SOLUTIONS 4.1

SOL 4.1.1 Option (A) is correct.
Since the field intensity exists in a closed surface and lines of field intensity makes a closed curve so the flux lines leaving the spherical surface equal to the total flux entering the surface and So the net flux

$$\Phi = \oint \boldsymbol{B} \cdot d\boldsymbol{S} = 0$$

According to divergence theorem

$$\oint \boldsymbol{B} \cdot d\boldsymbol{S} = \int (\nabla \cdot \boldsymbol{B}) dv$$

$$0 = \int \nabla \cdot \boldsymbol{B}\, dv$$

Since volume of the sphere will have certain finite value so,

$$\nabla \cdot \boldsymbol{B} = 0$$

or $\quad \nabla \cdot \boldsymbol{H} = 0$ at all points inside the sphere

SOL 4.1.2 Option (A) is correct.
The magnetic field intensity produced due to a small current element Idl is defined as

$$d\boldsymbol{H} = \frac{Id\boldsymbol{l} \times \boldsymbol{a}_R}{4\pi R^2}$$

where $d\boldsymbol{l}$ is the differential line vector and $\boldsymbol{a}_R$ is the unit vector directed towards the point where field is to be determined. So for the circular current carrying loop we have

$$d\boldsymbol{l} = ad\phi \boldsymbol{a}_\phi$$

$$\boldsymbol{a}_R = -\boldsymbol{a}_\rho$$

Therefore the magnetic field intensity produced at the centre of the circular loop is

$$\boldsymbol{H} = \int_{\phi=0}^{2\pi} \frac{Iad\phi \boldsymbol{a}_\phi \times (-\boldsymbol{a}_\rho)}{4\pi a^2}$$

$$= \frac{Ia}{4\pi a^2}[\phi]_0^{2\pi}(\boldsymbol{a}_z) = \frac{I}{2a}\boldsymbol{a}_z \text{ A/m}$$

SOL 4.1.3 Option (C) is correct.
As calculated in previous question the magnetic field intensity produced at the centre of the current carrying circular loop is

$$H = \frac{I}{2a}$$

So by symmetry the semicircular loop will produce the field intensity half to the field intensity produced by complete circular loop.

i.e. Field intensity at the centre of semicircular loop $= \frac{1}{2}H = \frac{I}{4a}$

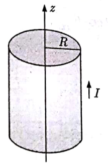
SOL 4.1.4 Option (B) is correct.
Since current in the wire is distributed over the outer surface so net enclosed current, I_{enc} for any Amperian loop inside the wire will be zero.
and as from Ampere's circuital law we have

$$\oint \boldsymbol{H} \cdot d\boldsymbol{l} = I_{enc}$$

So $\quad \oint \boldsymbol{H} \cdot d\boldsymbol{l} = 0 \qquad (I_{enc} = 0)$

or $\quad \boldsymbol{H} = 0$ for $r < R$

SOL 4.1.5 Option (B) is correct.
Consider the cylindrical wire is lying along z-axis as shown in the figure. As the current I is distributed over the outer surface of the cylinder so for an Amperian loop at a distance $r (> R)$ from the centre axis, enclosed current is equal to the total current flowing in the wire.



Now from Ampere's circuital law we have,

$$\int \boldsymbol{B} \cdot d\boldsymbol{l} = \mu_0 I_{enc}$$

$$B(2\pi r) = \mu_0 I \qquad (I_{enc} = I)$$

or $\quad \boldsymbol{B} = \frac{\mu_0 I}{2\pi r}\boldsymbol{a}_\phi$

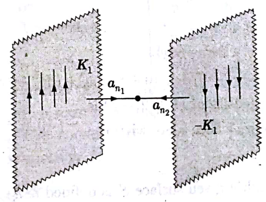
or $\quad B \propto \frac{1}{r}$

SOL 4.1.6 Option (B) is correct.
Consider one of the sheet carries the current density $\boldsymbol{K}_1$. So, the other sheet will have the current density $-\boldsymbol{K}_1$.
Magnetic flux density produced at any point P due to a current sheet is defined as

$$\boldsymbol{B} = \frac{\mu_0}{2}\boldsymbol{K} \times \boldsymbol{a}_n$$

where $\boldsymbol{K}$ is current density of the sheet and $\boldsymbol{a}_n$ is the unit vector normal to the sheet directed towards point P.
So for any point in the space between the sheets normal vector will be opposite in direction for the two sheets as shown in figure

i.e. $\quad \boldsymbol{a}_{n_2} = -\boldsymbol{a}_{n_1}$



Therefore, the resultant magnetic flux density at any point in the space between the two sheets will be

$$\boldsymbol{B} = \frac{\mu_0}{2}[\boldsymbol{K}_1 \times \boldsymbol{a}_{n_1} + (-\boldsymbol{K}_1) \times (-\boldsymbol{a}_{n_1})] = \mu_0 \boldsymbol{K}_1 \times \boldsymbol{a}_{n_1}$$

Since $\boldsymbol{a}_{n_1}$ is unit vector normal to the surface, and $\boldsymbol{K}_1$ is given current density. So the cross product will be a constant.

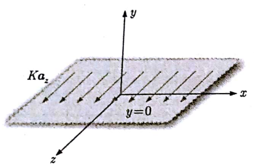
SOL 4.1.7 Option (A) is correct.
The magnetic flux density at any point is equal to the curl of magnetic vector potential $\boldsymbol{A}$ at the point.

i.e. $$\boldsymbol{B} = \nabla \times \boldsymbol{A} = \nabla \times (12\cos\theta \boldsymbol{a}_\theta)$$

$$= \frac{1}{r}\frac{\partial}{\partial r}(12r\cos\theta)\,\boldsymbol{a}_\phi = \frac{12\cos\theta}{r}\boldsymbol{a}_\phi$$

or, $$\boldsymbol{B} = \frac{12\cos 0}{3}\boldsymbol{a}_\phi = 4\boldsymbol{a}_\phi \qquad \text{(at } (3,0,\pi))$$

SOL 4.1.8 Option (B) is correct.
Consider the current sheet shown in the figure.



Magnetic field intensity produced at any point P due to a current sheet is defined as

$$\boldsymbol{H} = \frac{1}{2}\boldsymbol{K} \times \boldsymbol{a}_n$$

where $\boldsymbol{K}$ is current density of the sheet and $\boldsymbol{a}_n$ is the unit vector normal to the sheet directed towards point P.

So, for $y > 0$ $\quad \boldsymbol{H} = \frac{1}{2}(K\boldsymbol{a}_z) \times (\boldsymbol{a}_y) = -\frac{1}{2}K\boldsymbol{a}_x \quad (\boldsymbol{K} = K\boldsymbol{a}_z \text{ A/m}, \boldsymbol{a}_n = \boldsymbol{a}_y)$

and for $y < 0$ $\quad \boldsymbol{H} = \frac{1}{2}(K\boldsymbol{a}_z) \times (-\boldsymbol{a}_y) = \frac{K}{2}\boldsymbol{a}_x \quad (\boldsymbol{K} = K\boldsymbol{a}_z \text{ A/m}, \boldsymbol{a}_n = -\boldsymbol{a}_y)$

SOL 4.1.9 Option (B) is correct.
Magnetic flux density is defined as the curl of vector magnetic potential

i.e. $$\boldsymbol{B} = \nabla \times \boldsymbol{A}$$

$$= \begin{bmatrix} \boldsymbol{a}_x & \boldsymbol{a}_y & \boldsymbol{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x^2y & 2y^2 & -8xyz \end{bmatrix}$$

$$= (-8xz - 0)\,\boldsymbol{a}_x + (0 + 8yz)\,\boldsymbol{a}_y + (2y^2 - 2x^2)\,\boldsymbol{a}_z$$

So the net magnetic flux density at $(1, -2, -5)$ is

$$\boldsymbol{B} = 40\boldsymbol{a}_x + 80\boldsymbol{a}_y + 6\boldsymbol{a}_z \text{ wb/m}^2$$

SOL 4.1.10 Option (C) is correct.
Total magnetic flux through a given surface S is defined as

$$\Phi = \int_S \boldsymbol{B} \cdot d\boldsymbol{S}$$

where $d\boldsymbol{S}$ is the differential surface vector having direction normal to the surface

So, for the given surface $z = 4$, $0 \le x \le 1$, $-1 \le y \le 4$ we have

$$d\boldsymbol{S} = (dxdy)\,\boldsymbol{a}_z$$

and as calculated in previous question we have

$$\boldsymbol{B} = (-8xz - 0)\,\boldsymbol{a}_x + (0 + 8yz)\,\boldsymbol{a}_y + (2y^2 - 2x^2)\,\boldsymbol{a}_z$$

Therefore, the total magnetic flux through the given surface is

$$\Phi = \int_{y=-1}^{4}\int_{x=0}^{1}(2y^2 - 2x^2)\,(dxdy)$$

$$= 2 \times 1\int_{-1}^{4} y^2\,dy - 2 \times 5\int_0^1 x^2\,dx$$

$$= 2\left[\frac{y^3}{3}\right]_{-1}^{4} - 10\left[\frac{x^3}{3}\right]_0^1 = 2 \times \frac{65}{3} - \frac{10}{3}$$

$$= 40 \text{ wb}$$

SOL 4.1.11 Option (D) is correct.
The magnetic flux density at any point is equal to the curl of the vector magnetic potential at the point

i.e., $$\boldsymbol{B} = \nabla \times \boldsymbol{A}$$

$$= \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\phi)\,\boldsymbol{a}_z = \frac{1}{\rho}\frac{\partial}{\partial \rho}(2\rho)\,\boldsymbol{a}_z = \frac{2}{\rho}\boldsymbol{a}_z$$

The current density $\boldsymbol{J}$ in terms of magnetic flux density $\boldsymbol{B}$ is defined as

$$\boldsymbol{J} = \frac{1}{\mu_0}(\nabla \times \boldsymbol{B})$$

$$= \frac{1}{\mu_0}\left[-\frac{\partial}{\partial \rho}\left(\frac{2}{\rho}\right)\right]\boldsymbol{a}_\phi = \frac{2}{\mu_0\rho^2}\boldsymbol{a}_\phi$$

This current density would produce the required vector potential.

SOL 4.1.12 Option (B) is correct.
The current density for a given magnetic field intensity $\boldsymbol{H}$ is defined as

$$\boldsymbol{J} = \nabla \times \boldsymbol{H}$$

Given $$\boldsymbol{H} = (z\cos ay)\boldsymbol{a}_y + (z + e^y)\boldsymbol{a}_z$$

So $$\nabla \times \boldsymbol{H} = \begin{vmatrix} \boldsymbol{a}_x & \boldsymbol{a}_y & \boldsymbol{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (z + e^y) & z\cos ay & 0 \end{vmatrix}$$

$$= \left[-\frac{\partial}{\partial z}(z\cos ay)\boldsymbol{a}_x - \left(-\frac{\partial}{\partial z}(z + e^y)\right)\boldsymbol{a}_y + \left(\frac{\partial}{\partial x}z\cos ay - \frac{\partial}{\partial y}(z + e^y)\right)\right]\boldsymbol{a}_z$$

$$= -\cos ay\boldsymbol{a}_x + \boldsymbol{a}_y - e^y\boldsymbol{a}_z$$

or, $$\boldsymbol{J} = \nabla \times \boldsymbol{H} = -\cos ay\boldsymbol{a}_x + \boldsymbol{a}_y - e^y\boldsymbol{a}_z$$

Therefore the current density in the x-z plane is

$$\boldsymbol{J} = -\boldsymbol{a}_x + \boldsymbol{a}_y - \boldsymbol{a}_z \text{ A/m}^2 \qquad (y = 0 \text{ in } x\text{-}z \text{ plane})$$

SOL 4.1.13 Option (A) is correct.
In a source free region current density, $\boldsymbol{J} = 0$.
The current density at any point is equal to the curl of magnetic field intensity $\boldsymbol{H}$.

i.e. $$\boldsymbol{J} = \nabla \times \boldsymbol{H}$$

or $\qquad \nabla \times H = 0 \qquad (J = 0)$

and since the curl of a given vector field is zero so it can be expressed as the gradient of a scalar field

i.e. $\qquad H = \nabla f$

So A and R both are true and R is correct explanation of A.

SOL 4.1.14 Option (A) is correct.

As the beam is travelling in a_z direction so the field intensity produced by it will be in a_ϕ direction and using Ampere's circuital law at it's surface we have

$$H_\phi(2\pi a) = I_{enc}$$

$$H_\phi(2\pi a) = \int_0^a 2\left(1 - \frac{\rho}{a}\right) 2\pi\rho d\rho$$

$$H_\phi(2\pi a) = 4\pi\left[\frac{\rho^2}{2} - \frac{\rho^3}{3a}\right]_0^a$$

$$H_\phi(2\pi a) = \frac{2\pi a^2}{3}$$

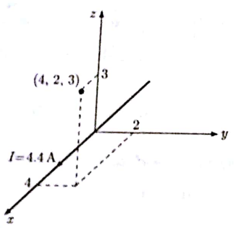
or
$$H = \frac{a}{3} a_\phi$$

SOL 4.1.15 Option (A) is correct.

