

CHAPTER 4

MAGNETOSTATIC FIELDS

4.1 INTRODUCTION

Magnetostatic fields are produced by the moving charges, (i.e., charges that are moving with constant velocity) or constant current flow. In this chapter we shall study the subject of magnetism that includes following topics:

- Concept of magnetic flux, magnetic flux density, and magnetic field intensity.
- Biot-Savart law, which defines the magnetic field at a point due to a differential current element.
- Ampere's circuit law, which defines the magnetic field in a loop.
- Magnetic field intensity due to various current distributions: straight line current, square current loop, solenoid, etc.
- Scalar and vector magnetic potential.

4.2 MAGNETIC FIELD CONCEPT

Steady magnetic fields are also called static magnetic fields or magnetostatic fields. The two opposite ends of a magnet are called its poles. If a magnet is floated freely, one pole will point towards the north pole and is called the north pole of the magnet, denoted by N . The other pole is the south pole, denoted by S , as shown in Figure 4.1. Following are some important terms related to magnetic field.

4.2.1 Magnetic Flux

Magnetic flux is the group of magnetic field lines emitted outward from the north pole of a magnet, as shown in Figure 4.1. It is measured in Weber and is denoted as Φ .

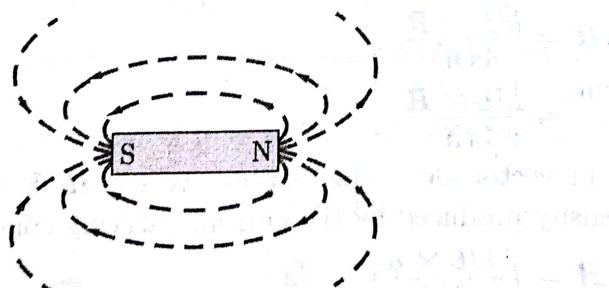


Figure 4.1: Magnetic Flux Lines from a Magnet

4.2.2 Magnetic Flux Density

Magnetic flux density is the amount of magnetic flux per unit area of a section, perpendicular to the direction of magnetic flux. It is a vector quantity and also known as the magnetic induction. The unit of magnetic flux density is Weber per squared metre (Wb/m^2) or Tesla (T). It is denoted by B . Mathematically,

$$B = \frac{d\Phi}{dS} a_n$$

where $d\Phi$ is a small amount of magnetic flux through small area dS of the section perpendicular to magnetic flux and a_n is the unit vector normal to the surface area. The above equation may be also expressed as

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{S}$$

i.e. the magnetic flux through any surface is the surface integral of the normal component of \mathbf{B} .

4.2.3 Magnetic Field Intensity

The degree to which a magnetic field can magnetise a material is called Magnetic field intensity or Magnetising force. It is a vector quantity and denoted by H . Its unit is Newton per Weber (N/Wb) or Ampere per metre (A/m).

4.2.4 Relation between Magnetic Field Intensity (H) and Magnetic Flux Density (B)

The magnetic field intensity is related to the magnetic flux density as

$$B = \mu H = \mu_0 \mu_r H$$

where, μ is the permeability of the medium, $\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$ is the permeability of free space, and μ_r is the relative permeability of the medium.

4.3 BIOT-SAVART'S LAW

When electric current flows through a conductor, it produces a magnetic field. Biot-Savart's law gives the magnetic field intensity produced due to a current element. According to the Biot-Savart's law, the magnetic field intensity dH produced at a point P due to a differential current element dL , shown in Figure 4.2, is given by

$$dH = \frac{IdL \sin \alpha}{4\pi R^2}$$

As we know, the magnitude of cross product $(IdL \times a_R)$ is equal to $(IdL \sin \alpha)$ so, in vector form the magnetic field intensity can be given as

$$\begin{aligned} dH &= \frac{IdL \times R}{4\pi R^2} \\ &= \frac{IdL \times R}{4\pi R^3} \end{aligned}$$

where a_R is the unit vector along the distance vector R . Hence, the total magnetic field intensity produced by the current carrying conductor is

$$\begin{aligned} H &= \int_L \frac{IdL \times a_R}{4\pi R^3} \\ &= \int_L \frac{IdL \times R}{4\pi R^3} \quad \dots(4.1) \end{aligned}$$

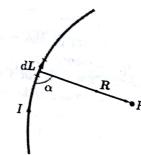


Figure 4.2: Illustration of Biot-Savart's Law

4.3.1 Direction of Magnetic Field Intensity

As the magnetic field intensity is the cross product of IdL and R so, its direction can be determined by either "Right-hand rule" or "Right handed screw rule".

1. Right-Hand Rule : The direction of dH can be determined by the right-hand rule with the right-hand thumb pointing in the direction of the current, the right-hand fingers encircling the wire in the direction of dH as shown in Figure 4.3(a).
2. Right Handed Screw Rule : The direction of dH can be determined also by the right handed screw rule, with the screw place along the wire and pointing in the direction of current flow, the direction of rotation of the screw in the direction of dH as shown in Figure 4.3(b).

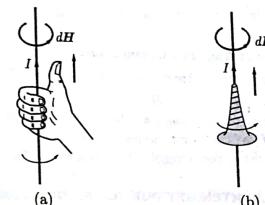


Figure 4.3 : Determination of Direction of Magnetic Field Intensity using (a) Right Hand Rule, (b) Right Handed Screw Rule

4.3.2 Conventional Representation of (H) or Current (I)

The direction of the magnetic field intensity (H) or current (I) is represented by a small circle with dot \odot if H or I is out of page or by a small circle with cross sign \otimes if H or I is into the page as shown in Figure 4.4.

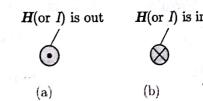


Figure 4.4 : Conventional Representation of H (or I) (a) Out of the Page (b) Into the Page

4.4 AMPERE'S CIRCUITAL LAW

Ampere's circuital law states that the line integral of the magnetic field intensity around any closed path is equal to the direct current enclosed by the path. The closed path on which Ampere's law is applied is known as *Amperian path* or *Amperian loop*. Following are the two mathematical forms of Ampere's circuital law:

Integral Form of Ampere's Circuital Law

If the total current enclosed by a closed loop L be I as shown in Figure 4.5, then from Ampere's circuital law the line integral of magnetic field intensity \mathbf{H} around the closed loop L is equal to I , i.e.

$$\oint_L \mathbf{H} \cdot d\mathbf{L} = I$$

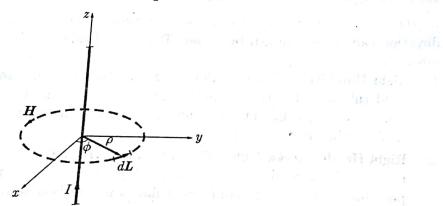


Figure 4.5: Illustration of Ampere's Circuital Law

Differential Form of Ampere's Circuital Law

In differential form Ampere's circuital law is defined as

$$\nabla \times \mathbf{H} = \mathbf{J}$$

i.e. the curl of the magnetic field intensity (\mathbf{H}) is equal to the current density (\mathbf{J}) at the point in space. If at any point, no current density exists, then the curl of the magnetic field is zero at that point.

