# HAND WRITTEN NOTES OF ELECTRONICS & COMMUNICATION ENGINEERING

SUBJECT:

# ELECTROMAGNETIC THEORY

7

E.M.T

Date
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20.

$W_t = 251W$
$d = 100m$
$A_{er} = 500\times10^{-4}$
$W_r = ?$

$W_r = \frac{251}{4\pi(100)^2}\times500\times10^{-4}$
$= 99\times10^{-6}$
$= 100\mu W$

19.

$W_r = \frac{W_t\ G_t}{4\pi d^2}A_{er}$

$W_r = 8\pi k \times 4 \times 5$ $\frac{1\times10\times1}{4\pi(1)^2} = \frac{10}{4\pi}$

$W_r = 0.8W$

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$\frac{E_1}{E_2} = \frac{d_2}{d_1}$

$\frac{1}{E_2} = \frac{4\times10^3}{2\times10^3}$

$E_2 = \frac{1}{2} = 0.5\ mV/m$

E.M.T 170

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Jai Durga Maa

classmate 170

Friday [redacted]

EMT

③

1. Static Electromagnetic fields - William Hayt (Theory) & Scham series (Problems)

IES = 30% - 40% Gate 10%

2. EM waves - Jordan Batmain
3. YI waves - John D Ryder
4. Guided waves - Jordan Batmain
5. Antennas - K.D. Prasad.

} 80% (gate) [15 question] (IES)

I Static Electro-Magnetic fields:

Tough : coordinate systems
Vector calculus

Easy: 1. Analogies E-H fields

$\bar{E}$ - Electric field Intensity volt/m
$\bar{H}$ - Magnetic field intensity amp/m
} strength of the field.

B - Magnetic flux density weber/$m^2$
D - Electric flux density colomb/$m^2$
} strength of the field

$\epsilon = \epsilon_0 \epsilon_R$ = Permittivity of the medium

Physically meaning :→ It is the ability of the material to hold or to allow or to permit $\bar{E}$ field. Farads/m

$$C = \frac{\epsilon A}{d}$$

The least value of $\epsilon$ is $\epsilon_0$ because $\epsilon_R \geq 1$. So every medium in the world including vacuum permits electric field.

Least Permittivity = Vaccum
Best " = Dielectric



$\mu = \mu_0 \mu_R$ = permeability of the medium

It is the ability of the material to hold or to allow or to permitt H field.

unit :- Henry/m

$\mu = \mu_0$ when $\mu_R = 1$

$\mu_R \geq 1$

$\mu_0$ = permeability of the vaccum.

$$\boxed{C = \frac{Q}{V}}$$

$$Q = C \times V$$

$$\frac{\text{coulombs}}{m^2} = \frac{\text{farads}}{m} \times \frac{\text{Volts}}{m}$$

$$\boxed{D = \epsilon E}$$

$$\boxed{L = \frac{\psi_m}{I}} \qquad \frac{\text{webers}}{m^2} = \frac{\text{Henry}}{m} \times \frac{\text{amps}}{m}$$

$$\boxed{B = \mu H}$$

cause of Electric field – Stationary point charge Q.

| | | | | | |
|---|---|---|---|---|---|
| coulombs | $Q$ | = point | | $I\,dl$ | Amp-m |
| C/m | $\rho_L$ | = line | | $I$ | Amp |
| C/m² | $\rho_s$ | = surface | | $\bar{K}$ | Amp/m |
| C/m³ | $\rho_v$ | = volume | | $\bar{J}$ | Amp/m² |

$\rho_L = \frac{dq}{dl}$ $\rho_v = \frac{dq}{dv}$ $\rho_s = \frac{dq}{ds}$ (5)

source: of Magnetic field. – DC current I flowing in a line

$\bar{K}$ = surface current Amp/m
$\bar{J}$ = current in a solid conductance Amp/m²
$I\bar{dl}$ = current element - I flowing in a wire of zero lengths (Amp-m)

$Q$ = colombs = $\rho_v \cdot dl \Rightarrow \frac{\text{colombs}}{m} \times m$

$\rho_s \cdot ds \Rightarrow \frac{\text{colombs}}{m^2} \times m^2$

$\rho_v \cdot dv \Rightarrow \frac{\text{colombs}}{m^3} \times m^3$

$I\,\bar{dl}$ = Amp-m
$\bar{K}\,ds = \bar{J}\,dv$
$\bar{J}\,dv$

Vector Calculus:

$\nabla$ is called as spatial derivative vector operator when operated on any quantity it gives an idea of the rate of change of the quantity in space.
It also specify the direction in which the quantity is changing.
$\nabla$ can be operated on scalar quantities and it can be operated on vector quantity.
If a scalar quantity f is considered and $\nabla$ is operated on it, it is called gradient of a scalar

eg scalar $f = 4x^2y + xz$

vector $\bar{A} = 8xyz\, a_x - 4x^2y\, a_y + 7z\, a_z$ (6)

- If $\nabla$ is operated on vector quantity then

  $\nabla \cdot \bar{A} \rightarrow$ Divergence of vector.

  $\nabla \times \bar{A} \rightarrow$ curl of a vector.

$\nabla$
- Scalar $f \rightarrow \nabla f$ (gradient of a scalar)
- Vector
  - $\nabla \cdot \vec{A}$ (Divergence of vector)
  - $\nabla \times \vec{A}$ (curl of a vector)

Vector identity

1. $\nabla \times \nabla f = 0$ curl (Grad. of scalar) $= 0$ ($\nabla \times \nabla = 0$)

(in curl $\nabla \times \bar{\nabla} = |\nabla||\nabla| \sin\theta$) when both the same $\theta = 0^\circ$.

2. $\nabla \cdot (\nabla \times \bar{A}) = 0$ Divergence (curl of vector) $= 0$

(orthogonal)

$\nabla \times A$ result in a vector perpendicular to $\nabla$ and $\bar{A}$ where the resultant vector is operated again with $\nabla$ as dot product the ans is 0.

$$A \times B = C \qquad C \perp (A \,\&\, B)$$

$$A \cdot C = 0 \quad \{ |A||C| \cos 90^\circ = 0 )$$

$$A \cdot (A \times B) = 0$$

3) $\nabla \times (\nabla \times \bar{A}) = \nabla(\nabla \cdot A) - A(\nabla \cdot \nabla)$

$$= \nabla(\nabla \cdot A) - \nabla^2 A$$

Summary:

1. $\nabla \times \nabla = 0$
2. $\nabla \cdot \nabla = \nabla^2$ = (scalar Laplacian operator)

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Outflow & Divergence:

consider a cause which have effects spread out from the cause. The effects are such that there outward dispersive and hence expand increasing their area of presence.

In the process as area increases the strength decreases Hence strength is called as density as shown below.

$$\text{Area} \uparrow \times \text{strength} \downarrow = \text{constt} \propto \text{cause.}$$

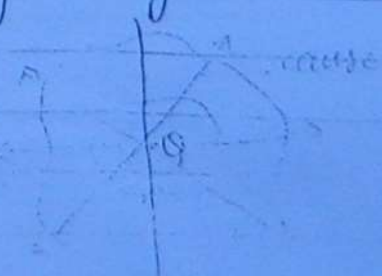
$$\text{Strength} = \frac{\text{constt}}{\text{Area}}$$

$$= \text{Density}$$

cause → Q (charge)
Effect → Electric flux $\psi_e$ (outward from the cause)
strength → Electric flux density (D)

If a cause is a charge of Q coloumbs then $\psi_e$ is the effect because of the cause which are called as electric flux then strength is called as flux density D.

The total effects can always being analyzed over a area completely enclosed the cause.



Hence strength x Area over any closed surface should be equal to the cause. 8

total effect $\boxed{\oint_{\text{closed}} D \cdot ds = Q}$ cause $\boxed{\text{Gauss Law}}$

Instead of if an open ckt is considered the effects analysed partial effects and they are called as flux through the surface.

$$\boxed{\int_{(\text{open})} D \cdot ds = \psi_e} = \text{constt} \quad \boxed{\text{Gauss's law}}$$

* Every closed surface has a volume

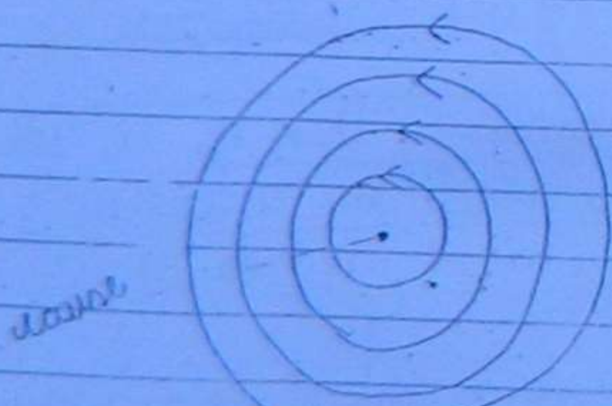
eg. $4\pi r^2$ for a sphere

Integrate with $r$ $\frac{4\pi r^3}{3}$

eg. $2\pi r h$ for cylinder

$\pi r^2 h$

Circulation & curl.



Consider a cause which has effect surrounding the cause in the circulatory manner. The effect are such that the strength decreases as it take longer length of circulation. Hence strength is constt per unit length.

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Strength ↓ x length ↑ = const ∝ cause

Strength of the effect − constt / length
= Intensity (per m)

cause − current I (Amp)
effect around the cause − Magnetic field.
Strength of the effect − magnetic field intensity. (H)

The cause is a current and the effect is circulatory magnetic field around the cause strength is magnetic field intensity H. Hence

Strength x length of circulation = constt ∝ cause

$$\boxed{\underbrace{\oint}_{\text{closed}} H \cdot dl = I}$$

Ampere's law
(Integral form)

Remark: closed line ⟶ open surface

eg $2\pi r \longrightarrow \pi r^2$ (circle)

eg. $4a \longrightarrow a^2$ (square)

Summary 1

outflow (closed surface) $= \psi_e = \phi$ ; Electric flux = colombs.

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circulation (closed line) $= I$ · Magnetic field intensity

$H =$ amp/m

$\{\because \oint H \cdot dl = I\}$

(c) $Q \longrightarrow$ cause $\longleftarrow I$ (Amp)

Gaussian — Ampere law

($C/m^2$) $D \longrightarrow$ strength $\longleftarrow H$ (Amp/m)

colomb's law — Lorentz's law

$F \rightarrow$ (N/C) (Volt/m) force $\leftarrow B$ (weber/$m^2$) (Tesla)

Divergence & Stoke's Theorem

$D \longrightarrow$ colomb/$m^2$ = outflow/area

$\nabla \cdot D \longrightarrow \frac{1}{m} \times \frac{C}{m^2}$ = Divergence = outflow/volume = $\frac{\text{charge}}{\text{volume}}$

$$\boxed{\nabla \cdot D = \rho_v}$$

Divergence physically stands for the outward flowing ability of cause

Strength of outflow at the cause which depends on the charge accumulation there. Hence charge density for

$H \longrightarrow$ amp/m = circulation/length . (11)

$\nabla \times H \longrightarrow \frac{1}{m} \times \frac{Amp}{m}$ = curl = $\frac{\text{circulation}}{\text{area}} = \frac{\text{current}}{\text{area}} = \frac{Amp}{m^2}$

$$\boxed{\nabla \times H = J}$$

closed surface (vector) $\xrightarrow{\nabla \cdot D}$ Volume (scalar)

Surface is a vector quantity and flux density is also a vector quantity.
Volume is a scalar quantity which every closed surface has. hence divergence or dot product is a vector transformation giving a scalar quantity hence

$\boxed{\nabla \cdot D = \rho_v}$ — Gauss law in (point form)

closed line (vector) $\xrightarrow{\nabla \times H}$ open surface (vector)

A line is a vector quantity and the enclosed surface by any line is also a vector quantity hence cross product is a vector transformation to another vector Hence

$\boxed{\nabla \times H = J}$ Ampere's law in point form.

$$\oint D \cdot ds = Q = \int \rho_v \, dv = \int (\nabla \cdot D) \, dv$$

$\boxed{\oint D \cdot ds = \int (\nabla \cdot D) \, dv}$ Divergence Theorem

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1 $\int D \cdot ds = \int (\nabla \cdot D)\, dv$ X

Because open surface doesnot have any volume.

2. $\oint D \cdot ds = \int (\nabla \times D)\, dv$ X

3. $\oint D \cdot ds = \int D\, dv$ X

Stoke's theorem:

$$\oint H \cdot dl = I = \int J \cdot ds = \int (\nabla \times H) \cdot ds$$

$$\boxed{\oint H \cdot dl = \int (\nabla \times H) \cdot ds}$$

$$\int_C H \cdot dl = \oint H \cdot dl$$

Identify the wrong statement

1. $\int H \cdot dl = \oint (\nabla \times H) \cdot ds$ X

2. $\int H \cdot dl = \int (\nabla \times H) \cdot ds$ X

3. $\oint H \cdot dl = \int (\nabla \cdot H)\, ds$ X

Maxwell's equation for static E/H fields.
static

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→ Time dependent → ~~E(t)/H(t)~~

→ Space dependent → E (x, y, z) or H (x, y, z)

E/H are the same at all the time
E/H are not the same at all point in space.
If E/H are not changing with

Maxwell's first Eq$^n$ is as it is Gauss Law without modification

1. $\oint D \cdot ds = Q$ (integral form) $\nabla \cdot D = \rho_v$ (Point form)

Maxwell's fourth Eq$^n$ is Ampere's law without modification for static field

4. $\oint H \cdot dl = I$ $\nabla \times H = J$

(Integral form) (Point form)

These two laws define the basic nature of E and H field respectively

Maxwell's third and second Eq$^n$ define what is not the nature of E and H fields i.e a divergent dispersive electric field can not be curly & circulatory.

Maxwell's second Eq$^n$ states that

$$\nabla \times E = 0$$

cross product and curled are always defined for intensity term (per meter terms)

As $D = \epsilon E$ and if $\epsilon$ is const through out the medium
i.e the medium is Homogeneous and isotropic.

$\nabla \times D = 0$ is also mathematically correct

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2 Maxwell's second Eqn.

$\oint E \cdot dl = 0$ (Integral form)

$\nabla \times E = 0$ (Point form)

↓

Irrotational nature

curl (vector) = 0

vector → Irrotational.

(Apply Stoke's theorem)

3 Magnetic field is a circulatory field which has effect around the cause and hence cannot have divergent effect from the cause. Hence

$\boxed{\nabla \cdot B = 0}$ Point form

→ solenoidal nature

$\nabla \cdot B = 0$

$\nabla \cdot (\mu H) = 0$

$\nabla \cdot H = 0$

If $\mu$ = const

= space indepent, Homogeneous, Isotropic

Apply divergence theroem in integral form we get

$$\oint B \cdot ds = \int (\cancel{\nabla \cdot B})\, dv = 0$$

$\boxed{\oint B \cdot ds = 0}$ Integral form

$\nabla \cdot B = 0$

solenoidal nature

Divergence (vector) = 0

vector → solenoid

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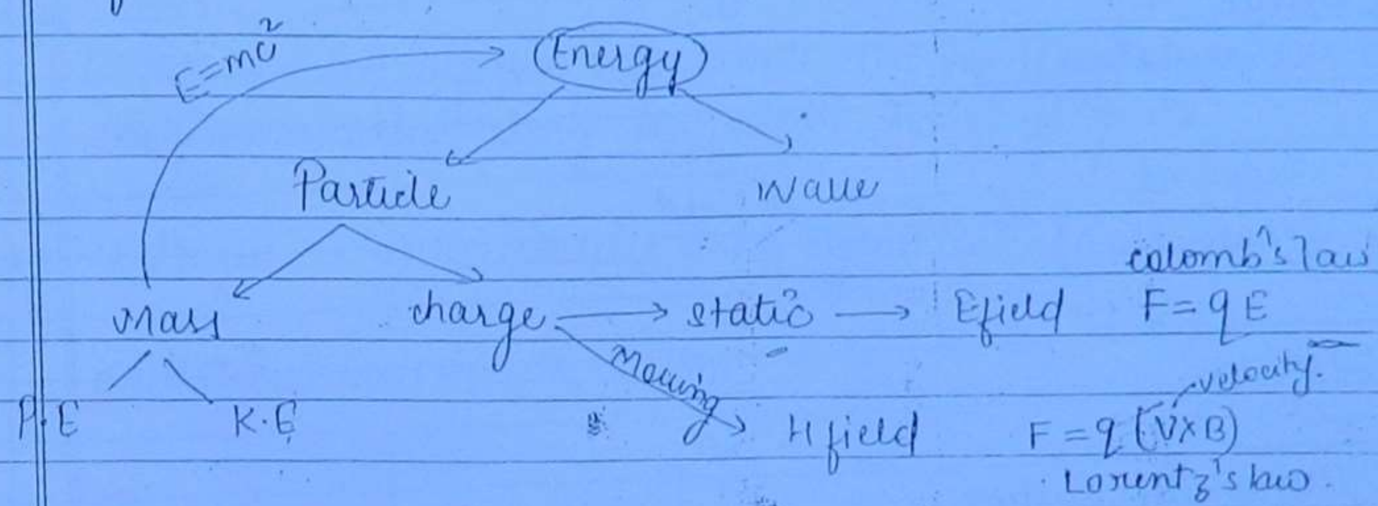
## Physical Interpretation of E/H




E field → Energy format that is around a moving charge and is felt by only other charge.

H field → Energy format that is around a moving charge and is felt by only other moving charge.

Summary:

$q \longrightarrow$ Stationary — E-field (Static) (Energy)

$q \longrightarrow$ moving
without acceleration
with uniform velocity
linearly with time

} H field (Static) (Energy)

$\frac{dq}{dt} = K = I$

= DC current

$q = kt$

q —moving→ E(t) Time varying fields
with acceleration → H(t) (Power)

Energy is varied with time i.e Power involved.

Voltage ————→ E field
(charge accumulation)

current ————→ H field.
(charge flow)

Monday

Coordinate System:
It is a way of ~~addressing~~ point or locating point in a 3D space from a pre-defined reference.

system of class - Cartesian coordinate system

Reference → 3 infinite mutually orthogonal planes.
It refers to planar symmetry.
eg:- sheet of charge, UPW, uniform Plane wave, Rectangular waveguide.

Reference - XY, YZ, ZX plane.

Parameter - x, y, z
unit vectors $a_x$, $a_y$, $a_z$

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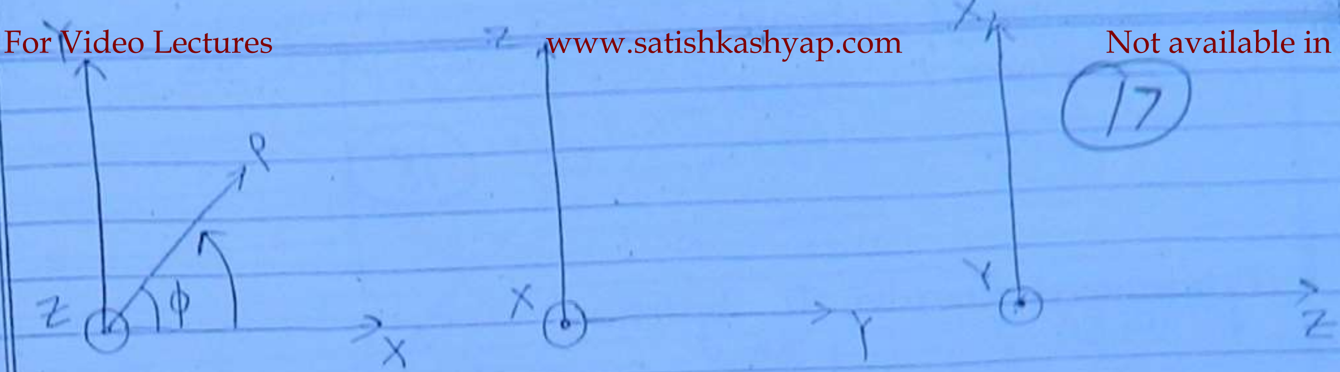
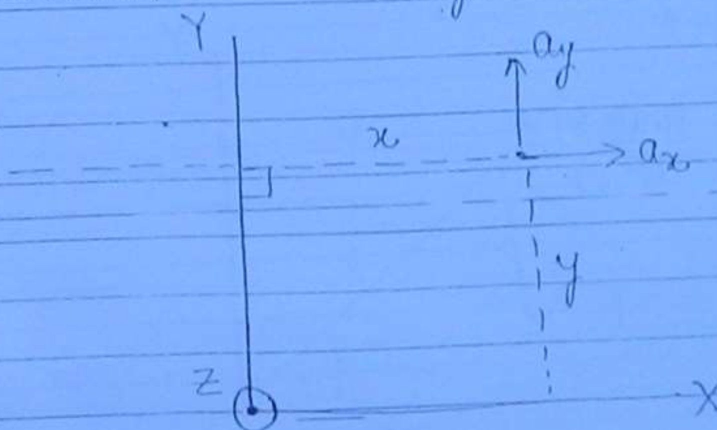



Right handed system represents the increasing order of the parameter along the right hand fingures.

eg: Angles:– anticlockwise – in cylinderical – around z axis

cartesian coordinate system:




X = The shortest distance or perpendicular distance from YZ plane.

- Rang $(-\infty, \infty)$
- It increases as we go along the x axis (It is called as $a_x$ direction)

Y = The shortest distance or perpendicular distance from ZX plane

Range is $(-\infty, +\infty)$

It increase as we go parallel to y axis ($a_y$)

z: The shortest distance or $\perp$ distance from the xy plane

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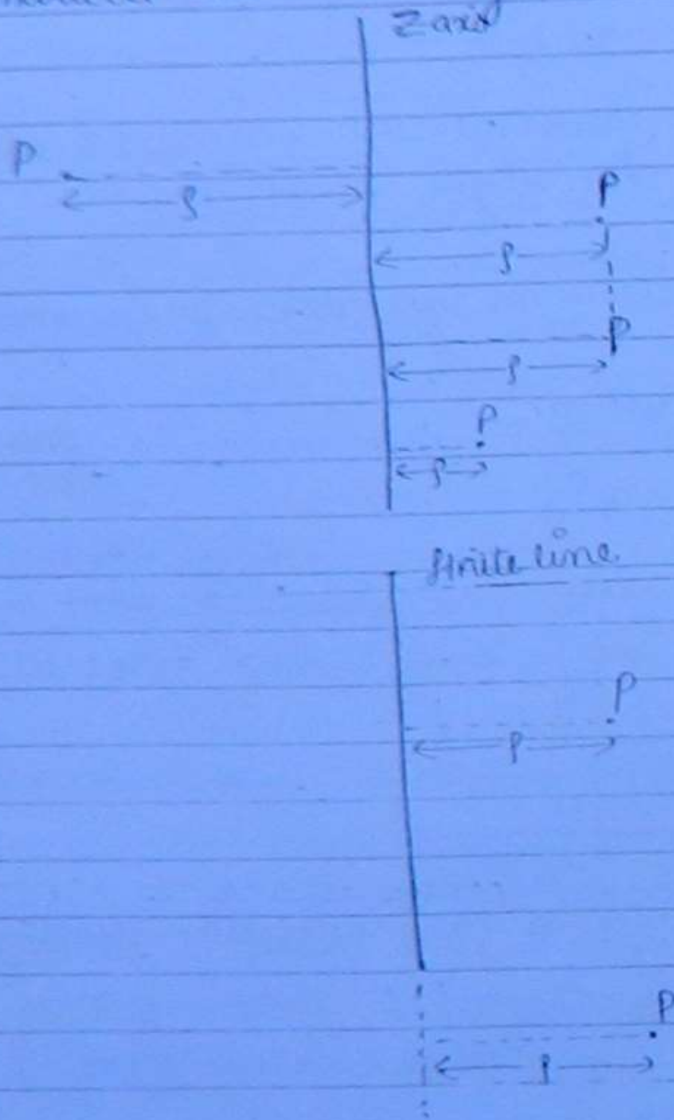
## Cylindrical Coordinate System




$\rho$ = [shortest distance or radial distance of the point from the reference z axis]

$\downarrow$

All the points with the same $\rho$ are on a concentric cylinder around the line

· Product → cos

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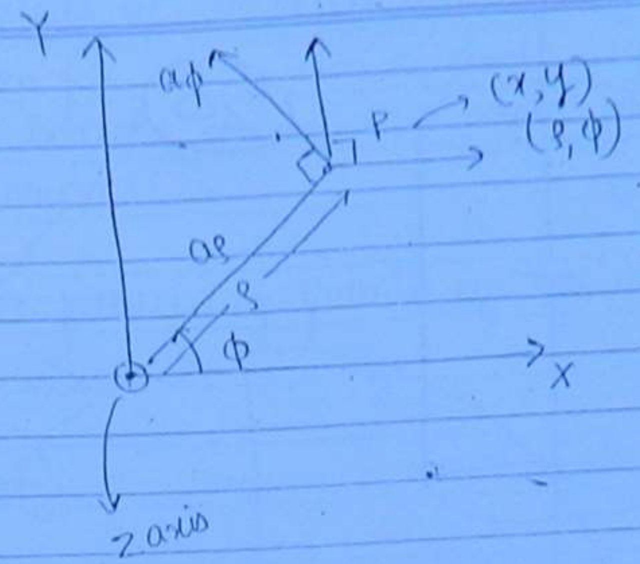
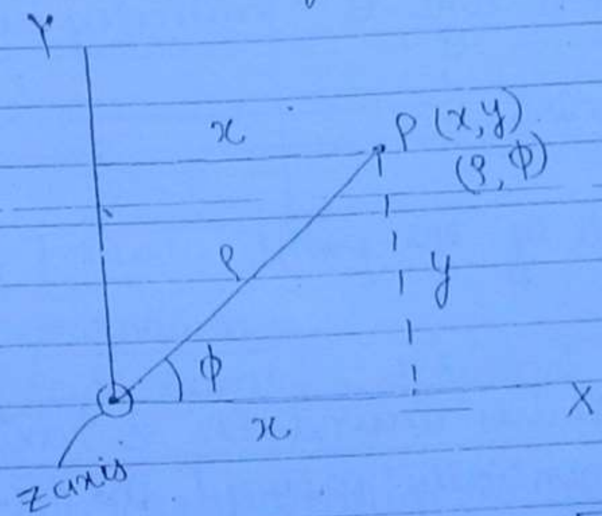



$a_\rho \cdot a_\phi = 0$

$a_x \cdot a_y = 0$

Point Transformation

cartesian ⟷ cylindrical




$x = \rho\cos\phi$ $\qquad$ $\rho = \sqrt{x^2+y^2}$

$y = \rho\sin\phi$ $\qquad$ $\phi = \tan^{-1}(y/x)$

$z = z$ $\qquad$ $z = z$

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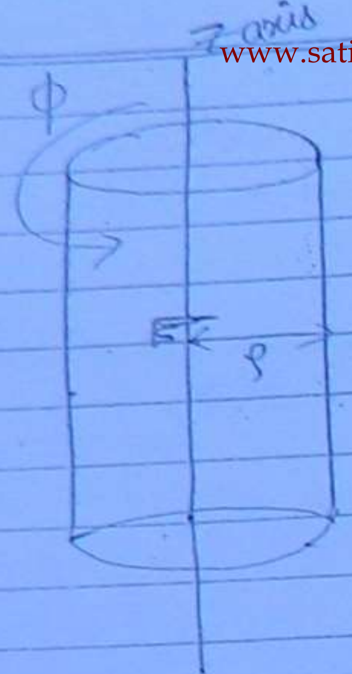
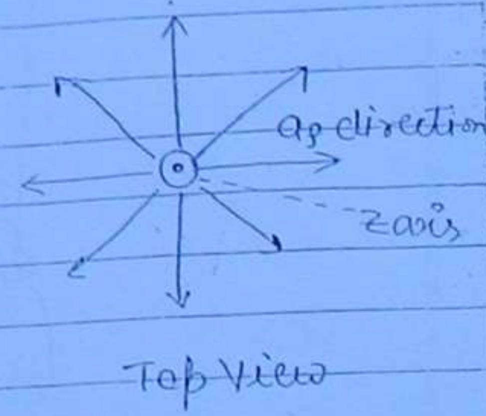


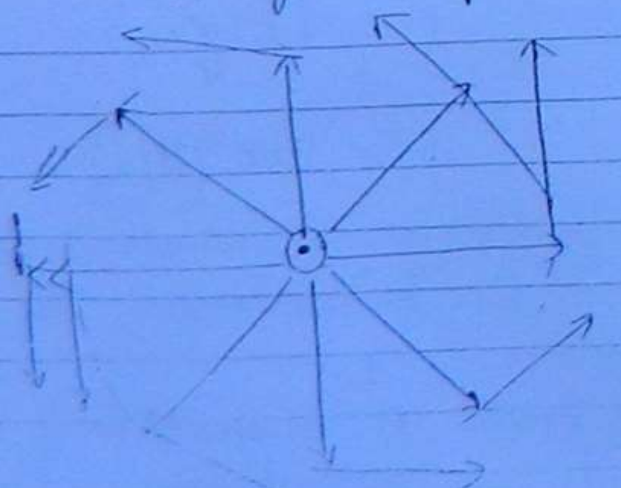




$\rho$ increases radially outward from the z axis
light from tubelight is radially out
electric field from a line is radically out.

Range of $\rho$ $(0 \to \infty)$

$\phi$ = orientation angle of the point around the reference z axis

$\phi$ increases anticlockwise around the z axis
- It is also tangentially around the z axis



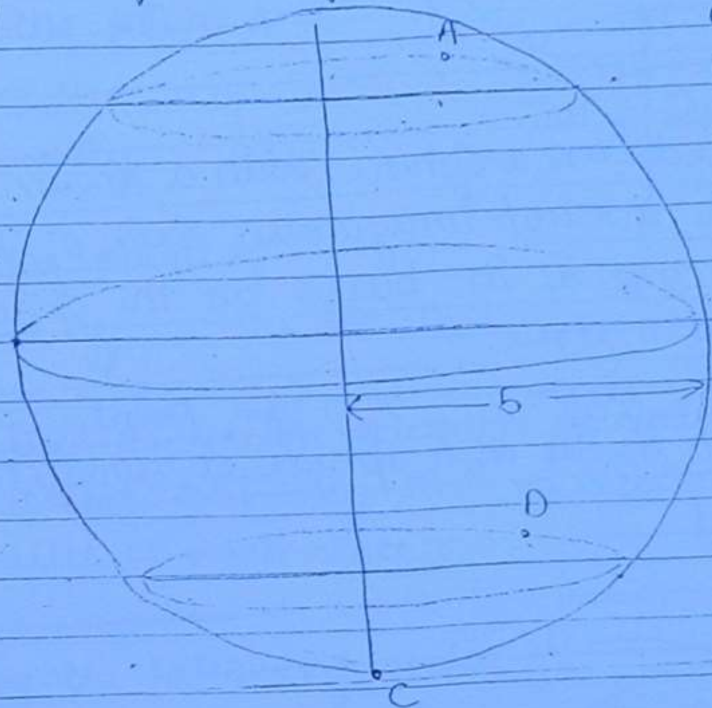
Range of

- $\phi$ is the orientation angle around the imaginary axis that locates a point on the $\theta$ circle
- Different values of $\phi$ can be seen in anticlock-wise direction for one given $\theta$ circle. Hence $a_\phi$ is out of the board, out of the paper and into the paper as shown.

(21)

Q Identify the following points in spherical coordinate system.

A = $(5, 30^\circ, \phi)$
B = $(5, 90^\circ, \phi)$
C = $(5, 180^\circ, \phi)$
D = $(5, 150, \phi)$




(i) $(5, 90^\circ, \phi)$
(ii) $(3, 30^\circ, \phi)$
(iii) $(3, 150, \phi)$
(iv) $(5, 30, \phi)$
(v) $(5, 180^\circ, \phi)$
(vi) $(5, 150^\circ, \phi)$
(vii) $(3, 90^\circ, \phi)$

Point Transformation
cylindrical $\rightarrow$ spherical

- For the point transformation imaginary axis in spherical coordinate system superimposed with z axis of cylindrical coordinates Hence $\phi$ is common variable
- Any point on the sphere is at any $\rho$ distance from the z axis and at any height h along the z axis

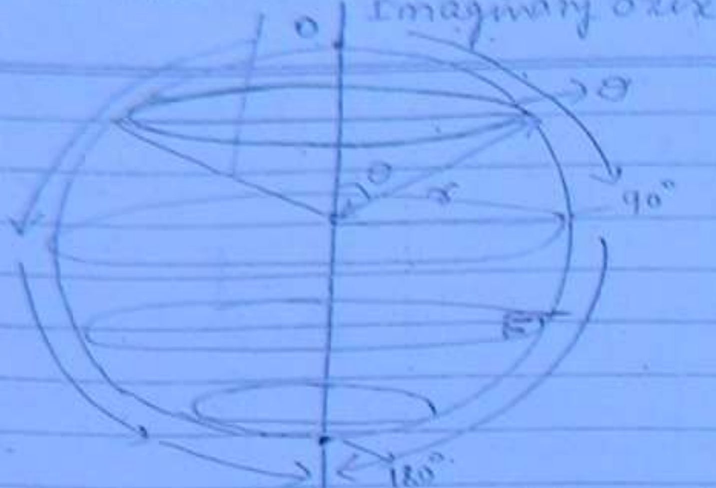
$\rho = r \sin\theta$



$\phi$ - $\phi$

θ = It is an angle that distributes / identifies the sphere into various circles.

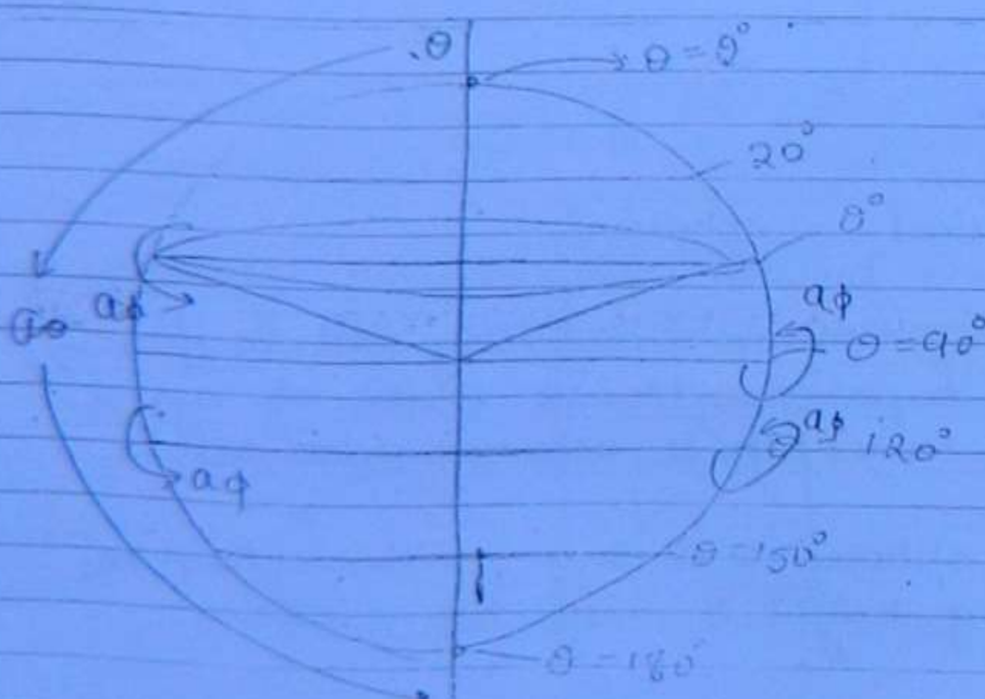





Identification of a circle on a sphere with a known angle θ rotate around the vertical imaginary axis of the sphere we obtained cone as its base on the sphere. that circle is called as θ circle

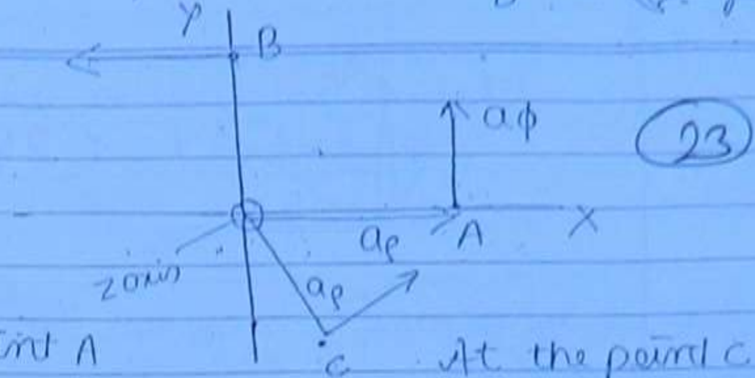
$r$ = sphere  $r, \theta$ = circle,  $r, \theta, \phi$ = point

The set of all possible circles on a sphere is complete in interval $\theta = [0, \pi]$




θ increases as we move on the sphere from its imaginary axis back to its imaginary axis as shown

Q In the diagram shown below identify $a_\rho$ and $a_\phi$ direction at A, B, C in terms of $a_x$ & $a_y$




Sol^n

At the point A

$a_\rho \to a_x$

$a_\phi \to a_y$.

At the point B

$a_\rho \to a_y$ $\quad a_\phi \to -a_x$

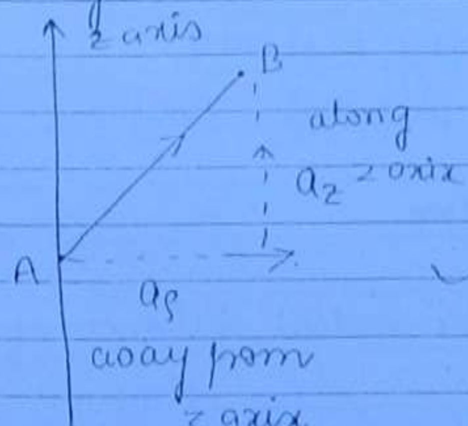
At the point C

$a_\rho \to a_x, -a_y$

$a_\phi \to a_x, a_y$

Any two $a_z$ vectors always parallel to each other and horizontal but any two $a_\rho$ vectors are not necessarily parallel to each other.

Q Identify the following vector




(a) = $a_\rho$

(b) = $a_z$

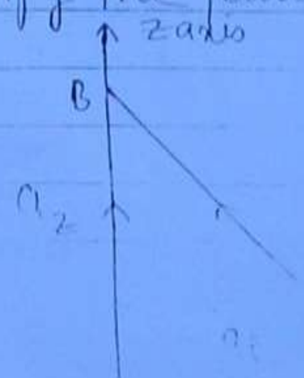
(c) = $a_\rho, a_z$ ✓

(d) $a_\rho, -a_z$

(e) $-a_\rho, a_z$

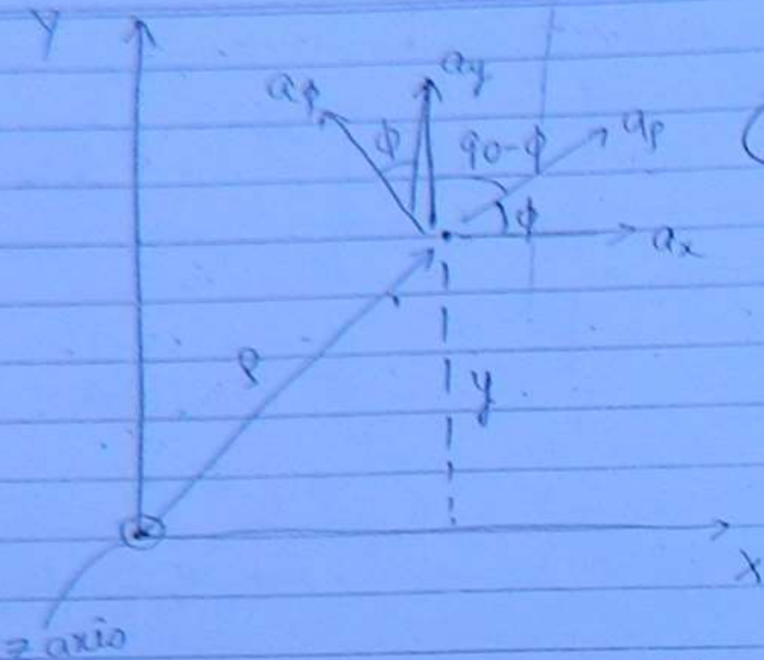
(f)

Q Identify the following vector




Ans $(-a_\rho, a_z)$

Summary:




(24)

$\phi$ - angle of the point decides $a_\rho$ direction with respect to $a_x$ and $a_y$.

$a_x \cdot a_\rho = |a_x|\ |a_\rho| \cos\phi = \cos\phi$

$a_y \cdot a_\rho = \sin\phi \qquad |a_y|\ |a_\rho| \cos(90^\circ - \phi)$

$a_y \cdot a_\phi = \cos\phi \qquad |a_y|\ |a_\phi| \cos\phi$

$a_x \cdot a_\phi = -\sin\phi \qquad |a_x|\ |a_\phi| \cos(90^\circ + \phi)$

Spherical coordinate system:

$r$ = shortest distance or radial distance of the point from the origin

If $r$ = constt, they are infinite points all located on a sphere concentric with the origin.

(iii) The direction has to obey the rule

$$\underset{\text{direct}}{E} \times \underset{\text{dir}^n}{H} = \text{propagation direction}$$

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12 W.B (b) & (d) are wrong because it is not in the form of $25e^{-0.2y}\sin(10^8t-y)\,a_z$

i) $\omega = 10^8$ $\beta = 1$ in $E(y,t) = 25\sin(10^8t-y)\,a_z$

$v_p' < 3\times10^8\,m/sec$

$$V_p' = \frac{\omega}{\beta} = 10^8 \quad i.e \quad \frac{3\times10^8}{\sqrt{\epsilon_r}} = V_p' \Rightarrow$$

$$\sqrt{\epsilon_r} = \frac{3\times10^8}{10^8} = 3$$

$$\boxed{\epsilon_r = 9}$$

∴ It is lossless dielectric

(ii) $f = \frac{\omega}{2\pi} = \frac{10^8}{2\pi} \Rightarrow \lambda = 2\pi$ $2\pi f = \omega$

(iii) H (vector)

$$H(y,t) = \frac{25}{\eta}\sin(10^8t-y)$$

$$\eta = \frac{120\pi}{\sqrt{9}} = 40\pi$$

$$H(y,t) = \frac{25}{40\pi}\sin(10^8t-y)\,a_x$$

$$a_z \times ? = a_y$$

b) correct

$\beta = 2 = \frac{2\pi}{\lambda}$

$\lambda = 3.14$

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(c) correct

(d) waves propagation is defined only w.r to electric field situation but never H field

Incorrect

W.B $E = 50 \sin(10^7 t + kz)\hat{j}$ V/m

which is correct

(a) wrong

(b) $f = \frac{3\times10^8}{\lambda} \Rightarrow \lambda = \frac{3\times10^8}{\frac{\omega}{2\pi}} \Rightarrow \lambda = \frac{3\times10^8\times2\pi}{\omega}$

$\Rightarrow \lambda = \frac{3\times10^8\times2\pi}{10^7} \Rightarrow \lambda = 30\times2\pi = 188.4$ m.

correct

(c) $k = \beta = \frac{2\pi}{\lambda} \Rightarrow \frac{1}{30} = \frac{2\pi}{\lambda}$ i.e 0.03

wrong

(d) wrong (because there is no exponential term only 50 is present so no attenuation.

7 W.B $\beta = \frac{2\pi}{\lambda}$ $v_p = \frac{3\times10^8}{\sqrt{\varepsilon_r}}$ (for lossless medium)

$v_p = \frac{3\times10^8}{\sqrt{81}} = \frac{3\times10^8}{9} = \frac{10^8}{3}$

$v_p = \frac{10^8}{3}$ $f = \frac{10^8\times6\pi\times3}{10^3}$

cylindrical coordinate system:

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- Reference → 1 infinite line
  It has axial symmetry
- eg: line charge, current carrying wire, cylindrical waveguide, coaxial cable.
- References → z axis
- Parameter → $\rho, \phi, z$
- unit vectors → $a_\rho, a_\phi, a_z$

Spherical Coordinate system:

- Reference → 1 point
  Point Symmetry
- eg of point symmetry :- point charge, antenna, current element $d\vec{l}$,
- Reference - origin
- Parameters - $r, \theta, \phi$
- unit vectors - $a_r, a_\theta, a_\phi$

best eg is assume yourself a point and locate the position of stars

☆ All coordinate systems are assumed to be following unit orthognal, orthonormal, right handed systems.

orthogonal:
orthogonality: The dot(·) product of any two different unit vectors of the same coordinate system is zero.

$a_\rho \cdot a_\phi = 0$

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$a_r \cdot a_\phi = 0$

$a_z \cdot a_\phi \neq 0$

$a_r \cdot a_r = 1$

$a_\phi \cdot a_\theta = 0$

Orthonormality:

The cross (x) product of any two different unit vector of the same coordinate system is always the third unit vector (obeying right hand rule)

$x \to y \to z$ $\qquad$ $\rho \to \phi \to z$ $\qquad$ $r \to \theta \to \phi$

$a_x \times a_y = a_z$ $\qquad$ $a_\rho \times a_\phi = a_z$ $\qquad$ $a_r \times a_\theta = a_\phi$

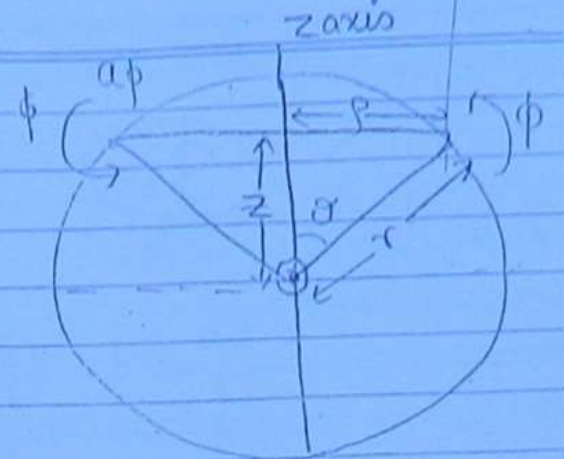
$a_y \times a_z = a_x$ $\qquad$ $a_\phi \times a_z = a_\rho$ $\qquad$ $a_\theta \times a_\phi = a_r$

$a_z \times a_x = a_y$ $\qquad$ $a_z \times a_\rho = a_\phi$ $\qquad$ $a_\phi \times a_r = a_\theta$

The cross (x) product of any two similar unit vector of the same coordinate system is zero.

eg $a_\rho \times a_\rho = 0$

$a_z \times a_z = 0$

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cartesian to spherical

$x = \rho\cos\phi \Rightarrow r\sin\theta\cos\phi$

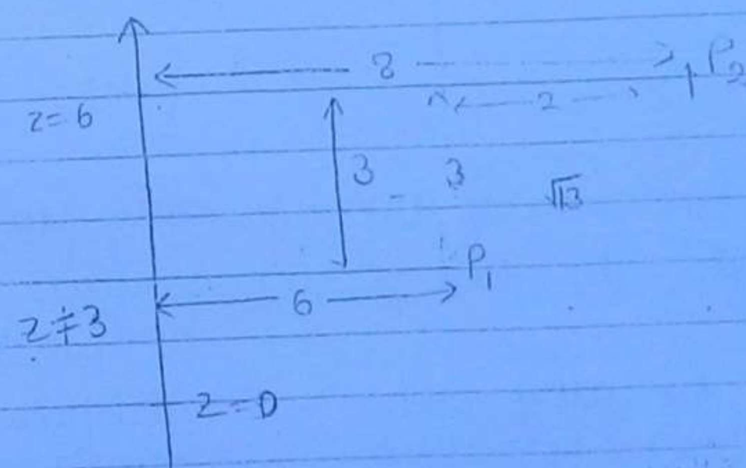
$y = \rho\sin\phi \Rightarrow r\sin\theta\sin\phi$

$z = z \quad r\cos\theta$

Q. calculate the distance b/w the two points becau without using point transformation:

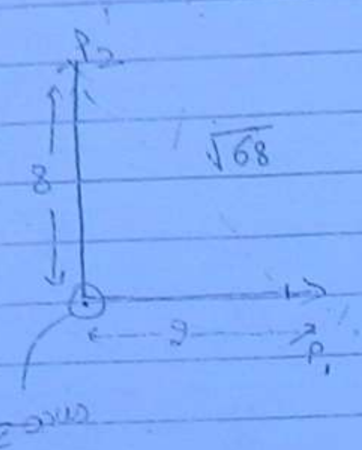
$(5, \pi/2, 3)$ and $(8, \pi/2, 5)$

sol$^n$

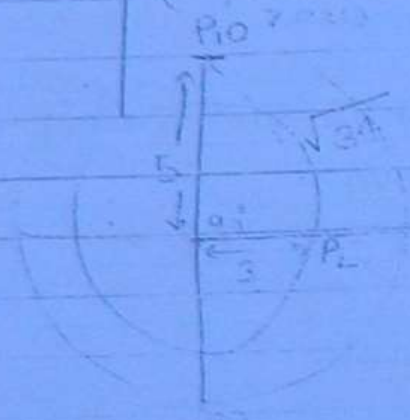



Q $(2, 0^\circ, 6)$ to $(6, 0^\circ, 2)$

(c) $(2, 0°, 5)$ To $(8, \pi/2, 5)$




(d) $(5, 0°, 0°)$ and $(3, 90°, 0°)$




Q Identify the locus of the following description
(a) Point (b) line, (c) surface (d) Volume

(a) $x = 5$, for all $y$ and $z$ surface

(b) $r=5$, for all $\theta$ & $\phi$

surface → closed. $r=5$ means on the surface

If $r\le 5$ then volume

(c) $\rho=5$, for all $\phi$ and $z$

surface → cylindrical closed surface.

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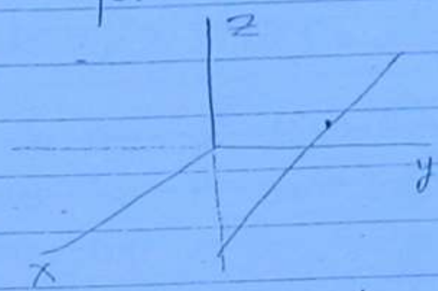
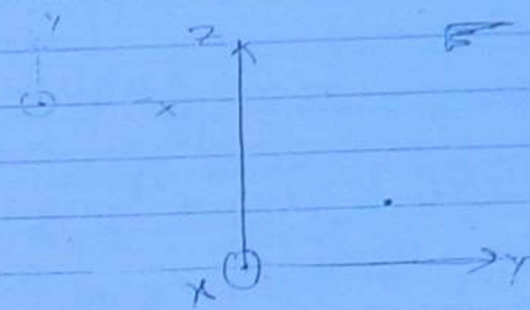
(d) $y=2$, $z=1$ for all $x$









(e) $\rho=5$, $z=3$ for all $\phi$

line = closed line

Summary:

Vector

1. 2 parameters - fixed
   1 parameter - variable
   — line definition (line is one dimensional)

2. 1 parameter - fixed
   2 parameter - variable
   — surface definition. (surface is 2 dimensional)

3. 3 parameters - fixed
   — point definition.

4. 3 parameters - variable
   — volume definition

Line, Surface, Volume Integrals

$d\vec{l}$

- Line element of zero length
- Its direction is in that direction in which the length changes.

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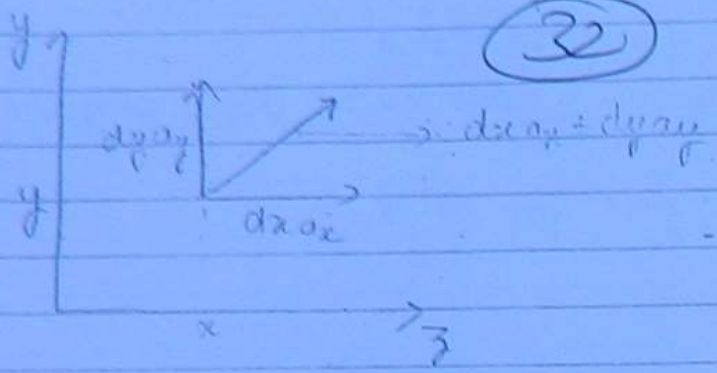
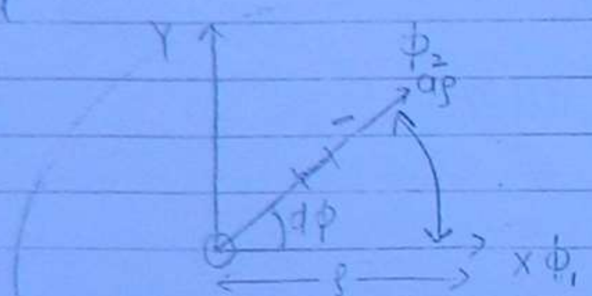



$d\vec{l} = dx\,a_x + dy\,a_y + dz\,a_z$

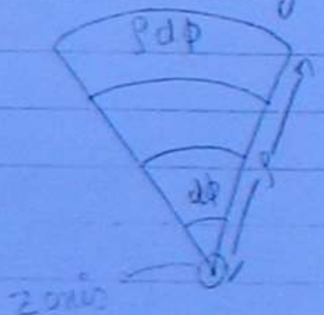
cylindrical




$\phi$- direction movement or change in length is always a curvature

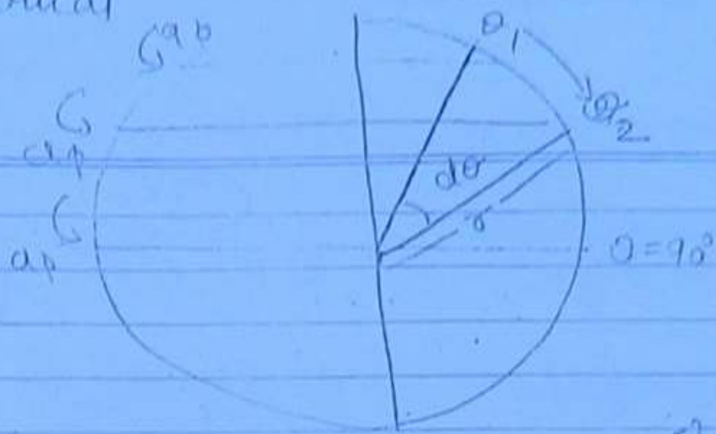
$= d\rho\,a_\rho + \rho\,d\phi\,a_\phi + dz\,a_z$

A curvature length is always dependent on curvature radius and angle of curvature




$\rho\,d\phi$ is always same

For spherical

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$$= dr\,a_r + r\,d\theta\,a_\theta + r\sin\theta\,d\phi\,a_\phi$$

movement in the $\phi$ direction can be at calculate various height on a sphere i.e it can be a various $\theta$ const circle the curvature at each circle is $r\sin\theta$ hence length in $\phi$ direction is $r(\sin\theta)\,d\phi$

(∵ cylinder has same curvature but sphere have different)

| Parameters | | | Scaling factors | | |
|---|---|---|---|---|---|
| $x$ | $y$ | $z$ | 1 | 1 | 1 |
| $\rho$ | $\phi$ | $z$ | 1 | $\rho$ | 1 |
| $r$ | $\theta$ | $\phi$ | 1 | $r$ | $r\sin\theta$ |
| $u$ | $v$ | $w$ | $h_1$ | $h_2$ | $h_3$ |

$$\boxed{dl = h_1\,du\,a_u + h_2\,dv\,a_v + h_3\,dw\,a_w}$$

Jai Durga Maa

Tuesday:

$\overline{ds}$

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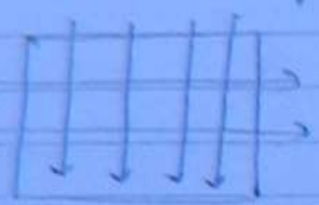
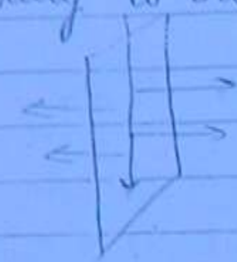
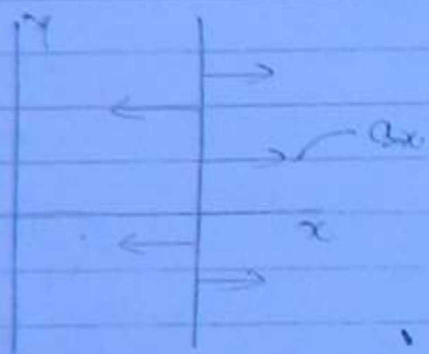
Surface direction is always normal to the plane of the surface.

eg: current crossing a surface cuts normally to the surface

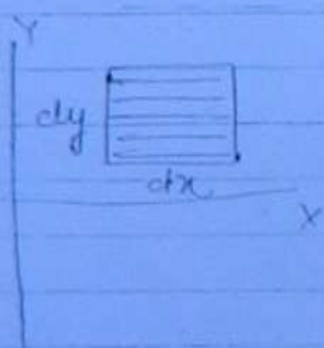




$x = 5 = \text{constt}$




$a_x$ direction




XY Plane
$z = a$ surface
$x, y \rightarrow$ variable

$a_z$ direction

$$\overline{ds} = dy\,dz\,a_x + dz\,dx\,a_y + dx\,dy\,a_z$$

Parameter - fixed - constt
Direction of the vector in the direction of the increase of the parameter

$$V = -L\frac{dI}{dt}$$

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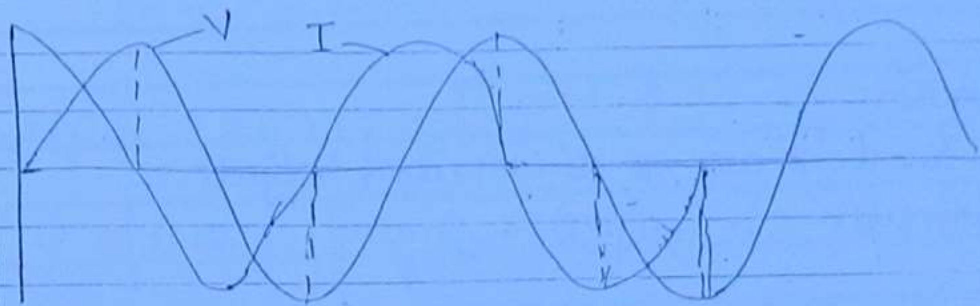
$$I = C\frac{dv}{dt}$$




1. The voltage is initially held on to the plates of the capacitor which gradually discharges through the inductor and hence accumulate on the other side due to the discharging current as $V = -L\frac{dI}{dt}$

Process goes on and the voltage and the current sustain each other alternately as $I = C\frac{dv}{dt}$. Hence a steady state oscillatory voltage and current exist without power dissipation.

$P = VI \quad \int_0^T \sin t \cos t = 0$ (No power consumption)

Practical w/f.

we need always a feedback

The entire working of the ckt and [illegible] steady state w/f is due [illegible] capacitor charge and dis[illegible] the capacitor [illegible] exponential [illegible] again the same function and hence V and I are said to be harmonic functions

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$$\frac{e^{j\theta} \pm e^{-j\theta}}{2}$$

V/I Harmonic functions

Harmonic function
- All harmonic functions are 2 dimensional objects
- Exist in 3 formats.
- These are

$A \sin\theta$ . $A \cos\theta$ .. $Ae^{j\theta}$

A – dimension 1 – Amplitude
$\theta$ – Dimension 2 – phase.

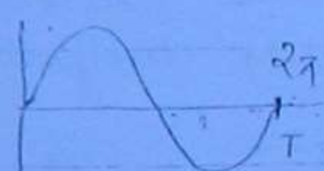
Properties of Harmonic funⁿ

1. $\theta$ – phase – should always linearly change with the variable

$\theta \propto t$ time Harmonic

$\theta = \omega t$

$t\omega = \frac{\theta}{\omega}$




$\frac{2\pi}{T} = \omega =$ phase shift const per unit time

$\theta \propto z$ space Harmonics

$\theta = \beta z$

Phase shift const per unit length

Generalized formula

$d\bar{s} = h_2 h_3\, dv\, dw\, a_u + h_3 h_1\, dw\, du\, a_v + h_1 h_2\, du\, dv\, a_w$

Given an electric field $\bar{D} = 4xy\,a_x + 2xy^2\,a_y + 3zx\,a_z$ $C/m^2$ find the flux $\psi$ crossing the surface

$y = 2, \quad 0 < x < 1$
$\quad -2 < z < 2$

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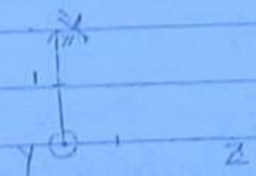
$y = 2,$




$4(x)(y)\,a_x + 2xy^2\,a_y + 3zx\,a_z$

$\psi = \int D \cdot ds = \iint \left[(4xy\,a_x) + (2xy^2\,a_y) + (3zx\,a_z)\right]\left[dx\,dz\,a_y\right]$

as $y = 2$. $\quad ds = dx\,dz\,a_y$

$\psi = \int_{x=0}^{1} \int_{z=-2}^{2} 2xy^2 \,\Big|\, dx\,dz$

$= y^2\, 2\left[\frac{x^2}{2}\right]_0^1 \left[z\right]_{-2}^{2} \Rightarrow 2y^2\left[\frac{1}{2}\right]\left[2+2\right]$

$\Rightarrow \frac{2y^2 \times 4}{2} = \frac{2 \times 4 \times 4}{2}$

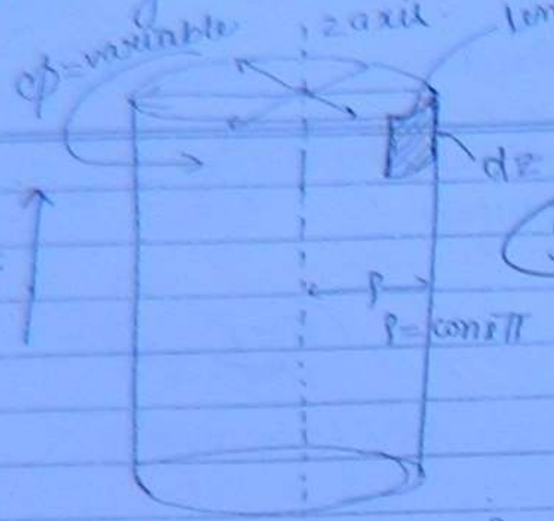
$\Rightarrow 16\,C$

Q Given $\bar{D} = 4xy\,a_x$
find the flux $\psi$ crossing the surface
$y = 2, \quad 0 < x < 1$
$\quad -2 < z < 2$

Sol $\quad ds = dx\,dz\,a_y$

$\psi = \int D \cdot ds = 0$

$\bar{ds}$ in cylindrical

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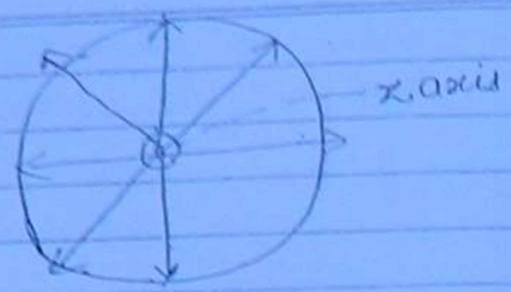



$\rho$ = constt surface (cylindrical)

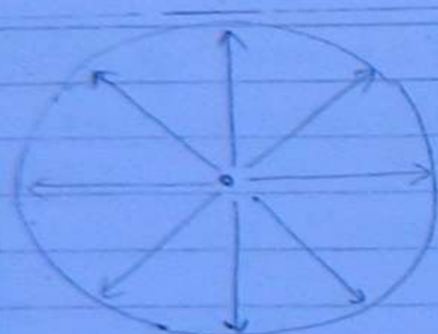
$a_\rho$ direction cuts the cylinder radially outwards.

$$\bar{ds} = \rho\, d\phi\, dz\, a_\rho + dz\, d\rho\, a_\phi + \rho\, d\rho\, d\phi\, a_z$$

$\bar{ds}$ in spherical coordinate sys

$r$ = constt

$a_r$ = direction radially outward. Hence $a_r$ is normal



$\theta$ and $\phi$ → dimension, length are $r\,d\theta$, $r\sin\theta\, d\phi$

$$\bar{ds} = r^2 \sin\theta\, d\theta\, d\phi\, a_r + r\sin\theta\, d\phi\, dr\, a_\theta + r\, dr\, d\theta\, a_\phi$$

eg. sphere

$$S = \int ds \Big|_{\substack{r = \text{constt} \\ \text{sphere}}}$$

$$\int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} r^2 \sin\theta\, d\theta\, d\phi$$

Q. If $D = 5(\rho-3)^2 a_\rho$ C/m²
Find the flux crossing the surface.
$\rho = 4 \quad 0<\phi<\pi \quad -5<z<5.$

$$\psi = \int D \cdot ds = \int 5(\rho-3)^2 a_\rho \, [\rho \, d\phi \, dz \, a_\rho]$$

$$= \int 5(\rho-3)^2 \rho \, d\phi \, dz$$

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$$= \int_{\phi=0}^{\pi} \int_{z=-5}^{5} 5(\rho-3)^2 \rho \, d\phi \, dz$$

$$= 5 \times 4 (1)^2 \int_{\phi=0}^{\pi} \int_{z=-5}^{5} d\phi \, dz \qquad [\phi]_0^{\pi} \; [z]_{-5}^{5}$$

$$= 20 [\pi - 0][5+5]$$

$$= 200\pi$$

$dv$

volume is a scalar triple product of the lengths in 3 dimensions.

$$dv = dx \, dy \, dz$$

$$= \rho \, d\rho \, d\phi \, dz$$

$$= r^2 \sin\theta \, dr \, d\theta \, d\phi$$

Divergence, Curl, Gradient

$$\bar{A} = A_u a_u + A_v a_v + A_w a_w$$

$$\nabla \cdot \bar{A} = \frac{1}{h_1 h_2 h_3} \left[ \frac{\partial}{\partial u}(h_2 h_3 A_u) + \frac{\partial}{\partial v}(h_3 h_1 A_v) + \frac{\partial}{\partial w}(h_1 h_2 A_w) \right]$$

$$\nabla\cdot\bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \quad [$$

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$$\nabla\cdot\bar{A} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z} \quad [\text{cylindrical}]$$

$$= \frac{\cancel{1}}{r^2\sin\theta}\frac{\partial}{\partial r}(r^2\sin\theta A_r) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(r\sin\theta A_\theta)$$

$$+ \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\phi}(rA_\phi)$$

$$\nabla\cdot\bar{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

[spherical]

curl:

$$\nabla\times\bar{A} = \frac{1}{h_1h_2h_3}\begin{vmatrix} h_1 a_u & h_2 a_v & h_3 a_w \\ \partial/\partial u & \partial/\partial v & \partial/\partial w \\ h_1 A_u & h_2 A_v & h_3 A_w \end{vmatrix}$$

Static Electric field:

- Fundamental of colombs law behind is the Gauss law.

