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Module 4

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Aristotelian Logic / Categorical Logic / Logic of Classes

Key Terms:

- Categories
- Class
- Subject
- Predicate
- Predication
- Categorical Proposition
- Universal
- Particular
- Opposition
- Syllogism

Limitations of Propositional Logic:

Consider this argument:

1. Fruits are nutritious food.
2. If fruits are nutritious food, fruits will boost our immunity.

   $\therefore$ Fruits will boost our immunity.

It is valid, and its validity is demonstrable in Propositional Calculus / Logic:

$$\left.\begin{array}{ll} 1. & F \\ 2. & F \supset L \\ & \therefore L \end{array}\right\} \text{M.P.}$$

But now consider:

1. Apples are nutritious food.
2. All nutritious food boost immunity.

   $\therefore$ Apples boost immunity .

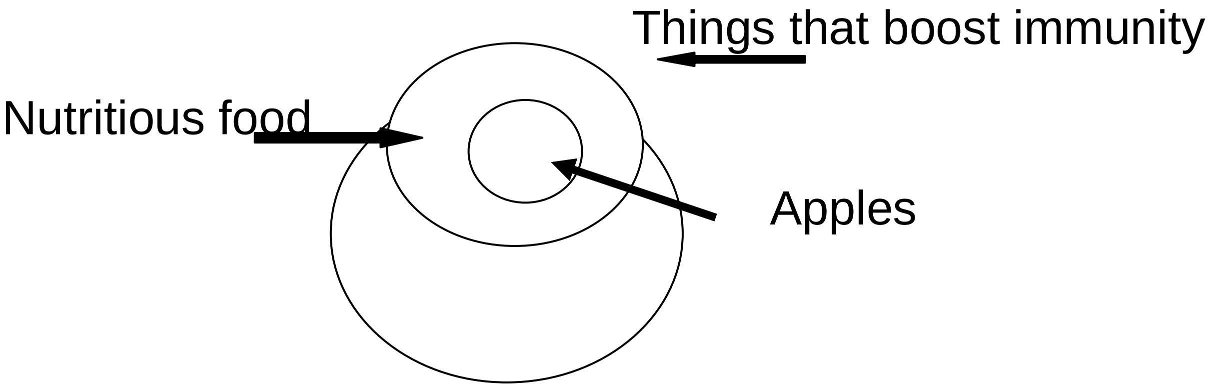
Valid. But when symbolized:

1. A

2. N / $\therefore$ I

Valid, but the validity is not evident in the symbolized form.

What you sense, however, is a logical relationship:



But in the symbolization mechanism of Propositional Logic no access to show the relation that makes the argument valid.

Similarly, consider :

1. Rabbits are not elephants
2. Bugs Bunny is a rabbit

Therefore, Bugs Bunny is not an elephant.

<u>Valid</u>. But when symbolized, it is:

1.~R

2. B / ∴~E

Its validity is not captured.

Why?

Often, logically significant information is given at the **sub-sentential level**: **Within** the structurally simple proposition

But these remain inaccessible to Propositional logic, because of its nature.

Limitations of Propositional Logic:

- Its basic unit for logical operation is the **structurally simple statements**
- It cannot further analyze the structurally simple statements to recover the logical relevant information from them. E.g., the logical relationships given in them.

So, we need to go beyond propositional logic to logics which allow further logical analysis of the structurally simple statements

- Aristotelian Logic of subject-predicate classes
- First order Predicate Logic

Aristotle's Logic of classes

Aristotle's logic is also known as Syllogistic Logic (Logic of Syllogisms)

Prior Analytics **by Aristotle:** Work in which his system of logic appeared.

1. His propositions: **Categorical propositions**
2. His structured arguments: **Syllogisms**. (Logic of Syllogisms)
3. A method of deduction for proving validity of arguments: Deductive Logic
4. A method of countermodels for proving invalidity

What are Categorical Propositions?

First, what is a **category**?

A **Category** in Aristotle's system stands for a **basic kind**, **type**, or **a class**.

Aristotle's examples: Things, Place, beings, time, Relation, etc : The basic kinds

More ordinary examples: Colors, Horses, Games, Indians, Nigerians etc: Intuitively formed **classes**

Class: A **collection** of items, things with a common characteristic.

E.g.: Class of Tennis players

Class of whales

Class of Fortune 500 companies

1. **Categorical propositions**: A proposition about categories or classes

A special kind of statements/ propositions that are about categories or classes and class membership.

Their typical structure:

Subject term (**Class 1**) – **verb (to be)** -Predicate term (**Class 2**)

Or, **Individual** –verb (to be)-Predicate term (**Class 2**)

3.**Format of categorical propositions**:

Subject term (1) verb 'to be' **(is, is not, are, are not)** predicate term (2)

There is a certain right sequence:

Subject-verb-Predicate

Examples:

Festivals are fun-events.

Marigolds are flowers.

Subject and Predicate Terms both must be Categorical or Class terms

Subject and Predicate terms must be Class terms:

Class indicating nouns.