According to Biot-savart law, magnetic field intensity at any point P due to the current element Idl is defined as

$$H = \int \frac{Idl \times R}{4\pi R^3}$$

where R is the vector distance of point P from the current element.



Now the current element carries a current of 4.4 A in $+a_x$ direction.

So we have, $\qquad R = (4a_x + 2a_y + 3a_z) - (xa_x)$

(Since on x-axis y- and z-component will be zero)

$$R = (4 - x) a_x + 2a_y + 3a_z$$

or
$$R = \sqrt{(4-x)^2 + 2^2 + 3^2}$$
$$= \sqrt{x^2 - 8x + 29}$$

and $\qquad Idl = 4.4 dx a_x \qquad$ (filament lies from $x = -\infty$ to $x = \infty$)

Therefore the magnetic field intensity is

$$H = \int_{-\infty}^{+\infty} \frac{(4.4a_x) \times [(4-x)a_x + 2a_y + 3a_z]}{4\pi(x^2 - 8x + 29)^{3/2}} dx$$

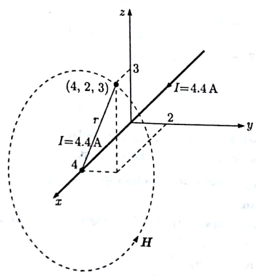
$$= \frac{4.4}{4\pi}(2a_z - 3a_y)\int_{-\infty}^{+\infty} \frac{dx}{(x^2 - 8x + 29)^{3/2}}$$

$$= \frac{4.4}{4\pi}(2a_z - 3a_y)\left[\frac{(2x-8)}{26(x^2 - 8x + 29)^{1/2}}\right]_{-\infty}^{+\infty}$$

$$= \frac{4.4}{26\pi}(2a_z - 3a_y)$$

$$= 0.1077a_z - 0.162a_y \; = 0.1a_z - 0.2a_y \text{ A/m}$$

ALTERNATIVE METHOD :



According to Ampere's circuital law, the line integral of magnetic field intensity H around a closed path is equal to the net current enclosed by the path.

Since we have to determine the magnetic field intensity at point (4, 2, 3) so we construct a circular loop around the infinite current element that passes though the point (4, 2, 3) as shown in the figure.

Now from Ampere's circuital law we have,

$$\oint H \cdot dl = I_{enc}$$

$$(2\pi r) H = 4.4 \qquad (I_{enc} = 4.4 \text{ A})$$

or
$$H = \frac{4.4}{2\pi \times \sqrt{13}} \qquad r = \sqrt{13} \text{ from figure.}$$

Now direction of the magnetic field intensity is defined as

$$a_\phi = a_l \times a_\rho$$

where $\qquad a_l \rightarrow$ unit vector in the direction of flow of current

$a_\rho \rightarrow$ unit vector normal to the line current directed toward the point.

So we have, $\qquad a_\phi = a_x \times \frac{[(4a_x + 2a_y + 3a_z) - (4a_x)]}{\sqrt{(4-4)^2 + 2^2 + 3^2}}$

$$= a_x \times \frac{(2a_y + 3a_z)}{\sqrt{13}}$$

$$= \frac{2a_z - 3a_y}{\sqrt{13}}$$

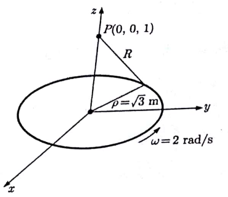
Therefore the magnetic field intensity at the point (4,2,3) is

$$H = \frac{4.4}{2\pi\sqrt{13}} \frac{(2a_z - 3a_y)}{\sqrt{13}}$$

$$= \frac{4.4}{26\pi}(2a_z - 3a_y)$$

$$= 0.1a_z - 0.2a_y \text{ A/m}$$

SOL 4.1.16 Option (A) is correct.



Since the uniformly charged disk is rotating with an angular velocity $\omega = 2\,\text{rad/s}$ about the z-axis so we have the current density

$$K = \rho_s \times (\text{angular velocity}) = \rho_s(\omega\rho) = 2\times 2\times\rho$$

or
$$\boldsymbol{K} = 4\rho\boldsymbol{a}_\phi$$

According to Biot-savart law, magnetic field intensity at any point P due to the current sheet element $\boldsymbol{K}dS$ is defined as

$$\boldsymbol{H} = \int_s \frac{\boldsymbol{K}dS\times\boldsymbol{a}_R}{4\pi R^2}$$

where $\boldsymbol{R}$ is the vector distance of point P from the current element.

Now from the figure we have

$$\boldsymbol{R} = \boldsymbol{a}_z - \rho\boldsymbol{a}_\rho$$

or
$$R = \sqrt{1+\rho^2}$$

and
$$\boldsymbol{a}_R = \frac{\boldsymbol{a}_z - \rho\boldsymbol{a}_\rho}{\sqrt{1+\rho^2}}$$

So the magnetic field intensity due to a small current element $\boldsymbol{K}dS$ at point P is

$$d\boldsymbol{H} = \frac{\boldsymbol{K}dS\times\boldsymbol{a}_R}{4\pi R^2} = \frac{(4\rho\boldsymbol{a}_\phi)\times(\boldsymbol{a}_z-\rho\boldsymbol{a}_\rho)}{4\pi(\rho^2+1)^{3/2}} = \frac{4\rho(\boldsymbol{a}_\rho+\rho\boldsymbol{a}_z)}{4\pi(\rho^2+1)^{3/2}}$$

On integrating the above over ϕ around the complete circle, the $\boldsymbol{a}_\rho$ components get cancelled by symmetry, leaving us with

$$\boldsymbol{H}(z) = \int_0^{2\pi}\int_0^{\sqrt{3}}\frac{4\rho^2\boldsymbol{a}_z}{4\pi(\rho^2+1)^{3/2}}(\rho d\rho d\phi)$$

$$= 2\int_0^{\sqrt{3}}\frac{\rho^3}{(\rho^2+1)^{3/2}}d\rho\boldsymbol{a}_z = 2\left[\sqrt{\rho^2+1}+\frac{1}{\sqrt{\rho^2+1}}\right]_0^{\sqrt{3}}\boldsymbol{a}_z$$

$$= 2\left[\frac{3+2(1-\sqrt{1+3})}{\sqrt{1+3}}\right]\boldsymbol{a}_z = \boldsymbol{a}_z\ \text{A/m}$$

SOL 4.1.17 Option (C) is correct.

The magnetic field intensity produced at any point P due to an infinite sheet carrying uniform current density $\boldsymbol{K}$ is defined as

$$\boldsymbol{H} = \frac{1}{2}(\boldsymbol{K}\times\boldsymbol{a}_n)$$

where $\boldsymbol{a}_n$ is the unit vector normal to the sheet directed toward the point P.

So the field intensity produced between the two sheets due to the sheet $\boldsymbol{K}_1 = 3\boldsymbol{a}_z$ located at $x = 2\,\text{m}$ is

$$\boldsymbol{H}_1 = \frac{1}{2}(3\boldsymbol{a}_z)\times(-\boldsymbol{a}_z) = -\frac{3}{2}\boldsymbol{a}_y\ \text{A/m} \qquad (\boldsymbol{a}_n = -\boldsymbol{a}_z)$$

and the field intensity produced between the two sheets due to the sheet $\boldsymbol{K}_2 = -3\boldsymbol{a}_z$ located at $x = -2\,\text{m}$ is

$$\boldsymbol{H}_2 = \frac{1}{2}(-3\boldsymbol{a}_z)\times(\boldsymbol{a}_z) = -\frac{3}{2}\boldsymbol{a}_y\ \text{A/m} \qquad (\boldsymbol{a}_n = \boldsymbol{a}_z)$$

Therefore the net magnetic field intensity produced at any point between the two sheets is

$$\boldsymbol{H} = \boldsymbol{H}_1 + \boldsymbol{H}_2 = -3\boldsymbol{a}_y$$

Since the magnetic field intensity at any point is the equal to the negative gradient of scalar potential at the point

i.e.
$$\boldsymbol{H} = -\nabla V_m$$

So for the field $\boldsymbol{H} = -3\boldsymbol{a}_y$ in the region between the two current carrying sheets, we have

$$-3\boldsymbol{a}_y = -\frac{dV_m}{dy}\boldsymbol{a}_y \quad \text{(the field has a single component in } \boldsymbol{a}_y \text{ direction)}$$

or
$$V_m = 3y + C_1 \qquad \text{where } C_1 \text{ is constant}$$

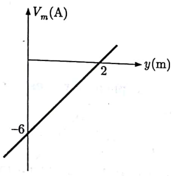
Putting $V_m = 0$ for point $P(1,2,5)$ (given), we have

$$0 = 3\times(2) + C_1$$

or
$$C_1 = -6$$

Thus,
$$V_m = (3y-6)\ \text{A}$$

and the graph of V_m versus y will be as plotted below



SOL 4.1.18 Option (B) is correct.

The magnetic flux density at any point is equal to the curl of the vector magnetic potential at the point

i.e.,
$$\boldsymbol{B} = \nabla\times\boldsymbol{A} \qquad (1)$$

Since $\boldsymbol{B} = \mu_0\boldsymbol{H} = -3\mu_0\boldsymbol{a}_y\ \text{Wb/m}^2$ (calculated in previous question)

As the magnetic flux density is in $\boldsymbol{a}_y$ direction so $\boldsymbol{A}$ is expected to be z -directed. Therefore from eq (1) we have

$$-\frac{\partial A_z}{\partial x} = -3\mu_0$$

or
$$A_z = 3\mu_0 x + C_2$$

Putting $A_z = 0$ at point $P(1,2,5)$ (given), we have

$$0 = 3\mu_0 + C_2$$

or
$$C_2 = -3\mu_0$$

So,
$$A_z = 3\mu_0(x-1) = -3\mu_0 \qquad \text{at origin } (0,0,0)$$

Thus, the magnetic vector potential at origin is

$$\boldsymbol{A} = -3\mu_0\boldsymbol{a}_z\ \text{Wb/m}$$

SOL 4.1.19 Option (C) is correct.

Magnetic dipole moment of a spherical shell of radius r having surface charge density ρ_S is given as

$$m = \frac{4\pi}{3}\rho_S \omega r^4 \qquad \text{where } \omega \text{ is angular velocity.}$$

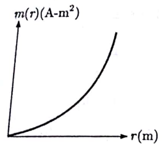
Since the total charge of 5 C is distributed over the volume of the sphere so, the magnetic dipole moment of the sphere is given as

$$m(r) = \int \frac{4\pi}{3}(\rho_v dr)\omega r^4 \qquad (\rho_S = \rho_v dr)$$

where ρ_v is uniformly distributed volume charge density of the sphere. Therefore, we have

$$m(r) = \frac{4\pi}{3}\rho_v \omega \frac{r^5}{5} = \frac{1}{5}Q\omega r^2 \qquad \left(\rho_v = \frac{Q}{\frac{4}{3}\pi r^3}\right)$$

$$= \frac{1}{5}\times(5)\times(4)\times r^2 = 4r^2 \text{ A-m}^2 \qquad (Q = 5\,\text{C},\ \omega = 4\,\text{rad/s})$$



SOL 4.1.20 Option (B) is correct.

The average magnetic field intensity over a sphere of radius r, due to steady currents within the sphere is defined as

$$H_{ave} = \frac{1}{4\pi}\frac{2m}{r^3} = \frac{1}{4\pi}\frac{2\times 4r^2}{r^3} = \frac{2}{\pi r} \qquad (m = 4r^2)$$

As the sphere is spinning about the z-axis so, the produced magnetic field will be in $\boldsymbol{a}_z$ direction as determined by right hand rule. Thus, we have

$$\boldsymbol{H}_{ave} = \frac{2}{\pi r}\boldsymbol{a}_z$$

SOL 4.1.21 Option (D) is correct.

Magnetic dipole moment of a conducting loop carrying current I is defined as :

$$\boldsymbol{m} = IS\boldsymbol{a}_n$$

where S is the area enclosed by the conducting loop and $\boldsymbol{a}_n$ is normal vector to the surface. So we have

$$\boldsymbol{m} = (7)(0.1)\,\boldsymbol{a}_n \qquad (I = 7\,\text{A}, S = 0.1\,\text{m}^2)$$

Now the given plane is

$$x + 3y - 1.5z = 3.5$$

For which we have the function

$$f = x + 3y - 1.5z$$

and the normal unit vector to the plane is,

$$\boldsymbol{a}_n = \frac{\nabla f}{|\nabla f|} = \frac{\boldsymbol{a}_x + 3\boldsymbol{a}_y - 1.5\boldsymbol{a}_z}{\sqrt{1^2 + 3^2 + (-1.5)^2}}$$

So the magnetic dipole moment of the coil is

$$\boldsymbol{m} = (0.7)\frac{(\boldsymbol{a}_x + 3\boldsymbol{a}_y - 1.5\boldsymbol{a}_z)}{3.5}$$

$$= 0.2\boldsymbol{a}_x + 0.6\boldsymbol{a}_y - 0.3\boldsymbol{a}_z \text{ A-m}^2$$

SOL 4.1.22 Option (C) is correct.

