SOLUTIONS 3.1

SOL 3.1.1 Option (A) is correct.

For a given current density, the total current that passes through a given surface is defined as

$$I = \int \boldsymbol{J} \cdot d\boldsymbol{S}$$

where $d\boldsymbol{S}$ is the differential surface area having the direction normal to the surface.

So we have $\quad d\boldsymbol{S} = \rho d\rho d\phi \boldsymbol{a}_z \quad$ for the plane $z = 2$

Therefore the total current crossing the plane $z = 2$, $\rho < 4$ is

$$I = \left(\frac{40}{\rho}\boldsymbol{a}_\rho - \frac{20\sin\phi}{(\rho^2+1)}\boldsymbol{a}_z\right)(\rho d\rho d\phi \boldsymbol{a}_z)$$

$$= \int_{\phi=0}^{2\pi}\int_{\rho=0}^{4}\left(-\frac{20\sin\phi}{\rho^2+1}\right)(\rho d\rho d\phi)$$

$$= -\left[\int_{\rho=0}^{4}\frac{20}{\rho^2+1}\rho d\rho\right]\underbrace{\left[\int_{\phi=0}^{2\pi}\sin\phi d\phi\right]}_{0}$$

$$= 0\text{ A}$$

SOL 3.1.2 Option (D) is correct.

From the equation of continuity we have the relation between the volume charge density, ρ_v and the current density, $\boldsymbol{J}$ as

$$\frac{\partial \rho_v}{\partial t} = -\nabla \cdot \boldsymbol{J}$$

Given the current density,

$$\boldsymbol{J} = \frac{40}{\rho}\boldsymbol{a}_\rho - \frac{20\sin\phi}{(\rho^2+1)}\boldsymbol{a}_z \text{ A/m}^2$$

So, we have the components $J_\rho = \frac{40}{\rho}$, $J_\phi = 0$ and $J_z = -\frac{20\sin\phi}{(\rho^2+1)}$

Therefore,

$$\frac{\partial \rho_v}{\partial t} = -\left[\frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho J_\rho) + \frac{1}{\rho}\frac{\partial J_\phi}{\partial \phi} + \frac{\partial J_z}{\partial z}\right]$$

$$= \left[\frac{1}{\rho}\frac{\partial}{\partial \rho}(40) + \frac{\partial}{\partial z}\left(\frac{-20\sin\phi}{\rho^2+1}\right)\right]$$

$$= 0$$

So, volume charge density will be constant with respect to time.

SOL 3.1.3 Option (A) is correct.

Given the current $I = 6$ A is flowing radially outward (in $\boldsymbol{a}_\rho$ direction) through the medium between the cylinders. So the current density in the medium between the cylinders is

$$\boldsymbol{J} = \frac{I}{2\pi\rho l}\boldsymbol{a}_\rho = \frac{6}{2\pi\rho \times 2}\boldsymbol{a}_\rho \qquad (l = 2\text{ m})$$

$$= \frac{3}{2\pi\rho}\boldsymbol{a}_\rho \text{ A/m}^2$$

For a given current density in a certain medium having conductivity σ, the electric field intensity is defined as

$$\boldsymbol{E} = \frac{\boldsymbol{J}}{\sigma} = \frac{1}{\sigma}\left(\frac{3}{2\pi\rho}\boldsymbol{a}_\rho\right)$$

$$= \frac{3}{2\pi \times 4 \times 10^{-2} \times 0.05} \qquad (\rho = 4 \times 10^{-2}\text{ m}, \sigma = 0.05\text{ S/m})$$

$$= 238.73\text{ V/m}$$

SOL 3.1.4 Option (A) is correct.

Voltage between the cylindrical surfaces is defined as the line integral of the electric field between the two surfaces

i.e. $\quad V = -\int \boldsymbol{E} \cdot d\boldsymbol{l}$

Now the electric field intensity in the medium between the two cylindrical surfaces as calculated in previous question is $\boldsymbol{E} = \frac{1}{\sigma}\left(\frac{3}{2\pi\rho}\boldsymbol{a}_\rho\right)$

and the differential displacement between the two cylindrical surfaces is $d\boldsymbol{l} = d\rho \boldsymbol{a}_\rho$

So the voltage between the cylindrical surfaces is

$$V = -\int_{\rho=3\times10^{-2}}^{5\times10^{-2}}\left(\frac{3}{2\pi\rho\sigma}\boldsymbol{a}_\rho\right)\cdot(d\rho\boldsymbol{a}_\rho) = -\frac{3}{2\pi\sigma}\ln\left(\frac{5}{3}\right)$$

$$= -4.88\text{ volt}$$

So, the voltage between them will be 4.88 volt.

SOL 3.1.5 Option (A) is correct.

As we have already calculated the voltage between the two cylindrical surfaces and the current flowing radially outward in the medium between the surfaces is given in the question. So the resistance between the cylindrical surface can be evaluated directly as

$$R = \frac{V}{I} = \frac{4.88}{6} = 0.813\,\Omega \qquad (V = 4.88\text{ volt}, I = 6\text{ A})$$

SOL 3.1.6 Option (C) is correct.

Since voltage between the cylindrical surfaces is $\quad V = 4.88$ volt

and current flowing in the medium is $\quad I = 6$ A

So, Power dissipated in the medium is

$$P = VI = (4.88) \times 6 = 29.28\text{ watt}$$

SOL 3.1.7 Option (D) is correct.

Consider a constant voltage is applied across the ends of the wire so, the electric field intensity throughout the wire cross section will be constant.

i.e. $\quad E = \frac{J_1}{\sigma_1} = \frac{J_2}{\sigma_2}$

where $\quad J_1$ is the current density in the material having conductivity σ_1.

J_2 is the current density in the material having conductivity σ_2.

So, the ratio of the current density is

$$\frac{J_1}{J_2} = \frac{\sigma_1}{\sigma_2}$$

i.e. it will be independent of both r and R.

SOL 3.1.8 Option (B) is correct.

Electric field intensity is defined as the negative gradient of the potential

i.e.
$$\begin{aligned} \boldsymbol{E} &= -\nabla V \\ &= -\left(\frac{\partial V}{\partial x}\boldsymbol{a}_x + \frac{\partial V}{\partial y}\boldsymbol{a}_y + \frac{\partial V}{\partial z}\boldsymbol{a}_z\right) \\ &= 500\boldsymbol{a}_y \text{ V/m} \end{aligned}$$

SOL 3.1.9 Option (D) is correct.
For a given electric field intensity $\boldsymbol{E}$ in a material having relative permittivity ε_r, the electric flux density is defined as :

$$\begin{aligned} \boldsymbol{D} &= \varepsilon_r \varepsilon_0 \boldsymbol{E} \\ &= \frac{8}{5} \times (8.85 \times 10^{-12}) \times (500\boldsymbol{a}_y) \qquad \varepsilon_r = 8/5 \\ &= 7.08\boldsymbol{a}_y \text{ nC/m}^2 \end{aligned}$$

SOL 3.1.10 Option (A) is correct.
For an applied electric field intensity $\boldsymbol{E}$ in a material having relative permittivity ε_r, the polarization of the material is defined as

$$\begin{aligned} \boldsymbol{P} &= \varepsilon_0(\varepsilon_r - 1)\boldsymbol{E} \\ &= (8.85 \times 10^{-12})(1.6 - 1) \times (500\boldsymbol{a}_y) \\ &= 8.85 \times 10^{-12} \times 0.6 \times 500\boldsymbol{a}_y \\ &= 2.66 \times 10^{-9}\boldsymbol{a}_y = 2.66\boldsymbol{a}_y \text{ nC/m}^2 \end{aligned}$$

SOL 3.1.11 Option (B) is correct.
Since the two regions is being separated by the plane $y = 0$, so the tangential and normal component of the electric field to the plane $y = 0$ are given as

$$\begin{aligned} \boldsymbol{E}_{1t} &= 50\boldsymbol{a}_x - 10\boldsymbol{a}_z \\ \boldsymbol{E}_{1n} &= 20\boldsymbol{a}_y \end{aligned}$$

From the boundary condition, the tangential component of electric field will be uniform.

i.e. $\boldsymbol{E}_{2t} = \boldsymbol{E}_{1t} = 50\boldsymbol{a}_x - 10\boldsymbol{a}_z$

and the normal component of the field is nonuniform and given as

$$\begin{aligned} \varepsilon_2 \boldsymbol{E}_{2n} &= \varepsilon_1 \boldsymbol{E}_{1n} \\ \boldsymbol{E}_{1n} &= \frac{\varepsilon_1}{\varepsilon_2}\boldsymbol{E}_{1n} = \frac{2}{5}(20\boldsymbol{a}_y) = 8\boldsymbol{a}_y \end{aligned}$$

So the electric field intensity in the second region is

$$\begin{aligned} \boldsymbol{E}_2 &= \boldsymbol{E}_{2t} + \boldsymbol{E}_{2n} = (50\boldsymbol{a}_x - 10\boldsymbol{a}_z) + (8\boldsymbol{a}_y) \\ &= 50\boldsymbol{a}_x + 8\boldsymbol{a}_y - 10\boldsymbol{a}_z \text{ kV/m} \end{aligned}$$

Therefore the electric flux density in the region 2 is

$$\begin{aligned} \boldsymbol{D}_2 &= \varepsilon_{r2}\varepsilon_0 \boldsymbol{E}_2 \\ &= 5 \times 8.85 \times 10^{-12}(50\boldsymbol{a}_x + 8\boldsymbol{a}_y - 10\boldsymbol{a}_z) \times 10^3 \\ &= 2.21\boldsymbol{a}_x + 0.35\boldsymbol{a}_y - 0.44\boldsymbol{a}_z \ \mu\text{C/m}^2 \end{aligned}$$

SOL 3.1.12 Option (D) is correct.
Energy density in the region having electric field intensity $\boldsymbol{E}_2$ is defined as

$W_E = \frac{1}{2}\varepsilon_{r2}\varepsilon_0 \boldsymbol{E}_2 \cdot \boldsymbol{E}_2$, where the relative permittivity of the medium is ε_{r2}

As calculated in previous question the electric field intensity is

$$\boldsymbol{E}_2 = 50\boldsymbol{a}_x + 8\boldsymbol{a}_y - 10\boldsymbol{a}_z \text{ kV/m}$$

So the energy density in the region 2 is

$$\begin{aligned} W_E &= \frac{1}{2} \times 5 \times \varepsilon_0[(50)^2 + (8)^2 + (10)^2] \times 10^6 \\ &= 59 \text{ mJ/m}^3 \end{aligned}$$

SOL 3.1.13 Option (B) is correct.
As the dielectric slab occupies the region $0 < z < d$ and the field intensity in the free space is in $+\boldsymbol{a}_z$ direction so, the field will be normal to the boundary of plane dielectric slab.

So from the boundary condition the field normal to the surface are related as

$$\begin{aligned} \varepsilon \boldsymbol{E}_{in} &= \varepsilon_0 \boldsymbol{E} \\ \boldsymbol{E}_{in} &= \frac{\varepsilon_0}{4\varepsilon_0}E_0\boldsymbol{a}_z = \frac{E_0}{4}\boldsymbol{a}_z \qquad (\varepsilon = 4\varepsilon_0) \end{aligned}$$

Therefore, $\boldsymbol{D}_{in} = \varepsilon \boldsymbol{E}_{in} = 4\varepsilon_0 \frac{E_0}{4}\boldsymbol{a}_z = \varepsilon_0 E_0 \boldsymbol{a}_z$

SOL 3.1.14 Option (C) is correct.
Total energy stored in a region having electric field is given as

$$\begin{aligned} W &= \frac{1}{2}\varepsilon_0 \int_v (\boldsymbol{E} \cdot \boldsymbol{E})\,dv \\ &= \frac{1}{2}\varepsilon_0 \int_v (\boldsymbol{E}_1 + \boldsymbol{E}_2) \cdot (\boldsymbol{E}_1 + \boldsymbol{E}_2)\,dv \qquad (\boldsymbol{E} = \boldsymbol{E}_1 + \boldsymbol{E}_2) \\ &= \frac{1}{2}\varepsilon_0 \int_v (E_1^2 + E_2^2 + 2\boldsymbol{E}_1 \cdot \boldsymbol{E}_2)\,dv \\ &= \frac{1}{2}\varepsilon_0 \int_v E_1^2\,dv + \frac{1}{2}\varepsilon_0 \int E_2^2\,dv + \int_v \varepsilon_0(\boldsymbol{E}_1 \cdot \boldsymbol{E}_2)\,dv \\ &= W_1 + W_2 + \int_v \varepsilon_0(\boldsymbol{E}_1 \cdot \boldsymbol{E}_2)\,dv \end{aligned}$$

SOL 3.1.15 Option (A) is correct.
Consider a neutral dielectric is placed in an electric field $\boldsymbol{E}$, due to which the dielectric gets polarized with polarization $\boldsymbol{P}$, the bound surface charge density of the dielectric be ρ_{pS} and the bound volume charge density be ρ_{pv}.
So the total bound charge by the dielectric is given as

$$Q_{bound} = \oint_S \rho_{pS}\,dS + \int_v \rho_{pv}\,dv$$

Since for a given polarization $\boldsymbol{P}$ of a dielectric material, the bound surface charge density over the surface of material is defined as

$$\rho_{pS} = \boldsymbol{P} \cdot \boldsymbol{a}_n$$

where $\boldsymbol{a}_n$ is the unit vector normal to the surface directed outward.
while the bound volume charge density inside the material is defined as

$$\rho_{pv} = -\nabla \cdot \boldsymbol{P}$$

So we have,
$$\begin{aligned} Q_{bound} &= \oint_S (\boldsymbol{P} \cdot \boldsymbol{a}_n)\,dS - \int_v \nabla \cdot \boldsymbol{P}\,dv \\ &= \oint_s \mathbf{P} \cdot d\mathbf{S} - \int_v \nabla \cdot \mathbf{P}\,dv. \qquad (dS\boldsymbol{a}_n = d\boldsymbol{S}) \end{aligned}$$

But according to the divergence theorem

$$\oint_s \boldsymbol{P} \cdot d\boldsymbol{S} = \int_v \nabla \cdot \boldsymbol{P}\,dv$$

Therefore, $Q_{bound} = 0$

SOL 3.1.16 Option (B) is correct.

Given the conductivity of material, $\sigma = 10^6 (\Omega m)^{-1}$

and conductance of the wire, $G = 10^6 (\Omega)^{-1}$

Since the conductance of a wire of length l having cross sectional area S is

$$G = \frac{\sigma S}{l}$$

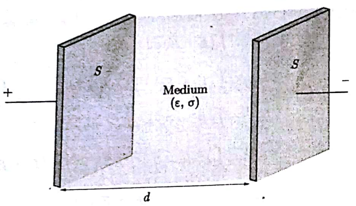
So we have, $$10^6 = \frac{10^6 \times \pi r^2}{l} \qquad (S = \pi r^2)$$

$$r = \sqrt{\frac{l}{\pi}}$$

SOL 3.1.17 Option (A) is correct.

