MAGNETOSTATICS:

BIOT - SAVART'S LAW:- $d\vec{H} = \dfrac{I\, \vec{dl} \times \hat{a}_R}{4\pi R^2}$, Direction of $\vec{dH}$ can be determined by Right-Hand Rule

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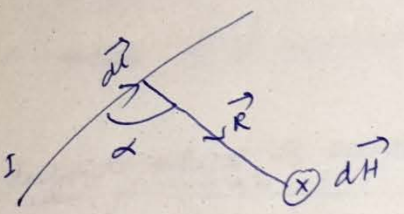
$$\vec{H} = \int_L \frac{I\, \vec{dl} \times \hat{a}_R}{4\pi R^2}$$



For Surface current density $(\vec{K} ds)$. , $\vec{K}$ is A/m.

$$\vec{H} = \iint_S \frac{\vec{K}\, ds \times \hat{a}_R}{4\pi R^2}$$

For volume current density $(\vec{J}\, dv)$, $\vec{J}$ is A/m^2

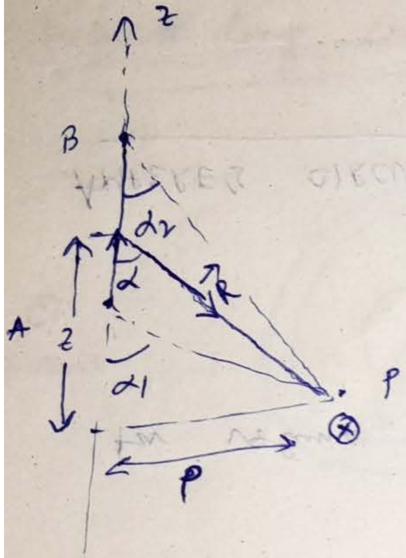
$$\vec{H} = \iiint_V \frac{\vec{J}\, dv \times \hat{a}_R}{4\pi R^2}$$

AMPERE'S CIRCUIT LAW:- $\oint \vec{H} \cdot \vec{dl} = I_{enc}$ — used to determine $\vec{H}$ for symmetrical current distribution.

$$\Rightarrow \iint_S (\nabla \times \vec{H}) \cdot \vec{ds} = \iint_S \vec{J} \cdot \vec{ds}$$

$$\Rightarrow \boxed{\nabla \times \vec{H} = \vec{J}}$$ — Magnetostatic field is not conservative.

Field due to a straight conductor



Element located at $\vec{dl}$ $(0, 0, z)$.

$$\vec{dH} = \frac{I\,\vec{dl} \times \vec{R}}{4\pi R^3}, \quad \vec{dl} = dz\,\hat{a}_z, \quad \vec{R} = \rho\,\hat{a}_\rho - z\,\hat{a}_z$$

$$\vec{dl} \times \vec{R} = \rho\, dz\, \hat{a}_\phi$$

$$\vec{H} = \int \frac{I\rho\, dz}{4\pi\left[\rho^2 + z^2\right]^{3/2}}\,\hat{a}_\phi, \quad \boxed{\hat{a}_\phi = \hat{a}_l \times \hat{a}_\rho}$$

$z = \rho \cot\alpha, \quad dz = -\rho\,\mathrm{Cosec}^2\alpha\, d\alpha,$

$$\vec{H} = -\frac{I}{4\pi}\int_{\alpha_1}^{\alpha_2} \frac{\rho^2\,\mathrm{Cosec}^2\alpha\, d\alpha}{\rho^3\,\mathrm{Cosec}^3\alpha}\,\hat{a}_\phi = -\frac{I}{4\pi\rho}\,\hat{a}_\phi \int_{\alpha_1}^{\alpha_2} \sin\alpha\, d\alpha$$

$$= \boxed{\frac{I}{4\pi\rho}\left(\mathrm{Cos}\,\alpha_2 - \mathrm{Cos}\,\alpha_1\right)\hat{a}_\phi}$$

- Semi-infinite conductor :- $\alpha_1 = 90^\circ, \alpha_2 = 0^\circ$, $\vec{H} = \dfrac{I}{4\pi\rho}\,\hat{a}_\phi$

- Infinite conductor :- $\alpha_1 = 180^\circ, \alpha_2 = 0^\circ$, $\vec{H} = \dfrac{I}{2\pi\rho}\,\hat{a}_\phi$