The magnetic field intensity, ($\boldsymbol{H}$) in the terms of magnetic vector potential, ($\boldsymbol{A}$) is defined as

$$\boldsymbol{H} = \frac{1}{\mu_0}(\nabla \times \boldsymbol{A})$$

$$= \frac{1}{\mu_0}[\nabla \times (6y - 2z)\,\boldsymbol{a}_x + 4xz\boldsymbol{a}_y]$$

$$= \frac{1}{\mu_0}[-8\boldsymbol{a}_x - 2\boldsymbol{a}_y + 6\boldsymbol{a}_z]$$

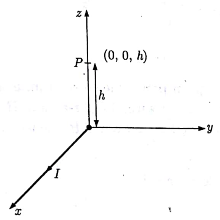
Since the electric current density at any point is equal to the curl of magnetic field intensity at that point.

i.e. $\boldsymbol{J} = \nabla \times \boldsymbol{H}$

So, we have the electric current density in the free space as

$$\boldsymbol{J} = \nabla \times \frac{1}{\mu_0}[-8\boldsymbol{a}_x - 2\boldsymbol{a}_y + 6\boldsymbol{a}_z] = 0$$

SOL 4.1.23 Option (B) is correct.



Consider the point P on z-axis is $(0,0,h)$ and current flowing in the current element is I in $\boldsymbol{a}_x$ direction. Since the magnetic field intensity at any point P due to a current element I is defined as

$$\boldsymbol{H} = \frac{I}{4\pi\rho}[\cos\alpha_2 - \cos\alpha_1]\boldsymbol{a}_\phi$$

where $\rho \rightarrow$ distance of point P from the current element.

$\alpha_1 \rightarrow$ angle subtended by the lower end of the element at P.

$\alpha_2 \rightarrow$ angle subtended by the upper end of the element at P.

So for the given current element along positive x-axis we have

$$\alpha_1 = 90°$$

$$\alpha_2 = 0°$$

Therefore, $$\boldsymbol{H} = \frac{I}{4\pi h}\boldsymbol{a}_\phi \qquad (\rho = h)$$

Now the direction of magnetic field intensity is defined as

$$\boldsymbol{a}_\phi = \boldsymbol{a}_l \times \boldsymbol{a}_\rho$$

where $\boldsymbol{a}_l$ is unit vector along the line current and $\boldsymbol{a}_\rho$ is the unit vector normal to the line current directed toward the point P.

So, $\quad a_\phi = a_x \times a_z = -a_y$

Therefore magnetizing force is

$$H = \frac{I}{4\pi h}(-a_y)$$

or $\quad H = \frac{I}{4\pi h} \qquad \ldots(1)$

Now consider the current flowing in the current element introduced along the positive y-axis is I in a_y direction. So, the magnetic field intensity produced at point P due to the current element along the positive y-axis is

$$H = \frac{I}{4\pi\rho}[\cos\alpha_2 - \cos\alpha_1]a_\phi$$

$$= \frac{I}{4\pi h}[\cos 0^\circ - \cos 90^\circ]a_\phi \qquad (\rho = h, \alpha_1 = 90^\circ, \alpha_2 = 0^\circ)$$

$$= \frac{I}{4\pi h}a_z \qquad (a_\phi = a_y \times a_x = a_z)$$

Therefore the resultant magnetic field intensity produced at point P due to both the current elements will be

$$H_{net} = \frac{I}{4\pi h}(-a_y + a_z)$$

or, $\quad H_{net} = \frac{I}{4\pi h}\sqrt{2}$

Thus, from equation (1) we have

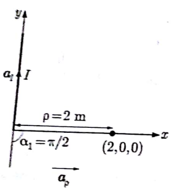
$$H_{net} = \sqrt{2}\,H$$

SOL 4.1.24 Option (A) is correct.

Consider the flux density at the given point due to semi infinite wire along y-axis is B_1 and the flux density due to wire along z-axis is B_2.

The magnetic flux density B produced at any point P due to a straight wire carrying current I is defined as

$$B = \frac{\mu_0 I}{4\pi\rho}[\cos\alpha_2 - \cos\alpha_1]a_\phi$$



where $\quad \rho \to$ distance of point P from the straight wire.

$\alpha_1 \to$ angle subtended by the lower end of the wire at P.

$\alpha_2 \to$ angle subtended by the upper end of the wire at P.

and the direction of the magnetic flux density is given as

$$a_\phi = a_l \times a_\rho$$

where a_l is unit vector along the line current and a_ρ is the unit vector normal to the line current directed toward the point P. So, we have

$$\rho = 2\,\text{m}$$

$$a_\phi = a_y \times (a_x) = -a_z \qquad (a_l = a_y, a_\rho = a_x)$$

$$\alpha_1 = \frac{\pi}{2}, \quad \alpha_2 = 0 \qquad \text{(as } y \text{ tends to } \infty)$$

Therefore the magnetic flux density produced at point P due to the semi infinite wire along y-axis is

$$B_1 = \frac{\mu_0(4)}{4\pi(2)}\left(\cos 0 - \cos\frac{\pi}{2}\right)(-a_z) = -\frac{\mu_0}{2\pi}a_z$$

Similarly we have the magnetic flux density produced at point P due to semi infinite wire along z-axis as

$$B_2 = -\frac{\mu_0}{2\pi}a_y$$

Thus, the net magnetic flux density produced at point P due to the L-shaped filamentary wire is

$$B = -\frac{\mu_0}{2\pi}a_y - \frac{\mu_0}{2\pi}a_z$$

$$= -2(a_y + a_z) \times 10^{-7}\ \text{Wb/m}^2$$

SOL 4.1.25 Option (A) is correct.

Using ampere's circuital law we have

$$\int B \cdot dl = \mu_0 I_{enc}$$

As the conductor carries current I which is uniformly distribute over the conductor cross section so, the current density inside the conductor is

$$J = \frac{I}{\pi R^2}$$

We construct an Amperian loop of radius $\rho(< R)$ inside the cylindrical wire for which the enclosed current is

$$I_{enc} = \left(\frac{I}{\pi R^2}\right)\pi\rho^2 = I\left(\frac{\rho^2}{R^2}\right)$$

and since the current is flowing along z-axis so using right hand rule we get the direction of magnetic flux density along $+a_\phi$.

Thus, from Amperes circuital law, we have

$$(B_\phi)(2\pi\rho) = I_{enc}$$

or $\quad B_\phi = \frac{\mu_0}{2\pi\rho}\left(\frac{I\rho^2}{R^2}\right)$

or $\quad B = \frac{\mu_0 I\rho}{2\pi R^2}a_\phi$

SOL 4.1.26 Option (A) is correct.

Similarly as calculated above we construct an Amperian loop of radius $\rho(> R)$ outside the cylinder for which the entire current flowing in the wire will be enclosed.

i.e. $\quad I_{enc} = I$

and from Ampere's circuital law we get,

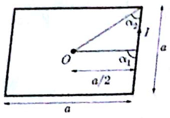
$$B_\phi(2\pi\rho) = \mu_0 I$$

$$B_\phi = \frac{\mu_0 I}{2\pi\rho}$$

So $\quad B \propto \frac{1}{\rho}$

SOL 4.1.27 Option (A) is correct.

Consider one half side of the square loop to determine the magnetic field intensity at the centre O as shown in the figure.



The magnetic field intensity H produced at any point P due to a straight wire carrying current I is defined as

$$H = \frac{I}{4\pi\rho}[\cos\alpha_2 - \cos\alpha_1]$$

where $\rho \rightarrow$ distance of point P from the straight wire.

$\alpha_1 \rightarrow$ angle subtended by the lower end of the wire at P.

$\alpha_2 \rightarrow$ angle subtended by the upper end of the wire at P.

So we have

$$\rho = a/2$$

$$\alpha_1 = \pi/2$$

and $$\alpha_2 = \pi/4$$

Therefore the magnetic field intensity produced at centre O due to the half side of the square loop is

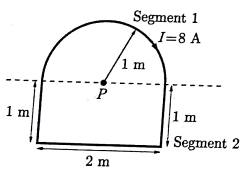
$$H_1 = \frac{I}{4\pi(a/2)}\left(\cos\frac{\pi}{4} - \cos\frac{\pi}{2}\right) = \frac{I}{2\sqrt{2}\,\pi a}$$

As all the eight half sides produces same field intensity at the centre of the loop so, net field intensity produced at the center due to the complete square loop is

$$H_{net} = 8\left(\frac{I}{2\sqrt{2}\,\pi a}\right) = \frac{2\sqrt{2}\,I}{\pi a}$$

SOL 4.1.28 Option (D) is correct.

For the shown current loop we divide the loop in two segments as shown in figure



Now the field intensity due to segment (1) (Semicircular loop) at point P can be given directly as calculated in Que.60

i.e. $$H_1 = \frac{I}{4a}$$ where a is radius of semicircular loop

or $$H_1 = \frac{8}{4(1)} = 2\text{ A/m} \qquad (a = 1\text{ m})$$

again for determining the field intensity due to segment (2) we consider it as the half portion of a complete square loop of side 2 m and since the field intensity due to a completer square loop of side a carrying current I can be directly given from previous question.

i.e. $$H = \frac{2\sqrt{2}\,I}{\pi a}$$

so the field intensity due to the half portion of square loop is

$$H_2 = \frac{1}{2}H = \frac{\sqrt{2}\,I}{\pi a}$$

or $$H_2 = \frac{\sqrt{2}(8)}{\pi(2)} = \frac{4\sqrt{2}}{\pi} \qquad (I = 8\text{ A}, a = 2\text{ m})$$

As determined by right hand rule the direction of field intensity produced at point P due to the two segments will be same (inward) therefore, the net magnetic field intensity produced at point P will be

$$H_{net} = H_1 + H_2 = 2 + \frac{4\sqrt{2}}{\pi} = 3.8\text{ A/m inward.}$$

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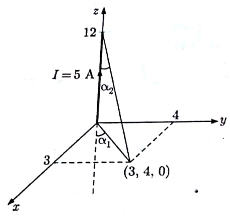
SOLUTIONS 4.2

SOL 4.2.1 Correct answer is 0.074 .



Magnetic field intensity at any point P due to a filamentary current I is defined as

$$\boldsymbol{H} = \frac{I}{4\pi\rho}[\cos\alpha_2 - \cos\alpha_1]\boldsymbol{a}_\phi$$

where $\rho \rightarrow$ distance of point P from the current filament.

$\alpha_1 \rightarrow$ angle subtended by the lower end of the element at P.

$\alpha_2 \rightarrow$ angle subtended by the upper end of the element at P.

From the figure we have

$$\rho = \sqrt{3^2+4^2} = 5$$

$$\alpha_1 = \pi/2 \Rightarrow \cos\alpha_1 = 0$$

and $$\cos\alpha_2 = \frac{12}{\sqrt{5^2+12^2}} = \frac{12}{13}$$

Now we Put these values to get,

$$\boldsymbol{H} = \frac{5}{4\pi\times 5}\left(\frac{12}{13}-0\right)\boldsymbol{a}_\phi \qquad (I = 5\text{ A})$$

$$= \frac{3}{13\pi}\boldsymbol{a}_\phi = 0.074\boldsymbol{a}_\phi \text{ wb/m}^2$$

SOL 4.2.2 Correct answer is 1.

According to Biot-savart law, magnetic field intensity at any point P due to the current element Idl is defined as

$$\boldsymbol{H} = \int\frac{Idl\times\boldsymbol{R}}{4\pi R^3}$$

where $\boldsymbol{R}$ is the vector distance of point P from the current element.

Here current is flowing in $\boldsymbol{a}_\phi$ direction

So the small current element

$$Idl = I\rho d\phi\boldsymbol{a}_\phi = 4\times 2d\phi\boldsymbol{a}_\phi = 8d\phi\boldsymbol{a}_\phi$$

and since the magnetic field to be determined at center of the loop so we have

$$R = 2\text{ m} \qquad (\text{radius} = 2\text{ m})$$

and

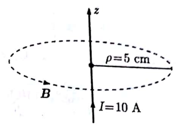
$$\boldsymbol{a}_R = -\boldsymbol{a}_\rho \qquad (\text{pointing towards origin})$$

Therefore the magnetic field intensity at origin is

$$\boldsymbol{H} = \int_0^{2\pi}\frac{(8d\phi\boldsymbol{a}_\phi)\times(-\boldsymbol{a}_\rho)}{4\pi(2)^2} = \int_0^{2\pi}\frac{8}{16\pi}d\phi\boldsymbol{a}_z = \frac{\boldsymbol{a}_z}{2\pi}[\phi]_0^{2\pi}$$

$$= \boldsymbol{a}_z \text{ A/m}$$

SOL 4.2.3 Correct answer is 4.



