SOLUTIONS 7.1

SOL 7.1.1 Option (C) is correct.

From the property of phasor, we know that the instantaneous electric field is the real part of $\{\boldsymbol{E}_s e^{j\omega t}\}$.

i.e.
$$\boldsymbol{E}(x,t) = \text{Re}\{\boldsymbol{E}_s e^{j\omega t}\} \qquad (1)$$

where $\boldsymbol{E}_s$ is the phasor form of electric field.

Given the electric field intensity in time domain,

$$\begin{aligned}\boldsymbol{E}(x,t) &= E_0 e^{-\alpha x}\sin(\omega t-\beta x)\boldsymbol{a}_y \\ &= E_0 e^{-\alpha x}\left[\frac{e^{j(\omega t-\beta x)}-e^{-j(\omega t-\beta x)}}{2j}\right]\boldsymbol{a}_y \\ &= -\frac{jE_0}{2}e^{-\alpha x}e^{j(\omega t-\beta x)}\boldsymbol{a}_y + C.C.\end{aligned}$$

where $C.C.$ is complex conjugate of the 1[st] part.

So, using the property of complex conjugates we get

$$\begin{aligned}\boldsymbol{E}(x,t) &= 2\,\text{Re}\left\{-\frac{jE_0}{2}e^{-\alpha x}e^{j(\omega t-\beta x)}\boldsymbol{a}_y\right\} \\ &= \text{Re}\{-jE_0 e^{-\alpha x}e^{-j\beta x}e^{j\omega t}\boldsymbol{a}_y\}\end{aligned}$$

Comparing it with equation (1), we get

$$\boldsymbol{E}_s = -jE_0 e^{-(\alpha+j\beta)x}\boldsymbol{a}_y \text{ V/m}$$

SOL 7.1.2 Option (A) is correct.

Wave equation for a plane wave propagating in $+\boldsymbol{a}_z$ direction is given as

$$\frac{\partial^2 f}{\partial t^2} - v_p^2\frac{\partial^2 f}{\partial z^2} = 0 \qquad \text{where } v_p \text{ is the velocity of wave propagation}$$

Now from Assertion (A) the electric field is

$$\boldsymbol{E} = E_0\sin(z)\cos(ct)\boldsymbol{a}_x$$

It represents the electric field of a plane wave if it satisfies the wave equation

i.e.
$$\frac{\partial^2 \boldsymbol{E}}{\partial t^2} - c^2\frac{\partial^2 \boldsymbol{E}}{\partial z^2} = 0 \qquad \text{where } c \text{ is velocity of wave in free space}$$

From the given expression of field intensity we have

$$\frac{\partial \boldsymbol{E}}{\partial t} = -cE_0\sin(z)\sin(ct)$$

or,
$$\frac{\partial^2 \boldsymbol{E}}{\partial t^2} = -c^2E_0\sin(z)\cos(ct)$$

and
$$\frac{\partial \boldsymbol{E}}{\partial z} = E_0\cos(z)\cos(ct)$$

or,
$$\frac{\partial^2 \boldsymbol{E}}{\partial z^2} = -E_0\sin(z)\cos(ct)$$

Thus, we get,
$$\frac{\partial^2 \boldsymbol{E}}{\partial t^2} - c^2\frac{\partial^2 \boldsymbol{E}}{\partial z^2} = 0$$

Since, the electric field $\boldsymbol{E}$ satisfies the wave equation so it represents the field of a plane wave.

Therefore, A and R both are true and R is the correct explanation of A.

SOL 7.1.3 Option (B) is correct.

Given the magnetic field intensity in free space is

$$\boldsymbol{H} = 0.1\cos(10^9 t-\beta y)\boldsymbol{a}_z \text{ A/m} \qquad (1)$$

The general equation of magnetic field intensity of the EM wave propagating in $\boldsymbol{a}_y$ direction is given as

$$\boldsymbol{H} = H_0\cos(\omega t-\beta y)\boldsymbol{a}_z \text{ A/m} \qquad (2)$$

Comparing equations (1) and (2) we get,

direction of wave propagation, $\quad \boldsymbol{a}_k = \boldsymbol{a}_y$

and angular frequency, $\quad \omega = 10^9 \text{ rad/sec}$

So, the phase constant of the wave is

$$\begin{aligned}\beta &= \frac{\omega}{c} = \frac{10^9}{3\times 10^8} \qquad (c \text{ is velocity of wave in free space}) \\ &= 3.33 \text{ rad/m}\end{aligned}$$

Now, electric field intensity in free space is defined as

$$\boldsymbol{E} = -\eta_0\boldsymbol{a}_k\times\boldsymbol{H}$$

where η_0 is intrinsic impedance in free space and $\boldsymbol{a}_k$ is direction of wave propagation.

So,
$$\begin{aligned}\boldsymbol{E} &= -377(\boldsymbol{a}_y)\times 0.1\cos(10^9 t-\beta y)\boldsymbol{a}_z \qquad (\eta_0 = 377\ \Omega) \\ &= -37.7\cos(10^9 t-3.33y)\boldsymbol{a}_x\end{aligned}$$

Therefore, electric field intensity of the wave at $y = 1$ cm at $t = 0.1$ ns is

$$\begin{aligned}\boldsymbol{E} &= -37.7\cos[(10^9)(10^{-10})-(3.33)(10^{-2})]\boldsymbol{a}_x \\ &= -37.6\boldsymbol{a}_x \text{ V/m}\end{aligned}$$

SOL 7.1.4 Option (D) is correct.

Given the instantaneous electric field in the free space is

$$\boldsymbol{E} = (5a_y-6a_x)\cos(\omega t-50z) \text{ V/m}$$

So, the phasor form of electric field intensity is

$$\boldsymbol{E}_s = (5\boldsymbol{a}_y-6\boldsymbol{a}_x)e^{-j50z} \text{ V/m}$$

The phasor form of magnetic field is given in the terms of electric field intensity as

$$\boldsymbol{H}_s = \frac{1}{\eta_0}(\boldsymbol{a}_k)\times(\boldsymbol{E})$$

where $\boldsymbol{a}_k$ is the unit vector in the direction of wave propagation and η_0 is the intrinsic impedance in free space.

So,
$$\begin{aligned}\boldsymbol{H}_s &= \frac{1}{\eta_0}(\boldsymbol{a}_z)\times(5\boldsymbol{a}_y-6\boldsymbol{a}_x)e^{-j50z} \text{ V/m} \qquad (\boldsymbol{a}_k = \boldsymbol{a}_z) \\ &= \frac{1}{\eta_0}(-5\boldsymbol{a}_x-6\boldsymbol{a}_y)e^{-j50z} \text{ V/m} \\ &= -\frac{1}{\eta_0}(5\boldsymbol{a}_x+6\boldsymbol{a}_y)e^{-j50z} \text{ V/m}\end{aligned}$$

SOL 7.1.5 Option (C) is correct.

For any electromagnetic wave propagating in a medium electric field leads magnetic field by an angle θ_n, where θ_n is the phase angle of intrinsic impedance given as

$$\tan 2\theta_n = \frac{\sigma}{\omega\varepsilon}$$

Now, for a perfect conductor

$$\sigma = \frac{1}{\rho} \approx \infty$$

i.e.
$$\tan 2\theta_n \approx \infty$$
$$2\theta_n = 90^\circ$$
$$\theta_n = 45^\circ$$

So, electric field leads magnetic field by 45° or in other words magnetic field lags electric field by 45°.

SOL 7.1.6 Option (A) is correct.

Given, the electric field intensity of the wave in phasor form
$$\boldsymbol{E}_s = (5\boldsymbol{a}_x + 10\boldsymbol{a}_z)e^{-j(4x-2z)}\ \text{V/m}$$
So we get the direction of wave propagation as
$$\boldsymbol{a}_k = \frac{4\boldsymbol{a}_x - 2\boldsymbol{a}_z}{|4\boldsymbol{a}_x - 2\boldsymbol{a}_z|} = \frac{4\boldsymbol{a}_x - 2\boldsymbol{a}_z}{\sqrt{20}} = \frac{2\boldsymbol{a}_x - \boldsymbol{a}_z}{\sqrt{5}}$$
Therefore, the phasor form of magnetic field intensity of the plane wave is given as
$$\boldsymbol{H}_s = \frac{1}{\eta_0}\boldsymbol{a}_k \times \boldsymbol{E}_s \qquad \text{where } \eta_0 \text{ is intrinsic impedance in free space}$$
$$= \frac{1}{120\pi}\left(\frac{2\boldsymbol{a}_x - \boldsymbol{a}_z}{\sqrt{5}}\right) \times (5\boldsymbol{a}_x + 10\boldsymbol{a}_z)e^{-j(4x-2z)}\ \text{V/m}$$
$$= -29.66e^{-j(4x-2z)}\ \text{mA/m}$$

SOL 7.1.7 Option (B) is correct.

The time average power density of the EM wave is given as
$$\mathcal{P}_{ave} = \frac{E^2}{2\eta_0}\boldsymbol{a}_k$$
where E is the magnitude of the electric field intensity of the wave, $\boldsymbol{a}_k$ is the unit vector in the direction of wave propagation and η_0 is the intrinsic impedance in the free space. So, we get
$$\mathcal{P}_{ave} = \frac{\sqrt{5^2+10^2}}{2(120\pi)}\left(\frac{4\boldsymbol{a}_x - 2\boldsymbol{a}_z}{\sqrt{20}}\right) = 148.9\boldsymbol{a}_x - 74.15\boldsymbol{a}_z\ \text{Watt/m}^2$$

SOL 7.1.8 Option (B) is correct.

As the given electric field vector has the amplitude
$$\boldsymbol{E}_0 = \left(-2\sqrt{3}\,\boldsymbol{a}_x + \sqrt{3}\,\boldsymbol{a}_y - \boldsymbol{a}_z\right)$$
So in the same direction the wave will be polarized.

SOL 7.1.9 Option (A) is correct.

From Maxwells' equation we have
$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$
Given $\qquad \boldsymbol{E} = 100\cos(\omega t - \beta z)\boldsymbol{a}_x$

or, $\qquad \nabla \times \boldsymbol{E} = 100\beta\sin(\omega t - \beta z)\boldsymbol{a}_y$

So, $\qquad -\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times \boldsymbol{E} = 100\beta\sin(\omega t - \beta z)\boldsymbol{a}_y$

Therefore the magnetic flux density vector is
$$\boldsymbol{B} = \int 100\beta\sin(\omega t - \beta z)\boldsymbol{a}_y\,dt$$
$$= \frac{100\beta}{\omega}\cos(\omega t - \beta z)\boldsymbol{a}_y$$
$$= \frac{100}{\sqrt{\mu_0\varepsilon_0}}\cos(\omega t - \beta z)\boldsymbol{a}_y \qquad \left(\beta = \frac{\omega}{\sqrt{\mu_0\varepsilon_0}}\right)$$
$$= 3 \times 10^{10}\cos(\omega t - \beta z)\boldsymbol{a}_y$$

SOL 7.1.10 Option (A) is correct.

Poynting vector in an EM field is defined as
$$\mathcal{P} = \boldsymbol{E} \times \boldsymbol{H}$$
where $\boldsymbol{E}$ is electric field intensity and $\boldsymbol{H}$ is the magnetic field intensity in the region.

Now, the electric field intensity in the region is given as
$$\boldsymbol{E} = 100\cos(\omega t - \beta z)\boldsymbol{a}_x$$
and as calculated in previous question the magnetic field intensity in the region is
$$\boldsymbol{B} = 3 \times 10^{10}\cos(\omega t - \beta z)\boldsymbol{a}_y$$
So, the poynting vector in the field is
$$\mathcal{P} = \boldsymbol{E} \times \frac{\boldsymbol{B}}{\mu_0} \qquad \left(\boldsymbol{H} = \frac{\boldsymbol{B}}{\mu_0}\right)$$
$$= [100\cos(\omega t - \beta z)\boldsymbol{a}_x] \times \frac{[3 \times 10^{10}\cos(\omega t - \beta z)\boldsymbol{a}_y]}{\mu_0}$$
$$= \frac{3 \times 10^{12}}{\mu_0}\cos^2(\omega t - \beta z)\boldsymbol{a}_z = 10^4\sqrt{\frac{\varepsilon_0}{\mu_0}}\cos^2(\omega t - \beta z)\boldsymbol{a}_z$$

SOL 7.1.11 Option (B) is correct.

Time average stored energy density in electric field is defined as
$$w_e = \frac{1}{4}\varepsilon_0\boldsymbol{E}_s \cdot \boldsymbol{E}_s^*$$
where $\boldsymbol{E}_s$ is the electric field intensity in phasor form and $\boldsymbol{E}_s^*$ is its conjugate.

Therefore, the average stored energy density in the region is
$$w_e = \frac{\varepsilon_0}{4}\left(5\sin\pi x e^{-j\pi/2}e^{-j\sqrt{3}\pi z}\boldsymbol{a}_y\right) \cdot \left(5\sin\pi x e^{+j\pi/2}e^{j\sqrt{3}\pi z}\boldsymbol{a}_y\right)$$
$$= \frac{25\varepsilon_0}{4}\sin^2\pi x$$

SOL 7.1.12 Option (B) is correct.

Given the electric field
$$\boldsymbol{E} = 10\sin\pi y\sin(6\pi \times 10^8 t - \sqrt{3}\,\pi x)\boldsymbol{a}_z\ \text{V/m}$$
In phasor form, $\quad \boldsymbol{E}_s = 10\sin\pi y e^{-j\pi/2}e^{-j\sqrt{\beta}\pi x}\boldsymbol{a}_z$

So, from Maxwell's equation, the magnetic flux density in the phasor form is given as
$$\boldsymbol{B}_s = \frac{1}{j\omega}(\nabla \times \boldsymbol{E}_s)$$
where $\omega = 6\pi \times 10^8$ as determined from the given expression of $\boldsymbol{E}$.

So, $\quad \boldsymbol{B}_s = -\frac{10\sqrt{3}\,\pi}{6\pi \times 10^8}(\sin\pi y)e^{-j\frac{\pi}{2}}e^{-j\sqrt{3}\pi x}\boldsymbol{a}_y + j\frac{10\pi}{6\pi \times 10^8}(\cos\pi y)e^{-j\frac{\pi}{2}}e^{-j\sqrt{3}\pi x}\boldsymbol{a}_y$

Therefore, the time average energy density stored in the magnetic field will be
$$w_m = \frac{1}{4\mu_0}(\boldsymbol{B}_s \cdot \boldsymbol{B}_s^*) \qquad \text{where } \boldsymbol{B}_s^* \text{ is the conjugate of } \boldsymbol{B}_s$$
or, $$w_m = \frac{10^{-9}}{144\pi}(25 + 50\sin^2\pi x)$$

SOL 7.1.13 Option (B) is correct.

The reflection coefficient of the wave propagating from medium 1 to medium 2 is defined as

$$\Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1}$$

where η_1 and η_2 are the intrinsic impedance of the two mediums respectively. So, the reflection coefficient for the wave propagating from free space to a dielectric medium is given as

$$\Gamma = \frac{\eta-\eta_0}{\eta+\eta_0}$$

where η is intrinsic impedance of the dielectric medium and η_0 is intrinsic impedance in free space. Since the intrinsic impedance of the dielectric medium is given as

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{4\varepsilon_0}} = \frac{\eta_0}{2}$$

So, we have
$$\Gamma = \frac{\eta_0/2-\eta_0}{\eta_0/2+\eta_0} = \frac{1/2-1}{1/2+1} = -\frac{1}{3}$$

Therefore, the magnitude of electric field of reflected wave is

$$E_r = \Gamma E_0 = -\frac{E_0}{3} \quad (E_0 \text{ is the magnitude of incident field})$$

SOL 7.1.14 Option (D) is correct.

Time period of wave propagating in a medium is given as :

$$T = \frac{2\pi}{\omega} \qquad \text{where } \omega \text{ is the angular frequency of the wave.}$$

Given the magnetic field intensity in the free space is

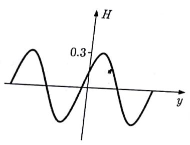
$$\boldsymbol{H} = 0.3\cos(\omega t - \beta y)\,\boldsymbol{a}_z \text{ A/m}$$

So, at $t = T/8$ the magnetic field intensity is

$$\boldsymbol{H} = 0.3\cos\left(\omega\frac{T}{8} - \beta y\right)\boldsymbol{a}_z = 0.3\cos\left(\frac{\pi}{4} - \beta y\right)\boldsymbol{a}_z \qquad (T = 2\pi/\omega)$$

or,
$$H = 0.3\cos(\beta y - \pi/4)$$

Therefore we get the plot of H versus y as shown below



SOL 7.1.15 Option (D) is correct.

Phase velocity of the medium, $v_p = 7.5 \times 10^7$ m/s

Relative permeability, $\mu_r = 4.8$

Conductivity $\sigma = 0$ (lossless medium)

Since phase velocity of an EM wave in a medium is defined as

$$v_p = \frac{c}{\sqrt{\mu_r\varepsilon_r}}$$

where c is the velocity of wave in air, μ_r is the relative permeability of the medium and ε_r is the relative permittivity of the medium. So, we have

$$7.5 \times 10^7 = \frac{3 \times 10^8}{\sqrt{(4.8)\varepsilon_r}} \qquad (c = 3 \times 10^8 \text{ m/s})$$

or,
$$\varepsilon_r = 3.33$$

Now the intrinsic impedance of the medium is given as

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}} \qquad (\sigma = 0)$$

$$= 377\sqrt{\frac{4.8}{3.33}} = 452.4\,\Omega \qquad \left(\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\,\Omega\right)$$

Given the electric field intensity in the phasor form is

$$\boldsymbol{E}_s = 5e^{j0.3x}\,\boldsymbol{a}_z \text{ V/m} \qquad (1)$$

and the general equation of electric field phasor of an EM wave propagating in $\boldsymbol{a}_x$ direction is

$$\boldsymbol{E}_s = E_0 e^{-j\beta x}\,\boldsymbol{a}_z \text{ V/m} \qquad (2)$$

So, comparing the equations (1) and (2) we get

direction of wave propagation, $\boldsymbol{a}_k = -\boldsymbol{a}_x$

and phase constant, $\beta = 0.3$ rad/m

and from the Maxwell's equation, the magnetic field phasor of the wave is given as

$$\boldsymbol{H}_s = \frac{1}{\eta}(\boldsymbol{a}_k) \times \boldsymbol{E}_s$$

where η is the intrinsic impedance of the medium and $\boldsymbol{a}_k$ is the unit vector in the direction of wave propagation.

