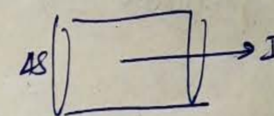
Currents. (Steady)

Current $I = \frac{dQ}{dt}$ (Amperes)

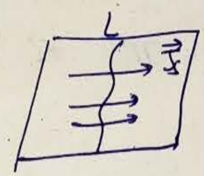
Current Density $\vec{J}$ A/m^2. If current ΔI flows through a surface ΔS normally,
(volume)

$$J = \frac{\Delta I}{\Delta S}, \quad \Delta I = \vec{J} \cdot \vec{\Delta S}$$

$$I = \int_S \vec{J} \cdot \vec{ds}$$



Surface current Density $\vec{J_s}$ A/m.

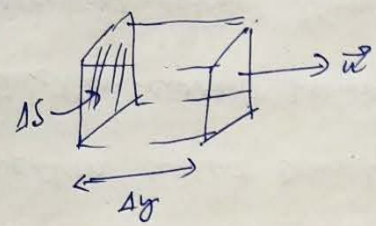


$$I = \int_L \vec{J_s} \cdot \vec{dl}$$

- Linear current Density does not exist.
- Convection current Density — Current flowing through an insulating medium/vaccum
- Conduction current Density — Satisfies OHM'S Law, does not involve conductors.

Displacement current Density — Radiation

· Flow of charge of volume density ρ_v, at velocity $\vec{u} = u_y \hat{a}_y$



$$\Delta I = \frac{\Delta Q}{\Delta t} = \rho_v \Delta S \frac{\Delta y}{\Delta t} = \rho_v \Delta S\, u_y$$

$$J_y = \frac{\Delta I}{\Delta S} = \rho_v u_y \Rightarrow \vec{J} = \rho_v \vec{u}.$$ (Convection current density) A/m^2

· When an impressed electric field $\vec{E}$ is applied, the force on an electron with charge '$-e$' is, $\vec{F} = -e\vec{E}$.

Since the 'e' is not in free-space, it will not experience acceleration.

It suffers constant collisions with atomic lattice and drifts from one atom to another

$\vec{u}$ = avg. drift velocity, τ = mean collision time

$$\frac{m\vec{u}}{\tau} = -e\vec{E}$$ (Avg. change in momentum must match the applied force).

$$\vec{u} = -\frac{e\tau}{m}\vec{E}$$ (drift velocity $\propto$ applied field)

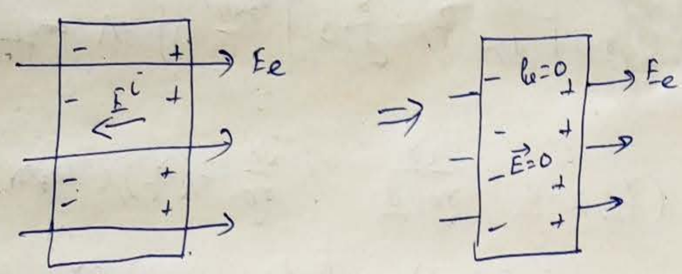
$\rho_v = -ne$, n = no. of electrons/vol.

$$\vec{J} = \rho_v \vec{u} = \frac{ne^2\tau}{m}\vec{E} = \sigma\vec{E}$$ (Conduction current density)

σ: conductivity of the conductor.

$\vec{J} = \sigma\vec{E}$ (Ohm's Law pt. form)

Conductors



Ei : Internal induced field

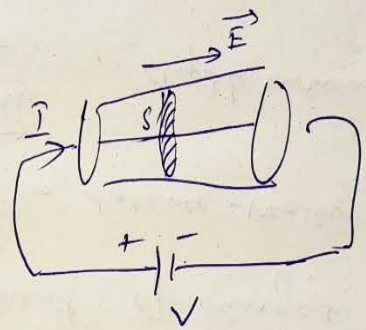
Ee : Externally applied field

charges accumulate on surface : Induced surface charge.

- A perfect conductor ($\sigma = \infty$) cannot contain an electrostatic field within it.

