Electromagnetic Engineering (EC 21006)
T U T O R I A L - V

ELECTROSTATICS

1. The line y=1, z = -3, carries charge 30nC/m while the plane x=1 carries charge 20nC$/m^2$. Find **E** at the origin.
2. For a spherical charge distribution

$$\rho_v = \begin{cases} \rho_0(a^2 - r^2), & r < a \\ 0, & r > a \end{cases}$$

(a) Find **E** and V for $r \geq a$ (b) Find **E** and V for $r \leq a$

(c) Find the total charge (d) Show that E is maximum when r = 0.145a.

3. Uniform charge density of 2C/m^3 exists in the volume $2 \leq x \leq 4$ m .Use Gauss' law to find

   D in all regions.
4. Volume charge density at a place varies as

$$\rho_v = \frac{\rho_0\, Sin(\pi r)}{r^2}$$ where ρ_0 is a constant.

   Find the surfaces on which electric field is zero.
5. Let the volume charge density is given by

$$\rho_v = 0, \quad \rho < 1\ mm$$
$$= 2\, Sin(2000\,\pi\rho) \quad nC/m^2,\ 1\ mm < \rho < 1.5\ mm$$
$$= 0, \quad \rho > 1.5\ mm$$

   Find the electric flux density everywhere.
6. Spherical surfaces at r = 2, 4 and 6 m carry uniform surface charge densities of $20\ nC/m^2$, $-4\ nC/m^2$ and ρ_{so} respectively.

   a) Find $\vec{D}$ at r =1, 3 and 5 m.

   b) Determine ρ_{so} such that $\vec{D} = 0$ at r = 7 m.
7. Suppose that an electric flux density in cylindrical coordinates is of the form $\vec{D} = D_\rho \hat{a}_\rho$. Describe the dependence of charge density ρ_v on coordinates ρ, ϕ and z if

a) $D_\rho = f(\phi, z)$

b) $D_\rho = \left(\frac{1}{\rho}\right) f(\phi, z)$

c) $D_\rho = f(\rho)$.

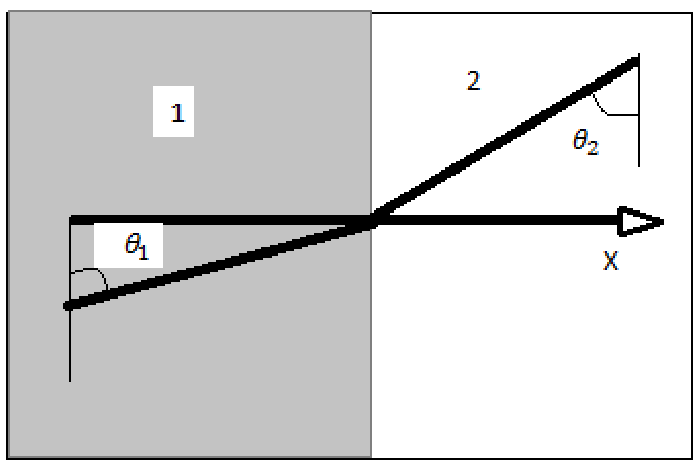
8. (a) A point charge Q lies at the origin. Show that div $\vec{D}$ is zero everywhere except at the origin.

(b) Replace the point charge with a uniform volume charge density ρ_{v0} for 0 < r < a. Relate ρ_{v0} to Q and a so that the total charge is the same. Find $\nabla \cdot \vec{D}$ everywhere.

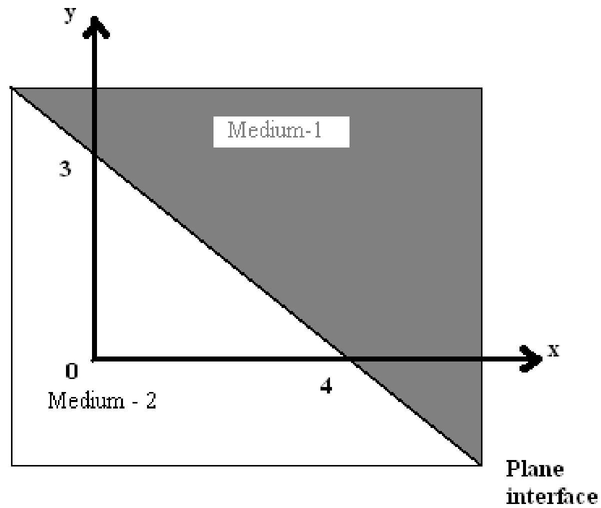
9. Find the polarization **P** in a dielectric material with $\epsilon_r = 2.8$ if $\boldsymbol{D} = 3.0 \times 10^{-7}\mathbf{a}\ \ C/m^2$.

10. Determine the value of **E** in a material for which electric susceptibility is 3.5 and $\boldsymbol{P} = 2.3 \times 10^{-7}\mathbf{a}\ \ C/m^2$.

11. Region 1, defined by $x < 0$, is free space, while region 2, $x > 0$, is a dielectric material for which $\epsilon_{r2} = 2.4$ as in the figure below . Given $\boldsymbol{D}_1 = 3\boldsymbol{a}_x - 4\boldsymbol{a}_y + 6\boldsymbol{a}_z\ C/m^2$. Find $\boldsymbol{E}_2$ and the angles $\theta_1 and\ \theta_2$.



12. A plane boundary of infinite extent in the z direction passes through the points (4,0,0) and (0,3,0) as indicated in the figure. The electric field intensity in medium 1 (ϵ_r =2.5) is $\boldsymbol{E} = 25\boldsymbol{a}_x + 50\boldsymbol{a}_y + 25\boldsymbol{a}_z\ V/m$. Determine the $\boldsymbol{E}$ field in medium 2 (ϵ_r =5).



13. If medium 2 in the above problem is a conductor, and the y component of the electric field intensity in medium 1 is 50 V/m, what are the other components of the $\boldsymbol{E}$ field? What is the free surface charge density on the conductor?

14. A conducting sphere of radius 'a' is concentrically placed in a dielectric (ϵ_1) sphere of radius $\boldsymbol{r}_1$ which in turn is embedded in a dielectric (ϵ_2) sphere of radius $\boldsymbol{r}_2$. Determine $\boldsymbol{D}$ and $\boldsymbol{P}$ in all regions if a charge of 5nC is placed on the conducting sphere. Find energy and the volume and surface bound charge densities.

15. In spherical co-ordinates $\vec{E} = \frac{2r}{r^2+a^2}\,\hat{a}_r\ V/m$.

    Find the potential at any point, using the reference

    a) $V = 0$ at infinity, b) $V = 0$ at $r = 0$, c) $V = 100$V at $r = a$

16. Determine the potential distribution in a coaxial cable when the inner conductor of radius 'a' is at a potential of $\boldsymbol{V}_0$ and the outer conductor of radius 'b' is grounded. The space between the conductors is filled with two concentric dielectrics. The permittivity of the inner dielectric is ϵ_1 and the same for the outer dielectric is ϵ_2. The dielectric interface is at radius 'c'. Determine

    a) the potential distribution,

    b) the $\boldsymbol{D}$ and $\boldsymbol{E}$ fields in each region,

    c) the surface charge density on the inner conductor, and

    d) the capacitance per unit length of the cable.

17. A dipole for which $\vec{p} = 10\,\epsilon_0 \hat{a}_z\ C \cdot m$ is located at the origin. What is the equation of the surface on which $E_z = 0$ but $|\vec{E}| \neq 0$?