(41)

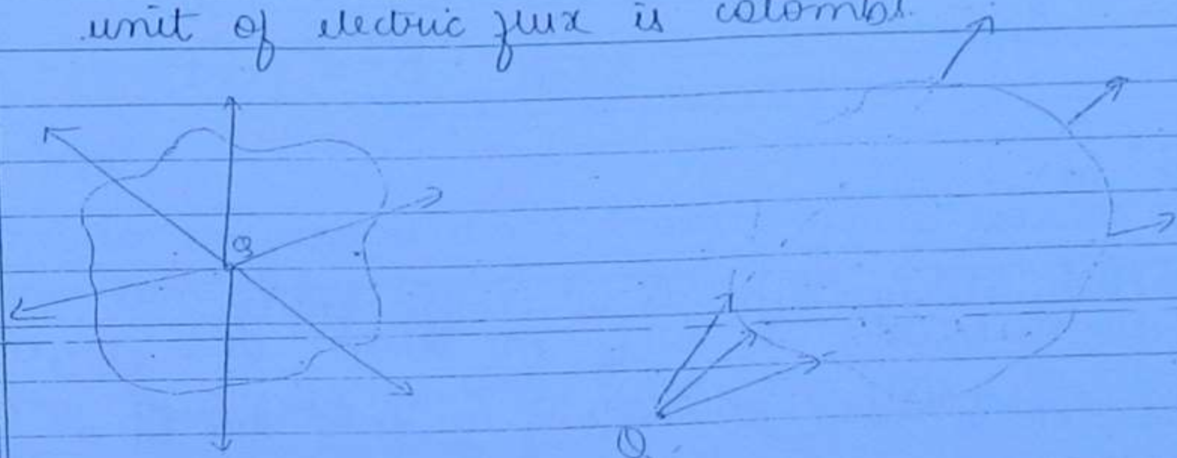
Gauss Law:

Statement: The net (Total) electric flux (Effects) leaving any closed surface is always equal to the charge (cause) enclosed in that volume

The complete effects from any cause are analysed by considering an encapsulating surface i.e closed surface. Hence.

$$\boxed{\psi_{e(total)} = Q}$$

unit of electric flux is colombs.



If the charge is inside the surface there are net flux line crossing the surface outwardly.
If the same charge is outside flux entering the volume or surface should be equal to flux leaving.

Summary: charge - source / sink for flux lines

Note: Gauss law never define for an open surface i.e for an open surface flux only exits through it, we cannot define entering / leaving flux classmate Date Page

$$\boxed{\oint D \cdot ds = Q}$$ Integral form of Gauss's law

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D = { Flux density displacement
Strength of flux = $\frac{\text{Flux}}{\text{area}} = \frac{C}{m^2}$ = Instantaneous flux at every point

$$\boxed{\int D \cdot dl = \frac{\psi}{4\pi}}$$ Not gauss law.

$$\boxed{\nabla \cdot D = \rho_v}$$ point form of Gauss law.

$\nabla \cdot D = \frac{\text{outflow}}{\text{volume}}$ = flux density = $\frac{\text{charge}}{\text{volume}} = \rho_v$

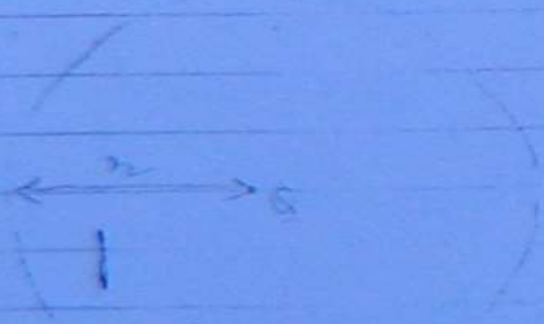
$$\frac{1}{m} \times \frac{C}{m^2} = \frac{C}{m^3}$$

Diverging ability always depends on charge density.

Gauss law: Application 1: Strength of the field due to a point charge Q

$\oint D \cdot ds = Q$

r = const , spherical surface
Equidist from the source



strength anywhere around the charge can be determined using gauss law

be used for any closed surface.
In this example we choose a symmetric spherical surface for applying Gauss law.

The choice of a sphere is because the surface is equidistant from the charge and hence the strength is is constt. and hence the integration converges to multiplication

$$D(r) \times \text{Area of the sphere} = Q$$

$$D(r) = \frac{Q}{4\pi r^2} \cdot a_r$$

C/m²

Choosen surface is an r = constt sphere having $a_r$ direction so by logic D also have same direction as $a_r \cdot a_r = 1$. Hence the field is radially outward and divergent from the cause or charge.

Colomb have a different measure of field strength which was in terms of force b/w charges per unit charge

$$E = \frac{F}{q}$$

He called it as intensity or electric field intensity with unit $\frac{\text{Newton}}{\text{Colomb}}$.

He also proved that charge having a mass should have force and hence we the field E and related

$$\frac{\bar{F}}{q} = \frac{\bar{D}}{\epsilon}$$

Hence $E = \frac{Q}{4\pi\epsilon r^2} \cdot a_r$

N/C

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Note 1 Electric field direction E is the direction of flux line, it is the direction of the repulsive force on a positive charge and hence it is always outward from a charge.

$+Q \xrightarrow{\text{repulsive}}$ test charge

$-Q \longleftarrow$ test charge

Static Magnetic fields.

Biot Savart's Law. [Ampere's law for current element]

- Biot Savart's law derived from Ampere's law.
- applying that a small length I carrying wire can be treated as the basic cause of magnetic fields.

Ampere's law = Ampere's circuital law

Biot savart's law states -

Imp-m. $I\,\overline{dl} \longrightarrow$ small current element - vector quantity

assumes - cause.

Imp. $H \longrightarrow$ magnetic field strength - field Intensity

- Effect.

Imp for Exam

$$\boxed{H(r) = \frac{I\,dl \times a_r}{4\pi r^2}}$$

The strength expression ... very similar to the ... field and ...

but the direction is not as if in electric field. 45

The direction of magnetic field is always current direction multiplied with radial direction to the point from the current.

current direction × radial dir$^n$ to point from the current

Intensity $H(r) = \frac{Amp \cdot m}{m^2} \times a_r$

$= \frac{Amp}{m^2}$

Lorentz's basic force Eq$^n$ defines the field strength in magnetic field as flux density $\left(\frac{weber}{m^2}\right)$. Hence as shown below

$$B = \frac{\bar{F}}{I\,dl} = \frac{\text{Force}}{\text{Basic cause}}$$

$$F = q(v \times B) \longrightarrow \text{Lorenz's force Eq}^n$$

$$dF = dq\left(\frac{dl}{dt} \times B\right) = I\,dl \times B$$

$q$ = charge

$v$ = velocity of the moving charge

$\frac{dl}{dt} = v_d$ = drift in a conductor of length $l$

Given $B(r) = \mu H$

$B(r) = \frac{F}{I\,dl} = \frac{\text{Newton}}{A \cdot m}$

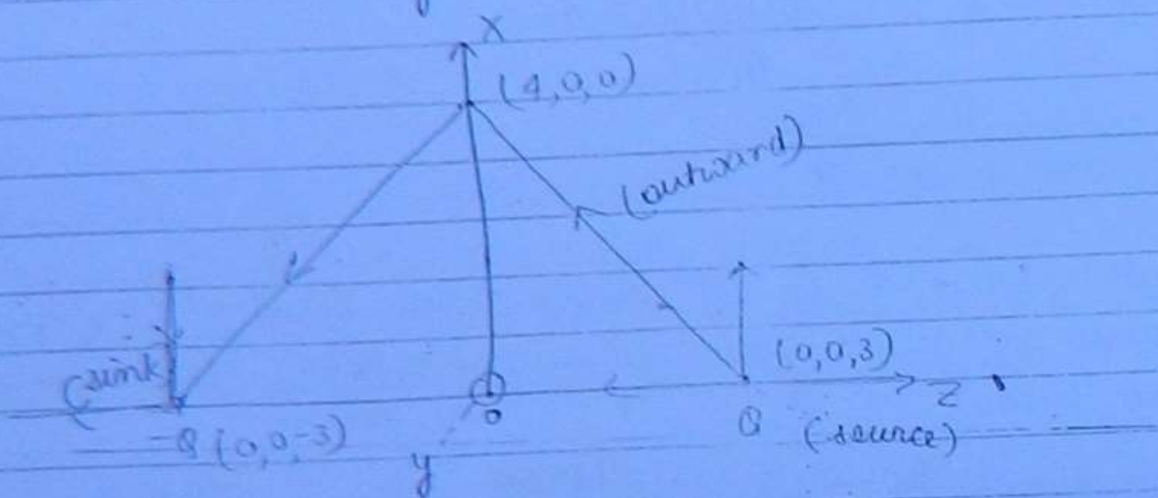
it relates current and force b/w currents (46)

Note: B's direction is physically the direction which a moving charges tracing when it enters a magnetic field.

Workbook:-

1. Static Electromagnetic fields

1




$$r = \sqrt{4^2+0^2+(-3)^2} = \sqrt{16+9} = 5$$

$$E_1 = \frac{Q}{4\pi\varepsilon_0 r^2}\frac{[4a_x - 3a_z]}{\sqrt{4^2+3^2}} \qquad \left\{ a_r = \hat{r} = \frac{\bar{r}}{|r|} \right\}$$

$$E_1 = \frac{Q}{4\pi\varepsilon_0 (5)^2}\frac{[4a_x - 3a_z]}{\sqrt{4^2+3^2}}$$

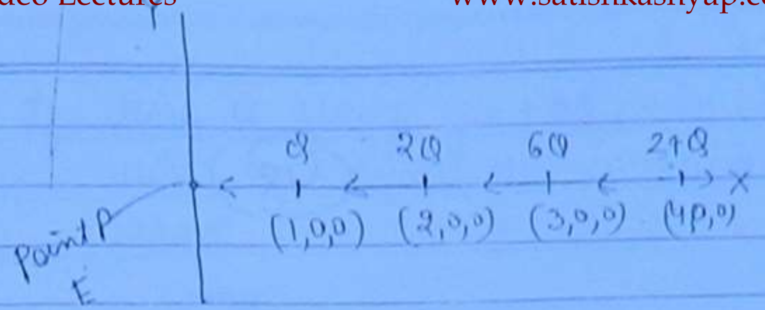
$E_1 = a_x, -a_z$

$E_2 = \cancel{-a_x} \; -a_z$

$$E_2 = \frac{Q}{4\pi\varepsilon_0 r^2}\frac{[4a_x + 3a_z]}{\sqrt{4^2+3^2}}$$

$$r = \sqrt{4^2 + \ldots (-3)^2} \quad 5$$

2. $\Rightarrow$ 71 (Q) 47




direction $(-a_x)$

$$E_T = \left( \frac{Q}{4\pi\varepsilon_0(1)^2} + \frac{2Q}{4\pi\varepsilon_0(2)^2} + \frac{6Q}{4\pi\varepsilon_0(3)^2} + \frac{24Q}{4\pi\varepsilon_0(4)^2} \right)(-a_x)$$

$$E_T = \frac{Q}{4\pi\varepsilon_0}\left[ 1 + \underbrace{\frac{1}{2}}_{\text{minimum}} + \frac{2}{3} + \underbrace{\frac{3}{2}}_{\text{maximum}} \right](-a_x)$$

as distance increases the charge is also increases.
so 4th one has strongest charge and the 2nd one
has the least charge contributed

4.
$$\left.\begin{array}{l} -5 < x < 5 \\ -5 < y < 5 \\ -5 < z < 5 \end{array}\right\} \text{cube}$$

$\cancel{2C} \longrightarrow (4,8,3)$

(Inside) $8C \longrightarrow (2,-1,-3)$ Entering flux is dominating

(Inside) $-12C \longrightarrow (-4,0,1)$

$-4C$ Ans

5. centre - origin

$d_1 = \sqrt{4^2+8^2+3} = \sqrt{16+64+9} = \sqrt{25+64} > 6$

$d_2 = \sqrt{2^2+(1)^2+(1)^2} = \sqrt{4+1+1} = \sqrt{6} < 6$ — 8C (Enclosed)

$d_3 = \sqrt{4^2+(1)^2} = \sqrt{16+1} = \sqrt{17} < 6$ — 12C (Enclosed)

$-4C$ Ans

6 $d_1 = \sqrt{2^2+1^2+1^2} > 6$

$d_2 = \sqrt{0^2+2^2+5^2} \not> 6$ — 86

$d_3 = \sqrt{6^2+3^2+2^2} > 6$

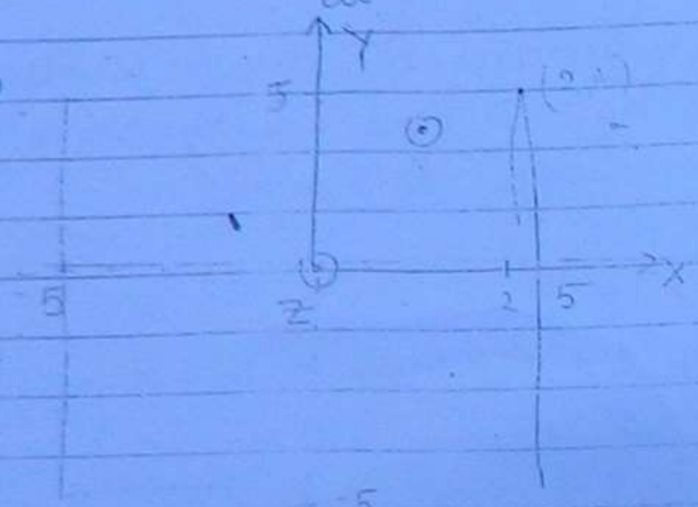
(48)

86 Ans

Extension Prob. – calculate the net flux leaving the surface $-5<x<5$, $-5<y<5$, $-5<z<5$ due to a line charge $\rho_L = 10\,nC/m$ located at $x=2,\ y=1$ for all z

Soln $x=2,\ y=4$ $\rho_l = \frac{dq}{dl}$

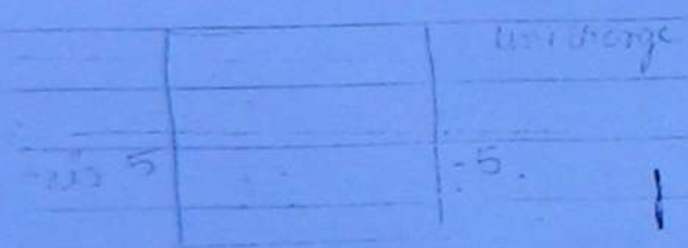
$\rho_L = 10\,nC/m$




flux leaving = charge enclosed = part of a line charge

How much part of the line charge is in the cube.

2nd view




Length of the line inside the cube = 10m

(-5 to 5 of z)

$Q = \rho_l \cdot l$

Q If line is along $y = x$ line in the $z = 4$ plane $100\sqrt{2}$

Hint - Repeat the question $z=0$, $y=x$ line

Hint $z=b$, $y=x$ line

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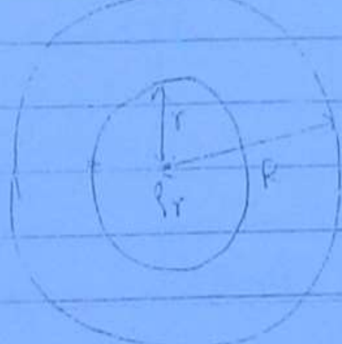
8. Colomb's law cannot apply because it is for point charge not for volume. We use $\oint D \cdot ds = Q$.

E/D inside the charge.

The gaussian surface considered is concentric sphere of $r < R$ so that the strength on the surface same everywhere and hence loop $\oint D \cdot ds = Q$.

D. area = charge




$\rho_v \times \frac{4}{3}\pi r^3$

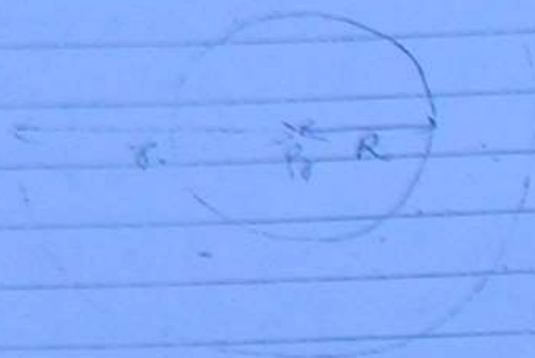
$C/m^3 \times m^3 = C$

$$D(r) = \frac{\text{charge inside}}{\text{Area}} = \frac{\rho_v \frac{4\pi r^3}{3}}{4\pi r^2} = \frac{\rho_v r}{3}$$

$$E(r) = \frac{\rho_v r}{3\epsilon}$$

$$\boxed{E(R/2) = \frac{\rho_v R}{6\epsilon}}$$

E at the centre is perfectly zero on the and maximum on the surface






$\oint D \cdot ds = Q$

$D(r) = \frac{\text{charge inside}}{\text{area}} = \rho_v \frac{4}{3}\pi r^3$

$= \frac{\rho_v \frac{4}{3}\pi R^3}{4\pi r^2} = \frac{\rho_v R^3}{3r^2}$

$E(r) = \frac{\rho_v R^3}{3r^2\epsilon}$

$E(2R) = \frac{\rho_v (2R)^3}{3r^2\epsilon} = \rho_v 8R$

$E(2R) = \frac{\rho_v R}{12\epsilon}$

$D(r) = \frac{\rho_v R^3}{3r^2}$

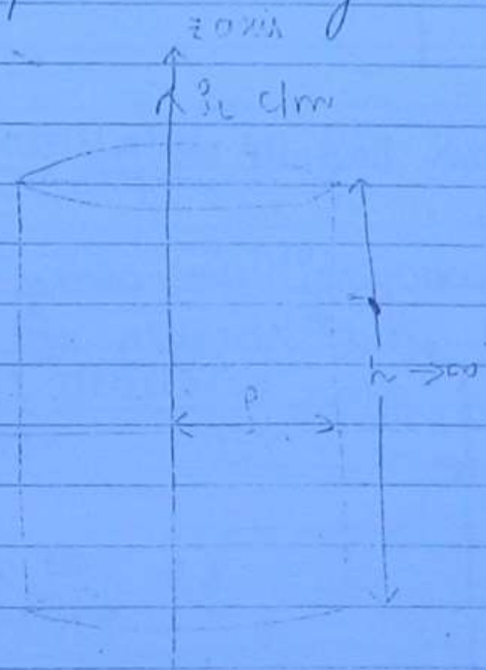
$E(r) = \frac{\rho_v (R)^3}{3\epsilon r^2}$

$E(2R) = \frac{\rho_v R^3}{3\epsilon (2R)^2} = \frac{\rho_v R^3}{12\epsilon R^2} \Rightarrow \frac{\rho_v R}{12\epsilon}$

Line charges & I carrying wires (57)

Gauss Law. Application 2: Strength E due to an infinite length line charge.

The application of gauss law involves chosing an cylindrical surface If $\rho$ = constt value. The surface have equi-distant nature from the charge and Hence D is constt everywhere.




$$\oint D \cdot ds = Q$$

$$D(\rho) \times \text{area} = \text{charge enclosed}$$

$$\boxed{D(\rho) = \frac{\rho_L h}{2\pi\rho h} = \frac{\rho_L}{2\pi\rho}}$$

The closed surface is a curved surface area hence the flux exits only through this surface with the top and bottom surfaces are ignored.

As the two surface is $\rho$ = const surface ds is $a_\rho$ directed
Hence by logic D is $a_\rho$ directed

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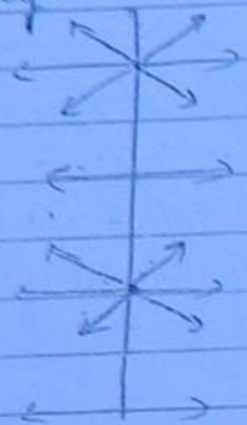
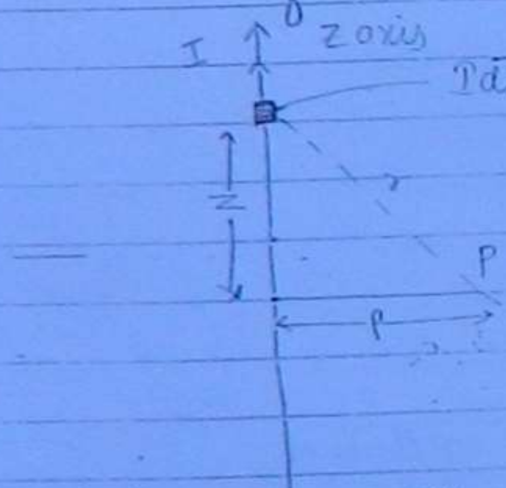
$a_\rho$ = radially outward from the line



Strength H due to an infinite length I carrying wire

consider a current carrying wire along the z axis
lets us calculate the field strength at $\rho$ distance from it




- For any infinite length or finite length.
- Identify a small incremental length element dl, such that $|dl| \to 0$
- Find the incremental strength dH due to this length.

$$dH = \frac{I dl}{4\pi r^2} \times a_r$$

3. When applied for a finite length current carrying wire

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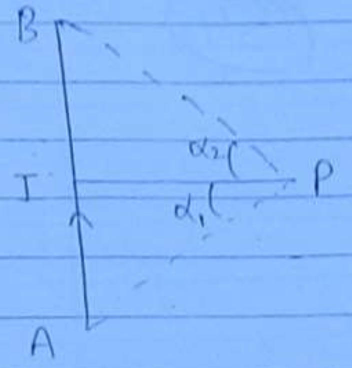



$$H = \frac{I}{4\pi\rho}\left(\sin\alpha_1 + \sin\alpha_2\right) a_\phi$$

* If E is to be calculated in line charge in Electrostatics

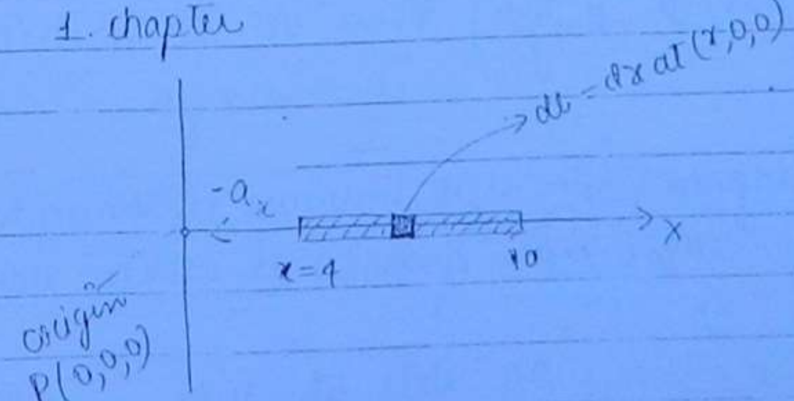
$$E(\rho) = \frac{\rho_L}{2\pi\epsilon\rho}\, a_\rho = \frac{D(\rho)}{\epsilon}$$

If B is to be calculated in I carrying wire in Magnetostatics.

$$B(\rho) = \mu H = \frac{\mu_0 I}{2\pi\rho}\, a_\phi$$

Work Book 1. chapter

3.




Let small length dl on this wire

find dH due to this length using

$$Q = \rho_L dx$$

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$$r = x$$

$$a_r = -a_x$$

$$dE = \frac{\rho_L dx}{4\pi\varepsilon_0 x^2}(-a_x)$$

$$E = \int_{x=4}^{10} \frac{\rho_L dx}{4\pi\varepsilon_0 x^2}(-a_x)$$

$$= \frac{\rho_L}{4\pi\varepsilon_0}\int_4^{10} \frac{1}{x^2} dx \quad (-a_x)$$

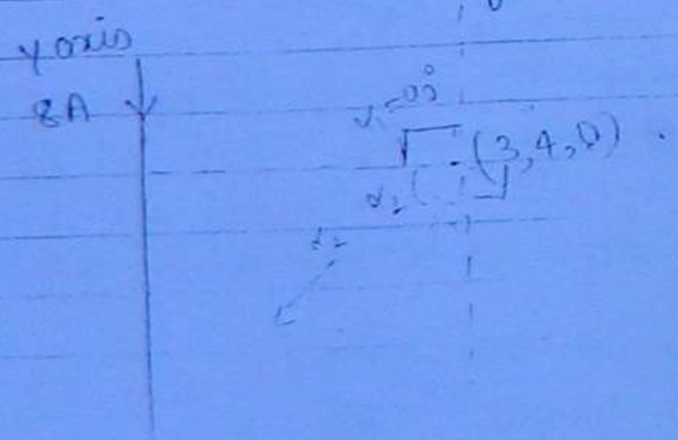
$$= \frac{\rho_L}{4\pi\varepsilon_0}\left[-x^{-2+1}\right]_4^{10} \Rightarrow \frac{\rho_L}{4\pi\varepsilon_0}\left[-\left[\frac{1}{10} - \frac{1}{4}\right]\right]$$

$$\Rightarrow \frac{\rho_L}{4\pi\varepsilon_0}\left[-\frac{1}{10} + \frac{1}{4}\right] \Rightarrow \frac{\rho_L}{4\pi\varepsilon_0}\left[\frac{-4+10}{10\times 4}\right] \Rightarrow \frac{\rho_L}{4\pi\varepsilon_0}\left(\frac{6}{40}\right)(-a_x)$$

10 Ans 3, 4

(1) Volume is same then no effect on flux $\phi_E$

(2) gauss law. the flux leaving the surface is equal to the cause i.e charge.

11. 

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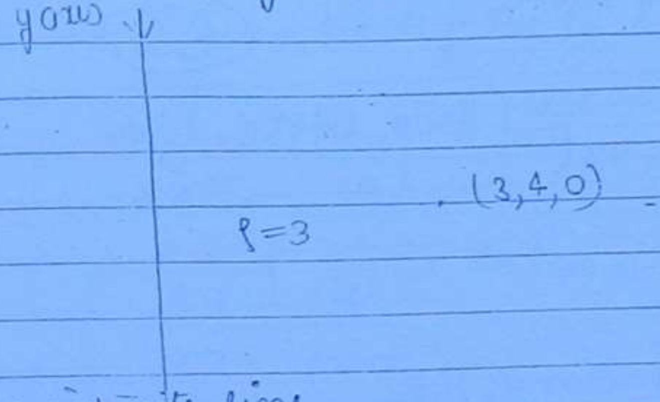
Q Repeat the same quest if the current was flowing on the entire y-axis.

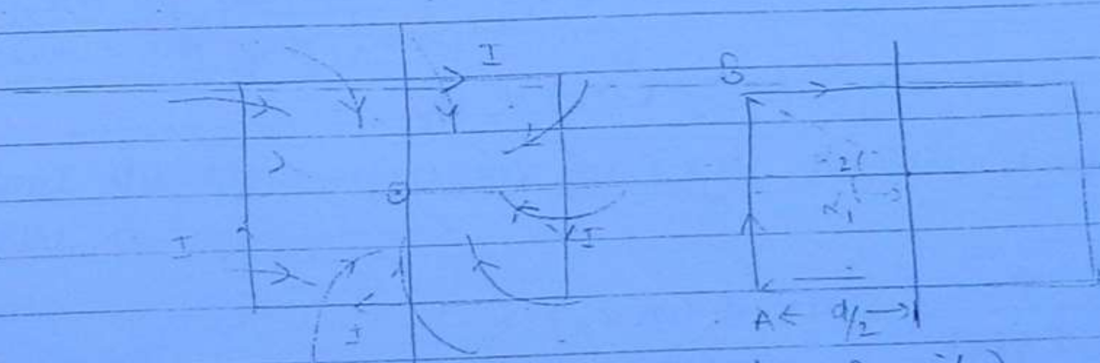



For infinite line

$$H = \frac{I}{2\pi\rho} = \frac{8}{2\pi(3)} a_z = \frac{1.33}{\pi} a_z$$

The field in L shape is stronger and in infinite line is weak as in L shape the cause come close.

W.B 12




H direction at the centre (due to AB side)

$$a_y \times a_x = -a_z$$

Note for a symmetric current distribution magnetic field at the geometric centre is always maximum.

H magnitude due to side AB

$$H = \frac{I}{4\pi\rho}(\sin\alpha_1 + \sin\alpha_2)$$

$$\frac{I}{4\pi\rho}(\sin 45^\circ + \sin 45^\circ)$$

Break the L-shape wire into two part and then calculate the H for y-axis and H for x-axis seprately and then take the vector sum.

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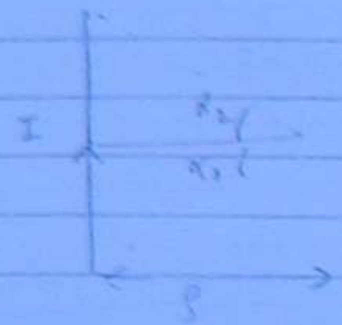



$$H = \frac{I}{4\pi\rho}\left(\sin\alpha_1 + \sin\alpha_2\right) a_\phi$$

$$H_{(y\text{-axis})} = \frac{8}{4\pi(3)}\left(\sin 90^\circ + \sin\alpha_2\right) a_\phi$$

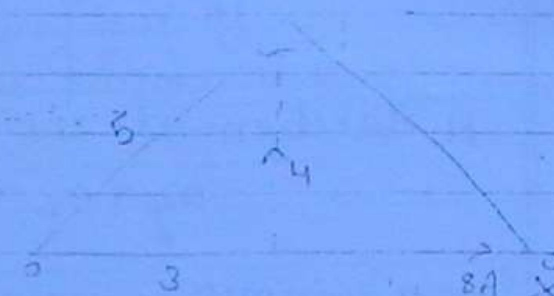
$$H_{(y\text{-axis})} = \frac{8}{4\pi(3)}\left(1 + \frac{4}{5}\right) a_\phi$$

direction current direction × radial dir$^n$ to the point from the current

$$\left(-a_y \times a_x\right) = a_z$$

$$x \rightarrow y \rightarrow z$$

$$H_{(x\text{-axis})} = \frac{8}{4\pi(4)}\left(\sin\alpha_1 + \sin\alpha_2\right)$$




$$= \frac{8}{16\pi}\left(9 + \frac{3}{5}\right)$$

$$= \frac{8}{16\pi}\left(\frac{8}{5}\right)$$

$$= \frac{64}{16\pi 5} \Rightarrow \frac{8}{2\times\pi\times 5} \quad \frac{8}{10\pi} = \frac{4}{5\pi}\, a_z$$

$x \rightarrow y \rightarrow z$

$$\frac{I}{4\pi(d/2)}\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)(-a_z)$$

H at the centre, totally due to 4 sides

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$$H = \frac{4\times I}{4\pi d/2}\left(\frac{2}{\sqrt{2}}\right)(-a_z)$$

$$= \frac{2I}{\pi d}\left(\frac{2}{\sqrt{2}}\right)(-a_z)$$

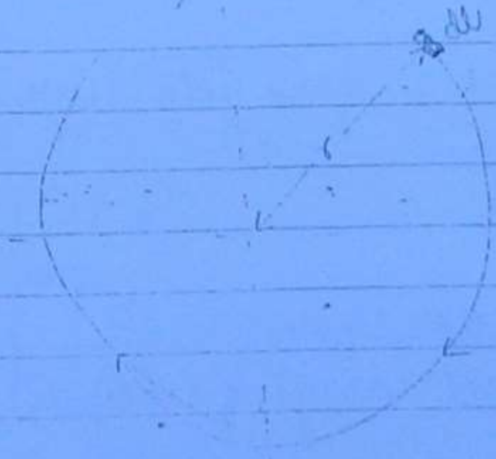
$$= \frac{4I}{\sqrt{2}\pi d}(-a_z)$$

$$= \frac{2\sqrt{2}\sqrt{2}\,I}{\cancel{\sqrt{2}}\pi d}(-a_z)$$

$$= \frac{2\sqrt{2}}{\pi d} I (-a_z)$$

$$H = \frac{2(\times 1.41)}{3.14}\frac{I}{d} = \frac{0.9\,I}{d}$$

Q Repeat the same question for circle - find the H at the centre. O



using Biot savart law.

$$dH = \frac{I\,dl \times a_r}{4\pi r^2}$$

$$dl = r\,d\phi$$

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$$H = \frac{I}{4\pi}\int \frac{r\,d\phi}{r^2}$$

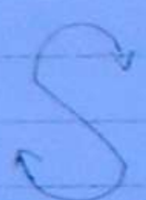
$$H = \frac{I}{4\pi}\int \frac{d\phi}{r}$$

$$H = \frac{I}{4\pi r}\int_0^{2\pi} d\phi$$

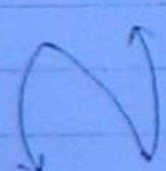
$$H = \frac{I}{4\pi r}(2\pi) \Rightarrow \frac{I}{2r} = \frac{I}{d}$$

1 clockwise current - H field direction into the paper.
2 All current carrying wires that are closed (square, circle) and have a finite area of enclosure are referred as magnetic dipole

clockwise current I flow side — South pole



anticlockwise current I flow side North pole



Atom is a example of dipole.
dipole → closed current carrying wire.

$\tan^{-1}(\infty) = \pi/2$
$\tan^{-1}(-\infty) = -\pi/2$
$\tan^{-1}(1) = 45$
$\tan^{-1}(0) = 0$

using Biot savart's law.

$$dH = \frac{I\,dz\,a_z}{4\pi(\rho^2+z^2)} \times (a_r)$$

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$$r = \sqrt{\rho^2+z^2}$$

$$a_r = \hat{r} = \frac{\bar{r}}{|r|}$$

$$\hat{r} = \frac{\bar{r}}{|r|} = \frac{(\rho a_\rho - z a_z)}{\sqrt{\rho^2+z^2}}$$

$$dH = \frac{I\,dz\,a_z}{4\pi(\rho^2+z^2)} \times \frac{(\rho a_\rho - z a_z)}{\sqrt{\rho^2+z^2}}$$

The total strength $\bar{H}$ is

$$H = \int_{z=-\infty}^{\infty} dH = \int \frac{\rho I\,dz}{4\pi(\rho^2+z^2)^{3/2}}\, a_z \times (\rho a_\rho - z a_z)$$

$$H = \int \frac{\rho I\,dz}{4\pi(\rho^2+z^2)^{3/2}}\, a_\phi$$

$$= \frac{I \cdot \rho}{4\pi}\int_{-\infty}^{\infty} \frac{dz}{(\rho^2+z^2)^{3/2}}$$

put $z = \rho \tan\theta$

$dz \quad \rho \sec^2\theta\, d\theta$

$z^2 + z^2 \tan^2\theta = \rho^2 \sec^2\theta$

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$$H = \int dH$$

$$= \frac{I \cdot \rho}{4\pi} \int_{-\pi/2}^{\pi/2} \rho \frac{\sec^2\theta}{\rho^3 \sec^3\theta} \, d\theta$$

$$= \frac{I \cdot \rho}{4\pi} \int_{-\pi/2}^{\pi/2} \frac{1}{\rho^2 \sec\theta} \, d\theta$$

$$= \frac{I \cdot \rho}{4\pi \rho^2} \int_{-\pi/2}^{\pi/2} \cos\theta$$

$$= \frac{I}{4\pi\rho} \left[\sin\theta\right]_{-\pi/2}^{\pi/2}$$

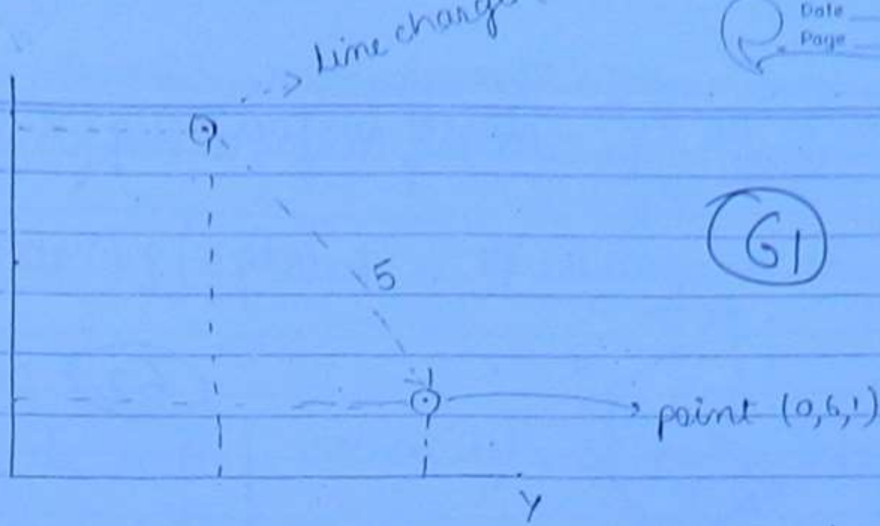
$$= \frac{I}{4\pi\rho} \left[\sin \pi/2 - \sin(-\pi/2)\right]$$

$$= \frac{I}{4\pi\rho} \left[1 - (-1)\right] \Rightarrow \frac{I}{4\pi\rho} \times 2 \Rightarrow \frac{I}{2\pi\rho}$$

$$\boxed{H = \frac{I}{2\pi\rho} a_\phi}$$

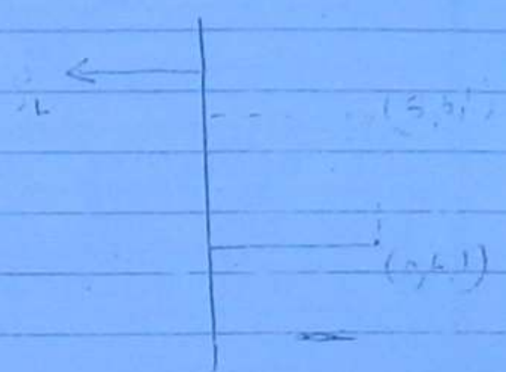
1. $a_\phi$ - The H direction which is $(a_z \times a_\rho)$ is circulatory around the current.

2. The expression is similar to $D = \frac{\rho_l}{2\pi\rho} a_\rho$




G1

$E = \frac{\rho_L}{2\pi\epsilon\rho} a_\rho$




$E \propto \frac{1}{\rho}$

$\rho$ between $(0,6,1)$ & $(7,3,5)$

$\rho = \sqrt{3^2+4^2} = 5$

$E = ?$ at $(5,6,1)$

$E \propto \frac{1}{\rho}$

$\rho = 5$ in both cases

E is the same

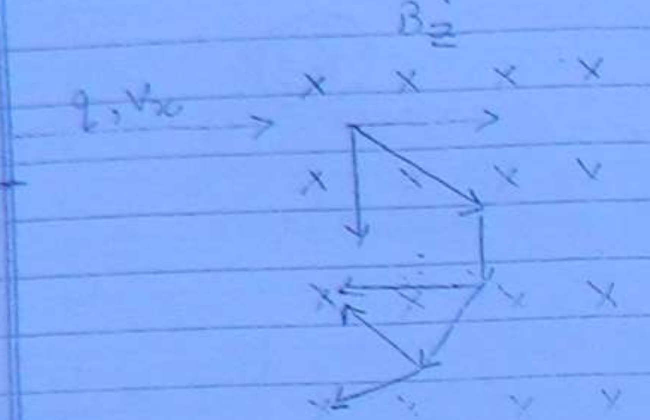
Note: If z axis is defined as $(0,0,z)$

Any point $(x,y,z)$ had a radial distance

$\rho = \sqrt{x^2+y^2}$

which is independent of z

Maxwell's III Eqn – closed surface integral of B.




$F_y = q(v_x \times B_z)$

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1 The nature of magnetic field line is always to form closed loop around the current as seen in H due to a line current carrying wire ( $a_\phi$ direction)

2. Any phenomena that is circulatory or closed never has a distinct starting / Ending point

There are no source / sink points for magnetic flux lines

$$\nabla \cdot B = 0$$

Because:

Hence divergence of magnetic flux density is zero everywhere because divergence needs a distinct Start or an End

Maxwell's III$^{rd}$ Eq in point form.

$$\boxed{\nabla \cdot B = 0}$$

Mathematically this property is called as Solenoidal nature of H. fields

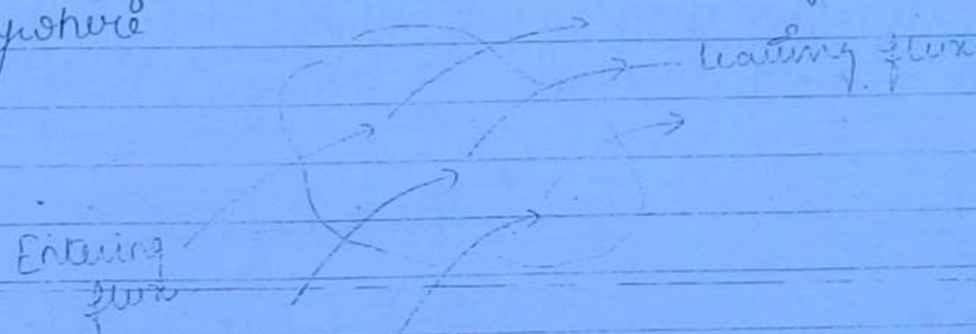
Applying divergence theorem for the point form. (63)

$$\oint B \cdot ds = \int (\nabla \cdot B) \cdot dv \quad \text{[Divergence theorem]}$$

$\therefore \ \nabla \cdot B = 0$

$$\boxed{\oint B \cdot ds = 0}$$ Maxwell's III Eq$^n$ - Integral form.

Magnetic Monopoles donot exist because entering flux into any closed surface should always be equal to leaving flux as flux cannot start from anywhere or terminate anywhere




Hence we conclude that only dipole exist in magnetic fields i.e the basic cause of magnetic fields is a current I which flows only in closed circuits.
It flows only when Both the polarities (dipoles) exist.

Summary.- E fields Vs H fields

1. Q - Basic cause in E-fields - scalar quantity. Idl - Point current - in H-field. elements - vector quantity.

2. $\rho_l$ → line charge density I → flows in a line

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4. $\bar{D}$ is called as flux density in E-fields

$\bar{B}$ is called as flux density in magnetic field.

It is always a measure of strength in terms of charge.

It is always a measure of strength - force.

It is always independent of $\varepsilon$

It is always $\mu$, dependent

$\bar{E}$ - field Intensity in E fields.

$\bar{H} \rightarrow$ field intensity in magnetic fields

It is always a measure of strength - force

It is always a measure of strength in terms of current (cause)

It is always $\frac{1}{\varepsilon}$ dependent

It is always independent $\mu$.

5. $F \propto E \propto \frac{1}{\varepsilon}\,(10^{-12})$ —— $F \times B \times \mu\,(10^{-7})$ ——

Strongest

E field is one of the strongest force

Weakest

B field is one of the weakest force in nature

Potential, Gradient, closed line Integral of E

Potential

1. A scalar measure of field strength of E field in terms of the energy at a point or in terms of work done to reach the point.

... work done to reach the point from a ... charge.

$E = \frac{F}{Q}$ $\quad V = \frac{W}{Q}$.

classmate Date Page

$V = \frac{W}{Q} = \frac{\text{Joule}}{\text{coloumb}} = \text{volts}$

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Note: If work done by the charge is the measure of potential and never work done on the charge.

Work = force · displacement

$W = F \cdot l$

$dW = F \cdot dl$

$dW = -Q\, E \cdot dl$

Note: work is done by the charge only when it goes against the field (against the repulsive force) (that's y -ve sign)

$W = -\int Q \cdot E \cdot dl$

$W = -Q \int \bar{E} \cdot dl$

$$\boxed{V = \frac{W}{Q} = -\int \bar{E} \cdot \bar{dl}}$$

$V \rightarrow$ Potential function of space but it is a scalar function.

$E = 4xy a_z - 4yz a_y$

$\rightarrow$ It is similar to intensity function which is an vector function

If the potential is evaluated b/w two distinct point with reference at B then $V_{AB}$ is called potential difference with b/w A & B.

(Potential difference) $V_{AB} = -\int_{ref\ B}^{at\ A} E \cdot dl$ (Potential difference b/w A & B)

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→ If ref B is assumed to be zero value; then
$V_A$ = absolute potential w.r.t B.

eg. Ground is taken zero in most Electric ckts
Infinite distance is zero potential.

14/7/11 Thursday

$$V = -\int \bar{E} \cdot \bar{dl}$$

$$= -\int \frac{Q}{4\pi\varepsilon_0 r^2} a_r \cdot dr\, a_r$$

When the field intensity is radially directed the potential calculation is simplified when $dl$ is $dr\, a_r$

$$V = -\int \frac{Q}{4\pi\varepsilon_0 r^2} dr$$

$$= -\frac{Q}{4\pi\varepsilon_0}\int \frac{dr}{r^2}$$

$$\boxed{V = \frac{Q}{4\pi\varepsilon_0 r}}$$

Note:- If E decreases as $\frac{1}{r^2}$ V decreases as $\frac{1}{r}$.
$E\left(\frac{1}{r^2}\right)$ → vector
$V\left(\frac{1}{r}\right)$

If $r$ = constt then $V$ = constt the locus of all these points forms a sphere around the charge. where the potential is constt this is called as equipotential surface. The family of equipotential surfaces graphically represent the changes in potential

(67)

if $r$ = constt
→ concentric sphere

Potential of a line charge

$$V = -\int \bar{E} \cdot d\bar{l}$$

$$= -\int \frac{\rho_L}{2\pi\epsilon_0 \rho} a_\rho \cdot d\rho \, a_\rho$$

{Always use natural log not a}

$$= -\frac{\rho_L}{2\pi\epsilon_0} \int \frac{d\rho}{\rho}$$

$$\boxed{V = \frac{\rho_L}{2\pi\epsilon_0} \ln\left(\frac{1}{\rho}\right)}$$

$$\left\{ -\int \frac{d\rho}{\rho} = \ln\left(\frac{1}{\rho}\right) \right\}$$

1. $E\left(\frac{1}{\rho}\right) \rightarrow$ vector

$V\left\{\ln\left(\frac{1}{\rho}\right)\right\} \rightarrow$ scalar

If $\rho$ = constt then $V$ = constt then we get concentric cylinders.

$$-\frac{1}{2}\left(\frac{x^{1/2+1}}{1/2+1}\right)\Bigg|_0^4 - 4\left[\frac{y^3}{3}\right]_0^1$$

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$$\Rightarrow -\frac{1}{2}\left|\frac{x^{3/2}}{3/2}\right|_0^4 - 4\left[\frac{1}{3}\right]$$

$$\Rightarrow -\frac{1}{2}\left[\times\frac{2}{3}\right]\left[(4)^{3/2}\right] - \frac{4}{3}$$

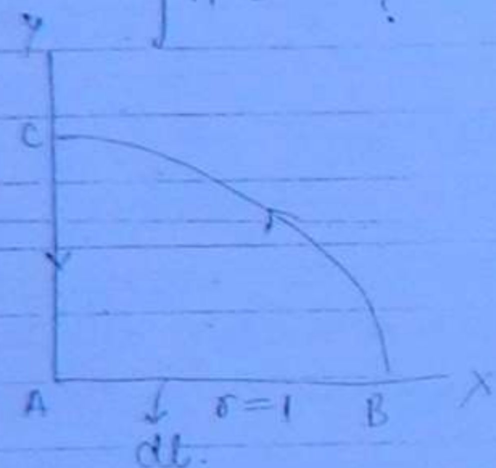
$$\Rightarrow \frac{-1}{3}\left[4^{3/2}\right] - \frac{4}{3}$$

$$\Rightarrow \frac{-8}{3} - \frac{4}{3} \Rightarrow -4V$$

Q 21 WB

$$\bar{A} = 2\rho\cos\phi\, a_\rho$$

$$\oint \bar{A}\cdot\bar{dl} = ?$$




$\oint A\cdot dl$ = 3 lines

$$\oint A\cdot dl = \oint_A^B \bar{A}\cdot\bar{dl} + \int_B^C \bar{A}\cdot\bar{dl} + \oint_C^A \bar{A}\cdot\bar{dl}$$

(1) $$\int_A^B \bar{A}\cdot\bar{dl} = \int_{\rho=0}^{1} 2\rho\, d\rho = 1$$

$dl = d\rho\, a_\rho$

$\phi = 0^\circ$

$\bar{A} = 2\rho\, d\rho$

2. B to C

$$dl = \rho \, d\phi \, a_\phi$$
$$\bar{A} = 2\rho \cos\phi \, a_\rho$$



$$\oint_B^C \bar{A} \cdot \bar{dl} = 0$$

3. C to A

$$dl = d\rho \, a_\rho$$
$$\phi = 90^\circ$$
$$\oint_B^C \bar{A} \cdot \bar{dl} = \int_B^C 2\rho \cos\phi$$
$$= 0.$$

**Potential Gradient**

Scalar $Eq^n$ of a surface $\xrightarrow{\text{gradient}}$ Vector Direction of the surface

In maths the gradient is used to find direction vector of any scalar surface $Eq^n$ i.e gradient is used to find a normal vector everywhere given to the surface

eg. Linear surface

$$f = 3x - 4y - 8z = 100$$
$$\nabla \cdot f = 3a_x - 4a_y - 8a_z$$

Non linear.

$$g = 4x^2y - 8xz = 100$$
$$\nabla \cdot g = (8xy - 8z)a_x + 4x^2 a_y - 8x a_z$$

$$V = -\int \bar{E} \cdot \bar{dl}$$

(70)

$$dv = -\bar{E} \cdot \bar{dl}$$

$$dv = -|E| \cdot |dl| \cos\theta$$

$$\frac{dv}{dl} = -E\cos\theta$$

case I

If $\theta = 90°$ i.e the change of potential per unit length is analysed orthogonal to or ⊥ to the electric field direction. The potential has the same value. Hence the locus of all the points ⊥ to E field constitutes equi-potential surface

$$\theta = 90°$$

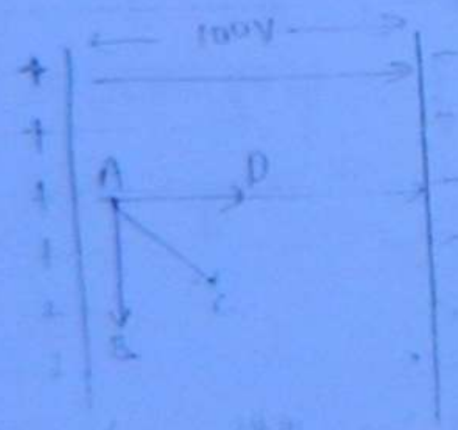
$$V = \text{const}$$

case II

If $\theta = 0°/180°$

$$\left.\frac{dv}{dl}\right|_{max} = |E|$$

→ The magnitude of the E field intensity is the maximum rate of change of potential per unit length.



case II

$\theta = 0^\circ$

$$\left.\frac{dV}{dl}\right|_{max} = -E$$

The direction of E field intensity is the direction in which potential decreases at a maximum rate

Hence every scalar can have a vector defined from a unique direction of change by maximum and the rate of change by maximum. This is called as gradient operation. If the vector is E the scalar is V, then

$$\boxed{E = -\nabla V}$$ Volts/mete

Potential gradient means E field intensity (E)

Potential Eqn $\xrightarrow{\text{Gradient}}$ vector Intensity

unit of electric field intensity (E) is $\frac{\text{Volts}}{\text{meter}}$

Formula for gradient operation

If $V$ = scalar function of space

$V(u, v, w)$

$$\boxed{\nabla \cdot V = \frac{1}{h_1}\frac{\partial V}{\partial u} a_u + \frac{1}{h_2}\frac{\partial V}{\partial v} a_v + \frac{1}{h_3}\frac{\partial V}{\partial w} a_w}$$

Q given the potential fun V = [illegible] $(x^2 - y^2)$ for all z find the Eqn of the [illegible] surface passing through the [illegible] ( [illegible] )

(3,1,1)

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Eqⁿ of equipotential surface is

$$V = 25(x^2-y^2) = k$$

(as voltage is constt on equipotential surface)

$$25(x^2-y^2) = k$$

$$x^2-y^2 = k'$$

$25(x^2-y^2) = k$ at $(3,1,1)$

$25(9-1) = k \Rightarrow 200$

$$25(x^2-y^2) = 200 \Rightarrow x^2-y^2 = 8$$

The potential funⁿ given in the question is in itself equipotential surface definition

Q given $V = \frac{4\cos\theta}{r^2}$. Find $\bar{E}$ at $(2, \pi/2, \pi/2)$

$$V = \frac{4\cos\theta}{r^2}$$

$$\nabla \cdot V = \frac{1}{h_1}\frac{\partial}{\partial r}\left(\frac{4\cos\theta}{r^2}\right)a_r + \frac{1}{h_2}\frac{\partial}{\partial\theta}\left(\frac{4\cos\theta}{r^2}\right)a_\theta + \frac{1}{h_3}$$

$$\nabla \cdot V = \frac{4}{1}\cos\theta\left(r^{-2+1}\right)(-2) + \frac{4}{r^3}(-\sin\theta)\,a_\theta$$

$$= -8\cos\theta\left(\frac{1}{r^3}\right) + \frac{4}{r^3}(-\sin\theta)$$

$$= -8\cos\pi/2\left(\frac{1}{8}\right) + \frac{4}{8}(-\sin\pi/2)$$

$$\frac{4}{}(-1 \qquad a_\theta$$

closed line integral of E - Maxwell's II Eq$^n$

$$\int E \cdot dl = \text{potential}$$

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$$\oint E \cdot dl = 0$$

→ Potential at a point in space is always unique at a point of time

→ Potential cannot be a multivalued function

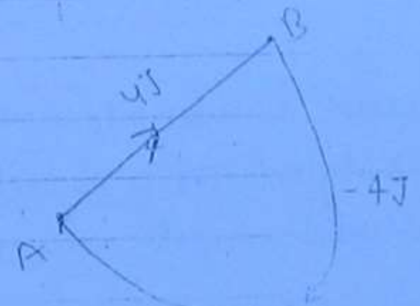
→ The work done in moving a charge in any closed loop is zero. i.e in a closed loop we sometime aquire energy sometimes loss energy. such that energy is conserved.

Hence E field is a conservative field.

→ E-field lines never forms closed loop, the lines are always outwardly divergent from a charge.

E field is an irrotational vector

$$\boxed{\nabla \times E = 0}$$




Work done in moving a charge b/w two points is independent of path of consideration

Note: $\oint E \cdot dl = 0$ - Maxwell's 2$^{nd}$ Eq$^n$ in integral form but not $\int E \cdot dl = -V$

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Similarly $\nabla \times E = 0$ — Maxwell's II eqn in point form.

but not $E = -\nabla V$

Note: To identify whether a given vector/field is a valid E or H put $\nabla \times E = 0$ — Valid Electric (E) field

put $\nabla \cdot B = 0$ — Valid Magnetic field.

— In static E/H only.

Potential, Vector Potential, Maxwell IV Eqn (Ampere's law)

Potential in Magnetic fields expressed as a scalar quantity is called as MMF (Magneto motive force)

$$V_m = \int H \cdot dl \quad \text{(amperes)}$$

$$H = \nabla V_m$$

Its unit is ampere but it is never similar to current It is equal to current when analysed for a closed path. As current flows only in closed circuits hence

$$\boxed{\oint H \cdot dl = I}$$

Ampere's law in integral form.

Maxwell IV Eqn in integral form.

but not $V_m = \int H \cdot dl$

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Statement of Ampere's Law:

The circulation of magnetic field intensity in any $\boxed{\text{closed}}$ $\boxed{\text{loop}}$ is always equal to the $\boxed{\text{current}}$ crossing the $\boxed{\text{surface}}$ enclosed.

→ circulation means the effects which are around the current.

→ current means the cause of the effects

→

$$\nabla \times H = \text{curl of } H = \frac{\text{circulation}}{\text{area}} = \frac{\text{current}}{\text{area}} = J \quad A/m^2$$

$$\boxed{\nabla \times H = J}$$

Ampere's law in point form
Maxwell's IV Eqn in point form

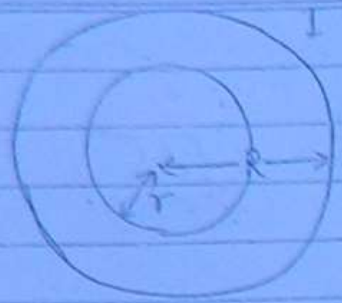
eg:

$$\oint H \cdot dl = I$$

$$H(\rho) = \frac{I}{2\pi\rho}$$

As the length of the circulation increases the strength of the effect is reduced. If the circulation is in a length of $2\pi\rho$ strength is $\frac{I}{2\pi\rho}$

26 W.B




given $r<R$

$H(r) = ?$

Case I $r<R$

To apply ampere's law consider a circular line concentric and symmetric with the current. so the strength (H) is constt everywhere

$$\oint H \cdot dl = I$$

$H(r) \times$ length of circulation $=$ current in the area

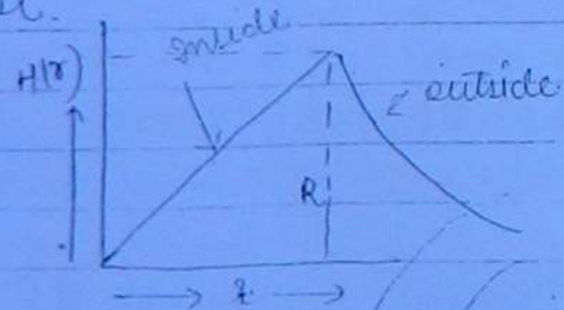
current density $\rightarrow \left(\frac{I}{\pi R^2}\right) \pi r^2 =$ current crossing the ~~area~~ formed by the closed line

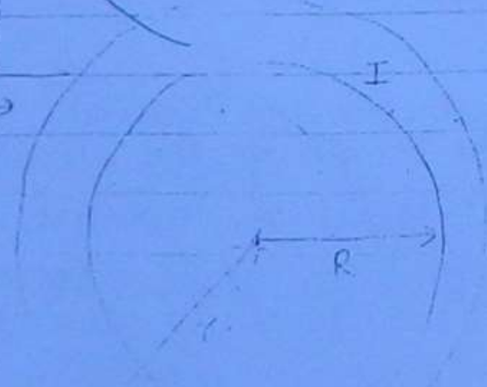
Total area

$$H(r) = \frac{\frac{I}{\pi R^2} \times \pi r^2}{2\pi r}$$

$$= \frac{I r^2}{2\pi R^2 r} \Rightarrow \frac{I r}{2\pi R^2}$$

Note 1. The field is zero at the centre of the conductor and $max^m$ of the conductor.




Case II $r>R$

$$H(r) = \frac{I}{2\pi r}$$

22 W.B

A sphere has the same geometry as that of point and hence a potential can be calculated on the same basis as.

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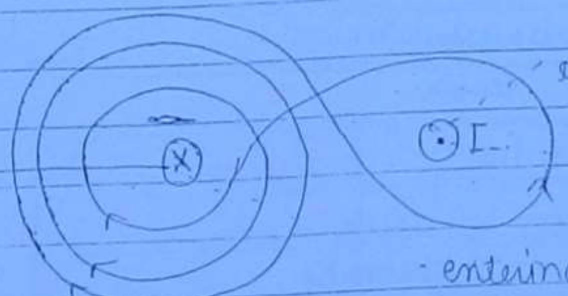
$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

$$V = \frac{2\times10^{-8}}{4\pi\varepsilon_0(10\text{ cm})} \Rightarrow \frac{2\times10^{-8}}{4\pi\times}$$

25 W.B

current clockwise




current anticlockwise - leaving current of the board

entering current to the board

$$I + I + I - (-I) = 4I$$

Vector Magnetic Potential $\bar{A}$

failure of MMF in certain regions

$$E = -\nabla V$$
$$\nabla \times E = 0$$
$$\nabla \times (-\nabla V) = 0$$

$$H = \nabla V_m$$

$$\nabla \times H = J$$

$$\nabla \times (\nabla V_m) = 0 = J$$

Hence $H = \nabla V_m$ definition is correct and exist only in those regions where there is no current density. i.e in free space or current free - regions only But not inside conductor.

If the curl of vector potential is B. as divergence of B is 0 everywhere this definition satisfied for any point in magnetic fields

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$$B = \nabla \times \bar{A}$$
$$\nabla \cdot B = 0$$

$$\nabla \cdot (\nabla \cdot A) = 0$$

$$\text{Div} \cdot (\text{curl of vector}) = 0$$

$\bar{A}$ has the unit of $-\frac{\text{weber}}{\text{meter}}$

From faraday's law.

$$\frac{\text{weber}}{\text{second}} = \text{volts} = \frac{\text{Joule}}{\text{colombs}}$$

$$\left\{ \because \text{volts} = \frac{W}{Q} = \frac{\text{Joule}}{\text{colombs}} \right\}$$

$$\text{weber} = \frac{\text{Joule}}{\text{colombs}} \times \text{sec} \Rightarrow \frac{\text{Joule}}{\frac{\text{colombs}}{\text{sec}}}$$

$$\text{weber} = \frac{\text{Joule}}{\text{Amp}}$$

$$\frac{\text{weber}}{\text{meter}} = \frac{\text{Joule}}{\text{Amp-meter}}$$

$$\bar{A} = \frac{W}{I \cdot \bar{dl}}$$

Hence potential signifies work or energy measured per basic source of the field same potential signifies the basic source of [illegible] current element $I\bar{dl}$ is a vector quantity

$$\bar{A} = \frac{W}{I\bar{dl}}$$

Note: If $\bar{E} = \frac{\bar{F}}{Q}$ $\quad \bar{B} = \frac{\bar{F}}{I\bar{dl}}$

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If $\bar{V} = \frac{W}{Q}$ $\quad \bar{A} = \frac{W}{I\bar{dl}}$

$E = \nabla V$ $\quad B = \nabla \times \bar{A}$

If $V = \frac{Q}{4\pi\epsilon_0 r}$ — point

Expression of A is obtained by duality of expression of V.

$$\boxed{\bar{A} = \frac{\mu_0 I\bar{dl}}{4\pi r}}$$ point current element.

