CHAPTER 6

TIME VARYING FIELDS AND MAXWELL EQUATIONS

6.1 INTRODUCTION

Maxwell's equations are very popular and they are known as Electromagnetic Field Equations. The main aim of this chapter is to provide sufficient background and concepts on Maxwell's equations. They include:

- Faraday's law of electromagnetic induction for three different cases: time-varying magnetic field, moving conductor with static magnetic field, and the general case of moving conductor with time-varying magnetic field.
- Lenz's law which gives direction of the induced current in the loop associated with magnetic flux change.
- Concept of self and mutual inductance
- Maxwell's equations for static and time varying fields in free space and conductive media in differential and integral form

6.2 FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION

According to Faraday's law of electromagnetic induction, emf induced in a conductor is equal to the rate of change of flux linkage in it. Here, we will denote the induced emf by V_{emf}. Mathematically, the induced emf in a closed loop is given as

$$V_{emf} = \frac{-d\Phi}{dt} = -\frac{d}{dt}\int_S \mathbf{B} \cdot d\mathbf{S} \qquad ...(6.1)$$

where Φ is the total magnetic flux through the closed loop, $\mathbf{B}$ is the magnetic flux density through the loop and S is the surface area of the loop. If the closed path is taken by an N-turn filamentary conductor, the induced emf becomes

$$V_{emf} = -N\frac{d\Phi}{dt}$$

6.2.1 Integral Form of Faraday's Law

We know that the induced emf in the closed loop can be written in terms of electric field as

$$V_{emf} = \oint_L \mathbf{E} \cdot d\mathbf{L} \qquad ...(6.2)$$

From equations (6.1) and (6.2), we get

$$\oint_L \mathbf{E} \cdot d\mathbf{L} = -\frac{d}{dt}\int_S \mathbf{B} \cdot d\mathbf{S} \qquad ...(6.3)$$

This equation is termed as the integral form of Faraday's law.

6.2.2 Differential Form of Faraday's Law

Applying Stoke's theorem to equation (6.3), we obtain

$$\int_S (\nabla \times \boldsymbol{E}) \cdot d\boldsymbol{S} = -\frac{d}{dt}\int_S \boldsymbol{B} \cdot d\boldsymbol{S}$$

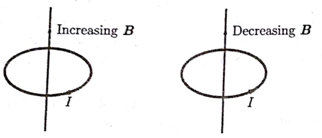
Thus, equating the integrands in above equation, we get

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

This is the differential form of Faraday's law.

6.3 LENZ'S LAW

The negative sign in Faraday equation is due to Lenz's law which states that the direction of emf induced opposes the cause producing it. To understand the Lenz's law, consider the two conducting loops placed in magnetic fields with increasing and decreasing flux densities respectively as shown in Figure 6.1.



(a) (b)

Figure 6.1: Determination of Direction of Induced Current in a Loop according to Lenz's Law (a) $\boldsymbol{B}$ in Upward Direction Increasing with Time (b) $\boldsymbol{B}$ in Upward Direction Decreasing with Time

METHODOLOGY: TO DETERMINE THE POLARITY OF INDUCED EMF

To determine the polarity of induced emf (direction of induced current), we may follow the steps given below.

Step 1: Obtain the direction of magnetic flux density through the loop. In both the Figures 6.1(a),(b) the magnetic field is directed upward.

Step 2: Deduce whether the field is increasing or decreasing with time along its direction. In Figure 6.1(a), the magnetic field directed upward is increasing, whereas in Figure 6.1(b), the magnetic field directed upward is decreasing with time.

Step 3: For increasing field assign the direction of induced current in the loop such that it produces the field opposite to the given magnetic field direction. Whereas for decreasing field assign the direction of induced current in the loop such that it produces the field in the same direction that of the given magnetic field. In Figure 6.1(a), using right hand rule we conclude that any current flowing in clockwise direction in the loop will cause a magnetic field directed downward and hence, opposes the increase in flux (i.e. opposes the field that causes it). Similarly in Figure 6.1(b), using right hand rule, we conclude that any current flowing in anti-clockwise direction in the loop will cause a magnetic field directed upward and hence, opposes the decrease in flux (i.e. opposes the field that causes it).

Step 4: Assign the polarity of induced emf in the loop corresponding to the obtained direction of induced current.

6.4 MOTIONAL AND TRANSFORMER EMFS

According to Faraday's law, for a flux variation through a loop, there will be induced emf in the loop. The variation of flux with time may be caused in following three ways:

6.4.1 Stationary Loop in a Time Varying Magnetic Field

For a stationary loop located in a time varying magnetic field, the induced emf in the loop is given by

$$V_{\text{emf}} = \oint_L \boldsymbol{E} \cdot d\boldsymbol{L} = -\int_S \frac{\partial \boldsymbol{B}}{\partial t} \cdot d\boldsymbol{S}$$

This emf is induced by the time-varying current (producing time-varying magnetic field) in a stationary loop is called *transformer emf.*

