Tensor Evolution

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Tensor Evolution

- Extension of LLVM Scalar Evolution (SCEV) for Tensors
  - Analysis and Optimization Technique

- Tensors are
  - multi-dimensional arrays
  - fundamental to Machine Learning models

Scalar Evolution (SCEV)

"Scalar Evolution is an LLVM analysis that is used to analyze, categorize and simplify expressions in loops. Many optimizations such as - generalized loop-strength-reduction, parallelization by induction variable (vectorization), and loop-invariant expression elimination - rely on ***SCEV*** *analysis. However, SCEV is also a complex topic."*

-- some Large Language Model

Scalar Evolution Analysis and Optimization

- SCEV analysis and opt

```
int foo(int *a, int n, int k){
  for (int i = 0; i < n; i++)
    a[i] = i*k;
}
```

```
$ opt -analyze -scalar-evolution foo.ll
```

```
1. Printing analysis 'Scalar Evolution Analysis' for function 'foo':
2. Classifying expressions for: @foo
3. ...
4. %mul = mul nsw i32 %i, %k
5.  -->  {0,+,%k}<%for.body>  Exits: ((-1 + %n) * %k)
6. ...
```

Tensor Evolution – Motivating Example 1

```
# PyTorch code.
# a and x are tensors
def forward(self, a, x):
  for _ in range(15):
    x = a + x
  return x
```

- Tensor Evolution Optimization

```
# PyTorch code.
# a and x are tensors
def forward(self, a, x):
  return 15*a+x
```

Mathematical Formulation

- Basic Recurrence (Tensor Evolution)
  - a constant or loop-invariant tensor T_c
  - a function τ_1 over natural number N that produces tensor of same shape as T_c
  - an element-wise operator + associative and commutative
  - τ defined as function τ (i) over N

$$\tau = \{ T_c, +, \tau_1 \} \qquad \text{eq. 1}$$

$$\{T_c, +, \tau_1\}(i) = T_c + \tau_1(0) + \tau_1(1) \ldots + \tau_1(i-1) \qquad \text{eq. 2}$$

Mathematical Formulation

- Chain of Recurrences (Tensor Evolution)
  - loop-invariant tensors Tc_0 , Tc_1 , Tc_2 , …, Tc_{i-1} ;
  - function τ_k defined over N,
  - operators $\odot_1$, $\odot_2$, …, $\odot_k$,
  - chain of evolution of tensor value represented by tuple

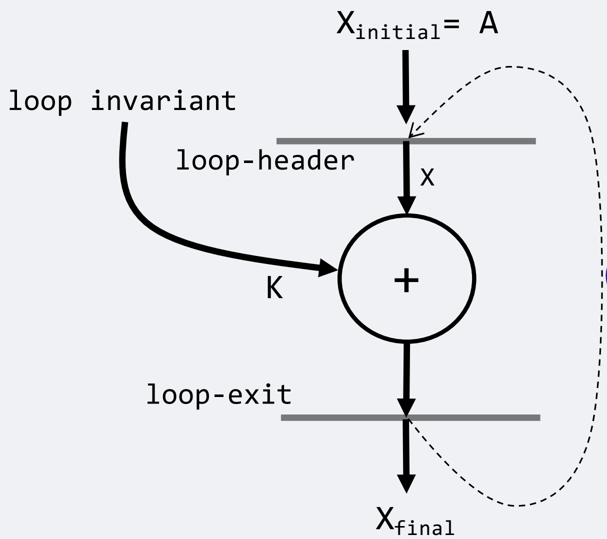
$$\tau = \{Tc_0 , \odot_1, Tc_1 , \odot_2, \ldots, \odot_k, \tau_k\} \qquad \text{eq. 1}$$

$$\tau\,(i) = \{Tc_0 , \odot_1, \{Tc_1 , \odot_2, \ldots, \odot_k, \tau_k\}\}(i) \qquad \text{eq. 2}$$

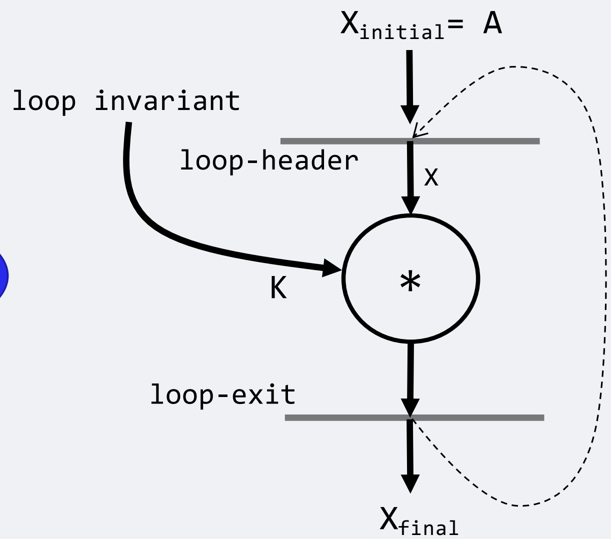
- Note: Operators could be same or different (+,-, *, tanh).

