Probability and Stochastic Process (MA20106)
Assignment– Probability

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1. Suppose A and B are events with $P(A) = 0.5, P(B) = 0.4$ and $P(A \cap B^c) = 0.2$. Then find $P(B^c|A \cup B)$.

2. There are two boxes, each containing two components. Each component is defective with probability 1/4, independent of all other components. Find the probability that exactly one box contains exactly one defective component.

3. 2000 cashew nuts are mixed thoroughly in flour. The entire mixture is divided into 1000 equal parts and each part is used to make one biscuit. Assume that no cashews are broken in the process. A biscuit is picked at random. Calculate the probability that it contains no cashew nuts.

4. A system comprising of n identical components works if at least one of the components works. Each of the components works with probability 0.8, independent of all other components. Then find the minimum value of n for which the system works with probability at least 0.97.

5. A system consisting of n components functions iff at least one of n components functions. Suppose that all the n components of the system function independently, each with probability $\frac{3}{4}$. If the probability of functioning of the system is $\frac{63}{64}$, then calculate the value of n.

6. Four persons $P, Q, R,$ and S take turns (in the sequence $P, Q, R, S, P, Q, R, S, P, \dots$) in rolling a fair die. The first person to get a six wins. Then find the probability that S wins.

7. There are two urns U_1 and U_2. U_1 contains four white and four black balls, and U_2 is empty. Four balls are drawn at random from U_1 and transferred to U_2. Then a ball is drawn at random from U_2. Find the probability that the ball drawn from U_2 is white.

8. Let E and F be two independent events with
$$P(E|F) + P(F|E) = 1, P(E \cap F) = \frac{2}{9} \text{ and } P(F) < P(E).$$
Then calculate $P(E)$.

9. In a colony all families have at least one child. The probability that a randomly chosen family from this colony has exactly k children is $(0.5)^k; k = 1, 2, \dots$. A child is either a male or a female with equal probability. Then find the probability that such a family consists of at least one male child and at least one female child.

10. Player P_1 tosses 4 fair coins and player P_2 tosses a fair die independently of P_1. Then find the probability that the number of heads observed is more than the number on the upper face of the die.

11. Let E, F and G be three events such that the events E and F are mutually exclusive, $P(E \cup F) = 1$, $P(E \cap G) = 1/4$, and $P(G) = 7/12$. Then find $P(F \cap G)$.

12. There are four urns labeled U_1, U_2, U_3 and U_4, each containing 3 blue and 5 red balls. The fifth urn, labelled U_5, containing 4 blue and 4 red balls. An urn is selected from these five urns and a ball is drawn at random from it. Given that the selected ball is red, find the probability that it came from the urn U_5.

13. Let E and F be two mutually disjoint events. Further, let E and F be independent of G. If $p = P(E) + P(F)$ and $q = P(G)$, then find $P(E \cup F \cup G)$.

14. Let A and B be two events with $P(A|B) = 0.3$ and $P(A|B^c) = 0.4$. Find $P(B|A)$ and $P(B^c|A^c)$ in terms of $P(B)$. If $1/4 \leq P(B|A) \leq 1/3$ and $1/4 \leq P(B^c|A^c) \leq 9/16$, then determine the value of $P(B)$.

15. Independent trails consisiting of rolling fair die is performed. Find the probability that 2 appears before 3 or 5.

16. A nonempty subset P is formed by selecting elements at random and without replacement from a set B consisting of $n(> 1)$ distinct elements. Another nonempty subset Q is formed in similar fashion from the original set B consisting of the same n elements. Then what is the probability that P and Q do not have any common element ?

17. (a) One coin is selected at random from two coins. The probability of obtaining head for one of them is $\frac{1}{3}$ and for the other it is $\frac{1}{2}$. If the selected coin is tossed and the head shows up, what is the probability that it is the fair coin?
(b) Let p denote the probability that the weather (either wet or dry) tomorrow will be the same as that of today. If the weather is dry today, show that P_n, the probability that it will be dry n days later, satisfies
$$P_n = (2p - 1)p_{n-1} + (1 - p), \quad n \geq 1.$$
Hence, or otherwise, determine the value of P_{50} for $p = \frac{3}{4}$.

18. For detecting a disease, a test gives correct diagnosis with probability 0.99. It is known that 1 % of a population suffers from this disease. If a randomly selected individual from this population tests positive, then what is the probability that the selected individual actually has the disease?

19. Let $U_1, U_2, \ldots, U_n$ be n urns such that urn U_k contains k white and k^2 black balls, $k = 1, 2, \ldots, n$. Consider the random experiment of selecting an urn and drawing a ball out of it at random. If the probability of selecting urn U_k is proportional to $(k + 1)$, then
(a) find the probability that the ball drawn is black.
(b) find the probability that urn U_n was selected, given that the ball drawn is white.

20. Let E and F be two events with $P(E) > 0$, $P(F|E) = 0.3$ and $P(E \cap F^c) = 0.2$. Then $P(E)$?

21. Two coins with probability of heads u and v, respectively, are tossed independently. If P(both coins show up tails)=P(both coins show up heads), then $u + v$ equals ?

22. (a) Student population of a university has 30% Asian, 40% American, 20% European and 10% African students. It is known that 40% of all Asian students, 50% of all American students, 60% of all European students and 20% all African students are girls. Find the probability that a girl chosen at random from the university is an Asian.

(b) Let A_1, A_2 and A_3 be pairwise independent events with $P(A_i) = \frac{1}{2}, \quad i = 1, 2, 3$. Suppose that A_3 and $A_1 \cup A_2$ are independent. Find the value of $P(A_1 \cap A_2 \cap A_3)$.

23. A person makes repeated attempts to destroy a target. Attempts are made in independent of each other. The probability of destroying the target in any attempt is 0.8. Given that he fails to destroy the target in the first five attempts, find the probability that the target is destroyed in the 8th attempt ?

24. Let E and F be two events with $P(E) = 0.7, \quad P(F) = 0.4, \quad P(E \cap F^c) = 0.4$. Then $P(F|E \cup F^c)$?