

**MA 20104 Probability and Statistics**  
**Hints/Solutions to Assignment No. 3**

1.

$$P(\text{Ruby wins}) = \frac{\binom{4}{2}\binom{3}{1} + \binom{4}{3}}{\binom{7}{3}} = \frac{22}{35} \approx 0.6286.$$

2. Suppose  $n$  missiles are fired and  $X$  is the number of successful hits.

Then  $X \sim \text{Bin}(n, 0.75)$ .

We want  $n$  such that  $P(X \geq 3) \geq 0.95$ , or  $P(X = 0) + P(X = 1) + P(X = 2) \leq 0.05$ . This is equivalent to  $10(9n^2 - 3n + 2) \leq 4^n$ .

The smallest value of  $n$  for which this is satisfied is  $n = 6$ .

3. Let  $X$  be the number of defectives in a sample of three items.

Then  $X \sim \text{Bin}(3, 0.1)$

The required probability  $= P(X \leq 1) = (0.9)^3 + \binom{3}{1}(0.9)^2(0.1) = 0.972$ .

4.  $np = 8, npq = 4$ . This gives  $n = 16, p = 0.5$ .

So  $P(X \geq 3) = 1 - P(X = 0) - P(X = 1) - P(X = 2)$

$$= 1 - \left(\frac{1}{2}\right)^{16} - 16\left(\frac{1}{2}\right)^{16} - 120\left(\frac{1}{2}\right)^{16} = \frac{65399}{65536} \approx 0.9979.$$

5. Let  $P_n$  denote the probability of an  $n$ -component system to operate effectively. Then

$$P_3 = 1 - (1-p)^3 \text{ and } P_6 = 1 - (1-p)^6 - \binom{6}{1}p(1-p)^5.$$

$$\text{Now } P_6 - P_3 \geq 0, \text{ if } \frac{9 - \sqrt{21}}{10} \leq p \leq 1.$$

6. This is mgf of a geometric distribution with  $p = \frac{2}{7}$ . Using memoryless property of the

$$\text{geometric distribution, the required probability} = P(X > 2) = \left(\frac{5}{7}\right)^2 = \frac{25}{49}.$$

7.

$$\begin{aligned} P(\text{returning a packet}) &= P(X > 1) = 1 - P(X = 0) - P(X = 1) \\ &= 1 - (0.99)^{10} - 10(0.99)^9(0.01), \text{ say.} \end{aligned}$$

Let  $Y$  denote the number of packets returned. Then  $Y \sim \text{Bin}(3, p)$ .

$$P(Y = 0) + P(Y = 1) = 0.9999455.$$

8. X has NB(3, 0.8) distribution.

$$P(X = k) = \binom{k-1}{2} (0.2)^{k-3} (0.8)^3, \quad k = 3, 4, \dots$$

$$\begin{aligned} \text{The required probability} &= P(X \geq 5) = 1 - P(X = 3) - P(X = 4) \\ &= 1 - (0.8)^3 - 3(0.2)(0.8)^3 = 0.1808. \end{aligned}$$

$$9. \text{ Required Probability} = \frac{\binom{5}{1} \binom{15}{3}}{\binom{20}{4}} = \frac{455}{969} = 0.4696.$$

10.

$$\begin{aligned} P(Y = 2000j) &= \frac{e^{-2}(2)^j}{j!}, \quad j = 0, 1, \dots, 9, \\ &= \sum_{j=10}^{\infty} \frac{e^{-2}(2)^j}{j!}, \quad j = 10. \end{aligned}$$

11. Let A be the event that person gets a cold and B denote the event that the drug is beneficial to him. Let X be the number of times an individual contracts the cold in a year. Then  $X|B \sim P(2)$ , and  $X|B^c \sim P(3)$ .

$$P(A^c | B) = P(X = 0 | B) = e^{-2}. \quad P(A^c | B^c) = P(X = 0 | B^c) = e^{-3}.$$

Using Bayes Theorem  $P(B | A^c) = 0.89$ .

12. Let X be the number of errors. Then  $X \sim P(300)$ . Let Y be the number of errors in 2% of the pages. Then  $Y \sim P(6)$ . The required probability is  $P(Y \leq 4) \cong 0.2851$ .

13. Use Geometric probability to evaluate. The required probability = 5/9.

14. This is MGF of discrete uniform distribution on points 1, 2, ..., 10. The variance is

$$\frac{N^2 - 1}{12} = \frac{99}{12} = 8.25.$$

15.  $X \sim U\left(\frac{3C}{4}, 2C\right)$ . Let b be the bid by the contractor and let the profit be P(X). Then

$$P(X) = \begin{cases} 0, & \text{if } X < b \\ b - C & \text{if } X \geq b \end{cases}. \quad EP(X) = \frac{4}{5C}(b - C)(2C - b) = g(b), \text{ say.}$$

$$g(b) \text{ is maximized at } b = \frac{3C}{2}.$$

16. Let  $X$  denote the life of a bulb. Then  $P(X > 100) = e^{-2}$ . Let  $Y$  denote the number of bulbs working after 100 hours. Then  $Y \sim \text{Bin}(10, e^{-2})$ . So  $P(Y \geq 2) \approx 0.4008$ .

