2. i) skin depth(δ)

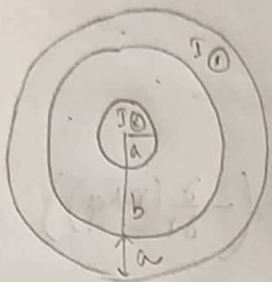
$$= \frac{1}{\sqrt{\pi f \sigma \mu}}$$

$\rho = 1.7 \times 10^{-8}\ \Omega m$
$\mu = 1.256 \times 10^{-6}\ H/m$

$$= \sqrt{\frac{\rho}{\pi f \mu}} \quad \left(\rho = \frac{1}{\sigma}\right)$$

For $f = 1GHz$ $\delta = 2.075\ \mu m$
For $f = 10GHz$ $\delta = 0.656\ \mu m$
For $f = 100GHz$ $\delta = 0.207\ \mu m$

ii)



It is said that current is restricted between a depth 3δ.
We came same current I flows through both

$$J = J_s \exp\left[-\frac{(1+j)d}{\delta}\right]$$

In the region $0 \le \rho \le a-3d$

$$B * 2\pi\rho = 0$$

Or $B = 0$

In the region $a-3\delta \le \rho \le \cancel{3\delta}\, a$

$$I = \int_{a-3\delta}^{a} J_s \exp\left[-\frac{(1+j)d}{\delta}\right] \cancel{\rho\, d\rho}\ 2\pi\rho\, d\rho$$

$$\text{or } I = 2\pi J_s \int_{a-3\delta}^{a} \exp\left[-\frac{(1+j)(a-\rho)}{\delta}\right] \rho\, d\rho$$

$$\text{or } I = 2\pi J_s\ \frac{j}{2} e^{-3(1+j)} \delta \left[\left\{e^{3(1+j)} - (4+3j)\right\}\delta - (1+j)\left\{e^{3(1+j)} - 1\right\} a\right]$$

$$I_{enc} = 2\pi J_s \left[\frac{j}{2}\delta\ \left\{\delta - (1+j)\rho\right\} e^{-\frac{(1+j)(a-\rho)}{\delta}} + \frac{1}{2} e^{-3(1+j)}\delta\ \left\{(3-4j)\delta - (1-j)a\right\}\right]$$

$$\frac{I_{enc}}{I} = \frac{j\left(\delta - (1+j)\rho\right)\exp\left[-\frac{(1+j)(a-\rho)}{\delta}\right] + e^{-3(1+j)}\left[(3-4j)\delta - (\right.}{j e^{-3(1+j)}\left[\left[-(4+3j) + e^{3(1+j)}\right]\delta - (1+j)\, a\left\{e^{3(1+j)} - 1\right\}\right]}$$

or $\dfrac{I_{enc}}{I} = \dfrac{[\delta-(1+j)\rho]\exp\left[-\dfrac{(1+j)(a-\rho)}{\delta}\right] + e^{-3(1+j)}[-(4+3j)\delta+(1+j)]}{e^{-3(1+j)}\left[\{e^{3(1+j)}-(4+3j)\}\delta - a(1+j)\{e^{3(1+j)}-1\}\right]}$

or $\dfrac{I_{enc}}{I} = \dfrac{e^{3(1+j)}[\delta-(1+j)\rho]\exp\left[\dfrac{-(1+j)(a-\rho)}{\delta}\right] + (1+j)-(4+3j)\delta}{\delta\{e^{3(1+j)}-(4+3j)\} - a(1+j)\{e^{3(1+j)}-1\}}$

$B\times 2\pi\rho = \mu_0 I_{enc}$

or $B = \dfrac{\mu_0 I}{2\pi\rho}\,\dfrac{e^{3(1+j)}[\delta-(1+j)\rho]\exp\left[\dfrac{-(1+j)(a-\rho)}{\delta}\right] + (1+j)-(4+3j)\delta}{\delta\{e^{3(1+j)}-(4+3j)\} - a(1+j)[e^{3(1+j)}-1]}$

In the region $a \le \rho \le a+b-3\delta$

$B\times 2\pi\rho = \mu_0 I$

or $B = \dfrac{\mu_0 I}{2\pi\rho}$

In the region $a+b-3\delta \le \rho \le a+b$

$$B = \frac{\mu_0 I}{2\pi\rho}\left[1 - \frac{e^{3(1+j)}[\delta-(1+j)\rho]\exp\left[\frac{-(1+j)(a+b-\rho)}{\delta}\right] + (1+j)-(4+3j)\delta}{\delta\{e^{3(1+j)}-(4+3j)\} - a(1+j)\{e^{3(1+j)}-1\}(a+b)}\right]$$

In the region $a+b \le \rho < \infty$

$B = 0$

With changing frequency the skin depth would change. However the expressions would remain same. Clearly the decay would be faster with increasing frequency and magnetic field would be confined more and more towards the surface and inbetween conductors.

iii) $\Psi_{ext} = \int \vec{B}\cdot d\vec{s}$

or $\Psi_{ext} = \dfrac{\mu_r\mu_0 I}{2\pi}\displaystyle\int_0^l\int_a^b \frac{1}{\rho}\, d\rho\, dz$

or $\Psi_{ext} = \dfrac{\mu_r\mu_0 I}{2\pi}\, l \ln\dfrac{b}{a}$

or $L_{ext} = \dfrac{\mu_r\mu_0 l}{2\pi}\ln\dfrac{b}{a} = \dfrac{\mu l}{2\pi}\ln\dfrac{b}{a}$ for length l of conductor.

