



STATISTICS FOR DATA SCIENCE

Random Variables

Prof. Uma D

Department of Computer Science and Engineering

STATISTICS FOR DATA SCIENCE

Continuous Random Variables

Prof. Uma D

- Continuous Random Variable
- Probability Density Function
- Cumulative Distribution Function
- Mean and Variance

- A continuous random variable is one which takes an **infinite number of possible** values.
- Continuous random variables are usually **measurements**.

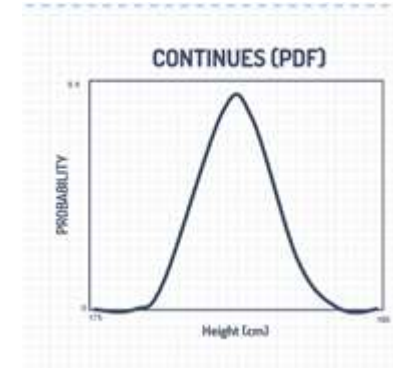
Examples

- height
- weight
- the amount of sugar in an orange
- the time required to run a mile.

STATISTICS FOR DATA SCIENCE

Probability Density Function

- A random variable is **continuous** if its probabilities are given by areas under a curve.
- The curve is called a **probability density function (pdf)** for the random variable. Sometimes the **pdf** is called the **probability distribution**.
- The function **$f(x)$** is the probability density function of X .
- Let X be a continuous random variable with probability density function $f(x)$. Then $\int_{-\infty}^{\infty} f(x) dx = 1$



STATISTICS FOR DATA SCIENCE

Continuous Random Variables

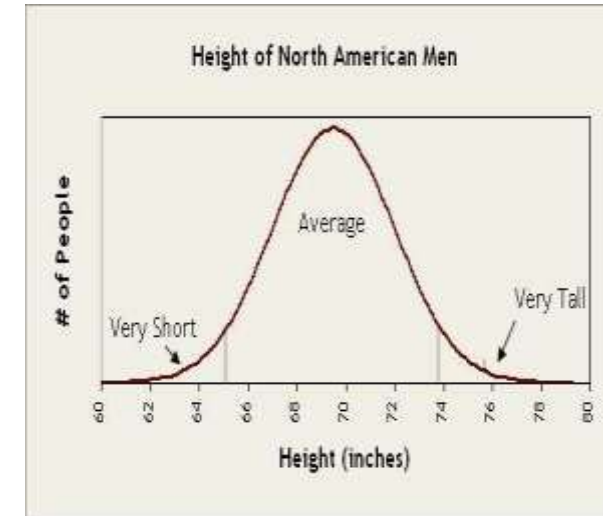
We model Continuous Random Variables with a **curve $f(x)$** called a **probability density function(pdf)**.

$f(x)$ is function that represents the height of the curve at point x .

Values where the curve is **high** are **more likely to occur**.

For Continuous Random Variables, probabilities are areas under the curve – hence found using integration.

Looks like a **smooth histogram**.



Probability Density Function of a C.R.V.

Probability Density Function of a C.R.V.

Cumulative Distribution Function of a C.R.V.

Percentile and Median of a C.R.V.

STATISTICS FOR DATA SCIENCE

Continuous Random Variables

Mean and Variance of a C.R.V.



Problem 1

Suppose for a random variable X:

$$f(x) = \begin{cases} cx^3 & \text{for } 2 \leq x \leq 4 \\ 0 & \text{otherwise.} \end{cases}$$

- a) What value of c makes this a legitimate probability distribution?
- b) What is $P(X > 3)$.
- c) Find $P(X \leq 2.7)$.
- d) What is the median of this distribution?
- e) Find mean and variance of this distribution.
- f) What is the cumulative distribution function?

STATISTICS FOR DATA SCIENCE

Continuous Random Variables



Solution:

a) What value of c makes this a legitimate probability distribution?

STATISTICS FOR DATA SCIENCE

Continuous Random Variables



Solution:

b) What is $P(X > 3)$.

STATISTICS FOR DATA SCIENCE

Continuous Random Variables

Solution:

c) Find $P(X \leq 2.7)$.



Solution:

d) What is the median of this distribution?

Solution:

e) Find mean and variance of this distribution.

Solution:

e) Find mean and variance of this distribution.

Solution:

f) What is the cumulative distribution function?

Problem 1 - Solution

a) $c = 1/60$

b) $P(X > 3) = 0.729$

c) $P(X \leq 2.7) = 0.155$

d) Median = 3.415

e) Mean = $248/75 = 3.3$

Variance = $11.2 - \text{sq}(3.3) = 0.31$

f) CDF = $(x^4 - 2^4) / 240$

Do It Yourself !!!

Let X be a random variable with PDF given by

$$f(x) = \begin{cases} x/250 & 20 \leq x \leq 30 \\ 0 & \text{otherwise} \end{cases}$$

- 1) Find $P(X \geq 25)$.
- 2) Find $E(X)$ and $\text{Var}(X)$.
- 3) Find CDF.
- 4) Find median.
- 5) Find 60th percentile.



THANK YOU

Prof. Uma D

Department of Computer Science and Engineering