MCQ 5.4.22 Which one of the following formulae is not correct for the boundary between two magnetic materials ?

(A) $\boldsymbol{B}_{n1} = \boldsymbol{B}_{n2}$

(B) $\boldsymbol{B}_2 = \sqrt{\boldsymbol{B}_{n2} + \boldsymbol{B}_{t2}}$

(C) $\boldsymbol{H}_1 = \boldsymbol{H}_{n1} + \boldsymbol{H}_{t1}$

(D) $\boldsymbol{a}_{n21} \times (\boldsymbol{H}_1 - \boldsymbol{H}_2) = \boldsymbol{K}$ where $\boldsymbol{a}_{n21}$ is a unit vector normal to the interface and directed from region 2 to region 1.

MCQ 5.4.23 Interface of two regions of two magnetic materials is current-free. The region 1, for which relative permeability $\mu_{r1} = 2$ is defined by $z < 0$, and region 2, $z > 0$ has $\mu_{r2} = 1$. If $\boldsymbol{B}_1 = 1.2\boldsymbol{a}_x + 0.8\boldsymbol{a}_y + 0.4\boldsymbol{a}_z$ T; then $\boldsymbol{H}_2$ is

(A) $1/\mu_0[0.6\boldsymbol{a}_x + 0.4\boldsymbol{a}_y + 0.4\boldsymbol{a}_z]$ A/m

(B) $1/\mu_0[1.2\boldsymbol{a}_x + 0.8\boldsymbol{a}_y + 0.8\boldsymbol{a}_z]$ A/m

(C) $1/\mu_0[1.2\boldsymbol{a}_x + 0.4\boldsymbol{a}_y + 0.4\boldsymbol{a}_z]$ A/m

(D) $1/\mu_0[0.6\boldsymbol{a}_x + 0.4\boldsymbol{a}_y + 0.8\boldsymbol{a}_z]$ A/m

MCQ 5.4.24 If $\boldsymbol{A}$ and $\boldsymbol{J}$ are the vector potential and current density vectors associated with a coil, then $\int \boldsymbol{A} \cdot \boldsymbol{J}\, dv$ has the units of

(A) flux-linkage (B) power

(C) energy (D) inductance

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SOLUTIONS 5.1

SOL 5.1.1 Option (C) is correct.

For a moving charge Q in the presence of both electric and magnetic fields, the total force on the charge is given by

$$\boldsymbol{F} = Q[\boldsymbol{E} + (\boldsymbol{v} \times \boldsymbol{B})]$$

where $\boldsymbol{E} \to$ electric field

$\boldsymbol{v} \to$ velocity of the charged particle

$\boldsymbol{B} \to$ magnetic flux density

So, at time $t = 0$ total force applied on the electron is

$$\boldsymbol{F}(0) = e[\boldsymbol{E} + (\boldsymbol{V}(0) \times \boldsymbol{B})]$$

Now we have

$$\boldsymbol{V}(0) \times \boldsymbol{B} = (200\boldsymbol{a}_x - 300\boldsymbol{a}_y - 400\boldsymbol{a}_z) \times (-3\boldsymbol{a}_x + 2\boldsymbol{a}_y - \boldsymbol{a}_z)$$
$$= 1100\boldsymbol{a}_x + 1400\boldsymbol{a}_y - 500\boldsymbol{a}_z$$

therefore the applied force on the electron is

$$\boldsymbol{F}(0) = (1.6 \times 10^{-19})[(0.1\boldsymbol{a}_x - 0.2\boldsymbol{a}_y + 0.3\boldsymbol{a}_z) \times 10^3 + 1100\boldsymbol{a}_x + 1400\boldsymbol{a}_y - 500\boldsymbol{a}_z]$$

$$m_e\boldsymbol{a}(0) = 1.6 \times 10^{-19}[(100 + 1100)\boldsymbol{a}_x + (1400 - 200)\boldsymbol{a}_y + (300 - 500)\boldsymbol{a}_z]$$

($\boldsymbol{F}(0) = m_e\boldsymbol{a}(0)$, where $\boldsymbol{a}(0)$ is acceleration of electron at $t = 0$)

$$\boldsymbol{a}(0) = \frac{1.6 \times 10^{-19}}{9.1 \times 10^{-31}} \times 200(6\boldsymbol{a}_x + 6\boldsymbol{a}_y - \boldsymbol{a}_z)$$
$$= 3.5 \times 10^{13}(6\boldsymbol{a}_x + 6\boldsymbol{a}_y - \boldsymbol{a}_z)\ \text{m/s}^2$$

SOL 5.1.2 Option (B) is correct.

Force $\boldsymbol{F}$ applied on a current element in the presence of magnetic flux density $\boldsymbol{B}$ is defined as

$$\boldsymbol{F} = I(\boldsymbol{L} \times \boldsymbol{B})$$

where $I \to$ current flowing in the element

$\boldsymbol{L} \to$ vector length of current element in the direction of current flowing

So,

$$\boldsymbol{F} = 3 \times 10^{-3}[2\boldsymbol{a}_z \times (\boldsymbol{a}_x + 3\boldsymbol{a}_y)]$$
$$= 6 \times 10^{-3}[\boldsymbol{a}_y - 3\boldsymbol{a}_x] = -18\boldsymbol{a}_x + 6\boldsymbol{a}_y\ \text{mN}$$

SOL 5.1.3 Option (A) is correct.

The magnitude of the force experienced by either of the loops will be same but the direction will be opposite.

So the force experienced by C_1 due to C_2 will be $-\boldsymbol{F}$.

SOL 5.1.4 Option (B) is correct.

Magnetic dipole moment of a coil carrying current I and having area S is given by

$$\boldsymbol{m} = IS\boldsymbol{a}_n$$

where $\boldsymbol{a}_n$ is normal vector to the surface of the loop.

Since the coil is lying in the plane $2x + 6y - 3z = 4$ so the unit vector normal to the plane of the coil is given as.

So, $$a_n = \frac{\nabla f}{|\nabla f|} = \frac{2\boldsymbol{a}_x + 6\boldsymbol{a}_y - 3\boldsymbol{a}_z}{\sqrt{2^2 + 6^2 + (-3)^2}} \qquad (f = 2x + 6y - 3z)$$

Therefore the magnetic dipole moment of the coil is

$$\boldsymbol{m} = (5)(1)\frac{(2\boldsymbol{a}_x + 6\boldsymbol{a}_y - 3\boldsymbol{a}_z)}{7} \qquad (I = 5\,\mathrm{A}, S = 1\,\mathrm{m}^2)$$

$$= \frac{5(2\boldsymbol{a}_x + 6\boldsymbol{a}_y - 3\boldsymbol{a}_z)}{7}$$

As the torque a magnetic field $\boldsymbol{B}$ on the loop having magnetic moment $\boldsymbol{m}$ is defined as

$$\boldsymbol{T} = \boldsymbol{m} \times \boldsymbol{B}$$

So the torque on the given coil is

$$\boldsymbol{T} = \left[\frac{5(2\boldsymbol{a}_x + 6\boldsymbol{a}_y - 3\boldsymbol{a}_z)}{7}\right] \times (6\boldsymbol{a}_x + 4\boldsymbol{a}_y + 5\boldsymbol{a}_z)$$

$$= 30\boldsymbol{a}_x - 20\boldsymbol{a}_y - 20\boldsymbol{a}_z \text{ N-m}$$

SOL 5.1.5 Option (B) is correct.

Magnetic dipole moment of a coil of area S carrying current I is defined as

$$\boldsymbol{m} = IS\boldsymbol{a}_n$$

where $\boldsymbol{a}_n$ is the unit vector normal to the surface of the loop.
and since from the given data we have

$$I = 10\,\mathrm{A}$$

$$S = \pi r^2 = \pi \times (1)^2 = \pi$$

$$\boldsymbol{a}_n = \boldsymbol{a}_z \qquad \text{(normal vector to the surface } z = 0)$$

So the magnetic moment of the circular current loop lying in the plane $z = 0$ is

$$\boldsymbol{m} = 10\pi\boldsymbol{a}_z$$

Now the torque on an element having magnetic moment $\boldsymbol{m}$ in the presence of magnetic flux density $\boldsymbol{B}$ is defined as

$$\boldsymbol{T} = \boldsymbol{m} \times \boldsymbol{B}$$

Therefore, the torque acting on the circular loop is

$$\boldsymbol{T} = (10\pi\boldsymbol{a}_z) \times (4\boldsymbol{a}_x - 4\boldsymbol{a}_y - 2\boldsymbol{a}_z) \quad (\boldsymbol{B} = 4\boldsymbol{a}_x - 4\boldsymbol{a}_y - 2\boldsymbol{a}_z)$$

$$= 10\pi(4\boldsymbol{a}_y + 4\boldsymbol{a}_x) = 40\pi(\boldsymbol{a}_x + \boldsymbol{a}_y)$$

SOL 5.1.6 Option (D) is correct.

Given the permeability, $\mu = 3\mu_0$ and magnetic flux density $= \boldsymbol{B}$

So the field intensity inside the material will be

$$\boldsymbol{H} = \frac{\boldsymbol{B}}{\mu} = \frac{\boldsymbol{B}}{3\mu_0}$$

Since the magnetization of a magnetic material is defined as

$$\boldsymbol{M} = \frac{\boldsymbol{B}}{\mu_0} - \boldsymbol{H}$$

where $\boldsymbol{B}$ and $\boldsymbol{H}$ are the flux density and field intensity inside the material. So we get

$$\boldsymbol{M} = \frac{\boldsymbol{B}}{\mu_0} - \frac{\boldsymbol{B}}{3\mu_0} = \frac{2\boldsymbol{B}}{3\mu_0}$$

SOL 5.1.7 Option (D) is correct.

As the magnetic flux density and magnetic field intensity inside a magnetic material are related as

$$\boldsymbol{B} = \mu_r \mu_0 \boldsymbol{H}$$

So, comparing it with given expression for magnetic flux density we get the relative permeability as

$$\mu_r = k = k - 1$$

Therefore, the magnetization vector inside the material is given as

$$\boldsymbol{M} = (\mu_r - 1)\boldsymbol{H} = (k - 1)\boldsymbol{H}$$

SOL 5.1.8 Option (B) is correct.

For the spherical cavity of magnetization $\boldsymbol{M}$, the flux density is given by

$$\boldsymbol{B}_{cavity} = \frac{2}{3}\mu_0\boldsymbol{M}$$

Since the cavity is hollowed. So net magnetic flux density at the center of cavity is

$$\boldsymbol{B}_{net} = \boldsymbol{B}_0 - \boldsymbol{B}_{cavity} = \boldsymbol{B}_0 - \frac{2}{3}\mu_0\boldsymbol{M}$$

and so the net magnetic field intensity at the center of cavity is

$$\boldsymbol{H}_{net} = \frac{1}{\mu_0}\boldsymbol{B}_{net} = \frac{1}{\mu_0}\left[\boldsymbol{B}_0 - \frac{2}{3}\mu_0\boldsymbol{M}\right]$$

$$= \frac{1}{\mu_0}\left[\mu_0\boldsymbol{H}_0 + \mu_0\boldsymbol{M} - \frac{2}{3}\mu_0\boldsymbol{M}\right]$$

$$= \left[\boldsymbol{H}_0 + \frac{\boldsymbol{M}}{3}\right] \qquad (\boldsymbol{B}_0 = \mu_0(\boldsymbol{H}_0 + \boldsymbol{M}))$$

SOL 5.1.9 Option (B) is correct.

In a magnetic medium the magnetic field intensity and magnetic flux density are related as

$$\boldsymbol{B} = \mu_0(1 + \chi_m)\boldsymbol{H}$$

So the magnetic flux density inside the medium is

$$\boldsymbol{H} = \frac{\boldsymbol{B}}{(1 + \chi_m)\mu_0} = \frac{4z}{3\mu_0}\boldsymbol{a}_z \qquad (\boldsymbol{B} = 4z\boldsymbol{a}_z\,\mathrm{T},\ \chi_m = 2)$$

Now the magnetization of a magnetic medium having magnetic field intensity $\boldsymbol{H}$ is given as

$$\boldsymbol{M} = \chi_m\boldsymbol{H}$$

$$= 2\left(\frac{4z}{3\mu_0}\right)\boldsymbol{a}_z$$

$$= \frac{8z}{3\mu_0}\boldsymbol{a}_z$$

The bound current density inside a medium having magnetization $\boldsymbol{M}$ is given as

$$\boldsymbol{J}_b = \nabla \times \boldsymbol{M}$$

$$= \nabla \times \left(\frac{8z}{3\mu_0}\boldsymbol{a}_z\right) = \frac{8}{3\mu_0}\boldsymbol{a}_y\,\mathrm{A/m^2}$$

SOL 5.1.10 Option (A) is correct.

Total current density inside a medium having magnetic flux density $\boldsymbol{B}$ is given as

$$\boldsymbol{J}_T = \frac{\nabla \times \boldsymbol{B}}{\mu_0} = \frac{1}{\mu_0}\left[\frac{\partial(4z)}{\partial z}\right]\boldsymbol{a}_y \qquad (\boldsymbol{B} = 4z\boldsymbol{a}_x\,\mathrm{T})$$

$$= \frac{4}{\mu_0}\boldsymbol{a}_y\,\mathrm{A/m^2}$$

SOL 5.1.11 Option (C) is correct.

Magnetic field intensity in 1^{st} medium is given

$$H_1 = 9a_x + 16a_y - 10a_z$$
$$= H_{1t} + H_{1n}$$

where H_{1t} and H_{1n} are respectively the tangential and normal components of the magnetic field intensity to the boundary interface in medium 1. From boundary condition we have

$$H_{1t} = H_{2t}$$

and $$\mu_2 H_{2n} = \mu_1 H_{1n}$$

where H_{2t} and H_{2n} are respectively the tangential and normal component of magnetic field intensity in medium 2. So we get the components in medium 2 as

$$H_{2t} = 9a_x - 10a_z$$

and $$H_{2n} = \frac{\mu_1}{\mu_2} H_{1n}$$
$$= \frac{\mu_{r1}\mu_0}{\mu_{r2}\mu_0} H_{1n}$$
$$= \frac{7}{6}(16a_y) = 18.67a_y$$

Therefore, the net magnetic field intensity in medium 2 is

$$H_2 = H_{2t} + H_{2n}$$
$$= 9a_x + 18.67a_y - 10a_z \text{ A/m}$$

SOL 5.1.12 Option (B) is correct.

