$\boxed{\text{Info}}$ Space and coordinate

The concepts about space and coordinate are very different. Space is object and coordinate is the description of the object. This ~~situation~~ relation looks like the relation between tiger and that large fierce animal. Tiger is just the name or label of that kind of animal, not the ~~essence of~~ animal itself. Coordinates are just the label of points in the space.

~~Space is a concept that is very simple.~~

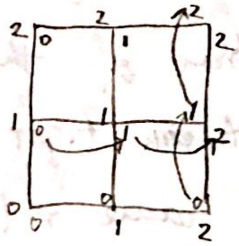
Space is a very simple concept. ~~Tha~~ It can be equal with the the concept of set. And point can be equal with the concept of element. Coordinates are the labels ~~of~~ or names of these elements.

Cartesian space and Cartesian coordinates are the basic concepts of analytical geometry. This kind of space can be labeled by continuous real number interval or the tuple of this kind of intervals. We know real number interval is a complicated mathematical object and the basic property of ~~real~~ continuous real number interval is continuity. That means Cartesian space, is a ~~kin~~ kind of continuous space and we can do calculus on it.

The continuity of the space also ~~te~~ tell us that the ~~dimensio~~ dimensionality of the space is the intrinsic property of the space. We can not label the points on the ~~high~~ ~~continuouuous~~

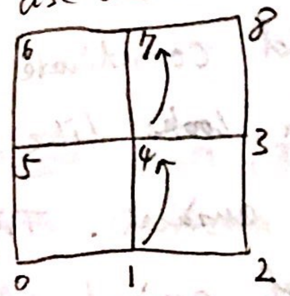
continuous space by lower dimensional real number tuple. We use a very simple/naive example to explain this.

use two dimensional tuple



0 to 1, 1 to 2
"continuous" change

use one dimensional tuple



1 to 4, 4 to 7
not "continuous" change

We must pay attention to ~~a~~ the truth that Cartesian space is just a kind of set and nothing except a set. The ~~set~~ elements of the set can be ~~labeld~~ labeled by continuous real number. Except that, Cartesian space does not have any kind of other structure. This means there is no geometry on the pure Cartesian space.

There is no geometry on a pure ~~Ca~~ Cartesian space, because the centre concept of a geometry has been lost. ~~The lost concept is called the distance between two point in the space (i.e. the measurement on the space). On the one hand, we need define the distance of any two points on of the space. On the other hand, this~~ The lost concept is the length of continuous curve on the space. (i.e. the measurement on the space). We also know that we can calculate the length of any ~~curve~~ curve if we have defined the distance of two points which are infinitely close to each other, $dl(x_i, x_f)$, x_i is the initial point and x_f is the finial point. We use a matrix to define it, called metric matrix

$$(g_{kl}(x_i)).$$

And the dl is defined as

$$dl(x_i, x_f) = g_{kl}(x_i)\, dx^k\, dx^l$$

where

$$dx^k = x_f^k - x_i^k.$$

The object that cover a Cartesian space ~~and~~ $\{x^i\}$ and a set of metric matrix $\{g_{ij}(x)\}$ is the most basic object of study of analytical geometry. The terminology "space" which is used ~~as~~ at geometry refers to this kind of object.

because the (g_{ij}) changed as a second order ~~can~~ contravariant tensor.

If a metric matrix (recisely speaking, metric tensor) is positive ~~defined~~ definite at ~~every~~ any point of the space, this kind of space iscalled Riemannian space. Ulteriorly, if the positive definite ~~matrix~~ matrices change to identity matrix under one coordinates, the space is called Euclidean space. Similarly, we can provide the concepts about pseudo-Riemannian space and Minkowski space.

END Info