As the medium between capacitor plates is conducting so it carries the resistive as well as capacitive property.

Consider the plates are separated by a distance d and the surface area of plates is S as shown in the figure.



So the total resistance of the medium between plates is

$$R = \frac{d}{\sigma S}$$

and capacitance of the capacitor is

$$C = \frac{\varepsilon S}{d}$$

Therefore the time constant of the capacitor will be

$$\tau = RC = \frac{\varepsilon}{\sigma}$$

SOL 3.1.18 Option (B) is correct.

For a given current density, the total current that passes through a given surface is defined as

$$I = \int \boldsymbol{J} \cdot d\boldsymbol{S}$$

where $d\boldsymbol{S}$ is the differential surface area having the direction normal to the surface.

Since the current density is independent of θ and ϕ so we can have directly

the current $$I = \boldsymbol{J} \cdot \boldsymbol{S} = J(4\pi r^2 \boldsymbol{a}_r)$$

$$= \frac{1}{r} e^{-10^3 t} 4\pi r^2$$

$$= 4\pi \times (6)^2 \times \frac{1}{6} \times e^{-10^3 \times 10^{-3}}$$

$$= 4\pi \times 6 \times e^{-1} = 24\pi e^{-1} = 27.7 \text{ A}$$

SOL 3.1.19 Option (C) is correct.

From the equation of continuity we have the relation between the volume charge density, ρ_v and the current density, $\boldsymbol{J}$ as

$$\frac{\partial \rho_v}{\partial t} = -\nabla \cdot \boldsymbol{J}$$

and since the current density have only the component in $\boldsymbol{a}_r$ direction so we have,

$$\frac{\partial \rho_v}{\partial t} = -\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 J_r)$$

$$\frac{\partial \rho_v}{\partial t} = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{1}{r} e^{-10^3 t}\right)$$

Integrating both sides we get,

$$\rho_v(r,t) = -\int \frac{1}{r^2} e^{-10^3 t} dt + f(r)$$

where $f(r)$ is the function independent of time.

$$\rho_v(r,t) = \frac{10^{-3}}{r^2} e^{-10^3 t} + f(r)$$

Now for $t \to \infty$ $\qquad \rho_v(r,t) = 0$

So, we put the given condition in the equation to get $f(r) = 0$

therefore $$\rho_v(r,t) = \frac{10^{-3}}{r^2} e^{-10^3 t}$$

i.e. $$\rho_v(r,t) \propto \frac{1}{r^2}$$

SOL 3.1.20 Option (D) is correct.

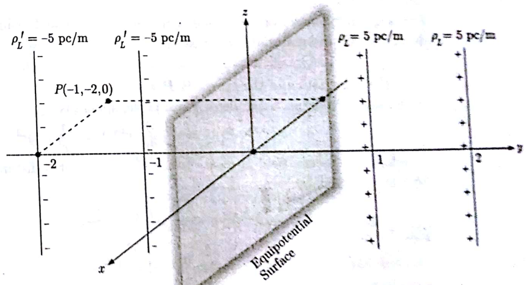
The velocity of charge density can be defined as the ratio of current density to the charge density in the region

i.e. $$\boldsymbol{v} = \frac{\boldsymbol{J}}{\rho_v} = \frac{(1/r)\, e^{-10^3 t} \boldsymbol{a}_r}{(10^{-3}/r^2)\, e^{-10^3 t}} = 10^3 r \boldsymbol{a}_r$$

So, at $r = 0.6$ m, $\boldsymbol{v} = 10^3 \times 0.6 \boldsymbol{a}_r = 600 \boldsymbol{a}_r$ m/s

SOL 3.1.21 Option (B) is correct.

The given problem can be solved easily by using image theory as the conducting surface $y = 0$ can be replaced by the equipotential surface in the same plane $y = 0$ and image of line charges ($\rho_L' = -5$ pC/m at $x = 0$, $y = -1$ and $x = 0$, $y = -2$) as shown in the figure



The work done to carry a unit positive charge from a point located at a distance a from the line charge with charge density ρ_L to another point located at a distance b from the line charge is defined as

$$V_{ab} = -\frac{\rho_L}{2\pi\varepsilon_0}\ln\left(\frac{b}{a}\right)$$

and since the surface $y=0$ has zero potential, so the potential at point P will be equal to the work done in moving a unit positive charge from the plane $y=0$ to the point P. So the potential at point P will be

$$V_P = -\sum\frac{\rho_L}{2\pi\varepsilon_0}\ln\left(\frac{b}{a}\right)$$

where a is the distance of the surface $y=0$ from the line charges while b is the distance of point P from the line charges.

So, $$V_P = -\frac{5\times10^{-12}}{2\pi\varepsilon_0}\left[-\ln\left(\frac{1}{2}\right)-\ln\left(\frac{\sqrt{2}}{1}\right)+\ln\left(\frac{\sqrt{10}}{1}\right)+\ln\left(\frac{\sqrt{17}}{2}\right)\right]$$

$$= -0.2 \text{ volt}$$

SOL 3.1.22 Option (A) is correct.

Electric field at a distance R from a line charge having uniform charge density ρ_L is defined as

$$\boldsymbol{E} = \frac{\rho_L}{2\pi\varepsilon_0}\frac{\boldsymbol{R}}{R^2}$$

So the net electric field intensity produced at the point P due to the four line charges discussed in previous question is given as

$$\boldsymbol{E} = \sum\frac{\rho_L}{2\pi\varepsilon_0}\frac{\boldsymbol{R}}{R^2}$$

where $\boldsymbol{R}$ is the distance of point P from the line charges

Therefore, $$\boldsymbol{E} = \frac{\rho_l}{2\pi\varepsilon_0}\left[\frac{(-1,-2,0)-(0,1,0)}{|(-1,-3,0)|^2}+\frac{(-1,-2,0)-(0,2,0)}{|(-1,-4,0)|^2}\right.$$
$$\left.-\frac{(-1,-2,0)-(0,-1,0)}{|(-1,-1,0)|^2}-\frac{(-1,-2,0)-(0,-2,0)}{|(-1,0,0)|^2}\right]$$
$$= \frac{5\times10^{-12}}{2\pi\varepsilon_0}\left[-\frac{(1,3,0)}{10}-\frac{(1,4,0)}{17}+\frac{(1,1,0)}{2}+\frac{(1,0,0)}{1}\right]$$
$$= 0.12\boldsymbol{a}_x - 0.0032\boldsymbol{a}_y = 0.12\boldsymbol{a}_x - 0.003\boldsymbol{a}_y \text{ V/m}$$

SOL 3.1.23 Option (C) is correct.

For a given polarization $\boldsymbol{P}$ inside a material, the bound volume charge density inside the material is defined as

$$\rho_{pv} = -\nabla\cdot\boldsymbol{P}$$

Since the polarization of the sphere is $\boldsymbol{P}(r) = 2r\boldsymbol{a}_r$

So the bound volume charge density inside the sphere is

$$\rho_{pv} = -\nabla\cdot\boldsymbol{P}(r) = -\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 2r) = -\frac{1}{r^2}\times 6r^2 = -6$$

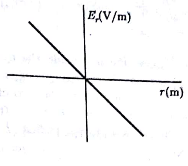
Therefore the electric field intensity inside the sphere at a distance r from the center is given by

$$\boldsymbol{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q_{enc}}{r^2}\boldsymbol{a}_r = \frac{1}{4\pi\varepsilon_0}\frac{\rho_{pv}\times\frac{4}{3}\pi r^3}{r^2}\boldsymbol{a}_r$$
$$= \frac{\rho_v r}{3\varepsilon_0}\boldsymbol{a}_r = -\frac{6r}{3\varepsilon_0}\boldsymbol{a}_r = -\left(\frac{2}{\varepsilon_0}\right)r\boldsymbol{a}_r$$

So the radial component of the electric field inside the sphere is

$$E_r = -\frac{2}{\varepsilon_0}r$$

which is linearly decreasing with a slope $\left(-\frac{2}{\varepsilon_0}\right)$ with respect to r as shown below :



SOL 3.1.24 Option (C) is correct.

For a given polarization $\boldsymbol{P}$ of a material, the bound surface charge density over the surface of material is defined as

$$\rho_{PS} = \boldsymbol{P}\cdot\boldsymbol{a}_n$$

So the bound surface charge density over the spherical surface is

$$\rho_{PS} = \boldsymbol{P}(r)\cdot\boldsymbol{a}_r \qquad (\boldsymbol{a}_n = \boldsymbol{a}_r)$$
$$= 2r = 2a \qquad \text{(at the spherical surface } r=a)$$

So, total bound surface charge over the sphere is

$$Q_{PS} = 2a\times4\pi a^2 = 8\pi a^3$$

and the bound volume charge density inside the sphere as calculated above is

$$\rho_{pv} = -6$$

So, total bound volume charge inside the sphere is

$$Q_{pv} = \rho_{pv}\left(\frac{4}{3}\pi a^3\right) = (-6)\times\left(\frac{4}{3}\pi a^3\right) = -8\pi a^3$$

Therefore the total bound charge in the sphere is

$$Q_{bound} = Q_{PS} + Q_{pv} = 8\pi a^3 - 8\pi a^3 = 0$$

According to Gauss law the outward electric field flux through a closed surface is equal to the charge enclosed by the surface and since the total bound charge for any point outside the sphere is zero So, the electric field intensity at any point outside the sphere is $\boldsymbol{E}=0$.

SOL 3.1.25 Option (C) is correct.

For a given polarization $\boldsymbol{P}$ of a material, the surface charge density over the surface of material is defined as

$$\rho_S = \boldsymbol{P}\cdot\boldsymbol{a}_n$$

where $\boldsymbol{a}_n$ is the unit vector normal to the surface directed outward of the material.

while the volume charge density inside the material is defined as

$$\rho_v = -\nabla\cdot\boldsymbol{P}$$

Since the the cylinder has uniform polarization $\boldsymbol{P}$.

So, volume charge density inside the sphere is

$$\rho_v = -\nabla\cdot\boldsymbol{P} = 0$$

and the surface charge density over the top and bottom surface of the cylinder is

$$\rho_S = \boldsymbol{P}\cdot\boldsymbol{a}_n = \pm P \qquad (+P \text{ at top surface and } -P \text{ at bottom surface})$$

So the total bound charge by the cylinder is

$$Q_{bound} = Q_S + Q_v$$
$$= \int_S \rho_S\, dS + \int_v \rho_v\, dv = [+P(\pi r^2) - P(\pi r^2)] + 0 = 0$$

SOL 3.1.26 Option (A) is correct.

As calculated above the volume charge density inside the cylinder is zero while the surface charge density at top and bottom surfaces are respectively $+P$ and $-P$, so the cylinder can be considered as the two circular plates (top and bottom surface) separated by a distance L. Since the separation between the plates is larger than the cross sectional radius ($L = 2r$) so the fringing field(electric field) will exist directed from the upper plate towards the lower plate.

SOL 3.1.27 Option (C) is correct.

The electric flux lines will be the same as the electric field intensity outside the cylinder but as the volume charge density is zero $\rho_v = 0$ inside the cylinder so $\oint \boldsymbol{D} \cdot d\boldsymbol{S} = 0$ and therefore the flux lines will be continuous.

SOL 3.1.28 Option (A) is correct.

Consider the charge densities of the two surface of the slab is $\rho_{s1}\ \mathrm{C/m^2}$ and $\rho_{s2}\ \mathrm{C/m^2}$ as shown in the figure.

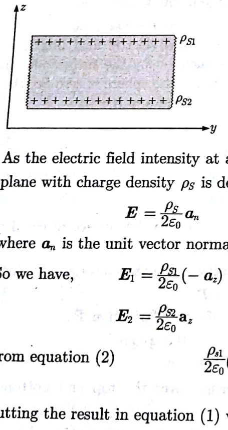
As the sum of the charge densities is $\rho_{so}\ \mathrm{C/m^2}$ so we have

$$\rho_{s1} + \rho_{s2} = \rho_{s0} \qquad \text{...(1)}$$

and since the electric field intensity inside the conducting slab must be zero so,

$$\boldsymbol{E}_1 + \boldsymbol{E}_2 = 0 \qquad \text{...(2)}$$

where $\boldsymbol{E}_1$ is field inside slab due to charge density ρ_{s1} and $\boldsymbol{E}_2$ is field inside slab due to ρ_{s2}



As the electric field intensity at any point P due to the uniformly charged plane with charge density ρ_S is defined as

$$\boldsymbol{E} = \frac{\rho_S}{2\varepsilon_0}\boldsymbol{a}_n$$

where $\boldsymbol{a}_n$ is the unit vector normal to the plane directed toward point P

So we have,
$$\boldsymbol{E}_1 = \frac{\rho_{S1}}{2\varepsilon_0}(-\boldsymbol{a}_z) \qquad (\boldsymbol{a}_n = -\boldsymbol{a}_z)$$
$$\boldsymbol{E}_2 = \frac{\rho_{S2}}{2\varepsilon_0}\boldsymbol{a}_z \qquad (\boldsymbol{a}_n = \boldsymbol{a}_z)$$

From equation (2)
$$\frac{\rho_{s1}}{2\varepsilon_0}(-\boldsymbol{a}_z) + \frac{\rho_{s2}}{2\varepsilon_0}\boldsymbol{a}_z = 0$$
$$\rho_{s1} = \rho_{s2}$$

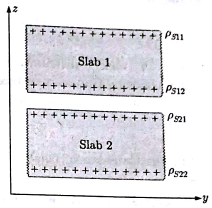
Putting the result in equation (1) we get
$$\rho_{s1} = \rho_{s2} = \frac{\rho_{s0}}{2}$$

SOL 3.1.29 Option (B) is correct.