So,
$$\boldsymbol{H}_s = \frac{1}{45.24}(-\boldsymbol{a}_x) \times (5e^{j0.3x}\,\boldsymbol{a}_z) \qquad (\boldsymbol{a}_k = -\boldsymbol{a}_x)$$

$$= \frac{5}{452.4}e^{j0.3x}\,\boldsymbol{a}_y = 11.05e^{j0.3x}\,\boldsymbol{a}_y \text{ mA/m}$$

and the angular frequency of the wave is given as

$$\omega = \beta v_p = (0.3)(7.5 \times 10^7) = 2.25 \times 10^7$$

So, the magnetic field intensity of the EM wave in time domain is

$$\boldsymbol{H}(x,t) = \mathrm{Re}\{\boldsymbol{H}_s e^{j\omega t}\} = 11.05\cos(\omega t + 0.3x)\,\boldsymbol{a}_y$$

$$= 11.05\cos(2.25 \times 10^7 t + 0.3x)\,\boldsymbol{a}_y \text{ mA/m}$$

SOL 7.1.16 Option (B) is correct.

Given the field intensities of the plane wave as

$$\boldsymbol{E}(x,t) = 900\cos(5 \times 10^6\pi t - \beta x)\,\boldsymbol{a}_y \text{ V/m}$$

$$\boldsymbol{H}(x,t) = 1.9\cos(5 \times 10^6\pi t - \beta x)\,\boldsymbol{a}_z \text{ V/m}$$

So, we get $|\boldsymbol{E}| = 900$, $|\boldsymbol{H}| = 1.9$, $\omega = 5 \times 10^6\pi$

Now, the intrinsic impedance in the medium is

$$\eta = \frac{|\boldsymbol{E}|}{|\boldsymbol{H}|} = \frac{900}{1.9} = 473.7$$

and phase constant of the wave in the medium is

$$\beta = \frac{\omega}{v_p} = \frac{5 \times 10^6\pi}{7 \times 10^7} = 0.224 \text{ m}^{-1}$$

Since, for a perfect dielectric $\sigma = 0$

Therefore,
$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \eta_0\sqrt{\frac{\mu_r}{\varepsilon_r}} \qquad (1)$$

and
$$\beta = \frac{\omega}{v_p} = \frac{\omega}{c}\sqrt{\mu_r\varepsilon_r} \qquad (2)$$

Comparing the equation (1) and (2) we get,

$$\mu_r = \left(\frac{\beta\eta c}{\omega\eta_0}\right) = \left[\frac{(0.224)(473.7)(3\times 10^8)}{(5\times 10^6\pi)(377)}\right]$$
$$= 5.37$$

Again from equation (1)

$$\varepsilon_r = \left(\frac{\eta_0}{\eta}\right)^2 \mu_r = \left(\frac{377}{473.7}\right)^2 \times 5.37 = 3.4$$

SOL 7.1.17 Option (A) is correct.

General equation of electric field intensity of a plane wave propagating in free space in $-\boldsymbol{a}_x$ direction having amplitude E_0 and frequency ω is given as :

$$\boldsymbol{E} = E_0\cos(\omega t + \beta x)\boldsymbol{a}_n$$

where β is phase constant of the wave and $\boldsymbol{a}_n$ is the unit vector in the direction of polarization of wave and since the EM wave is polarized in $+\boldsymbol{a}_z$ direction. So,

$$\boldsymbol{a}_n = \boldsymbol{a}_z$$

and we get, $$\boldsymbol{E} = E_0\cos\left(\omega t + \frac{\omega}{c}x\right)\boldsymbol{a}_z \qquad \left(\text{in free space } \beta = \frac{\omega}{c}\right)$$

Therefore, the magnetic field intensity of the wave is given as

$$\boldsymbol{H} = \frac{1}{\eta_0}(\boldsymbol{a}_k)\times(\boldsymbol{E})$$

where $\boldsymbol{a}_k$ is the unit vector in the direction of wave propagation and η_0 is the intrinsic impedance of the wave in the medium.

So, $$\boldsymbol{H} = \frac{1}{\eta_0}(-\boldsymbol{a}_x)\times\left[E_0\cos\left(\omega t + \frac{\omega}{c}x\right)\boldsymbol{a}_z\right] \qquad (\boldsymbol{a}_k = -\boldsymbol{a}_x)$$
$$= \frac{E_0}{\eta_0}\cos\left(\omega t + \frac{\omega}{c}x\right)\boldsymbol{a}_y$$

SOL 7.1.18 Option (A) is correct.

General equation of electric field intensity of a plane wave propagating in free space is given as :

$$\boldsymbol{E} = E_0\cos(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})\boldsymbol{a}_n \qquad (1)$$

where $\boldsymbol{a}_n$ is unit vector in direction of polarization, $\boldsymbol{k}$ is the wave number in the direction of wave propagation with amplitude $k = \beta = \frac{\omega}{c}$, and $\boldsymbol{r} = x\boldsymbol{a}_x + y\boldsymbol{a}_y + z\boldsymbol{a}_z$ is the position vector.

Since, the wave is propagating in the direction from origin to point (1,1,1).

So, $$\boldsymbol{k} = \left(\frac{\omega}{c}\right)\frac{(\boldsymbol{a}_x + \boldsymbol{a}_y + \boldsymbol{a}_z) - 0}{\sqrt{1^2+1^2+1^2}} = \frac{\omega}{c}\left(\frac{\boldsymbol{a}_x + \boldsymbol{a}_y + \boldsymbol{a}_z}{\sqrt{3}}\right)$$

and since the field is polarized parallel to x-z plane

So, $$\boldsymbol{a}_n = \frac{m\boldsymbol{a}_x + n\boldsymbol{a}_z}{\sqrt{m^2+n^2}} \qquad \text{where } m \text{ and } n \text{ are constants.}$$

Now, the electric field of wave is always perpendicular to the direction of propagation of EM wave. So, we have

$$\boldsymbol{k}\cdot\boldsymbol{a}_n = 0$$

$$\left[\frac{\omega}{c}\left(\frac{\boldsymbol{a}_x + \boldsymbol{a}_y + \boldsymbol{a}_z}{\sqrt{3}}\right)\right]\cdot\left[\frac{m\boldsymbol{a}_x + n\boldsymbol{a}_z}{\sqrt{m^2+n^2}}\right] = 0$$

$$m + n = 0$$

$$m = -n$$

Therefore, the unit vector in the direction of polarization of the wave is

$$\boldsymbol{a}_n = \frac{m\boldsymbol{a}_x + (-m)\boldsymbol{a}_z}{\sqrt{m^2+(-m)^2}} = \frac{\boldsymbol{a}_x - \boldsymbol{a}_z}{\sqrt{2}} \qquad (m = -n)$$

Putting all the values in equation (1), we get the electric field of the wave as

$$\boldsymbol{E} = E_0\cos\left[\omega t - \frac{\omega}{c}\left(\frac{\boldsymbol{a}_x + \boldsymbol{a}_y + \boldsymbol{a}_z}{\sqrt{3}}\right)\cdot(x\boldsymbol{a}_x + y\boldsymbol{a}_y + z\boldsymbol{a}_z)\right]\left(\frac{\boldsymbol{a}_x - \boldsymbol{a}_z}{\sqrt{2}}\right)$$
$$= E_0\cos\left[\omega t - \frac{\omega}{\sqrt{3}\,c}(x+y+z)\right]\left(\frac{\boldsymbol{a}_x - \boldsymbol{a}_z}{\sqrt{2}}\right)$$

SOL 7.1.19 Option (A) is correct.

For the microwave experiment the angular frequency is

$$\omega = 2\pi f = 2\pi\times 10\times 10^9 \qquad (f = 10\,\text{GHz})$$
$$= 2\pi\times 10^{10}$$

So, $$\frac{\sigma}{\omega\varepsilon} = \frac{6.25\times 10^7}{2\pi\times 10^{10}\times 1\times 8.85\times 10^{-12}}$$
$$= 1.12\times 10^8 >> 1$$

Therefore, the skin depth of the material is

$$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}} \qquad (\sigma/\omega\varepsilon >> 1)$$
$$= \sqrt{\frac{2}{2\pi\times 10^{10}\times 1\times 4\pi\times 10^{-7}\times 6.25\times 10^7}}$$
$$= 6.36\times 10^{-7}\,\text{m} = 0.636\,\mu\text{m}$$

Thus, for the successful experiment, width of coating must be greater than skin depth

i.e. $$t > 0.636$$
$$t > 0.64\,\mu\text{m}$$

SOL 7.1.20 Option (B) is correct.

Given, the electric field intensity of the plane wave

$$\boldsymbol{E} = 3\cos(10^7 t - 0.2y)\boldsymbol{a}_x + 2\sin(10^7 t - 0.2y)\boldsymbol{a}_z\ \text{V/m}$$

Comparing it with the general equation of electric field of a plane wave, we get

Angular frequency, $\omega = 10^7$

Phase constant, $\beta = 0.2$

So, the phase velocity of the propagating wave is

$$v_p = \frac{\omega}{\beta} = \frac{10^7}{0.2} = 5\times 10^7\,\text{m/s}$$

or, $$\frac{c}{\sqrt{\varepsilon_r}} = 5\times 10^7$$ $\longrightarrow$ How can this formula be used here ? ($\sigma \neq 0$).

where c is velocity of wave in air and ε_r is the relative permittivity of the medium.

So, $$\varepsilon_r = \left(\frac{3\times 10^8}{5\times 10^7}\right)^2 = 36$$

Therefore, permittivity of the medium is

$$\varepsilon = \varepsilon_r\varepsilon_0 = 36\varepsilon_0$$

Now, the complex permittivity of the medium is given as

$$\varepsilon_c = \varepsilon' - j\varepsilon''$$

where $$\varepsilon' = \varepsilon = 36\varepsilon_0$$

and $$\varepsilon'' = \frac{\sigma}{\omega} = \frac{2\times 10^7}{10^7} = 2$$

Thus, $$\varepsilon_c = (36\varepsilon_0 - j2)\ \text{F/m}$$

SOL 7.1.21 Option (A) is correct.

Conductivity of all the metals are in the range of mega siemens per meter and frequency of the visible waves are in the range of 10^{15} Hz. So, we can assume

$$\text{Conductivity of a metal} \approx 10^6\,\text{S/m}$$
$$\text{Frequency of a visible wave} \approx 10^{15}\,\text{Hz}$$

Now, the attenuation constant of a wave in a certain medium is given as :

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]}$$

Since for a metal, $\sigma >> \omega\varepsilon$

So,
$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}}\sqrt{\frac{\sigma}{\omega\varepsilon}} = \sqrt{\frac{\omega\mu\sigma}{2}}$$

Therefore, the skin depth of a metal is

$$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{2}{10^{15}\times 4\pi\times 10^{-7}\times 10^6}} = \sqrt{\frac{2}{4\pi\times 10^{14}}} = \sqrt{\frac{1}{2\pi}}\times 10^{-7} \approx 1\,\text{nm}$$

Thus, the skin depth is in the range of nanometers for a metal and that's why the wave (visible wave) can't penetrate inside the metal and the metals are opaque.

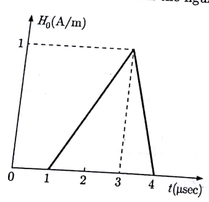
i.e. (A) and (R) both are true and (R) is the correct explanation of (A).

SOL 7.1.22 Option (D) is correct.

Since, the wave is propagating in free space so, the velocity of the wave is 3×10^8 m/s and the amplitude of magnetic field intensity in $z = 0$ plane is given as

$$\boldsymbol{H}_0 = \frac{\boldsymbol{E}_0}{\eta_0}$$

Therefore, the plot of magnetic field intensity H_0 versus time t in $z = 0$ plane is as shown in the figure below :



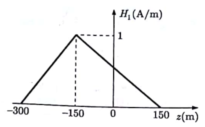
Since, the wave is propagating in $+\boldsymbol{a}_z$ direction so, an amplitude which exists in the plane $z = 0$ at any time t must exist in the plane

$$z = (1\times 10^{-6} - t)\times 3\times 10^8\,\text{m at } t = 1\,\mu\text{sec}.$$

So, the amplitude of H_0 will be equal to the H_1 at $t = 1\,\mu$sec for the plane

$$z = (10^6 - t)\times 3\times 10^8\,\text{m}$$

Thus, the plot of H_1 versus z will be as shown in figure below



SOL 7.1.23 Option (B) is correct.

Given the electric field intensity in phasor form

$$\boldsymbol{E}_s = E_0(\boldsymbol{a}_y - j\boldsymbol{a}_z)e^{-j\beta x}$$

So, the instantaneous expression of electric field intensity will be,

$$\boldsymbol{E} = \text{Re}\{E_0(\boldsymbol{a}_y - j\boldsymbol{a}_z)e^{j(\omega t-\beta x)}\}$$
$$= \text{Re}\{E_0(a_y - ja_z)[\cos(\omega t)+j\sin(\omega t)]\}e^{-j\beta x}$$
$$= E_0(a_y\cos(\omega t) + a_z\sin(\omega t))e^{-j\beta x}$$

Therefore, the magnitude of the field is

$$|\boldsymbol{E}| = \sqrt{(E_0\cos\omega t)^2 + (E_0\sin\omega t)^2}$$

or, $\quad |\boldsymbol{E}_1|^2 + |\boldsymbol{E}_2|^2 = E_0$

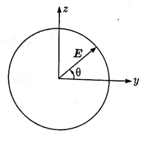
which is a circular equation i.e. the wave is circularly polarized.

Now, the instantaneous angle θ that the field $\boldsymbol{E}$ makes with y-axis is given as

$$\tan\theta = \frac{E_0\sin\omega t}{E_0\cos\omega t}$$

or, $\quad \theta = \omega t$

Therefore as the time increases, $\boldsymbol{E}$ rotates from y to z as shown in figure below :



and since the direction of wave propagation is in $+\boldsymbol{a}_x$ direction so, the rotation from y to z obeys the right hand rule. Thus, we conclude that the field is Right hand circularly polarized.

SOL 7.1.24 Option (A) is correct.

Given the phasor form of electric field intensity,

$$\boldsymbol{E}_s = 4(\boldsymbol{a}_z - j\boldsymbol{a}_z)e^{-j3y}$$

So, the electric field intensity of the reflected wave will be

$$\boldsymbol{E}_{rs} = \Gamma[4(a_z - ja_z)]e^{j3y}$$

where Γ is the reflection coefficient at the interface. Therefore,

$$\boldsymbol{E}_{rs} = 4(-\boldsymbol{a}_z + j\boldsymbol{a}_z)e^{j3y} \qquad \text{(for perfect conductor } \Gamma = -1)$$

and the instantaneous expression of the electric field of reflected wave will be

$$\boldsymbol{E} = \text{Re}\{4(-\boldsymbol{a}_z + j\boldsymbol{a}_z)(\cos\omega t + j\sin\omega t)\}e^{j3y}$$

$$= 4(-\cos(\omega t)\boldsymbol{a}_z - \sin(\omega t)\boldsymbol{a}_x)e^{j\beta y}$$

Therefore, the magnitude of the reflected field is

$$|\boldsymbol{E}| = \sqrt{(4\cos\omega t)^2 + (4\sin\omega t)^2}$$

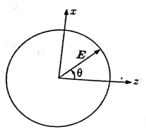
or, $$|\boldsymbol{E}_1|^2 + |\boldsymbol{E}_2|^2 = 4$$

which is a circular equation i.e. the wave is circularly polarized.
Now, the instantaneous angle θ that $\boldsymbol{E}$ makes with z-axis is given as

$$\tan\theta = \frac{-4\sin\omega t}{-4\cos\omega t}$$

$$\theta = \omega t$$

So, as time increases, electric field $\boldsymbol{E}$ rotates from z to x as shown in the figure below :



Since the direction of wave propagation is along $-\boldsymbol{a}_y$, so, the rotation from z to x follows left hand rule. Thus, we conclude that the EM wave is LHC (left hand circularly) polarized.

