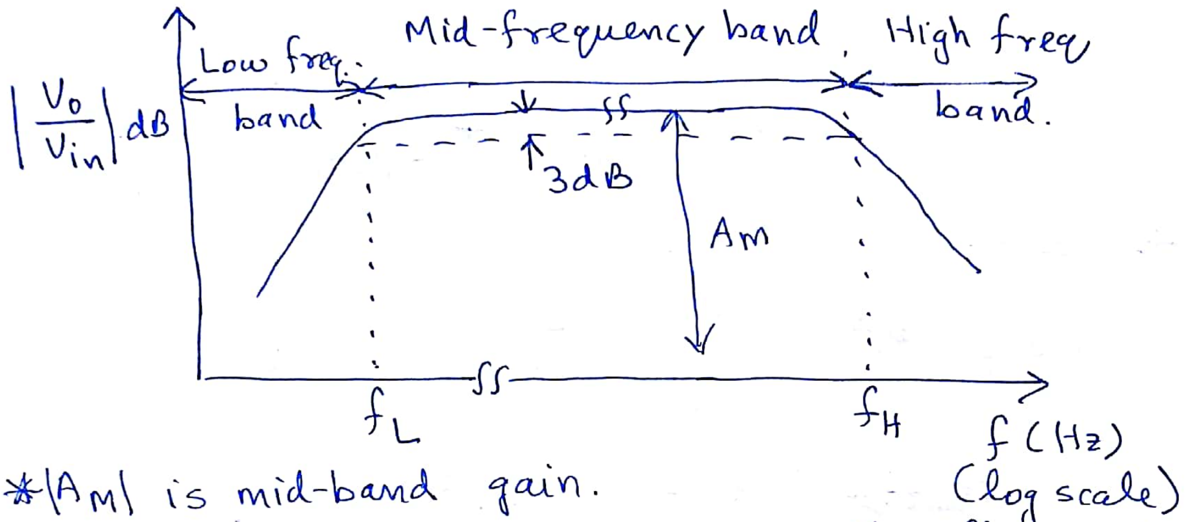
FREQUENCY RESPONSE

* Typical amplifier frequency response looks as follows:



* $|A_M|$ is mid-band gain.

* In low-frequency band gain rolls-off due to coupling & bypass capacitors.

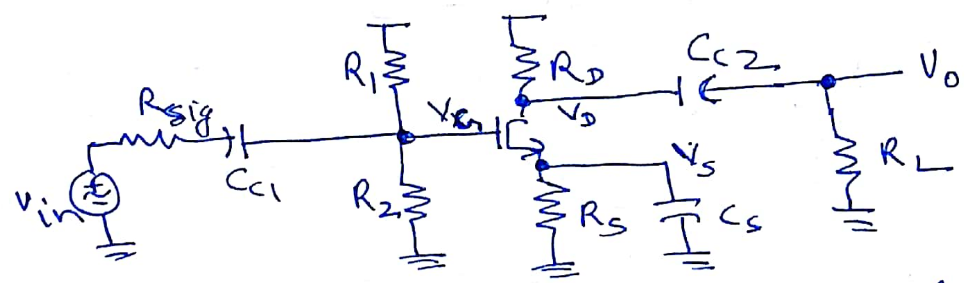
* In high-frequency band gain rolls-off due to internal parasitic capacitances of BJT & MOSFET.

* In mid-band gain is flat & all capacitors can be ignored/neglected. ⇒ coupling & bypass capacitors are considered as ∞ and parasitic capacitors as "0".

⇒ coupling & bypass capacitors ≈ ∞, i.e. short-circuit.

⇒ parasitic capacitors ≈ 0 i.e., open circuit

* Low-Frequency Response of CS Amplifier:-



⇒ Consider only C_{C1} and everything else as ∞ i.e., C_S and C_{C2} as perfect short.

Now, $V_g = V_{in} \dfrac{R_1 \| R_2}{(R_1 \| R_2) + R_{sig} + \frac{1}{sC_{c1}}}$

$$\Rightarrow V_g = V_{in} \frac{R_1 \| R_2}{R_1 \| R_2 + R_{sig}} \cdot \frac{s}{s + \frac{1}{C_{c1}(R_1 \| R_2 + R_{sig})}}$$

~~$\Rightarrow$ Zero at DC.~~

~~$\Rightarrow$ Pole at $\frac{1}{C_{c1}(R_1 \| R_2 + R_{sig})} = \omega_{PL1}$~~

Thus, $V_o = -g_m V_g \cdot (R_D \| R_L)$.