The direction of $\bar{A}$ is always the current direction itself

Q. The closed line integral of vector p. magnetic potential is

Sol$^n$ $\oint A \cdot dl = \frac{\text{weber}}{\text{meter}} \times \text{meter} = \text{weber}$.

Magnetic flux crossing the open surface formed by the closed line

$$\oint A \cdot dl = \int (\nabla \times A) \cdot ds$$

Note: $\oint B \cdot ds = 0$ = Entering flux = Leaving flux

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Maxwell III $Eq^n$

But $\int B \cdot ds = \psi_m$ = cutting flux

This is not Maxwell III $Eq^n$

15/7/11 Friday

Laplace / Poisson's Equation

The most common form of charge is a volume charge moving inside materials. Hence $\boxed{\rho_v = ne}$ for most application

~~where~~ $n$ = no. of carriers per unit volume

$e$ = charge of the carrier

Poisson $Eq^n$ relates in potential develop around any volume charge.

$$V \longrightarrow \rho_v$$

$$\nabla \cdot D = \rho_v$$

$$D = \epsilon E$$

$$E = -\nabla V$$

$$\nabla \cdot (\epsilon E) = \rho_v$$

$$\nabla \cdot \left(\epsilon\left(-\nabla V\right)\right) = \rho_v$$

$$V = \frac{-6r^5}{\epsilon_0}$$

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$$\nabla^2 V = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial r}\left(h_2 h_3 \frac{\partial}{\partial r}\left(\frac{-6r^5}{\epsilon_0}\right)\right)\right]$$

$$= \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial r}\left[r^2 \sin\theta \frac{\partial}{\partial r}\left(\frac{-6r^5}{\epsilon_0}\right)\right]$$

$$= \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial r}\left[\frac{r^2 \sin\theta}{\epsilon_0}(-6 \times 5\, r^4)\right]$$

$$= \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial r}\left[\frac{-30 r^6 \sin\theta}{\epsilon_0}\right]$$

$$= \frac{1}{r^2 \sin\theta}\left(\frac{-30 \sin\theta}{\epsilon_0}\right)\frac{\partial}{\partial r}\left[r^6\right]$$

$$= \frac{1}{r^2 \sin\theta}\left(\frac{-30 \sin\theta}{\epsilon_0}\right) 6 r^5$$

$$= \frac{-180 r^3}{\epsilon_0}$$

as $\nabla^2 V = \frac{-\rho_v}{\epsilon}$

$$\nabla^2 V = \frac{-180 r^3}{\epsilon_0} = \frac{-\rho_v}{\epsilon}$$

$$\rho_v = 180 r^3$$

step 2 $Q = \int \rho_v \, dv$

$$= \int_0^1 \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} 180 r^3 \, r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$180\left[\frac{8 r^{6}}{6}\right]_0^1 \left[-\cos\theta\right]_0^{\pi} \left[\phi\right]_0^{2\pi}$$

$$180 \times \frac{1}{6}(2\pi) = 120\pi$$

Alternate Methode

$$D \leftarrow E = -\nabla V$$

$$\downarrow$$

$$\oint D \cdot dl = Q$$

31 W.B

$$\nabla^2 V = -\frac{\rho_v}{\epsilon}$$

given $E$ & $V$ are zero — function zero — Intial condition

$$V' = E = \frac{dv}{d\rho}$$

$$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \cdot \frac{\partial v}{\partial \rho}\right) = + \frac{\left[10^{-8}(1+10\rho)\right]}{\frac{1}{36\pi \times 10^{9}}}$$

$$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \cdot \frac{\partial v}{\partial \rho}\right) = 360\pi(1+10\rho)$$

$$\rho\frac{\partial v}{\partial \rho} = \int 360\pi(\rho + 10\rho^2)\, d\rho$$

$$\rho \cdot \frac{\partial v}{\partial \rho} = 360\pi\left(\frac{\rho^2}{2} + \frac{10\rho^3}{3}\right)$$

$$V = \int_{\rho = 2cm}^{\rho = 5cm} 360\pi\left(\frac{\rho}{2} + \frac{10\rho^2}{3}\right) d\rho$$

$$V = 360x \int_{\rho=2cm}^{5cm} \left(\frac{\rho^2}{4} + \frac{10\rho^3}{9}\right)$$

$$V = 360x \left[\frac{\rho^2}{4} + \frac{10\rho^3}{9}\right]_{2\times10^{-2}}^{5\times10^{-2}}$$

$$V =$$

32. $\nabla^2 \cdot V = \frac{-\rho_v}{\epsilon}$ $\quad V = [20x^3 + 10y^4]$

$$\nabla^2 \cdot V = \frac{\partial}{\partial x}\left(\frac{\partial V}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial V}{\partial y}\right)$$

$$= \frac{\partial}{\partial x}\left[60x^2 \cancel{+40y^3}\right] + \frac{\partial}{\partial y}\left[40y^3\right]$$

$$\Rightarrow (120x + 120y)_{(2,0)} = \frac{-\rho_v}{\epsilon_0}$$

$$\Rightarrow 240 = \frac{-\rho_v}{\epsilon_0} \Rightarrow \boxed{-240\epsilon_0 = \rho_v}$$ Ans

34. $\phi \rightarrow$ 1 dimensional fn - Laplace

II derivative of $\phi = 0$

$$\nabla^2 \phi = 0$$

$\phi \rightarrow$ linear fun

$\phi \rightarrow$ changes at const rate (charge per unit length) $\frac{\phi_2 - \phi_1}{d} = \frac{\phi_3 - \phi_2}{2d}$

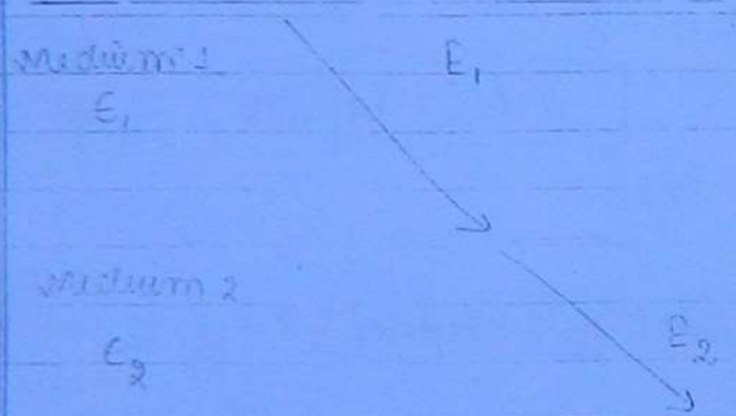
$$\frac{\phi_2 - \phi_1}{d} = \frac{\phi_3 - \phi_2}{2}$$ 84

$$2\phi_2 - 2\phi_1 = \phi_3 - \phi_2 \Rightarrow \phi_2 = \frac{2\phi_1 + \phi_3}{3}$$

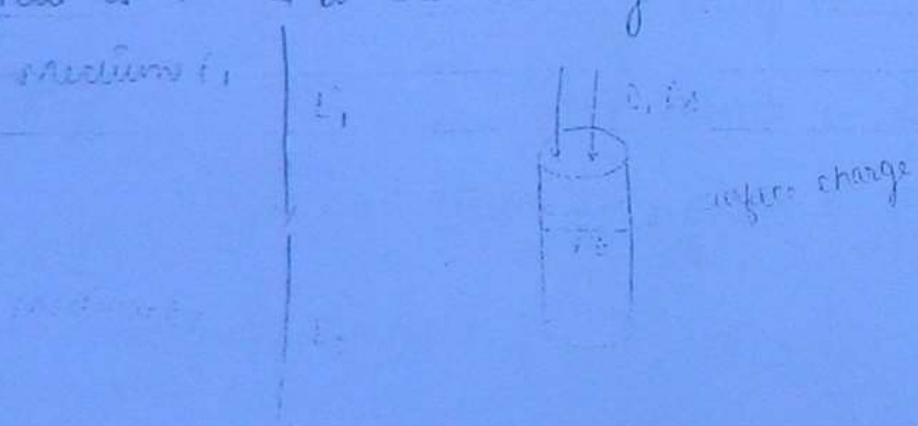
Imp — conv

**Boundary conditions (Dielectric)**

- If a field is known in one medium and the field is to be calculated in the adjacent medium we use boundary conditions.
- Boundary condition can be defined for only two types of fields for normal and tangential directions only.




case I  E field is normal to the Boundary



$$\boxed{\nabla^2 V = -\frac{\rho_v}{\epsilon}}$$ Poission's Eqn.

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If $\rho_v = 0$ charge free regions

eg. free space

$$\boxed{\nabla^2 V = 0}$$ Laplace Eqn

$\nabla^2$ – scalar laplacian operator

Note 1 Laplace and poission's are second order differential Eqn but donot have two solutions we always have a unique solution. This is called as <u>Uniqueness Theorem</u>.

Voltage at a point is unique it cannot have multiple value. This is the physical meaning of uniqueness theorem.

Laplace and Poission in Magnetic fields.

In magnetic field the same relationship exist b/w vector potencial $\bar{A}$ and current density J

$$\nabla \cdot D = \rho_v \longrightarrow \nabla \times H = J$$
$$D = \epsilon E \longrightarrow B = \mu H$$
$$E = -\nabla V \longrightarrow B = \nabla \times A$$

$\nabla \times (\nabla \times A) = \mu J$

$\nabla(\nabla \cdot A) - \nabla^2 A = \mu J$

{$(\nabla \cdot A) = 0$ as magnetic field doesnot have divergence
it has only curly nature)

$$\boxed{\nabla^2 A = -\mu J}$$

86

$$\boxed{\nabla^2 A = 0}$$ If $J = 0$ current free region

Formula for Laplace operation on scalar V
If $V = V(u, v, w)$ = scalar function of space

$$\nabla \cdot (\underbrace{\nabla V}_{\text{vector}}) = \underbrace{\nabla^2 V}_{\text{scalar}}$$

Divergence of vector gives scalar $(\nabla^2)$

$$\nabla^2 V = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u}\left(\frac{h_2 h_3}{h_1}\frac{\partial V}{\partial u}\right) + \frac{\partial}{\partial v}\left(\frac{h_3 h_1}{h_2}\frac{\partial V}{\partial v}\right) + \frac{\partial}{\partial w}\left(\frac{h_1 h_2}{h_3}\frac{\partial V}{\partial w}\right)\right]$$

$Q = \int \rho_v \, dv$ step 2.

$\nabla^2 V = -\frac{\rho_v}{\epsilon}$ step 1

V(r) only so it reduce the calculation

consider a cylinder symmetrically in both the medium. and use the $\oint D \cdot ds = Q$

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As there is no charge enclosed in the cylinder entering flux equals to the leaving flux because cylinder is consist of material and atoms has no charge inside it is electrically neutral.

so. $D_1 \Delta s = D_2 \Delta s$

$$D_1 = D_2$$

$$\epsilon E_1 = \epsilon E_2$$

If the Boundary has surface charge $\rho_s$ C/m²

$$D_2 \cdot \Delta s = D_1 \cdot \Delta s + \rho_s \cdot \Delta s$$

$$(D_2 - D_1) = \rho_s$$

Words statement of Boundary Condition

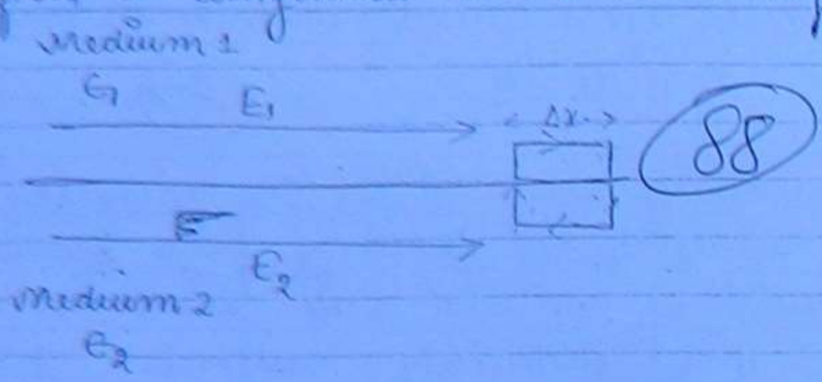
1. The normal component of flux density are continous on either sides if there is no surface charge else discontineus (not equal) (some gap) by amount equal to the surface charge density on the boundary

$$\boxed{D_{n1} = D_{n2}}$$

or. $\boxed{D_{n2} - D_{n1} = \rho_s}$

n-indicating that it is for normal

case II E field is tangential to the Boundary.



Using Maxwell $Eq^n$ (II) $\oint E \cdot dl = 0$

Take a closed line symmetrical in both the media that has $\Delta x$ length.

$$\oint E \cdot dl = 0$$

$$E_1 \Delta x - E_2 \Delta x = 0$$

$$\boxed{E_1 = E_2}$$

Statement
The tangential components of electric field intensity are always continuous.

36 W.B

$x = 0$

| $x < 0$ | $x > 0$ |
|---|---|
| $\epsilon_1 = 1.5\epsilon_0$ | $\epsilon_2 = 2.5\epsilon_0$ |
| $E_1 = 2a_x - 3a_y + a_z$ | |
| $E_{n_1} = 2a_x$ | |
| $E_{t_1} = -3a_y + a_z$ | $E_{t_2} = -3a_y + a_z$ |

$\therefore |E_{t_1} \quad E_{t_2}|$

$x =$ const surface. 89
Normal is always $a_x$
direction " " $a_x$

$\{ x-5=0$ (take gradient) $1a_x$

$D_{n_1} = \epsilon E$
$= 1.5\epsilon_0 \, 2a_x$
$= 3\epsilon_0 a_x$

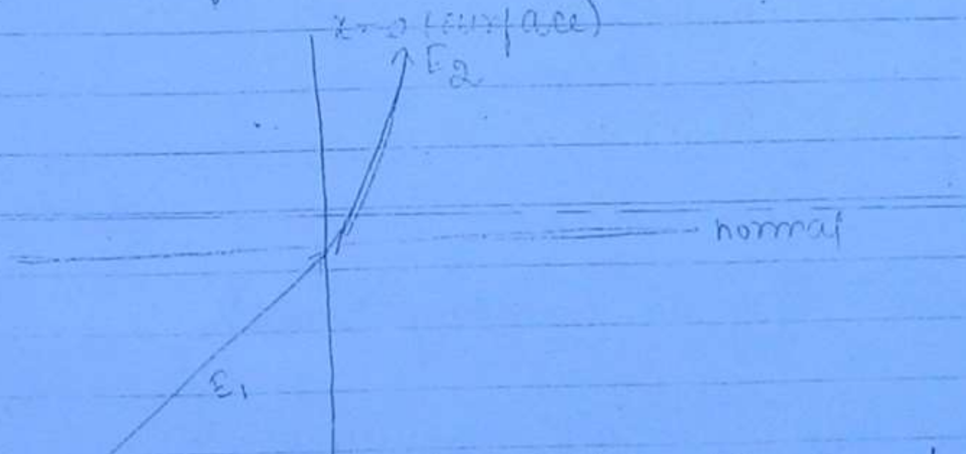
$\longrightarrow D_{n_2} = 3\epsilon_0 a_x$

$E_{n_2} = \dfrac{D_{n_2}}{\epsilon} = \dfrac{3\epsilon_0 a_x}{2.5\epsilon_0} = 1.2a_x$

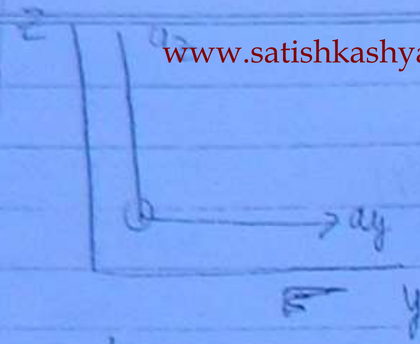
$E_2 = 1.2a_x - 3a_y + a_z$ Ans

$D_1 = \epsilon_0(3a_x - 4.5a_y + 1.5a_z)$ $\quad D_2 = \epsilon_0(3a_x - 7.5a_y + 2.5a_z)$

Note:




As seen in the diagram the field in the second medium shifting away from the normal which can be understood the decrease normal components in $E_2$.
It can also be understood that the field is shifting towards the boundary increase tangential component as seen in $D_2$




yz plane

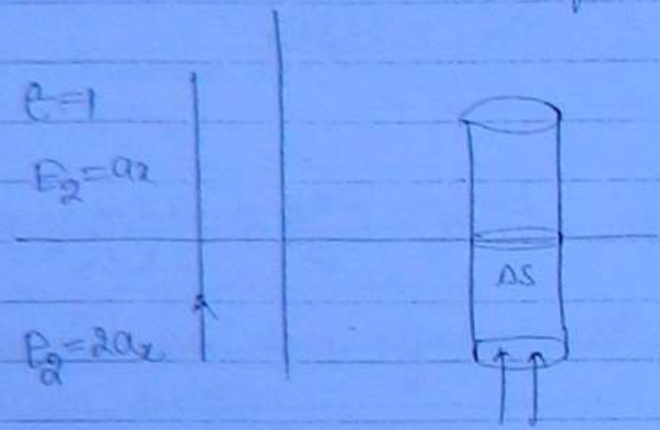
x = 0 plane

$a_x \to$ normal

$a_y, a_z \to$ tangential

90

97 W.B




$2 \cdot 2\epsilon_0 \cdot \Delta s$ = Entering

$1 \cdot 1\epsilon_0 \cdot \Delta s$ = leaving

$\rho_s \cdot \Delta s = -3\epsilon_0 \Delta s$

$\rho_s = -3\epsilon_0$

sink at the boundary

Extension : Magnetic Boundary conditions

1. $B_{n_1} = B_{n_2}$ Apply Maxwell III Eq$^n$

Always $\oint B \cdot ds = 0$

2. $H_{t_1} = H_{t_2}$ Apply Maxwell IV Eq$^n$

not always $\oint H \cdot dl = I$

$H_{t_2} - H_{t_1}$ $I$

$\bar{K}$ = surface current density Amp/m

Note: D and $\rho_s$ have same units $C/m^2$; $\bar{H}$ and $\bar{K}$ have the same units. A/m

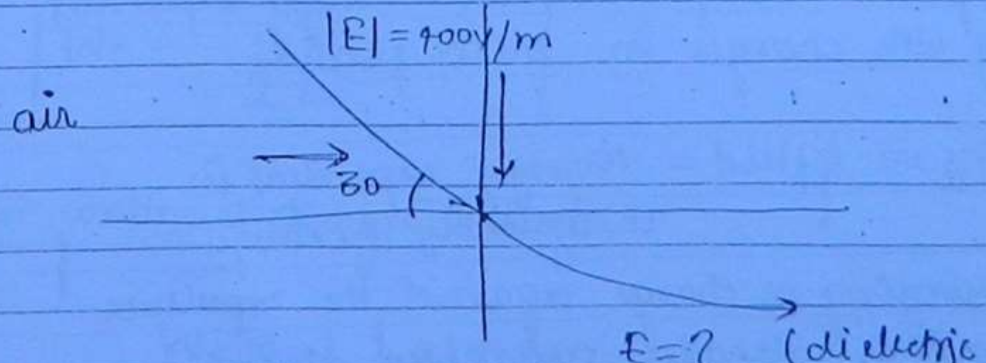
38 W.B

$z=0$

| $z<0$ | $z>0$ |
|---|---|
| $\mu_{r1} = 2$ | $\mu_{R2} = 1$ |
| $\bar{B}_1 = 1.2\bar{a}_x + 0.8a_y + 0.4a_z$ | $B_2$ |
| $B_{n1} = 0.4a_z$ <br> $\{\because z=0\}$ | $H_2 = ?$ <br> $B_{n2} = 0.4a_z$ |
| $B_{t1} = 1.2a_x + 0.8a_y$ | |
| $H_{t1} = \frac{B_{t1}}{\mu_{R1}}$ | $H_{t2} = \frac{1}{\mu_0}(0.6a_x + 0.4a_y)$ |
| $= \frac{1}{2\mu_0}(1.2a_x + 0.8a_y)$ | $H_{n2} = \frac{1}{\mu_0}(0.4a_z)$ |
| $= \frac{1}{\mu_0}(0.6a_x + 0.4a_y)$ | |

$\left(H = \frac{B}{\mu}\right)$

39 W.B




cos is tangential as it is adjacent

sin is taken as normal

$E_{t_1} = 400\cos 30^\circ = 400 \times \frac{\sqrt{3}}{2} = 200\sqrt{3} = E_{t_2}$

$E_{n_1} = 400 \sin 30^\circ = 200$

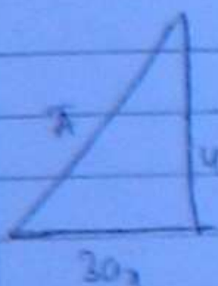
92

$D_{n_1} = 200\epsilon_0 = D_{n_2}$

$E_{n_2} = \frac{200\epsilon_0}{\epsilon_r}$ $\{\epsilon_r = 20\epsilon_0\}$

$E_{n_2} = \frac{200\epsilon_0}{20\epsilon_0} = 10$

(magnitude) $E_2 = \sqrt{(200\sqrt{3})^2 + (10)^2} \approx 200\sqrt{3}$




$|A| = \sqrt{3^2 + 4^2}$

Energy Density in a electric field.

Consider a system of n discrete point charges having a electric field E and the total energy $W_E$.
The total energy can be equal to the energy expanded in assembling the charges in their positions

Total energy in E field = Energy expanded in assembly the charges

It involves bringing a charge against the repulsive forces of the quanta already assembled charges.

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$$W_E \begin{cases} W_1 = 0 \\ W_2 = -Q_2 V_{21} \\ W_3 = -Q_3 V_{31} - Q_3 V_{32} \\ W_4 = -Q_4 V_{41} - Q_4 V_{42} - Q_4 V_{43} \\ \vdots \\ W_n = -Q_n V_{n1} - Q_n V_{n2} \cdots - Q_n V_{n(n-1)} \end{cases}$$

$V_{21}$ = potential of 2 due to charge 1

The total energy $W_E$ is the sum of all the energy

The subscript can be interchange without changing the meaning Hence

$Q_2 \cdot V_{21} = Q_2 \cdot \frac{Q_1}{4\pi\varepsilon_0 r_{21}}$

$Q_1 \cdot V_{12} = \frac{Q_1 \cdot Q_2}{4\pi\varepsilon_0 r_{12}}$

$$W_E \begin{cases} W_1 = 0 \\ W_2 = -Q_1 V_{12} \\ W_3 = -Q_1 V_{13} - Q_2 V_{23} \\ W_4 = -Q_1 V_{14} - Q_2 V_{24} - Q_3 V_{34} \\ \vdots \\ W_n = -Q_1 V_{1n} \cdots\cdots - Q_{n-1} V_{n-1n} \end{cases}$$

$$2W_E = -Q_1 V_1 - Q_2 V_2 - Q_3 V_3 \cdots\cdots Q_n V_n$$

Total Energy $\boxed{W_E = -\frac{1}{2} \sum_{i=1}^{n} Q_i V_i}$

for continous charges & E fields

$$W_E = -\frac{1}{2} \int \rho_v V \, dv \qquad \{ Q = \rho_v dv \}$$

$$= -\frac{1}{2} \int (\nabla \cdot D) V \, dv$$

$$W_E = \frac{1}{2}\int D(\nabla V)\,dv$$

$$W_E = \int \frac{1}{2}(D\cdot E)\,dv$$

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$$\frac{dW_E}{dv} = \frac{1}{2} D\cdot E$$

$$\boxed{\frac{dW_E}{dv} = \frac{1}{2}\epsilon E^2}$$

$\frac{dW_E}{dv}$ = strength of the energy at every point in the E field.

$$\frac{dW_E}{dv} = \frac{1}{2}\epsilon E^2$$

Note 1. $D\cdot E = \frac{\text{Joule}}{m^3} = \frac{\text{Newton} \times \text{meter}}{m^3} = \frac{\text{Newton}}{m^2}$

$\frac{N}{C}\cdot\frac{C}{m^2} \Rightarrow \frac{\text{Newton}}{m^2}$ = Pressure.

2\. $W_E = \frac{1}{2}CV^2$ is similar to $\frac{1}{2}\epsilon E^2$

Extension:- $\frac{dW_H}{dv} = \frac{1}{2}\mu H^2$

$= \frac{1}{2} B\cdot H$

It is similar to $\frac{1}{2}LI^2$

Ohm's law and Continuity Eqn.

$I = \frac{Q}{t} = \frac{Ne}{\frac{l}{v}} = \frac{NevA}{lA}$

95

$I = neAv$

$$I = \frac{dQ}{dt} = \frac{d}{dt}\int \rho_v\, dv = \int \frac{\partial \rho_v}{\partial t}\, dv$$

$\rho$ = continious volume charge movement

the current can be considered as the current crossing the closed surface formed by the volume charge. Hence in this case

$$I = \oint J \cdot ds$$

Note: Generally $\oint J \cdot ds = 0$ (loop integral) means law of conservation of charge because entering charge ~~is equal to~~ the leaving charge.

Apply divergence theorem

$$\oint J \cdot ds = \int (\nabla \cdot J)\, dv$$

Hence by comparison

$$\boxed{\nabla \cdot J = \frac{\partial \rho_v}{\partial t}}$$ continuity Eqn.

This is called as continuity Eqn which is the definition of current in field theory.

- outflow of current depends on movement of volume charges density

$\frac{\partial J}{\partial x}$

Apply for flow.

$$\nabla \cdot J = \frac{\partial \rho_v}{\partial t}$$

$$\nabla \cdot J = \frac{\partial J}{\partial x} = \frac{\partial \rho_v}{\partial t}$$

$$\partial J = \partial \rho_v \cdot \frac{\partial x}{\partial t} \qquad \left\{ \because \frac{\partial x}{\partial t} = v_d \text{ drift velocity} \right\}$$

$$\boxed{J = \rho_v v_d}$$

$v_d \propto E$

$v_d = \mu E$

$\mu$ → mobility (ability to move)

$J = \rho_v \mu E$ —

In point form

$\boxed{J = \sigma E}$ ohm's law & continuity Eq'n in

Here $\boxed{\sigma = \rho_v \mu}$ conductivity

Hence conductivity is ability to allow current in to the medium. It is the product of availibility and ability to move.

case 1 $\sigma \to \infty$ very good conductor

$$\frac{J}{\sigma} = E$$

$$\frac{J}{\infty} = E$$

$$E = 0$$

- E field cannot exist inside a good conductor
- only flow exist but not accumulation of charge - Hence only current exist but not E field.

$$V = \int D \cdot dl = const$$

97

Every conductor is an equipotential surface
Potential diff. b/w any 2 points is zero.

For conductor surface as $\sigma = \infty$ on the surface

1. E along the surface = $E_{tan} = 0 = E_t$
(Electric field never be parallel) to the conductor surface

2. $D_{normal}$ to the surface = $D_{normal} \neq 0$

$$\boxed{D_n = \rho_s}$$

case 2. $\sigma = 0$ very good dielectric

$J = \sigma E \Rightarrow J = 0 \times E$

$J = 0$

- E field can exist inside a good dielectric
- not flow exist but accumulation exists Hence only E field exist

$$\nabla \cdot J = \frac{\partial \rho_v}{\partial t}$$

$$J = \sigma E$$

$$\nabla \cdot (\sigma E) = \frac{\partial \rho_v}{\partial t}$$

$$\sigma\ \nabla\cdot\left(\frac{D}{\epsilon}\right) = \frac{\partial \rho_v}{\partial t}$$

$$\boxed{\frac{\partial \rho_v}{\partial t} = \frac{\sigma}{\epsilon}\rho_v}$$

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The solution of the $Eq^n$ is - an exponentially decaying function( derivative back the same function is exponential) hence

$$\rho_v(t) = \rho_{v0}\cdot e^{-\frac{t\sigma}{\epsilon}}$$

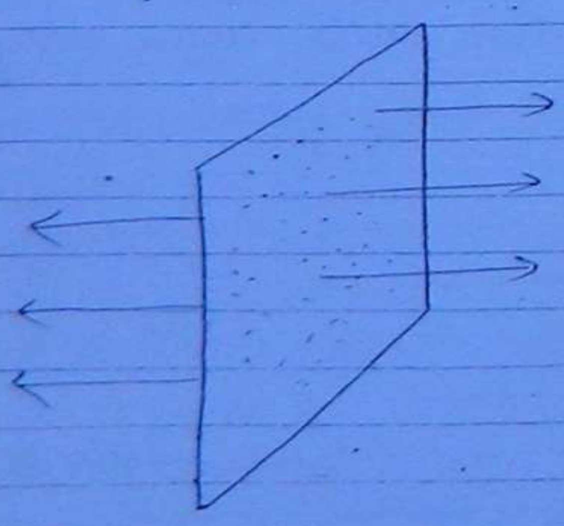
The $Eq^n$ shows the any charge placed in any medium exponentially spread into ~~the~~ medium with the time constt depending on $(\epsilon/\sigma)$

This ~~so~~ time constt is called as relaxation time or Average spreading time

$$\boxed{\frac{\epsilon}{\sigma} = \text{Relaxation time}}$$

If $\sigma=\infty$, good conductor $\frac{\epsilon}{\sigma}=0$

Sheet charges of $\rho_s$ $C/m^2$ & uniform fields



$$D = \frac{\rho_s}{2}$$

$E\left(\frac{1}{r^2}\right)$

$E\left(\frac{1}{\rho}\right)$

$E$(uniform)

$E(r)$

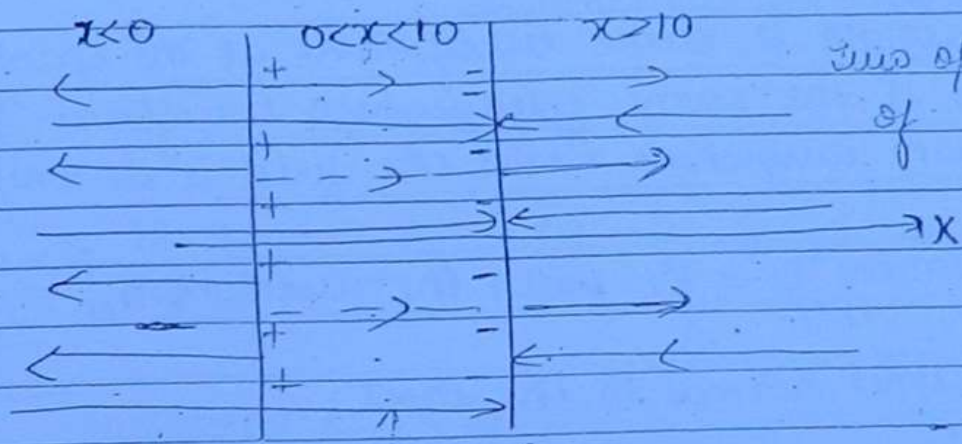
$$I\,\bar{dl} = \bar{K}\,ds = \bar{J}\,dv$$

$$H = \frac{K}{2} \times a_N$$

99

$$D = \frac{\rho_s}{2}\,a_N$$

7/11




two opposite field of equal strength cancel each other

$+\rho_s$ uniform $-\rho_s$

$$E_1 = \left|\frac{\rho_s}{2\epsilon_0}\right|(-a_x)$$

$$E_2 = \left|\frac{\rho_s}{2\epsilon_0}\right|(a_x)$$

left side $E = 0$

Right side two fields because of two sheets.

$$E_1 = \left|\frac{\rho_s}{2\epsilon_0}\right| a_x$$

$$E_2 = \left|\frac{\rho_s}{2\epsilon_0}\right|(-a_x)$$

$E_0 = 0$ on right also.

field in b/w.

$$E = \frac{\rho_s}{\epsilon_0}\,a_x$$

- consider an infinite sheet of charge density $\rho_s$ C/m$^2$ the field can be found Normal to the sheet and only to the right or left of the sheet.

Tcb

- The sheet is being infinitely large the lines are not divergent but are parallel to themselves.

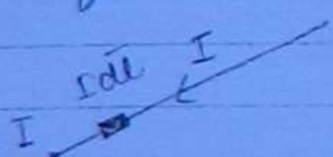
Laser lines are best example

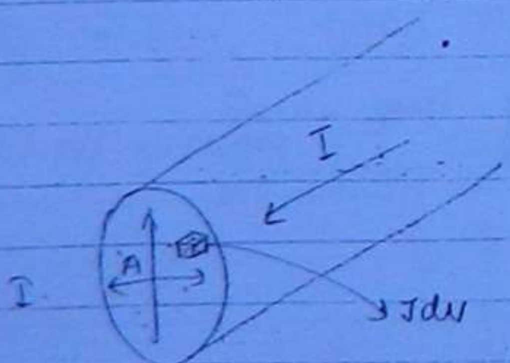
- Hence the strength is same everywhere as the spacing b/w the lines is the same everywhere the flux density is same everywhere Hence the field is a uniform field.

Hence the density : $D = \frac{\rho_s}{2}$ with the direction $a_N$

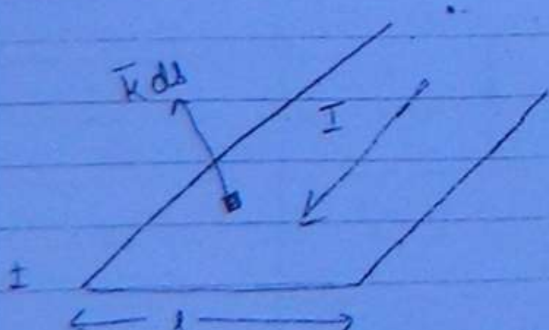
where $a_N$ = unit normal to the sheet.

~~sheet of currents $\bar{K}$ A/m~~




$$J = \frac{I}{A} = \frac{\text{Ampere}}{m^2}$$




$$K = \frac{I}{l} = \frac{\text{Amp}}{\text{meter}}$$

Two infinite sheets of charge density equivalent opposite nature have field only b/w the sheets and cancel out everywhere else

(101)

W.B 3 infinite sheets — $18\,nC/m^2 \pm a_x$ at $x=4$

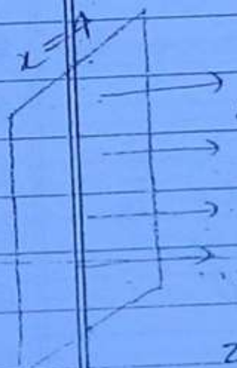
$9\,nC/m^2 \pm a_y$ at $y=3$

$-24\,nC/m^2 \pm a_z$ at $z=0$

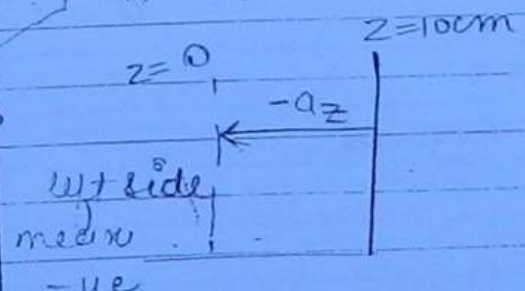
$$E = \frac{D}{\epsilon} = \frac{\rho_s}{2\epsilon} a_N$$

$$E = E_1 a_x + E_2 a_y + E_3 a_z$$

$$E_1 = -\frac{18}{2\epsilon_0} a_x, \quad E_2 = \frac{+9}{2\epsilon_0} a_y, \quad E_3 = \frac{-24}{2\epsilon_0} a_z$$

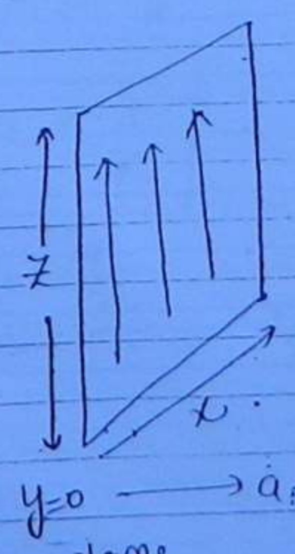



18 W.B




$$E = \frac{\rho_s}{2\epsilon} = \frac{20\times10^{-9}}{2\times\frac{1}{36\pi\times10^{9}}} = -360\pi\, a_z \text{ V/m}$$

19 W.B




$\bar{K} = 30\hat{k} = 30a_z$ mA/m

H at $(1, 20, -2) = ?$

$$H = \frac{\bar{K}}{2} \times a_N$$

$$= \frac{30 \times a_N}{2}$$

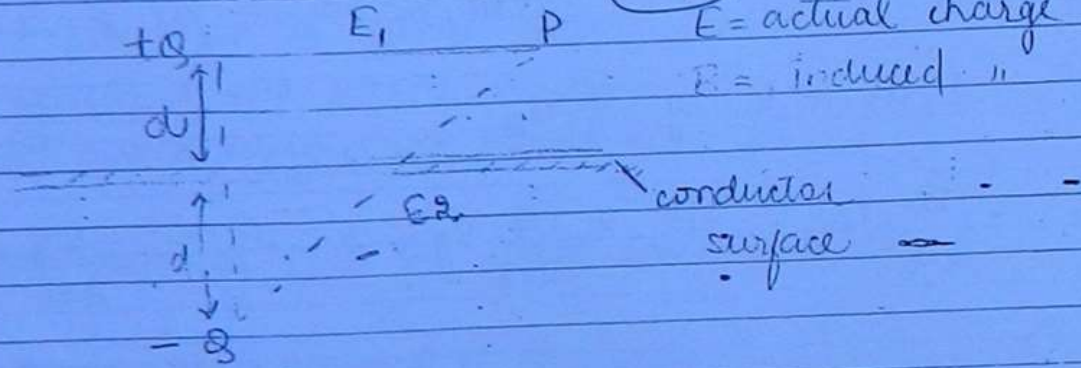
current direction

$$= 15(a_z \times a_y)$$

$$= \cancel{-15}\ -15\, a_x$$

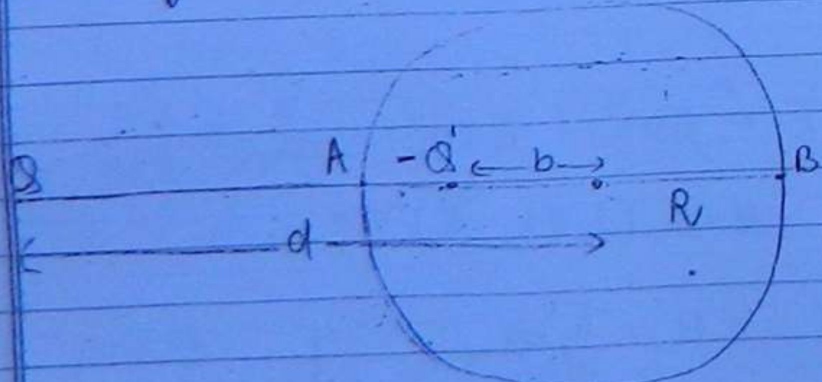
When charge is brought to the near a conductor surface. The charges inside a conductor are realigned due to the field of the charge that is the charges are displaced. Hence the final field anywhere is sum of the field of the actual charge and the field due to this induced charge.

(102)




This induced charge is represented with a image charge following the dymaics of a mirror. Hence for a flat conductor the charges are equal the image has opposite sign.

9 w.r.t Image should always be with coaxial.




Let induced charge = image charge = $-Q'$

Let the image be at 'b' distance

Every conductor is a equipotential surface

$$V_A = \frac{Q}{4\pi\varepsilon_0(d-R)} - \frac{Q'}{4\pi\varepsilon_0(R-b)} = 0$$

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Let find potential at B also so that eqn become simple,

$$V_B = \frac{Q}{4\pi\varepsilon_0(d+R)} - \frac{Q'}{4\pi\varepsilon_0(R+b)} = 0$$

Potential at A and B are zero because the sphere is given a grounded sphere.

$$\frac{Q}{Q'} = \frac{d-R}{R-b} = \frac{d+R}{R+b}$$

29 W.B

$$W_E = \frac{1}{2}\sum_{i=1}^{n} Q_i V_i$$

$$W_E = \frac{1}{2}\sum Q_i \frac{Q}{4\pi\varepsilon_0 r}$$

$$\boxed{W_E \propto \frac{1}{r}}$$

$$\frac{W_1}{W_2} = \frac{1}{2}$$

other method

Q — 1/2 — Q — 1/2 — Q

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$W_1 = 0$ $\quad W_2 = \frac{Q\,Q}{4\pi\varepsilon_0\left(\frac{1}{2}\right)}$ $\quad W_3 = \frac{Q\cdot Q}{4\pi\varepsilon_0(1)} + \frac{Q\,Q}{4\pi\varepsilon_0\left(\frac{1}{2}\right)}$

$W_{E_1} = \frac{5Q^2}{4\pi\varepsilon_0}$ $\quad W_{E_2} = \frac{5Q^2}{8\pi\varepsilon_0}$

28 W.B

$1\,\mu C$ at $(-2,1,5)$ $\quad 4\,\mu C$ at $(1,3,-1)$

$W_1 = 0$ $\quad W_2 = \frac{Q_1 \cdot Q_2}{4\pi\varepsilon_0 r}$

$W_2 = \frac{1\times 4\times 10^{-12}}{4\pi\varepsilon_0\sqrt{3^2+2^2+6^2}}$ $\quad W_E = W_1 + W_2 = 5.14\,mJ$

We can also use $W_E = \frac{1}{2}\sum_{i=1}^{2} Q_i V_i$

40 W.B

$D_n = \rho_s$ as the surface charge density is

$\rho_s = 2\sqrt{(1)^2+(\sqrt{3})^2}$

$2\sqrt{4}$

$\rho_s = 2\times 2$

$\rho_s = 4$ Ans

$\oint D \cdot ds = 0$ $\oint B \cdot ds = 0$

i) $Q = 0$

ii) $+Q, -Q$.

105

Magnetic dipole in magnetic field.

- Whenever a current flows in a closed line it is treated as Magnetic dipole.
- finite Area.

Dipole moment $m = I \times area = I \cdot A$

Electric dipole in Electric fields

- Whenever two charges of equal and opposite are separated by a finite distance

Electric - dipole

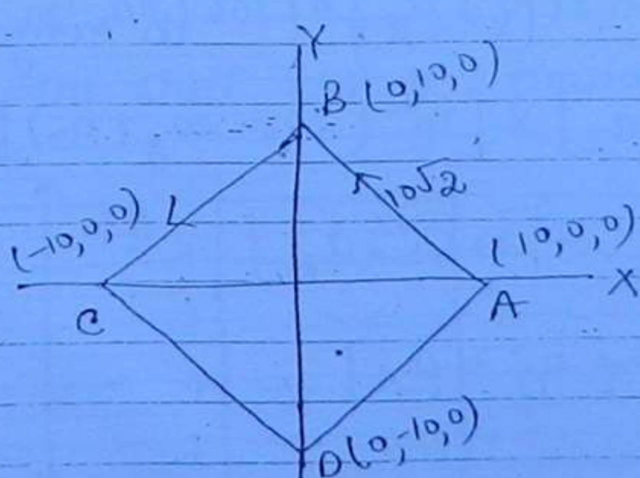
finite distance

Dipole moment $p = Q d$

The importance of dipole moment is it inside the torque or the moment of the dipole in an external field such that magnetic torque is equal $= T = M \times B$

Electric torque is equal $T = P \times E$

6 W.B




$M = I \times Area$

$= 0.01 \times (10\sqrt{2})^2$

$= 2$

- M's direction is the direction of the area surface of the loop

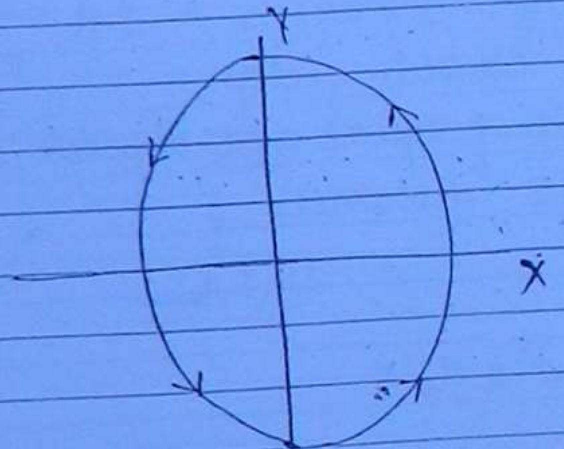
Surface $z = 0$, xy plane

Direction $= \pm a_z$

.106

current is anticlockwise as per RHS thumb direction
M direction (+ve $+a_z$) Ans $+2a_z$

45 W.B.



Magnetic dipole moment (Torque) $M = I \times A$

$= I \times (\pi r^2)$

$= (0.1) \times \pi \times (10^{-3})^2 \, a_z$

$$\text{Torque} = M \times B$$
$$= \left[(0.1) \times \pi \times (10^{-3})^2 a_z\right] \times \left[10^{-5}(2a_x - 2a_y + a_z)\right]$$
$$= \left[10^{-7}\pi \, a_z\right] \times \left[10^{-5}(2a_x - 2a_y + a_z)\right]$$
$$= \left[2 \times 10^{-12}\pi a_y + 2 \times 10^{-12}\pi a_x\right]$$
$$= 2 \times 10^{-12}\pi\left[a_y + a_x\right]$$

Capacitors & Inductors:



Capacitors -

Ability to hold E field confining it into a small region

$$C = \text{Farads} = \frac{\oint D\cdot ds}{\int E\cdot dl} = \frac{\epsilon \oint E\cdot ds}{\int E\cdot dl} = \frac{Q}{V}$$

- It is always measure in terms of charge utilize and the potential develop by this charge. because potential is accumulation and hence the measure of holding ability.

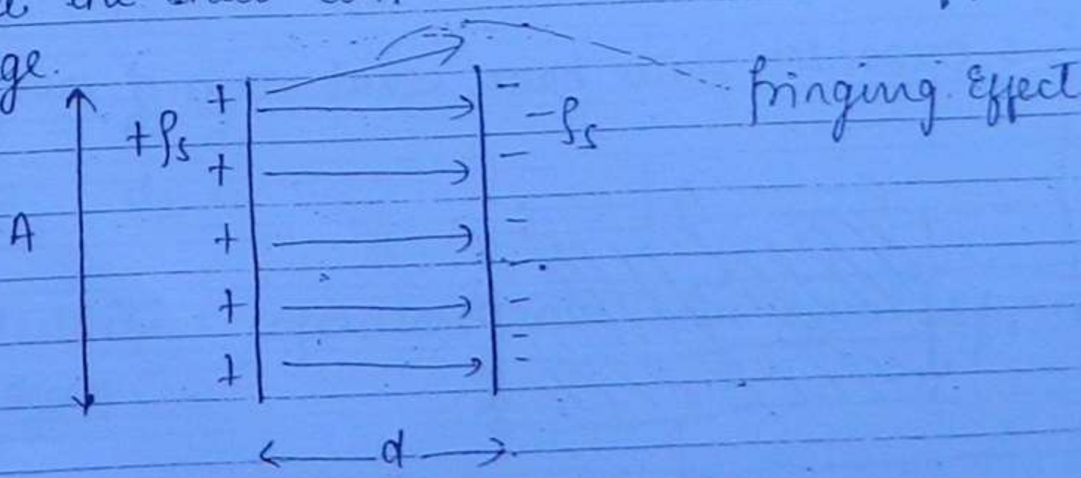
- The best examples of capacitor are geometry involving

  eg. Parallel plates
  concentric cylinders
  concentric spheres.

Parallel Plate capacitors.

- Two sheet charge of Area A
- seperation d $(A >> d)$

hence the sheet can be considered an infinite sheet of charge.




$$C = \frac{Q}{V} \qquad Q = \rho_s A$$

$$V = Ed = \frac{\rho_s}{\epsilon} d$$

so

$$\boxed{C = \frac{\epsilon A}{d}}$$

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$$C = \frac{\epsilon \oint E \cdot ds}{\int E \cdot dl}$$

$E =$ const. for uniform field.

$$C = \frac{\epsilon E A}{E \cdot d} = \frac{\epsilon A}{d}$$

Total energy held by the field

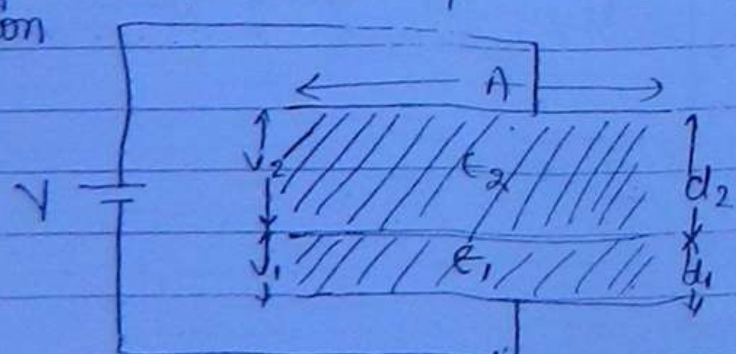
$$W_E = \left(\frac{1}{2} \epsilon E^2\right)(Ad)$$

$$= \frac{1}{2} \frac{\epsilon A}{d} (Ed)^2$$

$$W_E = \frac{1}{2} C V^2$$

Multiple dielectrics in parallel plate capacitor

(i) Dielectric & capacitor plates have equal area of cross section



Page

$\epsilon_1$ and $\epsilon_2 \longrightarrow$ series

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$C_1 = \frac{\epsilon_1 A}{d_1}$ $\quad C_2 = \frac{\epsilon_2 A}{d_2}$

voltage divides b/w the dielectrics.

$$\boxed{V_1 + V_2 = V}$$

$$C_{eq} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{(in series)}$$

$$= \frac{C_1 + C_2}{C_1 C_2}$$

$$C_{eq} \Rightarrow \frac{C_1 C_2}{C_1 + C_2}$$

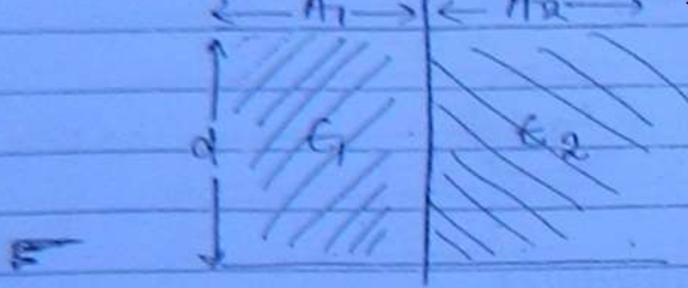
$$C_{eq} = \frac{\frac{\epsilon_1 A}{d_1} \cdot \frac{\epsilon_2 A}{d_2}}{\frac{\epsilon_1 A}{d_1} + \frac{\epsilon_2 A}{d_2}}$$

$C_1 V_1 = C_2 V_2$ (as charge is common and voltage divides in series)

$$\frac{V_1}{V_2} = \frac{C_2}{C_1}$$

$$\frac{V_1}{V_2} = \frac{\epsilon_2}{d_2} \cdot \frac{d_1}{\epsilon_1}$$

re(Γi) Dielectrics & capacitor plates have equal seperation / thickness

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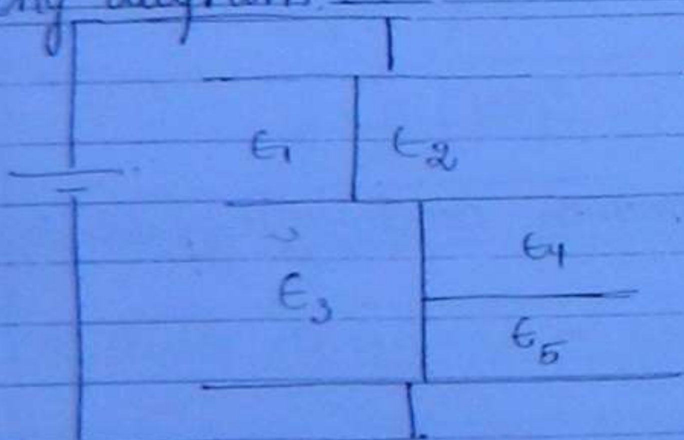
$C_1\ C_2 \rightarrow$ are in shunt parallel.

voltage applied to plates = voltage b/w the dielectrics

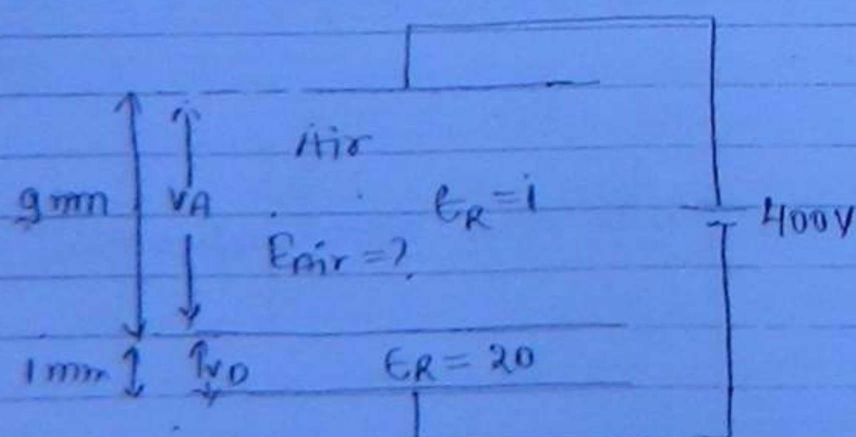
$$C_{eq} = C_1 + C_2$$
$$= \frac{\epsilon_1 A_1}{d} + \frac{\epsilon_2 A_2}{d}$$

Identify the equivalent capacitance in the following diagram.




$C_{eq} = (C_1 \text{ parallel } C_2)$ series $(C_3 \text{ parallel } (C_4 \text{ series } C_5))$

18 W.B



$V_A + V_D = 400$

$\frac{V_A}{V_D} = \frac{20}{1} \times \frac{9}{1}$

$\frac{V_A}{V_D} = 180 \quad V_A = 180 V_D$

$180 V_D + V_D = 400$

$181 V_D = 400$

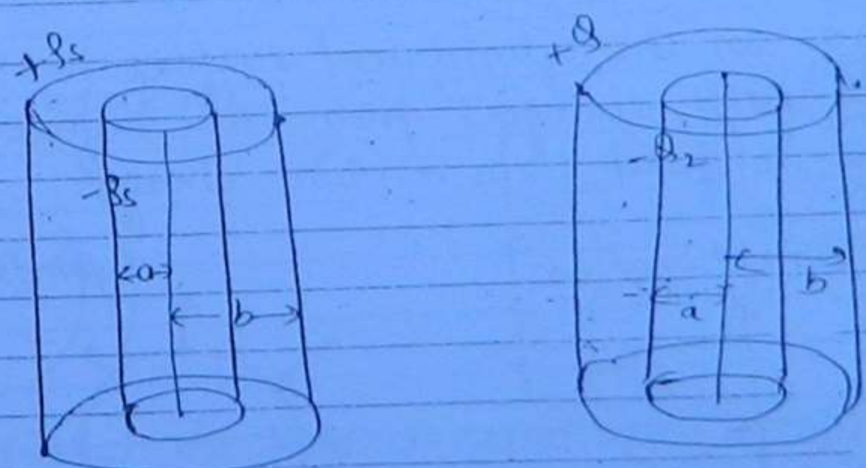
$V_D = 2.21$

$V_A + 2.21 = 400$

$V_A = 397.8 V$

$E = \frac{V_A}{d} = \frac{397.8 V}{9 mm} \approx 44 \frac{KV}{m}$

concentric cylinders:




(i) If two concentric cylinders have equal and opposite charge densities that means the charges are unequal on their surfaces. Hence a field or flux leaving exist outside the cylinder also.

ii If two concentric cylinders have equal and opposite charge the densities are unequally spread but net flux outside of cylinder is zero. Hence the field is confined b/w the cylinders

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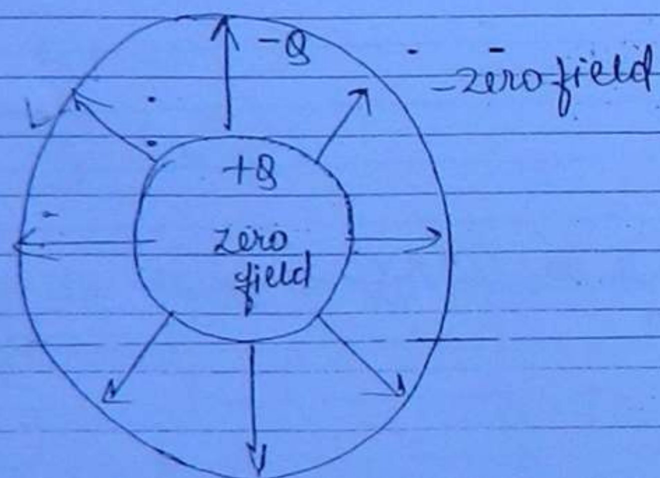
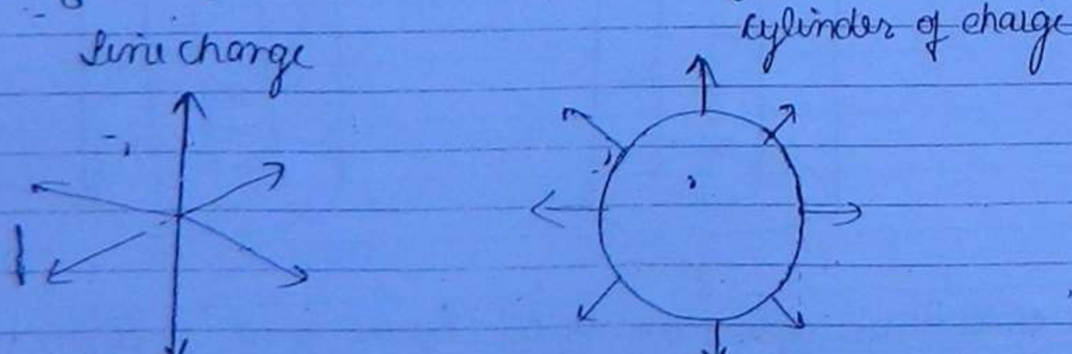
$$\rho_{s_1} = \frac{Q}{2\pi bh}$$

$$\rho_{s_2} = \frac{-Q}{2\pi ah}$$




A charge on the cylindrical surface has the same radial field divergent from the surface similar to that of a field from a line charge.




$E \propto \frac{1}{\rho}$ $E \propto \frac{1}{\rho}$

Inductors:

- Ability to hold magnetic field (H) confined into a small region is called as Inductance.

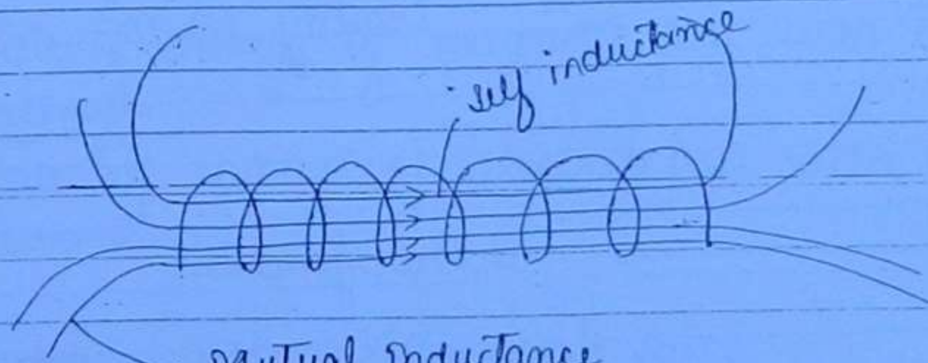
(IB)

- Henry $L = \dfrac{\int B \cdot ds}{\oint H \cdot dl} = \dfrac{\mu \int H \cdot ds}{\oint H \cdot dl} = \dfrac{\psi_m}{I} = \dfrac{\text{flux}}{\text{current}}$

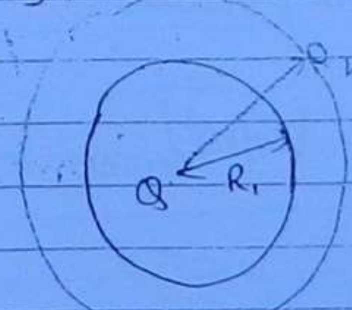
Inductance is a measure of the confined flux and the current utilize for this confinement.

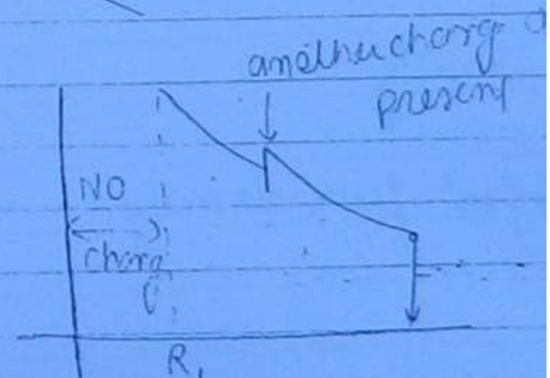
- Geometries :- Solenoids
  - concentric cylinders
  - Toroids.

} All the have some or other circular dielectric




47 W.B




- The graph shows that the field is zero upto $R_1$ distance means there is no charge enclosed upto $R_1$ so that charge is Hollow sphere of charge $Q_1$ and radius $R_1$

51. $J \cdot A = \dfrac{\text{Amp}}{m^2} \times \dfrac{\text{weber}}{m}$

$\dfrac{\text{weber}}{m} = \dfrac{\text{Joule}}{\text{Amp-m}}$

$$\frac{weber}{m} = \frac{W}{I\,dl}$$

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$$J \cdot A = \frac{Amp}{m^2} \times \frac{weber}{m} = \frac{Amp}{m^2} \times \frac{Joule}{Amp\text{-}m} = \frac{Joule}{m^3}$$

Summary 1

Scalar function $\xrightarrow{\nabla\text{- gradient}}$ Vector
Intensity per m

eg. voltage V $\xrightarrow{\nabla \cdot V}$ E $\frac{volts}{m}$

vector function $\xrightarrow{\nabla \times \text{curl}}$ vector function
Intensity per m Density per $m^3$

eg. Intensity

Vector function $\longrightarrow$ Scalar function

Density $\longrightarrow$ (per $m^3$) volume

$c/m^2$ flux density $\xrightarrow{\nabla \cdot D}$ $\rho_v$ $(c/m^3)$

Summary 2. (115)

Maxwell's Equations: Not maxwell's Eqn.

| Integral form | Point form | open integrals. |
| --- | --- | --- |
| 1. $\oint D \cdot ds = Q$ | 1 $\nabla \cdot D = \rho_v$ | 1. $\int D \cdot ds = \psi_e$ = colomb |
| 2 $\oint E \cdot dl = 0$ ✓ | 2. $\nabla \times E = 0$ | 2. $-\int E \cdot dl = V$ = volts |
| 3. $\oint B \cdot ds = 0$ | 3 $\nabla \cdot B = 0$ | 3. $\int B \cdot ds = \psi_m$ = weber |
| 4. $\oint H \cdot dl = I$ ✓ | 4. $\nabla \times H = J$ | 4. $\int H \cdot dl = V_m$ = amps |

Note: - Rate of change of magnetic flux with time is EMF voltage.

Rate of change of electric flux with the time is current.