SOL 7.1.25 Option (C) is correct.
Given the electric field intensity of incident wave,

$$\boldsymbol{E}_{is} = 10\boldsymbol{a}_z e^{-j(6y+8z)}$$

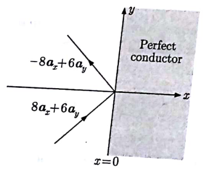
So, the direction of wave propagation is

$$K = 6\boldsymbol{a}_y + 8\boldsymbol{a}_z$$

Since the wave is incident on the perfect conductor so, the magnitude of the reflected wave is given as,

$$\boldsymbol{E}_{r0} = -\boldsymbol{E}_{i0} = -10\boldsymbol{a}_z \qquad (\Gamma = -1 \text{ for perfect conductor})$$

The direction of wave propagation of reflected wave will be along $(6\boldsymbol{a}_y - 8\boldsymbol{a}_z)$ as shown in figure below :



Therefore, the field intensity of the reflected wave is

$$\boldsymbol{E}_{rs} = -10\boldsymbol{a}_z e^{-j(6y-8z)}$$

Thus, the net electric field intensity of the total wave in free space after reflection will be

$$\boldsymbol{E}_s = \boldsymbol{E}_{is} + \boldsymbol{E}_{rs} = 10\boldsymbol{a}_z e^{-j(6y+8z)} + [-10\boldsymbol{a}_z e^{-j(6y-8z)}]$$

$$= 10\boldsymbol{a}_z e^{-j6y}(e^{-j8z} - e^{-8z}) = -j20\boldsymbol{a}_z e^{-j6y}\sin 8x \text{ V/m}$$

SOL 7.1.26 Option (A) is correct.
Given, the electric field intensity of the incident wave,

$$\boldsymbol{E}_{is} = 25\boldsymbol{a}_x e^{-j(6z+8y)} \text{ V/m}$$

So, the direction of the wave propagation is

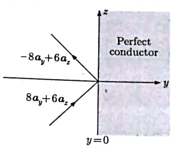
$$\boldsymbol{k} = 6\boldsymbol{a}_z + 8\boldsymbol{a}_y$$

Since the wave is incident on a perfect conductor so, the magnitude of the electric field of the reflected wave is

$$\boldsymbol{E}_{r0} = -\boldsymbol{E}_{i0} \qquad \text{(reflection coefficient, } \Gamma = -1)$$

$$= -25\boldsymbol{a}_x$$

The reflected wave will propagate in $6\boldsymbol{a}_z - 8\boldsymbol{a}_y$ direction as shown in figure below:



So, we get the electric field intensity of reflected wave as

$$\boldsymbol{E}_{rs} = -25\boldsymbol{a}_x e^{-j(6z-8y)} \text{ V/m}$$

Since, the magnetic field intensity of a plane wave in terms of electric field intensity is defined as

$$\boldsymbol{H} = \frac{1}{\eta_0}(\boldsymbol{a}_k \times \boldsymbol{E})$$

where $\boldsymbol{a}_k$ is unit vector in the direction of wave propagation and η_0 is the intrinsic impedance of free space. So, the magnetic field intensity of the reflected wave is given as

$$\boldsymbol{H}_{rs} = \frac{1}{\eta_0}(\boldsymbol{a}_k \times \boldsymbol{E}_{rs})$$

where, $$\boldsymbol{a}_k = \frac{\boldsymbol{k}}{k} = \frac{6a_z - 8a_y}{|6a_z - 8a_y|} = (0.6a_z - 0.8a_y)$$

So, we get $$\boldsymbol{H}_{rs} = \frac{1}{120\pi}[(0.6\boldsymbol{a}_z - 0.8\boldsymbol{a}_y) \times (-25\boldsymbol{a}_x e^{-j(6z-8y)})]$$

$$= \frac{1}{120\pi}[(-15\boldsymbol{a}_y - 20\boldsymbol{a}_z)e^{-j(6z-8y)}]$$

$$= -\left(\frac{\boldsymbol{a}_y}{8\pi} + \frac{\boldsymbol{a}_z}{6\pi}\right)e^{-j(6z-8y)} \text{ A/m}$$

SOL 7.1.27 Option (B) is correct.
The general expression for phasor form of electric field vector is

$$\boldsymbol{E}_s = E_0 e^{-j(\beta_x x + \beta_y y + \beta_z z)}$$

Comparing the given field with this expression we get,

$$\beta_x x + \beta_y y + \beta_z z = 0.01\pi(-3x + \sqrt{3}\,y - 2z)$$

So, the propagation vector is

$$\boldsymbol{k} = \nabla(\beta_x x + \beta_y y + \beta_z z) = 0.01\pi(-3\boldsymbol{a}_x + \sqrt{3}\,\boldsymbol{a}_y - 3\boldsymbol{a}_z)$$

Therefore, the direction of the propagation of the wave is

$$a_k = \frac{\boldsymbol{k}}{k} = \frac{-3\boldsymbol{a}_x + \sqrt{3}\,\boldsymbol{a}_y - 2\boldsymbol{a}_z}{\sqrt{9+3+4}}$$
$$= \frac{1}{4}(-3\boldsymbol{a}_x + \sqrt{3}\,\boldsymbol{a}_y - 2\boldsymbol{a}_z)$$

SOL 7.1.28 Option (D) is correct.

From the given expression of the field vector, we have the propagation vector,

$$\boldsymbol{k} = \frac{\pi}{25}(\sqrt{3}\,\boldsymbol{a}_x - 2\boldsymbol{a}_y - 3\boldsymbol{a}_z)$$

So the phase constants along x, y and z-axes are

$$\beta_x = \frac{\sqrt{3}\,\pi}{25};\ \beta_y = -\frac{2\pi}{25};\ \beta_z = -\frac{3\pi}{25}$$

Therefore, the apparent wave lengths along the three axes are

$$\lambda_x = \frac{2\pi}{\beta_x} = \frac{2\pi}{\left(\frac{\sqrt{3}\,\pi}{25}\right)} = \frac{50}{\sqrt{3}} = 28.87\text{ m}$$

$$\lambda_y = \frac{2\pi}{|\beta_y|} = \frac{2\pi}{\left|\left(\frac{-2\pi}{25}\right)\right|} = +25\text{ m}$$

$$\lambda_z = \frac{2\pi}{|\beta_z|} = \frac{2\pi}{\left|\left(\frac{-3\pi}{25}\right)\right|} = +\frac{50}{3} = 16.7\text{ m}$$

SOL 7.1.29 Option (B) is correct.

As determined in previous question, the propagation vector of the plane wave is

$$\boldsymbol{k} = \frac{\pi}{25}(\sqrt{3}\,\boldsymbol{a}_x - 2\boldsymbol{a}_y - 3\boldsymbol{a}_z)$$

Therefore, the direction of wave propagation is

$$a_k = \frac{\boldsymbol{k}}{k} = \frac{\sqrt{3}\,\boldsymbol{a}_x - 2\boldsymbol{a}_y - 3\boldsymbol{a}_z}{4}$$

So the phase constant along the direction of wave propagation is

$$\beta = \boldsymbol{k} \cdot a_k = 0.16\pi$$

Therefore, the angular frequency of the propagating wave is

$$\omega = v_p\beta = (3\times10^8)\times(0.16\pi)\ \text{(In free space } v_p = 3\times10^8\text{ m/s)}$$
$$= 1.51\times10^8\text{ rad/sec}$$

So, for the determined values of apparent phase constants in previous question, the apparent phase velocities are given as

$$v_{px} = \frac{\omega}{\beta_x} = \frac{1.51\times10^8}{\left(\frac{\sqrt{3}\,\pi}{25}\right)} = 6.93\times10^8\text{ m/s}$$

$$v_{py} = \frac{\omega}{\beta_y} = \frac{1.51\times10^8}{\left|-\frac{2\pi}{25}\right|} = 6\times10^8\text{ m/s}$$

and

$$v_{pz} = \frac{\omega}{\beta_z} = \frac{1.51\times10^8}{\left|-\frac{3\pi}{25}\right|} = 4\times10^8\text{ m/s}$$

SOL 7.1.30 Option (D) is correct.

The necessary condition for the vector field $\boldsymbol{E} = \boldsymbol{E}_0 e^{-j\beta}$ to represent the electric field intensity of a uniform plane wave is

$$\boldsymbol{k} \cdot \boldsymbol{E}_0 = 0$$

where $\boldsymbol{k}$ is the propagation vector of the wave and $\boldsymbol{E}_0$ is the amplitude of the electric field intensity of the plane wave. Now, we check all the given options for this condition.

(A) From given data we have

$$\boldsymbol{k} = \sqrt{3}\,\boldsymbol{a}_y + \boldsymbol{a}_z$$
$$\boldsymbol{E}_0 = -j\boldsymbol{a}_x - 2\boldsymbol{a}_y + j\sqrt{3}\,\boldsymbol{a}_z$$

So,
$$\boldsymbol{k} \cdot \boldsymbol{E}_0 = -2\sqrt{3} + j\sqrt{3} \neq 0$$

(B) From given data we have

$$\boldsymbol{E}_0 = \boldsymbol{a}_x - j2\boldsymbol{a}_y - \sqrt{3}\,\boldsymbol{a}_z$$
$$\boldsymbol{k} = \boldsymbol{a}_x + \sqrt{3}\,\boldsymbol{a}_z$$

So,
$$\boldsymbol{k} \cdot \boldsymbol{E}_0 = 1 - 3 \neq 0$$

(C) From given data we have

$$\boldsymbol{E}_0 = \left(\sqrt{3} + j\frac{1}{2}\right)\boldsymbol{a}_x + \left(1 + \frac{j\sqrt{3}}{2}\right)\boldsymbol{a}_y - j\sqrt{3}\,\boldsymbol{a}_z$$
$$\boldsymbol{k} = \sqrt{3}\,\boldsymbol{a}_x + 3\boldsymbol{a}_y + 2\boldsymbol{a}_z$$

So,
$$\boldsymbol{k} \cdot \boldsymbol{E}_0 = 3 + \frac{j\sqrt{3}}{2} + 3 + \frac{j3\sqrt{3}}{2} - j2\sqrt{3} \neq 0$$

(D) From given data we have

$$\boldsymbol{E}_0 = \left(-\sqrt{3} - j\frac{1}{2}\right)\boldsymbol{a}_x + \left(1 - j\frac{\sqrt{3}}{2}\right)\boldsymbol{a}_y + j\sqrt{3}\,\boldsymbol{a}_z$$
$$\boldsymbol{k} = \sqrt{3}\,\boldsymbol{a}_x + 3\boldsymbol{a}_y + 2\boldsymbol{a}_z$$

So,
$$\boldsymbol{k} \cdot \boldsymbol{E}_0 = -3 - j\frac{\sqrt{3}}{2} + 3 - j\frac{3\sqrt{3}}{2} + j2\sqrt{3} = 0$$

So the vector represents electric field vector of a uniform plane wave.

SOL 7.1.31 Option (D) is correct.

For the field vectors $\boldsymbol{E}_s$ and $\boldsymbol{H}_s$ defined as

$$\boldsymbol{E}_s = \boldsymbol{E}_0 e^{-j\beta}$$

and

$$\boldsymbol{H}_s = \boldsymbol{H}_0 e^{-j\beta}$$

The condition that it represents the field vectors of a uniform plane wave is

$$\boldsymbol{E}_0 \cdot \boldsymbol{H}_0 = 0,\quad \boldsymbol{E}_0 \cdot \boldsymbol{k} = 0 \text{ and } \boldsymbol{H}_0 \cdot \boldsymbol{k} = 0$$

where $\boldsymbol{k}$ is the propagation vector of the plane wave.

Now, we check the all given pairs for this condition

In Option (D)
$$\boldsymbol{E}_0 = -j\boldsymbol{a}_x - 2\boldsymbol{a}_y + j\sqrt{3}\,\boldsymbol{a}_z$$
$$\boldsymbol{H}_0 = \boldsymbol{a}_x - j2\boldsymbol{a}_y - \sqrt{3}\,\boldsymbol{a}_z$$

and
$$\boldsymbol{k} = \sqrt{3}\,\boldsymbol{a}_x + \boldsymbol{a}_z$$

So
$$\boldsymbol{E}_0 \cdot \boldsymbol{H}_0 = -j + j4 - j3 = 0$$
$$\boldsymbol{E}_0 \cdot \boldsymbol{k} = -j\sqrt{3} + j\sqrt{3} = 0$$
$$\boldsymbol{H}_0 \cdot \boldsymbol{k} = \sqrt{3} - \sqrt{3} = 0$$

Therefore, it represents the field vectors of a uniform plane wave.

SOL 7.1.32 Option (A) is correct.

For a propagating electromagnetic wave, the field satisfies the following Maxweell's equation.

$$\nabla \cdot \boldsymbol{E} = 0$$
$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \neq 0$$

Now, we check the condition for the given fields as below.

$$\boldsymbol{P} = 60\sin(\omega t + 10x)\boldsymbol{a}_z$$

So, $\nabla \cdot \boldsymbol{P} = 0$

and $\nabla \times \boldsymbol{P} = -600\cos(\omega t + 10x)\boldsymbol{a}_y \neq 0$

i.e. $\boldsymbol{P}$ is a possible EM field.

again, $$\boldsymbol{Q} = \frac{10}{\rho}\cos(\omega t - 2\rho)\boldsymbol{a}_\phi$$

So, $\nabla \cdot \boldsymbol{Q} = 0$

and $\nabla \times \boldsymbol{Q} = \frac{1}{\rho}\frac{\partial}{\partial \rho}[10\cos(\omega t - 2\rho)]\boldsymbol{a}_z \neq 0$

i.e. $\boldsymbol{Q}$ is a possible EM field

$$\boldsymbol{R} = 3\rho^2\cot\phi\,\boldsymbol{a}_\rho + \frac{1}{\rho}\cos\phi\,\boldsymbol{a}_\phi$$

So, $\nabla \cdot \boldsymbol{R} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(3\rho^2\cot\phi)\frac{\sin\phi}{\rho} \neq 0$

i.e. $\boldsymbol{R}$ is not a possible EM field.

$$\boldsymbol{S} = \frac{1}{r}\sin\theta\sin(\omega t - 6r)\boldsymbol{a}_\phi$$

So, $\nabla \cdot \boldsymbol{S} = \frac{1}{r^2\sin\theta}\sin(\omega t - 6r)\frac{\partial(\sin^2\phi)}{\partial r} \neq 0$

i.e. $\boldsymbol{S}$ is not an EM field.

Thus, the possible EM fields are $\boldsymbol{P}$ and $\boldsymbol{Q}$.

****

SOLUTIONS 7.2

SOL 7.2.1 Correct answer is 0.2 .

Given the magnetic field intensity,

$$\boldsymbol{H} = 10\cos(6\times 10^7 t - ky)\boldsymbol{a}_z \text{ A/m}$$

Comparing it with the general equation of magnetic field.

$$\boldsymbol{H} = H_0\cos(\omega t - ky)\boldsymbol{a}_z \text{ A/m}$$

We get, $\omega = 6\times 10^7$

So, the wave no is,

$$k = \frac{\omega}{c} = \frac{6\times 10^7}{3\times 10^8} = 0.2 \text{ (c is the velocity of wave in free space)}$$

SOL 7.2.2 Correct answer is 2.

Given magnetic field intensity in the non magnetic medium is

$$\boldsymbol{H} = 1.5\cos(10^9 t - 5z)\boldsymbol{a}_x \text{ A/m}$$

Comparing it with the general equation of magnetic field intensity

$$\boldsymbol{H} = H_0\cos(\omega t - \beta z)\boldsymbol{a}_x \text{ A/m}$$

We get, $\omega = 10^9$ rad/sec

and $\beta = 5$.

So, the phase velocity of the wave in the medium is given as

$$v_P = \frac{\omega}{\beta} = \frac{10^9}{5} = 2\times 10^8 \text{ m/s}$$

SOL 7.2.3 Correct answer is 0.5 .

Wavelength of an electromagnetic wave with phase constant β in a medium is defined as

$$\lambda = \frac{2\pi}{\beta}$$

So, the phase constant of the wave in terms of wavelength can be given as

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{12.6} = 0.5 \text{ rad/m} \qquad (\lambda = 12.6 \text{ m})$$

SOL 7.2.4 Correct answer is 2.25 .

Given the electric field intensity in the nonmagnetic material as

$$\boldsymbol{E} = 8\cos(4\times 10^8 t - 2x)\boldsymbol{a}_y \text{ V/m}$$

Comparing it with the general equation of electric field

$$\boldsymbol{E} = E_0\cos(\omega t - \beta x)\boldsymbol{a}_y \text{ A/m}$$

We get, $\omega = 4\times 10^8$ rad/s

and $\beta = 2$ rad/m

So, the phase velocity of the wave in the medium is given by

$$v_p = \frac{\omega}{\beta} = 2\times 10^8 \text{ m/s}$$

Since the medium is non magnetic so, $\mu = \mu_0$ and the relative permittivity of the medium is given as

$$\varepsilon_r = \left(\frac{c}{v_p}\right)^2 = \left(\frac{3 \times 10^8}{2 \times 10^8}\right)^2 = 2.25$$

SOL 7.2.5 Correct answer is 12.57 .

The general equation of electric field intensity of an EM wave propagating in $\boldsymbol{a}_x$ direction in a medium is given as

$$\boldsymbol{E} = E_0 \cos(\omega t - \beta x)\boldsymbol{a}_y \text{ A/m}$$

Comparing it with the given expression of electric field intensity, we get

$$\omega = 5 \times 10^8 \text{ rad/s}$$

So, the time period of the EM wave is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{5 \times 10^8} = 12.57 \text{ ns}$$

SOL 7.2.6 Correct answer is 3.93 .

The general equation of electric field intensity of an EM wave propagating in $\boldsymbol{a}_x$ direction in a medium is given as

$$\boldsymbol{E} = E_0 \cos(\omega t - \beta x)\boldsymbol{a}_y \text{ A/m}$$

Comparing it with the given expression of electric field intensity, we get

$$\omega = 4 \times 10^8 \text{ rad/s}$$

So, the time period of the wave in air is given as

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{4 \times 10^8}$$
$$= 15.71 \text{ ns}$$

Since in one time period the wave travels its one wavelength (λ) so, time taken by the wave to travel $\lambda/4$ distance is

$$t = \frac{T}{4} = 3.93 \text{ ns}$$

SOL 7.2.7 Correct answer is 251.33 .

Intrinsic impedance of any material is given as

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

where μ is permeability, σ is conductivity and ε is permittivity of the medium.

Since the given material is lossless, nonmagnetic and dielectric so, we have

$$\sigma = 0 \qquad \text{(lossless)}$$
$$\mu = \mu_0 \qquad \text{(non magnetic)}$$

and

$$\varepsilon = \varepsilon_r \varepsilon_0 = (2.25)\varepsilon_0 \qquad (\varepsilon_r = 2.25)$$

Therefore the intrinsic impedance of the material is

$$\eta = \sqrt{\frac{j\omega\mu_0}{0 + j\omega(2.25)\varepsilon_0}}$$
$$= \frac{\eta_0}{1.5} = \frac{377}{1.5} = 251.3\,\Omega \qquad \left(\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\,\Omega\right)$$

SOL 7.2.8 Correct answer is 0.8 .