Magnetic flux density in any medium in terms of magnetic field intensity is defined as

$$B = \mu H$$

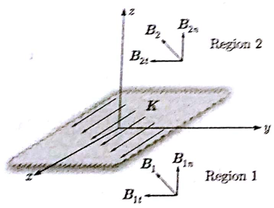
where μ is the permeability of the medium. So, the magnetic flux density in medium 2 is given as

$$B_2 = \mu_2 H_2 = \mu_{r2}\mu_0 H_2$$
$$= 6 \times (4\pi \times 10^{-7}) \times (9a_x + 18.67a_y - 10a_z) \quad (\mu_{r2} = 6)$$
$$= (6.8a_x + 14.1a_y - 7.5a_z) \times 10^{-5} \text{ wb/m}^2$$

SOL 5.1.13 Option (D) is correct.

The magnetic flux density in region $z < 0$ is given as

$$B = 4a_y + 3a_z \text{ Wb/m}^2$$



Now we consider the flux density in region 1 is B_1. So, we have

$$B_1 = 4a_y + 3a_z$$

Therefore the tangential component B_{1t} and normal component B_{1n} of the magnetic flux density in region 1 are

$$B_{1t} = 4a_y$$

and $$B_{1n} = 3a_z$$

From the boundary condition the tangential and normal components of magnetic flux density in two mediums are related as

$$B_{1n} = B_{2n}$$
$$B_{2t} - B_{1t} = \mu_0 K$$

where B_{2t} and B_{2n} are respectively the tangential and normal components of the magnetic flux density in region 2 and K is the current density at the boundary interface.

So, we get $$B_{2n} = B_{1n} = 3a_z \qquad (B_{1n} = 3a_z)$$

and $$B_{2t} = 4a_y + \mu_0(4a_x) \qquad (B_{1t} = 4a_y,\ K = 4a_x \text{ A/m})$$
$$= 4a_y + 4\mu_0 a_x$$

Therefore the net flux density in region 2 $(z > 0)$ is

$$B_2 = B_{2t} + B_{2n} = 4a_y + 4\mu_0 a_x + 3a_z$$

SOL 5.1.14 Option (D) is correct.

As the surface boundary of the slab is parallel to yz-plane so the given magnetic flux density will be tangential to the surface.

i.e. $$B_{to} = B_0$$

and $$H_{to} = \frac{B_{to}}{\mu_0} = \frac{B_0}{\mu_0}$$

Since the tangential component of magnetic field intensity is uniform at the boundary of the magnetic material So, magnetic field intensity inside the material is

$$H_{in} = H_{to} = \frac{B_0}{\mu_0}$$

Therefore, the flux density inside the material is

$$B_{in} = \mu H_{in} = \mu_r \mu_0 \frac{B_0}{\mu_0} = \mu_r B_0$$

SOL 5.1.15 Option (D) is correct.

From boundary condition the normal component of flux density is uniform at boundary

i.e. $$B_{1n} = B_{2n}$$
$$B_1 \sin\theta_1 = B_2 \sin\theta_2$$

and the tangential component of field intensity is uniform

i.e. $$H_{1t} = H_{2t}$$
$$H_1 \cos\theta_1 = H_2 \cos\theta_2$$

SOL 5.1.16 Option (B) is correct.

Relation between θ_1 and θ_2 at boundary of region (1) and region (2) as

$$\mu_1 \tan\theta_1 = \mu_2 \tan\theta_2$$

and at the interface between region (2) and region (3) is

$$\mu_2 \tan\theta_3 = \mu_3 \tan\theta_4; \qquad \text{since } \theta_2 = \theta_3$$

So, combining the two eq. we get,

$$\mu_1 \tan\theta_1 = \mu_3 \tan\theta_4$$

Thus, θ_4 will be independent of μ_2 only.

SOL 5.1.17 Option (A) is correct.

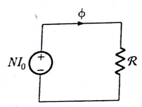
From the analogy between electrical and magnetic circuits, we have the following relations,

$$\mathcal{F} \text{ (magnetomotive force)} \rightarrow V \text{ (voltage)}$$
$$\phi \text{ (magnetic flux)} \rightarrow I \text{ (current)}$$
$$\mathcal{R} \text{ (Reluctance)} \rightarrow R \text{ (Resistance)}$$

Now, magnetomotive force,

$$\mathcal{F} = NI_0$$

and so, the electrical analog of the magnetic circuit is

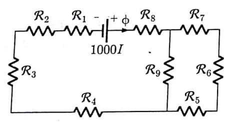


SOL 5.1.18 Option (A) is correct.

For drawing the electrical analog replace the coil by a source (magnetomotive force) and each section of the core by a reluctance. In the shown magnetic material there are 9 sections so we draw the reluctance for each of them and we get the magnetomotive force as

$$\mathcal{F} = 1000I \qquad (N = 1000)$$

So the equivalent circuit is



SOL 5.1.19 Option (A) is correct.

Consider the particle carries a total charge Q.

Since for a moving charge Q in the presence of both electric and magnetic fields, the total force on the charge is given by

$$\boldsymbol{F} = Q[\boldsymbol{E} + (\boldsymbol{v} \times \boldsymbol{B})]$$

where $\boldsymbol{E} \rightarrow$ electric field

$\boldsymbol{v} \rightarrow$ velocity of the charged particle

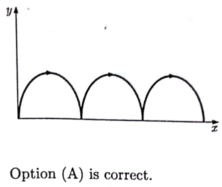
$\boldsymbol{B} \rightarrow$ magnetic flux density

So initially the magnetic force on the particle will be zero as the particle is released at rest ($\boldsymbol{v} = 0$). Therefore the electric field will accelerate the particle in y-direction and as it picks up speed (consider the velocity is $\boldsymbol{v} = k\boldsymbol{a}_y$, k is very small) a magnetic force develops which will be given by

$$\boldsymbol{F} = \boldsymbol{v} \times \boldsymbol{B}$$

since the magnetic field is in $\boldsymbol{a}_z$ direction while the beam has the velocity in $\boldsymbol{a}_y$ direction so the magnetic force will be in $\boldsymbol{a}_x$ $(\boldsymbol{a}_y \times \boldsymbol{a}_z)$ direction.

Therefore the magnetic force will pull the charged particle around to the right and as the magnetic force will be always perpendicular to both the velocity of particle and electric field. So the particle will initially goes up in the y-direction and then following a curve path lowers down towards the z-axis.



SOL 5.1.20 Option (A) is correct.

For a moving charge Q in the presence of both electric and magnetic fields, the total force on the charge is given by

$$\boldsymbol{F} = Q[\boldsymbol{E} + (\boldsymbol{v} \times \boldsymbol{B})]$$

where $\boldsymbol{E} \rightarrow$ electric field

$\boldsymbol{v} \rightarrow$ velocity of the charged particle

$\boldsymbol{B} \rightarrow$ magnetic flux density

Since initially the velocity of the charge (at the time of injection) is $v_0 = 2\boldsymbol{a}_y$ m/s

and for the region $y > 0$ magnetic flux density is $\boldsymbol{B} = 3\boldsymbol{a}_z$ wb/m^2.

so there will be no any velocity component in $+\boldsymbol{a}_z$ direction caused by the field (since the magnetic field is in $\boldsymbol{a}_z$ direction).

i.e. $\qquad v_z = 0$

So we consider the velocity of the point charge in the region $y > 0$ at a particular time t as

$$\boldsymbol{v} = v_y \boldsymbol{a}_y + v_z \boldsymbol{a}_z$$

Therefore we have the force applied by the field on the charge particle at time t as

$$\boldsymbol{F} = Q[(v_y \boldsymbol{a}_y + v_z \boldsymbol{a}_z) \times (3\boldsymbol{a}_z)]$$

$$m\left[\frac{dv_y}{dt}\boldsymbol{a}_y + \frac{dv_z}{dt}\boldsymbol{a}_z\right] = Q[-3v_y \boldsymbol{a}_z + 3v_z \boldsymbol{a}_y]$$

So, we get $\qquad \dfrac{dv_y}{dt} = \dfrac{3Q}{m}v_z$

and $\qquad \dfrac{dv_z}{dt} = -\dfrac{3Q}{m}v_y$

From the two relations we have

$$\frac{d^2 v_z}{dt^2} + \left(\frac{3Q}{m}\right)^2 v_z = 0$$

$$v_z = A_1 \cos\left(\frac{3Q}{m}t\right) + B_1 \sin\left(\frac{3Q}{m}t\right)$$

where A_1 and B_1 are constants.

and since at $t = 0$, $v_z = 0$ (since charge was injected with a velocity in $\boldsymbol{a}_y$ direction)

Putting the condition in the expression we get $A_1 = 0$

and so we have $\qquad v_z = B_1 \sin\left(\dfrac{3Q}{m}t\right) = B_1 \sin t \qquad Q = 2\,\text{C},\ m = 6\,\text{kg}$

Again, $\dfrac{dv_z}{dt} = -\dfrac{3Q}{m} v_y$

so

$$v_y = -\left(\frac{m}{3Q}\right)\frac{dv_z}{dt}$$
$$= -B_1 \cos\left(\frac{3q}{m}t\right) = -B_1 \cos t$$

and since at $t = 0$, $v_y = 2\,\text{m/s}$

Putting the condition in the expression we get,

$$2 = -B_1 \cos 0$$
$$B_1 = -2$$

So, we have, $v_z = -2\sin t \rightarrow \dfrac{dz}{dt} = -2\sin t$

$$v_y = 2\cos t \rightarrow \frac{dy}{dt} = 2\cos t$$

Solving the equations we get,

$$z = 2\cos t + C_2$$

and $y = 2\sin t + C_3$

and since at $t = 0$, $y = z = 0$ (charge is located at origin at the time of injection)

Putting the condition in the expression we get

$$C_2 = -2 \text{ and } C_3 = 0$$

So we have $z = 2\cos t - 2 \Rightarrow z + 2 = 2\cos t$

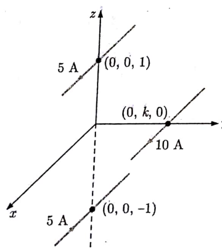
and $y = 2\sin t$

Therefore the equation of the path that the charged particle will follow is

$$y^2 + (z+2)^2 = 4$$

This is the equation of a circle centred at $(0, 0, -2)$.

SOL 5.1.21 Option (A) is correct.



Net magnetic flux density arising from the two current filaments $-5\boldsymbol{a}_z$ and $5\boldsymbol{a}_z$ A at the location of third filament is given by

$$\boldsymbol{B} = \boldsymbol{B}_1 + \boldsymbol{B}_2 \qquad (1)$$

where $\boldsymbol{B}_1$ and $\boldsymbol{B}_2$ are the magnetic flux density produced by the current filaments $5\boldsymbol{a}_z$ and $-5\boldsymbol{a}_z$ respectively. Since the magnetic flux density produced at a distance ρ from a straight wire carrying current I is defined as

$$\boldsymbol{B} = \frac{\mu_0 I}{2\pi\rho}\boldsymbol{a}_\phi$$

and the direction of the magnetic flux density is given as

$$\boldsymbol{a}_\phi = \boldsymbol{a}_l \times \boldsymbol{a}_\rho$$

where $\boldsymbol{a}_l$ is unit vector along the line current and $\boldsymbol{a}_\rho$ is the unit vector normal to the line current directed toward the point P. So, the magnetic flux density produced by the current filament $5\boldsymbol{a}_z$ is

$$\boldsymbol{B}_1 = \frac{5\mu_0}{2\pi(\sqrt{1+k^2})}\left[\boldsymbol{a}_z \times \left(\frac{k\boldsymbol{a}_y - \boldsymbol{a}_z}{\sqrt{1+k^2}}\right)\right]$$
$$= \frac{5\mu_0}{2\pi(1+k^2)}(k\boldsymbol{a}_z + \boldsymbol{a}_y)$$

Similarly the magnetic flux density produced by the current filament $(-5\boldsymbol{a}_z)$ is

$$\boldsymbol{B}_2 = \frac{\mu_0 \times (5)}{2\pi(\sqrt{1+k^2})}\left[(-\boldsymbol{a}_z) \times \left(\frac{k\boldsymbol{a}_y + \boldsymbol{a}_z}{\sqrt{1+k^2}}\right)\right]$$
$$= \frac{5\mu_0}{2\pi(1+k^2)}(-k\boldsymbol{a}_z + \boldsymbol{a}_y)$$

Therefore from equation (1), we get the net magnetic flux density experienced by the third filamentary current of $10\,\boldsymbol{a}_z$ A as

$$\boldsymbol{B} = \frac{5\mu_0}{2\pi(1+k^2)}[k\boldsymbol{a}_z + \boldsymbol{a}_y - k\boldsymbol{a}_z + \boldsymbol{a}_y]$$
$$= \frac{5\mu_0}{2\pi(1+k^2)}(2\boldsymbol{a}_y)$$
$$= \frac{5\mu_0}{\pi(1+k^2)}\boldsymbol{a}_y$$

As the force experienced by a current element $I\boldsymbol{dl}$ in the presence of magnetic flux density $\boldsymbol{B}$ is defined as

$$d\boldsymbol{F} = I\boldsymbol{dl} \times \boldsymbol{B}$$

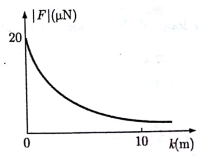
where I is the current flowing in the element and $\boldsymbol{dl}$ is the differential vector length of the current element in the direction of flow of current.

