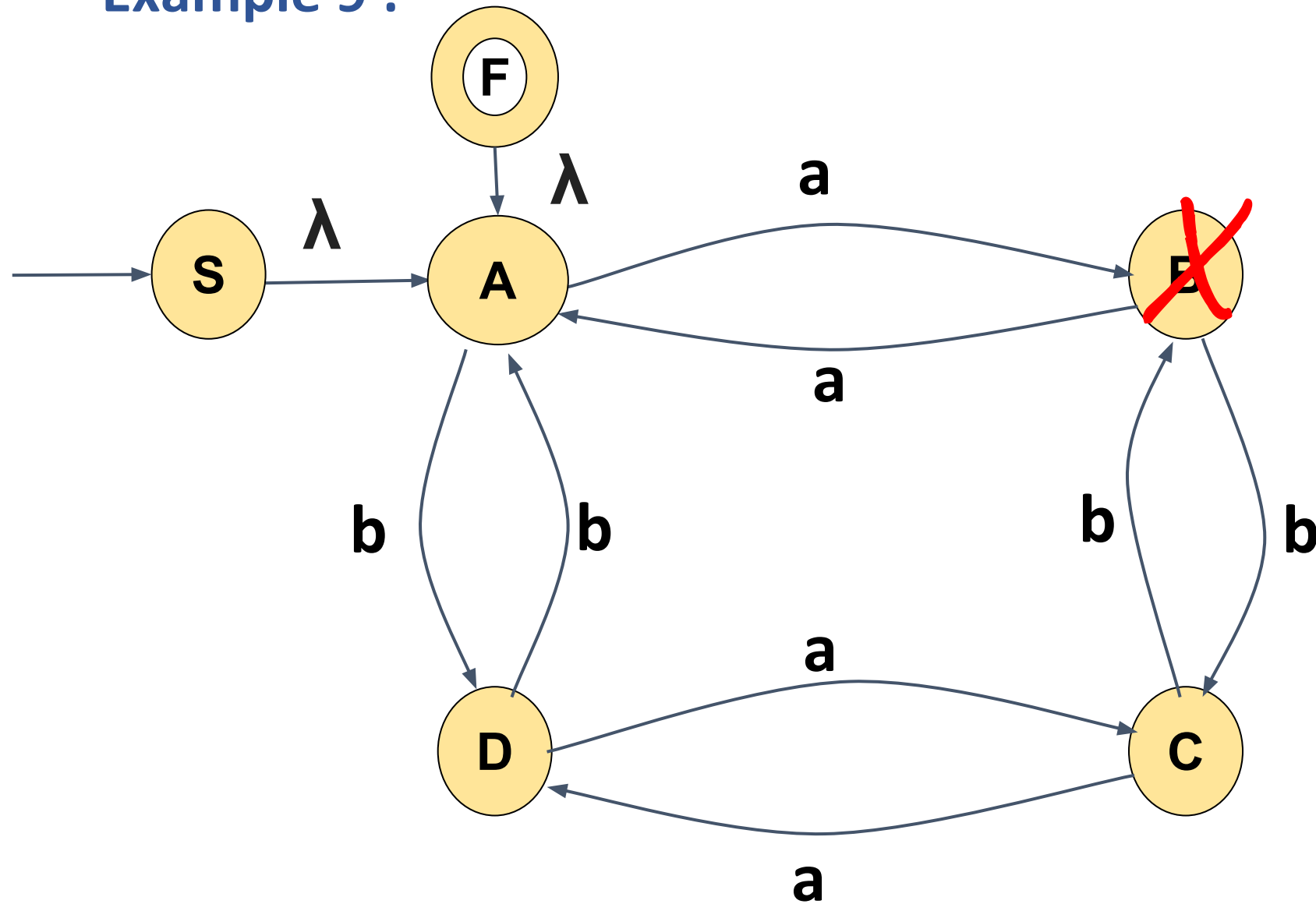


### Example 9 :

Consider the same example: Order of elimination: B,C, D ,A (Adding new start state and new final state will remain same)