As the slabs are conducting so net electric field inside the slab must be zero. and since the electric field intensity at any point P due to the uniformly charged plane with charge density ρ_S is defined as

$$\boldsymbol{E} = \frac{\rho_S}{2\varepsilon_0}\boldsymbol{a}_n$$

where $\boldsymbol{a}_n$ is the unit vector normal to the plane directed toward point P



So, the net electric field intensity inside slab 1 is

$$\frac{\rho_{S11}}{2\varepsilon_0}(-\boldsymbol{a}_z) + \frac{\rho_{S12}}{2\varepsilon_0}\boldsymbol{a}_z + \frac{\rho_{S21}}{2\varepsilon_0}\boldsymbol{a}_z + \frac{\rho_{S22}}{2\varepsilon_0}\boldsymbol{a}_z = 0$$

($\boldsymbol{a}_n = -\boldsymbol{a}_z$ for ρ_{S11} while $\boldsymbol{a}_n = \boldsymbol{a}_z$ for rest of the charge densities)

$$-\rho_{s11} + \rho_{s12} + \rho_{s21} + \rho_{s22} = 0 \qquad \text{...(1)}$$

and the net electric field intensity inside slab 2 is

$$\frac{\rho_{S11}}{2\varepsilon_0}(-\boldsymbol{a}_z) + \frac{\rho_{S12}}{2\varepsilon_0}(-\boldsymbol{a}_z) + \frac{\rho_{S21}}{2\varepsilon_0}(-\boldsymbol{a}_z) + \frac{\rho_{S22}}{2\varepsilon_0}\boldsymbol{a}_z = 0$$

($\boldsymbol{a}_n = \boldsymbol{a}_z$ for ρ_{S22} while $\boldsymbol{a}_n = -\boldsymbol{a}_z$ for rest of the charge densities)

$$-\rho_{s11} - \rho_{s12} - \rho_{s21} + \rho_{s22} = 0 \qquad \text{...(2)}$$

Solving eq. (1) and eq (2) we get,

$$\rho_{s11} = \rho_{s22} \text{ and } \rho_{s12} = -\rho_{s21}$$

SOL 3.1.30 Option (A) is correct.

As all the four surfaces form the boundaries of the conductors extending away from the region between them so, the medium outside the defined region is conductor and so the field intensity outside the region will be zero. Now the electric potential in the non conducting region is given as

$$V = 5xy$$

So the electric field intensity in the region is

$$\boldsymbol{E} = -\nabla V = -5y\boldsymbol{a}_x - 5x\boldsymbol{a}_y$$

From the conductor-free space boundary condition we have the surface charge density on the boundary surface defined as

$$\rho_s = \varepsilon_0 E_n$$

where E_n is the normal component of the electric field intensity in the free space.

So, the surface charge density on the surface $x = 0$ is

$$\rho_s = \varepsilon_0(-5y) \text{ (the normal component } E_n = -5y \text{ for the surface } x = 0)$$
$$= -5\varepsilon_0 y$$

SOL 3.1.31 Option (B) is correct.
Again as discussed in above question, the surface charge density on the surface $y=0$ will be given by

$$\rho_s = \varepsilon_0 E_n$$

and since the field component normal to surface $y=0$ is

$$E_n = -5x$$

So, the surface charge density on the surface $y=0$ is

$$\rho_s = -5\varepsilon_0 x$$

SOL 3.1.32 Option (A) is correct.
From the symmetry associated with the charge distribution the electric field must be radially directed. Then choosing Gaussian surfaces which are cylinders having the same axis ($\rho=0$) as the conductors and of length l, we get

$$(2\pi\rho l)E_\rho = 0 \qquad \text{for } \rho < 2\,\text{m}$$

(Since there is no charge enclosed by the Gaussian surface)

Thus $$E_\rho = 0 \qquad \text{for } \rho < 2\,\text{m}$$

Now, since the field inside the conductor $2 < \rho < 3\,\text{m}$ is zero; there cannot be any charge on the surface $\rho = 2\,\text{m}$.

i.e. $$\rho_S = 0 \qquad \text{at } \rho = 2\,\text{m}$$

and all the charge associated with the inner conductor resides on the surface $\rho = 3\,\text{m}$.

i.e. $$\rho_S = \frac{10\,\text{C/m}}{2\pi(3)} = \frac{5}{3\pi}\,\text{C/m}^2 \qquad \text{at } \rho = 3\,\text{m}$$

Proceeding further we have

$$2\pi\rho l E_\rho = \frac{1}{\varepsilon_0}(10\,\text{C/m})l \qquad \text{for } 5 < \rho < 6\,\text{m}$$

where l is length of the cylinder.

So $$\boldsymbol{E} = \frac{10}{2\pi\varepsilon_0\rho}\boldsymbol{a}_\rho \qquad \text{for } 5 < \rho < 6\,\text{m}$$

This the field produced by the inner conductor but the fact is that the field inside the conductor $5 < \rho < 6\,\text{m}$ is zero that gives

$$[\rho_S]_{at\,\rho=5\,\text{m}} = \varepsilon_0[\boldsymbol{E}]_{at\,\rho=5} \cdot (-\boldsymbol{a}_\rho)$$
$$= \varepsilon_0\left(\frac{10}{2\pi\varepsilon_0(5)}\boldsymbol{a}_\rho\right) \cdot (-\boldsymbol{a}_\rho) = -\frac{10}{2\pi(5)} = -\frac{1}{\pi}\,\text{C/m}^2$$

and $$[\rho_S]_{at\,\rho=6\,\text{m}} = \frac{1}{2\pi(6)}\{6\,\text{C/m} - [\rho_S]_{at\,\rho=5}2\pi(5)\}$$
$$= \frac{1}{12\pi}(6+10) = \frac{4}{3\pi}$$

SOL 3.1.33 Option (B) is correct.
From the boundary condition for the charge carrying interface, the tangential component of electric field on either side of the surface will be same.

i.e. $$\boldsymbol{E}_{1t} = \boldsymbol{E}_{2t}$$

while the normal components are related as

$$\boldsymbol{E}_{1n} - \boldsymbol{E}_{2n} = \frac{\rho_s}{\varepsilon_0}$$

now as the field intensity in the region $z < 0$ is

$$\boldsymbol{E}_2 = 2\boldsymbol{a}_x + 3\boldsymbol{a}_y - 2\boldsymbol{a}_z$$

So the tangential component, $$\boldsymbol{E}_{2t} = 2\boldsymbol{a}_x + 3\boldsymbol{a}_y$$

and the normal component, $$\boldsymbol{E}_{2n} = -2\boldsymbol{a}_z$$

Therefore the field components in region ($z > 0$) are

$$\boldsymbol{E}_{1t} = \boldsymbol{E}_{2t} = 2\boldsymbol{a}_x + 3\boldsymbol{a}_y$$

and $$\boldsymbol{E}_{1n} = \boldsymbol{E}_{2n} + \frac{\rho_s}{\varepsilon_0} = \left[-2 + \frac{2\times10^{-9}}{8.85\times10^{-12}}\right]\boldsymbol{a}_z = 224\boldsymbol{a}_z$$

So the net field intensity in the region $z > 0$ is

$$\boldsymbol{E}_1 = \boldsymbol{E}_{1t} + \boldsymbol{E}_{1n} = 2\boldsymbol{a}_x + 3\boldsymbol{a}_y + 224\boldsymbol{a}_z$$

SOL 3.1.34 Option (B) is correct.
As the dielectric slab occupies the region $0 < y < 1\,\text{m}$ and the electric field in the free space is directed along $\boldsymbol{a}_y$ so, the field will be normal to both the boundary surfaces $y=0$ and $y=1$.
So from the boundary condition the field normal to the interface of dielectrics are related as

$$\varepsilon\boldsymbol{E}_i = \varepsilon_0\boldsymbol{E} \qquad (\text{where } \boldsymbol{E}_i \text{ is the field inside the dielectric})$$
$$\boldsymbol{E}_i = \frac{\varepsilon_0}{4\varepsilon_0}(1+y)^2\boldsymbol{E} = \frac{(1+y)^2}{4}(4\boldsymbol{a}_y) \qquad \text{since } \varepsilon = \frac{4\varepsilon_0}{(1+y)^2}$$
$$= (1+y)^2\boldsymbol{a}_y$$

So the polarization inside the dielectric is

$$\boldsymbol{P} = \varepsilon\boldsymbol{E}_i - \varepsilon_0\boldsymbol{E}_i$$
$$= \frac{4\varepsilon_0}{(1+y)^2}\boldsymbol{E}_i - \varepsilon_0\boldsymbol{E}_i = \left(\frac{4\varepsilon_0}{(1+y)^2} - \varepsilon_0\right)(1+y)^2\boldsymbol{a}_y$$
$$= [4-(1+y)^2]\varepsilon_0\boldsymbol{a}_y$$

Now for a given polarization $\boldsymbol{P}$ inside a dielectric material, the surface charge density over the surface of dielectric is defined as

$$\rho_{pS} = \boldsymbol{P}\cdot\boldsymbol{a}_n$$

where $\boldsymbol{a}_n$ is the unit vector normal to the surface directed outward of the dielectric.
So, the bound surface charge density at $y=0$ is

$$[\rho_{ps}]_{at\,y=0} = \boldsymbol{P}\cdot(-\boldsymbol{a}_y) \qquad (\boldsymbol{a}_n = -\boldsymbol{a}_y)$$
$$= [4-(1+y^2)]\varepsilon_0(-1) = -(4-1)\varepsilon_0 = -3\varepsilon_0 \qquad y=0$$

and the surface charge density at $y = 1\,\text{m}$ is

$$[\rho_{ps}]_{at\,y=1} = \boldsymbol{P}\cdot(\boldsymbol{a}_y) \qquad (\boldsymbol{a}_n = \boldsymbol{a}_y)$$
$$= [4-(1+y)^2]\varepsilon_0(1) = (4-4)\varepsilon_0 = 0$$

SOL 3.1.35 Option (A) is correct.
As calculated in previous question, polarization inside the dielectric is

$$\boldsymbol{P} = [4-(1+y)]\varepsilon_0\boldsymbol{a}_y$$

Since for a given polarization $\boldsymbol{P}$ inside a material, the bound volume charge density inside the material is defined as

$$\rho_{pv} = -\nabla\cdot\boldsymbol{P}$$

So the volume charge density inside the dielectric is

$$\rho_v = -\frac{\partial}{\partial y}[4-(1+y)^2]\varepsilon_0$$
$$= 2(1+y)\varepsilon_0$$

So when we move from $y=0$ to $y=1\,\text{m}$, the volume charge density will be linearly increasing.

SOL 3.1.36 Option (A) is correct.

As the charge is being located at origin so the field intensity due to it will be in radial direction and normal to the surface of the dielectric material. Therefore the flux density will be uniform(as from boundary condition) and at any point r inside the dielectric flux density will be

$$D = \frac{Q}{4\pi r^2}a_r$$

Now it is given that electric field intensity at any point inside the dielectric is

$$E = \frac{Q}{4\pi\varepsilon_0 b^2}a_r$$

and since in a medium of permittivity $\varepsilon = \varepsilon_r\varepsilon_0$ the flux density is defined as

$$D = \varepsilon_r\varepsilon_0 E$$

So for the given field we have

$$\frac{Q}{4\pi r^2}a_r = \varepsilon_r\varepsilon_0\left(\frac{Q}{4\pi\varepsilon_0 b^2}a_r\right)$$

$$\varepsilon_r = \frac{b^2}{r^2}$$

SOL 3.1.37 Option (C) is correct.

The electric field between the plates carrying charge densities $+\rho_{S0}$ and $-\rho_{S0}$ is defined as

$$E = \frac{\rho_{S0}}{\varepsilon}$$

where ε is the permittivity of the medium between the plates.

Now consider that near the plate 1 permittivity is ε_1 and near the plate 2, permittivity is ε_2. So at any distance x from plate 1 permittivity is given by

$$\varepsilon = \varepsilon_1 + \left(\frac{\varepsilon_2-\varepsilon_1}{d}\right)x \text{(Since the permittivity is linearly increasing)}$$

So the field intensity at any point in the medium will be

$$E = \frac{\rho_{S0}}{\varepsilon_1 + \left(\frac{\varepsilon_2-\varepsilon_1}{d}\right)x}$$

Therefore the potential difference between the plates will be

$$V = \int_0^d \frac{\rho_{S0}}{\varepsilon_1 + \left(\frac{\varepsilon_2-\varepsilon_1}{2}\right)x}dx$$

$$= \left[\frac{\rho_{S0}}{\left(\frac{\varepsilon_2-\varepsilon_1}{d}\right)}\ln\left[\varepsilon_1 + \left(\frac{\varepsilon_2-\varepsilon_1}{d}\right)x\right]\right]_0^d = \frac{\rho_{S0}d}{\varepsilon_2-\varepsilon_1}\ln\left(\frac{\varepsilon_2}{\varepsilon_1}\right)$$

SOL 3.1.38 Option (A) is correct.

The capacitor of a parallel plate capacitor is defined as

$$C = \frac{\varepsilon S}{d}$$

So, the capacitance in 1[st] dielectric region will be

$$C_1 = \frac{\varepsilon_1 S}{1} = \frac{3\varepsilon_0 S}{1}$$

and the capacitance in 2[nd] dielectric region

$$C_2 = \frac{\varepsilon_2 S}{3} = \frac{2\varepsilon_0 S}{3}$$

Therefore the voltage drop in 1[st] dielectric region is

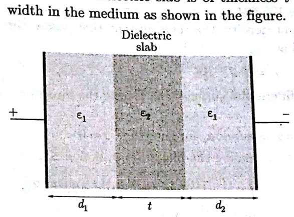
$$V_1 = \frac{C_2}{C_1 + C_2}\text{V} \qquad \text{(where } V \text{ is total voltage drop)}$$

$$= \frac{(2\varepsilon_0 S/3)}{3\varepsilon_0 S + \frac{2}{3}\varepsilon_0 S}(9\text{ Volt}) = \frac{18}{11}\text{ Volt}$$

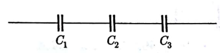
and similarly, $\qquad V_2 = \frac{C_1}{C_1 + C_2}\text{V} = \frac{3\varepsilon_0 S}{3\varepsilon_0 S + 2\varepsilon_0\frac{S}{3}}(9) = \frac{81}{11}\text{ Volt}$

SOL 3.1.39 Option (A) is correct.

Consider the dielectric slab is of thickness t and d_1, d_2 are the remaining width in the medium as shown in the figure.



Now the capacitance of the whole configuration will be considered as the three capacitors (capacitance in the three regions) connected in series as shown in the figure



So, $\qquad C_1 = \frac{\varepsilon_1 S}{d_1}, \quad C_2 = \frac{\varepsilon_2 S}{t}$ and $C_3 = \frac{\varepsilon_1 S}{d_2}$

The equivalent capacitance, is defined as

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{t}{\varepsilon S} + \frac{(d_1+d_2)}{\varepsilon S}$$

Since t; $(d_1 + d_2)$ will be constant although if the dielectric slab is moved leftward or rightward so the equivalent capacitance will be constant. But if the slab is pulled outward then the capacitance will change as the effective surface area of the capacitance due to dielectric slab changes.

SOL 3.1.40 Option (C) is correct.

