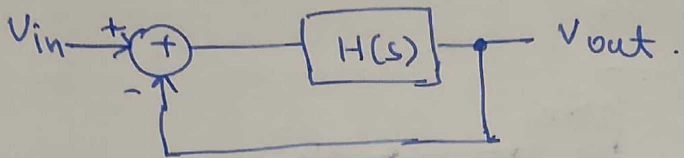
OSCILLATORS



* In the above feedback system, where feedback factor, $\beta = 1$, if the amplifier experiences so much phase-shift at high frequencies that the overall feedback becomes positive, then oscillation may occur.

Note:- Oscillation may occur at any β. However, as you know oscillation may occur better as $\beta \to 1$. ⇒ Remember phase-margin reduces as $\beta \to 1$.

* $$\frac{V_{out}}{V_{in}}(s) = \frac{H(s)}{1 + H(s)}.$$

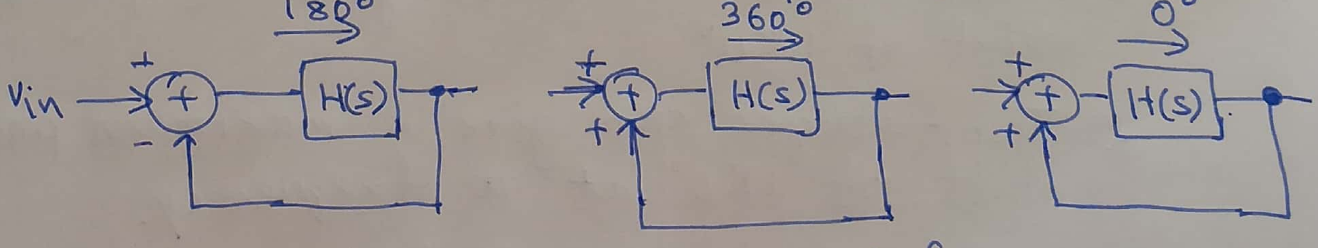
Barkhausen Criteria for oscillation :-

$$|H(j\omega_0)| \geq 1$$

$$\angle H(j\omega_0) = 180°.$$

⇒ At ω_0 forward path gives a phase-shift of 180° and then the negative feedback, i.e., summing node gives another 180° phase-shift ⇒ overall 360° phase-shift in the loop results in positive-feedback or oscillation.

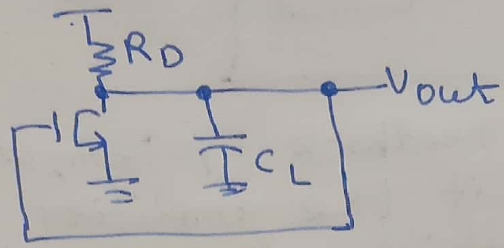
Various Views Of Feedback System :-



* The above mentioned phase-shifts are happening at the oscillation frequency, ω_0.
* The summing node gives DC phase-shift.

②

RING OSCILLATORS :-

* 

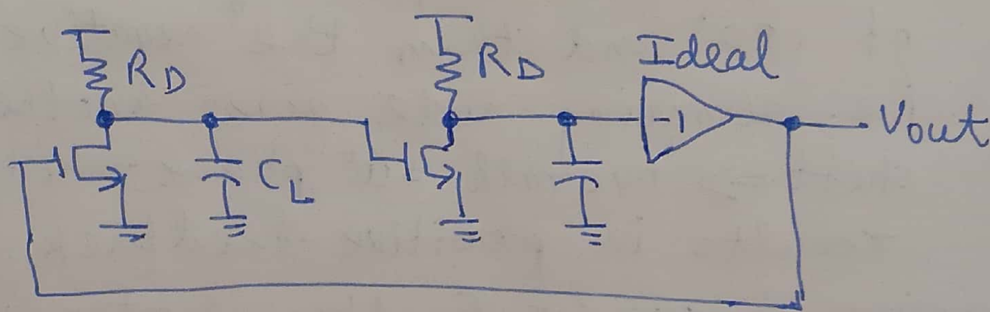
The above single-stage CS amplifier has ~~~~ negative feedback at DC. ~~~~ Can it oscillate.

* Note:- For oscillation to occur we need
  - → DC phase-shift of 180°. i.e. negative feedback.
  - → Frequency dependent phase-shift of 180°.

* The maximum frequency dependent phase-shift that single-stage CS amplifier can give is 90° at ∞ frequency.

  ⇒ No oscillation.

* How about two-stage CS-amplifier in negative-feedback?

