Example : Consider all families with two children and assume that boys and girls are equally likely.

(i) If a family is chosen at random and is found to have a boy what is the prob that the other one is also a boy ?

(ii) If a child is chosen at random from these families and is found to be a boy what is the prob that the other child in that family is also a boy ?

Soln. (i) $\Omega = \{(b,b), (b,g), (g,b), (g,g)\}$

$A \rightarrow$ family has a boy

$B \rightarrow$ second child is also a boy

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{1/4}{3/4} = \frac{1}{3}$$

(ii) $\Omega = \{ b_b, b_g, g_b, g_g \}$

$A \rightarrow$ child is a boy $\quad P(A) = \frac{1}{2}$

$B \rightarrow$ child has a brother $\quad P(A \cap B) = \frac{1}{4}$

$$P(B|A) = \frac{1/4}{1/2} = \frac{1}{2}$$

2. These are two types of tubes in an electronic gadget. It will cease to function iff one of each kind is defective. The prob that there is defective tube of the first kind is 0.1; ...... the

second kind is 0.2. It is known that two tubes are defective. What is the prob. that the gadget still works?

<u>Sol</u>^{n}: $A \to$ two tubes are defective

$B \to$ gadget still works

$$P(A) = (0.1)^2 + (0.2)^2 + 2(0.1)(0.2)$$
$$= 0.09$$

$$P(A \cap B) = (0.1)^2 + (0.2)^2 = 0.05$$
$$P(B|A) = 5/9.$$

3. All the bolts in a machine come from either factory A or factory B. (both have same chance). The % of defective bolts is 5% from A & 1% from B. Two bolts are inspected.

(i) If the first is found to be good what is the prob that the second is also good?

(ii) If the first is found to be defective what is the prob that the second is also defective?

Sol^n (i) $G_1 \rightarrow$ first bolt is good

$G_2 \rightarrow$ second bolt is also good.

$$P(G_1) = P(\text{first is good} \mid A)\, P(A) + P(\text{first is good} \mid B)\, P(B)$$

$$= (0.95) \cdot \frac{1}{2} + (0.99) \cdot \frac{1}{2} = 0.97$$

$$P(G_1 \cap G_2) = P(\text{both are good} \mid A)\, P(A) + P(\text{both are good} \mid B)\, P(B)$$

$$= (0.95)^2 \cdot \frac{1}{2} + (0.99)^2 \cdot \frac{1}{2}$$

$$P(G_2 | G_1) = \frac{P(G_1 \cap G_2)}{P(G_1)} = 0.9704 > 0.97$$

(ii) $D_1 \rightarrow$ first bolt is defective

$D_2 \rightarrow$ second . . . . .