Force per unit meter length experienced by the third filament is

$$\boldsymbol{F} = \int_{x=0}^{1}(10\boldsymbol{a}_x\,dx) \times \frac{5\mu_0}{\pi(1+k^2)}\boldsymbol{a}_y \qquad (\boldsymbol{dl} = dx\boldsymbol{a}_x)$$
$$= \frac{10 \times 5 \times 4\pi \times 10^{-7}}{\pi(1+k^2)}\boldsymbol{a}_z = \frac{20\boldsymbol{a}_z}{(1+k^2)}\,\mu\text{N}$$

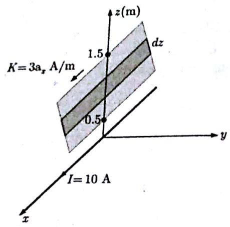
or, $$F = \frac{20}{(1+k^2)}\,\mu\text{N}$$

Thus, the graph between F and k will be as shown in the figure below :



SOL 5.1.22 Option (B) is correct.

Consider the strip is formed of many adjacent strips of width dz each carrying current Kdz.



Since the magnetic flux density produced at a distance ρ from a straight wire carrying current I is defined as

$$B = \frac{\mu_0 I}{2\pi\rho}$$

So the magnetic flux density produced by each differential strip is

$$dB = \frac{\mu_0(Kdz)}{2\pi z}\boldsymbol{a}_y \qquad (I = Kdz)$$

(Using right hand rule we get the direction of the magnetic flux density along $\boldsymbol{a}_y$)

Therefore the net magnetic flux density produced by the strip on the current filament is

$$\boldsymbol{B} = \int_{z=0.5}^{1.5}\frac{3\mu_0\boldsymbol{a}_y}{2\pi z}dz = \frac{3\mu_0}{2\pi}\ln\left(\frac{1.5}{0.5}\right)\boldsymbol{a}_y \qquad (K = 3\,\text{A/m})$$

$$= 6.6\times10^{-7}\boldsymbol{a}_y\ \text{wb/m}^2$$

As the force experienced by a current element $I\boldsymbol{dl}$ in the presence of magnetic flux density $\boldsymbol{B}$ is defined as

$$d\boldsymbol{F} = I\boldsymbol{dl}\times\boldsymbol{B}$$

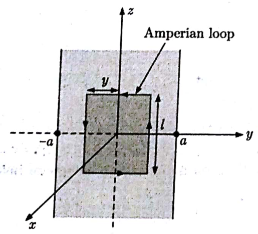
where I is the current flowing in the element and $\boldsymbol{dl}$ is the differential vector length of the current element in the direction of flow of current. So the force exerted on the filament per unit length is

$$\boldsymbol{F} = \int I\boldsymbol{dl}\times\boldsymbol{B}$$

$$= \int_{x=0}^{1}(10dx\,\boldsymbol{a}_x)\times(6.6\times10^{-7}\boldsymbol{a}_y) = 6.6\boldsymbol{a}_z\,\mu\text{N/m}$$

SOL 5.1.23 Option (A) is correct.

Consider a rectangular Amperian loop of dimension $(l)\times(2y)$ inside the slab as shown in the figure below.



As from the Amperes circuital law, we have

$$\int \boldsymbol{H}\cdot\boldsymbol{dl} = I_{enc}$$

So for the Amperian loop inside the slab we get

$$\boldsymbol{H}(2l) = (2y\times l)(J_0) \qquad \text{for } -a\le y\le a$$

(Net magnetic field intensity along the edge $2y$ will be cancelled due to symmetry)

Therefore the magnetic field intensity (magnetizing factor) at any point inside the slab is

$$\boldsymbol{H} = J_0 y\boldsymbol{a}_z$$

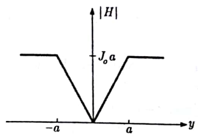
or

$$H = J_0|y| \qquad (\text{for } |y|\le a)$$

and the magnetic field intensity (magnetizing factor) at any point outside the slab is

$$H = J_0 a \qquad (\text{for } |y| > a)$$

Thus, the plot of H versus y will be as shown below



SOL 5.1.24 Option (A) is correct.

Force on any dipole having moment $\boldsymbol{m}$ due to a magnetic flux density $\boldsymbol{B}$ is defined as

$$\boldsymbol{F} = \nabla(\boldsymbol{m}\cdot\boldsymbol{B})$$

Since the magnetic moment of the dipole is given as

$$\boldsymbol{m} = m_0\boldsymbol{a}_x \qquad (1)$$

and as calculated in previous question the magnetic field intensity produced due to the slab is

$$\boldsymbol{H} = J_0 y\boldsymbol{a}_z$$

So we get the magnetic flux density produced due to the slab as

$$\boldsymbol{B} = \mu_0\boldsymbol{H} = \mu_0 J_0 y\boldsymbol{a}_z \qquad (2)$$

Therefore from equation (1) and (2) we get

$$\boldsymbol{m}\cdot\boldsymbol{B} = 0$$

Thus the force acting on the dipole is

$$\boldsymbol{F} = 0$$

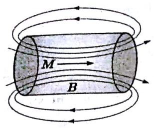
SOL 5.1.25 Option (A) is correct.

Magnetic flux density inside a magnetic material is defined as

$$\boldsymbol{B} = \mu_0(\boldsymbol{H}+\boldsymbol{M})$$

So, $\boldsymbol{B}$ and $\boldsymbol{M}$ will be in same direction inside the cylinder.

Now as the magnetic field lines are circular so outside the cylinder it will make a loop. Thus, the magnetic field lines will be as shown below



SOL 5.1.26 Option (A) is correct.

Volume current density inside a magnetic material is equal to the curl of its magnetization M

i.e. $\qquad \boldsymbol{J} = \nabla \times \boldsymbol{M}$

So volume current density due inside the circular cylinder is

$$\boldsymbol{J} = \frac{1}{\rho}\frac{\partial}{\partial \rho}\rho(5\rho^2)\,\boldsymbol{a}_z = 15\rho\boldsymbol{a}_z$$

or $\qquad J \propto \rho$

SOL 5.1.27 Option (B) is correct.

As calculated above the volume current density inside the cylinder is

$$\boldsymbol{J} = 15\rho\boldsymbol{a}_z$$

So, we can get the flux density by Ampere's circuital law as

$$\oint \boldsymbol{B}\cdot d\boldsymbol{l} = \mu_0 I_{enc}$$

$$(B)(2\pi\rho) = \mu_0 I_{enc}$$

$$B = \frac{\mu_0}{2\pi\rho} I_{enc}$$

Now the enclosed current in the loop is

$$I_{enc} = \int_S \boldsymbol{J}\cdot d\boldsymbol{S}$$

$$= \int_0^{\rho}\int_0^{2\pi}(15\rho)(\rho d\rho d\phi)$$

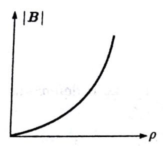
$$= 2\pi \times 15\left[\frac{\rho^3}{3}\right]_0^{\rho}$$

$$= 10\pi\rho^3$$

So, the magnetic flux density inside the cylinder is

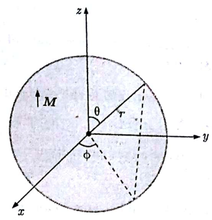
$$B = \frac{\mu_0}{2\pi\rho} I_{enc} = 5\mu_0\rho^2 \qquad (I = 10\pi\rho^3)$$

Thus the plot of magnetic flux density B versus ρ is as shown below



SOL 5.1.28 Option (D) is correct.

Let the magnetized sphere be of radius r, centered at origin and the magnetization be M in $\boldsymbol{a}_z$ direction as shown in figure.



Volume current density inside a material is equal to the curl of magnetization M

i.e. $\qquad \boldsymbol{J} = \nabla \times \boldsymbol{M}$

So the volume current density inside the cylinder is

$$\boldsymbol{J} = \nabla \times (M\boldsymbol{a}_z) = 0$$

and since the surface current density in terms of magnetization is defined as

$$\boldsymbol{K} = \boldsymbol{M}\times\boldsymbol{a}_n \qquad \text{where } \boldsymbol{a}_n \text{ is unit vector normal to the surface.}$$

So the surface current density on the sphere is

$$\boldsymbol{K} = (M\boldsymbol{a}_z)\times(\boldsymbol{a}_r) \qquad (\boldsymbol{a}_n = \boldsymbol{a}_r)$$

$$= M\sin\theta\boldsymbol{a}_\phi \qquad \text{...(1)}$$

Now, consider a rotating spherical shell of uniform surface charge density σ, that corresponds to a surface current density at any point (r,θ,ϕ). So we have

$$\boldsymbol{K} = \sigma\omega R\sin\theta\boldsymbol{a}_\phi \qquad \text{...(2)}$$

where $\qquad \omega \rightarrow$ angular velocity of spherical shell across z-axis

$R \rightarrow$ radius of the sphere.

and the magnetic flux density produced inside the rotating spherical shell is defined as

$$\boldsymbol{B} = \frac{2}{3}\mu_0\sigma\omega R \qquad \text{...(3)}$$

Comparing the eq.(1) and eq.(2) we get

$$M = \sigma\omega R$$

Putting this value in eq.(3) we get the magnetic flux density for the magnetized sphere as

$$\boldsymbol{B} = \frac{2}{3}\mu_0\boldsymbol{M} \qquad (M = \sigma\omega R)$$

SOL 5.1.29 Option (B) is correct.

The surface current density of a material in terms of its magnetization is defined as

$$\boldsymbol{K} = \boldsymbol{M}\times\boldsymbol{a}_n \qquad \text{where } \boldsymbol{a}_n \text{ is unit vector normal to the surface.}$$

So, the surface current density of the cylinder is

$$\boldsymbol{K} = (M\boldsymbol{a}_z)\times(\boldsymbol{a}_\rho) = M\boldsymbol{a}_\phi \qquad (\boldsymbol{M} = M\boldsymbol{a}_z, \boldsymbol{a}_n = \boldsymbol{a}_\rho)$$

Therefore the surface current density is directed along $\boldsymbol{a}_\phi$ as shown in option (B).

SOL 5.1.30 Option (B) is correct.

Since the magnetic flux density inside a magnetic material is defined as

$$B = \mu_0(H + M)$$

So, we have the magnetic field intensity inside the material as

$$H = \frac{1}{\mu_0}B - M$$

and outside the material the magnetic field intensity is

$$H = \frac{1}{\mu_0}B$$

So the field lines outside the material will be same as for the case of magnetic flux density shown earlier. Whereas inside the material the direction of magnetic field intensity will be opposite to the direction of magnetization. Thus the sketch of the field intensity will be same as shown in the option (B).

SOL 5.1.31 Option (A) is correct.

Consider the wire is lying along z-axis. So at any point inside the wire (at a distance $\rho < a$ from it's axis) magnetic field intensity will be determined as

$$\int H \cdot dl = I_{enc} \qquad \text{(Ampere's circuital law)}$$

$$H(2\pi\rho) = I\left(\frac{\pi\rho^2}{\pi a^2}\right) \qquad \text{(for Amperian loop of radius } \rho\text{)}$$

or, $$H = \frac{I\rho}{2\pi a^2}a_\phi$$

The direction of the magnetic field intensity is determined using right hand rule.

Now the stored energy in the magnetic field H is defined as

$$W_m = \int_v \frac{1}{2}\mu_0 H^2 dv$$

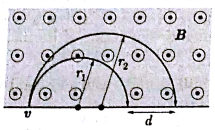
So the stored energy in the internal magnetic filed per unit length (over the unit length in z-direction) will be

$$W_m = \int_{z=0}^{1}\int_{\phi=0}^{2\pi}\int_{\rho=0}^{a}\frac{\mu_0 I^2\rho^2}{2(2\pi a^2)^2}\rho d\rho d\phi dz = \frac{\mu_0 I^2}{16\pi}$$

Therefore, the energy per unit length depends only on I and is uniform for the uniform current.

SOL 5.1.32 Option (A) is correct.

Consider the path followed by the two particles are the curvatures having radii r_1 and r_2 as shown in figure. So at balanced condition centrifugal force will be equal to magnetic force.



Therefore for the first charged particles

$$\frac{mv^2}{r_1} = Bqv \Rightarrow r_1 = \frac{mv}{Bq}$$

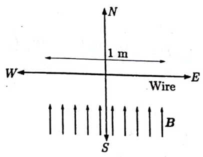
and $$\frac{(2m)v^2}{r_2} = Bqv \Rightarrow r_2 = \frac{mv}{Bq}$$

So the distance between the two particles at releasing end is

$$d = 2r_2 - 2r_1 = 2\left(\frac{2mv}{Bq}\right) - 2\left(\frac{mv}{Bq}\right) = \frac{2mv}{Bq}$$

SOL 5.1.33 Option (A) is correct.

The wire is oriented in east-west direction and magnetic field is directed northward as shown in the figure.

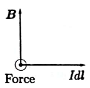


Since the direction of gravitational force will be into the paper(toward the earth) so for counteracting the gravitational force, applied force must be outward.

Now the force experienced by a current element Idl in a magnetic field B is

$$F = \oint_L (Idl) \times B$$

As the magnetic field B is directed toward north therefore, using right hand rule for cross vector we conclude that for producing the outward force current must flow from west to east as shown in the figure below.



SOL 5.1.34 Option (A) is correct.

Consider the current flowing in the wire is I. So the magnetic force applied by the field B_0 on the wire is

$$F_m = ILB_0 \qquad \text{where } L \text{ is length of the wire}$$

At balanced condition the magnetic force will be equal to the gravitational force :

$$F_m = mg$$

where m is the mass of the wire and g is acceleration due to gravity.

So comparing the two results we get the current flowing in the wire as

$$I = \frac{mg}{LB_0}$$

Since $B_0 = 0.6 \times 10^{-4}\,\text{Wb/m}^2$, $m = 0.3\,\text{kg}$ and $L = 1\,\text{m}$

Therefore $$I = \frac{(0.3) \times 9.8}{(1) \times (0.6 \times 10^{-4})} = 49\,\text{kA} \qquad (g = 9.8\,\text{m/s})$$

SOL 5.1.35 Option (B) is correct.

Given the B-H curve for the material,

$$B = 2\mu_0 HH$$

The work done per unit volume in magnetizing a material from 0 to B_0 that has non uniform permeability is defined as

$$w_m = \int_0^{B_0} \boldsymbol{H} \cdot d\boldsymbol{B}$$

Now for determining $d\boldsymbol{B}$. we can express

$$\boldsymbol{B} = 2\mu_0 H^2 \boldsymbol{a}_H$$

where $\boldsymbol{a}_H$ is the unit vector in direction of $\boldsymbol{H}$.