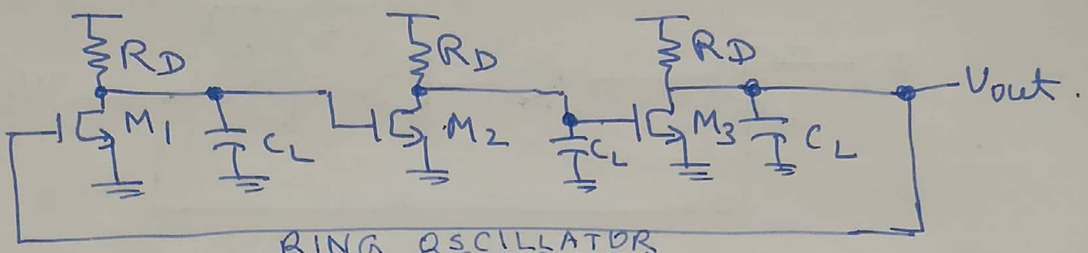


⇒ Ideal signal inversion give DC phase shift of 180°

⇒ Two-stages can give a maximum phase-shift of 180° at ∞ frequency.

⇒ No Oscillation

* How about 3-stage CS-amplifier??



RING OSCILLATOR

Note:- All stages are decoupled. Hence,

$$H(s) = -\frac{A_0^3}{\left(1+\frac{s}{\omega_0}\right)^3}$$

where, $A_0 = -g_m R_D$.

At ω_{osc} the phase-shift is $180°$ ⇒ each stage gives a phase-shift of $60°$. Thus,

$$\tan^{-1}\left(\frac{\omega_{osc}}{\omega_0}\right) = 60°$$

$$\Rightarrow \omega_{osc} = \sqrt{3}\,\omega_0 .$$

Thus, minimum DC gain A_0 required is,

$$\frac{A_0^3}{\left[\sqrt{1+\left(\frac{\omega_{osc}}{\omega_0}\right)^2}\right]^3} = 1.$$

$$\Rightarrow A_0 = 2.$$

But we want loop gain ≥ 1 ⇒ $A_0 > 2$.

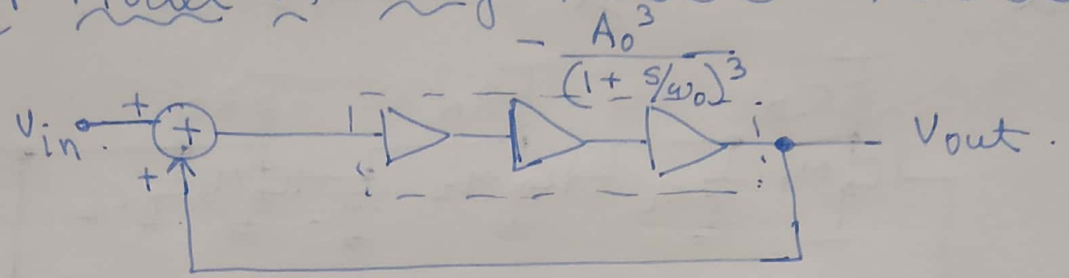
* If loop-gain $=1$ is sufficient for oscillation at ω_{osc} why do we say loop-gain > 1.

⇒ In order to make sure oscillation is sustained as temperature, voltage, and process (PVT) changes.

⇒ Remember gain is a function of temp., voltage, and process. Why??

⇒ Isn't g_m PVT dependent??

④ Linear Model of Ring Oscillator (3-stage)



If $A_0 < 2 \Rightarrow$ no oscillation.

Now,

$$\frac{V_{out}}{V_{in}}(s) = \frac{-A_0^3/\left(1+\frac{s}{\omega_0}\right)^3}{1+\frac{A_0^3}{\left(1+\frac{s}{\omega_0}\right)^3}}$$

$$\Rightarrow \frac{V_{out}}{V_{in}}(s) = \frac{-A_0^3}{\left(1+s/\omega_0\right)^3 + A_0^3}.$$

