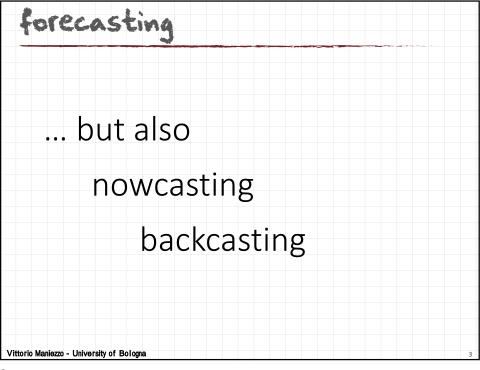
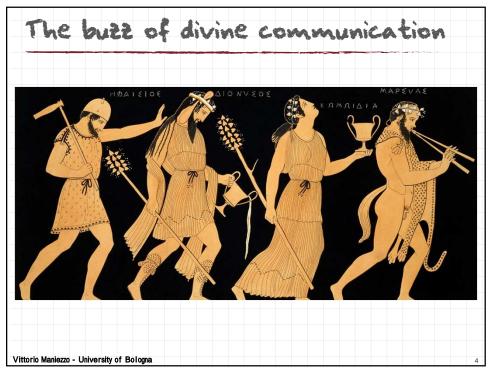


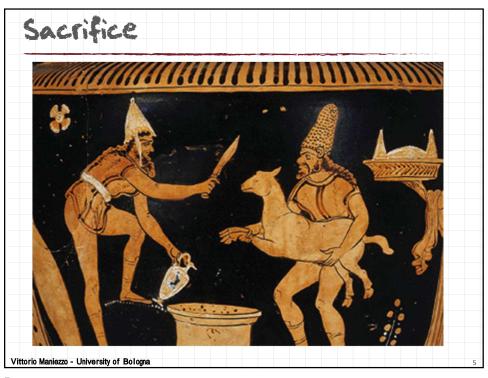
Introduction Forecasting 101

2

Vittorio Maniezzo - University of Bologna

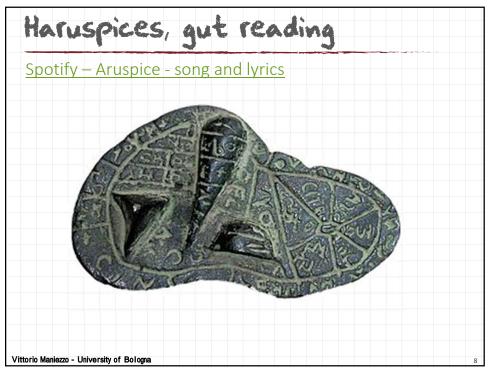


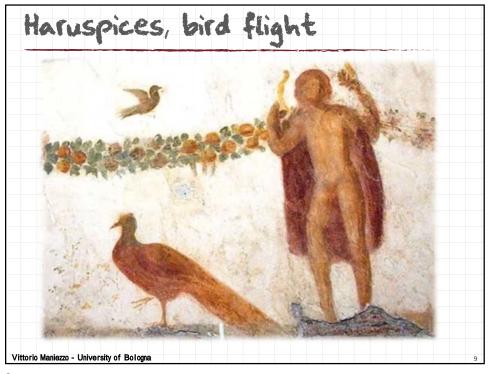


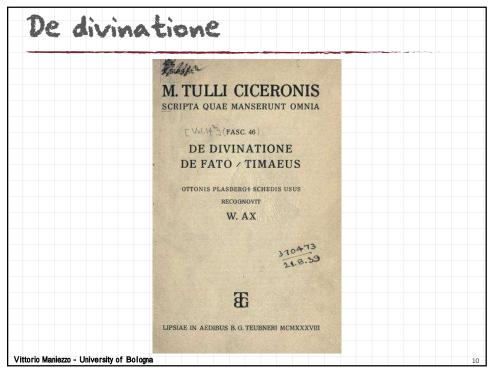






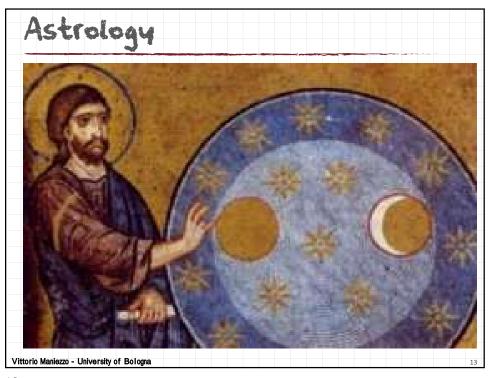


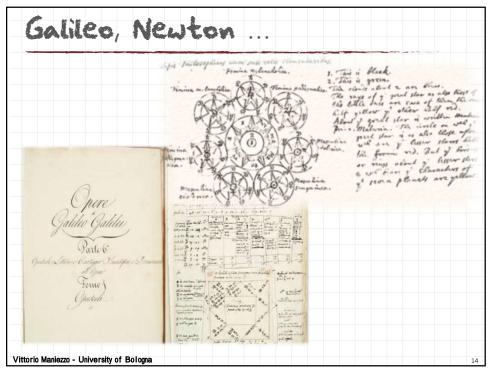




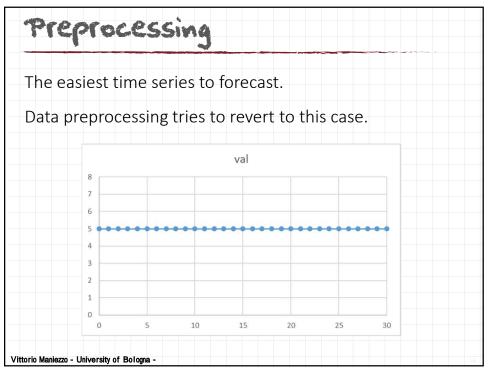


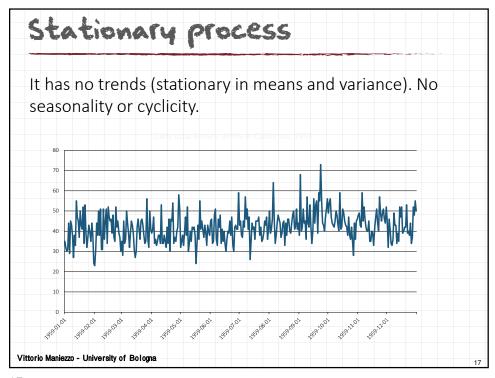


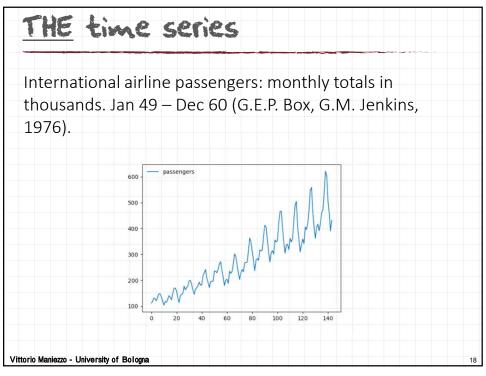












STL decomposition