So, $$\frac{d\boldsymbol{B}}{dH} = 4\mu_0 H\boldsymbol{a}_H = 4\mu_0 \boldsymbol{H}$$

and $$w_m = \int_0^{H_0} \boldsymbol{H} \cdot (4\mu_0 \boldsymbol{H}) = 4\mu_0 \left[\frac{H^3}{3}\right]_0^{H_0} = \frac{4\mu_0 H_0^3}{3}$$

SOL 5.1.36 Option (B) is correct.

As discussed earlier, the path of electron will be parallel to the input beam but in opposite direction. So the ejected electrons will be flowing in the $-\boldsymbol{a}_y$ direction.

SOL 5.1.37 Option (A) is correct.

As the permeability of the medium varies from μ_1 to μ_2 linearly. So at any distance z from one of the plate near to which permeability is μ_1, the permeability is given as

$$\mu = \mu_1 + \frac{(\mu_2 - \mu_1)}{d} z \qquad (1)$$

The magnetic flux density between the two parallel sheets carrying equal and opposite current densities is defined as

$$B = \mu K$$

where K is the magnitude of the current density of the sheets.

Therefore the flux per unit length between the two sheets is

$$\frac{\phi}{l} = \int_0^d B dz \quad \text{where } d \text{ is the separation between the two sheets.}$$

$$= \int_0^d \mu K dz = K \int_0^d \left[\mu_1 + \frac{(\mu_2 - \mu_1)}{d} z\right] dz \qquad \text{(from equation (1))}$$

$$= K\left[\mu_1 z + \frac{\mu_2 - \mu_1}{d}\left(\frac{z^2}{2}\right)\right]_0^d = K\left(\frac{\mu_1 + \mu_2}{2}\right) d$$

SOL 5.1.38 Option (B) is correct.

Given the field intensity inside the slab is

$$\boldsymbol{H} = 4\boldsymbol{a}_x + 2\boldsymbol{a}_y$$

So the magnetic flux density inside the slab is given as

$$\boldsymbol{B} = \mu \boldsymbol{H} \quad \text{where } \mu \text{ is the permeability of the material.}$$

$$= 2\mu_0 (4\boldsymbol{a}_x + 2\boldsymbol{a}_y) \qquad (\mu = 2\mu_0)$$

Therefore the magnetization of the material is

$$\boldsymbol{M} = \frac{\boldsymbol{B}}{\mu_0} - \boldsymbol{H}$$

$$= 8\boldsymbol{a}_x + 4\boldsymbol{a}_y - (4\boldsymbol{a}_x + 2\boldsymbol{a}_y) = 4\boldsymbol{a}_x + 2\boldsymbol{a}_y$$

Now the magnetization surface current density at the surfaces of a magnetic material is defined as

$$\boldsymbol{K}_m = \boldsymbol{M} \times \boldsymbol{a}_n$$

where $\boldsymbol{a}_n$ is the unit vector normal to the surface directed outward of the material So, at $z = 0$ magnetization surface current density is

$$[\boldsymbol{K}_m]_{\text{at } z=0} = \boldsymbol{M} \times (-\boldsymbol{a}_z) \qquad (\boldsymbol{a}_n = -\boldsymbol{a}_z)$$

$$= 4\boldsymbol{a}_y - 2\boldsymbol{a}_x$$

and at $z = d$, the magnetization surface current density is

So, $$[\boldsymbol{K}_m]_{\text{at } z=d} = \boldsymbol{M} \times (\boldsymbol{a}_z) \qquad (\boldsymbol{a}_n = \boldsymbol{a}_z)$$

$$= (4\boldsymbol{a}_x + 2\boldsymbol{a}_y) \times (\boldsymbol{a}_z) = -4\boldsymbol{a}_y + 2\boldsymbol{a}_x$$

SOL 5.1.39 Option (A) is correct.

As calculated in the previous question the magnetization vector of the material is

$$\boldsymbol{M} = 4\boldsymbol{a}_x + 2\boldsymbol{a}_y$$

The magnetization volume current density inside a magnetic material is equal to the curl of magnetization.,

i.e. $$\boldsymbol{J}_m = \nabla \times \boldsymbol{M}$$

Therefore the magnetization volume current density inside the slab is

$$\boldsymbol{J}_m = \begin{vmatrix} \boldsymbol{a}_x & \boldsymbol{a}_y & \boldsymbol{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 4 & 2 & 0 \end{vmatrix} = 0$$

SOL 5.1.40 Option (A) is correct.

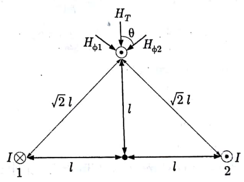
As ⊗ shows the direction into the paper while ⊙ shows the direction out of the paper. So the wire of length l carries current $2I$ that flows out of the paper.

The Magnetic field intensity produced at a distance ρ from an infinite straight wire carrying current I is defined as

$$H = \frac{I}{2\pi\rho}$$

So the magnetic field intensity produced at the top wire due to the infinite wire carrying current inward is

$$H_{\phi 1} = \frac{I}{2\pi(\sqrt{2}\,l)} \qquad (\rho = \sqrt{2}\,l)$$



and the magnetic field intensity at top wire due to the infinite wire carrying current outward is

$$H_{\phi 2} = \frac{I}{2\pi(\sqrt{2}\,l)} \qquad (\rho = \sqrt{2}\,l)$$

Therefore the resultant field intensity at the wire of length l is

$$H_T = (H_{\phi 1} + H_{\phi 2})\cos\theta$$

$$= \frac{2I}{2\pi(\sqrt{2}\,l)} \times \frac{1}{\sqrt{2}} = \frac{I}{2\pi l}$$

Since the force exerted on a current element Idl by a magnetic field H is defined as

$$dF = (\mu H)(Idl)$$

So the force experienced by the wire of length l is

$$F = (\mu H_T)(2I)l = \mu\left(\frac{I}{2\pi l}\right)(2I)l = \frac{\mu I^2}{\pi}$$

SOL 5.1.41 Option (C) is correct.

Since the boundary surface of the two medium is $z = 0$, so the normal component B_{1n} and tangential component B_{1t} of magnetic flux density in medium 1 are

$$B_{1n} = a_z$$

and $$B_{1t} = 0.4a_x + 0.8a_y$$

As the normal component of magnetic flux density is uniform at the boundary of two medium So, the normal component of magnetic flux density in the medium 2 is

$$B_{2n} = B_{1n} = a_z \qquad (1)$$

Now for determining tangential component of field in medium 2, we first calculate tangential component of magnetic field intensity in medium 1 which is given as

$$H_{1t} = \frac{B_{1t}}{\mu_1} \quad \text{where } \mu_1 \text{ is the permeability of medium 1.}$$

$$= \frac{1}{4\mu_0}(0.4a_x + 0.8a_y) = \frac{0.1a_x + 0.2a_y}{\mu_0} \qquad (\mu_1 = 4\mu_0)$$

Again from the boundary condition the tangential component of magnetic field intensity in the two mediums are related as

$$a_n \times (H_{1t} - H_{2t}) = K$$

where H_{2t} and H_{1t} are the tangential components of magnetic field intensity in medium 2 and medium 1 respectively, K is the surface current density at the boundary interface of the two mediums and a_n is the unit vector normal to the boundary interface. So we have

$$a_z \times \left[\frac{0.1a_x + 0.2a_y}{\mu_0} - (H_{2tx}a_x + H_{2ty}a_y)\right] = \frac{1}{\mu_0}(0.2a_x - 0.4a_y)$$

$$\left(\frac{0.1}{\mu_0} - H_{2tx}\right)a_y - \left(\frac{0.2}{\mu_0} - H_{2ty}\right)a_x = \frac{1}{\mu_0}(0.2a_x - 0.4a_y)$$

Comparing the x and y-components we get

$$H_{2tx} = \frac{0.1}{\mu_0} + \frac{0.4}{\mu_0} = \frac{0.5}{\mu_0}$$

and $$H_{2ty} = \frac{0.2}{\mu_0} + \frac{0.2}{\mu_0} = \frac{0.4}{\mu_0}$$

Therefore the tangential component of magnetic field intensity in medium 2 is

$$H_{2t} = \frac{0.5}{\mu_0}a_x + \frac{0.4}{\mu_0}a_y$$

and the tangential component of magnetic flux density in medium 2 is

$$B_{2t} = \mu_2 H_{2t} = a_x + 0.8a_y$$

Thus the net magnetic flux density in medium 2 is

$$B_2 = B_{2t} + B_{2n} = a_x + 0.8a_y + a_z$$

****

SOLUTIONS 5.2

SOL 5.2.1 Correct answer is 5.

For a moving charge Q in the presence of both electric and magnetic fields, the total force on the charge is given by

$$F = Q[E + (v \times B)]$$

where $E \rightarrow$ electric field

$v \rightarrow$ velocity of the charge

$B \rightarrow$ magnetic flux density

Since the electron beam follows its path without any deflection so the net force applied by the field will be zero

i.e. $$Q[E + (v \times B)] = 0$$

$$15a_y + v \times 3a_z = 0$$

As the electric field is directed along a_y and magnetic field is directed along a_z so the velocity of beam will be in a_x direction (perpendicular to both of the field).

Consider the velocity of the beam is $V = ka_x$

So we have

$$15a_y + ka_x \times 3a_z = 0$$

$$15a_y - 3ka_y = 0$$

$$k = \frac{15}{3} = 5\text{ m/s}$$

So, the velocity of the beam will be 5 m/s along the x-axis.

SOL 5.2.2 Correct answer is -2.

The magnetic field intensity produced at any point in the free space will be the vector sum of the field intensity produced by all the current sheets.

Since, the magnetic field intensity produced at any point P due to an infinite sheet carrying uniform current density K is defined as

$$H = \frac{1}{2}(K \times a_n)$$

where a_n is the unit vector normal to the sheet directed toward the point P. So in the region $0 < z < 1$ magnetic field intensity due to K_2 and K_3 will be cancelled as the unit normal vector to the two sheets will be opposite to each other.

Therefore in this region magnetic field intensity will be produced only due to the current density $K_1 = 4a_x$ which is given as

$$H = \frac{1}{2}K_1 \times a_n = \frac{1}{2}(4a_x) \times (a_z) \qquad (a_n = a_z)$$

$$= -2a_y\text{ A/m}$$

SOL 5.2.3 Correct answer is -14.

As the conducting filament is located along the line $y = 0$, $z = 0.2$ m which is in the region $0 < z < 1$ m, so, the net magnetic field intensity produced on the conducting filament by the current sheets is

$$H = -2a_y\text{ A/m} \qquad \text{(as determined in previous question)}$$

or, $$\boldsymbol{B} = \mu_0 \boldsymbol{H} = -2\mu_0 \boldsymbol{a}_y$$

Now the force experienced by a current element $\boldsymbol{Idl}$ in the presence of magnetic flux density $\boldsymbol{B}$ is defined as

$$d\boldsymbol{F} = I\boldsymbol{dl} \times \boldsymbol{B}$$

where I is the current flowing in the element and $\boldsymbol{dl}$ is the differential vector length of the current element in the direction of flow of current.

So force per unit length experienced by the conducting filament is

$$\frac{d\boldsymbol{F}}{dl} = 7\boldsymbol{a}_x \times (-2\mu_0 \boldsymbol{a}_y) \qquad (I = 7\,\text{A},\ \boldsymbol{dl} = dl\boldsymbol{a}_x)$$

$$= -14\mu_0 \boldsymbol{a}_z\ \text{N/m}$$

SOL 5.2.4 Correct answer is 2249.

In a magnetic medium the magnetization in terms of magnetic field intensity is defined as

$$\boldsymbol{M} = \chi_m \boldsymbol{H}$$

where χ_m is magnetic susceptibility given as

$$\chi_m = \mu_r - 1 = 1.3 \qquad (\text{relative permeability, } \mu_r = 2.3)$$

and since the magnetic field intensity in terms of magnetic flux density is given as

$$\boldsymbol{H} = \frac{\boldsymbol{B}}{\mu} = \frac{\boldsymbol{B}}{\mu_0 \mu_r} \qquad (\boldsymbol{B} = 5\boldsymbol{a}_z\ \text{mwb/m}^2)$$

$$= \frac{5 \times 10^{-3}}{4\pi \times 10^{-7} \times 2.3}\boldsymbol{a}_z = 1730\boldsymbol{a}_z\ \text{A/m} \qquad (\mu_r = 2.3)$$

So the magnetization inside the medium is

$$\boldsymbol{M} = \chi_m \boldsymbol{H} = 2249\ \text{A/m}$$

SOL 5.2.5 Correct answer is 2.19 .

Magnetic flux density in a medium in terms of magnetic filed intensity is defined as

$$\boldsymbol{B} = \mu \boldsymbol{H} = \mu_r \mu_0 \boldsymbol{H}$$

$$= (4/\pi)(4\pi \times 10^{-7})(2\rho^2 \boldsymbol{a}_\phi) \qquad (\mu_r = 4/\pi,\ \boldsymbol{H} = 2\rho^2 \boldsymbol{a}_\phi\ \text{A/m})$$

$$= 32 \times 10^{-7} \rho^2 \boldsymbol{a}_\phi$$

Again the magnetic flux density inside a magnetizing material is defined as

$$\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M})$$

where $\boldsymbol{M}$ is the magnetization of the material. So, we have

$$\boldsymbol{M} = \frac{\boldsymbol{B}}{\mu_0} - \boldsymbol{H}$$

$$= \left[\frac{32 \times 10^{-7}\rho^2}{4\pi \times 10^{-7}} - 2\rho^2\right]\boldsymbol{a}_\phi$$

$$= 2\rho^2\left[\frac{4}{\pi} - 1\right]\boldsymbol{a}_\phi$$

At $\rho = 2$ $$\boldsymbol{M} = 2.19\boldsymbol{a}_\phi\ \text{A/m}$$

SOL 5.2.6 Correct answer is 22.87 .