$$\Rightarrow V_o = \underbrace{-g_m (R_D \| R_L) \frac{R_1 \| R_2}{R_1 \| R_2 + R_{sig}}}_{|A_M|} \cdot \frac{s}{s + \frac{1}{C_{c1}(R_1 \| R_2 + R_{sig})}}$$

$$\Rightarrow V_o = -g_m (R_D \| R_L) \frac{R_1 \| R_2}{(R_1 \| R_2 + R_{sig})} \left[\frac{s\, C_{c1}(R_1 \| R_2 + R_{sig})}{1 + s C_{c1}(R_1 \| R_2 + R_{sig})} \right]$$

$\Rightarrow$ Zero at DC.

$\Rightarrow$ Pole at $\omega_{PL1} = \dfrac{1}{C_{c1}[R_1 \| R_2 + R_{sig}]}$.

* Consider only C_s and C_{c1} and C_{c2} as short

$$V_g = V_{in} \left[\frac{R_1 \| R_2}{R_{sig} + (R_1 \| R_2)} \right]$$

Now, $\dfrac{V_o}{V_g} = -\dfrac{\text{Impedance at Drain}}{\left[\text{Impedance at Source} + \frac{1}{g_m}\right]}$

$$\Rightarrow \frac{V_o}{V_g} = -\frac{R_D \| R_L}{\left(\frac{1}{sC_s} \| R_s\right) + \frac{1}{g_m}}.$$

$$\Rightarrow \frac{V_o}{V_g} = -\frac{R_D \parallel R_L}{\frac{\frac{1}{sC_s} R_s}{\frac{1}{sC_s} + R_s} + \frac{1}{g_m}}$$

$$\Rightarrow \frac{V_o}{V_g} = -\frac{(R_D \parallel R_L)}{\frac{1}{g_m} + \frac{R_s}{sC_s} \cdot \frac{sC_s}{sR_sC_s + 1}}$$

$$\Rightarrow \frac{V_o}{V_{in}} = -V_{in} \frac{(R_1 \parallel R_2)}{(R_1 \parallel R_2 + R_{sig})} \cdot \frac{R_D \parallel R_L}{\frac{1}{g_m} + \frac{R_s}{1 + sC_sR_s}}$$

$$\Rightarrow \frac{V_o}{V_{in}} = -V_{in} \frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_{sig}} \frac{(R_D \parallel R_L) g_m (1 + sC_sR_s)}{(1 + g_mR_s) + sC_sR_s}$$

$$\Rightarrow \frac{V_o}{V_{in}} = -V_{in} \frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_{sig}} \cdot \frac{g_m(R_D \parallel R_L)}{1 + g_mR_s} \cdot \frac{(1 + sC_sR_s)}{1 + \frac{sC_sR_s}{1 + g_mR_s}}$$

Thus, Zero at $\frac{1}{R_sC_s}$

Pole at $\frac{1 + g_mR_s}{C_sR_s}$

What is mid-band gain here? Isn't it $\frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_{sig}} \cdot g_m(R_D \parallel R_L)$

* Consider only C_{C2} while C_{C1} and C_s are shorts

$$\frac{V_o}{V_g} = -g_m \, R_D \parallel \left[\frac{1}{sC_{C2}} + R_L\right]$$

$$\Rightarrow \frac{V_o}{V_{in}} = -\frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_{sin}} \frac{g_m R_D \left(\frac{1}{sC_{C2}} + R_L\right)}{R_D + R_L + \frac{1}{sC_{C2}}}$$

$$\Rightarrow \frac{V_o}{V_{in}} = -g_m \cdot \frac{R_1 \parallel R_2}{R_{sig} + R_1 \parallel R_2} \cdot \frac{R_D R_L}{R_D + R_L} \cdot \frac{\frac{1}{sC_{C2} R_L} + 1}{1 + \frac{1}{sC_{C2}(R_D + R_L)}}$$

$$\Rightarrow \frac{V_o}{V_{in}} = -g_m \frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_{sig}} \cdot (R_D \parallel R_L) \frac{1 + sC_{C2} R_L}{1 + sC_{C2}(R_D + R_L)} \cdot \frac{R_D + R_L}{R_L}$$

Thus,

$$\frac{V_o}{V_{in}} = -g_m \frac{R_1 \| R_2}{(R_1 \| R_2) + R_{sig}} \cdot (R_D \| R_L) \frac{1 + sC_{c2}R_L}{1 + sC_{c2}(R_D + R_L)} \cdot \frac{R_D + R_L}{R_L} \cdot \frac{R_L}{R_L + \frac{1}{sC_{c2}}}$$

$$\Rightarrow \frac{V_o}{V_{in}} = \underbrace{-g_m \frac{R_1 \| R_2}{R_1 \| R_2 + R_{sig}} (R_D \| R_L)}_{|A_M|.} \frac{sC_2 (R_D + R_L)}{\left[1 + sC_{c2}(R_D + R_L)\right]}.$$

* Zero at DC
* Pole at $\omega_{PL3} = \frac{1}{C_{c2}(R_D + R_L)}$.