STL stands for *Seasonal and Trend decomposition using Loess* (locally estimated scatterplot smoothing)

STL models a time series as a combination of

- Trend μ_t
- Seasonality s_t
- Random shock r_t .

The combination can be:

Additive: $y_t = \mu_t + s_t + r_t$

Multiplicative : $y_t = \mu_t \times s_t \times r_t$

Mixed: $y_t = \mu_t \times s_t + r_t$

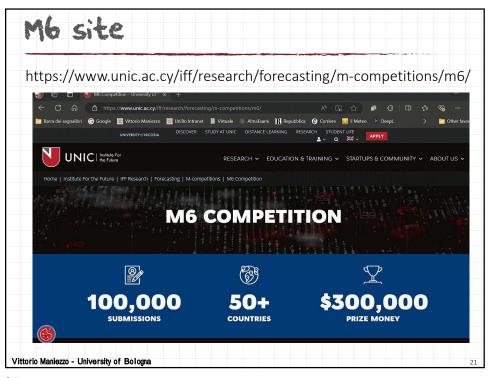
Vittorio Maniezzo - University of Bologna

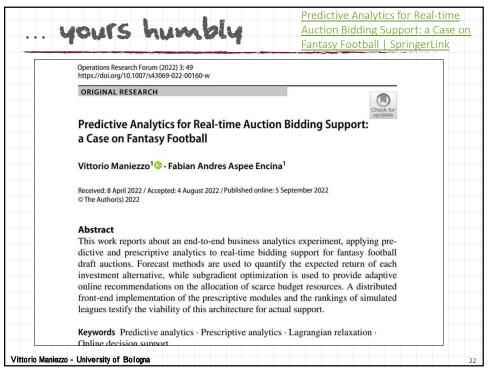
19

The M competitions

A series of open competitions to evaluate and compare the accuracy of different time series forecasting methods.