Given

Magnetic field intensity, $H = 70\ \text{A/m}$

Total magnetic flux in the bar, $\Phi = 4.2\ \text{mWb}$

Cross sectional area of bar, $S = 2\ \text{m}^2$

So we have the magnetic flux density in the bar

$$B = \frac{\Phi}{S} = \frac{4.2 \times 10^{-3}}{2}$$

$$= 2.1\ \text{mwb/m}^2$$

Since the magnetic field intensity and magnetic flux density are related as

$$B = \mu_0(1 + \chi_m)H$$

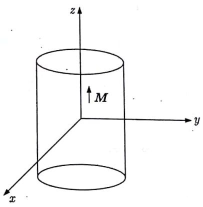
So, we have $$2.1 \times 10^{-3} = (4\pi \times 10^{-7})(1 + \chi_m)(70)$$

$$(1 + \chi_m) = \frac{(2.1 \times 10^{-3})}{70(4\pi \times 10^{-7})}$$

$$\chi_m = \left(\frac{3 \times 10^{-5}}{4\pi \times 10^{-7}} - 1\right)$$

$$= (23.87 - 1) = 22.87$$

SOL 5.2.7 Correct answer is 8.8 .



Volume current density inside a material is equal to the curl of magnetization $\boldsymbol{M}$

i.e. $$\boldsymbol{J} = \nabla \times \boldsymbol{M}$$

So the volume current density inside the cylinder is

$$\boldsymbol{J} = \nabla \times (0.7\boldsymbol{a}_z) = 0 \qquad (\boldsymbol{M} = 0.7\boldsymbol{a}_z\ \text{A/m})$$

and since the surface current density in terms of magnetization is defined as

$$\boldsymbol{K} = \boldsymbol{M} \times \boldsymbol{a}_n \qquad \text{where } \boldsymbol{a}_n \text{ is unit vector normal to the surface.}$$

So the surface current density of the cylinder is

$$\boldsymbol{K} = (0.7\boldsymbol{a}_z) \times \boldsymbol{a}_\rho = 0.7\boldsymbol{a}_\phi \qquad (\boldsymbol{M} = 0.7\boldsymbol{a}_z\ \text{A/m},\ \boldsymbol{a}_n = \boldsymbol{a}_\rho)$$

Therefore the current flowing in cylinder is just similar to a solenoid and the field intensity produced due to a solenoid at any point inside it is given as

$$B = \mu_0 K = \mu_0 nI$$

where n is the no. of turns per unit length of the solenoid and I is the current flowing in the solenoid.

Thus, the magnetic flux density inside the cylinder is (direction is determined by right hand rule)

$$\boldsymbol{B} = 0.7\mu_0 \boldsymbol{a}_z = 8.8 \times 10^{-7}\boldsymbol{a}_z \qquad (K = 0.7)$$

SOL 5.2.8 Correct answer is 0.356 .

From Snells law we have the relation between the incidence and refracted angle of magnetic flux lines as

$$\frac{\tan\alpha_1}{\tan\alpha_2} = \frac{\mu_{r1}}{\mu_{r2}}$$

where μ_{r1} and μ_{r2} are relative permeability of the two medium.

$$\frac{\tan 75^\circ}{\tan \alpha_2} = \frac{600}{1} \qquad \text{(relative permeability of air = 1)}$$

$$\tan \alpha_2 = \frac{\tan 75^\circ}{600}$$

$$\alpha_2 = \tan^{-1}\left[\frac{\tan 75^\circ}{600}\right]$$

$$= 0.356^\circ$$

SOL 5.2.9 Correct answer is 5.

The magnetic stored energy per unit volume of the plate for a given uniform flux density (uniform permeability) is defined as

$$w_m = \frac{1}{2}\boldsymbol{H}\cdot\boldsymbol{B}$$

Given $\quad \boldsymbol{B} = (3\boldsymbol{a}_x + 4\boldsymbol{a}_y) \times 10^{-3}\ \text{Wb/m}^2$

So we have, $\quad \boldsymbol{H} = \frac{\boldsymbol{B}}{\mu} = \left(\frac{3\boldsymbol{a}_x + 4\boldsymbol{a}_y}{4\mu_0}\right) \times 10^{-3}\ \text{A/m}$

and therefore $\quad w_m = \frac{1}{2}\boldsymbol{H}\cdot\boldsymbol{B}$

$$= \frac{1}{2}\left[\frac{9+16}{4\mu_0}\right] \times 10^{-6} = \frac{25 \times 10^{-6}}{8 \times 4\pi \times 10^{-7}}$$

$$= 2.49\ \text{J/m}^3$$

now the separation between the plates is given as $d = 2\ \text{m}$

Thus magnetic energy stored per unit area of the plate is

$$W_m/A = w_m \times d$$

$$= (2.49) \times 2 = 5\ \text{J/m}^2$$

SOL 5.2.10 Correct answer is 157.1 .

Internal inductance of a loop of radius r is defined as

$$L_{in} = \frac{\mu_0}{8\pi}(2\pi r) = \frac{4\pi \times 10^{-7} \times 2\pi x \times 50 \times 10^{-2}}{8\pi} \qquad (r = 50\ \text{cm})$$

$$= 157.1\ \text{nH}$$

SOL 5.2.11 Correct answer is 1.6 .

Given that,

the magnetic flux density, $\quad B = 0.4\ \text{T}$

no. of turns of coil, $\quad N = 200$

length of magnetic core, $\quad l = 15\ \text{cm} = 15 \times 10^{-2}$

permeability of the core, $\quad \mu_r = 150$

So, current required to produce the given magnetic field is

$$i = \frac{Bl}{\mu N}$$

$$= \frac{(0.4)(15 \times 10^{-2})}{(150\mu_0)(200)} = 1.6\ \text{A}$$

SOL 5.2.12 Correct answer is -3.

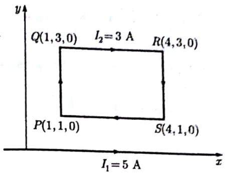
The magnetic flux density produced at a distance ρ from an infinitely long straight wire carrying current I is defined as

$$B = \frac{\mu_0 I}{2\pi\rho}$$

So the magnetic flux density produced by the straight wire at side QR of the loop is (direction of magnetic flux density is determined by right hand rule)

$$\boldsymbol{B}_{QR} = \frac{\mu_0 I_1}{2\pi(3)}\boldsymbol{a}_z \qquad (\rho = 3)$$

$$= \frac{5\mu_0}{6\pi}\boldsymbol{a}_z \qquad (I_1 = 5\ \text{A})$$



Force experienced by a current element $I\boldsymbol{dl}$ in the presence of magnetic flux density $\boldsymbol{B}$ is defined as

$$d\boldsymbol{F} = I\boldsymbol{dl} \times \boldsymbol{B}$$

where I is the current flowing in the element and $\boldsymbol{dl}$ is the differential vector length of the current element in the direction of flow of current.

So the force exerted by wire on the side QR of the square loop is

$$\boldsymbol{F}_{QR} = \int_Q^R I_2\boldsymbol{dl} \times \boldsymbol{B}_{QR}$$

where I_2 is the current flowing in the square loop as shown in the figure. So, we get

$$\boldsymbol{F}_{QR} = \int_{x=1}^{4}(3dx\boldsymbol{a}_x) \times \left(\frac{5\mu_0\boldsymbol{a}_z}{6\pi}\right) \qquad (I_2 = 3\ \text{A}, \boldsymbol{dl} = dx\boldsymbol{a}_x)$$

$$= \frac{5\mu_0}{2\pi}[4-1](-\boldsymbol{a}_y)$$

$$= \frac{-5 \times 4\pi \times 10^{-7} \times 3}{2\pi}\boldsymbol{a}_y$$

$$= -3 \times 10^{-6}\boldsymbol{a}_y\ \text{N} = -3\boldsymbol{a}_y\ \mu\text{N}$$

SOL 5.2.13 Correct answer is 6.

Total force on the loop will be the vector sum of the forces applied by the straight wire on all the sides of the loop. The forces on sides PQ and RS will be equal and opposite due to symmetry and so we have

$$\boldsymbol{F}_{PQ} + \boldsymbol{F}_{RS} = 0$$

Therefore the total force exerted on the conducting loop by the straight wire is

$$\boldsymbol{F}_{\text{total}} = \boldsymbol{F}_{QR} + \boldsymbol{F}_{SP} \qquad (1)$$

where $\boldsymbol{F}_{QR}$ and $\boldsymbol{F}_{SP}$ are the forces exerted by the straight wire on the sides QR and SP of the conducting loop respectively.

As calculated in previous question we have

$$\boldsymbol{F}_{QR} = -3 \times 10^{-6}\boldsymbol{a}_y\ \text{N}$$

Similarly we get the force exerted by the wire on the side SP of the loop as

$$\boldsymbol{F}_{SP} = \int_S^P I_2\boldsymbol{dl} \times \boldsymbol{B}_{SP}$$

where $\boldsymbol{B}_{SP}$ is the magnetic flux density produced by the wire on the side SP . So, we get

$$B_{SP} = \frac{\mu_0 I_1}{2\pi(1)} a_z \qquad (\rho = 1)$$

$$= \frac{5\mu_0}{2\pi} a_z \qquad (I_1 = 5\text{ A})$$

$$F_{SP} = \int_{1}^{4} 3(-dx a_x) \times \frac{5\mu_0}{2\pi} a_z \qquad (I_2 = 3\text{ A}, dl = -dx a_x)$$

$$= 9 \times 10^{-6} a_y \text{ N}$$

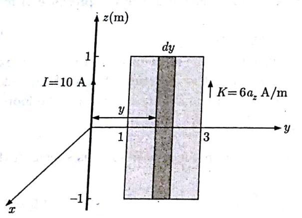
Thus, from equation (1), the total force exerted by the straight wire on the conducting loop is

$$F_{total} = -3 \times 10^{-6} a_y + 9 \times 10^{-6} a_y$$

$$= 6 \times 10^{-6} a_y \text{ N}$$

SOL 5.2.14 Correct answer is -26.4.

Consider the strip as made up of many adjacent strips of width dy, each carrying current Kdy



Since the magnetic flux density produced at a distance ρ from a straight wire carrying current I is defined as

$$B = \frac{\mu_0 I}{2\pi\rho}$$

So the magnetic flux density produced at distance y from the current filament located along z-axis as shown in the figure will be

$$B = \frac{\mu_0 I}{2\pi y}(-a_x) \quad \text{(Direction is determined using right hand rule)}$$

$$= -\frac{10\mu_0}{2\pi y} a_x$$

As the force experienced by a current element Idl in the presence of magnetic flux density B is defined as

$$dF = Idl \times B$$

and since the length of strip is $l = 2$ m so, the force exerted on the width dy of strip is given by

$$dF = l(Kdy) \times B$$

Therefore the net force exerted on the strip is

$$F = \int_{y=1}^{3} (2)(6a_z) \times \left(-\frac{10\mu_0}{2\pi y} a_x\right) dy \qquad (l = 2\text{ m}, K = 6a_z)$$

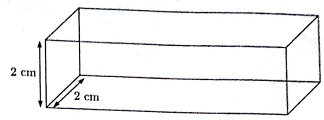
$$= -\frac{60\mu_0}{\pi} a_y [\ln y]_1^3$$

$$= -26.4 a_y \ \mu\text{N}$$

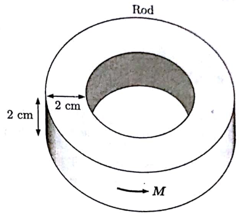
SOL 5.2.15 Correct answer is 4.

Let the circular ring being placed such that magnetization M is in a_ϕ direction and the ring is centered at origin.

So, we have $\quad M = 4a_\phi$



Rod



Circular Ring

As the surface current density of a material in terms of its magnetization is defined as

$$K = M \times a_n \qquad \text{where } a_n \text{ is unit vector normal to the surface.}$$

So the surface current density of the ring is

$$K = 4a_\phi \times (-a_\rho) = 4a_z \qquad (M = 4a_\phi,\ a_n = -a_\rho)$$

and since the volume current density inside a material is equal to the curl of magnetization M

i.e. $\quad J = \nabla \times M$

So the volume current density inside the ring is

$$J = \nabla \times (4a_\phi) = 0 \qquad (M = 4a_\phi)$$

Now from Ampere's circuital law we have

$$\oint_L B \cdot dl = \mu_0 I_{enc}$$

and for determining the field inside the circular ring, the current present on the inner surface of ring will be considered only. So we get

$$(B)(2\pi\rho) = \mu_0(K)(2\pi\rho)$$

Therefore the magnetic flux density inside the circular ring is

$$B = (\mu_0)(4) = 4\mu_0 \text{ wb/m}^2 \qquad (K = 4\text{ A/m})$$

ALTERNATIVE METHOD :

Magnetic flux density inside a magnetic material is defined as

$$B = \mu_0 M$$

and since the magnetization of the rod is $M = 4$ A/m so, we can have directly the magnetic flux density inside the ring as

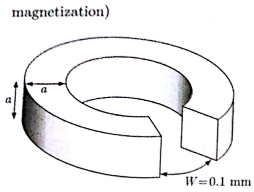
$$B = 4\mu_0 \text{ Wb/m}^2$$

SOL 5.2.16 Correct answer is 50.04 .

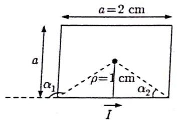
As calculated above for the complete circular ring, magnetic flux density inside the ring is

$$B = 4\mu_0 \boldsymbol{a}_\phi \text{ wb/m}^2$$

(Magnetic flux density will be directed along the assumed direction of magnetization)



Now we calculate the flux density contributed by the gap at its centre when it was the complete ring. The gap has its cross section in form of a square loop as shown in figure below



As calculated in previous question the surface current density of the ring is

$$K = 4\text{ A/m}$$

and since the width of the gap(square loop) is w so, net current in the loop is

$$I = Kw = 4w$$

Now the magnetic flux density at any point P due to a filamentary current I is defined as

$$\boldsymbol{H} = \frac{I}{4\pi\rho}[\cos\alpha_2 - \cos\alpha_1]\boldsymbol{a}_\phi$$

where $\rho \rightarrow$ distance of point P from the current filament.