$\frac{\text{weber}}{\text{sec}}$ = volts [Faraday's law]

$\frac{\text{colomb}}{\text{sec}}$ = amps chapter-II

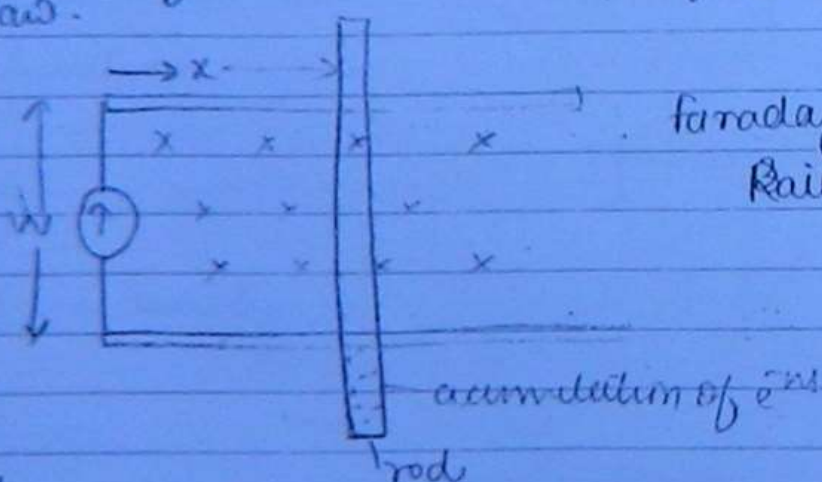
## Time varying fields & Maxwell's Eqn (116)

- Maxwell's $I^{st}$ and $III^{rd}$ Eqn i.e surface integrals are unmodified and are consistent for time varying fields also. However the line integral are modified.

Faraday's Law and Maxwell's II Eqn

$$\oint E\cdot dl = 0 \quad \& \quad \nabla\times E = 0$$

Faraday's law states that the EMF is produced when in a closed conductor and the magnetic flux through the area of the conductor changes with the time. Produced voltage opposes the changing flux. ~~this is~~ Lenz law.




Faraday's sliding Rail experiment.

Hence $\boxed{V = \oint E\cdot dl = -\frac{d\psi_m}{dt}}$

$$V = -\frac{d}{dt}(B \cdot A) \qquad (F_y = q(V_x \times B_z))$$

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$$= -BW\frac{dx}{dt}.$$

$$V = -B \cdot W \cdot V$$

$$V = \oint E \cdot dl = \frac{-d}{dt}\int B \cdot dl$$

$$\int(\nabla \times E)\,ds = \oint E \cdot dl = \int -\frac{\partial B}{\partial t} \cdot dl.$$

$$\oint E \cdot dl = \int -\frac{\partial B}{\partial t} \cdot ds \quad \& \quad \nabla \times E = -\frac{\partial B}{\partial t}$$

$$\boxed{\nabla \times E = -\frac{\partial B}{\partial t}}$$ Hence modified

Note: Potential at a point is unique at a time but it can change at various time. Hence the modification

Jai Durga Mata

Date ____ Page ____

## Inconsistency of Ampere's law — Maxwell's IV Eqn.

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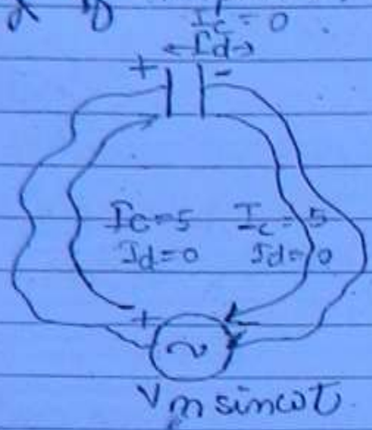


$$I = \frac{c\,dv}{dt}$$

$$I = C \cdot w \cdot V_m \sin(\omega t + 90^\circ)$$

$$V_m \sin\omega t = \left(\frac{1}{j\omega c}\right) I \qquad \left\{ e^{j90} = j \right\}$$

$$V = IX.$$

It is clear from the eqn that a current flows in the wire and on the place of the capacitor but there is no current b/w capacitor plates hence ampere's law fails to explain the closed nature of current in the ckt

Maxwell added a term $I_d$ or $J_d$ which is said to flow b/w the capacitor plates making the ckt closed

Applying continuity Eqn.

$$\nabla \cdot J_d = \frac{\partial \rho_v}{\partial t}$$

$$\nabla \cdot J_d = \frac{\partial}{\partial t}(\nabla \cdot D)$$

$$\nabla \cdot J_d = \nabla \cdot \frac{\partial D}{\partial t}$$

$\oint H \cdot dl$

$\nabla \times H = J_c + J_d$

Date
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$I_d = \int \frac{\partial D}{\partial t} \cdot dl$ $J_d = \frac{\partial D}{\partial t}$ 119

The term $J_d$ is called as displacement current density and it accepted as a current format but it is not due to a moving electron it is due to a time varying electric flux line.

Physical interpretation of flow in a capacitor.

When a time varying voltage is given to the capacitor the plates are alternately charge and discharge in accordance to the polarity changes. This is itself a continuous process and hence the wires always have a $J_c$ or $I_c$

Between the plates of the capacitor there is an electric flux changing due to charge and discharge of the plates. This is a form of current this is $J_d$ or $I_d$.

Mathematically

$$|J_c| = |J_d|$$

In this example

Summary:

1. $\nabla \times H = \frac{\partial D}{\partial t} + J_c$

$= \epsilon \frac{\partial E}{\partial t} + \sigma E$ (Time varying electric field

2. $\nabla \times E = -\frac{\partial B}{\partial t} = -\frac{\mu \partial H}{\partial t}$

Time varying electric field produces a orthogonal space varying magnetic field and vice versa

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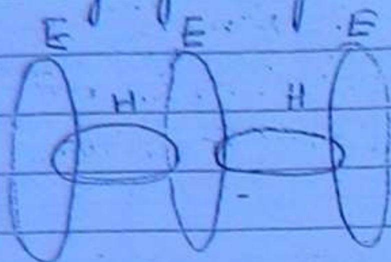
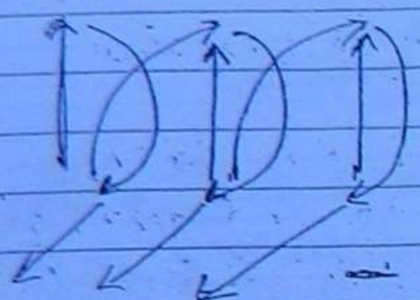
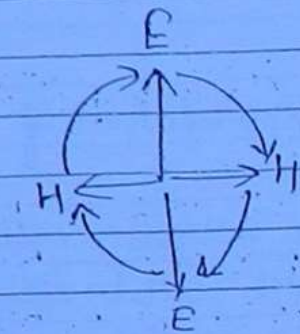


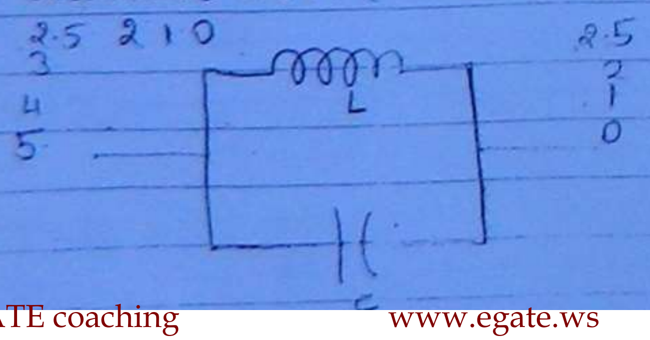



Electric field or accumulation is a form of energy which gives rise to flow or magnetic field which is also a form of energy. Thus to support each other and hence the energy flows continously like a wave.

For this propagation of energy in E and H format $\sigma$, $\epsilon$, and $\mu$ called as material constt. or permitting abilities are very crucial

$$\sigma = \frac{v}{m} , \quad \epsilon = \frac{F}{m} , \quad \mu = \frac{H}{m}$$

Ideal oscillator ckts: (Ideal LC ckt as an oscillator)



2. Each derivative of a harmonic is back again the same function orthogonally shifted i.e by 90°. such that II^nd derivative is back the same function with negation with -ve sign.

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$$A\sin\omega t \xrightarrow{I\,deri} \omega A\sin(\omega t+90^\circ) \xrightarrow{II^{nd}} \omega^2 A\sin(\omega t+180^\circ) \xrightarrow{III^{rd}} \omega^3 A\sin(\omega t+270^\circ)$$

$$A\cos\omega t \xrightarrow{I^{st}\,deri} \omega A\cos(\omega t+90^\circ) \xrightarrow{II} \omega^2 A\cos(\omega t+180^\circ) \xrightarrow{III} \omega^3 A\cos(\omega t+270^\circ)$$

$$Ae^{j\omega t} \longrightarrow \omega A e^{j(\omega t+90^\circ)} \longrightarrow \omega^2 A e^{j(\omega t+180^\circ)} \longrightarrow \omega^3 A e^{j(\omega t+270^\circ)}$$

$$j = e^{j90^\circ} \qquad -1 = e^{j180} \qquad -j = e^{j270}$$

The discussed LC ckt excellently obeys the harmonic sol^n as shown

$$V = -L\frac{dI}{dt}$$

$$I = C\frac{dV}{dt}$$

$$V = -L\frac{d}{dt}\left(C\frac{dV}{dt}\right) \Rightarrow -LC\frac{d^2V}{dt^2}$$

$$\frac{d^2V}{dt^2} = \left(\frac{-1}{LC}\right)V$$

$$\omega^2 = \frac{1}{LC}$$

$$\omega = \frac{1}{\sqrt{LC}}$$

charging and discharging rate of the capacitor decides the frequency of the oscillation Hence

$$\therefore w = \frac{1}{\sqrt{LC}}$$

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eg:

$$\nabla \times H = \epsilon \frac{\partial E}{\partial t}$$

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$

These are called as EM wave eqn which are second order differential eqn of time and space Hence EM waves are harmonics of time and space. as the second order derivative in space this is same as second order derivative in time

$$\nabla \times (\nabla \times H) = \epsilon \frac{\partial}{\partial t} (\nabla \times E)$$

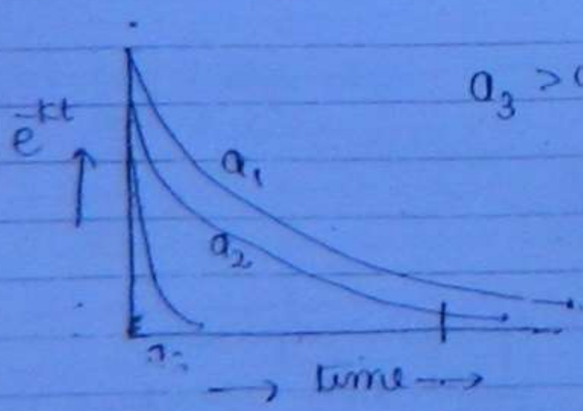
$$-\nabla^2 H = -\mu\epsilon \frac{\partial^2 H}{dt^2}$$

$$\boxed{\nabla^2 H = \mu\epsilon \frac{\partial^2 H}{\partial t^2}}$$

$$\boxed{\nabla^2 E = \mu\epsilon \frac{\partial^2 E}{\partial t^2}}$$

Types of exponential function $(e^{-kt}$ Vs $t)$

case i) $K = a$ - +ve real no.

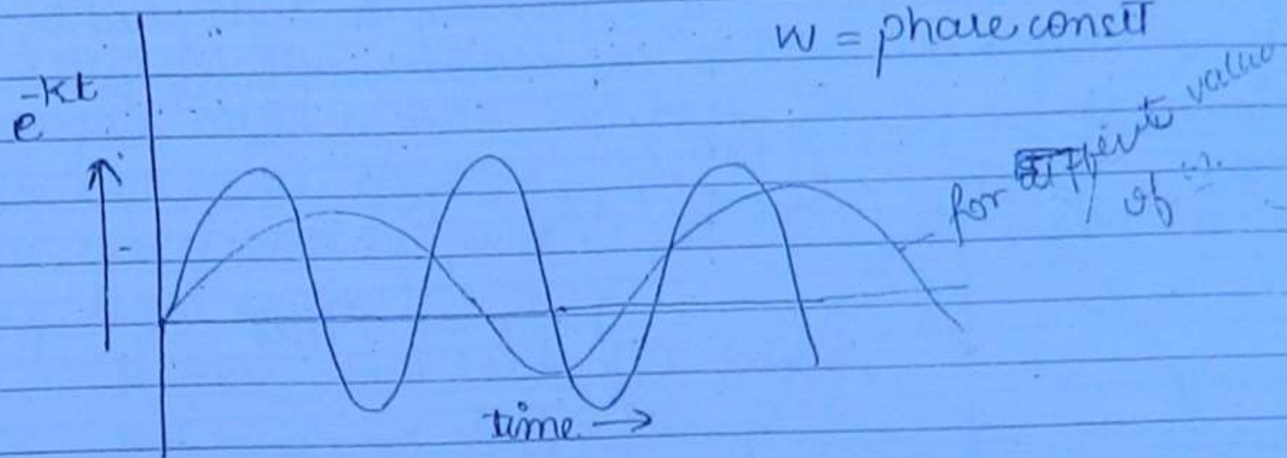



$a_3 > a_2 > a_1$

$a$ = decay constt.

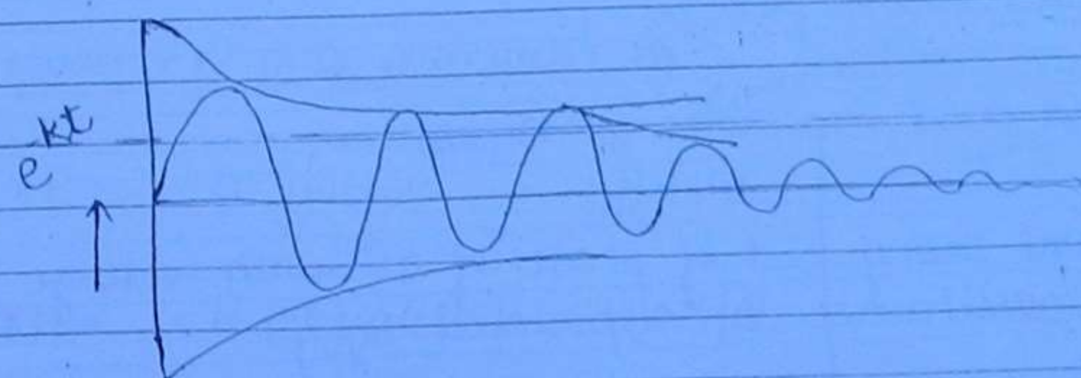
case(ii) $K = j\omega$ = any imaginary no. (123)

$w$ = phase const

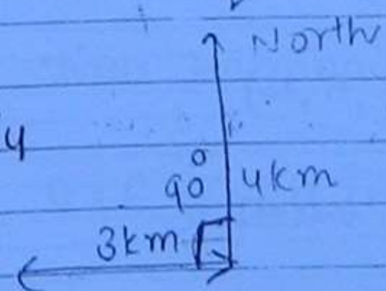


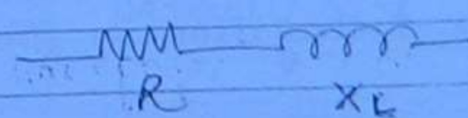

case(iii) $K = a + j\omega$ = any complex no.

$$e^{-kt} = \left(e^{-at}\right)\left(e^{-j\omega t}\right)$$




$3+j4$




$R + jX_L$

$V = IZ$

$I(R+jX)$

j always represents shifting factor b/w two independent domains or b/w two orthogonal dimensions of any phenomenon. If a dimension is multiplied by j it will translate to the second orthogonal dimension.

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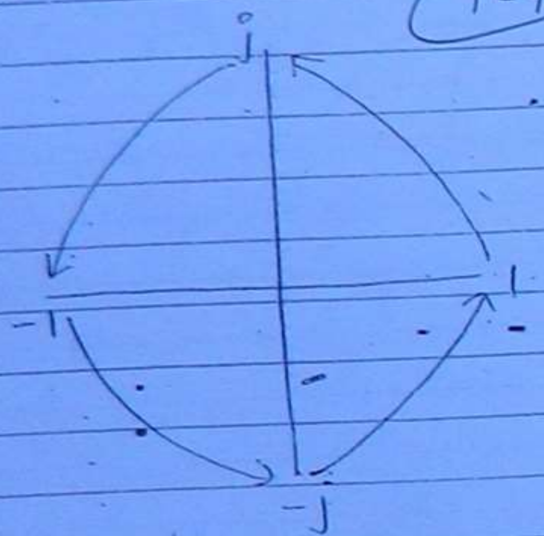


$1 \times j = 1\angle 90^\circ = j$

$j \times j = -1 = 1\angle 180^\circ$

$-1 \times j = -j = 1\angle 270^\circ$

$-j \times j = 1\angle \cancel{360^\circ} = 1\angle 0^\circ$

eg. i) North and Earth journey

ii) Resistance and Reactance in the ckt

If current being out of phase of 90° with voltage then inductance is represented by j

EM wave Propagation in materials $(\sigma, \epsilon, \mu)$

Every EM wave to propogation always a needs a harmonic source at one end.

$$\left.\begin{aligned} E_s &= E_0 e^{j\omega t} \\ H_s &= H_0 e^{j\omega t} \end{aligned}\right\} \omega = \text{frequency of the source}$$

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}.$$

Sound waves are particle waves

$$\nabla \times H = \epsilon \frac{\partial E}{\partial t} + \sigma E$$ (125)

As seen in the maxwell's eq$^n$ every EM wave to propogate needs a time harmonic source which is $E_s$ and $H_s$ as shown.

The source can be crystal oscillator, x-ray gun, or any light source

$Hz - KHz \longrightarrow$ Audio freq. waves

$MHz \longrightarrow$ Radio freq. "

$GHz - 10^{12} \longrightarrow$ Microwave

$10^{15} - 10^{18} \longrightarrow$ light

$10^{20} - 10^{22} \longrightarrow$ x-rays, Gammarays.

$10^{25} \longrightarrow$ cosmic rays.

Put source Eq$^n$ into Maxwell's Eq$^n$.

-90° phase shift

$$\boxed{\begin{aligned} \nabla \times E &= -\mu j w H_0 e^{j\omega t} = -j\omega \mu H \quad &— (1) \\ \nabla \times H &= \sigma E + \epsilon j \omega E = (\sigma + j\omega \epsilon) E \quad &— (2) \end{aligned}}$$

Time Harmonic format of Maxwell Eq$^n$.

$$\nabla \times E = -\mu j \omega H_0 e^{j\omega t} = -j\omega \mu H$$

$$H = \frac{\nabla \times E}{-j\omega\mu}$$

Put H in (2) eq$^n$

$$\nabla \times \left(\frac{\nabla \times E}{-j\omega\mu}\right) = \sigma E + \epsilon j \omega E$$

$$\nabla(\nabla \cdot E) - \nabla^2 E = -j\omega\mu(\sigma + j\omega\epsilon) E$$

when medium is considered as source free medium or charge free medium or Homogeneous then

$\nabla \cdot E = 0$ or $\nabla \cdot H = 0$ always.

$$\boxed{\begin{aligned} \nabla^2 E &= j\omega\mu(\sigma + j\omega\epsilon)E \\ \nabla^2 H &= j\omega\mu(\sigma + j\omega\epsilon)H \end{aligned}} \Bigg\} \text{Helmholtz's Eq}^n$$

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The two eq$^n$s have second order derivative with space giving the same function again i.e E and H are Harmonic in space and the eq$^n$s are called as Helmholtz's Eq$^n$

uniform plane wave

let $j\omega\mu(\sigma + j\omega\epsilon) = \gamma^2$

$\nabla \longrightarrow \frac{\partial}{\partial z} a_z$ only.

This called as uniform plane wave assumed to be from a uniform planar source

Let propagation be assumed in z direction only.

$E \longrightarrow a_x$ only.

$H \longrightarrow a_y$ only.

$$\boxed{\begin{aligned} &\frac{\partial^2 E_x}{\partial z^2} - \gamma^2 E_x = 0 \quad \text{--- (3)} \\ &\frac{\partial^2 H_y}{\partial z^2} - \gamma^2 H_y = 0 \quad \text{--- (4)} \end{aligned}}$$

Propagation Eq$^n$s for a plane wave.

Solution are -

$$\boxed{\begin{aligned} E(z)_x &= (c_1 e^{-\gamma z} + c_2 e^{\gamma z})a_x \\ H(z)_y &= (c_3 e^{-\gamma z} + c_4 e^{\gamma z})a_y \end{aligned}}$$

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$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

The two solns because there are two possibilities of wave travelling in +z direction from the planar source.

Therefore $e^{-\gamma z}$ is the solution of the decaying exponential fcn +ve z and hence assuming our material to the right of the source we choose this soln.

$$E(t) = e^{j\omega t} \qquad E(z) = e^{-\gamma z}$$

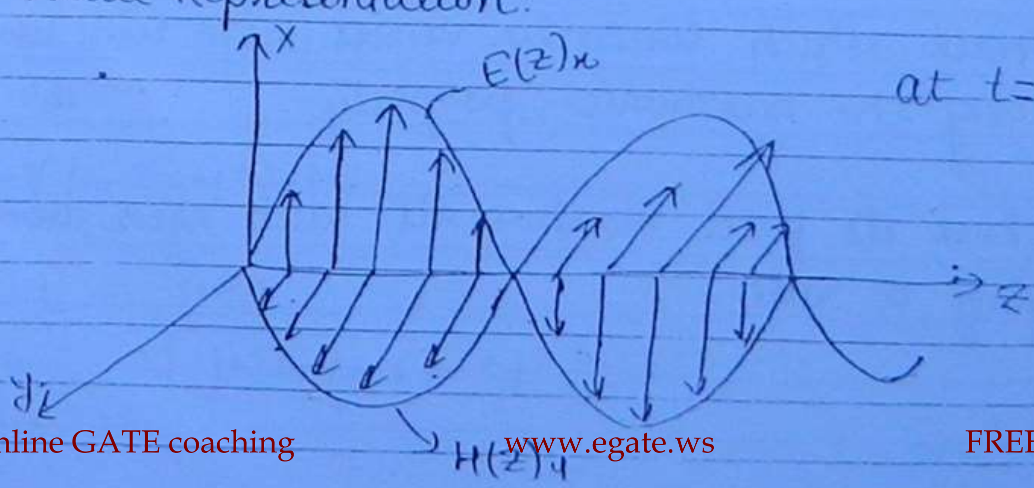
Hence the final wave solution is the product soln of time and space harmonics of E and H

$$\boxed{\begin{aligned} E(z,t)_x &= E_0 e^{-\gamma z} e^{j\omega t} \; a_x \\ H(z,t)_y &= H_0 e^{-\gamma z} \cdot e^{j\omega t} \; a_y \end{aligned}}$$

final wave solution

EM wave Representation:

at $t = t_1$



at $t=t_2$ the wave shifts and propagate further.

Propogation constt ($\gamma$) 128

It is a constt of the exponential funⁿ that decides the shape of the exponential funⁿ as $z$ increases. Hence $\gamma$ decides the course of propogation in the medium

Let $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \alpha + j\beta$.

Let $\gamma$ any two dimensional funⁿ

$$\boxed{\begin{aligned} E(z,t)_x &= \left(E_o e^{-\alpha z}\right) e^{j(\omega t - \beta z)} a_x \\ H(z,t)_y &= \left(H_o e^{-\alpha z}\right)\left( e^{j\boxed{(\omega t - \beta z)}} a_y \end{aligned}}$$

field dirⁿ

Phase

amplitude

Harmonic

- Every electromagnetic (EM) wave have amplitude which exponetialy decays at $\alpha$ rate Hence $\alpha$ is called as attenuation constt.

$\alpha$ = attenuation constt per meter

- It has a phase which linearly varies with time and space following the harmonic property

- Every wave has its field components with their directions obeying a rule.

| | | | |
|---|---|---|---|
| E | × H | = | Propagation |
| direction | direction | | direction |

(129)

## Interinsic wave Impedance ($\eta_i$)

→ Effect $\gamma = \alpha + j\beta$

volt $E \rightleftharpoons H$ current

$$\eta = \frac{E}{H}$$

cause $= \eta = R + jX$

When E is converted into H and H to E the rate of transformation or the slope of transformation is called as $\eta$. In the process of transformation there is loss in amplitude and a change in phase. Hence $\eta$ has resistance and reactance. Hence every medium has $\gamma$

Resistance → attenuation
Reactance → phase.

$a_z \quad a_x$

$$\nabla \times E = -j\omega\mu H$$

$$\frac{\partial E}{\partial z} = -j\omega\mu H$$

$$-\gamma E = -j\omega\mu H$$

$$\frac{E}{H} = \frac{j\omega\mu}{\sqrt{j\omega\mu(\sigma + j\omega\epsilon)}}$$

$$\boxed{\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}}$$

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chapter 2.

W.B

100 turns —— $(t^3 - 2t) = \psi_m$ m weber

$$V = -\frac{d}{dt}(\psi_m)$$

$$= -\frac{d}{dt}\left(100 \times (t^3 - 2t) \times 10^{-3}\right)$$

$$V = -0.1\,(3t^2 - 2)\Big|_{t=2s}$$

$$V = -1\,V$$

2 W.B

B — uniform → (decrease linearly)
↓ with space ↓ with time

$$V = -\frac{d}{dt}(BA) = -A(-k) = DC\ voltage.$$

decreasing means slope is -ve means dc voltage

3 W.B

$I_c$

$$V = I\left(\frac{1}{j\omega C}\right)$$

$$|I| = \omega C \cdot V$$

$$= \omega \frac{\epsilon A}{d} V$$

$I_d$

$$J_A = \frac{\epsilon \partial E}{\partial t}$$

$$I_d = \frac{\epsilon \partial E}{\partial t} \cdot A$$

$$= j\omega E \cdot \epsilon A$$

$$= j\omega \frac{\epsilon A}{d} \cdot V$$

$$I_c = I_d$$

$e = 2.71$

$I = \frac{W \epsilon A V}{d} \Rightarrow 2 \times \pi \times 3.6 \times 10^{9} \times \frac{1}{36\pi \times 10^{9}} \times \frac{10^{-4}}{10^{-3}} \times 0.5$

$\Rightarrow 10mA$ Ans

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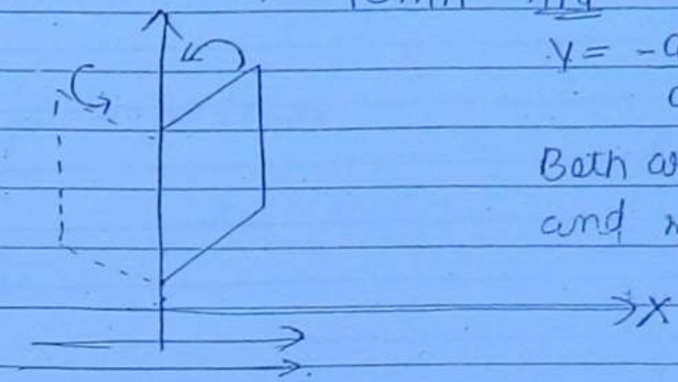
4 W.B



$V = -\frac{d(BA)}{dt}$

Both area is changing and magnetic field

$B(t)x.$

$20m \xrightarrow{\text{medium}} \frac{1}{e}$ times Intial value.

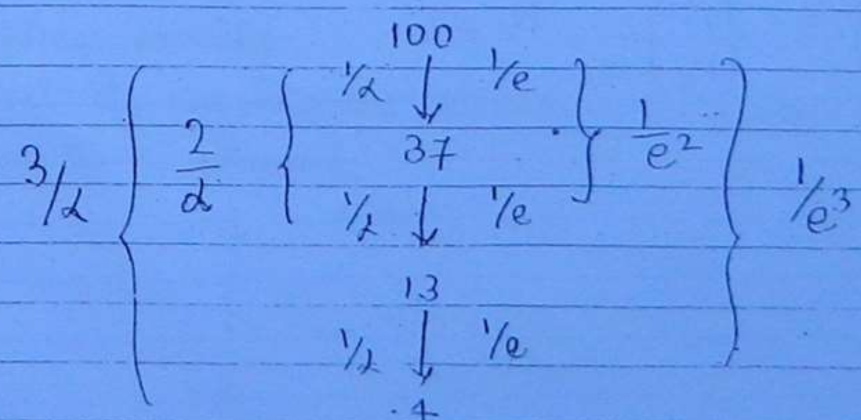
$|E(z,t)| = E_0 e^{-\alpha z} = |E(z)|$

$z = \frac{1}{\alpha} = 20$

$E\left(\frac{1}{\alpha}\right) = \frac{E_0}{e} = 0.37 E_0 \quad \left\{\alpha = \frac{1}{20}\right\}$

$\beta = \frac{2\pi}{\lambda} = \frac{\theta}{z} \Rightarrow \frac{\pi/6}{20} \Rightarrow \frac{\pi}{120}\ m^{-1}$

$\gamma = \alpha + j\beta \Rightarrow \frac{1}{20} + j\frac{\pi}{120}$



7 W.3

$100 \longrightarrow 2 Q + 5m$

(132)

$$|E| = E_0 e^{-\alpha z}$$

$$20 = 100 e^{-\alpha(5)}$$

$$e^{5\alpha} = \frac{100}{20} = 5$$

$$5\alpha = \ln(5)$$

$$\alpha = \frac{\ln(5)}{5}$$

$$40 = 100 e^{-\alpha z}$$

$$e^{\alpha z} = \frac{100}{40} = 2.5$$

$$\alpha z = \ln(5/2)$$

$$z = \frac{5 \ln(5/2)}{\ln(5)} = 2.8466$$

Note: If $\left(\frac{P_1}{P_2}\right)$ = Power Ratio

$$10 \log\left(\frac{P_1}{P_2}\right) = \left(\frac{P_1}{P_2}\right) \text{ dB.}$$

$$\ln\left(\frac{P_1}{P_2}\right) = \left(\frac{P_1}{P_2}\right) \text{ Nepers.}$$

Wednesday.

EM wave propagation in free space ($\sigma = 0$, $\epsilon = \epsilon_0$, $\mu = \mu_0$)

$\epsilon_R = \mu_R = 1$

$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = j\omega\sqrt{\mu_0\epsilon_0}$

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$\Rightarrow \alpha + j\beta = j\omega\sqrt{\mu_0\epsilon_0}$

$\alpha = 0$

$\beta = \omega\sqrt{\mu\epsilon}$

EM waves never attenuate in free space.
They propagate with phase shift only.
They are permitting abilities even for free space for
E/H Hence propagation.

$\eta = \sqrt{\dfrac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\dfrac{\mu_0}{\epsilon_0}} =$ real no.

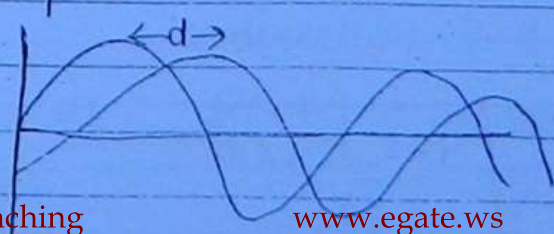
$= \dfrac{E}{H}$

E and H are orthogonal in space ie $E_x$ & $E_y$ But are in phase in time

$$\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} = \sqrt{\frac{4\pi \times 10^{-7}}{\frac{1}{36\pi \times 10^{9}}}} = \sqrt{4\pi \times 36\pi \times 10^{2}} = 120\pi = 377\Omega$$

$v_p$ = Phase velocity

Phase velocity defines the distance an in phase point move for a given time elapsed.



$$\frac{d}{t} = \frac{\lambda}{T} = \lambda f$$

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$\lambda \longrightarrow 2\pi \longrightarrow T$

$$v_p = \frac{\lambda d}{2\pi T} = \frac{\omega}{\beta}$$

$$v_p = \frac{\omega}{\omega\sqrt{\mu_0\epsilon_0}} = \frac{1}{\sqrt{\mu_0\epsilon_0}} \Rightarrow \frac{1}{\sqrt{4\pi\times10^{-7}\times\frac{1}{36\pi\times10^{9}}}} \Rightarrow 3\times10^{8}\ m/s$$

case (ii) EM wave propagation in Ideal dielectrics / Lossless dielectrics / perfect dielectrics ($\sigma = 0$, $\epsilon = \epsilon_0\epsilon_R$, $\mu = \mu_0$
($\epsilon_R >> 1$)

Most dielectrics have $\mu_R = 1$
It is rare to have a magnetic with dielectric

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = j\omega\sqrt{\mu_0\epsilon_0\epsilon_R}$$

$$\Rightarrow \alpha + j\beta = j\omega\sqrt{\mu_0\epsilon_0\epsilon_R}$$

$$\sigma = 0$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0\epsilon_R}} = \text{real no.}$$

$$= \frac{E}{H}$$

$$\eta = \sqrt{\frac{4\times10^{-7}}{\frac{1}{36\pi\times10^{9}}}} = \sqrt{\frac{4\pi\times36\pi\times10^{2}}{\sqrt{\epsilon_r}}}$$

$= \frac{120\pi}{\sqrt{\epsilon_r}} = \frac{377\Omega}{\sqrt{\epsilon_r}}$

135

$$V_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu_0\epsilon_0\epsilon_r}}$$

chapter - 2

W.B

$$J_c = \nabla \times H$$

$$J_d = \epsilon\frac{\partial E}{\partial t}$$

Magnetic current is due to $J_b$ = Bound current density.

It exists whenever a material is magnetized.

$J_b$ = Bound current density.

- Ferromagnetic
- spinning / revolving electrons

W.B (i) Propagation direction

$\sin(\omega t - \beta z)$ $e^{-\alpha z}$

$e^{j(\omega t - \beta z)}$

$+z$

$\sin(\omega t + 4x)$

$\downarrow$

x direction $(-a_x)$

(ii) $\beta = 4$ as $E(x,t) = 25\sin(\omega t + 4x)a_y$

$\beta = 4 = \frac{2\pi}{\lambda} =$

$\lambda = \frac{\pi}{2}$

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(iii) $f = \frac{3\times10^8}{\pi/2} = \frac{600}{\pi}$ MHz $\left(f = \frac{c}{\lambda}\right)$

(iv) $H(x,t) \Rightarrow$ $\nabla\times E = -\mu\frac{\partial H}{\partial t}$

$$H = \frac{-1}{\mu}\int \nabla\times E\, dt$$

note

$$H(x,t) = \left(\frac{25}{120\pi}\right)\sin(\omega t+4x)\,(-a_z)$$

① ② ③

$a_y \times (?) = (-a_x)$

$-a_z$

for a given field component of a wave the other field component is calculated with three rules.

i) Amplitude should obey the rule

$$\frac{E}{H} = \eta$$

ie $\frac{|E|}{|H|} = |\eta|$

ii) The phase should be the same in the same harmonic with free space or ideal dielectric

18 W.B $\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0 \times 8}} = \frac{120\pi}{2} = 188.4$

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20 W.B $H(z,t)$ $\quad \frac{E}{H} = 120\pi$

$E(z,t) = (3.77)\cos(4\times10^7 t - \beta z)(a_y)$ $\quad E = 120\pi\times .10$

21 W.B $\lambda$ — free space - 2cm

$\lambda$ — dielectric - 1cm

$\epsilon_R = ?$

$\beta = \omega\sqrt{\mu_0\epsilon_0\epsilon_R} = \frac{2\pi}{\lambda}$

$\lambda \propto \frac{1}{\sqrt{\epsilon_r}}$

$\frac{\lambda_1}{\lambda_2} = \frac{2}{1} = \sqrt{\frac{\epsilon_0\epsilon_R}{\epsilon_0}}$ $\quad \boxed{\epsilon_R = 4.}$

$\frac{\lambda_1}{\lambda_2} = \frac{\sqrt{\epsilon_0\epsilon_R}}{\sqrt{\epsilon_0(1)}}$

$\frac{2}{1} = \sqrt{\epsilon_r}$

$\epsilon_r = 4.$

22 W.B $\vec{E}_i = 20e^{-j5z}\vec{a}_x - 25e^{-j5z}a_y$ V/m

$H = \frac{20}{120\pi}e^{-j5z}a_y + \frac{25}{120\pi}e^{-j5z}(-a_x)$

$H = \frac{20}{120\pi}e^{-j5z}a_y + \frac{25}{120\pi}e^{-j5z}(a_x)$

24 W.B $360^\circ$ —— $\lambda$

$2\pi$ —— $\lambda$

$\underbrace{3\,mm}_{90^\circ}$ —— thickness

$\frac{\lambda}{4} = 3mm$

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$\lambda$ — $2\pi$

$\frac{\lambda}{4}$ — $\frac{\pi}{2}$

$\lambda = 12mm$

$\lambda f = v_p = \frac{3\times10^8}{\sqrt{\epsilon_R}}$

$\sqrt{\epsilon_R} = \frac{3\times10^8}{12\times10^{-3}\times10\times10^9}$

$\epsilon_R = 6.25$

$E = A\cos(\omega t - \frac{\omega}{c}z)a_y$

$\cancel{H} = \frac{A}{\sqrt{\frac{\mu_0}{\epsilon_0}}}\cos(\omega t - \beta z)a_y$ $\quad c = \cancel{3\times10^8}$

$H = \frac{A}{\sqrt{\frac{\mu_0}{\epsilon_0}}}\cos(\omega t - \beta z)(-a_x)$

$H = \frac{A}{\sqrt{\frac{\mu_0}{\epsilon_0}}}\cos(\omega t - \beta z)(-a_x)$

$H = -A\sqrt{\frac{\epsilon_0}{\mu_0}}\cos\omega(t - \frac{z}{c})a_x$

$H = -jA\sqrt{\frac{\epsilon_0}{\mu_0}}\sin\omega(t - \frac{z}{c})a_x$

best u. 12 4, 6, 12, 13 14, 22, 24, 25, 25

classmate Date Page

9B $\gamma$ = complex $\rightarrow$ $\gamma = \alpha + j\beta$. $\gamma = j\beta$. {$\because \alpha = 0$ for free space},

$\rightarrow \nabla^2 E - \gamma^2 E = 0$. 139

$$\boxed{E(z) = e^{-\gamma z}}$$

Imaginary $\rightarrow$ $\gamma = j\beta$ free space

$\rightarrow \nabla^2 E + \beta^2 E = 0$ $\quad \beta = K$

$\nabla^2 E + K^2 E = 0$

$$E(z) = E_0 e^{-j\beta z} \qquad (c)$$

4B $E(x,z,t)_y = 25 \sin(\omega t - 3x + 4z) a_y$

Propagation $\rightarrow$ $(3a_x - 4a_z)$

$$H(x,z,t) = \frac{25}{120\pi} \sin(\omega t - 3x + 4z)\left(\frac{4a_x + 3a_z}{5}\right)$$

$ay \times (?) = 3a_x - 4a_z$ $\qquad \sqrt{9+16} = \sqrt{25} = 5$

$H \rightarrow 3a_z + 4a_x$

direction should always be a unit factor

$$H(x,z,t) = \frac{5}{120\pi} \sin(\omega t - 3x + 4z)(4a_x + 3a_z)$$

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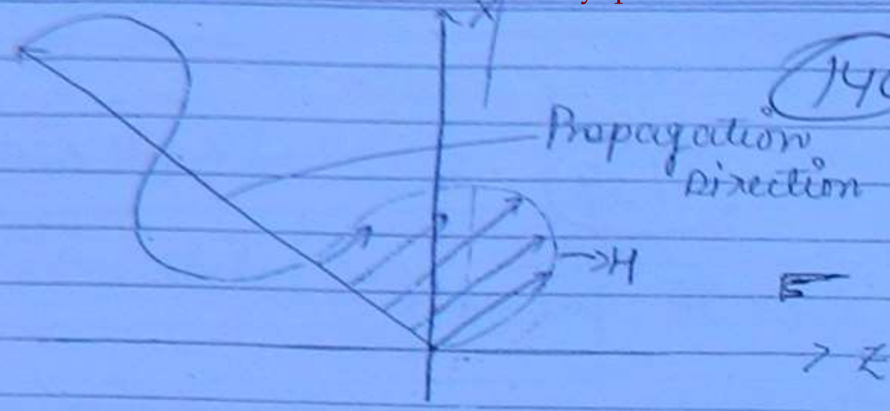


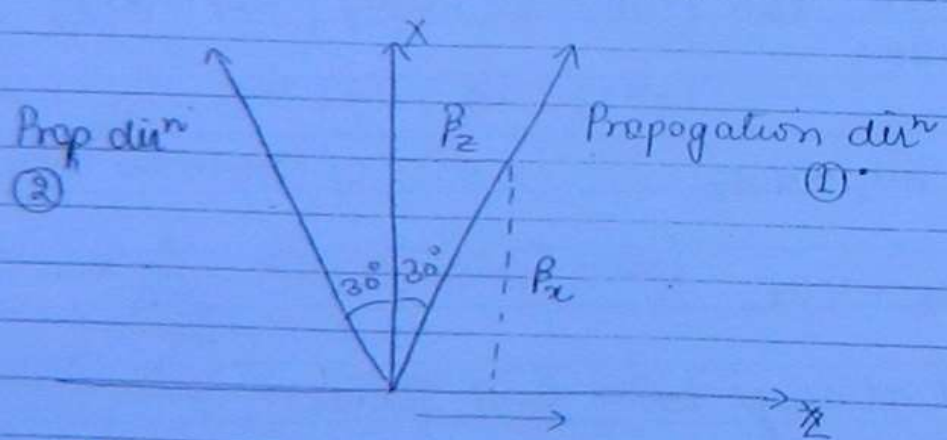

H ⊥ P ⊥ E ⊥ H

Note $E(x, z, t)_y$

— $H(x,z,t)_{(x,z)}$ is notation used to represent this wave and satisfies $E \perp H \perp P \perp E$.

Extention of the question $\boxed{\beta = \sqrt{\beta_x^2 + \beta_z^2} = 5}$

9 W.B




$\left.\begin{array}{l} E(x,z,t) \\ H(x,z,t) \end{array}\right]$ $\tan 30° = \dfrac{\beta_z}{\beta_x} = \dfrac{1}{\sqrt{3}}$

$\beta_x = \sqrt{3}\,\beta_z$

$e^{j(\omega t - \beta_x x - \beta_z z)}$ $\beta_x$

$e^{j(\omega t - \sqrt{3}\beta_z x - \beta_z z)}$

Conclusion from Problems:

1. Material has properties (141) $\theta, \sigma, \epsilon, \mu$

1. Transmission line has primary constt. $R, G, C, L$

$\sigma = ℧/m$

$\epsilon\, \mu$ = Reactive

$R, G = \Omega, ℧$

$L, C$ = Reactive

$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$

$= \sqrt{(j\omega\mu)(\sigma + j\omega\epsilon)}$

$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$

$\eta = \sqrt{\dfrac{j\omega\mu}{\sigma + j\omega\epsilon}}$

$Z_0 = \sqrt{\dfrac{R+j\omega L}{G+j\omega C}}$

$\gamma \cdot \eta = j\omega\mu$

$\mu = \dfrac{|\gamma \cdot \eta|}{w}$

$\gamma/\eta = \sigma + j\omega\epsilon$

$\sigma = \text{Real}\left[\gamma/\eta\right]$

$\epsilon = \dfrac{\text{Imag}\left[\gamma/\eta\right]}{w}$

2. $\eta = \dfrac{E_x}{H_y} = \sqrt{\dfrac{j\omega\mu}{\sigma + j\omega\epsilon}} = |\eta|\angle\theta$ = complex

$E_x$ & $H_y$ are always orthogonal in space but have phase shift in time in general materials (when $\eta$ is complex).

3. $\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$

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Square on both the sides

$$\underbrace{\alpha^2 - \beta^2}_{\text{Real}} + \overbrace{j2\alpha\beta}^{\text{Imaginary}} = \overbrace{j\omega\mu\sigma}^{\text{Imag.}} - \underbrace{\omega^2\mu\epsilon}_{\text{Real}}$$

$$\boxed{\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right)}}$$

$$\boxed{\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right)}}$$

Loss Tangent and dissipation factor $\left[\frac{\sigma}{\omega\epsilon}\right]$

In the value of $\gamma$ is completely dependent on the Term $\frac{\sigma}{\omega\epsilon}$ If it is quite large then $\alpha$ is very large and hence attenuation is very high

good conductor $\left\{ \frac{\sigma}{\omega\epsilon} >> 1 \right.$ $\Rightarrow \alpha\uparrow \ \beta\uparrow \Rightarrow$ Propagation is difficult
Attenuation is high

The term is quite large for a good conductor because $\sigma \gg \omega\epsilon$

Hence electromagnetic (EM) waves cannot travel inside good conductor

eg. $\frac{\sigma}{\omega\epsilon} >> 1$ for earth only when $\omega$ upto MHz

$$\boxed{\omega < 10^6}$$

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let $\sigma = 10^{-6}$

$w = 10^{6}$

$\epsilon = 10^{-12}$

$$\frac{\sigma}{\omega\epsilon} = \frac{10^{-6}}{\omega \times 10^{-12}} >> 1$$

Whether a object is a good conductor or not depends on $\sigma$ as well as on frequency of the wave.

eg. $\frac{\sigma}{\omega\epsilon} >> 1$ for earth only when $w$ (freq.) upto MHz only.

Earth is a good conductor for low frequency audio and radio (AF / RF) freq.

eg. $\frac{\sigma}{\omega\epsilon} >> 1$ for Human Body upto light frequency. only.

- $\frac{\sigma}{\omega\epsilon}$ is called as loss tangent and dissipation factor
- $\frac{\sigma}{\omega\epsilon}$ is unitless because it is $\left|\frac{J_c}{J_d}\right|$

$$\left|\frac{J_c}{J_d}\right| = \frac{\sigma E}{\epsilon \frac{\partial E}{\partial t}} = \frac{\sigma E}{\epsilon \cdot j\omega E} = \left|\frac{\sigma}{j\omega\epsilon}\right| = \frac{\sigma}{\omega\epsilon}$$

- $J_c >> J_d$ very good conductor

Case (iii) EM wave propagation in good conductors

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

$$\sigma >> \omega\epsilon$$

$$= \sqrt{j\omega\mu\sigma}$$

$= \sqrt{\omega\mu\sigma \angle 90^\circ} = \sqrt{\omega\mu\sigma} \angle 45^\circ$

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...° means ...al and ...maginary ...are same

$\cos 45^\circ + j\sin 45^\circ = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$

$\gamma = \sqrt{\frac{\omega\mu\sigma}{2}} + j\sqrt{\frac{\omega\mu\sigma}{2}} = \alpha + j\beta$

$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$

$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$

$\eta = \sqrt{\frac{j\omega\mu}{\sigma}}$

$\eta = \sqrt{\frac{\omega\mu}{\sigma}\angle 90^\circ} \Rightarrow \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ$

$\eta = \sqrt{\frac{\omega\mu}{2\sigma}} + j\sqrt{\frac{\omega\mu}{2\sigma}}$

$R = X = \sqrt{\frac{\omega\mu}{2\sigma}}$

...ote: 1 As the resistance is equal to the reactance the rate of Amplitude loss is equal to the rate of phase change

$\gamma = \alpha + j\beta$

$\eta = R + jX$

2. $\eta$'s phase $= 45^\circ$

$E_x / H_y$ are out of phase by $45^\circ$ in conductors

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Deapth of Penetration or skin depth:

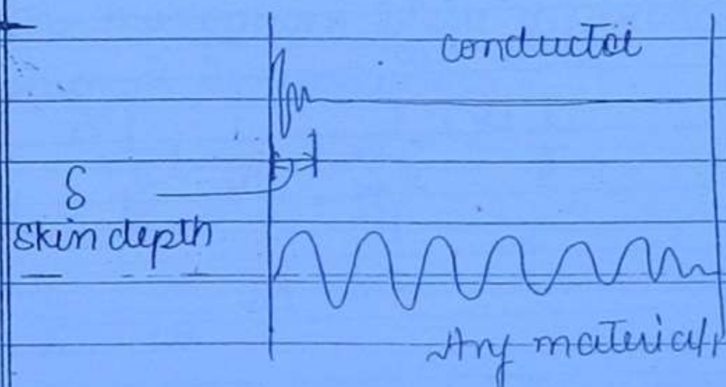
An exponentially decaying wave never reaches exact zero value but is approximately small after travelling $\frac{1}{\alpha}$ distance. The $\frac{1}{\alpha}$ distance is called as skin depth.

Hence skin depth is defined as:

A distance travelled by the wave where the amplitude becomes $\frac{1}{e}$ times i.e 37% of intial value.

$$\boxed{\text{skin depth } \delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}}}$$

100 $\xrightarrow{\frac{1}{e}}$ 37




$$\delta = \sqrt{\frac{2\sigma}{\omega\mu\sigma^2}} = \frac{1}{\sigma}\sqrt{\frac{2\sigma}{\omega\mu}} = \frac{1}{\sigma R} = \rho$$

$R = R_\delta$ = skin Resistance

27. N.B.

$v_p$ = phase velocity

$3\times10^8$ —— free space

$\frac{\sigma}{\omega\epsilon} = 0$

$\frac{3\times10^8}{\sqrt{\epsilon_R}} \longrightarrow$ Dielectric

$10^5 - 10^6 \longrightarrow$ Materials

few m/s $\longrightarrow$ conductor (weak)

$$V_\beta = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right)}}$$

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$$v_p = \frac{1}{\left(\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right)}\right)}.$$

Q. Repeat the same question for the same data if the freq. is increased by 4 times.

Sol$^n$

$\delta = 4\,\mu m$

frequency $= 800\,kHz$

.. ~~Stamens~~ freq. = 4 times

$V_p = \omega\delta$

$V_p' = \omega'\delta'$

$= 4\omega\,\frac{\delta}{2}$

$V_p' = 2V_p = 10\,m/s$

$\omega = 2\pi f$

$\omega' = 4\omega$.

$\frac{\delta'}{\delta} \propto \frac{1}{\sqrt{f}} = \frac{1}{\sqrt{4}}$ $\frac{1}{2}\delta$

$\delta' = \frac{1}{2}\delta$

$\delta = \sqrt{\frac{2}{\omega\mu\sigma}}$ $\qquad \delta' = \frac{\delta}{2}$

(w-8)

$1.73 = \sqrt{3} = \frac{\sigma}{\omega\epsilon}$ $\qquad \frac{\sigma}{\omega\epsilon} \gg 1$

E/H — phase — ??

$$\eta\text{'s phase} = \sqrt{\frac{j\omega\mu}{\sigma+j\omega\epsilon}} = \sqrt{\frac{\omega\mu\angle 90^\circ}{K\angle\tan^{-1}\left(\frac{\omega\epsilon}{\sigma}\right)}}$$

$$= \sqrt{K'\angle 90^\circ - \tan^{-1}\left(\frac{\omega\epsilon}{\sigma}\right)}$$

$$= \frac{90-\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)}{2} = \frac{90-30}{2} = 30^\circ$$

Phase diff b/w E and H

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↑ $0^\circ$ — free space

$\frac{\sigma}{\omega\epsilon} = 0$

↑ $0^\circ$ — Dielectrics.

↑ $22\frac{1}{2} - 30^\circ$ — Materials

$45^\circ$ — conductors

Note: The maximum phase difference b/w E and H is $45^\circ$. which is in conductors.

29 WB $J_c = J_d$ $I_c = I_d$

$$\frac{J_c}{J_d} = \frac{\sigma}{\omega\epsilon} = 1$$

$$\frac{\sigma}{2\pi f\epsilon} = 1 \Rightarrow f = \frac{\sigma}{2\pi\epsilon} \Rightarrow \frac{10^{-2}}{2\pi\epsilon_0\epsilon_R} \Rightarrow \frac{10^{-2}}{2\times\pi\times4\times\frac{1}{36\pi\times10^{9}}}$$

$$\Rightarrow 4.5\times10^{7} = 45\,MHz \text{ ans}$$

30 WB $$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} \Rightarrow \sqrt{\frac{2}{2\pi f\mu\sigma}}$$

$$\delta_1 = \sqrt{\frac{1}{f_1}}$$

$$\delta_2 = \sqrt{\frac{1}{f_2}}$$

$$\frac{\delta_1}{\delta_2} = \sqrt{\frac{f_2}{f_1}} \text{ or } \frac{\delta_2}{\delta_1} = \sqrt{\frac{f_1}{f_2}} \Rightarrow \delta_2 = 25\sqrt{\frac{10^6}{4\times10^6}} = \frac{25}{2}$$

$$= 12.50\,cm$$

$$\frac{J_c}{J_d} = \frac{I_c}{I_d} = \frac{\sigma}{\omega\epsilon}$$

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$$I_d = \frac{\omega\epsilon}{\sigma \times I_c} \Rightarrow \frac{2\pi \times 50 \times \frac{1}{36\pi \times 10^9}}{58}$$

$$I_d = 4.8 \times 10^{-11} A \quad \text{Ans}$$

Power, Power Density, Poynting vector

E/H → Energy formats
→ time
→ Power EH wave

$$E \times H = \frac{\text{volt}}{m} \times \frac{\text{amp}}{m} = \frac{\text{watts}}{m^2}$$

$$\cancel{\nabla \times H = \sigma E + \epsilon \frac{\partial E}{\partial t}}$$

$$(\nabla \times H) \cdot E = \sigma E^2 + \epsilon \frac{\partial E}{\partial t} \cdot E$$

$$H \cdot (\nabla \times E) - \nabla \cdot (E \times H) = \sigma E^2 + \epsilon \frac{\partial E}{\partial t} \cdot E$$

$$-\nabla (E \times H) - \sigma E^2 = \epsilon \frac{\partial E}{\partial t} \cdot E + \mu \frac{dH}{\partial t} \cdot H$$

$$-\nabla \cdot (E \times H) - \sigma E^2 = \frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon E^2 + \frac{1}{2}\mu H^2\right)$$

$$\left\{ \because \; x\,dx = d\left(\frac{x^2}{2}\right) \right\}$$

$$\int -\nabla \cdot (E \times H)\, dv - \int \sigma E^2\, dv = \int \frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon E^2 + \frac{1}{2}\mu H^2\right) dv$$

$$\oint (E \times H) \cdot ds + \underbrace{\int \sigma E^2 dv}_{\text{power given to the medium}} = -\frac{\text{Energy}}{\text{Unit time}}$$

$\frac{320}{\sqrt{2}}$ = $V_{rms}$

$320 \sin(100\pi t)$ → Instantaneous

230 V, 50 Hz

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$$\oint (E \times H) \cdot ds + \int \sigma E^2 dV = -$$ Power crossing the closed surface

The right hand side is a power term that is crossing the surface enclosing the EM wave source. Hence

$E \times H$ = power crossing per unit area.
= power density = strength of the power at any instance in the EM wave.

It is called the poynting vector of the EM wave which indicates the power

$$\boxed{E(z,t)_x \times H(z,t)_y = P(z,t)_z}$$

Power is carried by the EM wave in the propagation direction due to both the energy formats. the EM wave always have a finite power with it.

$$P(z,t)_z = \left(E_0 e^{-\alpha z}\right) e^{j(\omega t - \beta z)} \left(H_0 e^{-\alpha z}\right) e^{j(\omega t - \beta z)}$$

= Instantaneous Poynting Vector
= Power density at an instance of time at an instance of (point) of space

Instantaneous never useful we always want or an avg.

Time Averaged Poynting Vector

$$P(z)_{avg} = \frac{(E_0 e}{\sqrt{2}}$$

The average of any harmonic is never its radional traditional avg which always gives a zero value. It is called as R.M.S which is amplitude by $\sqrt{2}$ for any harmonic

Similarly for the harmonic power density the time average operation gives. $P(z)_{avg}$

$$P(z)_{avg} = E(RMS) \times H(RMS) \qquad 150$$

$$P(z)_{avg} = \left(\frac{E_0 e^{-\alpha z}}{\sqrt{2}}\right) \cdot \left(\frac{H_0 e^{-\alpha z}}{\sqrt{2}}\right) = \frac{1}{2} E_0 H_0 e^{-2\alpha z}$$

$$V_{avg} = \frac{1}{T}\int_0^T V(t)\, dt$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T V^2(t)\, dt} = \frac{V_{max}}{\sqrt{2}}$$

Average power of the wave exponentially decays at $2\alpha$ rate

$$\left.\begin{aligned} P(z)_{avg} &= \frac{1}{2}\frac{E_0^2}{\eta} e^{-2\alpha z} \\ &= \frac{1}{2}\eta H_0^2 e^{-2\alpha z} \end{aligned}\right\} \frac{1}{2}(E \times H^*)$$

As the power decays in the wave the medium acquire this power in the form of ohmic power. Hence

$$\sigma E^2 = J \cdot E$$

is the ohmic power dissipated to the medium per unit volume

The time average poynting vector can also be represented as

$$P(z)_{avg} = \frac{1}{2} E \times H^*$$

where a conjugate operation means the phases and the harmonic are cancelled and only amplitudes are used in power calculation

{$\therefore V^2(t)$ means → conjugate}

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In free space when $\alpha = 0$

$$P_{avg} = \frac{1}{2}\frac{E_0^2}{\eta} = \frac{1}{2}\eta H_0^2 = \frac{1}{2}E_0 H_0$$

B- Total Power Lost = (Intial Power − final Power) × Area

$$\text{Total Power Lost} = (\underset{\text{Density}}{\text{Intial Power}} - \underset{\text{Density}}{\text{final Power}}) \times \text{Area}$$

$$= \left(\frac{1}{2} \times \frac{200 \times 200}{80} - \frac{1}{2} \times \frac{\left(200 \times e^{-0.4 \times 0.6}\right)^2}{\eta}\right) \times \text{Area}$$

$$= \left(\frac{1}{2} \times \frac{(200)^2}{80} - \frac{1}{2}\frac{\left(200 \times e^{-0.4 \times 0.6}\right)^2}{\eta}\right) \times 4 \times 10^{-4}$$

$$= 0.0381$$

formula = $P(z)_{avg} = \frac{1}{2}\frac{E_0^2}{\eta} e^{-2\alpha z}$

for intialaly $z = 0$ and for final $z = 60\,cm$

If $\sigma = 0.8$ and $\eta$ = unknown

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Total Power lost $= \int (\sigma E^2)\, dV$

$= \int \sigma \left(E_0 e^{-\alpha z}\right)^2 dA\, dz$

$= \sigma \cdot E_0^2 A \int_{z=0}^{0.6} e^{-2\alpha z}\, dz$

$= 0.8 \times (200)^2 \times (4\times10^{-4}) \left[ e^{-2\times0.4\times0.6} - 1 \right]$

$= -4.8795$

B (i) $H = 50 \sin(\omega t - \beta z) a_x + 150 \sin(\omega t - \beta z) a_y$

$= 50 \sin(\omega t - \beta z)(a_x + 3a_y)$

$= 50\sqrt{10} \sin(\omega t - \beta z) \left(\frac{a_x + 3a_y}{\sqrt{10}}\right)$

$P_{avg} = \frac{1}{2} \eta H_0^2$

$\eta = 120\pi$

$P_{avg} = \frac{1}{2} (50\sqrt{10})^2 \times 120\pi$

$= \frac{1}{2} (2500\times10) \times 120\pi$

$= 4.71\times10^6$

$H = 50 \sin(\omega t - \beta z) a_x + 150 \cos(\omega t - \beta z) a_x$

$= 50 \sin(\omega t - \beta z) a_x [1 + 3j]$

$= 50(1+3j) \sin(\omega t - \beta z) a_x$

Orthogonality in time has the same effect as the orthogonality in space.

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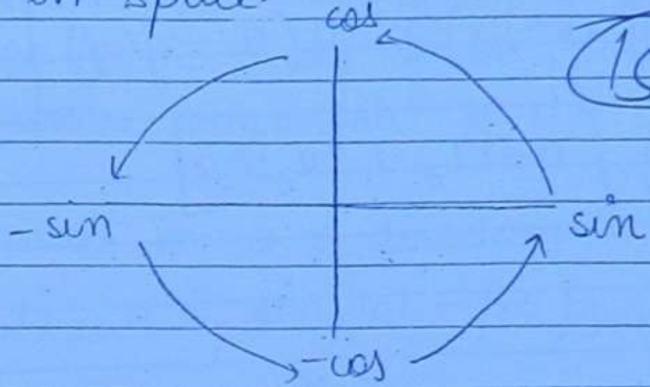


$H_0 = 50\sqrt{10}$.

