- Guided TEM wave Propagation: Transmission lines

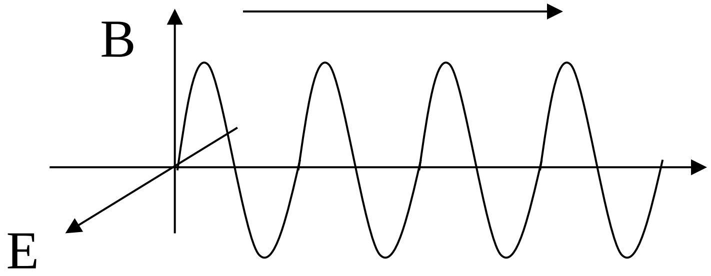
    1. Examples of Transmission Lines
    2. Transmission line parameters, equations
    3. Telegrapher's Equations
    4. Wave propagations
    5. Characteristic Impedance
    6. Special cases of lossless line

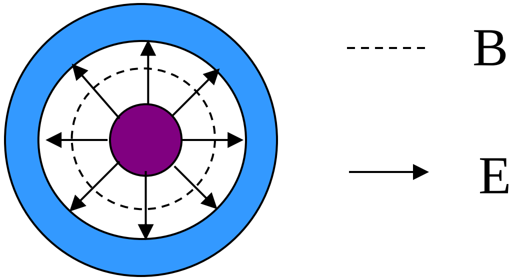
Guided Wave propagation requires the presence of guiding structures – metal or dielectrics. Here we consider metal guided wave propagation.

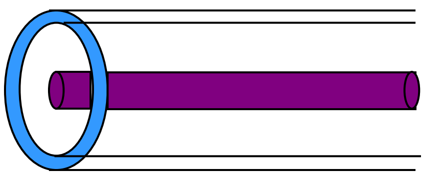
For **TEM** (Transverse Electro- Magnetic) wave, the Electric fields and the Magnetic fields are perpendicular to each other and lie in the plane perpendicular to the direction of propagation

Conventional **transmission lines** support TEM wave propagation with the presence of at least two metallic guiding structures.

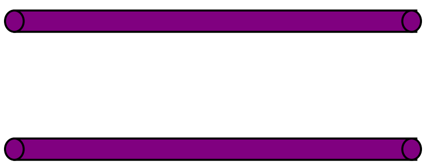
- Types of transmission lines
  - Transverse electromagnetic (TEM) transmission lines



