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Resiliency of the Limit Order Book☆

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Abstract

This study contributes to our understanding of the liquidity replenishment process in limit order markets. A measure of resiliency is proposed and quantified for different liquidity shocks through the impulse response functions generated from a high frequency vector autoregression. The model reveals a rich set of liquidity dynamics for the Australian equity market. Resiliency is found to be consistently high across all large stocks, consistent with competition for liquidity provision coming from computerized algorithms. For other stocks, some variation in resiliency is observed indicating more selective participation by these liquidity providers. The prevalence of order splitting strategies is not surprising, as it has become a mechanism traders use to benefit from the resiliency of liquidity.

JEL Classification: G12; G14; C32

Keywords: Resiliency, Liquidity, Limit order book, Liquidity shocks.

1. Introduction

While few would argue against the wisdom of the words 'no single measure can capture all aspects of liquidity', adopted measures have traditionally relied upon information from visible limit orders. An important but often missing element in completing the liquidity picture is the measurement and understanding of latent sources of liquidity. Biais et al. (1995) first documented the existence of potential liquidity outside the limit order book which they attribute to the presence of traders who do not have exposed orders but are actively monitoring the market for favourable order placement opportunities. They find that traders quickly submit limit orders when liquidity provision is attractive and follow 'defensive strategies' (Harris, 1996) by cancelling liquidity in reaction to order flow likely to originate from informed traders.

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Latent sources of liquidity can be captured by examining the resiliency dimension of liquidity (Kyle, 1985). However, different perceptions of resiliency exist and there is no consensus among academics on how it should be defined and measured. Kyle (1985) takes the price recovery perspective, defining resiliency as the rate at which pricing errors caused by temporary order-flow shocks are corrected in the market. On the other hand, Garbade (1982) takes the liquidity replenishment perspective, describing resiliency as the speed of replenishment of the limit order book. 'A market is resilient if new orders pour in promptly in response to a temporary order imbalance'. This perspective is also adopted by Large (2007), measuring a resilient limit order book as one that reverts promptly back to its normal shape but the analysis restricts the cause of liquidity depletion to the presence of large trades. This study defines resiliency as the time required for liquidity to recovery from a set of common liquidity shocks. These shocks are represented by 'specific order events' related to both trade and cancellation activities. Our view on resiliency captures both perspectives. We can quantify the level of resiliency in prices as well as different dimensions of limit order book liquidity. An important difference between the two perspectives is that liquidity shocks have a permanent impact on prices but a transient impact on our liquidity variables.

A number of developments have increased the relevance of resiliency as a dimension of liquidity. Improvements in exchange technology witnessed in the last two decades have resulted in substantial increases in both the speed and level of automation of the trading process. The relative ease at which market participants can actively monitor and respond to changes in market conditions challenges the value of liquidity measures based solely on 'displayed' liquidity. Another important development has been the decision by many exchanges to facilitate market participants hiding a portion of their order flow. Hidden order strategies have become popular among traders who are cautious about exposing their full trading intentions for fear others may react by withdrawing liquidity or trading ahead of their order. Recent academic studies (Pardo and Pascual, 2012; De Winne and D'hondt, 2007) have supported the view of traders able detect and predict hidden liquidity by monitoring limit order activity.

Order splitting has become a common practice where large orders are not fully exposed, but are typically split into a number of smaller child orders and their execution dynamically managed over specified trading horizons. Understanding resiliency is important for the development of these optimal execution strategies. Obizhaeva and Wang (2013) and Alfonsi et al. (2010) study this problem in a limit order market and determines that the optimal strategy is most dependent on the resiliency of the order book as opposed to their static properties such as the spread, depth or instantaneous price impact. The models of Obizhaeva and Wang (2013) and Alfonsi et al. (2010) assume the existence of an order book resilience effect where the initial impact of trading dissipates over time as new orders arrive to replenish the book. However, few studies have examined the extent to which this effect has been empirically observed and the liquidity dynamics that generates these features has been largely unexplored.

Resiliency is also relevant for regulators and market operators. There is continued debate over the optimality of market structures relying solely on endogenous liquidity provision with a growing number of academic studies (Venkataraman and Waisburd, 2007; Bessembinder et al., 2011; Anand and Venkataraman, 2012) providing support for the use of designated market makers within electronic limit order markets. The 'flash crash' of May 6, 2010 in which the prices of some US securities took a precipitous decline, renewed concerns over the resiliency of limit order book markets and their capacity to absorb liquidity shocks. This study sheds light on the issue by quantifying the effects of common liquidity shocks on the limit order book. The Australian equity market is part of a growing number of markets operating without designated market makers, providing a natural setting for examining these concerns.

We propose a high frequency vector autoregression (VAR) specification to capture short term liquidity dynamics and provide insights into the liquidity replenishment mechanism of the limit order book. The model incorporates relevant characteristics of the order arrival process including variables representing different dimensions of the limit order book. Resiliency is quantified through the impulse response functions which measure the time profile of different liquidity variables in a dynamic system generated by specific liquidity shocks. If the limit order book lacks resiliency, then liquidity shocks are accompanied by a slow rate of recovery to equilibrium levels.

We document a number of dynamic interactions between different dimensions of liquidity that may be useful for the enhancement of optimal execution strategies. Depth at the best prices deteriorates in response to a liquidity shock that results in a widening of the spread. Conversely, spreads tend to widen in response to a negative shock to depth at the best prices. Significant interactions are observed along two dimensions of limit order book depth. Firstly, shocks to the depth on one side of the limit order book affects depth on the other side at the best prices. Secondly, shocks to the depth behind the best prices affects depth at the best prices on the same side of the limit order book.

Our investigation also examines the resiliency of the limit order book arising from different liquidity shocks and their effects on the liquidity replenishment process. Examining resiliency along each dimension of liquidity reveals that spreads and depth at the best prices have similar rates of recovery while resiliency is significantly weaker for depth away from the best prices. A similar level of resiliency is observed from liquidity shocks having the same impact on the limit order book regardless of whether the source of the shock is due to market orders or order cancellations. The presence of trading does not slow the rate at which new limit orders arrive to replenish liquidity and affects only the price levels at which liquidity is offered.

The price-time priority rules of limit order markets results in a 'first mover advantage' in the supply of liquidity (Biais et al., 1995). This creates significant incentives for liquidity suppliers to make investments in trading technology to increase the speed and efficiency in which they can identify and respond to liquidity imbalances in the limit order book. While concerns have been expressed over the benefits and implications of a 'technology arms race', competition among liquidity suppliers could have a positive effect on the resiliency of the market by increasing the speed in which liquidity imbalances are corrected.

2. Related Literature

Few theoretical models of limit order books consider the concept of market resiliency. Foucault et al. (2005) provide one such model containing a number of specific predictions regarding market resiliency. The resiliency of the limit order book increases with the proportion of patient traders and the waiting cost, but decreases in the order arrival rate. However, their model only allows an examination of spread resiliency. Rosu (2009) also develops a dynamic model of the limit order book that provides a condition for a resilient limit order book, that patient sellers arrive faster than impatient buyers.

This paper fits in with existing literature that examines resiliency using VAR based approaches as it provides a flexible framework to capture dynamic relationships. Pioneered by Hasbrouck (1991), the VAR framework has been extensively used to examine the price impact of trading and this literature also provides insights into price resiliency. Hasbrouck (1991) finds that an unexpected trade has a positive, concave and persistent impact on prices and the full price impact is only revealed after a protracted lag due to microstructure imperfections. Dufour and Engle (2000) extend the Hasbrouck (1991) bivariate model of trades and quotes by incorporating information on the duration between trades. They find that time plays an important informational role in the dynamics between trades and quotes. High trading activity results in a greater price impact as liquidity suppliers infer a greater presence of informed traders during times of higher trading activity. Engle and Patton (2004) incorporate both bid and ask quotes in an error correction model to examine price impact. Their empirical findings support both an asymmetric impact between buys and sells and error correcting behaviour in the spread. A large spread tends to lead to a fall in the ask price and a rise in the bid price. Hautsch and Huang (2012) propose a cointegrated VAR model of quotes and order book depths to examine the price impact of limit orders. They find that limit orders have permanent price effects and the magnitude of the effect depends on the aggressiveness and size of the order as well as the state of the limit order book. This confirms that the market reacts to the trading intentions revealed by limit order submissions.

Fewer studies have utilized VAR models to examine limit order book resiliency. Hmaied et al. (2006) investigate the dynamics of market liquidity of Tunisian stocks through a joint model of depths, spreads and volatility. They find significant interactions between the variables and their impulse response function analysis reveals that liquidity shocks are absorbed more quickly for frequently traded stocks. Coppejans et al. (2004) analyse the dynamics of liquidity in the limit order book on the Swedish index futures market and find that increases in market liquidity measured by order book depth has a positive effect of lowering volatility. A liquidity clustering effect is also observed across bid and ask side depth and increases in depth on one side of the market leads to a rise in depth on the other side. In the empirical study by Danielsson and Payne (2002), a VAR model is adopted to jointly estimate the dynamic effects of spreads, depth, volume and volatility of the DEM/USD exchange rate traded on Reuters D2000-2 FX electronic broking system. Focussing on the determination of order book depth, they find that both increased volatility and wider spreads leads to decreased depth. In times of high volatility, market participants supply less liquidity and at worse terms. The effect of volume on depth depends on the side that initiated the trading. After market buy activity, buy side depth increases while sell side depth is reduced.

Our model setup has three distinguishing characteristics. Firstly, the impulse responses are generated from liquidity shocks representing order events that consume order book liquidity. The impulse responses have a clear interpretation which contrasts with earlier studies on resiliency (Hmaied et al., 2006) where they were generated from orthogonalized shocks. Secondly, resiliency can be quantified for each liquidity variable permitting an examination of whether resiliency differs along different dimensions. Thirdly, the inclusion of a variable to capture the duration between order events facilitates the measurement of resiliency in both event time and wall clock time which is of interest to practitioners.

Empirical studies have also adopted other approaches in examining market resiliency. Degryse et al. (2005) use an event study approach, analysing the resiliency of the Paris Bourse by observing the behaviour of variables such as the spread, depth and duration at the best quotes within a window around the submission of an aggressive order. Large (2007) proposed an intensity model to quantify the resiliency of a single London Stock Exchange stock that treats order events as a multivariate point process.

This study is also related to empirical studies on order submission strategies in limit order book markets (Griffiths et al., 2000; Ranaldo, 2004). While we do not explicitly model the determinants of order choice, our study complements this literature as the dynamics of liquidity provision is a consequence of the order subsmission strategies of traders and how they respond to earlier order events and information contained in the limit order book.

2.1. Australian Securities Exchange Institutional Details

The Australian Securities Exchange (ASX) operates a continuous electronic order-driven market with auctions to open and close trading. In contrast with the NYSE and Nasdaq, there are no designated market makers and liquidity is supplied solely by traders who submit limit orders. Limit orders awaiting execution are consolidated in a centralized limit order book which is transparent to all market participants and their position in the bid or ask queue based on strict price-time priority rules. Modifying order volume downwards does not affect order priority. However, modifying order volume upwards automatically generates an additional order for the increase in volume while the original order maintains existing price-time priority in the limit order book. Modifying order price causes the order to move to the lowest time priority for all orders at the new price level unless the order becomes marketable, in which case it is matched against an existing order in the limit order book. The ASX goes through several market phases through the day. Table 1 provides an overview of the flow of trading on ASX equities [1].

[1] See Australian Securities Exchange (2008) for a comprehensive description of the Integrated Trading System (ITS) and the trading rules of the ASX that are relevant to the sample used in this study.

Table 1: Market Trading Schedule on ASX Equities

Market Phase	Time	Description
Market Pre-Open	7:00 am - 10:00 am	Orders can be entered, amended or delete but no matching takes place.
Market Opening	10:00 am - 10:09 am	Staggered call auctions open continuous trading.
Normal Trading	10:00 am - 4:00 pm	Orders can be entered, amended or deleted. Orders that can be matched are executed.
Pre CSPA[a]	4:00 pm - 4:10 pm	Trading ceases but brokers can enter, amend and delete orders in preparation of the closing price auction.
CSPA	4:10 pm - 4:12 pm	Call auction establishes market closing price.
Adjust / Adjust ON	4:12 p m - 6:50 pm	Only orders remaining in the queue can be deleted or amended.
Purge Orders	6:50 pm - 6:59 pm	Orders that have expired or too far away from the market are purged.

[a] CSPA stands for closing single price auction.