$P_{avg} = \frac{1}{2} \times 120\pi \times (50\sqrt{10})^2 = 4.71 \times 10^6$

(ii) $H = 50 \sin(\omega t - \beta z) a_x + 150 \cos(\omega t - \beta z) a_y$

$H = 50 \sin(\omega t - \beta z)(a_x + j3 a_y)$

$E \times a_x = a_z$
$\times a_y = a_z$

$P_{avg} = \frac{1}{2}(E \times H^*)$

$E(z,t) = (50 \times \eta) \sin(\omega t - \beta z)(-a_y)$

$\{\because E \times a_x = a_z\}$

$+ (150 \times \eta) \cos(\omega t - \beta z)(a_x)$

$\{\because E \times a_y = a_z\}$

$E(z,t) = (50 \times \eta) \sin(\omega t - \beta z)(3j a_x - a_y)$

$H(z,t) = 50 \sin(\omega t - \beta z)(a_x + j3 a_y)$

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$$P_{avg} = \frac{1}{2}(E \times H^*)$$

$$P_{avg} = \frac{1}{2}\left[50\eta\,(3j a_z - a_y) \times 50\,(a_z - 3j a_y)\right]$$

$$P_{avg} = \frac{1}{2}\left[50 \times 50\eta\left[0 + a_z + 9a_z + 0\right]\right]$$

$$P_{avg} = \frac{1}{2} \times 50 \times 50 \times \eta \times 10$$

$$\vec{E} = (a_x + j a_y)\,e^{jkz - j\omega t} \qquad \bar{H} = \left(\frac{k}{\omega\mu}\right)(a_y + j a_x)\,e^{jkz - j\omega t}$$

$$E = (a_x + j a_y) \qquad H = \left(\frac{k}{\omega\mu}\right)(a_y + j a_x)$$

$$\frac{1}{2}(E \times H^*) = \frac{k}{2\omega\mu}\left(a_x + j a_y\right) \times \left(a_y - j a_x\right)$$

$$= \frac{k}{2\omega\mu}\left[a_z + 0 + 0 - a_z\right] = 0$$

= Null vector

Average power crossing = Density × Area

$$= \frac{1}{2}(E_0 H_0) \times \text{Area}$$

$$= \frac{1}{2}\left(\frac{50 \times 5}{12\cancel{\pi}}\right) \times \cancel{\pi}\left(\sqrt{24}\right)^2$$

$$= \frac{250}{2}\ \text{Watt}$$

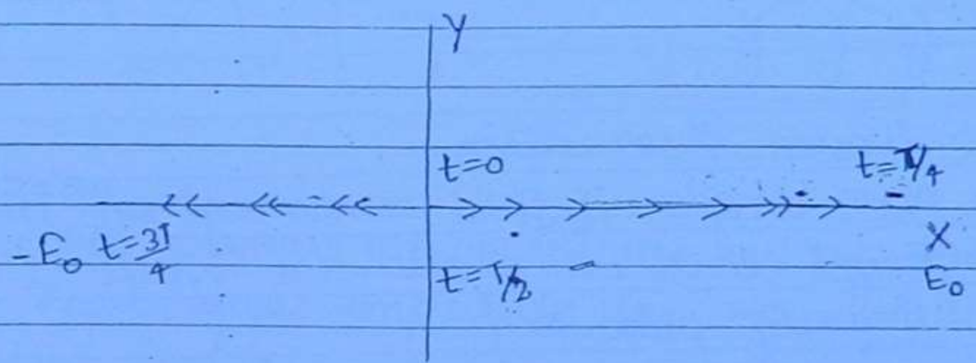
## Wave Polarization



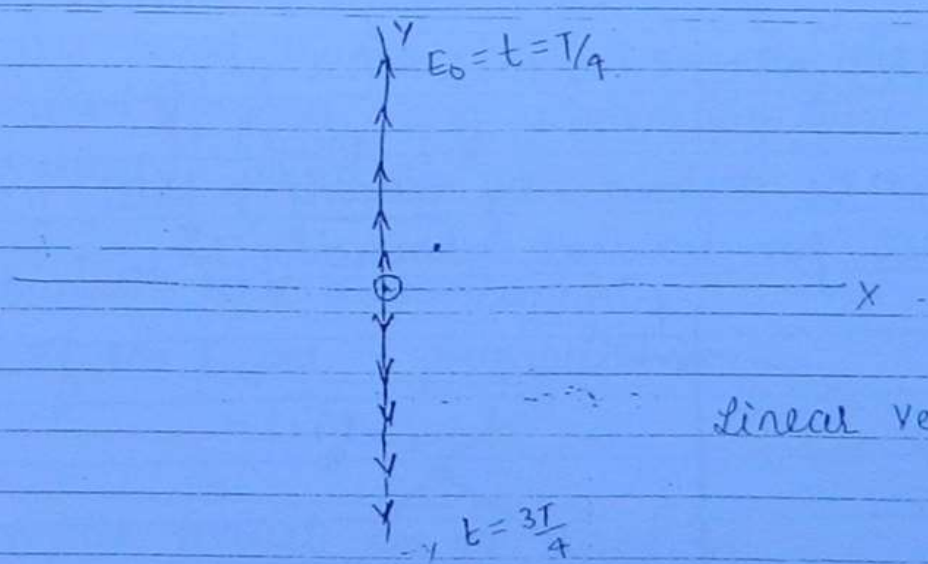
It defines the orientation of the E field in the plane wave. It is the study of the possible relative orientations of the planar components of the E-field.

case (i) Wave travel in $+z$ direction

$$E(z,t)_x = E_0 \sin(\omega t - \beta z)a_x$$




Linear Horizontal Polarization




Linear vertical Polarization

case (ii)

$$E(z,t) = E_{x0}\sin(\omega t - \beta z)a_x + E_{y0}\sin(\omega t - \beta z)a_y$$

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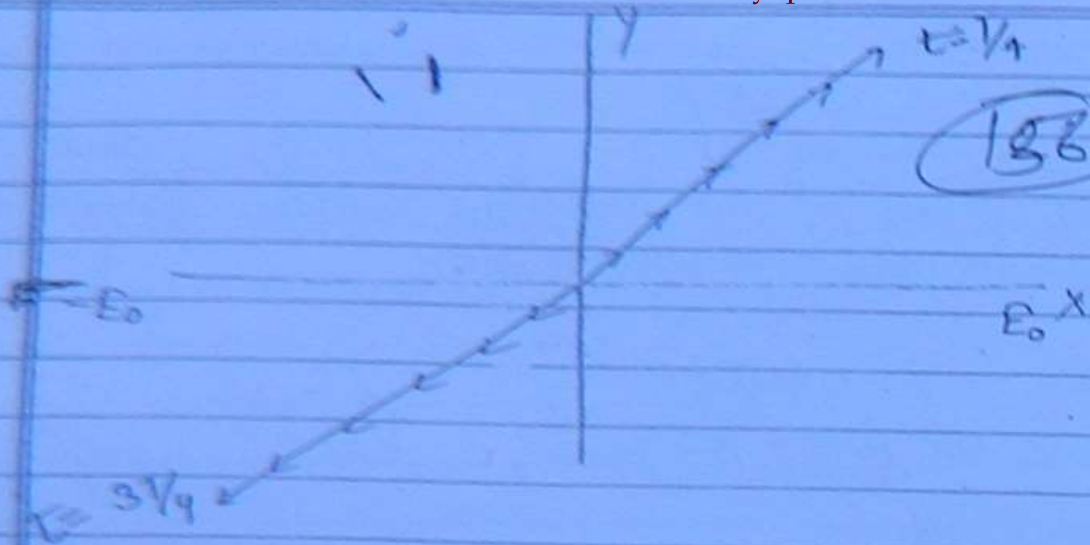
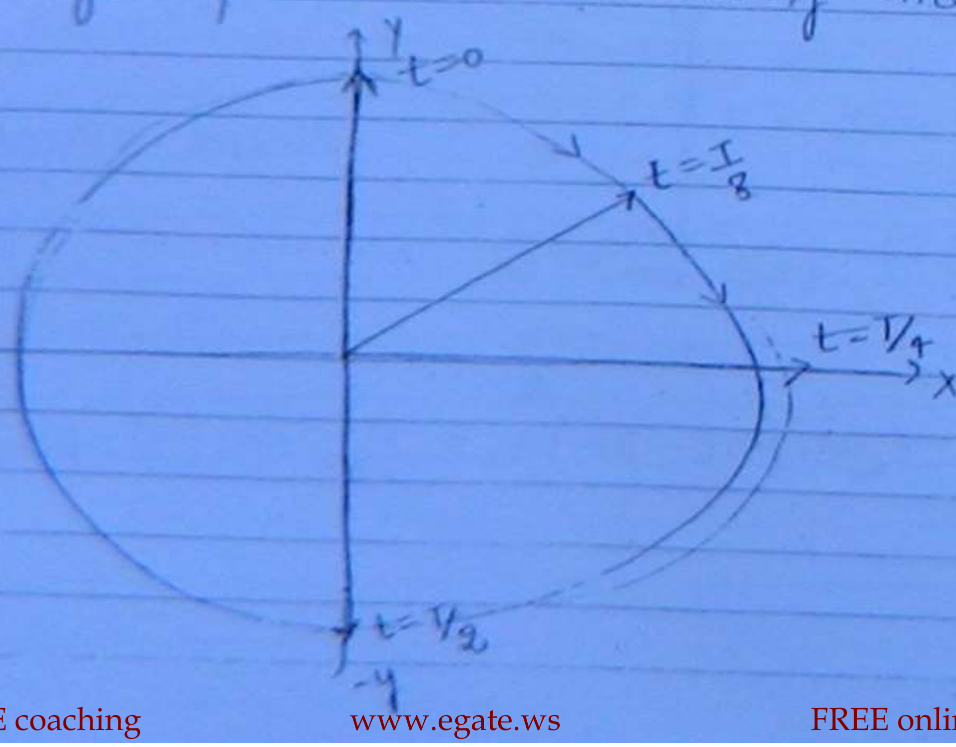



Summary for Linear Polarization
If the wave has single E field component or two planar component both in phase the wave is said to be linearly polarized.

e(iv) The wave travel in +z direction

$$E(z,t) = E_0 \sin(\omega t - \beta z)a_x + E_0 \cos(\omega t - \beta z)a_y$$

Polarization can be defined - by tracing locus E field on the (z=0) surface for various advancing time.



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$t = 0 \rightarrow E = E_y \ , \ E_x = 0$

$t = T/8 \rightarrow E_x = \dfrac{E_0}{\sqrt{2}} \ , \ E_y = \dfrac{E_0}{\sqrt{2}}$

$t = \dfrac{T}{4} \rightarrow E = E_x \ , \ E_y = 0$

$t = T/2 \rightarrow E = E_y \ , \ E_x = 0$

Summary

If the two planar component are out of phase by $90^\circ$ and have equal amplitude the var wave is circularly polarized.

Sence of Rotation - Left or Right

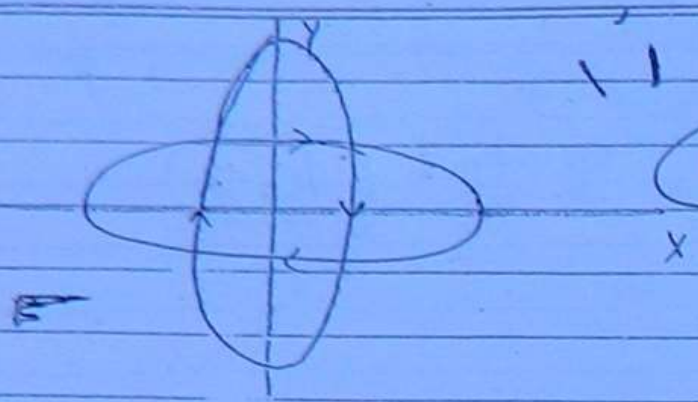
If the left hand thumb direct towards propagation direction and closed fingures along advancing time the wave is left circularly polarized.

ont: clockwise - time advancement
out of the paper - propagation
- LEFT -

ue.(4) The wave travel in +z direction

$$E(z,t) = E_{x0} \sin(\omega t - \beta t) a_x + E_{y0} \cos(\omega t - \beta z) a_y$$

$$\text{locus} = \left(\frac{X}{E_{x0}}\right)^2 + \left(\frac{Y}{E_{y0}}\right)^2 = 1$$

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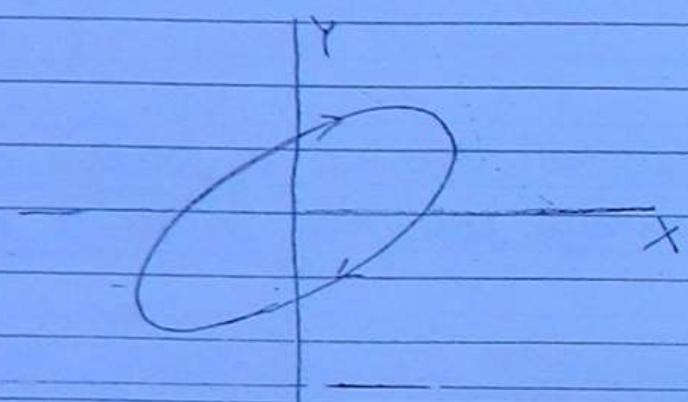
(vi) the wave Travel +z direction

$$E(z,t) = E_{x0}\sin(\omega t-\beta z)a_x + E_{y0}\sin(\omega t-\beta z+\theta^\circ)a_y$$




Summary (of ellipitcal Polarization)
If the planar component has any phase and unequal amplitude the wave is ellipitcally polarized

Every ellipse has Axial Ratio [AR]

$$\text{Axial Ratio [AR]} = \frac{\text{major axis}}{\text{minor axis}}$$

(Range) $AR = (1, \infty)$

circle linear

42 W.B (i) Amplitudes are unequal and both are in phase
same phase and two planar → linear

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(ii) $25 \sin(\omega t + 4x)(a_y + j a_z)$

$25 \sin(\omega t + 4x) a_y + 25 \cos(\omega t + 4x) a_z$ (amplitude are same)
circular.

(iii) $25 \sin(\omega t + 4x + 60^\circ)(a_y + j a_z)$

Polarization is always relative phase difference b/w the planar components.
(circular)

(iv) $25 \sin(\omega t + 4x)(a_y + (1+j) a_z)$

$25 \cancel{\sin(\omega t + 4x)} a_y + \cancel{25 \sin(\omega t + 4x) a_z}$
$+ 25 \cancel{\sin(\omega t + 4x)} j a_z$

$\Rightarrow 25 \cancel{\sin}(\omega t + \cancel{4x}) a_y + \cancel{25 \cos}(\cancel{\omega t + 4x}) a_z + 25 \sin(\omega t + 4x) a_z$

$$j = e^{j90}$$

$$(1+j) = \sqrt{2} e^{j45}$$

$25 \sin(\omega t + 4x)(a_y + \sqrt{2} e^{j45} a_z)$ Elliptical

(v) $25 \sin(\omega t + 4x)(a_y + 2e^{j60} a_z)$ Elliptical

(vi) $25 \sin(\omega t + 4x)((1-j) a_y + (1+j) a_z)$

$25 \sin(\omega t + 4x)(\sqrt{2} e^{-j45} a_y + \sqrt{2} e^{j45} a_z)$

circular

44 $E(t) = [E_1 \cos\omega t \, a_x - E_2 \sin\omega t \, a_y] \, e^{-jkz}$

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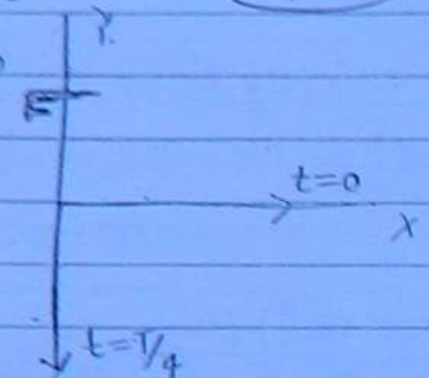
at $t=0$ $E_x$ $E_y = 0$

at $t = \frac{T}{4}$ $E_y$ $E_x = 0$



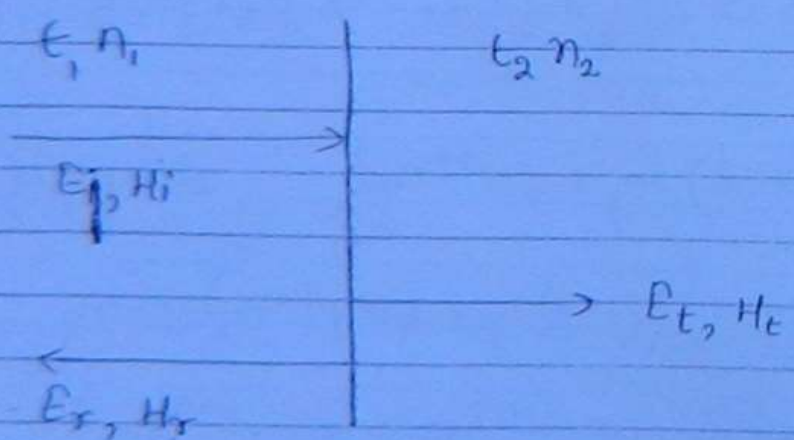

Elliptically polarized → left

Table.

| $E_x$ | $E_y$ | $P_z$ | Polarization |
|---|---|---|---|
| sin | cos | +z | LCP |
| sin | cos | −z | RCP |
| cos | sin | +z | RCP |
| sin | − cos | +z | RCP |

Reflection / Transmission of Plane Waves

Normal Incidence → Dielectric - Dielectric



Put $\tau = (1+\Gamma')$

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$$1-\Gamma' = \frac{\eta_1}{\eta_2}(1+\Gamma')$$

$$\Rightarrow \boxed{\Gamma' = \frac{\eta_2-\eta_1}{\eta_2+\eta_1}}$$

$$\eta_1 = \sqrt{\frac{\mu_0}{\epsilon_1}} \qquad \eta_2 = \sqrt{\frac{\mu_0}{\epsilon_2}}$$

for dielectric-dielectric

$$\boxed{\Gamma' = \frac{\sqrt{\epsilon_1}-\sqrt{\epsilon_2}}{\sqrt{\epsilon_1}+\sqrt{\epsilon_2}}}$$

Extension 1. $1+\Gamma'_H = \tau_H$

2. $\Gamma'_H = -\Gamma'_E = \dfrac{\eta_1-\eta_2}{\eta_1+\eta_2}$

3. $1+\Gamma'_p = \tau_p$

$$\Gamma'_p = \Gamma'_E \cdot \Gamma'_H = -\Gamma'^2_E = -\Gamma'^2_H$$

Summary:

If $\epsilon_1 > \epsilon_2$ $\Gamma'_E = +ve$

$$1+\Gamma'_E = \tau_E = \frac{E_t}{E_i} > 1$$

$\Gamma'_H = -ve$

$1+\Gamma'_H = \tau_H$ $\quad H_t < 1$ $\quad \boxed{\tau_p < 1 \text{ Always}}$

45 $\epsilon_1 = \epsilon_0 \qquad \epsilon_{r} = 4.0$ 162

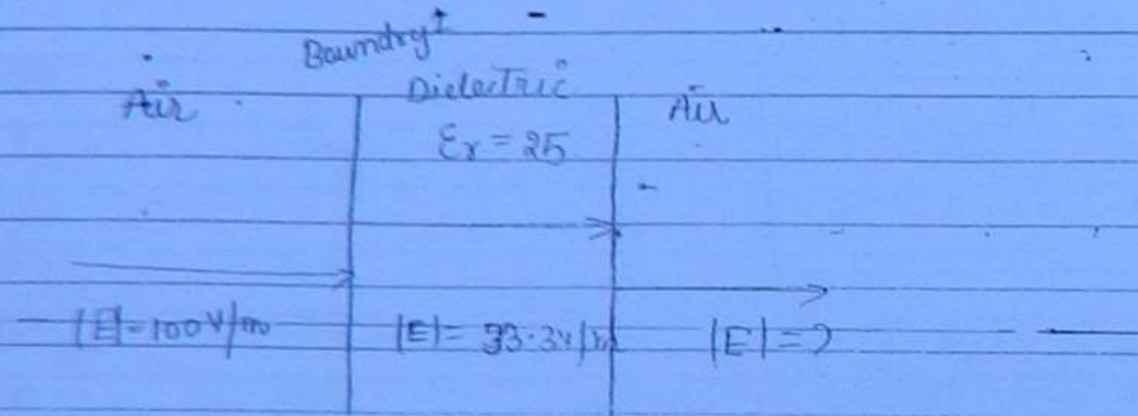
$$\Gamma_E = \frac{1-\sqrt{4}}{1+\sqrt{4}} = \frac{-1}{3} \longrightarrow \tau_E = 1 + \left(\frac{-1}{3}\right) = \frac{2}{3}$$

$$\Gamma_H = \frac{+1}{3} \longrightarrow \tau_H = 1 + \frac{1}{3} = \frac{4}{3}$$

$$\Gamma_P = \frac{-1}{9}$$

$$\tau_P = 1 - \frac{1}{9} = \frac{8}{9} \longrightarrow \tau_P = \tau_E \cdot \tau_H = \frac{8}{9}$$

Q




Sol

Boundary 1

$$\Gamma_E = \frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}} = \frac{1-5}{1+5} \Rightarrow \frac{-2}{3}$$

$$\tau_E = 1 + \Gamma_E = 1 - \frac{2}{3} = \frac{1}{3}$$

$$\tau_E = \frac{1}{3} = \frac{E_t}{E_i}$$

$$E_t = \frac{1}{3} \times 100 = 33.3 \text{ V/m}$$

Boundary 2

$$\Gamma_E = \frac{5-1}{5+1} = \frac{4}{6} = \frac{2}{3}$$

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$E_t > E_i$ ~~or E~~

For any interface E incident and E transmitted follow the relationship which depends on $\epsilon_1$ and $\epsilon_2$. such that.

$$E_t > E_i \quad \& \quad E_t < E_i$$

it depends on $\epsilon_1$ & $\epsilon_2$.

If the transmitted E flied is greater than E incident It has to be companisted $H_t <$ H incident so that the transmitted power is always less than incident power.

$$H_t < H_i \quad \text{if} \quad E_t > E_i$$
$$H_t > H_i \quad \text{if} \quad E_t < E_i$$

$$P_t < P_i \text{ always}$$

$$E_i = \eta_1 H_i$$
$$E_t = \eta_2 H_t$$
$$E_r = -\eta_1 H_r$$

$$a_x \times a_y = a_z$$
$$a_x \times -a_y = -a_z$$
$$-a_x \times a_y = -a_z$$

when the propagation direction changes only E or H will negate its sign i.e either E or H undergoes a phase reversal but not go both.

Applying Boundary Condition

$$E_{t1} = E_{t2}$$ (when propagation is normal the both fields are tangential)

$$E_i + E_r = E_t \checkmark$$

$$H_{t1} = H_{t2}$$

$H_i + H_r = H_t$

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divide the eq$^n$ $E_i + E_r = E_t$ by $E_i$

$$\frac{E_i}{E_i} + \frac{E_r}{E_i} = \frac{E_t}{E_i}$$

the fraction of the field component reflecting back into the first medium it is measured by the term $\frac{E_r}{E_i}$ = Reflection coefficient ($\Gamma$) and similarly the transmitted power $\frac{E_t}{E_i}$ = transmission coefficient ($\tau$)

$$1 + \frac{E_r}{E_i} = \frac{E_t}{E_i}$$

$$\frac{E_r}{E_i} = \Gamma \quad , \quad \frac{E_t}{E_i} = \tau$$

$$\boxed{1 + \Gamma = \tau}$$

Again using the same analysis $H_i + H_r = H_t$

$$\frac{E_i}{\eta_1} - \frac{E_r}{\eta_1} = \frac{E_t}{\eta_2}$$

$$E_i - E_r = \frac{\eta_1}{\eta_2} E_t$$

divide the above eq by $E_i$

$$1 - \frac{E_r}{E_i} = \frac{\eta_1}{\eta_2} \frac{E_t}{E_i}$$

$$1 - \Gamma = \frac{\eta_1}{\eta_2} \tau$$

$$T_E = 1 + \Gamma_E \Rightarrow 1 + \frac{2}{3} = \frac{5}{3}$$

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$$T_E = \frac{5}{3} = \frac{E_t}{E_i}$$

$$E_t = \frac{5}{3} \times 33.3 = 55.5 \text{ V/m}$$

46WB

$$E_i = E_o \cos(\omega t - \beta z)\, a_y$$

$$\frac{\omega}{\beta} = \frac{3 \times 10^9 \pi}{10\pi} = 3 \times 10^8$$

It means $\epsilon_R = 1$

$$\Gamma_E = \frac{1-2}{1+2} = \frac{-1}{3}$$

$$T_E = 1 + \Gamma_E = 1 + \frac{1}{3} = \frac{2}{3}$$

(ω is a source property) It doesn't change from medium to medium

$$T_E = \frac{E_t}{E_i}$$

$$E_t = \frac{2}{3} E_o \cos(\omega t - 2\beta z)\, a_y \quad \underline{\text{Ans}}$$

$$\beta =$$

Note
- Among the properties of a wave $\alpha, \beta, \gamma, \eta, \lambda, v_p$ all these properties are material dependent.
- They change from medium to medium
- ∴ $E_t$ should be ajusted accordingly.
- ω – source property and never changes in any medium
- For $E_r$ calculations all parameters are same except propagation direction

Q $E_i \longrightarrow H_r = ?$ 166

Air - Dielectric

$E_i \xrightarrow{\eta_1} H_i \xrightarrow{\Gamma_H} H_r$

$\downarrow \Gamma_E$

$E_r$

$\downarrow -\eta_1$

$H_r$

17 W.B

$$\frac{E_t}{E_r} = -2 \Rightarrow \frac{E_t}{E_i \times \frac{E_r}{E_i}} \Rightarrow \frac{\tau}{\Gamma} \Rightarrow \frac{1+\Gamma}{\Gamma} = -2$$

$$\Gamma = \frac{-1}{3}$$

$$\Gamma_E = \frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}} = \frac{1 - \frac{\sqrt{\epsilon_2}}{\sqrt{\epsilon_1}}}{1 + \frac{\sqrt{\epsilon_2}}{\sqrt{\epsilon_1}}} = \frac{-1}{3} \Rightarrow \frac{4}{3} = \frac{2}{3}\sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$= 4$ Ans

8 W.B

$$\frac{\sigma}{\omega\epsilon} >> 1 \Rightarrow \frac{5}{2\pi \times 25KHz \times 80 \times \frac{1}{36\pi \times 10^9}} \Rightarrow$$

$\frac{\sigma}{\omega\epsilon} >> 1$ so it is a conductor

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}} \text{ (for good conductor)}$$

$$\alpha = \sqrt{\frac{2\pi \times 25 \times 10^3 \times 4\pi \times 10^{-7} \times 5}{2}}$$

$$\alpha = 0.702$$

$$10 = 100\, e^{-\alpha z}$$

$$\ln\left(\frac{10}{100}\right) = -0.70\, z$$

$$\frac{-2.302}{-0.70} = z$$

$$z = 3.28\, m$$

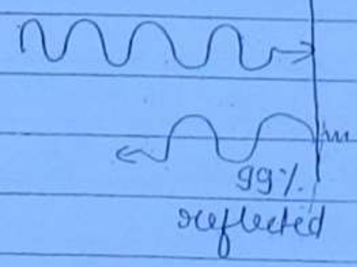
50 W

Dielectric - Conductor Interface.

(167)

Dielectric | Conductor

$\eta_1$ | $\eta_2$




1% travel.

$\Gamma_H = +1$

$\Gamma_E = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \simeq -1$

$\eta_1 = 377/\sqrt{\epsilon_r}$

$\eta_2 = \sqrt{\dfrac{j\omega\mu}{\sigma}} \simeq 0$

$E_i - E_r$

$\Gamma_E = -1 = \dfrac{E_r}{E_i}$

Note:- (1)

- when $\Gamma_E$ is very close to -1 $\Gamma_H$ is very close to +1.
- $\Gamma_E$ being -ve means the reflected electric field is changing its sign and Hence cancels with the incident electric field. and Hence at the Boundary electric field is zero and magnetic field is maximum

Note (2) when the wave is normally incident the fields are obiviously tangential and Hence E field should be zero along the conductor surface as $\boxed{E_{tan} = 0}$

(C) Electric field is minimum and magnetic field is maximum.

48 W B

Air $= \eta_1 = 377$

thick Magnetic $\eta_2 = \sqrt{\dfrac{\mu}{\epsilon}} = 1$ as $\mu^* = \epsilon^*$

$\Gamma = \dfrac{1 - 377}{1 + 377} \simeq -1$

(Real parts are also equal if complex is equal.)

(B) Oblique Incidence (Inclined at any angle)

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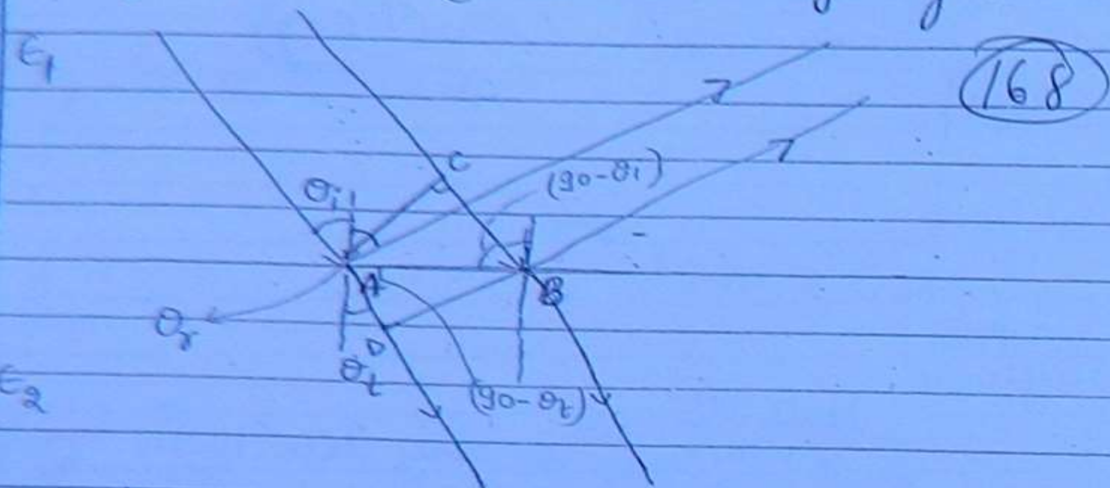



Snell's law.

- Relate the $\theta_i$, $\theta_r$, $\theta_t$ angles

Proof.

A/C $\xrightarrow{D}$ Equidistance from the source i.e A/C are in phase points

CB = distance travelled by the wave in medium 1

B/D are in-phase points

AD = distance travelled by the wave in medium 2.

The ratio of the distance in different medias should be equal to the ratio of the velocities in those medium

$$\frac{CB}{AD} = \frac{v_1}{v_2}$$

$$\frac{AB\cos(90-\theta_i)}{AB\cos(90-\theta_t)} = \frac{v_1}{v_2} = \frac{\frac{1}{\sqrt{\mu_0\epsilon_1}}}{\frac{1}{\sqrt{\mu_0\epsilon_2}}} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$$\boxed{\frac{\sin\theta_i}{\sin\theta_t} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}}$$

Refractive index of medium 2 w.r.t to medium 1

Law (2) However angle of incidence is equal to angle of reflection so

$$\boxed{\theta_i = \theta_r}$$

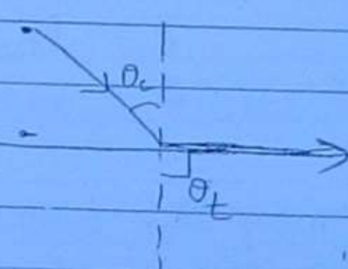


Critical Angle and Total internal Reflection (TIR)

If $\theta_t = 90^\circ$; the wave in the II medium is along the surface not into the medium

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{\sin\theta_i}{\sin 90^\circ} = \sin\theta_i = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$$\sin\theta_c = \sin\theta_i = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$



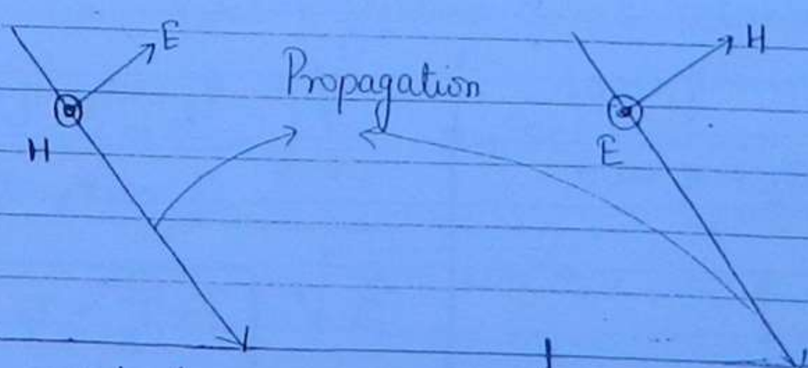

$\theta_i = \theta_c$ = critical angle

If $\theta_i > \theta_c$

Total internal reflection takes place and zero transmissions

S and P- polarized waves:

Medium 1




Medium 2

vertical polarization

P- polarization

Horizontal Polarization

S-polarization

In oblique incidence electric field orientation is crucial in boundary condition and hence subsequent $\Gamma$ and $\tau$ calculation

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## S-Polarized : Boundary condition

$$E_{t_1} = E_{t_2}$$

$$E_i + E_r = E_t$$

$$\boxed{1 + \Gamma_s = \tau_s}$$

## P-polarized : Boundary condition

$$E_{t_1} = E_{t_2}$$

$$E_i \cos\theta_i + E_r \cos\theta_r = E_t \cos\theta_t$$

~~Apply magnetic boundary condition in the same way.~~ we can derive for $\Gamma$ and $\tau$

$$\boxed{1 + \Gamma_p = \tau_p \frac{\cos\theta_t}{\cos\theta_i}}$$

$$\Gamma_s = \frac{\eta_2 \sec\theta_t - \eta_1 \sec\theta_i}{\eta_2 \sec\theta_t + \eta_1 \sec\theta_i}$$

$$\boxed{1 + \Gamma_s = \tau_s}$$

$$\Gamma_p = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$\boxed{1 + \Gamma_p = \tau_p \frac{\cos\theta_t}{\cos\theta_i}}$$

Substituting the parameter of dielectric - dielectric interface and converting the incident angle into transmitted angle

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$$\Gamma_s' = \frac{\sqrt{\epsilon_1}\cos\theta_i - \sqrt{\epsilon_2}\cos\theta_t}{\sqrt{\epsilon_1}\cos\theta_i + \sqrt{\epsilon_2}\cos\theta_t}$$

$$= \frac{\cos\theta_i - \sqrt{\frac{\epsilon_2}{\epsilon_1}}\sqrt{1 - \frac{\epsilon_1}{\epsilon_2}\sin^2\theta_i}}{\cos\theta_i + \sqrt{\frac{\epsilon_2}{\epsilon_1}}\sqrt{1 - \frac{\epsilon_1}{\epsilon_2}\sin^2\theta_i}}$$

$$\boxed{\Gamma_s' = \frac{\cos\theta_i - \sqrt{\frac{\epsilon_2}{\epsilon_1} - \sin^2\theta_i}}{\cos\theta_i + \sqrt{\frac{\epsilon_2}{\epsilon_1} - \sin^2\theta_i}}}$$

$$\boxed{\Gamma_p' = \frac{\frac{\epsilon_2}{\epsilon_1}\cos\theta_i - \sqrt{\frac{\epsilon_2}{\epsilon_1} - \sin^2\theta_i}}{\frac{\epsilon_2}{\epsilon_1}\cos\theta_i + \sqrt{\frac{\epsilon_2}{\epsilon_1} - \sin^2\theta_i}}}$$

Let $\Gamma_s' = 0$ i.e zero Reflection for s polarized wave

$$0 = \frac{\cos\theta_i - \sqrt{\frac{\epsilon_2}{\epsilon_1} - \sin^2\theta_i}}{\cos\theta_i + \sqrt{\frac{\epsilon_2}{\epsilon_1} - \sin^2\theta_i}}$$

$$\cos^2\theta_i = \frac{\epsilon_2}{\epsilon_1} - \sin^2\theta_i$$

$$\frac{\epsilon_2}{\epsilon_1} = 1$$

$$\boxed{\epsilon_1 = \epsilon_2}$$

It is impossible for any in incident angle to have zero reflections for s-polarized wave

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let $\Gamma_p = 0$

$$0 = \frac{\epsilon_2}{\epsilon_1}\cos\theta_i - \sqrt{\frac{\epsilon_2}{\epsilon_1} - \sin^2\theta_i}$$

$$\left(\frac{\epsilon_2}{\epsilon_1}\right)^2 \cos^2\theta_i = \frac{\epsilon_2}{\epsilon_1} - \sin^2\theta_i$$

$$\left(\frac{\epsilon_2}{\epsilon_1}\right)^2 (1 - \sin^2\theta_i) = \frac{\epsilon_2}{\epsilon_1} - \sin^2\theta_i$$

$$\sin^2\theta_i \left(1 - \left(\frac{\epsilon_2}{\epsilon_1}\right)^2\right) = \frac{\epsilon_2}{\epsilon_1} - \left(\frac{\epsilon_2}{\epsilon_1}\right)^2$$

$$\sin^2\theta_i = \frac{\frac{\epsilon_2}{\epsilon_1}\left(1 - \frac{\epsilon_2}{\epsilon_1}\right)}{\left(1 + \frac{\epsilon_2}{\epsilon_1}\right)\left(1 - \frac{\epsilon_2}{\epsilon_1}\right)}$$

$$\boxed{\sin\theta_i = \frac{\sqrt{\epsilon_2}}{\sqrt{\epsilon_1 + \epsilon_2}}}$$

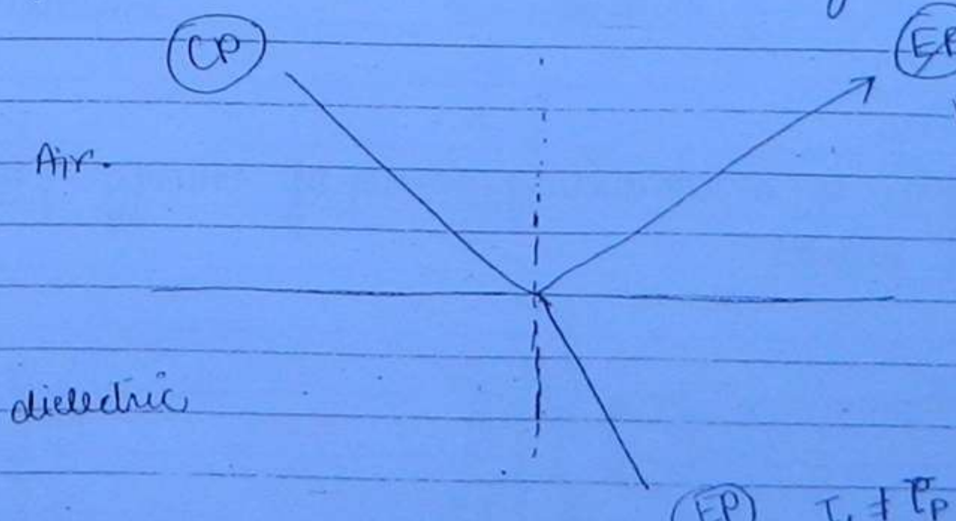
$$\boxed{\tan\theta_i = \sqrt{\frac{\epsilon_2}{\epsilon_1}}}$$

Zero Reflection & complete transmission is possible for the p-polarized wave. at a specific angle called as Brewster's angle

Brewster's Angle :

$$\boxed{\tan\theta_B = \sqrt{\frac{\epsilon_2}{\epsilon_1}}}$$

| Total Internal Reflection<br>critical Angle | Zero Reflection<br>Brewster angle |
| --- | --- |
| $\sin\theta_c = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$ | 1. $\tan\theta_B = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$ |
| $\epsilon_1 > \epsilon_2$ is the condition | No such restriction. Any two media can have. |
| All incidence angle is greater than critical angle have the same phenomena. All $\theta_i > \theta_c$ have the total internal reflection | 2. At exactly one single angle $\theta_i = \theta_B$ - zero reflection - |
| either s or p polarized both can have critical angle | 3. only p polarized wave can have Brewster's angle. |




EP LP

$\Gamma_s \neq \Gamma_p$

If $\Gamma_p = 0$

EP $\tau_s \neq \tau_p$

Note: When the incident wave is circularly polarized with equal s,p components the reflected wave must be elliptically polarized with unequal s and p components as $\Gamma_s' \neq \Gamma_p'$.

The transmitted is also ellipitcally as

$T_s \neq T_p$

If $\Gamma_p' = 0$ the only chance for the reflected wave to be linearly polarized is when $\Gamma_p' = 0$ so that the reflected wave has only s component. However the transmitted wave is always elliptically polarized.

Hence at Brewster angle of incidence the reflected is linearly polarized and transmitted is ellipitically polarized.

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Energy Propagation

High frequency $> 10^{13} Hz - 10^{15} Hz$ — Ex: light, xrays — E/H wave, $\sigma = 0$, free space

$(10^3 - 10^9)$ KHz MHz GHz — wireless comm. Broadcast; wired point-point

← Antenna →

Low frequency $<$ Hz - KHz — eg Power waves, 230V AC — V/I waves, $\sigma = \infty$, conductors

Transmission line:

A transmission line is a conducting mean of energy propagation using V-I waves.

$\xrightarrow[\text{Medium}]{\text{EM wave}}$ Harmonic source $w$ $\left.\begin{matrix} E_s \\ H_s \end{matrix}\right\} e^{j\omega t} \longrightarrow$ Propagation Space Harmonic $\left.\begin{matrix} E(z) \\ H(z) \end{matrix}\right\} e^{-\gamma z}$

$\sigma, \epsilon, u$

$$\gamma = \sqrt{j\omega u(\sigma + j\epsilon\omega)}$$

$$\eta = \sqrt{\frac{j\omega u}{(\sigma + j\epsilon\omega)}}$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$1 + \Gamma = \tau$$

Med 1 | Med 2

$\xrightarrow[\text{Transmission line}]{\text{VI wave}}$ Harmonic source $\omega$ $\left.\begin{matrix} V_s \\ I_s \end{matrix}\right\} \longrightarrow$ Propagation length Harmonic $\left.\begin{matrix} V(x) \\ I(x) \end{matrix}\right\} e^{-\gamma x}$

$R, G, C, L$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$Z_0 = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$1 + \Gamma = \tau$$

Line | Load

$Z_0$ | $Z_L$

Geometries of a Transmission line.

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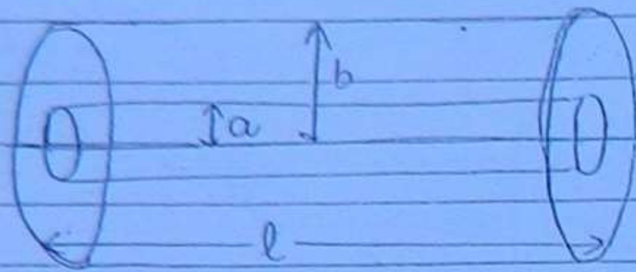
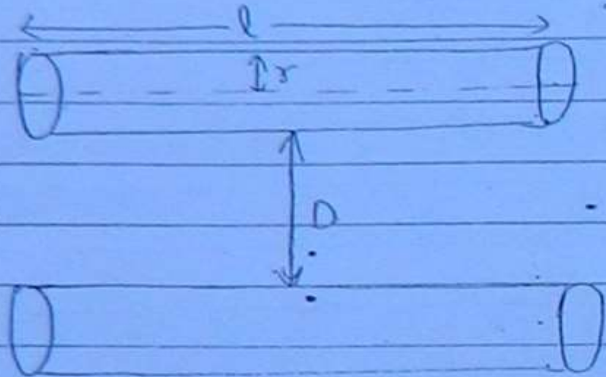



coaxial cable




$D >> r$

Parallel line

Primary Constants of a line ( R, L, G, C)

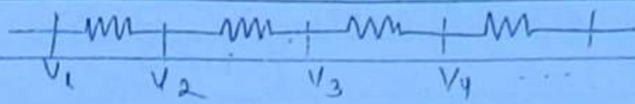
Resistance (R)

- It is the resistance of the conducting material along the length of the line

$$\boxed{R = \frac{\rho l}{A}}$$ — distributed

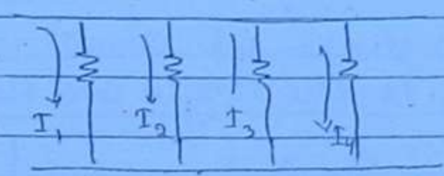
$$R = \frac{R}{l} = \frac{\text{ohm}}{\text{metre}}$$

- The primary constt is the resistance per unit. length i.e the instantaneous value as the study is made on a per unit length basis. extending it for any length.
- Resistance causes voltage decay all along the line as it appears in series.




conductance (G)

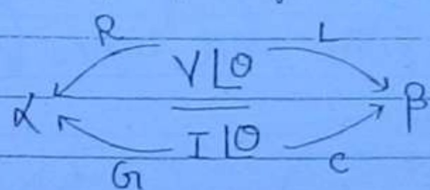
It is the conductance of the dielectric (insulating material) b/w the lines




- Conductance b/w the line leads to <u>current leakage</u> and it is distributed everywhere b/w the lines
- Hence the primary constt is G per unit length

$$G = \frac{G}{l} = \frac{\text{mho}}{\text{meter}} \quad \text{—— distributed}$$

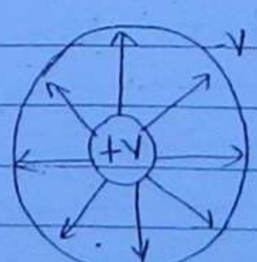
- It is the shunt property b/w the lines

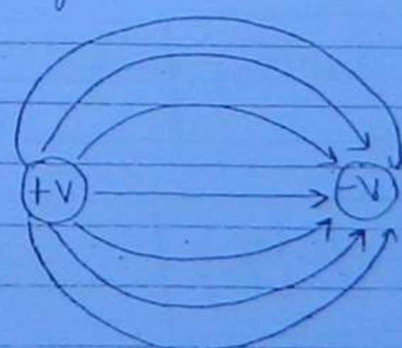



Both R and G cause attenuation on the line.

capacitance (c)

The voltages on the line have charge accumulation and hence have E-field surrounding the line and have capacitance



When we give voltage ... line then electric field

" " current ... then get magnetic field

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$$\boxed{C = \frac{2\pi\epsilon_0 l}{\ln\left(\frac{b}{a}\right)}}$$ coaxial cable

$$\boxed{C = \frac{\pi\epsilon_0 l}{\ln\left(\frac{D}{r}\right)}}$$ parallel wire

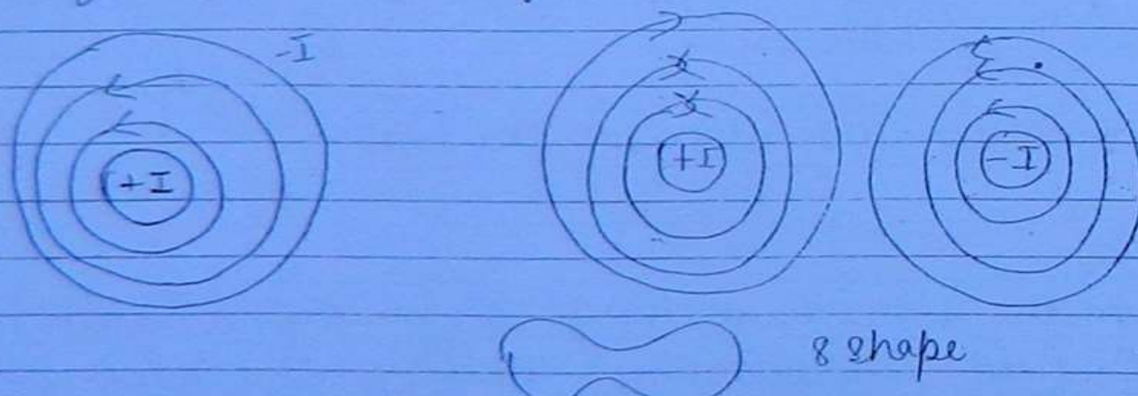
- The capacitance is proportional to the length and Hence it is said to be distributed.
- The primary constt c is $\frac{C}{l}$

$$\frac{C}{l} = \frac{\text{farad}}{\text{length}} \quad \text{— (distributed) —}$$

- The capacitance appears in shunt b/w the lines

Inductance (L)

The currents on the line have a charge flow and hence a H field surrounding the line and hence inductance




$$\boxed{L = \frac{\mu_0 \cdot l}{2\pi} \ln(b/a)}$$ — coaxial cable

$$\boxed{L = \frac{\mu_0 \cdot l}{\pi} \ln\left(D/r\right)}$$ — parallel wire

Inductance is propotional to the length and hence is distributed on the line

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the primary const therefore is $L = \frac{L}{l}$

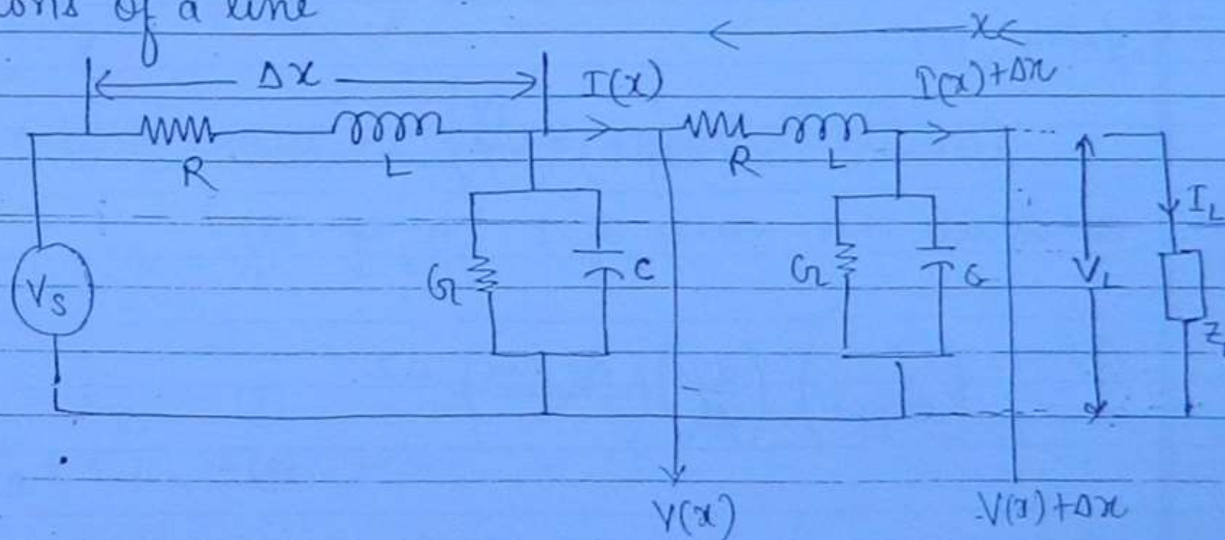
$$L = \frac{L}{l} = \frac{\text{Henry}}{\text{meter}}$$

Inductance comes in series due to the flowing current.

$$\boxed{LC = \mu_0 \epsilon_0}$$

Distributed inductance and distributed conductance product is $\mu_0 \epsilon_0$ for any transmission line

VI Equations of a line




x — always starts from the load and increases towards the source.

i.e at $x = 0$ $V = V_L$
$I = I_L$

at $x = l$, $V = V_s$
$I = I_s$

Jai Durga Maa

classmate

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$Z$ of the line of $\Delta x$ length $= (R+j\omega L)\,\Delta x$ } Impedence in series

$Y$ " " " $= (G+j\omega C)\,\Delta x$ } Admittance in shunt

$\Delta V = I(x)\,(R+j\omega L)\cdot\Delta x$

$$\frac{dV}{dx} = I\cdot(R+j\omega L) \quad \text{——} \quad (1)$$

$\Delta I = V(x)\,(G+j\omega C)\,\Delta x$ $\qquad I = \frac{V}{R} = V \times \text{Admittance}$

$$\Delta V = I(x)\,(R+j\omega L)\cdot\Delta x \quad \text{——} \quad (2)$$

from (1)

$$I = \frac{1}{(R+j\omega L)}\left(\frac{dV}{dx}\right)$$

substitute this value of $I$ in eq (2)

$$\Delta V = \left(\frac{1}{\cancel{R+j\omega L}}\right)\left(\frac{dV}{\cancel{dx}}\right)\left(\cancel{R+j\omega L}\right)\cdot\Delta x$$

$$\frac{d}{dx}\left(\left(\frac{1}{R+j\omega L}\right)\frac{dV}{dx}\right) = V\cdot(G+j\omega C)$$

$$\frac{d^2V}{dx^2} = (R+j\omega L)(G+j\omega C)\,V \quad \text{——} \quad (3)$$

$$\frac{d^2I}{dx^2} = (R+j\omega L)(G+j\omega C)\,I \quad \text{——} \quad (4)$$

In transmission ...

hence the V-I Eq$^n$ are harmonic functions of length of the line since energy propagates along the length of a line

$\sqrt{(R+j\omega L)(G+j\omega C)} = \gamma$ (per m) $(m^{-1})$ $\left(\frac{ohm}{m}\right)\times\left(\frac{mho}{m}\right)$

$$\frac{d^2V}{dx^2} - \gamma^2 V = 0 \quad \text{— (3)}$$

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$$\frac{d^2I}{dx^2} - \gamma^2 I = 0 \quad \text{— (4)}$$

Solutions of (3) & (4)

$$V(x) = \left(C_1 e^{-\gamma x} + C_2 e^{\gamma x}\right)$$

$$I(x) = \left(C_3 e^{-\gamma x} + C_4 e^{\gamma x}\right)$$

Using Boundary conditions called intial conditions

$$x=0, \quad V = V_L, \quad I = I_L$$

$$V_L = C_1 + C_2 \quad \text{— (7)}$$
$$I_L = C_3 + C_4 \quad \text{— (8)}$$

using eq (1)

$$\frac{dv}{dx} = I\,(R+j\omega L)$$

$$-\gamma C_1 e^{-\gamma x} + \gamma C_2 e^{\gamma x} = I\,(R+j\omega L)$$

at $x=0$

$$-\gamma C_1 + \gamma C_2 = I_L\,(R+j\omega L)$$

$$(C_2 - C_1) = \frac{I_L\,(R+j\omega L)}{\gamma}$$

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$$C_2 - C_1 = \frac{I_L(R+j\omega L)}{\sqrt{(R+j\omega L)(G+j\omega C)}}$$

$$C_2 - C_1 = I_L \cdot \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$C_2 - C_1 = I_L \times Z_0 \quad \text{—} \quad (9)$$

Again using $dI = V(x)(G+j\omega C)$

$$\frac{dI}{dx} = V(x)(G+j\omega C)$$

$$-\gamma C_3 e^{-\gamma x} + \gamma C_4 e^{\gamma x} = V(G+j\omega C)$$

at $x=0$

$$\gamma(C_4 - C_3) = V(G+j\omega C)$$

$$C_4 - C_3 = \frac{V(G+j\omega C)}{\sqrt{(R+j\omega L)(G+j\omega C)}}$$

$$\frac{V_L}{Z_0} = C_4 - C_3 \quad \text{—} \quad (10)$$

On solving (7) (8) (9) (10) we get the values of $C_1, C_2, C_3, C_4$

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$$C_1 = \frac{V_L - I_L Z_0}{2} \qquad C_2 = \frac{V_L + I_L Z_0}{2}$$

$$C_3 = \frac{I_L - V_L/Z_0}{2} \qquad C_4 = \frac{I_L + V_L/Z_0}{2}$$

$$V(x) = C_1 e^{-\gamma x} + C_2 e^{\gamma x}$$

Put $C_1$ & $C_2$ in the above Eqn.

$$V(x) = \frac{V_L - I_L Z_0}{2} e^{-\gamma x} + \frac{V_L + I_L Z_0}{2} e^{\gamma x}$$

$$= \frac{V_L}{2}\left[\left(1 - \frac{Z_0}{Z_L}\right) e^{-\gamma x} + \left(1 + \frac{Z_0}{Z_L}\right) e^{\gamma x}\right]$$

$$= \frac{V_L}{2Z_L}\left[(Z_L - Z_0) e^{-\gamma x} + (Z_L + Z_0) e^{\gamma x}\right]$$

$$\boxed{V(x) = \frac{V_L}{2Z_L}(Z_L + Z_0)\left[e^{\gamma x} + \frac{(Z_L - Z_0)}{(Z_L + Z_0)} e^{-\gamma x}\right]}$$

$$I(x) = \left(C_3 e^{-\gamma x} + C_4 e^{\gamma x}\right)$$

$$= \frac{I_L - \frac{V_L}{Z_0}}{2} e^{-\gamma x} + \frac{I_L + \frac{V_L}{Z_0}}{2} e^{\gamma x}$$

$$= \frac{I_L}{2}\left[\left(1 - \frac{Z_L}{Z_0}\right) e^{-\gamma x} + \left(1 + \frac{Z_L}{Z_0}\right) e^{\gamma x}\right]$$

$$\boxed{I(x) = \frac{I_L(Z_L + Z_0)}{2Z_0}\left[e^{\gamma x} + \left(\frac{Z_0 - Z_L}{Z_0 + Z_L}\right) e^{-\gamma x}\right]}$$

forward rel wave
source - load
direction

rx direction
load - source
[illegible]

1. Every transmission line has two waveforms on it the forward wave and the reflected wave.

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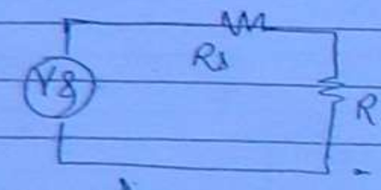
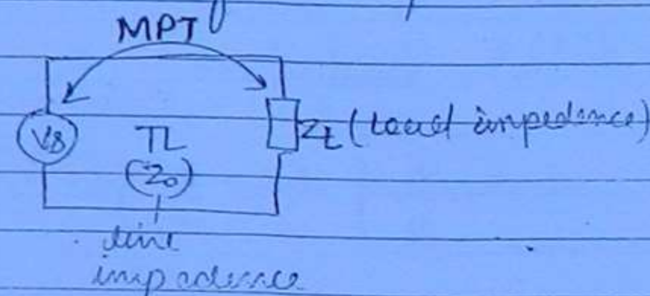
2. The reflected wave becomes zero only under one condition i.e $Z_L = Z_0$. In any other condition the reflected wave is due to unabsorbed power in the load.

3. If $Z_L = Z_0$ the complete source power is transmitted to the load. Maximum power transfer takes place



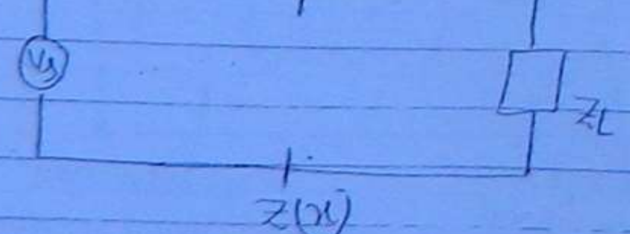

MPT = Maximum Power ~~Transformation~~

characteristic Impedence:

$Z_0$ is a unique impedence of the line which depends on its primary constt and it is the impedence with which the load should be terminated for maximum power transfer and hence called as characteristic impedence

Impedence on the line and Input impedence

Every transmission line has impedence anywhere on the line which is due to the load at one end and primary constt R, L, G, C



$$Z(x) = \frac{V(x)}{I(x)}$$

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$$Z_{in} = \text{Input impedence} = \left.\frac{V(x)}{I(x)}\right|_{x=l}$$

$$V(x) = \frac{V_L(Z_L+Z_0)}{2Z_L}\left[e^{\gamma x} + \left(\frac{Z_L-Z_0}{Z_L+Z_0}\right)e^{-\gamma x}\right]$$

$$= \frac{V_L}{Z_L}\left[\left(\frac{Z_L+Z_0}{2}\right)e^{\gamma x} + \left(\frac{Z_L-Z_0}{2}\right)e^{-\gamma x}\right]$$

$$= \frac{V_L}{Z_L}\left[Z_L\left(\frac{e^{\gamma x}+e^{-\gamma x}}{2}\right) + Z_0\left(\frac{e^{\gamma x}-e^{-\gamma x}}{2}\right)\right]$$

$$= V_L \times \frac{Z_L}{Z_L}\left(\frac{e^{\gamma x}-e^{-\gamma x}}{2}\right) + \frac{V_L}{Z_L}\times Z_0\left(\frac{e^{\gamma x}-e^{-\gamma x}}{2}\right)$$

$$= V_L \cosh(\gamma x) + I_L Z_0 \sinh(\gamma x)$$

Similarly

$$I(x) = I_L \cosh(\gamma x) + \frac{V_L}{Z_0}\sin(\gamma x)$$

$$\boxed{Z(x) = \frac{V(x)}{I(x)} = \frac{V_L\cosh(\gamma x) + I_L Z_0 \sinh(\gamma x)}{I_L\cosh(\gamma x) + \frac{V_L}{Z_0}\sinh(\gamma x)}}$$

$$Z(x) = \frac{I_L\left(\frac{V_L}{I_L}\cosh(\gamma x) + Z_0\sinh(\gamma x)\right)}{I_L\left(\cosh(\gamma x) + \frac{Z_L}{Z_0}\sinh(\gamma x)\right)}$$

$$\boxed{Z(x) = Z_0 \left[ \frac{Z_L \cosh(\gamma x) + Z_0 \sinh(\gamma x)}{Z_0 \cosh(\gamma x) + Z_L \sinh(\gamma x)} \right]}$$

Mismatched line

Line 1 $\quad \longleftarrow Z(x) \longrightarrow$

$Z_{in}$ —+—+—+—+—+—+—+— $Z_L$

Line 2 $\quad Z_0 \longleftarrow Z_0 \longrightarrow Z_0 \qquad y = Z_L = Z_0$

Matched line

If a line is terminated with its characteristic impedance the impedence anywhere on the line and at the I/p is also the same such a line is called ~~as~~ matched line

Short circuit and open circuit line

If $Z_L = 0 \Rightarrow$ short ckt line

then $Z_{in} = Z_{sc} = Z_0 \tanh(\gamma x)$

If $Z_L = \infty \Rightarrow$ open circuit line

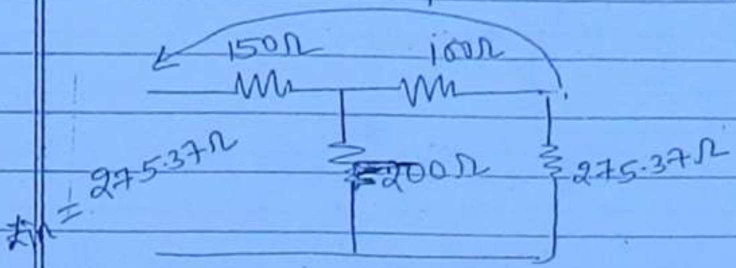
then $Z_{in} = Z_{oc} = Z_0 \coth(\gamma x)$

where $x$ = the complete length = length of line

$$\boxed{Z_0 = \sqrt{Z_{sc} \cdot Z_{oc}}}$$

Note: characteristic Impedence is always the geometric mean of short ckt and open circuit input impedence.

eg.

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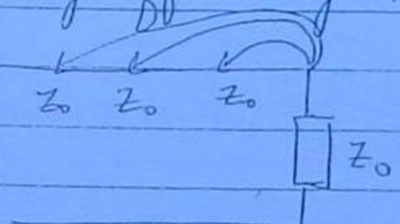
$Z_{sc} = 150 + (100 || 200) = 216.66\Omega$

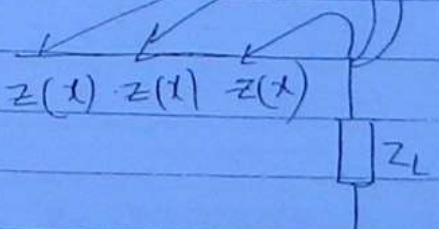
$Z_{oc} = 350\Omega$

$Z_0 = \sqrt{350 \times 216.66} = 275.37\Omega$

For any discrete N/w (3-4 elements N/w) if its end is terminated with $Z_0$ its I/p impedence is also the same i.e the N/w behaves like a mirror and the O/p is reflected in the I/p. similarly for a continous RLGC N/w or a distributed transmission line if the termination is $Z_0$ the loading effect anywhere is $Z_0$ as shown.




Hence the I/p impedence is same everywhere instead if the load is $Z_L$ the loading affect is different at different points on the line



NTL (N/w and transmission line)

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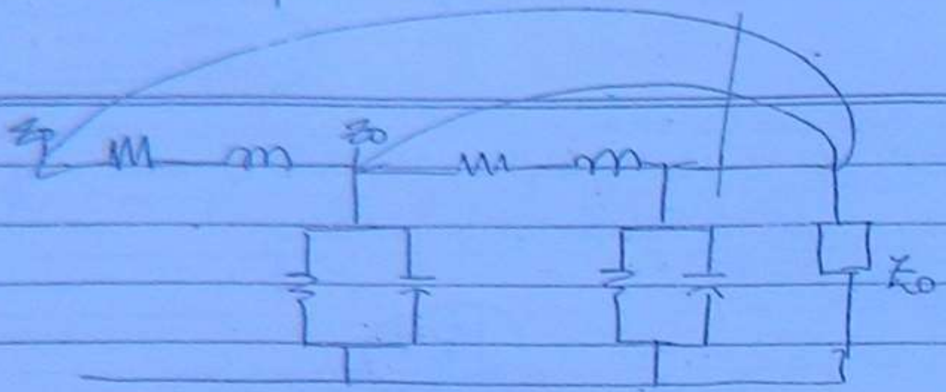
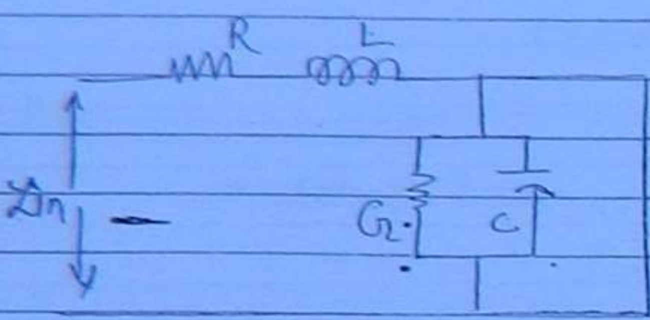
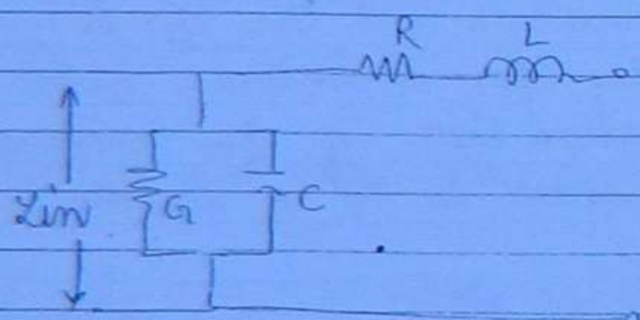



$Z_0 = \sqrt{Z_{sc} \cdot Z_{oc}}$




$Z_{sc} = (R + j\omega L)$




$Z_{oc} = \dfrac{1}{G + j\omega C}$

$Z_0 = \sqrt{Z_{sc} \cdot Z_{oc}} = \sqrt{\dfrac{R + j\omega L}{G + j\omega C}}$

Lossless Line & Distortionless Line:

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$\gamma$ = propagation constt.

$= \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$

If $\alpha = 0$ — No attenuation on the line
No loss on the line
Lossless line

series — No resistance
shunt — No leakage

(i) $R = G = 0$

$\gamma = j\omega\sqrt{LC} = 0 + j\beta$

$$\boxed{\beta = \omega\sqrt{LC}}$$

(ii) $\omega L >> R$ , $\omega C >> G$

- High freq. lines
- Inductive / capacitive lines are also lossless.

