Electromagnetic Engineering (EC 21006)
T U T O R I A L - VI

MAGNETOSTATICS

1. A certain current density is given by
$$\vec{J} = 100e^{-2z}(\rho\,\hat{a}_\rho + \hat{a}_z)\ A/m^2$$
Find the total current passing through each of these surfaces
   a) $z = 0$, $0 \le \rho \le 1$, in the $\hat{a}_z$ direction
   b) $z = 1$, $0 \le \rho \le 1$, in the $\hat{a}_z$ direction
   c) closed cylinder defined by $0 \le z \le 1$, $0 \le \rho \le 1$, in an outward direction.

2. Let $\vec{J} = 400\frac{Sin\theta}{(r^2+4)}\hat{a}_r\ A/m^2$
   a) Find the total current flowing through that portion of the spherical surface $r = 0.8$, bounded by $0.1\pi < \theta < 0.3\pi, \theta < \phi < 2\pi$
   b) Find the average value of $\vec{J}$ over the defined area.

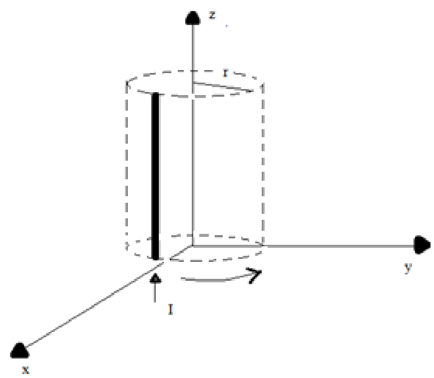
3. The current density in a certain region is approximated by
$$\vec{J} = (0.1/\mathrm{r})\ exp(-10^6 t)\ \hat{a}_r\ \ A/m^2\ .$$
   a) At $t = 1\mu s$, how much current is crossing the surface $r = 5$?
   b) Repeat for $r = 6$.
   c) Find $\rho_v(r,t)$ assuming that $\rho_v \to 0$ as $t \to \propto$.
   d) Find an expression for the velocity of the charge density.

4. A solid wire of conductivity σ_1 and radius a has a jacket of material having conductivity σ_2 and its inner and outer radii are respectively a and b. Find the ratio of current densities in the two materials.

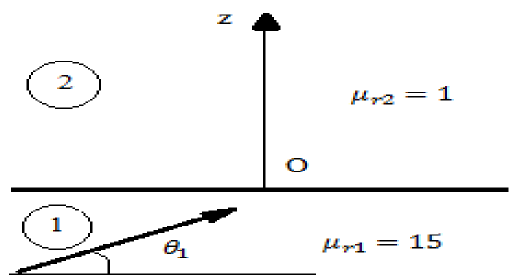
5. A conducting filament carries current I from point A (0, 0, *a)* to point B (0, 0, *b).* Show that at point *P(x, y,* 0),
$$\mathbf{H} = \frac{I}{4\pi\sqrt{x^2 + y^3}}\left[\frac{b}{\sqrt{x^2 + y^2 + b^2}} - \frac{a}{\sqrt{x^2 + y^2 + a^2}}\right]\mathbf{a}_y$$

6. A conductor of length 4m with current held at 10A in the $\boldsymbol{a}_y$ direction lies along the y-axis between y = $\pm$2m. If the field $\mathbf{B} = 0.05\boldsymbol{a}_x$ (T), find the work done in moving the conductor parallel to itself at a constant speed so that its center finally resides at x = z = 2m.

7. Find the work and power required to move the conductor (shown below) one full turn in the positive direction at a rotational frequency of N revolutions per minute, if $\boldsymbol{B} = B_o\boldsymbol{a}_r$ (where B_o is a positive constant).



8. In region 1 of figure given below, $\boldsymbol{B}_1 = 1.2\boldsymbol{a}_x + 0.8\boldsymbol{a}_y + 0.4\boldsymbol{a}_z$ (T). Find $\boldsymbol{H}_2$ at z = +0 and the angles between the field vectors and the tangent to the interface.



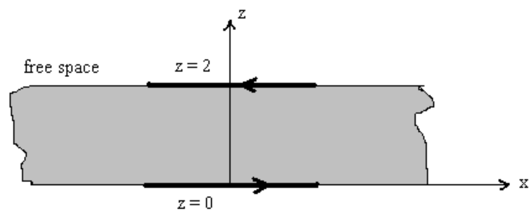
9. A current sheet, $\boldsymbol{K} = 9.0\boldsymbol{a}_y$ A/m, is located at z=0, the interface between region 1, z < 0, with $\mu_{r1} = 4$, and region 2, z > 0, $\mu_{r2} = 3$. Given that $\boldsymbol{H}_2 = 14.5\boldsymbol{a}_x + 8.0\boldsymbol{a}_z$ (A/m), find$\boldsymbol{H}_1$.

10. In a certain material for which μ = 6.5μ$_o$ and within which $\mathbf{H}=10\mathbf{a}_x+25\mathbf{a}_y-40\mathbf{a}_z$ A/m. Find
    a) the magnetic susceptibility χ_m of the material,
    b) relative permeability
    c) the magnetic flux density **B,**
    d) the magnetization vector **M**
    e) the free current density $\vec{J}$
    f) the bound current density $\vec{J_b}$
    g) the total current density $\vec{J_T}$
    h) the magnetic energy density.