2.2. Data Description

Data containing the complete record of all order events is obtained from the Australian Equities Tick History (AETHS) database[2] for a sample of 30 ASX listed stocks covering a three month period from September to November 2009. To provide for adequate representation of stocks with different levels of liquidity, the 30 stocks chosen covered different industries with precisely 10 stocks in the large, mid and small capitalization cateogry as classified by S&P/ASX indices on the first trading day of the sample period.

An order event is defined in the data as one of the following:

1. ENTER event refers to the arrival of a new order to the market.
2. AMEND event refers to the modification of an existing order.
3. TRADE event occurs when a buy or sell order is matched against an existing order in the order book.
4. DELETE event refers to the deletion of an existing limit order.

Each order event is timestamped to the nearest millisecond. Trade direction can be directly determined from submitted orders which is a significant advantage over other tick by tick datasets where trade initiation is inferred via a trade classification algorithm. The complete record of time-stamped order events in conjunction with classified trades allows a full reconstruction of the limit order book. Split transactions are common and occur when a single marketable order is filled against multiple existing limit orders, at potentially different prices. Split transactions, identified as trades with the same order identifier and occurring at the same millisecond timestamp, are consolidated to a single transaction. Trade sizes are aggregated and a volume-weighted average price (VWAP) is calculated and recorded as the trade price of the consolidated trade.

[2]Data supplied by Securities Industry Research Centre of Asia-Pacific (SIRCA) on behalf of the ASX. See http://www.sirca.org.au/.

To remove obvious data errors, the data was filtered to ensure the bid-ask spread is positive and that every observed order event matches against the corresponding change in the limit order book. All priority crossing and off market trades were removed as the focus of the study is on liquidity that can be consumed in the limit order book. All limit order book records prior to 10:15am and after 3:45pm are also discarded as they may be contaminated with effects from the opening and closing call auctions. Table 2 presents summary statistics of the companies included in our study. As expected, larger capitalization stocks have significantly more trading and limit order activities and a lower duration between order events.

3. Method

This section presents the econometric methodology used to examine the dynamics of liquidity. The first methodological choice is the sampling frequency. Typically, this choice is between event time or wall clock time. Studies of liquidity using wall-clock time include Hmaied et al. (2006) and Coppejans et al. (2004) while Hasbrouck (1991), Dufour and Engle (2000), Engle and Patton (2004), Hautsch and Huang (2012) and Degryse et al. (2005) prefer event time. Studies based on wall clock time suffers from two drawbacks. Firstly, wall clock time necessitates choosing an interval length for time aggregation. The appropriate interval length would likely vary among stocks in the sample depending on liquidity and quote activity. For instance, Large (2007) finds that using data on a London stock Exchange listed stock, when the order book does replenish after a large trade, it does so fairly quickly and ‘too fast to be captured by 5min sampling’. An inappropriate choice on interval length could adversely affect the results. Secondly, the time aggregration of data containing all order events occurring within that interval results in a potential loss of information and contemporaneous dependencies in the dynamics we are seeking to examine. The dataset contains order events that were submitted to the market and are recorded at irregularly spaced time intervals making order event time a natural choice. Specifying the model in order event time allows a precise definition of the exact source of the liquidity shock which cannot be achieved with a VAR approach using time aggregated data.

3.1. Variable Definition

The vector of endogenous variables in the model is given by:

$$x_t = \{p_t^a, p_t^b, x_t^b, x_t^s, v_t^{b,1}, v_t^{b,25}, v_t^{a,1}, v_t^{a,25}, d_t\}' \tag{1}$$

Each increment in t represents an order event described in Section 2.2 that impacts the first five price levels of the limit order book. The values contained in the variables at each time t represents what is observed immediately following the arrival of the tth order event. In other words, the variables at time t incorporates the information from the tth order event.

Table 3 provides a brief description of each variable. p_t^b and p_t^a are the logarithms of the best bid and ask prices respectively. The trading process is endogenized through the trading indicator variables which allows us to distinguish between trading and order cancellation events. The choice of including two separate trade indicator variables, x_t^b and x_t^s which

Table 2: Sample: Descriptive Statistics

This table provides summary statistics on trade and order book data. The sample contains 30 companies listed on the ASX. The top 10 companies are large capitalization stocks, the following 10 are mid capitalization stocks and the bottom 10 companies are small capitalization stocks. Order book depth is measured in thousands of shares. L1 denotes the first (topmost) price level of the limit order book. L2-5 denotes order book price levels 2 to 5. The mean values of bid and ask prices, volumes and duration between order events (measured in seconds) are reported. The sample period covers every trading day from 1st September to 30 November 2009.

Size Category	Stocks Trades	# Buy Trades	# Sell Events	# Order Price	L1 Bid Price	L1 Ask Volume	L1 Bid Volume	L2-5 Bid Volume	L1 Ask Volume	L2-5 Ask (secs)	Duration
Large	ANZ	2178	1945	36511	22.94	22.95	6.360	30.557	6.506	26.854	0.54
	BHP	2863	2540	40852	38.41	38.42	5.654	28.541	6.273	30.918	0.48
	CBA	2436	2376	41718	51.43	51.45	1.793	7.470	1.745	6.611	0.47
	MQG	1709	1659	26506	51.74	51.76	0.952	3.463	0.946	3.175	0.75
	ORG	1169	1014	17900	15.91	15.92	3.479	19.014	3.686	17.231	1.10
	QBE	1580	1372	24268	22.93	22.95	4.157	18.278	3.377	13.671	0.82
	RIO	2310	2099	32372	63.95	63.97	1.233	4.112	1.451	5.222	0.61
	WES	1474	1548	23509	27.02	27.03	2.075	9.661	2.182	9.582	0.84
	WOW	1405	1245	22024	28.76	28.78	2.693	13.685	2.705	11.022	0.90
	WPL	1893	1781	29869	49.88	49.90	0.986	4.479	1.021	4.362	0.66
Mid	BBG	646	616	10598	10.80	10.81	2.032	9.897	2.127	8.504	1.87
	BEN	505	529	8657	8.94	8.95	3.964	16.573	4.115	17.198	2.29
	BLD	575	604	9133	5.83	5.84	9.838	47.676	10.147	45.583	2.17
	CTX	568	628	8923	11.16	11.18	3.829	17.811	3.420	10.490	2.22
	DJS	573	495	8148	5.55	5.56	14.724	80.144	14.447	76.665	2.43
	GFF	465	379	6075	1.61	1.62	63.246	197.948	60.926	170.142	3.26
	HVN	587	545	9238	4.24	4.25	22.250	92.376	24.623	92.883	2.14
	JHX	687	717	10731	7.45	7.46	4.861	21.753	4.795	22.882	1.84
	MTS	440	409	6682	4.54	4.55	33.053	195.439	30.960	161.215	2.96
	UGL	572	650	9267	14.23	14.24	1.397	4.209	1.560	4.526	2.13
Small	BWP	188	175	2583	1.75	1.76	12.211	65.231	13.920	56.471	7.64
	CAB	255	268	4177	5.97	5.98	4.704	17.252	4.783	19.950	4.73
	EQN	266	280	3879	3.72	3.73	14.821	61.614	15.505	62.151	5.10
	IRE	311	349	5713	8.10	8.12	1.652	5.346	1.624	5.182	3.46
	MAH	203	163	2322	0.63	0.63	205.973	1187.783	197.279	1102.765	8.51
	NXS	125	103	1663	0.34	0.35	857.584	4016.943	668.653	3360.231	11.88
	SKE	104	93	1095	2.18	2.19	2.882	13.406	3.212	13.846	17.92
	SUL	88	89	945	5.36	5.38	1.127	3.479	1.425	4.122	20.83
	TPI	259	242	3501	1.50	1.50	37.283	150.240	33.808	130.523	5.65
	WTF	303	248	4900	5.81	5.83	3.343	8.502	3.592	9.917	4.04

Table 3: Variable Definition

Variable	Description
p_t^a	Log of the ask price ($)
p_t^b	Log of the bid price ($)
x_t^b	Buy trade dummy variable
x_t^s	Sell trade dummy variable
$v_t^{b,1}$	Log depth at the best bid price (thousands)
$v_t^{b,25}$	Log of the cumulative depth from 2nd to 5th bid price step (thousands)
$v_t^{a,1}$	Log depth at the best ask price (thousands)
$v_t^{a,25}$	Log of the cumulative depth from 2nd to 5th ask price step (thousands)
d_t	Log of duration since the previous order book event (seconds)
s_t	Spread in logs ($s_t = p_t^a - p_t^b$)
q_t	Mid-quote ($q_t = 0.5(p_t^a + p_t^b)$)

identify the occurrence of a buy and sell trade respectively, allows the model to capture potential asymmetric effects that have been found by Engle and Patton (2004) and Hautsch and Huang (2012). The discreteness of these variables did not introduce difficulties in estimation but residuals are heteroskedastic and White standard errors are used for statistical inference.

The v_t variables represent the volume of waiting limit orders in the market. Volume at the best prices and volume behind the market are defined separately. There are strong reasons for including depth information beyond the best prices. Firstly, evidence suggests that there is information content in the limit order book beyond the best prices. Cao et al. (2009) find that order book information behind the market is moderately informative of price discovery based on the Hasbrouck information share measure (Hasbrouck, 1995). Secondly, a trader's intention may be to execute a large order that is unable to be filled based on visible liquidity at the best prices. The volume of standing limit orders behind the best prices will influence execution strategy as it determines the cost of immediate execution. $v_t^{b,1}$ is defined as the log depth available at the first occupied bid price level (L1) and $v_t^{b,25}$ is defined as the log cumulative depth from the second to the fifth price level (L2-5) of the limit order book. $v_t^{b,1}$ has a natural interpretation as the volume of trading necessary to move the price by at least one price level while the sum of $v_t^{b,1}$ and $v_t^{b,25}$ represents the volume of trading required to move the price by at least five price levels[3]. Separate variables are defined to measure volume on the bid and ask side of the limit order book. Hautsch and Huang (2012) consider price impacts on liquid assets where price gaps, defined as levels in the limit order book with no volume can be safely assumed not to occur. However, this study examines a representative sample of small capitalization stocks where price gaps are observed more often. Our definition of depth behind the market based on the aggregated volume of limit

[3]Hautsch and Huang (2012) finds that the most significant price effects are for limit orders submitted up to the third price level. Defining $v_t^{b,25}$ to the fifth level of the limit order book should suffice as an adequate representation of order book depth behind the market.

orders at a fixed price distance away from the best prices implicitly controls for price gaps that occur in the limit order book.

This paper follows Hautsch and Huang (2012) in modelling quotes and depths in logarithms[4]. Volume variables are characterised by occassional spikes and the logarithmic transformation reduces the impact large volumes may have on the estimation and allows an interpretation of the coefficients as elasticities.

Lastly, the duration variable is endogenized to recognize the important role of time in modeling the joint dynamics of quotes and depths. It is defined as the time elapsed between order book events[5]. The informational role of time between trades has been well established. Under the Easley and O'Hara (1992) model, the time between trades indicates the likelihood of an information event and the subsequent presence of informed trading. The likelihood of informed trading not only affects quotes but also the willingness to supply liquidity. Easley and O'Hara (1992) make an equally important prediction that spreads decrease as the time between trades increases. The role of time is empirically validated by Dufour and Engle (2000). Endogenizing duration plays another important role in our model in facilitating translation from order event time to wall clock time.

3.2. Model Specification

We consider a high frequency cointegrated VAR that jointly models bid and ask quotes and limit order book depths. The model choice closely resembles that of Hautsch and Huang (2012). The cointegrated VAR in VEC form is given by

$$\Delta x_t = \mu + \alpha\beta' x_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta x_{t-i} + D_0 z_t + \epsilon_t \tag{2}$$

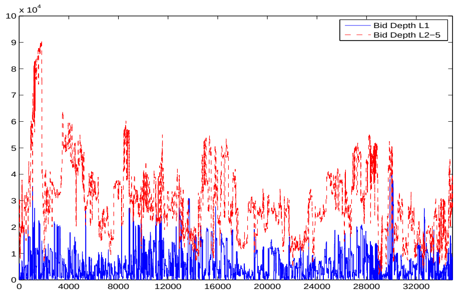
where z_t is a vector containing a set of diurnal dummies that controls for intraday periodicities in the data series. We divide the trading day into six intervals, the first representing order events from 10:15am to 11:00am, then one for every hour till 3:00pm and the last from 3:00pm to 3:45pm.