For $Z_0$ $\quad Z_0 = \sqrt{\dfrac{R+j\omega L}{G+j\omega C}}$

If $R = G = 0$

$$\boxed{Z_0 = \sqrt{\frac{L}{C}}} = \text{real};$$

$$Z_0 = \sqrt{\frac{\mu_0}{2\pi}\ln(b/a)\frac{\ln(b/a)}{2\pi\epsilon_0}}$$ — for Coaxial cable

$$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \cdot \frac{\ln(b/a)}{2\pi}$$

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$$Z_0 = \frac{120\pi}{2\pi} \ln(b/a) \Rightarrow 60 \ln(b/a) \qquad \left\{ \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi \right\}$$

$$\boxed{Z_0 = 60 \ln(b/a)}$$ — for air filled coaxial line cable

$$\boxed{Z_0 = \frac{60}{\sqrt{\epsilon_r}} \ln(b/a)}$$ — for dielectric filled coaxial cable

for parallel line

$$\boxed{Z_0 = 120 \ln(D/r)}$$ — for air filled parallel line

$$\boxed{Z_0 = \frac{120}{\sqrt{\epsilon_r}} \ln(D/r)}$$ for dielectric filled parallel line

phase velocity $V_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu_0 \epsilon}}$

$$\boxed{V_p = \frac{1}{\sqrt{\mu_0 \epsilon}}}$$

Distortionless line

Distortionless simple Harmonic

$\sin\theta = \dfrac{e^{j\theta} - e^{-j\theta}}{2j}$ $\quad \cos\theta = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$

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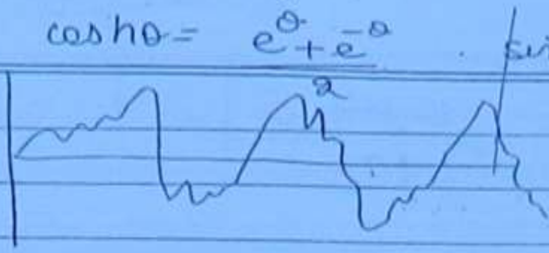
$\cosh\theta = \dfrac{e^{\theta} + e^{-\theta}}{2}$ $\quad \sinh\theta = \dfrac{e^{\theta} - e^{-\theta}}{2}$



Distorted or multi-Harmonic

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If $\beta \propto w$

$\boxed{\beta \text{ is a linear function of } w \Rightarrow \text{the line is distortionless}}$

Phase-shift rates with space & time should be linear

Rise time of the wave = fall time of the wave.

Series arm time constt = shunt arm time constt

$$\frac{L}{R} = \frac{C}{G} \Rightarrow LG = RC$$

Workbook - chapter 3 (Transmission line)

1. $$Z_{in} = Z_0 \left[ \frac{Z_L \cosh(j\beta l) + Z_0 \sinh(j\beta l)}{Z_0 \cosh(j\beta l) + Z_L \sinh(j\beta l)} \right]$$

$$\cosh(j\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2} = \cos(\theta)$$

$$Z_{in} = Z_0 \left[ \frac{Z_L \cos(\beta l) + j Z_0 \sin(\beta l)}{Z_0 \cos(\beta l) + Z_L \sin(\beta l)} \right]$$

$$\sinh(j\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2} = j\sin\theta$$

e

Page

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(i) $\lambda/8 = l$

$$\beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{8} \Rightarrow \frac{\pi}{4}$$

$$Z_{in} = Z_0 \left[ \frac{Z_L \cos\frac{\pi}{4} + jZ_0 \sin\pi/4}{Z_0 \cos\pi/4 + jZ_L \sin\pi/4} \right]$$

$$Z_{in} = Z_0 \left[ \frac{Z_L + jZ_0}{Z_0 + jZ_L} \right]$$

(ii) $\lambda/4 = l$

$$\beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \pi/2.$$

$$Z_{in} = Z_0 \left[ \frac{Z_L \cos\pi/2 + jZ_0 \sin\pi/2}{Z_0 \cos\pi/2 + Z_L \sin\pi/2} \right]$$

$$Z_{in} = Z_0 \left[ \frac{jZ_0}{Z_L} \right] = \frac{Z_0^2}{Z_L}$$

(iii) $\lambda/2 = l$

$$\beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi$$

$$Z_{in} = Z_0 \left[ \frac{Z_L \cos\pi + jZ_0 \sin\pi}{Z_0 \cos\pi + Z_L \sin\pi} \right]$$

$$= Z_0 \left[ \frac{-Z_L}{-Z_0} \right]$$

$$Z_{in} = Z_L$$

(iv) $\lambda = l$

$$\beta l = \frac{2\pi}{\lambda} \times \lambda = 2\pi$$

$$Z_{in} = Z_0 \left[ \frac{Z_L \cos(2\pi) + jZ_0 \sin(2\pi)}{Z_0 \cos(2\pi) + Z_L \sin(2\pi)} \right]$$

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$$Z_{in} = Z_L$$

Conclusion: The I/p impedence $Z_{in}$ and the behaviour of the line always depends on in term $\beta l$ Hence length to wavelength relationship is crucial for any line Hence $\beta l$ is called as Electrical length of the line
units = Radians or degrees.

2 w.8

$Z_0 = 50\Omega$ $\quad$ $Z_0 \neq Z_L$ (Not a matched line)

$Z_L = j50\Omega$

$50\Omega$ = Inductive load → line is not matched
($j50\Omega$)

$$Z_{in} = 50\left(\frac{j50 + j50}{50 + j50}\right) \Rightarrow 50 \times j50 \left(\frac{2}{50 + j50}\right) \Rightarrow \frac{50 \times j50 \times 2}{50(1+j)}$$

$Z_{in} = \infty$ $\quad$ O.C.

3. $$Z_{in} = \frac{50(j50)}{j50} = \frac{50 \times 50}{j50} = \frac{50}{j}$$

$$\frac{1}{j} = \frac{j^4}{j} = j^3 = (-j)$$

$Z_{in} = -j50$ (capacitive)

(4) $Z_{in} = Z_L$

$Z_{in} = j50$ (Inductive)

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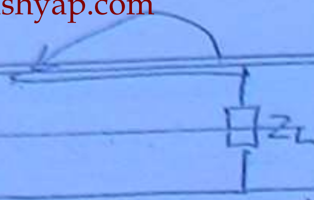
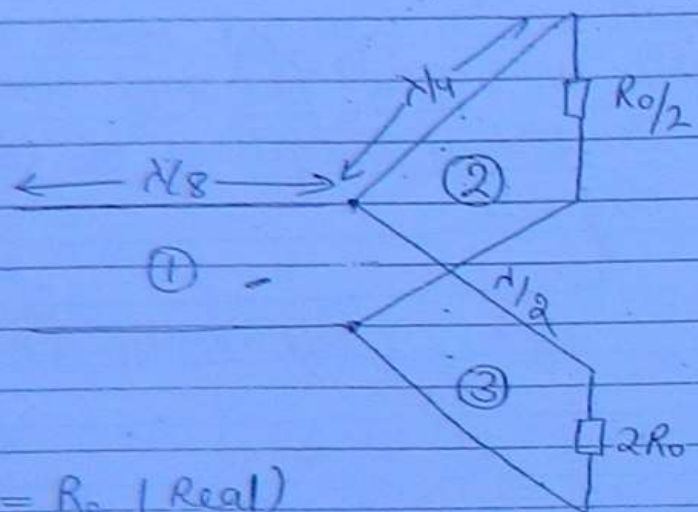
$$Z_{in} = \frac{Z_0^2}{Z_L} \quad \backslash 1$$



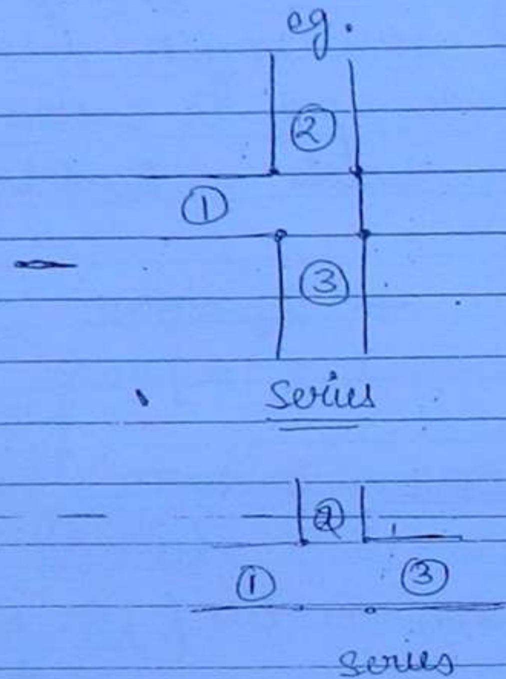
- A $\lambda/4$ line at one end offers an exact opposite impedance to the impedance at the other end. Hence it is called as impedance inverter
- Similarly a $\lambda/2$ line this is simple impedance reflector

W.B



$Z_0 = R_0$ (Real)
means lossless line

eg.



$$Z_{L_1} = \left( Z_{in2} \parallel Z_{in3} \right)$$

Load of first line is input impedance for ② and ③ that are in shunt

$$Z_{in2} = \frac{Z_0^2}{Z_L} \quad (\text{for } \lambda/4 \text{ line})$$

$$= \frac{R_0^2}{R_0/2} = 2R_0$$

$$Z_{in3} = Z_L \quad (\text{for } \lambda/2 \text{ line})$$

$$= 2R_0$$

$Z_{L1} = (2R_0 \| 2R_0) = R_0$

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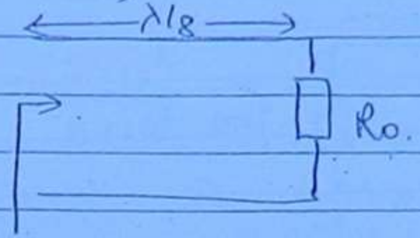
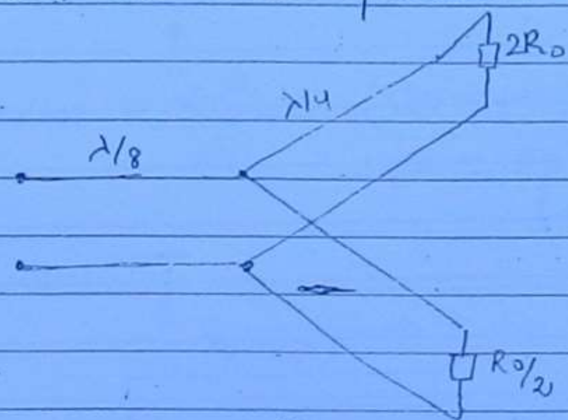



$Z_{in} = R_0$

Repeat the same question with the load interchange.




$Z_0 = R_0$ (Real)

$Z_{in} = (Z_{in2} \| Z_{in3})$

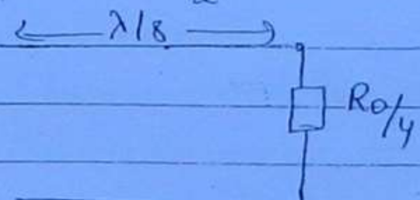
$Z_{in2} = \frac{Z_0^2}{Z_L}$ (for λ/4) line

$= \frac{R_0^2}{2R_0} = \frac{R_0}{2}$

$Z_{in} = (R_0/2 \| R_0/2)$

$= R_0/4$

$Z_{in3} = Z_L = \frac{R_0}{2}$




$Z_{in} = R_0 \left( \frac{Z_L + jZ_0}{Z_0 + jZ_L} \right)$

$Z_{in1} = R_0 \left( \frac{\frac{R_0}{4} + jR_0}{R_0 + j\frac{R_0}{4}} \right) \Rightarrow R_0 \left( \frac{1+j4}{4+j} \right)$

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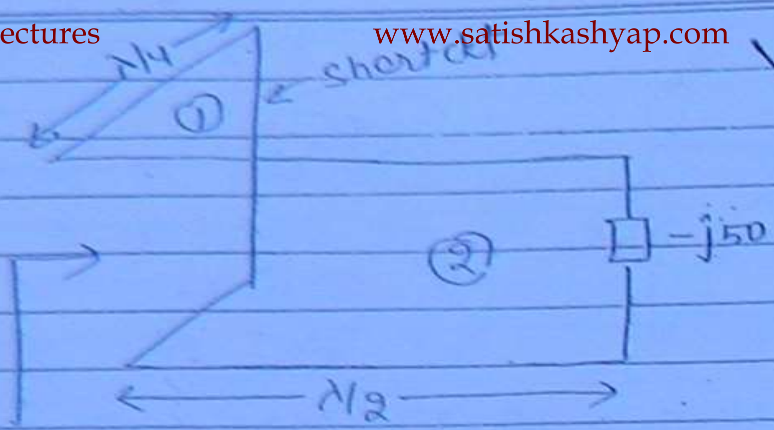



$Y_{in} = ?$

$$Y_{in} = Y_{in1} + Y_{in2}$$

$$Y_{in1} = \frac{Z_L}{Z_0^2} \quad (\text{for } l = \lambda/4)$$
$$= -j50 \quad 0$$

$$Y_{in2} = \frac{1}{Z_L} = \frac{1}{-j50} = \frac{j}{50}$$

$$Y_{in} = 0 + \frac{j}{50} = j(0.02)$$

other method

$$Y_{in} = \frac{1}{Z_{in}}$$

$$Z_{in1} = \infty = O/C$$
$$Z_{in2} = Z_L = -j50$$

$$Z_{in} = (\infty \parallel -j50)$$
$$= -j50$$

$$Y_{in} = \frac{1}{Z_{in}} = \frac{1}{j50} = \frac{j}{50} = j(0.02)$$

$$\boxed{\begin{array}{ll} 1 & R \parallel 0 = 0 \\ 2. & R \parallel \infty = R \end{array}}$$

9. If $f = 5\times10^6$ Hz (197)

$$\lambda = \frac{V_p}{f} = \frac{2\times10^8}{5\times10^6} = \frac{200}{5} = 40.$$

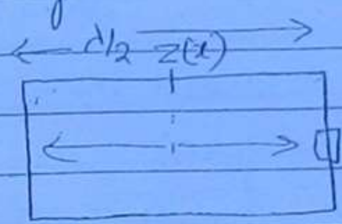
$$\beta l = \frac{2\pi}{\lambda}\times10 \Rightarrow \frac{2\times\pi}{40}\times10 \Rightarrow \frac{\pi}{2}$$

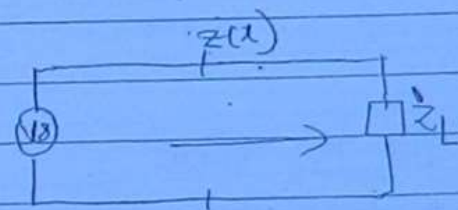
$$l = \frac{\lambda}{4}$$

$$Z_{in} = \frac{Z_0^2}{Z_L} = \frac{(30+j40)^2}{(30-j40)}$$

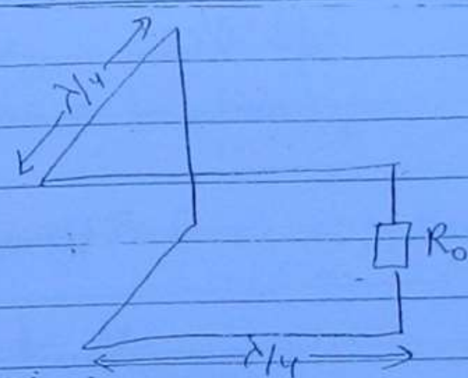
6/7/14 Tuesday

7

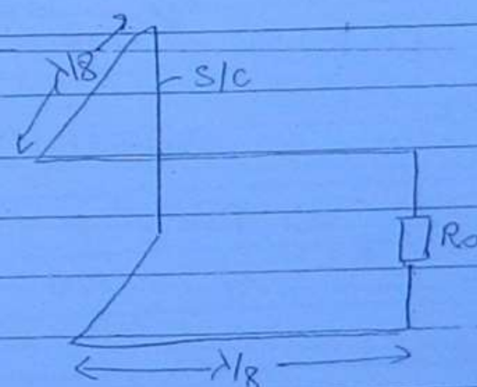



(i)




(ii)




(i) $Z$ at centre $= \infty \parallel R_0 = R_0$

$$\frac{Z_0^2}{Z_L} = \frac{Z_0^2}{0} = \infty \left(\text{for } \lambda/4 \text{ S.C}\right)$$

$$\frac{Z_0^2}{Z_L} = \frac{R_0^2}{R_0} = R_0 \left(\text{for } R_0\right)$$

(ii) at $l = \frac{\lambda}{4}$ = means $\lambda/8$

$$Z \text{ at centre} = jR_0 \parallel R_0 = \frac{jR_0\times R_0}{jR_0+R_0} = \frac{jR_0}{j+1}$$

$$Z = Z_0\left[\frac{Z_L + jZ_0}{Z_0 + Z_L}\right] = R_0\left[\frac{0+jR_0}{R_0+0}\right] = jR_0$$

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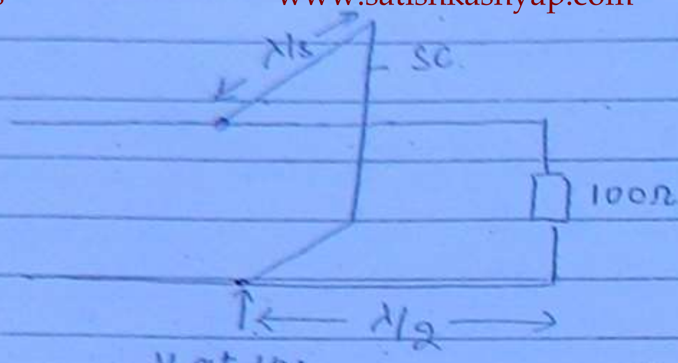



$$Y_{in} = Y_{in_1} + Y_{in_2}$$

$$Z_{in_1} = Z_0 \frac{(Z_L + jZ_0)}{(Z_0 + jZ_L)} = \frac{Z_0(0 + jZ_0)}{(Z_0 + j(0))} = jZ_0 = j50.$$

$$Y_{in_1} = \frac{1}{Z_{in_1}} = \frac{1}{j50} \Rightarrow \frac{-j}{50} \Rightarrow -j(0.02) \qquad \frac{1}{100} = 0.01$$

alternate method: for $\lambda/8$ line

$$Z_{sc} = Z_0 \tanh(j\beta l) \qquad l = \lambda/8$$
$$= jZ_0 \tan(\beta l) \qquad \beta l = \pi/4$$
$$= jZ_0$$

$$Y_{in_2} = \frac{1}{Z_L} = \frac{1}{100}$$

8- $l = 10m$

$V_p = 2 \times 10^8 \, m/s$

$f = 10 MHz$

$$\lambda = \frac{2\times10^8}{10\times10^6} = \frac{v_p}{f} = 20\,m$$

$$\beta l = \frac{2\pi}{\lambda}\times 10 \Rightarrow \frac{2\pi\times10}{20} = \pi \quad (\lambda/2 \text{ line})$$

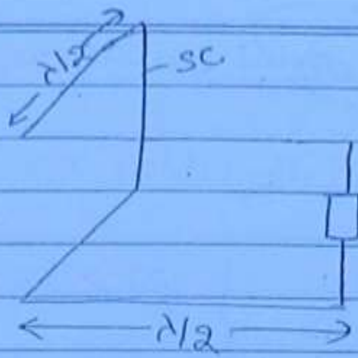
$$Z_{in} = Z_L$$

$$Z_{in} = (30 - j40)$$

$Z = R_0 \left[\frac{R_0 + jR_0}{(R_0 + jR_0)}\right] \Rightarrow R_0\left(\frac{1+1}{2}\right) = R_0$



(iii)




$Z$ at the centre $= 0 \| R_0 = 0$

$Z = Z_L = 0$ (for $\lambda/2$)

$Z = Z_L = R_0$

10 $l = \pi/4$

$\beta l = \frac{2\pi}{\lambda} \times \left(\frac{\pi}{4}\right)$

$\beta l = \pi/4 \times \frac{\lambda}{2\pi} = \frac{\lambda}{8}$

$l = \lambda/8$

It is short circuited at one end:

$Z_{in} = j60\Omega$

$Z_0 = ?$

$Z_{SC} = jZ_0 \tan\beta l$

$j60 = jZ_0 \tan \pi/4$

$jZ_0 = j60\Omega \Rightarrow Z_0 = 60\Omega$

$Z_{in} = Z_0 \frac{(Z_L + jZ_0)}{(Z_0 + jZ_L)}$

$j60 = Z_0\left(\frac{0 + jZ_0}{Z_0 + j0}\right)$

$j60 = jZ_0$

$Z_0 = 60$

11 $l = \lambda/8$ $f$ $f \propto \frac{1}{\lambda}$

$f' = 2f$

$\lambda' = \lambda/2 \quad = \lambda = 2\lambda'$

$l = \frac{2\lambda'}{8} = \frac{\lambda'}{4}$

when $l = \lambda/4$ — short ckt at one end.

$Z_{in} = \infty$ as $Z_{in} = \frac{Z_0^2}{Z_L} = \frac{Z_0^2}{0} = \infty$

12. $f = 12\,MHz$

$Z_{in} = j60$

$j60 = j\omega L$

$L = \dfrac{60}{2\times\pi\times 12\times 10^6}$

$L = \dfrac{30}{\pi 12\times 10^6}$

$L = \dfrac{2.5}{\pi}\ \mu H$

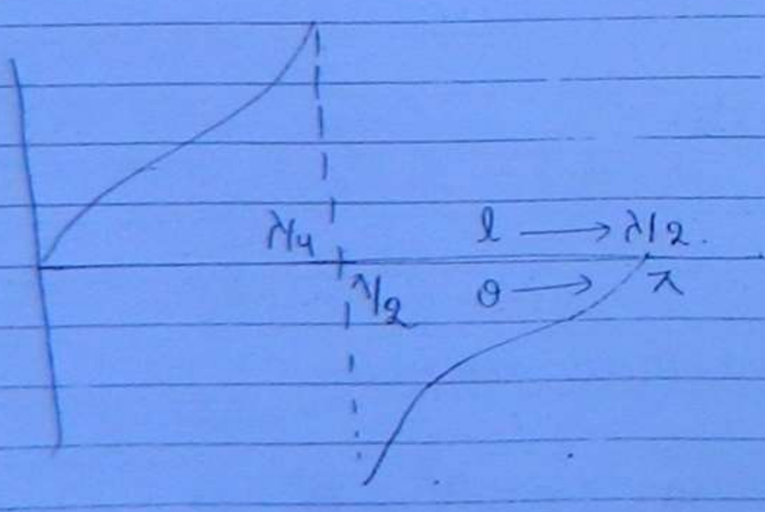
Tabular Coloum:

short circuit

$Z_{sc} = jZ_0 \tan\beta l$

$\left.\begin{array}{l} 0<\beta l<\pi/2 \\ 0<l<\lambda/4 \end{array}\right\}$ $Z_{in}$ = Purely inductive

$\left.\begin{array}{l} \lambda/4<l<\lambda/2 \\ \pi/2<\beta l<\pi \end{array}\right\}$ $Z_{in}$ = Purely capacitive



2. open circuit line

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$Z_{OC} = Z_0 \coth(j\beta l) = -jZ_0/\tan\beta l$

$\left.\begin{matrix} 0 < \beta l < \pi/2 \\ 0 < l < \lambda/4 \end{matrix}\right\}$ $Z_{in}$ = Purely capacitive

$\left.\begin{matrix} \lambda/4 < l < \lambda/2 \\ \pi/2 < \beta l < \pi \end{matrix}\right\}$ $Z_{in}$ = Purely Inductive

13 (B) $Z_0 = ?$

$C$ = capacitance

$V = c = 3\times10^8$ m/sec

$\epsilon_R$

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{L\times C}{C\times C}} = \frac{1}{C}\sqrt{LC} = \frac{1}{C\times V} = \frac{1}{C\times c}$$

as $v_p = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu_0\epsilon_0\epsilon_R}}$

$$Z_0 = \frac{1}{\frac{3\times10^8\times C}{\sqrt{\epsilon_R}}} \Rightarrow \frac{\sqrt{\epsilon_r}}{3\times10^8\times C}$$

Note: Every lossless line is by default distortionless but the vice versa is not true because distortionless is not lossless and but lossless is always distortionless.

(i) $LG = RC$

$R = G = 0$

(ii) $\beta = \omega\sqrt{LC}$

$\beta \propto \omega$

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Note: (1) $Z_0 = \text{real} = \sqrt{\frac{L}{C}}$

$\Rightarrow \alpha = 0$

$\beta = \omega\sqrt{LC}$

} Lossless Line

gate (2) If $w = 0$ for DC line (It is not necessary, whenever $Z_0$ = real)

$\gamma = \sqrt{RG} = \alpha + j0$

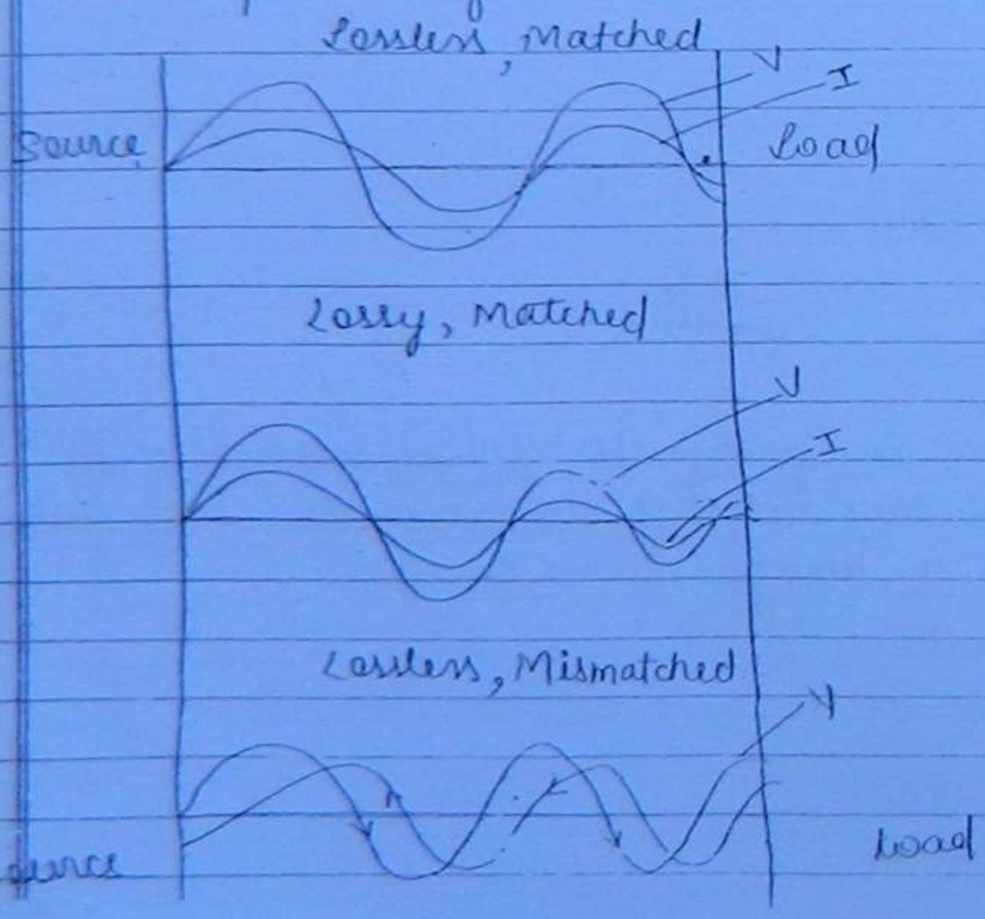
$Z_0 = \sqrt{\frac{R}{G}}$

$= \text{real}$

} DC line

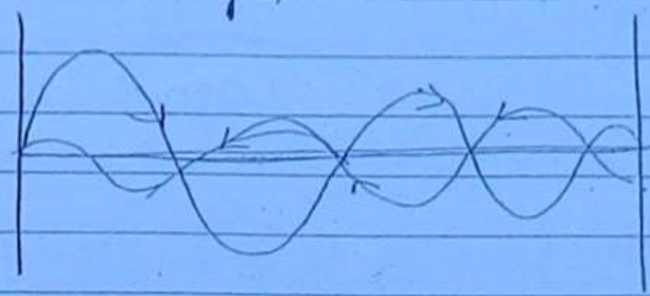
Note: Loss is a line property. It is R, L, G, C dependent.
- It is attenuation on the line.

But ~~match is a load property~~. It is $Z_L$ dependent.
- It is power reflections at the load.



Lossy, mismatched





Reflection coefficient, Standing wave, SWR.

$$V(x) = \frac{V_L(Z_L+Z_0)}{2Z_L}\left[e^{\gamma x} + \left(\frac{Z_L-Z_0}{Z_L+Z_0}\right)e^{-\gamma x}\right]$$

consider lossless line

$$= \underbrace{V_0 e^{j\beta x}}_{\text{forward wave}} + \underbrace{V_0\left(\frac{Z_L-Z_0}{Z_L+Z_0}\right)e^{-j\beta x}}_{\text{reflected wave.}}$$

$\frac{V_r}{V_f}$ = Reflection coefficient $= \left(\frac{Z_L-Z_0}{Z_L+Z_0}\right)e^{-j2\beta x}$

for the voltage anywhere on the line $= \Gamma(x)$

At the load $x=0$

$\Gamma$ at the load $= \frac{Z_L-Z_0}{Z_L+Z_0} = |\Gamma|\angle\theta$

ratio of the $= \left|\frac{V_r}{V_f}\right| \angle\theta$

$|\Gamma| = \left|\frac{V_r}{V_f}\right|$ = fraction of the reflected voltage with respect to forward voltage

$\Gamma$ can be a complex no when $Z_L$ is any complex impedence. Hence $\Gamma$ is

$|\Gamma| \angle\theta$

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Hence $= \left|\frac{V_r}{V_f}\right| \angle\theta$

Hence $|\Gamma|$ stands for the fraction of the reflected voltage w.r.t to the forward voltage.
It is the ratio of the amplitude of reflected to forward voltages.

$|\Gamma| = 0 \leq |\Gamma| \leq 1$

$|\Gamma| = 0$ when $Z_L = Z_0$ (Matched case)

$|\Gamma| = 1$ complete mismatch
~~No power absorption at the load~~
(all reflect back)

$\theta \rightarrow$ the phase difference b/w $V_r$ and $V_f$

$\Gamma \rightarrow$ It is a measure of mismatch b/w the expected impedence $Z_0$ and the actual impedence $Z_L$ at the load.

$\Gamma(x) \rightarrow$ It is a measure of mismatch anywhere on the line i.e it is b/w the expected impedence $Z_0$ and the actual impedence $Z(x)$ anywhere on the line

$$\boxed{\Gamma(x) = \frac{Z(x) - Z_0}{Z(x) + Z_0}}$$

Note 1 $\Gamma'_I$ = Reflection coefficient for currents

$= -\Gamma'_V$

$$\boxed{\Gamma'_I = -\Gamma'_V}$$

$$I(x) = I_0 e^{j\beta x} + I_0 \left(\frac{Z_0 - Z_L}{Z_0 + Z_L}\right) e^{-j\beta x}$$

2. $$I(x) = I_0 e^{j\beta x} - I_0 \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) e^{-j\beta x}$$

If forward voltage, forward current – are in phase, ref voltage, ref current – are out of phase by 180°

Four Cases of complete Mismatch on the line ($\Gamma' = 1$)

case(i) $Z_L = jR_0$ = pure Inductive load

$Z_0 = R_0$ = Lossless line

$$\Gamma' = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{jR_0 - R_0}{jR_0 + R_0} = \frac{j-1}{j+1} = \frac{\sqrt{2}\angle 135^\circ}{\sqrt{2}\angle 45^\circ} = 1\angle 90^\circ$$

$= j$

Note Inductance cannot consume any real power. Hence the complete voltage reflects back with a phase shift of 90° such that if $V_f$ = sin then $V_r$ = cos

case(ii) $Z_L = -jR_0$ = pure capacitive load

$Z_0 = R_0$ = Lossless line

$$\Gamma' = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{-jR_0 - R_0}{-jR_0 + R_0} = \frac{-(j+1)}{-j+1} = \frac{j+1}{j-1} =$$

$= 1\angle -9$

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case(iii) $Z_L = 0$ (short circuit line)

$Z_0 = R_0$ (loss less line)

$$\Gamma = \frac{Z_L - R_0}{Z_L + R_0} = \frac{0 - R_0}{0 + R_0} = -1 = -1\angle 180°$$

$|\Gamma| = 1$ (No power consumed)

Note: A short circuit does not consume any real power such that if the forward voltage is sin the reflected voltage is − sin so the net voltage is 0.

$V_f = \sin$

$V_r = -\sin$

Net $V = 0$

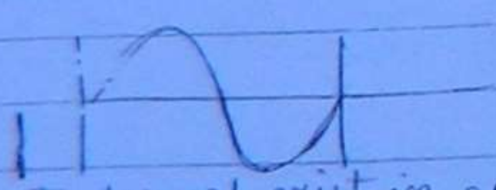
voltage does not exist in short ckt.

case (iv) $Z_L = \infty$ (open circuit line)

$Z_0 = R_0$ (lossless line)

$$\Gamma = \frac{Z_L - R_0}{Z_L + R_0} = \frac{1 - \frac{R_0}{Z_L}}{1 + \frac{R_0}{Z_L}} = 1 \Rightarrow 1\angle 0°$$

$V_f = \sin$ , $V_r = \sin$ $|\Gamma| = 1$



current does not exist in open ckt only voltage exist

Standing Waves & SWR

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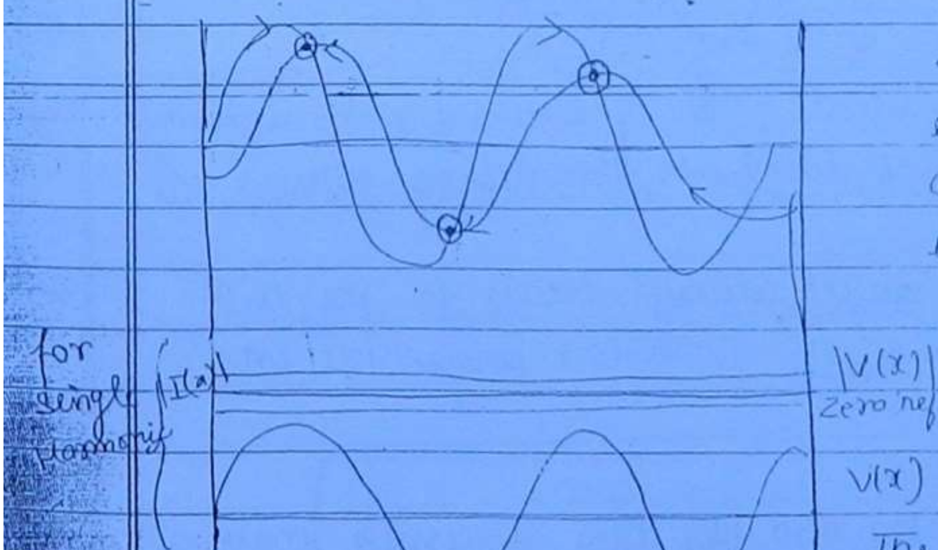
$$V(x) = V_0 e^{j\beta x} + V_0 |\Gamma| e^{j\theta} \cdot e^{-j\beta x}$$
$$= V_0 e^{j\beta x} + V_0 |\Gamma| e^{-j(\beta x - \theta)}$$

$$= A \sin \psi_1 + B \sin \psi_2$$

- If there were two waveforms they have unequal amplitudes and different phases depending on reflection coefficient
- If the magnitude plot $|V(x)|$ is plotted for only the forward wave existing matched line. It is a constt-valued straight line

$A \sin\theta$ vs $\theta$ $\rightarrow$ Harmonic plot

Instead $A$ vs $\theta$ $\rightarrow$ Amplitude plot




- If two w/fs travelling in opposite directions and a phase diff. b/w them interfer among themselves. there is a chance of periodic addition and cancellation of amplitude this never occur for wave in same dir$^n$

The two w/fs are the maxima when they come inphase and a minima when they go out of phase.

$$|V(x)| = A + B = V_0 + V_0|\Gamma| = |V(x)|_{max} = V_{max}$$

$$|V(x)| = A - B =$$

$$\beta x - (-\beta x + \theta) = 2n\pi \qquad n = 0, 1, 2$$

$$\boxed{2\beta x_{max} = 2n\pi + \theta}$$ Position of voltage maxima

$V(x) = (A-B) = V_0 - V_0|\Gamma| = |V(x)|_{min} = V_{min}$

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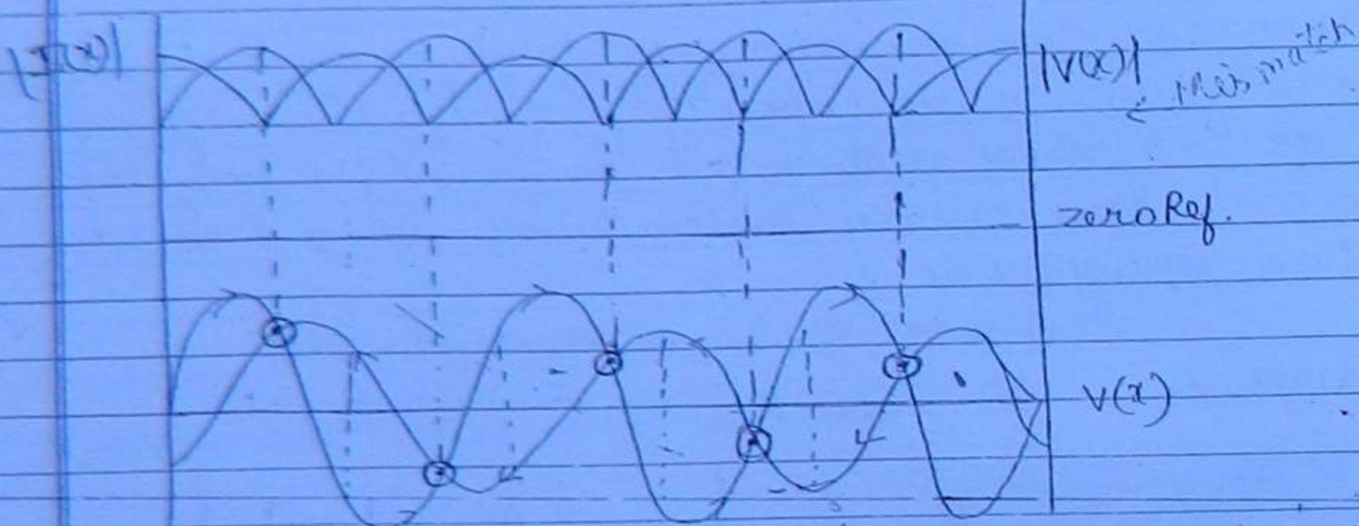
$\beta x - (-\beta x + \theta) = (2n+1)\pi$

$\boxed{2\beta x_{min} = (2n+1)\pi + \theta}$ Position of voltage minima




The resultant amplitude plot of two w/fs travelling in opposite direction is called as standing wave formation

Interference b/w n-source always leads to points of permanent maxima's or minima's anywhere in space.

1. On a standing wave the gap b/w two successive maximas is $\lambda/2$

$\beta x$ — $2\pi$ → Harmonic

On the Harmonic of $2\pi$ period $\beta x$ repeats such than $x$ repeats for $\lambda$.

On the standing wave of $2\pi$ period $2\beta x_{max}$ repeats hence $x_{max}$ repeats for $\lambda/2$.

$$2\beta x_{max} = 2\pi \quad \cdot \text{ Standing wave.}$$

$$2 \times \frac{2\pi}{\lambda} x_{max} = 2\pi$$

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$$\frac{2\pi}{(\lambda/2)} x_{max} = 2\pi .$$

Note 2. current also forms an identical standing wave but the voltage maximas coincide with current minimas.

$$I(x) = I_0 e^{j\beta x} - I_0 |\Gamma| e^{-j\beta x} e^{j\theta}$$

Note 3 At the points of voltage maxima and current minima. the impedence should have been maximum i.e $Z_{max} = \frac{V_{max}}{I_{min}}$

$$Z_{min} = \frac{V_{min}}{I_{max}}$$

SWR → Standing wave Ratio

It can be defined for both CSWR or VSWR.

$$VSWR = \frac{V_{max}}{V_{min}} = \frac{V_0 + V_0|\Gamma|}{V_0 + V_0|\Gamma|} = \boxed{\frac{1+|\Gamma|}{1-|\Gamma|} = SWR}$$

$$\boxed{|\Gamma| = \frac{SWR - 1}{SWR + 1}}$$

SWR = Measure of mismatch on the line

Range = $[1, \infty]$

SWR = 1 ⇒ Matched Line

⇒ $V_{max} = V_{min}$

⇒ No standing wave.

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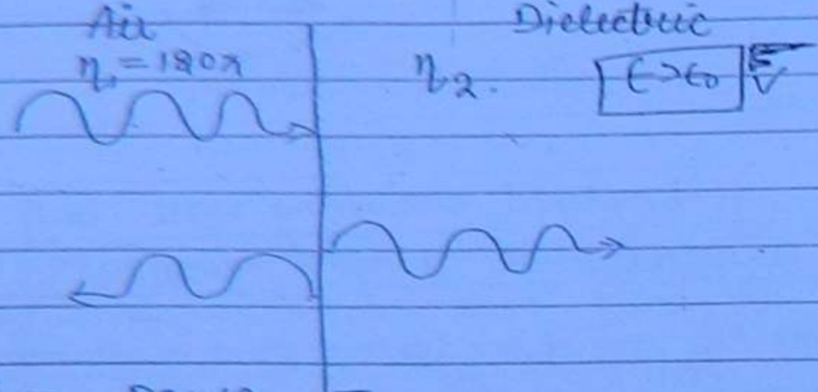
SWR = ∞ ⟹ completely mismatch

⟹ $|\Gamma| = 1$

⟹ $V_{min}$ or $I_{min}$ is zero.

25 W.B




given ESWR = 5

$$|\Gamma| = \frac{5-1}{5+1} = \frac{2}{3}$$

$$\Gamma = \frac{\eta_2 - 120\pi}{\eta_2 + 120\pi}$$

$$\underbrace{\Gamma}_{\text{Real}} = |\Gamma|\angle\theta = \frac{\eta_2 - \overbrace{\eta_1}^{120\pi}}{\underbrace{\eta_2}_{\frac{120\pi}{\sqrt{\epsilon_R}}} + \eta_1}$$

Real

$\theta = 0^\circ$ or $180^\circ$

SWR always gives $|\Gamma|$ which is $\Gamma = |\Gamma|\angle\theta$

In this problem $\Gamma$ is real has $\eta_1, \eta_2$ are real and hence $\Gamma$ is $\pm\frac{2}{3}$

$\theta = 0^\circ$ or $180^\circ$

$\eta_2 < \eta_1$ = -ve

$$\eta_2 = \frac{\eta_1}{\sqrt{\epsilon_r}}$$

as $\epsilon > \epsilon_0$ so $\Gamma = -ve.$

$$-\frac{2}{3} = \frac{\eta_2 - 120\pi}{\eta_2 + 120\pi} \qquad 211$$

$$\eta_2 = 24\pi\,\Omega$$

Note

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

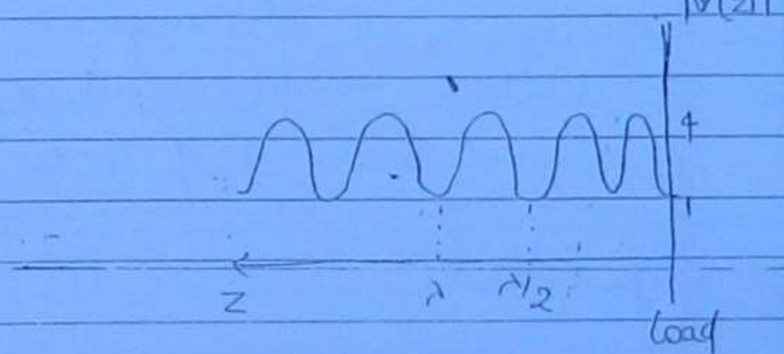
If $\sigma = 0$ $\eta = \sqrt{\frac{\mu}{\epsilon}} = \eta_{max}$

So $\eta \le 120\pi$

26 WB $Z_0 = 50\Omega$ Real

$$|\Gamma| = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Real. Real (resistive)




$$|\Gamma| = \frac{4-1}{4+1} = \frac{3}{5}$$

$\pm\frac{3}{5}$ $\theta = 0°$ or $180°$

voltage is minimum at load $Z_L < Z_0$

$$2\beta x_{min} = (2n+1)\pi + \theta$$

$$0 = 0 + \pi + \theta$$

$$\theta = -\pi = 180 \qquad \Gamma\text{'s phase} = 180°$$

$$-\frac{3}{5} = \frac{Z_L - 50}{Z_L + 50} \Rightarrow -3Z_L - 150 = 5Z_L - 250$$

$$100 = 8Z_L$$

$$Z_L = \frac{100}{8} = \frac{50}{4} = \frac{25}{2} = 12.5\Omega$$

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Note 1 $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$

$2\beta x = 2n\pi + \theta$

If $Z_L$ = resistive, $Z_0$ = lossless line } Real

$\Gamma$ = Real

If $Z_L > Z_0$ then & $\Gamma$ = positive

voltage maxima occur at the load.

$x_{max} = 0$ if $n = 0$

eg: O/c line $Z_L = \infty$. $|\Gamma| = 1$

If $Z_L < Z_0$ & $\Gamma$ = negative

voltage minima occur at the load.

$x_{min} = 0$ if $n = 0$

eg S/c line $Z_L = 0$ $|\Gamma| = -1$

Note 2. If $Z_L, Z_0$ are real

$$SWR = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+\left|\frac{Z_L - Z_0}{Z_L + Z_0}\right|}{1-\left|\frac{Z_L - Z_0}{Z_L + Z_0}\right|} = \frac{1+\frac{Z_L - Z_0}{Z_L + Z_0}}{1-\frac{Z_L - Z_0}{Z_L + Z_0}}$$

If $Z_L > Z_0$

$$\boxed{SWR = \left|\frac{Z_L}{Z_0}\right|}$$

$$\boxed{SWR = \left|\frac{Z_0}{Z_L}\right|}$$

Jai Durga Maa

classmate Date Page

Wednesday

V.B

$VSWR = 5$

$\Gamma = |\Gamma| \angle \theta$

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$|\Gamma| = \frac{5-1}{5+1} = \frac{2}{3}$

17 cm — 27 cm

$\beta = \frac{2\pi}{\lambda}$

$\frac{\lambda}{2} = 10 \Rightarrow \lambda = 20$

| $1^{st}$ min | $2^{nd}$ min | $3^{rd}$ min |
|---|---|---|
| 7 — | 17 — | 27 |
| $n=0$ | $n=1$ | $n=2$ |

$2\beta x_{min} = (2n+1)\pi + \theta$

$2 \times \frac{2\pi}{(20)}(7) = \pi + \theta$

$\theta = \frac{2 \times 7\pi}{10} - \pi$

$\theta = \frac{2 \times 7\pi}{5} - \pi$

$= \frac{14\pi - 5\pi}{5} =$

$\theta = \frac{8\pi}{20}$

B

$VSWR = 5 \qquad Z_0 = 50\Omega$

$|\Gamma| = \frac{5-1}{5+1} = \frac{2}{3}$

$\lambda/4$ — $I^{st}$ voltage maxima

$\Rightarrow |\Gamma| = \frac{Z_L - Z_0}{Z_L + Z_0}.$

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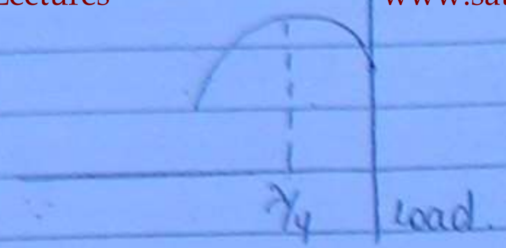



If maxima is at $\lambda/4$ the minima is at the load.
means $Z_L < Z_0$ means $|\Gamma| = -ve$

$$-\frac{2}{3} = \frac{Z_L - Z_0}{Z_L + Z_0} \Rightarrow \frac{Z_L - 50}{Z_L + 50}$$

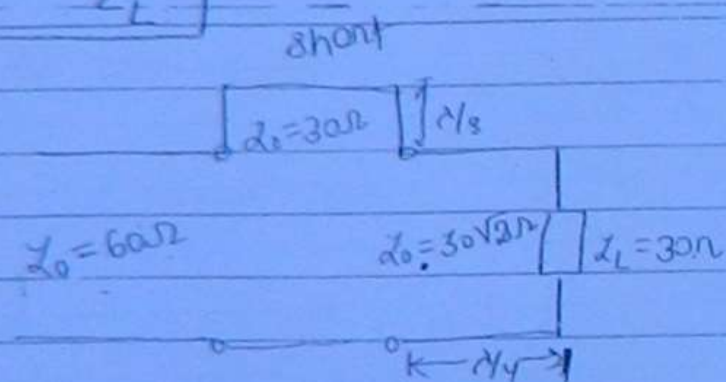
$$-2Z_L - 100 = 3Z_L - 3 \times 150$$

$$50 = 5Z_L$$

$$Z_L = 10\,\Omega$$

$$\boxed{VSWR = \frac{Z_0}{Z_L}}$$

48 W-B




② → $Z_{in} = \frac{Z_0(Z_L + jZ_0)}{(Z_0 + jZ_L)} = \frac{30\,(0 + j30)}{(30 + j0)}$
(for S.C $\lambda/8$)

$$Z_{in} = j30$$

③ → $Z_{in} = \frac{Z_0^2}{Z_L} = \frac{(30\sqrt{2})^2}{30} = \frac{900 \times 2}{30} = 30 \times 2 = 60\,\Omega$

(2) and (3) are in series
$(j30+60)$

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51 W.B

phase $= -150^\circ$ $\beta = \frac{2\pi}{150} = \frac{\pi}{75}$

$n=0$

$2\beta x_{max} = (2n\pi)+\theta$

$2\times\frac{2\pi}{150}\times x_{max} = (2\times0\times\pi)+\theta = -\frac{5\pi}{6}$

$n=1$

$2\beta x_{max} = (2\pi)+\theta$

$2\times\frac{2\pi}{150}\times x_{max} = 2\pi - \frac{5\pi}{6} = \frac{7\pi}{6}$

$x_{max} = \frac{7\times25}{4} = 43m$

$1^{st}$ max ——— 43 m
$2^{n}$ max ——— 43+75 = 118 m
$3^{rd}$ max ——— 118+75 = 193 m
4 max ——— 193+75 = 268m
5 max ——— 268+75 = 343m
6 max ——— 343+75 = 418m
7 max ——— 418+75 = 493m

$\Gamma$ at input

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$Z_{in} = Z_{in_1} \parallel Z_{in_2}$

$=$

$Z_{in_1} = \frac{Z_0^2}{Z_L} = \frac{(50)^2}{100} = \frac{2500}{100} = 25$

$Z_{in_2} = \frac{Z_0^2}{Z_L} = \frac{(50)^2}{200} = \frac{2500}{200} = \frac{25}{2}$

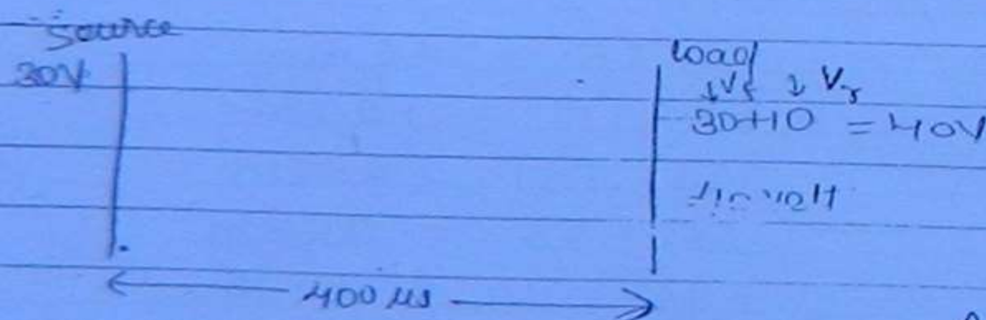
$Z_{in} = 25 \parallel \frac{25}{2} \Rightarrow \frac{1}{25} + \frac{2}{25} = \frac{3}{25} = \frac{25}{3}$

$\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} \Rightarrow \Gamma = \frac{\frac{25}{3} - 50}{\frac{25}{3} + 50} = \frac{25 - 150}{25 + 150} = \frac{-135}{175}$

$\Gamma = \frac{-135}{175}$

$Z_{in} = \frac{Z_0^2}{Z_L} = \frac{(50)^2}{25} \times 3 = \frac{2500 \times 3}{25} = 300$

$\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{300 - 50}{300 + 50} = \frac{250}{350} = \frac{5}{7}$.




$\Gamma = \frac{V_r}{V_F} = \frac{+10}{30} = \frac{1}{3}$

A+B = then
$\Gamma$ = +ve

$\frac{1}{3} = \frac{Z_L - Z_0}{Z_L - Z_0}$.

$$\frac{1}{3} = \frac{Z_L - 50}{Z_L + 50} \Rightarrow Z_L + 50 = 3Z_L - 150$$

$$\Rightarrow 200 = 2Z_L$$

$$Z_L = 100.$$

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Steady state current $= I = \frac{30}{100} = \frac{3}{10} = \cdot 3$ Amp

29 W.B Two sources are there

30 W.B $Z_0 = 50\Omega$ $Z_L = 40 + j30\Omega$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{40 + j30 - 50}{40 + j30 + 50}$$

$$\Gamma = \frac{j30 - 10}{j30 + 90}$$

$$\Gamma = \frac{\sqrt{900 + 100}}{\sqrt{900 + 8100}} = \frac{\sqrt{1000}}{\sqrt{9000}}$$

$$\Gamma = \sqrt{1/9} = \frac{1}{3}$$

$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 1/3}{1 - 1/3}$$

$$= \frac{3+1}{3} \times \frac{3}{3-1} = \frac{4}{2} = 2$$

31 W.B $Z_0 = 50\Omega$ $\Gamma = 0.268$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \Rightarrow 0.268 = \frac{Z_L - 50}{Z_L + 50}$$

$$\Rightarrow Z_L = 86$$

Transmitted power $= \underbrace{(1+\Gamma)}_{\text{voltage}} \underbrace{(1-\Gamma)}_{\text{current}} \cdot \frac{V_0^2}{Z_L}$

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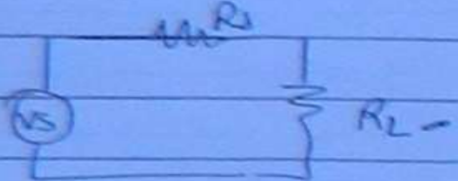
Transmitted power $= (1+\Gamma)V_L \, (1-\Gamma)\dfrac{V_L}{Z_L}$

$= (1+0.268)$

$= 2.428$

$P_{max} = V_{max} \times I_{min}$

$= I_{max} \times V_{min}$




for maximum voltage

N.B

$VSWR = 3 = \dfrac{V_r}{V_f}$

$|\Gamma_E| = \dfrac{VSWR - 1}{VSWR + 1} = \dfrac{3-1}{3+1} = \dfrac{2}{4} = \dfrac{1}{2}$

$(\Gamma_P) = -\Gamma_E^2$

$= \dfrac{1}{4} - 25\%$

N.B

$\Gamma = 0.6 \angle 60^\circ$

$|\Gamma| = 0.6$

$l = \lambda/8$

$\Gamma(x) = \dfrac{Z_L - Z_0}{Z_L + Z_0} e^{-j2\beta x}$

$\Gamma(x) = \Gamma_L \, e^{-j2 \times \frac{2\pi}{\lambda} \times \frac{\lambda}{8}}$

$\Gamma'(x) = 0.6\, e^{j60}\, e^{-j90}$

$= 0.6\, e^{j(-30)}$

$\Gamma'(x) = 0.6 \angle -30^\circ$

(219)

35 W.B $V_f = 25 \sin(\beta x - 75^\circ)$

$\Gamma' = 0.6 \angle -30^\circ$

$\Gamma' = \frac{V_r}{V_f} \Rightarrow \frac{25 \sin(\beta x - 75^\circ)}{V_r} = 0.6 \angle -30^\circ$

$V_r \Rightarrow \frac{25 \sin(\beta x - 75^\circ)}{0.6 \angle -30}$

$0.6 = \frac{V_r}{V_f} \Rightarrow V_r = 25 \times 0.6 = 15$

$V_r = \cancel{25}\ 15 \sin(-\beta x - 105^\circ)$

$\Gamma'_{phase} = V_{r\,phase} - V_{f\,phase}$

$V_{r\,phase} = \Gamma'_{phase} + V_{f\,phase}$

$= -30^\circ - 75^\circ$

$= -105^\circ$

36 $2\beta x_{min} = (2n+1)\pi + \theta$

for $n = 0$

$2 \times \frac{2\pi}{\lambda} x_{min} = (\pi) - 30^\circ$

$\frac{4\pi}{\lambda} x_{min} = \pi - \pi/6$

$x_{min} = \frac{5\pi}{6} \times \frac{\lambda}{4\pi}$

$x_{min} = \frac{5\lambda}{24}$

# Impedance Matching Techniques



Loads are typically high impedences of the order of. 100Ω to a few kΩ where as line impedance are few. 10 to 100Ω. Hence ~~intermediate~~ intermittent devices are placed b/w the load and the line this is called as impedence matching devices

Impedance Matching techniques.

- Active (Bias)
  - CB amp
  - CC amp.
- Passive
  - shunt
    - Stub Matching
  - series
    - λ/4 quiater wave transformer

λ/4 quater wave transformer

A quater wave transformer is a λ/4 length line placed in series b/w the line and the load.

The load of the $Z_0$ line = $Z_{in}$ of the $Z_0'$ line of λ/4 length.

$$= \frac{Z_0'^2}{Z_L}$$

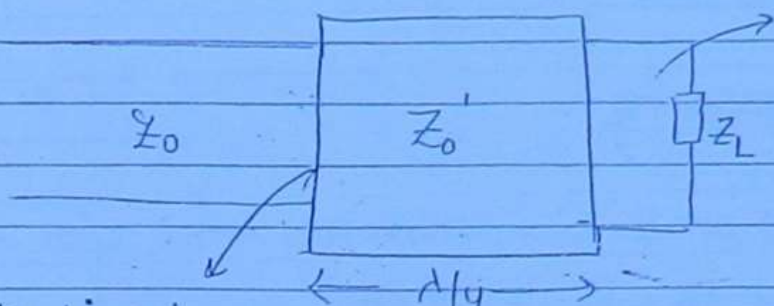
$$= Z_0$$

Hence $Z_0'$ should be such that it is the geometric mean of line and load impedances. so that the impedance inverter offers the exact low impedance to the line due to high impedance load.

Hence the line is perfectly matching ∴ the matching device

However the $Z_0'$ matching device this is not matched to $Z_L$ load






The load is not completely matched to the matching line

The line is matched to the matching device

eg: 50 Ω – line
100 Ω – load.

$$\Gamma = \frac{100-50}{100+50} = \frac{1}{3}.$$

with matching device

$$Z_0' = \sqrt{50 \times 100} = 71$$

$$\Gamma = \frac{100-71}{100+71} = < \frac{1}{3}.$$

The mismatch b/w the matching device and the load is certainly less than the mismatch without the device hence an improvement

Disadvantage.

1. As the transformer unit is $\lambda/4$ it cannot be used over a wide range of frequencies

2. The technique is useful only for resistive loads or lossless line.

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So $Z_0' =$ real and lossless.

$$Z_0' = \sqrt{Z_0 \cdot Z_L}$$

Shunt – stub Matching

Step 1. Identify a length on the line from the load side where the impedence is $Z_0 \pm jX = Z(x)\Big|_{x=l_1}$

$$Y(x)\Big|_{x=l_1} = Y_0 \pm jB.$$

This $x$ value is called as $l_s$ or position of the stub

Mathematically

$$\text{Real}\left[\frac{Z_L\cos\beta x + jZ_0\sin\beta x}{\cancel{Z_0\cos\beta x + jZ_L\sin\beta x}}\right] = 1$$

$x = l_1$

Step 2 Rationalize the eq$^n$.

$$\frac{Z_L\cos\beta x + jZ_0\sin\beta x}{Z_0\cos\beta x + jZ_L\sin\beta x} \times \frac{Z_0\cos\beta x - jZ_L\sin\beta x}{Z_0\cos\beta x - jZ_L\sin\beta x}$$

$\Rightarrow Z_L Z_0 \cos^2\beta x$

$$x = l_1 = \frac{\lambda}{2\pi}\tan^{-1}\sqrt{\frac{Z_L}{Z_0}}$$

Step 2: At this position an equal and opposite reactance is placed such that $y(l_s) = Y_0 \pm jB$ 223

as $\mp jB$ added to it

$$y(l_s) = Y_0 \pm jB \mp jB \quad \text{such that}$$

$$y(l_s) = Y_0$$

So that $\mp jB$ is a stub Reactance.

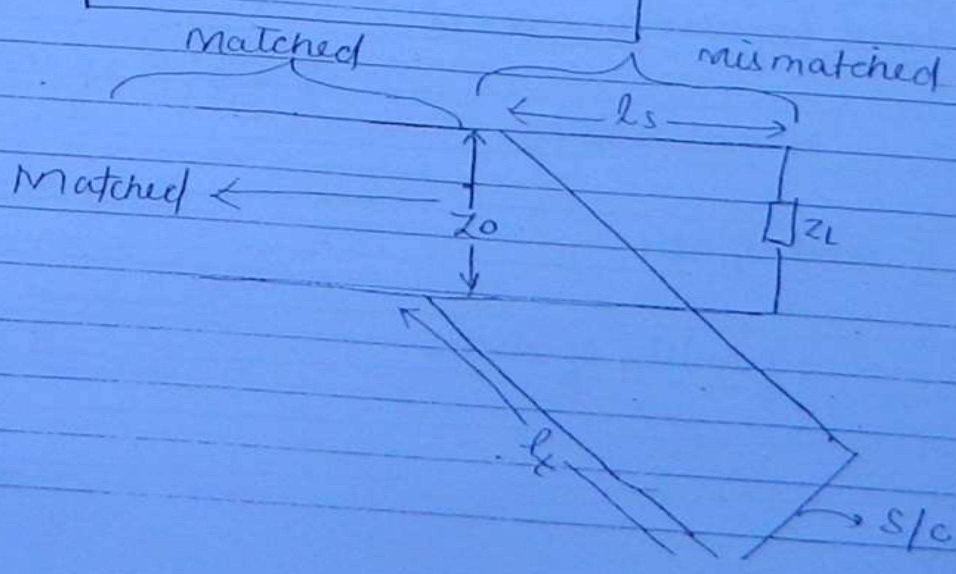
Step 3: This reactance is provided by a short circuited line of $l_t$ length. because every short circuit line has

$$Z_{sc} = jZ_0 \tan\beta l$$

Hence a stub is short circuit line of $l_t$ length. which is precalculated such that

$$Z_0 \tan(\beta l_t) = -\text{Imag}\left[Z_0\left[\frac{Z_L\cos\beta l_s + jZ_0\sin\beta l_s}{Z_0\cos\beta l_s + jZ_L\sin\beta l_s}\right]\right]$$

$$\boxed{l_t = \frac{\lambda}{2\pi}\tan^{-1}\frac{\sqrt{Z_L Z_0}}{Z_L - Z_0}}$$



Note: The stub should always be place as close as to the load as possible

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Disadvantage:
$l_s$ and $l_t$ are $\lambda$ dependent hence the stub has to be moved each time of frequency changes.

Note: The stub length can be adjusted by moving a short to and fro on the stub on Hence short circuited stub are preferred over open circuited stub.