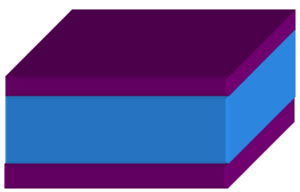




Coaxial line

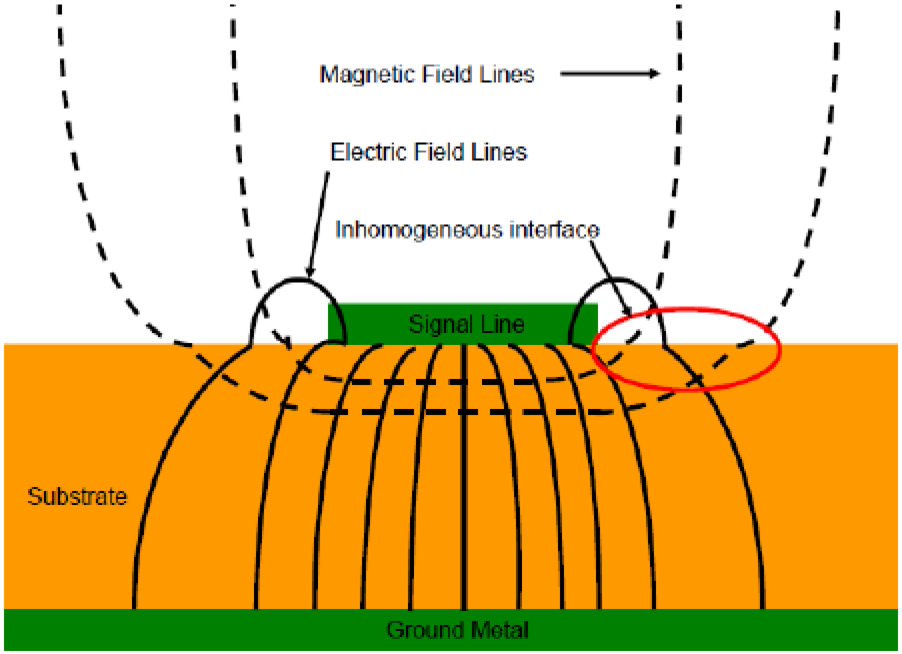


Two-wire line

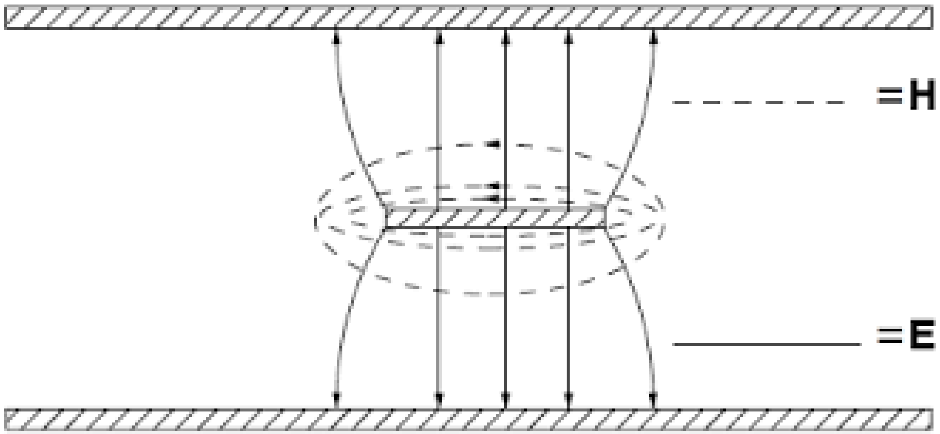


Parallel-plate line

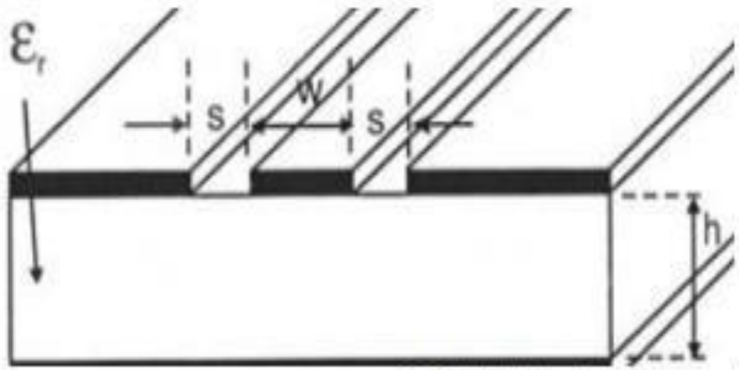
Microstrip Transmission Line & Field distribution

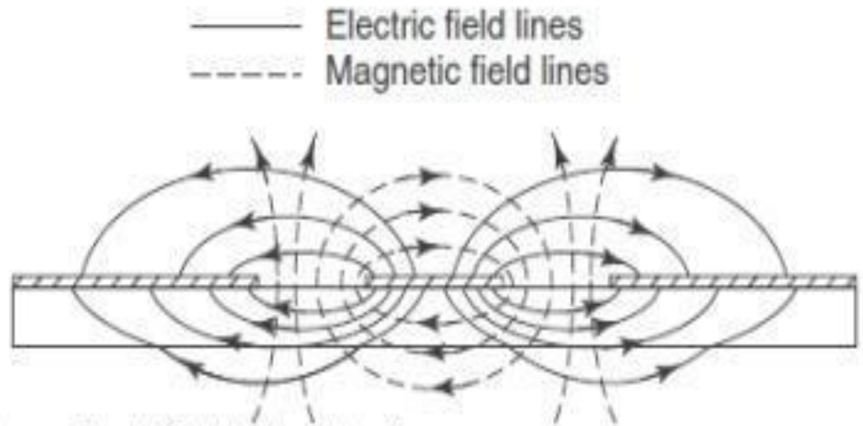


Stripline Transmission Line & Field distribution



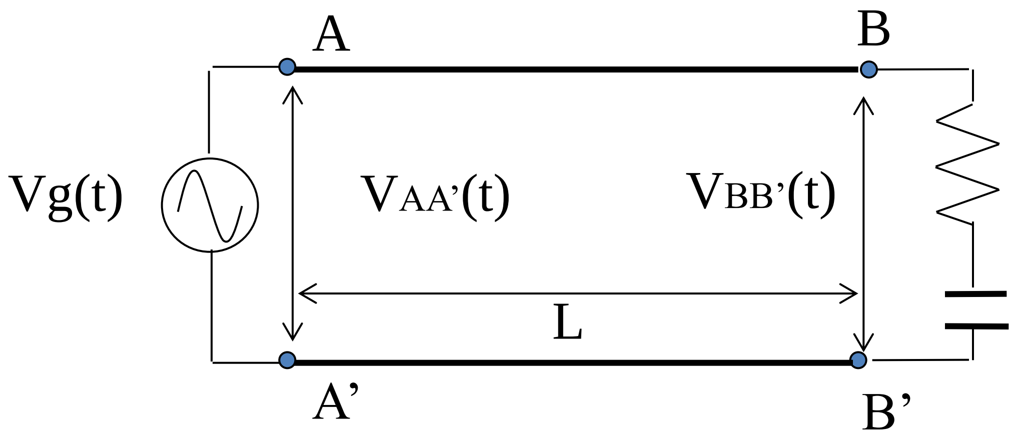
Co-Planar Waveguide (CPW) Transmission Line & Field distribution