Given, the potential field in free space

$$V = \frac{40}{r^3}\cos\theta\sin\phi$$

So, the potential at point P $\left(r = 2, \theta = \frac{\pi}{3}, \phi = \frac{\pi}{2}\right)$ is given as

$$V_P = \frac{40}{(2)^3}\cos\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{2}\right) = 2.5\text{ Volt}$$

Now, as the conducting surface is equipotential, so, the potential at any point on the conducting surface will be equal to the potential at point P.

i.e. $\qquad V = V_P = 2.5\text{ Volt}$

or $\qquad \frac{40}{r^3}\cos\theta\sin\phi = 2.5$

$16\cos\theta\sin\phi = r^3$ This is the equation of the conducting surface.

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SOLUTIONS 3.2

SOL 3.2.1 Correct answer is 125.664 .

For a given current density, the total current that passes through a given surface is defined as

$$I = \int \boldsymbol{J} \cdot d\boldsymbol{S}$$

where $d\boldsymbol{S}$ is the differential surface area having the direction normal to the surface.

So we have $d\boldsymbol{S} = \rho d\rho d\phi \boldsymbol{a}_z$ for the plane $z = 0$

Therefore, the current passing the plane $z = 0$, $0 \le \rho \le 2$ is

$$I = \int_{\phi=0}^{2\pi}\int_{\rho=0}^{2} [10e^z(\rho^2\boldsymbol{a}_\rho + \boldsymbol{a}_z)] \cdot (\rho d\rho d\phi \boldsymbol{a}_z)$$

$$= 10\int_0^{2\pi}\int_0^2 e^z \rho dz d\phi$$

$$= 10\int_0^{2\pi}\int_0^2 \rho d\rho d\phi \qquad (z = 0)$$

$$= 10\left[\frac{\rho^2}{2}\right]_0^2 \times [\phi]_0^{2\pi}$$

$$= 10 \times 2 \times 2\pi = 40\pi \text{ A} = 125.664 \text{ A}$$

SOL 3.2.2 Correct answer is 1.5708 .

For a given current density, the total current that passes through a given surface is defined as

$$I = \int \boldsymbol{J} \cdot d\boldsymbol{S}$$

where $d\boldsymbol{S}$ is the differential surface area having the direction normal to the surface.

So, we have $d\boldsymbol{S} = (r\sin\theta d\phi)(dr)\boldsymbol{a}_\theta$ for the surface $\theta = 90°$

Therefore, the total current crossing the surface $\theta = 90°, 0 < \phi < 2\pi$, $0 < r < 1\,\text{m}$ is

$$I = \int (r\cos^2\theta \boldsymbol{a}_r + r^2\sin\theta \boldsymbol{a}_\theta - r^2\boldsymbol{a}_\phi) \cdot (r\sin\theta d\phi dr \boldsymbol{a}_\theta)$$

$$= \int_{r=0}^{1}\int_{\phi=0}^{2\pi} r^3\sin^2\theta d\phi dr \qquad \text{at } \theta = 90°$$

$$= [\phi]_0^{2\pi} \times \left[\frac{r^4}{4}\right]_0^1 = 2\pi \times \frac{1}{4} = \frac{\pi}{2} = 1.5708 \text{ A}$$

SOL 3.2.3 Correct answer is 5.026 .

For a given current density, the total current flowing through a cross section is defined as

$$I = \int \boldsymbol{J} \cdot d\boldsymbol{S}$$

where $d\boldsymbol{S}$ is the differential cross sectional area vector having the direction normal to the cross section.

So we have $d\boldsymbol{S} = \rho d\rho d\phi \boldsymbol{a}_z$ (since the cylindrical wire is lying along z-axis)

Therefore the total current flowing through the wire (cross section) is

$$I = \left(\frac{50}{\rho}\boldsymbol{a}_z\right) \cdot (\rho d\rho d\phi \boldsymbol{a}_z)$$

$$= \int_{\rho=0}^{16\times10^{-3}}\int_{\phi=0}^{2\pi}\left(\frac{50}{\rho}\right)(\rho d\rho d\phi)$$

$$= 50 \times [\rho]_0^{16\times10^{-3}} \times [\phi]_0^{2\pi}$$

$$= 50 \times 16 \times 10^{-3} \times 2\pi = 5.026 \text{ Å}$$

SOL 3.2.4 Correct answer is 6.25 .

Since hydrogen atom contains a single electron (−ve charge) and a single proton (+ve charge). So, the dipole moment due to one atom of the hydrogen will be

$p = qd$ where q is electronic charge and d is effective length

i.e. $q = 1.6 \times 10^{-19}\,\text{C}$ and $d = 7.1 \times 10^{-16}\,\text{m}$

So, $p = (1.6 \times 10^{-19}) \times (7.1 \times 10^{-16})$

and since the polarization in a material is defined as the dipole moment per unit volume.

Therefore $P = np$ where n is the number of atoms per unit volume.

i.e. $n = 5.5 \times 10^{19}\,\text{atoms/cm}^3$

$= 5.5 \times 10^{25}\,\text{atoms/m}^3$

So, $P = (5.5 \times 10^{25}) \times (1.6 \times 10^{-19} \times 7.1 \times 10^{-16})$

$= 6.25 \times 10^{-9}\,\text{C/m}^2 = 6.25\,\text{nC/m}^2$

SOL 3.2.5 Correct answer is 1.0177 .

When an electric field E is applied to a material with dielectric constant ε_r then the polarization of the material is defined as

$$P = \varepsilon_0(\varepsilon_r - 1)E$$

So, $$\varepsilon_r - 1 = \frac{P}{\varepsilon_0 E} = \frac{6.25 \times 10^{-9}}{8.85 \times 10^{-12} \times 40 \times 10^3} = 1.7655 \times 10^{-2}$$

$$\varepsilon_r = 1 + 0.0177 = 1.0177$$

SOL 3.2.6 Correct answer is 2.

Given $\boldsymbol{D} = 2\boldsymbol{P} \Rightarrow \boldsymbol{P} = \boldsymbol{D}/2$

If the polarization of a dielectric material placed in an electric field $\boldsymbol{E}$ is $\boldsymbol{P}$, then the electric flux density in the material is defined as

$$\boldsymbol{D} = \varepsilon_0\boldsymbol{E} + \boldsymbol{P}$$

$$= \varepsilon_0\boldsymbol{E} + \boldsymbol{D}/2$$

or $$\boldsymbol{D} = 2\varepsilon_0\boldsymbol{E} \qquad \text{.........(1)}$$

and since the relation between the electric field, $\boldsymbol{E}$ and flux density, $\boldsymbol{D}$ inside a dielectric material with dielectric constant ε_r is defined as

$$\boldsymbol{D} = \varepsilon_0\varepsilon_r\boldsymbol{E}$$

So, comparing the result with equation (1) we get, $\varepsilon_r = 2$.

SOL 3.2.7 Correct answer is 3.

Energy on a dipole with moment $\boldsymbol{p}$ in an electric field $\boldsymbol{E}$ is defined as

$$W_E = -\boldsymbol{p} \cdot \boldsymbol{E}$$

$$= -(-2\boldsymbol{a}_z + 3\boldsymbol{a}_y) \cdot (1.5\boldsymbol{a}_z - \boldsymbol{a}_z)$$

$$= -(-3 + 0) = 3 \text{ J}$$

SOL 3.2.8 Correct answer is 1.95 .

Resistance of a conductor of length l and having uniform cross sectional area S is

$$R = \frac{l}{\sigma S} \qquad \text{where } \sigma \text{ is the conductivity of the conductor}$$

Given the conductivity, $\sigma = 5 \times 10^6\,(\Omega\text{m})^{-1}$

the length of the conductor, $l = 8\,\text{m}$

side of the square cross section, $a = 3\,\text{cm}$

and radius of the bored hole, $r = 0.5\,\text{cm}$

So, the net cross sectional area is

S= area of square cross section(bar) − area of circular cross section(hole)

or $$S = a^2 - \pi r^2 = (3)^2 - \pi(0.5)^2 = (9 - \tfrac{\pi}{4})\,\text{cm}^2$$

The total resistance between the square ends is given as

$$R = \frac{l}{\sigma S} = \frac{8}{(5 \times 10^6) \times \left[(9 - \frac{\pi}{4}) \times 10^{-4}\right]}$$

$$= 1.948 \times 10^{-3}\,\Omega = 1.95\,\text{m}\Omega$$

SOL 3.2.9 Correct answer is 924.6 .

The two materials of composite bar will behave like two wires of resistance R_L(resistance due to lead) and R_C(resistance due to copper) connected in parallel.

As from the previous question we have the resistance due to the lead is

$$R_L = 1.948\,\text{m}\Omega$$

and since the area of the cross section filled with copper is equal to the area of the cross section defined by hole so we have

Cross sectional area $S_C = \frac{\pi}{4}\,\text{cm}^2$

Length of the bar $l = 8\,\text{m}$

and conductivity of the copper, $\sigma_C = \frac{1}{\text{resistivity of the copper}}$

$$= \frac{1}{1.72 \times 10^{-8}}$$

So the resistance due to copper is

$$R_C = \frac{l}{S_C \sigma_C} = \frac{8}{(\frac{\pi}{4} \times 10^{-4})(\frac{1}{1.72 \times 10^{-8}})} = 1.76\,\text{m}\Omega$$

Therefore the equivalent resistance of the composite bar is

$$R = R_C \,||\, R_L = \frac{(1.948) \times (1.76)}{1.948 + 1.76}$$

$$= 924.62 \times 10^{-6}\,\Omega = 924.6\,\mu\Omega$$

SOL 3.2.10 Correct answer is 8.9 .

Given the radii of spherical shell as

$$a = 1\,\text{cm} = 0.01\,\text{m}$$

$$b = 2\,\text{cm} = 0.02\,\text{m}$$

The capacitance of a spherical capacitor having inner and outer radii a and b respectively is defined as

$$C = \frac{4\pi\varepsilon_r\varepsilon_0}{\left(\frac{1}{a} - \frac{1}{b}\right)} = \frac{4\pi \times 4 \times 8.85 \times 10^{-12}}{\frac{1}{0.01} - \frac{1}{0.02}} = 8.9\,\text{pF}$$

SOL 3.2.11 Correct answer is 7.26 .

Since the dielectric has been removed from the portion defined by $(\frac{\pi}{2} < \phi < \pi)$ so the composite capacitor will have the dielectric filled only in $\frac{3}{4}$th portion of the total capacitor and so the configuration can be treated as the two capacitors connected in parallel with each other.

The capacitance of the portion carrying air($\varepsilon_r = 1$) as the medium between the spherical shells

$$C_1 = \frac{1}{4} \times \frac{4\pi\varepsilon_0}{\frac{1}{a} - \frac{1}{b}} = \frac{1}{4} \times \frac{4\pi \times 8.85 \times 10^{-12}}{\frac{1}{0.01} - \frac{1}{0.02}}$$

$$= 0.56 \times 10^{-12} = 0.56\,\text{pF}$$

The capacitance of the portion carrying dielectric($\varepsilon_r = 4$) as the medium between the spherical shells

$$C_2 = \frac{3}{4} \times C$$

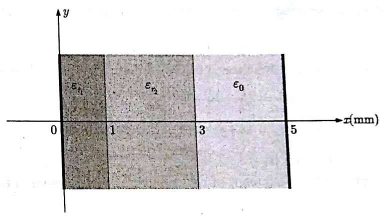
where C is the capacitance if no any portion of dielectric was removed as already calculated in previous question.

So we have $$C_2 = \frac{3}{4} \times 8.9 \times 10^{-12} = 6.7 \times 10^{-12} = 6.7\,\text{pF}$$

Therefore the equivalent capacitance of the composite capacitor is,

$$C_{eq} = C_1 + C_2 = 0.56 + 6.7 = 7.26\,\text{pF}$$

SOL 3.2.12 Correct answer is 3.05 .



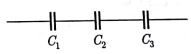
Capacitance of a parallel plate capacitor is defined as

$$C = \frac{\varepsilon S}{d}$$

where S is the surface area of the parallel plates

d is the separation between the plates

Here, the three different regions will be treated as the three capacitors connected in series as shown below



So the capacitance of the region 1 is $$C_1 = \frac{\varepsilon_{r1}\varepsilon_0 S}{0.001} = 2500\varepsilon_0 S$$

Capacitance of the region 2 is $$C_2 = \frac{\varepsilon_{r2}\varepsilon_0 S}{0.002} = 2000\varepsilon_0 S$$

Capacitance of the region 3 is $$C_3 = \frac{\varepsilon_0 S}{0.002} = 500\varepsilon_0 S$$

Therefore the equivalent capacitance of the whole configuration is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{\varepsilon_0 S}\left(\frac{1}{2500} + \frac{1}{2000} + \frac{1}{500}\right)$$

So, $C_{eq} = 3.45 \times 10^2 \varepsilon_0 S$

The capacitance per square meter of surface area will be

$$C_{eq}' = \frac{C_{eq}}{S} = 3.45 \times 10^2 \varepsilon_0 = 3.05\,\text{nF/m}^2$$

SOL 3.2.13 Correct answer is 143.

Capacitance between the two cylindrical surfaces is defined as

$$C = \frac{2\pi\varepsilon l}{\ln(b/a)}$$

Where $l \rightarrow$ length of the cylinder

$a \rightarrow$ inner radius of the cylinder

$b \rightarrow$ outer radius of the cylinder

Since, the medium between the conducting cylinders includes the dielectric layer($\varepsilon_r = 4$) from $\rho = 4\,\text{cm}$ to $\rho = 6\,\text{cm}$ and air($\varepsilon_r = 1$) from $\rho = 6\,\text{cm}$ to $\rho = 8\,\text{cm}$, so the configuration can be treated as the two capacitance connected in series.

Now for the dielectric layer ($\varepsilon_r = 4$) from $\rho = 4\,\text{cm}$ to $\rho = 6\,\text{cm}$, capacitance is

$$C_1 = \frac{2\pi\varepsilon_0\varepsilon_r 1}{\ln(6/4)} = \frac{8\pi\varepsilon_0}{\ln(1.5)} \qquad (l = 1\,\text{m})$$

and for the air medium ($\varepsilon_r = 1$) from $\rho = 6\,\text{cm}$ to $\rho = 8\,\text{cm}$, capacitance is

$$C_2 = \frac{2\pi\varepsilon_0 \times 1}{\ln(8/6)} = \frac{2\pi\varepsilon_0}{\ln(4/3)}$$

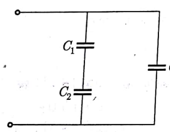
So, the equivalent capacitance of the configuration is evaluated as

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{\ln(1.5)}{8\pi\varepsilon_0} + \frac{\ln(4/3)}{2\pi\varepsilon_0}$$

$$C_{eq} = 143\,\text{pF}$$

SOL 3.2.14 Correct answer is 2.76 .