* So low-frequency poles are :-

$$\omega_{PL1} = \frac{1}{C_{c1}\left[(R_1 \| R_2) + R_{sig}\right]}$$

$$\omega_{PL2} = \frac{1 + g_m R_s}{C_s R_s}.$$

$$\omega_{PL3} = \frac{1}{C_{c2}(R_D + R_L)}.$$

* Isn't this same as the pole obtained by associating a pole with each node with a subtle variant???

What is the variant $\Rightarrow$ Pole is defined as product of capacitance times the equivalent resistance across the capacitor.

* There are effectively 3 poles and 3 zeros at low frequency and the transfer function

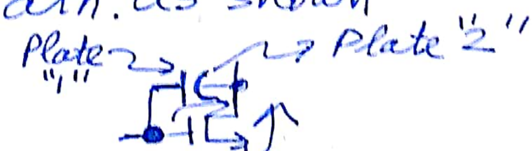
can be given by,

$$\frac{V_o}{V_{sig}} = -\frac{R_1 || R_2}{R_1 || R_2 + R_{sig}}\, g_m \left(R_D' || R_L\right) \cdot \frac{s}{s+\omega_{PL1}} \cdot \frac{s+\omega_{ZL2}}{s+\omega_{PL2}} \cdot \frac{s}{s+\omega_{PL3}}$$

where, ω_{PL1}, ω_{PL2}, and ω_{PL3} are already defined and $\omega_{ZL2} = \frac{1}{C_S R_S}$, $\omega_{ZL1} = 0$, and $\omega_{ZL2} = 0$.

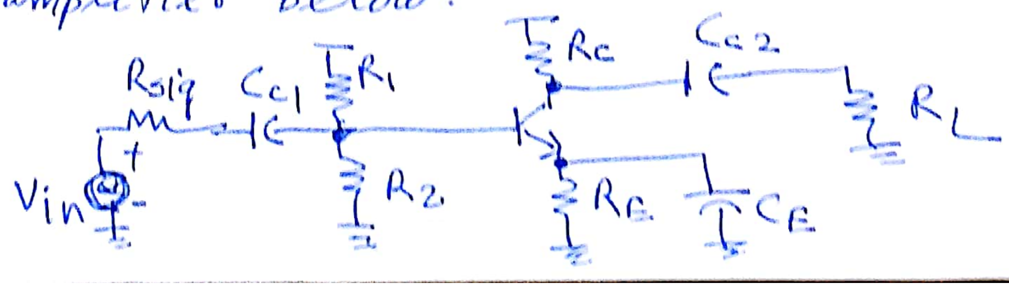
NOTES:-

* Finding the poles by inspection can be done by associating a pole with each node.
* If there is a floating capacitor use Miller Effect to decouple it provided the two nodes of the floating capacitor are such that signal goes from one plate of capacitor to other plate also through another path. as shown below:-



* If capacitor is the only path for a floating capacitor then "DO NOT APPLY" Miller Effect. Instead find the equivalent resistance between the two plates of capacitor.

Exercise:- Please apply the same technique of BJT. amplifier below:-



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Prove that,

(i) $|A_M| = \dfrac{R_1 \| R_2 \| r_\pi}{R_1 \| R_2 \| r_\pi + R_{sig}} \, g_m (R_C \| R_L)$.

(ii) $\omega_{PL1} \doteq \dfrac{1}{C_{C1}\left[(R_1 \| R_2 \| r_\pi) + R_{sig}\right]}$.

(iii) $\omega_{PL2} \cong \dfrac{1 + g_m R_E}{C_E R_E}$

(iv) $\omega_{PL3} = \dfrac{1}{C_{C2}(R_C + R_L)}$

(v) $\omega_{ZL1} = 0$

(vi) $\omega_{ZL2} = \dfrac{1}{C_E R}$

(vii) $\omega_{ZL3} = 0$.