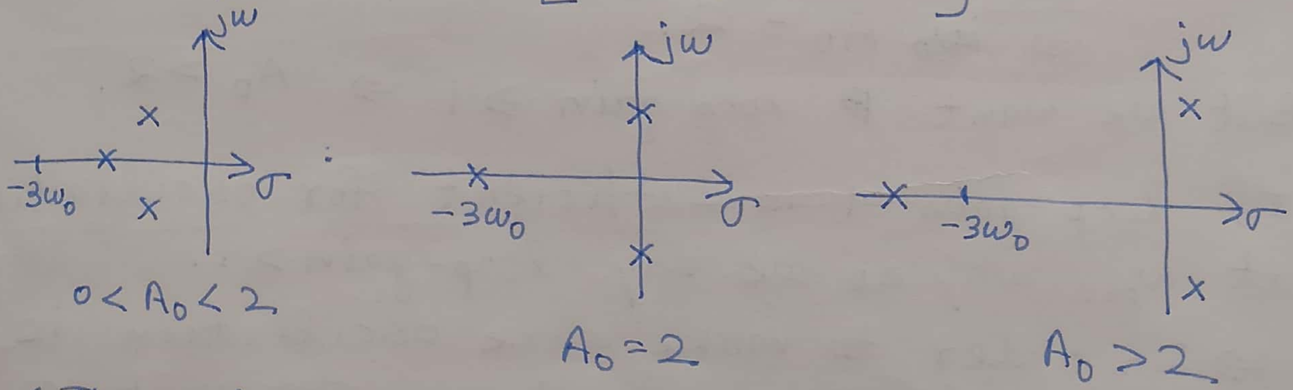
The denominator can be expanded as,

$$\left(1+\frac{s}{\omega_0}\right)^3 + A_0^3 = \left(1+\frac{s}{\omega_0}+A_0\right)\left[\left(1+\frac{s}{\omega_0}\right)^2 - \left(1+\frac{s}{\omega_0}\right)A_0 + A_0^2\right]$$

The 3-poles are,

$$s_1 = (-A_0 - 1)\,\omega_0$$

$$s_{2,3} = \left[\frac{A_0(1 \pm j\sqrt{3})}{2} - 1\right]\omega_0.$$



* The pole, s_1 gives a decaying exponential which can be ignored as it would eventually die-out.

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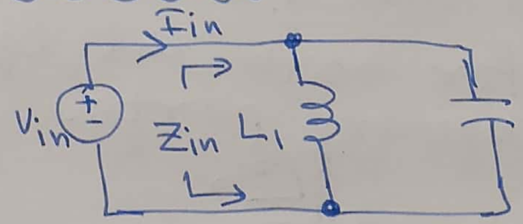
* Thus, neglecting the effect of s_1, we can express the output waveform as,

$$V_{out} = a \exp\left[\frac{A_0-2}{2}\omega_0 t\right] \cdot \cos\left[\frac{A_0}{2}\sqrt{3}\,\omega_0 t\right]$$

* If $A_0 > 0 \Rightarrow V_{out}$ tends to $\infty \Rightarrow$ but is limited by supply voltage.
* If $A_0 = 2 \Rightarrow$ oscillation sustains.

LC OSCILLATORS :-



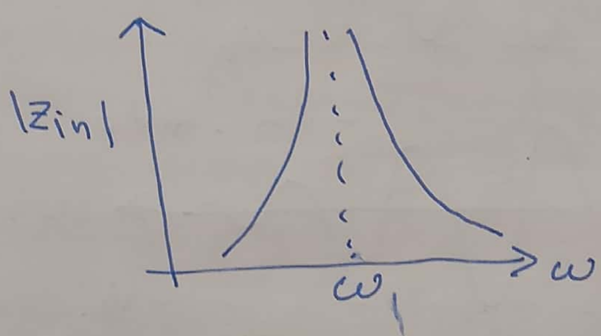
Here,

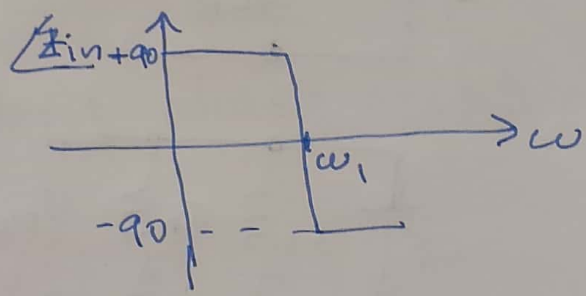
$$Z_{in} = (L_1 s) \parallel \left(\frac{1}{sC_1}\right)$$

$$\Rightarrow Z_{in} = \frac{jL_1\omega}{1 - L_1 C_1 \omega^2}$$

$\Rightarrow Z_{in} \to \infty$ at $\omega_1 = 1/\sqrt{L_1 C_1}$ = resonance frequency.

$\Rightarrow$ Vin does not supply any current.