The equivalent arrangement of the capacitor can be drawn in form of circuit as below



For which the capacitances are calculated as below

$$C_1 = \frac{\varepsilon_0 S/2}{d/2} = \frac{\varepsilon_0 S}{d} = \frac{\varepsilon_0 \times 10 \times 10^{-4}}{4 \times 10^{-3}} = \frac{\varepsilon_0}{4}$$

$$C_2 = \frac{\varepsilon_r\varepsilon_0 S/2}{d/2} = \frac{\varepsilon_r\varepsilon_0 S}{d} = \frac{3\varepsilon_0}{4} \qquad \varepsilon_r = 3$$

$$C_3 = \frac{\varepsilon_0 S/2}{d} = \frac{\varepsilon_0 \times 10 \times 10^{-4}}{2 \times 4 \times 10^{-3}} = \frac{\varepsilon_0}{8}$$

Therefore the equivalent capacitance of the capacitor is

$$C_{eq} = C_3 + \frac{C_1 C_2}{C_1 + C_2} = \frac{\varepsilon_0}{8} + \frac{\frac{\varepsilon_0}{4}\frac{3\varepsilon_0}{4}}{\frac{\varepsilon_0}{4} + \frac{3\varepsilon_0}{4}} = \frac{\varepsilon_0}{4} = 2.76\,\text{pF}$$

SOL 3.2.15 Correct answer is 0.

For a given polarization $\boldsymbol{P}$ inside a material, the bound surface charge density over the surface of material is defined as

$$\rho_{ps} = \boldsymbol{P} \cdot \boldsymbol{a}_n$$

where $\boldsymbol{a}_n$ is the unit vector normal to the surface directed outward.

while the bound volume charge density inside the material is defined as

$$\rho_{pv} = -\nabla \cdot \boldsymbol{P}$$

Since the component of polarization of rod along y-axis is $P_y = 2y^2 + 3$. So, the polarization of the material is $\boldsymbol{P} = (2y^2 + 3)\,\boldsymbol{a}_y$. and the charge density on the surface of the rod is $\rho_{ps} = \boldsymbol{P} \cdot \boldsymbol{a}_n$

At $y = 0$(top surface) $\qquad \rho_{S1} = (2y^2 + 3)\,\boldsymbol{a}_y \cdot (-\boldsymbol{a}_y) = -3$

At $y = 5$(bottom surface) $\qquad \rho_{S2} = (2y^2 + 3)\,\boldsymbol{a}_y \cdot (\boldsymbol{a}_y) = 53$

and since the polarization has no radial component so no charge will be stored on its curvilinear surface and so the total bound surface charge on the surface of the rod is

$$Q_{PS} = \int \rho_{PS}\,ds = \rho_{S1} S + \rho_{S2} S \;(S \text{ is the cross sectional area})$$

$$= -3S + 53S = 50S$$

Now, the bound volume charge density inside the material is

$$\rho_{pv} = -\nabla \cdot \boldsymbol{P} = -\nabla \cdot (2y^2 + 3)\,\boldsymbol{a}_y = -4y$$

So the total bound volume charge stored inside the material will be

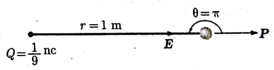
$$Q_{pv} = \int \rho_{pv}\,dv$$

$$= \int_0^5 (-4y)\,S\,dy = -4S\left[\frac{y^2}{2}\right]_0^5 - 50S$$

So, Total bound charge

$$Q_{bound} = Q_S + Q_v = 50S - 50S = 0$$

SOL 3.2.16 Correct answer is 2.



Electric field produced by the point charge at a distance r is

$$\boldsymbol{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\boldsymbol{a}_r$$

So, the induced dipole moment in the neutral atom due to the electric field $\boldsymbol{E}$ produced by the point charge will be

$$\boldsymbol{p} = \alpha\boldsymbol{E} = \frac{\alpha q}{4\pi\varepsilon_0 r^2}\boldsymbol{a}_r$$

and since the electric field intensity produced due to a dipole having moment p at a distance r from the dipole is defined as

$$\boldsymbol{E}_{dip} = \frac{p}{4\pi\varepsilon_0 r^3}(2\cos\theta\,\boldsymbol{a}_r + \sin\theta\,\boldsymbol{a}_\theta)$$

where θ is the angle formed between the distance vector $\boldsymbol{r}$ and dipole moment $\boldsymbol{p}$

So the field produced by the induced dipole at the point charge is

$$E_{dip} = \frac{2p}{4\pi\varepsilon_0 r^3} = \frac{2\left(\frac{\alpha q}{4\pi\varepsilon_0 r^2}\right)}{4\pi\varepsilon_0 r^3} = \frac{2\alpha q}{(4\pi\varepsilon_0)^2 r^5} \qquad (\theta = \pi \text{ as shown in the figure})$$

Therefore the force experienced by the point charge due to the field applied by induced dipole is

$$F = qE_{dip} = 2\alpha\left(\frac{q}{4\pi\varepsilon_0}\right)^2 \frac{1}{r^5}$$

$$= 2\alpha\left(\frac{1}{9} \times 10^{-9} \times 9 \times 10^9\right)^2 \times \frac{1}{(1)^5} = 2\alpha \text{ N}$$

So, $\frac{F}{\alpha} = 2 \text{ N}$

SOL 3.2.17 Correct answer is 0.16 .

Electric field intensity produced due to a dipole having moment p, at a distance r from the dipole is defined as

$$\boldsymbol{E}_{dip} = \frac{p}{4\pi\varepsilon_0 r^3}(2\cos\theta \boldsymbol{a}_r + \sin\theta \boldsymbol{a}_\theta)$$

where θ is the angle formed between the distance vector $\boldsymbol{r}$ and dipole moment $\boldsymbol{p}$

So the electric field intensity produced due to dipole P_1 at P_2 is

$$\boldsymbol{E}_1 = \frac{p_1}{4\pi\varepsilon_0 r^3}\boldsymbol{a}_\theta = \frac{2 \times 10^{-9}}{4\pi\varepsilon_0 \times (1)^3}\boldsymbol{a}_\theta \qquad (\theta = \pi/2)$$

Therefore the torque on P_2 due to P_1 is

$$\boldsymbol{T} = \boldsymbol{p}_2 \times \boldsymbol{E}_1$$

Taking the magnitude only we have the torque on P_2 is

$$T = p_2 E_1 \sin\theta = (9 \times 10^{-9})\left(\frac{2 \times 10^{-9}}{4\pi\varepsilon_0}\right)\sin 90^\circ$$

$$= 1.62 \times 10^{-7} \text{ N-m} = 0.16 \,\mu\text{N-m}$$

SOL 3.2.18 Correct answer is 0.324 .

Electric field intensity produced due to a dipole having moment p, at a distance r from the dipole is defined as

$$\boldsymbol{E}_{dip} = \frac{p}{4\pi\varepsilon_0 r^3}(2\cos\theta \boldsymbol{a}_r + \sin\theta \boldsymbol{a}_\theta)$$

where θ is the angle formed between the distance vector $\boldsymbol{r}$ and dipole moment $\boldsymbol{p}$

So the electric field intensity produce due to dipole P_2 at P_1 is

$$\boldsymbol{E}_2 = -\frac{p_2}{4\pi\varepsilon_0 r^3}2\boldsymbol{a}_r = -\frac{9 \times 10^{-9}}{4\pi\varepsilon_0 (1)^3} \times 2\boldsymbol{a}_r \qquad (\theta = \pi)$$

Therefore the torque on P_1 due to P_2 is

$$\boldsymbol{T} = \boldsymbol{p}_1 \times \boldsymbol{E}_2$$

Considering the magnitude only we have the torque on P_1 is

$$T = p_1 E_2 \sin\theta = 2 \times 10^{-9} \times \left(\frac{-9 \times 10^{-9}}{4\pi\varepsilon_0} \times 2\right) \qquad (\theta = \pi/2)$$

$$= 3.24 \times 10^{-7} \text{ N-m} = 0.324 \,\mu\text{N-m}$$

SOL 3.2.19 Correct answer is 0.

Since the spherical shell is of inner radius $r = 2$ m so region inside the sphere will have no polarization and therefore the total charge enclosed inside the shell for $r < 2$ m will be zero.

i.e. $Q_{enc} = 0$

According to Gauss law the total outward electric flux from a closed surface is equal to the charge enclosed by the surface and since the total enclosed charge for $r < 2$ m is zero so the electric field intensity at $r = 1$ m will be zero.

SOL 3.2.20 Correct answer is 0.

Since the total bound charge by a polarized neutral dielectric is zero as discussed earlier. So for any point outside the spherical shell the total enclosed charge(bound charge) will be zero and as discussed in the previous question, according to Gauss law the electric field intensity at any point outside the spherical shell will be zero.

So, for the surface $r = 7$ $\qquad Q_{enc} = 0$

Therefore the electric field intensity is $\qquad E = 0$

SOL 3.2.21 Correct answer is -1.

As we have to find electric field at $r = 5$ m so we determine first the charge enclosed by the surface $r = 5$ m which will be equal to the sum of the volume charge stored in the region $2 \le r \le 5$ m and the surface charge stored at $r = 2$ m.

Since for a given polarization $\boldsymbol{P}$ of a dielectric material, the bound volume charge density inside the material is defined as

$$\rho_{pv} = -\nabla \cdot \boldsymbol{P}$$

So the bound volume charge density inside the dielectric defined in the region $2 \le r \le 6$ m will be

$$\rho_{pv} = -\nabla \cdot \boldsymbol{P}(r) = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{5}{r}\right) = -\frac{5}{r^2}$$

so the total bound volume charge in the region $2 \le r \le 5$ m is

$$Q_{pv} = \int \rho_{pv}\, dv = \int_{r=2}^{5} -\frac{5}{r} \times 4\pi r^2 dr$$

$$= -20\pi[r]_2^5 = -60\pi$$

Now for a given polarization $\boldsymbol{P}$ inside a dielectric material, the bound surface charge density over the surface of dielectric is defined as

$$\rho_{PS} = \boldsymbol{P} \cdot \boldsymbol{a}_n$$

where $\boldsymbol{a}_n$ is the unit vector normal to the surface pointing outward of the material.

So the bound surface charge density at $r = 2$ m is

$$\rho_{PS} = \boldsymbol{P}(r) \cdot (-\boldsymbol{a}_r) \qquad (\boldsymbol{a}_n = -\boldsymbol{a}_r)$$

Therefore the total bound surface charge over the surface $r = 2$ m is

$$Q_{PS} = -\frac{5}{r} \times 4\pi r^2 \qquad (\text{for spherical surface } S = 4\pi r^2)$$

$$= -\frac{5}{2} \times 4\pi \times 2^2 = -40\pi \qquad r = 2 \text{ m}$$

So, the total enclosed charge by the surface $r = 5$ m is

$$Q_{enc} = Q_{pv} + Q_{ps} = -60\pi - 40\pi = -100\pi$$

So the electric field intensity at $r = 5$ m will be,

$$\boldsymbol{E} = \frac{1}{4\pi\varepsilon_0} \times \frac{Q_{enc}}{r^2}\boldsymbol{a}_r = \frac{1}{4\pi\varepsilon_0} \times \frac{-100\pi}{5^2}\boldsymbol{a}_r = -\frac{1}{\varepsilon_0}\boldsymbol{a}_r \qquad ...(1)$$

Since, from the given problem, we have the electric field at $r = 5$ as

$$E = \frac{k}{\varepsilon_0} a_r \qquad ...(2)$$

Thus, by comparing equations (1) and (2), we get

$$k = -1$$

SOL 3.2.22 Correct answer is 27.

Since the electric field intensity at any point inside a conductor is always zero, so the electric flux density at a distance r from the center of the spherical conductor can be given as

$$D = \begin{cases} 0, & r < 1 \\ \dfrac{Q}{4\pi r^2} a_r, & r > 1\,\text{m} \end{cases}$$

Alternate Sol^n: (Mine) (Proprietary)

Take $r=\infty$ as a surface & have 2 Cap in series. (Spherical).

from $r=1$ to $r=2$.

" $r=2$ to $r=\infty$.

and $C_{eq} = \frac{C_1 C_2}{C_1+C_2}$

Energy $= \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$.

where $Q = 3\,\text{mC}$ is the total charge carried by the conductor. and since the dielectric material surrounding the spherical conductor has permittivity $\varepsilon_r = 3$, so the electric field intensity at a distance r from the center of the sphere is

$$E = \begin{cases} 0, & r < 1\,\text{m} \\ \dfrac{Q}{4\pi\varepsilon_r\varepsilon_0 r^2} a_r & 1 < r < 2\,\text{m} \\ \dfrac{Q}{4\pi\varepsilon_0 r^2} a_r & r > 2\,\text{m} \end{cases}$$

So, the total energy of the configuration is

$$W_E = \frac{1}{2}\int D \cdot E\, dv$$

$$= \frac{1}{2}\left[\int_0^1 0\,dr + \int_1^2 \left(\frac{Q}{4\pi r^2}\right)\left(\frac{Q}{4\pi\varepsilon_r\varepsilon_0 r^2}\right)(4\pi r^2\, dr) + \int_2^\infty \left(\frac{Q}{4\pi r^2}\right)\left(\frac{Q}{4\pi\varepsilon_0 r^2}\right)(4\pi r^2\, dr)\right]$$

$$= \frac{1}{2}\frac{Q^2}{(4\pi)^2}4\pi\left\{\frac{1}{\varepsilon}\int_1^2 \frac{1}{r^2}dr + \frac{1}{\varepsilon_0}\int_2^\infty \frac{1}{r^2}dr\right\}$$

$$= \frac{Q^2}{8\pi}\left\{\frac{1}{\varepsilon_0\varepsilon_r}\left[-\frac{1}{r}\right]_1^2 + \frac{1}{\varepsilon_0}\left[-\frac{1}{r}\right]_2^\infty\right\} = \frac{Q^2}{8\pi\varepsilon_0}\left\{\frac{1}{3}\times\frac{1}{2}+\frac{1}{2}\right\}$$

$$= \frac{(3\times 10^{-3})^2 \times 9 \times 10^9}{2} \times \frac{8}{12}$$

$$= 2.7 \times 10^4\,\text{J} = 27\,\text{kJ}$$

SOL 3.2.23 Correct answer is 9.