Given,

Frequency of the wave propagation, $f = 0.5 \text{ MHz} = 0.5 \times 10^6 \text{ Hz}$

Conductivity of medium, $\sigma = 3 \times 10^7 \text{ S/m}$

Relative permeability of medium, $\mu_r = \varepsilon_r \approx 1$

So, the angular frequency of the wave propagation is

$$\omega = 2\pi f = 2\pi \times 0.5 \times 10^6 = \pi \times 10^6$$

and we get

$$\frac{\sigma}{\omega\varepsilon} = \frac{3 \times 10^7}{\pi \times 10^6 \times 8.85 \times 10^{-12}}$$
$$= 0.1 \times 10^{13} >> 1$$

Therefore, the phase constant of the propagating wave is given as

$$\beta = \sqrt{\frac{\omega\mu\sigma}{2}} \qquad (\sigma/\omega\varepsilon >> 1)$$
$$= \sqrt{\frac{\pi \times 10^6 \times 4\pi \times 10^{-7} \times 3 \times 10^7}{2}}$$
$$= 7695.29 \text{ rad/m}$$

So, the wavelength of the radio wave in the medium is

$$\lambda = \frac{2\pi}{\beta} = 0.8 \text{ mm}$$

SOL 7.2.9 Correct answer is 9.08 .

Given the magnetic field intensity of the plane wave in free space is

$$\boldsymbol{H}_s = (2 + j5)(4\boldsymbol{a}_y + 2j\boldsymbol{a}_z)e^{-j\beta x} \text{ A/m}$$

From the Maxwell's equation, the maximum electric field intensity of the plane wave is given as

$$|\boldsymbol{E}|_{max} = \eta_0 |\boldsymbol{H}|_{max}$$

where η_0 is intrinsic impedance in air and $|\boldsymbol{H}|_{max}$ is the maximum magnetic field intensity of the plane wave.

Now, the maximum magnetic field intensity of the plane wave is given as

$$|\boldsymbol{H}|_{max} = \sqrt{\boldsymbol{H}_s \cdot \boldsymbol{H}_s^*}$$

where $\boldsymbol{H}_s^*$ is the complex conjugate of the magnetic field phasor.

So,

$$|\boldsymbol{H}|_{max} = \sqrt{[(2 + j5)(4\boldsymbol{a}_y + 2j\boldsymbol{a}_z)] \cdot [(2 - j5)(4\boldsymbol{a}_y - 2j\boldsymbol{a}_z)]}$$
$$= \sqrt{(2 + j5)(2 - j5)[(4)(4) - (2j)(2j)]}$$
$$= \sqrt{29 \times 20} = 24.1 \text{ A/m}$$

Therefore, the maximum electric field intensity of the plane wave is

$$|\boldsymbol{E}|_{max} = \eta_0 |\boldsymbol{H}|_{max} = 377 \times 24.1 = 9.08 \text{ kV/m}$$

SOL 7.2.10 Correct answer is 60.

For an EM wave propagating in two mediums, the wavelengths of the wave in two mediums are related as

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

where λ_1 and λ_2 are the wavelengths of EM wave in two mediums with permittivity ε_1 and ε_2 respectively. So, the wavelength of plane wave in free space is given as

$$\frac{\lambda_0}{\lambda} = \sqrt{\frac{1}{\varepsilon_r}}$$
$$\lambda_0 = \lambda\sqrt{\varepsilon_r}$$

where λ is the wavelength of the wave in the medium with relative permittivity ε_r.

So,

$$\lambda_0 = 20\sqrt{9} = 60 \text{ cm} \qquad (\lambda = 20 \text{ cm}, \varepsilon_r = 9)$$

SOL 7.2.11 Correct answer is 20.

Given,

Conductivity of the glass, $\sigma = 10^{-12} \text{ S/m}$

and relative permittivity of the glass, $\varepsilon_r = 2.25$

So, the permittivity of glass is

$$\varepsilon = \varepsilon_0 \varepsilon_r = 2.25\varepsilon_0$$

Therefore, the time taken by the charge to flow out to the surface is

$$\tau \approx \frac{\varepsilon}{\sigma} = \frac{(2.25) \times (8.85 \times 10^{-12})}{10^{-12}}$$

$$= 19.9 \approx 20 \text{ sec}$$

SOL 7.2.12 Correct answer is 0.52 .

Given the electric field intensity of the propagating wave,

$$\boldsymbol{E} = E_0 e^{-x/3} \sin(10^8 t - \beta x)\boldsymbol{a}_y \text{ V/m} \quad (1)$$

The general equation of electric field intensity of plane wave propagating in $\boldsymbol{a}_x$ direction is given by

$$\boldsymbol{E} = E_0 e^{-\alpha x} \sin(\omega t - \beta x)\boldsymbol{a}_y \text{ V/m} \quad (2)$$

Comparing equation (1) and (2) we get,

$$\alpha = \frac{1}{3} \text{ NP/m and } \omega = 10^8 \text{ rad/sec}$$

So, the attenuation constant of a propagating wave is given as

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}$$

Let $$x_0 = \sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2}$$

Therefore, $$\alpha = \omega\sqrt{\frac{\mu_0\varepsilon_0}{2}\mu_r\varepsilon_r(x_0 - 1)}$$

or, $$(x_0 - 1) = \frac{2\alpha^2}{\omega^2 \mu_0\varepsilon_0\mu_r\varepsilon_r}$$

Now, we put $\alpha = 1/3$, $\mu_r = \varepsilon_r = 4$, $\omega = 10^8$ so, we get

$$x_0 - 1 = \frac{2 \times (1/3)^2 c_0^2}{(10^8)^2 \times (4)^2} \qquad \left(c_0 = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = 3 \times 10^8 \text{ m/s}\right)$$

$$x_0 - 1 = 2\left(\frac{1/3 \times 3 \times 10^8}{10^8 \times 4}\right)^2$$

$$x_0 = \frac{1}{8} + 1$$

$$x_0 = \frac{9}{8}$$

$$\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} = \frac{9}{8}$$

$$\frac{\sigma}{\omega\varepsilon} = \sqrt{\frac{81}{64} - 1}$$

Thus, $$\text{loss tangent} = \frac{\sigma}{\omega\varepsilon} = 0.52$$

SOL 7.2.13 Correct answer is 8.33 .

From the field intensity we get,

$$\omega = 10^9\pi$$

and it is given that, $\mu_r = 0.5$, $\sigma = 0.01$ S/m, $\varepsilon_r = 8$.

So, the phase constant,

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega t}\right)^2} + 1\right]}$$

$$= 10^9\pi\sqrt{\frac{\mu_0\varepsilon_0(8)(0.5)}{2}\left[\sqrt{1 + \left(\frac{0.01}{10^9\pi 8\varepsilon_0}\right)^2} + 1\right]}$$

$$= 20.95$$

Let the distance travelled by the wave be z to have a phase shift of $10°$.

So, $$\beta z = 10° = \frac{10\pi}{180} \text{ rad}$$

$$z = \frac{\pi}{18 \times (20.95)} = 8.33 \text{ mm}$$

SOL 7.2.14 Correct answer is 542.

The attenuation constant of a propagating wave in a medium is defined as

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}$$

Now, from the given data we have

$\mu_r = 0.5$, $\sigma = 0.01$ S/m, $\varepsilon_r = 8$.

So, $$\alpha = 10^9\pi\sqrt{\frac{\mu_0\varepsilon_0(8)(0.5)}{2}\left[\sqrt{1 + \left(\frac{0.01}{10^9\pi 8\varepsilon_0}\right)^2} - 1\right]}$$

$$= 0.9425$$

Initially the amplitude of the electric field $= 0.5$

So, after travelling distance z amplitude of wave $= 0.5e^{-\alpha z}$.

Therefore, the distance travelled by the wave for which the amplitude of the wave reduced by 40% is evaluated as

$$0.5e^{-\alpha z} = 0.5 \times \frac{60}{100} \qquad \text{(amplitude reduces to 60\%)}$$

$$e^{-(0.9425)z} = 0.6$$

or, $$z = \frac{1}{0.9425}\ln\left(\frac{1}{0.6}\right) = 542 \text{ mm}$$

SOL 7.2.15 Correct answer is 0.796 .

Skin depth (δ) of any medium is defined as the reciprocal of attenuation constant (α) of a plane wave in the medium

i.e. $$\delta = \frac{1}{\alpha}$$

The attenuation constant of the plane wave in the medium is given as

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}$$

Now, $$\frac{\sigma}{\omega\varepsilon} = \frac{2}{2\pi f\varepsilon_r\varepsilon_0} = \frac{2}{2\pi \times 50 \times 10^3 \times 80 \times 8.85 \times 10^{-12}}$$

$$= 8991.8 >> 1$$

i.e. $$\frac{\sigma}{\omega\varepsilon} >> 1$$

So, $$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}} \qquad (\sigma/\omega\varepsilon >> 1)$$

$$= \sqrt{\frac{2\pi \times 50 \times 10^3 \times 4 \times 4\pi \times 10^{-7} \times 2}{2}} = 0.4\pi$$

Therefore, $$\delta = \frac{1}{\alpha} = \frac{1}{0.4\pi} = 0.796 \text{ m}$$

SOL 7.2.16 Correct answer is 0.06 .

Velocity of the wave in free space is

$$c = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 3 \times 10^8 \text{ m/s}$$

So, the velocity of the wave in dielectric 1 is

$$v_{P1} = \sqrt{\frac{\mu_0}{4\varepsilon_0}} = \frac{c}{2}$$

The velocity of wave in dielectric 2 is

$$v_{P2} = \sqrt{\frac{\mu_0}{9\varepsilon_0}} = \frac{c}{3}$$

The velocity of wave in dielectric 3 is

$$v_{P3} = \sqrt{\frac{\mu_0}{3\varepsilon_0}} = \frac{c}{\sqrt{3}}$$

Therefore, the time t taken by the wave to strike the interface at $x = 5\text{ m}$ is

$$\begin{aligned} t &= t_1 + t_2 + t_3 \\ &= \frac{6}{3 \times 10^8} + \frac{3}{c/2} + \frac{2}{c/3} \\ &= (0.02 + 0.02 + 0.02) \times 10^{-6} \\ &= 0.06\,\mu\text{sec} \end{aligned}$$

SOL 7.2.17 Correct answer is 0.99 .

Given

Frequency of the propagating wave, $f = 50\text{ MHz} = 50 \times 10^6\text{ Hz}$

Skin depth of the dielectric medium, $\delta = 0.32\text{ mm} = 0.32 \times 10^{-3}\text{ m}$

Permittivity of dielectric, $\mu = 6.28 \times 10^{-7}$

So, the conductivity of the dielectric medium is given as

$$\begin{aligned} \sigma &= \frac{1}{\pi f \mu \delta^2} = \frac{1}{(3.14) \times (50 \times 10^6) \times (6.28 \times 10^{-7}) \times (0.32 \times 10^{-3})^2} \\ &= 0.99 \times 10^5\text{ S/m} \end{aligned}$$

SOL 7.2.18 Correct answer is -60.

Frequency of the wave, $f = 8\text{ GHz} = 8 \times 10^9\text{ Hz}$

Distance travelled by the wave, $z = 0.175\text{ mm} = 0.175 \times 10^{-3}\text{ m}$

Permittivity of dielectric, $\mu = 6.28 \times 10^{-7}$

and as calculated in previous question the conductivity of the dielectric medium is

$$\sigma = 0.99 \times 10^5\text{ S/m}$$

So, the attenuation constant of the wave in the dielectric medium is

$$\begin{aligned} \alpha &= \sqrt{\pi f \mu \sigma} \\ &= \sqrt{(3.14) \times (8 \times 10^9) \times (6.28 \times 10^{-7}) \times (0.99 \times 10^5)} \\ &= 3.95 \times 10^4\text{ NP/m} \end{aligned}$$

Therefore, the reducing factor of the field intensity in dB after travelling distance z is

$$20\log_{10} e^{-\alpha z} = 20\log_{10} e^{-(3.95 \times 10^4) \times (0.175 \times 10^{-3})} = -60\text{ dB}$$

SOL 7.2.19 Correct answer is 53.31 .

Given, the magnetic field intensity of the EM wave propagating in free space,

$$\boldsymbol{H} = 0.1\cos(\omega t - \beta y)\boldsymbol{a}_z\text{ A/m}$$

So, the time average power density of the EM wave is given as

$$\mathcal{P}_{ave} = \frac{1}{2}\eta_0 H^2 \boldsymbol{a}_z$$

where η_0 is the intrinsic impedance in free space and H is the magnitude of magnetic field intensity in free space.

So,

$$\begin{aligned} \mathcal{P}_{ave} &= \frac{1}{2}(120\pi)(0.1)^2\boldsymbol{a}_z \qquad (\eta_0 = 120\pi,\ H = 0.1) \\ &= 0.6\pi\boldsymbol{a}_z \end{aligned}$$

Therefore, the total power passing through the square plate of side 20 cm is given as

$$\begin{aligned} P_{total} &= \int \mathcal{P}_{ave} \cdot d\boldsymbol{S} \\ &= \mathcal{P}_{ave} \cdot S\boldsymbol{a}_n \end{aligned}$$

where S is the area of the square plate given as

$$S = (0.2)^2 = 0.04\text{ m}^2 \qquad \text{(Side of square} = 0.2\text{ m)}$$

and $\boldsymbol{a}_n$ is the unit vector normal to the plate given as

i.e.

$$\boldsymbol{a}_n = \frac{\boldsymbol{a}_x + \boldsymbol{a}_y}{\sqrt{2}}$$

So,

$$\begin{aligned} P_{total} &= (0.6\pi\boldsymbol{a}_z) \cdot \left[0.04\left(\frac{\boldsymbol{a}_x + \boldsymbol{a}_y}{\sqrt{2}}\right)\right] \\ &= 0.05331\text{ Watt} \\ &= 53.31\text{ mW} \end{aligned}$$

SOL 7.2.20 Correct answer is 10.025 .

Given, the electric field intensity of the incident wave,

$$\boldsymbol{E}_{is} = 5e^{-j5y}\boldsymbol{a}_z\text{ V/m}$$

So, we get the phase constant of the wave as

$$\beta_1 = 5$$

or,

$$\frac{\omega}{c}\sqrt{\mu_{r1}\varepsilon_{r1}} = 5 \qquad \left(\beta = \frac{\omega}{v_p}\right)$$

$$\frac{\omega}{c}\sqrt{(4)(1)} = 5$$

$$\omega = \frac{5c}{2}$$

Now, the intrinsic impedance of the lossless medium is given as

$$\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}} = 2\sqrt{\frac{\mu_0}{\varepsilon_0}} = 2\eta_0 = 754$$

and the intrinsic impedance of lossy medium is

$$\eta_2 = |\eta_2|\angle\theta_{\eta_2}$$

where, the magnitude of the intrinsic impedance is given as

$$\begin{aligned} |\eta_2| &= \frac{\sqrt{\mu_2/\varepsilon_2}}{\left[1 + \left(\frac{\sigma_2}{\omega\varepsilon_2}\right)^2\right]^{1/4}} = \frac{\sqrt{\mu_0/4\varepsilon_0}}{\left[1 + \left(\frac{0.1}{\frac{5c}{2}4\varepsilon_0}\right)\right]^{1/4}} \\ &= \frac{60\pi}{(15.18)^{1/4}} = 95.48 \end{aligned}$$

and the phase angle of the intrinsic impedance is

$$\tan 2\theta_{\eta_2} = \frac{\sigma_2}{\omega\varepsilon_2} = 3.77$$

or

$$\theta_{\eta_2} = 37.57°$$

So, the reflection coefficient of the wave is given as

$$\begin{aligned} \Gamma &= \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{95.48\angle 37.57° - 754}{95.48\angle 37.57° + 754} \\ &= 0.1886\angle 171.08° \end{aligned}$$

Therefore, the standing wave ratio is

$$\begin{aligned} S &= \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.8186}{1 - 0.8186} \\ &= 10.025 \end{aligned}$$

SOL 7.2.21 Correct answer is 12.5 .

As calculated in previous question we have the propagation vector from the given data as

$$\boldsymbol{k} = 0.04\pi(-2\boldsymbol{a}_x - 3\boldsymbol{a}_y + \sqrt{3}\,\boldsymbol{a}_z)$$

and the direction of wave propagation is

$$\boldsymbol{a}_k = \frac{\boldsymbol{k}}{k} = \frac{0.04\pi(-2\boldsymbol{a}_x - 3\boldsymbol{a}_y + \sqrt{3}\,\boldsymbol{a}_z)}{|0.04\pi(-2\boldsymbol{a}_x - 3\boldsymbol{a}_y + \sqrt{3}\,\boldsymbol{a}_z)|}$$

$$= \frac{(-2\boldsymbol{a}_x - 3\boldsymbol{a}_y + \sqrt{3}\,\boldsymbol{a}_z)}{\sqrt{(-2)^2 + (-3)^2 + (\sqrt{3})^2}}$$

$$= \frac{-2\boldsymbol{a}_x - 3\boldsymbol{a}_y + \sqrt{3}\,\boldsymbol{a}_z}{4}$$

Therefore, the phase constant along the direction of propagation is

$$\beta = \boldsymbol{k} \cdot \boldsymbol{a}_k$$

$$= \left[0.04\pi(-2\boldsymbol{a}_x - 3\boldsymbol{a}_y + \sqrt{3}\,\boldsymbol{a}_z)\right] \cdot \left(\frac{-2\boldsymbol{a}_x - 3\boldsymbol{a}_y + \sqrt{3}\,\boldsymbol{a}_z}{4}\right)$$

$$= 0.16\pi$$

So, the wavelength along the direction of wave propagation is

$$\lambda = \frac{2\pi}{\beta} = 12.5\text{ m}$$

SOL 7.2.22 Correct answer is 24.

