ELECTROMAGNETIC ENGINEERING – (EC21006) - Spring-2012

Tutorial – 2 (Gradient & Line Integrals)

27.01.2012 Submission Date: 05.02.2012

1) Determine the gradient of the following fields and compute its value at the specified point:

   a) $V = e^{(2x+3y)}\cos(5z)$ at (0.1,-0.2.0.4)

   b) $T = 5\rho e^{-2z}\sin(\varphi)$ at (2,π/3,0)

   c) $Q = \frac{\sin\theta\sin\varphi}{r^2}$ at (1,π/6,π/2)

2) Given a scalar function $f(x,y,z) = 12xy + z$, find $\int \vec{\nabla} f.\vec{dl}$ from (0,0,0) to (1,1,0) along :

   a) a straight line joining the two points

   b) the path $y = x^2$

   c) the path $y = x^3$

3) Compute $\oint_C \vec{A}.\vec{dl}$ around the closed curve given below in Fig.1 if $\vec{A} = (x-y)\hat{a}_x + (x+y)\hat{a}_y$

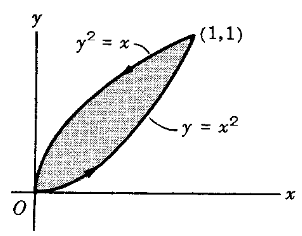

Fig.1

4) If the atmospheric pressure over a region is given by the scalar field:

$$p(\rho,\varphi,z) = 40\left(2 - e^{-\frac{\rho^2}{3}}\right)e^{-\frac{z}{6000}}\left\{2 - \frac{1+\sin\left(\frac{\varphi}{2}\right)}{2+\rho^3}.\right\}$$

Find the direction of the wind velocity at the point P(3,π/2,200) assuming that difference in pressure is the sole cause for wind flow .

5) The maximum rate of change of a function is given by
$\vec{V} = (y^2z^3\cos(x) - 4x^3z)\hat{a}_x + 2z^3y\sin(x)\,\hat{a}_y + (3y^2z^2\sin(x) - x^4)\hat{a}_z$.

a) Find the corresponding scalar function.

b) Find the unit vector in the direction of the maximum space rate of change of the scalar function.

6) Given $\vec{A} = 2(x+4y)\hat{a}_x + 8x\hat{a}_y$. Find $\int \vec{A}.\vec{dl}$ from the origin to (4,2,0) along:

a) straight line joining the two points.

b) the path $x^2 = 8y$

c) straight lines from (0,0,0) to (1,0,1) and then from (1,0,1) to (4,2,0).

d) Are the results of part (a), (b) and (c) the same? Justify.

e) What would be the result if the path of integration is that of Fig.1?

7) Find the gradient of $W = \cos(\theta)\sin(\varphi)\ln(r) + r^2\varphi$

a) Find the component of $\vec{\nabla}W$ along $\hat{a}_z$.

b) Find the component of $\vec{\nabla}W$ tangential to r = 2.

c) Find the directional derivative of W along the straight line joining the origin to (2,2,2) at (2,2,2).

8) Find the work done by the electric field $\vec{E} = 2\rho^2\cos(3\varphi)\,\hat{a}_\rho + 2\rho^2\sin(3\varphi)\hat{a}_\varphi$ in moving an electron from (1,10°,4) to (4,10°,2) along a straight line.

9) Let $\vec{A} = 5e^{-r/4}\hat{a}_r + e^{-r/4}\cos(\theta)\,\hat{a}_\theta + 5\cos(\varphi)\hat{a}_\varphi$ then find $\int \vec{A}.\vec{dl}$ from (2,π/4,π/2) to (2,3π/4,3π/2) along::

a) Straight line joining the points

b) shortest curved paths over the sphere from (2,π/4,π/2) to (2,π/4,3π/2) and then from (2,π/4,3π/2) to (2,3π/4,3π/2)

c) Are the results obtained in (a) and (b) same? Justify.

10) Given $\vec{B} = \rho^{-1}\hat{a}_\rho + \frac{1}{\rho}\cos(\varphi)\hat{a}_\varphi + z\hat{a}_z$. Find $\int \vec{B}.\vec{dl}$ from (8,π,0) to (2,π/2,2) along:

a) straight line joining the points.

b) straight lines from (8,π,0) to (8,π,2), then from (8,π,2) to (2,π,2) and the path described by $\rho = 2, z = 2$ from (2,π,2) to (2,π/2,2).

c) Does the results in (a) and (b) vary? Justify.

11) The height of a certain hill (in meters) is given by

$h(x,y) = 10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12)$

where y is the distance (in meter) north of Kharagpur and x is the distance(in meter) east of Kharagpur.

a) Where is the top of the hill located?

b) How high is the hill?

c) How steep is the slope at a point 1 meter north and 1 meter east of Kharagpur? In what direction is the slope steepest at that point?

12) Let $\vec{R}$ be the separation vector from a fixed point (x',y',z') to the point (x,y,z) and let R be its length. Show that

a) $\nabla(R^2) = 2\vec{R}$

b) $\nabla\left(\frac{1}{R}\right) = -\frac{\hat{a}_R}{R^2}$

c) What is the general formula for $\nabla(R^n)$?

13) A certain scalar field is given by $V = -\frac{Q\cos\theta}{r^2}$ $(r \neq 0)$, where Q is a constant.

a) Find the gradient of this field.

b) For a given r, at what value of θ are the r and θ components of this gradient field equal?

14) Determine the rate of change of the scalar field $f(x,y,z) = xy + 2z^2$ at (1,1,1) in the direction of the vector $\hat{a}_x - 2\hat{a}_y + \hat{a}_z$.

15) Show that if a vector field is expressed in spherical co-ordinate as

$\vec{F}(r,\theta,\varphi) = \frac{K}{r^2}\hat{a}_r$

Then $\oint_C \vec{F}.\vec{dl} = 0$

for any closed contour C.