- Recurrences
  - Algebraic properties
  - Computationally reducible at any iteration point

Tensor Evolution – Basic Recurrence



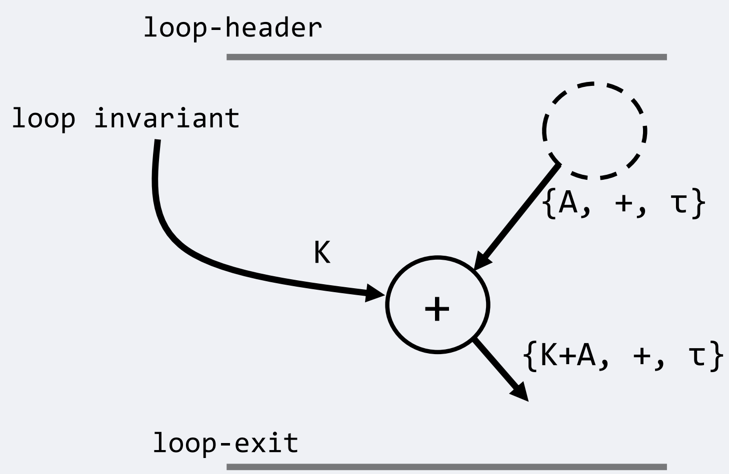
X = {A, +, K}



X = {A, *, K}

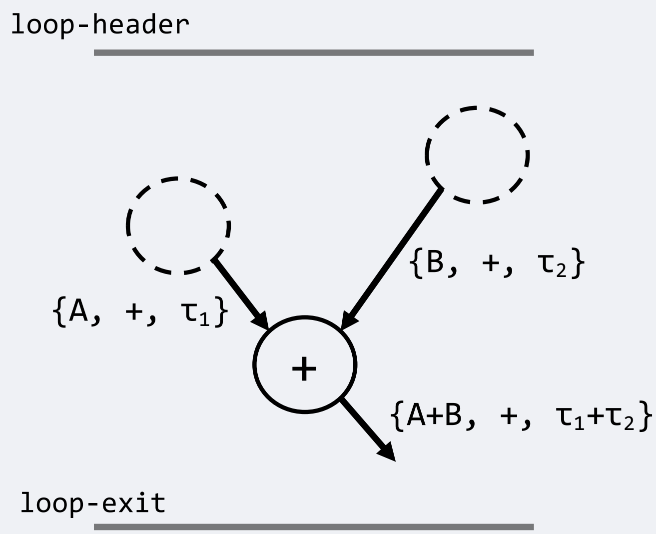
Tensor Evolution

- Lemma: Add a constant (LIV) tensor



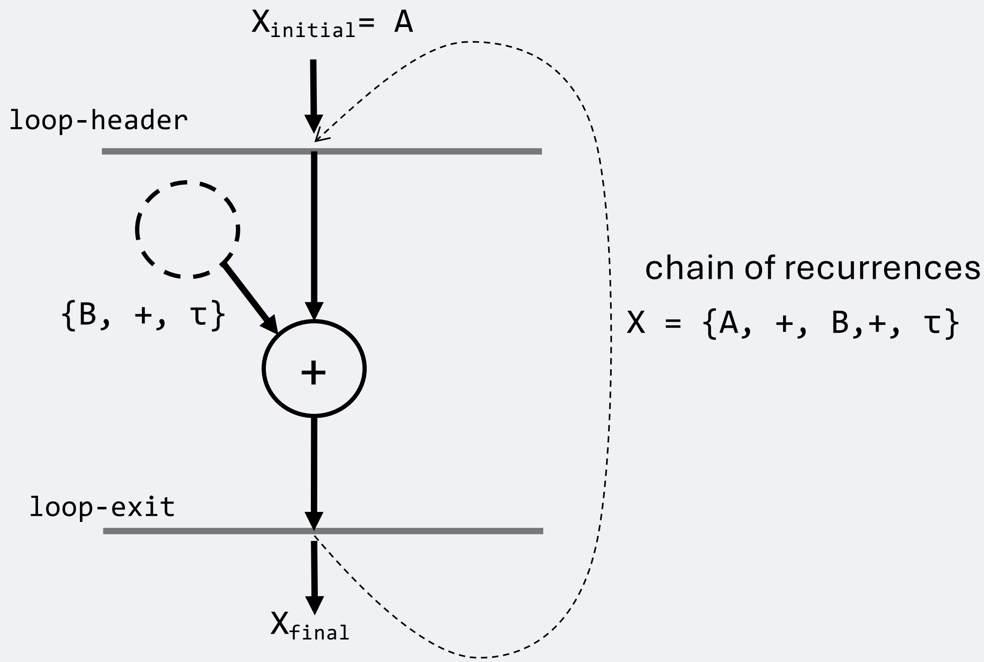
Tensor Evolution

- Lemma: Add two TEVs



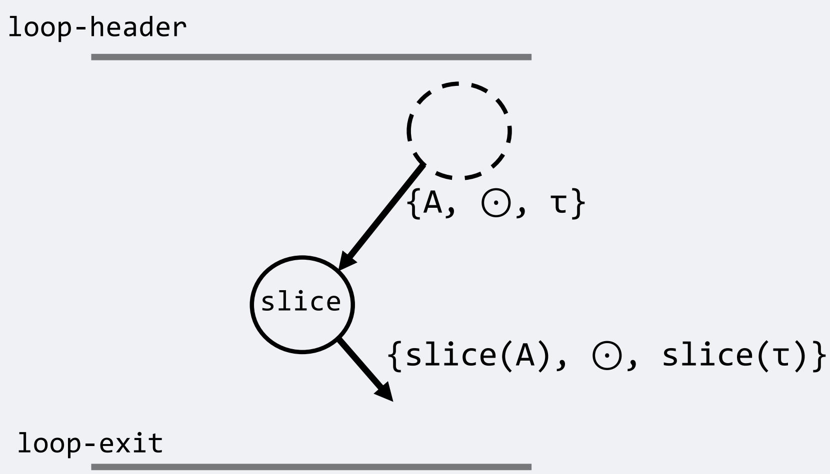
Tensor Evolution

- Lemma: TEV inject into TEV



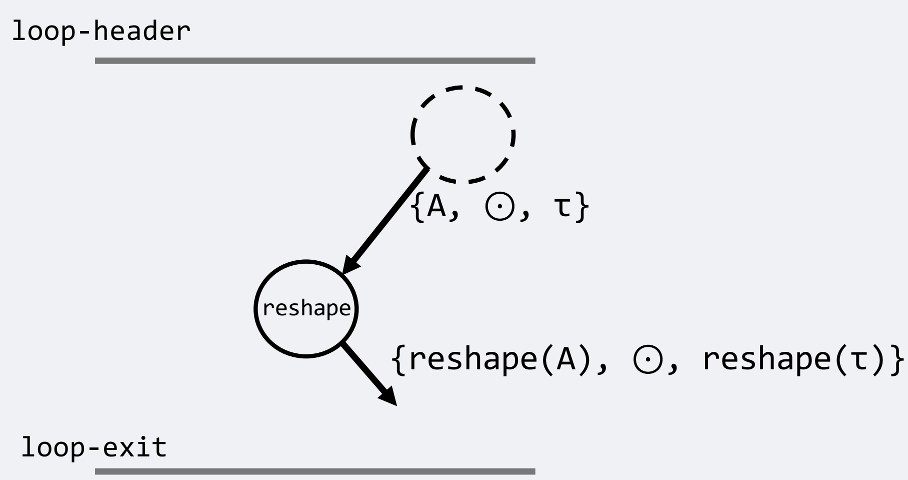
Tensor Evolution

- Lemma: Slice



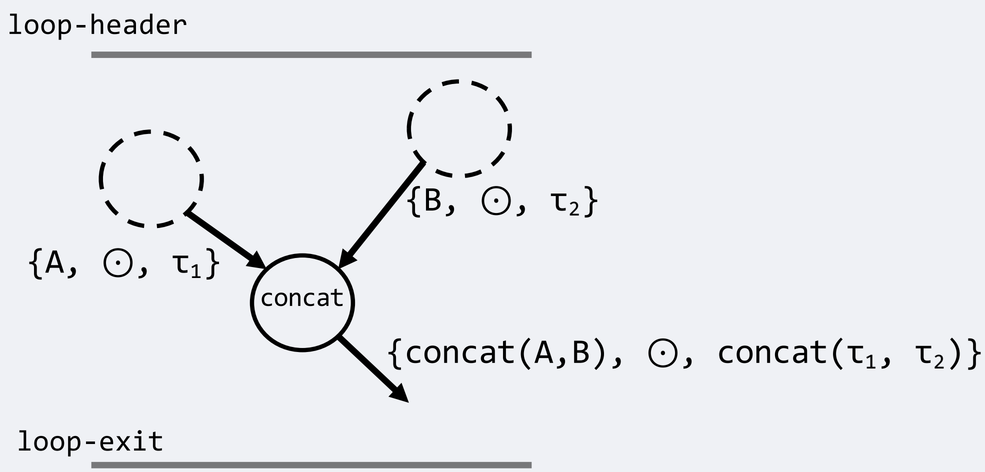
Tensor Evolution

- Lemma: Reshape



Tensor Evolution

- Lemma: Concat



Concat Lemma Proof

- Claim: $C(\{A, \odot, \tau_1\}, \{B, \odot, \tau_2\}) \Rightarrow \{C(A, B), \odot, C(\tau_1, \tau_2)\}$