	No.	name	Years	Num. of time series	Prizes
	1	M Competition	1982	1001	
	2	M2 Competition	1993	29	
	3	M3 Competition	2000	3003	
	4	M4 Competition	2020	100,000	\$27,000
	5	M5 Competition	2021 - 2022	Around 42,000 hierarchical timeseries provided by Walmart	\$100,000
	6	M6 Competition	2022 - 2024	Real time financial forecasting	\$300,000
Vitt	orio Maniezzo - Unive	rsity of Bologna			





Fantasy sports

Fantasy sports, among which fantasy football, are more than 60 years old and permit fans to create an own league for their sport of liking.

A participant acts as owner and manager of her / his own team

- first bidding for the players to enter the team (<u>draft</u> phase)
- then deciding for each match who will be used among the acquired players (<u>lineup</u> phase).

A team does not need to be composed of players who actually play together in real-world events

The team effectiveness is computed after the actual performance of the players in real world sport events.

There exist agencies that compute for each player and for each event a score in "fantasy points". The success level of a fantasy team on a match day is given by the sum of the fantasy points gained by the used players in the actual real world corresponding matches.

Vittorio Maniezzo - University of Bologna

23

Fantasy leagues

Each manager lists his fantasy team into a contest (a league), to be ranked against all other teams that entered the contest.

Leagues can be freely organized by groups of friends or whatever, but often there is an entry fee in order to win contest prizes.

Fantasy sports generate an economically relevant industry, moving hundreds of millions of dollars worldwide.

Entry fees for structured contests range from free to over \$10,000 per entry, some contests pay out prizes up to one-million dollars.

Participants can invest money into decision making support tools, leading to an estimate of multimillion annual spending to purchase additional information and decision-making tools

Vittorio Maniezzo - University of Bologna

The draft problem

The draft problem for Italian fantasy football requires, as other fantasy football leagues, to recruit 23 (25) players with the following roles:

- 3 goalkeepers (3),
- 7 defenders (8),
- 8 midfielders (8) among which at least 1 attacking midfielder and possibly at least 3 pure midfielders,
- 5 forwards (6).

The lineup at each match will consist of 11 players chosen among these 23, according to the allowed formations (4-4-2, 5-3-1, 4-3-2-1 and the like).

Vittorio Maniezzo - University of Bologna

25

The draft problem

In the draft phase, to acquire players, the manager enters an auction with the other managers of his league, where player names will be called and bids will be placed on the currently auctioned player.

Each manager has a maximum budget B, usually 260 F.M. (FantaMillions), but also 300 or 500 (or whatever agreed upon) could be allowed.

Vittorio Maniezzo - University of Bologna

I : index s is:	et of all aucti	ioned players, x_i	: binary decision var	iable associated w	ith each player $i \in I$. An I	P formulation
	(P)	$z_P = max$	$\sum_{i\in I} p_i x_i$		(1)	
		s.t.	$\sum_{i \in G} x_i = 3$		(2)	
			$\sum_{i \in \mathcal{D}} x_i = 7$		(3)	
			$\sum_{i\in M} x_i = 8$		(4)	
			$\sum_{i\in T} x_i \ge 1$		(5)	
			$\sum_{i \in M \setminus T} x_i \ge 3$		(6)	
			$\sum_{i\in\mathcal{F}}x_i=5$		(7)	
			$\sum_{i \in I} c_i x_i \le B$		(8)	
			$x_i \in \{0,1\},$	$i \in I$	(9)	
of goalkee midfielde	epers (2), the	number of defer the number of fo	nders (3) , the numb	er of midfielders (4	, subject to constraints or 4,6) , the number of attac 4 and $T \subseteq M$. Constraint	king

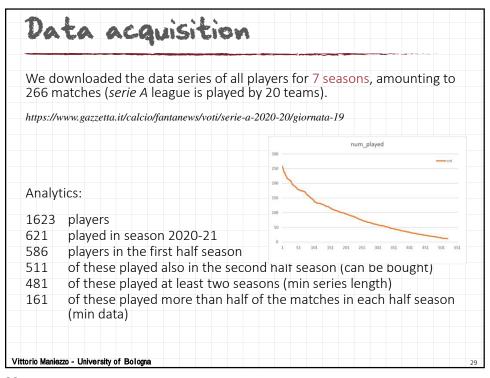
Sharpe's index

Purchasing a player is an investment, having the double objective of maximizing profit and minimizing risk.