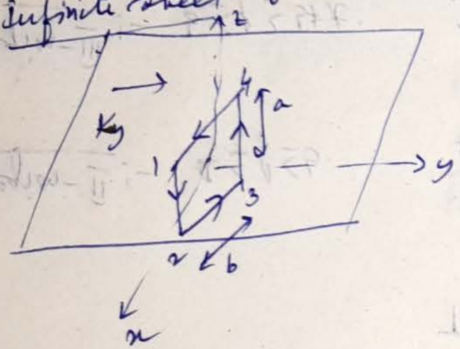
• Infinite line of current:

$$I = \oint H_\phi \hat{a}_\phi \cdot \rho\, d\phi\, \hat{a}_\phi = H_\phi \int \rho\, d\phi = H_\phi \cdot 2\pi\rho$$

$$\Rightarrow H_\phi = \frac{I}{2\pi\rho}\hat{a}_\phi$$

(choose an "Amperian path" where $\vec{H}$ is constant).

• Infinite sheet of current:



$$\oint \vec{H} \cdot \vec{dl} = I_{enc} = K_y b.$$

Consider the (infinite) filament of current along $\hat{a}_y$

Here above the sheet, $\hat{a}_\rho = +\hat{a}_z$

& below the sheet, $\hat{a}_\rho = -\hat{a}_z$.

Thus, $\hat{a}_\phi = \hat{a}_y \times \hat{a}_\rho \Rightarrow +\hat{a}_x$, above the sheet

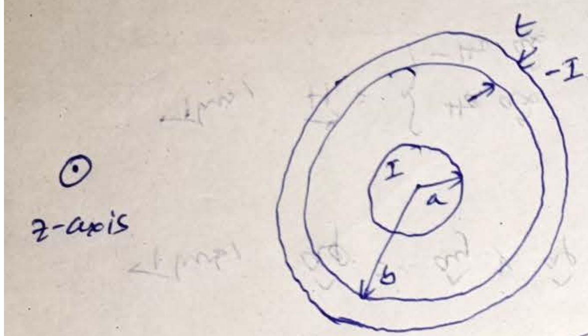
& $-\hat{a}_x$, below the sheet.

$$\oint \vec{H} \cdot \vec{dl} = 2H_0 b = K_y b \Rightarrow H_0 = \frac{1}{2} K_y.$$

Thus, $\vec{H} = \begin{cases} H_0 \hat{a}_x, & z > 0 \\ -H_0 \hat{a}_x, & z < 0 \end{cases}$

Thus, $\boxed{\vec{H} = \frac{1}{2} \vec{K} \times \hat{a}_n}$, $\hat{a}_n$ = unit normal vector directed from current sheet to the pt. of interest.

· Infinite long coaxial transmission line.



Region-I :- $0 \le \rho \le a$.

$\oint \vec{H} \cdot \vec{dl} = \int \vec{J} \cdot \vec{ds}$, $\vec{J} = \frac{I}{\pi a^2} \hat{a_z}$, $\vec{ds} = \rho \, d\phi \, d\rho \, \hat{a_z}$

$I_{enc} = \int \vec{J} \cdot \vec{ds} = \frac{I}{\cancel{\pi} a^2} \int_{\phi=0}^{2\pi} \int_{\rho=0}^{a} \rho \, d\phi \, d\rho = \frac{I \rho^2}{a^2}$.

Thus, $H_\phi \, 2\pi\rho = \frac{I\rho^2}{a^2} \Rightarrow H_\phi = I\rho / 2\pi a^2$

Region-II :- $a \le \rho \le b$. $H_\phi \cdot 2\pi\rho = I \Rightarrow H_\phi = I/2\pi\rho$.

$\vec{J} = -\frac{I}{\pi\left[(b+t)^2 - b^2\right]} \hat{a_z}$

Region-III :- $b \le \rho \le b+t$ $I_{enc} = I + \int \vec{J} \cdot \vec{ds}$,

$\vec{ds} = \rho \, d\rho \, d\phi$, $\phi = (0, 2\pi)$, $\rho = (b, \rho)$.