  - Where C is the concatenation operation, $\odot$ is an element-wise operator.
- Recap: $\{A, \odot, \tau_1\}(i) = A \odot \tau_1(1) \odot \tau_1(2) \odot \ldots \odot \tau_1(i)$ – (1)
  - Where A is a loop-invariant tensor, $\odot$ is an operator, and $\tau_1(i)$ represents evolving tensors.
- Now, consider LHS: $C(\{A, \odot, \tau_1\}(i), \{B, \odot, \tau_2\}(i))$
- $[A \odot \tau_1(1) \odot \ldots \odot \tau_1(i)] \,||\, [B \odot \tau_2(1) \odot \ldots \odot \tau_2(i)]$
  - Where || is the concatenation operator
- Stacking along axis = 0 to preserve structure for simplicity, we get
  - $C(A, B) \odot C(\tau_1(1), \tau_2(1)) \odot \ldots \odot C(\tau_1(i), \tau_2(i))$
  - From (1), the above is equivalent to $\{C(A, B), \odot, C(\tau_1, \tau_2)\}(i)$
- $C(\{A, \odot, \tau_1\}, \{B, \odot, \tau_2\}) = \{C(A, B), \odot, C(\tau_1, \tau_2)\}$

An example for intuition

- Given $A = [1, 2]$, $\tau_1(i) = [i, i]$
- $B = [3, 4]$, $\tau_2(i) = [2i, 2i]$, $\odot = +$
- LHS: $C(\{A, +, \tau_1\}, \{B, +, \tau_2\})(i)$
- $= C([1 + i, 2 + i], [3 + 2i, 4 + 2i])$
- $= [1+i, 2+i, 3+2i, 4+2i]$

- RHS: $\{C(A, B), +, C(\tau_1, \tau_2)\}(i)$
- $= [1, 2, 3, 4] + [i, i, 2i, 2i]$
- $= [1+i, 2+i, 3+2i, 4+2i]$

TeV Injection Lemma Proof

Rewrite Rule:

$\{A, +, \{B, +, \tau\}\} \rightarrow \{A, +, B, +, \tau\}$

The LHS is essentially a CR, examining right-to-left, $\{B, +, \tau\}$ is a BR with B as the loop inv tensor evolving with value τ

1. By definition, a TEV follows:

$\{B, +, \tau\}(i) = B + \tau(1) + \tau(2) + \ldots + \tau(i)$

2. Expanding LHS:

$\{A, +, \{B, +, \tau\}\}(i) = A + \{B, +, \tau\}(1) + \{B, +, \tau\}(2) + \ldots + \{B, +, \tau\}(i)$