For internal inductance of second conductor

$$d\lambda = \frac{d\psi}{I} I_{enc}$$

$$\frac{I_{enc}}{I} \approx 1 - \frac{\rho \exp\left[-\frac{(1+j)(a+b-\rho)}{\delta}\right]}{(b+a)\left[1-e^{-3(1+j)}\right]}$$

$$B = \frac{\mu_0}{2\pi\rho} I_{enc}$$

$$\text{or } B = \frac{\mu_0 I}{2\pi\rho} \frac{I_{enc}}{I}$$

$$d\lambda = B\, d\rho\, dz \frac{I_{enc}}{I}$$

$$\text{or } d\lambda = \frac{\mu_r\mu_0 I}{2\pi\rho} \left(\frac{I_{enc}}{I}\right)^2 d\rho\, dz$$

$$\text{or } L_{int2} = \int_{a+b-3\delta}^{a+b} \frac{\mu_r\mu_0 l}{2\pi\rho} \left(\frac{I_{enc}}{I}\right)^2 d\rho$$

$$\text{or } L_{int2} = \frac{\mu_r\mu_0 l}{2\pi} \int_{a+b-3\delta}^{a+b} \frac{1}{\rho}\left(1 - \frac{\rho \exp\left[-\frac{(1+j)(a+b-\rho)}{\delta}\right]}{(a+b)\left[1-e^{-3(1+j)}\right]}\right)^2 d\rho$$

$$\text{or } L_{int2} = \frac{\mu_r\mu_0 l}{2\pi} \int_{a+b-3\delta}^{a+b} \frac{1}{\rho}\left[1 - \frac{2\rho \exp\left[-\frac{(1+j)(a+b-\rho)}{\delta}\right]}{(a+b)\left[1-e^{-3(1+j)}\right]} + \frac{\rho^2 \exp\left(-\frac{2(1+j)(a+b-\rho)}{\delta}\right)}{(a+b)^2\left(1-e^{-3(1+j)}\right)^2}\right]$$

$$\text{or } L_{int2} = \frac{\mu_r\mu_0 l}{2\pi} \int_{a+b-3\delta}^{a+b} \frac{1}{\rho} - \frac{2\exp\left(-\frac{(1+j)(a+b-\rho)}{\delta}\right)}{(a+b)\left[1-e^{-3(1+j)}\right]} + \frac{\rho \exp\left(-\frac{2(1+j)(a+b-\rho)}{\delta}\right)}{(a+b)^2\left(1-e^{-3(1+j)}\right)^2}$$

$$\text{or } L_{int2} = \frac{\mu_r\mu_0 l}{2\pi}\left[\ln\frac{a+b}{a+b-3\delta} - \frac{2(1-j)\left[1-e^{-3(1+j)}\right]\delta}{2(a+b)\left[1-e^{-3(1+j)}\right]} + \frac{j/8\,\delta\, e^{-6(1+j)}}{(a+b)^2\left(1-e^{-3(1+j)}\right)^2}\left[\left\{e^{6(1+j)} - (7+6j)\right\}\delta - 2(1+j)(a+b)\left(e^{6(1+j)}-1\right)\right]\right]$$

$$\text{or } L_{int2} \approx \frac{\mu_r\mu_0 l}{2\pi}\left[\ln\left(\frac{a+b}{a+b-3\delta}\right) - \frac{\delta(1-j)}{a+b} + \frac{\delta j\, e^{-6(1+j)}}{8(a+b)^2\left[1-e^{-3(1+j)}\right]^2}\left(-2(1+j)(a+b)\left(e^{6(1+j)}-1\right)\right)\right]$$

$$\text{or } L_{int2} \approx \frac{\mu_r\mu_0 l}{2\pi}\left[-\ln\left(1-\frac{3\delta}{a+b}\right) + \frac{\delta(1-j)}{4(a+b)} \frac{1-e^{-6(1+j)}}{\left[1-e^{-3(1+j)}\right]^2}\right]$$

$$\text{or } L_{int2} \approx \frac{\mu_0 l}{2\pi} \ln\left(\frac{a+b}{a+b-3\delta}\right) \text{ or } -\frac{\mu_0 l}{2\pi}\ln\left(1-\frac{3\delta}{a+b}\right)$$