Finance literature contains different functions to consider them both, we adapted Sharpe's index (ratio, Sharpe, 1966) to our case. In finance, it is defined as the difference between the returns of the investment and the risk-free return, divided by the standard deviation of the investment (i.e., its volatility).

- risk-free return was the *minimum forecast profit* among players with the same role.
- volatility was measured as the average negative deviation, for each player.

Vittorio Maniezzo - University of Bologna



Proficiency forecast

We finally moved to forecasting future player performance (fantasy points).

The horizon of interest was half a season.

We downloaded all data at the end of the first half season 2020-2021, our training set, and forecast the second half season, which was unknown to

The draft problem would be sufficiently solved by forecasting the average profit for each player (its p_i). As we cover also the lineup problem, we tried however to get an as detailed as possible forecast.

It is notoriously implausible to reliably forecast detailed sports results, however, averages and maybe trends are more amenable to forecast.

We worked on series of rolling averages over three fantasy point values, the least window size that provided significant results.

We then moved on to forecasting trying different methods.

Vittorio Maniezzo - University of Bologna

Autoregressive models AR(p)

In autoregressive models (of order p, AR(p)) the future value of a variable (added to a random component and to a constant) is assumed to be a linear combination of the last p observations

$$y_t = c + \sum_{i=1}^{p} \varphi_i y_{t-i} + \varepsilon_t$$

Feasible only for stationary processes! It only models a linear dependency.

where:

 y_t value at time t

 ε_t random error at time t

 φ_i (i = 1, ..., p) model parameters

c constant

Vittorio Maniezzo - University of Bologna

31

Moving Average models MA(q)

In moving average models (of order q, MA(q)) the future value of a variable is assumed to be equal to the average of the observations, μ , added to a linear combination of the last q errors:

$$y_t = \mu + \sum_{j=1}^q \theta_j \varepsilon_{t-j} + \varepsilon_t$$

Where:

 μ average of the series

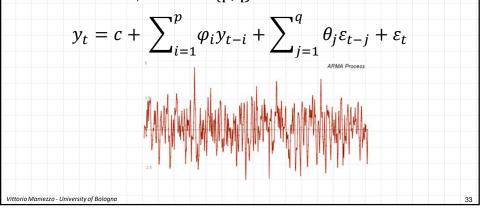
 θ_i (j = 1, ..., q) model parameters

Random errors are usually assumed to have a normal distribution (mean 0, variance σ^2)

ARMA models

Autoregressive models (AR) and moving average models (MA) can be combined in a more general class, the ARMA models.

In mathematics, an ARMA(p,q) model is:



33

SARIMA models

An extension, proposed by Box, Jenkins (1970), of ARIMA models which can be applied also to data with seasonality.

A seasonal diff is included.

Model identification can be complex.

Notationally, they are denoted by $ARIMA(p, d, q)(P, D, Q)_m$

where:

- p,d,q are ARIMA parameters referring to periods (ex., week),
- *P,D,Q* the same parameters referring to seasons (ex., trimester)
- m number of period in one season (ex., weeks in a trimester.
 Multiple seasonalities can be addressed).

Vittorio Maniezzo - University of Bologna

SARIMAX

Seasonal Autoregressive Integrated Moving-Average with Exogenous Regressors (SARIMAX)

SARIMAX is an extension of SARIMA that also includes the modeling of exogenous variables.

Exogenous variables (also called covariates or predictors or externals) are parallel input sequences that have observations at the same time steps as the original series.

The primary series is referred to as endogenous data to contrast it from the exogenous sequence(s).

The observations for exogenous variables are included in the model directly at each time step and are <u>not</u> modeled in the same way as the primary endogenous sequence (as an AR, MA, etc. process).

Vittorio Maniezzo - University of Bologna

35

Simple Exponential Smoothing

Simple Exponential Smoothing (SES), also *single exponential smoothing*, models the next time step as an exponentially weighted linear function of observations at prior time steps. It requires a single parameter, α , called the smoothing factor or smoothing coefficient.