6.4.2 Moving Loop in Static Magnetic Field

When a conducting loop is moving in a static field, an emf is induced in the loop. This induced emf is called *motional emf* and given by

$$V_{\text{emf}} = \oint_L \boldsymbol{E}_m \cdot d\boldsymbol{L} = \oint_L (\boldsymbol{u} \times \boldsymbol{B}) \cdot d\boldsymbol{L}$$

where $\boldsymbol{u}$ is the velocity of loop in magnetic field. Using Stoke's theorem in above equation, we get

$$\nabla \times \boldsymbol{E}_m = \nabla \times (\boldsymbol{u} \times \boldsymbol{B})$$

6.4.3 Moving Loop in Time Varying Magnetic Field

This is the general case of induced emf when a conducting loop is moving in time varying magnetic field. Combining the above two results, total emf induced is

$$V_{\text{emf}} = \oint_L \boldsymbol{E} \cdot d\boldsymbol{L} = -\int_S \frac{\partial \boldsymbol{B}}{\partial t} \cdot d\boldsymbol{S} + \oint_L (\boldsymbol{u} \times \boldsymbol{B}) \cdot d\boldsymbol{L}$$

or,

$$V_{\text{emf}} = \underbrace{-\int_S \frac{\partial \boldsymbol{B}}{\partial t} \cdot d\boldsymbol{S}}_{\text{transformer emf}} + \underbrace{\oint_L (\boldsymbol{u} \times \boldsymbol{B}) \cdot d\boldsymbol{L}}_{\text{motional emf}}$$

Using Stoke's theorem, we can write the above equation in differential form as

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} + \nabla \times (\boldsymbol{u} \times \boldsymbol{B})$$

6.5 INDUCTANCE

An inductance is the inertial property of a circuit caused by an induced reverse voltage that opposes the flow of current when a voltage is applied. A circuit or a part of circuit that has inductance is called an inductor. A device can have either self inductance or mutual inductance.

6.5.1 Self Inductance

Consider a circuit carrying a varying current which produces varying magnetic field which in turn produces induced emf in the circuit to oppose the change in flux. The emf induced is called emf of self-induction because the change in flux is produced by the circuit itself. This phenomena is called self-induction and the property of the circuit to produce self-induction is known as *self inductance.*

Self Inductance of a Coil

Suppose a coil with N number of turns carrying current I. Let the current induces the total magnetic flux Φ passing through the loop of the coil. Thus, we have

$$N\Phi \propto I$$

or
$$N\Phi = LI$$

or
$$L = \frac{N\Phi}{I}$$

where L is a constant of proportionality known as self inductance.

Expression for Induced EMF in terms of Self Inductance

If a variable current i is introduced in the circuit, then magnetic flux linked with the circuit also varies depending on the current. So, the self-inductance of the circuit can be written as

$$L = \frac{d\Phi}{di} \qquad \text{...(6.4)}$$

Since, the change in flux through the coil induces an emf in the coil given by

$$V_{emf} = -\frac{d\Phi}{dt} \qquad \text{...(6.5)}$$

So, from equations (6.4) and (6.5), we get

$$V_{emf} = -L\frac{di}{dt}$$

6.5.2 Mutual Inductance

Mutual inductance is the ability of one inductor to induce an emf across another inductor placed very close to it. Consider two coils carrying current I_1 and I_2 as shown in Figure 6.2. Let $\boldsymbol{B}_2$ be the magnetic flux density produced due to the current I_2 and S_1 be the cross sectional area of coil 1. So, the magnetic flux due to $\boldsymbol{B}_2$ will link with the coil 1, that is, it will pass through the surface S_1. Total magnetic flux produced by coil 2 that passes through coil 1 is called mutual flux and given as

$$\Phi_{12} = \int_{S_1} \boldsymbol{B}_2 \cdot d\boldsymbol{S}$$

We define the mutual inductance M_{12} as the ratio of the flux linkage on coil 1 to current I_2, i.e.

$$M_{12} = \frac{N_1\Phi_{12}}{I_2}$$

where N_1 is the number turns in coil 1. Similarly, the mutual inductance M_{21} is defined as the ratio of flux linkage on coil 2 (produced by current in coil 1) to current I_1, i.e.

$$M_{21} = \frac{N_2\Phi_{21}}{I_1}$$

The unit of mutual inductance is Henry (H). If the medium surrounding the circuits is linear, then

$$M_{12} = M_{21}$$

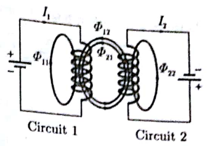


Figure 6.2 : Mutual Inductance between Two Current Carrying Coils

Expression for Induced EMF in terms of Mutual Inductance

If a variable current i_2 is introduced in coil 2 then, the magnetic flux linked with coil 1 also varies depending on current i_2. So, the mutual inductance can be given as

$$M_{12} = \frac{d\Phi_{12}}{di_2} \qquad \text{...(6.6)}$$

The change in the magnetic flux linked with coil 1 induces an emf in coil 1 given as

$$(V_{emf})_1 = -\frac{d\Phi_{12}}{dt} \qquad \text{...(6.7)}$$

So, from equations (6.6) and (6.7) we get

$$(V_{emf})_1 = -M_{12}\frac{di_2}{dt}$$

This is the induced emf in coil 1 produced by the current i_2 in coil 2. Similarly, the induced emf in the coil 2 due to a varying current in the coil 1 is given as

$$(V_{emf})_2 = -M_{21}\frac{di_1}{dt}$$

6.6 MAXWELL'S EQUATIONS

The set of four equations which have become known as Maxwell's equations are those which are developed in the earlier chapters and associated with them the name of other investigators. These equations describe the sources and the field vectors in the broad fields to electrostatics, magnetostatics and electro-magnetic induction.