$\alpha_1 \rightarrow$ angle subtended by the lower end of the filament at P.

$\alpha_2 \rightarrow$ angle subtended by the upper end of the filament at P.

So the flux density at center of the square loop produced due to one side of the loop is

$$\boldsymbol{B}_{sq1} = \frac{\mu_0 I}{4\pi \times (10^{-2})} \times \left(\frac{2}{\sqrt{2}}\right) \qquad (\rho = 1\text{ cm}, \alpha_1 = 135°, \alpha_2 = 45°)$$

Summing the flux density produced due to all the four sides of loop, we get total magnetic flux density produced by the square loop as

$$\boldsymbol{B}_{sq} = 4 \times \left(\frac{\mu_0 I\sqrt{2}}{4\pi(10^{-2})}\right) = \frac{\sqrt{2}\,\mu_0(4w)}{\pi} \times 10^2 \qquad (I = Kw)$$

$$= \frac{\sqrt{2}\,\mu_0 \times (4)(0.1 \times 10^{-3}) \times 10^2}{\pi} \qquad (w = 0.1\text{ mm})$$

$$= \frac{4\sqrt{2} \times 10^{-2}}{\pi}\mu_0 \boldsymbol{a}_\phi$$

Therefore at the centre of the gap the net magnetic flux density will reduce by this amount of the flux density. Thus at the centre of the gap the net magnetic flux density at the centre of the loop will be

$$\boldsymbol{B}_{net} = \boldsymbol{B} - \boldsymbol{B}_s$$

$$= 4\mu_0 - \frac{4\sqrt{2} \times 10^{-2}}{\pi}\mu_0$$

$$= \mu_0\left(4 - \frac{4\sqrt{2} \times 10^{-2}}{\pi}\right)$$

$$= 50.04 \times 10^{-7}\text{ wb/m}^2$$

SOL 5.2.17 Correct answer is 0.33 .

The magnetic flux density produced at any point P due to an infinite filamentary current I is defined as

$$B = \frac{\mu_0 I}{2\pi\rho}$$

where ρ is the distance of point P from the infinite current filament.

Now consider a small area dS of the coil located at a distance x from the current filament. The magnetic flux density produced on it due to the current filament along y-axis is

$$B = \frac{\mu_0 I}{2\pi x} \qquad (\rho = x)$$

Since the flux density will be normal to the surface of the coil as determined by right hand rule therefore, the total magnetic flux passing through the coil is

$$\psi_m = \int \boldsymbol{B} \cdot d\boldsymbol{S} = \int_{y=0}^{1}\int_{x=2}^{6}\left(\frac{\mu_0 I}{2\pi x}\right)(dxdy) = \frac{\mu_0 I}{2\pi}\ln 3$$

As the mutual inductance in terms of total magnetic flux ψ_m is defined as

$$M = \frac{N\psi_m}{I}$$

where $I \rightarrow$ current flowing in the element that produces the magnetic flux.

$N \rightarrow$ Total no. of turns of the coil that experiences the magnetic flux.

Thus the mutual inductance between the current filament and the loop is

$$M = \frac{1500}{I}\left(\frac{\mu_0 I}{2\pi}\ln 3\right) = 0.33\text{ mH} \qquad N = 1500$$

SOL 5.2.18 Correct answer is 4.

Since the two conducting plates of width $w = 2$ m carry a uniform current of $I = 4$ A each so, the surface current density of each plate is

$$K = \frac{I}{w} = \frac{4}{2} = 2\text{A/m}$$

Now consider the first plate carrying current in $+\boldsymbol{a}_z$ direction is located at $y = 0$ and the second plate carrying current in $-\boldsymbol{a}_z$ direction is located at $y = d$, where d is a very small separation between the plates.

Since the magnetic field intensity produced at any point P due to an infinite sheet carrying uniform current density $\boldsymbol{K}$ is defined as

$$\boldsymbol{H} = \frac{1}{2}(\boldsymbol{K} \times \boldsymbol{a}_n)$$

where $\boldsymbol{a}_n$ is the unit vector normal to the sheet directed toward the point P. So, the magnetic field intensity produced at the second plate due to the first plate is

$$H_{12} = \frac{1}{2}(2\boldsymbol{a}_z \times \boldsymbol{a}_y) = -\boldsymbol{a}_x \qquad (K_1 = 2\boldsymbol{a}_z, \boldsymbol{a}_n = \boldsymbol{a}_y)$$

Now the force per meter, exerted on the 2nd plate due to the 1st plate will be

$$F_{12} = \int_0^1 \int_0^2 (K_2 \times B_{12})\, dS$$

where $K_2 \rightarrow$ current density of the 2nd plate

$B_{12} \rightarrow$ magnetic flux density produced at the 2nd plate due to 1st plate

So,
$$F_{12} = \int_0^1 \int_0^2 (-2\boldsymbol{a}_z) \times (\mu_0 H_{12})\, dydz \qquad (K_2 = -2\boldsymbol{a}_z, B_{12} = \mu_0 H_{12})$$

$$= \int_0^1 \int_0^2 (-2\boldsymbol{a}_z) \times (-\mu_0 \boldsymbol{a}_x)\, dydz = 4\mu_0 \boldsymbol{a}_y$$

As the force applied by first plate on the 2nd plate is in $\boldsymbol{a}_y$ direction so it is a repulsive force. Therefore the repulsive force between the plates is $4\mu_0$.

SOL 5.2.19 Correct answer is 10.05 .

No, of turns, $n = 20{,}000$ turns/meter
Relative permeability, $\mu_r = 100$
cross sectional area, $S = 0.04\ \text{m}^2$
Current in the solenoid, $I = 100 \times 10^{-3}$ A
So, its self inductance will be,

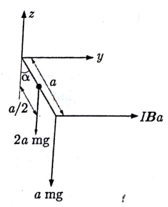
$$L' = \mu_0 \mu_r n^2 S$$
$$= (4\pi \times 10^{-7}) \times (100) \times (20{,}000)^2 \times (0.04)$$
$$= 2.011 \times 10^3$$

Therefore the energy stored per unit length in the field is

$$W'_m = \frac{1}{2} L' I^2 = \frac{1}{2} \times 2.011 \times 10^3 \times 10^{-2} = 10.05\ \text{J/m}$$

SOL 5.2.20 Correct answer is 0.7854 .

Consider the square loop has side a. Now, when the loop is situated in the field $B = 1.96\ \text{Wb/m}^2$. Suppose it swings with an angle α. So in the new position the torque must be zero. Gravitational forces acting on all the sides of loop will be down wards and the force due to magnetic field will be in horizontal direction as shown in the figure.



So, in balanced condition,from the shown figure we have

$$a\, mg \sin\alpha(a) + 2a\, mg \sin\alpha\left(\frac{a}{2}\right) = IBa$$

$$\tan\alpha = \frac{IB}{2mg} = \frac{(2)(1.96)}{(2)(0.2)(9.8)}$$

$$\alpha = \tan^{-1}(1) = \pi/4 = 0.7854$$

SOL 5.2.21 Correct answer is -40.

At any point in between the two parallel shuts the net magnetic flux density produced by the two sheets is given as

$$\boldsymbol{B} = \boldsymbol{B}_1 + \boldsymbol{B}_2$$

where $\boldsymbol{B}_1$ is the flux density produced by the lower sheet and $\boldsymbol{B}_2$ is the flux density produced by upper sheet.

Now the magnetic flux density produced at point P due to a plane sheet having current density $\boldsymbol{K}$ is defined as

$$\boldsymbol{B} = \frac{\mu}{2} \boldsymbol{K} \times \boldsymbol{a}_n$$

where $\boldsymbol{a}_n$ is the unit vector normal to the sheet and directed toward point P . So, the flux density produced by lower sheet is

$$\boldsymbol{B}_1 = \frac{\mu}{2}(4\boldsymbol{a}_x) \times a_z \qquad (\boldsymbol{K} = 4\boldsymbol{a}_x, \boldsymbol{a}_n = \boldsymbol{a}_z)$$

and the flux density produced by the lower sheet is

$$\boldsymbol{B}_2 = \frac{\mu}{2}(-4\boldsymbol{a}_x) \times (-a_z) \qquad (\boldsymbol{K} = -4\boldsymbol{a}_x, \boldsymbol{a}_n = -\boldsymbol{a}_z)$$

So the net magnetic flux density produced in the region between the two sheets is

$$\boldsymbol{B} = \frac{\mu}{2}(4\boldsymbol{a}_x) \times a_z + \frac{\mu}{2}(-4\boldsymbol{a}_x) \times (-\boldsymbol{a}_z)$$
$$= -4\mu \boldsymbol{a}_y$$

where μ is the permeability of the medium.

Therefore the flux density in region 1 is

$$\boldsymbol{B}_{region\,1} = -4\mu_1 \boldsymbol{a}_y = -8\mu_0 \boldsymbol{a}_y \qquad (\mu_1 = 2\mu_0)$$

and the flux density in region 2 is

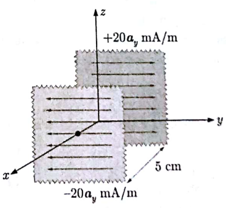
$$\boldsymbol{B}_{region\,2} = -4\mu_2 \boldsymbol{a}_y = -16\mu_0 \boldsymbol{a}_y \qquad (\mu_2 = 4\mu_0)$$

So the net flux per unit length in the region between the two sheets is

$$\frac{\phi}{l} = (\boldsymbol{B}_{region\,1})\ (\text{width of region 1}) + (\boldsymbol{B}_{region\,2})\ (\text{width of region 2})$$
$$= (-8\mu_0 \boldsymbol{a}_y)(1) + (-16\mu_0 \boldsymbol{a}_y)(2)$$
$$= -40\mu_0 \boldsymbol{a}_y\ \text{Wb/m}$$

SOL 5.2.22 Correct answer is 25.1327 .

Consider the sheets as shown in figure that having the surface current densities $+20\ \text{mA/m}\boldsymbol{a}_y$ and $-20\ \text{mA/m}\boldsymbol{a}_y$



So the field intensity between the plates will be given as

$$\boldsymbol{H} = \boldsymbol{H}_1 + \boldsymbol{H}_2$$

where $\boldsymbol{H}_1$ is the field intensity produced by the sheet located at $x = 0$ and $\boldsymbol{H}_2$ is the field intensity produced by the sheet located at $x = 5\,\text{cm}$.
Now the magnetic field intensity produced at point P due to a plane sheet having current density $\boldsymbol{K}$ is defined as

$$\boldsymbol{H} = \frac{1}{2}\boldsymbol{K} \times \boldsymbol{a}_n$$

where $\boldsymbol{a}_n$ is the unit vector normal to the sheet directed toward point P. So, the magnetic field intensity in the region between the plates is

$$\boldsymbol{H} = \underbrace{\frac{1}{2}\boldsymbol{K}_1 \times \boldsymbol{a}_{n1}}_{\text{at } x=0} + \underbrace{\frac{1}{2}\boldsymbol{K}_2 \times \boldsymbol{a}_{n2}}_{\text{at } x=5\,\text{cm}}$$

$$= \frac{1}{2}(20 \times 10^{-3}\boldsymbol{a}_y) \times (\boldsymbol{a}_x) + \frac{1}{2}(-20 \times 10^{-3}\boldsymbol{a}_y) \times (-\boldsymbol{a}_x)$$

$$= -20 \times 10^{-3}\boldsymbol{a}_z$$

and magnetic flux density in the region between the sheets is

$$\boldsymbol{B} = \mu\boldsymbol{H} = -40\mu_0 \times 10^{-3}\boldsymbol{a}_z \qquad (\mu = 2\mu_0)$$

Therefore the stored magnetic energy per unit volume in the region is

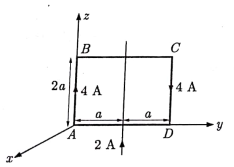
$$w_m = \frac{1}{2}\boldsymbol{H} \cdot \boldsymbol{B} = \frac{1}{2}(800\mu_0 \times 10^{-6})$$

$$= 400 \times 4\pi \times 10^{-7} \times 10^{-6} = 160\pi\ \text{J/m}^3$$

Since the separation between plates is $d = 5\,\text{cm}$. So, stored energy per unit area between the plates is

$$W_m/A = w_m \times d = (160\pi) \times (0.05)$$
$$= 8\pi\ \text{J/m}^2 = 25.1327\ \text{J/m}^2$$

SOL 5.2.23 Correct answer is 6.4 .
Consider the square loop is of side $2a$ as shown in the figure



Since the sides BC and AD crosses the straight wire so no force will be experienced by the sides, while the flux density produced by the straight wire at sides AB and CD will be equal in magnitude.
Now the magnetic flux density produced at a distance ρ from a straight wire carrying current I is defined as

$$B = \frac{\mu_0 I}{2\pi\rho}$$

So the magnetic flux density produced by the straight wire at the two sides of the loop is

$$B = \frac{\mu_0(2)}{2\pi(a)} = \frac{\mu_0}{\pi a} \qquad (I = 2\,\text{A}, \rho = a)$$

Since the force exerted on a current element Idl by a magnetic field $\boldsymbol{B}$ is defined as

$$d\boldsymbol{F} = (I d\boldsymbol{l}) \times \boldsymbol{B}$$

Therefore the force experienced by side AB of length $2a$ is

$$\boldsymbol{F}_1 = [4(2a)\boldsymbol{a}_z] \times \left[\frac{\mu_0}{\pi a}\boldsymbol{a}_x\right] = \frac{8\mu_0}{\pi}(-\boldsymbol{a}_y) \qquad (I = 4\,\text{A})$$

Similarly force experienced by side CD is

$$\boldsymbol{F}_2 = [4(2a)(-\boldsymbol{a}_z)] \times \left[\frac{\mu_0}{\pi a}(-\boldsymbol{a}_x)\right] = \frac{8\mu_0}{\pi}(\boldsymbol{a}_y)$$

Thus the net force experienced by the loop is

$$\boldsymbol{F} = \boldsymbol{F}_1 + \boldsymbol{F}_2 = \frac{16\mu_0}{\pi}(\boldsymbol{a}_y)$$
$$= 16 \times 4 \times 10^{-7}\boldsymbol{a}_y$$
$$= 6.4\boldsymbol{a}_y\ \mu\text{N}$$

SOL 5.2.24 Correct answer is 50.6 .
According to Snell's law the permeability of two mediums are related as

$$\mu_0 \tan\theta_1 = \mu \tan\theta_2$$
$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{15\mu_0}{\mu_0}$$
$$\tan\theta_1 = 15\tan\theta_2 \qquad \text{...(1)}$$

Now, the given flux density in medium 2 is

$$\boldsymbol{B}_2 = 1.2\boldsymbol{a}_y + 0.8\boldsymbol{a}_z$$

So the normal and tangential component of the magnetic flux density in medium 2 is

$$\boldsymbol{B}_{2n} = 0.8\boldsymbol{a}_z$$

and

$$\boldsymbol{B}_{2t} = 1.2\boldsymbol{a}_y$$

From the figure we have

$$\tan\theta_2 = \frac{B_{2n}}{B_{2t}} = \frac{0.8}{1.2} = \frac{2}{3}$$

or

$$\theta_2 = \tan^{-1}(2/3)$$

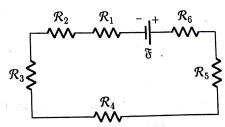
from equation (1)

$$\tan\theta_1 = 15\tan\theta_2$$
$$\tan\theta_1 = 10$$
$$\theta = \tan^{-1}(10)$$

Thus the angular deflection is

$$\theta_1 - \theta_2 = \tan^{-1}(10) - \tan^{-1}(2/3)$$
$$= 50.6°$$

SOL 5.2.25 Correct answer is 358098.62 .
For calculating total reluctance of the circuit, we have to draw the electrical analog of the circuit. In the given magnetic circuit, there are total six section for which six reluctance has been drawn below.