CPW Electric-E and Magnetic-H field distribution

Transmission line parameters, equations



$V_{BB'}(t) = V_{AA'}(t)$

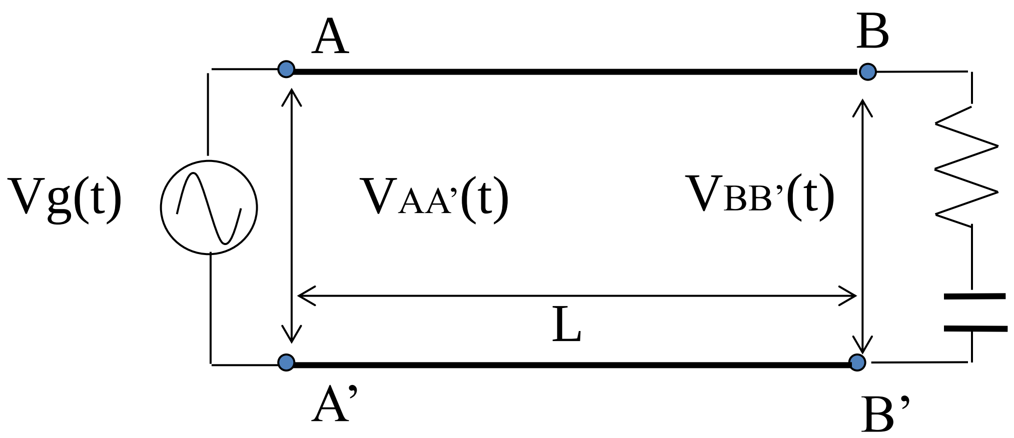
$V_{AA'}(t) = V_g(t) = V_0\cos(\omega t)$,

Low frequency circuits:

$$\boxed{V_{BB'}(t) = V_{AA'}(t)}$$

Approximate result

$$V_{BB'}(t) = V_{AA'}(t-t_d) = V_{AA'}(t-L/c)$$
$$= V_0\cos(\omega(t-L/c)),$$

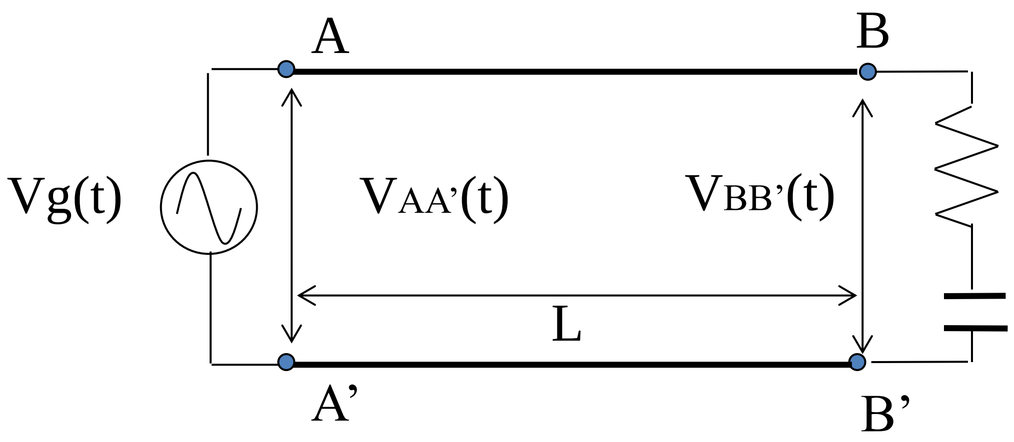


Recall: $f\lambda = c$, and $\omega = 2\pi f$

$$
\begin{aligned}
V_{BB'}(t) &= V_{AA'}(t-t_d) = V_{AA'}(t-L/c) \\
&= V_0\cos(\omega(t-L/c)) \\
&= V_0\cos(\omega t - 2\pi L/\lambda),
\end{aligned}
$$

If $\lambda \gg L$, $V_{BB'}(t) \approx V_0\cos(\omega t) = V_{AA'}(t)$,

If $\lambda \le L$, $V_{BB'}(t) \neq V_{AA'}(t)$, **the circuit theory has to be replaced**.



e. g: f = 1GHz, L = 1cm

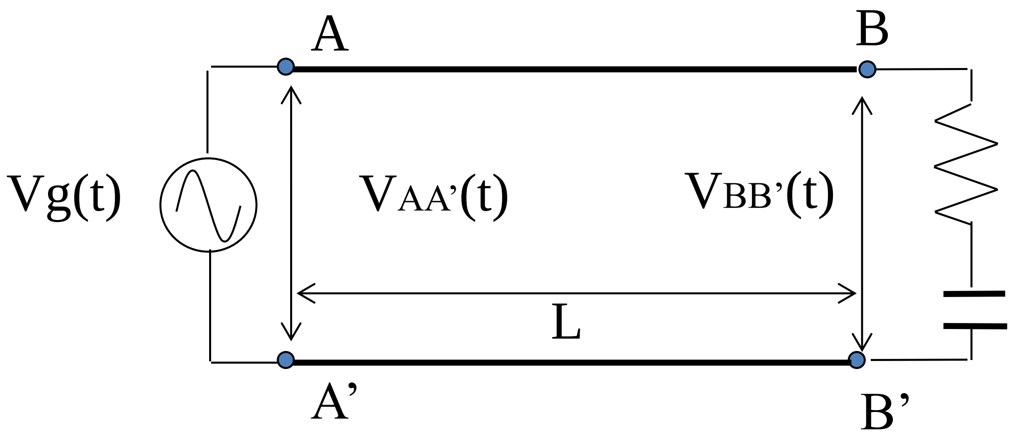
Time delay $\Delta t = L/c = 1\text{cm} / 3\text{x}10^{10}\ \text{cm/s} = 30\ \text{ps}$