4.5 MAGNETIC FIELD INTENSITY DUE TO VARIOUS CURRENT DISTRIBUTIONS

Similar to the different charge distribution discussed in previous chapters, we can have three types of current density distribution given as:

1. Line current density, I , given in Ampere,
2. Surface current density, K , given in Ampere per meter (A/m), and
3. Volume current density, J , given in Ampere per squared metre (A/m^2)

These current densities are related to each other as

$$IdL \equiv Kds \equiv Jdv$$

Thus, in terms of the distributed current sources, equation (4.1) becomes

$$\mathbf{H} = \int_L \frac{IdL \times \mathbf{a}_R}{4\pi R^2} \quad (\text{Line current})$$

$$\mathbf{H} = \int_S \frac{Kds \times \mathbf{a}_R}{4\pi R^2} \quad (\text{Surface current})$$

$$\mathbf{H} = \int_V \frac{Jdv \times \mathbf{a}_R}{4\pi R^2} \quad (\text{Volume current})$$

Now, we obtain a more generalised expression for some typical current distributions.

4.5.1 Magnetic Field Intensity due to a Straight Line Current

Consider a straight current carrying filamentary conductor of finite length AB located along the z -axis as shown in Figure 4.6. The current flows from point A to point B . The magnetic field intensity produced due to the straight line current is given by

$$\mathbf{H} = \frac{I}{4\pi\rho} (\cos\alpha_2 - \cos\alpha_1) \mathbf{a}_\phi \quad \dots(4.2)$$

where α_1 and α_2 are the angles subtended at point P by lower end A and upper end B respectively.

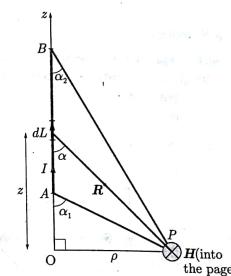


Figure 4.6: Magnetic Field Intensity due to a Straight Line Current

4.5.2 Magnetic Field Intensity due to an Infinite Line Current

A special case to the above expression is found when the conductor is infinite in length. For this case, point A is at $(0, 0, -\infty)$ while B is at $(0, 0, \infty)$. So, the angle subtended by its lower and upper ends respectively becomes

$$\alpha_1 = 180^\circ$$

$$\text{and} \quad \alpha_2 = 0^\circ$$

Therefore, equation (4.2) reduces to

$$\mathbf{H} = \frac{I}{2\pi\rho} \mathbf{a}_\phi$$

This expression can also be derived using Ampere's circuital law.

4.5.3 Magnetic Field Intensity due to a Square Current Carrying Loop

Consider a square loop of side $2a$ located in the $z=0$ plane and carrying a current I in the anti-clockwise direction as shown in Figure 4.7. The net magnetic field intensity at the origin due to the square current carrying loop is given by

$$\mathbf{H} = \frac{\sqrt{2}I}{\pi a} \mathbf{a}_z$$

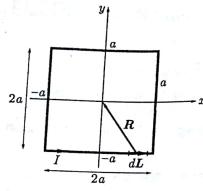
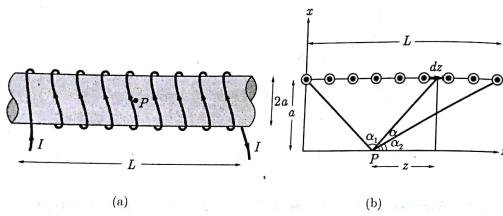


Figure 4.7: Magnetic Field Intensity due to Square Current Carrying Loop

4.5.4 Magnetic Field Intensity due to a Solenoid

Consider a short solenoid of length L and radius a as shown in Figure 4.8(a). It has n turns per metre of its length and it carries a current I . The cross-section of solenoid is shown in the Figure 4.8(b). Magnetic field intensity at point P due to the solenoid is given by

$$\mathbf{H} = \frac{nI}{2} [\cos \alpha_2 - \cos \alpha_1] \mathbf{a}_z$$

Figure 4.8: (a) Solenoid with n Turns per Unit Length, (b) Cross Section of Solenoid

4.5.5 Magnetic Field Intensity due to an Infinite Sheet of Current

Consider a thin infinite current carrying conductor plane having a uniform surface current density K . The magnetic field intensity due to the infinite sheet of current is given by

$$\mathbf{H} = \frac{1}{2} K \times \mathbf{a}_n$$

where \mathbf{a}_n is the unit vector normal to the surface directed towards the point of interest.

4.6 MAGNETIC POTENTIAL

Just like an electric potential, we can define a potential associated with magnetostatic field. In fact, the magnetic potentials are of two types:

1. Magnetic Scalar Potential, and
2. Magnetic Vector Potential

4.6.1 Magnetic Scalar Potential

From Ampere's law we know

$$\nabla \times \mathbf{H} = \mathbf{J}$$

If the current density \mathbf{J} is zero in some region of space, then we have

$$\nabla \times \mathbf{H} = 0$$

Since the curl of magnetic field intensity is zero, so we can write the magnetic field \mathbf{H} as the gradient of scalar quantity as

$$\mathbf{H} = -\nabla V_m \quad \dots(4.3)$$

where, V_m is called the *magnetic scalar potential*. Its unit is Ampere. Equation (4.3) can be expressed in integral form as

$$(V_m)_{z,y} = - \int_y^z \mathbf{H} \cdot d\mathbf{L}$$

Following are some important points related to magnetic scalar potential:

POINTS TO REMEMBER

1. In source free region ($\mathbf{J} = 0$), the magnetic scalar potential satisfies the Laplace's equation, i.e.,
- $\nabla^2 V_m = 0$
2. A scalar magnetic potential can't be uniquely defined. It means magnetostatic field is generally non-conservative and becomes conservative only when $\mathbf{J} = 0$
3. In the presence of steady currents which are sources of \mathbf{B} , a scalar magnetic potential can't be defined. Thus, the magnetic scalar potential is defined only in regions of space in the absence of currents.