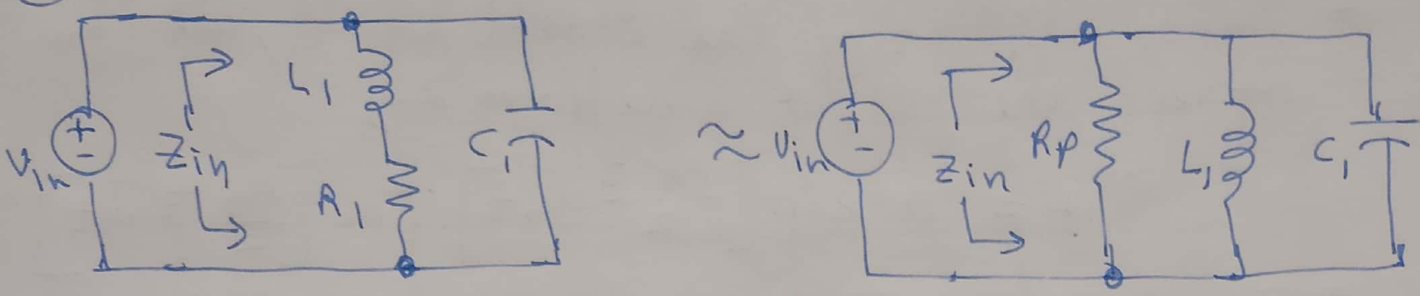




* It is difficult to get $Z_{in} = \infty$ at ω_1. as ~~wires have~~ inductors have finite resistance, resulting in a lossy tank.

Why is a parallel LC circuit called an LC-tank??

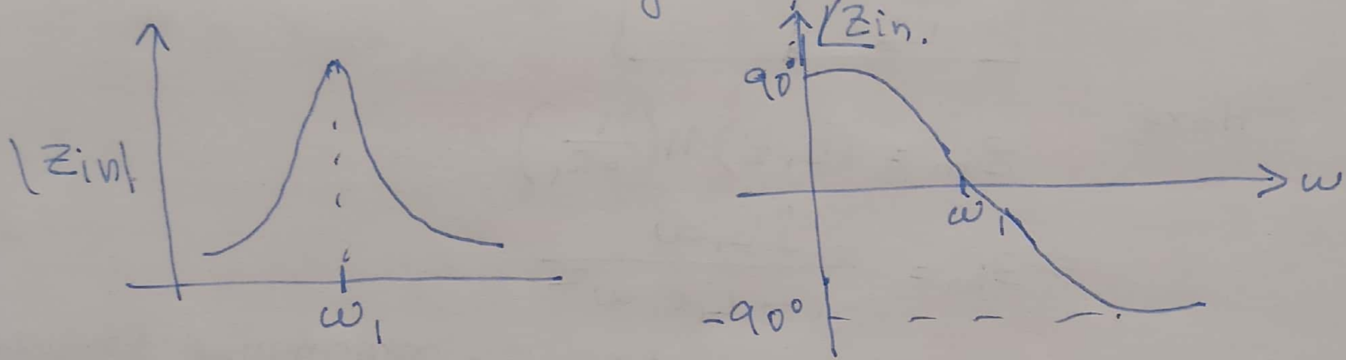
(6)

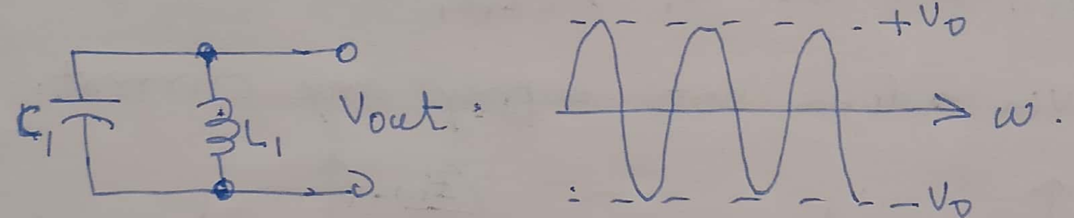


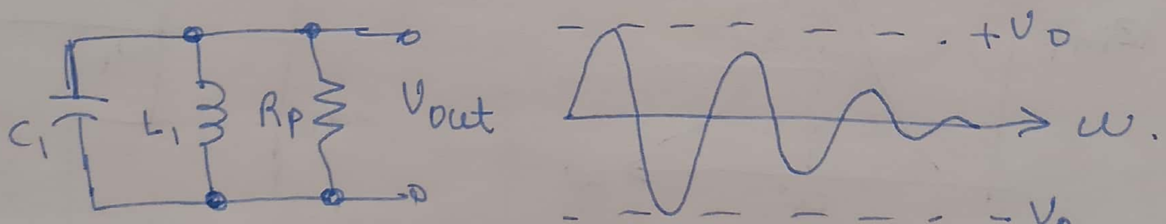
if, $R_p = \frac{L_1^2 \omega^2}{R_1}$ around the resonante frequency.

