STATISTICS FOR DATA SCIENCE

Normal Distribution

Prof. Uma D

Department of Computer Science and Engineering

STATISTICS FOR DATA SCIENCE

Normal Distribution

Prof. Uma D

STATISTICS FOR DATA SCIENCE

Topics to be covered...

- Problems
- Linear Function of a Normal Random Variable
- Linear Function of a Independent Normal Random Variable
- Two independent Normally distributed random variables

Standardizing Normally Distributed Random Variables

We can convert a Random Variable X having a Normal distribution with any mean and Standard deviation in to the Random variable that has a Standard Normal Distribution.

$$X \sim N(\mu, \sigma^2)$$

Standardizing X : using a basic linear transformation:

$$z = (x - \mu) / \sigma$$

Example

Reading Z Table to Find Area

Find area under the normal curve:

a) To the left of z = -0.49

b) To the left of z = 0.49

c) To the right of z = 0.49

d) Between z = 0.40 and z = 1.30

e) Between z = -1.50 and z = 0.90

Example

Let Z ~ N(0, 1) . Find a constant c for which

a) $P(Z \geq c) = 0.1587$

b) $P(c \leq Z \leq 0) = 0.4772$

c) $P(-c \leq Z \leq c) = 0.8664$

Solution

a) **P(Z ≥c) = 0.1587**

=> Area to left of c = 1 – 0.1587 = 0.8413

=> **c = 1.00**

b) **P(c ≤ Z ≤ 0) = 0.4772**

Area to left of 0 = 0.5

=>Area to left of c = 0.5 – 0.4772 = 0.0228

=> **c = -2.00**

c) **P(-c ≤ Z ≤ c) = 0.8664**

P(0 ≤ Z ≤ c) = 0.8664/2 = 0.4332

Area to right of c = 0.5 – 0.4332 = 0.0668 => Area to left of -c = 0.0668 => c=-1.50

Area to left of c = 1 – 0.0668 = 0.9332 => **c = 1.50**

Solution

Problems

X has a normal distribution with mean 5 and standard deviation 2. Find P(x>7).

Example

If $X \sim N(2, 9)$, compute:

a) $P(X \geq 2)$

b) $P(1 \leq X < 7)$

c) Find the median of X.

d) Find 75^{th} percentile of X.

Solution

a) $P(X \geq 2)$

b) $P(1 \leq X < 7)$

Solution

c) Find the median of X.

d) Find 75^{th} percentile of X.

STATISTICS FOR DATA SCIENCE

Linear Function of a Normal Random Variable

Linear Function of a Independent Normal Random Variable

Two independent normally distributed random variables

Sum/ Difference of two independent normally distributed random variables is normal.

Example

Let X_1 be a normal random variable with mean 2 and variance 3,and let X_2 be a normal random variable with mean 1 and variance 4. Assume that X_1 and X_2 are independent.

What is the distribution of the linear combination $Y = 2X_1 + 3X_2$?

Example

A light fixture holds two light bulbs. Bulb A is a type whose lifetime is normally distibuted with mean 800 hours and standard deviation 100 hours. Bulb B has a lifetime that is normally distibuted with mean 900 hours and standard deviation 150 hours. Assume the lifetimes of the bulbs are independent.

1) What is the probability Bulb B lasts longer than bulb A?

2) What is the probability Bulb B lasts 200 hours more than bulb A?

3)Another light fixture holds only one bulb. A bulb of type A is installed, and when it burns out, a bulb of type B is installed.

What is the probability that the total lifetime of the two bulbs is more than 2000 hours?

Example

A light fixture holds two light bulbs. Bulb A is a type whose lifetime is normally distibuted with mean 800 hours and standard deviation 100 hours. Bulb B has a lifetime that is normally distibuted with mean 900 hours and standard deviation 150 hours. Assume the lifetimes of the bulbs are independent.

Example

1)What is the probability Bulb B lasts longer than bulb A?

Example

2)What is the probability Bulb B lasts 200 hours more than bulb A?

Example

3)Another light fixture holds only one bulb. A bulb of type A is installed, and when it burns out, a bulb of type B is installed.

What is the probability that the total lifetime of the two bulbs is more than 2000 hours?

Example

Do It Yourself!!!

The lifetime of a battery is in a certain application is normally distributed with mean 16 hours, standard deviation 2 hours.

a) What is the probability that a battery will last more than 19 hours?

b) Find the 10^{th} percentile of the lifetimes.

c) A particular battery lasts 14.5 hours. What percentile is its lifetime on?

THANK YOU

Prof. Uma D

Department of Computer Science and Engineering