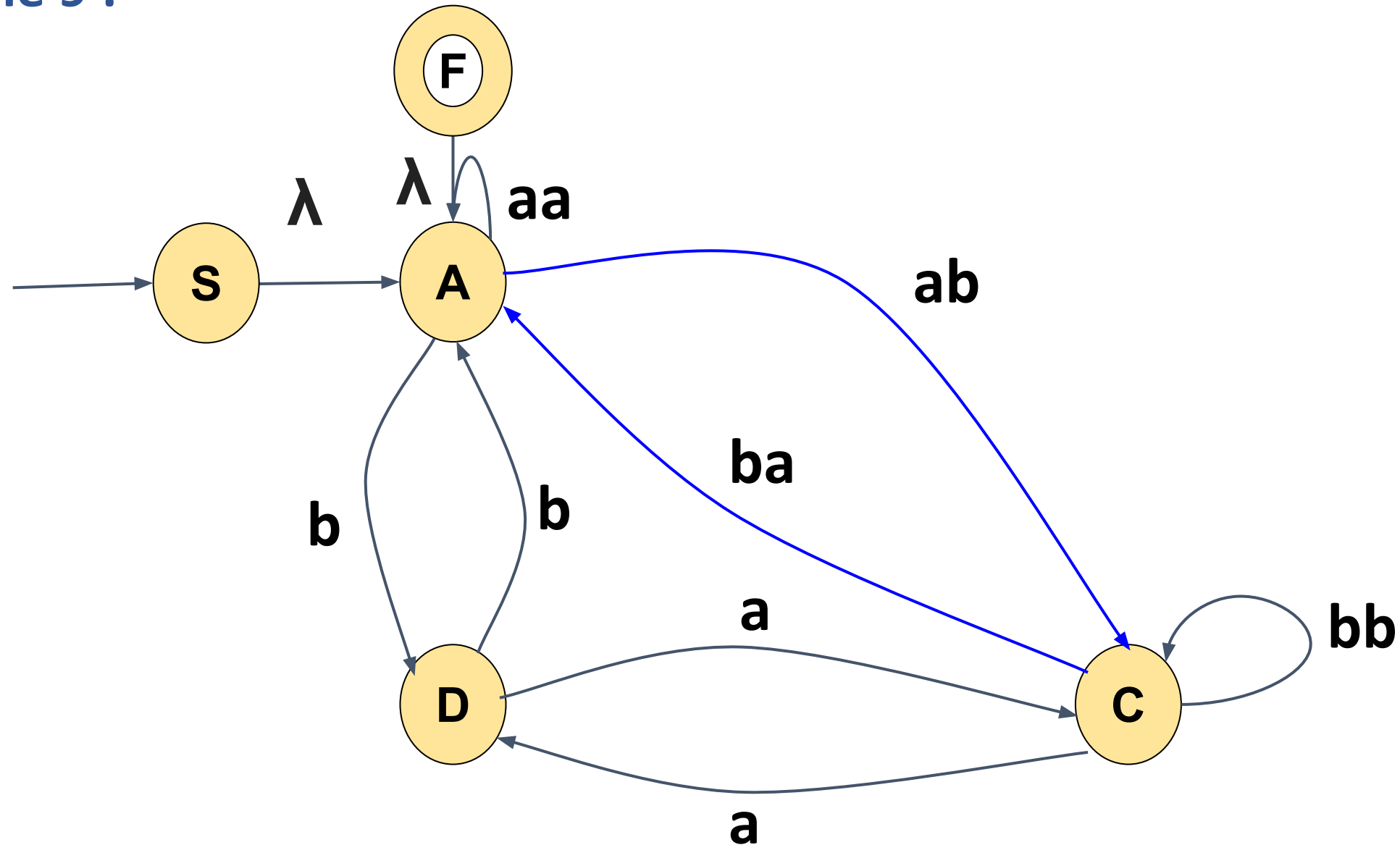
### Example 9 :