From the given expression of magnetic field vector we get,

$$\beta_x x + \beta_y y + \beta_z z = 0.04\pi(\sqrt{3}\,x - 2y - 3z)$$

So, the propagation vector of the plane wave is

$$\boldsymbol{k} = \nabla(\beta_x x + \beta_y y + \beta_z z)$$

$$= 0.04\pi(\sqrt{3}\,\boldsymbol{a}_x - 2\boldsymbol{a}_y - 3\boldsymbol{a}_z)$$

and the direction of wave propagation is

$$\boldsymbol{a}_k = \frac{\boldsymbol{k}}{k} = \frac{0.04\pi(\sqrt{3}\,\boldsymbol{a}_x - 2\boldsymbol{a}_y - 3\boldsymbol{a}_z)}{|0.04\pi(\sqrt{3}\,\boldsymbol{a}_x - 2\boldsymbol{a}_y - 3\boldsymbol{a}_z)|}$$

$$= \frac{(\sqrt{3}\,\boldsymbol{a}_x - 2\boldsymbol{a}_y - 3\boldsymbol{a}_z)}{4}$$

Therefore, the phase constant along the direction of wave propagation is

$$\beta = \boldsymbol{k} \cdot \boldsymbol{a}_k = 0.16\pi$$

Since the wave is propagating in free space so it's phase velocity will be

$$v_p = 3 \times 10^8\text{ m/s}$$

or,
$$\frac{\omega}{\beta} = 3 \times 10^8$$

So, the frequency of the plane wave is

$$f = \frac{(3 \times 10^8)(0.16\pi)}{2\pi} = 2.4 \times 10^7\text{ Hz} \qquad (\omega = 2\pi f)$$

$$= 24\text{ MHz}$$

SOL 7.2.23 Correct answer is 17.9 .

Since, 20% of the energy in the incident wave is reflected at the boundary. So, we have,

$$|\Gamma|^2 = \frac{20}{100}$$

or,
$$|\Gamma| = \sqrt{0.2} = \pm 0.447$$

where Γ is the reflection coefficient at the medium interface. Therefore, we have

$$\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \pm 0.447$$

$$\frac{\eta_0\sqrt{\frac{\mu_{r2}}{\varepsilon_{r2}}} - \eta_0\sqrt{\frac{\mu_{r1}}{\varepsilon_{r1}}}}{\eta_0\sqrt{\frac{\mu_{r2}}{\varepsilon_{r2}}} + \eta_0\sqrt{\frac{\mu_{r1}}{\varepsilon_{r1}}}} = \pm 0.447$$

$$\frac{\sqrt{\frac{\mu_{r2}}{\mu_{r2}^3}} - \sqrt{\frac{\mu_{r1}}{\mu_{r1}^3}}}{\sqrt{\frac{\mu_{r2}}{\mu_{r2}^3}} + \sqrt{\frac{\mu_{r1}}{\mu_{r1}^3}}} = \pm 0.447 \qquad (\varepsilon_{r1} = \mu_{r1}^3, \varepsilon_{r2} = \mu_{r2}^3)$$

$$\frac{\mu_{r1} - \mu_{r2}}{\mu_{r1} + \mu_{r2}} = \pm 0.447$$

$$\frac{\mu_{r1}}{\mu_{r2}} = \frac{1 \pm 0.447}{1 \mp 0.447}$$

$$\frac{\mu_{r1}}{\mu_{r2}} = 2.62 \text{ or } 0.38$$

So,
$$\frac{\varepsilon_{r2}}{\varepsilon_{r1}} = \left(\frac{\mu_{r2}}{\mu_{r1}}\right)^3 = 0.056 \text{ or } 17.9$$

SOL 7.2.24 Correct answer is 2.25 .

Intrinsic impedance of 1st medium is

$$\eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_1}}$$

and the intrinsic impedance of 2nd medium is

$$\eta_2 = \sqrt{\frac{\mu_0}{\varepsilon_2}}$$

So, the reflection coefficient at the interface of the two medium is given as

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$\Gamma = \frac{\sqrt{\frac{\mu_0}{\varepsilon_2}} - \sqrt{\frac{\mu_0}{\varepsilon_1}}}{\sqrt{\frac{\mu_0}{\varepsilon_2}} + \sqrt{\frac{\mu_0}{\varepsilon_1}}}$$

or,
$$\frac{1}{5} = \frac{\sqrt{\frac{1}{\varepsilon_2}} - \sqrt{\frac{1}{\varepsilon_1}}}{\sqrt{\frac{1}{\varepsilon_2}} + \sqrt{\frac{1}{\varepsilon_1}}} \qquad \left(\text{given } \Gamma = \frac{1}{5}\right)$$

$$\frac{1}{5} = \frac{1 - \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}}{1 + \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}}$$

$$\frac{5+1}{5-1} = \frac{2}{2\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}} \qquad \text{(By rationalization)}$$

$$\frac{6}{4} = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}}$$

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{9}{4} = 2.25$$

****

SOLUTIONS 7.3

SOL 7.3.1 Option (A) is correct.
Given magnetic field intensity in the non magnetic medium is

$$\boldsymbol{H} = 3\cos(\omega t - kz)\boldsymbol{a}_z \text{ A/m}$$

The negative coefficient of z in $(\omega t - kz)$ shows that the wave is propagating in $+\boldsymbol{a}_z$ direction.

SOL 7.3.2 Option (C) is correct.
Attenuation constant for a plane wave with angular frequency ω in a certain medium is given as

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]} \qquad (1)$$

Since for a poor conductor, conductivity is very low

i.e. $\sigma << \omega\varepsilon$

or, $\frac{\sigma}{\omega\varepsilon} << 1$

So, in equation (1) using binomial expansion we get,

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[1+\frac{1}{2}\left(\frac{\sigma}{\omega\varepsilon}\right)^2 - 1\right]} \qquad (\sigma/\omega\varepsilon << 1)$$

$$= \omega\sqrt{\frac{\mu\varepsilon}{2}}\frac{1}{\sqrt{2}}\frac{\sigma}{\omega\varepsilon} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$$

Therefore, the skin depth of the poor conductor is

$$\delta = \frac{1}{\alpha} = \frac{2}{\sigma}\sqrt{\frac{\varepsilon}{\mu}}$$

which is independent of frequency (ω).

SOL 7.3.3 Option (A) is correct.

SOL 7.3.4 Option (A) is correct.

SOL 7.3.5 Option (B) is correct.

SOL 7.3.6 Option (C) is correct.

SOL 7.3.7 Option (A) is correct.

SOL 7.3.8 Option (C) is correct.

SOL 7.3.9 Option (B) is correct.

SOL 7.3.10 Option (A) is correct.

SOL 7.3.11 Option (A) is correct.

SOL 7.3.12 Option (A) is correct.

SOL 7.3.13 Option (A) is correct.

SOL 7.3.14 Option (C) is correct.

SOL 7.3.15 Option (D) is correct.

SOL 7.3.16 Option (C) is correct.

SOL 7.3.17 Option (C) is correct.
Power radiated from any source is constant.

SOL 7.3.18 Option (C) is correct.
Given, the electric field intensity of the propagating wave

$$\boldsymbol{E} = \boldsymbol{a}_x\sin(\omega t - \beta z) + \boldsymbol{a}_y\sin(\omega t - \beta z + \pi/2)$$

So, we conclude that the wave is propagating along $\boldsymbol{a}_z$ direction and the field components along $\boldsymbol{a}_x$ and $\boldsymbol{a}_y$ are equal.

i.e. $E_x = E_y$

Therefore, the wave is circularly polarized. Now we will determine the field is either right circular or left circular. The angle between the electric field $\boldsymbol{E}$ and x-axis is given as

$$\theta = \tan^{-1}\left(\frac{\cos\omega t}{\sin\omega t}\right) = \frac{\pi}{2} - \omega t$$

So, with increase in time the tip of the field intensity moves from y to x-axis and as the wave is propagating in $\boldsymbol{a}_z$ direction therefore, the wave is left hand circularly polarized.

SOL 7.3.19 Option (B) is correct.

We have $$\frac{\partial^2 E_x}{\partial Z^2} = c^2\frac{\partial^2 E_x}{\partial t^2}$$

As the field component E_x changes with z so, we conclude that the EM wave is propagating in z- direction.

SOL 7.3.20 Option (D) is correct.
Intrinsic impedance of a medium is given as

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

Since, copper is good conductor i.e. $\sigma >> \omega\varepsilon$ so, we get

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}}\angle 45°$$

Thus, the impedance will be complex with an inductive component.

SOL 7.3.21 Option (C) is correct.
The depth of penetration or skin depth is defined as

$$\delta = \frac{1}{\sqrt{\pi f\mu\sigma}}$$

i.e. $\delta \propto \frac{1}{\sqrt{f}}$

or, $\delta \propto \sqrt{\lambda}$ $\qquad (\lambda = c/f)$

So, the depth of penetration (skin depth) increases with increase in wavelength.

SOL 7.3.22 Option (A) is correct.
Given, the electric field intensity of the wave

$$\boldsymbol{E}(z,t) = E_o e^{j(\omega t+\beta z)}\boldsymbol{a}_x + \varepsilon_0 e^{j(\omega t+\beta z)}\boldsymbol{a}_y$$

Generalizing $\qquad \boldsymbol{E}(z) = \boldsymbol{a}_x E_1(z) + \boldsymbol{a}_y E_2(z)$ ...(1) ...(2)

Comparing (1) and (2) we can see that $E_1(z)$ and $E_2(z)$ are in space quadrature but in time phase so, their sum $\boldsymbol{E}$ will be linearly polarized along a line that makes an angle ϕ with x-axis as shown below.

SOL 7.3.23 Option (C) is correct.
The Skin depth of a conductor is defined as

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

So, statement 2 and 3 are correct while are incorrect.

SOL 7.3.24 Option (C) is correct.
For circular polarization the two orthogonal field components must have the same magnitude and has a phase difference of $90°$.
So, all the three statements are necessary conditions.

SOL 7.3.25 Option (A) is correct.
Velocity of light in any dielectric medium is defined as

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0 \varepsilon_0 \varepsilon_r}} = \frac{c}{\sqrt{\varepsilon_r}}$$

where c is velocity of light in vacuum and ε_r is dielectric constant of the medium.
Since $\varepsilon_r > 1$, so $v < c$. Therefore, both A and R are true and R is correct explanation of A.

SOL 7.3.26 Option (D) is correct.
The poynting vector is the instantaneous power flow per unit area in an EM wave and defined as

$$\mathcal{P} = \boldsymbol{E} \times \boldsymbol{H}$$

So, $\boldsymbol{E} \times \boldsymbol{H}$ is rate of energy flow (power flow) per unit area.

SOL 7.3.27 Option (C) is correct.
Poynting vector represents the instantaneous power density vector associated with the EM field at a given point.
i.e.
$$\mathcal{P} = \boldsymbol{E} \times \boldsymbol{H}$$

SOL 7.3.28 Option (B) is correct.
Given, the electric field intensity of the wave in free space,

$$\boldsymbol{E} = 50\sin(10^7 t + kz)\boldsymbol{a}_y \text{ V/m}$$

Comparing it with the general expression of electric field defined as

$$\boldsymbol{E} = E_0 \sin(\omega t - \beta z)\boldsymbol{a}_y \text{ V/m}$$

We get,
(1) The wave propagates in $-\boldsymbol{a}_z$ direction along z-axis.
(2) The wavelength is given as

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8 \times 2\pi}{(10^7)} = 188.5 \text{ m}$$

(3) Wave number, $k = \frac{2\pi}{\lambda} = \frac{2\pi}{188.5} = 0.033$
(4) The wave doesn't attenuate as it travels.

SOL 7.3.29 Option (B) is correct.
An electromagnetic wave incident on a conducting medium has the depth of penetration (skin depth) defined as

$$\delta = \frac{1}{\alpha}$$

i.e. inversely proportion to attenuation constant.

SOL 7.3.30 Option (C) is correct.
The gyro frequency is the frequency whose period is equal to the period of revolution of an electron in its circular orbit under the influence of earth's magnetic field. So, the radio wave at frequency near f_g is attenuated by the earth's magnetic field. (Since, there is a resonance phenomena and oscillating electron receive more and more energy from incident wave.)

SOL 7.3.31 Option (A) is correct.
An EM wave propagating in free space consists of electric and magnetic field intensity both perpendicular to direction of propagation.

SOL 7.3.32 Option (C) is correct.
Skin depth (δ) is the distance through which the wave amplitude decreases to a factor e^{-1} or $1/e$.

SOL 7.3.33 Option (C) is correct.
The depth of penetration of wave (skin depth) in a lossy dielectric (conductor) is given as

$$\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

So, the skin depth increases when
(1) permeability decreases
(2) conductivity decreases
(3) frequency decreases
Since, the wavelength of the wave is given as

$$\lambda = \frac{v_p}{f} \quad \text{i.e.} \quad \lambda \propto \frac{1}{f}$$

So, as λ increases, f decreases and therefore, skin depth increases.

SOL 7.3.34 Option (B) is correct.
For a good conductor,

$$\alpha = \beta = \sqrt{\pi f \mu \sigma}$$

Since, the skin depth is defined as

$$\delta = \frac{1}{\alpha} \text{ or } \delta = \frac{1}{\beta} \qquad (\alpha = \beta)$$

Now, the phase constant of the wave is given as

$$\beta = \frac{2\pi}{\lambda}$$

So, we have $\quad \delta = \frac{1}{\beta} = \frac{\lambda}{2\pi} \quad$ It is defined for a good conductor.

SOL 7.3.35 Option (D) is correct.
The polarization of a uniform plane wave described the time varying behaviour of the electric field intensity vector so for polarization the field vector must be transverse to the propagation of wave.
i.e. Transverse nature of electromagnetic wave causes polarization.

SOL 7.3.36 Option (B) is correct.
Fields are said to be circularly polarized if their components have same magnitudes but they differ in phase by $\pm 90°$.

****

SOLUTIONS 7.4

SOL 7.4.1 Option (C) is correct.

Electric field of the propagating wave in free space is given as

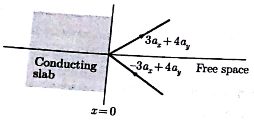
$$\boldsymbol{E}_i = (8\boldsymbol{a}_x + 6\boldsymbol{a}_y + 5\boldsymbol{a}_z)\, e^{j(\omega t + 3x - 4y)} \text{ V/m}$$

So, it is clear that wave is propagating in the direction $(-3\boldsymbol{a}_x + 4\boldsymbol{a}_y)$. Since, the wave is incident on a perfectly conducting slab at $x = 0$. So, the reflection coefficient will be equal to -1.

i.e.

$$\boldsymbol{E}_{r_o} = (-1)\,\boldsymbol{E}_{i_o} = -8\boldsymbol{a}_x - 6\boldsymbol{a}_y - 5\boldsymbol{a}_z$$

Again, the reflected wave will be as shown in figure below :



i.e. the reflected wave will be along the direction $3\boldsymbol{a}_x + 4\boldsymbol{a}_y$. Thus, the electric field of the reflected wave will be

$$\boldsymbol{E}_r = (-8\boldsymbol{a}_x - 6\boldsymbol{a}_y - 5\boldsymbol{a}_z)\, e^{j(\omega t - 3x - 4y)} \text{ V/m}$$

SOL 7.4.2 Option (A) is correct.

Given, the electric field intensity of the EM wave as

$$\boldsymbol{E} = 10(\boldsymbol{a}_y + j\boldsymbol{a}_z)\, e^{-j25x}$$

So, we conclude that the wave is propagating in $\boldsymbol{a}_x$ direction and the y and z-components of the field are same. Therefore, the wave is circularly polarized.

Now, the angle formed by the electric field with the z-axis is given as

$$\theta = \omega t$$

So, with increase in time the tip of the field magnitude rotates from z to y -axis and as the wave is propagating in $\boldsymbol{a}_x$ direction so, we conclude that the wave is left circular (i.e., left circular polarization).

The phase constant of the field is given as

$$\beta = \frac{\omega}{c}$$

$$25 = \frac{2\pi f}{c} \qquad (\beta = 25)$$

$$f = \frac{25 \times c}{2\pi} = \frac{25 \times 3 \times 10^8}{2 \times 3.14}$$

$$= 1.2\,\text{GHz}$$

SOL 7.4.3 Option (C) is correct.

Intrinsic impedance of EM wave

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{4\varepsilon_0}} = \frac{120\pi}{2} = 60\pi$$

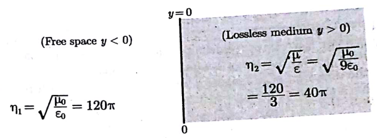
Time average power density of the EM wave is given as

$$P_{ave} = \frac{1}{2}EH = \frac{1}{2}\frac{E^2}{\eta}$$

$$= \frac{1}{2 \times 60\pi} = \frac{1}{120\pi} \qquad (E = 1\,\text{V/m})$$

SOL 7.4.4 Option (A) is correct.

In the given problem



Reflection coefficient at the medium interface is given as

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{400\pi - 120\pi}{40\pi + 120\pi} = -\frac{1}{2}$$

As, given the electric field component of the incident wave is

$$\boldsymbol{E}_i = 24\cos(3 \times 10^8 - \beta y)\,\boldsymbol{a}_x$$

So, we conclude that the incident wave is propagating along $\boldsymbol{a}_y$ direction and the angular frequency of the wave is

$$\omega = 3 \times 10^8 \text{ rad/s}$$

So, the phase constant of the wave is given as

$$\beta = \frac{\omega}{c} = \frac{3 \times 10^8}{3 \times 10^8} = 1$$

Therefore, the reflected wave will be propagating in $-\boldsymbol{a}_y$ direction and its electric field component is given as

$$\boldsymbol{E}_r = \Gamma \boldsymbol{E}_{i0}\cos(3 \times 10^8 + y) \qquad (\beta = 1\,\text{rad/m})$$

where E_{i0} is the maximum value of the field component of incident wave.

i.e. $\boldsymbol{E}_{i0} = 24\boldsymbol{a}_x$

So, we have

$$\boldsymbol{E}_{r0} = -\frac{1}{2}\left[24\cos(3 \times 10^8 + y)\boldsymbol{a}_x\right]$$

$$= -12\cos(3 \times 10^8 + y)\,\boldsymbol{a}_x$$

Therefore, the magnetic field component of the reflected wave is given as

$$\boldsymbol{H}_r = \frac{1}{\eta}(\boldsymbol{a}_k \times \boldsymbol{E}_r)$$

where η_1 is the intrinsic impedance of medium 1, and $\boldsymbol{a}_k$ is the unit vector in the direction of wave propagation. So, we get

$$\boldsymbol{H}_r = \frac{1}{120\pi}\left[\boldsymbol{a}_y \times \left(-12\cos(3 \times 10^8 + y)\,\boldsymbol{a}_x\right)\right]$$

$$= \frac{1}{10\pi}\cos(3 \times 10^8 + y)\,\boldsymbol{a}_z$$

SOL 7.4.5 Option (C) is correct.

