



# STATISTICS FOR DATA SCIENCE

## Factors affecting Margin of Errors

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## Factors affecting Margin of Errors

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### Margin of Error

For Large Samples

CI for  $\mu$ :  $\bar{X} \pm Z_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$

CI for  $p$ :  $\tilde{p} \pm Z_{\alpha/2} \sqrt{\frac{\tilde{p}(1-\tilde{p})}{\tilde{n}}}$

CI for  $\mu$ :  $(-\infty, \bar{X} + Z_{\alpha} \cdot \frac{s}{\sqrt{n}})$

$\tilde{n} = n+4$   
 $\tilde{p} = \frac{X+2}{\tilde{n}}$

For Small Samples

$\bar{X} \pm t_{n-1, \alpha/2} \cdot \frac{s}{\sqrt{n}}$  (2-sided)

$(-\infty, \bar{X} + t_{n, \alpha} \cdot \frac{s}{\sqrt{n}})$

$(\bar{X} - t_{n, \alpha} \cdot \frac{s}{\sqrt{n}}, +\infty)$

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## Factors affecting Margin of Errors

MoE varies  $\Rightarrow$  CI varies

For  $\mu$ :

$$MoE = \underbrace{Z_{\alpha/2}}_{\substack{\text{As } n \uparrow \text{ SE } \downarrow}} * \underbrace{\frac{s}{\sqrt{n}}}_{\text{SE}}$$

Factors that affect MoE

CL  
n  
s

MoE  $\Rightarrow$  LESS

As  $n \uparrow$ , SD  $\Rightarrow$  stabilized

As CL  $\uparrow$

Z  $\uparrow$

MoE  $\uparrow$

MoE  $\downarrow$  ✓

MoE  $\uparrow$

As n  $\uparrow$

As s  $\uparrow$

## Margin of Errors in Confidence Intervals

$(1-\alpha) \times 100\%$  Confidence Interval is given by

Large

CI for  $\mu$ :  $\bar{X} \pm MoE$

$$\bar{X} \pm Z_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

Small  
 $\bar{X} \pm MoE$

$$\bar{X} \pm t_{n-1, \alpha/2} * \frac{s}{\sqrt{n}}$$

CI for  $p$ :

$$\hat{p} \pm MoE$$

$$\hat{p} \pm \sqrt{\frac{p(1-p)}{n}} \quad (\text{Traditional})$$

$$\tilde{p} \pm \sqrt{\frac{\tilde{p}(1-\tilde{p})}{\tilde{n}}} \quad (\text{Modern})$$

$\tilde{n} = n+4$      $\tilde{p} = \frac{x+2}{\tilde{n}}$

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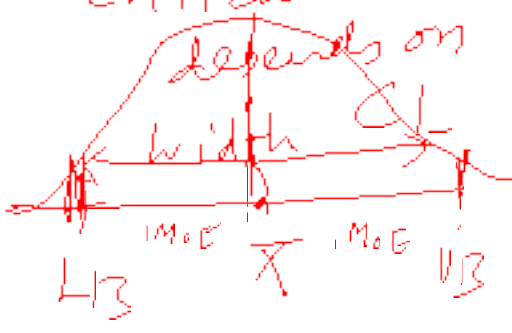
## Margin of Errors in Confidence Intervals

CL for  $\mu$ :  $MoE = Z_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$

critical value depends on width of CL

As CL  $\uparrow$ , MoE  $\uparrow$

width of the CL =  $2 \cdot MoE$



A normal distribution curve with mean  $\bar{x}$ . The confidence interval is shown as a horizontal line segment centered at  $\bar{x}$ , extending from  $L3$  to  $U3$ . The width of the interval is labeled as  $2 \cdot MoE$ . The margin of error is labeled as  $MoE$  on both sides of the mean.