17.  $P(X > 5) = \frac{1}{4}P(X > 5 | A) + \frac{3}{4}P(X > 5 | B) = 0.25 e^{-1} + 0.75 e^{-2.5} \approx .1535$ .

18. Let  $X$  denote the life of a motherboard. Then  $f_X(x) = \frac{1}{2}e^{-x/2}$ ,  $x > 0$ . Let  $P(X)$  be the profit.

Then

$$P(x) = \begin{cases} 3000, & x < 1/2 \\ 5000, & x \geq 1/2 \end{cases}.$$

$$\begin{aligned} EP(X) &= 3000P(X < 1/2) + 5000P(X \geq 1/2) \\ &= 3000(1 - e^{-1/4}) + 5000 e^{-1/4} \\ &= 3000 + 2000 e^{-1/4} = 4557.60 \end{aligned}$$

19.  $P(\text{system fails before time } t) = 1 - \exp\{-t \sum_{i=1}^n \lambda_i\}$ .

$P(\text{only component } j \text{ fails before time } t | \text{ system fails before time } t)$

$$= \frac{(1 - \exp\{-\lambda_j t\}) \exp\{-t \sum_{\substack{i=1 \\ i \neq j}}^n \lambda_i\}}{1 - \exp\{-t \sum_{i=1}^n \lambda_i\}}.$$

20. Let  $X$  denote the life of an AC. Then  $f_X(x) = \frac{1}{100}e^{-x/100}$ ,  $x > 0$ .

$P(X > 100) = e^{-1}$ . Let  $Y$  denote the number of AC s working after 100 hours. Then  $Y \sim \text{Bin}(5, e^{-1})$ .  $P(Y \geq 2) \approx 0.6054$ .

21. The required probability =  $P(X < 15) = 1 - e^{-15/10} \approx 0.7769$ .

22. Let  $X$  denote the lead time for orders of diodes. Then  $E(X) = \frac{r}{\lambda} = 20$ ,  $V(X) = \frac{r}{\lambda^2} = 100$ .

Then  $r = 4$ ,  $\lambda = 1/5$ . So  $f_X(x) = \frac{1}{6.5^4}e^{-x/5}x^3$ ,  $x > 0$ .

The reqd probability =  $P(X < 15) = 1 - 13e^{-3} = 0.3528$ .

23. Let  $X$  denote the life of an electronic equipment. For manufacturer 'A',

$E(X) = \frac{r}{\lambda} = 4$ ,  $V(X) = \frac{r}{\lambda^2} = 8$ . Then  $r = 2$ ,  $\lambda = 1/2$ . So  $f_{X|A}(x) = \frac{x}{4}e^{-x/2}$ ,  $x > 0$ .

For manufacturer 'B',  $E(X) = \frac{r}{\lambda} = 2$ ,  $V(X) = \frac{r}{\lambda^2} = 4$ . Then  $r = 1$ ,  $\lambda = 1/2$ . So

$$f_{X|B}(x) = \frac{1}{2} e^{-x/2}, \quad x > 0. \text{ Given } P(A) = 0.75, \quad P(B) = 0.25.$$

$$\begin{aligned} P(X > 12) &= 0.75 P(X > 12 | A) + 0.25 P(X > 12 | B) = 0.25 \times 7e^{-6} + 0.25 \times e^{-6} \\ &= 2e^{-6} \approx 0.005. \end{aligned}$$

24. Let X denote the printing time. For Printer I,  $f_{X|I}(x) = \frac{1}{3} e^{-x/3}$ ,  $x > 0$ . For Printer II,

$$E(X) = \frac{r}{\lambda} = 2, \quad V(X) = \frac{r}{\lambda^2} = 2. \text{ Then } r = 2, \lambda = 1. \text{ So } f_{X|II}(x) = x e^{-x}, \quad x > 0.$$

For Printer III,  $f_{X|III}(x) = \frac{1}{4}$ ,  $0 < x < 4$ . Given  $P(I) = 0.3$ ,  $P(II) = 0.3$ ,  $P(III) = 0.4$ .

$$\begin{aligned} P(X < 1) &= 0.3 P(X < 1 | I) + 0.3 P(X < 1 | II) + 0.4 P(X < 1 | III) \\ &= 0.3(1 - e^{-1/3}) + 0.3(1 - 2e^{-1}) + 0.4(0.25) \approx 0.2643. \end{aligned}$$

25.  $P(X \leq 100 | X \geq 90) = 0.15$  gives  $\alpha = 0.0000855$ .

$$P(X > 80) = \exp\{-0.0000855 \times 80^2\} = 0.5786.$$