4.6.2 Magnetic Vector Potential

From law of conservation of magnetic flux density, we know that the divergence of flux density is zero, i.e.,

$$\nabla \cdot \mathbf{B} = 0$$

Since, the divergence of the curl of any vector is zero. So, vector \mathbf{B} can be expressed as the curl of another vector field, (say \mathbf{A}), i.e.

$$\mathbf{B} = \nabla \times \mathbf{A}$$

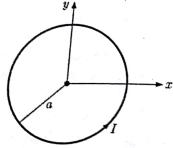
The vector field \mathbf{A} so defined is called the "vector magnetic potential". Its unit is weber per meter (Wb/m). Magnetic vector potential satisfies the Poisson's equation, i.e.

$$\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}$$

EXERCISE 4.1

- MCQ 4.1.1** Magnetic field intensity \mathbf{H} exists inside a certain closed spherical surface. The value of $\nabla \cdot \mathbf{H}$ will be
 (A) 0 at each point inside the sphere.
 (B) 0 at the center of the sphere only.
 (C) 0 at the outer surface of the sphere only.
 (D) Can't be determined as \mathbf{H} is not given.

- MCQ 4.1.2** A circular loop of radius a , centered at origin and lying in the xy plane, carries current I as shown in the figure.



- The magnetic field intensity at the centre of the loop will be
 (A) $\frac{I}{2a} \mathbf{a}_z$ (B) $-\frac{I}{2a} \mathbf{a}_z$
 (C) $\frac{I}{4a} \mathbf{a}_z$ (D) $\frac{2I}{a} \mathbf{a}_z$

- MCQ 4.1.3** In the free space a semicircular loop of radius a carries a current I . What will be the magnitude of magnetic field intensity at the centre of the loop?
 (A) $\frac{I}{a}$ (B) $\frac{2I}{a}$
 (C) $\frac{I}{4a}$ (D) $\frac{4I}{a}$

Common Data For Q. 4 and 5 :

A long cylindrical wire of cross sectional radius R carries a steady current I distributed over its outer surface.

- MCQ 4.1.4** Magnetic field intensity inside the wire at a distance $r (< R)$ from its center axes will be
 (A) non uniform
 (B) zero
 (C) uniform and depends on r only
 (D) uniform and depends on both r and R

- MCQ 4.1.5** The magnetic flux density outside the wire at a distance $r (> R)$ from its center axes will be proportional to

- (A) r
 (C) r/R
 (B) $1/r$
 (D) $1/R$

MCQ 4.1.6

- Two infinite current carrying sheets are placed parallel to each other in free space such that they carry current in the opposite direction with the same surface current density. The magnetic flux density in the space between the sheets will be
 (A) zero
 (B) constant
 (C) linearly increasing from one sheet to other
 (D) none of these

MCQ 4.1.7

- In a spherical co-ordinate system magnetic vector potential at point (r, θ, ϕ) given as $\mathbf{A} = 12 \cos \theta \mathbf{a}_\phi$. The magnetic flux density at point $(3, 0, \pi)$ will be
 (A) $4\mathbf{a}_\phi$ (B) 0
 (C) $4\mathbf{a}_\phi$ (D) $36\mathbf{a}_\phi$

MCQ 4.1.8

- An infinite plane current sheet lying in the plane $y = 0$ carries a linear current density $\mathbf{K} = K\mathbf{a}_x$ A/m. The magnetic field intensity above ($y > 0$) and below ($y < 0$) the plane will be

- | | |
|--|--|
| (A) $\frac{K}{2} \mathbf{a}_x$
(B) $-\frac{K}{2} \mathbf{a}_x$
(C) $-2Ka_x$
(D) $-\frac{K}{2} \mathbf{a}_y$ | $y > 0$
$y < 0$
$-\frac{K}{2} \mathbf{a}_x$
$\frac{K}{2} \mathbf{a}_x$
$2Ka_x$
$\frac{K}{2} \mathbf{a}_y$ |
|--|--|

Common Data For Q. 9 and 10 :

In a Cartesian system, vector magnetic potential at a point (x, y, z) is defined as

$$\mathbf{A} = 2x^2 y \mathbf{a}_z + 2y^2 x \mathbf{a}_z - 8xyz \mathbf{a}_w \text{ wb/m}$$

MCQ 4.1.9

- The magnetic flux density at point $(1, -2, -5)$ will be
 (A) $40\mathbf{a}_x + 6\mathbf{a}_z$ wb/m²
 (C) $-40\mathbf{a}_x - 80\mathbf{a}_y - 6\mathbf{a}_z$ wb/m²

- (B) $40\mathbf{a}_x + 80\mathbf{a}_y + 6\mathbf{a}_z$ wb/m²
 (D) $80\mathbf{a}_y - 6\mathbf{a}_z$ wb/m²

MCQ 4.1.10

- The total magnetic flux through the surface $z = 4$, $0 \leq x \leq 1$, $-1 \leq y \leq 4$ will be

- (A) 20 wb (B) $-10/3$ wb
 (C) 40 wb (D) $130/3$ wb

MCQ 4.1.11

- The current density that would produce the magnetic vector potential $\mathbf{A} = 2\mathbf{a}_\phi$ in cylindrical coordinates is

- | | |
|--|---|
| (A) $\frac{1}{\mu_0 r^2} \mathbf{a}_\phi$
(C) $\frac{2}{\mu_0 r} \mathbf{a}_\phi$ | $\frac{2r^2}{\mu_0} \mathbf{a}_\phi$
$\frac{2}{\mu_0 r^2} \mathbf{a}_\phi$ |
|--|---|

- MCQ 4.1.12** Magnetic field intensity produced due to a current source is given as

$$\mathbf{H} = (z \cos ay) \mathbf{a}_x + (z + e^y) \mathbf{a}_z$$

The current density over the xz plane will be

- (A) $(\mathbf{a}_x - \mathbf{a}_y - \mathbf{a}_z) A/m^2$
- (B) $-\mathbf{a}_x + \mathbf{a}_y - \mathbf{a}_z$
- (C) $-2\mathbf{a}_x + \mathbf{a}_y - 2\mathbf{a}_z$
- (D) $\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z$

- MCQ 4.1.13** Assertion (A) : In a source free region, magnetic field intensity can be expressed as a gradient of scalar function.

Reason (R) : Current density for a given magnetic field intensity is defined as

$$\mathbf{J} = \nabla \times \mathbf{H}$$

- (A) A and R both are true and R is correct explanation of A.
- (B) A and R both are true and R is not the correct explanation of A.
- (C) A is true but R is false.
- (D) R is true but A is false.