- The disadvantage is partly removed using a double stub matching which is $l_{s_1}$ and $l_{s_2}$ both fixed. $l_{t_1}$ and $l_{t_2}$ both are variable. Hence it can be used for a wide range of frequencies

Smith chart – circle Diagram:
- It is a rectangular graph. —— $\Gamma_r$ vs $\Gamma_i$
- Polar plot $|\Gamma|$ vs $\theta$

- Calculate $\Gamma$, VSWR but known $\left(\frac{Z_L}{Z_0}\right)$ i.e $Z_L$ & $Z_0$ should be known.

$\frac{Z_L}{Z_0}$ = Normalized load Impedence = $R+jX$

– Dividing by $Z_0$ – Normalization

- It is graph $\Gamma_r$ and $\Gamma_i$ axis

Graph consist of → 2 families of circles

Date

$R + jX$
- → Constt R circle
- → Constt X circle

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$$\Gamma_r + j\Gamma_i = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{\frac{Z_L}{Z_0} - 1}{\frac{Z_L}{Z_0} + 1} = \frac{R + jX - 1}{R + jX + 1}$$

$$\Gamma_r + j\Gamma_i = \frac{(R-1) + jX}{(R+1) + jX}$$

Constt R circle Eq$^n$:

$$\left(\Gamma_r - \frac{R}{R+1}\right)^2 + \Gamma_i^2 = \left(\frac{1}{R+1}\right)^2$$

Constt X circle Eq$^n$

$$\left(\Gamma_r - 1\right)^2 + \left(\Gamma_i - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2$$

Properties of constt R circle:

i) Centres $\left(\frac{R}{R+1}, 0\right)$

(ii) Radius $\left(\frac{1}{R+1}\right)$

(iii) pass through $(1,0)$

(iv) Range $R\,[0, \infty]$

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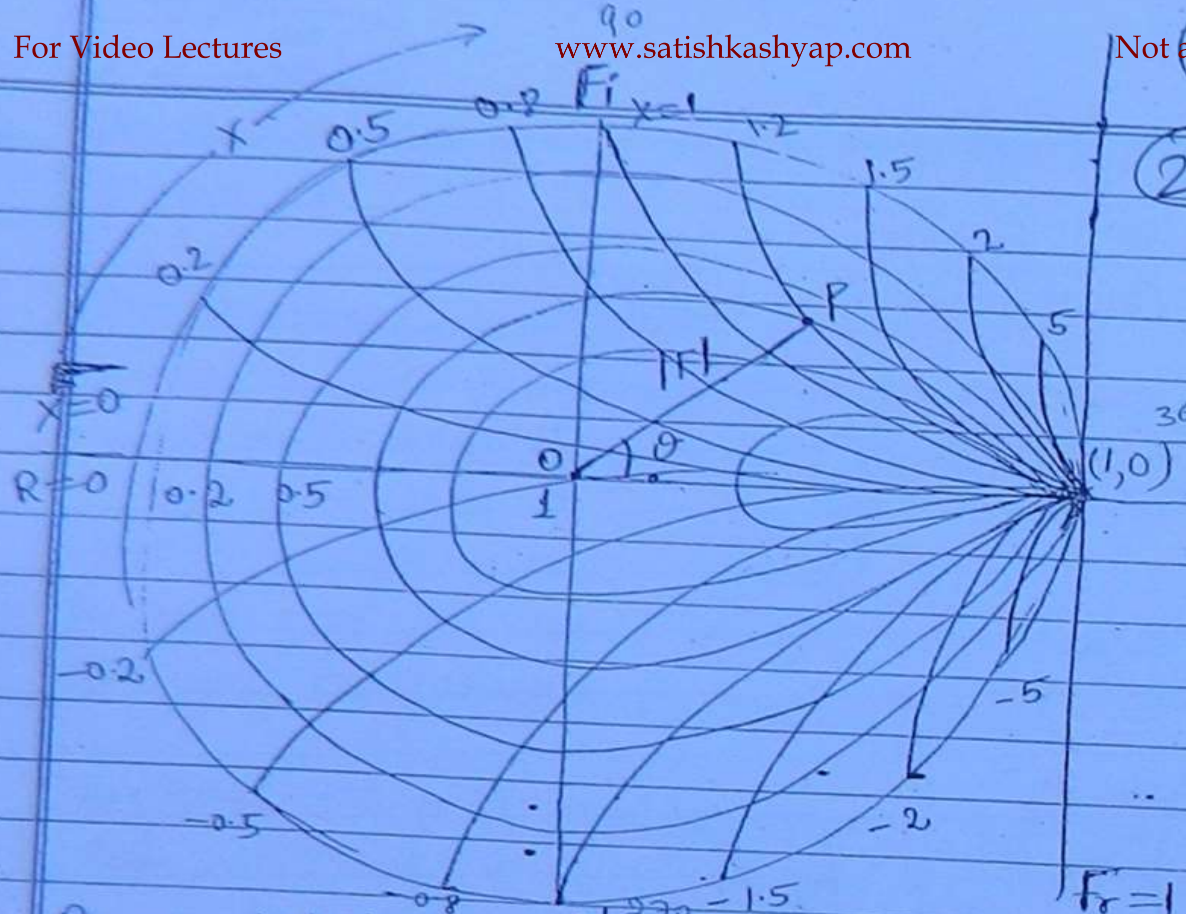


- R = 0 circle is the biggest possible circle and it is the boundary of the smith chart.
The centre is at at the origion and radius = 1.

R = 1 ~~so~~ circle passes through the origin and centre is $\left(\frac{1}{2}\right)$ and radius is also $\left(\frac{1}{2}\right)$

As R increases the circle.

$R = \infty$ is the point (1,0)

They all have common tangent. $\Gamma_{real} = 1$ axis
They all concurrent circles with their centres on the $\Gamma$ real axis
All the circles with R 0 to 1 line in four quadrants
The circles with R>1 lie only in the $1^{st}$ and $4^{th}$ quadrant

Properties of constt x circles:

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(i) centre $(1, \frac{1}{x})$

(ii) Radius $(\frac{1}{x})$

(iii) will pass through (1,0)

(iv) Range of x $[-\infty, \infty]$

Note: 1. All the ct x circles have common tangent the real axis. They are all concurrent circle.

2. Circles with x 0 to 1 lie in the first and second quadrant circles with $x \geq 1$ lie in the ~~3rd and fourth~~ only in the first quadrant

- If we traverse on the R=0 circle clockwise the x increases inductively.

Similarly if we traverse the x=0 circle horizontally R increases.

- A common tangent of R-circles contains the centres of x circles and the viceversa. Hence R and x form a 'orthogonal famil of circles.

(1) eg $Z_L = jR_0$ , $Z_0 = R_0$

$$\frac{Z_L}{Z_0} = \frac{jR_0}{R_0} = j \Rightarrow 0+j = R+jx$$

$R=0$

$\Gamma = j$ $x=1$

(2) $Z_L = -jR_0$ , $Z_0 = R_0$

$$\frac{Z_L}{Z_0} = \frac{-jR_0}{R_0} = 0-j = R+jx$$

$R=0$

$x=-1$

$\Gamma = -j$

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3. $Z_L = 0 \quad Z_0 = R_0$

$\frac{Z_L}{Z_0} = \frac{0}{R_0} = 0 + 0 = R + jX$

$R = 0, \; X = 0 \quad \Gamma = -1$

4. $Z_L = \infty \quad Z_0 = R_0$

$\frac{Z_L}{Z_0} = \frac{\infty}{R_0} = \infty = R + jX$

$R = \infty, \; X = \infty \quad \Gamma = 1$

5. $Z_L = R_0, \; Z_0 = R_0$

$\frac{Z_L}{Z_0} = \frac{R_0}{R_0} = 1 + 0j = R + 0jX$

$R = 1, \; X = 0, \quad \Gamma = 0$

Procedure for Calculation of $\Gamma$

1) Identify the intersection of the R & x circle. Called pt P and joini'd the origin O.

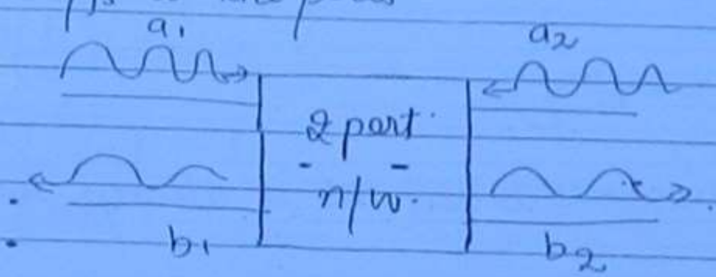
- OP length $= |\Gamma|$
- Phase of $\{\Gamma\}$ is the inclination angle of the segment OP with positive $\Gamma$ real axis is a phase of $\Gamma$

Thursday

S- parameters - Scattering Matrix of a TL (229)

Regular two port N/w models relate the voltage and currents across the two ports but at high frequencies these models fail to explain because we do not have V- and I at high frequencies Hence S parameters are used.

Static Matrix relates the incident w/fs at the ports to the outward w/fs at the ports




$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$b_1 = S_{11}a_1 + S_{12}a_2$$

$$b_2 = S_{21}a_1 + S_{22}a_2$$

$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0}$ = Reflection coeff. at port 1

$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0}$ Reflection coeff at port 2.

$S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0}$ Transmission coefficient from port 2 to port 1

$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$ Transmission coefficient from port 1 to port 2

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Note 1. If $S_{11} = S_{22}$
Symmetric Network

2. If $S_{11} = S_{22} = 0$

Symmetric and matched

3. $S_{12} = S_{21}$
Reciprocal N/W
eg. (Linear N/Ws, Power supply)

If the n/w is linear elements based. It is always reciprocal.

4. $|S_{12}| = |S_{21}| = 1$
Lossless N/W.

W.B

$Z_0 = 100\Omega$
$Z_L = 500\Omega$
$\Gamma = \frac{500-100}{500+100} = \frac{400}{600} = \frac{2}{3}$

W.B.

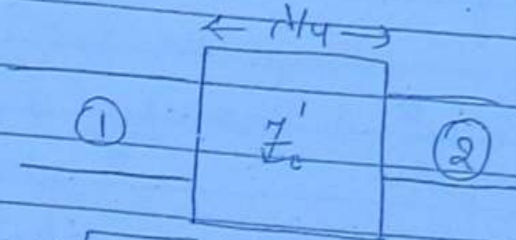
$Z_L = 225$ ohms. $Z_0 = 256$ ohm

$Z_0' = \sqrt{225 \times 256}$ as $Z_0' = \sqrt{Z_0 Z_L}$

43 W.B

$50\Omega \rightarrow Z_0$ $\quad L_1$

$72\Omega \rightarrow Z_L$ $\quad L_2$




$$Z_0' = \sqrt{Z_0 Z_L} = \sqrt{50 \times 72} = 60$$

$$60 \ln\left(\frac{b}{a}\right) = 60$$

$$\ln(b/a) = 1$$

$$(b/a) = e^{(1)}$$

$$b = 27$$

52 (W.B) 1. The impedence on any line, λ/2 values of length Hence a circle diagram Repeats for 360° and Hence one revolution around the smith chart λ/2 distance

# Waveguide

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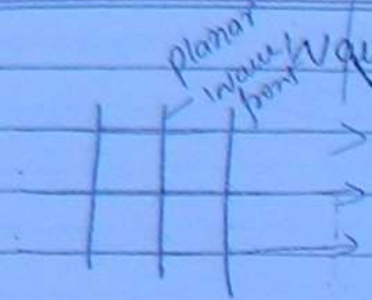
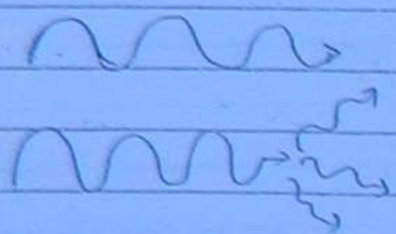


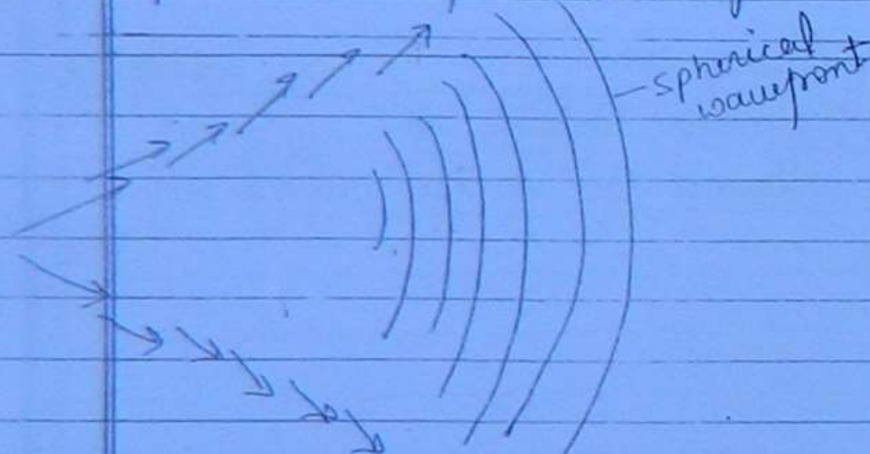

uniform. plane wave



— uniform

→ Expanding wave. (Dispersive wave)

uniform plane plane wave travel in straight line and have a planar wavefront hence the strength every-where can be assumed to be same.
However most EM waves come from various sources which have dispersive nature and hence they form spherical wavefront as they travel forward as shown.



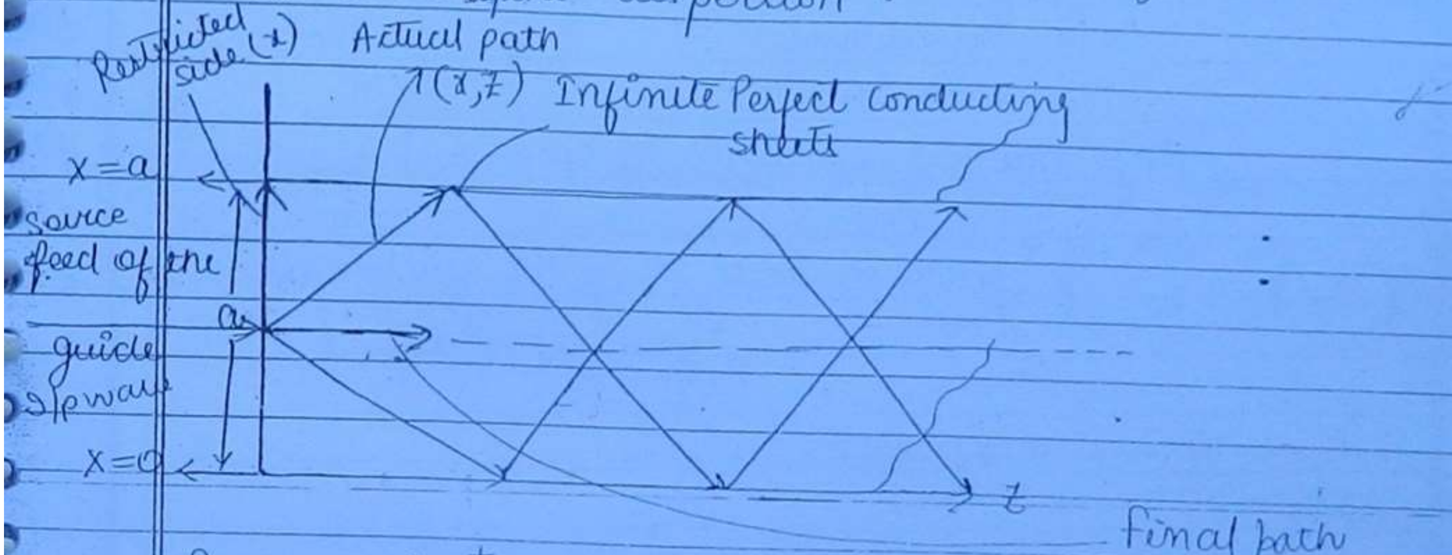

this is the cause of diffusion and diffraction properties of EM waves. this is called as Huygen's Principle

Hence waveguides are used confined electromagnetic wave within specific boundary. we can use parallel plane waveguide and Rectangular waveguide with Parallel Plane waveguide one dimensional confinement Rectangular waveguide for two dimensional.

– Parallel plane W/G – $E(x,z,t) / H(x,z,t)$
Reactangalar W/G $E(x,y,z,t) / H(x,y,z,t)$

Parallel Plane Waveguides

- The wave is assumed to travel in z but also dispersive in x only.
- Two infinitely large conducting states confine the wave and avoid the dispersion




- Every conductor should have $E_{tang}=0$

$$\boxed{E_{tan}=0}$$

i.e $E(x)$ at $x=0$
$E(x)$ at $x=a$
should not have component parallel to the conductor

$$\boxed{E(x)_{tan}=0}$$

at $x=0$ & $a$

Wave Eqns in Waveguide

$$\nabla^2 E = \gamma^2 E$$

$$\nabla^2 H = \gamma^2 H$$

Solving Helmholtz's Eqn for x & z dimensions

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Assumption:

$\gamma$ = free space propagation constt b/w the guides

$\gamma = j\omega\sqrt{\mu_0\epsilon_0}$

E(z)/H(z) → Propagation in the guide axis
No Boundaries
No Restriction
Any natural Harmonic of space-z

$\Big\} e^{-\bar{\gamma} z}$

$\gamma = \gamma_z = \bar{\gamma}$ = propagation constt along the guide axis

$E(z) = e^{-\bar{\gamma} z}$

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial z^2} = -\omega^2\mu_0\epsilon_0 E$$

$$\frac{\partial^2 E}{\partial x^2} + \bar{\gamma}^2 E = -\omega^2\mu_0\epsilon_0 E$$

$$\frac{\partial^2 E}{\partial x^2} = -(\omega^2\mu_0\epsilon_0 + \bar{\gamma}^2)E$$

Hence the wave is following the harmonic solution in x-dimension also such that

$$\underbrace{\bar{\gamma}^2 + \omega^2\mu_0\epsilon_0}_{} = \gamma_x^2$$

Propagation constt in restricted dimension (x-dim)

$$\frac{\partial^2 E}{\partial x^2} + \gamma_x^2 E = 0$$

$(D^2 + m^2 = 0)$ (Trignometric Harmonic)

$(D^2 - m^2 = 0)$ Natural Harmonic

classmate

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Solution of the Eq^n.

$$E(x) = c_1 \sin(\gamma_x x) + c_2 \cos(\gamma_x \cdot x)$$

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Applying boundary conditions

at $x = 0$ $E(x)_{tan} = 0$

$$E(x) = 0 + c_2$$

$$\Rightarrow \boxed{c_2 = 0}$$

Imp

Conclusion - In the restricted dimension the wave solution is always a trignometric harmonic such that in tangential component of a waves Electric field should be sin harmonic only.

at $x = a$

$$E(x) = c_1 \sin(\gamma_x \cdot a) = 0$$

$$\gamma_x \cdot a = m\pi$$

$$m = 0, 1, 2, 3, \ldots$$

$$\gamma_x = \frac{m\pi}{a}$$

Hence $$\boxed{\bar{\gamma} = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2 \mu_0 \epsilon_0}}$$

Concept 1:

cut-off frequency of a guide

$$\bar{\gamma} = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2 \mu_0 \epsilon_0}$$

$$\left(\frac{m\pi}{a}\right)^2 > \omega^2\mu_0\epsilon_0$$

$$\bar{\gamma} = \text{real} = \alpha + j0$$

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No propagation – No wave.

The $\bar{\gamma}$ is either real or imaginary but never complex

Hence

Hence the wave is not propagating but is exponentially removed by the guidwalls

$$\omega^2\mu_0\epsilon_0 > \left(\frac{m\pi}{a}\right)^2$$

$$\bar{\gamma} = 0 + j\bar{\beta}$$

No attenuation and propagation constt along the guide axis exist

Hence every waveguide has the minimum cut off frequency below which there cannot be propagation

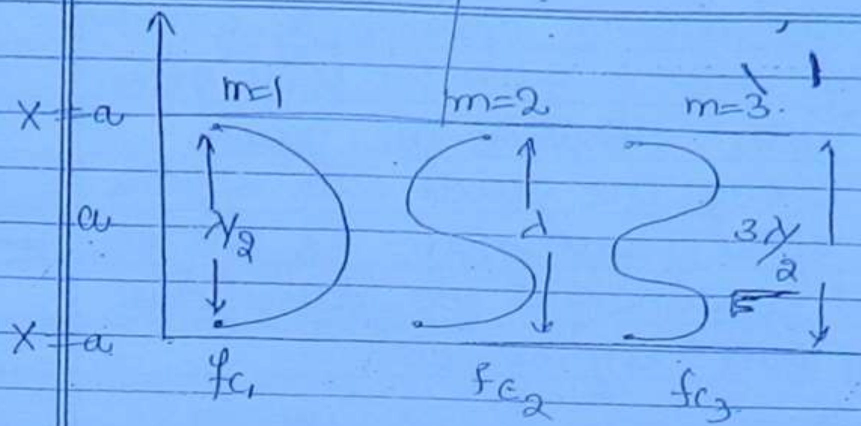
Hence $$\frac{m\pi}{a} = \sqrt{\omega^2\mu_0\epsilon_0}$$

$$\frac{m\pi}{a\sqrt{\mu_0\epsilon_0}} = \omega = \frac{m\pi c}{a}$$

$$\boxed{f_c = \frac{mc}{2a}}$$

$$\lambda_c = \frac{c}{f}$$

$$\boxed{\lambda_c = \frac{2a}{m}}$$




At exact cut off frequency there is no propagation along the guide axis but the wave resonates b/w the guide walls

Concept 2. Wave Angle or tilt angle

$$\bar{\gamma} = j\bar{\beta} = j\sqrt{\omega^2\mu_0\epsilon_0 - \left(\frac{m\pi}{a}\right)^2}$$

$\bar{V}_p$ = phase velocity along the guide axis

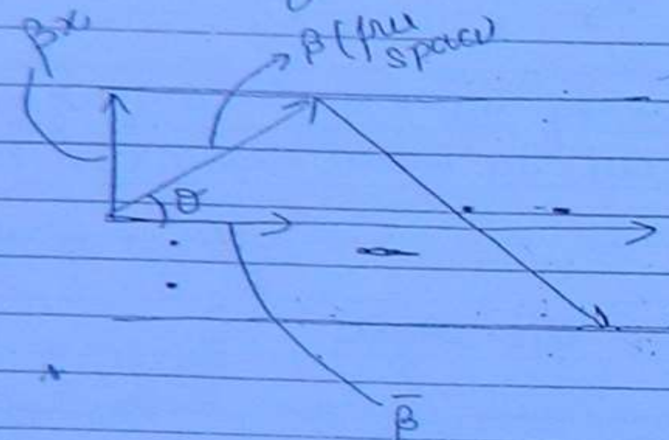
$$= \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\omega^2\mu_0\epsilon_0 - \left(\frac{m\pi}{a}\right)^2}}$$

$$\bar{V}_p = \frac{1}{\sqrt{\mu_0\epsilon_0 - \left(\frac{m\pi}{a\omega}\right)^2}}$$

$$= \frac{1}{\sqrt{\mu_0\epsilon_0}\left(\sqrt{1 - \left(\frac{m\pi c}{a\omega}\right)^2}\right)}$$

$$\boxed{\bar{V}_p = \frac{c}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}}}$$

the phase velocity along the guide axis is apparently greater than free space velocity because when the wave goes in a zig-zag path with inclination the guide axis the wavelength along the guide axis appears longer than the free space wavelength. i.e the phase shift rate and its dynamics are altered due to multiple reflections and zig zag path

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$\bar{v}_p > c$

$$\bar{\beta} = \beta \cos\theta$$

$$\frac{2\pi}{\bar{\lambda}} = \frac{2\pi}{\lambda}\cos\theta$$

$$\bar{\lambda} = \frac{\lambda}{\cos\theta}.$$

$$\bar{v}_p = \frac{c}{\cos\theta}$$

$\because \bar{\lambda} = \bar{c}/f$

$\bar{c} = \bar{\lambda} f$

$\bar{c} = \frac{\lambda}{\cos\theta} f = \frac{c}{\cos\theta}$

Hence by comparison.

$$\boxed{\sin\theta = \frac{\omega_c}{\omega}}$$

where $\theta$ = tilt angle with the guide axis

No two frequencies will ~~never~~ ever have the same tilt angle and hence no two frequencies will never ever have the same velocity.

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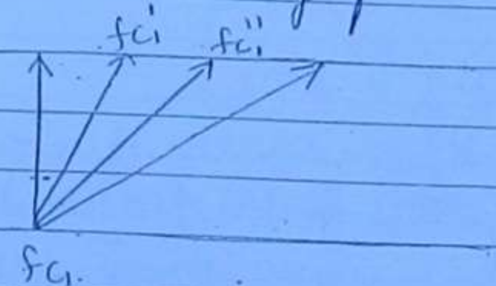
At $\omega = \omega_c$ $\theta = 90^\circ$

No propagation, only Resonance

$\omega > \omega_c$ any finite tilt angle.



concept 3. Group velocity ($\bar{v}_g$) along the guide axis

Dispersion Relationship = $\beta = \sqrt{\omega^2 \mu_0 \epsilon_0 - \left(\frac{m\pi}{a}\right)^2}$

When $\beta$ is propotional to $\omega$.
like in uniform plane waves, lossless transmission lines
velocity is correctly $\frac{\omega}{\beta}$.

$\beta \propto \omega$ (dispersion less)
$v_p = \frac{\omega}{\beta}$
} one ray

But when $\beta$ is the non linear function of $\omega$ like in waveguides velocity is correctly $\frac{d\omega}{d\beta}$ but not $\frac{\omega}{\beta}$.
this is called as dispersion condition

$\beta = \sqrt{\omega^2 \mu_0 \epsilon_0 - \left(\frac{m\pi}{a}\right)^2}$

perime beam

$v_p \neq \frac{\omega}{\beta} = \frac{d\omega}{d\beta}$
} multiple ray

$$V_g = \frac{d\omega}{d\beta}$$

This velocity is called as group velocity. This has to be used has dispersive ~~medium~~ (group of rays or group of frequencies)

$y = mx$ $\qquad\qquad$ $y = mx^2$

$\frac{y}{x} = \frac{dy}{dx} = m$ $\qquad\qquad$ $\frac{y}{x} = mx$

linear fun$^n$ $\qquad\qquad$ $\frac{dy}{dx} = 2mx$

non linear fun$^n$.

rivation $V_g = \frac{d\omega}{d\bar{\beta}}$

$$\frac{d\bar{\beta}}{d\omega} = \frac{1}{2\sqrt{\omega^2\mu_0\epsilon_0 - \left(\frac{m\pi}{a}\right)^2}} \quad 2\omega\sqrt{\mu_0\epsilon_0}$$

$$\frac{d\bar{\beta}}{d\omega} = \frac{\sqrt{\mu_0\epsilon_0}}{\sqrt{\mu_0\epsilon_0 - \left(\frac{m\pi}{a\omega}\right)^2}}$$

$$\frac{d\bar{\beta}}{d\omega} = \frac{\mu_0\epsilon_0}{\sqrt{\mu_0\epsilon_0}\left(1 - \left(\frac{m\pi}{a\omega\sqrt{\mu_0\epsilon_0}}\right)^2\right.}$$

$$\frac{d\bar{\beta}}{d\omega} = \frac{\sqrt{\mu_0\epsilon_0}}{\left[1 - \left(\frac{m\pi}{a\omega\sqrt{\mu_0\epsilon_0}}\right)^2\right]}$$

$$\frac{d\bar{\beta}}{d\omega} = \frac{c}{\left[1-\left(\frac{m\pi c}{a\omega}\right)^2\right]} \quad (24)$$

$$= \frac{1}{c\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}}$$

$$\bar{v}_g = c\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2} = c\cos\theta$$

$$\boxed{v_p \cdot v_g = c^2}$$

- Hence group velocity defines a physical rate at which the wave is travelling or the energy is propagated along the guide axis.

- The rate of change of phase with space i.e $\lambda$ is longer than in free space

$$\bar{\lambda} > \lambda$$

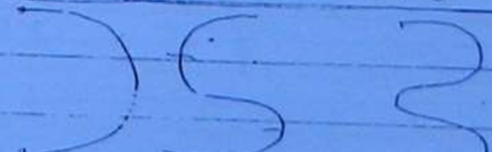
Hence $\bar{v}_p > c$

but $v_g < c$ because it represents the delay in time.

## Concept 4 Modes of operation

m - stands for connection mechanism for the feed.. It is an integer standing for the no. wired feed connection. Mathematically it is a the no. of half cycles the wave complete b/w the guide wall. or the no. of maximas b/w the guide wall

-m- m=1 m=2 m=3



Relation between $\lambda$, $\lambda_c$, and $\lambda_g$

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$$\bar{\lambda} = \lambda_g = \frac{\lambda}{\cos\theta} = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{\lambda}{\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}}$$

Squaring both side

$$1-\left(\frac{\lambda}{\lambda_c}\right)^2 = \left(\frac{\lambda}{\lambda_g}\right)^2$$

$$\frac{1}{\lambda^2} = \frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2}$$

$\lambda = \frac{c}{f}$ (freespace) $\quad \lambda_c = \frac{2a}{m}$ (cut off freq.)

Transverse electric, TM, TEM waves.

Consider a dispersive beam of EM waves which are $E(x,z,t)_{(x,y,z)}$ $H(x,z,t)_{(x,y,z)}$

Let us use time harmonic maxwell's $Eq^n$

$$\nabla \times H = j\omega\epsilon E \qquad \nabla \times E = -j\omega\mu H$$

$$\begin{vmatrix} a_x & a_y & a_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{vmatrix}$$

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_x$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y$$

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$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_y$$

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$$\nabla \times H = -j\omega\epsilon E$$

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = j\omega\epsilon E_x$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H}{\partial y} = j\omega\epsilon E_z.$$

All y derivatives are zero. and all z derivatives are the same function with $\bar{\gamma}$ scaling. as.

$$\left.\begin{matrix} E(z) \\ H(z) \end{matrix}\right\} e^{-\bar{\gamma}z} \quad \text{any natural harmonic.}$$

$$\nabla \times H = -j\omega\epsilon E \qquad \nabla \times E = -j\omega\mu H$$

$$\bar{\gamma} H_y = j\omega\epsilon E_x \qquad \bar{\gamma} E_y = -j\omega\mu H_x$$

$$-\bar{\gamma} H_x - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y \qquad -\bar{\gamma} E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y$$

$$\frac{\partial H_y}{\partial x} = j\omega\epsilon E_z \qquad \frac{\partial E_y}{\partial x} = -j\omega\mu H_z$$

$$-\bar{\gamma} H_x - \frac{\partial H_z}{\partial x} = j\omega\epsilon\left(-\frac{j\omega\mu H_x}{\bar{\gamma}}\right)$$

$$-\bar{\gamma} H_x - \frac{\partial H_z}{\partial x} = \frac{\omega^2\epsilon\mu H_x}{\bar{\gamma}}$$

multiply $\bar{\gamma}$ both side

$$-\bar{\gamma}^2 H_z - \bar{\gamma}\frac{\partial H_z}{\partial x} = \omega^2 \epsilon \mu H_x$$

$$-\bar{\gamma}\frac{\partial H_z}{\partial x} = \omega^2 \epsilon \mu H_x + \bar{\gamma}^2 H_x$$

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$$-\bar{\gamma}\frac{\partial H_z}{\partial x} = (\omega^2 \epsilon \mu + \bar{\gamma}^2) H_x$$

$$-\bar{\gamma}\frac{\partial H_z}{\partial x} = \gamma_x^2 H_x$$

Eqn 1 $$\boxed{\frac{\partial H_z}{\partial x} = -\frac{\gamma_x^2}{\gamma} H_x}$$

Eqn 2 $$\boxed{\frac{\partial E_z}{\partial x} = -\frac{\gamma_x^2}{\gamma} E_x}$$

Eqn 3 $$\boxed{\frac{\partial H_z}{\partial x} = \frac{\gamma_x^2}{j\omega\mu} E_y}$$

Eqn 4 $$\boxed{\frac{\partial E_z}{\partial x} = \frac{-\gamma_x^2}{j\omega\epsilon} H_y}$$

Note: The axial field components are the central components upon which the fields in the other direction also depend upon these components.

Note: If the axial component of magnetic field is zero i.e the field mechanism is such that magnetic field is not directed along the guide axis the other dependent component is also vanish hence the wave becomes

If $H_z = 0$ we are left with $E_x$, $H_y$, $E_z$

and the wave is $H(x, z, t)_y$ $E(x, z, t)_{(x,z)}$

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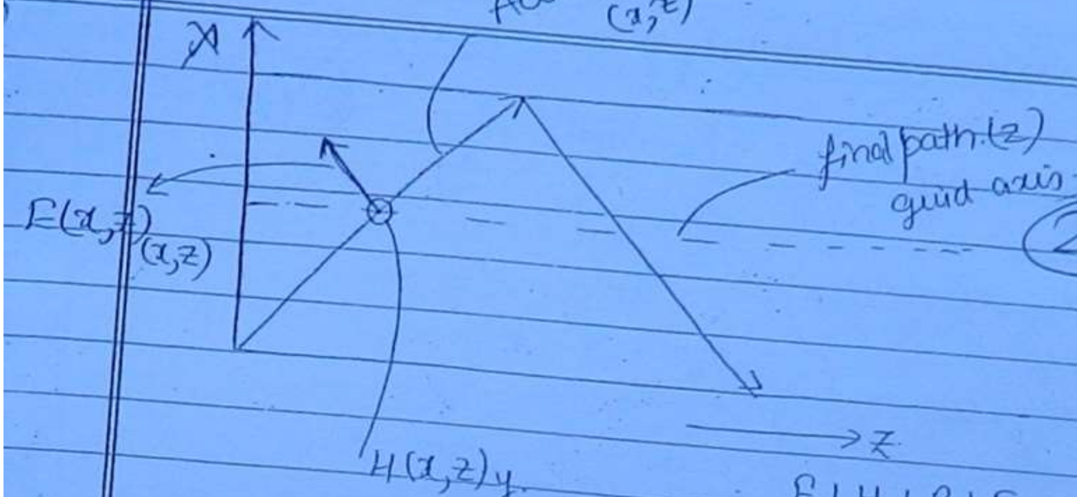


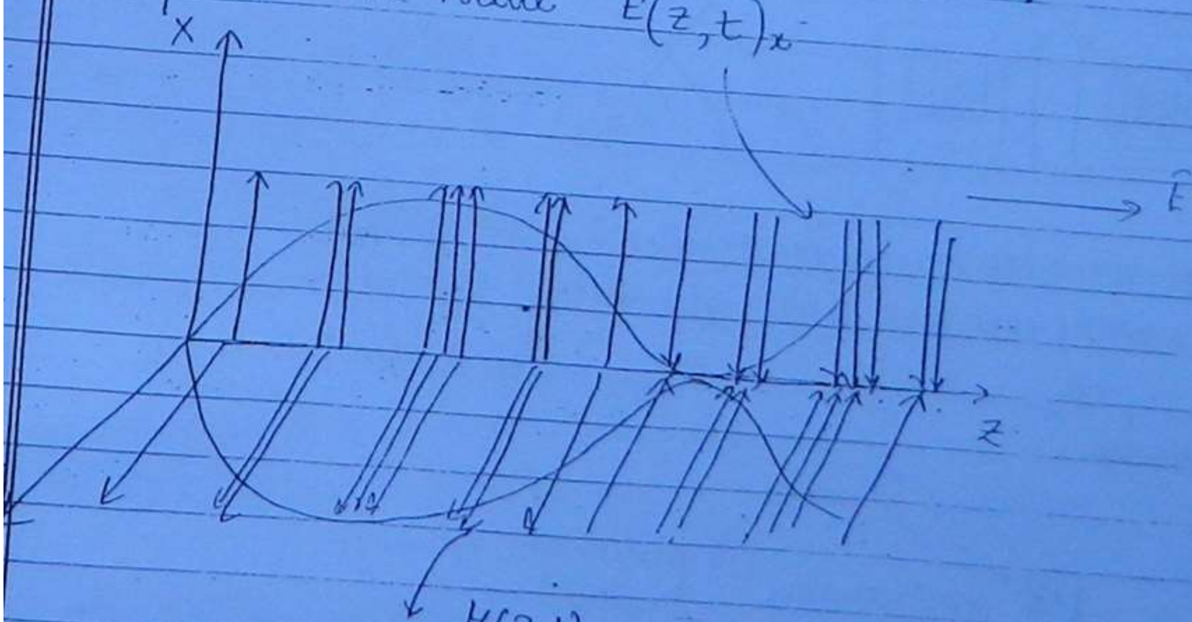

$E \perp H \perp P \perp E$

The magnetic field is not only perpendicular to the Electric field and the actual propagation but it is also ⊥ to the guide axis. Hence this wave is called as transverse Magnetic wave. (TM wave)

Similarly if $E_z = 0$ the components we are left are $H_x$ $E_y$ $H_z$ and the wave is

$E(x,z,t)_y$ , $H(x,z,t)_{(x,z)}$. This wave is called transverse electric wave or (TE wave)

uniform Plane wave $E(z,t)_x$



$E_x = H_y$ $E_y = H_x$ $\frac{E_z \; H_z}{90°}$ $\frac{E_y \; H_y}{90}$ $F_z \; H_z$

x x y = z

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TE wave solutions in Parallel plane waveguide

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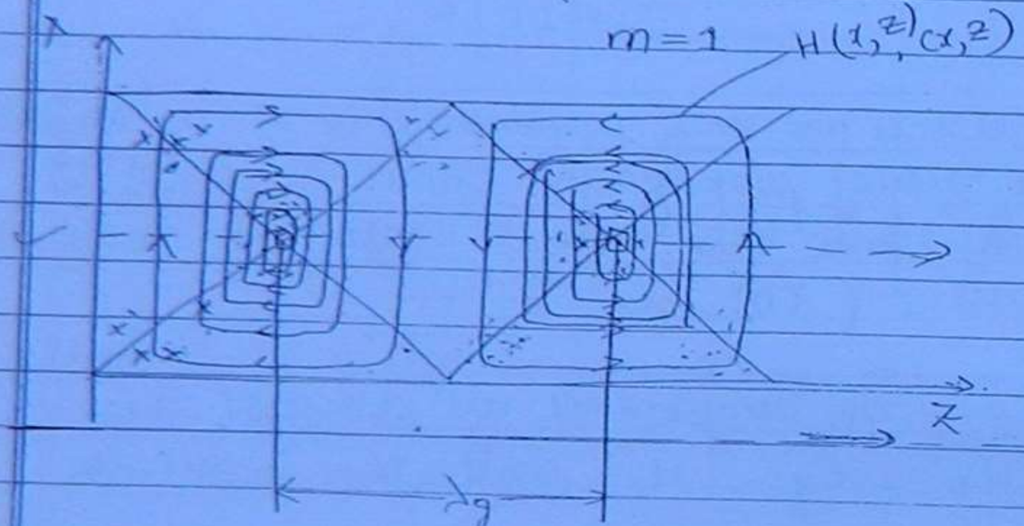
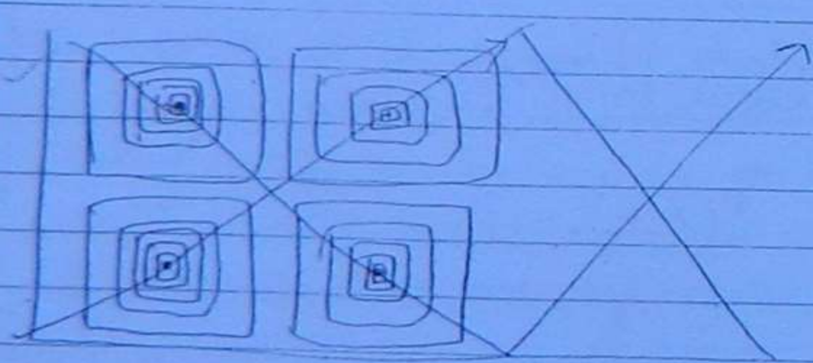
$$E(x,z,t)_y = E_{y0} \sin\left(\frac{m\pi}{a}x\right) \cdot e^{-\gamma z} e^{j\omega t} \quad a_y$$

$$H(x,z,t)_x = H_{x0} \sin\left(\frac{m\pi}{a}x\right) e^{-\gamma z} e^{j\omega t} \quad a_x$$

$$H(x,z,t)_z = H_{z0} \cos\left(\frac{m\pi}{a}x\right) e^{-\gamma z} e^{j\omega t} \quad a_z$$




If m = 0 EM wave donot exist in TE format
i-e m = 1 is the least possibility for TE existance
$TE_{10}$, $TE_{20}$, $TE_{30}$ ..... $TE_{m0}$ representation for TE waves.

$TE_{m0}$

feed mechanism side

guides are not placed in

TM wave solutions in parallel plane W/G

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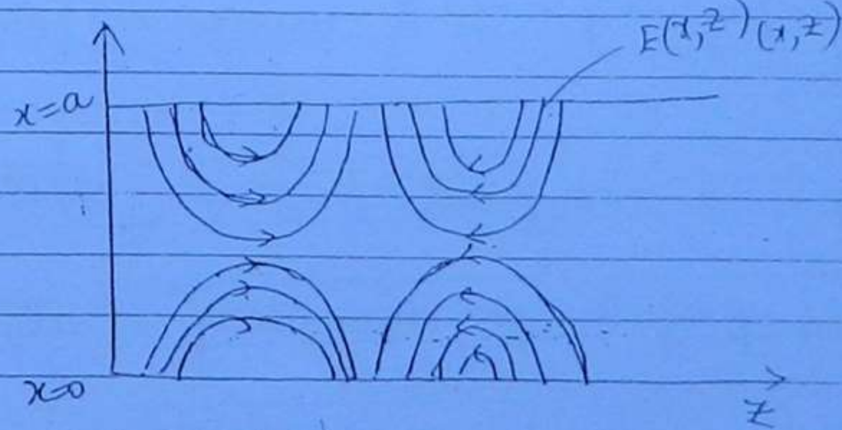
$$E(x,z,t)_x = E_{x0} \cos\left(\frac{m\pi}{a}x\right) e^{-\gamma z} e^{j\omega t} a_x$$

$$E(x,z,t)_z = E_{z0} \sin\left(\frac{m\pi}{a}x\right) e^{-\gamma z} e^{j\omega t} a_z$$

$$H(x,z,t)_y = H_{y0} \cos\left(\frac{m\pi}{a}x\right) e^{-\gamma z} e^{j\omega t} a_y$$

The $E(x)_x$ directed in x or $E(x)_z$ directed in z will follow trigonometric harmonic such that $E(x)_{tang}$ should be equal to sin harmonic as the guide walls are $x=0$ and $x=a$ surfaces. $E(x)$ directed in $x$ is normal component. $E(x)$ directed in y or $E(x)$ directed in z is tangential component. Hence $E(x)_z$ = directed in sin harmonic in $x$.




In the TM operations if $m=0$ only $E(x)$ and $H(y)$ components are left out and the wave become.

$$E(z,t)_x = E_0 e^{-\gamma z} e^{j\omega t} a_x$$

$$H(z,t)_y = H_0 e^{-\gamma z} e^{j\omega t} a_y$$

This wave is called as transverse Electromagnetic wave

It has neither $E_z$ or $H_z$ and has $m=0$ condition.

$TM_{10}, TM_{20}, TM_{30} \ldots TM_{m0}$
TM modes representation

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Properties of TEM waves:

(i) They travel only along the guide axis i.e they are not dispersive in x-side.

(ii) Their wave angle is zero always.

(iii) $w_c = 0$ because $m=0$. No cut off freq. Any freq. can travel as TEM.

Summary:

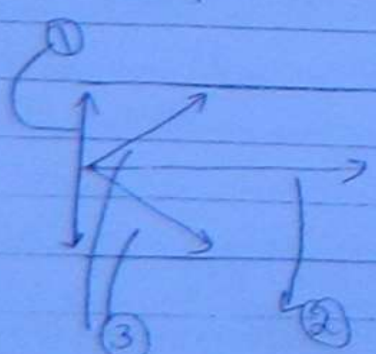
1. $E(x,t) / H(x,t)$

Wave at cut off frequency, $f_{c_1}, f_{c_2} \cdots f_{cm}$ $\theta = 90°$, $\bar{\gamma} = 0$, Not propagating; It is resonating between the guide walls.

2. $E(z,t) / H(z,t) \longrightarrow$ TEM wave, $\theta = 0°$, $\bar{\gamma} = j\omega\sqrt{\mu\epsilon_0}$

$m = 0$, $w_c = 0$; perfectly travelling along $z$ axis, No modes

3. $E(x,z,t) / H(x,z,t)$ — TE / TM — modes $m, w_c, \bar{\gamma}, \gamma, \gamma_x$



Q Identify the following field lines belonging to mode

(i) $TE_{10}$ (ii) $TM_{10}$, (iii) $TE_{20}$ (iv) $TM_{20}$ (v) TEM (vi) TM or TEM (vii) TE or TM.

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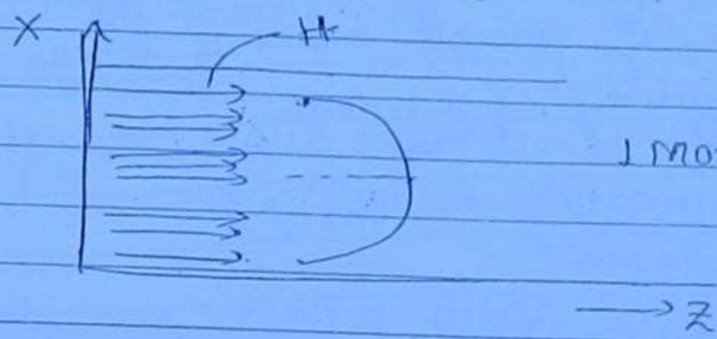
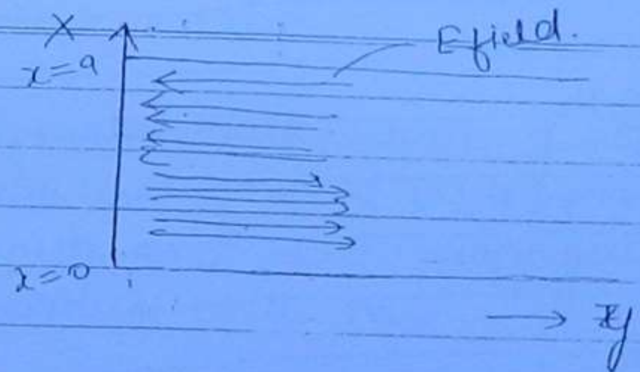



1 maxima = m=1.

TE $\begin{cases} E_y \\ H_x \\ H_z \end{cases}$ $\left.\begin{matrix} H_y \\ E_x \\ E_z \end{matrix}\right\}$ TM.

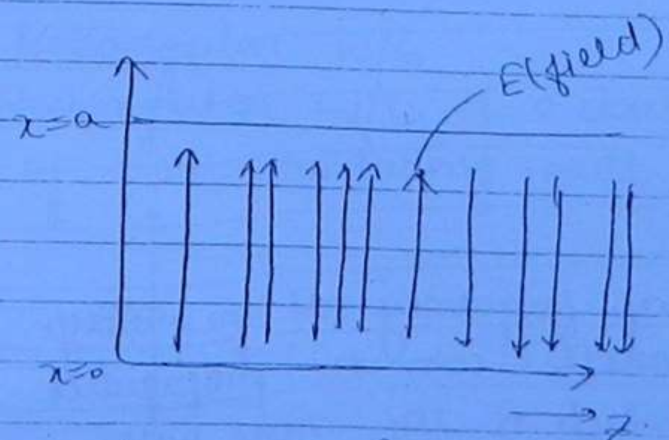
$H(x)_z \longrightarrow TE_{10}$

(ii)




$E(x)_y = TE_{20}$.

(iii)




$E(z)_x \rightarrow$ TM or TEM

No. of maxima is b/w the guidewalls but not along the guide axis.

field (varying) Director

field (propagation) Director

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Q For a parallel plate waveguide of 1 cm sepration calculate the total no. of possible modes of energy Transfer for all frequency $f \leq 40GHz$

Sol $$f_c = \frac{mc}{2a} = \frac{m \times 3 \times 10^8}{2 \times 10^{-2}} = 40 \times 10^9$$

$$m = \frac{40 \times 10^9 \times 2 \times 10^{-2}}{3 \times 10^8}$$

$$m = \frac{80 \times 10^7}{3 \times 10^8}$$

$$m = 26.6 \times 10^{-1}$$

$$m = 2.66$$

1 cm $\longrightarrow$ $f_{c_1} = \frac{c}{2a} = 15GHz$

15 ——— $\infty$ $TE_{10}/TM_{10}$

30 ——— $\infty$ $TE_{20}/TM_{20}$

0 ——— $\infty$ —— TEM

5 modes ans.

Dominant Mode

In a mode of a lower cut of frequency all frequency permissible in the higher modes can be propagated hence dominant mode which is the more with lowest cut off frequency can be used for all frequencies the guide support

Dominant mode for parallel plane wave guide, TEM mode

Note: Wave impedence of TE waves or TM waves is defined only with respect to the transverse component to the guide axis. The total components are not considered.

$$\eta_{TE} = \text{wave impedence of TE waves} = \frac{E_y}{H_x} = \frac{E_T}{H_T\cos\theta} = \frac{120\pi}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$E_T$ $H_T \rightarrow$ total fields.

$E_x, E_y, H_x, H_y \longrightarrow$ transverse to guide axis

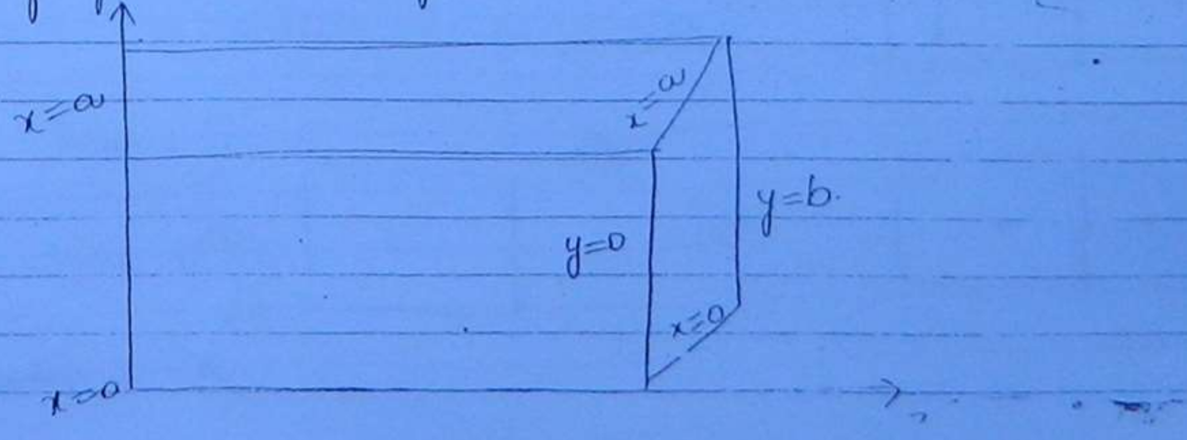
$$\eta_{TM} = \frac{E_x}{H_y} = \frac{E_T\cos\theta}{H_T} = 120\pi\cos\theta$$

$$\eta_{TEM} = 120\pi$$

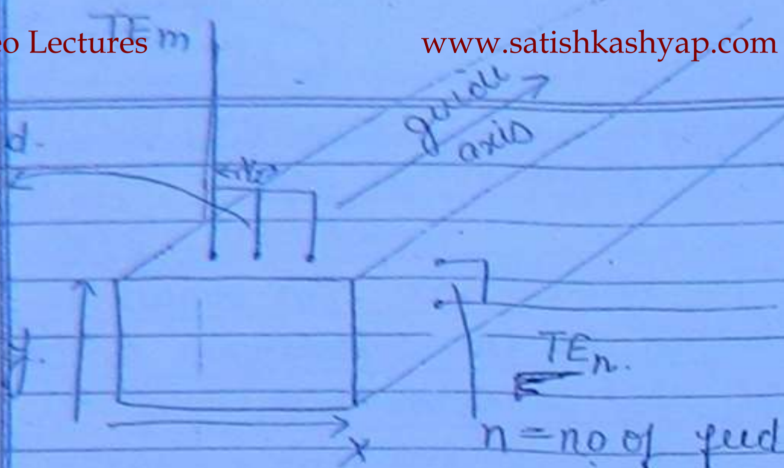
Paravallel Plate wave guide may not be a practical W/G But ground wave propugation over a conducting earth and duct propagation are excellent example of parallel plate W/G.

Rectangular W/G

Rectangular W/G is a closed and confined structure out of four conducting wall.



no. of feed in points

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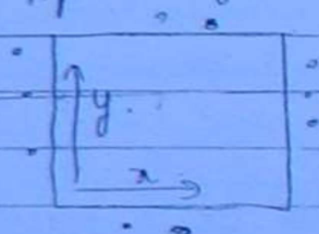
n = no of feed points

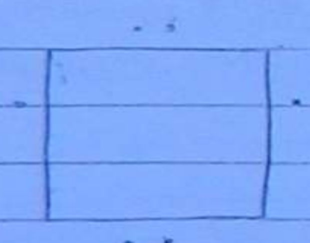
$E_z = 0 \longrightarrow$ TE feed. } Horizontal/vertical
$E_x$ or $E_y$ exists

$H_z = 0 \longrightarrow$ TM feed. } Axial
$H_z$ or $H_y$ exist

Q Identify the field connection in the following cases.

① 

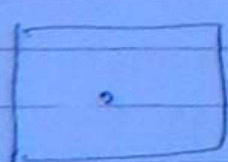

$TE_{13}$

② 

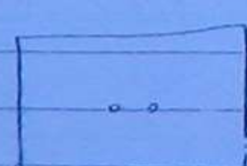
$TE_{21}$

③ 

$TE_{02}$

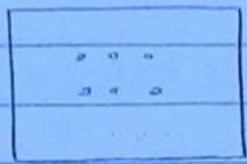
④ 

$TM_{11}$

⑤ 

$TM_{21}$

(6)



$TM_{32}$

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$TE_{32} \rightarrow$ Doesnot exist

$\underbrace{TM_{m0} \text{ \& } TM_{0n}}$ $\rightarrow$ Evanscent modes Non-existing modes in Rectangular W/G.

chapter 4 waveguide (workbook)

1. $f_c = \frac{mc}{2a}$

mode = 1 $f_c = \frac{3\times10^8}{2\times2.5\times10^{-2}} = \frac{30\times10^9}{5} = 6\times10^9$ Hz

$\sin\theta = \left(\frac{f_c}{f}\right)$

$\theta = \sin^{-1}\left(\frac{6\times10^9}{22\times10^9}\right)$

mode = 3 $f_c = \frac{3\times3\times10^8}{5\times10^{-2}} = \frac{90\times10^9}{5} = 18\times10^9$ Hz

$\sin\theta = \left(\frac{f_c}{f}\right)$

$\theta = \sin^{-1}\left(\frac{18}{22}\right)$

2. $f_c = \frac{mc}{2a} \Rightarrow a = \frac{mc}{2\times f_c} \Rightarrow \frac{3\times3\times10^8}{2\times36\times10^9} = \frac{1}{80\times}$

$= 0.0125$ m $\Rightarrow 1.25$ cm

2) $\sin\theta = \frac{f_c}{f}$

$\sin\theta = \frac{10}{36}$

$\sin\theta = \frac{10}{9}$

3) $f_c = \frac{m \times c}{2a} = \frac{\cancel{3} \times 10^8}{2 \times \cancel{3} \times 10^{-2}} = \frac{10^{10}}{2} = 0.5 \times 10^{10}$
$= 5 \times 10^9$

(Normal condition) X

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$f_c = \frac{1 \times 3 \times 10^8}{2 \times 3 \times 3 \times 10^{-2}} = \frac{3 \times 10^{10}}{18} = \frac{10^{10}}{6} =$
$= 0.166 \times 10^{10}$
$= \cancel{1.6 \times 10^9}$

$\sin\theta = \left(\frac{\cancel{1.6 \times 10^9}}{\cancel{2 \times 10^9}}\right)$

$\cancel{\theta = \sin^{-1}(0.8)}$

$\sin\theta = \frac{10^{10}}{6} \times \frac{1}{2 \times 10^9}$

$\theta = \sin^{-1}\left(\frac{5}{6}\right)$ Ans

Propagation along the guide axis

$$\bar{\gamma} = \sqrt{\underbrace{\left(\frac{m\pi}{a}\right)^2}_{\gamma_x} + \underbrace{\left(\frac{n\pi}{b}\right)^2}_{\gamma_y} - \underbrace{\omega^2\mu_0\epsilon_0}_{\gamma \text{ of freespace}}}$$

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If $\bar{\gamma} = 0$ $\quad w_c = \left(\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}\right) c$

$$f_c = \left(\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}\right) \frac{c}{2}$$

$$\sin\theta = \frac{\omega_c}{\omega}$$

$$\bar{v}_p = \frac{c}{\cos\theta}$$

$$\bar{v}_g = c\cos\theta$$

$$\frac{1}{\lambda^2} = \frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2}$$

$$\eta_{TE} = \frac{\eta_0}{\cos\theta} \qquad \eta_0 = 120\pi$$

$$\eta_{TM} = \eta_0 \cos\theta$$

TM wave solutions in Reactangular Waveguide

$(H_x, H_y, E_x, E_y, E_z)$ $\quad H_z = 0$

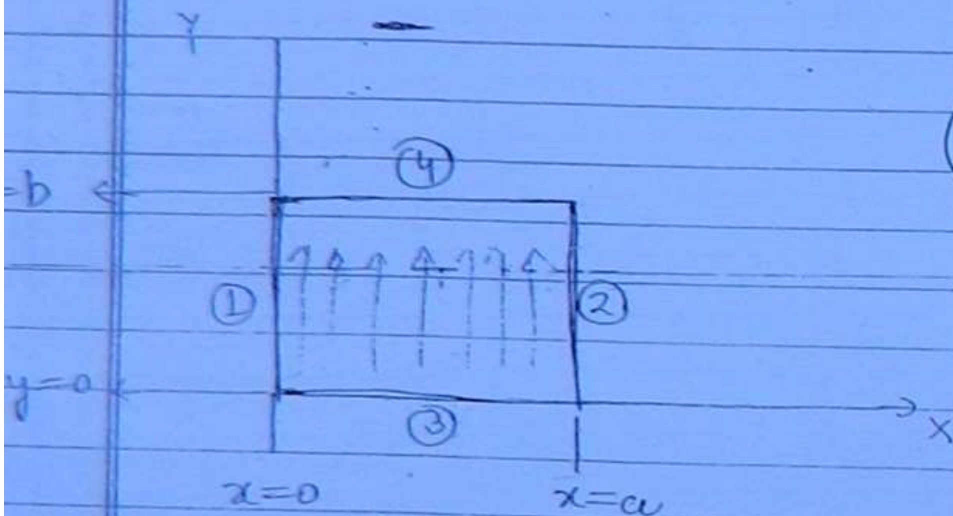
$$E_z = E(x,y,z,t)_z = E_{z_0} \sin\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-\gamma z} e^{j\omega t} a_z$$

$$E_x = E(x,y,z,t)_x = E_{x0}\left(\frac{m\pi}{a}\right) \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-\gamma z} e^{j\omega t} a_x$$

$$E_y = E(x,y,z,t)_y = E_{y0}\left(\frac{n\pi}{b}\right) \sin\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) e^{-\gamma z} e^{j\omega t} a_y$$

$$H_x = H(x,y,z,t)_x = H_{x0}\left(\frac{n\pi}{b}\right) \sin\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) e^{-\gamma z} e^{j\omega t} a_x$$

$$H_y = H(x,y,z,t)_y = H_{x0}\left(\frac{m\pi}{a}\right) \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-\gamma x} e^{j\omega t} a_y$$

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$E(t) / H(t) \longrightarrow e^{j\omega t} \longrightarrow$ source Harmonic

$E(z) / H(z) \longrightarrow e^{-\gamma z} \longrightarrow$ Harmonic Natural

$E(x,y) / H(x,y) \longrightarrow$ sin or cos Harmonic trignometric

$E(x,y)$ i.e the wave in the restricted dimensions should be strictly a trignometric harmonic following the rule

$E(x \text{ or } y)_{tang}$ = parallel or tangent to the guide wall should be an harmonic

$E(x \text{ or } y)\Big|_{tang} = sin$

If $m = 0$ & $n \neq 0$

$TM_{on}$ mode does not exist

$TM_{mo}$ " " "

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The non existant modes for a waveguide are called as evanescent modes.

If two different modes have the same cut off frequency. they are said to be degenerate modes

$TM_{11} \rightarrow$ is the least fc possible TM operation

TE wave solutions in Rectangular w/G ($E_z = 0$)

$(H_x, H_y, E_x, E_y, H_z)$

$E(x, y, z, t)_{(x,y)}$

$H(x, y, z, t)_{(x,y,z)}$

$H_z = H(x, y, z, t)_z = H_{z0} \cos\left(\frac{m\pi}{a} x\right) \cos\left(\frac{n\pi}{b} y\right) e^{-\gamma z} e^{j\omega t} a_z$

$E_x = E(x, y, z, t)_x = E_{x0}\left(\frac{n\pi}{b}\right) \cos\left(\frac{m\pi}{a} x\right) \sin\left(\frac{n\pi}{b} y\right) e^{-\gamma z} e^{j\omega t} a_x$

$E_y = E(x, y, z, t)_y = E_{y0}\left(\frac{m\pi}{a}\right) \sin\left(\frac{m\pi}{a} x\right) \cos\left(\frac{n\pi}{b} y\right) e^{-\gamma z} e^{j\omega t} a_y$

$H_x = H(x, y, z, t)_x = H_{x0}\left(\frac{m\pi}{a}\right) \sin\left(\frac{m\pi}{a} x\right) \cos\left(\frac{n\pi}{b} y\right) e^{-\gamma z} e^{j\omega t} a_x$

$H_y = H(x, y, z, t)_y = H_{y0}\left(\frac{n\pi}{b}\right) \cos\left(\frac{m\pi}{a} x\right) \sin\left(\frac{n\pi}{b} y\right) e^{-\gamma z} e^{j\omega t} a_y$

$E(x, \text{or } y)_{tan} = \sin$ $H(x \text{ or } y)_{tan} = \cos$

$E(x \text{ or } y)_{normal} = \cos$ $H(x \text{ or } y)_{nor} = \sin$

E field can never be paranallel to the guide wall at the Boundaries

E field always starts normally from the guide walls.

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If $m \neq 0$ & $n = 0$

$E(x, z, t)_y = E_{y0}' \sin\left(\frac{m\pi}{a} x\right) e^{-\gamma z} e^{j\omega t} a_y$

$H(x, z, t)_x = H_{x0}' \sin\left(\frac{m\pi}{a} x\right) e^{-\gamma z} e^{j\omega t} a_x$

$H(x, z, t)_z = H_{z0}' \cos\left(\frac{m\pi}{a} x\right) e^{-\gamma z} e^{j\omega t} a_z$

If $n \neq 0$ & $m = 0$

$E(y, z, t)_x$

$H(y, z, t)_y$

$H(y, z, t)_z$

$TE_{10}$ or $TE_{01}$ can exist and has the least cut off frequency

Dominant mode for TE or for the guide in waves

$TE_{10}$ or $TE_{01}$

$TE_{10}$ ; $f_c = \frac{c}{2a}$

$TE_{01}$ ; $f_c = \frac{c}{2b}$

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If $a > b$ $TE_{10}$ dominant mode

$a < b$ $TE_{01}$ " "

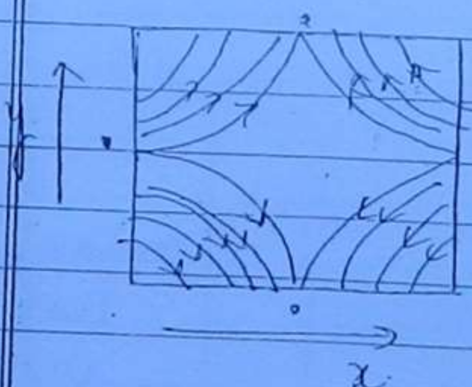
The broad side dimensions of the guide decides the dominant mode.

