MA 20104 Probability and Statistics
Hints/Solutions to Assignment No. 5

1. The range of $Y = y$ is interval $[1, 4)$. For $0 < y < 1$, there are two inverse images; $x = \sqrt{y}$ and $x = -\sqrt{y}$. So for $0 < y \leq 1$,
$$f_Y(y) = \frac{2(\sqrt{y}+1)}{9}\frac{1}{2\sqrt{y}} + \frac{2(-\sqrt{y}+1)}{9}\frac{1}{2\sqrt{y}} = \frac{2}{9\sqrt{y}}$$
For $1 < y < 4$, there is only one inverse image $x = \sqrt{y}$
So for $1 < y < 4$,
$$f_Y(y) = \frac{2(\sqrt{y}+1)}{9}\frac{1}{2\sqrt{y}} = \frac{1}{9}\left(1 + \frac{1}{\sqrt{y}}\right)$$

2. The range of $Y = y$ is interval $[0, \frac{9}{4})$. There are two inverse images; $x = \sqrt{y}$ and $x = -\sqrt{y}$. So for $0 < y \leq \frac{1}{4}$, we get
$$f_Y(y) = \frac{1}{2}\frac{1}{2\sqrt{y}} + \frac{1}{2}\frac{1}{2\sqrt{y}} = \frac{1}{2\sqrt{y}}.$$
For $\frac{1}{4} < y < \frac{9}{4}$, we get
$$f_Y(y) = \frac{1}{2}\frac{1}{2\sqrt{y}}\left(\frac{3}{2} - \sqrt{y}\right) + \frac{1}{2}\frac{1}{2\sqrt{y}}\left(3 - \frac{3}{2} - \sqrt{y}\right).$$
So we have
$$f_Y(y) = \begin{cases} \dfrac{1}{2\sqrt{y}} & if \quad 0 \leq y \leq \dfrac{1}{4} \\ \dfrac{3}{4\sqrt{y}} - \dfrac{1}{2} & if \quad \dfrac{1}{4} < y < \dfrac{9}{4} \\ 0 & elsewhere \end{cases}$$

3. The range of $Y = y$ is interval $(0, 1]$. There are two inverse images; $x = \sin^{-1} y$ and $x = \pi - \sin^{-1} y$.
So we get

$$f_Y(y)=\frac{2}{\pi^2}\left(\frac{\sin^{-1}y}{\sqrt{(1-y^2)}}+\frac{\pi-\sin^{-1}y}{\sqrt{(1-y^2)}}\right)=\frac{2}{\pi\sqrt{(1-y^2)}}\quad ,0<y\le 1.$$

4. Apply direct pmf approach.
5. Apply pdf approach.
6. Apply pdf approach.
7. The range of $Y=y$ is $(0,\infty)$. For $y>0$, there are two inverse images $x=\sqrt{\frac{2y}{m}}$ and $x=-\sqrt{\frac{2y}{m}}$. So we have
$$f_Y(y)=c\frac{2y}{m}\ e^{-\frac{2by}{m}}\frac{1}{\sqrt{2my}}+c\frac{2y}{m}\ e^{-\frac{2by}{m}}\frac{1}{\sqrt{2my}}.$$
After simplification
$$f_Y(y)=\frac{2c}{m}.\sqrt{\frac{2y}{m}}\ e^{-\frac{2by}{m}}\ ,\qquad y>0.$$
8. The range of $Y=y$ is interval $[0,3)$. For $0<y\le 1$, there are two inverse images; $x=y$ and $x=-y$. So for $0<y\le 1$,
$$f_Y(y)=\frac{1+y}{4}+\frac{1-y}{4}=\frac{1}{2}$$
For $1<y<3$, there is only one inverse image $x=y$
So for $1<y<3$,
$$f_Y(y)=\frac{3-y}{4}.$$
9. The range of $Y=y$ is $(-\infty,\infty)$. For $y>0, x=4y^2$ and for $y<0, x=-y^2$. So the pdf of Y is
$$f_Y(y)=\begin{cases}-\dfrac{2y}{\sqrt{2\pi}}e^{-\frac{y^4}{2}}, & y<0\\ \quad x & \\ \dfrac{8y}{\sqrt{2\pi}}e^{-8y^4}, & y\ge 0\end{cases}.$$

10. Apply direct pmf approach.
11. Apply pdf approach.
12. Apply pdf approach.