Hautsch and Huang (2012) argue that order book depth is an inventory variable that has potentially non-stationary properties at high frequencies. They estimate a cointegrated VAR allowing for nonstationarity of order book depth and find evidence of cointegration relationships between spreads and depths[6]. In this study, order book depth variables are not defined in the same manner to Hautsch and Huang (2012). Depth behind the market is summarized into a single variable for the bid and the ask side. Some general patterns emerge by examining the time series and sample autocorrelation functions of our order book depth variables. Figure 1 provides these time series for a single stock BHP on a representative trading day. There are strong positive comovements between order book volume levels

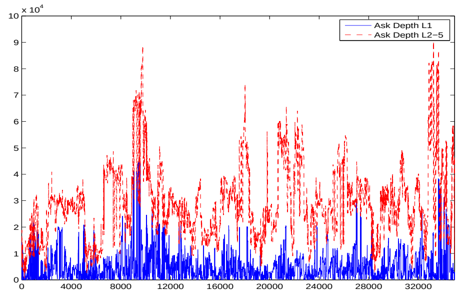
[4]All depth variables are scaled by 1,000 before taking logs.

[5]Duration is measured in seconds with precision to the nearest millisecond.

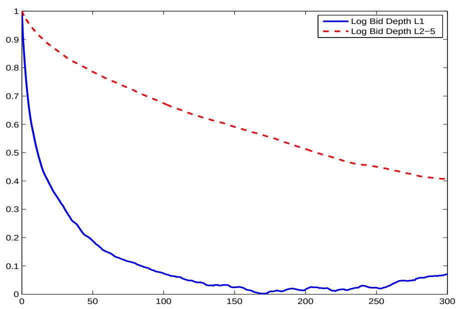
[6]Hautsch and Huang (2012) estimate (2) placing only stationarity restrictions on the buy and sell trade dummies.



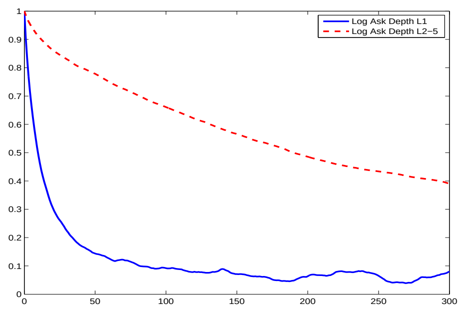
(a) Time Series of Bid Depths



(b) Time Series of Ask Depths



(c) Autocorrelation functions of Bid Depths



(d) Autocorrelation functions of Ask Depths

Figure 1: Plots of Order Book Depth
Time series and autocorrelation function plots of order book depth for BHP Billiton (BHP) on Monday 07 Sep 2009. The solid line displays L1 depth and the dotted line displays cumulative L2-5 depth.

at the best prices and volume levels behind the market. The sample autocorrelations of our v_t variables indicates a persistent but potentially stationary process with a lower level of persistence observed for order book depth at the best prices relative to depth behind the market[7]. This is an intuitive result as depth at the best prices inherits an additional source of variability from trading. The stationary properties of each endogenous variable are examined in Table 4 which reports the results of the Augmented Dickey-Fuller (ADF) test conducted on our full sample of stocks for each of the 65 trading days. The results are consistent with the generally accepted view that the log bid and ask price series (p_t^b and p_t^a) are non-stationary processes but the spread defined as $s_t = logP_t^a - logP_t^b$ (Engle and Patton, 2004) is a stationary process. There is strong support of stationarity in the other remaining variables. In particular, the $v_t^{b,25}$ and $v_t^{a,25}$ series representing depth beyond the best prices reject the null hypothesis of a unit root on over 95% of daily samples for mid-capitalization stocks and on over 75% of samples for small capitalization stocks. Overall, the results suggest an adequate specification of the cointegrating matrix β is given by

$$\beta = \begin{pmatrix} 1 & 0 & \dots & \dots & 0 \\ -1 & 0 & \dots & \dots & 0 \\ 0 & 1 & 0 & \dots & \vdots \\ \vdots & \vdots & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \vdots \\ 0 & \dots & \dots & \dots & 1 \end{pmatrix}$$

where the first column represents the log spread and the other co-integrating vectors account for the remaining $I(0)$ variables.

The model is recast to represent the dynamics in spreads and mid-quote returns. This representation is appealing as the spread is explicitly modeled and the interactions between the spread and depth can be directly observed.

The following rotation matrix R given by:

$$R = \begin{pmatrix} 1 & -1 & 0 & \dots & 0 \\ 0.5 & 0.5 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & & & \ddots & \\ 0 & \dots & & \dots & 1 \end{pmatrix}$$

applied to Equation (2) achieves the transformation from changes in the bid and ask quotes into changes in the spread and mid-quote returns.

$$R\Delta x_t = R\mu + R\alpha\beta' x_{t-1} + \sum_{i=1}^{p-1} R\Gamma_i R^{-1} R\Delta x_{t-i} + RD_0 z_t + R\epsilon_t \tag{3}$$

[7]The rate of decay of the ACF for L2-5 depth indicates that this could alternatively be modelled as a fractionally integrated process. However, at the time of writing the properties of fractionally integrated processes have not been fully developed and is left for future research (Johansen and Nielsen (2012)).

Table 4: Stationarity Tests on Endogenous Variables

Augmented Dickey-Fuller (ADF) tests were conducted on the sample of 30 selected stocks for each of the 65 trading days. Rejections of the null hypothesis of a unit root provides evidence of stationarity. The chosen lag length for the test is 30. The table reports both the number of rejections and the proportion of rejections at the 1% significance level. A minimum sample size of 500 per trading day was set for conducting the ADF test. This resulted in 628 ADF tests conducted for small capitalization stocks out of a total of 650 samples.

	Large Cap Stocks		Mid Cap Stocks		Small Cap Stocks	
Variable	Number	(%)	Number	(%)	Number	(%)
p_t^a	211	32.5%	303	46.6%	285	45.4%
p_t^b	214	32.9%	294	45.2%	277	44.1%
s_t	650	100.0%	649	99.8%	606	96.5%
Δq_t	650	100.0%	650	100.0%	628	100.0%
x_t^b	650	100.0%	650	100.0%	626	99.7%
x_s^b	650	100.0%	650	100.0%	627	99.8%
$v_t^{b,1}$	650	100.0%	649	99.8%	612	97.5%
$v_t^{b,25}$	650	100.0%	620	95.4%	494	78.7%
$v_t^{a,1}$	650	100.0%	650	100.0%	615	97.9%
$v_t^{a,25}$	650	100.0%	625	96.2%	483	76.9%
d_t	650	100.0%	650	100.0%	623	99.2%

With further manipulation, this can be re-specified as:

$$y_t = A_0 + \sum_{i=1}^{p} A_i y_{t-i} + B_0 z_t + u_t \qquad (4)$$

where $y_t = \{s_t, \Delta q_t, x_t^b, x_t^s, v_t^{b,1}, v_t^{b,25}, v_t^{a,1}, v_t^{a,25}, d_t\}'$ and A_p is a q x q matrix having a second column of zeros. This is a stationary VAR(p) system with coefficient restrictions which can be estimated equation by equation without loss of efficiency.

Estimation was conducted for all thirty stocks. The large number of observations in our data permits a separate estimation of Equation (4) for each Monday to Friday trading week from September to November 2009 resulting in a total of 360 separate VAR models[8]. To control for the absence of trading during overnight periods, a set of pre-sample values are determined on every trading day and incorporated into the estimation. Separate estimation for each trading week provides a balance between retaining a sufficient number of observations while addressing possible structural instabilities in the liquidity dynamics over time[9].

The lag length chosen for the estimation depended upon the market capitalization of the stock. Larger stocks tended to have higher quoting activity and slower decaying ACF profiles necessitating a higher lag order to capture the underlying dynamics. Ten lags for

[8]There are 12 Monday to Friday trading weeks or 60 trading days from September to November 2009 resulting in a total of 30 x 12 = 360 separate VAR estimations.

[9]Hautsch and Huang (2012) report estimates on each trading day as a strategy to reduce the impact of possible structural breaks on their results.

each endogenous variable ($p = 10$) was chosen for large capitalization stocks while eight lags ($p = 8$) and six lags ($p = 6$) were chosen for mid and small capitalization stocks respectively. In determining the appropriate lag length, the choice was guided by both residual diagnostic tests and information criteria. At the chosen lags, Ljung-Box serial correlation tests reported almost no remaining serial correlation across all estimations. The findings presented below are robust to the choice of lag order in the VAR specification.

3.3. Estimation Results

In addition to facilitating the construction of impulse response functions for measuring resiliency, the results presented in this section from estimating Equation (4) capture a rich set of liquidity dynamics and interactions between different liquidity variables. For reasons of brevity, this section reports the cross-sectional summary statistics of individual model estimation results. Tables 5 to 8 present results for each co-efficient group, defined as the co-efficient on all lags of a variable in each equation of the VAR specification. For each co-efficient group, the median sum of co-efficient values and the number and proportion of co-efficient groups that are positive (negative) and significantly different from zero. Using a White heteroskedasticity consistent covariance estimator, joint Wald tests are conducted to examine significance on the entire variable group.

Table 5 reports summary statistics of the estimation results on the bid depth equation. The results provide evidence of significant 'liquidity clustering' effects documented in Coppejans et al. (2004). This effect is consistent with Admati and Pfleiderer (1988) that uninformed traders act strategically in timing their trades in during high liquidity periods, but is also consistent with information effects of Biais et al. (1995). Liquidity declines in times of high information asymmetry as traders cancel their orders to avoid adverse selection. This can lead to both a reduction in limit order book depth and a widening of the bid-ask spread. Panel A of Table 5 shows that best bid depth reacts to both best ask depth, bid depth beyond the best prices and the log spread. In contrast, Panel B in Table shows that the dynamics of order book depth behind the market reacts only to depth at the best prices on the same side of the limit order book.

Analogous estimation results on the ask side depth equation is presented in Table 6. A deterioration in order book depth at the best prices can have a detrimental effect on the depth of the entire limit order book while a deterioration in depth behind the best prices has a lesser impact, affecting only the depth on the same side of the limit order book.

Table 5 and 6 established a negative association between order book depth and past values of the log spread. The estimation results on the spread equation reported in Table 7 indicates the existence of a two way Granger causality relationship between these two variables. The spread also responds negatively to decreased depth on both sides of the limit order book and this relationship is relatively stronger for depth at the best prices than depth behind the best prices. This is consistent with the predictions of Kyle (1985) and Glosten and Milgrom (1985) that times of greater information asymmetry leads to both wider spreads and lower depths.

Table 8 highlights the existence of significant relations involving the duration between order book events. Firstly, the duration between order events decreases with the presence of

Table 5: Estimation Results: Bid Market Depth

The table presents a summary of the VAR estimation results on $v_t^{b,1}$ and $v_t^{b,25}$ for each firm-trading week. Panel A reports the estimation results of the L1 bid depth equation ($v_t^{b,1}$). Panel B reports the estimation results of L2-5 bid depth equation ($v_t^{b,25}$). Column 2 reports the median sum of coefficient estimates across all firm-trading weeks. Column 3 (5) reports the number of firm-trading weeks with positive (negative) and significant joint Wald test statistics for the entire variable group. Column 4 (6) reports the percentage of firm-trading weeks that are positive (negative) and significant. There are a total of 30 x 12 = 360 firm-trading weeks in our sample. The significance level chosen is 1%.

Panel A: $v_t^{b,1}$ equation

Variable	Median Sum of Coeffs	# Pos and Sig	% Pos and Sig	# Neg and Sig	% Neg and Sig
s_t	-80.97710	1	0.3%	328	91.1%
Δq_t	-0.86399	33	9.2%	71	19.7%
x_t^b	-0.06074	4	1.1%	114	31.7%
x_t^s	-0.01409	17	4.7%	51	14.2%
$v_t^{b,1}$	0.90372	360	100.0%	0	0.0%
$v_t^{b,25}$	0.01998	221	61.4%	6	1.7%
$v_t^{a,1}$	0.02127	292	81.1%	0	0.0%
$v_t^{a,25}$	0.00566	87	24.2%	8	2.2%
d_t	0.00182	65	18.1%	16	4.4%

Panel B: $v_t^{b,25}$ equation

Variable	Median Sum of Coeffs	# Pos and Sig	% Pos and Sig	# Neg and Sig	% Neg and Sig
s_t	-1.34920	56	15.6%	98	27.2%
Δq_t	0.00993	40	11.1%	22	6.1%
x_t^b	0.00475	29	8.1%	4	1.1%
x_t^s	0.00661	46	12.8%	9	2.5%
$v_t^{b,1}$	0.00274	205	56.9%	7	1.9%
$v_t^{b,25}$	0.97712	360	100.0%	0	0.0%
$v_t^{a,1}$	0.00051	69	19.2%	26	7.2%
$v_t^{a,25}$	0.00037	16	4.4%	18	5.0%
d_t	0.00010	16	4.4%	9	2.5%

Table 6: Estimation Results: Ask Market Depth

The table presents a summary of the VAR estimation results on $v_t^{a,1}$ and $v_t^{a,25}$ for each firm-trading week. Panel A reports the estimation results of the L1 ask depth equation ($v_t^{a,1}$). Panel B reports the estimation results of L2-5 ask depth equation ($v_t^{a,25}$). Column 2 reports the median sum of coefficient estimates across all firm-trading weeks. Column 3 (5) reports the number of firm-trading weeks with positive (negative) and significant joint Wald test statistics for the entire variable group Column 4 (6) reports the percentage of firm-trading weeks that are positive (negative) and significant. There are a total of 30 x 12 = 360 firm-trading weeks in our sample. The significance level chosen is 1%.