* How do you obtain that??

$\Rightarrow$ Write Zin for both and equate real and imaginary parts.



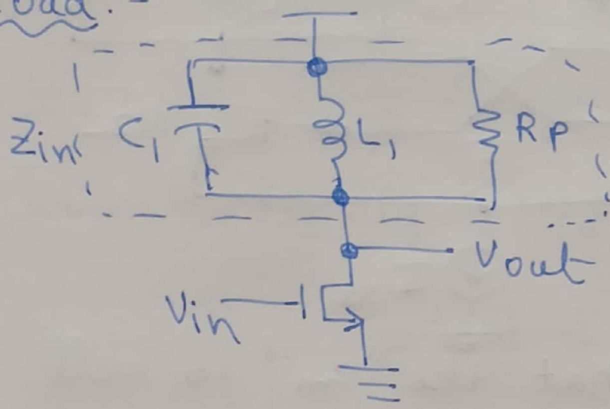




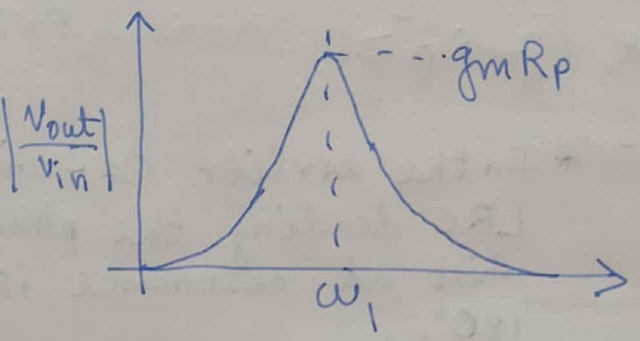
Time-Domain Behavior

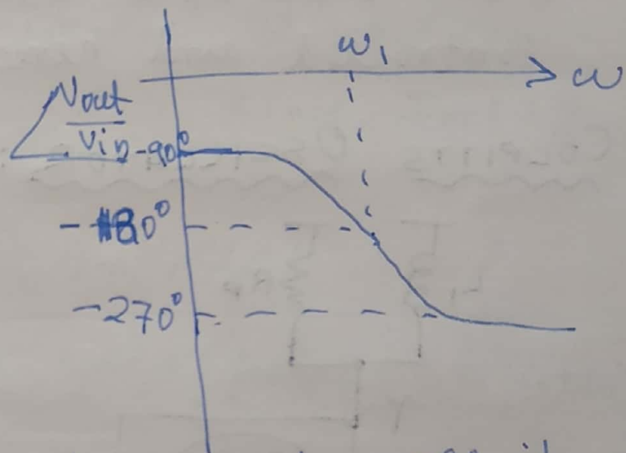
CROSS-COUPLED OSCILLATOR :-

Common-Source Amplifier with LC-tank Load :-

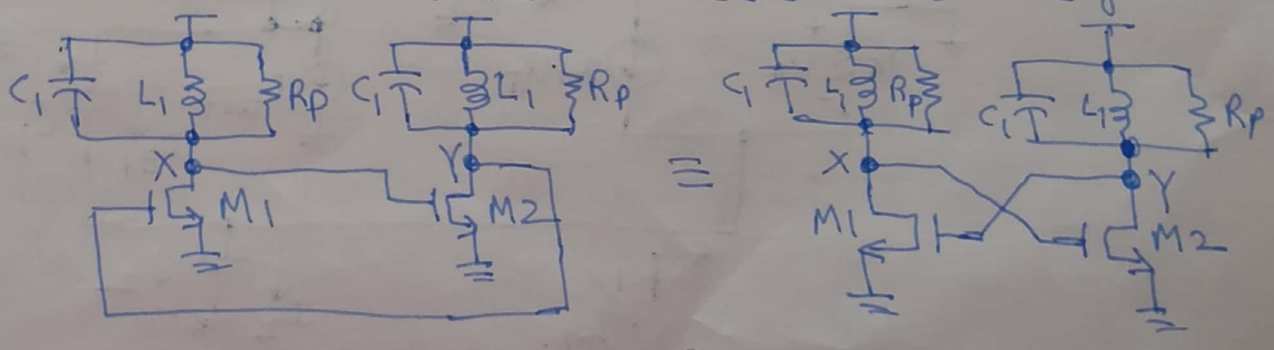


$$\frac{V_{out}}{V_{in}} = -g_m Z_{in}(s).$$





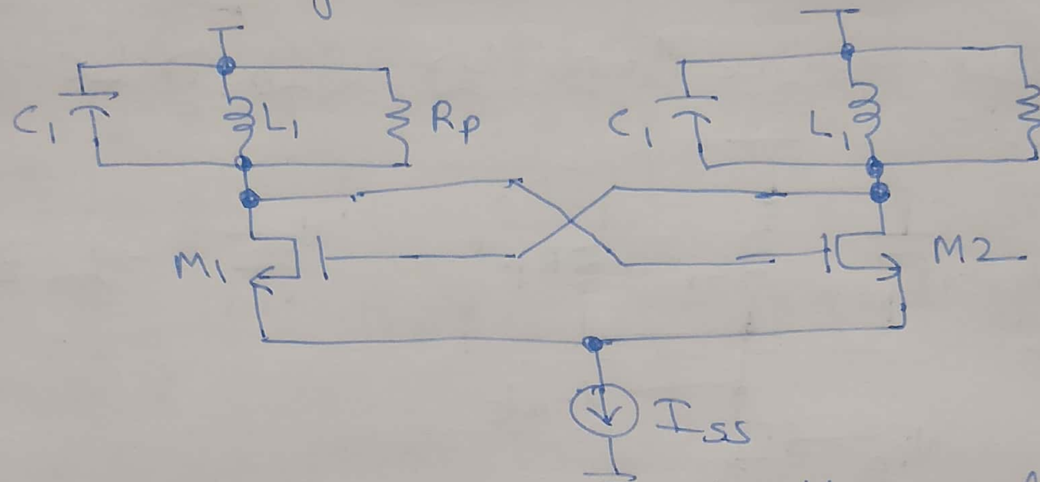