According to Ampere's circuital law, the line integral of magnetic field intensity H around a closed path is equal to the net current enclosed by the path.

Since we have to determine the magnetic field intensity due to the infinite line current at $\rho = 5$ cm so we construct a circular loop around the line current as shown in the figure.

Now from Ampere's circuital law we have

$$\oint_L \boldsymbol{B}\cdot d\boldsymbol{l} = \mu_0 I_{enc}$$

or $$B(2\pi\rho) = \mu_0\times 10 \qquad (I_{enc} = 10\text{ A})$$

Therefore we have the magnetic flux density at $\rho = 5$ cm as

$$B = \frac{4\pi\times 10^{-7}\times 10}{2\pi\times 5\times 10^{-2}} = 4\times 10^{-5}\text{ wb/m}^2$$

SOL 4.2.4 Correct answer is -20.

According to Ampere's circuital law the contour integral of magnetic field intensity in a closed path is equal to the current enclosed by the path.

i.e. $$\oint \boldsymbol{H}\cdot d\boldsymbol{l} = I_{enc}$$

Now using right hand rule, we obtain the direction of the magnetic field intensity in the loop as it will be opposite to the direction of $\boldsymbol{L}$.

So, $$\oint \boldsymbol{H}\cdot d\boldsymbol{l} = -I_{enc} = -20\text{ A}$$

(10 A is not inside the loop. So it won't be considered.)

SOL 4.2.5 Correct answer is -1.5915 .

The magnetic field intensity produced at a distance ρ from an infinitely long straight wire carrying current I is defined as

$$H = \frac{I}{2\pi\rho}$$

As determined by right hand rule, the direction of magnetic field intensity will be same(in $-\boldsymbol{a}_y$ direction) due to both the current source. So, at point P the net magnetic field intensity due to both the current carrying wires will be

$$\boldsymbol{H} = \boldsymbol{H}_1 + \boldsymbol{H}_2$$

$$= \frac{I}{2\pi(4)}(-\boldsymbol{a}_y) + \frac{I}{2\pi(1)}(-\boldsymbol{a}_y)$$

$$= -\frac{5(8)}{8\pi}\boldsymbol{a}_y = -\frac{5}{\pi}\boldsymbol{a}_y = -1.5915\boldsymbol{a}_y \qquad (I = 8\text{ A})$$

SOL 4.2.6 Correct answer is 6.

Since the current flows from Q_1 and terminates at Q_2 and the charge Q_2 is located at the surface of the contour so the actual current is not enclosed by the closed path and the circulation of the field is given as

$$\int \boldsymbol{B}\cdot d\boldsymbol{l} = \mu_0[I_d]_{enc} \qquad ([I_c]_{enc} = 0)$$

and
$$[I_d]_{enc} = \frac{d}{dt}\left[\int \varepsilon_0\boldsymbol{E}_1\cdot d\boldsymbol{S} + \int \varepsilon_0\boldsymbol{E}_2\cdot d\boldsymbol{S}\right]$$

where $\boldsymbol{E}_1$ is the electric field intensity produced by charge Q_1 while $\boldsymbol{E}_2$ is the field intensity produced by charge Q_2.

So,
$$[I_d]_{enc} = \frac{d}{dt}\left[\varepsilon_0\left(\frac{Q_1}{8\varepsilon_0}\right) + \varepsilon_0\left(\frac{Q_2}{2\varepsilon_0}\right)\right] = \frac{1}{8}\frac{dQ_1}{dt} + \frac{1}{2}\frac{dQ_2}{dt} \qquad (1)$$

As the current flows from Q_1 and terminates at Q_2 so the rate of change in the net charges is given as

$$-\frac{dQ_1}{dt} = \frac{dQ_2}{dt} = 16\text{ A}$$

Therefore from equation (1) we have the enclosed displacement current as

$$[I_d]_{enc} = \frac{1}{8}(-16) + \frac{1}{2}(16) = 6\text{ A}$$

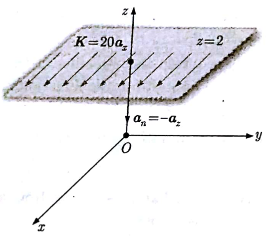
Thus, the circulation of magnetic flux density around the closed loop is

$$\int \boldsymbol{B}\cdot d\boldsymbol{l} = \mu_0(6)$$
$$= 6\mu_0\text{ Wb/m}$$

or
$$\frac{1}{\mu_0}\int \boldsymbol{B}\cdot d\boldsymbol{l} = 6$$

or
$$\int \boldsymbol{H}\cdot d\boldsymbol{l} = 6$$

SOL 4.2.7 Correct answer is 10.



Magnetic field intensity at any point P due to an infinite current carrying sheet is defined as

$$\boldsymbol{H} = \frac{1}{2}\boldsymbol{K}\times\boldsymbol{a}_n$$

where $\boldsymbol{K}$ is the current density and $\boldsymbol{a}_n$ is the unit vector normal to the current sheet directed toward the point P.

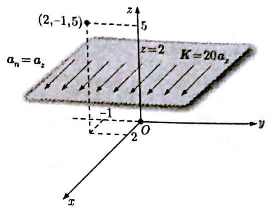
Since we have to determine the magnetic field intensity at origin so from the figure we have

$$\boldsymbol{a}_n = -\boldsymbol{a}_z$$

Therefore the magnetic field intensity at the origin is

$$\boldsymbol{H} = \frac{1}{2}(20\boldsymbol{a}_x)\times(-\boldsymbol{a}_z) = 10\boldsymbol{a}_y\text{ A/m} \qquad (\boldsymbol{K} = 20\boldsymbol{a}_x)$$

SOL 4.2.8 Correct answer is -10.



Magnetic field intensity at any point P due to an infinite current carrying sheet is defined as

$$\boldsymbol{H} = \frac{1}{2}\boldsymbol{K}\times\boldsymbol{a}_n$$

where $\boldsymbol{K}$ is the current density of the infinite sheet and $\boldsymbol{a}_n$ is the unit vector normal to the current sheet directed toward the point P.

Since we have to determine the magnetic field intensity at point $(2, -1, 5)$ which is above the plane sheet as shown in figure, so we have,

$$\boldsymbol{a}_n = +\boldsymbol{a}_z$$

Therefore the magnetic field intensity at the point $(2, -1, 5)$ is

$$\boldsymbol{H} = \frac{1}{2}(20\boldsymbol{a}_x\times\boldsymbol{a}_z) = -\frac{20}{2}\boldsymbol{a}_y = -10\boldsymbol{a}_y\text{ A/m} \qquad (\boldsymbol{K} = 20\boldsymbol{a}_x)$$

SOL 4.2.9 Correct answer is -0.13.

Consider the current density at $\rho = 8$ cm is $\boldsymbol{J}$ directed along $+\boldsymbol{a}_z$.

Now the magnetic field for $\rho > 8$ cm must be zero.

i.e.
$$\boldsymbol{H} = 0 \qquad (\text{for } \rho > 8\text{ cm})$$

So from Ampere's circuital law we have

$$\int \boldsymbol{H}\cdot d\boldsymbol{l} = I_{enc} = 0$$

Since for the region $\rho > 8$ cm the Amperian loop will have all the current distributions enclosed inside it.

i.e.
$$I_{enc} = 14\times10^{-3} + 2\times(2\pi\times0.5\times10^{-2}) - 0.8\times(2\pi\times0.25\times10^{-2}) + J(2\pi\times8\times10^{-2})$$
$$= 6.43\times10^{-2} + J(16\pi\times10^{-2})$$

So we have
$$6.43\times10^{-2} + J(16\pi\times10^{-2}) = 0 \qquad (I_{enc} = 0)$$

or
$$J = -\frac{6.43\times10^{-2}}{16\pi\times10^{-2}}$$

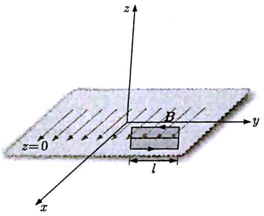
or
$$\boldsymbol{J} = -0.13\boldsymbol{a}_z\text{ A/m}$$

SOL 4.2.10 Correct answer is -2.

For determining the magnetic field at any point above the plane $z=0$, we draw a rectangular Amperian loop parallel to the y-z plane and extending an equal distance above and below the surface as shown in the figure.

From Ampere's circuital law,

$$\oint \boldsymbol{B}\cdot d\boldsymbol{l} = \mu_0 I_{enc}$$



Since the infinite current sheet is located in the plane $z=0$ so, the z-component of the magnetic flux density will be cancelled due to symmetry and in the closed Amperian loop the integral will be only along y-axis. Thus we have

$$B(2l) = \mu_0 I_{enc}$$

$$2Bl = \mu_0 Kl \qquad (I_{enc} = Kl)$$

As determined by right hand rule, the magnetic flux density above the plane $z=0$ will be in $-\boldsymbol{a}_y$ direction. So we have the flux density above the current sheet as

$$\boldsymbol{B} = -\frac{\mu_0\times 4}{2}\boldsymbol{a}_y = -2\mu_0\boldsymbol{a}_y \text{ wb/m}^2 \qquad (K=4\text{ A/m})$$

ALTERNATIVE METHOD :

The magnetic flux density produced at any point P due to an infinite sheet carrying uniform current density $\boldsymbol{K}$ is defined as

$$\boldsymbol{B} = \frac{1}{2}\mu_0(\boldsymbol{K}\times \mathbf{a}_n)$$

where $\boldsymbol{a}_n$ is the unit vector normal to the sheet directed toward the point P.

So, magnetic flux density at any point above the current sheet $\boldsymbol{K} = 4\boldsymbol{a}_x$ is

$$\boldsymbol{B} = \frac{1}{2}\mu_0(4\boldsymbol{a}_x)\times(\boldsymbol{a}_z) = -2\mu_0\boldsymbol{a}_y \text{ wb/m}^2 \qquad (\boldsymbol{a}_n = \boldsymbol{a}_z)$$

SOL 4.2.11 Correct answer is -4.

Magnetic flux density at a certain point is equal to the curl of magnetic vector potential at the point.

i.e. $\qquad \boldsymbol{B} = \nabla\times\boldsymbol{A}$

So from the above determined value of magnetic flux density $\boldsymbol{B}$ we have,

$$\nabla\times\boldsymbol{A} = -2\mu_0\boldsymbol{a}_y \text{ wb/m}^2 \qquad (1)$$

Since $\boldsymbol{A}$ is parallel to $\boldsymbol{K}$ so the vector potential $\boldsymbol{K}$ will depend only on z. Hence, we have

$$\boldsymbol{A} = A(z)\,\boldsymbol{a}_x$$

From equation (1) we have,

$$-2\mu_0\boldsymbol{a}_y = \begin{vmatrix} \boldsymbol{a}_x & \boldsymbol{a}_y & \boldsymbol{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A(z) & 0 & 0 \end{vmatrix}$$

$$-2\mu_0\boldsymbol{a}_y = -\frac{\partial A(z)}{\partial z}\boldsymbol{a}_y$$

or $\qquad A(z) = 2\mu_0 z$

So, $\qquad \boldsymbol{A} = 2\mu_0 z\boldsymbol{a}_x$

Therefore the vector magnetic potential at $z=-2$ is

$$\boldsymbol{A} = -4\mu_0\boldsymbol{a}_x \text{ wb/m}$$

SOL 4.2.12 Correct answer is 12.

Current density at any point in a magnetic field is defined as the curl of magnetic field intensity at the point.

i.e. $\qquad \boldsymbol{J} = \nabla\times\boldsymbol{H}$

Since the magnetic field intensity in the free space is given as

$$\boldsymbol{H} = 2\rho^2\boldsymbol{a}_\phi$$

Therefore the current density is

$$\boldsymbol{J} = \frac{1}{\rho}\frac{\partial(\rho)(2\rho^2)}{\partial\rho}\boldsymbol{a}_z = \frac{1}{\rho}\frac{\partial}{\partial\rho}(2\rho^3)\,\boldsymbol{a}_z$$

$$= 6\rho\boldsymbol{a}_z = 12\boldsymbol{a}_z \text{ A/m}^2 \qquad (\rho = 2\text{ m})$$

SOL 4.2.13 Correct answer is 6.

Given that the cylindrical wire located along z-axis produces a magnetic field intensity, $\boldsymbol{H} = 3\rho\boldsymbol{a}_\phi$.

So, applying the differential form of Ampere's circuital low we have the current density with in the conductor as

$$\boldsymbol{J} = \nabla\times\boldsymbol{H}$$

$$= \frac{1}{\rho}\begin{vmatrix} \boldsymbol{a}_\rho & \rho\boldsymbol{a}_\phi & \boldsymbol{a}_z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix} = \frac{1}{\rho}\begin{vmatrix} \boldsymbol{a}_\rho & \rho\boldsymbol{a}_\phi & \boldsymbol{a}_z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ 0 & 3\rho^2 & 0 \end{vmatrix}$$

$$= \frac{1}{\rho}\frac{\partial}{\partial\rho}(3\rho^2)\boldsymbol{a}_z = 6\boldsymbol{a}_z \text{ A/m}^2$$

SOL 4.2.14 Correct answer is 5.5 .