The electric potential at the centre of sphere will be equal to the work done to carry a unit charge from infinity to the centre of the sphere (the line integral of the electric field intensity from infinity to the center of the sphere)

i.e. $$V = -\int_\infty^0 E \cdot dl$$

Since the sphere has uniform charge density $\rho_v = 0.6\,\text{nC/m}^3$ embedded in it, so the electric field intensity at a distance r from the center of the sphere can be given as

$$E = \begin{cases} \dfrac{\rho_v r}{3\varepsilon_r\varepsilon_0} a_r, & r < R \\ \dfrac{\rho_v R^3}{3\varepsilon_0 r^2} a_r, & r > R \end{cases}$$

where R is the radius of the sphere i.e. $R = \frac{1}{\sqrt{\pi}}$ m

So, the potential at the centre of sphere will be

$$V = -\int_\infty^0 E \cdot dl \quad \text{(where differential displacement is } dl = dr a_r\text{)}$$

$$= -\int_\infty^{1/\sqrt{\pi}} \frac{\rho_v}{3\varepsilon_0 r^2}\left(\frac{1}{\sqrt{\pi}}\right)^3 dr - \int_{1/\sqrt{\pi}}^0 \frac{\rho_v r}{3\varepsilon} dr$$

$$= -\frac{\rho_v}{3\varepsilon_0}\left(\frac{1}{\sqrt{\pi}}\right)^3 \times \left[-\frac{1}{r}\right]_\infty^{1/\sqrt{\pi}} - \frac{\rho_v}{3\varepsilon_r\varepsilon_0}\left[\frac{r^2}{2}\right]_{1/\sqrt{\pi}}^0$$

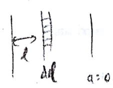
$$= \frac{\rho_v}{3\varepsilon_0}\left(\frac{1}{\sqrt{\pi}}\right)^3 \times \sqrt{\pi} + \frac{\rho_v}{3\varepsilon_r\varepsilon_0} \times \frac{1}{2\pi} = \frac{\rho_v}{3\varepsilon_0\pi} + \frac{\rho_v}{12\varepsilon_0\pi}$$

$$= \frac{5\rho_v}{12\pi\varepsilon_0} = \frac{5\rho_v}{3\times 4\pi\varepsilon_0} = \frac{5 \times 0.6 \times 10^{-9} \times 9 \times 10^9}{3} \qquad (\varepsilon_r = 2)$$

$$= 9\,\text{volt} \qquad \rho_v = 0.6\,\text{nC/m}^3$$

SOL 3.2.24 Correct answer is 45.1 .

Alternate sol^n:



a=0   a=0.1m

$dC = \frac{\varepsilon A}{dl}$.

$\frac{1}{C} = \int \frac{1}{dC} = \int_0^{0.1} \frac{dl}{\varepsilon A}$

Consider the surface charge density on the parallel plates is $\pm\rho_S$ so the electric flux density between the plates is defined as

$$D = \rho_S a_n$$

where a_n is the unit vector normal to the surface of plates directed from one plate toward the other plate.

Since permittivity changes from layer to layer, but the field is normal to the surface so electric flux density D will be uniform throughout the plate separation as from boundary condition.

So the electric field intensity at any point between the parallel plates is

$$E = \frac{D}{\varepsilon_0\varepsilon_r} = \frac{\rho_S a_n}{2\varepsilon_0(1+100a^2)} \qquad \varepsilon_r = 2(1+100a^2)$$

Therefore the voltage between the plates can be evaluated by taking the line integral of electric field from one plate to the other plate

i.e. $$V = -\int E \cdot dl = -\int_{a=0}^{0.1}\left(\frac{\rho_S a_n}{2\varepsilon_0(1+100a^2)}\right)\cdot(da) \qquad (dl = da)$$

$$= \frac{\rho_S}{2\varepsilon_0}\frac{1}{100}\int_0^{0.1}\frac{da}{(0.1)^2+a^2} \qquad \text{(the direction of } a \text{ is along } a_n\text{)}$$

$$= \frac{\rho_S}{2\varepsilon_0} \times \frac{1}{100} \times \frac{1}{0.1}\left[\tan^{-1}\left(\frac{a}{0.1}\right)\right]_0^{0.1}$$

$$= \frac{\rho_S}{2\varepsilon_0} \times \frac{1}{10}\left[\frac{\pi}{4} - 0\right] = \frac{\rho_S \pi}{80\varepsilon_0}$$

Now charge stored at the parallel plates is

$$Q = (\rho_S)(S) \qquad \text{where } S \text{ is surface area of the plates}$$

$$= \rho_s \times (0.2) \qquad S = 0.2\,\text{m}^2$$

So, the capacitance of the capacitor is evaluated as

$$C = \frac{Q}{V} = \frac{\rho_s \times (0.2)}{(\rho_s\pi)/80\varepsilon_0} = \frac{16\varepsilon_0}{\pi}$$

$$= 4.51 \times 10^{-11} = 45.1\,\text{pF}$$

SOL 3.2.25 Correct answer is 3.64 .

For the two wire transmission line consists of the cylinders of radius b and separated by a distance $2h$ (centre to centre), the capacitance per unit length between them is defined as

$$\therefore \quad C' = \frac{\pi\varepsilon}{\cosh^{-1}(h/b)}$$

Here, $2h = 2\,\text{cm}$ and $b = 0.2\,\text{cm}$

So, $C' = \dfrac{\pi \times 2 \times 8.85 \times 10^{-12}}{\cosh^{-1}(1/0.2)} = 3.64 \times 10^{-11}\,\text{F/m} \quad (\varepsilon_r = 2)$

So the charge per unit length on each wire will be,

$$Q = C' V_o = 3.64 \times 10^{-11} \times 100 \qquad (V_o = 100\,\text{V})$$
$$= 3.64 \times 10^{-9}\,\text{C/m}$$
$$= 3.64\,\text{nC/m}$$

SOL 3.2.26 Correct answer is 41.1 .

Consider the oil rises to a height h in the space between the tubes.
So, the capacitance of the tube carrying oil partially will be treated as the two capacitors connected in parallel.
Since the capacitance between the two cylindrical surfaces is defined as

$$C = \frac{2\pi\varepsilon l}{\ln(b/a)}$$

Where $l \rightarrow$ length of the cylinder
$a \rightarrow$ inner radius of the cylinder
$b \rightarrow$ outer radius of the cylinder

So the capacitance of the portion carrying oil ($\chi_e = 1$) as the medium between the cylindrical surfaces is

$$C_{oil} = \frac{2\pi\varepsilon_r\varepsilon_0 h}{\ln(3/1)} = \frac{4\pi\varepsilon_0 h}{\ln(3)} \qquad (\varepsilon_r = \chi_e + 1 = 2)$$

and the capacitance of the portion carrying air($\varepsilon_r = 1$) as the medium between the cylindrical surfaces is

$$C_{air} = \frac{2\pi\varepsilon_0(1-h)}{\ln(3/1)}$$

Therefore the equivalent capacitance of the tube carrying oil to the height h is

$$C = C_{oil} + C_{air} = 2\pi\varepsilon_0 \frac{(1+h)}{\ln(3)}$$

Since the energy stored in a capacitor is defined as

$$W_E = \frac{1}{2} C V^2 \qquad \text{where } V \text{ is the applied voltage to the capacitor}$$

So the net upward force due to the capacitance is given by

$$F = \frac{dW_E}{dh} = \frac{1}{2} V^2 \frac{dC}{dh} = \frac{1}{2} V^2 \frac{2\pi\varepsilon_0}{\ln(3)}$$

and net downward force on the oil due to gravity will be

$$F = mg = (0.01\,\text{gm/cm}^3) \times \pi(b^2 - a^2)\, h \times g$$
mass density $= 0.01\,\text{gm/cm}^3$
$$= \frac{0.01}{10^{-6}} \times \pi(9-1) \times 10^{-6} \times h \times g = 0.08\pi h g$$

Since in equilibrium both the upward and downward forces are equal

So, $$0.08\pi h g = \frac{1}{2} V^2 \frac{2\pi\varepsilon_0}{\ln(3)}$$

$$0.08\pi h \times (9.8) = \frac{1}{2} \times (2 \times 10^3)^2 \times \frac{2\pi \times 8.85 \times 10^{-12}}{\ln(3)}$$

$$h = \frac{1}{2} \times \frac{(2 \times 10^3)^2 \times 2 \times 8.85 \times 10^{-12}}{0.08 \times 9.8 \times \ln(3)}$$
$$= 4.11 \times 10^{-5}\,\text{m}$$
$$= 41.1\,\mu\text{m}$$

SOL 3.2.27 Correct answer is 0.0796.

From the symmetry associated with the charge distribution the electric field must be radially directed. As, there is no charge enclosed by the surface $r = 2\,\text{m}$ so we get

$$E_r = 0 \qquad \text{for } r < 2\,\text{m}$$

Now from the conductor-free space boundary condition we have the surface charge density on the boundary surface defined as

$$\rho_s = \varepsilon_0 E_n$$

where E_n is the normal component of the electric field intensity in the free space.

So the charge density at $r = 2\,\text{m}$ is

$$\rho_{S1} = E_r = 0$$

Therefore the total charge will be concentrated over the outer surface which is given as

$$\rho_{S2} = \frac{Q}{4\pi r^2} = \frac{9}{4\pi(3)^2} = \frac{1}{4\pi}\,\text{C/m}^2 = 0.0796\,\text{C/m}^2$$

SOL 3.2.28 Correct answer is 8.

As the dielectric slab occupies the region $x > 0$ and the electric field in the free space is directed along $\boldsymbol{a}_x$ so, the field will be normal to the boundary surface, $x = 0$ of the dielectric slab.
So from the boundary condition the field normal to the interface of dielectrics are related as

$$\varepsilon_r\varepsilon_0 \boldsymbol{E}_i = \varepsilon_0 \boldsymbol{E} \qquad (\text{where } \boldsymbol{E}_i \text{ is the field inside the dielectric})$$
$$\boldsymbol{E}_i = \frac{\boldsymbol{E}}{\varepsilon_r} = \frac{10\boldsymbol{a}_x}{5} = 2\boldsymbol{a}_x \qquad (\varepsilon_r = 5)$$

So, the polarization inside the dielectric is

$$\boldsymbol{P} = (\varepsilon - \varepsilon_0)\boldsymbol{E}_i = (5\varepsilon_0 - \varepsilon_0)\boldsymbol{E}_i = 8\varepsilon_0 \boldsymbol{a}_x$$

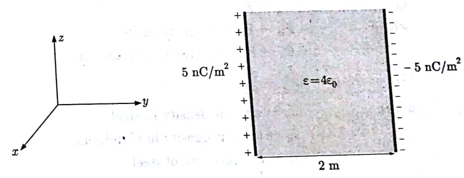
Since, from the given problem, we have

$$\boldsymbol{P} = k\varepsilon_0 \boldsymbol{a}_x$$

So, we get $$k = 8$$

SOL 3.2.29 Correct answer is 283.

Consider the parallel sheets arrangement as shown in the figure.



Electric field intensity at any point P due to the uniformly charged plane with charge density ρ_S is defined as

$$\boldsymbol{E} = \frac{\rho_S}{2\varepsilon} \boldsymbol{a}_n$$

where $\boldsymbol{a}_n$ is the unit vector normal to the plane directed toward point P and ε is the permittivity of the medium.

So the field intensity inside the dielectric due to the left sheet will be

$$E_1 = \frac{5 \times 10^{-9}}{2\varepsilon}(a_y) \qquad (a_n = a_y)$$

and again the field intensity inside the dielectric due to right sheet will be

$$E_2 = \frac{-5 \times 10^{-9}}{2\varepsilon}(-a_y) = \frac{5 \times 10^{-9}}{2\varepsilon}a_y \qquad (a_n = -a_y)$$

so the net field intensity inside the dielectric will be

$$E = E_1 + E_2 = \frac{5 \times 10^{-9}}{\varepsilon}a_y$$

Since the field intensity is uniform inside the dielectric So potential difference between the plates will be directly given as

$$V = E \times \text{(distance between the plates)}$$

$$= \frac{5 \times 10^{-9}}{4\varepsilon_0} \times 2$$

$$= 2.824 \times 10^2 \text{ Volt} = 283 \text{ kV} \qquad (\varepsilon = 4\varepsilon_0)$$

SOL 3.2.30 Correct answer is 8.85 .

Assume that the surface charge densities on the plates is $\pm\rho_{S0}$ so the electric field intensity between the plates will be

$$E = \frac{\rho_{S0}}{\varepsilon_0}$$

and the potential difference between the plates will be given by

$$V = E \times \text{(Distance between plates)}$$

$$5 \times 10^3 = \left(\frac{\rho_{S0}}{\varepsilon_0}\right) \times (0.5 \times 10^{-2})$$

Therefore the surface charge density is

$$\rho_{S0} = \frac{(8.85 \times 10^{-12}) \times (5 \times 10^3)}{(0.5 \times 10^{-2})} = 8.85\,\mu\text{C}$$

SOL 3.2.31 Correct answer is 1.10 .

Since, the wire is coated with aluminum So,the configuration can be treated as the two resistance connected in parallel and therefore, the field potential will be same across both the material or we can say that the field intensity will be same inside both material.

i.e. $E_{st} = E_{al}$

where $E_{st} \to$ Field intensity in steel

$E_{al} \to$ Field intensity in aluminum.

or, $\frac{J_{st}}{\sigma_{st}} = \frac{J_{al}}{\sigma_{al}}$

where $J_{st} \to$ current density in steel

$J_{al} \to$ current density in aluminum

$\sigma_{st} \to$ conductivity of steel

$\sigma_{al} \to$ conductivity of aluminum

So, we get, $\frac{J_{st}}{J_{al}} = \frac{\sigma_{st}}{\sigma_{al}} = \frac{2 \times 10^6}{3.8 \times 10^7} = \frac{1}{19}$

$$J_{al} = 19J_{st} \qquad \text{...(1)}$$

Now, the total current through the wire is given as,

$$I = J_{st}(\pi a^2) + J_{al}(\pi b^2 - \pi a^2)$$

where $a \to$ cross sectional radius of inner surface (steel wire)

$b \to$ cross sectional radius of outer surface (with coating)

Since, thickness of coating is

$$t = 2 \times 10^{-3}$$

So, $b = a + t = (2 \times 10^{-3}) + (2 \times 10^{-3}) = (4 \times 10^{-3})$

Therefore, we get,

$$80 = J_{st}\pi(4 \times 10^{-6}) + J_{al}[\pi(16 \times 10^{-6}) - \pi(4 \times 10^{-6})]$$

or, $80 = J_{st}\pi(4 \times 10^{-6}) + 19J_{st}[\pi(12 \times 10^{-6})]$ (from eq.(1))

So, $J_{st} = \frac{80}{232\pi \times 10^{-6}} = 1.10 \times 10^5 \text{ A/m}^2$

SOL 3.2.32 Correct answer is 67.8 .

As calculated in previous question the electric field between the two dielectrics having surface charge densities ρ_S and $-\rho_S$ is

$$E = \frac{\rho_S}{\varepsilon}$$

where ε is the permittivity of the medium between the sheets.