*To derive ω_{PL2} and ω_{ZL2} we do the following:-

$$I_B = V_{sig} \cdot \frac{R_B}{R_B + R_{sig}} \cdot \frac{1}{(R_B \| R_{sig}) + (\beta+1)\left[\dfrac{1}{g_m} + \dfrac{\frac{1}{sC_E} \cdot R_E}{R_E + \frac{1}{sC_E}}\right]}$$

where, $R_B = R_1 \| R_2$.

Thus,

$$\frac{V_o}{V_{sig}} = \frac{R_B}{R_B + R_{sig}} \cdot \frac{\beta \, R_C \| R_L}{R_B \| R_{sig} + (\beta+1)\left[\dfrac{1}{g_m} + \dfrac{R_E}{1 + sC_E R_E}\right]}$$

$$\Rightarrow \frac{V_o}{V_{sig}} = \frac{R_B}{R_B + R_{sig}} \cdot \frac{\beta (R_C \| R_L)\, g_m (1 + sC_E R_E)}{(R_B \| R_{sig})\, g_m (1 + sC_E R_E) + (\beta+1)(1 + sC_E R_E) + (\beta+1)\, g_m R_E}$$

MOSFET CAPACITANCE :-

* Gate Capacitance :-

$$C_{gs} = C_{gd} = \frac{1}{2} WL C_{ox} \quad \ldots\ldots \text{Triode.}$$

$$\left.\begin{array}{l} C_{gs} = \frac{2}{3} WL C_{ox} \\ C_{gd} \cong 0 \end{array}\right\} \text{saturation.}$$

$$C_{gs} = C_{gd} \cong 0 \quad \ldots\ldots \text{Cut off.}$$

~~$C_{gb} = WL C_{ox}$ .... Cut off.~~

Due to overlap of gate with source & drain the above values get modified as follows:-

$$C_{gs} = C_{gd} = \frac{1}{2} WL C_{ox} + C_{ov} W \quad \ldots \text{Triode.}$$

$$\left.\begin{array}{l} C_{gs} = \frac{2}{3} WL C_{ox} + C_{ov}.W \\ C_{gd} = C_{ov} W \end{array}\right\} \text{Saturation.}$$

$$C_{gs} = C_{gd} = C_{ov} W \quad \ldots\ldots\ldots \text{Cut off.}$$

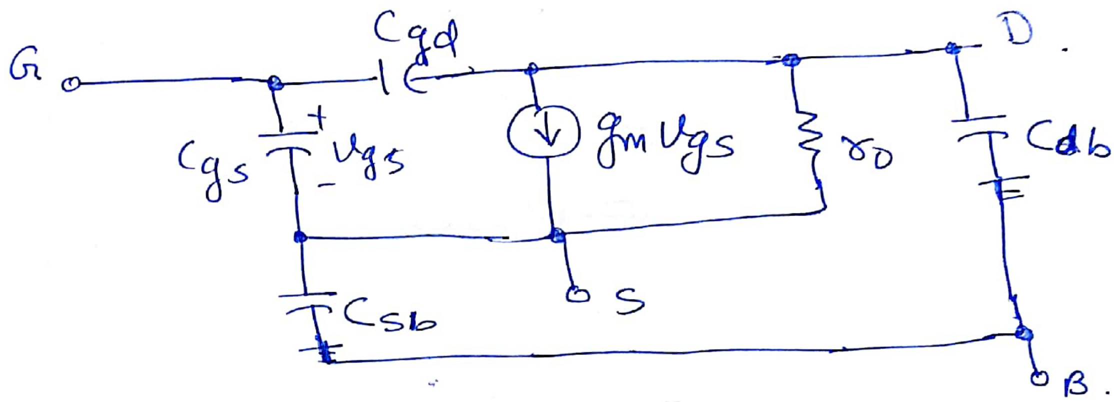
* Junction Capacitance :-

$$C_{sb} = \frac{C_{sbo}}{\sqrt{1 + \frac{V_{SB}}{V_0}}}$$

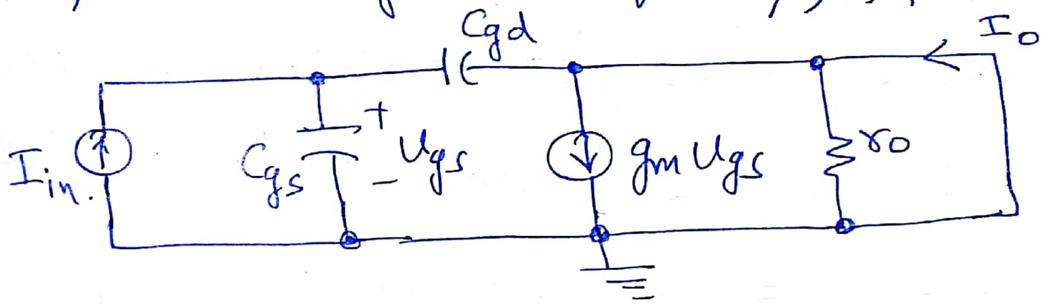
and
$$C_{Db} = \frac{C_{Dbo}}{\sqrt{1 + \frac{V_{DB}}{V_0}}}$$

where, a grading coefficient of $m = 1/2$ is assumed.