Magnetic dipole moment of a conducting loop carrying current I is defined as :

$$m = IS$$

where S is the area enclosed by the conducting loop. So we have

$$m = 7\times(\pi\times 0.5^2) = 5.5 \qquad (I = 7\text{ A}, R = 0.5\text{ m})$$

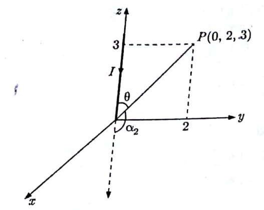
The direction of the moment is determined by right hand rule as when the curl of fingers lies along the direction of current,then the thumb indicates the direction of moment.

So, $\qquad \boldsymbol{m} = 5.5\boldsymbol{a}_z$ A-m^2

SOL 4.2.15 Correct answer is 0.73 .

Since the current is flowing in the $-\boldsymbol{a}_z$ direction

So, $\qquad Idl = 10dz(-\boldsymbol{a}_z)$



Magnetic field intensity at any point P due to a filamentary current I is defined as

$$\boldsymbol{H} = \frac{I}{4\pi\rho}[\cos\alpha_2 - \cos\alpha_1]\boldsymbol{a}_\phi$$

where $\rho \rightarrow$ distance of point P from the current filament.

$\alpha_1 \rightarrow$ angle subtended by the lower end of the element at P.

$\alpha_2 \rightarrow$ angle subtended by the upper end of the element at P.

Now from the figure we have,

$$\rho = 2$$

$$\alpha_2 = \pi - \theta$$

or $$\cos\alpha_2 = \cos(\pi - \theta)$$

$$= -\cos\theta = -\frac{3}{\sqrt{2^2+3^2}} = -\frac{3}{\sqrt{13}}$$

and $$\alpha_1 = 0 \qquad \text{(angle subtended by end } z = \infty)$$

or $$\cos\alpha_1 = \cos 0 = 1$$

So, $$\boldsymbol{H} = \frac{I}{4\pi\rho}[\cos\alpha_2 - \cos\alpha_1]\boldsymbol{a}_\phi$$

$$= \frac{I}{4\pi \times 2}\left[1 - \left(-\frac{3}{\sqrt{13}}\right)\right]\boldsymbol{a}_\phi = \frac{(10)}{8\pi} \times \left(1 + \frac{3}{\sqrt{13}}\right)\boldsymbol{a}_\phi$$

Now the direction of magnetic field intensity is defined as

$$\boldsymbol{a}_\phi = \boldsymbol{a}_l \times \boldsymbol{a}_\rho$$

where $\boldsymbol{a}_l$ is unit vector along the line current and $\boldsymbol{a}_\rho$ is the unit vector normal to the line current directed toward the point P.

So we have $$\boldsymbol{a}_\phi = (-\boldsymbol{a}_z) \times (\boldsymbol{a}_y) = \boldsymbol{a}_x$$

Therefore, $$\boldsymbol{H} = \frac{10}{8\pi}\left(1 + \frac{3}{\sqrt{13}}\right)(\boldsymbol{a}_x)$$

$$= 0.73\boldsymbol{a}_x \text{ A/m}$$

SOL 4.2.16 Correct answer is 2.8648 .

As the magnetic field intensity at the center of the triangle produced by all the three sides will be exactly equal so we consider only one side lying along x-axis that carries 4 A current flowing in $+\boldsymbol{a}_x$ direction as shown in the figure.

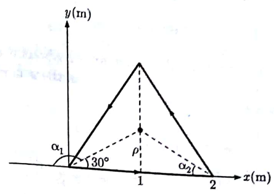
Now the magnetic field intensity at any point P due to a filamentary current I is defined as

$$\boldsymbol{H} = \frac{I}{4\pi\rho}[\cos\alpha_2 - \cos\alpha_1]\boldsymbol{a}_\phi$$

where $\rho \rightarrow$ distance of point P from the current filament.

$\alpha_1 \rightarrow$ angle subtended by the lower end of the element at P.

$\alpha_2 \rightarrow$ angle subtended by the upper end of the element at P.



From the figure we have

$$\tan 30° = \frac{\rho}{1} \Rightarrow \rho = \frac{1}{\sqrt{3}}$$

$$\alpha_1 = \pi - \pi/6 = \frac{5\pi}{6} \Rightarrow \cos\alpha_1 = \cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$$

and $$\alpha_2 = 30° \Rightarrow \cos\alpha_2 = \cos 30° = \frac{\sqrt{3}}{2}$$

So the magnetic field intensity produced by one side of the triangle at centre of the triangle is

$$\boldsymbol{H}_1 = \frac{4}{4\pi \times \frac{1}{\sqrt{3}}}[\cos\alpha_2 - \cos\alpha_1]\boldsymbol{a}_\phi$$

$$= \frac{\sqrt{3}}{\pi}\left[\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right]\boldsymbol{a}_\phi = \frac{3}{\pi}\boldsymbol{a}_\phi$$

Now the direction of magnetic field intensity is determined as

$$\boldsymbol{a}_\phi = \boldsymbol{a}_l \times \boldsymbol{a}_\rho$$

where $\boldsymbol{a}_l$ is unit vector along the line current and $\boldsymbol{a}_\rho$ is the unit vector normal to the line current directed toward the point P.

and since the line current is along x-axis so we have

$$\boldsymbol{a}_\phi = \boldsymbol{a}_x \times \boldsymbol{a}_y = \boldsymbol{a}_z \qquad (\boldsymbol{a}_l = \boldsymbol{a}_x,\ \boldsymbol{a}_\rho = \boldsymbol{a}_y)$$

Therefore the net magnetic field intensity due to all the three sides of triangle is

$$\boldsymbol{H} = 3\boldsymbol{H}_1 = 3 \times \left(\frac{3}{\pi}\right)\boldsymbol{a}_z = \frac{9}{\pi}\boldsymbol{a}_z \text{ A/m} \qquad (\boldsymbol{a}_\phi = \boldsymbol{a}_z)$$

$$= 2.8648\boldsymbol{a}_z \text{ A/m}$$

SOL 4.2.17 Correct answer is -0.75 .

According to Biot-savart law, magnetic field intensity at any point P due to the current sheet element $\boldsymbol{K}dS$ is defined as

$$\boldsymbol{H} = \int_s \frac{\boldsymbol{K}dS \times \boldsymbol{a}_R}{4\pi R^2}$$

where $\boldsymbol{R}$ is the vector distance of point P from the current element.

Now we consider a point $(0, y, z)$ on the current carrying sheet, from which we have the vector distance of point (3,0,0)

$$\boldsymbol{R} = (3\boldsymbol{a}_x + 0\boldsymbol{a}_y + 0\boldsymbol{a}_z) - (0\boldsymbol{a}_x + y\boldsymbol{a}_y + z\boldsymbol{a}_z) = (3\boldsymbol{a}_x - y\boldsymbol{a}_y - z\boldsymbol{a}_z)$$

or $$\boldsymbol{a}_R = \frac{3\boldsymbol{a}_x - y\boldsymbol{a}_y - z\boldsymbol{a}_z}{\sqrt{3^2 + y^2 + z^2}} = \frac{3\boldsymbol{a}_x - y\boldsymbol{a}_y - z\boldsymbol{a}_z}{\sqrt{9 + y^2 + z^2}}$$

Therefore the magnetic field intensity due to the current sheet is

$$H = \int_{z=-2}^{2}\int_{y=-\infty}^{\infty} \frac{4a_y \times (3a_x - ya_y - za_y)}{4\pi(9+y^2+z^2)^{3/2}} dydz \qquad (K = 4a_y)$$

$$= \int_{z=-2}^{2}\int_{y=-\infty}^{\infty} \frac{4(-za_x - 3a_z)}{4\pi(9+y^2+z^2)^{3/2}} dydz$$

We note that the x component is anti symmetric in z about the origin (odd parity). Since the limits are symmetric, the integral of the x component over z is zero. So we are left with

$$H = \int_{-2}^{2}\int_{-\infty}^{\infty} \frac{-12a_z}{4\pi(9+y^2+z^2)^{3/2}} dydz$$

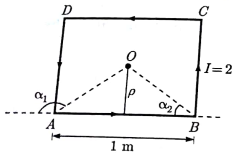
$$= -\frac{3}{\pi}a_z \int_{-2}^{2}\left[\frac{y}{(z^2+9)\sqrt{9+y^2+z^2}}\right]_{-\infty}^{+\infty} dz$$

$$= -\frac{3}{\pi}a_z \int_{-2}^{2}\frac{2}{z^2+9}dz = -\frac{6}{\pi}a_z\left[\frac{1}{3}\tan^{-1}\left(\frac{z}{3}\right)\right]_{-2}^{2}$$

$$= -\frac{2}{\pi} \times (2) \times (0.59)\, a_z$$

$$= -0.75 a_z \text{ A/m}$$

SOL 4.2.18 Correct answer is 2.26 .



As all the four sides of current carrying square loop produces the same magnetic field at the center so we consider only the line current AB for which we determine the magnetic field intensity at the center.
Now the magnetic field intensity at any point P due to a filamentary current I is defined as

$$H = \frac{I}{4\pi\rho}[\cos\alpha_2 - \cos\alpha_1]a_\phi$$

where $\rho \rightarrow$ distance of point P from the current filament.

$\alpha_1 \rightarrow$ angle subtended by the lower end of the filament at P.

$\alpha_2 \rightarrow$ angle subtended by the upper end of the filament at P.

From the figure, we have

$$\rho = \frac{1}{2}\text{ m}, \quad \alpha_2 = 45^\circ \text{ and } \alpha_1 = 180^\circ - 45^\circ$$

So the magnetic field intensity at the centre O due to the line current AB is

$$H_1 = \frac{I}{2\pi R}[\cos\alpha_2 - \cos\alpha_1]$$

$$= \frac{1}{2\pi \times (1/2)}[\cos 45^\circ - \cos(180^\circ - 45^\circ)]$$

$$= \frac{1}{\pi} \times \frac{2}{\sqrt{2}} = \frac{\sqrt{2}}{\pi} \text{ A/m}$$

and the magnetic flux density produced by the line current AB is

$$B_1 = \mu_0 H_1 = 4\pi \times 10^{-7} \times \frac{\sqrt{2}}{\pi}$$

$$= 5.66 \times 10^{-7} \text{ wb/m}^2$$

Therefore the net magnetic flux density due to the complete square loop will be four times of B_1
i.e.

$$B = 4B_1 = 4 \times (5.66 \times 10^{-7})$$

$$= 2.26 \times 10^{-6} \text{ wb/m}^2$$

SOL 4.2.19 Correct answer is 0.2 .
According to Biot-savart law, magnetic field intensity at any point P due to the current element Idl is defined as

$$H = \int \frac{Idl \times a_R}{4\pi R^2}$$

where R is the vector distance of point P from the current element.
As the cross product of two parallel lines is always zero so the straight segments will produce no field at P. Therefore the net magnetic field produced at point P will be only due to the two circular section.
i.e.

$$H = H_{CD} + H_{AB}$$

or

$$H = \left[\int_0^{\pi/2} \frac{(I\rho d\phi a_\phi) \times (-a_\rho)}{4\pi\rho^2}\right]_{\text{at } \rho = 1\text{ m}} + \left[\int_0^{\pi/2} \frac{I\rho d\phi(-a_\phi) \times (-a_\rho)}{4\pi\rho^2}\right]_{\text{at } \rho = 2\text{ m}}$$

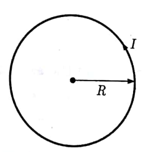
$$= \int_0^{\pi/2} \frac{Ia_z}{4\pi(1)}d\phi - \int_0^{\pi/2} \frac{Ia_z}{4\pi(2)}d\phi$$

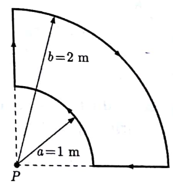
$$= \frac{3.2}{4\pi} \times \left[1 - \frac{1}{2}\right] \times \left(\frac{\pi}{2}\right)a_z = 0.2 \text{ A/m}$$

ALTERNATIVE METHOD :

The magnetic field intensity produced at the center of a circular loop of radius R carrying current I is defined as

$$H = \frac{I}{2R}$$





and since the straight line will not produce any field at point P so due to the two quarter circles having current in opposite direction, magnetic field at the center will be

$$H = \frac{1}{4}\left[\frac{I}{2a} - \frac{I}{2b}\right]$$

where $a \rightarrow$ inner radius

$b \rightarrow$ outer radius

$$H = \frac{1}{4}\left[\frac{3(2)}{2 \times 1} - \frac{3(2)}{2 \times 2}\right] = 0.2 \text{ A/m}$$

SOL 4.2.20 Correct answer is 0.82 .