So electric field in slab 1 is $E_1 = \frac{\rho_S}{\varepsilon} = \frac{\rho_S}{2\varepsilon_0}$

and electric field in slab 2 is $E_2 = \frac{\rho_S}{\varepsilon} = \frac{\rho_S}{4\varepsilon_0}$

Since the electric field between the sheets is uniform so the potential difference between the plates will be

$$V = \sum E \times \text{(distance)}$$

$$= E_1(1\text{ m}) + E_2(2\text{ m})$$

$$= \frac{\rho_S}{2\varepsilon_0}(1) + \frac{\rho_S}{4\varepsilon_0}(2) = \frac{\rho_s}{\varepsilon_0}$$

$$= \frac{0.6 \times 10^{-9}}{8.85 \times 10^{-12}} = 67.8 \text{ Volt}$$

****

SOLUTIONS 3.3

SOL 3.3.1 Option (B) is correct.
According to boundary condition the tangential components of electric field are uniform

i.e. $$\mathbf{E}_{1t} = \mathbf{E}_{2t} = \mathbf{E}_{3t} \qquad \text{...(1)}$$

but the normal component of electric fields are non uniform and defined as

$$\varepsilon_1 \mathbf{E}_{1n} = \varepsilon_2 \mathbf{E}_{2n} = \varepsilon_3 \mathbf{E}_{3n}$$

Since $$\varepsilon_1 = \varepsilon_3 \qquad \text{(Given)}$$

So, $$\mathbf{E}_{m} = \mathbf{E}_{3n} \neq \mathbf{E}_{2n} \qquad \text{...(2)}$$

and as the net electric field is given by

$$\mathbf{E} = \mathbf{E}_t + \mathbf{E}_n \text{ (sum of tangential and normal component)}$$

Therefore by combining the results of eq (1) and (2) we get

$$\mathbf{E}_1 = \mathbf{E}_3 \neq \mathbf{E}_2$$

SOL 3.3.2 Option (A) is correct.

SOL 3.3.3 Option (C) is correct.

SOL 3.3.4 Option (D) is correct.

SOL 3.3.5 Option (A) is correct.

SOL 3.3.6 Option (A) is correct.

SOL 3.3.7 Option (D) is correct.

SOL 3.3.8 Option (B) is correct.

SOL 3.3.9 Option (C) is correct.

SOL 3.3.10 Option (A) is correct.

SOL 3.3.11 Option (B) is correct.

SOL 3.3.12 Option (C) is correct.

SOL 3.3.13 Option (A) is correct.

SOL 3.3.14 Option (A) is correct.

SOL 3.3.15 Option (A) is correct.

SOLUTIONS 3.4

SOL 3.4.1 Option (D) is correct.
The capacitance of a parallel plate capacitor is defined as

$$C = \frac{\varepsilon_o A}{d} = \frac{8.85 \times 10^{-12} \times 10^{-4}}{10^{-3}} = 8.85 \times 10^{-13}$$

The charge stored on the capacitor is

$$Q = CV$$
$$= 8.85 \times 10^{-13} = 4.427 \times 10^{-13}$$

Therefore, the displacement current in one cycle

$$I = \frac{Q}{T} = fQ$$
$$= 4.427 \times 10^{-13} \times 3.6 \times 10^9 = 1.59 \text{ mA} \qquad (f = 3.6\text{ GHz})$$

SOL 3.4.2 Option (C) is correct.
The electric field of the EM wave in medium 1 is given as

$$\mathbf{E}_1 = 2\mathbf{a}_x - 3\mathbf{a}_y + 1\mathbf{a}_z$$

Since the interface lies in the $x = 0$ plane so, the tangential and normal components of the field intensity in medium 1 are

$$\mathbf{E}_{1t} = -3\mathbf{a}_y + \mathbf{a}_y \text{ and } \mathbf{E}_{1n} = 2\mathbf{a}_x$$

From the boundary condition, tangential component of electric field is uniform. So, we get the tangential component of the field intensity in medium 2 as

$$\mathbf{E}_{2t} = \mathbf{E}_{1t} = -3\mathbf{a}_y + \mathbf{a}_y$$

Again from the boundary condition the for normal component of electric flux density are uniform

i.e. $$\mathbf{D}_{1n} = \mathbf{D}_{2n}$$

or $$\varepsilon_1 \mathbf{E}_{1n} = \varepsilon_2 \mathbf{E}_{2n}$$

So, we get

$$1.5\varepsilon_o 2\mathbf{a}_x = 2.5\varepsilon_o \mathbf{E}_{2n}$$

or $$\mathbf{E}_{2n} = \frac{3}{2.5}\mathbf{a}_x = 1.2\mathbf{a}_x$$

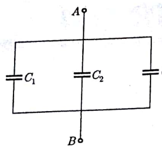
Thus, the net electric field intensity in the medium 2 is

$$\mathbf{E}_2 = \mathbf{E}_{2t} + \mathbf{E}_{2n} = -3\mathbf{a}_y + \mathbf{a}_z + 1.2\mathbf{a}_x$$

SOL 3.4.3 Option (D) is correct.
The surface charge density on a conductor is equal to the electric flux density at its boundary.

i.e. $$\sigma = D = \varepsilon E = 80\varepsilon_0 E \qquad (\varepsilon = 80\varepsilon_o)$$
$$= 80 \times 8.854 \times 10^{-12} \times 2 = 1.41 \times 10^{-9} \text{ C/m}^2$$

SOL 3.4.4 Option (B) is correct.
The configuration shown in the figure can be considered as the three capacitors connected in parallel as shown below



Now, consider the distance between the two plate is d and the total surface area of the plates is S. So, for the three individual capacitors the surface area is $S/3$ and the separation is d. Therefore, we get,

$$C_1 = \frac{\varepsilon_0\varepsilon_1(S/3)}{d}$$

$$C_2 = \frac{\varepsilon_0\varepsilon_2(S/3)}{d}$$

$$C_3 = \frac{\varepsilon_0\varepsilon_3(S/3)}{d}$$

Since, the three capacitance are in parallel So, the equivalent capacitance is

$$C_{eq} = C_1 + C_2 + C_3$$

$$= \frac{\varepsilon_0\varepsilon_1(S/3)}{d} + \frac{\varepsilon_0\varepsilon_2(S/3)}{d} + \frac{\varepsilon_0\varepsilon_3(S/3)}{d}$$

$$= \left(\frac{\varepsilon_0 S}{d}\right)\left(\frac{\varepsilon_1+\varepsilon_2+\varepsilon_3}{3}\right) = \left(\frac{\varepsilon_1+\varepsilon_2+\varepsilon_3}{3}\right)C \qquad \left(C = \frac{\varepsilon_0 S}{d}\right)$$

SOL 3.4.5 Option (D) is correct.

The electric field is equal to the negative gradient of electric potential at the point.

i.e. $$\boldsymbol{E} = -\nabla V$$

Given, electric potential

$$V = 4x + 2$$

So, $$\boldsymbol{E} = -4\boldsymbol{a}_x \text{ V/m}$$

SOL 3.4.6 Option (B) is correct.

The angle formed by the electric field vector in two mediums are related as

$$\frac{\tan\alpha_1}{\tan\alpha_2} = \frac{\varepsilon_1}{\varepsilon_2}$$

So, for the given field vectors we have,

$$\frac{\tan 60^\circ}{\tan\alpha_2} = \frac{3}{\sqrt{3}}$$

$$\tan\alpha_2 = 1$$

or $$\alpha_2 = \tan^{-1}(1) = 45^\circ$$

SOL 3.4.7 Option (A) is correct.

The tangential component of electric field on conducting surface is zero (since the surface conducts current) So, under static condition we have,

$$-\nabla V = \boldsymbol{E} = 0$$

or $$V = \text{constant}$$

i.e. the conducting surface is equipotential.

So, (A) and (R) both true and R is correct explanation of A.

SOL 3.4.8 Option (A) is correct.

Since, the electric field is incident normal to the slab. So, the electric field intensity($\boldsymbol{E}_i$) inside the slab is given as

$$\varepsilon\boldsymbol{E}_i = \varepsilon_0\boldsymbol{E}_0$$

$$\boldsymbol{E}_i = \frac{\varepsilon_0(6\boldsymbol{a}_z)}{3\varepsilon_0} = 2\boldsymbol{a}_z$$

Therefore, the polarization inside the slab is given as

$$\boldsymbol{P}_i = \varepsilon_0 X_e \boldsymbol{E}_i$$

where X_e is electric susceptibility defined as $X_e = \varepsilon_r - 1$. So, we have

$$\boldsymbol{P}_i = \varepsilon_0(3-1)\boldsymbol{E}_i = 4\varepsilon_0\boldsymbol{a}_z$$

SOL 3.4.9 Option (A) is correct.

Due to both the line charge and concentric circular conductors, the equipotential surfaces are circular (cylinder) i.e. concentric equipotential lines.

The flux lines due to both the configurations (line charge and concentric circular conductors) are in straight radial direction.

SOL 3.4.10 Option (A) is correct.

Capacitance of 1st plate is given as

$$C_1 = \frac{\varepsilon S_1}{d} = \frac{\varepsilon(1\times 1)}{d} = \frac{\varepsilon}{d}$$

The capacitance of 2nd plate is

$$C_2 = \frac{\varepsilon S_2}{d} = \frac{\varepsilon(2\times 2)}{d} = \frac{4\varepsilon}{d}$$

So, the ratio of capacitances is

$$\frac{C_2}{C_1} = 4$$

SOL 3.4.11 Option (D) is correct.

Consider the dielectric material with permittivity ε_1 is replaced by a dielectric material with permittivity ε_2.

The capacitance of parallel plate capacitor is defined as

$$C = \frac{\varepsilon S}{d}$$

i.e. the capacitance depends on the permittivity of the medium and so, due to the replacement of the material between the plates the capacitance changes.

Now, the charge is kept constant

i.e. $$Q_1 = Q_2$$

or, $$C_1V_1 = C_2V_2$$

So, due to the change in capacitance voltage on the capacitor changes and therefore the electric field intensity between the plates changes.

The stored energy in the capacitance is defined as

$$W = \frac{Q^2}{2C}$$

As total stored charge Q is kept constant while capacitance changes so, the stored energy in the capacitance also changes.

Thus, all the three given quantities changes due to the replacement of material between the plates.

SOL 3.4.12 Option (C) is correct.
According to continuity equation we have

$$\nabla \cdot J = -\frac{\partial \rho_v}{\partial t}$$

As for electrostatic field $\frac{\partial \rho_v}{\partial t} = 0$ so, we get

$$\nabla \cdot J = 0$$

SOL 3.4.13 Option (A) is correct.
Electrostatic fields only.

SOL 3.4.14 Option (C) is correct.
Surface or sheet resistivity is defined as resistance per unit surface area. So, the unit of surface resistivity is Ohm/sq. meter.

SOL 3.4.15 Option (B) is correct.
Since a conducting surface is equipotential so no electric field component exists tangential to the surface and therefore the electric field lines are normal to a conducting surface boundary.

SOL 3.4.16 Option (B) is correct.
Surface resistance of a metal is defined as

$$R_s \approx \sqrt{\frac{\omega\mu}{2\sigma}} = \sqrt{\frac{2\pi f\mu}{2\sigma}}$$

So, as frequency (f) increases the surface resistance increases.

SOL 3.4.17 Option (A) is correct.
When we determine force using method of images then in this method, the conducting surface is being removed and an additional distribution of charge is being introduced symmetrical to the existing charge distribution.

SOL 3.4.18 Option (B) is correct.
The conducting surface is equipotential and since the potential at infinity is zero so, the potential every where on a conducting surface of infinite extent is zero.
Since the conducting surface is equipotential so displacement density on a conducting surface is normal to the surface.
So A and R both true but R is not correct explanation of A.

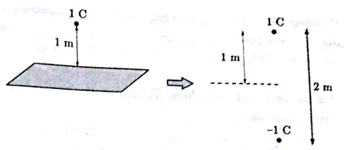
SOL 3.4.19 Option (A) is correct.
Capacitance, $C = 5\,\text{pF} = 5 \times 10^{-2}\,\text{F}$
Charge on capacitance,

$$Q = 0.1\,\mu\text{C} = 0.1 \times 10^{-6}\,\text{C}$$

The energy stored in the capacitor is defined as

$$W = \frac{Q^2}{2C} = \frac{(0.1 \times 10^{-6})^2}{2 \times 5 \times 10^{-12}} = 1\,\text{mJ}$$

SOL 3.4.20 Option (C) is correct.
Consider the charge of 1 C is placed near a grounded conducting plate at a distance of 1 m as shown in figure.

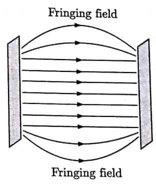


Using image of the charge we have one negative charge opposite side of the plate at the same distance as shown in the figure and the force between them is

$$F = \frac{(1)(-1)}{4\pi\varepsilon_0 r^2} = \frac{-1}{4\pi\varepsilon_0 (2)^2} = \frac{-1}{16\pi\varepsilon_0}\,\text{N}$$

Negative sign indicates that the direction of force is attractive.

SOL 3.4.21 Option (B) is correct.
Fringing field has been shown below in the figure



The capacitance of a parallel plate capacitor is given as

$$C = \varepsilon_0 \varepsilon_r \frac{A}{d}$$

It is valid only when the fringing is not taken into account. Now, the fringing field can be ignored only when the separation d between the plates is much less than the plate dimensions. So, for the fringing field taken under consideration, A/d is tending towards infinity.

SOL 3.4.22 Option (B) is correct.
The capacitance of a solid infinitely conducting sphere is defined as

$$C = 4\pi\varepsilon_0 R.$$

where R is radius of the solid sphere.

SOL 3.4.23 Option (A) is correct.
The electric potential produced by a point charge Q at the a distance r from it is defined as

$$V = \frac{Q}{4\pi\varepsilon r}$$

where ε is permittivity of the medium. So, the electric potential produced by the point charge $+10\,\mu\text{C}$ at the centre of the sphere is

$$V = \frac{Q}{4\pi\varepsilon_o r} = \frac{10 \times 10^{-6}}{4\pi\varepsilon_0 (5 \times 10^{-2})} \qquad (\text{Given } r = 5\,\text{cm})$$

As the surface of sphere is grounded so, the total voltage on the spherical capacitor will be equal to the potential at its centre as calculated above. Now, the capacitance of the isolated sphere is defined as

$$C = 4\pi\varepsilon a$$

where a is the radius of the sphere. Therefore, the induced charge stored on the sphere is given as

$$Q_{ind} = CV = (4\pi\varepsilon_0 a)\frac{(10\times 10^{-6})}{4\pi\varepsilon_0(5\times 10^{-2})}$$

$$= \frac{(2\times 10^{-2})\times(10\times 10^{-6})}{(5\times 10^{-2})} \qquad \text{(Given } a = 2\text{ cm)}$$

$$= 4\times 10^{-6}\,\text{C} = 4\,\mu\text{C}$$

SOL 3.4.24 Option (C) is correct.