Panel A: $v_t^{a,1}$ equation

Variable	Median Sum of Coeffs	# Pos and Sig	% Pos and Sig	# Neg and Sig	% Neg and Sig
s_t	-83.65490	2	0.6%	328	91.1%
Δq_t	0.76446	83	23.1%	35	9.7%
x_t^b	-0.01893	12	3.3%	48	13.3%
x_t^s	-0.06731	8	2.2%	120	33.3%
$v_t^{b,1}$	0.02152	303	84.2%	1	0.3%
$v_t^{b,25}$	0.00461	75	20.8%	12	3.3%
$v_t^{a,1}$	0.90607	360	100.0%	0	0.0%
$v_t^{a,25}$	0.01943	228	63.3%	3	0.8%
d_t	0.00268	80	22.2%	5	1.4%

Panel B: $v_t^{a,25}$ equation

Variable	Median Sum of Coeffs	# Pos and Sig	% Pos and Sig	# Neg and Sig	% Neg and Sig
s_t	-1.54550	52	14.4%	91	25.3%
Δq_t	-0.03196	27	7.5%	40	11.1%
x_t^b	0.00517	29	8.1%	6	1.7%
x_t^s	0.00939	47	13.1%	3	0.8%
$v_t^{b,1}$	0.00050	70	19.4%	28	7.8%
$v_t^{b,25}$	0.00044	28	7.8%	14	3.9%
$v_t^{a,1}$	0.00329	228	63.3%	8	2.2%
$v_t^{a,25}$	0.97620	360	100.0%	0	0.0%
d_t	0.00026	21	5.8%	2	0.6%

Table 7: Estimation Results: Spread

The table presents a summary of the VAR estimation results on s_t for each firm-trading week. Column 2 reports the median sum of coefficient estimates across all firm-trading weeks. Column 3 (5) reports the number of firm-trading weeks with positive (negative) and significant joint Wald test statistics for the entire variable group. Columns 4 (6) reports the percentage of firm-trading weeks that are positive (negative) and significant. There are a total of 30 x 12 = 360 firm-trading weeks in our sample. The significance level chosen is 1%.

Variable	Median Sum of Coeffs	# Pos and Sig	% Pos and Sig	# Neg and Sig	% Neg and Sig
s_t	0.91140	360	100.0%	0	0.0%
Δq_t	0.00001	11	3.1%	12	3.3%
x_t^b	0.00003	168	46.7%	5	1.4%
x_t^s	0.00003	173	48.1%	1	0.3%
$v_t^{b,1}$	-0.00001	0	0.0%	313	86.9%
$v_t^{b,25}$	0.00000	18	5.0%	123	34.2%
$v_t^{a,1}$	-0.00001	0	0.0%	322	89.4%
$v_t^{a,25}$	-3.53E-06	7	1.9%	146	40.6%
d_t	-7.40E-07	24	6.7%	65	18.1%

Table 8: Estimation Results: Duration between Order Events

The table presents a summary of the VAR estimation results on d_t for each firm-trading week. Column 2 reports the median sum of coefficient estimates across all firm-trading weeks. Column 3 (5) reports the number of firm-trading weeks with positive (negative) and significant joint Wald test statistics for the entire variable group. Column 4 (6) reports the percentage of firm-trading weeks that are positive (negative) and significant. There are a total of 30 x 12 = 360 firm-trading weeks in our sample. The significance level chosen is 1%.

Variable	Median Sum of Coeffs	# Pos and Sig	% Pos and Sig	# Neg and Sig	% Neg and Sig
s_t	-113.16950	13	3.6%	301	83.6%
Δq_t	-0.49495	41	11.4%	64	17.8%
x_t^b	-1.19630	4	1.1%	348	96.7%
x_t^s	-1.29720	0	0.0%	359	99.7%
$v_t^{b,1}$	0.04743	316	87.8%	7	1.9%
$v_t^{b,25}$	0.01643	136	37.8%	68	18.9%
$v_t^{a,1}$	0.05627	330	91.7%	2	0.6%
$v_t^{a,25}$	0.02094	160	44.4%	51	14.2%
d_t	0.49564	360	100.0%	0	0.0%

trading relative to other order events. The increased intensity of limit order activities reflects traders reacting to the information content inferred from the observed trade. Secondly, the duration between order events is positively related to existing depth in the limit order book, particularly at the best prices. A low level of existing order book liquidity tends to increase the intensity of limit order activities. This is consistent with the existence of an order book resilience effect where a depleted limit order book entices liquidity provision through the submission of new limit orders (Biais et al., 1995; Hedvall and Niemeyer, 1996; Degryse et al., 2005).

4. Measuring Resiliency

To quantify resiliency, we examine the effect of liquidity shocks on y_t by the following impulse response function:

$$I(h;\delta) \quad = \quad E[y_{t+h}|y_t + \delta_y, y_{t-1}, ...] - E[y_{t+h}|y_t, y_{t-1}, ...] \tag{5}$$

where the shock vector δ_y measures the change in the values of y_t from the occurrence of a liquidity shock and h is the number of future time steps. Hence, $I(h;\delta)$ measures the expected shift in the values of y_t from a liquidity shock captured by the VAR model.

4.1. Identifying Liquidity Shocks

The application of Equation (5) requires a set of liquidity shocks to be defined and the pre-shock state of the system to be initialized. For each liquidity shock, all continuous variables in y_t are initialized to its long run equilibrium value. The buy-sell trade indicator variables are set to zero, simulating a quiet period in which no trading is observed. We consider five scenarios representing liquidity shocks commonly observed by market participants in the limit order book. With little loss of generality, the liquidity shocks defined are negative shocks that result in a withdrawal of visible liquidity on the bid side of the limit order book. Analogous scenarios could equally be defined on the ask side. Recognising the high proportion of quote to trade activity in the market, liquidity shocks are not restricted to the occurrence of large trades (Large, 2007) but are also represented by order cancellations.

1. Market Order (MO): Arrival of a sell market order that reduces the volume of waiting limit orders at the best bid by one half.
2. Order Cancellation (OC): Arrival of an instruction to cancel an existing limit order at the best bid reducing the volume of waiting limit orders at the best bid by one half.
3. Aggressive Market Order (AMO): Arrival of a sell market order with volume exactly equal to the volume of waiting limit orders at the best bid. This scenario represents a trader who monitors the limit order book and chooses to limit the size of their market order to the quantity available at the best bid. By definition, this removes all the L1 bid depth and increases the bid-ask spread. To determine the new level of the spread s_t, the current best bid price p_t^b is initialized to its average value over the estimation period. The current best ask p_t^a is then inferred based on p_t^b and the equilibrium value

of the spread s_t. As the market order eliminates all existing volume at the best bid, p_t^b is reduced by one price level and the spread s_t and Δq_t are re-computed at the new best bid. We denote s_t^* and Δq_t^* to be the re-computed values of the spread and change in mid-quote respectively. The construction of δy_t requires some additional information on the volume of the limit order book behind the best prices. For the purposes of illustration, we assume that the initial state of the limit order book is such that the Level 1 bid volume (best bid) is equivalent to the Level 2 bid volume and Level 5 bid volume is equivalent to Level 6 bid volume. In this case, the sell market order has no effect on the values of all depth variables.

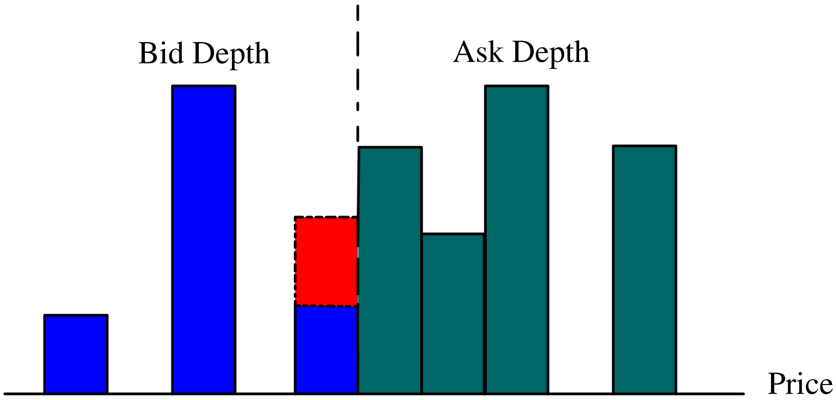
4. Aggressive Order Cancellation (AOC): Arrival of an instruction to cancel an existing limit order with volume equal to the volume of waiting limit orders at the best bid. This would occur if the cancelled limit order represented the only limit order at the best bid. The same assumptions and procedures are adopted as described in the scenario AMO.
5. Order Cancellation Behind The Market (OCBM): Arrival of an instruction to cancel an existing bid limit order that reduces the volume of waiting limit orders between L2 and L5 by one half. This would occur if the trader previously submitted a bid limit order with volume exactly equal to one half of the current cumulative volume between L2 and L5.

Table 9: Shock Vectors Representing Liquidity Shocks

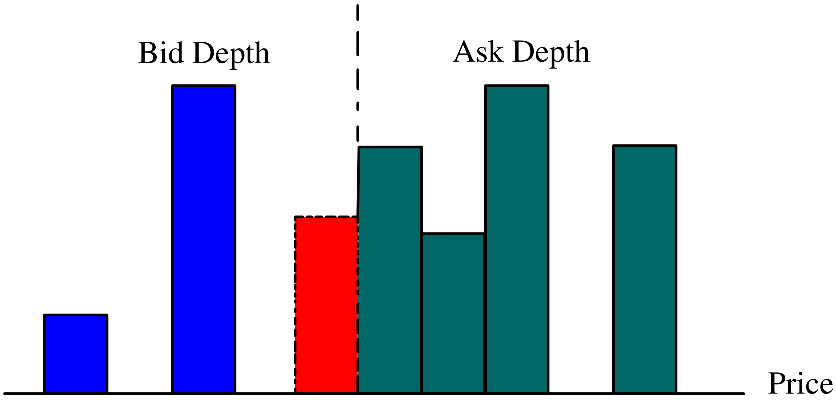
The table presents the shock vectors constructed to represent the five liquidity shock scenarios. All continuous variables are initialized to its long run equilibrium values. The buy-sell trade indicator variables are set to zero, simulating a quiet period in which no trading is observed.

δ_y	1 (MO)	2 (OC)	3 (AMO)	4 (AOC)	5 (OCBM)
s_t	0	0	$s_t^* - s_t$	$s_t^* - s_t$	0
Δq_t	0	0	$\Delta q_t^* - \Delta q_t$	$\Delta q_t^* - \Delta q_t$	0
x_t^b	0	0	0	0	0
x_t^s	1	0	1	0	0
$v_t^{b,1}$	-0.69	-0.69	0	0	0
$v_t^{b,25}$	0	0	0	0	-0.69
$v_t^{a,1}$	0	0	0	0	0
$v_t^{a,25}$	0	0	0	0	0
d_t	0	0	0	0	0

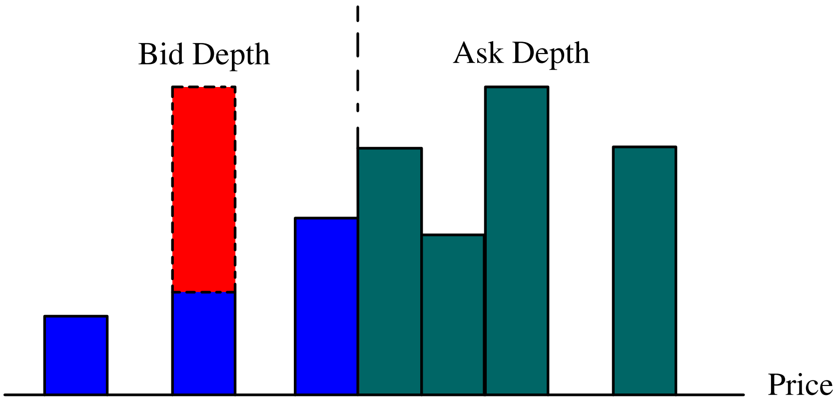
Table 9 summarizes the shock vectors representing each liquidity shock. The choice to model liquidity dynamics in order event time enables us to construct shock vectors δ_y for the above scenarios that capture all the contemporaneous relationships between the variables. Hautsch and Huang (2012) also estimate their co-integrated VAR model in order event time and construct a set of scenarios for estimating market impact. However, the scenarios differ from ours as their focus is on examining price effects from incoming limit orders.