CL for  $\mu$ :  $MoE = Z_{\alpha} \cdot \frac{s}{\sqrt{n}}$

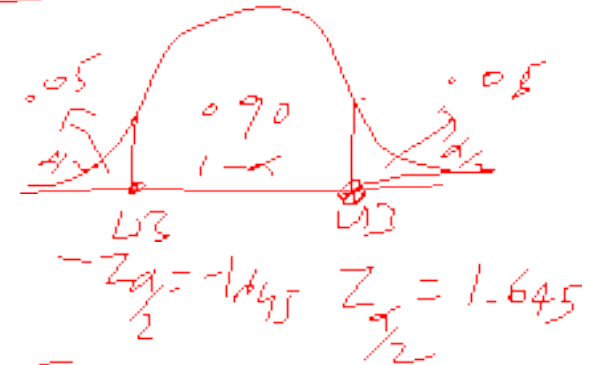
For a CL 90%



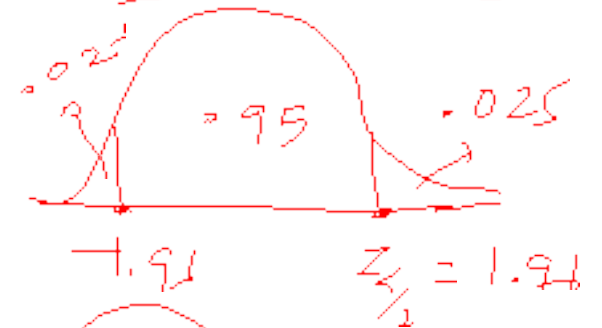
### Confidence Levels

0-100%

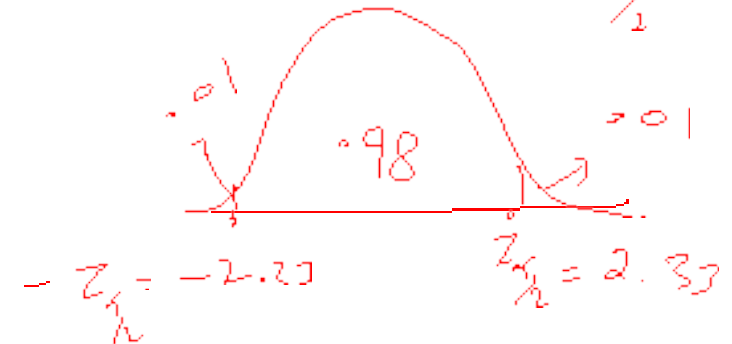
90%



95%



98%

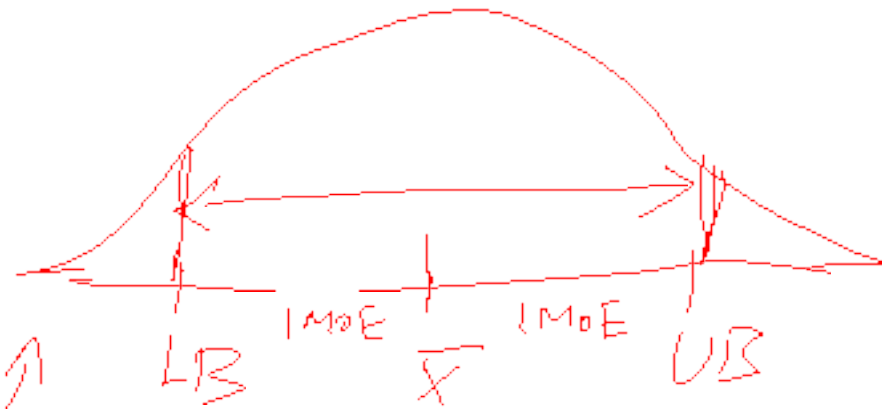


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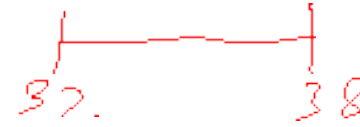
## Margin of Errors in Confidence Intervals

Width of  
the CI }  $= 2 * MoE$

As CL  $\uparrow$  MoE  $\uparrow$   
CL  $\Rightarrow$  Width



Multiplication  
depends on  
CL



95%  
 $\downarrow$

Small  $\left[ \begin{array}{l} t_{n-1, \alpha/2} \text{ (2-sided) Large} \\ t_{n-1, \alpha} \text{ (1-sided)} \end{array} \right.$

$\left[ \begin{array}{l} \text{Critical} \\ \text{(2-sided) value} \end{array} \right] Z_{\alpha/2}$   
 $\left[ \begin{array}{l} \text{(1-sided)} \end{array} \right] Z_{\alpha}$

$$MoE = z_{\alpha/2} \cdot \left( \frac{s}{\sqrt{n}} \right)$$

As  $CL \uparrow \Rightarrow z_{\alpha/2} \uparrow \Rightarrow MoE \uparrow \Rightarrow$  Larger  
in der  $CI$

Sample Size

When  $n \uparrow$ ,  $\left( \frac{s}{\sqrt{n}} \right) \Rightarrow$  decreases  
 $\rightarrow$  uncertainty of sample  $\downarrow$



As  $n \uparrow$ ,  
MoE  $\downarrow$

CI :  $\bar{X} \pm \overset{\text{Less}}{\text{MoE}} \Rightarrow \text{—————}$

$MoE = Z_{\alpha/2} * \left( \frac{s}{\sqrt{n}} \right) \rightarrow \text{uncertainty will be reduced as } n \uparrow$

Reduced

Uncertainty of sample  $\left(\frac{s}{\sqrt{n}}\right) \uparrow$

As  $s \uparrow$   
sample s.d.,

$$MoE = Z_{\alpha/2} + \frac{s}{\sqrt{n}}$$

$\Rightarrow MoE \uparrow$

|-----|  
wider C.I

As  $n \uparrow$ , MoE  $\downarrow$

$\Rightarrow$  CI will be narrow.



$$MoE = Z_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

Sample S.D  $\Rightarrow s$

As CL  $\uparrow$ , MoE  $\uparrow$   
As  $n \uparrow$ , MoE  $\downarrow$   
As  $s \uparrow$ , MoE  $\uparrow$

As  $s \uparrow \Rightarrow \frac{s}{\sqrt{n}} \uparrow \Rightarrow MoE \uparrow \Rightarrow \text{Wider CI}$

$s \downarrow \Rightarrow \frac{s}{\sqrt{n}} \downarrow \Rightarrow MoE \downarrow \Rightarrow \text{narrow CI}$   
 $\swarrow$   
when  $n \uparrow$



As  $CL \uparrow$   $MoE \uparrow \Rightarrow$  Wider CI

As  $n \uparrow$   $MoE \downarrow \Rightarrow$  Narrow CI

As  $s \uparrow$   $MoE \uparrow \Rightarrow$  Wider CI



# THANK YOU

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