### 1. Eliminate B

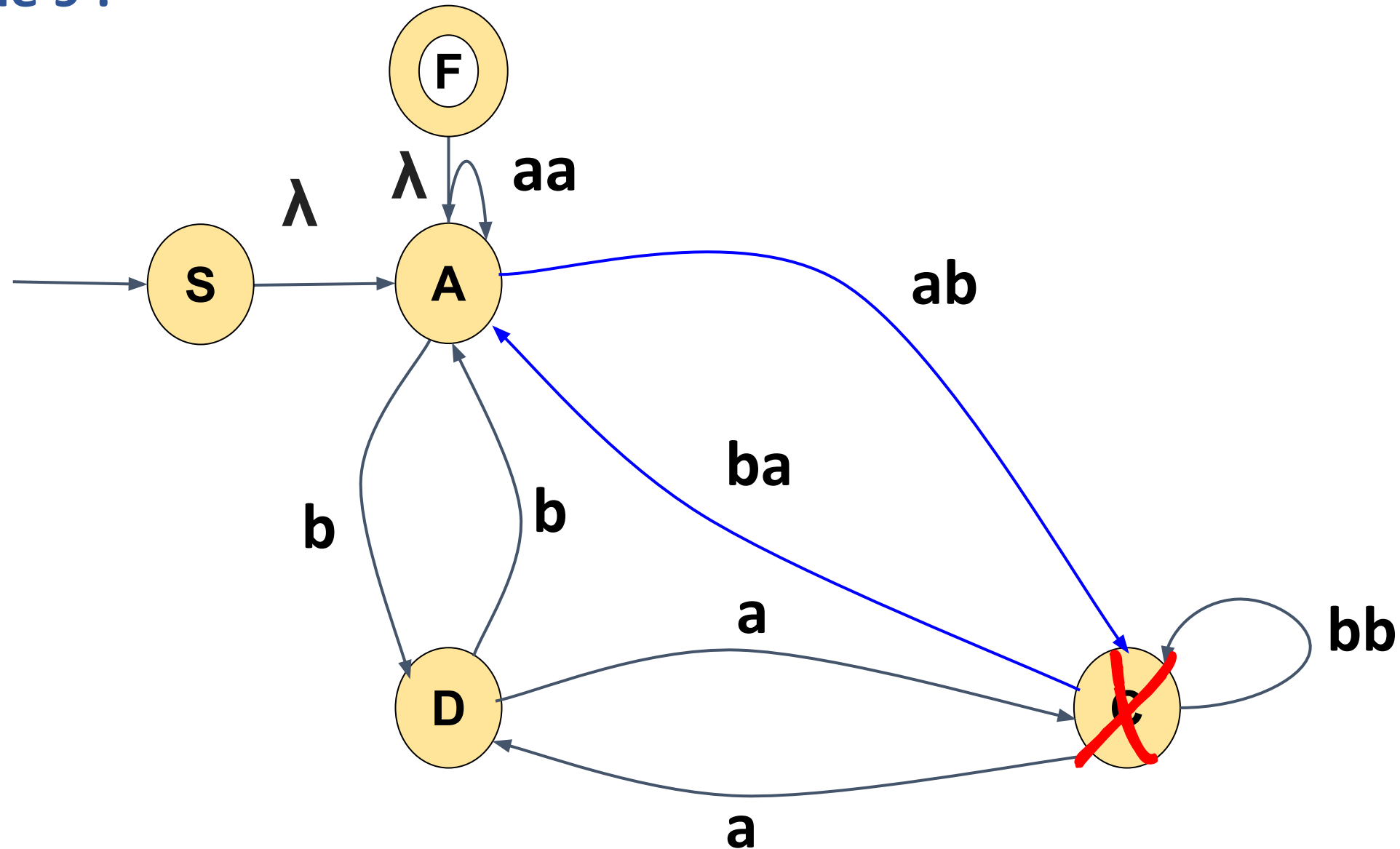


### Example 9 :



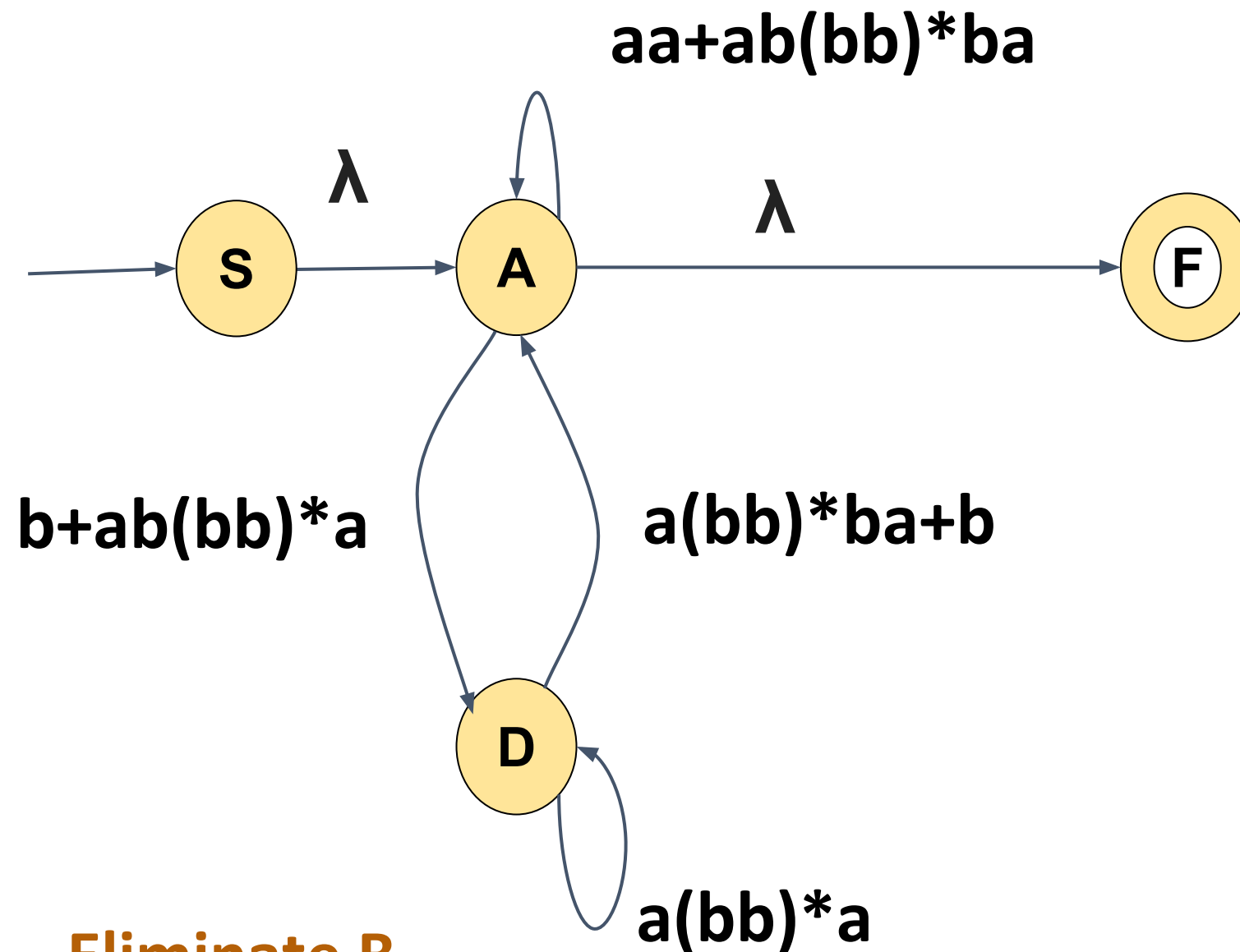