11. In a certain region for which χ_m = 19, $\mathbf{H}=5x^2yz\mathbf{a}_x+10xy^2z\mathbf{a}_y-15xyz^2\mathbf{a}_z$ A/m.
    How much energy is stored in 0 < x < 1, 0 < y < 2,-1 < z < 2?

12. Region $0 \le z \le 2m$ is filled with an infinite slab of magnetic material ($\mu = 2.5\mu_o$).If the surface of the slab at z = 0 and z = 2, respectively, carry surface currents 30$\mathbf{a}_x$ A/m and -40$\mathbf{a}_x$ A/m as in figure below, calculate **H** and **B** for

a) z < 0,
b) 0 < z < 2, and
c) z > 2.



13. A toroid coil of 2000 turns is wound over a magnetic ring with inner radius of 10mm, outer radius of 15mm, height of 10mm, and relative permeability of 500; a very long, straight conductor passing through the center of the toroid carries a dc current. Determine the mutual inductance between the toroid and the straight conductor.

14. Two perfectly conducting cylindrical surfaces of length l are located at ρ = 3 and ρ = 5 cm. The total current passing radially outward through the medium between the cylinders is 3A dc.
a) Find the electric field voltage and resistance between the cylinders if a conducting material having $\sigma = 0.05\ S/m$ is present for 3 < ρ < 5 cm.
b) Find the total power dissipated in the system.

15. A balanced coaxial cable contains three coaxial conductors of negligible resistance. Assume a solid inner conductor of radius a, an intermediate conductor of inner radiusb_i, outer radius b_0, and an outer conductor having inner and outer radii c_i and c_0, respectively. The intermediate conductor carries current I in the positive a_z direction and is at potential V_0. The inner and outer conductors are both at zero potential and carry currents I/2 (in each) in the negative a_z direction. Assuming that the current distribution in each conductor is uniform, find: (a) J in each conductor; (b) H everywhere; (c) E everywhere.

16. When x, y, and z are positive and less than 5, a certain magnetic field intensity may be expressed as $H = [x^2yz/(y+1)]\boldsymbol{a_x} + 3x^2z^2\boldsymbol{a_y} - [xyz^2/(y+1)]\boldsymbol{a_z}$. Find the total current in the $\boldsymbol{a_x}$ direction that crosses the strip x = 2, $1 \le y \le 4$, $3 \le z \le 4$, by a method utilizing: (a) a surface integral; (b) a closed line integral.

17. a) The magnetic flux density in a region of free space is given as

$\vec{B} = -3x\,\hat{a}_x + 5y\,\hat{a}_y - 2z\,\hat{a}_z$

Find the total force on the rectangular loop if it lies in the plane *z* = 0 and is bounded by *x* = 1 , *x* = 3, *y* = 2 and *y* = 5, all dimension in cm.

b) The loop is now subjected to the $\vec{B}$ field produced by two current sheets $\vec{K}_1 = 400\,\hat{a}_y\,A/m$ at *z* = 2 and $\vec{K}_2 = 300\,\hat{a}_z\,A/m$ at *y* = 0 in free space. Find the vector torque on the loop, referred to an origin

i) at (0,0,0)

ii) at the center of the loop.

18. Calculate values for H_ϕ, B_ϕ and M_ϕ at $\rho = c$ for a coaxial cable with *a* = 2.5 mm and *b* = 6 mm if it carries a current *I* = 12 A in the center conductor, and $\mu = 3\mu H/m$ for $2.5\,mm < \rho < 3.5\,mm$, $\mu = 5\mu H/m$ for $3.5\,mm < \rho < 4.5\,mm$ and $\mu = 10\mu H/m$ for $4.5\,mm < \rho < 6\,mm$. Use *c* = : (a) 3 mm; (b) 4 mm; (c) 5 mm.

19. Conducting planes in air at *z* = 0 and *z* = *d* carry surface currents of $\pm K_0 \boldsymbol{a_x}$ A/m.

a) Find the energy stored in the magnetic field per unit length (0 < *x* < 1) in a width *w*(0 < *y* < *w*).

b) Calculate the inductance per unit length of this transmission line from $W_H = \frac{1}{2}\,LI^2$, where *I* is the total current in a width w in either conductor.

c) Calculate the total flux passing through the rectangle 0 < *x* < 1, 0 < *z* < *d*, in the plane *y*=0, and from this result again find the inductance per unit length.

20. A coaxial cable has conductor dimensions of 1 and 5 mm. The region between conductors is air for $0 < \phi < \frac{\pi}{2}$ and $\pi < \phi < \frac{3\pi}{2}$ and a non-conducting material having $\mu_r = 8$ for $\frac{\pi}{2} < \phi < \pi$ and $\frac{3\pi}{2} < \phi < 2\pi$. Find the inductance per meter length.

21. A rectangular coil is composed of 150 turns of a filamentary conductor. Find the mutual inductance in free space between this coil and an infinite straight filament on the z axis if the four corners of the coil are located at:

a) (0,1,0), (0,3,0), (0,3,1) and (0,1,1)

b) (1,1,0), (1,3,0), (1,3,1) and (1,1,1).