The equation is simply

$$y_{t+1} = \alpha y_t + (1 - \alpha) y_{t-1}$$

That, when projected back in time, corresponds to

$$y_{t+1} = \alpha y_t + \alpha (1 - \alpha) y_{t-1} + \alpha (1 - \alpha)^2 y_{t-2} + \dots$$

The parameter controls the rate at which the influence of the observations at prior time steps decay exponentially. Values close to 1 mean that the model pays attention mainly to the most recent observations, whereas values close to 0 mean that more of the history is taken into account.

Vittorio Maniezzo - University of Bologna



NN for forecast

Neural networks (NN) are very effective in identifying non linear models.

Even with input data deriving from complex, nonlinear processes, NN are able to combine linear and nonlinear models to get whichever accuracy level is requested (in prediction):

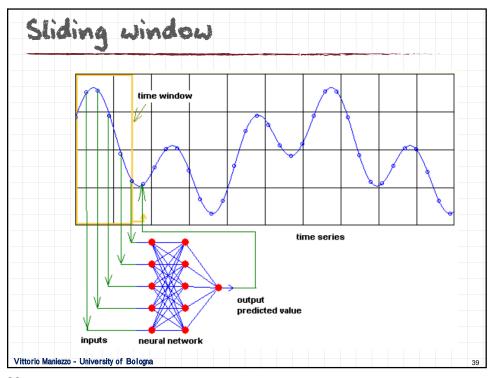
- NN can learn patterns in linear time series
- NN can learn patterns in nonlinear time series
- NN can generalize linear and nonlinear patterns

NN are nonparametric, they do not make hypotheses on noise distribution (gaussian or else).

NN identify a model from input data, essentially defining a model of the generating process.

Vittorio Maniezzo - University of Bologna

38

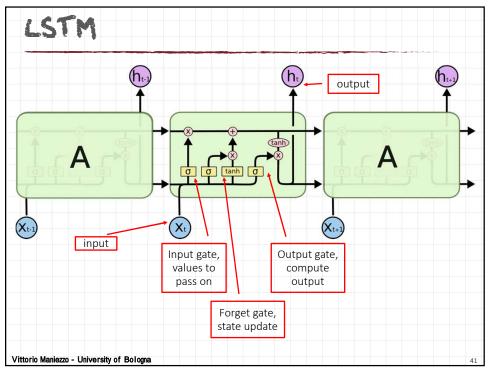


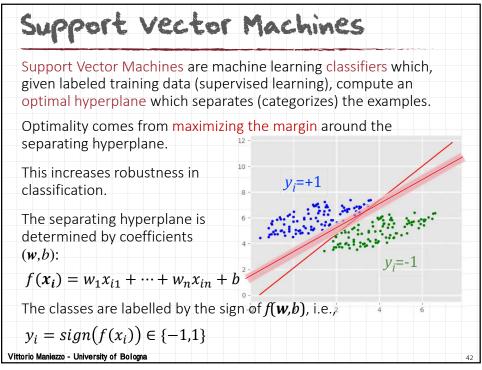
Recurrent NN

The most effective neural model for forecasting so far.

- They have at least one backward connection (feedback loop)
- They can keep activations even with no input → they have an internal state.
- Even with one single neuron, we can input a time series and get a series in the output.
- They can approximate any dynamical system.
- Very complex mathematical analysis.
- Very complex learning algorithms, hard to control. The most used one is Backpropagation Through Time (BTT).

Vittorio Maniezzo - University of Bologna



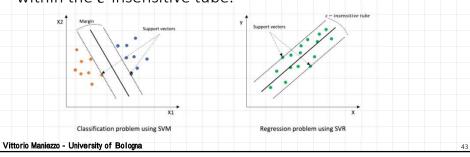


Support Vector Regression

SVR uses hyperplane and margin too, but with differences in their definitions.

The margin, called the ε -insensitive tube, is the error tolerance of the model. This tube allows some deviation of the data points from the hyperplane.

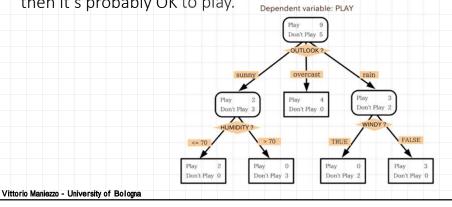
The hyperplane is the best fit possible to the data that fall within the ϵ -insensitive tube.