### 1. Eliminate B

### Example 9 :



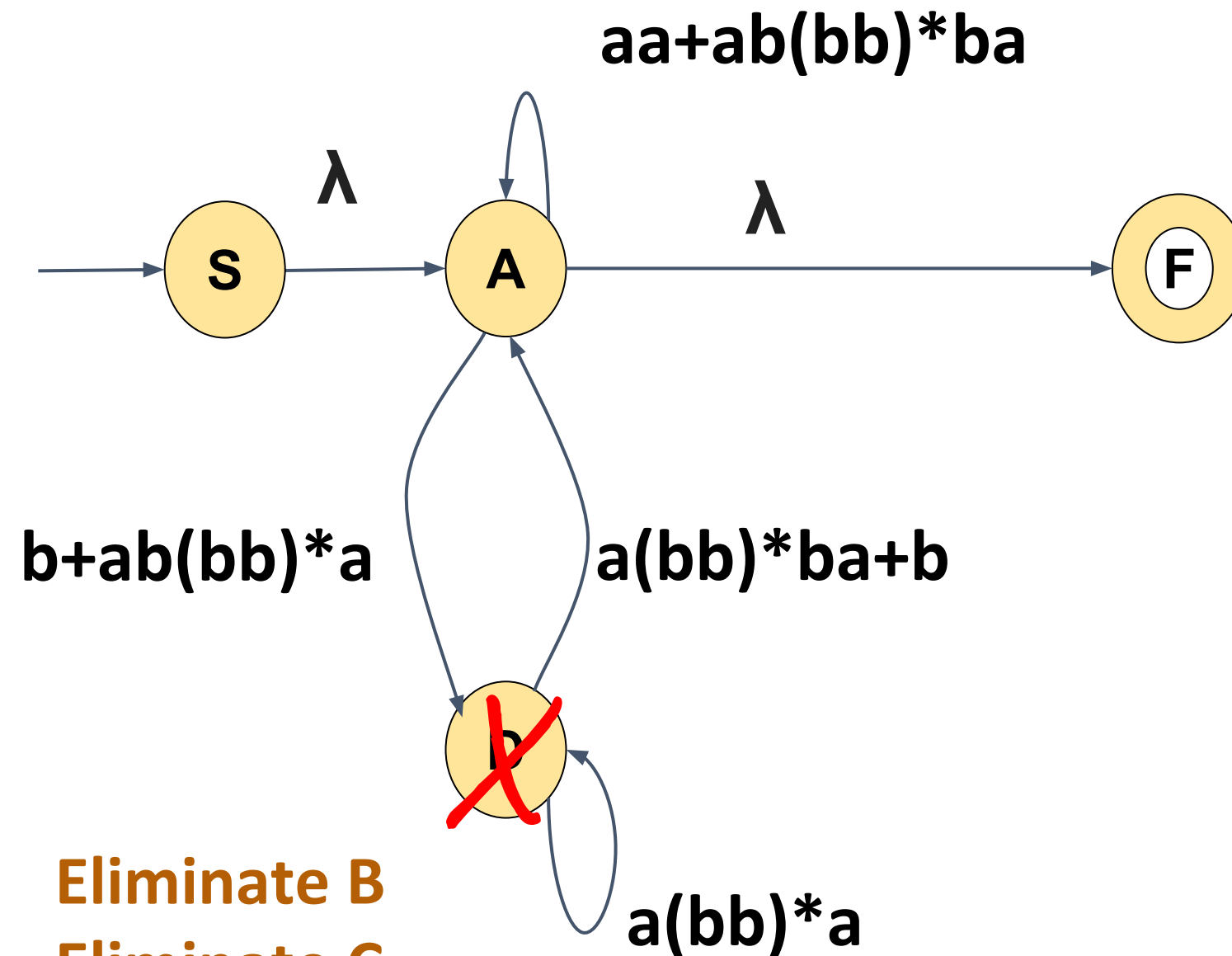
1. Eliminate B
2. Eliminate C