E.g. Festivals, Marigolds, Fun-events, Flowers.

Categorical Propositions must contain a subject **term** & a predicate **term** (connected by a verb)

Terms: Words. **Linguistic entities,** but their reference is in the world outside of language.
E.g.: Rooms, white, good, color.

Subject: What or whom the proposition is about (**non-linguistic**).

Subject term: The **word or expressi**on that refers to the subject (**linguistic**)
Occupies the first term position in a standard form proposition.

May be an individual : Dr.Manmohan Singh

Or may be a class term (universal): Laptops

Predicate: Tells about the subject

Predicate Term: Linguistic entity which sometimes is an adjective, sometimes a class term.

Example:

1. **White** is a **color.**
2. **Rooms** are **good**
3. **Dr. Manmohan Singh** is an **Indian citizen**.
4. **Horses** are **mammals**

►**Predication: The act of** ascription of a property to a subject through language

►Attributing a predicate term about a subject term

E.g. **“X is F”** means: The property of ‘being F’ or ‘F-ness’ is predicated to X.

‘**White is a color**’ ⟶ The property of ‘being a color’ is predicated to ‘White’ .

Predication is an analytical tool to go inside a structurally simple proposition.

Given a categorical statements, we may try the lens of

Class relationship to observe the logical relationship given within:

X is F may be understood as **X class** is a **member** of the **F class**.

'**White is a color**' : White class is a member of the Color class.

Aristotelian predicates are **precise**: No borderline cases in their domain of application

Predicates may be **precise**, or **vague** predicates

Vague predicates: Predicates which allow borderline between membership and non-membership

E.g. 'Being old'. Now consider:

Mr Santosh who is 98 yrs old: **Very** old

Sheena who is 15: Not old

Tridib, who is 5: **Not at all** old

Mrs Ganesan, who is 45: ??

Is the statement 'Mrs Ganesan is old' (X is F) true?

Is it false?

Is it neither true nor false? Yes, that seems to be the case. But that is not permissible in a 2-valued system.

This counterexample to bivalence arises because 'being old' (F-ness) is a **vague predicate** : True of some people, False of some, and neither T nor F for borderline cases.

Aristotle's predicates are precise, their predication is also not vague.

Aristotle's categorical propositions are of **two basic kinds**:

(a) Each single assertion is either **Affirmation** (*kataphasis*) of a single predicate term of a single subject term : Affirmation of class relationship

1. *Laptops* are *machines*.
2. *Dr.Manmohan Singh* is an *Indian.*

(b) Or, **Denial** (*apophasis*) of a single predicate term of a single subject term: Denial of class relationship

1. *People* are not *machines*.
2. *Rafael Nadal* is not a *Japanese (person)*.

Aristotle called it the **Quality** of Categorical Statements: **Affirmative** and **Negative**

Affirmative: if subject class inclusion in predicate class is affirmed

E.g.: <u>Festivals</u> <u>are</u> <u>fun-events</u>.

Negative: if subject class inclusion in predicate class is denied

E.g.: <u>Roses</u> <u>are not</u> <u>fruits</u>.

But a standard form categorical proposition must also have: **Quantity**

<u>**Quantity: Universal** or **particular**</u>

Universal: If **all** members of the subject class is referred to

e.g. **All** Festivals are fun-events

Particular: If the subject term refers to only **some** members of a class

e.g.: <u>**Some**</u> fruits are sour.

So, **there is a standard form** for Categorical Statements.

Every standard form categorical proposition must have both **Quality and Quantity**

When combined:

QUANTITY		QUALITY	
		Affirmative	Negative
	Universal	✓	✓
	Particular	✓	✓

Basic 4 types of Standard form categorical proposition types

1. Universal (quantity) affirmative (quality): **A**

ALL roses ARE flowers

2. Universal Negative: **E**

NO rabbits ARE elephants

3. Particular affirmative: **I**

SOME dresses ARE full-sleeved apparels.

4. Particular negative: **O**

SOME trains ARE NOT punctual

A, I ⟶ **A**FF**I**RMO OR ‘I affirm’ (Latin)

E,O ⟶ N**E**G**O** or ‘I deny’ (Latin)

In Schematic form, where A and B are two categories or classes :

‘**A’: All As are Bs**

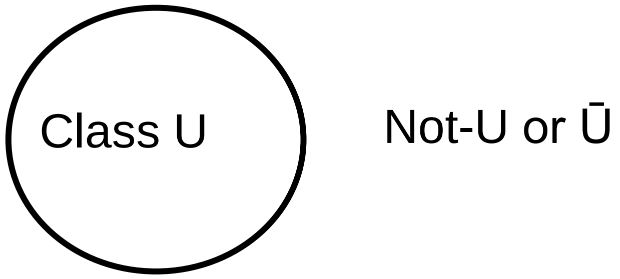
‘E’: No A s are Bs.

‘I ’: Some As are Bs

‘O’: Some As are not Bs.