(a) Liquidity Shock - Scenario MO and OC



(b) Liquidity Shock - Scenario AMO and AOC



(c) Liquidity Shock - Scenario OCBM

Figure 2: Illustration of Liquidity Shocks

The figures depict the effect of each liquidity shock on a hypothetical limit order book. Scenario MO and OC reduces the volume of limit orders at the best bid by one half. Scenarios AMO and AOC removes all the volume at the best bid and increases the bid-ask spread. Scenario OCBM reduces the cumulative L2-5 volume of bid limit orders by one half.

4.2. Impulse Responses

To compute the impulse responses, it is beneficial to consider the companion form for the VAR(p) process given in (4)

$$Y_t = \mu + AY_{t-1} + Bz_t + U_t \tag{6}$$

where

$$Y_t = \begin{pmatrix} y_t \\ y_{t-1} \\ \vdots \\ \vdots \\ y_{t-p+1} \end{pmatrix} \mu = \begin{pmatrix} A_0 \\ 0 \\ \vdots \\ \vdots \\ 0 \end{pmatrix} B = \begin{pmatrix} B_0 \\ 0 \\ \vdots \\ \vdots \\ 0 \end{pmatrix} U_t = \begin{pmatrix} u_t \\ 0 \\ \vdots \\ \vdots \\ 0 \end{pmatrix}$$

and

$$A = \begin{pmatrix} A_1 & \dots & \dots & A_{p-1} & A_p \\ I_k & 0 & \dots & \dots & 0 \\ \vdots & I_k & & & \vdots \\ \vdots & \vdots & & & \vdots \\ 0 & \dots & & I_k & 0 \end{pmatrix}$$

Express (6) as a vector moving average (VMA) process by repeated substitution of Y.

$$Y_t = M_t + \sum_{i=0}^{t-1} A^i U_{t-i} \tag{7}$$

where $M_t = \sum_{i=0}^{t-1} A^i\mu + A^t Y_0 + \sum_{i=0}^{t-1} A^i B z_{t-i}$ contains a deterministic trend, initial condition and the effect of the exogenous variables. Let $J = [I_K 0 \dots 0]$ be a selection matrix such that $JY_t = y_t$ and $U_t = J'u_t$.

$$y_t = JM_t + \sum_{i=0}^{t-1} JA^i J' u_{t-i} \tag{8}$$

The linear impulse-response function can be estimated by

$$\hat{f}(h;\delta) = J\hat{A}^h J'\delta_y \tag{9}$$

and the asymptotic distribution of the impulse response function follows from Lutkepohl (1990)

$$\sqrt{T}(\hat{f} - f) \to N(0, G_h \Sigma_\alpha G_h') \tag{10}$$

where $\alpha = vec(A_1, \ldots, A_p)$, $G_h = \partial vec(f)/\partial vec(A_1, \ldots, A_p)'$ and $\hat{\alpha}$ is a consistent estimator such that

$$\sqrt{T}(\hat{\alpha} - \alpha) \to N(0, \Sigma_\alpha). \tag{11}$$

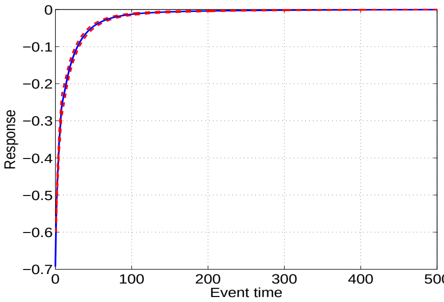
Confidence intervals are constructed using the diagonal elements of $G_h \hat{\Sigma}_\alpha G_h'$ and a White consistent estimator used for computing $\hat{\Sigma}_\alpha$. Due to the restrictions placed on the coefficients of lagged Δq_t in our specification, $\alpha = vec(A_1, \ldots, A_p^*)$ is actually estimated where A_p^* is A_p with the second column of zeros removed.

4.3. Responses to Liquidity Shocks

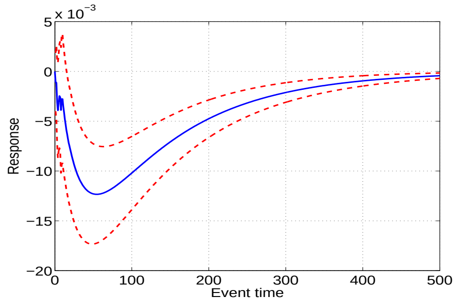
This section presents the impulse responses generated by the VAR specification for the set of liquidity shocks defined in Section 4.1. The impulse responses illustrates the dynamic effects and adjustment path to equilibrium of each variable. Space constraints prevent us from presenting the impulse responses for all stocks and estimation periods. To illustrate the effects of different liquidity shocks, impulse responses are presented for BHP Billiton (BHP) over the week beginning 07 September 2009. However, the impulse response profiles are remarkably consistent and the findings qualitatively similar across the whole sample of stocks. Results for other stocks and estimation periods are available on request.

The dynamic effects of a market sell order are displayed in Figure 3. The immediate effect is a reduction in depth available at the best bid which recovers quickly after the event. However, the impact of a liquidity shock also affects the ask side of the limit order book. Liquidity at the best ask deteriorates in response to a reduction in depth at the best bid which tends to occur quickly prior to recovery. These effects are consistent with the theoretical predictions of Parlour (1998) and Rosu (2009). Since the lower bid depth increases the execution probability of outstanding limit orders, traders are more likely to submit bid limit orders than market orders to achieve immediate execution. Hence, the impact of a liquidity shock affects both sides of the limit order book. Liquidity at the best ask deteriorates in response to a reduction in depth at the best bid. The deterioration in liquidity occurs quite rapidly, within 10 order events before recovering. On the other hand, traders who wish to sell are more willing to submit sell market orders with some traders responding to the reduced bid order depth by switching their standing limit orders to market orders. A deterioration in depth observed at the best prices also affects liquidity behind the market on the same side of the limit order book but to a significantly lesser degree.

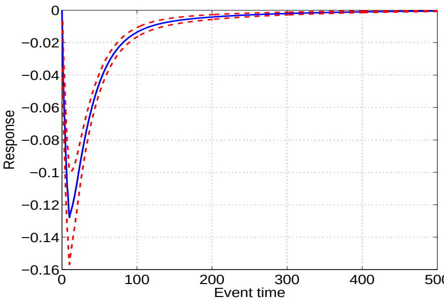
Given an order cancellation instruction defined in scenario OC has the same effect on the limit order book as scenario MO, does a market order have a different impact on future liquidity? Figure 4 displays the impulse response profiles of a bid order cancellation of the same volume as a sell market order. There are no noticeable differences observed in the rate of recovery between the two shocks with the same 'liquidity clustering' effects present and liquidity recovering just as quickly. This is a somewhat surprising finding as one may expect the perception of prevailing information asymmetry associated with trading to be greater than from order cancellations. The most noticeable differences actually occur in the trade and duration variables. Market orders significantly increase quote activity relative to order cancellations. This is consistent with two effects. First, traders respond to the occurrence



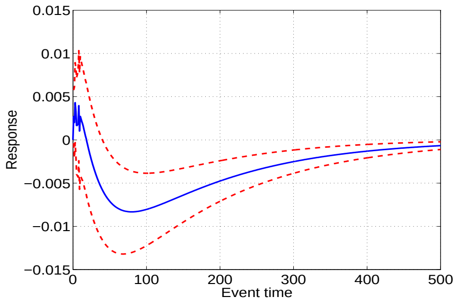
(a) Response of L1 Bid Depth



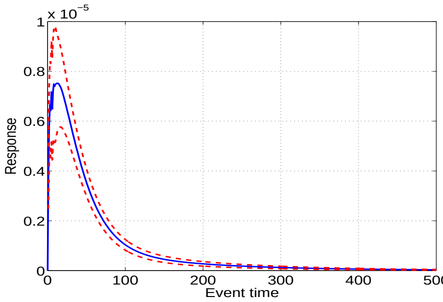
(b) Response of cumulative L2-5 Bid Depth



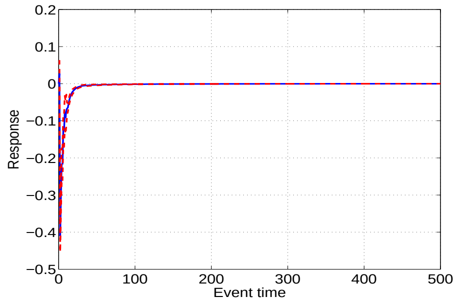
(c) Response of L1 Ask Depth



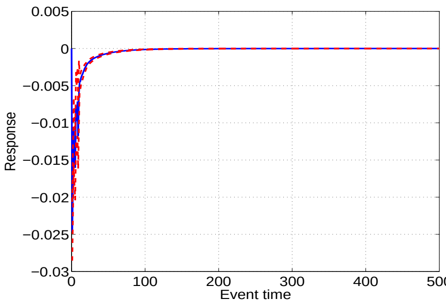
(d) Response of cumulative L2-5 Ask Depth