- MCQ 4.1.14** An electron beam of radius a travelling in \mathbf{a}_z direction, the current density is given as

$$\mathbf{J} = 2\left(1 - \frac{\rho}{a}\right) \mathbf{a}_z \quad \text{For } \rho < a$$

The magnetic field intensity at the surface of the beam will be

- (A) $\frac{a}{3} \mathbf{a}_\phi$
- (B) $\frac{a}{6} \mathbf{a}_\phi$
- (C) $\frac{2\pi a^2}{3} \mathbf{a}_\phi$
- (D) $\frac{a^2}{3} \mathbf{a}_\phi$

- MCQ 4.1.15** If there is a current filament on the x -axis carrying 4.4 A in \mathbf{a}_z direction then what will be the magnetic field intensity at point $(4, 2, 3)$?

- (A) $0.1(\mathbf{a}_x - 2\mathbf{a}_y) A/\text{m}$
- (B) $1.76\mathbf{a}_x - 1.62\mathbf{a}_y A/\text{m}$
- (C) $(-1.077\mathbf{a}_x + 1.62\mathbf{a}_y) A/\text{m}$
- (D) $-0.1(2\mathbf{a}_x - \mathbf{a}_y) A/\text{m}$

- MCQ 4.1.16** In the plane $z = 0$ a disk of radius $\sqrt{3} \text{ m}$, centered at origin carries a uniform surface charge density $\rho_s = 2 \text{ C/m}^2$. If the disk rotates about the z -axis at an angular velocity $\omega = 2 \text{ rad/s}$ then the magnetic field intensity at the point $P(0, 0, 1)$ will be

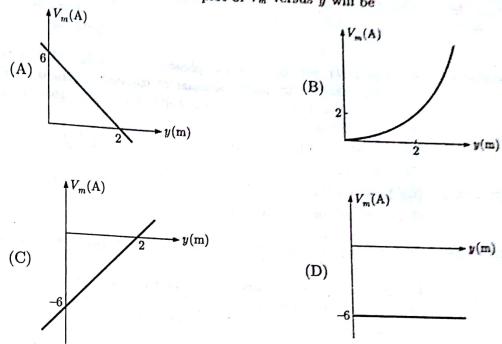
- (A) $\mathbf{a}_x A/\text{m}$
- (B) $2\mathbf{a}_x A/\text{m}$
- (C) $\mathbf{a}_y A/\text{m}$
- (D) $2\mathbf{a}_y A/\text{m}$

Common Data For Q. 17 and 18 :

In a Cartesian system two parallel current sheets of surface current density $K_1 = 3\mathbf{a}_x A/\text{m}$ and $K_2 = -3\mathbf{a}_x A/\text{m}$ are located at $x = 2 \text{ m}$ and $x = -2 \text{ m}$ respectively. The net vector and scalar potential due to the sheets are zero at a point $P(1, 2, 5)$.

- MCQ 4.1.17**

Consider the scalar potential at any point (x, y, z) in the region between the two planar sheets is V_m . The plot of V_m versus y will be



- MCQ 4.1.18**

The vector potential at origin will be

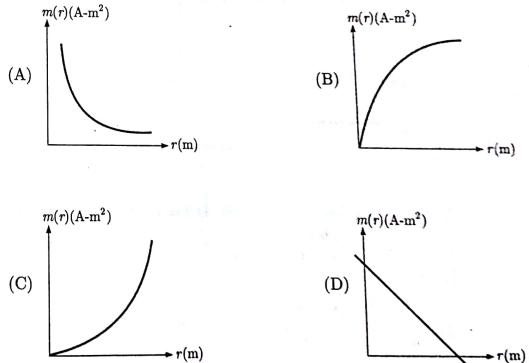
- (A) $3\mu_0 \mathbf{a}_x \text{ Wb/m}$
- (B) $-3\mu_0 \mathbf{a}_x \text{ Wb/m}$
- (C) 0
- (D) -3 Wb/m

Common Data For Q. 19 and 20 :

A uniformly charged solid sphere of radius r is spinning with angular velocity $\omega = 4 \text{ rad/s}$ about the z -axis. The sphere is centered at origin and carries a total charge 5 C which is uniformly distributed over its volume.

- MCQ 4.1.19**

The plot of magnetic dipole moment of the sphere, $m(r)$ versus the radius of the sphere, r will be



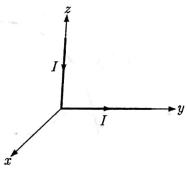
- MCQ 4.1.20** The average magnetic field intensity within the sphere will be
 (A) $\frac{2}{\pi r} \mathbf{a}_\theta$ (B) $\frac{2}{\pi r} \mathbf{a}_z$
 (C) $\frac{2}{r} \mathbf{a}_\theta$ (D) $\frac{1}{2\pi r} \mathbf{a}_z$

- MCQ 4.1.21** A rectangular coil, lying in the plane $x+3y-1.5z=3.5$ carries a current 7 A such that the magnetic moment of the coil is directed away from the origin. If the area of the rectangular coil is 0.1 m^2 then the magnetic moment of the coil will be
 (A) $-0.2\mathbf{a}_x - 0.2\mathbf{a}_y + 0.3\mathbf{a}_z \text{ A}\cdot\text{m}^2$
 (B) $2\mathbf{a}_x + 6\mathbf{a}_y - 3\mathbf{a}_z \text{ A}\cdot\text{m}^2$
 (C) $1.4\mathbf{a}_x + 4.2\mathbf{a}_y - 2.1\mathbf{a}_z \text{ A}\cdot\text{m}^2$
 (D) $0.2\mathbf{a}_x + 0.6\mathbf{a}_y - 0.3\mathbf{a}_z \text{ A}\cdot\text{m}^2$

- MCQ 4.1.22** Vector magnetic potential in a certain region of free space is $\mathbf{A} = (6y - 2z)\mathbf{a}_x + 4xz\mathbf{a}_y$. The electric current density at any point (x, y, z) will be
 (A) $(-8\mathbf{a}_x + 2\mathbf{a}_y + 6\mathbf{a}_z) \text{ A/m}^2$
 (B) $(3\mathbf{a}_x + \mathbf{a}_z) \text{ A/m}^2$
 (C) 0
 (D) $\frac{1}{\mu_0}(8\mathbf{a}_x + 2\mathbf{a}_y - 6\mathbf{a}_z) \text{ A/m}^2$

- MCQ 4.1.23** Magnetizing force at any point P on z -axis due to a semi infinite current element placed along positive x -axis is H . If one more similar current element is placed along positive y -axis then the resultant magnetizing force at the point P will be
 (A) $H/\sqrt{2}$
 (B) $\sqrt{2}H$
 (C) $2H$
 (D) $-\sqrt{2}H$

- MCQ 4.1.24** A L-shaped filamentary wire with semi infinite long legs making an angle 90° at origin and lying in $y-z$ plane as shown in the figure.