### Example 9 :



1. Eliminate B
2. Eliminate C

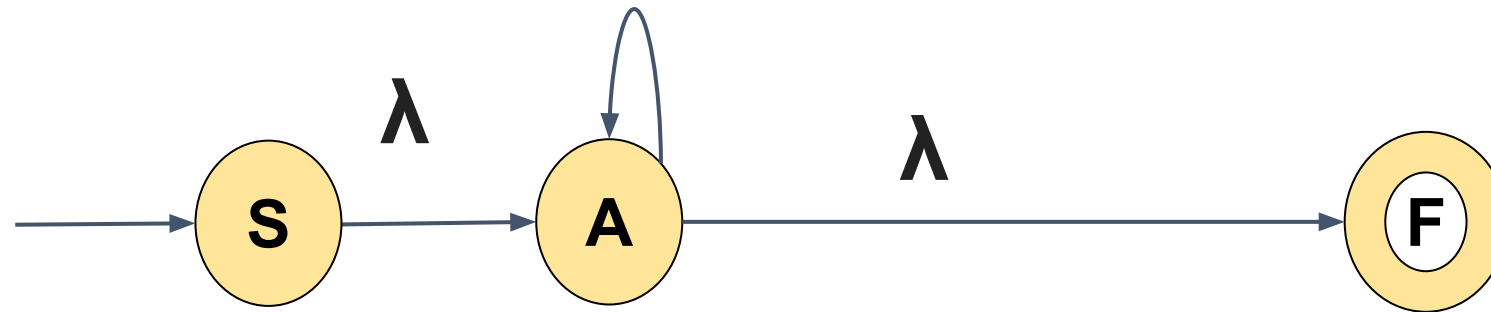
### Example 9 :