Representation of Categorical Propositions:

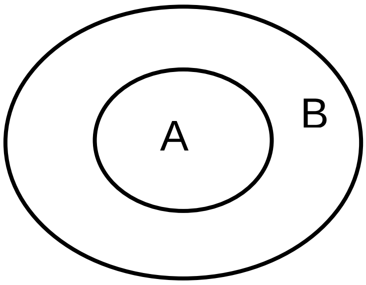
For a single class U, one may try:



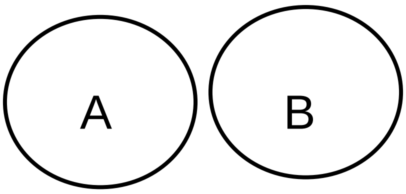
For any two classes / categories, A and B:

One may use **Euler's Diagram**: Eulerian circles. Circles within circles.

All As are Bs :



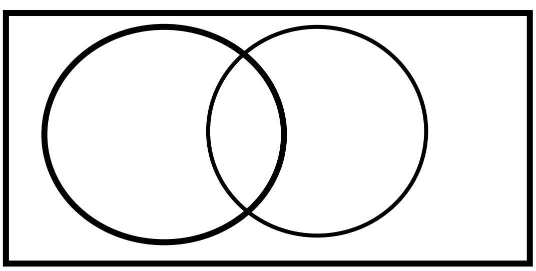
No As are Bs:



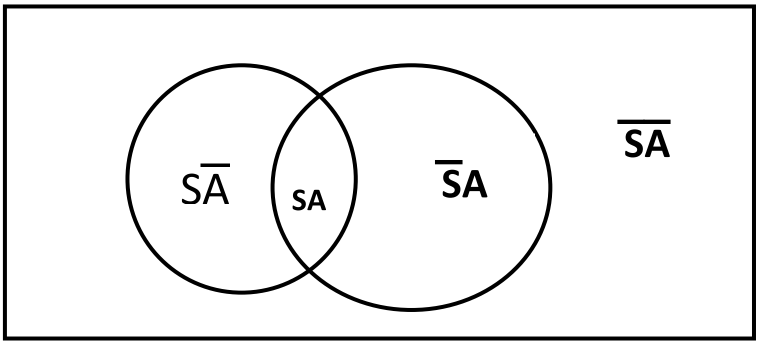
But, we shall not use any of these. We shall use **Venn Diagram to represent Categorical Statements.**

Venn Diagram: Intersecting circles, set within Universe of Discourse

U.D.



Universe of Discourse (U.D).: A presumed context. The Universal Set.

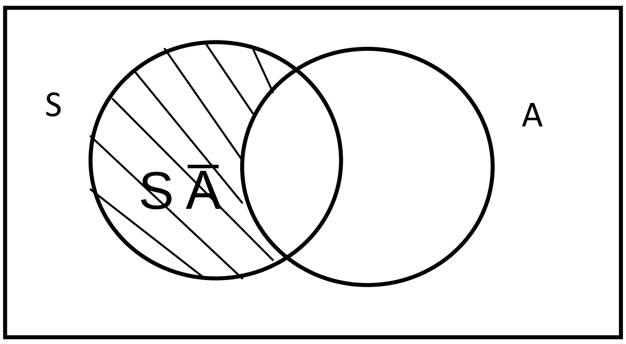


Regions in Venn Diagram

Using this, we may represent:

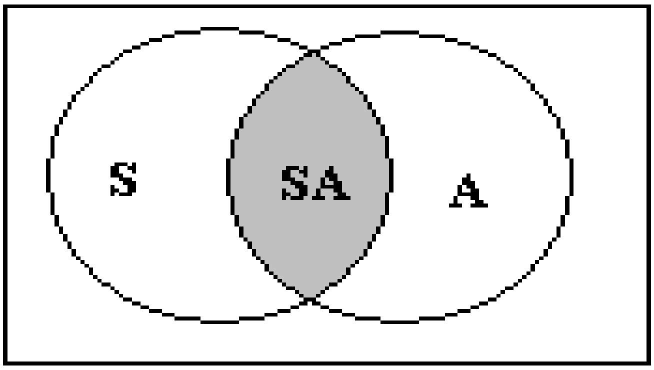
UNIVERSAL AFFIRMATIVE : **All S are A.**

$$\Longrightarrow S\bar{A} = \emptyset$$



UNIVERSAL NEGATIVE :

No S are A



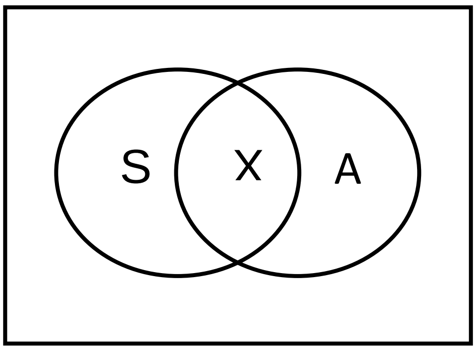
SA = Ø

PARTICULAR AFFIRMATIVE:

Some S are A ⟹ SA ≠ Ø

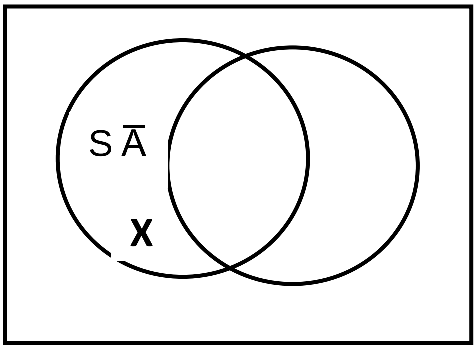
We put a 'X' in the non-empty area to indicate **there is at least one member** there

'Some': At least one



PARTICULAR NEGATIVE :

Some S are not A ⟹ $S\bar{A} \neq \emptyset$



► Remember: Without U.D., a Venn Diagram does not make sense.

Universe of discourse (UD): A presumed context.

DeMorgan (1846): Introduced the idea of (i) Restricted UD and (ii) Unrestricted UD, and the importance of each

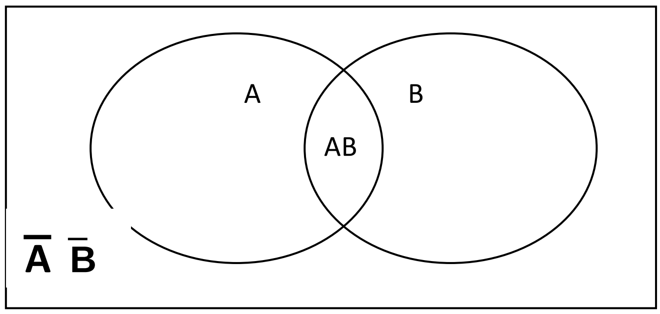
Boole (1854) : There is an assumed or expressed limit within which the subject of its operations are confined.

Venn (1881): Venn used Euler's diagram, but added to it.

(i) Diagram must have intersecting regions

(ii) Shading to indicate empty regions within the overlapping regions

(iii) And the UD.

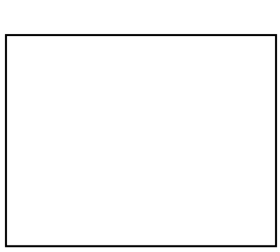


Lewis Carroll: Without Universe of discourse, we'll run into difficulty if we ever have to express:

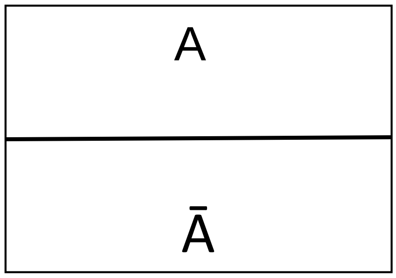
No not-A is not-B.

<u>Lewis Carroll's diagram</u> (1886):

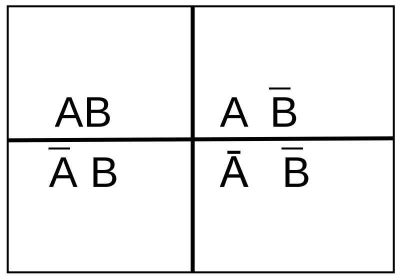
Universe of discourse:



For 1 category ‘A’:

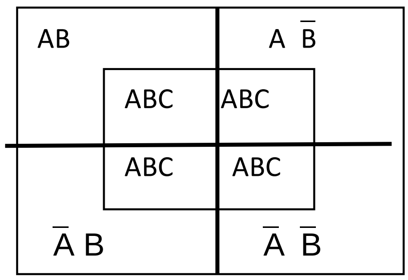


For 2 categories A and B:



Empty cell shown as ‘0’, occupied cell as ‘1’.

For 3 categories, A,B,C:



But we shall use Venn Diagram.

Statements which are not in this standard form but are potentially categorical in nature, may be reformulated and rearranged if they are to be used in Categorical logic.

<u>Translating non-standard propositions into standard form categorical propositions</u>

►Propositions may come in varied form. Need to identify the potential, non-standard categorical statements, which may be recast as standard categorical statements.

1. A statement in which one can sense the class terms, but the predicate term is an adjective, not really a class term.

Remedy: Replace predicate by <u>closest</u> class term

E.g. No trains are <u>punctual</u>.

<u>Std form</u>: No trains are <u>punctual vehicles.</u>

2. When the verb is not the right kind of verb for a standard form categorical proposition.

Remedy: Replace verb with a <u>suitable</u> ‘to be’ verb.

E.g. Some colors <u>soothe</u> the eyes

Std form: Some colors are ….(a class term) that soothe the eyes

Std form: Some colors are shades that soothe the eyes

3. When terms are not in the right sequence.

Remedy: Arrange them in subject-verb-predicate **(S-V-P)** sequence

E.g.: Hotels are all booked.