The magnetic field intensity at any point P due to an infinite filamentary current I is defined as

$$H = \frac{I}{2\pi\rho}$$

where ρ is the distance of point P from the infinite current filament.

Now the two semi infinite lines will be in combination treated as a single infinite line for which magnetic field intensity at point P will be

$$H_1 = \frac{I}{2\pi R} \quad (R \text{ is the length of point } P \text{ from line current})$$

$$= \frac{4}{2\pi \times 2} = \frac{1}{\pi} \qquad (I = 4\,\text{A}, R = 2\,\text{m})$$

As the magnetic field intensity produced at the center of a circular loop of radius R carrying current I is defined as

$$H = \frac{I}{2R}$$

So magnetic field produced at point P due to the semi circular segment is

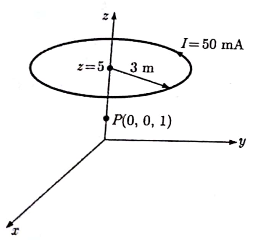
$$H_2 = \frac{1}{2} \times \frac{I}{2R} = \frac{1}{2}$$

Therefore net magnetic field intensity produced at point P is

$$H = H_1 + H_2 = \frac{1}{\pi} + \frac{1}{2}$$

$$= 0.82\,\text{A/m}$$

SOL 4.2.21 Correct answer is 1.8 .



Magnetic field intensity produced at any point P on the axis of the circular loop carrying current I is defined as

$$H = \frac{I\rho^2}{2(\rho^2 + h^2)^{3/2}}$$

where h is the distance of point P from the centre of circular loop and ρ is the radius of the circular loop.

From the figure we have

$$\rho = 3\,\text{m} \quad \text{and} \quad h = 5 - 1 = 4\,\text{m}$$

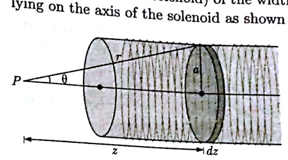
and using right hand rule we conclude that the magnetic field intensity is directed along $+\boldsymbol{a}_z$. So the magnetic field intensity produced at point P is

$$\boldsymbol{H} = \frac{50 \times 10^{-3}(3)^2}{2(3^2 + 4^2)^{3/2}}\boldsymbol{a}_z$$

$$= \frac{9 \times 50 \times 10^{-3}}{2 \times 125}\boldsymbol{a}_z = 1.8\boldsymbol{a}_z\,\text{mA/m}$$

SOL 4.2.22 Correct answer is 4.

Assume the cylindrical tube is of radius a for which we have to determine the magnetic field intensity at the axis of solenoid. Now we consider a small ring (small section of solenoid) of the width dz at a distance z from point P lying on the axis of the solenoid as shown in the figure.



The total current flowing in the loop of the ring will be

$$dI = nIdz \qquad \text{where } n \text{ is the no of turns per unit length}$$

Since magnetic field intensity produced at any point P on the axis of the circular loop carrying current I is defined as

$$H = \frac{I\rho^2}{2(\rho^2 + h^2)^{3/2}}$$

where h is the distance of point P from the centre of circular loop and ρ is the radius of the circular loop.

So we have the magnetic field intensity due to the ring as

$$dH = \frac{(nIdz)\,a^2}{2(a^2 + z^2)^{3/2}} \qquad (\rho = a, h = z)$$

From the figure we have

$$z = a\cot\theta \Rightarrow dz = -\frac{a}{\sin^2\theta}d\theta$$

and

$$\sin\theta = \frac{a}{r} = \frac{a}{\sqrt{a^2 + z^2}} \Rightarrow \frac{1}{(a^2 + z^2)^{3/2}} = \frac{\sin^3\theta}{a^3}$$

The total magnetic field intensity produced at point P due to the solenoid is

$$H = \int_{z=-\infty}^{\infty} \frac{(nIdz)\,a^2}{2(a^2 + z^2)^{3/2}} = \frac{nI}{2}\int_{\theta=0}^{\pi} \frac{a^2\sin^3\theta}{a^3\sin^2\theta}(-a\,d\theta)$$

$$= -\frac{nI}{2}\int_{\theta=\pi}^{0} \sin\theta\,d\theta = \frac{nI}{2}(\cos 0 - \cos\pi)$$

$$= nI = 1000 \times 4 \times 10^{-3} = 4\,\text{A/m} \qquad (n = 1000, I = 4\,\text{mA})$$

SOL 4.2.23 Correct answer is -3.

As calculated in the previous question the magnetic field intensity inside a long solenoid carrying current I is defined as

$$H = nI \qquad \text{where } n \text{ is no. of turns per unit length}$$

and since using right hand rule we conclude that the direction of magnetic field intensity will be right wards $(+\boldsymbol{a}_y)$ due to outer solenoid and left wards $((-\boldsymbol{a}_y))$ due to inner solenoid. So the resultant magnetic field intensity produced inside the inner solenoid will be

$$\boldsymbol{H} = \boldsymbol{H}_1 + \boldsymbol{H}_2 = n_1 I(-\boldsymbol{a}_y) + n_2 I\boldsymbol{a}_y$$

where n_1 and n_2 are the no. of turns per unit length of the inner and outer solenoids respectively.

So

$$\boldsymbol{H} = -(3 \times 10^{-3})(2000)\,\boldsymbol{a}_y + (3 \times 10^{-3})(1000)\,\boldsymbol{a}_y$$

$$= 3 \times 10^{-3}(-1000)\,\boldsymbol{a}_y = -3\boldsymbol{a}_y\,\text{A/m}$$

SOL 4.2.24 Correct answer is 3.

Since no any magnetic field is produced at any point out side a solenoid so in the region between the two solenoids field will be produced only due to the outer solenoid.

i.e. $$\boldsymbol{H} = n_2 I \boldsymbol{a}_y$$
$$= 1000 \times 3 \times 10^{-3} \boldsymbol{a}_y = 3\boldsymbol{a}_y \text{ A/m}$$

SOL 4.2.25 Correct answer is 0.

Since no any magnetic field is produced at any point out side a solenoid so, at any point outside the outer solenoid, the net magnetic field intensity produced due to the two solenoids will be zero.

SOL 4.2.26 Correct answer is 12.5 .

Since the current density inside the wire is given by
$$J \propto \rho$$
So we have, $$J = k\rho \qquad \text{where } k \text{ is a constant.}$$
and the total current flowing in the wire is given by
$$I_0 = \int_S \boldsymbol{J} \cdot d\boldsymbol{S}$$
or $$5 \times 10^{-3} = \int_0^{2\times 10^{-2}} k\rho(2\pi\rho)\, d\rho \qquad (I_0 = 5\,\text{mA})$$
$$5 \times 10^{-3} = \frac{2\pi k(2\times 10^{-2})^3}{3}$$
So we have $$k = \frac{3 \times 5 \times 10^{-3}}{2\pi \times 8 \times 10^{-6}} = \frac{15}{16\pi} \times 10^3$$
Now for the Amperian loop at $\rho = 1\,\text{cm}$ enclosed current is
$$I_{enc} = \int_S \boldsymbol{J} \cdot d\boldsymbol{S}$$
$$= \int_{\rho=0}^{1\times 10^{-2}} k\rho(2\pi\rho)\, d\rho$$
$$= \left(\frac{15}{16\pi} \times 10^3\right) \times 2\pi\left[\frac{\rho^3}{3}\right]_0^{1\times 10^{-2}}$$
$$= \frac{15}{8} \times \frac{1}{3} \times 10^{-3} = \frac{15}{24} \times 10^{-3}$$
So from Ampere's circuital law we have
$$\oint_L \boldsymbol{B} \cdot d\boldsymbol{l} = \mu_0 I_{enc}$$
$$B(2\pi\rho) = \frac{\mu_0 15}{24} \times 10^{-3}$$
Therefore the magnetic flux density at $\rho = 1\,\text{cm}$ is
$$B = \frac{15}{24} \times \frac{10^{-3}}{2\pi(1 \times 10^{-2})} \times 4\pi \times 10^{-7}$$
$$= 1.25 \times 10^{-8}$$
$$= 12.5\,\text{nWb/m}^2$$

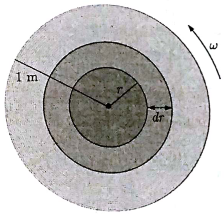
SOL 4.2.27 . Correct answer is -259.

Current density at any point in a magnetic field is defined as the curl of magnetic field intensity at the point.

i.e. $$\boldsymbol{J} = \nabla \times \boldsymbol{H}$$
So the current density component in $\boldsymbol{a}_z$ direction is

$$J_z = (\nabla \times \boldsymbol{H})_z$$
$$= \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}$$
$$= -\left(\frac{xz^2}{(y+1)^2} + 6x^2 z\right)\boldsymbol{a}_z$$
Therefore the total current passing through the surface $x = 2\,\text{m}, 1 \le y \le 4\,\text{m}$, $3 \le z \le 4\,\text{m}$ is
$$I = \int_S \boldsymbol{J}_z \cdot d\boldsymbol{S}$$
$$= -\int_{z=3}^{4}\int_{y=1}^{4}\left(\frac{xz^2}{(y+1)^2} + 6x^2 z\right)dydz \qquad (d\boldsymbol{S} = dydz\boldsymbol{a}_z)$$
$$= -\int_3^4\int_1^4\left(\frac{2z^2}{(y+1)^2} + 24z\right)dydz \qquad (x = 2\,\text{m})$$
$$= -\int_3^4\left[\frac{-2z^2}{y+1} + 24zy\right]_1^4 dz = -\int_3^4\left(\frac{3}{5}z^2 + 72z\right)dz$$
$$= -259\text{ A}$$

SOL 4.2.28 Correct answer is 1.5708 .



Magnetic dipole moment of a conducting loop carrying current I is defined as :
$$m = IS$$
where S is the area enclosed by the conducting loop. So for a ring of radius r, magnetic dipole moment
$$m = I(\pi r^2).$$
Now as the charged disk(charge density, $\rho_S = 20\,\text{C/m}^2$) is rotating with angular velocity $\omega = 0.1\,\text{rad/s}$ so, the current in the loop is given as
$$dI = \rho_S \omega r dr$$
Therefore the magnetic dipole moment is
$$m = \int dI(\pi r^2)$$
$$= \int_{r=0}^{1} (\rho_S \omega r dr)(\pi r^2)$$
$$= \rho_S \omega \pi \int_0^1 r^3\, dr$$
$$= 20 \times 0.1 \times \pi\left[\frac{r^4}{4}\right]_0^1$$
$$= \frac{\pi}{2} = 1.5708\text{ A-m}^2$$

SOL 4.2.29 Correct answer is 1.39 .

Magnetic flux density across the toroid at a distance r from it's center is defined as

$$B = \frac{\mu_0 NI}{2\pi r}\,\boldsymbol{a}_\phi$$

where $N \rightarrow$ Total no. of turns

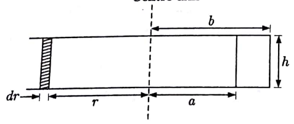
$I \rightarrow$ Current flowing in the toroid

So, the total magnetic flux across the toroid is given by the surface integral of the flux density

i.e. $$\phi = \int_S \boldsymbol{B}\cdot d\boldsymbol{S}$$

where $d\boldsymbol{S}$ is differential surface area vector.

Consider a width dr of toroid at a distance r from it's center as shown in figure



So we have the total magnetic flux across the toroid as

$$\phi = \int_{r=1\,\text{m}}^{2}\left(\frac{\mu_0 NI}{2\pi r}\boldsymbol{a}_\phi\right)(hdr\boldsymbol{a}_\phi) \qquad (d\boldsymbol{S} = hdr\boldsymbol{a}_\phi)$$

$$= \frac{4\pi\times10^{-7}\times10^5\times10\times10}{2\pi}\ln\left(\frac{2}{1}\right) \qquad (N = 10^5,\ I = 10\,\text{A})$$

$$= 1.39\ \text{Wb}$$

SOL 4.2.30 Correct answer is -4.31 .

As determined in previous question the magnetic flux density across the toroid at a distance r from it's center is

$$\boldsymbol{B} = \frac{\mu_0 NI}{2\pi r}\boldsymbol{a}_\phi$$

So at the mean radius,

$$r = \frac{a+b}{2} = 1.5\,\text{m}$$

we have, $$\boldsymbol{B} = \frac{\mu_0 NI}{3\pi}\boldsymbol{a}_\phi \qquad (r = 1.5\,\text{m})$$

Therefore the total magnetic flux is

$$\phi' = \int \boldsymbol{B}\cdot d\boldsymbol{S} = \int_{r=1}^{2}\left(\frac{\mu_0 NI}{3\pi}\boldsymbol{a}_\phi\right)(hdr\boldsymbol{a}_\phi) \qquad (d\boldsymbol{S} = hdr\boldsymbol{a}_\phi)$$

$$= \frac{4\pi\times10^{-7}\times10^5\times10\times10}{3\pi}[r]_1^2 \qquad (N = 10^5,\ I = 10\,\text{A})$$

$$= 1.33\ \text{Wb}$$

Thus, the percentage of error is

$$\%\,\text{error} = \frac{\phi'-\phi}{\phi}\times100\% \qquad (\phi = 1.39\ \text{wb as calculated above})$$

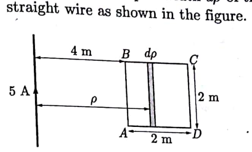
$$= \frac{1.33-1.39}{1.39}\times100\% = -4.31\%$$

SOL 4.2.31 Correct answer is 0.81 .