- If the current flowing in the wire is $I = 4 \text{ A}$ then the magnetic flux density at $(2\text{m}, 0, 0)$ will be
 (A) $-2 \times 10^{-7}(\mathbf{a}_y + \mathbf{a}_z) \text{ Wb/m}^2$
 (B) $2 \times 10^{-7}(\mathbf{a}_y + \mathbf{a}_z) \text{ Wb/m}^2$
 (C) $-4 \times 10^{-7}(\mathbf{a}_y + \mathbf{a}_z) \text{ Wb/m}^2$
 (D) $4 \times 10^{-7}(\mathbf{a}_y + \mathbf{a}_z) \text{ Wb/m}^2$

Common Data For Q. 25 and 26 :

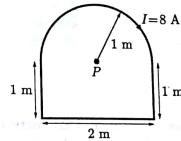
An infinitely long straight conductor of cylindrical cross section and of radius R carries a current I , which is uniformly distributed over the conductor cross section.

- MCQ 4.1.25** If the conductor is located along z -axis then the magnetic flux density at a distance $\rho (< R)$ from the cylindrical axis will be
 (A) $\frac{\mu_0 I_\rho}{2\pi R^2} \mathbf{a}_\phi$ (B) $\frac{\mu_0 I}{2\pi \rho} \mathbf{a}_\phi$
 (C) $\frac{\mu_0 I}{2\pi} \mathbf{a}_\phi$ (D) $\frac{\mu_0 I \rho^2}{R^2} \mathbf{a}_\phi$

- MCQ 4.1.26** Magnetic flux density at a distance $\rho (> R)$ from the cylindrical axis will be proportional to
 (A) $\frac{1}{\rho}$ (B) $\frac{1}{\rho^2}$
 (C) ρ (D) ρ^2

- MCQ 4.1.27** The magnitude of the magnetic field intensity produced at center of a square loop of side a carrying current I is
 (A) $\frac{2\sqrt{2}I}{\pi a}$ (B) $\frac{\sqrt{2}I}{\pi a}$
 (C) $\frac{I}{\sqrt{2}\pi a}$ (D) $\frac{8I}{\pi a}$

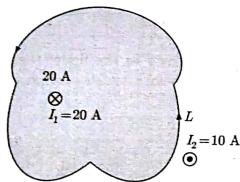
- MCQ 4.1.28** For the single turn loop of current shown in the figure the magnetic field intensity at the center point P of the semi circular portion will be



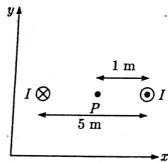
- (A) 5.8 A/m outward
 (B) 5.8 A/m inward
 (C) 3.8 A/m outward
 (D) 3.8 A/m inward

EXERCISE 4.2

- MCQ 4.2.1** A conducting filament carries a current 5 A from origin to a point $(0, 0, 12)$. Magnetic field intensity at point $(3, 4, 0)$ due to the filament current will be _____ wb/m^2 .
- MCQ 4.2.2** A circular conducting loop of radius 2 m, centered at origin in the plane $z = 0$ carries a current of 4 A in the a_x direction. What will be the magnetic field intensity at origin in a_x direction?
- MCQ 4.2.3** A long straight wire placed along z -axis carries a current of $I = 10 \text{ A}$ in the $+a_z$ direction. The magnetic flux density at a distance $\rho = 5 \text{ cm}$ from the wire will be _____ $\times 10^{-5} \text{ wb/m}^2$
- MCQ 4.2.4** For the currents and the closed path shown in the figure, $\oint \mathbf{H} \cdot d\mathbf{l} = \text{_____ Ampere}$.



- MCQ 4.2.5** Two infinitely long wires separated by a distance 5 m, carry currents I in opposite direction as shown in the figure. If $I = 8 \text{ A}$, then the magnetic field intensity at point P is _____ A/m in a_y direction.

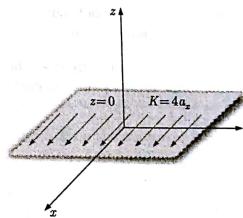


- MCQ 4.2.6** Two point charges Q_1 and Q_2 are located at $(0, 0, 0)$ and $(1, 1, 1)$ respectively. A current of 16 A flows from the point charge Q_1 to Q_2 along a straight wire connected between them. What will be the value of $\oint \mathbf{H} \cdot d\mathbf{l}$ (in A/m) around the closed path formed by the triangle having the vertices $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$?

- Common Data For Q. 7 and 8 :**
An infinite current sheet with uniform current density $K = 20 a_z \text{ A/m}$ is located in the plane $z = 2$.
- MCQ 4.2.7** The magnetic field intensity at origin will be _____ A/m in a_x direction.
- MCQ 4.2.8** Magnetic field intensity at point $(2, -1, 5)$ will be _____ A/m in a_x direction.
- MCQ 4.2.9** In the free space two cylindrical surfaces $\rho = 0.5 \text{ cm}$ and $\rho = 0.25 \text{ cm}$ carries the uniform surface current densities $2 a_z \text{ A/m}$ and $-0.8 a_z \text{ A/m}$ respectively and a current filament on the entire z -axis carries a current of 14 mA in the $+a_z$ direction. The surface current density on the cylindrical surface at $\rho = 8 \text{ cm}$ which will make the net magnetic field $H = 0$ for $\rho > 8 \text{ cm}$ will be _____ A/m in a_z direction.

Common Data For Q. 10 and 11 :

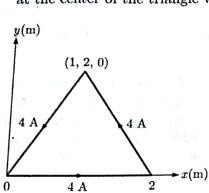
An infinite current sheet with uniform surface current density $K = 4 a_z \text{ A/m}$ is located at $z = 0$ as shown in figure.



- MCQ 4.2.10** Magnetic flux density at any point above the current sheet ($z > 0$) will be _____ $\times \mu_0 a_y \text{ wb/m}^2$.
- MCQ 4.2.11** The vector magnetic potential at $z = -2$ will be _____ $\times \mu_0 a_z \text{ wb/m}^2$.
- MCQ 4.2.12** In the free space, magnetic field intensity at any point (ρ, ϕ, z) is given by $H = 2\rho^2 a_\phi \text{ A/m}$. The current density at $\rho = 2 \text{ m}$ will be _____ A/m^2 in a_z direction.
- MCQ 4.2.13** Magnetic field intensity produced at a distance ρ from an infinite cylindrical wire located along entire z -axis is $3\rho a_z \text{ A/m}$. The current density within the conductor will be _____ A/m^2 in a_z direction.
- MCQ 4.2.14** A circular loop of wire with radius $R = 0.5 \text{ m}$ is located in plane $x = 0$, centered at origin. If the loop carries a current $I = 7 \text{ A}$ flowing in clockwise as viewed from negative x -axis then, its magnetic dipole moment will be _____ A-m^2 in a_z direction.
- MCQ 4.2.15** In the free space, the positive z -axis carries a filamentary current of 10 A in the $-a_z$ direction. Magnetic field intensity at a point $(0.2, 3)$ due to the

MCQ 4.2.16

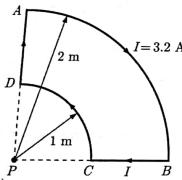
filamentary current will be _____ A/m in a_z direction.