Std form: All hotels are accommodations that are booked.

4. When quantity term is missing

Remedy: Provide a suitable quantity term

E.g. Festivals are fun-events.

Std form: All festivals are fun-events.

5. When a quantity term is used, but it is a non-standard quantity term.

Remedy: Replace it with a suitable standard form quantity term:

E.g. A dolphin is intelligent.

Std form: All dolphins are intelligent animals.

E.g.Two vaccines are being developed.

Std form: Some vaccines are cures that are being developed.

More complex translations:

6. **Exclusive statements**.

E.g. Only button mushrooms are edible.

Std form: All mushrooms which are edible are button mushrooms.

Or, No non-button mushrooms are edible mushrooms.

7.Exceptive statements:

E.g. All except the students are outsiders.

Std form: All non-students are people who are outsiders **and** no students are people who are outsiders.

Categorical logic uses the lens of class inclusion, or class membership to further analyze the subject-predicate relationship.

E.g. All As are Bs mean all the members of A class are also members of the B class.

Traditional Square of Opposition

When the subject term and the predicate terms are identical,

E.g. All Indians are Asians

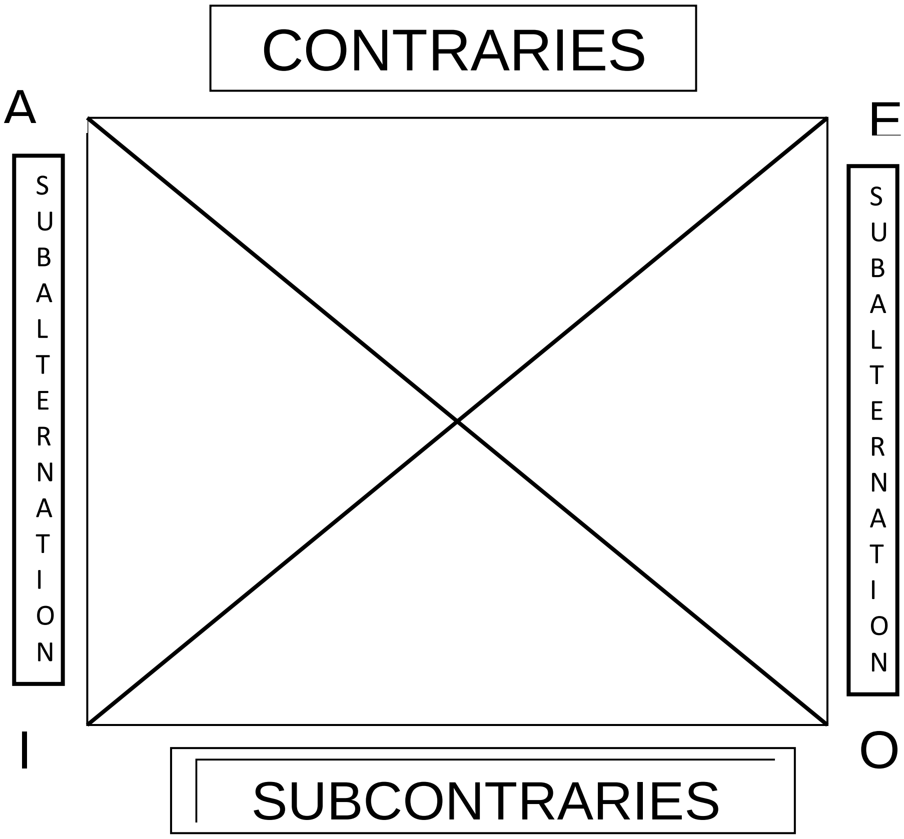
No Indians are Asians

Some Indians are Asians

Some Indians are not Asians

Categorical Logic represents the logical relationship among the A,E,I,O by a Square of Opposition

Traditional Square of Opposition



Opposition: Logical differences, logical relationships that form the basis of immediate inferences.

Aristotle: Inferences can be of **two types**: Immediate and Mediated.

Immediate inference: Inferences from a single premise, without the mediation of any other concept or statement.

Traditional Square of opposition

Supposed to provide ground for **immediate inference** for categorical propositions **with same subject and predicate terms.**

Contradictories: Quality and Quantity both differ. **Exactly one of the pair can be true, exactly one can be false**.

A – O, E - I

Contraries: Quantity same (Universal), Only quality differs. **They cannot both be true (if one true, other is false), but both may be false**.

A - E

Subcontraries: Quantity same (Particular), only quality differs. **Both cannot be false, but both may be true.**

I –O

Subalternation: **A - I, E - O**

Only differing quantity. **Truth flows down** from Universal to Particular, **falsity climbs up.**

If Universal is true, corresponding particular is true**.**

If particular is false, corresponding universal is false

Immediate inferences based on <u>Square of opposition</u>:

1. If A is true: What must be the truth-values of the E, I, O?

   E is false (Contrary)
   I is true (subalternation)
   O is false (Contradiction)

2. If E is true:

   A is false (Contrary)
   O is true (subalternation)
   I is false (Contradiction)

3. If I is true:

   E is false (Contradiction)
   A is undetermined.
   O is undetermined.