$\vec{E} = -\nabla V = 0$, $V_{ab} = 0$ (Equipotential)

- Gauss's Law $\Rightarrow$ $\rho_v = 0$ inside the conductor.



Note:–

$V = -\int \vec{E} \cdot \vec{dl}$

i.e. '–' sign indicates opposite direction of $\vec{E}$.

The normal to the surface would also be in the same direction, opposite to $\vec{J}$.

Thus the negative sign cancels.

- For finite conductivity σ,

$$R = \frac{V}{I} = \frac{\int_l \vec{E} \cdot \vec{dl}}{\iint_S \sigma \vec{E} \cdot \vec{ds}}$$

- Power P (watts) : Rate of change of energy W (Joules) or Force × velocity.

$$P = \iiint_v \rho_v dv \, \vec{E} \cdot \vec{u} = \iiint_v \vec{E} \cdot \rho_v \vec{u} \, dv = \iiint_v \vec{E} \cdot \vec{J} \, dv \quad \text{(Joule's Law)}$$

Power Density w_p (W/m^3) $= \frac{dP}{dv} = \vec{E} \cdot \vec{J} = \sigma |\vec{E}|^2$

Dielectrics.

- When an electric field $\vec{E}$ is applied, the +ve charge (nuclei) is displaced from its equilibrium position in the direction of $\vec{E}$ by the force $\vec{F_+} = Q\vec{E}$.

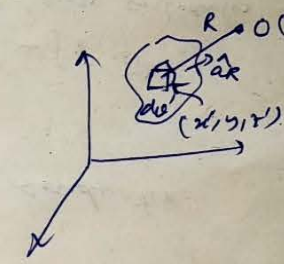
  The −ve charge (electron cloud) is displaced in the opposite direction.

  and the dielectric is "polarized".

- A dipole results from the displacement of charges

  Dipole moment $\vec{p} = Q\vec{d}$, $\vec{d}$ is distance vector from $-Q$ to $+Q$

  Polarization $\vec{P} = \lim_{\Delta v \to 0} \dfrac{\sum_{k=1}^{N} Q_k \vec{d_k}}{\Delta v}$, dipole moment / volume (C/m^2)



- Potential at an exterior pt. 'O' is $dV = \dfrac{\vec{P}\cdot\hat{a}_R \, dv'}{4\pi\varepsilon_0 R^2}$

$R^2 = (x-x')^2 + (y-y')^2 + (z-z')^2$

Note, $\nabla'\left(\frac{1}{R}\right) = \dfrac{\hat{a}_R}{R^2}$, $\dfrac{\vec{P}\cdot\hat{a}_R}{R^2} = \vec{P}\cdot\nabla'\left(\frac{1}{R}\right) = \nabla'\cdot\left(\frac{\vec{P}}{R}\right) - \dfrac{\nabla'\cdot\vec{P}}{R}$, using the vector identity,
$\nabla'\cdot f\vec{A} = f\,\nabla'\cdot\vec{A} + \vec{A}\cdot\nabla' f$

Thus $V = \iiint_{v'} \dfrac{1}{4\pi\varepsilon_0}\left[\nabla'\cdot\frac{\vec{P}}{R} - \frac{1}{R}\nabla'\cdot\vec{P}\right] dv'$

$= \iint_{S'} \dfrac{\vec{P}\cdot\hat{a}_n'}{4\pi\varepsilon_0 R} ds' + \iiint_{v'} \dfrac{1}{4\pi\varepsilon_0 R} \times (-\nabla'\cdot\vec{P})\, dv'$ = Potential due to <u>bound</u> Surface charge + Potential due to <u>bound</u> volume charge

$\rho_{ps} = \vec{P}\cdot\hat{a}_n$

$\rho_{pv} = -\nabla\cdot\vec{P}$

Total positive bound charge on surface 's' $Q_b = \oiint \vec{P} \cdot \vec{ds} = \oiint \rho_{ps}\, ds$

" bound charge inside 's' $-Q_b = \iiint_V \rho_{pv}\, dv = -\iiint_V \nabla \cdot \vec{P}\, dv$

. Total charge for electrically neutral dielectric $= \oiint_S \rho_{ps}\, ds + \iiint_V \rho_{pv}\, dv = 0.$