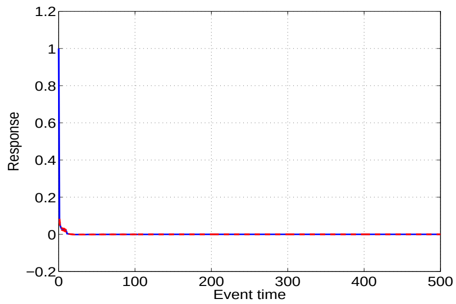
(e) Response of Log Spread



(f) Response of Log Duration



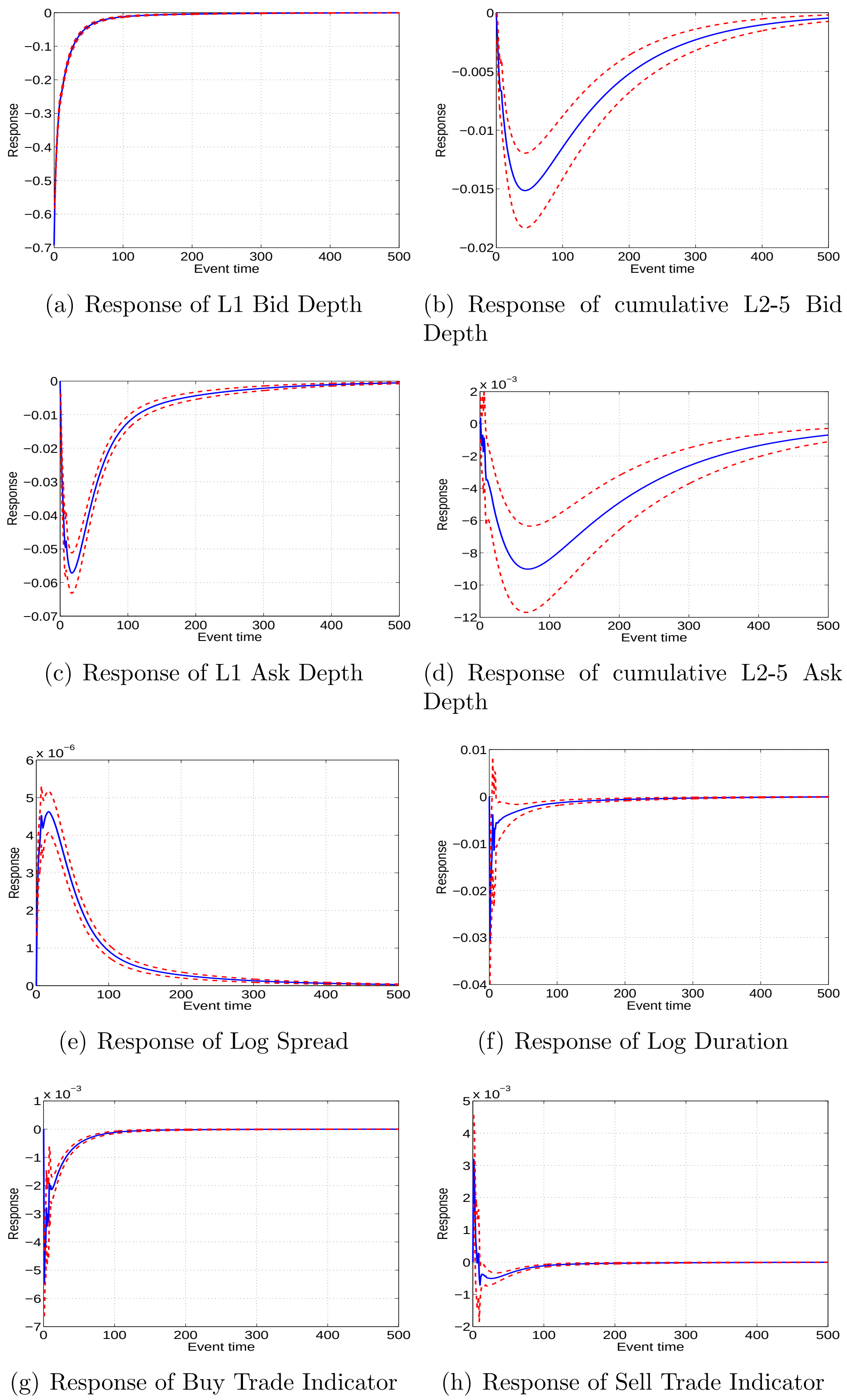
(g) Response of Buy Trade Indicator



(h) Response of Sell Trade Indicator

Figure 3: Response to a Liquidity Shock - Scenario MO

This figure presents the impulse responses resulting from the arrival of a sell market order that reduces the volume of waiting limit orders at the best bid by one half. Dashed lines represent 95% confidence intervals. The impulse responses presented are for BHP estimated over the trading week beginning Monday 07 September 2009.



(a) Response of L1 Bid Depth

(b) Response of cumulative L2-5 Bid Depth

(c) Response of L1 Ask Depth

(d) Response of cumulative L2-5 Ask Depth

(e) Response of Log Spread

(f) Response of Log Duration

(g) Response of Buy Trade Indicator

(h) Response of Sell Trade Indicator

Figure 4: Response to a Liquidity Shock - Scenario OC

This figure presents the impulse responses resulting from the arrival of a cancellation instruction on an existing buy limit order at the best bid reducing the volume of waiting limit orders at the best bid by one half. Dashed lines represent 95% confidenct intervals. The impulse responses presented are for BHP estimated over the trading week beginning Monday 07 September 2009.

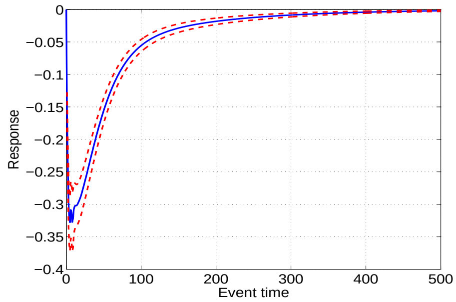
of trading by modifying their orders. Secondly, autocorrelation is observed in trading and a sell market order increases the arrival probability of another sell market order relative to an order cancellation event.

The immediate effect on the limit order book from a liquidity shock defined by scenario AMO is an increase in the bid-ask spread. Figure 5 demonstrates the resiliency of the spread. An increase in the bid ask spread induces fresh price improving limit orders such that the effect dissipates substantially within 50 order book updates. Despite no direct impacts on order book depth, an aggressive market order has significant effects on the future provision of liquidity through the submission of limit orders. Firstly, depth at the best prices on both sides of the order book deteriorates in response to a widening of the spread. Secondly, a deterioration in order book depth behind the market is also observed but with less severity and a slower recovery rate. Hence, sell market orders that consume all liquidity at the best bid also affect liquidity well beyond the best prices. Thirdly, there is a significant increase in the frequency of order book updates in the immediate aftermath of an aggressive market order as traders respond by revising and withdrawing their orders. However, the deterioration in liquidity occurs rapidly for order book depth at the best prices. After 10 order events, L1 bid depth begins to recover and the time to recovery is comparatively slower than in scenario MO with the effect of the shock largely dissipated after 150 order events.

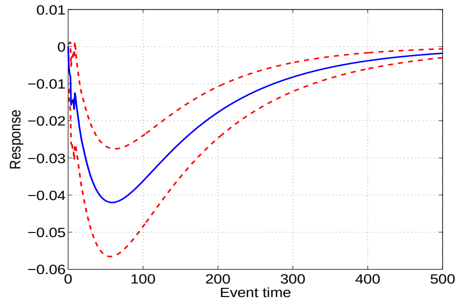
Figure 6 displays the impulse response profiles of an aggressive order cancellation and a comparison with Figure 5 highlights similarities in the rate of recovery of spreads and depths. Consistent with comparisons between scenarios 1 and 2, no evidence is found that the liquidity replenishment process differs between trading and order cancellation events. The most noticeable differences are again in the duration and trade indicator variables. A bid limit order cancellation that increases the bid ask spread reduces the likelihood of a sell trade due to the increased cost of immediate execution.

Figure 7 shows that limit order cancellations behind the market can have negative effects on depth at the best prices. Depth at the best prices on both sides of the limit order book deteriorate in response to the shock although a larger response is observed on the side experiencing the shock. The recovery profile of the impulse response for L2-5 bid depth indicates a larger number of order events is required for recovery to occur. A comparison of the impulse responses of $v_t^{b,25}$ for scenario OCBM relative to $v_t^{b,1}$ for Scenario MO provides evidence that depth recovers faster at the top of the book than behind the market.

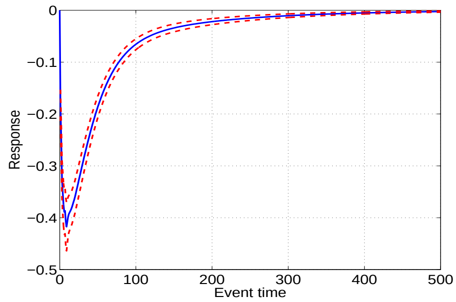
Figure 8 displays the cumulative price impact on the bid and ask side of the limit order book for different liquidity shocks. The impulse responses exhibit the same quantitative features as documented by Hasbrouck (1991), Engle and Patton (2004) and Hautsch and Huang (2012) Market orders have a higher price impact than cancellation of existing orders and the adjustment process occurs relatively quickly reaching its permanent level after approximately 40 lags. This confirms that traders do perceive market orders to carry greater private information but reflect this in the price levels at which they are willing to supply liquidity as opposed to a reluctance to provide liquidity after observing the shock.



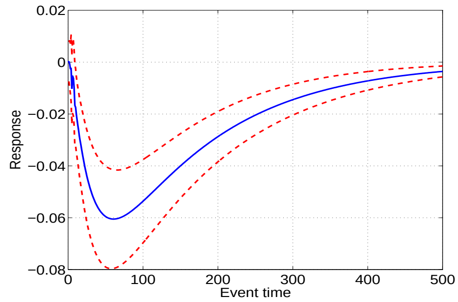
(a) Response of L1 Bid Depth



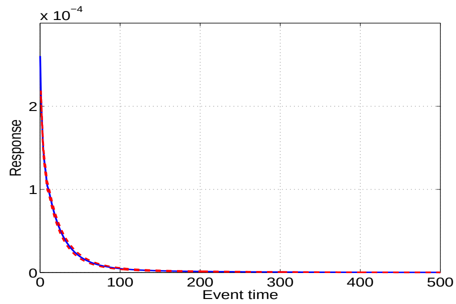
(b) Response of cumulative L2-5 Bid Depth



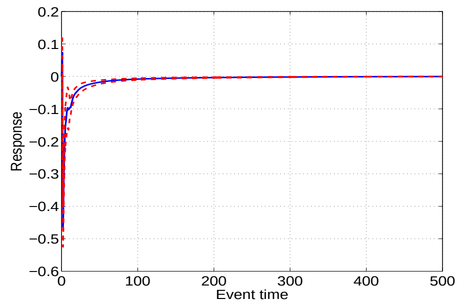
(c) Response of L1 Ask Depth



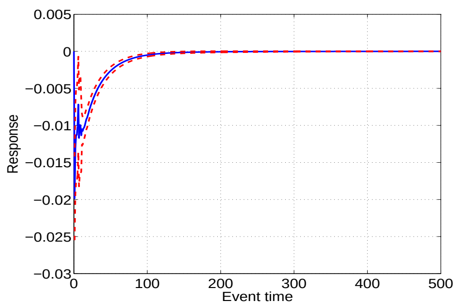
(d) Response of cumulative L2-5 Ask Depth



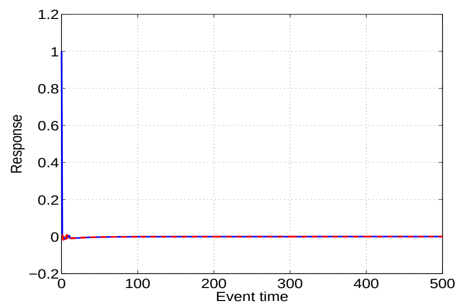
(e) Response of Log Spread



(f) Response of Log Duration



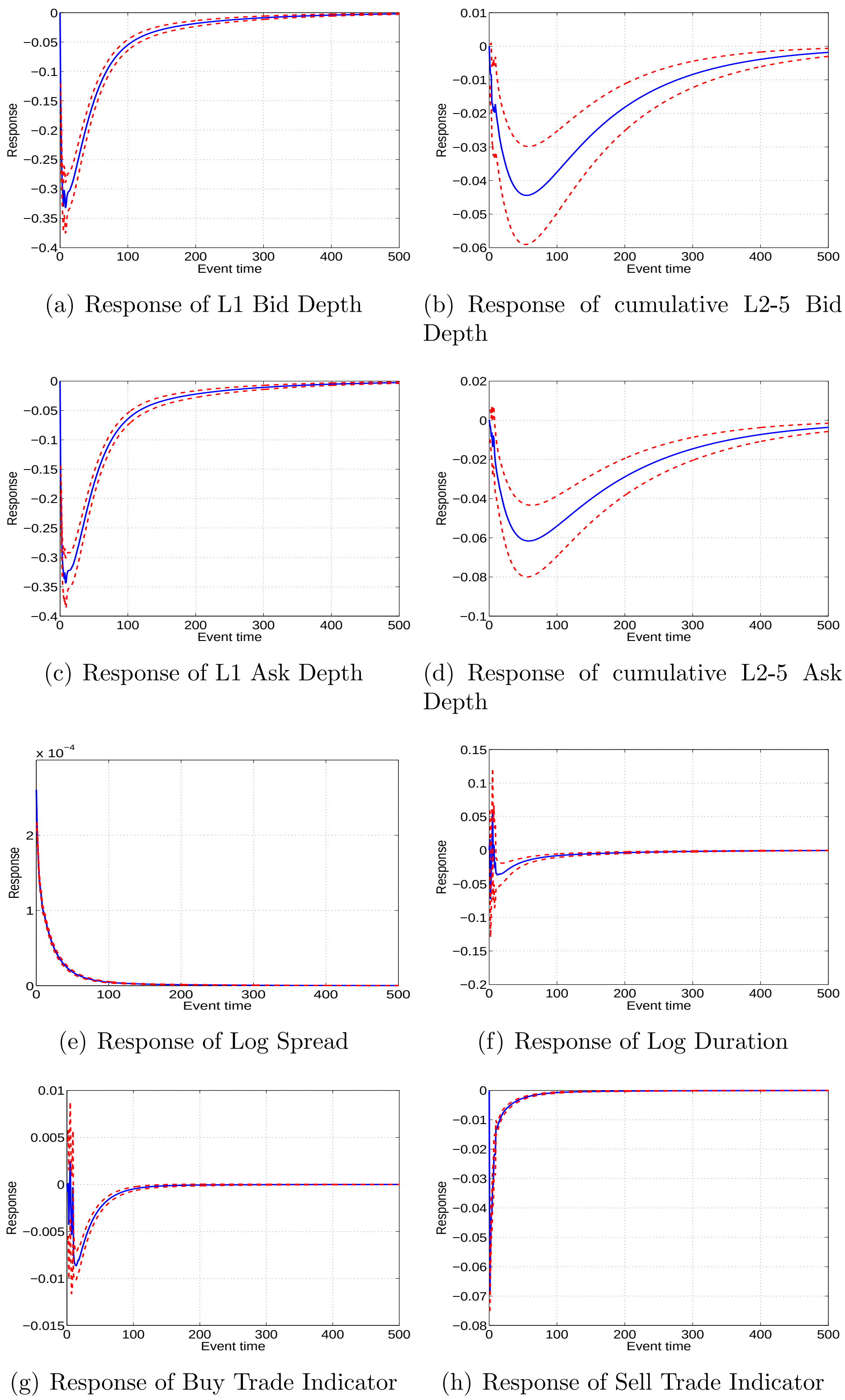
(g) Response of Buy Trade Indicator



(h) Response of Sell Trade Indicator

Figure 5: Response to a Liquidity Shock - Scenario AMO

This figure presents the impulse responses resulting from the arrival of an aggressive sell market order that precisely removes all the volume of waiting limit orders at the best bid. This increases the spread but the state of the limit order book is such that even though the market order 'shifts' the limit order book, the depth at L1 and the cumulative depth at L2-5 remains unchanged. Dashed lines represent 95% confidence intervals. The impulse responses presented are for BHP estimated over the trading week beginning Monday 07 September 2009.