$$P(D_1) = P(D_1|A)\,P(A) + P(D_1|B)\,P(B)$$

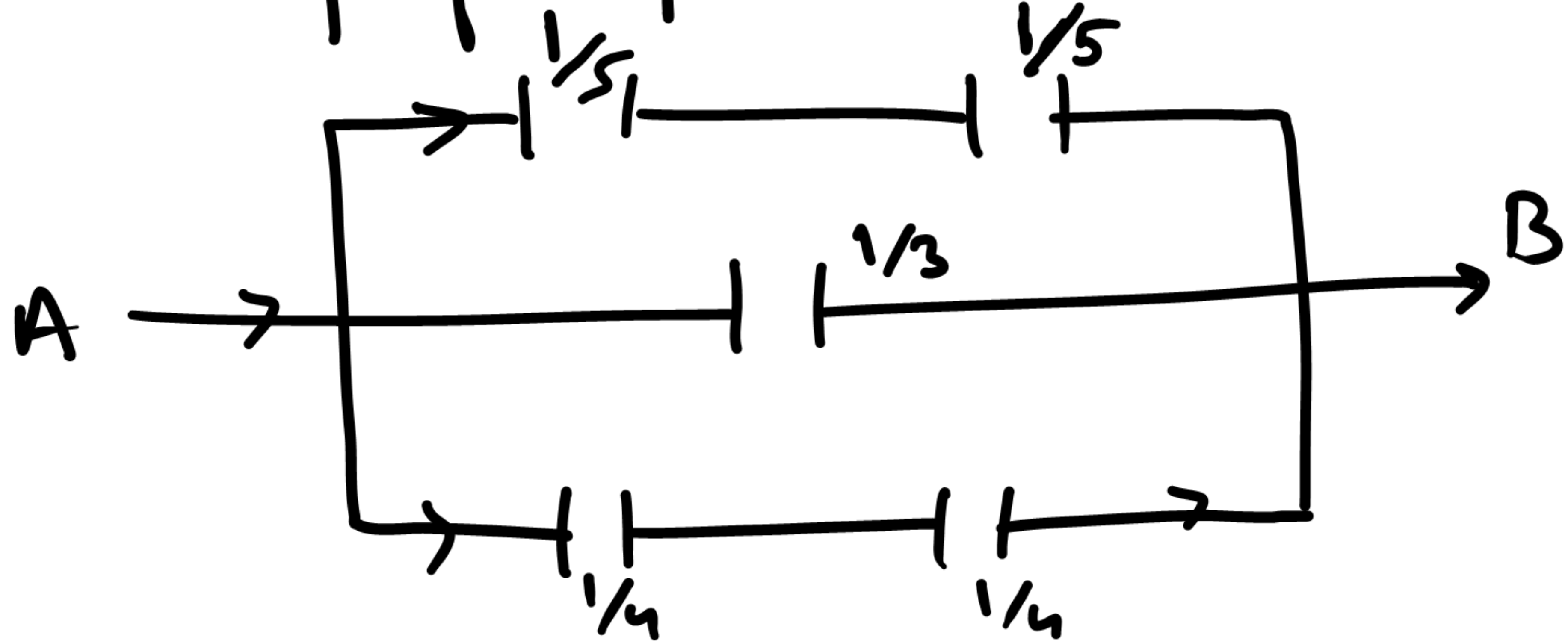
$$= (0.05) \cdot \frac{1}{2} + (0.01) \cdot \frac{1}{2} = 0.03$$

$$P(D_1 \cap D_2) = (0.05)^2 \cdot \frac{1}{2} + (0.01)^2 \cdot \frac{1}{2}$$

$$P(D_2 \mid D_1) = \frac{13}{300} > 0.03$$

Probability of repetition increases!!

4.



An electric network looks as in the above figure, where the numbers indicate the probabilities of failure for various links, which are all independent. What is the prob that the circuit is closed?

<u>Sol</u>

Sol^n Denote the three paths by E_1, E_2, E_3 as working.

$$P\left(\bigcup_{i=1}^{3} E_i\right) = 1 - P\left(\left(\bigcup E_i\right)^c\right)$$

$$= 1 - P\left(\bigcap_{i=1}^{3} E_i^c\right)$$

$$= 1 - \prod_{i=1}^{3} P(E_i^c) = \frac{379}{400}$$

$$P(E_1^c) = 1 - \left(\frac{4}{5}\right)^2, \quad P(E_2^c) = \frac{1}{3}$$

$$P(E_3^c) = 1 - \left(\frac{3}{4}\right)^2$$

Random Variables

A random variable is a real-valued function defined on the sample space

Let $(\Omega, \mathcal{B}, P)$ be a probability space. A random variable

$$X : \Omega \longrightarrow \mathbb{R}$$

$\mathcal{B} \longrightarrow \mathcal{C}$ $\qquad$ $P(A)$

$P \longrightarrow Q$ $\qquad$ $A \to E \in \mathcal{C}$

$P(A) \longrightarrow Q(E)$

$$(\Omega, \mathcal{B}, P) \xrightarrow{X} (\boxed{\mathbb{R}}, \mathcal{C}, Q)$$

<u>Example</u>: Suppose a fair coin is tossed once. $\Omega = \{H, T\}$

$$\mathcal{B} = \{\phi, \{H\}, \{T\}, \Omega\}$$

$P(\phi) = 0$, $P(\{H\}) = \frac{1}{2}$, $P(\{T\}) = \frac{1}{2}$

$P(\Omega) = 1$

$X \to$ no. of heads.

$X(H) = 1, \quad X(T) = 0$

$\mathcal{C} = \{ \phi, \{1\}, \{0\}, \{0,1\} \}$

$Q(\{1\}) = \frac{1}{2}, \quad Q(\{0\}) = \frac{1}{2}$

$Q(\phi) = 0, \quad Q(\{0,1\}) = 1$

$P(X=1) = \frac{1}{2}, \quad P(X=0) = \frac{1}{2}$