Phase shift $\Delta\varphi = 2\pi f \Delta t = 0.06\ \pi$ $V_{BB'}(t) = V_{AA'}(t)$

f = 10GHz, L = 1cm

Time delay $\Delta t = L/c = 1\text{cm} / 3\text{x}10^{10}\ \text{cm/s} = 30\ \text{ps}$

Phase shift $\Delta\varphi = 2\pi f \Delta t = 0.6\ \pi$ $V_{BB'}(t) \neq V_{AA'}(t)$



- time delay

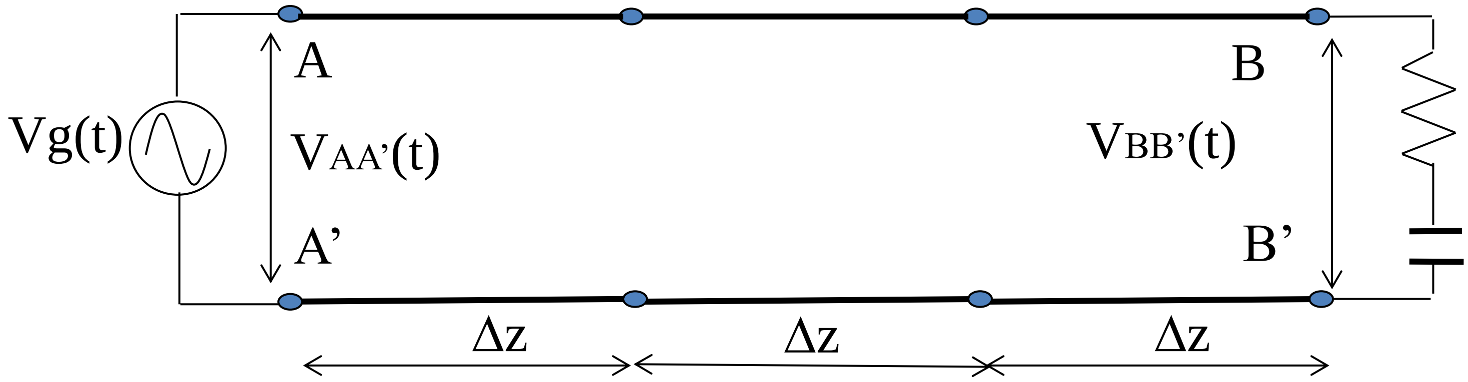
  $V_{BB'}(t) = V_{AA'}(t-t_d) = V_{AA'}(t-L/v_p)$,

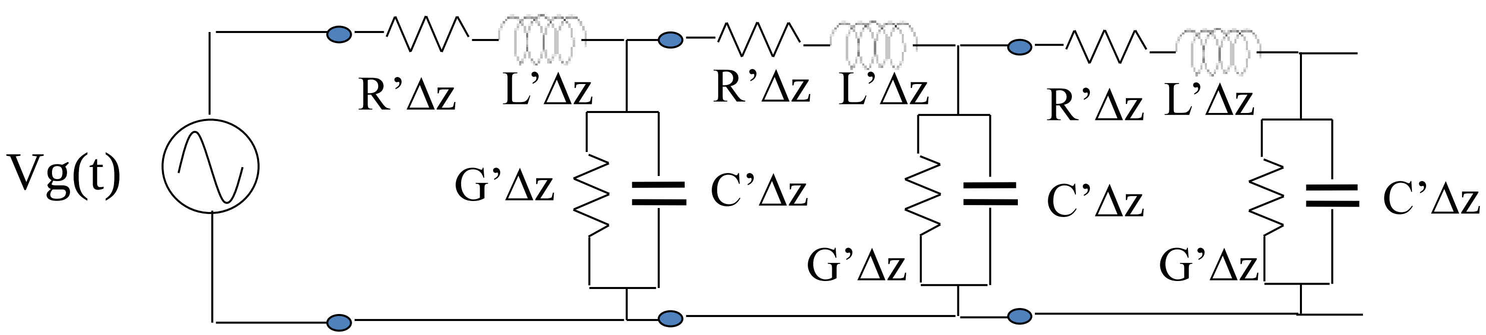
- Reflection: the voltage has to be treated as wave, some bounce back