High-Frequency MOSFET Model:-



* Speed of a transistor is quantified by, unity current-gain frequency, f_T.



Here, $I_o = g_m V_{gs}$.

and $I_{in} = V_{gs}\, s\,[C_{gs} + C_{gd}]$.

$$\therefore \frac{I_o}{I_{in}} = \frac{g_m}{s(C_{gs}+C_{gd})}.$$

$$\Rightarrow f_T = \frac{g_m}{2\pi(C_{gs}+C_{gd})} = \frac{\mu_n C_{ox}\frac{W}{L} V_{ov}}{2\pi(C_{gs}+C_{gd})}.$$

$$= \frac{\sqrt{2\mu_n C_{ox}\frac{W}{L} I_D}}{2\pi(C_{gs}+C_{gd})} = \frac{2I_D}{2\pi V_{ov}(C_{gs}-C_{gd})}$$

BJT High-Frequency Model :-

* C.B junction is reverse-biased. ⇒ depletion capacitance, $C_\mu = \dfrac{C_{\mu o}}{\left(1 + \frac{V_{CB}}{V_o}\right)^m}$.

  where, m is grading coefficient, (0.2 to 0.5).

* EB junction is forward biased, the depletion layer capacitance is given by,

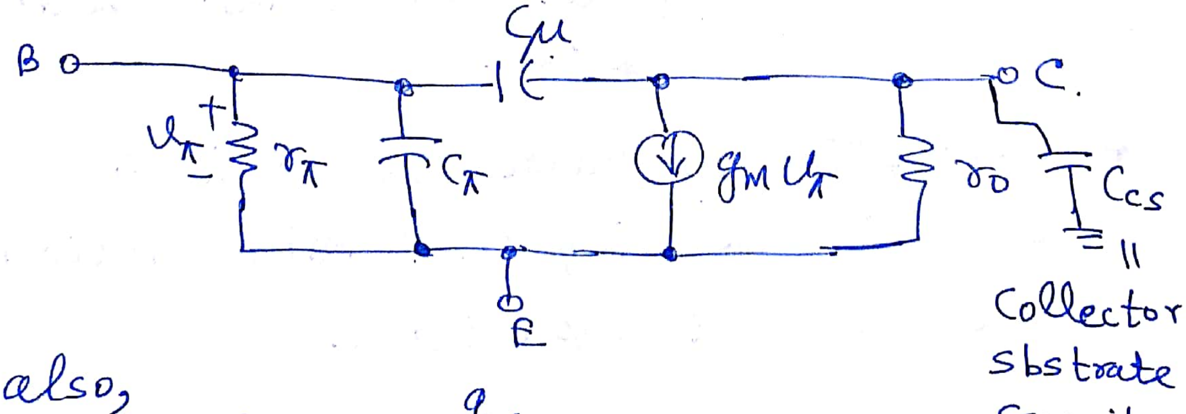
$$C_{je} \simeq 2 C_{jeo}.$$

  where, C_{jeo} is the value of C_{je} at 0 bias of EB-junction

* Minority carriers in base move by diffusion. Some charge is stored in base when it is forward biased ⇒ some capacitance. We call that as diffusion capacitance, C_{de}.

Thus, $C_\pi = C_{je} + C_{de}$.

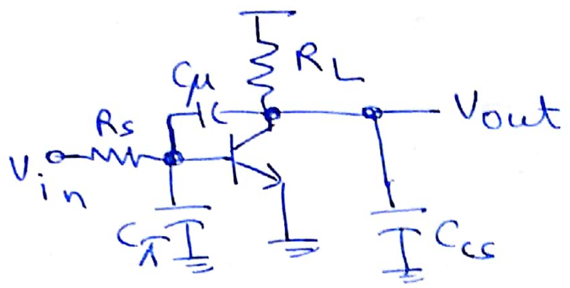
where, $C_{de} = \tau_F \dfrac{I_C}{V_T}$ and τ_F is a device constant.