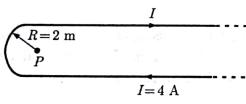
MCQ 4.2.17 A current sheet $K = 4a_z$ A/m flows in the region $-2 < z < 2$ in the plane $x = 0$. Magnetic field intensity at point $P(3,0,0)$ due to the current sheet will be _____ A/m in a_z direction.

MCQ 4.2.18 A square conducting loop of side 1 m carries a steady current of 2 A. Magnetic flux density at the center of the square loop will be _____ $\times 10^{-6}$ wb/m²

MCQ 4.2.19 A filamentary conductor is formed into a loop ABCD as shown in figure. If it carries a current of 3.2 A what is the magnitude of magnetic field intensity (in A/m) at point P?



MCQ 4.2.20 The magnetic field intensity at point P due to the steady current configurations shown in figure will be _____ A/m.



MCQ 4.2.21 In the plane $z = 5$ m a thin ring of radius, $a = 3$ m is placed such that z -axis passes through its center. If the ring carries a current of 50 mA in a_z direction then the magnetic field intensity at point $(0, 0, 1)$ will be _____ mA/m in a_z direction.

MCQ 4.2.22

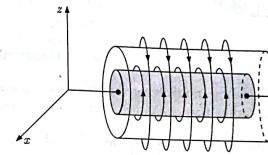
An infinite solenoid (infinite in both directions) consists of 1000 turns per unit length wrapped around a cylindrical tube. If the solenoid carries a current of 4 mA what will be the magnetic field intensity along its axis?

Common Data For Q. 23 to 25 :

The two long coaxial solenoids of radius a and b carry current $I = 3$ mA but in opposite directions. Solenoids are placed along y -axis as shown in figure. The inner solenoid has 2000 turns per unit length and outer solenoid has 1000 turns per unit length.

MCQ 4.2.23

Magnetic field intensity inside the inner solenoid will be _____ A/m directed along a_y .



MCQ 4.2.24

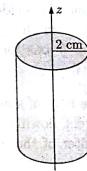
The magnetic field intensity in the region between the two solenoids will be _____ A/m directed along a_y .

MCQ 4.2.25

What will be the magnetic field (in A/m) outside the outer solenoid?

MCQ 4.2.26

A long cylindrical wire lying along z -axis carries a total current $I_0 = 5$ mA as shown in the figure. The current density inside the wire at a distance ρ from its axis is given by $J \propto \rho$. If the cross sectional radius of the wire is 2 cm, what will be the magnetic flux density (in wb/m²) at $\rho = 1$ cm?



MCQ 4.2.27

Magnetic field intensity is given in a certain region as

$$H = \frac{x^2yz}{1+x} a_x + 3x^2z^2 a_y - \frac{xyz^2}{y+1} a_z \text{ A/m}$$

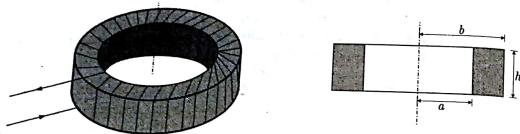
What is the total current (in Ampere) passing through the surface $x = 2$ m, $1 \leq y \leq 4$ m, $3 \leq z \leq 4$ m in a_z direction?

MCQ 4.2.28

A phonograph record of radius 1 m carries a uniform surface charge density $\rho_s = 20$ C/m². If it is rotating with an angular velocity $\omega = 0.1$ rad/s; what will be the magnetic dipole moment (in A - m²)

Common Data For Q. 29 and 30 :

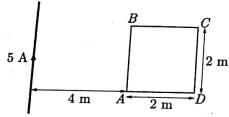
A circular toroid with a rectangular cross section of height $h = 10 \text{ m}$, carries a current $I = 10 \text{ A}$ flowing in 10^5 turns of closely wound wire around it as shown in figure. The inner and outer radii of toroid are $a = 1 \text{ m}$ and $b = 2 \text{ m}$ respectively.



MCQ 4.2.29 What will be the total magnetic flux (in weber) across the circular toroid ?

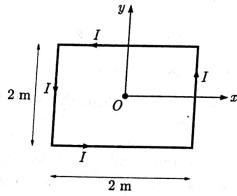
MCQ 4.2.30 If the magnetic flux is found by multiplying the cross sectional area by the flux density at the mean radius then what will be the percentage of error ?

MCQ 4.2.31 An infinitely long straight wire carrying current 5 A and a square loop of side 2 m are coplanar as shown in the figure. The distance between side AB of square loop and the straight wire is 4 m . What will be the total magnetic flux (in μwb) crossing through the rectangular loop ?



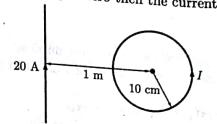
MCQ 4.2.32 A 0.5 m square loop is lying in $x-y$ plane such that one of its side is parallel to y -axis and the centre of the loop is 0.3 m away from the y -axis. How much Ampere current must flow through the entire y -axis for which the magnetic flux through the loop is $5 \times 10^{-5} \text{ Tesla m}^2$?

MCQ 4.2.33 Consider a filamentary wire is bent to form a square loop of side 2 m lying in the $x-y$ plane as shown in the figure. If the current flowing in the wire is $I = 1 \text{ A}$ then the magnetic flux density at the center of the loop will be $---$ $\times 10^{-7} \text{ A}_z \text{ Wb/m}^2$



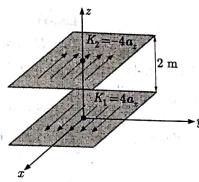
MCQ 4.2.34

An infinitely long straight wire carrying a current 20 A and a circular loop of the wire carrying a current I are coplanar as shown in the figure. The radius of the circular loop is 10 cm and the distance of the centre of the loop from the straight wire is 1 m . If the net magnetic field intensity at the centre of the loop is zero then the current $I =$ _____ Ampere.



MCQ 4.2.35

Two perfect conducting infinite parallel sheets separated by a distance 2 m carry uniformly distributed surface currents with equal and opposite densities $4a_z$ and $-4a_z$ respectively as shown in figure. The medium between the two sheets is free space. The magnetic flux between the sheets per unit length along the direction of current will be $k\mu_0 a_z \text{ Wb/m}$ such that the value of k is _____.