The intrinsic impedance of the wave is defined as

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}$$

where μ is permeability and ε is permittivity of the medium.
Now, the reflection coefficient at the medium interface is given as

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

Substituting values for η_1 and η_2 we have

$$\tau = \frac{\sqrt{\frac{\mu_o}{\varepsilon_o\varepsilon_r}} - \sqrt{\frac{\mu_o}{\varepsilon_o}}}{\sqrt{\frac{\mu_o}{\varepsilon_o\varepsilon_r}} + \sqrt{\frac{\mu_o}{\varepsilon_o}}} = \frac{1-\sqrt{\varepsilon_r}}{1+\sqrt{\varepsilon_r}} = \frac{1-\sqrt{9}}{1+\sqrt{9}} \qquad (\varepsilon_r = 9)$$

$$= -0.5$$

or,

$$|\Gamma| = 0.5$$

SOL 7.4.6 Option (A) is correct.
Since, the wave is propagating in a direction making an angle 90° with positive y-axis. So, the y-component of propagation constant will be zero. As the direction of propagation makes an angle 30° with positive x-axis so, we have the propagation constant of the wave as

$$\gamma = \beta\cos 30° x \pm \beta \sin 30° y$$

where β is the phase constant of the wave. So, we get

$$\gamma = \frac{2\pi}{\lambda}\frac{\sqrt{3}}{2}x \pm \frac{2\pi}{\lambda}\frac{1}{2}y = \frac{\pi\sqrt{3}}{\lambda}x \pm \frac{\pi}{\lambda}y$$

Now, in all the given options the direction of electric field of the wave is given along $\boldsymbol{a}_y$. So, considering that direction we get the field intensity of the wave as

$$\boldsymbol{E} = \boldsymbol{a}_y E_0 e^{j(\omega t - \gamma)} = \boldsymbol{a}_y E_0 e^{j\left[\omega t - \left(\frac{\pi\sqrt{3}}{\lambda}x \pm \frac{\pi}{\lambda}y\right)\right]}$$

SOL 7.4.7 Option (D) is correct.
Since, the given field intensity have components in $\boldsymbol{a}_x$ and $\boldsymbol{a}_y$ direction so, the magnitude of the field intensity of the plane wave is

$$|H|^2 = H_x^2 + H_y^2 = \left(\frac{5\sqrt{3}}{\eta_0}\right)^2 + \left(\frac{5}{\eta_0}\right)^2 = \left(\frac{10}{\eta_0}\right)^2$$

So, the time average power density of the EM wave is given as

$$P_{ave} = \frac{\eta_0 |H|^2}{2} = \frac{\eta_0}{2}\left(\frac{10}{\eta_0}\right)^2 = \frac{50}{\eta_0} \text{ watts}$$

SOL 7.4.8 Option (D) is correct.
The Brewster angle is given as

$$\tan\theta_n = \sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}$$

$$\tan 60° = \sqrt{\frac{\varepsilon_{r2}}{1}}$$

or

$$\varepsilon_{r2}' = 3$$

SOL 7.4.9 Option (A) is correct.
The reflection coefficient at the medium interface is given as

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\sqrt{\frac{\mu_o}{\varepsilon_o\varepsilon_r}} - \sqrt{\frac{\mu_o}{\varepsilon_o}}}{\sqrt{\frac{\mu_o}{\varepsilon_o\varepsilon_r}} + \sqrt{\frac{\mu_o}{\varepsilon_o}}} = \frac{1+\sqrt{\varepsilon_r}}{1+\sqrt{\varepsilon_r}} = \frac{1-\sqrt{4}}{1+\sqrt{4}} = -\frac{1}{3}$$

So, the transmitted power is

$$P_t = (1 - |\Gamma|^2) P_i$$

$$P_t = \left(1 - \frac{1}{9}\right)P_i = \frac{8}{9}P_i$$

or,

$$\frac{P_t}{P_i} = \frac{8}{9}$$

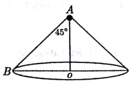
SOL 7.4.10 Option (D) is correct.

$$\sin\theta = \frac{1}{\sqrt{\varepsilon_r}} = \frac{1}{\sqrt{2}}$$

or

$$\theta = 45° = \frac{\pi}{4}$$

The configuration is shown below. Here A is point source.



Now $\quad AO = 1$ m
From geometry $\quad BO = 1$ m
Thus, $\quad$ area $= \pi r^2 = \pi \times OB = \pi$ m²

SOL 7.4.11 Option (C) is correct.
Given, the electric field of the EM wave in medium 1 as

$$\boldsymbol{E}_1 = 4\boldsymbol{a}_x + 3\boldsymbol{a}_y + 5\boldsymbol{a}_z$$

As the medium interface lies in the plane $x = 0$ so, the tangential and normal components of the electric field are

$$\boldsymbol{E}_{1t} = 3\boldsymbol{a}_y + 5\boldsymbol{a}_z$$

and

$$\boldsymbol{E}_{1n} = 4\boldsymbol{a}_x$$

Now, from the boundary condition we know that the tangential component of electric field is uniform. So, we get

$$\boldsymbol{E}_{2t} = \boldsymbol{E}_{1t} = 3\boldsymbol{a}_y + 5\boldsymbol{a}_z$$

Again from the boundary condition the normal component of displacement vector are equal.

i.e. $\quad \boldsymbol{D}_{2n} = \boldsymbol{D}_{1n}$

or $\quad \varepsilon_2 \boldsymbol{E}_{2n} = \varepsilon_1 \boldsymbol{E}_{1n}$

or $\quad 4\varepsilon_o \boldsymbol{E}_{2n} = 3\varepsilon_o 4\boldsymbol{a}_x$

or $\quad \boldsymbol{E}_{2n} = 3\boldsymbol{a}_x$

Thus, the net electric field intensity in medium 2 is

$$\boldsymbol{E}_2 = \boldsymbol{E}_{2t} + \boldsymbol{E}_{2n} = 3\boldsymbol{a}_x + 3\boldsymbol{a}_y + 5\boldsymbol{a}_z$$

SOL 7.4.12 Option (C) is correct.
From the expression of the magnetic field intensity of the EM wave, we have
Angular frequency, $\omega = 50{,}000$
Phase constant, $\quad \beta = 0.004$
So, the phase constant of the wave is given as

$$v_p = \frac{\omega}{\beta} = \frac{5 \times 10^4}{4 \times 10^{-3}} = 1.25 \times 10^7 \text{ m/s}$$

SOL 7.4.13 Option (C) is correct.
Refractive index of glass $n_g = \sqrt{\mu_r \varepsilon_r} = 1.5$
Frequency $\quad f = 10^{14}$ Hz

$$c = 3 \times 10^8 \text{ m/sec}$$

The wavelength of the 10^{14} Hz beam of light is

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{10^{14}} = 3 \times 10^{-6}$$

So, wavelength of the light beam in glass is given as

$$\lambda_g = \frac{\lambda}{n_g} = \frac{3 \times 10^{-6}}{1.5} = 2 \times 10^{-6}\,\text{m}$$

SOL 7.4.14 Option (A) is correct.

The time average poynting vector of the EM wave is defined as

$$\mathcal{P}_{ave} = \frac{1}{2}\text{Re}[\boldsymbol{E}_s \times \boldsymbol{H}_s^*]$$

where, $\boldsymbol{E}_s$ is the phasor form of the electric field intensity and $\boldsymbol{H}_s^*$ is the complex conjugate of the phsor form of magnetic field intensity. So, we have

$$\boldsymbol{E}_s \times \boldsymbol{H}_s^* = (\boldsymbol{a}_x + j\boldsymbol{a}_y)\, e^{jkz - j\omega t} \times \frac{k}{\omega\mu}(-j\boldsymbol{a}_x + \boldsymbol{a}_y)\, e^{-jkz + j\omega t}$$

$$= \boldsymbol{a}_z\left[\frac{k}{\omega\mu} - (-j)(j)\frac{k}{\omega\mu}\right] = 0$$

Thus,
$$\mathcal{P}_{ave} = \frac{1}{2}\text{Re}[\boldsymbol{E}_s \times \boldsymbol{H}_s^*] = 0$$

SOL 7.4.15 Option (D) is correct.

We have
$$\text{VSWR} = \frac{E_{max}}{E_{min}} = 5 = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

or
$$|\Gamma| = \frac{2}{3}$$

As the wave is normally incident on the interface so, the reflection coefficient will be real (either positive or negative). Now, for a wave propagating from medium 1 to medium 2 having permittivities ε_1 and ε_2 respectively.

(i) If $\varepsilon_2 > \varepsilon_1$, the reflection coefficient is negative

(ii) If $\varepsilon_2 < \varepsilon_1$ then, the reflection coefficient is positive.

Since, the given EM wave is propagating from free space to the dielectric material with $\varepsilon > \varepsilon_0$, therefore

$$\Gamma = -\frac{2}{3}$$

or,
$$\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -\frac{2}{3}$$

or,
$$\frac{\eta_2 - 120\pi}{\eta_2 + 120\pi} = -\frac{2}{3}$$

So,
$$\eta_2 = 24\pi$$

SOL 7.4.16 Option (B) is correct.

The skin depth (δ) of a material is related to the operating frequency (f) as

$$\delta \propto \frac{1}{\sqrt{f}}$$

Therefore,
$$\frac{\delta_2}{\delta_1} = \sqrt{\frac{f_1}{f_2}}$$

$$\frac{\delta_2}{25} = \sqrt{\frac{1}{4}}$$

or
$$\delta_2 = \sqrt{\frac{1}{4}} \times 25 = 12.5\text{ cm}$$

SOL 7.4.17 Option (D) is correct.

The intrinsic impedance of a medium with permittivity ε and permeability μ is defined as

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}$$

So, the reflection coefficient at the boundary interface of the two mediums is given as

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\sqrt{\frac{\mu_o}{\varepsilon_o\varepsilon_r}} - \sqrt{\frac{\mu_o}{\varepsilon_o}}}{\sqrt{\frac{\mu_o}{\varepsilon_o\varepsilon_r}} + \sqrt{\frac{\mu_o}{\varepsilon_o}}}$$

$$= \frac{1 - \sqrt{\varepsilon_r}}{1 + \sqrt{\varepsilon_r}} = \frac{1 - \sqrt{4}}{1 + \sqrt{4}} \qquad \text{since } \varepsilon_r = 4$$

$$= \frac{-1}{3} = 0.333\angle 180^\circ$$

SOL 7.4.18 Option (B) is correct.

We have
$$E(z,t) = 10\cos(2\pi \times 10^7 t - 0.1\pi z)$$

So, we get
$$\omega = 2\pi \times 10^7 t$$
$$\beta = 0.1\pi$$

Therefore, the phase velocity of the wave is given as

$$v_p = \frac{\omega}{\beta} = \frac{2\pi \times 10^7}{0.1\pi} = 2 \times 10^8\text{ m/s}$$

SOL 7.4.19 Option (C) is correct.

We have
$$E = (0.5\boldsymbol{a}_x + \boldsymbol{a}_y e^{j\frac{\pi}{2}})\, e^{j(\omega t - kz)}$$

So, its components along x and y-axis are

$$|E_x| = 0.5 e^{j(\omega t - kz)}$$

and
$$|E_y| = e^{j\frac{\pi}{2}} e^{j(\omega t - kz)}$$

i.e.
$$|E_x| \neq |E_y|$$

Since, the components are not equal and have the phase difference of $\pi/2$ so, we conclude that the EM wave is elliptically polarized.

SOL 7.4.20 Option (A) is correct.

Loss tangent of a medium is defined as

$$\tan\delta = \frac{\sigma}{\omega\varepsilon}$$

where σ is the conductivity ε is permittivity of the medium and ω is operating angular frequency. So, we get

$$\tan\delta = \frac{1.7 \times 10^{-4}}{2\pi \times 3 \times 10^9 \times 78\varepsilon_o} \qquad (\omega = 2\pi f)$$

$$= \frac{1.7 \times 10^{-4} \times 9 \times 10^9}{3 \times 10^9 \times 39} = 1.3 \times 10^{-5}$$

SOL 7.4.21 Option (A) is correct.

The required condition is

$$|I_c| = |I_d|$$

i.e. the conduction current equals to the displacement current. So, we get

$$|J_c| = |J_d|$$
$$|\sigma E| = |j\omega\varepsilon E|$$

or,
$$\sigma = 2\pi f\varepsilon_o\varepsilon_r \qquad (\omega = 2\pi f,\ \varepsilon = \varepsilon_r\varepsilon_0)$$

or,
$$f = \frac{\sigma}{2\pi \times \varepsilon_o\varepsilon_r} = \frac{2\sigma}{4\pi\varepsilon_o\varepsilon_r}$$

$$= \frac{9 \times 10^9 \times 2 \times 10^{-2}}{4} = 45 \times 10^6 = 45\text{ MHz}$$

SOL 7.4.22 Option (B) is correct.

VSWR (voltage standing wave ratio) of the transmission line is defined as

$$S = \frac{1+|\Gamma|}{1-|\Gamma|}$$

where Γ is the reflection coefficient of the transmission line. So, we get

$$3 = \frac{1+|\Gamma|}{1-|\Gamma|} \qquad (\text{VSWR} = 3)$$

or $$|\Gamma| = 0.5$$

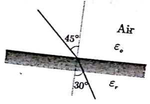
Therefore, the ratio of the reflected power strength to the incident power is given as

$$\frac{P_r}{P_i} = |\Gamma|^2 = 0.25$$

Thus, 25% of incident power is reflected.

SOL 7.4.23 Option (C) is correct.

The fig is as shown below :



As per snell law

$$\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{1}{\varepsilon_r}}$$

or $$\frac{\sin 30°}{\sin 45°} = \frac{1}{\sqrt{\varepsilon_r}}$$

$$\frac{\frac{1}{2}}{\frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{\varepsilon_r}}$$

or $$\varepsilon_r = 2$$

SOL 7.4.24 Option (B) is correct.

Since, the phase constant is defined as

$$\beta = \frac{2\pi}{\lambda} = \omega\sqrt{\mu\varepsilon}$$

So, the wavelength in terms of permittivity of the medium can be given as

$$\lambda = \frac{2\pi}{\omega\sqrt{\mu\varepsilon}}$$

or, $$\lambda \propto \frac{1}{\sqrt{\varepsilon}}$$

So, we get $$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

SOL 7.4.25 Option (C) is correct.

A scalar wave equation must satisfy following relation

$$\frac{\partial^2 E}{\partial t^2} - v_p^2\frac{\partial^2 E}{\partial z^2} = 0 \qquad ...(1)$$

where $$v_p = \frac{\omega}{\beta} \qquad (\text{Phase velocity of the wave})$$

Basically ω is the multiply factor of frequency, f and β is multiply factor

of z or x or y.

So, we can conclude that expression given in option (C) does not satisfy equation (1) (i.e. the wave equation).

SOL 7.4.26 Option (D) is correct.

In a lossless dielectric ($\sigma = 0$) medium, impedance is given by

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}$$

where μ is permeability and ε is permittivity of the medium. So, we get

$$\eta = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}}$$

$$= 120\pi \times \sqrt{\frac{\mu_r}{\varepsilon_r}}$$

$$= 120\pi \times \sqrt{\frac{2}{8}} = 188.4\,\Omega$$

SOL 7.4.27 Option (A) is correct.

Given, the electric field intensity of the EM wave as

$$\boldsymbol{E} = 24e^{j(\omega t+\beta z)}\boldsymbol{a}_y \text{ V/m}$$

Now, the time average poynting vector for the EM wave is defined as

$$\mathcal{P} = \frac{1}{2}(\boldsymbol{E}_s \times \boldsymbol{H}_s^*) = \frac{|\boldsymbol{E}_s|^2}{2\eta}\boldsymbol{a}_k \qquad \left(\boldsymbol{H}_s = \frac{\boldsymbol{E}_s}{\eta}\right)$$

where η is the intrinsic impedance of the medium and $\boldsymbol{a}_k$ is the direction of wave propagation. Since, from the given expression of the field intensity we conclude that the wave is propagating along $-\boldsymbol{a}_z$ So, we have

$$\mathcal{P} = \frac{(24)^2}{2 \times 120\pi}(-\boldsymbol{a}_z) = -\frac{2.4}{\pi}\boldsymbol{a}_z \qquad (\boldsymbol{a}_k = -\boldsymbol{a}_z,\ |\boldsymbol{E}| = 24 \text{ V/m})$$

SOL 7.4.28 Option (B) is correct.

Given the propagation constant of the wave

$$\gamma = \alpha + j\beta = 0.1\pi + j0.2\pi$$

So, we get $$\beta = 0.2\pi$$

or, $$\frac{2\pi}{\lambda} = 0.2\pi$$

Therefore, wavelength of the propagating wave is

$$\lambda = \frac{2}{0.2} = 10 \text{ m}$$

SOL 7.4.29 Option (A) is correct.