1. Eliminate B
2. Eliminate C
3. Eliminate D

### Example 9 :

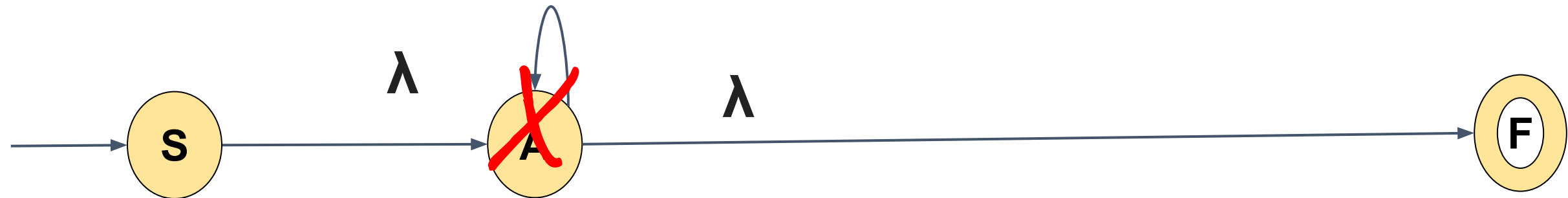
$aa+ab(bb)^*ba + (b+ab(bb)^*a)(a(bb)^*a)^*(a(bb)^*ba+b$



1. Eliminate B
2. Eliminate C
3. Eliminate D

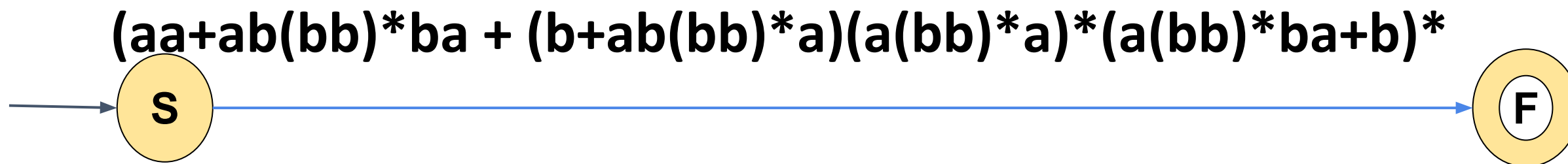
### Example 9 :

$(aa+ab(bb)^*ba + (b+ab(bb)^*a)(a(bb)^*a)^*(a(bb)^*ba+b)$



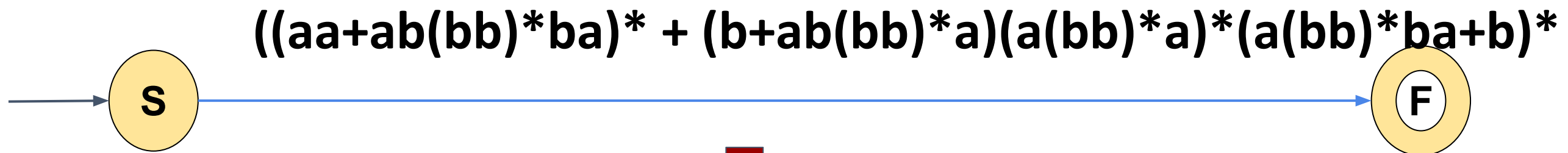
1. Eliminate B
2. Eliminate C
3. Eliminate D
4. Eliminate A