EXERCISE 4.3

- MCC 4.3.1 Assertion (A) :** For a static magnetic field the total number of flux lines entering a given region is equal to the total no. of flux lines leaving the region.
Reason (R) : An isolated magnetic charge doesn't exist.

(A) Both A and R one true and R is the correct explanation of A.
 (B) Both A and R are true but R is not the correct explanation of A.
 (C) A is true but R is false.
 (D) A is false but R is true.

Match List-I with List-II and select the correct answer using the codes given below. (Notations have their usual meaning).

List-I	List-II
a. Ampere's law	1. $\nabla \cdot D = \rho_v$
b. Conservative nature of magnetic field	2. $\int_S \mathbf{J} \cdot d\mathbf{S} = \int_L \mathbf{H} \cdot d\mathbf{l}$
c. Gauss's law	3. $\nabla \cdot B = 0$
d. Non existence of magnetic monopole	4. $\int E \cdot dl = 0$

Codes :

	a	b	c	d
(A)	3	1	4	2
(B)	2	1	4	3
(C)	2	4	1	3
(D)	3	4	2	1

- The source which doesn't cause a magnetic field is

 - (A) A charged disk rotating at uniform speed
 - (B) An accelerated charge
 - (C) A charged sphere spinning along its axis
 - (D) A permanent magnet

- MCQ 4.3.4** The correct configuration that represents magnetic flux lines of a magnetic dipole is



- MCQ 4.3.5** The correct configuration that represents current I and magnetic field intensity H is



- MCQ 4.3.7** If the magnetic field, $H = 4a_r$, A/m, flux density in free space is
 (A) $1.6\pi a_r \mu_0 \text{ wb/m}^2$ (B) $16\pi a_r \mu_0 \text{ wb/m}^2$
 (C) $1.6\pi a_r \mu_0 \text{ wb/m}$ (D) $160\pi a_r \mu_0 \text{ wb/m}$

- MCQ 4.3.8** $\oint (\nabla \times \mathbf{H}) \cdot d\mathbf{S}$

 - (A) zero
 - (B) I_{enc}
 - (C) I
 - (D) $\oint \mathbf{H} \cdot d\mathbf{S}$

- MCQ 4.3.10** The unit of scalar magnetic potential is
(A) Ampere
(C) Amp/m

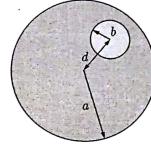
- MCQ 4.3.12** $\nabla \times \mathbf{A}$ is
 (A) \mathbf{H}
 (B) \mathbf{B}
 (C) \mathbf{J}
 (D) 0
- MCQ 4.3.13** Torque has the unit of
 (A) N-m
 (B) N/m
 (C) N-m²
 (D) N
- MCQ 4.3.14** $\oint \mathbf{B} \cdot d\mathbf{s}$ is
 (A) zero
 (B) Q
 (C) H
 (D) J
- MCQ 4.3.15** Absolute permeability of free space is
 (A) $4\pi \times 10^{-7} \text{ A/m}$
 (B) $4\pi \times 10^{-7} \text{ H/m}$
 (C) $4\pi \times 10^{-7} \text{ F/m}$
 (D) $4\pi \times 10^{-7} \text{ H/m}^2$

EXERCISE 4.4**Common Data For Q. 1 and 2 :**

An infinitely long uniform solid wire of radius a carries a uniform dc current of density J .

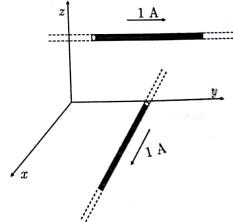
- MCQ 4.4.1** The magnetic field at a distance r from the center of the wire is proportional to
 (A) r for $r < a$ and $1/r^2$ for $r > a$
 (B) 0 for $r < a$ and $1/r$ for $r > a$
 (C) r for $r < a$ and $1/r$ for $r > a$
 (D) 0 for $r < a$ and $1/r^2$ for $r > a$

- MCQ 4.4.2** A hole of radius b ($b < a$) is now drilled along the length of the wire at a distance d from the center of the wire as shown below.

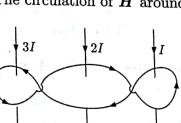


- The magnetic field inside the hole is
 (A) uniform and depends only on d
 (B) uniform and depends only on b
 (C) uniform and depends on both b and d
 (D) non uniform

- MCQ 4.4.3** Two infinitely long wires carrying current are as shown in the figure below. One wire is in the $y-z$ plane and parallel to the y -axis. The other wire is in the $x-y$ plane and parallel to the x -axis. Which components of the resulting magnetic field are non-zero at the origin?



- (A) x, y, z components
 (B) x, y components
 (C) y, z components
 (D) x, z components

- | | | | |
|-------------------|---|-------------------|---|
| MCQ 4.4.4 | A flux of 1.2 mWb exerts in a magnet having a cross-section of 30 cm^2 . The flux density in tesla is
(A) 4
(B) 0.4
(C) 2.5
(D) 40 | MCQ 4.4.11 | What is the magnetic field intensity vector \mathbf{H} between two parallel sheets with separation ' d ' along z-axis both sheets carrying surface current $K = K_y a_y$?
(A) $-k_y a_y$
(B) $+k_y a_y$
(C) $-k_y a_z$
(D) Zero |
| MCQ 4.4.5 | The magnetic flux density B and the vector magnetic potential A are related as
(A) $B = \nabla \times A$
(B) $A = \nabla \times B$
(C) $B = \nabla \cdot A$
(D) $A = \nabla \cdot B$ | MCQ 4.4.12 | Current density (J), in cylindrical coordinate system is given as :
$J(\rho, \phi, z) = \begin{cases} 0 & \text{for } 0 < \rho < a \\ J_0(\rho/a^2) a_z & \text{for } a < \rho < b \end{cases}$ <p>where a_z is the unit vector along z-coordinate axis. In the region, $a < r < b$, what is the expression for the magnitude of magnetic field intensity (H)?
 (A) $\frac{J_0}{\rho^2} (\rho^3 - a^3)$
 (B) $\frac{J_0}{\rho^2} (\rho^3 + a^3)$
 (C) $\frac{J_0}{3a^2 \rho} (\rho^3 - a^3)$
 (D) $\frac{J_0}{2\pi\rho} (\rho^3 - a^3)$</p> |
| MCQ 4.4.6 | Consider the following statements relating to the electrostatic and magnetostatic field :
1. The relative distribution of charges on an isolated conducting body is dependent on the total charge of the body.
2. The magnetic flux through any closed surface is zero.
Which of the above statements is/are correct ?
(A) Neither 1 nor 2
(B) 1 only
(C) 2 only
(D) Both 1 and 2 | MCQ 4.4.13 | Which one of the following concepts is used to find the expression of radiated E and H field due to a magnetic current element ?
(A) Concept of vector magnetic potential
(B) Concept of scalar electric potential
(C) Concept of scalar magnetic potential
(D) Concept of vector electric potential |
| MCQ 4.4.7 | The line integral of the vector potential A around the boundary of a surface S represents which one of the following?
(A) Flux through the surface S
(B) Flux density in the surface S
(C) Magnetic field intensity
(D) Current density | MCQ 4.4.14 | The circulation of \mathbf{H} around the closed contour C , shown in the figure is