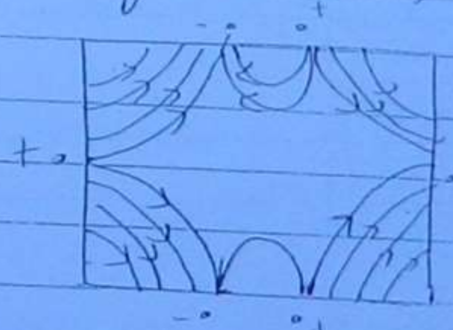
TEM waves cannot exist in rectangular waveguide

In all single conductor guides TEM wave cannot exist

eg: Rectangular w/G and Cylindrical w/G.

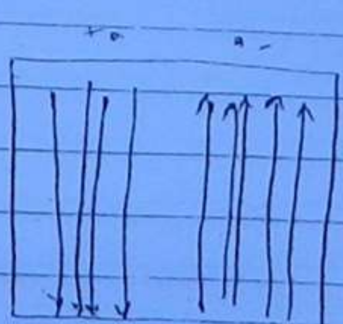
TE waves in Rectangular wave guide $E_x$, $E_y$



normal- Terminate normal enter

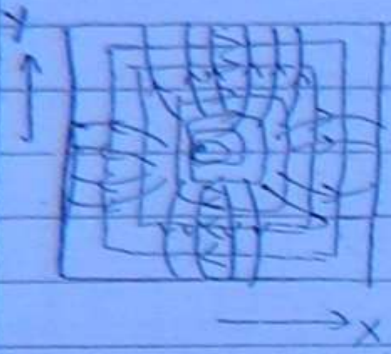
$TE_{11}$ $TE_{21}$

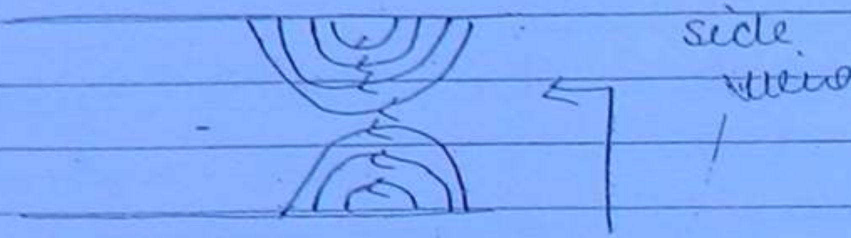


$E_y$

TM waves in Rectangular waveguide

$E_x, E_y, E_z$




$TM_{11}$

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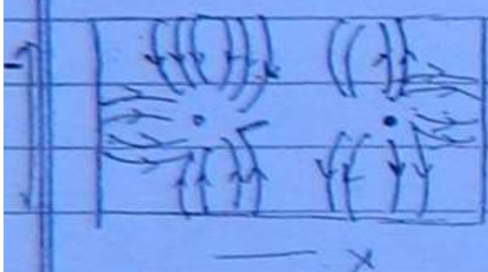
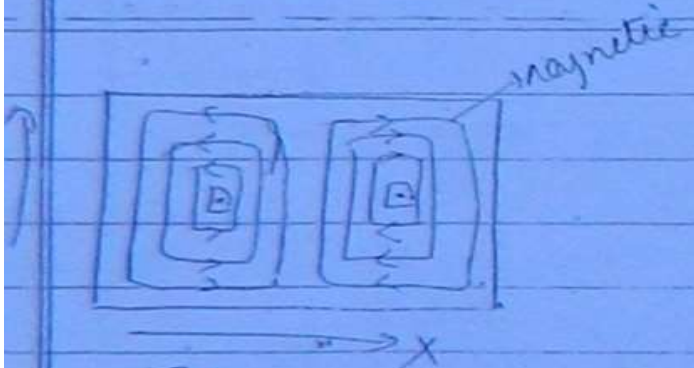
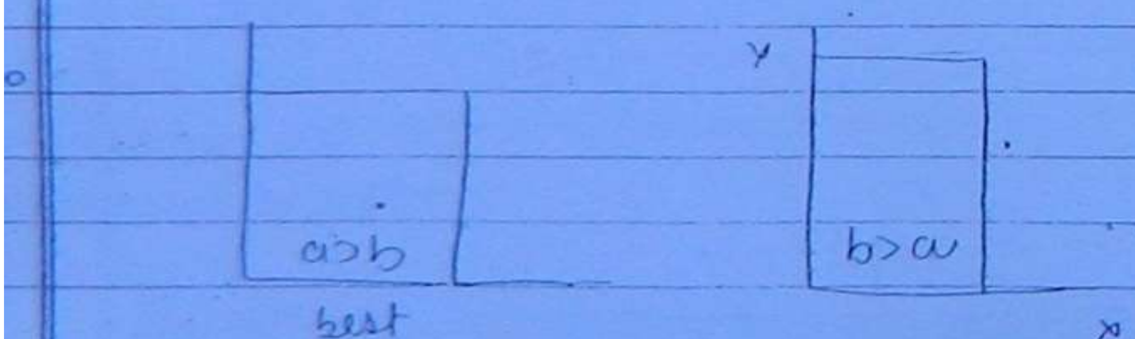


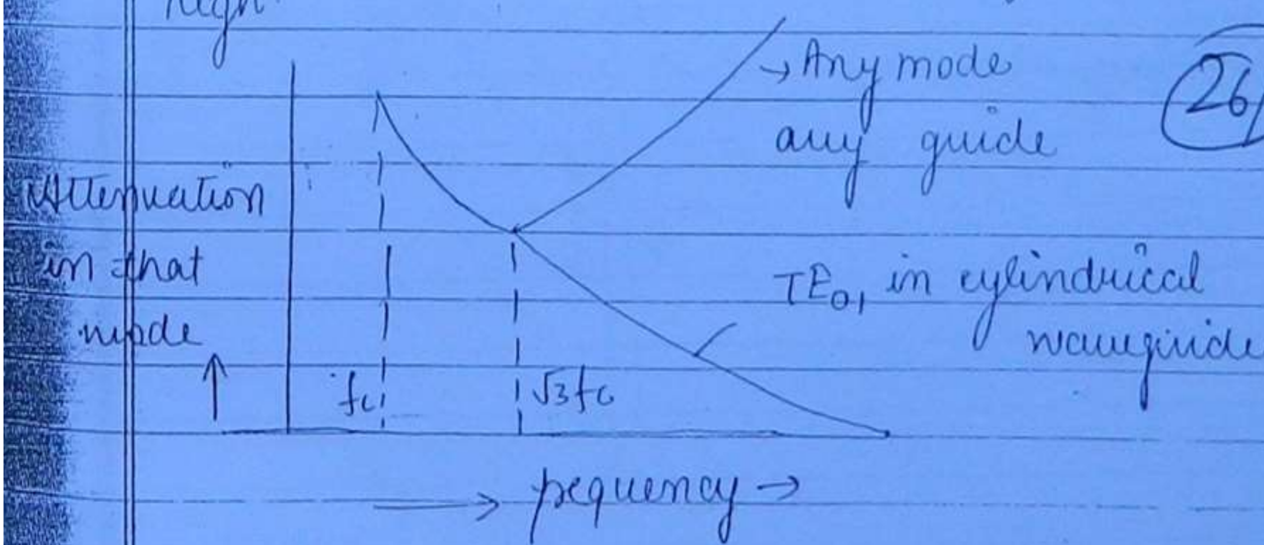

$TM_{21}$




$TM_{21}$



Higher frequencies are not preferred for lower modes i.e. if the frequency is quite larger than cut off. attenuation in a non conducting walls is very high.

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# 5. Basics of Antennas.

(1) Hertzian Diople
Half wave dipole

5) classification of Antenna

2. Basic Definations & Terminology

3. Antennas Arrays.

4. FRIIS free space propagation $Eq^n$

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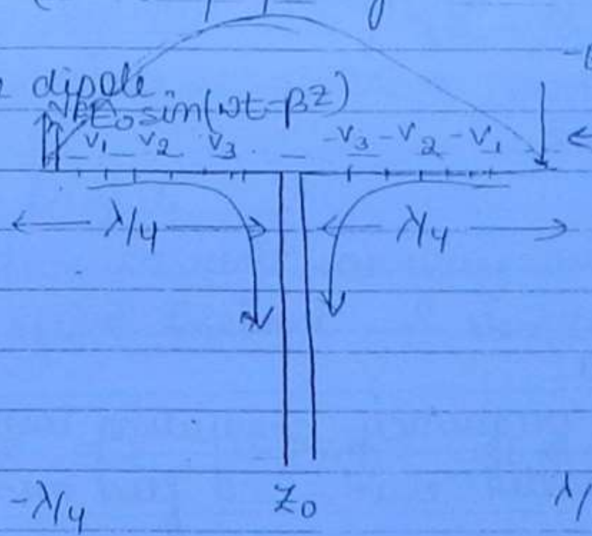
$$\underset{\text{waves}}{V/I} \xrightarrow[\text{Antenna}]{\text{Radiation}} \underset{\text{waves}}{E/H}$$

$$\xleftarrow[\text{Induction}]{}$$

Transmission line (TL) $KHz - MHz - GHz$ freespace
(Audio frequency - microwave)

Half wave dipole ← centre feed dipole




$f = 100 MHz$

$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{10^8}$

$\lambda = 3$

$\lambda/2 = 1.5 m$

- A centre fed half wave dipole is a transmission line opened out by $\lambda/4$ on either side i.e the frequency to be received decides the length of the antenna.

- An EM wave of this frequency when travels along the length of the antenna, axis of the antenna induces voltage all along the conducting length such that at edges of the antenna equal and opposite voltages and hence maximum potential differences at any time.

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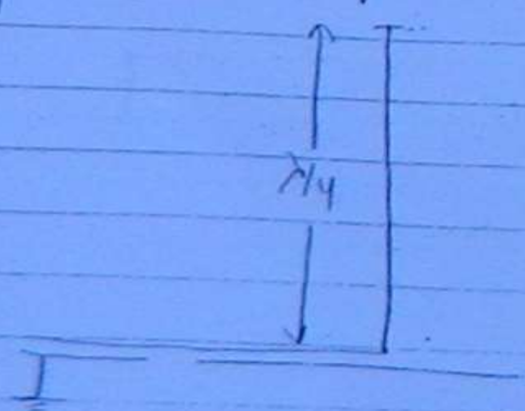
These voltages progressively decrease as we come to the centre or the feed point

These voltages drive current which are maximum at the centre and 0 at the edges whose graph is shown. This is called as asymptotic current distribution i.e

$$|I(z)| = I_0 \cos \beta z$$

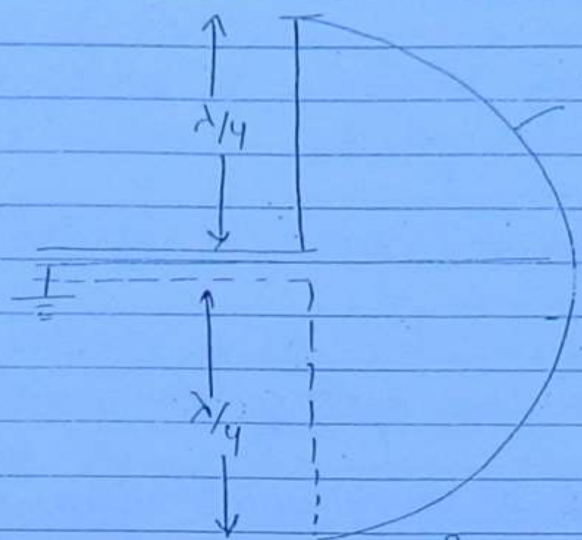
$$z = \frac{-\lambda}{4} \text{ to } \lambda/4$$

Quarter wave Monopole:
vertically grounded - low frequency conducting Earth
It is a single wire grounded [illegible] and feed mechanism



900 MHz
1800 MHz } - GSM

$\lambda$ = cm
100 MHz
$\lambda$ = 3 m




ground is giving reflection
5 - (-5) = 10 V
reflection.

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The image of the antenna in the earth makes it behave lyk an half wave dipole.

Note: for the entire working of the half wave dipole the wave should be travelling along the axes of the antenna i.e the antenna should be correctly aligned with the propogation of the wave. The wave polarization should be the same as antenna polarization.

Hertzian Diople

(length) dl length - current carrying wire

$I(t) = I_m \cos \omega t$ → Oscillatory current element

$dl \to 0 \Rightarrow dl \ll \lambda \quad dl < \lambda/30$

- It Radiates & produces EM waves

If an oscillatory time harmonic current is the send in a wire it has electromagnetic waves all arounded. It can be proved for an $H(z)$ dipole Hertzian dipole as shown Below

$I\,dl \to I(t)\,dl$

$A = \frac{\mu_0 I\,dl}{4\pi r}$ 2 $B = \nabla \times H$

$\nabla \times H = \epsilon \frac{\partial E}{\partial t}$

$I\,dl \longrightarrow I(t)\,dl$

1. $\bar{A} = \dfrac{\mu_0 I dl}{4\pi r}$

2. $B = \nabla \times \vec{A}$

3. $\nabla \times H = \epsilon \dfrac{\partial E}{\partial t}$

$E = \dfrac{1}{\epsilon}\int (\nabla \times H)\, dt$

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If the derivation is carried out in spherical coordinates it can be proved that radiated waves exist all around hence radiation is defined as the production and propagation of EM waves around any I(t) time harmonic current.

Tuesday.

Hertzian dipole:

Time Harmonic (I in wire)

space Harmonic r directed prop.

$$E(r,\theta,\phi,t)_\theta = -\frac{I_m dl \sin\theta\, \omega}{4\pi \epsilon_0 c^2 r} \cdot \sin\omega t \cdot e^{-j\beta r}\, a_\theta$$

$$H(r,\theta,\phi,t)_\phi = -\frac{I_m dl \sin\theta\, \omega}{4\pi c r} \cdot \sin\omega t \cdot e^{-j\beta r}\, a_\phi$$

Radiation of a Hertzian Dipole

$$E(z,t)_x = E_0 e^{j\omega t} e^{-\gamma z} a_x$$

$$H(z,t)_y = H_0 e^{j\omega t} e^{-\gamma z} a_y$$

uniform plane wave
TEM wave.

Radiated wave travels radially outward in the r direction and have a time harmonic same as the current. and an amplitude which dependends on various [illegible]

Properties of Radiation

i. The amplitudes of radiated waves decreases as $\frac{1}{r}$ as the wave disperses radially outward.

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2. This is not due to attenuation but it is dispersion.

3. The amplitude of the radiation is not the same in all the directions around the antenna i.e radiation is directive or angle dependent.

4. The amplitude of the radiation is dependent on the length to wavelength relationship of the dipole. $dl \times 2\pi f \left(\frac{2\pi c}{\lambda}\right)$

Total Radiated Power from a Hertzian dipole.

$$W_r = \iint \underbrace{P_{avg} \cdot ds}_{\text{Power density} \times \text{Enclosing area}} \rightarrow \text{any enclosing spherical surface.}$$

$$\iint \frac{1}{2}\frac{E_0^2}{\eta} \cdot ds$$

$$W_r = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \frac{1}{2} \times \left(\frac{I_m\, dl \sin\theta \cdot \omega}{4\pi\epsilon_0 c^2 r}\right)^2 \times \frac{1}{120\pi} \times r^2 \sin\theta\, d\theta\, d\phi$$

$$\boxed{W_r = I_{rms}^2\, 80\pi^2 \left(\frac{dl}{\lambda}\right)^2}$$

Hence an Hertzian dipole dissipates power as a resistor which dissipates heat but in an antenna the dissipated power is in the form of radiated EM waves. Hence

$$\boxed{\text{radiation Resistance } R_r = 80\pi^2\left(\frac{dl}{\lambda}\right)^2}$$

for Hertzian diople $dl << \lambda$

Radiation Resistance

Radiation resistance is a measure of radiated power for an given i/p current. It should be as large as possible for practical antennas.

adiation of Half wave Dipole:

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- Array or group of Hertzian dipoles where dl is from $-\lambda/4$ to $\lambda/4$

- $|I(l)| = I_m = I_0 \cos\beta l$

Assymptotic current distribution

$$\boxed{E(r,\theta,\phi,t)_\theta = \left[\frac{60 I_m}{r} \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right] \sin\omega t\, e^{-j\beta r} a_\theta}$$

$$\boxed{H(r,\theta,\phi,t)_\phi = \left[\frac{I_m}{2\pi r} \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right] \sin\omega t\, e^{-j\beta r} a_\phi}$$

Note: $\frac{E_\theta}{H_\phi} = 120\pi$ for any EM wave

(Hertzian Dipole) $\frac{E_\theta}{H_\phi} = \frac{1}{\epsilon c} = \frac{1}{\epsilon \frac{1}{\sqrt{\mu\epsilon}}} = \sqrt{\frac{\mu}{\epsilon}} = 120\pi$

Total Radiated Power from a half wave dipole

$$= \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \frac{1}{2} \times \left(\frac{60 I_m}{r} \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right)^2 \times \frac{1}{120\pi} \times r^2 \sin\theta \, d\theta\, d\phi$$

$= I_{rms}^2 (73)$

$$\boxed{W_r = I_{rms}^2 (73)}$$

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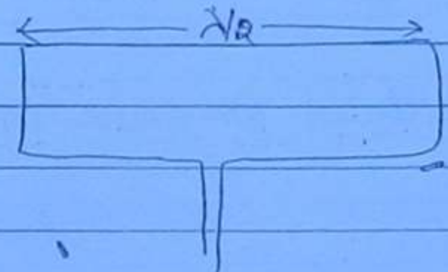
Radiation Resistance of Half wave Dipole = $73\,\Omega$

" " Quarter wave Monopole = $36.5\,\Omega$

" " Folded Dipole = $2^2 \times 73$

= $292\,\Omega$




Basic Terms and definitions:

1. Isotropic Antenna
2. Radiation Power density
3. " " Intensity.
4. Gain
5. ~~Effecient~~ Effective length
6. " Area
7. Radiation Pattern

Isotropic Antenna

It is also called omnidirectional antenna and radiates power in all directions uniformly.

It's E field is independent of $\theta$ and $\phi$

2. Radiation Power Density:

It is the strength of the radiated EM wave anywhere around the antenna.

Power/Area $\frac{dW_r}{ds}$ = watts/$m^2$ = Poynting vector of the EM wave

$$= \frac{1}{2}\frac{E_0^2}{\eta}(r,\theta,\phi)$$

$$= V(r,\theta,\phi)$$

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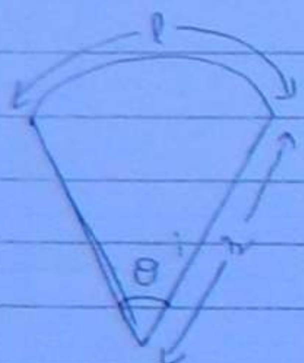
3. Radiation Power Intensity

It is the strength of the radiated EM wave in any direction from the antenna

$$= \psi(\theta,\phi) = \frac{\text{Power}}{\text{direction}} = \frac{\text{Power}}{\text{solid angle}} = \frac{dW_r}{d\Omega} = \frac{\text{watts}}{\text{steradian}}$$



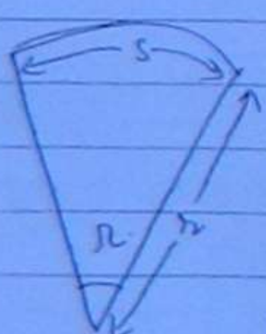

$l = \theta r$

$\theta = 1$ radian

$l = r$

$\theta = 6.28$ radians. $(2\pi) = 6.28$

$c = 6.28r$




$S = \Omega r^2$

$\Omega = 1$ steradian

$S = r^2$

$\Omega = 12.56$ steradian $(4\pi = 12.56)$

TSA $= 12.56\, r^2$

One steradian is a solid angle subtended by a cone whose base curvature is equal to square of radius of curvature

Hence $\boxed{ds = d\Omega\, r^2 = r^2 \sin\theta\, d\theta\, d\phi}$

$$\boxed{d\Omega = \sin\theta\, d\theta\, d\phi}$$

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(Density)

$$\psi(\theta,\phi) = \frac{dW_r}{d\Omega} \Rightarrow \frac{dW_r}{ds}\cdot\frac{ds}{d\Omega} \Rightarrow v(r,\theta,\phi)\cdot r^2$$

$$= \frac{1}{2}\frac{E_0^2(\theta,\phi)}{\eta}$$

eg: $v(r,\theta,\phi) = v_{avg}$ = Average Rad. Power

$$= \frac{W_r}{4\pi r^2}$$

$\psi(\theta,\phi) = \psi_{avg} = \frac{W_r}{4\pi}$ = Average Power per direction

4. Gain → $G_D$ - Directive Gain
→ $G_p$ - Power Gain
→ D - Directivity

$G_D$ - Directive Gain

$G_D$ = Radiation Intensity of the given antenna in a given direction / Radiation intensity of isotropic antenna

$$G_D = \frac{\psi(\theta,\phi)}{\psi_{avg}} = \frac{\psi(\theta,\phi)}{\frac{W_r}{4\pi}} = \frac{4\pi\,\psi(\theta,\phi)}{\iint \psi(\theta,\phi)\, d\Omega}$$

$$G_D = \frac{4\pi\,\psi(\theta,\phi)}{\iint \psi(\theta,\phi)\,d\Omega}$$

$G_p$ – Power gain

$$G_p = \frac{4\pi\,\psi(\theta,\phi)}{W_{IN}} = \frac{4\pi\,\psi(\theta,\phi)}{W_r}\cdot\frac{W_r}{W_{IN}}$$

$$\boxed{G_p = G_D \times \text{Efficiency of Radiation}}$$

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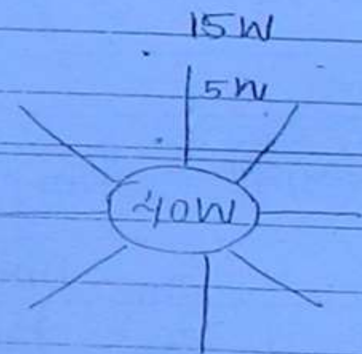
$$\boxed{G_p = G_D \times \frac{W_r}{W_r + W_e}}$$

$$\boxed{G_p = G_D \times \left(\frac{R_r}{R_r + R_e}\right)}$$




D = 3

$W_e$ = loss power
$R_r$ = loss resistance
$W_{IN}$ = total i/p power

Directivity (D)

$D = G_D\big|_{max}$ = Maximum value of directing gain

(ow) Antenna chapter 5 (Work Book)

W.B $D \longrightarrow G_D \longrightarrow \psi(\theta,\phi) \longrightarrow U(r,\theta,\phi) \longrightarrow E(r,\theta,\phi)$

for Hertzian dipole $|E| = \frac{E_1 \sin\theta}{r}$

$$U(r,\theta,\phi) = \frac{1}{2}\cdot\frac{E_1^2 \sin^2\theta}{r^2\,\eta}$$

6 W-B $\lambda = 492\ m$

$\frac{\lambda}{4} = 123\ m$

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$l = 124\ m$

It is a quarter monopole.

$R_r = 36.5\ \Omega$

7 W-B 6dB.

$10 \log \quad = 6\ dB.$

$\log G_D = \frac{6}{10}\ dB.$

$G_D = 10^{0.6}.$

Antenna is passive element. So whenever i/p is given in lossless antenna the same power is transmitted.

10 W.B. $\psi(\theta,\phi) = \psi_{avg} = \frac{4\pi}{8} = \frac{W_r}{4\pi} = \frac{100}{4\times\pi} = \frac{50}{2\pi} = 7.96W$

$U(r,\theta,\phi) = U_{avg} = \frac{W_r}{4\pi r^2} = \frac{100}{4\pi\times(10\times10^3)^2} = \frac{100}{4\pi\times10^8} =$

$0.079 \times 10^{-6}$

$= 0.08\ \mu W$

12 W B $40\pi = W_{IN}$

Efficiency $= \frac{W_r}{W_{in}}$

Efficiency $\eta = 90\%$

$\frac{90}{100} = \frac{W_r}{40\pi}$

$36\pi = W_r$

$W_r = \frac{40\pi \times 90}{100}$

$\psi_{max} = 150\ W/\Omega$

$W_r = 36\pi$

$D = G_D|_{max} = \frac{4\pi\,\psi(\theta,\phi)|_{max}}{} = \frac{4\pi\times150}{} = \frac{150}{a}$

$10 \log (16.67) = 12.2 \, dB$

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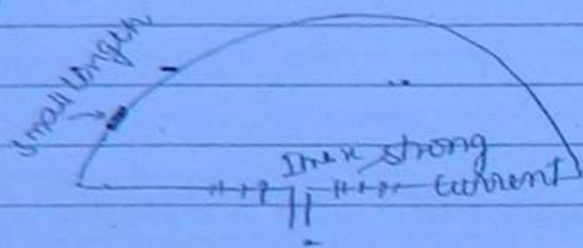
5. Effective length ($l_{eff}$)

Half wave Dipole

$I(z)$ — asymptotic — $W_r$ — $l_{phy}$
($0$ — $I_{max}$)

$I_{avg}$ — uniform — $W_r$ — $l_{eff}$




It is the length required to radiate the power $W_r$ with uniform currents, assuming the same power is radiated with non uniform currents over the physical length

For a half wave dipole.

$$\boxed{I_{avg} = \frac{2 I_{max}}{\lambda}}$$

$$I_{avg} = \frac{\int_0^{T/2} I_m \sin \omega t \, dt}{T/2} = \frac{2 I_{max}}{\lambda}$$

where $I_{max}$ = current at the centre

The same relationship is share for effective length and physical length.

$$\boxed{L_{eff} = \frac{2 \, l_{phy}}{\lambda}}$$

$G_D = \omega_4$

$$\psi(\theta, \phi) = \frac{1}{2} \frac{E_1^2 \sin^2\theta}{\eta}$$

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$$G_D = \frac{4\pi \cdot \frac{1}{2} \frac{E_1^2 \sin^2\theta}{\eta}}{\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \frac{1}{2}\frac{E_1^2}{\eta} \sin^2\theta \cdot \sin\theta \, d\theta \, d\phi}$$

$$= \frac{4\pi \cdot \sin^2\theta}{2\pi \int_0^{\pi} \sin^3\theta \, d\theta}$$

$$= \frac{2 \sin^2\theta}{\left(\frac{4}{3}\right)} = \frac{3}{2} \sin^2\theta$$

$$G_D = \frac{3}{2} \sin^2\theta$$

$$D = \frac{3}{2} = 1.5$$

Repeat the same proplem for a half wave dipole.

$$D \rightarrow G_D \rightarrow \psi(\theta, \phi) \rightarrow U(r, \theta, \phi) \rightarrow E(r, \theta, \phi)$$

for half wave dipole $\quad |E| = E_1 \frac{\cos(\pi/_2 \cos\theta)}{\sin\theta}$

$$U(r, \theta, \phi) = \frac{1}{2} \frac{E_1^2}{r^2 \eta} \frac{\cos^2(\pi/_2 \cdot \cos\theta)}{\sin^2\theta}$$

For a half wave dipole Directivity = "1.63"

2 W.S

4 – half wave dipoles.

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$W_r = I_{rms}^2 \cdot 73$ (for one half wave dipole)

$= \left(\frac{1}{2 \times \sqrt{2}}\right) \cdot 73$

$W_r = \frac{1^2}{2} \times \frac{1}{\sqrt{2}} \cdot 73 \times 4$ (4 dipole)

$W_r =$ 36.5 watt.

4.W.B

$\underbrace{\frac{\lambda}{2} - \frac{\lambda}{4} - \frac{\lambda}{8} \text{ upto } \frac{\lambda}{10}}_{\text{Half wave dipole}}$

$R_r = 73\Omega$ – $\lambda/2$ dipole

$R_r = 18.25\Omega$ – $\lambda/8$ dipole

$R_l = 1.5$ (given)

$\text{Efficiency} = \frac{18.25}{19.25} = 89\%$

Electrically short dipoles

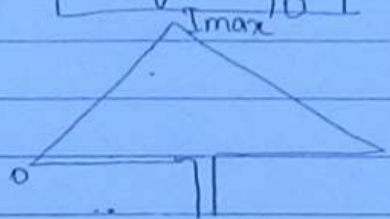
For electrically short dipole i.e length is less than $\frac{\lambda}{10}$.

$$\boxed{l_{phy} < \frac{\lambda}{10}}$$

The currents are non uniform but they are linear

i.e $I(z) \propto z$

$\boxed{l_{phy} < \frac{\lambda}{10}}$ $\boxed{l_{phy} > \frac{\lambda}{30}}$ 277



$I(z)$ = linear — $W_r$ — Phys.

$(I(z) \propto z)$

$$\boxed{I_{avg} = \frac{I_{max}}{2}}$$

$$\boxed{L_{eff} = \frac{l_{phy}}{2}}$$

Hertzian Dipole length is $\boxed{l_{phy} < \frac{\lambda}{30}}$

for an the Hertzian dipole the currents are uniform as a length is very small and hence

$I(z) = I_{max}$
$=$ uniform

$$\boxed{L_{eff} = l_{phy}}$$

As Hertzian dipole $= R_r = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2$

for any Antenna $R_r = 80\pi^2 \left(\frac{L_{eff}}{\lambda}\right)^2$

Effective length assume uniform current everywhere hence $dl$ can be replaced with $L_{eff}$ for any antenna.

case(i) Electrically short Dipole:

$$R_r = 80\pi^2 \left(\frac{L_{phy}}{2\lambda}\right)^2$$

$$\boxed{R_r = 20\pi^2 \left(\frac{L_{phy}}{\lambda}\right)^2}$$

case(ii) Electrically short Monople:

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$$R_r = 10\pi^2 \left(\frac{L_{phy}}{\lambda}\right)^2$$

case(iii) Electrically short monopole: vertically grounded conducting surface.

$$R_r = 10\pi^2 \left(\frac{2h}{\lambda}\right)^2 \qquad L_{phy} = 2h$$

$h$ = height of the monopole.

$$R_r = 40\pi^2 \left(\frac{h}{\lambda}\right)^2$$

Radiation Pattern

It is a pattern graph it is a polar plot of radiation intensity showing the regions where the radiation strength is finite eg: Hertzian Dipole has $E = \frac{E_1 \sin\theta}{r}$

$$\psi = \psi_0 \sin^2\theta$$

If $\theta = 0^\circ / 180^\circ$ $\quad \psi = 0$
$\hookrightarrow \theta_{NP} \longrightarrow$ null points

If $\theta = 90^\circ / 270^\circ$ $\quad \psi = \psi_{max}$

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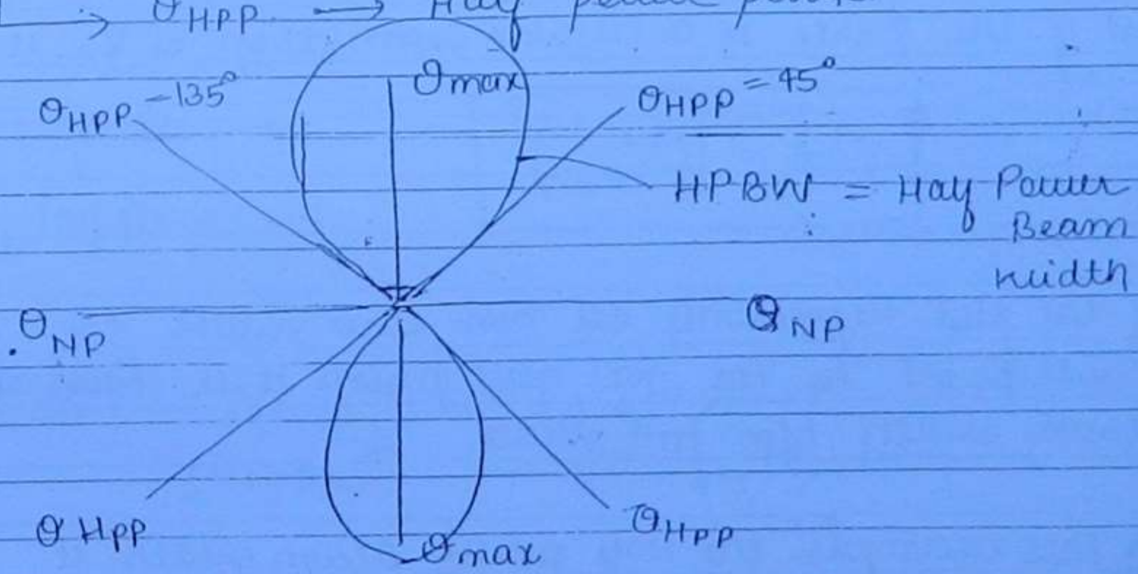
$\hookrightarrow \theta = \theta_{max}$

If $\theta = 45^\circ / ~~35^\circ~~ / 135^\circ / 225^\circ / 315$ $\quad \psi = \frac{\psi_0}{2}$
$\hookrightarrow \theta_{HPP} \longrightarrow$ Half power points




- The Hertzian dipole has radiation intensity depending upon θ but independent of φ. Hence the polar plot has half power beam width defined for θ half power points enclosing the maxima.
- It is the same for all φ Hence the beam is circular in the top view.

If the antenna is $\theta$ and $\phi$ dependent it has half power beam width in $\theta$ side and the half power beam width in $\phi$ side such that Beam solid angle is given by.

$$HPBW = \theta_{HPBW}$$
$$HPBW = \phi_{HPBW}$$

Beam solid angle $\Omega_A$

$$\theta_{HPBW} \times \phi_{HPBW}$$

= Steredians

= radians²

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But if the $\phi$ side is a circular beam then $\Omega_A$ is

$$\boxed{\Omega_A = (\theta_{HPBW})^2}$$

If the plot represents all non-zero values from the null point to the next null point it is called as beam width b/w full Nulls.

In this example the Half power Beam width is 90° and the beam width b/w full null is 180°

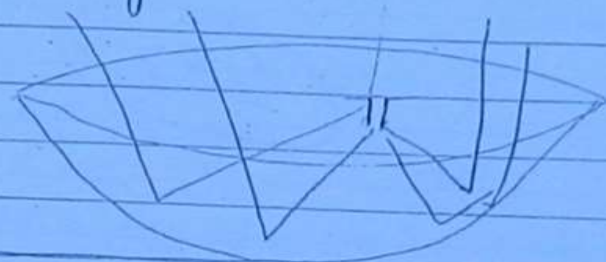
$$\boxed{D \propto \frac{1}{\Omega_A}}$$

$$D = \frac{4\pi}{\Omega_A}$$

If $D=1$ isotropic

Effective Area of an Antenna. $(A_e)$ or Capture Area

$$A_e = \frac{\text{Power Induced}}{\text{Poynting vector of the EM wave}} = \frac{\text{watts}}{\text{watts}/m^2}$$



MW – GHz – Dipole

$\lambda = cm$

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$$\boxed{A_e = \frac{\lambda^2}{4\pi} D}$$

$44\,dB = G$

$10 \log G = 44$

$\log G = 4.4$

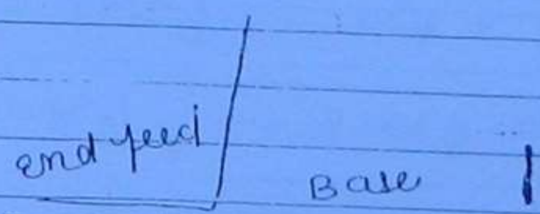
$D = 10^{4.4}$

1 Radian $= 57^\circ$

$3.14 \times 57^\circ = 180^\circ$

$\pi$ Radian $= \pi 0$

$$10^{4.4} = \frac{4\pi}{(\theta_{HPBW})^2} = \frac{4 \times 3.14 \times (57)^2}{(\theta_{HPBW})^2}$$




50 m long.

$f = 600\,kHz$

$\lambda = 500\ m$

$d = \frac{\lambda}{10}$

$$R_r = 40\pi^2\left(\frac{h}{\lambda}\right)^2 = 40\pi^2\left(\frac{50}{500}\right)^2 = \frac{2\pi^2}{5} = 4\Omega$$

125 m — 125 m — 36.5 Ω

λ/4

50 m ——— 4 Ω

λ/10

13 W.B

1 m — l

10 MHz = f

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$$\lambda = \frac{3\times10^8}{10\times10^6} = \frac{c}{f} = 30m$$

$$l = \frac{\lambda}{30}$$

$$R_r = 80\pi^2\left(\frac{dl}{\lambda}\right)^2 = 80\pi^2\left(\frac{\lambda}{\lambda 30}\right)^2 = 0.88\Omega$$

length is small than wavelength the radiating ~~ability it become poor~~.

25 W.B

$$l = 0.03\lambda$$

$$R_r = 80\pi^2\left(\frac{dl}{\lambda}\right)^2$$

$$= 80\pi^2\left(\frac{0.03\lambda}{\lambda}\right)^2 \Rightarrow 80\pi^2\times(0.03)^2$$

$$\Rightarrow 0.072\pi^2\Omega$$

26 W.B

$$l = 5\ m$$

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{3\times10^6} = 100\ m$$

$$R_r = 80\pi^2\left(\frac{dl}{\lambda}\right)^2$$

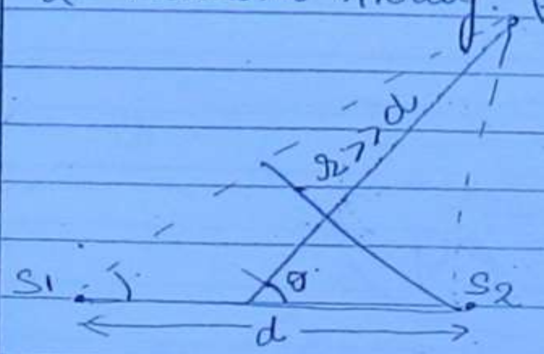
$$= 80\pi^2\left(\frac{5}{100}\right)^2 \Rightarrow$$

$$= 0.197\Omega \approx 2\Omega$$

Wednesday.

Array of Isotropic sources.

2- Element Array.




To analyse the radiation at a far-zone i,e $r >> d$.

$$E_T = E_1 + kE_1 e^{j\psi}$$

If $|I_1| = |I_2|$ in S1 & S2

$K = 1$.

$\psi$ = phase diff b/w the radiated waves from $s_1$ and $s_2$

$\psi$ = phase difference in currents in sources

+

path difference due to inclination

$2\pi$ — $\lambda$

? — $d\cos\theta$

$$\boxed{\psi = \alpha + \beta d\cos\theta}$$

$$E_T = E_0 (1 + \cos\psi + j\sin\psi)$$

$$|E_T| = |E_0| \sqrt{(1+\cos\psi)^2 + \sin^2\psi}$$

$$= |E_0| \sqrt{1 + \cos^2\psi + 2\cos\psi + \sin^2\psi}$$

$$= |E_0| \sqrt{1 + 2\cos\psi}$$

$$= E_0 \sqrt{2 \cdot 2\cos^2 \psi/2}$$

$$\boxed{|E_T| = 2E_0 \cos\psi/2}$$

Note: The individually isotropic antennas as an array are not isotropic due to interference of the radiation

$$E_T = 2E_0 \cos \psi/2$$
$$\psi = \alpha + \beta d \cos\theta$$

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case (i) $d = \lambda/2$, $\alpha = 0°$

$$E_T = 2E_0 \cos \psi/2$$

$$\psi = 0 + \frac{2\pi}{\lambda} \times \frac{\lambda}{2} \cos\theta$$

$$\psi = \pi \cos\theta$$

$$E_T = 2E_0 \cos\left(\frac{\pi}{2}\cos\theta\right)$$

$\theta = 0°$ or $180°$

$$E_T = 0$$

$\theta_{NP}$

$\theta = 90°$ or $270°$

$$E_T = 2E_0 \cos\left(\pi/2 \times \cos 90°\right)$$
$$E_T = 2E_0$$

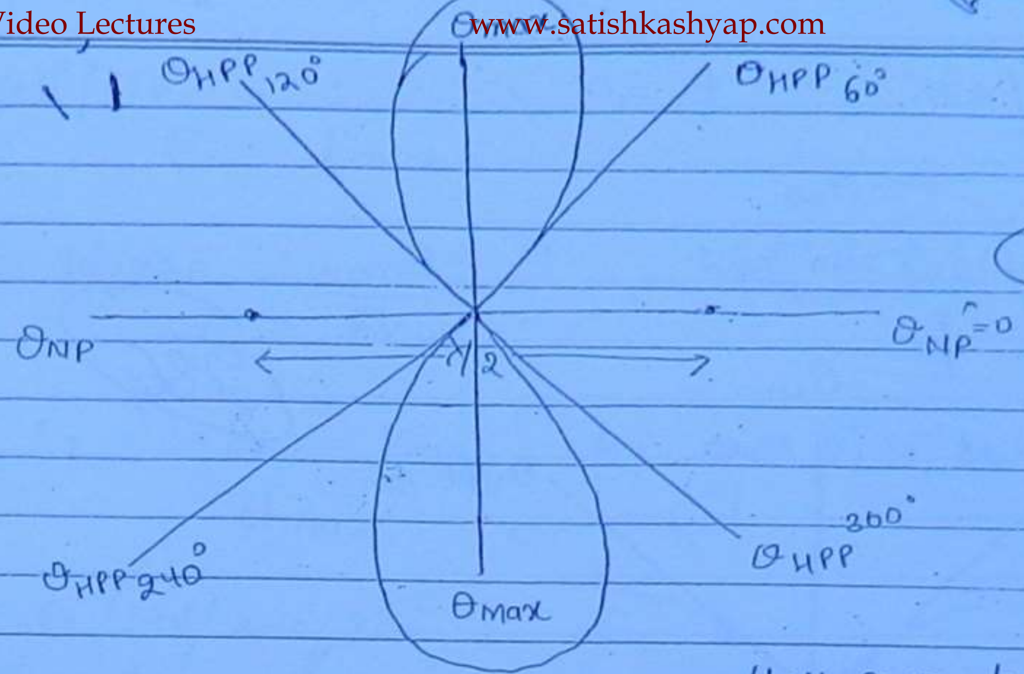
$$E_T = E_{max} = 2E_0$$

$\theta_{max}$

$\theta = 60° / 120° / 240° / 300°$

$$E_T = \frac{E_{max}}{\sqrt{2}}$$

$\theta_{HPP}$



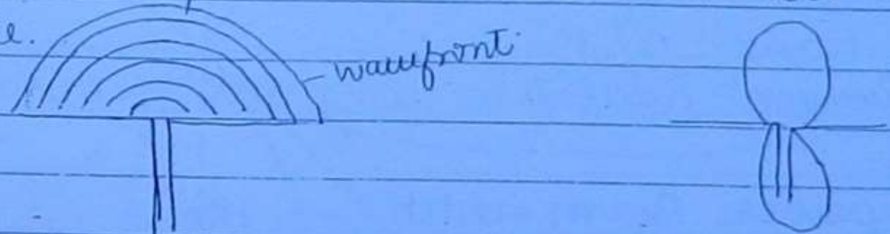

Half power beam width $= 60°$

## Broad side Array.

i.e $\theta_{max} = 90°/270°$

Maximum Radiation normal to the axis of the array.

i) The broad side array and a half wave dipole always have identical radiation pattern as the currents are maximum at the centre.




ii) $d = \lambda/2$ $\alpha = \pi$

$$E_T = 2E_0 \cos\left(\alpha + \pi\cos\theta\right) \qquad \psi = \pi + \frac{2\pi}{\lambda}\times\frac{\lambda}{2}\cos\theta$$

(Half power beam width is 60° for the broadside array)

$$E_T = 2E_0 \sin\left(\pi/_2 \cos\theta\right)$$

$$\left\{\cos\psi/_2 = \cos\left(\frac{\pi + \pi\cos\theta}{2}\right) = \cos\left[\pi/_2 + \frac{\pi}{2}\cos\theta\right] = \sin\left[\pi/_2\cos\theta\right]\right.$$

If $\theta = 0°/180°$ $E_T = E_{max}$

$\theta_{max}$

If $\theta = 90/270°$ $E_T = 0$

$\theta_{NP}$

If $\theta = 60°/120°/240°/300°$ $E_T = \frac{E_{max}}{\sqrt{2}}$

$\theta_{HPP}$

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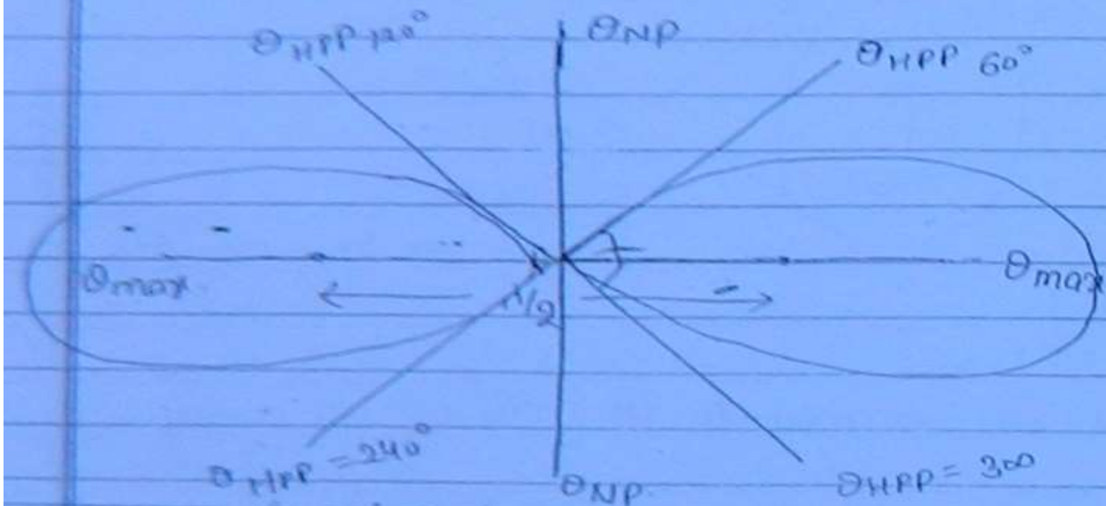


End fire Array

i.e. $\theta_{max} = 0°/180°$

maximum Radiation along axis of the array.

Half power Beamwidth = 120°

(iii) $d = \lambda$ , $\alpha = 0°$

$E_T = 2E_0 \cos \psi/2$

$\psi = 0 + \frac{2\pi}{\lambda} \times \lambda \cos\theta$

$\psi = 2\pi \cos\theta$

$E_T = 2E_0 \cos\left(\frac{2\pi \cos\theta}{2}\right)$

$E_T = 2E_0 \cos(\pi \cos\theta)$

$\theta = 0^\circ/180^\circ$

$E = E_{max}$

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$\theta = 90^\circ/270$

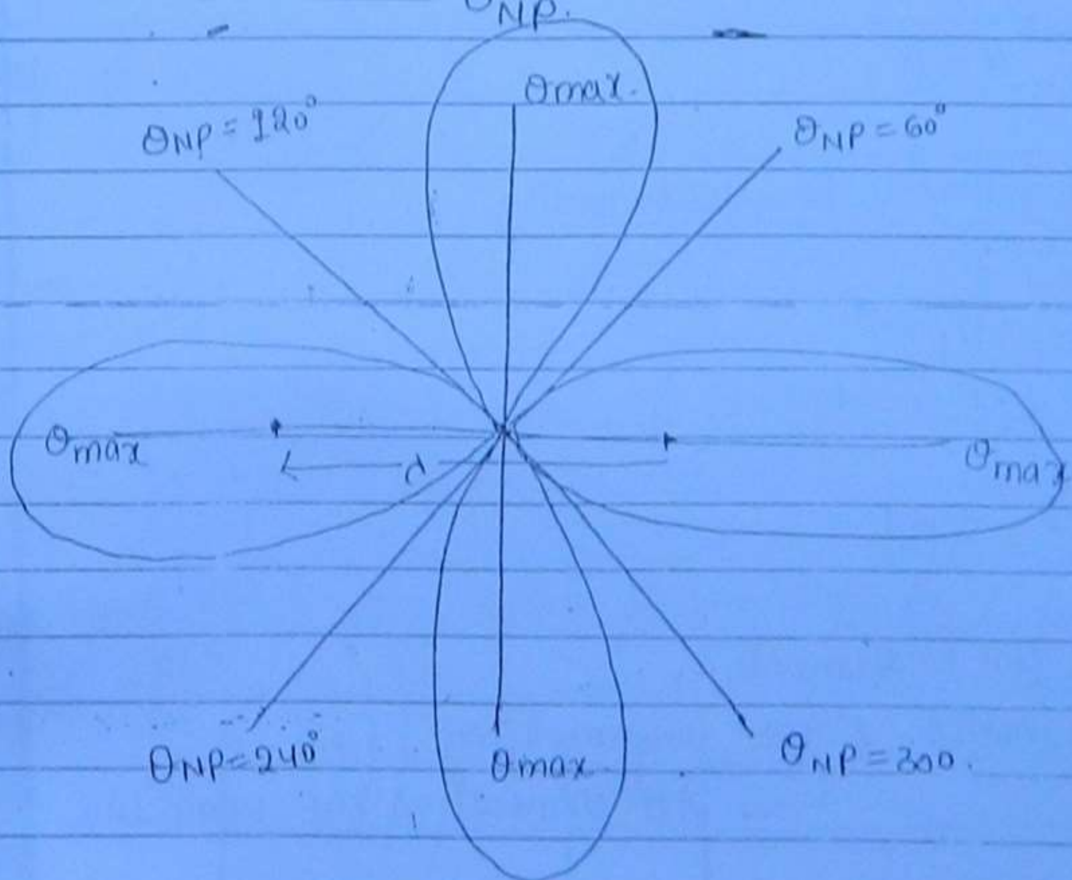
$E = E_{max}$

$E_T = 2E_0 = E_{max}$ for $\theta = 0^\circ/90^\circ/180^\circ/270^\circ$

$\theta_{max}$.

$\theta = 60^\circ/120^\circ/240^\circ/300^\circ$ $\quad E_T = 0.$

$\theta_{NP}$.




**Scanning Array.**

It is designed to have maxima in specific direction by changing $\alpha$ in the source.

Generalization or Designed principle

$\psi \to 0 \Rightarrow E_{max}$ = Max Radiation

$$\alpha + \beta d \cos\theta_{max} = 0$$

$$\boxed{\cos\theta_{max} = \frac{-\alpha}{\beta d}}$$

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If $\theta_{max} = 90°/270°$

$\Rightarrow$ Broad side array

$\Rightarrow \alpha = 0$

In phase currents Broadside array.

If $\theta_{max} = 0°/180°$

$\Rightarrow$ End fire Array.

$\Rightarrow \alpha = \pm\beta d$

Extension for N elements

N-elements - Linear, uniform Array

- All elements on the same line
- Equal spacing (d)
  Equal phase diff. progressively ($\alpha$) between the elements.

$$E_T = E_0 + E_0 e^{j\psi} + E_0 e^{j2\psi} + E_0 e^{j3\psi} \ldots E_0 e^{j(N-1)\psi}.$$

$$E_T = E_0 \left[ 1 + e^{j\psi} + e^{j2\psi} + E_0 e^{j3\psi} \ldots\ldots e^{j(N-1)\psi} \right]$$

Geometric Progression
first element $= 1 = a$
common factor $e^{j\psi} = r$

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$$E_T = E_0 \left( \frac{1 - e^{jN\psi}}{1 - e^{j\psi}} \right) = E_0 \left( \frac{1 - \cos N\psi - j\sin N\psi}{1 - \cos\psi - j\sin\psi} \right)$$

$$E_T = \frac{E_0 \sin\left(N\frac{\psi}{2}\right)}{\sin(\psi/2)}$$

If $\psi \to 0$

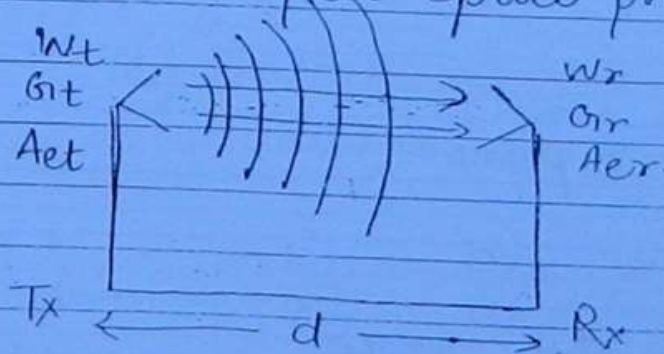
$$E_T = E_0 \left(\frac{N\psi}{2}\right) \left(\frac{2}{\psi}\right) = N E_0$$

$$E_T = N E_0$$

If $N = 2$.

$$E_T = 2E_0 \cos \psi/2$$

FRISS - free - space propagation eqn.




Power density at the reciever $= \dfrac{W_t G_t}{4\pi d^2}$

Power Received $= \dfrac{W_t G_t}{4\pi d^2} A_{er}$

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$$A_{er} = \frac{\lambda^2}{4\pi} G_r$$

(Power Received) $$\boxed{W_r = \frac{W_t G_t \cdot G_r}{\left(\frac{4\pi d}{\lambda}\right)^2}}$$ FRIIS - free space propagation Eq$^n$.

$\left(\frac{4\pi d}{\lambda}\right)^2$ = loss due to spatial attenuation - due to dispersion

Power density at the reciever $= \frac{W_t G_t}{4\pi d^2} = \frac{1}{2}\frac{E_0^2}{\eta} = \frac{E_{rms}^2}{\eta} = \frac{E_{rms}^2}{120\pi}$

$$E_{rms}^2 = \frac{30 W_t \cdot G_t}{d}$$

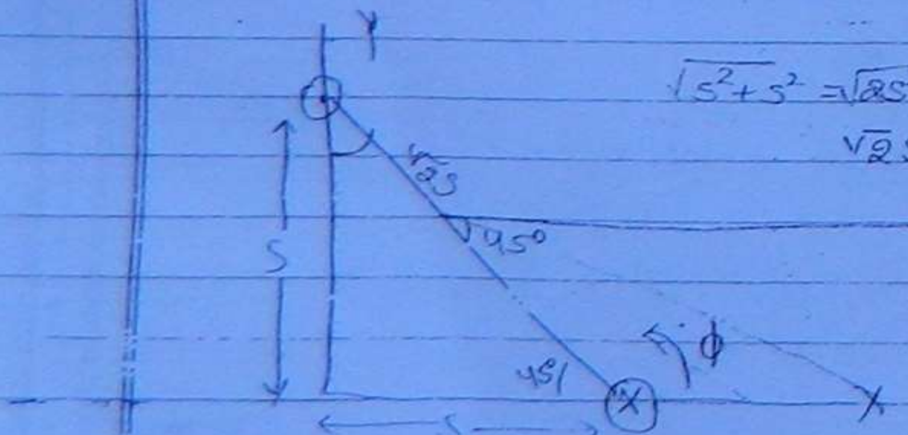
~~(RMS E field~~ at the reciever) $$\boxed{\cancel{E_{rms}} = \sqrt{\frac{30 W_t \cdot G_t}{d}}}$$

$$\boxed{W_r(dBW) = W_t(dBW) + G_t(dB) + G_r(dB) - L_s(dB)}$$

16 W.B

$\theta = \pi/2$ plane
$Z = r\cos\theta = 0$ } X-Y plane.

$\sqrt{s^2+s^2} = \sqrt{2s^2}$ $\theta = 45°$
$\sqrt{2}s$



$\alpha = \pi$

$d = \sqrt{2}s$ $\theta = 45°$ (291)

$$E_T = 2E_0 \cos\left(\frac{\alpha + \beta d \cos\theta}{2}\right)$$

$$E_T = 2E_0 \cos\left(\frac{\pi}{2} + \frac{2\pi}{\lambda} \cdot \frac{\sqrt{2}s}{2} \cos 45°\right)$$

$$\frac{E_T}{E_0} = 2\cos\left(\frac{\pi}{2} + \frac{\cancel{2}\pi}{\lambda} \cdot \frac{\sqrt{2}s}{\cancel{2}} \times \frac{1}{\sqrt{2}}\right)$$

$$\frac{E_T}{E_0} = 2\cos\left(\frac{\pi}{2} + \frac{\pi s}{\lambda}\right)$$

$$\frac{E_T}{E_0} = 2\sin\left(\frac{\pi s}{\lambda}\right)$$

$+3dB \uparrow$ 2 (double)

$-3dB \downarrow$ 2 (i.e half)

$5km \rightarrow d$

$10\log(\text{value}) = 3dB$

$\log(\text{value}) = 0.3$

$\text{value} = 10^{0.3} = 2$

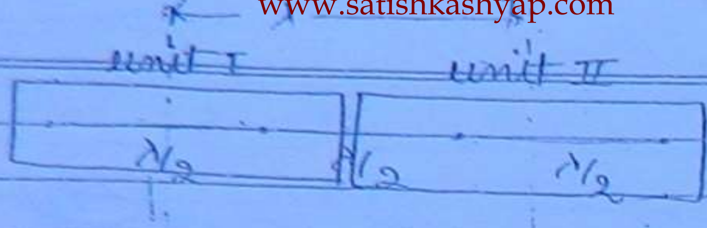
$E \propto \frac{1}{d}$ $\frac{E_1}{E_2} = \frac{d_2}{d_1}$

$$E_2 = \frac{E_1}{\sqrt{2}}$$

$$d_2 = \sqrt{2}d_1 = \sqrt{2} \times 5 = 7km$$

$$7km - 5km = 2km$$

14 W.B






Multiplication of patterns (Symmetric Arrays)

Two symmetrics units (same)

unit : 2 Element array.

$d = \lambda/2$

$E_T = 2E_0 \cos(\pi/2 \cos\theta)$

The Two units forming a group.

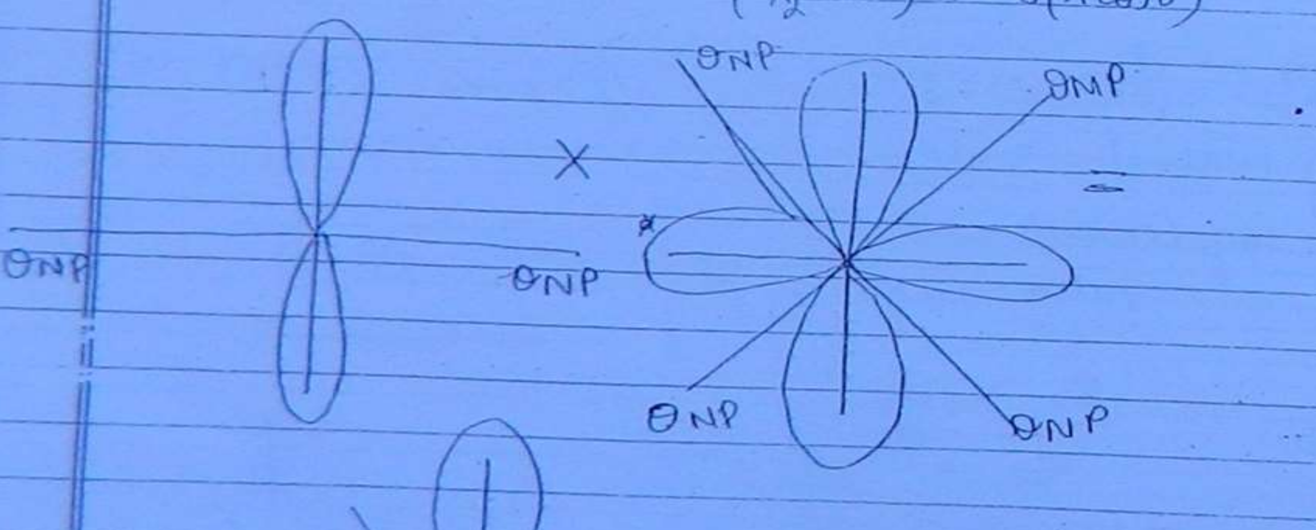
Group → combination of units

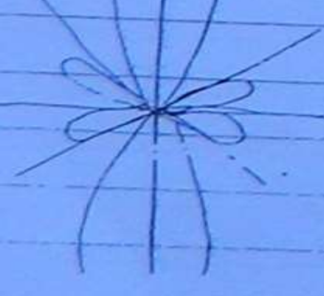
group → 2 units ⇒ $d = d$ ; $\alpha = 0$

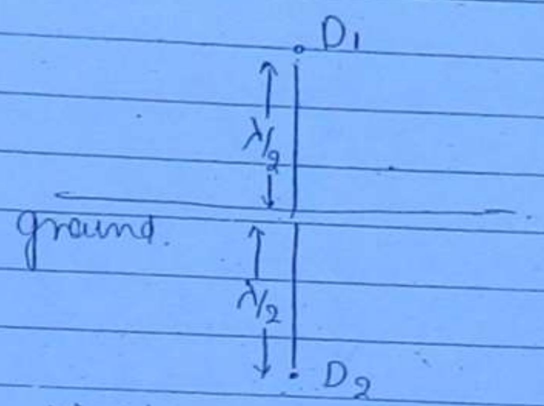
$E_T = 2E_0 \cos(\pi \cos\theta)$

Resultant Pattern = unit Pattern × Group Pattern

$= \cos(\pi/2 \cos\theta) \times \cos(\pi\cos\theta)$



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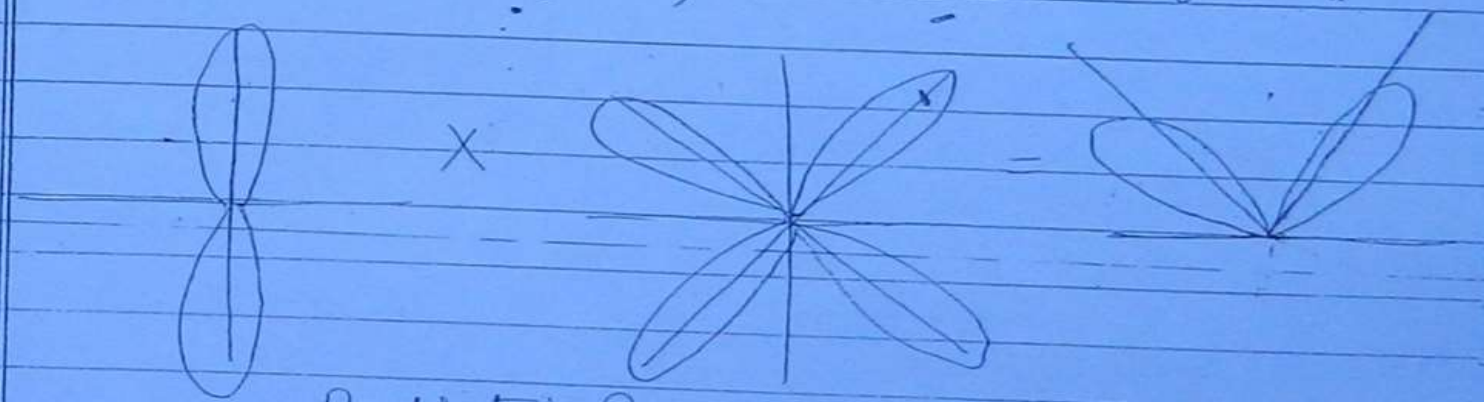
unit. Dipole $\lambda/2$

$$E_T = 2E_0 \cos\left(\pi/_2 \cos\theta\right)$$

group. – 2 dipoles – d – $\alpha = \pi$

$$E_T = 2E_0 \sin(\pi \cos\theta)$$ at $60^\circ$ we get major lobe.



Radiation Pattern above the ground.

N.B $\cos\theta_{max} = \dfrac{-\alpha}{\beta d}$

$$\cos 60^\circ = \frac{-\alpha}{\frac{2\pi}{\lambda} \times \lambda/4}$$

$$\frac{1}{2} = -\frac{\alpha}{\pi/2}$$

$$-\alpha = \pi/_2 \times \frac{1}{2}$$

$$-\alpha = \frac{\pi}{4} \text{ radians}$$