For internal inductance of first conductor

$d\lambda = d\Psi \frac{I_{enc}}{I}$

Since the expression is very complicated we can make approximations

$\delta << \rho$

$a - \rho \approx \delta$

$$\frac{I_{enc}}{I} = \frac{e^{3(1+j)}\left[\delta - (1+j)\rho\right] \exp\left[\frac{-(1+j)(a-\rho)}{\delta}\right] + (1+j) - (4+3j)\delta}{\delta\{e^{3(j+1)} - (4+3j)\} - a(1+j)(e^{3(j+1)} - 1)}$$

$$\text{or } \frac{I_{enc}}{I} \approx \frac{-e^{3(1+j)}(1+j)\rho \exp\left[\frac{-(1+j)(a-\rho)}{\delta}\right] + (1+j)}{-a(1+j)\left[e^{3(j+1)} - 1\right]}$$

$$\text{or } \frac{I_{enc}}{I} \approx \frac{e^{+3(1+j)}\rho \exp\left[-\frac{(1+j)(a-\rho)}{\delta}\right] + 1}{a\left[e^{+3(j+1)} - 1\right]}$$

$$\text{or } \frac{I_{enc}}{I} \approx \frac{\rho \exp\left[-\frac{(1+j)(a-\rho)}{\delta}\right] - 1}{a\left[1 - e^{-3(1+j)}\right]} \approx \frac{\rho \exp\left[-\frac{(1+j)(a-\rho)}{\delta}\right]}{a\left[1 - e^{-3(1+j)}\right]}$$

$$B \approx \frac{\mu_r \mu_0 I_{enc}}{2\pi\rho} \approx \frac{\mu_r \mu_0 I}{2\pi a} \frac{\exp\left[-\frac{(1+j)(a-\rho)}{\delta}\right] - 1/\rho}{1 - e^{-3(1+j)}}$$

$$\text{or } B \approx \frac{\mu_r \mu_0 I}{2\pi a(1 - e^{-3(1+j)})} \exp\left[-\frac{(1+j)(a-\rho)}{\delta}\right]$$

$$d\lambda = B\, d\rho\, dz \frac{I_{enc}}{I}$$

$$\text{or } d\lambda = \frac{\mu_r \mu_0 I\, d\rho\, dz}{2\pi a^2 (1 - e^{-3(1+j)})^2} \rho \exp\left[-\frac{2}{\delta}(1+j)(a-\rho)\right]$$

$$\text{or } \lambda = \frac{\mu_r \mu_0 I l}{2\pi a^2 (1 - e^{-3(1+j)})^2} \int_{a-3\delta}^{a} \rho \exp\left[-2\frac{(1+j)(a-\rho)}{\delta}\right] d\rho$$

or $L_{int1} = \frac{\mu_r \mu_0 l\, e^{-6(1+j)}\delta}{2\pi a^2 [1-e^{-3(1+j)}]^2} \left[\frac{j\delta}{8}\{e^{6(1+j)} - (7+6j)\}\delta \right.$

$\left. - 2(1+j)a\,(e^{6(1+j)}-1)\right]$

or $L_{int1} \approx \frac{\mu_r \mu_0 l}{2\pi a^2} \frac{e^{-6(1+j)}\delta}{[1-e^{-3(1+j)}]^2} \quad -2(1+j)a\,[e^{6(1+j)}-1]$

or $L_{int1} \approx \frac{\mu_r \mu_0 l\delta}{2\pi a}[1-e^{-6(1+j)}](1+j)$

or $L_{int1} \approx 0$ (since $\delta \ll a$)

∴ Thus the net internal inductance is

$L_{int} = L_{int1} + L_{int2}$

or $L_{int} \approx \frac{\mu_0 l}{2\pi} \ln\left(\frac{a+b}{a+b-3\delta}\right)$

or $L_{int} \approx \frac{\mu l}{2\pi} \ln\left(\frac{a+b}{a+b-3\delta}\right)$

For different frequencies the values of inductances can be determined. We assume the wire thickness is a μm.

For $f = 1\,GHz$

$L_{int} = \frac{\mu l}{2\pi} \ln\left(\frac{3.5a}{3.5a-3\delta}\right) = \left(1.02\ \frac{nH}{m}\right) l$

$L_{ext} = \frac{\mu l}{2\pi} \ln\frac{b}{a} = \frac{\mu l}{2\pi} \ln 2.5 = \left(0.183\ \frac{\mu H}{m}\right) l$

For $f = 10\,GHz$

$L_{int} = \frac{\mu l}{2\pi} \ln\left(\frac{3.5a}{3.5a-3\delta}\right) = \left(0.32\ \frac{nH}{m}\right) l$

$L_{ext} = \frac{\mu l}{2\pi} \ln 2.5 = \left(0.183\ \frac{\mu H}{m}\right) l$

For $f = 100\,GHz$

$L_{int} = \frac{\mu l}{2\pi} \ln\left(\frac{3.5a}{3.5a-3\delta}\right) = \left(0.101\ \frac{nH}{m}\right) l$

$L_{ext} = \frac{\mu l}{2\pi} \ln 2.5 = \left(0.183\ \frac{\mu H}{m}\right) l$