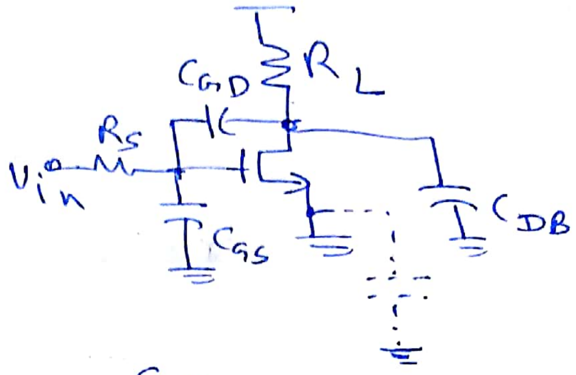


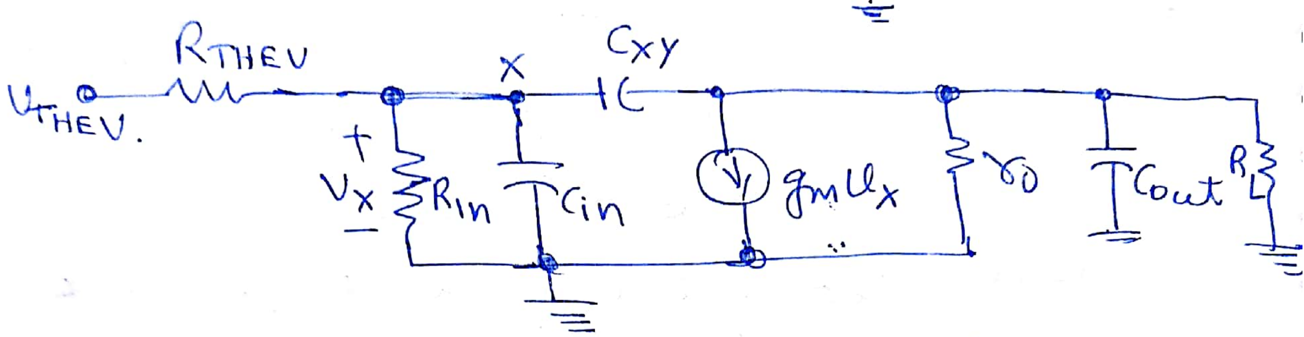
Here, also,

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

UNIFIED MODEL :-



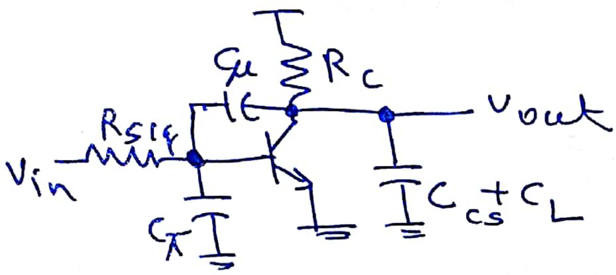


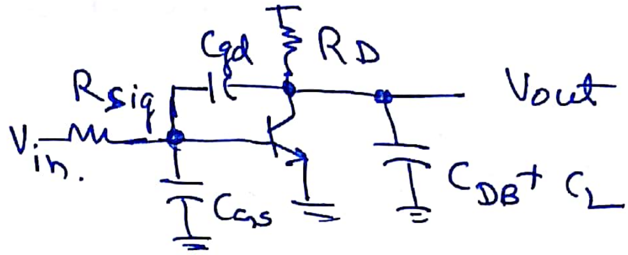


* Obtaining high-frequency response of amplifier
  (i) By direct analysis i.e. drawing small-signal model and obtaining the transfer function in s-domain.
  (ii) Associating a pole with each node
  ⇒ Use Miller effect to decouple floating capacitors.
  ⇒ Only get information about poles. but no information about zeroes.

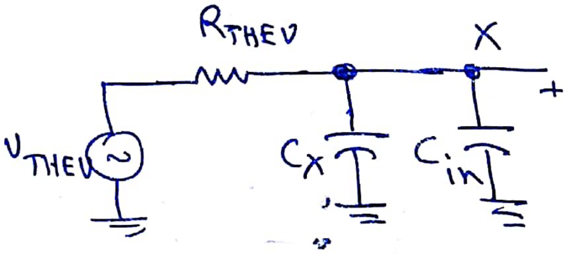
* Decoupling by Miller effect is done by finding DC gain and using that information to get the required capacitors.

