INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Date........February 2009 ....FN / AN Time : 2 Hrs. Full Marks ...30....... No. of Students ...92.....

Spring Semester, ... Deptt. of Electronics & Electrical Communication Engg. Sub. No. EC21006

2nd Year B.Tech. (H) / B.Arch. (H) / M.Sc. Sub. Name .... Electromagnetic Engineering

Instruction Answer ALL Questions. All symbols and variables have their usual meaning.

Note 1: The numbers in square brackets at the right-hand side of the text indicate the provisional allocation of maximum marks per question or sub-section of a question.

Note2: You may need: $\varepsilon_0 = 8.854\times 10^{-12}\,F/m, \mu_0 = 4\pi\times 10^{-7}\,H/m$.

Q.Ia Given a 60 μC point charge located at the origin. Find the total electric flux passing through (i) that portion of the sphere r=26 cm bounded by $0<\theta<\pi/2$ and $0<\phi<\pi/2$; (ii) the closed surface defined by ρ=26 cm and z = ±26 cm; (iii) the closed surface defined by ρ=26 cm, z=0 and z = 26cm. **[03]**

Q.Ib Given the electric flux density, $\overline{\mathcal{D}} = \hat{r}\, 0.3\, r^2$ nC/m^2 in free space: find (i) find the total charge within the sphere r=3; (ii) find the total electric flux leaving the sphere r = 4. **[02]**.

Q.Ic Given the potential field in cylindrical coordinates, $V = \dfrac{100}{z^2+1}\rho\cos\phi$ Volts, and point P at ρ = 3 m, φ = 60°, z = 2 m, find values at P for (i) the direction of steepest rise in V (ii) $\overline{\mathcal{E}}$; (iii) volume charge density ρ_v in free space. **[03]**

Q.Id Given $V = r z^2 \cos 2\phi$. Compute the directional derivative of V along the direction $2\hat{r} - \hat{z}$ and evaluate it at (1, π/2, 2). **[02]**

Q.Ie The electric field of a travelling electromagnetic wave is given by
$E(z,t) = 10\sin(18\pi\times 10^8 t - 8.5\pi z - \pi/6)$ V/m.
Determine (i) the direction of wave propagation, (ii) the wave frequency f, (iii) its wavelength λ; (iv) its wave velocity v. **[02]**

Q.I (f) (i) Given $\overline{\mathbf{Q}} = \hat{r}\, A\, r$ in spherical coordinates, calculate the flux of $\overline{\mathbf{Q}}$ (using the appropriate surface integral) through a spherical surface of radius a, centred at the origin. [02]

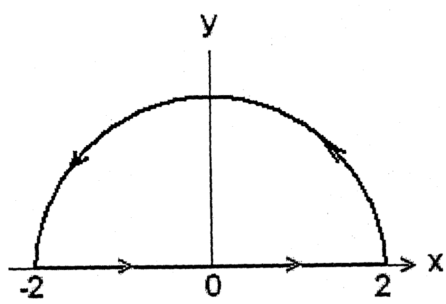
(ii) Verify the divergence theorem by calculating the volume integral of the div of $\overline{\mathbf{Q}}$ of Q.1f (i) over the appropriate volume. [01]

Q.II Verify Stokes' theorem for the vector field
$\overline{B} = \hat{r}\, r\cos\phi + \hat{\phi}\sin\phi$ by evaluating:

(a) $\oint_C \overline{B}\cdot d\overline{l}$ over the semicircular contour shown in the adjacent figure.

(b) $\iint_S \overline{\nabla}\times\overline{B}\cdot d\overline{a}$ over the surface of the above path

[05]



P.T.O.

Q.III (a) (i) Using the relationship $\nabla\cdot(\overline{P}\times\overline{Q})\equiv\overline{Q}\cdot\nabla\times\overline{P}-\overline{P}\cdot\nabla\times\overline{Q}$ and the appropriate Maxwell's equations and the equations for energy stored per unit volume ($w_e=\frac{1}{2}\overline{\mathcal{E}}\cdot\overline{\mathcal{D}};w_m=\frac{1}{2}\overline{\mathcal{B}}\cdot\overline{\mathcal{H}}$) derive the power balance relationship at a point and then at a region in space. **[03]**

(ii) From the results of (a) above obtain an interpretation for the Poynting vector. [01]

(iii) What does divergence of the Poynting vector represent? [01]

(b) (i) A permanent magnet is set up to create the static $\mathcal{H}$– field in a region of free space. A pair of charged stationary plates is set up to create a static $\mathcal{E}$– field that is not everywhere in the same direction as the magnetic field in the region. There is no movement. The divergence of the Poynting vector is integrated throughout a finite spherical volume (radius=a) within the region. Discuss briefly the result of this integration. **[03]**

(ii) Explain how the concept of current has been extended following Maxwell. **[02]**

(iii) Using the appropriate Maxwell's equations show that the normal components of $\mathcal{B}$ and $\mathcal{D}$ and the tangential components of $\mathcal{H}$ and $\mathcal{E}$ are continuous across a plane boundary between two different media in which there are no surface conduction currents nor any surface charges. [05]

Note 3: You may need

Cylindrical coordinates:

$$\overline{\nabla}w=\hat{\rho}\frac{\partial w}{\partial\rho}+\hat{\phi}\frac{1}{\rho}\frac{\partial w}{\partial\phi}+\hat{z}\frac{\partial w}{\partial z}$$

$$\overline{\nabla}\cdot\overline{A}=\frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial\rho}+\frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi}+\frac{\partial A_z}{\partial z}$$

$$\overline{\nabla}\times\overline{A}=\hat{\rho}\left(\frac{1}{\rho}\frac{\partial A_z}{\partial\phi}-\frac{\partial A_\phi}{\partial z}\right)+\hat{\phi}\left(\frac{\partial A_\rho}{\partial z}-\frac{\partial A_z}{\partial\rho}\right)+\hat{z}\left(\frac{1}{\rho}\frac{\partial(\rho A_\phi)}{\partial\rho}-\frac{1}{\rho}\frac{\partial A_\rho}{\partial\phi}\right)$$

Spherical Coordinates

$$\overline{\nabla}w=\hat{r}\frac{\partial w}{\partial r}+\hat{\theta}\frac{1}{r}\frac{\partial w}{\partial\theta}+\hat{\phi}\frac{1}{r\sin\theta}\frac{\partial w}{\partial\phi}$$

$$\overline{\nabla}\cdot\overline{A}=\frac{1}{r^2}\frac{\partial}{\partial r}(r^2A_r)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta\sin\theta)+\frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$\overline{\nabla}\times\overline{A}=\hat{r}\frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(A_\phi\sin\theta)-\frac{\partial A_\theta}{\partial\phi}\right]+\hat{\theta}\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi}-\frac{\partial}{\partial r}(rA_\phi)\right]+\hat{\phi}\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta)-\frac{\partial A_r}{\partial\theta}\right]$$