For a given cross sectional area S and length of the core l reluctance is defined as

$$\mathcal{R} = \frac{l}{\mu S}$$

Where μ is permeability of the medium in core

So, we have
$$\mathcal{R}_1 = \frac{5\times10^{-2}}{(1000\mu_0)(5\times10^{-4})} = \frac{1}{10\mu_0}$$
$$\mathcal{R}_2 = \frac{5\times10^{-2}}{(1000\mu_0)(10\times10^{-4})} = \frac{1}{20\mu_0}$$
$$\mathcal{R}_3 = \frac{6\times10^{-2}}{(1000\mu_0)(10\times10^{-4})} = \frac{3}{50\mu_0}$$
$$\mathcal{R}_4 = \frac{14\times10^{-2}}{(1000\mu_0)(10\times10^{-4})} = \frac{7}{50\mu_0}$$
$$\mathcal{R}_5 = \mathcal{R}_3 = \frac{3}{50\mu_0}$$
$$\mathcal{R}_6 = \frac{4\times10^{-2}}{(1000\mu_0)(10\times10^{-4})} = \frac{1}{25\mu_0}$$

Since all the reluctance are connected in series so total reluctance of the magnetic circuit is

$$\mathcal{R}_T = \mathcal{R}_1+\mathcal{R}_2+\mathcal{R}_3+\mathcal{R}_4+\mathcal{R}_5+\mathcal{R}_6$$
$$= \frac{1}{10\mu_0}+\frac{1}{20\mu_0}+\frac{3}{50\mu_0}+\frac{7}{50\mu_0}+\frac{3}{50\mu_0}+\frac{1}{25\mu_0}$$
$$= \frac{9}{20\mu_0} = 358098.62$$

SOL 5.2.26 Correct answer is 27.9 .

For a given reluctance $\mathcal{R}$ of a magnetic circuit, the self, inductance is defined as

$$L = \frac{N^2}{\mathcal{R}} \qquad \text{Where } N \text{ is no. of turns of coil}$$

Then,
$$L = \frac{(100)^2}{(9/20\mu_0)} \qquad \left(\mathcal{R}_T = \frac{9}{20\mu_0}\right)$$
$$= 2.79\times10^{-2}$$
$$= 27.9\,\text{mH}$$

SOL 5.2.27 Correct answer is 0.64 .

Give that

No. of turns of coil, $N = 50$

Length of the core, $l = 0.6\,\text{m}$

Relative permeability, $\mu_r = 600$

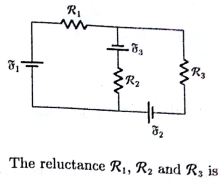
Inductance of the coil, $L = 0.2\,\text{mH} = 0.2\times10^{-3}\,\text{H}$

So, the cross sectional area of core is

$$S = \frac{Ll}{\mu N^2} = \frac{(0.2\times10^{-3})(0.6)}{(600\mu_0)(50)^2}$$
$$= 6.366\times10^{-5}\,\text{m}^2$$
$$= 0.64\,\text{cm}^2$$

SOL 5.2.28 Correct answer is 62.8 .

Since the core is ideal so it's reluctance will be zero and so the electrical analog for the magnetic circuit will be as shown below.



The reluctance $\mathcal{R}_1$, $\mathcal{R}_2$ and $\mathcal{R}_3$ is produced by the air gap.

$$\mathcal{R}_1 = \frac{l_1}{\mu_0 S_1} = \frac{4\times10^{-2}}{\mu_0(100\times10^{-4})} = \frac{4}{\mu_0}$$
$$\mathcal{R}_2 = \frac{2\times10^{-2}}{\mu_0(100\times10^{-4})} = \frac{2}{\mu_0}$$
$$\mathcal{R}_3 = \frac{2\times10^{-2}}{\mu_0(100\times10^{-4})} = \frac{2}{\mu_0}$$

So, the total reluctance seen by coil N_1 is

$$\mathcal{R}_T = \mathcal{R}_1+\mathcal{R}_2||\mathcal{R}_3$$
$$= \frac{4}{\mu_0}+\frac{1}{\mu_0} = \frac{5}{\mu_0}$$

and the self inductance of coil will be

$$L_1 = \frac{N_1^2}{\mathcal{R}_T} = 62.8\,\text{mH}$$

SOL 5.2.29 Correct answer is 23.6 .

The total reluctance of the magnetic circuit as seen from the coil N_2 is

$$\mathcal{R}_T = (\mathcal{R}_1||\mathcal{R}_2)+\mathcal{R}_3$$
$$= \left(\frac{4}{\mu_0}||\frac{2}{\mu_0}\right)+\frac{2}{\mu_0} \qquad \text{(as calculated above)}$$
$$= \frac{4}{3\mu_0}+\frac{2}{\mu_0} = \frac{10}{3\mu_0}$$

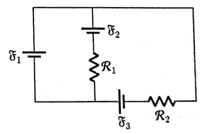
Therefore the self inductance of the coil N_2 is

$$L_2 = \frac{N_2^2}{\mathcal{R}_T} = \frac{(250)^2}{(10/3\mu_0)} = 23.6\,\text{mH}$$

SOL 5.2.30 Correct answer is 78.54 .

Since the coil N_1 and N_2 are directly connected through ideal core so entire flux produced by N_2 will link with N_1.

The electrical analog of the magnetic circuit is shown below where the reluctance $\mathcal{R}_1$ and $\mathcal{R}_2$ are the reluctance due to air gap.



So, the reluctance seen by coil N_2 is

$$\mathcal{R}_1 = \frac{l}{\mu_0 S} = \frac{(4\times10^{-3})}{\mu_0(2000\times10^{-6})} = \frac{2}{\mu_0}$$

Consider the current flowing in coil N_2 is i_2. So, the total flux produced by

$N_2 i_2$ is

$$\phi_2 = \frac{N_2 i_2}{\mathcal{R}_1} = \frac{500 i_2}{(2/\mu_0)} = 250\mu_0 i_2$$

Since the entire flux will link with N_1 So mutual induction between N_1 and N_2 is

$$M = L_{12} = \frac{N_1 \phi_2}{i_2} = \frac{(250)(250\mu_0 i_2)}{i_2} = 78.54 \text{ mH}$$

SOL 5.2.31 Correct answer is 0.

As the coil N_1 and N_2 are directly connected through an ideal core so entire flux will produced by N_2 will link with N_1 and so flux linked with N_3 will be zero. Therefore the mutual inductance between N_3 and N_2 is zero.

SOL 5.2.32 Correct answer is 6.2 .

Given, the expression for magnetization curve,

$$B = \frac{1}{3}H + H^2 \,\mu\text{Wb/m}^2$$

The energy stored per unit volume in a magnetic material having linear magnetic flux density is defined as

$$w_m = \int_{H=0}^{H_0} \mathbf{H} \cdot d\mathbf{B}$$

Since, magnetic field intensity varied from 0 to 210 A/m So, we have

$$w_m = \int_{H=0}^{210} H dB$$

Since, $$\frac{dB}{dH} = \frac{1}{3} + 2H$$

So, putting it in equation we get,

$$w_m = \int_{H=0}^{210} H\left(\frac{1}{3} + 2H\right) dH$$

$$= \left[\frac{H^2}{6} + \frac{2H^3}{3}\right]_0^{210}$$

$$= 6.18 \times 10^6 \text{ J/m}^3$$

$$= 6.2 \text{ MJ/m}^3$$

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SOLUTIONS 5.3

SOL 5.3.1 Option (A) is correct.

Force applied by a magnetic field $\boldsymbol{B}$ on a moving charge with velocity $\boldsymbol{v}$ is defined as

$$\boldsymbol{F} = \boldsymbol{v} \times \boldsymbol{B}$$

Since the direction of velocity $\boldsymbol{v}$ and $\boldsymbol{B}$ are perpendicular to each other as obtained from the shown figure so the resultant force will be perpendicular to both of them.

i.e. the force on the moving charged particle will be in upward direction. and as the particle is also deflected in upward direction with the applied force so it gives the conclusion that the particle will be positively charged.

SOL 5.3.2 Option (D) is correct.

Since a magnet bar must have south and north pole i.e. a single pole charge can't exist. So, a magnetic point charge doesn't exit.

SOL 5.3.3 Option (C) is correct.

The boundary condition for the current interface holds the following results.

(1) normal component of magnetic flux density is continuous.

i.e. $\boldsymbol{B}_{1n} = \boldsymbol{B}_{2n}$

(2) Tangential component of magnetic field intensity is continuous.

i.e. $\boldsymbol{H}_{1t} = \boldsymbol{H}_{2t}$

So, (A) and (B) are wrong statement. Now, we check the statement (C).

Consider the magnetic field intensity in 1st medium is $\boldsymbol{H}_1$ and magnetic field intensity in 2nd medium is $\boldsymbol{H}_2$. So, it's tangential component will be equal

i.e. $\boldsymbol{H}_{1t} = \boldsymbol{H}_{2t}$ (tangential component)

Since scalar magnetic potential difference is defined as the line integral of magnetic field intensity

i.e. $V_1 - V_2 = \oint \boldsymbol{H} \cdot d\boldsymbol{l} = I$

and since there is no current density at boundary.

So, we have $V_1 - V_2 = 0$ or $V_1 = V_2$ i.e. magnetic scalar potential will be same in both medium.

SOL 5.3.4 Option (B) is correct.

SOL 5.3.5 Option (C) is correct.

A diamagnetic material carries even no. of electrons inside it's atom.

Number of electron in carbon atom is six.

Which is even so it is a diamagnetic material rest of the material having odd no. of electrons.

SOL 5.3.6 Option (A) is correct.

A paramagnetic material have an odd no. of electrons and since atomic no. of Al is 13, which is odd. So, it is a paramagnetic material.

So, A and R both true and A is correct explanation of R.

SOL 5.3.7 Option (A) is correct.

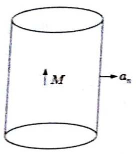
SOL 5.3.8 Option (B) is correct.

SOL 5.3.9 Option (A) is correct.

SOL 5.3.10 Option (A) is correct.

SOL 5.3.11 Option (B) is correct.

SOL 5.3.12 Option (C) is correct.



Volume current density inside a material is equal to the curl of magnetization M

i.e. $$J = \nabla \times M$$

and the surface current density in terms of magnetization is defined as

$$K = M \times a_n$$

where a_n is unit vector normal to the surface. Consider the cylinder is placed along z-axis

So, $$a_n = a_\rho \text{ and } M = Ma_z$$

Therefore the volume current density inside the cylinder is

$$J = \nabla \times (Ma_z) = 0 \qquad (M \text{ is not the function of } z)$$

and the surface current density of the cylinder is

$$K = Ma_z \times a_\rho = Ma_\phi$$

So the current flowing in cylinder is just similar to a solenoid and the field intensity produced due to a solenoid at any point outside it is zero. Thus we have the magnetic field intensity outside the cylinder as

$$H_{outside} = 0$$

SOL 5.3.13 Option (B) is correct.

Force applied on a moving charge in the presence of electric and magnetic field is defined as

$$F = F_e + F_m = q(E + v \times B)$$

where F_e and F_m are the electric and magnetic forces applied on the charge so it is clear that the moving charge experiences both the electric and magnetic forces. The electric force is applied in a uniform direction (in direction of electric field) i.e. it is an accelerating force while, the magnetic force is applied in the normal direction of both the magnetic field and velocity of the charged particle i.e. it is a deflecting force. Therefore, both the options are correct but R is not the correct explanation of A.

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SOLUTIONS 5.4

SOL 5.4.1 Option (A) is correct.

From boundary condition we have the following relation between the magnetic field intensity in the two mediums :

$$\mu_1 H_{1n} = \mu_2 H_{2n} \qquad (1)$$

and $$(H_1 - H_2) \times a_{n12} = K \qquad (2)$$

where H_1 and H_2 are the magnetic field intensity in the two mediums, a_{n12} is the unit vector normal to the interface of the mediums directed from medium 1 to medium 2 and K is the surface current density at the interface of the two mediums.