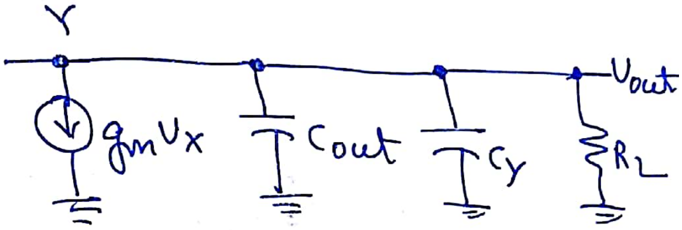
High Frequency Response of CS & CE Amplifier





⇓ Miller approx & Unified Model.





CE-stage

$$V_{THEV} = V_{in} \frac{r_\pi}{r_\pi + R_{sig}}$$

$$R_{THEV} = R_{sig} \parallel r_\pi$$

$$C_X = C_\mu (1 + g_m R_L)$$

$$C_Y = C_\mu \left(1 + \frac{1}{g_m R_L}\right)$$

$R_L = R_C$
$C_{XY} = C_\mu$
$C_{out} = C_S + C_L$.
$C_{in} = C_\pi$

CS-stage

$V_{THEV} = V_{in}$.

$R_{THEV} = R_{sig}$.

$$C_X = C_{GD}(1 + g_m R_L)$$

$$C_Y = C_{GD}\left(1 + \frac{1}{g_m R_L}\right)$$

$R_L = R_D$
$C_{XY} = C_{GD}$
$C_{out} = C_{DB} + C_L$.
$C_{in} = C_{GS}$.

Thus,

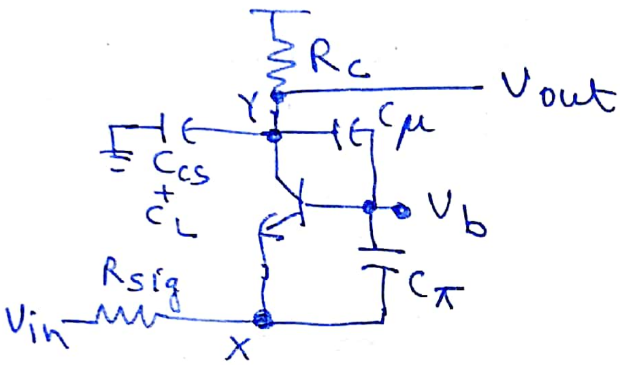
$$\omega_{P1} = \omega_{pin} = \frac{1}{R_{THEV}\left[C_{in} + (1 + g_m R_L) C_{XY}\right]}$$

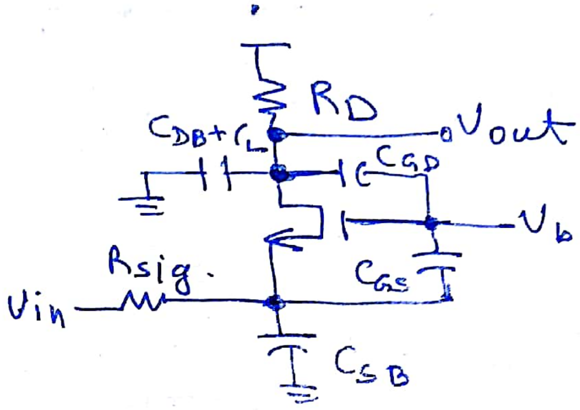
$$\omega_{P2} = \omega_{pout} = \frac{1}{R_L\left[C_{out} + \left(1 + \frac{1}{g_m R_L}\right) C_{XY}\right]}$$

DOMINANT POLE:- Whichever is the low-frequency of the two becomes the dominant pole.

HIGH-FREQUENCY RESPONSE OF CG and CB AMPLIFIER

* For simplicity of analysis we assume $r_o = \infty$ for BJT & MOS.





$$\omega_{P1} = \omega_{PX} = \frac{1}{\left(R_S \| \frac{1}{g_m}\right) C_X}$$

$$\omega_{P2} = \omega_{PY} = \frac{1}{R_L C_Y}$$

CB - stage

$C_X = C_\pi$

$C_Y = C_\mu + C_{CS}$

CG - stage

$C_X = C_{GS} + C_{SB}$

$C_Y = C_{DB} + C_{GD} + C_L$.

* If there was "r_o" would you apply Miller effect to decouple it. Why or why not??

* Input-pole is very close to f_T and therefore is the non-dominant pole as it is a high-frequency pole.