Skin depth of the conducting medium at frequency, $f_1 = 10$ MHz is given as

$$\delta = \frac{1}{\sqrt{\pi f_1\mu\sigma}}$$

or $$10^{-2} = \frac{1}{\sqrt{\pi \times 10 \times 10^6 \times \mu\sigma}} \qquad (f_1 = 10 \text{ MHz})$$

or, $$\mu\sigma = \frac{10^{-3}}{\pi}$$

Now, phase velocity at another frequency ($f_2 = 1000$ MHz) is

$$v_p = \sqrt{\frac{4\pi f_2}{\mu\sigma}}$$

Putting $\mu\sigma = 10^{-3}/\pi$ in the above expression, we get

$$v_p = \sqrt{\frac{4 \times \pi \times 1000 \times 10^6 \times \pi}{10^{-3}}} \simeq 6 \times 10^6 \text{ m/s}$$

SOL 7.4.30 Option (C) is correct.
Reflected power P_r of a plane wave in terms of incident power P_i is defined as

$$P_r = |\Gamma|^2 P_i \quad (1)$$

where, Γ is the reflection coefficient at the medium interface given as

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \quad (2)$$

where η_1 and η_2 are the intrinsic impedance of the two mediums (air and glass) respectively. Since, the refractive index of the glass is 1.5

i.e. $$\eta_2 = c\sqrt{\mu_2 \varepsilon_2} = 1.5 \quad (3)$$

where $$\mu_2 = \mu_0 \quad \text{(Permeability of glass)}$$
$$\varepsilon_2 = \varepsilon_r \varepsilon_0 \quad \text{(Permittivity of glass)}$$

So, putting these values in equation (3) we get

$$\sqrt{\varepsilon_r} = 1.5$$

and $$\eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{\eta_0}{1.5}$$

Therefore, from equation (2) we have

$$\Gamma = \frac{\frac{\eta_0}{1.5} - \eta_0}{\frac{\eta_0}{1.5} + \eta_0} = \frac{1 - 1.5}{1 + 1.5} = -\frac{1}{5} \quad \text{(for free space } \eta_1 = \eta_0\text{)}$$

Thus, from equation (1) the reflected power is given as

$$P_r = \left(\frac{1}{5}\right)^2 \times P_i$$

or, $$\frac{P_r}{P_i} = 4\%$$

SOL 7.4.31 Option (A) is correct.
Skin depth of a material is defined as

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

Putting the given values in the expression, we get

$$\delta = \frac{1}{\sqrt{3.14 \times 1 \times 10^9 \times 4\pi \times 10^{-7} \times 10^6}} = 15.9\,\mu\text{m}$$

SOL 7.4.32 Option (C) is correct.
The energy density in a medium having electric field intensity $\boldsymbol{E}$ is defined as

$$w_E = \frac{1}{2}\varepsilon |E|^2 \qquad \text{where } \varepsilon \text{ is permittivity of the medium.}$$

So, due to the field $E = 100\sqrt{\pi}$ V/m in free space, the energy density is

$$w_E = \frac{1}{2}(8.85 \times 10^{-12})(100\sqrt{\pi})^2$$
$$= 1.39 \times 10^{-7}\ \text{J/m}^3 = 139\ \text{nJ/m}^3$$

SOL 7.4.33 Option (C) is correct.
For a uniform plane wave propagating in free space, the fields $\boldsymbol{E}$ and $\boldsymbol{H}$ are every where normal to the direction of wave propagation $\boldsymbol{a}_k$ and their direction are related as

$$\boldsymbol{a}_k \times \boldsymbol{a}_E = \boldsymbol{a}_H$$

i.e. the angel between electric field ($\boldsymbol{a}_E$) and magnetic field vector ($\boldsymbol{a}_H$) is always $90°$.

SOL 7.4.34 Option (B) is correct.
The incidence angle of an EM wave for which there is no reflection is called Brewster's angle. For the vertically polarized wave (parallel polarized wave) the Brewster angle is defined as

$$\tan\theta_{B\|} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

So, for the given dielectric medium we get

$$\tan\theta_{B\|} = \sqrt{\frac{9}{4}}$$

or, $$\theta_{B\|} = \tan^{-1}\left(\frac{3}{2}\right)$$

SOL 7.4.35 Option (B) is correct.
Given, the electric field component of the EM wave propagating in free space,

$$\boldsymbol{E} = 10\cos(10^7 t + kz)\boldsymbol{a}_y\ \text{V/m}$$

The general equation of electric field component of an EM wave propagating in $\boldsymbol{a}_z$ direction is given as

$$\boldsymbol{E} = E_0\cos(\omega t + kz)\boldsymbol{a}_y\ \text{V/m}$$

So, we conclude that the EM wave is propagating in $\boldsymbol{a}_z$ direction.

$$\omega = 10^7\ \text{rad/s}$$

or $$2\pi f = 10^7$$
$$f = \frac{10^7}{2\pi}$$

So, $$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{10^7} \times 2\pi = 188.5\ \text{m}$$

i.e. wavelength of the wave is

wave amplitude, $E_0 = 10$ V/m

wave number, $$k = \frac{2\pi}{\lambda} = \frac{2\pi}{60\pi} = \frac{1}{30}$$
$$= 0.033\ \text{rad/m}$$

The wave doesn't attenuate as it travels. So, statement (2) and (3) are correct.

SOL 7.4.36 Option (B) is correct.
The incidence angle of a plane wave for which there is no reflection is called Brewster's angle. For the parallel polarized wave, Brewster's angle is given as

$$\tan\theta_{B\|} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

where ε_1 and ε_2 are the permittivity of two mediums respectively.
So, for the given parallel polarized plane wave the incidence angle (θ_i) for no reflection is given as

$$\tan\theta_i = \sqrt{\frac{\varepsilon_0}{81\varepsilon_0}}$$

or, $$\theta_i = \tan^{-1}\left(\frac{1}{9}\right)$$

Therefore, the angle α for no reflection is

$$\alpha = 90° - \theta_i$$
$$= 83.66°$$

SOL 7.4.37 Option (C) is correct.

Given, the characteristic impedance of air,

$$\eta = 360\,\Omega$$
$$E_x = 3\sin(\omega t - \beta z)\ \text{V/m}$$
$$E_y = 6\sin(\omega t - \beta z + 75^\circ)\ \text{V/m}$$

So, the time average power per unit area is

$$\mathcal{P}_{ave} = \frac{1}{2}\frac{|E|^2}{\eta} = \frac{1}{2}\times\frac{(|E_x|^2+|E_y|^2)}{360}$$
$$= \frac{1}{2}\times\frac{(3^2+6^2)}{360}$$
$$= 6.25\times10^{-2}\ \text{W/m}^2$$
$$= 62.5\ \text{mW/m}^2$$

SOL 7.4.38 Option (A) is correct.

Operating frequency $\quad f = 3\ \text{GHz} = 3\times10^9\ \text{Hz}$

Medium parameters, $\quad \mu = 4\pi\times10^{-7}\ \text{H/m}$

$$\varepsilon = 10^{-9}/36\pi$$
$$\sigma = 5.8\times10^7\ \text{S/m}$$

So, we have intrinsic impedance defined as

$$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}} = \frac{\sqrt{\frac{4\pi\times10^{-7}}{(10^{-9}/36\pi)}}}{\left[1+\left(\frac{5.8\times10^7}{2\pi\times3\times10^9\times\frac{10^{-9}}{36\pi}}\right)^2\right]^{1/4}}$$
$$= 2.02\times10^{-2}\,\Omega$$

The phase angle of intrinsic impedance is given as

$$\theta_\eta = \frac{1}{2}\tan^{-1}\left(\frac{\sigma}{\omega\varepsilon}\right) = \frac{1}{2}\times\tan^{-1}\left(\frac{5.8\times10^7}{2\pi\times3\times10^9\times\frac{10^{-9}}{36\pi}}\right)$$
$$= \frac{1}{2}\times\frac{\pi}{2} = \frac{\pi}{4}$$

So, $\quad \eta = |\eta|e^{j\theta_\eta} = 0.02e^{j\pi/4}\,\Omega$

SOL 7.4.39 Option (D) is correct.

Given, the electric field of a plane wave,

$$\boldsymbol{E} = 50\sin(10^8 t + 2z)\boldsymbol{a}_y\ \text{V/m}$$

Comparing it with the general expression electric field of a plane wave travelling in $\boldsymbol{a}_z$ direction given as

$$\boldsymbol{E} = E_0\sin(\omega t - \beta z)\boldsymbol{a}_y$$

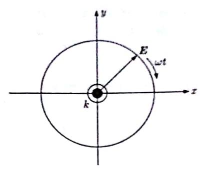
We get the direction of propagation of the given plane wave is $-\boldsymbol{a}_z$.

SOL 7.4.40 Option (C) is correct.

Given the electric field,

$$\boldsymbol{E} = (\boldsymbol{a}_x + j\boldsymbol{a}_y)e^{-j\beta z}$$

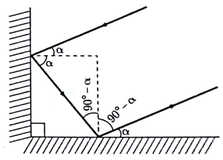
So, it is clear that y-component of field leads the x-component by 90° and the wave propagates in z-direction. The components are same. So, the tip of electric field traverse in circular path in the clockwise direction and wave propagates in z-direction as shown in figure.



Therefore, it is negative circularly polarized wave or (left hand polarized wave).

SOL 7.4.41 Option (D) is correct.

Consider the reflector is of angle $\theta = 90^\circ$ for which the incident and reflected wave is shown in figure.



So, it is clear that the incident and reflected wave both makes same angle α with the x-axis i.e. reflected wave in same direction.

SOL 7.4.42 Option (A) is correct.

Since, after reflection the phase of both x and y components will be reversed so the reflected wave will be also right circularly polarised.

SOL 7.4.43 Option (C) is correct.

Given,

Electric field intensity of the wave $\quad \boldsymbol{E} = 10\cos(6\pi\times10^8 t - bz)\boldsymbol{a}_x$

Permeability of medium, $\quad \mu = \mu_0$

Permittivity of medium, $\quad \varepsilon = 81\varepsilon_0$

From the expression of the electric field, we get the angular frequency as

$$\omega = 6\pi\times10^8$$

The phase velocity of the wave is given as

$$v_p = \frac{1}{\sqrt{\mu\varepsilon}}$$
$$= \frac{1}{\sqrt{\mu_0\times81\varepsilon_0}} = \frac{3\times10^8}{9} = \frac{10^8}{3} \quad \left(c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = 3\times10^8\ \text{m/s}\right)$$

So, the phase constant of the EM wave is

$$\beta = \frac{\omega}{v_p} = \frac{6\pi\times10^8}{10^8/3}$$
$$= 18\pi\ \text{rad/m}$$

SOL 7.4.44 Option (A) is correct.

Given, the phase velocity of the plane wave in dielectric is 0.4 times its value in free space

i.e. $$v_p = 0.4c \qquad (1)$$

Since, the phase velocity of a medium having permittivity ε and permeability μ is defined as

$$v_p = \frac{1}{\sqrt{\mu\varepsilon}}$$

So, putting it in equation (1) we get

$$\frac{1}{\sqrt{\mu_0\varepsilon_r\varepsilon_0}} = 0.4c \qquad (\mu = \mu_0\mu_r, \varepsilon = \varepsilon_0)$$

$$\varepsilon_r = \left(\frac{1}{0.4}\right)^2 = 6.25 \qquad \left(c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}\right)$$

SOL 7.4.45 Option (B) is correct.

Given, the electric field in free space,

$$\boldsymbol{E}(x,t) = 60\cos(\omega t - 2x)\boldsymbol{a}_y \text{ V/m}$$

So, we get the magnitude of the electric field as

$$E_0 = 60$$

The time average power density in the electric field is given as

$$\mathcal{P}_{ave} = \frac{1}{2}\frac{E_0^2}{\eta_0} = \frac{1}{2}\times\frac{(60)^2}{120\pi}$$

Therefore, the average power through the circular area of radius 4 m is

$$P_{ave} = (\mathcal{P}_{ave})\times(\pi r^2)$$

$$= \frac{1}{2}\times\frac{(60)^2}{120\pi}\times\pi(4)^2 = 240 \text{ W}$$

SOL 7.4.46 Option (C) is correct.

The relation between electric and magnetic field of the reflected, transmitted and incident wave is given below.

$$E_i = \eta_1 H_i$$

$$E_r = -\eta_1 H_r$$

$$E_t = \eta_2 H_t$$

So, (1) and (3) are correct while (2) is incorrect.

SOL 7.4.47 Option (D) is correct.

From snell's law,

$$n_1\sin\theta_1 = n_2\sin\theta_2$$

$$\sqrt{\mu_0\varepsilon_1}\sin\theta_1 = \sqrt{\mu_0\varepsilon_0}\sin\theta_2$$

$$\sqrt{\mu_0(2\varepsilon_2)}\sin 60° = \sqrt{\mu_0\varepsilon_2}\sin\theta_2$$

$$\sin\theta_2 = \left(\sqrt{2}\times\frac{\sqrt{3}}{2}\right) = \sqrt{1.5} > 1$$

which is not possible so there will be no transmitted wave.

SOL 7.4.48 Option (C) is correct.

(1) Consider $\boldsymbol{E}_1$ is x-component and $\boldsymbol{E}_2$ is y-component so, when $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$ will be in same phase. The wave will be linearly polarized.
(a → 1)

(2) When $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$ will have any arbitrary phase difference then it will be elliptically polarized.
(d → 2)

(3) When $\boldsymbol{E}_1$ leads $\boldsymbol{E}_2$ by 90° then ωt increases counter clockwise and so the wave is right circularly polarized.
(c → 3)

(4) When $\boldsymbol{E}_1$ lags $\boldsymbol{E}_2$ by 90° then the tip of field vector $\boldsymbol{E}$ will traverse circularly in clockwise direction and left circularly polarized.
(b → 4)

SOL 7.4.49 Option (A) is correct.

(a) Propagation constant for a perfect conductor is

$$\gamma = \alpha + j\beta$$

where $\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$ a → 1

(b) Radiation intensity of an antenna is defined as

$$U(\theta,\phi) = r^2\mathcal{P}_{ave}$$

$$= r^2\frac{|E|^2}{2\eta} = \left(\frac{r^2}{2\eta}\right)|E|^2 \qquad \text{b} \to 2$$

(c) Wave impedance of an EM wave is defined as

$$E_\theta = \eta H_\phi$$

$$\eta = \frac{E_\theta}{H_\phi} \qquad \text{c} \to 3$$

SOL 7.4.50 Option (D) is correct.

An incident wave normal to a perfect conductor is completely reflected in the reverse direction. The magnetic field intensity of reflected wave is same as the incident wave whereas the electric field intensity of reflected wave has the 180° phase difference in comparison to the incident field. ($\Gamma = -1$ for conducting surface).

SOL 7.4.51 Option (A) is correct.

Given,

Electric field intensity, $\boldsymbol{E} = E_x\boldsymbol{a}_x + E_y\boldsymbol{a}_y$

The direction of wave propagation, $\boldsymbol{a}_k = \boldsymbol{a}_z$

So, the magnetic field intensity of the EM wave is given as

$$\boldsymbol{H} = \frac{\boldsymbol{a}_k}{\eta}\times\boldsymbol{E}$$

where, η is the intrinsic impedance of the medium. Putting the expression for electric field in equation, we get

$$\boldsymbol{H} = \frac{\boldsymbol{a}_z}{\eta}\times(E_x\boldsymbol{a}_x + E_y\boldsymbol{a}_y) = \frac{1}{\eta}(E_x\boldsymbol{a}_y - E_y\boldsymbol{a}_x)$$

SOL 7.4.52 Option (A) is correct.

In a uniform plane wave the field intensities are related as

$$E = \eta H$$

where η is intrinsic impedance given as

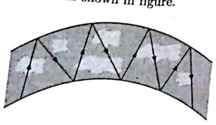
$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

Assume the medium is perfectly dielectric ($\sigma = 0$). So, we get

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}$$

or, $$\frac{E}{H} = \sqrt{\frac{\mu}{\varepsilon}}$$

SOL 7.4.53 Option (B) is correct.
The higher frequency (microwave) signal is continuously refracted on the ground as shown in figure.



This phenomenon is called ducting.

SOL 7.4.54 Option (D) is correct.
Given, the magnetic field intensity of a plane wave,

$$\boldsymbol{H} = 0.5e^{-0.1x}\cos(10^6 t - 2x)\boldsymbol{a}_z \quad (1)$$

The general expression for magnetic field intensity of a plane wave travelling in positive x-direction is

$$\boldsymbol{H} = H_0 e^{-\alpha x}\cos(\omega t - \beta x)\boldsymbol{a}_z \quad (2)$$

Comparing the equation (1) and (2) we get,

Wave frequency, $\omega = 10^6\ \text{rad/sec}$

Wavelength, $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{2} = 3.14\ \text{m}$

and the wave travels in $+x$-direction.
Since, the magnetic field intensity points toward $\boldsymbol{a}_z$ direction and the wave propagates in $+\boldsymbol{a}_x$ direction. So, direction of electric field intensity will be

$$\boldsymbol{a}_E = -\boldsymbol{a}_k \times \boldsymbol{a}_H = -(\boldsymbol{a}_x \times \boldsymbol{a}_z) = \boldsymbol{a}_y$$

Therefore, the wave is polarized in $\boldsymbol{a}_y$ direction (direction of electric field intensity).