4. If O is true:

   E is false (Contradiction)
   A is undetermined.
   O is undetermined

Immediate inferences based on Square of opposition:

If A is false:

O is true (Contradiction)

E is undetermined

I is undetermined

If E is false:

I is true (Contradiction)

A is undetermined

O is undetermined

If I is false:

E is true(Contradiction)

O is true (subcontrary)

A is false (subalternation)

If O is false:

A is true(Contradiction)

I is true (subcontrary)

E is false (subalternation)

There are other **immediate inferences** in categorical logic:

- Conversion
- Obversion
- Contraposition

But we shall not discuss these

Aristotle's Mediated inferences: Syllogism

Aristotle's theory of Inference: Known as **The Syllogistic**

A syllogism is a very specific kind of deductive argument.

Its FORM:

1. It is composed **exclusively** of standard form categorical propositions

2. Two premises, with exactly one term in common: Between the two premises, three class terms. A common class term

3. One conclusion, with **exactly one term from each premise.**

Therefore:

Total: In a syllogism

- **Exactly 3** standard form categorical statements,
- **Exactly 3** class terms
- Each term will occur **<u>exactly twice</u>** in the syllogism.

Example:

1. All <u>rectangles</u> are <u>figures that are 4-sided</u>
2. All <u>squares</u> are <u>rectangles.</u>

3.Therefore, all <u>squares</u> are <u>figures which are 4-sided</u>.

Predicate term in conclusion: MAJOR Term (P)

Subject term in conclusion: MINOR Term (S)

Common term in premises: MIDDLE Term (M)

Premise with major term: **Major premise**

Premise with minor term: **Minor premise**

The proper order of categorical propositions in a syllogism is:

Major premise

Minor premise

Conclusion

The syllogistic form is considered to be the backbone of ***Dialectic*** (Art of examining knowledge claims and refuting spurious claims)

And a differentiator from of ***Rhetoric*** (Art of persuasive speech).

Aristotle: In contrast to genuine *sullogismois* whose conclusion follow of necessity from premises, there are also:

Contentious (*eristikos***) or Sophistical arguments:** Which only *appear* to establish their conclusions**,** and are actually dubious, cleverly disguised invalid, or unsound arguments.

Case 1. Invalid arguments that only appear to be valid.

When put in syllogistic form, their invalidity becomes apparent. Aristotle had a method of marking their invalidity.

Case 2. Genuine sullogismoi the premises of which are only apparently, but not really acceptable.

Example. From Aristotle's time:

P1. Whatever you have not lost, you still have.

P2. You have not lost horns.

Con. So, you still have horns.

Problem is the superficial plausibility of P1.

Consider other instances where the need for Syllogistic form is :

Case 1: Acceptable?

Some birds are geese.
No geese are felines.
Therefore, some birds are not felines

Not acceptable.

Standard Sequence:

Major premise

Minor premise

Conclusion

Case 2.: Acceptable?

No unmarried males are responsible renters.

Some tenants are bachelors.

Therefore, some tenants are not responsible renters.

No. 4 – term fallacy.

Restrict number of terms to 3 by replacing with synonyms,.

Case 3: Acceptable?

No dolphins are non-mammals

Some amphibians are not non-pets

So, some non-mammals are pets.

No. 5 – term fallacy. Replace by appropriate immediate inferences. Reduce terms to 3.

Another kind with suppressed premise:

Enthymemes: Incomplete syllogisms, when one or more of the propositions in a categorical syllogism is left unstated.

Remedy: To supply the suitable or obvious missing premise.

Case 4.

Since knives are sharp objects, no knives are objects allowed in the hand luggage on a plane.

Std form:

No sharp objects are objects allowed in the hand luggage on a plane.

All knives are sharp objects.

Hence, no knives are objects allowed in the hand luggage on a plane.

Problem with enthymeme: Their unclear position due to suppressed premise.

The rigid structure of Syllogisms is to distinguish them from the bogus arguments used in Rhetorics / Sophistry

His insistence on:

Validity by virtue of Syllogistic Form

Traditional Scheme of Validity of Syllogism: By a system of Mood and Figure of a Syllogism, and the distribution of the terms.

1. **Mood of a syllogism**: Determined by the type of the standard form categorical statements the syllogism contains.

E.g.

1. All subscribers are persons who are listed

2.No persons who are listed are tax-evaders

Therefore, no tax-evaders are subscribers

Mood of this syllogism is: AEE

Example:

Some humans are creatures who have a long life span.

All humans are mammals.

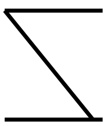
Therefore, some mammals are creatures who have a long life span.

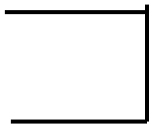
Mood of this syllogism is: IAI

2.Figure of a syllogism: Refers to the **positions** of the major and minor term with respect to the position of the middle term in the premises.