(A) 0
(B) $2l$
(C) $4l$
(D) $6l$ |
| MCQ 4.4.8 | An infinitely long straight conductor located along z-axis carries a current I in the +ve z-direction. The magnetic field at any point P in the $x-y$ plane is in which direction?
(A) In the positive z-direction
(B) In the negative z-direction
(C) In the direction perpendicular to the radial line OP (in $x-y$ plane) joining the origin O to the point P
(D) Along the radial line OP | MCQ 4.4.15 | The unit of magnetic flux density is
(A) gauss
(B) tesla
(C) bohr
(D) weber/sec |
| MCQ 4.4.9 | A 5 A current enters a right circular cylinder of 5 cm radius. What is the linear surface current density at the end surface?
(A) $(50/\pi) \text{ A/m}$
(B) $(100/\pi) \text{ A/m}$
(C) $(1000/\pi) \text{ A/m}$
(D) $(2000/\pi) \text{ A/m}$ | MCQ 4.4.16 | The magnetic flux density created by an infinitely long conductor carrying a current I at a radial distance R is
(A) $\frac{\mu_0 I}{2\pi R}$
(B) $\frac{1}{2\pi R}$
(C) $\frac{\mu_0 I}{2\pi R^3}$
(D) $\frac{4\pi R^2 I}{3}$ |
| MCQ 4.4.10 | What is the value of the magnetic vector potential due to an infinitesimally small current element, evaluated at infinite distance from it ?
(A) Infinity
(B) Unity
(C) Zero
(D) Any number between zero and infinity depending on the strength of the current element | | |

MCQ 4.4.17 Match List I with List II and select the correct answer using the codes given below the lists.

- | List-I | List-II |
|----------------------------|-----------------|
| a. Work | 1. Ampere/metre |
| b. Electric field strength | 2. Weber |
| c. Magnetic flux | 3. Volt/metre |
| d. Magnetic field strength | 4. Joule |

Codes :

	a	b	c	d
(A)	4	3	2	1
(B)	1	3	2	4
(C)	4	2	3	1
(D)	1	2	3	4

- MCQ 4.4.18** A long straight wire carries a current $I = 1\text{ A}$. At what distance is the magnetic field 1 Am^{-1} ?
 (A) 1.59 m
 (B) 0.159 m
 (C) 0.0159 m
 (D) 0.00159 m

- MCQ 4.4.19** How much current must flow in a loop radius 1 m to produce a magnetic field 1 mAm^{-1} ?
 (A) 1.0 mA
 (B) 1.5 mA
 (C) 2.0 mA
 (D) 2.5 mA

- MCQ 4.4.20** Assertion (A) : Knowing magnetic vector potential \mathbf{A} at a point, the flux density \mathbf{B} at the point can be obtained.
 Reason (R) : $\nabla \cdot \mathbf{A} = 0$.
 (A) Both A and R are true and R is the correct explanation of A
 (B) Both A and R are true and R is not the correct explanation of A
 (C) A is true but R is false
 (D) A is false but R is true

- MCQ 4.4.21** The magnetic vector potential \mathbf{A} obeys which equations ?
 1. $\mathbf{B} = \nabla \times \mathbf{A}$ 2. $\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}$
 3. $\mathbf{A} = \int \frac{\mu_0 I dl}{4\pi R}$

- Select the correct answer using the code given below :
 (A) 1 and 2 (B) 2 and 3
 (C) 1 and 3 (D) 1, 2 and 3

- MCQ 4.4.22** A long straight wire carries a current $I = 10\text{ A}$. At what distance is the magnetic field $H = 1\text{ Am}^{-1}$?
 (A) 1.19 m
 (B) 1.39 m
 (C) 1.59 m
 (D) 1.79 m

MCQ 4.4.23 What is the magnetic field due to an infinite linear current carrying conductor ?

- (A) $H = \frac{\mu_0 I}{2\pi r}\text{ A/m}$
 (B) $H = \frac{I}{2\pi r}\text{ A/m}$
 (C) $H = \frac{\mu_0 I}{2r}\text{ A/m}$
 (D) $H = \frac{I}{r}\text{ A/m}$

- MCQ 4.4.24** Equation $\nabla \cdot \mathbf{B} = 0$ is based on
 (A) Gauss's Law
 (B) Lenz's Law
 (C) Ampere's Law
 (D) Continuity Equation

- MCQ 4.4.25** Plane $y = 0$ carries a uniform current density $30a_x\text{ mA/m}$. At $(1, 20, -2)$, what is the magnetic field intensity ?
 (A) $-15a_x\text{ mT}$
 (B) $15a_x\text{ mT}$
 (C) $18.85a_y\text{ mA/m}$
 (D) $25a_x\text{ mA/m}$

- MCQ 4.4.26** Which one of the following is not the valid expression for magnetostatic field vector \mathbf{B} ?
 (A) $\mathbf{B} = \nabla \times \mathbf{A}$
 (B) $\mathbf{B} = \nabla \times \mathbf{A}$
 (C) $\nabla \cdot \mathbf{B} = 0$
 (D) $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$

- MCQ 4.4.27** Which one of the following statements is correct ? Superconductors are popularly used for
 (A) generating very strong magnetic field
 (B) reducing $\vec{J}^2 R$ losses
 (C) generating electrostatic field
 (D) generating regions free from magnetic field

- MCQ 4.4.28** Assertion (A) : $\oint \mathbf{B} \cdot d\mathbf{S} = 0$ where, \mathbf{B} = magnetic flux density, $d\mathbf{S}$ = vector with direction normal to surface elements $d\mathbf{S}$.

Reason (R) : Tubes of magnetic flux have no sources or sinks.

- (A) Both A and R are true and R is the correct explanation of A
 (B) Both A and R are true but R is NOT the correct explanation of A
 (C) A is true but R is false
 (D) A is false but R is true

- MCQ 4.4.29** Plane defined by $z = 0$ carry surface current density $2a_z\text{ A/m}$. The magnetic intensity ' H_y ' in the two regions $-\alpha < z < 0$ and $0 < z < \alpha$ are respectively

- (A) a_y and $-a_y$
 (B) $-a_y$ and a_y
 (C) a_z and $-a_z$
 (D) $-a_z$ and a_z