. If the dielectric region contains free charge ρ_v (volume density),

total volume charge density $\rho_t = \rho_v + \rho_{pv} = \nabla \cdot (\varepsilon_0 \vec{E})$

Hence, $\rho_v = \rho_t - \rho_{pv} = \nabla \cdot (\varepsilon_0 \vec{E}) + \nabla \cdot \vec{P} = \nabla \cdot \underbrace{(\varepsilon_0 \vec{E} + \vec{P})}_{\vec{D}} = \nabla \cdot \vec{D}$

Thus, $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$

The net effect of dielectric, is to increase the flux density by amount $\vec{P}$

($\vec{P} = 0$, for free space).

. $\vec{P} = \chi_e \varepsilon_0 \vec{E}$, χ_e: electric susceptibility

(For linear dielectric materials).

Thus, $\vec{D} = \varepsilon_0 (1 + \chi_e) \vec{E} = \varepsilon_0 \varepsilon_r \vec{E}$, $\varepsilon_r = 1 + \chi_e$ (relative permittivity).

$\varepsilon_0 = 10^{-9} \times \frac{1}{36\pi}$ F/m

Relaxation Time

$$I_{out} = \oint_S \vec{J} \cdot \vec{ds} = -\frac{dQ_{in}}{dt}$$ (Principle of charge conservation)

Net outward current flow through the surface

Time rate of decrease of charge within a given volume.

} provided no accumulation of charge at the pt.

$$\oint_S \vec{J} \cdot ds = \iiint_V (\nabla \cdot \vec{J}) dv = -\frac{d}{dt} \iiint_V \rho_v dv \Rightarrow \boxed{\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t}}$$ — CONTINUITY EQN.

- Steady currents; $\frac{\partial \rho_v}{\partial t} = 0 \Rightarrow \boxed{\nabla \cdot \vec{J} = 0} \Longrightarrow$ KCL

(Total charge leaving a volume = Total charge entering)

- $\vec{J} = \sigma \vec{E}$ (ohm's law), $\nabla \cdot \vec{E} = \frac{\rho_v}{\varepsilon}$ (Gauss's law)

Thus, Continuity Eqn., $\nabla \cdot \sigma \vec{E} = \sigma \frac{\rho_v}{\varepsilon} = -\frac{\partial \rho_v}{\partial t} \Rightarrow \frac{\partial \rho_v}{\partial t} + \frac{\sigma}{\varepsilon}\rho_v = 0$

$\ln \rho_v = -\frac{\sigma t}{\varepsilon} + \ln \rho_{v0} \Rightarrow \rho_v = \rho_{v0} e^{-t/T_r}$, $T_r = \varepsilon/\sigma$ (relaxation time constant)

At, $t=0$, ρ_{v0} is initial charge density. charge will vanish from any interior pt. and appear at the surface.

Eg.)

Copper, $\sigma = 5.8 \times 10^7$ S/m, $\varepsilon_r = 1$

$T_r = 1.53 \times 10^{-19}$ sec.

(rapid decay)

Quartz, $\sigma = 10^{-17}$ S/m, $\varepsilon_r = 5.0$

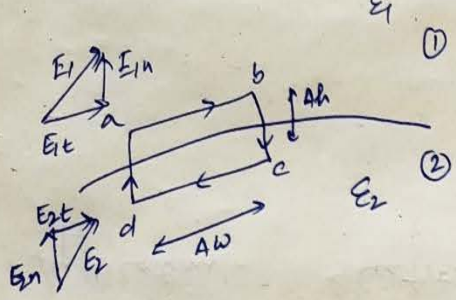
$T_r = 51.2$ days

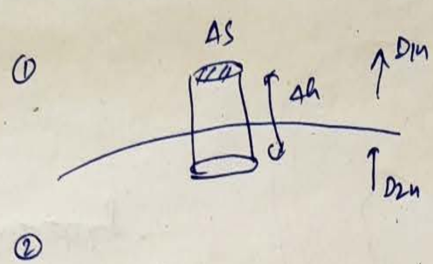
(very large relaxn. time)

Boundary Conditions.