Syllogisms can have *exactly four* possible figures. Shown below.

M P   P M





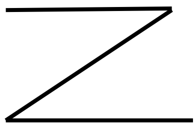
S M   S M

$\therefore S \rightarrow P$ $\therefore S \rightarrow P$

First Figure   **Second Figure**

--

M P   P M



M S   M S

$\therefore S \rightarrow P$ $\therefore S \rightarrow P$

Third Figure   **Fourth Figure**

Total: 64 possible Moods

4 possible figures

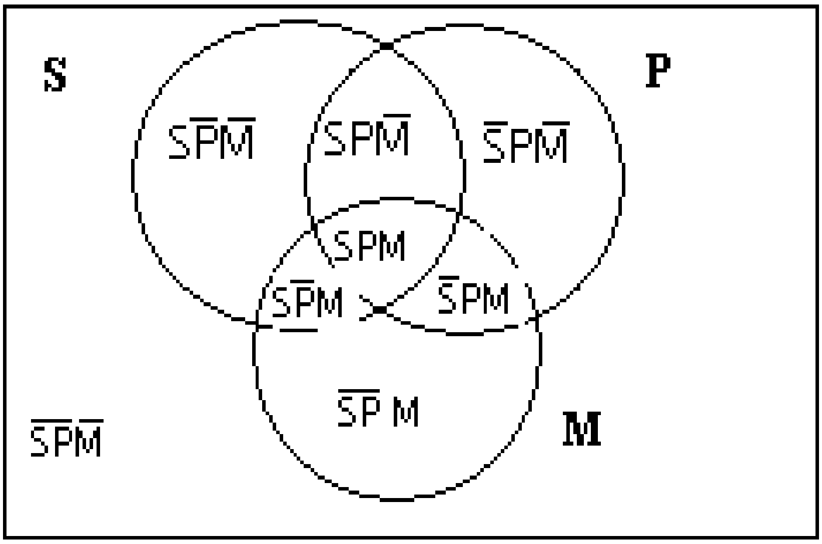
256 possible Syllogistic Form

<u>But only 15 forms were found Valid</u>.

Our method: By Venn Diagram

- Three overlapping circles, **S (Minor Term), P (Major term), and M (Middle term).**
- In this, we shall only try to plot the premises. Conclusion is **NOT** to be plotted.
- Syllogism is **valid** *iff* the same diagram in which the premises are plotted also shows the truth of conclusion.
- Syllogism is **invalid** *iff* the same diagram in which the premises are plotted does NOT show the truth of conclusion.

With respect to S,P,M, the areas within this diagram may be marked as:



Example:

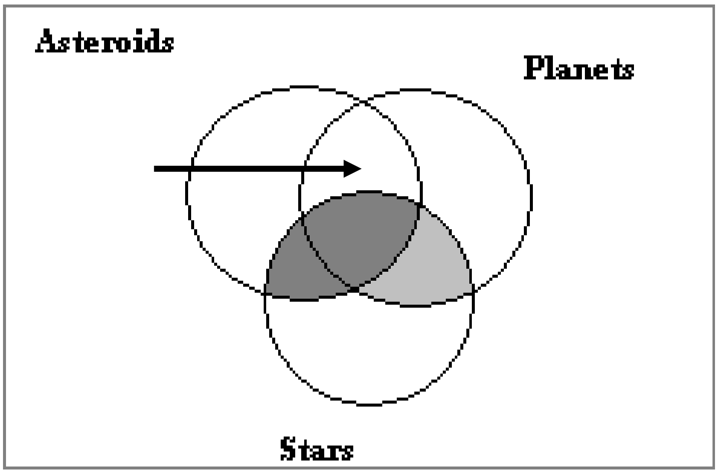
No star is a planet

No asteroid is a star (M)

No asteroid (S) is a planet (P)

No star is a planet
No asteroid is a star (M)

No asteroid (S) is a planet (P)



Even after all the shading there is still an unshaded area left in the Asteroid-Planet intersection area, which should have been shaded.

That area is marked by the arrow.