- power loss: due to reflection and some other loss mechanism,

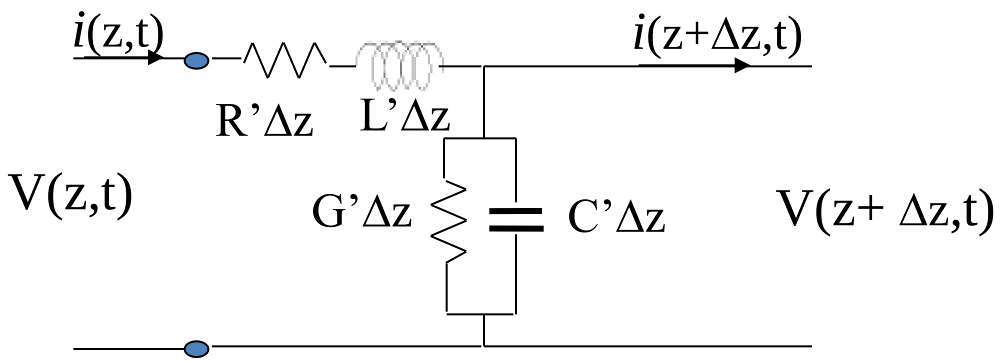
- Dispersion: in material, the velocity of wave propagation V_p could be different for different wavelength

- **Lumped-element Model**
  - Represent transmission lines as parallel-wire configuration





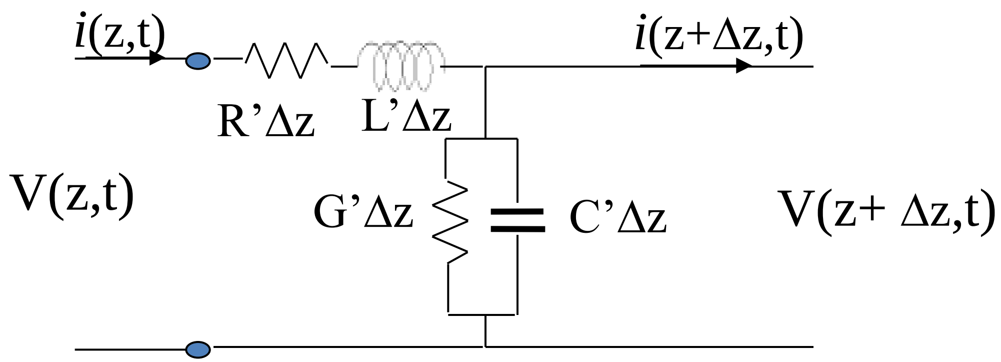
- Transmission line equations for an infinitesimally small segment



$$V(z,t) = R'\Delta z\, i(z,t) + L'\Delta z\, \partial i(z,t)/\partial t + V(z+\Delta z,t),$$

$$i(z,t) = G'\Delta z\, V(z+\Delta z,t) + C'\Delta z\, \partial V(z+\Delta z,t)/\partial t + i(z+\Delta z,t),$$

Using standard KCL & KVL, valid for such smaller segments



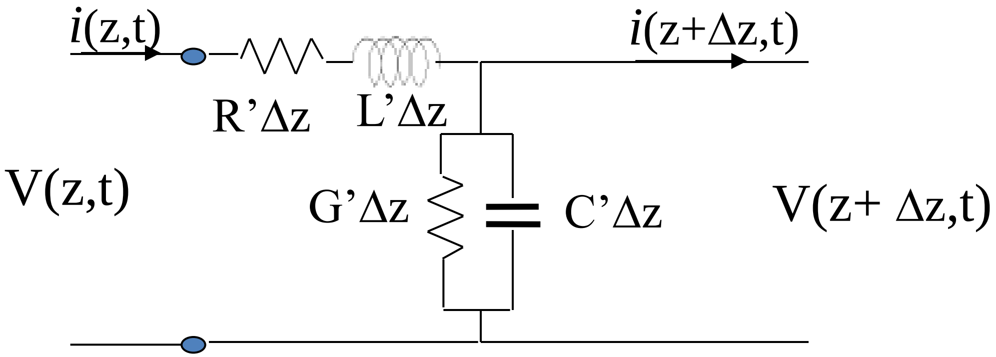
$V(z,t) = R'\Delta z\, i(z,t) + L'\Delta z\, \partial i(z,t)/\partial t + V(z+\Delta z,t),$

$\Rightarrow$ $-V(z+\Delta z,t) + V(z,t) = R'\Delta z\, i(z,t) + L'\Delta z\, \partial i(z,t)/\partial t$

$\Rightarrow$ $-\partial V(z,t)/\partial z = R'\, i(z,t) + L'\, \partial i(z,t)/\partial t,$

Rewrite V(z,t) and i(z,t) as phasors, for sinusoidal V(z,t) and i(z,t):

$V(z,t) = \mathrm{Re}(\tilde{V}(z)\, e^{j\omega t}),$ $\quad$ $i(z,t) = \mathrm{Re}(\tilde{i}(z)\, e^{j\omega t}),$