- Dielectric– Dielectric boundary.

$$\vec{E_1} = \vec{E}_{1t} + \vec{E}_{1n}$$
$$\vec{E_2} = \vec{E}_{2t} + \vec{E}_{2n}$$





$$\oint \vec{E}\cdot\vec{dl} = 0$$

$$\Rightarrow E_{1t}\,\Delta w - \cancel{E_{1n}\frac{\Delta h}{2}} - \cancel{E_{2n}\frac{\Delta h}{2}} - E_{2t}\,\Delta w + \cancel{E_{2n}\frac{\Delta h}{2}} + \cancel{E_{1n}\frac{\Delta h}{2}} = 0$$

$$\Rightarrow \boxed{E_{1t} = E_{2t}}$$

$$\oint\!\!\oint_S \vec{D}\cdot\vec{dS} = Q_{enc}.$$

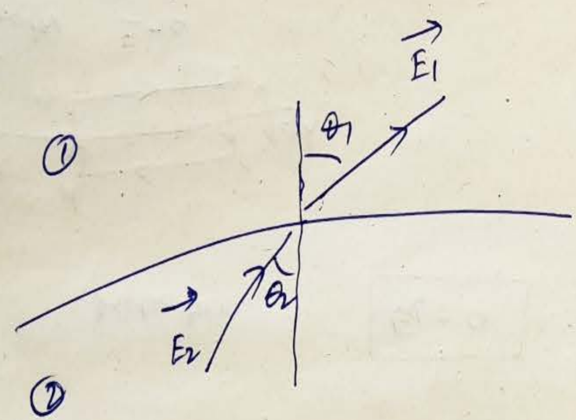
$$\Rightarrow \Delta Q = \rho_s\,\Delta S = D_{1n}\,\Delta S - D_{2n}\,\Delta S$$

$$\Rightarrow \boxed{D_{1n} - D_{2n} = \rho_s}$$, ρ_s = free-surface charge density at the boundary.

<u>Note</u> : $\vec{D}$ is directed from region 2 to region 1.

In absence of free charge, $D_{1n} = D_{2n}$.

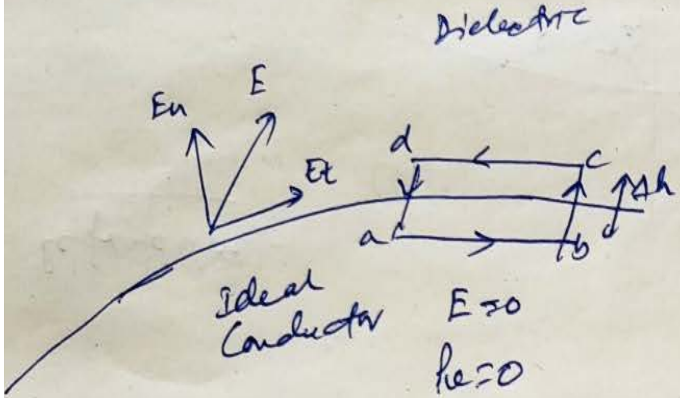
<u>Refraction</u>:-

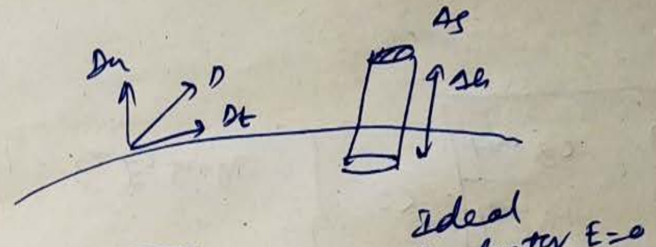


$$\left.\begin{aligned} E_1 \sin\theta_1 &= E_2 \sin\theta_2 \\ \varepsilon_1 E_1 \cos\theta_1 &= \varepsilon_2 E_2 \sin\theta_2 \end{aligned}\right\} \Rightarrow \boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}}$$

- <u>Conductor Dielectric boundary.</u>

$\boxed{E_t = 0}$





$\boxed{D_n = \rho_s}$

Electric field must approach a conductor surface normally.

Applications:- Electrostatic screening/ Shielding