SOL 7.4.55 Option (D) is correct.
From Maxwell's equation, For a varying magnetic field $\boldsymbol{B}$, the electric field intensity $\boldsymbol{E}$ is defined as

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

Since, the magnetic flux density $\boldsymbol{B}$ in terms of magnetic vector potential is given as

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}$$

So, from the two equations we have

$$\boldsymbol{E} = -\frac{\partial \boldsymbol{A}}{\partial t} \quad (\text{For } \nabla V = 0)$$

Given, $\boldsymbol{A} = \boldsymbol{a}_z A_z \sin(\omega t - \beta z)$

So,
$$\boldsymbol{E} = -\frac{\partial}{\partial t}[\boldsymbol{a}_z A_z \sin(\omega t - \beta z)]$$
$$= -\boldsymbol{a}_z \omega A_z \cos(\omega t - \beta z)$$

SOL 7.4.56 Option (A) is correct.
Given, the magnetic field intensity of the wave propagating in free space,

$$\boldsymbol{H}(z,t) = -\frac{1}{6\pi}\cos(\omega t + \beta z)\boldsymbol{a}_y$$

So, we conclude as

direction of propagation, $\boldsymbol{a}_k = -\boldsymbol{a}_z$
direction of magnetic field, $\boldsymbol{a}_H = \boldsymbol{a}_y$
So, the direction of electric field intensity is given as

$$\boldsymbol{a}_E = \boldsymbol{a}_H \times \boldsymbol{a}_k$$
$$= \boldsymbol{a}_y \times (-\boldsymbol{a}_z) = -\boldsymbol{a}_x$$

and the electric field amplitude is given as,

$$E = \eta_0 H$$
$$= (120\pi)\left(-\frac{1}{6\pi}\cos(\omega t + \beta z)\right)$$
$$= -20\cos(\omega t + \beta z)$$

So, the electric field vector of EM wave is

$$\boldsymbol{E}(z,t) = 20\cos(\omega t + \beta z)\boldsymbol{a}_x$$

SOL 7.4.57 Option (A) is correct.
Given, the electric field intensity of EM wave in phase form as

$$\boldsymbol{E}_s = 10e^{-j4y}\boldsymbol{a}_x$$

So, we get
the phase constant, $\beta = 4\ \text{rad/m}$
Since, the wave is propagating in free space, therefore, the angular frequency ω of the wave is given as

$$\omega = c\beta$$
$$= (3 \times 10^8)(4) = 4 \times 3 \times 10^8\ \text{rad/s}$$

SOL 7.4.58 Option (A) is correct.
A and R both true and R is correct explanation of A.

SOL 7.4.59 Option (B) is correct.
Skin depth of a material is defined as

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

Since, conductivity of the material is $\sigma = 0$.
So, we get $\delta \rightarrow$ infinity

SOL 7.4.60 Option (B) is correct.
(1) In a conducting medium as the wave travels its amplitude is attenuated by the factor $e^{-\alpha z}$ (i.e. attenuated exponentially).
(2) Conducting medium doesn't behave as on open circuit to the EM field.
(3) In lossless dielectric ($\sigma = 0$) relaxation time is defined as

$$T_r = \frac{\varepsilon}{\sigma} \rightarrow \infty$$

(4) In charge free region ($\rho_v = 0$). Poissions equation is generalised as

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon}$$
$$\nabla^2 V = 0$$

which is Laplace equation. Therefore only statement 2 is incorrect.

SOL 7.4.61 Option (B) is correct.
For a given electric field in free space the average power density is defined as

$$\mathcal{P}_{ave} = \frac{1}{2}\frac{|E|^2}{\eta_0} = \frac{1}{2}\frac{(60\pi)^2}{120\pi} = 15\pi\ \text{Watt/m}^2$$

SOL 7.4.62 Option (B) is correct.

Given, $E = 120\pi\cos(\omega t - \beta z)\boldsymbol{a}_x$

Since, the wave is propagating in $\boldsymbol{a}_z$ directions so; the magnetic flux density of the propagating wave is

$$H = \frac{\boldsymbol{a}_k \times \boldsymbol{E}}{\eta_0} = \frac{\boldsymbol{a}_z \times [120\pi\cos(\omega t - \beta z)\boldsymbol{a}_x]}{\eta_0}$$

$$= \cos(\omega t - \beta z)\boldsymbol{a}_y \qquad (\boldsymbol{a}_k = \boldsymbol{a}_z)$$

Therefore, the average power density of an EM wave is defined as

$$\mathcal{P}_{ave} = \frac{1}{2}\mathrm{Re}\{\boldsymbol{E} \times \boldsymbol{H}^*\}$$

$$= \frac{1}{2}[(120\pi\cos(\omega t - \beta z)\boldsymbol{a}_x) \times (\cos(\omega t - \beta z)\boldsymbol{a}_y)]$$

$$= 60\pi\boldsymbol{a}_z$$

SOL 7.4.63 Option (A) is correct.

Given, the electric field intensity is

$$\boldsymbol{E} = 10\sin(3\pi \times 10^8 t - \pi z)\boldsymbol{a}_x + 10\cos(3\pi \times 10^8 t - \pi z)\boldsymbol{a}_y$$

So, the magnetic field intensity is given as

$$\boldsymbol{H} = \frac{\boldsymbol{a}_k \times \boldsymbol{E}}{\eta_0} \qquad \text{(Direction of propagation is } \boldsymbol{a}_k = \boldsymbol{a}_z)$$

$$= \frac{10}{377}\sin(3\pi \times 10^8 t - \pi z)\boldsymbol{a}_y + \frac{10}{377}\cos(3\pi \times 10^8 t - \pi z)(-\boldsymbol{a}_x)$$

SOL 7.4.64 Option (D) is correct.

(1) For a perfect conducting medium the transmission coefficient is zero but a medium having finite conductivity transmission coefficient has some finite value. So it doesn't behave like an open circuit to the electromagnetic field.

(2) Relaxation time in a medium is defined as

$$T_r = \frac{\varepsilon}{\sigma}$$

Which in turn given the values in the range of 10^{-20} sec. While the radio frequency wave has the time period 'T' in the range of nsec to psec. (10^{-9} to 10^{-12}) So the relation time at radio frequency/microwave frequency is much less than the period.

(3) For a lossless dielectric ($\sigma = 0$) and so,

$$T_r = \frac{\varepsilon}{\sigma} \to \infty$$

(4) Intrinsic impedance of a perfect dielectric ($\sigma = 0$) is

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} \text{ which is a pure resistance.}$$

So, the statement (2), (3) and (4) are correct.

SOL 7.4.65 Option (A) is correct.

In free space electrons and photon both have the same velocity 3×10^8 m/s. So, the velocity of electromagnetic waves is same as velocity of light. So A and R both are true and R is the correct explanation of A.

SOL 7.4.66 Option (C) is correct.

From Maxwell's equation for an EM field, the divergence of the magnetic flux density is zero.

i.e.

$$\nabla \cdot \boldsymbol{B} = 0$$
$$\nabla \cdot (\nabla \times \boldsymbol{A}) = 0$$
$$\text{div curl } \boldsymbol{A} = 0$$

SOL 7.4.67 Option (A) is correct.

Electric field intensity due to the current element is defined as

$$\boldsymbol{E} = \frac{\boldsymbol{J}}{\sigma} = \frac{I}{\pi b^2 \sigma}\boldsymbol{a}_z$$

The magnetic flux density due to the current element is given as

$$\boldsymbol{H} = \frac{I}{2\pi b}\boldsymbol{a}_\phi$$

So, the poynting vector of the field is

$$\mathcal{P} = \boldsymbol{E} \times \boldsymbol{H}$$

$$= -\frac{I^2}{2\pi^2 b^3 \sigma}\boldsymbol{a}_\rho = -\frac{I^2}{2\sigma\pi^2 b^3}\boldsymbol{i}_r$$

SOL 7.4.68 Option (D) is correct.

All the three statements are correct.

SOL 7.4.69 Option (C) is correct.

Wavelength of a plane wave in any medium is defined as

$$\lambda = \frac{v_p}{f}$$

where v_p = phase velocity

f = frequency of the wave

Since, $v_p = \frac{c}{\sqrt{\varepsilon_r}}$

So, $\lambda \propto \frac{1}{\sqrt{\varepsilon_r}}$

$$\frac{\lambda_{air}}{\lambda_{dielectric}} = \sqrt{\frac{\varepsilon_{r,dielectric}}{\varepsilon_{r,air}}}$$

$$\frac{2}{1} = \sqrt{\frac{\varepsilon_r}{1}}$$

$$\varepsilon_r = 4$$

SOL 7.4.70 Option (D) is correct.

The velocity of an EM wave in free space is given as

$$v_c = C = 3 \times 10^8 \text{ m/s}$$

and the characteristic impedance (intrinsic impedance) is given as

$$Z_c = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi$$

So both the terms are independent of frequency of the wave i.e. remain unchanged.

SOL 7.4.71 Option (D) is correct.

Given, electric field intensity

$$\boldsymbol{E} = 5\cos(10^9 t + 30z)\boldsymbol{a}_x$$

So, we conclude that,

$$\omega = 10^9, \text{ and } \beta = 30$$

and since $\beta = \frac{\omega}{v_p}$

$$\beta = \frac{\omega}{(c/\sqrt{\varepsilon_r})} \quad \text{(For non magnetic medium } v_p = \frac{c}{\sqrt{\varepsilon_r}}\text{)}$$

$$\varepsilon_r = \left(\frac{\beta c}{\omega}\right)^2 = \left(\frac{30 \times 3 \times 10^8}{10^9}\right)^2 = 81$$

SOL 7.4.72 Option (C) is correct.

For attenuation of the wave the medium must have some finite conductivity σ. In the given wave equation the term $\mu\sigma\frac{\partial E}{\partial t}$ involves σ so this term is responsible for the attenuation of the wave.

SOL 7.4.73 Option (B) is correct.

The statement 1, 3 and 4 are correct while statement 2 is incorrect as Gauss's law is applicable only for symmetrical geometry.

SOL 7.4.74 Option (D) is correct.

In a Good conductor
$$\beta = \sqrt{\pi f \mu \sigma}$$
So,
$$\text{phase velocity } v_p = \frac{\omega}{\beta} = 2\sqrt{\frac{\pi f}{\mu\sigma}}$$

SOL 7.4.75 Option (B) is correct.

Given, the electric field intensity of the plane wave is
$$E(t) = [E_1 \cos\omega t a_x - E_2 \sin\omega t a_y] e^{-jkz}$$
Since the components of the field are
$$|E_x| = E_1$$
and
$$|E_y| = E_2$$
i.e.
$$|E_x| \neq |E_y|$$
So, the wave is elliptically polarized.

SOL 7.4.76 Option (D) is correct.

For a lossy dielectric, skin depth is defined as
$$\delta = \frac{\lambda}{2\pi}$$
So, as the wavelength increases the depth of penetration of wave also increases.

i.e. Reason (R) is correct.

The Skin depth is the depth by which electric field strength reduces to $\frac{1}{e} = 37\%$ of its original value i.e. Assertion (A) is false.

SOL 7.4.77 Option (D) is correct.

The electromagnetic equation in terms of vector potential A is given as
$$\nabla^2 A - \mu\varepsilon\frac{\partial^2 A}{\partial t^2} = -\mu J$$

SOL 7.4.78 Option (A) is correct.

The wavelength of an EM wave propagating in a waveguide is defined as
$$\lambda = \frac{\lambda'}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$
where λ' is the wavelength of the wave in unbounded medium(free space), f_c is the cutoff frequency of the waveguide and f is the operating frequency. Now, for a propagating wave in the waveguide, the operating frequency is higher than the cutoff frequency.

i.e.
$$f\ f_c \Rightarrow f_c/f < 1$$
Putting it in equation (1) we get

$$\lambda < \lambda'$$
i.e. Wavelength of a propagating wave in a wave guide is smaller than the free space wavelength.

SOL 7.4.79 Option (B) is correct.

For a lossless dielectric medium
$$\sigma = 0$$
and propagation constant,
$$\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$
$$\alpha + j\beta = j\omega\sqrt{\mu\varepsilon}$$
$$\beta = \omega\sqrt{\mu\varepsilon} \quad \text{i.e.} \quad \beta \propto \sqrt{\varepsilon_r}$$

SOL 7.4.80 Option (C) is correct.

For a lossless medium ($\sigma = 0$) intrinsic impedance is defined as
$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}\sqrt{\frac{\mu_r}{\varepsilon_r}}$$
$$60\pi = 120\pi\sqrt{\frac{1}{\varepsilon_r}}$$
$$\varepsilon_r = 4$$

SOL 7.4.81 Option (C) is correct.

A field is said to be conservative if the curl of the field is zero.

SOL 7.4.82 Option (A) is correct.

Given, the magnetic field intensity,
$$H = 0.5e^{-0.1x}\sin(10^6 t - 2x) a_z \text{ A/m}$$
Comparing it with general expression of magnetic field intensity of wave propagating in a_x direction given as
$$H = H_0 e^{-\alpha x}\sin(\omega t - \beta x) a_z$$
We get

(i) the direction of wave propagation is a_x

(ii) $\alpha = 0.1$, $\beta = 2$

So, propagation constant $\gamma = \alpha + j\beta = 0.1 + j2$

(iii)phase velocity, $v_p = \frac{\omega}{\beta} = \frac{10^6}{2} = 5 \times 10^5$ m/s

(iv) $a_H = a_z$, $\quad a_k = a_x$

So, direction of polarization,
$$a_E = -(a_k \times a_H) = -(a_x \times a_z) a_y \text{ i.e. wave is polarized along } a_y.$$

SOL 7.4.83 Option (C) is correct.

Skin depth of any conducting medium is defined as
$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$
So, at a given frequency $\omega = 2\pi f$
$$\delta \propto \frac{1}{\sqrt{\mu}} \text{ and } \delta \propto \frac{1}{\sqrt{\sigma}}$$

SOL 7.4.84 Option (A) is correct.

Given, the electric field intensity of the plane wave,
$$E = 10\sin(10\omega t - \pi z) a_x + 10\cos(\omega t - \pi z) a_y$$

So, the field components are

$$E_x = 10\sin(10\omega t - \pi z)$$
$$E_y = 10\cos(\omega t - \pi z)$$

and since, $|E_x| = |E_y|$ So the polarization is circular.

SOL 7.4.85 Option (D) is correct.

In free space electric field intensity is defined as

$$\boldsymbol{E} = -\eta_0(\boldsymbol{a}_k \times \boldsymbol{H})$$

where $\boldsymbol{a}_k$ is unit vector in the direction of propagation.

Given, $\boldsymbol{H} = 0.10\cos(4\times 10^7 t - \beta z)\boldsymbol{a}_x$ A/m

So, the direction of propagation, $\boldsymbol{a}_k = \boldsymbol{a}_z$

and we have,

$$\boldsymbol{E} = -377[\boldsymbol{a}_z \times (0.10\cos(4\times 10^7 t - \beta z)\boldsymbol{a}_x)] \quad (\eta_0 = 377\,\Omega)$$
$$= -37.7\cos(4\times 10^7 t - \beta z)\boldsymbol{a}_y$$

SOL 7.4.86 Option (B) is correct.

Given the electric field in medium A is

$$\boldsymbol{E} = 100\cos(\omega t - 6\pi x)\boldsymbol{z}$$

In medium A, $\varepsilon_r = 4$, $\mu_r = 1$, $\sigma = 0$

In medium B, $\varepsilon_r = 9$, $\mu_r = 4$, $\sigma = 0$

So,

(a) intrinsic impedance of medium 'B' is

$$\eta_B = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{4\mu_0}{9\varepsilon_0}} = \frac{2}{3}\times 120\pi = 80\pi \qquad (a \to 2)$$

(b) Intrinsic impedance of medium 'A' is

$$\eta_A = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{4\varepsilon_0}} = \frac{1}{2}\times 120\pi = 60\pi$$

So, reflection coefficient,

$$\Gamma = \frac{\eta_B - \eta_A}{\eta_B + \eta_A} = \frac{80\pi - 60\pi}{80\pi + 60\pi} = \frac{1}{7} \qquad (b \to 3)$$

(c) Transmission coefficient,

$$\tau = \frac{2\eta_B}{\eta_B + \eta_A} = \frac{2\times 80\pi}{80\pi + 60\pi} = \frac{8}{7} \qquad (c \to 4)$$

(d) Phase shift constant of medium A is given from the field equation as

$$\beta = 6\pi \qquad (d \to 1)$$

SOL 7.4.87 Option (B) is correct.

Average power density in an EM wave is defined as

$$\mathcal{P}_{ave} = \frac{1}{2}\mathrm{Re}(\boldsymbol{E}_s \times \boldsymbol{H}_s^*)$$
$$= \frac{1}{2}\times 50 \times \frac{5}{12\pi} = 3.316$$

So, the average power crossing a circular area of radius $\sqrt{24}$ m is

$$P_{ave} = \mathcal{P}_{ave}(\pi r^2)$$
$$= (3.316)\left(\pi(\sqrt{24})^2\right) = 250 \text{ Watt}$$

SOL 7.4.88 Option (A) is correct.

Electric field amplitude, $E_0 = 1$ V/m

Skin depth, $\delta = 10$ cm $= 0.1$ m

So, the attenuation constant of the wave in the conductor is

$$\alpha = \frac{1}{\delta} = 10$$

Now, the electric field intensity after travelling a distance z inside a conductor is

$$E = E_0 e^{-\alpha z}$$

where, E_0 is the field intensity at the surface of the conductor. So, the distance travelled by the wave for which amplitude of electric field changes to $(1/e^2)$ (V/m) is given as

$$E = \frac{E_0}{e^2}$$
$$E_0 e^{-10z} = \frac{E_0}{e^2}$$
$$10z = 2$$
$$z = 20 \text{ cm}$$

Alternatively, since the skin depth is the distance in which the wave amplitude decays to $(1/e)$ of its value at surface. So, for the amplitude to be $1/e^2$ of the field at its surface the wave penetrates a length of $2\delta = 20$ cm.

So A and R both are true and R is correct explanation of A.

SOL 7.4.89 Option (C) is correct.

For any media having conductivity, $\sigma = 0$. the intrinsic impedance is given as

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_r}{\varepsilon_r}}\eta_0$$

For media 1, $\eta_1 = \sqrt{\frac{2}{8}}(377) = 188\,\Omega$

For media 2, $\eta_2 = \sqrt{\frac{9}{1}}(377) = 1131\,\Omega$

for media 3, $\eta_3 = \sqrt{\frac{4}{4}}(377) = 377\,\Omega$

SOL 7.4.90 Option (A) is correct.

For an EM wave a medium incident on another medium, reflection coefficient is defined as

$$\Gamma = \frac{E_r}{E_i} = -\frac{H_r}{H_i}$$

and

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{2Z - Z}{2Z + Z} = \frac{1}{3}$$

So,

$$\frac{E_r}{E_i} = -\frac{H_r}{H_i} = \frac{1}{3}$$
$$\frac{E_i}{E_r} = 3 \text{ and } \frac{H_i}{H_r} = -3$$

SOL 7.4.91 Option (A) is correct.

For a perfect conductor conductivity, $\sigma = \infty$

So, the skin depth of the perfect conductor is

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = 0$$

****