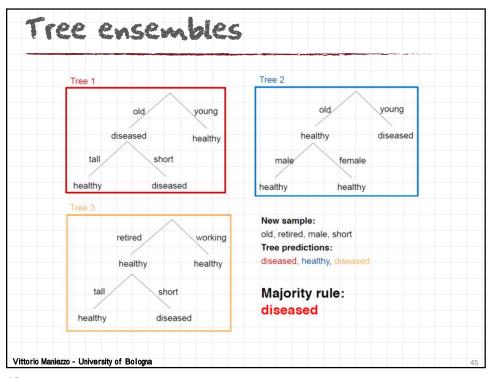


43

Decision tree

Example of a decision tree. In this case, the tree advises us, based upon weather conditions, whether to play ball or not. For example, if the outlook is sunny and the humidity is less than or equal to 70, then it's probably OK to play.





In boosting we train the individual models in a sequential way. Each individual model learns from mistakes made by the previous model. AdaBoost, Gradient Boost etc. are different types of boosting methods. Sequential Ensemble Method (Boosting) Model 1 Weight 1 Weight 2 Weight ... Weight N Vittorio Maniezzo - University of Bologna

Bagging (parallel)

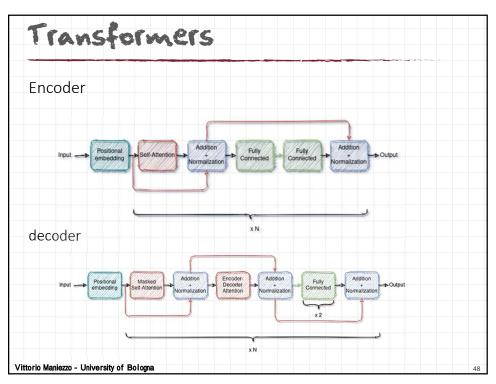
Bagging: decision trees depend onn the data on which they are trained. If the training data is changed (e.g. a tree is trained on a subset of the training data) the resulting decision tree and the predictions can be quite different.

Bagging creates N learners (decision trees) and produces N new training data sets by random sampling with replacement from the original set. The final prediction is the average of prediction from N decision trees.

Random Forest is a bagging method. It also does a random selection of features rather than using all features to grow trees.

Vittorio Maniezzo - University of Bologna

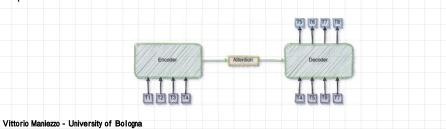
47



A transformer model

One can use an encoder-decoder transformer where the encoder takes as input the history of the time series while the decoder predicts the future values.

The decoder is linked with the encoder using an attention mechanism that learns to "attend" to the most useful part of the time series historical values before making a prediction.



49

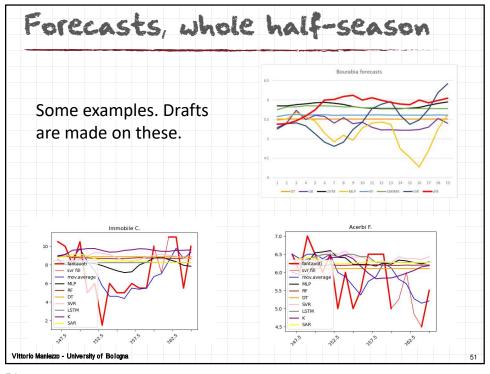
Fantasy football: forecasts

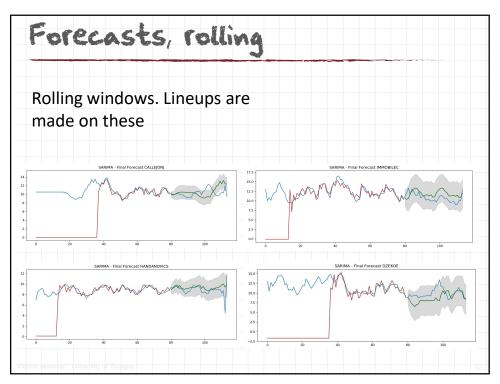
We applied and compared 6 univariate forecast methods:

- SARIMA (seasonality allowed, turned out to be useless)
- MLP (rolling window, one hidden layer)
- SVR (the same as in filling)
- LSTM
- Decision Tree (sklearn)
- Random forest

We recorded the results and eventually drafted a team based on the forecast of each of them.