Given electric field $\quad E = E_0\sin\omega t$

The conduction current is defined as

$$J_c = \sigma(E) = \sigma E_0\sin\omega t$$

where σ is conductivity and E is electric field intensity. and the displacement current density is

$$J_d = \frac{\partial D}{\partial t} = \varepsilon\frac{\partial E}{\partial t}$$

$$= \varepsilon E_0(\omega\cos\omega t) = \varepsilon\omega E_0\sin\left(\frac{\pi}{2}-\omega t\right)$$

So the phase difference between J_c and J_d is $90°$.

SOL 3.4.25 Option (B) is correct.

Method of images are used for the charge distribution at a distance from the grounded plane conductor.

SOL 3.4.26 Option (C) is correct.

Given

Capacitance of condenser, $\quad C = 0.005\,\mu\text{F} = 5\times 10^{-9}\,\text{F}$

Supply voltage, $\quad V = 500\,\text{V}$

Permittivity of oil, $\quad \varepsilon_r = 2.5 \quad$ (immersed oil)

So, the energy stored in condenser before immersion is

$$W = \frac{1}{2}CV^2 = \frac{1}{2}\times 5\times 10^{-9}\times(500)^2$$

$$= 6.25\times 10^{-4}\,\text{J}$$

After immersing the condenser in oil the capacitance changes while the total charge remains same.

i.e. $\quad Q_{after\ immersion} = Q_{before\ immersion} = (5\times 10^{-9})(500)$

$$= 2.5\times 10^{-6}\,\text{Coulomb}$$

The capacitance of the condenser after immersion is

$$C_{after\ immersion} = \varepsilon_r C$$

$$= (2.5)(5\times 10^{-9}) = 1.25\times 10^{-8}\,\text{F}$$

Therefore, the stored energy in the condenser immersed in oil is

$$W = \frac{Q^2}{2C_{(after\ immersion)}} = \frac{(2.5\times 10^{-6})}{2(1.25\times 10^{-8})} = 2.5\times 10^{-4}\,\text{J}$$

SOL 3.4.27 Option (B) is correct.

Given,

Capacitance, $\quad C = 3\,\mu\text{F} = 3\times 10^{-6}\,\text{F}$

Current, $\quad I = 2\,\mu\text{A} = 2\times 10^{-6}\,\text{A}$

Charging time, $\quad t = 6\,\text{sec}$

So, the total charge stored on capacitor is

$$Q = \text{Charge transferred}$$

$$= It = (2\times 10^{-6})(6)$$

Therefore, the voltage across the charged capacitor is

$$V = \frac{Q}{C} = \frac{(2\times 10^{-6})(6)}{3\times 10^{-6}}$$

$$= 4\,\text{Volt}$$

SOL 3.4.28 Option (C) is correct.

Given, the total charge on capacitor $= V$

(1) Electric field between the plates will be given as

$$E = -\nabla V$$

which is independent of permittivity of the material filled in capacitor so E will be constant.

(2) The displacement flux density inside the capacitor is given as

$$D = \varepsilon E$$

As E is constant while permittivity is doubled so D will also be doubled.

(3) The charge stored on the plates is given as

$$Q = CV$$

where V is constant but capacitance C will be doubled as it is directly proportional to the permittivity given as

$$C = \varepsilon\frac{S}{d}$$

So, the charge on plates will be get doubled.

(4) As discussed already, the capacitance will get doubled.

Therefore, the statements 2 and 4 are correct.

SOL 3.4.29 Option (B) is correct.

Since, resistance doesn't store any energy. So, the energy stored in the coil is only due to inductance and given as

$$W = \frac{1}{2}LI^2$$

where L is the inductance and I is the current flowing in the circuit. At the fully charged condition, inductor is short circuit and therefore, current through the circuit is

$$I = \frac{V}{R} = \frac{50}{5} = 10\,\text{A}$$

So, the energy stored in the field (in the inductor) is

$$W = \frac{1}{2}(0.4)(10)^2 = 20\,\text{Joules}$$

SOL 3.4.30 Option (C) is correct.

The normal component of electric flux density (D) across a dielectric-dielectric boundary is given as

$$D_{1n} - D_{2n} = \rho_s$$

where ρ_s is the surface charge density at the interface.

So, the normal components of electric flux density across a dielectric-dielectric boundary is dependent on the magnitude of surface charge density.

SOL 3.4.31 Option (C) is correct.
Statement 1, 2 and 4 are correct while statement 3 is incorrect.

SOL 3.4.32 Option (B) is correct.
The capacitance of an insulated conducting sphere of radius R in vacuum is

$$C = 4\pi\varepsilon_0 R$$

SOL 3.4.33 Option (C) is correct.
Maximum withstand voltage is the value that the dielectric between capacitor plates can toterate without any electrical breakdown. Maximum withstand voltage is larger for any dielectric material than that for free space (air). Since the maximum withstand voltage across the capacitor filled with air is V so the maximum withstand withstand voltage for the composite capacitor will be also V as the capacitors are connected in parallel. Now, the capacitance before filling the dielectric is

$$C = \frac{\varepsilon_0 A}{d}$$

and after filling the dielectric

$$C_{eq} = C_1 + C_2$$

$$= 4\varepsilon_0\frac{A/2}{d} + \varepsilon_0\frac{A/2}{d} = \frac{5\varepsilon_0 A}{2d}$$

So, the stored charge Q_1 after filling dielectric is determined as below

$$\frac{Q}{Q_1} = \frac{C}{C_{eq}} \qquad \text{(Since voltage is constant)}$$

or,

$$Q_1 = \frac{Q\frac{5\varepsilon_0 A}{2d}}{\frac{\varepsilon_0 A}{d}} = 2.5Q$$

Therefore, the maximum withstand voltage of the capacitor is V and charge is $2.5Q$.

SOL 3.4.34 Option (D) is correct.
Since, the potential on both sides of plate will be same (Consider the potential is V). So, the charge densities on the two sides is determined as below :

$$\rho_2 = C'_2 V$$

and

$$\rho_1 = C'_1 V$$

where C'_2 and C'_1 are the capacitance per unit area of the capacitance formed by the region d_1 and d_2.

Therefore,

$$\frac{\rho_1}{\rho_2} = \frac{C'_1 V}{C'_2 V} = \frac{\frac{\varepsilon_0}{d_1}V}{\frac{\varepsilon_0}{d_2}V} = \frac{d_2}{d_1}$$

SOL 3.4.35 Option (C) is correct.
Electric flux density in a polarized dielectric is defined as

$$\boldsymbol{D} = \boldsymbol{P} + \varepsilon_0\boldsymbol{E}$$

SOL 3.4.36 Option (A) is correct.
Image theory is applicable only for static charge distribution (electrostatic field).

SOL 3.4.37 Option (D) is correct.
The equivalent capacitance of series connected capacitance has the value less than the smallest capacitance here the smallest capacitance is C_6 so the total capacitance is less then C_6

i.e. $$C_{eq} < C_6$$

or $$C_{eq} \approx C_6$$

SOL 3.4.38 Option (C) is correct.
Given, the electric field intensity in medium 1.

$$\boldsymbol{E}_1 = 5\boldsymbol{a}_x - 2\boldsymbol{a}_y + 3\boldsymbol{a}_z$$

Since, the medium interface lies in plane $z = 0$.
So, we get the field components as

$$\boldsymbol{E}_{1t} = 5\boldsymbol{a}_x - 2\boldsymbol{a}_y$$

and

$$\boldsymbol{E}_{1n} = 3\boldsymbol{a}_z$$

Now, From the boundary condition for electric field we have

$$\boldsymbol{E}_{1t} = \boldsymbol{E}_{2t}$$

$$\varepsilon_1\boldsymbol{E}_{1n} = \varepsilon_2\boldsymbol{E}_{2n}$$

So, the field components in medium 2 are

$$\boldsymbol{E}_{2t} = \boldsymbol{E}_{1t} = 5\boldsymbol{a}_x - 2\boldsymbol{a}_y$$

$$\boldsymbol{E}_{2n} = \frac{\varepsilon_1}{\varepsilon_2}\boldsymbol{E}_{1n} = 6\boldsymbol{a}_z$$

Therefore, the net electric field intensity in medium 2 is given as

$$\boldsymbol{E}_2 = \boldsymbol{E}_{2t} + \boldsymbol{E}_{2n} = 5\boldsymbol{a}_x - 2\boldsymbol{a}_y + 6\boldsymbol{a}_z$$

So, the z-component of the field intensity in medium 2 is

$$\boldsymbol{E}_{2z} = 6\boldsymbol{a}_z$$

SOL 3.4.39 Option (A) is correct.
Electric flux density, $D = 1\,\text{C/m}^2$
Relative permittivity, $\varepsilon_r = 5$
Since, the normal component of flux density is uniform at the boundary surface of two medium so, the flux density inside the slab is

$$D = 1\,\text{C/m}^2$$

Therefore, the polarization of the slab is given as

$$P = \left(\frac{\varepsilon_r - 1}{\varepsilon_r}\right)D = \frac{4}{5}\times 1 = 0.8$$

SOL 3.4.40 Option (B) is correct.
The capacitance of a isolated spherical capacitor of radius R is defined as

$$C = 4\pi\varepsilon_0 R$$

Since the two spheres are identical and separated by a distance very much larger then R. So, it can be assumed as the series combination of capacitances. Therefore, the net capacitance between two spheres is given as

i.e. $$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{(4\pi\varepsilon_0 R)(4\pi\varepsilon_0 R)}{4\pi\varepsilon_0 R + 4\pi\varepsilon_0 R} = 2\pi\varepsilon_0 R$$

SOL 3.4.41 Option (C) is correct.
For steady current in an arbitrary conductor the current density is given as

$$\boldsymbol{J} = \frac{I}{A}$$

and since I is constant So, J is constant and therefore $\nabla \times J = 0$
So, the current density is solenoidal. i.e. Assertion (A) is true.
The reciprocal of resistivity is conductivity. i.e. Reason (R) is false.

SOL 3.4.42 Option (A) is correct.
Since, the displacement current density is defined as

$$J_d = \frac{\partial \boldsymbol{D}}{\partial t}$$

So, it is generated by a change in electric flux and therefore the displacement current has only A.C. components as derivative of D.C. components is zero.
i.e. A and R both are true and R is correct explanation of A.

SOL 3.4.43 Option (B) is correct.
Dielectric constant, $\varepsilon_r = 5$
Flux density, $D = 2\,\text{C/m}^2$
So, the polarization of the medium is given as

$$P = \left(\frac{\varepsilon_r - 1}{\varepsilon_r}\right) D = \frac{4}{5} \times 2 = 1.6\,\text{C/m}^2$$

SOL 3.4.44 Option (C) is correct.
The ohm's law in point form in field theory is expressed as below

$$V = RI \qquad \text{(For constant voltage)}$$

$$\boldsymbol{E} l = \frac{\rho l}{A} \boldsymbol{J} A$$

where l is length integral and A is the cross sectional area. So, we get

$$\boldsymbol{E} = \rho \boldsymbol{J}$$

$$\boldsymbol{E} = \frac{\boldsymbol{J}}{\sigma}$$

i.e. $\boldsymbol{J} = \sigma \boldsymbol{E}$

SOL 3.4.45 Option (C) is correct.
Displacement current density is defined as

$$\boldsymbol{J}_d = \varepsilon \frac{\partial \boldsymbol{E}}{\partial t}$$

and the conduction current density is defined as

$$\boldsymbol{J}_c = \sigma \boldsymbol{E}$$

for a dielectric ε must be larger while conductivity must tend to zero.
So, we get $J_d >> J_c$
i.e. displacement current is much greater than conduction current.

SOL 3.4.46 Option (B) is correct.
Conduction current, $I_c = 1\,\text{A}$
Operating frequency, $f = 50\,\text{Hz}$
Medium permittivity, $\varepsilon = \varepsilon_0$
Permeability $\mu = \mu_0$
Conductivity, $\sigma = 5.8 \times 10\,\text{mho/m}$
The ratio of conduction current density to the displacement current density is

$$\frac{J_c}{J_d} = \frac{\sigma}{\omega\varepsilon}$$

or, $$\frac{I_c/A}{I_d/A} = \frac{\sigma}{\omega\varepsilon} \qquad (A \text{ is cross sectional area})$$

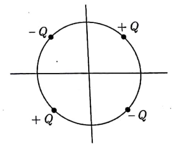
$$I_d = \frac{\omega\varepsilon}{\sigma} I_c = \frac{2\pi \times 50 \times \varepsilon_0}{5.8 \times 10}(1) = 4.8 \times 10^{-11}\,\text{A}$$

SOL 3.4.47 Option (D) is correct.
When there is no charge in the interior of a conductor, the electric field intensity is zero according to Gauss's law the total outward flux through a closed surface is equal to the charge enclosed.
Now if any charge is introduced inside a closed conducting surface then an electric field will be setup and the field exerting a force on the charges and making them move to the conducting surface. So all the charges inside a conductor is distributed over its surface. Therefore the outward flux through any closed surface constructed inside the conductor must vanish.
A is false but R is true.

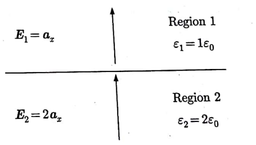
SOL 3.4.48 Option (D) is correct.
When the method of images is used for a system consisting of a point charge between two semi infinite conducting planes inclined at an angle ϕ, the no. of images is given by

$$N = \left(\frac{360^\circ}{\phi} - 1\right)$$

Here the angle between conducting planes is $\phi = 90^\circ$.
So, $N = 3$
and since all the images lie $\boldsymbol{a}_n$ a circle so we have the image charges as shown in figure.



SOL 3.4.49 Option (B) is correct.
Consider the two dielectric regions as shown below.



Since the field is normal to the interface So, the normal components of the fields are,

$$E_{1n} = 1 \text{ and } E_{2n} = 2$$

From boundary condition we have

$$\varepsilon_1 E_{1n} - \varepsilon_2 E_{2n} = \rho_s$$

(where ρ_s is surface charge density on the interface).

$$(\varepsilon_0)(1) - (2\varepsilon_0)(2) = \rho_s$$

$$\rho_s = -3\varepsilon_0$$

SOL 3.4.50 Option (B) is correct.

The stress is called the force per unit area which is directly proportional to the electric field intensity and electric field intensity is inversely proportional to the permittivity of dielectric material.

i.e. $$E \propto \frac{1}{\varepsilon}$$

So, ratio of stress is $$\frac{E_1}{E_2} = \frac{1/\varepsilon_0}{1/\varepsilon} = \frac{1/\varepsilon_0}{1/5\varepsilon_0} = 5$$

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