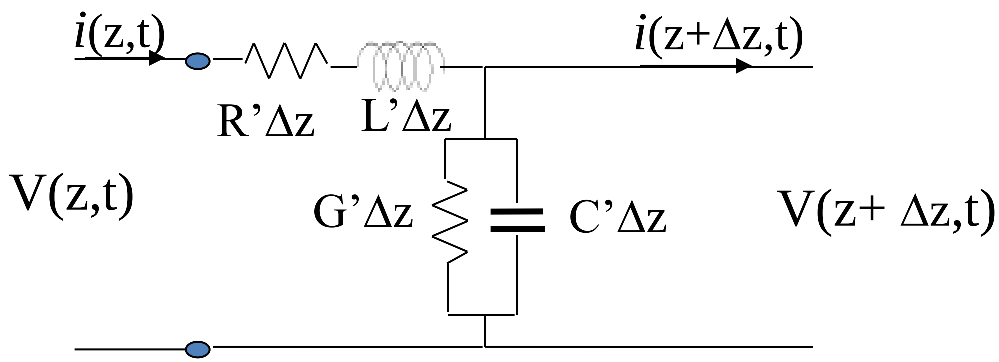


Recall:

$$d i(t)/dt = \mathrm{Re}(d\,\tilde{i}\, e^{j\omega t})/dt = \mathrm{Re}(\tilde{i}\; j\omega e^{j\omega t}),$$

$$-\,\partial V(z,t)/\partial z = R'\, i(z,t) + L'\, \partial\, i(z,t)/\partial t,$$

$$\Rightarrow \boxed{-\,d\tilde{V}(z)/dz = R'\,\tilde{i}(z) + j\omega L'\,\tilde{i}(z),}$$



$i(z,t) = G'\Delta z\, V(z+\Delta z,t) + C'\Delta z\, \partial V(z+\Delta z,t)/\partial t + i(z+\Delta z,t)$,

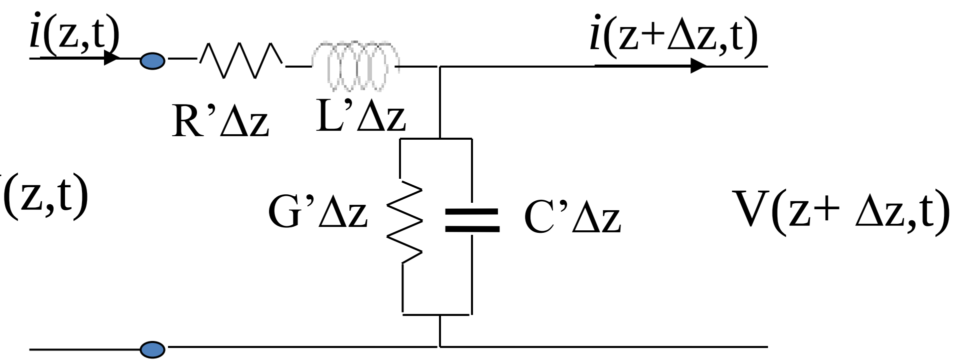
$\Rightarrow$ $-\, i\,(z+\Delta z,t) + i\,(z,t) = G'\Delta z\, V(z+\Delta z,t) + C'\Delta z\, \partial V(z+\Delta z,t)/\partial t$

$\Rightarrow$ $-\, \partial\, i(z,t)/\partial z = G'\, V(z,t) + C'\, \partial V(z,t)/\partial t$,

Rewrite V(z,t) and *i*(z,t) as phasors, for sinusoidal V(z,t) and *i*(z,t):

$V(z,t) = \mathrm{Re}(\, \tilde{V}(z)\, e^{j\omega t}),\qquad i\,(z,t) = \mathrm{Re}(\, \tilde{i}\,(z)\, e^{j\omega t})$,

- Transmission line equations



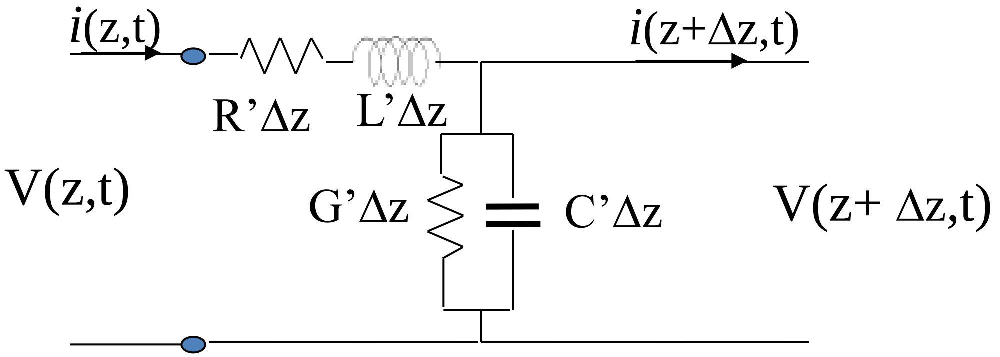
Recall:

$$dV(t)/dt = \mathrm{Re}(d\,\tilde{V}\,e^{j\omega t})/dt = \mathrm{Re}(\tilde{V} j\omega e^{j\omega t}),$$

$$-\,\partial i(z,t)/\partial z = G'\,V(z,t) + C'\,\partial\,V(z,t)/\,\partial t,$$

$$\Rightarrow \boxed{-\,d\,\tilde{i}(z)/dz = G'\,\tilde{V}(z) + j\omega C'\,\tilde{V}(z),}$$