$= I\left[1 - \frac{\rho^2 - b^2}{t^2 + 2bt}\right]$

$H_\phi = \frac{I}{2\pi\rho}\left[1 - \frac{\rho^2 - b^2}{t^2 + 2bt}\right]$.

Region-IV :- $\rho \ge b+t$. $I_{enc} = 0$, $\vec{H} = 0$.

Magnetic Flux Density ($\vec{B}$)

- $\vec{B} = \mu_0 \vec{H}$, μ_0 = Permeability of free space
(tesla) $= 4\pi \times 10^{-7}$ H/m

- Magnetic Flux (ψ_m) $= \iint_S \vec{B} \cdot \vec{ds}$
(webers)

- In electrostatics $\psi_e = \oint\!\!\oint \vec{D} \cdot \vec{ds} = Q_{enc}$ (Electric flux lines are not closed)
Flux through a closed surface.

In magnetostatics $\psi_m = \oint\!\!\oint \vec{B} \cdot \vec{ds} = 0$ (Magnetic flux lines close upon themselves) as no existence of magnetic isolated poles.

$$\Rightarrow \iiint (\nabla \cdot \vec{B})\, dv = 0$$

$$\Rightarrow \boxed{\nabla \cdot \vec{B} = 0}$$

Magnetic Potentials.

- $\nabla \times \vec{H} = \vec{J}$ $\boxed{\nabla \cdot \vec{B} = 0}$

In the region where $\vec{J} = 0$, $\nabla \times \vec{H} = 0 \Rightarrow \vec{H} = -\nabla V_m$.

V_m: Magnetic Scalar potential.

$\nabla \cdot \vec{B} = 0 \Rightarrow \nabla^2 V_m = 0$ (Laplace Eqn).

- Also $\vec{B} = \nabla \times \vec{A}$ as, $\nabla \cdot (\nabla \times \vec{A}) = 0$, $\vec{A}$: Magnetic Vector potential.

$\phi = \iint_S \vec{B} \cdot \vec{ds} = \oint_L \vec{A} \cdot \vec{dl}$

Biot-Savart's Law:- $\vec{B} = \frac{\mu_0}{4\pi} \int_L \frac{I \vec{dl}' \times \vec{R}}{R^3}$

Note, $\nabla\left(\frac{1}{R}\right) = -\frac{\vec{R}}{R^3}$, Thus, $\vec{B} = -\frac{\mu_0}{4\pi} \int_L I \vec{dl}' \times \nabla\left(\frac{1}{R}\right)$

using, $\nabla \times (f \vec{F}) = f(\nabla \times \vec{F}) + (\nabla f) \times \vec{F}$, choose, $f = \frac{1}{R}$, $\vec{F} = \vec{dl}'$

Also, $\nabla \times \vec{dl}' = 0$

we have, $\vec{dl}' \times \nabla\left(\frac{1}{R}\right) = \frac{1}{R} \nabla \times \vec{dl}' - \nabla \times \left(\frac{\vec{dl}'}{R}\right)$, Thus, $\vec{B} = \frac{\mu_0}{4\pi} \int_L I \, \nabla \times \left(\frac{\vec{dl}'}{R} \right)$

$$= \nabla \times \int_L \frac{\mu_0 I \, \vec{dl}'}{4\pi R}.$$

Hence, $\boxed{\vec{A} = \int_L \frac{\mu_0 I \, \vec{dl}'}{4\pi R}}$

vector Laplacian.

$$\nabla \times \underbrace{\nabla \times \vec{A}}_{\vec{B} = \mu_0 \vec{H}} = \nabla(\nabla \cdot \vec{A}) - \overrightarrow{\nabla^2 A}$$

Then, $\overrightarrow{\nabla^2 A} = -\mu_0 \nabla \times \vec{H} + \nabla(\nabla \cdot \vec{A})$

Choose $\boxed{\nabla \cdot \vec{A} = 0}$ — Coulomb's Gauge. Cond^n

Then, $\boxed{\overrightarrow{\nabla^2 A} = -\mu_0 \nabla \times H = -\mu_0 \vec{J}}$ (vector Poisson's Eq^n.)

Thus, $\nabla^2 A_x = -\mu_0 J_x$, $\nabla^2 A_y = -\mu_0 J_y$, $\nabla^2 A_z = -\mu_0 J_z$