(a) Response of L1 Bid Depth

(b) Response of cumulative L2-5 Bid Depth

(c) Response of L1 Ask Depth

(d) Response of cumulative L2-5 Ask Depth

(e) Response of Log Spread

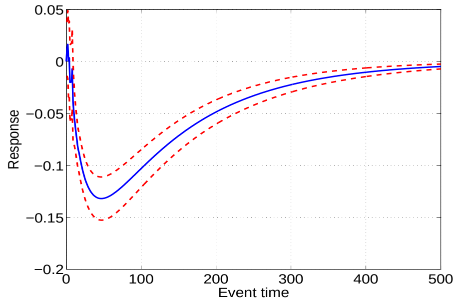
(f) Response of Log Duration

(g) Response of Buy Trade Indicator

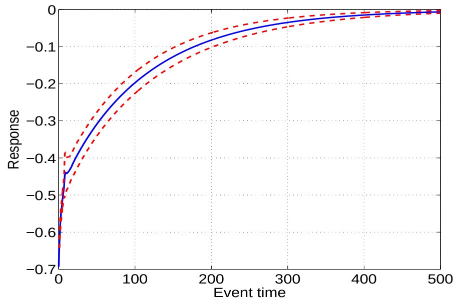
(h) Response of Sell Trade Indicator

Figure 6: Response to a Liquidity Shock - Scenario AOC

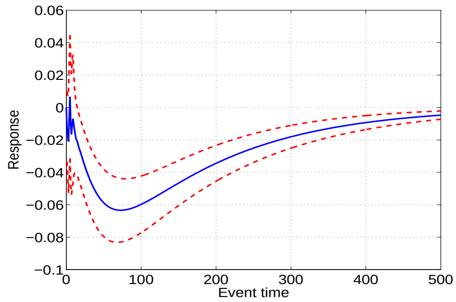
This figure presents the impulse responses resulting from the arrival of an aggressive cancellation instruction that precisely removes all the volume of waiting limit orders at the best bid. This increases the log spread but the state of the limit order book is such that even though the market order 'shifts' the limit order book, the depth at L1 and the cumulative depth at L2-5 remains unchanged. Dashed lines represent 95% confidence intervals. The impulse responses presented are for BHP estimated over the trading week beginning Monday 07 September 2009.



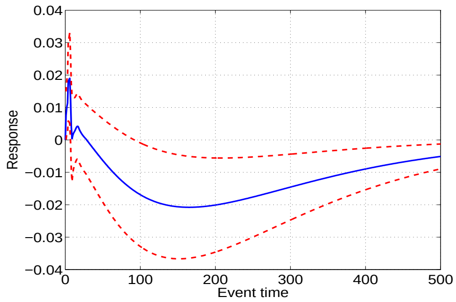
(a) Response of L1 Bid Depth



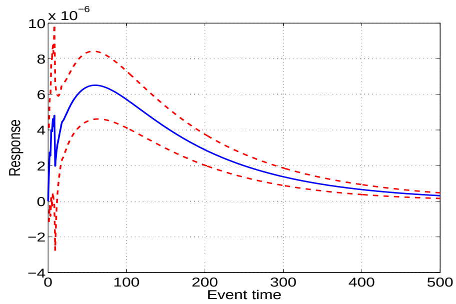
(b) Response of cumulative L2-5 Bid Depth



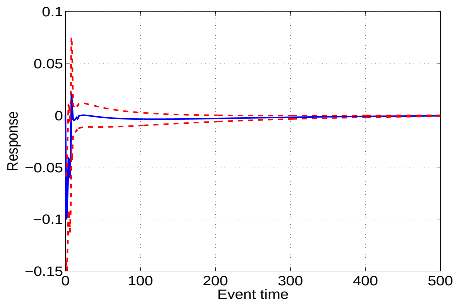
(c) Response of L1 Ask Depth



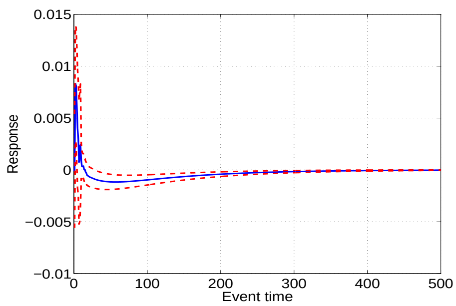
(d) Response of cumulative L2-5 Ask Depth



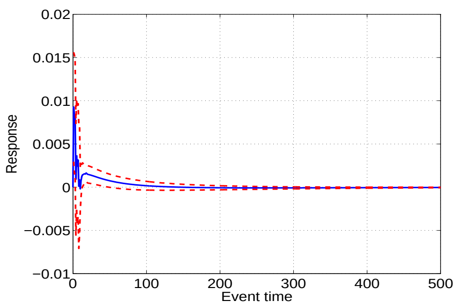
(e) Response of Log Spread



(f) Response of Log Duration



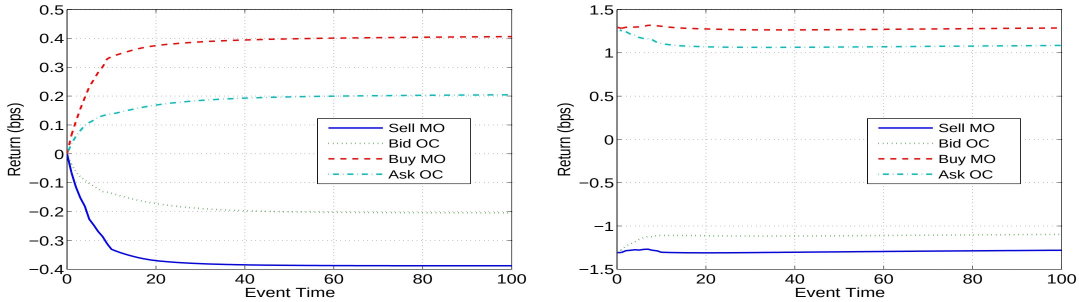
(g) Response of Buy Trade Indicator



(h) Response of Sell Trade Indicator

Figure 7: Response to a Liquidity Shock - Scenario OCBM

This figure presents the impulse responses resulting from the arrival of a cancellation instruction that reduces the volume of waiting limit orders between L2 and L5 by one half. Dashed lines represent 95% confidence intervals. The impulse responses presented are for BHP estimated over the trading week beginning Monday 07 September 2009.



(a) Scenario MO and OC

(b) Scenario AMO and AOC

Figure 8: Cumulative Price Impact of Trading vs Order Cancellations
This figure presents the cumulative price impact of market orders relative to order cancellations. The impulse responses presented are for BHP estimated over the trading week beginning Monday 07 September 2009.

4.4. Time to Recovery

The impulse responses illustrate the dynamic adjustment path of a liquidity shock with the number of limit order book events. While this may provide an appropriate cross-sectional comparison of the dynamic responses to shocks across stocks, practitioners are particularly interested in determining the speed at which a stock recovers from shocks in wall clock time. The speed of recovery measured in wall clock time is a function of the frequency of order book events which varies substantially between stocks. Dufour and Engle (2000) present an approach to translate impulse responses to wall clock time in their bivariate model of trades and quotes. Assuming strong exogeneity in the trade arrival process, the authors employ a Weibull Autoregressive Conditional Duration model to simulate the future stochastic path of time durations to shift from event time to wall clock time.

Resiliency is endogenized in our dynamic system and resiliency measured in wall-clock time can be estimated using the forecasts of the d_t variable in our VAR specification. However, forecasts obtained are on the log transformed durations and consideration is required on how to convert forecast of d_t into a raw duration forecast. Arino and Franses (2000) consider the issue of forecasting the levels of a log-transformed time series in a vector autoregressive system and provide expressions for the unbiased h-step ahead forecasts under a Gaussian VAR. However, a recent paper by Bardsen and Lutkepohl (2011) found that under a controlled simulation experiment, the unbiased forecast was inferior to the naive forecast obtained from simply applying the exponential transformation. In light of this finding, the naive forecast is used to recover the durations from the log transformed d_t variable.

Table 10 provides a broader picture of the resiliency on the Australian equity market. Half-life estimates and 90% recovery times are computed to summarize the resiliency of the liquidity variable directly impacted by each liquidity shock. The half-life is defined as the estimated number of time periods for the effect of a liquidity shock to be reduced by one half. The 90% recovery times are defined analogously but reflect the estimated time for the initial effect to have dissipated by at least 90%. Table 10 shows that the rate of recovery of liquidity shocks is high and robust across the sample of large capitalization stocks but vary

Table 10: Time to Recovery

The table summarizes the resiliency of each liquidity variable across all firm-trading weeks. Resiliency is measured as the time taken for the effect of a shock represented by each scenario to dissipate by a certain percentage (%). The chosen levels are 50% (commonly referred to as the half-life) and 90%. Estimates of the time taken are obtained from the estimated impulse response functions for each stock in our sample. There are two units of measurement. The half-life in order event time measures the number of order events observed from the time of the liquidity shock until the effect of the shock on the liquidity variable declines by one half. The half-life in wall-clock time is measured in seconds by applying the exponential transformation to forecasts of d_t from the VAR specification. For each variable, the minimum, maximum and median half-life is computed for the stocks in our sample.

Panel A: Half-life Estimates

			Order Event Time			Time in Seconds		
Scenario	Variable	Size Category	Min	Max	Median	Min	Max	Median
MO	$v_t^{b,1}$	Large	6	13	8	0.4	1.8	0.9
		Mid	5	34	10	0.5	38.3	2.3
		Small	2	25	9	0.5	52.0	5.3
OC	$v_t^{b,1}$	Large	6	9	7	0.5	1.7	0.9
		Mid	5	32	7	0.7	38.7	2.4
		Small	3	26	9	1.8	65.5	7.4
AMO	s_t	Large	7	31	14	0.5	3.6	1.7
		Mid	4	17	8	0.7	21.2	1.9
		Small	3	28	9	1.8	39.7	5.2
AOC	s_t	Large	6	25	13	0.6	3.4	1.8
		Mid	4	16	7	0.8	26.7	2.3
		Small	3	26	9	1.8	132.5	6.1
OCBM	$v_t^{b,25}$	Large	9	63	19	0.8	8.6	2.8
		Mid	10	724	59	1.7	857.2	19.0
		Small	5	547	44.5	3.1	695.7	34.7