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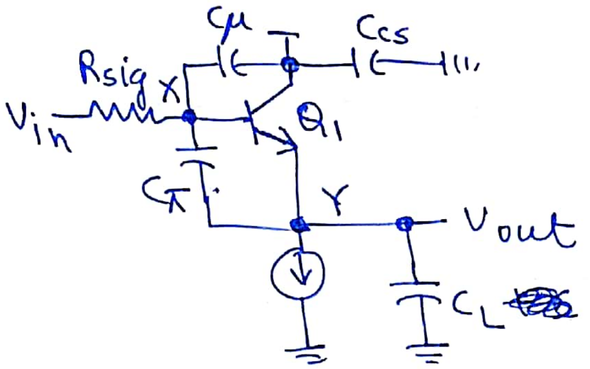
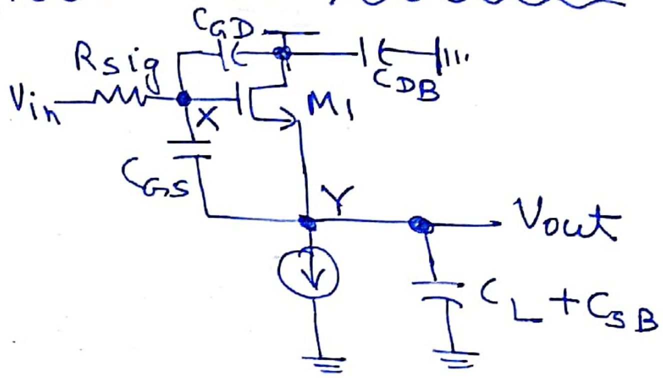
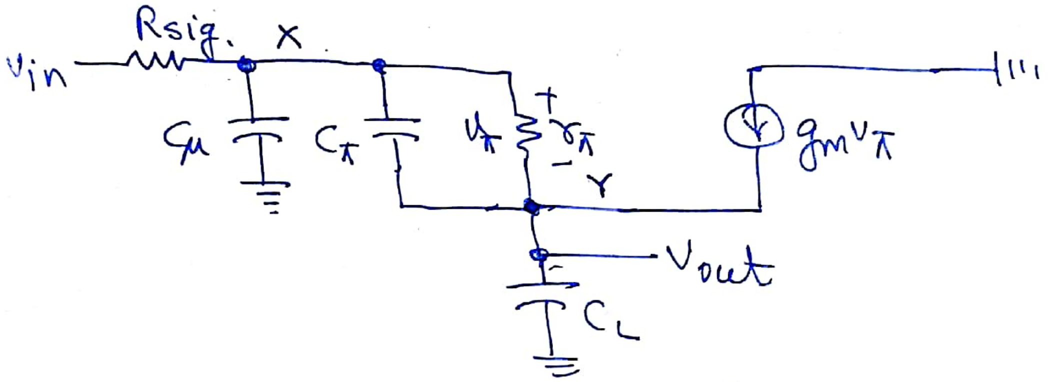
HIGH FREQUENCY RESPONSE OF FOLLOWERS







Applying KCL @ node X we get,

$$\frac{V_{out} + V_\pi - V_{in}}{R_S} + (V_{out} + V_\pi) sC_\mu + \frac{V_\pi}{r_\pi} + V_\pi sC_\pi = 0$$

Applying KCL @ node Y we get,

$$\frac{V_\pi}{r_\pi} + V_\pi sC_\pi + g_m V_\pi = V_{out} sC_L.$$

$$\Rightarrow V_\pi = \frac{V_{out}\, sC_L}{\frac{1}{r_\pi} + g_m + sC_\pi}$$

Thus,

$$\frac{V_{out}}{V_{in}}(s) = \frac{1 + \frac{C_\pi}{g_m} s}{as^2 + bs + 1} \quad \ldots \left[\text{if } r_\pi >> \frac{1}{g_m}\right]$$

where, $a = \frac{R_S}{g_m}(C_\mu C_\pi + C_\mu C_L + C_\pi C_L)$

$$b = R_S C_\mu + \frac{C_\pi}{g_m} + \left(1 + \frac{R_S}{r_\pi}\right)\frac{C_L}{g_m}.$$

$$\Rightarrow \quad \omega_z = \frac{g_m}{C_\pi} \approx f_T.$$

For source follower we can set $r_\pi = \infty$ and obtain,

$$\frac{V_{out}}{V_{in}} = \frac{1 + \frac{C_{GS}}{g_m} s}{as^2 + bs + 1}$$

where, $a = \frac{R_s}{g_m} \left[C_{GD} C_{GS} + C_{GD}(C_{SB} + C_L) + C_{GS}(C_{SB} + C_L) \right]$

and, $b = R_S C_{GD} + \frac{C_{GD} + C_{SB} + C_L}{g_m}$.