* If we connect output to input will it oscillate.
  - ⇒ DC phase-shift of 180°
  - ⇒ Frequency dependent phase shift of $-90°$
  - ⇒ total phase-shift of 270°.
* To make it oscillate connect two stages:-



Here, $(g_m R_p)^2 > 1$

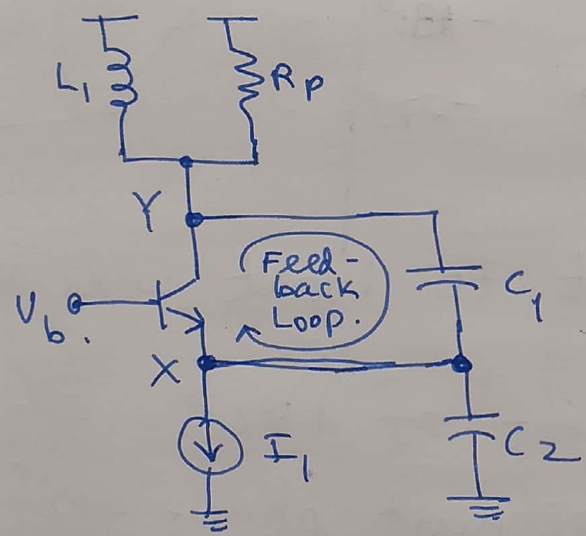
or, $[g_m(R_p \| r_o)]^2 > 1$

A better design is as follows:-



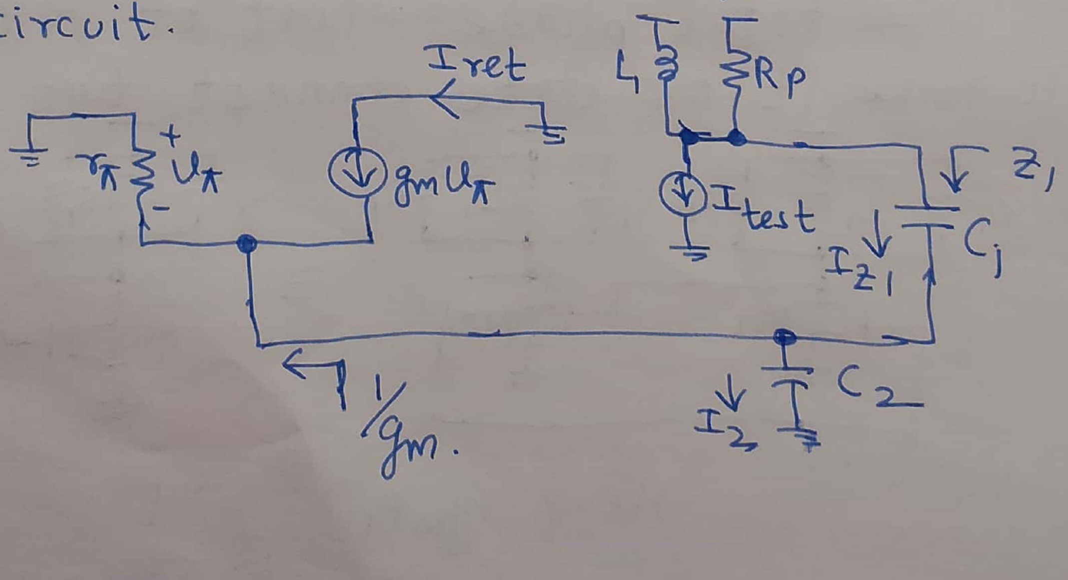
which makes sure that the "g_m" is well controlled and hence, $(g_m R_p)^2 \geq 1$ across PVT.