### Example 9 :



1. Eliminate B
2. Eliminate C
3. Eliminate D
4. Eliminate A

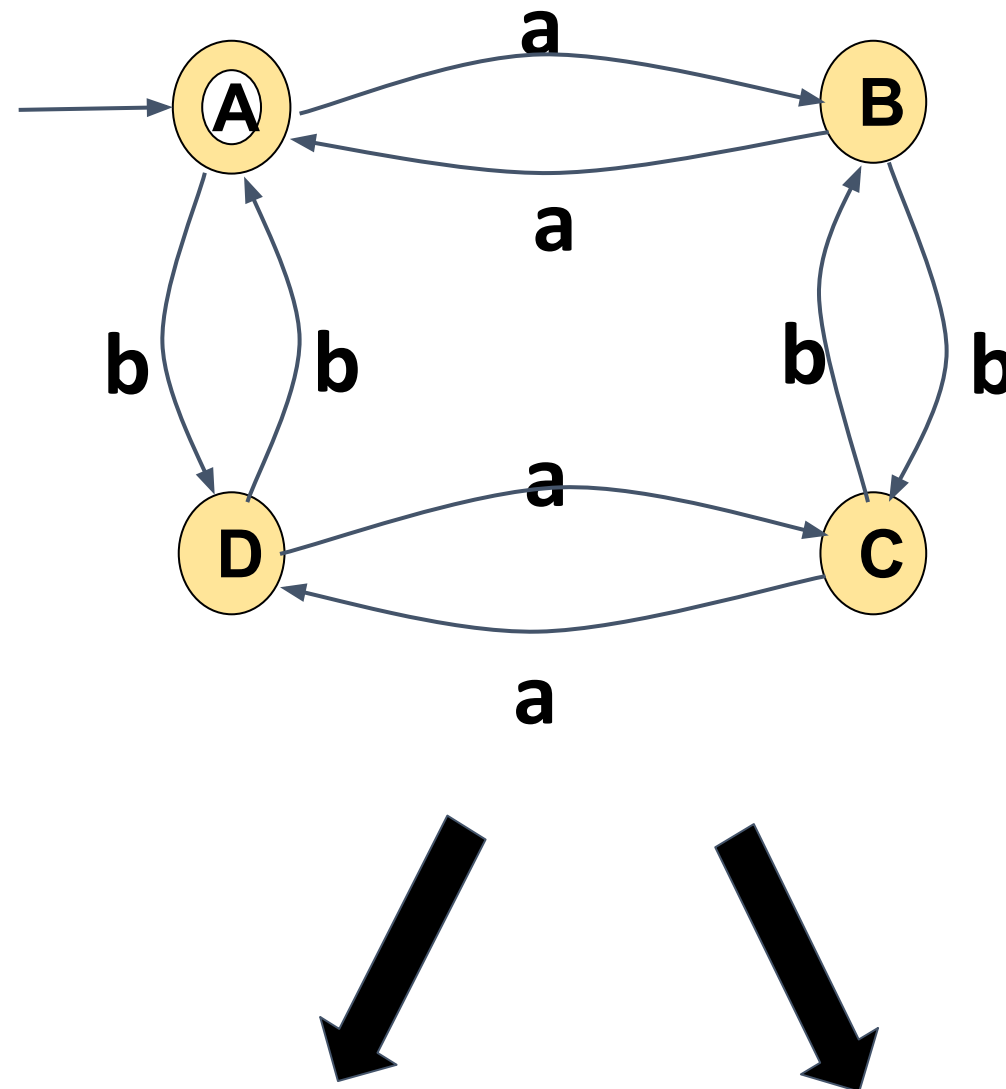
### Example 9 :



$$\text{RE} = ((aa+ab(bb)^*ba) + (b+ab(bb)^*a)(a(bb)^*a)^*(a(bb)^*ba+b))^*$$



### Example 9 :



$(aa+bb + (ab+ba+(aa+bb)^*(ab+ba))^*$

Eliminate : B,D,C,A

$((aa+ab(bb)^*ba)^* +$

$(b+ab(bb)^*a)(a(bb)^*a)^*(a(bb)^*ba+b))^*$

Eliminate B,C,D,A



# THANK YOU

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