Vittorio Maniezzo - University of Bologna





Deployment: Lagrangian relaxation

The complexity is due to the budgetary restriction. Associating a Lagrangian penalty λ with it, and relaxing the constraint, we have the formulation LP:

(LP)
$$z_{LP} = max \quad \sum_{i \in I} (p_i - \lambda c_i) x_i + \lambda B$$
 (10)
s.t. (2), (3), (4), (5), (6), (7), (9)

 $\lambda \geq 0$

Problem LP can be decomposed into four subproblems, defined on G, D, M and F, which can all be solved by inspection.

(11)

Having relaxed only one inequality constraint, the optimization of the Lagrangian dual can be expected to be quite fast and effective.

Vittorio Maniezzo - University of Bologna

53

Auction support on the edge

LP could easily be solved by state of the art MIP solvers, but we want to support real time bidders, likely young people using a smartphone.

Currently, smartphones (mobile devices in general) have enough power to support advanced optimization, but the efficiency of mobile MIP solvers is still far from that of the desktop ones.

On the contrary, Lagrangian optimization in our case is much less computationally demanding and can be easily implemented using web languages such as javascript.

Vittorio Maniezzo - University of Bologna

Deployment

Forecast results were persisted and are made available to mobile optimization clients, which implement the bidding support.

Each client runs its own optimization code, a subgradient optimization of the Lagrangian dual of problem *LP* paired with a Lagrangian heuristic.

Before the auction begins, a first optimization determines a target ideal team, as a function of the available budget.

As the auction proceeds, problem data can be updated and the optimizer restarted over the new values, adapting the incumbent Lagrangian multipliers and heuristic solution.

Reoptimizations run in a couple of seconds CPU time.

Vittorio Maniezzo - University of Bologna

55

Lineup

For each match, 11 out of the 23 players must be selected.

The selected players must conform to one of the allowed formations: 3-4-3, 3-4-1-2, ..., 4-4-1-1, 4-2-3-1.

Lineups are defined:

- Selecting for each formation the player with the highest expected profits in the role,
- then selecting the formation that maximizes the total expected profit.

Nonconstant forecasts permit different lineups for different matches.

Vittorio Maniezzo - University of Bologna

Comput. results: leagues

We computed a team for each forecast algorithm and for 4 budget levels.

We benchmarked the teams against one composed by the maximum cost players for each role, as of end Jan, 2021 (not all of them available for drafting, having been in Italy for too short), and against teams optimized projecting past profit means (no forecast, no sharpe's index).

Vittorio Maniezzo - University of Bologna

57

Results: forec		
Team	avg points	avg position
bestteam_400_RF	56.75	1.33
bestteam_400_SVR	54.75	1.79
best_400_MLP_NO	50.17	3.21
best_400_SAR_NO	47.83	3.96
bestteam_400_MLP	44.33	4.96
bestteam_400_SAR	42.17	5.75
best_400_RF_NO	36.92	7.29
bestteam_400_LSTM	34.25	8.38
best_400_SVR_NO	33.67	8.71
best_300_SVR_NO	31.25	10.17
bestteam_300_LSTM	30.50	10.46
best_300_MLP_NO	26.50	12.17
best_300_SAR_NO	24.00	12.83
bestteam_300_SVR	21.33	14.08
best_400_LSTM_NO	19.17	14.92
bestteam_300_RF	16.42	16.17
bestteam_300_SAR	14.67	16.83
best_300_RF_NO	11.50	18.04
bestteam_300_MLP	9.33	18.96
best 300 LSTM NO	7.33	20.00

team		avg points	avg position
bestteam_SVR		11.42	1.21
expensive Team	For the record:	10.58	1.79
bestteam RF	3-4-3 Handanovic S.	7.33	3.25
bestteam MLP	Gosens R.	6.67	3.83
bestteam SAR	Hakimi A.	5.33	5.08
bestteam LSTM	Hernandez T.	4.50	5.83
	Insigne L. Berardi D. Ilicic J. Joao Pedro G. Ronaldo C. Lukaku R. Muriel L.	(no statistic	al significance)