$= A + (B + \tau(1)) + (B + \tau(2)) + \ldots + (B + \tau(i))$

$= A + iB + (\tau(1) + \tau(2) + \ldots + \tau(i))$

4. Recognizing the pattern, we get:

$\{A, +, iB, +, \tau\}(i) = A + iB + \tau(1) + \tau(2) + \ldots + \tau(i)$

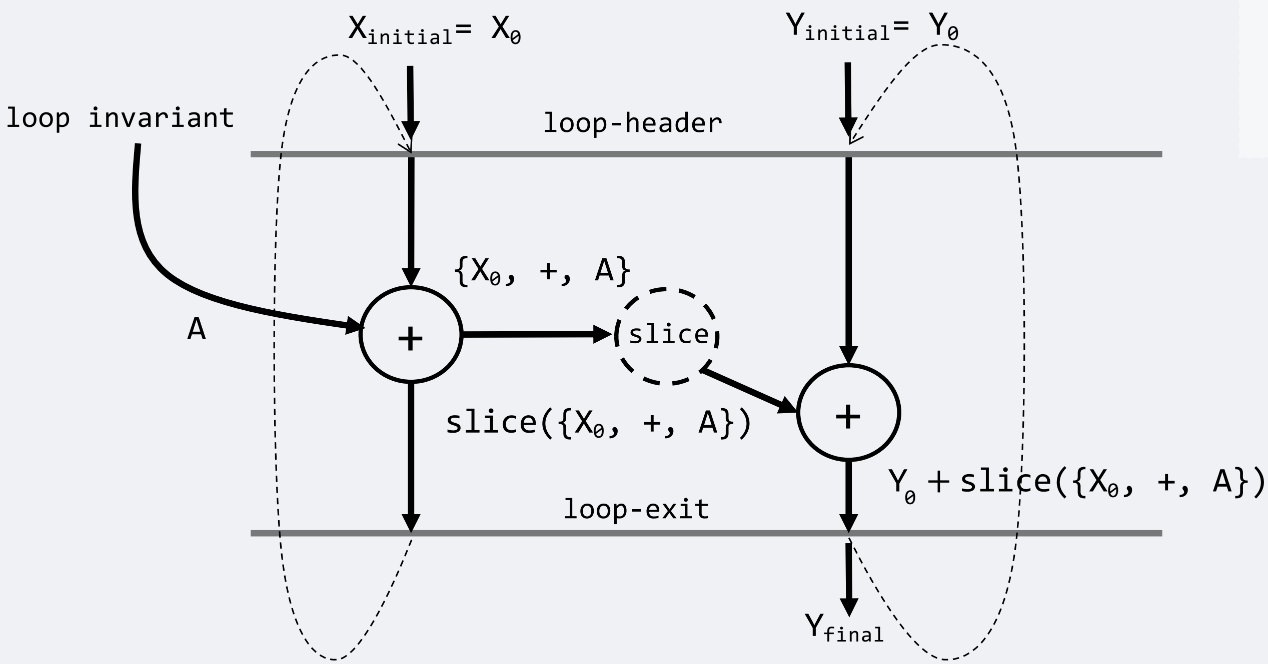
Now, it is easy to see that this is indeed a flattened recurrence as τ changes over loop iterations with A, B being loop invariant tensors.

Tensor Evolution

- Lemmas – Rewrite Rules
- Used for building TEV ‘available’ expressions and simplifications

operator	TEV expression	rewrite rule
slice	slice({A, +, τ})	{slice(A), +, slice(τ)}
reshape	reshape({A, $\odot$, τ})	{reshape(A), $\odot$, reshape(τ)}
concat	concat({A, $\odot$, τ_1},{B, $\odot$, τ_2})	{concat(A,B), $\odot$, concat(τ_1, τ_2)}
add K	K + {A, +, τ}	{K+A, +, τ}
add TEVs	{A, +, τ_1} + {B, +, τ_2}	{A+B, +, τ_1+τ_2}
mul	K * {A, +, τ}	{K*A, +, K*τ}
inject TEV	{A, +, {B, +, τ}}	{A, +, B, +, τ}

TEV Pass - Opt




```
# PyTorch code.
def forward(self, a, x, y):
  for _ in range(15):
     x = x + a
     …
     z = x[1,:]
     y = y + z
  return y
```

Evaluation of Y_k

$Y_k = \{Y_0, +, S(\{X_0, +, A\})\}_k$

➔ $Y_k = \{Y_0, +, S(\{X_0, +, A\})\}_k$

➔ $Y_k = \{Y_0, +, \{S(X_0), +, S(A)\}\}_k$

➔ $Y_k = \{Y_0, +, S(X_0), +, S(A)\}_k$

➔ $Y_k = Y_0 + k*S(X_0) + k*(k+1)/2*S(A)$

TEV Pass - Opt

Evaluation of Y_k

$Y_k = \{Y_0, +, S(\{X_0, +, A\})\}_k$

➔ $Y_k = \{Y_0, +, S(\{X_0, +, A\})\}_k$

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```
# PyTorch code.
def forward(self, a, x, y):
  for _ in range(15):
     x = x + a
     …
     z = x[1,:]
     y = y + z
  return y
```

```
# PyTorch code.
def forward(self, a, x, y):
  return  y + 15*x[1,:] + 15*(15+1)/2*a[1,:]
```

Conclusion

- TEV is extension of SCEV to Tensors
- Construction of TEV expressions and rewrite-lemmas
  - Complex optimizations on top of TEV (much like SCEV LSR etc)
- Prototyped in internal-compiler
- Potential opt for MLIR lower CFG dialects
  - Looking forward to collaboration and discussions

Thank you

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