It is NOT really established that no asteroid is a planet

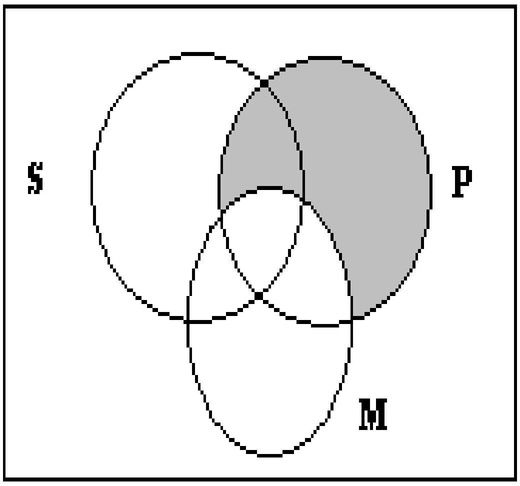
Therefore, the syllogism is **INVALID**

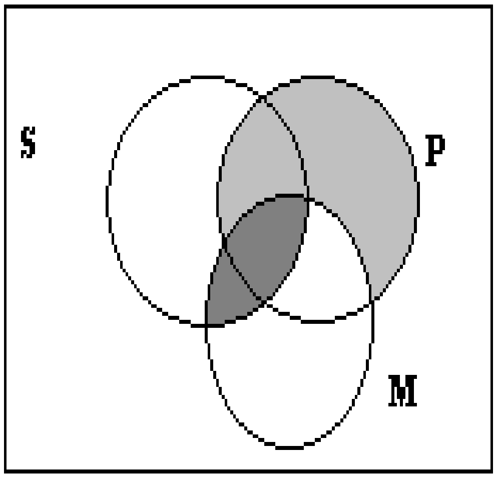
Example:

All contributors for the charity event (P) are successful industrialists (M)

No successful industrialists are lazy persons.

Therefore, no lazy persons (S) are contributors for the charity event (P).





The conclusion should show SP Intersection area as completely shaded.

That is exactly what the diagram by plotting the premises show.

Syllogism is **Valid.**

Note:

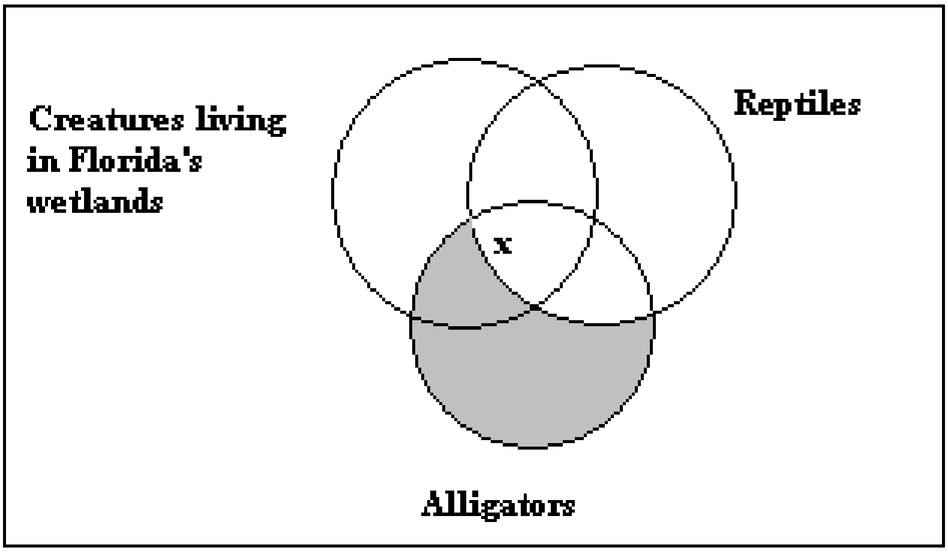
- When in the premises, universal and particular propositions are both present, plot the universal first.

Example:

All alligators are reptiles

So, some creatures that live in Florida's wetlands are reptiles

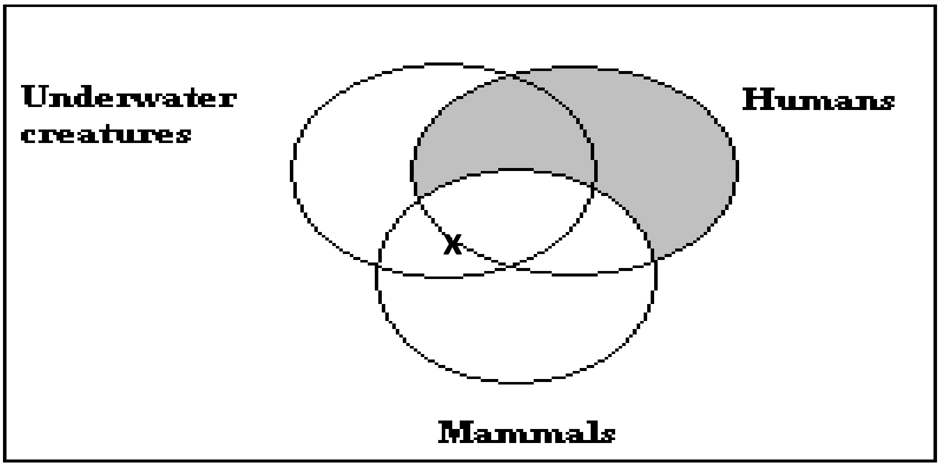
Some alligators are creatures that live in Florida's wetlands



Valid.

Note:

If in doubt which of the two regions the 'x' belongs to, put it on the border of both the regions



Sometimes when clearer information from the premises is not forthcoming about its exact position, you may have to put an 'x' on the border of two possible regions in all fairness.

It will depend on the syllogism whether such unclarity makes the syllogism valid or invalid.