COLPITTS OSCILLATOR :-



* In the earlier CS-with LRC loading the phase-shift at resonance is 180°.
* However, for oscillation we need total phase shift of 360° at the oscillation frequency.
* Can we instead realize a CB topology to get oscillation.

Lets, break the loop at Y and analyze the small signal circuit.



* Ignoring early effect, we need to make sure that I_{ret}/I_{test} must exhibit a $360°$ phase-shift and at least unity gain at oscillation frequency.

* $$Z_1 = \frac{1}{C_1 s} + \left(\frac{1}{g_m} \parallel \frac{1}{C_2 s}\right) = \frac{1}{C_1 s} + \frac{1}{C_2 s + g_m}$$

* Current flowing through C_1 is equal to:-

$$I_{Z_1} = -I_{test}\frac{\frac{L_1 R_p s}{L_1 s + R_p}}{\frac{L_1 R_p s}{L_1 s + R_p} + \frac{1}{C_1 s} + \frac{1}{C_2 s + g_m}}.$$

Thus, $V_x = I_{Z_1}\left(\frac{1}{sC_2} + \frac{1}{g_m}\right)$. Since, $I_{ret} = g_m V_\pi$,

we obtain,

$$\frac{I_{ret}}{I_{test}} = \frac{g_m R_p L_1 s^2}{L_1 C_1 C_2 R_p s^3 + [g_m R_p L_1 C_1 + L_1(C_1 + C_2)]s^2 + [g_m L_1 + R_p(C_1 + C_2)] + g_m R_p}$$

* Loop-gain should be unity, so we set the gain function to unity. Thus,

$$\cancel{\frac{I_{ret}}{I_{test}} \Rightarrow L_1 C_1 C_2 R_p s^3 + L_1(C_1 + C_2)s^2 + [g_m L_1 + R_p(C_1 + C_2)]s + g_m R_p}$$

$$\Rightarrow L_1 C_1 C_2 R_p s^3 + L_1(C_1 + C_2)s^2 + [g_m L_1 + R_p(C_1 + C_2)] + g_m R_p = 0$$

At oscillation frequency both real & imaginary part of above equation should go to "0". Hence,

$$-L_1(C_1 + C_2)\omega_1^2 + g_m R_p = 0$$

and, $$-L_1[C_1 C_2 R_p]\omega_1^3 + [g_m L_1 + R_p(C_1 + C_2)]\omega_1 = 0$$

Hence,

$$\omega_1^2 = \frac{C_1 + C_2}{C_1 C_2 L_1} + \frac{g_m}{R_p C_1 C_2}$$

Neglecting the 2nd term above we get,

$$\omega_1 = \sqrt{\frac{1}{L_1 \frac{C_1 C_2}{C_1 + C_2}}}$$

* Using the above equation in resonance we obtain,

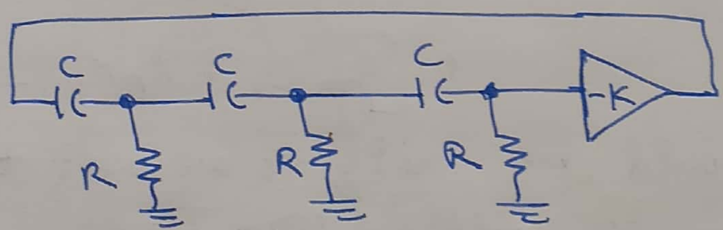
$$g_m R_p = \frac{(C_1 + C_2)^2}{C_1 C_2}$$