- **Telegrapher's equation** in phasor domain for volatge



$$\begin{cases} -\,d\tilde{V}(z)/dz = R'\,\tilde{i}(z) + j\omega L'\,\tilde{i}(z), \\ \\ -\,d\,\tilde{i}(z)/dz = G'\,\tilde{V}(z) + j\omega C'\,\tilde{V}(z), \end{cases}$$

Take d /dz on both sides

$$\Rightarrow \quad -\,d^2\tilde{V}(z)/dz^2 = R'\,d\tilde{i}(z)/dz + j\omega L'\,d\tilde{i}(z)/dz,$$

$$\begin{cases} -\,d\tilde{V}(z)/dz = R'\,\tilde{i}(z) + j\omega L'\,\tilde{i}(z), \\[2ex] -\,d\,\tilde{i}(z)/dz = G'\,\tilde{V}(z) + j\omega C'\,\tilde{V}(z), \end{cases}$$

$$-\,d^2\tilde{V}(z)/dz^2 = R'\,d\tilde{i}(z)/dz + j\omega L'\,d\tilde{i}(z)/dz,$$

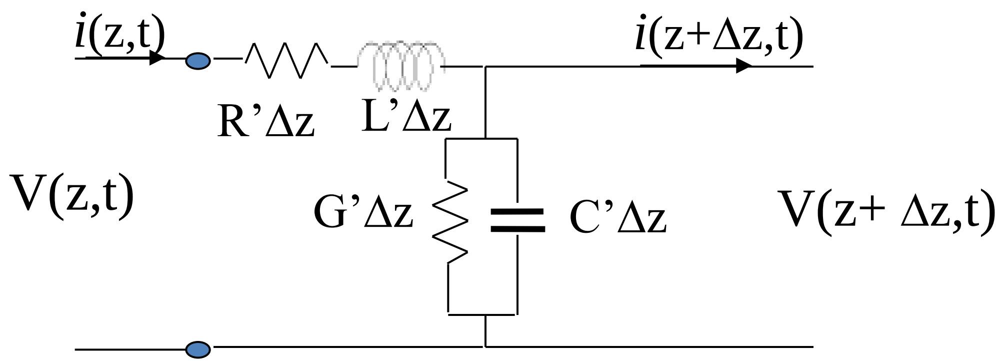
substitute to obtain

$$d^2\tilde{V}(z)/dz^2 = (R' + j\omega L')\,(G' + j\omega C')\tilde{V}(z), \quad \text{or}$$

$$\boxed{d^2\tilde{V}(z)/dz^2 - (R' + j\omega L')\,(G' + j\omega C')\tilde{V}(z) = 0,}$$

$$d^2\tilde{V}(z)/dz^2 - \gamma^2\tilde{V}(z) = 0,$$

$$\boxed{\gamma^2 = (R' + j\omega L')\,(G' + j\omega C'),} \qquad \text{Propagation const.}$$



$$\begin{cases} -d\tilde{V}(z)/dz = R'\,\tilde{i}(z) + j\omega L'\,\tilde{i}(z), \\ -d\,\tilde{i}(z)/dz = G'\,\tilde{V}(z) + j\omega C'\,\tilde{V}(z), \end{cases}$$

Take d /dz on both sides

$$\Rightarrow \quad -d^2\,\tilde{i}(z)/dz^2 = G'\,d\tilde{V}(z)/dz + j\omega C'\,d\tilde{V}(z)/dz,$$

- **Telegrapher's equation** in phasor domain for current

$$\begin{cases} -\,d\tilde{V}(z)/dz = R'\,\tilde{i}(z) + j\omega L'\,\tilde{i}(z), \\ -\,d\,\tilde{i}(z)/dz = G'\,\tilde{V}(z) + j\omega C'\,\tilde{V}(z), \end{cases}$$

$$-\,d^2\,i(z)/dz^2 = G'\,dV(z)/dz + j\omega C'\,dV(z)/dz,$$

Substitute to obtain

$$d^2\,\tilde{i}(z)/dz^2 = (R' + j\omega L')\,(G' + j\omega C')\,\tilde{i}(z), \qquad \text{or}$$

$$\boxed{d^2\,\tilde{i}(z)/dz^2 - (R' + j\omega L')\,(G' + j\omega C')\,\tilde{i}(z) = 0,}$$

$$d^2\,\tilde{i}(z)/dz^2 - \gamma^2\tilde{i}(z) = 0,$$

$$\boxed{\gamma^2 = (R' + j\omega L')\,(G' + j\omega C'),} \qquad \text{Propagation constant}$$

- **Wave equations**

$$
\begin{cases}
d^2\tilde{V}(z)/dz^2 - \gamma^2\tilde{V}(z) = 0, \\
d^2\,\tilde{i}(z)/dz^2 - \gamma^2\tilde{i}(z) = 0,
\end{cases}
$$

$$\gamma = \alpha + j\beta,$$

$$\alpha = \mathrm{Re}\sqrt{(R' + j\omega L')(G' + j\omega C')}\ ,$$ Attenuation constant

$$\beta = \mathrm{Im}\sqrt{(R' + j\omega L')(G' + j\omega C')}\ ,$$ Propagation constant