The magnetic flux density produced at a distance ρ from a straight wire carrying current I is defined as

$$B = \frac{\mu_0 I}{2\pi\rho}$$

Now consider a strip of width $d\rho$ of the square loop at a distance ρ from the straight wire as shown in the figure.



Total magnetic flux crossing the strip is

$$d\psi_m = B(2d\rho) \qquad (\text{area of strip} = 2d\rho)$$

$$= \frac{\mu_0 I}{2\pi\rho}(2d\rho)$$

So, the flux crossing the complete square loop is

$$\psi_m = \int d\psi_m$$

$$= \int_{\rho=4}^{6}\frac{\mu_0 I}{2\pi\rho}(2d\rho)$$

$$= \frac{\mu_0 I}{\pi}[\ln\rho]_4^6$$

$$= \frac{4\pi\times10^{-7}\times5}{\pi}\left(\ln\frac{6}{4}\right)$$

$$= 8.11\times10^{-7}\ \text{Weber} = 0.81\,\mu\text{Wb}$$

SOL 4.2.32 Correct answer is 208.5 .

As calculated in previous question the total flux crossing through the square loop due to the straight conducting element is

$$\psi_m = \int_{\rho=a}^{b}\frac{\mu_0 I}{2\pi\rho}(Ld\rho)$$

where I is the current carried by the conductor, L is the side of the square loop and a, b are the distance of the two sides of square loop from the conductor.

So we have $$L = 0.5\,\text{m}$$

$$a = 0.3 - \frac{0.5}{2} = 0.05\,\text{m}$$

$$b = 0.3 + \frac{0.5}{2} = 0.55\,\text{m}$$

Thus, $$\psi_m = \int_{\rho=0.05}^{0.55}\frac{\mu_0 I}{2\pi\rho}(0.5d\rho)$$

$$= \frac{\mu_0 I}{4\pi}[\ln\rho]_{0.05}^{0.55} = \frac{\mu_0 I}{4\pi}(\ln 11)$$

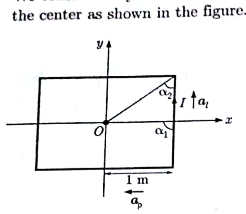
Therefore the current that produces the net flux $\psi_m = 5\times10^{-5}\,\text{Tm}^2$ is

$$I = \frac{4\pi}{\mu_0(\ln(1))}\times5\times10^{-5}$$

$$= 208.5\ \text{A}$$

SOL 4.2.33 Correct answer is 5.6569 .

We consider only the half side of the loop to determine the flux density at the center as shown in the figure.



The magnetic flux density $\boldsymbol{B}$ produced at any point P due to a straight wire carrying current I is defined as

$$\boldsymbol{B} = \frac{\mu_0 I}{4\pi\rho}[\cos\alpha_2 - \cos\alpha_1]\boldsymbol{a}_\phi$$

where $\rho \rightarrow$ distance of point P from the straight wire.

$\alpha_1 \rightarrow$ angle subtended by the lower end of the wire at P.

$\alpha_2 \rightarrow$ angle subtended by the upper end of the wire at P.

and the direction of the magnetic flux density is given as

$$\boldsymbol{a}_\phi = \boldsymbol{a}_l \times \boldsymbol{a}_\rho$$

where $\boldsymbol{a}_l$ is unit vector along the line current and $\boldsymbol{a}_\rho$ is the unit vector normal to the line current directed toward the point P.

Therefore, the magnetic flux density produced at centre O due to the half side of the square loop is

$$\boldsymbol{B}_1 = \frac{\mu_0 I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\boldsymbol{a}_\phi$$

where $\rho = 1\,\text{m}$ $\quad \alpha_1 = \frac{\pi}{2}$ and $\cos\alpha_2 = \frac{1}{\sqrt{1^2+1^2}} = \frac{1}{\sqrt{2}}$

Thus, $$\boldsymbol{B}_1 = \frac{(4\pi\times10^{-7})(1)}{4\pi(1)}\left(\frac{1}{\sqrt{2}} - 0\right)\boldsymbol{a}_z \qquad (\boldsymbol{a}_\phi = \boldsymbol{a}_y \times (-\boldsymbol{a}_x) = \boldsymbol{a}_z)$$

$$= \frac{10^{-7}}{\sqrt{2}}\boldsymbol{a}_z\ \text{Wb/m}^2$$

As all the half sides of the loop will produce the same magnetic flux density at the centre so, the net magnetic flux density produced at the centre due to whole square loop will be

$$\boldsymbol{B} = 8\boldsymbol{B}_1 = 4\sqrt{2}\times10^{-7}\boldsymbol{a}_z\ \text{Wb/m}^2$$

$$= 5.6569\times10^{-7}\boldsymbol{a}_z\ \text{Wb/m}^2$$

SOL 4.2.34 Correct answer is 0.6366 .

Using right hand rule we conclude that the field intensity produced at centre of the loop by the loop wire and the straight wire are opposing each other, so, the field intensity at the centre of the loop will be zero if

$$H_{wire} = H_{loop} \qquad ...(1)$$

where H_{wire} is the field intensity produced at the center of loop due to the straight wire and H_{loop} is the field intensity produced at the center of loop due to the current in the circular loop.

Since the magnetic field intensity produced at a distance ρ from an infinitely long straight wire carrying current I is defined as

$$H = \frac{I}{2\pi\rho}$$

So we have $$H_{wire} = \frac{I}{2\pi(1)} = \frac{20}{2\pi} = \frac{10}{\pi} \qquad (I = 20\,\text{A}, \rho = 1\,\text{m})$$

and as calculated earlier the field intensity produced by circular loop at its center is

$$H_{loop} = \frac{I}{2a} \qquad \text{where } a \text{ is the radius of the loop}$$

or, $$H_{loop} = \frac{I}{2(10\times10^{-2})} = \frac{10I}{2} = 5I \qquad (a = 10\,\text{cm})$$

So putting the values in eq. (1) we get

$$\frac{10}{\pi} = 5I$$

Thus, $$I = \frac{2}{\pi} = 0.6366\ \text{A}$$

SOL 4.2.35 Correct answer is -8.

The flux density due to infinite current carrying sheet is defined as

$$\boldsymbol{B} = \frac{\mu_0}{2}\boldsymbol{K}\times\boldsymbol{a}_n$$

where $\boldsymbol{K}$ is surface current density and $\boldsymbol{a}_n$ is unit vector normal to the surface directed toward the point where flux density is to be determined

So, for the sheet in $z = 0$ plane,

$$\boldsymbol{B}_1 = \frac{\mu_0}{2}(4\boldsymbol{a}_x)\times(\boldsymbol{a}_z) = -2\mu_0\boldsymbol{a}_y \qquad (\boldsymbol{a}_n = \boldsymbol{a}_z)$$

and for the sheet in $z = 2\,\text{m}$ plane

$$\boldsymbol{B}_2 = \frac{\mu_0}{2}(-4\boldsymbol{a}_x)\times(-\boldsymbol{a}_z) = -2\mu_0\boldsymbol{a}_y \qquad (\boldsymbol{a}_n = -\boldsymbol{a}_z)$$

Therefore, the net flux density between the sheets is

$$\boldsymbol{B} = \boldsymbol{B}_1 + \boldsymbol{B}_2 = -4\mu_0\boldsymbol{a}_y$$

Thus the magnetic flux per unit length along the direction of current is

$$\psi_m/l = \boldsymbol{B}\times(\text{Distance between the plates})$$

$$= -8\mu_0\boldsymbol{a}_y\ \text{Wb/m}$$

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SOLUTIONS 4.3

SOL 4.3.1 Option (A) is correct.
It is not possible to have an isolated magnetic poles (or magnetic charges). If we desire to have an isolated magnetic dipole by dividing a magnetic bar successively into two, we end up with pieces each having north and south poles. So an isolated magnetic charge doesn't exist.
That's why the total flux through a closed surface in a magnetic field must be zero.

i.e. $\int B \cdot dS = 0$

or more clear, we can write that for a static magnetic field the total number of flux lines entering a given region is equal to the total number of flux lines leaving the region.
So, (A) and (R) are both true and R is correct explanation of A.

SOL 4.3.2 Option (C) is correct.

SOL 4.3.3 Option (B) is correct.
The Magnetic field are caused only by current carrying elements and given as

$$B = \frac{\mu_0 Idl \times R}{4\pi R^3}$$

Since an accelerated electron doesn't form any current element(Idl) so it is not a source of magnetic field.

SOL 4.3.4 Option (C) is correct.
According to right hand rule if the thumb points in the direction of outward or inward current then rest of the fingers will curl along the direction of magnetic flux lines, This condition is satisfied by the configuration shown in option (C).

SOL 4.3.5 Option (C) is correct.
According to right hand rule if the thumb points in the direction of current then rest of the fingers will curl along the direction of magnetic field lines. This condition is satisfied by the configuration shown in option (C).

SOL 4.3.6 Option (D) is correct.
Since the magnetic flux density is defined as

$$B = \nabla \times A$$

and $\nabla \times B = \mu_0 J$

Now using the vector identity, we have

$$\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A$$

or, $\nabla \times B = \nabla(\nabla \cdot A) - \nabla^2 A$

or, $\mu_0 J = \nabla(\nabla \cdot A) - \nabla^2 A$

As the vector potential is always divergence free so we get,

$$\nabla^2 A = -\mu_0 J$$

SOL 4.3.7 Option (A) is correct.

SOL 4.3.8 Option (A) is correct.

SOL 4.3.9 Option (A) is correct.

SOL 4.3.10 Option (A) is correct.

SOL 4.3.11 Option (A) is correct.

SOL 4.3.12 Option (B) is correct.

SOL 4.3.13 Option (A) is correct.

SOL 4.3.14 Option (A) is correct.

SOL 4.3.15 Option (B) is correct.

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SOLUTIONS 4.4

SOL 4.4.1 Option (C) is correct.

For $r > a$,

$$I_{enclosed} = (\pi a^2)\,J$$

$$\oint \boldsymbol{H} \cdot d\boldsymbol{l} = I_{enclosed}$$

$$H(2\pi r) = (\pi a^2)\,J$$

$$H = \frac{I_o}{2\pi r} \qquad I_o = (\pi a^2)\,J$$

i.e.

$$H \propto \frac{1}{r}, \quad \text{for } r > a$$

For $r < a$,

$$I_{enclosed} = \frac{J(\pi r^2)}{\pi a^2} = \frac{Jr^2}{a^2}$$

So,

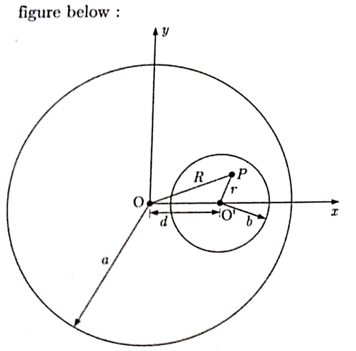
$$\oint \boldsymbol{H} \cdot d\boldsymbol{l} = I_{enclosed}$$

$$H(2\pi r) = \frac{Jr^2}{a^2}$$

$$H = \frac{Jr}{2\pi a^2} \quad \Rightarrow \quad H \propto r, \quad \text{for } r < a$$

SOL 4.4.2 Option (B) is correct.

Assume that the cross section of the wire lies in the x-y plane as shown in figure below :



Since, the hole is drilled along the length of wire. So, it can be assumed that the drilled portion carriers current density of $-J$.

Now, for the wire without hole, magnetic field intensity at point P will be given as

$$H_{\phi 1}(2\pi R) = J(\pi R^2) \quad \Rightarrow \quad H_{\phi 1} = \frac{JR}{2}$$

Since, point O is at origin and the cross section of the wire located in x-y plane. So, in vector form the field intensity due to the current carrying wire without considering the hole is given as

$$\boldsymbol{H}_1 = \frac{J}{2}(x\boldsymbol{a}_x + y\boldsymbol{a}_y) \qquad (1)$$

Again, only due to the hole magnetic field intensity at point P will be given as

$$(H_{\phi 2})(2\pi r) = -J(\pi r^2)$$

$$H_{\phi 2} = \frac{-Jr}{2}$$

Again, if we take O' at origin then in vector form

$$\boldsymbol{H}_2 = \frac{-J}{2}(x'\boldsymbol{a}_x + y'\boldsymbol{a}_y) \qquad (2)$$

where x' and y' denotes point 'P' in the new co-ordinate system.