Now, the magnetic field intensity in medium 1 is

$$H_1 = 3a_z + 30a_y \text{ A/m}$$

As the interface lies in the plane $z = 0$ so, we have

$$H_{1n} = 3a_z$$

From equation (1), the normal component of the field intensity in medium 2 is given as

$$H_{2n} = \frac{H_{1n}}{2} = 1.5a_z$$

Therefore, the net magnetic field intensity in medium 2 can be considered as

$$H_2 = 1.5a_z + Aa_y + Ba_x \qquad (3)$$

where A and B are the constants. So, from equation (2) we have

$$[(3a_z + 30a_y) - (1.5a_z + Aa_y + Ba_x)] \times a_z = 10a_x$$

$$[1.5a_z + (30 - A)a_y - Ba_x] \times a_z = 10a_x$$

$$0 - (30 - A)a_x - Ba_y = 10a_x$$

Comparing the components in the two sides we get

$$30 - A = 0 \Rightarrow A = 30$$

and $$-B = 10 \Rightarrow B = -10$$

Putting these values in equation (3) we get the magnetic field intensity in medium 2 as

$$H_2 = 1.5a_z + 30a_y - 10a_x \text{ A/m}$$

SOL 5.4.2 Option (B) is correct.

Given,

the magnetic moment $\quad m = 2.5 \text{ A-m}^2$

Mass of magnet, $\quad \text{mass} = 6.6 \times 10^{-3} \text{ kg}$

density of steel, $\quad \text{density} = 7.9 \times 10^3 \text{ kg/m}^3$

So, the net volume of the magnet bar is

$$v = \frac{\text{mass}}{\text{density}} = \frac{6.6 \times 10^{-3}}{7.9 \times 10^3}$$

$$= 0.835 \times 10^{-6} \text{ m}^3$$

Now, the magnetization of the magnet is defined as the magnetic moment per unit volume so, we get magnetization of the magnet bar as

$$M = \frac{m}{v} = \frac{2.5}{0.835 \times 10^{-6}}$$
$$= 3 \times 10^{6}\ \text{A/m}$$

SOL 5.4.3 Option (A) is correct.
Given,

Magnetic field intensity, $\boldsymbol{H} = \frac{5\boldsymbol{a}_z}{\mu}$ A/m

Current element, $Idl = 4 \times 10^{-4} \boldsymbol{a}_y$ A-m

So, the magnetic flux density is given as

$$\boldsymbol{B} = \mu \boldsymbol{H} = 5\boldsymbol{a}_z\ \text{A/m}$$

Since, the force exerted on a current element Idl placed in a magnetic field $\boldsymbol{B}$ is defined as

$$\boldsymbol{F} = (Idl) \times \boldsymbol{B}$$

So, putting all the values we get,

$$\boldsymbol{F} = (4 \times 10^{-4} \boldsymbol{a}_y) \times (5\boldsymbol{a}_z)$$
$$= -2 \times 10^{-3} \boldsymbol{a}_z\ \text{N} = -2\boldsymbol{a}_z\ \text{mN}$$

SOL 5.4.4 Option (C) is correct.
The electrical analogy of the magnetic field are listed below :

Electrical field		Magnetic field
EMF (Electromotive Force)	→	MMF (Magnetomotive Force)
Electric current	→	Magnetic flux
Resistance	→	Reluctance
Conductivity	→	Permeability

So, for the given match list we get, $A \to 3$, $B \to 2$, $C \to 4$, $D \to 1$.

SOL 5.4.5 Option (C) is correct.
At the surface of discontinuity (interface between two medium) the normal component of magnetic flux density are related as

$$B_{1n} = B_{2n}$$

i.e. normal component of magnetic flux density is uniform at the surface of discontinuity.
Statement 1 is correct
At the boundary interface between two mediums, the normal component of the electric flux density is related as

$$D_{2n} - D_{1n} = \rho_s \qquad \text{i.e. discontinuous}$$

where ρ_S is surface charge density at the interface. If the interface is charge free ($\rho_S = 0$) then, the equation changes to

$$D_{2n} = D_{1n} \qquad \text{i.e. continuous}$$

So, the normal component of flux density at the surface of discontinuity may or may not be continuous.

SOL 5.4.6 Option (C) is correct.
Magnetic current is composed of both displacement and conduction components.

SOL 5.4.7 Option (C) is correct.

Torque exerted on a loop with dipole moment $\boldsymbol{M}$ in a magnetic field $\boldsymbol{B}$ is defined as

$$\boldsymbol{T} = \boldsymbol{M} \times \boldsymbol{B}$$

SOL 5.4.8 Option (B) is correct.
Biot savart's law gives the magnetic flux density as defined below

$$\boldsymbol{B} = \frac{\mu_0 \int Idl \times \boldsymbol{R}}{4\pi R^2} \qquad (b \to 3)$$

Displacement current is determined by using maxwell's equation as

$\nabla \times \boldsymbol{H} = \boldsymbol{J}_c + \boldsymbol{J}_d$ where J_d is displacement current density $(c \to 1)$

Time average power flow in a field wave is determined by poynting vector as

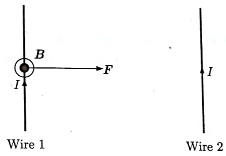
$$\mathcal{P}_{ave} = \frac{1}{2} \boldsymbol{E}_s \times \boldsymbol{H}_s \qquad (d \to 2)$$

Using Gauss's law line charge distribution can be determined. $(a \to 4)$

SOL 5.4.9 Option (B) is correct.
Magnetic energy density in a magnetic field is defined as

$$w_m = \frac{1}{2} \boldsymbol{J} \cdot \boldsymbol{A}$$

SOL 5.4.10 Option (B) is correct.
Consider the two wires carrying current as shown below :



The force exerted due to the wire 2 at wire 1 is given as

$$\boldsymbol{F} = (Idl) \times (\boldsymbol{B})$$

where Idl is the small current element of the wire 1 and $\boldsymbol{B}$ is magnetic flux density produced by wire 2 at wire 1. As determined by right hand rule the magnetic flux density produced due to wire 2 at wire 1 is out of the paper. Which will be towards wire 2. In the similar way the force due to wire 1 at wire 2 will be toward wire 1 i.e. attractive and perpendicular to the wire.

SOL 5.4.11 Option (B) is correct.
From the boundary condition for magnetic field, we have the following derived condition as

$$\mu_1 \boldsymbol{H}_{n1} = \mu_2 \boldsymbol{H}_{n2}$$

and $$\boldsymbol{H}_{t1} = \boldsymbol{H}_{t2}$$

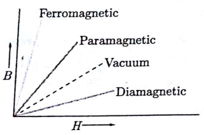
SOL 5.4.12 Option (C) is correct.
The magnetic flux density B and magnetic field intensity H in a medium with permeability μ are related as

$$B = \mu H = \mu_r \mu_0 H$$

Now, for the different magnetic material relative permeability μ_r are listed below :

Free space (vacuum)	$\mu_r = 1$
Diamagnetic	$\mu_r \lesssim 1$
Paramagnetic	$\mu_r \gtrsim 1$
Ferromagnetic	$\mu_r >> 1$

So, the B-H curve for the respective material has been shown below (depending on their slopes μ).



SOL 5.4.13 Option (A) is correct.
When a dielectric material is placed in an electric field then the electric dipoles are created in it. This phenomenon is called polarization of the dielectric material.
So, we conclude that both the statement are correct and statement (2) is correct explanation of (1).

SOL 5.4.14 Option (B) is correct.
For an inhomogeneous magnetic material, magnetic permeability is a variable and so, it has some finite gradient. Now, from Maxwell's equation we know

$$\nabla \cdot \boldsymbol{B} = 0$$

Since, $\boldsymbol{B} = \mu \boldsymbol{H}$

So, $\nabla \cdot \boldsymbol{B} = \nabla \cdot (\mu \boldsymbol{H})$

$$0 = \nabla \cdot \mu + \nabla \cdot \boldsymbol{H}$$

In the above equation $\nabla \cdot \mu$ have some finite value therefore,

$$\nabla \cdot \boldsymbol{H} \neq 0 \qquad \text{(in inhomogeneous medium)}$$

SOL 5.4.15 Option (C) is correct.
Force on a current element Idl kept in a magnetic field $\boldsymbol{B}$ is defined as

$$\boldsymbol{F} = \oint_L Idl \times \boldsymbol{B}$$
$$= [(10)(2)\boldsymbol{a}_z] \times [0.05\boldsymbol{a}_x] = 1.0\boldsymbol{a}_y \text{ N}$$

SOL 5.4.16 Option (C) is correct.
Magnetic energy density in a magnetic field is defined as

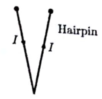
$$w_m = \frac{1}{2}\boldsymbol{J} \cdot \boldsymbol{A}$$

where $\boldsymbol{J}$ is the current density and $\boldsymbol{A}$ is the magnetic vector potential.

SOL 5.4.17 Option (D) is correct.
The force on a moving charge q with the velocity $\boldsymbol{v}$ in a region having magnetic field $\boldsymbol{B}$ and electric field $\boldsymbol{E}$ is defined as

$$F = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$$

SOL 5.4.18 Option (B) is correct.
The currents in the hairpin shaped wire flows as shown in the figure.



As the direction of current are opposite so the force acting between them is repulsive, and So it tend to a straight line.

SOL 5.4.19 Option (C) is correct.
Given, the Lorentz force equation,

$$\boldsymbol{F} = e(\boldsymbol{v} \times \boldsymbol{B})$$

If the particle is at rest then $\boldsymbol{v} = 0$ and so there will be no any deflection in particle due to the magnetic field.

SOL 5.4.20 Option (D) is correct.
Force acting on a small point charge q moving in an EM wave is defined as

$$\boldsymbol{F} = q\boldsymbol{E} + q(\boldsymbol{v} + \boldsymbol{B})$$

So, for $q = 1$

$$\boldsymbol{F} = \boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}$$

SOL 5.4.21 Option (B) is correct.
Given,
Current flowing in the conductor, $I = 5$ A
Magnetic flux density, $\boldsymbol{B} = 3\boldsymbol{a}_x + 4\boldsymbol{a}_y$
Since, the force experienced by a current carrying element Idl placed in a magnetic field $\boldsymbol{B}$ is defined as

$$d\boldsymbol{F} = (Idl) \times \boldsymbol{B}$$

As the current flowing in $\boldsymbol{a}_z$ direction so, we have

$$dl = dl\boldsymbol{a}_z$$

and the force experienced by the conductor is

$$d\boldsymbol{F} = (5dl\boldsymbol{a}_z) \times (3\boldsymbol{a}_x + 4\boldsymbol{a}_y)$$

Therefore, the force per unit length experienced by the conductor is

$$\frac{d\boldsymbol{F}}{dl} = 15\boldsymbol{a}_y - 20\boldsymbol{a}_x$$
$$= -20\boldsymbol{a}_x + 15\boldsymbol{a}_y \text{ N/m}$$

SOL 5.4.22 Option (B) is correct.
From the boundary condition for magnetic field we have the following relation :
Normal component of magnetic flux density is continuous
i.e. $\boldsymbol{B}_{n1} = \boldsymbol{B}_{n2}$
Any field vector is the sum of its normal and tangential component to any surface
i.e. $\boldsymbol{H}_1 = \boldsymbol{H}_{n1} + \boldsymbol{H}_{t1}$
When the interface between two medium carries a uniform current $\boldsymbol{K}$ then the tangential component of magnetic field intensity is not uniform.
i.e. $\boldsymbol{H}_{t1} - \boldsymbol{H}_{t2} = \boldsymbol{K}$
or, $\boldsymbol{a}_{n21} \times (\boldsymbol{H}_1 - \boldsymbol{H}_2) = \boldsymbol{K}$

But, $$B_2 \neq \sqrt{B_{n2} + B_{t2}}$$

SOL 5.4.23 Option (A) is correct.

Given, the magnetic flux density in medium 1 is

$$\boldsymbol{B}_1 = 1.2\boldsymbol{a}_x + 0.8\boldsymbol{a}_y + 0.4\boldsymbol{a}_z$$

and the interface lies in the plane $z = 0$.

So, the tangential and normal components of magnetic flux density in the two mediums are respectively :

$$\boldsymbol{B}_{1t} = 1.2\boldsymbol{a}_x + 0.8\boldsymbol{a}_y$$

and $$\boldsymbol{B}_{1n} = 0.4\boldsymbol{a}_z$$

Now, from the boundary condition of current free interface, we have the following relations between the components of field in two mediums.

$$\boldsymbol{B}_{1n} = \boldsymbol{B}_{2n}$$

and $$\frac{\boldsymbol{B}_{1t}}{\mu_1} = \frac{\boldsymbol{B}_{2t}}{\mu_2}$$

Therefore, we get the field components in medium 2 as

$$\boldsymbol{B}_{2n} = \boldsymbol{B}_{1n} = 0.4\boldsymbol{a}_z$$

and $$\boldsymbol{B}_{2t} = \boldsymbol{B}_{1t}\left(\frac{\mu_2}{\mu_1}\right) = \frac{1}{2}(1.2\boldsymbol{a}_x + 0.8\boldsymbol{a}_y) = (0.6\boldsymbol{a}_x + 0.4\boldsymbol{a}_y)$$

Thus, the net magnetic flux density in region 2 is

$$\boldsymbol{B}_2 = \boldsymbol{B}_{2n} + \boldsymbol{B}_{2t}$$
$$= 0.6\boldsymbol{a}_x + 0.4\boldsymbol{a}_y + 0.4\boldsymbol{a}_z$$

So, the magnetic field intensity in region 2 is

$$\boldsymbol{H}_2 = \frac{\boldsymbol{B}_2}{\mu_2} = \frac{1}{\mu_0}(0.6\boldsymbol{a}_x + 0.4\boldsymbol{a}_y + 0.4\boldsymbol{a}_z)\ \text{A/m}$$

SOL 5.4.24 Option (C) is correct.

Energy stored in a magnetic field is defined as

$$W_m = \frac{1}{2}\int \boldsymbol{A}\cdot\boldsymbol{J}\,dv$$

So, $\int \boldsymbol{A}\cdot\boldsymbol{J}\,dv$ has the units of energy.

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