$$\Rightarrow g_m R_p = \frac{C_2}{C_1}\left[1 + \frac{C_1}{C_2}\right]^2$$

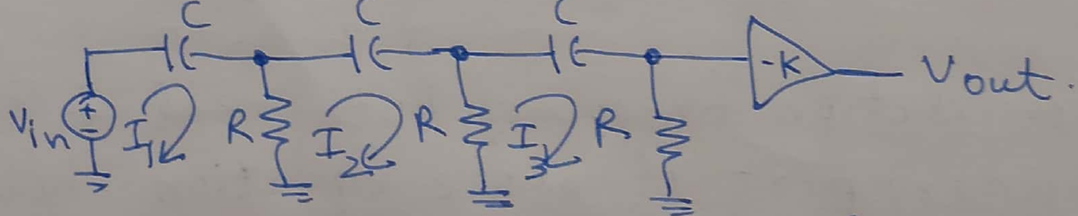
The above is minimum if $C_2/C_1 = 1$. Hence,

$$g_m R_p \geq 4.$$

CR-Phase SHIFT OSCILLATOR :-



* The above is a CR-phase-shift oscillator which incorporates a phase-shifting network.
* In order to analyze the above circuit we break the loop and find the loop-gain as follows:



Applying KVL in the first three loops we get,

$$I_1\left(R + \frac{1}{sC}\right) - I_2 R = V_{in} \quad \ldots (1)$$

$$-I_1 R + I_2\left(2R + \frac{1}{sC}\right) - I_3 R = 0 \quad \ldots (2)$$

and, $-I_2 R + I_3\left(2R + \frac{1}{sC}\right) = 0 \quad \dots \quad (3)$

Also, $V_{out} = -K I_3 R \quad \dots \quad (4)$

Loop-gain, $$A\beta = \frac{V_{out}}{V_{in}} = \frac{-K}{(1-5\alpha^2) + j(\alpha^3 - 6\alpha)}$$

where, $\alpha = \frac{1}{\omega RC}$.

Since loop-gain is a real quantity,

$$\alpha^3 - 6\alpha = 0$$

$$\Rightarrow \alpha = \text{[redacted]} \sqrt{6}$$

$$\Rightarrow \omega_0 = \frac{1}{RC\sqrt{6}}$$

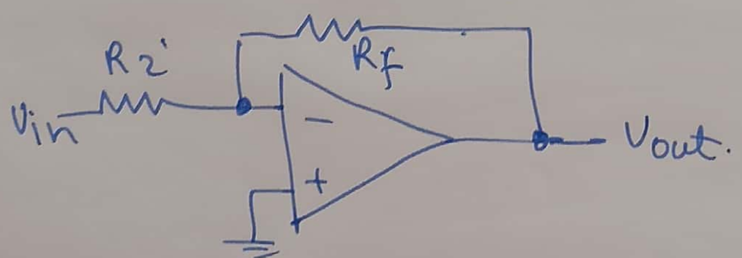
$$\Rightarrow f_0 = \frac{1}{2\pi RC\sqrt{6}}$$

At this frequency loop-gain should be ≥ 1.

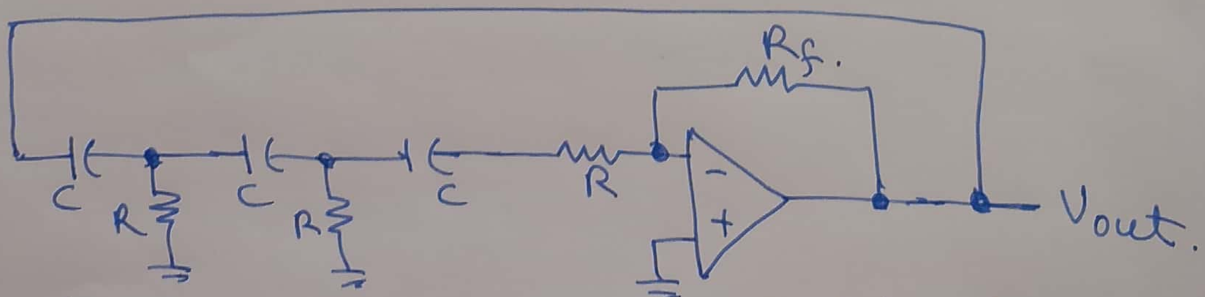
Thus,
$$|A\beta| = \frac{K}{29} > 1$$

$$\Rightarrow K > 29.$$

The gain K can be realized as follows using op-amp:-



Thus, the overall circuit is,



Note:- The last resistor is absorbed in R_i. Why??