Now the relation between two co-ordinate system will be

$$x = x' + d \quad \text{and} \quad y = y'$$

So, putting it into equation (2) we have

$$\boldsymbol{H}_2 = \frac{-J}{2}[(x-d)\,\boldsymbol{a}_x + y\boldsymbol{a}_y]$$

Therefore, the net magnetic field intensity at point P is

$$\boldsymbol{H}_{net} = \boldsymbol{H}_1 + \boldsymbol{H}_2 = \frac{J}{2}d\boldsymbol{a}_x$$

i.e. the magnetic field inside the hole will depend only on d.

SOL 4.4.3 Option (D) is correct.

Due to 1 A current wire in x-y plane, magnetic field be at origin will be in x direction as determined by right hand rule.

Due to 1 A current wire in y-z plane, magnetic field be at origin will be in z direction as determined by right hand rule.

Thus, x and z-component is non-zero at origin.

SOL 4.4.4 Option (B) is correct.

The total flux, $\Phi = 1.2\text{ mWb} = 1.2 \times 10^{-3}\text{ Wb}$

Cross sectional area, $A = 30\text{ cm}^2 = 30 \times 10^{-4}\text{ m}^2$

So, the flux density is given as

$$B = \frac{\Phi}{A} = \frac{1.2 \times 10^{-3}}{30 \times 10^{-4}} = 0.4\text{ Tesla}$$

SOL 4.4.5 Option (A) is correct.

The relation between magnetic flux density $\boldsymbol{B}$ and vector potential $\boldsymbol{A}$ is given as

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}$$

SOL 4.4.6 Option (D) is correct.

For an isolated body the charge is distributed over its region which depends on the total change and the curvature of the body. Thus Statement 1 is correct

Since the magnetic flux lines form loop so the total magnetic flux through any closed surface is zero. Thus Statement 2 is correct.

SOL 4.4.7 Option (A) is correct.

The magnetic flux density in terms of vector potential is defined as

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}$$

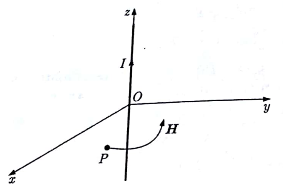
$$\int \boldsymbol{B} \cdot d\boldsymbol{S} = \int (\nabla \times \boldsymbol{A})\,d\boldsymbol{S}$$

$$\Phi = \oint A \cdot dl$$

i.e. the line integral of vector potential $\boldsymbol{A}$ around the boundary of a surface S is equal to the flux through the surface S.

SOL 4.4.8 Option (C) is correct.

Consider the current element along z-axis as shown in the figure.



Using right hand rule we get the direction of magnetic field directing normal to radial line OP.

SOL 4.4.9 Option (A) is correct.

For the given circular cylinder, consider the surface current density is K. So, the total current I through the cylinder is given as

$$K(2\pi r) = I$$

where r is radius of circular cylinder.

So, $$K = \frac{I}{2\pi r} = \frac{5}{2\pi(5\times 10^{-2})} = \frac{50}{\pi}\ \text{A/m}$$

SOL 4.4.10 Option (C) is correct.

Magnetic vector potential of an infinitesimally small current element is defined as

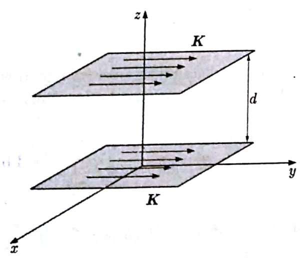
$$\boldsymbol{A} = \int \frac{\mu_0}{4\pi}\frac{Idl}{R}$$

where R is the distance from current element. Given that $R \to \infty$

So $$\boldsymbol{A} = 0$$

SOL 4.4.11 Option (D) is correct.

Consider the two parallel sheets are separated by a distance 'd' as shown in the figure below



The two sheets carries surface currents

$$\boldsymbol{K} = K_y \boldsymbol{a}_y$$

At any point between them the magnetic field intensity is given as

$$\boldsymbol{H} = \frac{1}{2}\boldsymbol{K}\times(\boldsymbol{a}_{nu} + \boldsymbol{a}_{nl})$$

where $\boldsymbol{a}_{nu}$ is the normal vector to the upper plate and $\boldsymbol{a}_{nl}$ is normal vector to the lower plate both directs toward the point between them

i.e. $$\boldsymbol{a}_{nu} = -\boldsymbol{a}_z \quad \text{and} \quad \boldsymbol{a}_{nl} = \boldsymbol{a}_z$$

So, $$\boldsymbol{H} = \frac{1}{2}K_y\boldsymbol{a}_y\times(-\boldsymbol{a}_z + \boldsymbol{a}_z) = 0$$

SOL 4.4.12 Option (C) is correct.

For the given current distribution, the current enclosed inside the cylindrical surface of radius ρ for $a < \rho < b$ is

$$I_{enc} = \int_a^{\rho}\left(J_0\frac{\rho}{a^2}\right)(2\pi\rho d\rho)$$
$$= \frac{2\pi J_0}{3\rho^2}(\rho^3 - a^3)$$

and the magnetic field intensity is given as

$$\int \boldsymbol{H}\cdot dl = I_{enc}$$
$$H(2\pi\rho) = \frac{2\pi J_0}{3a^2}(\rho^3 - a^3)$$
$$H = \frac{J_0(\rho^3 - a^3)}{3a^2\rho}$$

SOL 4.4.13 Option (A) is correct.

The radiated $\boldsymbol{E}$ and $\boldsymbol{H}$ field are determined by following steps

(1) Determine magnetic field intensity $\boldsymbol{H}$ from the expression

$$\boldsymbol{B} = \mu\boldsymbol{H} = \nabla\times\boldsymbol{A}$$

(2) then determine $\boldsymbol{E}$ from the expression

$$\nabla\times\boldsymbol{H} = \varepsilon\frac{\partial \boldsymbol{E}}{\partial t}$$

So, the concept of vector magnetic potential is used to find the expression of radiated $\boldsymbol{E}$ and $\boldsymbol{H}$ field.

SOL 4.4.14 Option (C) is correct.

Using right hand rule, we conclude that the direction of field intensity is same as determined for the two correct elements $3I$ and $2I$ while it is opposite for the current element I. Therefore, from the ampere's circuital law, we get the circulation of $\boldsymbol{H}$ around the closed contour as

$$\oint \boldsymbol{H}\cdot dl = I_{enclosed} = 2I + 3I - I = 4I$$

SOL 4.4.15 Option (B) is correct.

The unit of magnetic flux density $(\boldsymbol{B})$ is Tesla or Wb/m^2

SOL 4.4.16 Option (A) is correct.

From Ampere's circuital law, the circulation of magnetic field intensity in a closed path is equal to the current enclosed by the path

i.e. $$\oint \boldsymbol{H}\cdot dl = I_{enc}$$

So, for the current I the circulation at a radial distance R is given as

$$H(2\pi R) = I$$

or $$H = \frac{I}{2\pi R}$$

Therefore, the magnetic flux density at the radial distance R is

$$B = \mu_0 H = \frac{\mu_0 I}{2\pi R}$$

SOL 4.4.17 Option (A) is correct.
Unit of work is Joule.
Unit of electric field strength ($\boldsymbol{E}$) is volt/meter.
Unit of magnetic flux is Weber.
Unit of magnetic field strength is Ampere/meter.
So, in the match list we get, $A \to 4$, $B \to 3$, $C \to 2$, $D \to 1$.

SOL 4.4.18 Option (B) is correct.
Magnetic field intensity at a distance r from a long straight wire carrying current I is defined as

$$H = \frac{I}{2\pi r}$$

$$1 = \frac{1}{2\pi r}$$

$$r = \frac{1}{2\pi} = 1.59 \text{ m}$$

SOL 4.4.19 Option (C) is correct.
Consider the current flowing in the loop is I and since the magnetic field intensity is maximum at the centre of the loop given as

$$H = \frac{I}{2r}$$

where r is radius of the loop. So, the current that must flow in the loop to produce the magnetic field $H = 1 \text{ mA/m}$ is

$$I = 2rH = 2 \times 1 \times 1 = 2 \text{ mA}$$

SOL 4.4.20 Option (B) is correct.
Magnetic flux density $\boldsymbol{B}$ in terms of vector potential $\boldsymbol{A}$ is defined as

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}$$

So, $\boldsymbol{B}$ can be easily obtained from $\boldsymbol{A}$ also we know $\nabla \cdot \boldsymbol{A} = 0$ but it is not the explanation of Assertion (A).
i.e. A and R both are true but R is not the correct explanation of A.

SOL 4.4.21 Option (D) is correct.
(1) Magnetic flux density in terms of vector potential is given as

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}$$

(2) Poission's equation for magnetic vector potential is

$$\nabla^2 \boldsymbol{A} = -\mu_0 \boldsymbol{J}$$

(3) Magnetic vector potential for a line current is defined as

$$\boldsymbol{A} = \int \frac{\mu_0 I d\boldsymbol{l}}{4\pi R}$$

So, all the statements are correct.

SOL 4.4.22 Option (C) is correct.
Magnetic field intensity due to a long straight wire carrying current I at a distance r from it is defined as

$$H = \frac{I}{2\pi r}$$

$$1 = \frac{10}{2\pi r}$$

$$r = \frac{10}{2\pi} = 1.59 \text{ m}$$

SOL 4.4.23 Option (B) is correct.
Magnetic field intensity due to an infinite linear current carrying conductor is defined as

$$\oint \boldsymbol{H} \cdot d\boldsymbol{l} = I_{enc}$$

$$H(2\pi r) = I \quad \Rightarrow \quad H = \frac{I}{2\pi r}$$

SOL 4.4.24 Option (A) is correct.
The net outward magnetic flux through a closed surface is always zero as magnetic flux lines has no source or sinks.
i.e.

$$\oint \boldsymbol{B} \cdot d\boldsymbol{S} = 0 \qquad (1)$$

Now, from Gauss's law we have

$$\int (\nabla \cdot \boldsymbol{B}) dv = \oint \boldsymbol{B} \cdot d\boldsymbol{S} = 0 \qquad (2)$$

So, comparing the equation (1) and (2) we get

$$\nabla \cdot \boldsymbol{B} = 0$$

SOL 4.4.25 Option (A) is correct.
Given, the plane $y = 0$ carries a uniform current density $30\boldsymbol{a}_z$ mA/m and since the point A is located at $(1, 20, -2)$ so, unit vector normal to the current sheet is

$$\boldsymbol{a}_n = \boldsymbol{a}_y$$

Therefore, the magnetic field intensity

$$\boldsymbol{H} = \frac{1}{2}\boldsymbol{K} \times \boldsymbol{a}_n = \frac{1}{2}(30\boldsymbol{a}_z) \times (\boldsymbol{a}_y) = -15\boldsymbol{a}_x \text{ mA/m} \qquad ($$

$\boldsymbol{K} = 30\boldsymbol{a}_z$ mA/m)

SOL 4.4.26 Option (A) is correct.
The magnetic flux density at any point is curl of the magnetic vector potential at that point.
i.e. $\boldsymbol{B} = \nabla \times \boldsymbol{A}$
From the Maxwell's equation, the divergence of magnetic flux density is zero.
i.e. $\nabla \cdot \boldsymbol{B} = 0$
Again from the Maxwell's equation, the curl of the magnetic field intensity is equal to the current density.
i.e. $\nabla \times \boldsymbol{H} = \boldsymbol{J}$
or, $\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J}$ $\qquad (\boldsymbol{B} = \mu_0 \boldsymbol{H})$
The expression given in option (A) is incorrect
i.e. $\boldsymbol{B} \neq \nabla \cdot \boldsymbol{A}$

SOL 4.4.27 Option (A) is correct.
Superconductors are popularly used for generating very strong magnetic field.

SOL 4.4.28 Option (A) is correct.

As the magnetic flux lines have no source or sinks i.e. it forms a loop. So the total outward flux through a closed surface is zero.

i.e. $\oint \boldsymbol{B} \cdot d\boldsymbol{S} = 0$

SOL 4.4.29 Option (A) is correct.

The magnetic field intensity due a surface current density $\boldsymbol{K}$ is defined as

$$\boldsymbol{H} = \frac{1}{2}\boldsymbol{K} \times \boldsymbol{a}_n$$

Where $\boldsymbol{a}_n$ is unit normal vector to the current carrying surface directed toward the point of interest.

Given that, $\boldsymbol{K} = 2\boldsymbol{a}_x$.

and since the surface carrying current is in plane $z = 0$.

So, for $-\alpha < z < 0$ $\quad \boldsymbol{a}_n = -\boldsymbol{a}_z$

and $\quad \boldsymbol{H}_1 = \frac{1}{2}(2\boldsymbol{a}_x) \times (-\boldsymbol{a}_z) = \boldsymbol{a}_y$

For $0 < z < x$, $\quad \boldsymbol{a}_n = \boldsymbol{a}_z$

and $\quad \boldsymbol{H}_2 = \frac{1}{2}(2\boldsymbol{a}_x) \times (\boldsymbol{a}_z) = -\boldsymbol{a}_y$

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