Solving the second order differential equation

$$
\begin{cases}
\tilde{V}(z) = V_0^{+} e^{-\gamma z} + V_0^{-} e^{\gamma z} \\
\tilde{i}(z) = I_0^{+} e^{-\gamma z} + I_0^{-} e^{\gamma z}
\end{cases}
$$

- Wave equations

$$\begin{cases} \tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \\ \tilde{i}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \end{cases}$$

<u>where:</u>

V_0^+ and V_0^- are determined by boundary conditions.

I_0^+ and I_0^- are related to V_0^+ and V_0^- by **characteristic impedance** Z_0.

- **Characteristic impedance** Z_0

$$\begin{cases} \tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \\ \tilde{i}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \end{cases}$$

<u>recall:</u>

$$- d\tilde{V}(z)/dz = R' \tilde{i}(z) + j\omega L' \tilde{i}(z),$$

$$\Rightarrow \quad \gamma V_0^+ e^{-\gamma z} - \gamma V_0^- e^{\gamma z} = (R' + j\omega L') \tilde{i}(z),$$

$$\Rightarrow \quad \tilde{i}(z) = \frac{\gamma}{(R' + j\omega L')} (V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z})$$

$$\Rightarrow \quad I_0^+ = \frac{\gamma}{(R' + j\omega L')} V_0^+ \qquad I_0^- = \frac{-\gamma}{(R' + j\omega L')} V_0^-$$

- Characteristic impedance Z_0

$$I_0^+ = \frac{\gamma}{(R' + j\omega L')} V_0^+$$

$$I_0^- = \frac{-\gamma}{(R' + j\omega L')} V_0^-$$

Define characteristic impedance Z_0

$$Z_0 \equiv \frac{V_0^+}{I_0^+} = \frac{(R' + j\omega L')}{\gamma}$$

$$= \sqrt{\frac{(R' + j\omega L')}{(G' + j\,\omega C')}}$$

<u>recall:</u>

$$\gamma = \sqrt{(R' + j\omega L')\,(G' + j\omega C')}$$

- <u>Example, a loss-less air filled coaxial line :</u>

  R’ = 0 Ω, G’ = 0 /Ω, Z_0 = 50Ω, β = 20 rad/m, f = 700 MHz

  L’ = ? and C’ = ?

<u>solution:</u>

$$Z_0 = \sqrt{\frac{(R' + j\omega L')}{(G' + j\,\omega C')}} = \sqrt{\frac{j\omega L'}{j\,\omega C'}} = 50\Omega$$

$$\gamma = \sqrt{(R' + j\omega L')\,(G' + j\omega C')} = j\omega\sqrt{L'C'}$$

$$\gamma = \alpha + j\beta,$$

$$\beta = \omega\sqrt{L'C'} = 20 \text{ rad/m}$$

- <u>**lossless** transmission line :</u>

$$\gamma = \alpha + j\beta,$$

$$= \sqrt{(R' + j\omega L')\,(G' + j\omega C')}$$

If R'<< j ωL' and G' << jωC',

$$\gamma = \sqrt{(R' + j\,\omega L')\,(G' + j\omega C')}$$

$$= j\omega\sqrt{L'C'}$$

$$\boxed{\begin{aligned} \alpha &= 0 \\ \beta &= \omega\sqrt{L'C'} \end{aligned}}$$

lossless line

- <u>lossless transmission line :</u>

$$Z_0 = \sqrt{\frac{(R' + j\omega L')}{(G'+j\,\omega C')}} = \sqrt{\frac{j\omega L'}{j\,\omega C'}}$$

$$\boxed{Z_0 = \sqrt{\frac{L'}{C'}}}$$

lossless line

$$\boxed{\begin{aligned} \alpha &= 0 \\ \beta &= \omega\sqrt{L'C'} \end{aligned}}$$

$$\beta = 2\pi/\lambda \quad \Rightarrow \quad \lambda = 2\pi/\beta = \frac{1}{\omega\sqrt{L'C'}}$$

$$V_p = \omega/\beta = \frac{1}{\sqrt{L'C'}}$$

- <u>For TEM transmission line :</u>

$L'C' = \mu\varepsilon$

$$V_p = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}}$$

$$\beta = \omega\sqrt{L'C'} = \omega\sqrt{\mu\varepsilon}$$

- <u>summary :</u>

$$\begin{cases} \tilde{V}(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z} \\ \tilde{i}(z) = I_0^+ e^{-j\beta z} + I_0^- e^{j\beta z} \end{cases}$$

$$Z_0 = \sqrt{\frac{L'}{C'}}$$

$$V_p = \frac{1}{\sqrt{L'C'}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}}$$

$$\beta = \omega\sqrt{L'C'} = \omega\sqrt{\mu\varepsilon}$$