Panel B: 90% Recovery Estimates

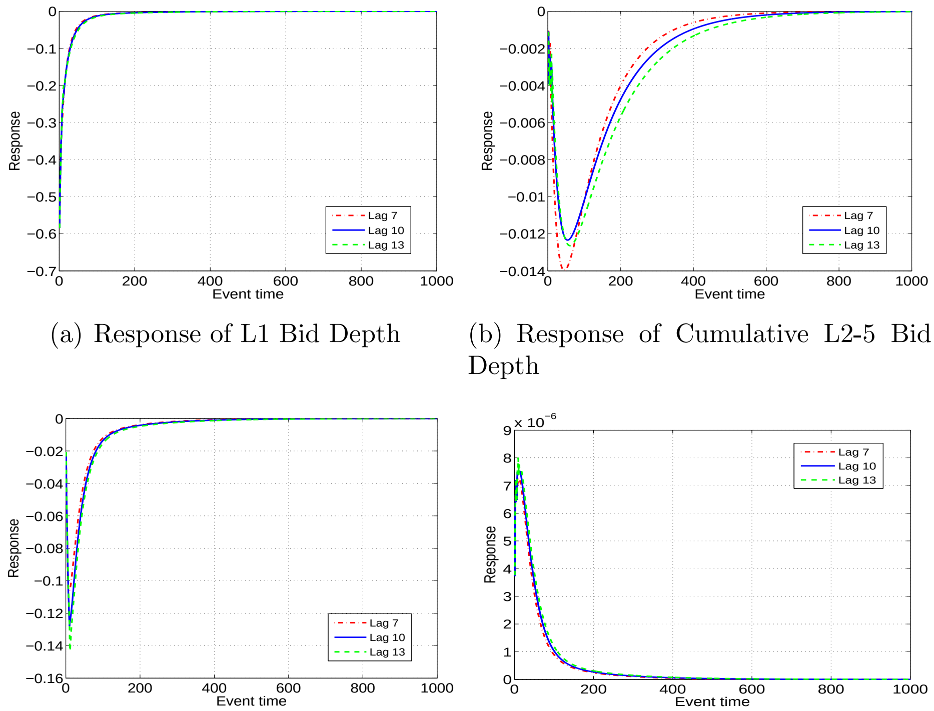
			Order Event Time			Time in Seconds		
Scenario	Variable	Size Category	Min	Max	Median	Min	Max	Median
MO	$v_t^{b,1}$	Large	30	62	44	2.5	11.0	5.2
		Mid	25	222	54	4.7	310.3	16.4
		Small	4	161	48.5	2.0	301.2	35.0
OC	$v_t^{b,1}$	Large	28	52	36.5	2.5	10.6	4.4
		Mid	23	221	46	4.1	297.1	14.8
		Small	12	146	45	7.9	308.8	36.3
AMO	s_t	Large	45	134	74	2.7	17.4	9.7
		Mid	20	80	40	4.9	125.5	10.8
		Small	11	126	37	8.2	178.1	24.6
AOC	s_t	Large	44	129	71	2.8	17.1	9.4
		Mid	20	80	39	5.3	128.7	11.3
		Small	10	109	37.5	8.2	194.3	25.9
OCBM	$v_t^{b,25}$	Large	52	340	107	5.3	49.4	13.9
		Mid	43	1000	254.5	7.5	2791.5.7	84.8
		Small	23	1000	192.5	12.4	2299.4	145.0

more significantly among smaller capitalization stocks. For large capitalization stocks, the median time for depth at the best bid to recover by one-half from a market sell order defined by Scenario MO is less than a second while the effect of the shock has dissipated by at least 90% after 5 seconds. For other liquidity shocks, the half-life estimates range between 1 and 3 seconds with no half-life estimates exceeding 9 seconds. For mid and small capitalization stocks, there is significantly more variability in the degree of resiliency. The median half life estimates from different liquidity shocks represented by all scenarios with the exception of OCBM range between 2 and 7 seconds while actual half-life estimates vary from less than a second to over 2 minutes. For order cancellations behind the market defined by scenario OCBM, significantly less resiliency is observed. Table 10 confirms the findings that while spreads and depth at the best prices have similar levels of resiliency, the recovery rate of depth behind the market occurs at a much slower pace. This is particularly prominent in smaller capitalization stocks where less activity is observed behind the market. When resiliency is measured in order event time, the recovery rates of the spread in scenarios AMO and AOC appear higher in smaller stocks relative to large capitalization stocks. This is one of the predictions of Foucault et al. (2005), that resiliency decreases in the order arrival rates. Smaller stocks tend to have lower arrival rates and limit order traders with a larger expected waiting time become more aggressive in their price improvements. On the whole, median half life estimates typically occur within 10 seconds and 90% recovery times within 40 seconds of a negative liquidity shock.

It is difficult to make direct comparisons of the level of resiliency with those documented in the empirical literature as liquidity shocks are defined differently. Coppejans et al. (2004) report on the Swedish stock index futures market that all of the impact occurs within the first ten minutes following the liquidity event. Hmaied et al. (2006) show for Tunisian stocks, the median time to recovery depends on the nature of the shock but ranges from 15 to 50 minutes. However, these studies use 10 and 15 minute sampling intervals respectively in their analysis. Large (2007) finds that for a London Stock Exchange stock, the order book replenishes reliably after a large trade only 40% of the time but if it does, it has a half life of around 20 seconds. Degryse et al. (2005) reports that on Euronext Paris, the bid-ask spread and depth return to pre-order levels after an aggressive order within 20 best limit order updates. In relation to these findings, the speed of order book replenishment appears to be faster than previously documented.

5. Robustness

To examine the robustness of our results to lag order misspecification in the VAR model, all estimations were repeated at different lag lengths from $p = 2$ to $p = 15$. At lower lags than the original lag p chosen for the estimation, a greater proportion of the estimated models contained significant serial correlation in the residuals indicating an underspecified VAR model. At higher lags than the original lag choice p, some of the estimated impulse responses had a distinct jagged appearance, evidence of potential overparameterization. For these alternative specifications, the dynamic relationships revealed in the estimated coefficients are present across all lag lengths and the profile of the impulse responses are remarkably similar. Figure 9 compares the estimated impulse response profiles from a liquidity shock defined by scenario 1 for our representative stock, BHP at two alternative lag lengths $p = 7$ and $p = 13$. Table 11 provides a comparison of the median half-life estimates across the entire sample against two alternative VAR specifications at three fewer and three greater lags than the original lag choice. The half-life estimates are robust to alternative lag order specifications and there is no indication of a lag truncation bias from underspecification.



(a) Response of L1 Bid Depth

(b) Response of Cumulative L2-5 Bid Depth

(c) Response of L1 Ask Depth

(d) Response of Log Spread

Figure 9: Robustness of Impulse Responses to Lag Order Mis-specification
This figure compares the impulse response profiles resulting from the arrival of a sell market order defined by Scenario MO under two alternative VAR specifications, $p = 5$ and $p = 15$. The impulse responses presented are for BHP estimated over the trading week beginning Monday 07 September 2009.

All the liquidity shocks defined in Section 4.1 affected the bid side of the limit order book. Table 12 reports the recovery estimates from the impulse responses generated by re-defining liqudity shocks to impact the ask side of the limit order book. The results did not reveal any qualitative differences from our main findings.

6. Concluding Remarks

Motivated by both the prevalence of order splitting strategies and concerns over the resiliency of market structures relying solely on endogenous liquidity provision, this study examines empirically the resiliency of a selection of ASX listed stocks. Resiliency is quantified by estimating a high frequency VAR specification to capture short term liquidity dynamics under a set of common liquidity shocks.

Table 11: Robustness of Half-life Estimates to Lag Order Mis-specification

The table presents a summary of half-life estimates computed at two alternative lag specifications. The two alternative lag specifications are three lags fewer and three lags greater than the original lag choice. Only the median half-life estimate is presented across stocks in the same size category. The half-life in order event time measures the number of order events observed from the time of the liquidity shock until the effect of the shock on the liquidity variable declines by one half. The half-life in wall-clock time is measured in seconds by applying the exponential transformation to forecasts of d_t from the VAR specification.

Scenario	Size Category	Order Event Time			Time In Seconds		
	Lag Adjustment	-3	0	3	-3	0	3
1	Large	10	8	8	0.9	0.8	0.8
	Mid	10	9.5	8	2.5	2.2	2.0
	Small	10	9	7	6.2	5.1	4.7
2	Large	7	7	7	0.9	0.9	0.8
	Mid	8.5	7	7	2.8	2.4	2.2
	Small	10	9	7	7.3	7.4	6.5
3	Large	14	14	13	1.7	1.7	1.6
	Mid	8	8	8	2.1	1.9	1.9
	Small	9	9	8	5.0	5.1	5.0
4	Large	13	13	11	1.9	1.8	1.7
	Mid	8	7	7	2.6	2.3	2.3
	Small	9	9	8	6.2	6.1	5.7
5	Large	20.5	19	18	2.9	2.8	2.4
	Mid	59	59	58	18.8	19.0	18.2
	Small	43	44.5	41.5	31.6	34.7	33.5

Table 12: Time to Recovery - Ask Side Shocks

The table summarizes the resiliency of each liquidity variable across all firm-trading weeks. Resiliency is measured as the time taken for the effect of a shock represented by each scenario to dissipate by a certain percentage (%). The chosen levels are 50% (commonly referred to as the half-life) and 90%. Estimates of the time taken are obtained from the estimated impulse response functions for each stock in our sample. There are two units of measurement. The half-life in order event time measures the number of order events observed from the time of the liquidity shock until the effect of the shock on the liquidity variable declines by one half. The half-life in wall-clock time is measured in seconds by applying the exponential transformation to forecasts of d_t from the VAR specification. For each variable, the minimum, maximum and median half-life is computed for the stocks in our sample.

Panel A: Half-life Estimates

			Order Event Time			Time in Seconds		
Scenario	Variable	Size Category	Min	Max	Median	Min	Max	Median
MO	$v_t^{b,1}$	Large	6	16	9	0.4	2.5	0.9
		Mid	5	26	10.5	0.6	19.0	2.5
		Small	2	36	10	0.5	44.9	5.1
OC	$v_t^{b,1}$	Large	6	9	7	0.5	1.7	0.9
		Mid	5	24	7	0.9	29.8	2.4
		Small	3	32	9	2.0	40.3	6.9
AMO	s_t	Large	8	30	14	0.5	3.7	1.7
		Mid	4	21	8	0.7	34.5	2.0
		Small	3	26	9	1.6	29.5	5.2
AOC	s_t	Large	7	23	13	0.5	3.2	1.7
		Mid	4	21	7	0.7	42.3	2.3
		Small	3	21	9	1.9	32.1	6.2
OCBM	$v_t^{b,25}$	Large	10	74	20	1.0	15.0	2.7
		Mid	10	1000	44	1.9	657.4	14.5
		Small	5	1000	47	3.7	1912.7	39.2

Panel B: 90% Recovery Estimates

			Order Event Time			Time in Seconds		
Scenario	Variable	Size Category	Min	Max	Median	Min	Max	Median
MO	$v_t^{b,1}$	Large	32	69	44	2.6	12.4	5.2
		Mid	32	158	55.5	5.5	240.3	16.8
		Small	5	127	49	2.3	216.4	36.9
OC	$v_t^{b,1}$	Large	27	53	38	2.6	11.2	4.6
		Mid	29	148	48	5.4	249.9	15.3
		Small	17	128	45	8.4	206.3	37.1
AMO	s_t	Large	46	134	74	2.6	17.4	9.7
		Mid	21	85	40	4.2	135.2	10.2
		Small	9	116	40	6.6	163.2	25.3
AOC	s_t	Large	45	127	72	2.7	16.9	9.5
		Mid	20	84	38.5	4.1	143.9	10.6
		Small	9	117	38.5	6.8	167.7	27.0
OCBM	$v_t^{b,25}$	Large	56	447	112	6.1	90.6	14.9
		Mid	46	1000	220	9.2	2843.1	74.8
		Small	20	1000	209	14.9	2807.9	172.0

Our empirical findings largely mitigate concerns on the capacity of the limit order book to absorb common liquidity shocks. We show that although liquidity shocks that directly impact one dimension of liquidity has detrimental effects on other dimensions of liquidity, the order book typically recovers quickly after the event. The speed at which the limit order book is replenished is more consistent with liquidity being provided by computerized algorithms as opposed to human traders. These liquidity suppliers seem to play a pivotal role in supporting the market's 'self-correcting' ability (Coppejans et al., 2004) and the widespread use of order splitting strategies by institutional investors to minimize transaction costs.

The success of order splitting strategies relies on limit order books exhibiting a sufficient degree of resilience. Automated liquidity provision appears pervasive across large capitalization stocks with a consistently high level of resiliency observed for all firm-trading weeks. The variation in resiliency observed across smaller stocks is consistent with the findings of Anand and Venkataraman (2012) that liquidity provision in smaller stocks can be 'sparse and opportunistic'. This presents additional challenges for investors implementing order splitting strategies and indicates that smaller stocks may indeed benefit from the presence of designated liquidity providers.

Our findings also has implications for the current debate over high frequency trading. Critics argue that high frequency traders, with access to faster trading than other investors gain an unfair advantage in the market place. However, price-time priority rules incentivizes competing liquidity providers to invest in trading technology to increase the speed at which they can identify and respond to liquidity imbalances in the limit order book which appears to have had a positive effect on the resiliency of the market.

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