6.6.1 Maxwell's Equations for Time Varying Fields

The four Maxwell's equation include Faraday's law, Ampere's circuital law, Gauss's law, and conservation of magnetic flux. There is no guideline for giving numbers to the various Maxwell's equations. However, it is customary to call the Maxwell's equation derived from Faraday's law as the first Maxwell's equation.

Maxwell's First Equation : Faraday's Law

The electromotive force around a closed path is equal to the time derivative of the magnetic displacement through any surface bounded by the path.

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \qquad \text{(Differential form)}$$

or
$$\oint_L \boldsymbol{E} \cdot d\boldsymbol{L} = -\frac{\partial}{\partial t}\int_S \boldsymbol{B} \cdot d\boldsymbol{S} \qquad \text{(Integral form)}$$

Maxwell's Second Equation: Modified Ampere's Circuital law

The magnetomotive force around a closed path is equal to the conduction plus the time derivative of the electric displacement through any surface bounded by the path. i.e.

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t} \qquad \text{(Differential form)}$$

$$\oint_L \boldsymbol{H} \cdot d\boldsymbol{L} = \int_S \left(\boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}\right) \cdot d\boldsymbol{S} \qquad \text{(Integral form)}$$

Maxwell's Third Equation : Gauss's Law for Electric Field

The total electric displacement through any closed surface enclosing a volume is equal to the total charge within the volume. i.e.,

$$\nabla \cdot \boldsymbol{D} = \rho_v \qquad \text{(Differential form)}$$

or, $$\oint_S \boldsymbol{D} \cdot d\boldsymbol{S} = \int_v \rho_v dv \qquad \text{(Integral form)}$$

This is the Gauss' law for static electric fields.

Maxwell's Fourth Equation : Gauss's Law for Magnetic Field

The net magnetic flux emerging through any closed surface is zero. In other words, the magnetic flux lines do not originate and end anywhere, but are continuous. i.e.,

$$\nabla \cdot \boldsymbol{B} = 0 \qquad \text{(Differential form)}$$

or, $$\oint_S \boldsymbol{B} \cdot d\boldsymbol{S} = 0 \qquad \text{(Integral form)}$$

This is the Gauss' law for static magnetic fields, which confirms the non-existence of magnetic monopole. Table 6.1 summarizes the Maxwell's equation for time varying fields.

Table 6.1: Maxwell's Equation for Time Varying Field

S.N.	Differential form	Integral form	Name
1.	$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$	$\oint_L \boldsymbol{E} \cdot d\boldsymbol{L} = -\frac{\partial}{\partial t}\int_S \boldsymbol{B} \cdot d\boldsymbol{S}$	Faraday's law of electromagnetic induction
2.	$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$	$\oint_L \boldsymbol{H} \cdot d\boldsymbol{L} = \int_S \left(\boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}\right) \cdot d\boldsymbol{S}$	Modified Ampere's circuital law
3.	$\nabla \cdot \boldsymbol{D} = \rho_v$	$\oint_S \boldsymbol{D} \cdot d\boldsymbol{S} = \int_v \rho_v dv$	Gauss' law of Electrostatics
4.	$\nabla \cdot \boldsymbol{B} = 0$	$\oint_S \boldsymbol{B} \cdot d\boldsymbol{S} = 0$	Gauss' law of Magnetostatic (non-existence of magnetic mono-pole)

6.6.2 Maxwell's Equations for Static Fields

For static fields, all the field terms which have time derivatives are zero, i.e.

$$\frac{\partial \boldsymbol{B}}{\partial t} = 0$$

and $$\frac{\partial \boldsymbol{D}}{\partial t} = 0$$

Therefore, for a static field the four Maxwell's equations described above reduces to the following form.

Table 6.2: Maxwell's equation for static field

S.N.	Differential Form	Integral form	Name
1.	$\nabla \times \boldsymbol{E} = 0$	$\oint_L \boldsymbol{E} \cdot d\boldsymbol{L} = 0$	Faraday's law of electromagnetic induction
2.	$\nabla \times \boldsymbol{H} = \boldsymbol{J}$	$\oint_L \boldsymbol{H} \cdot d\boldsymbol{L} = \int_S \boldsymbol{J} \cdot d\boldsymbol{S}$	Modified Ampere's circuital law
3.	$\nabla \cdot \boldsymbol{D} = \rho_v$	$\oint_S \boldsymbol{D} \cdot d\boldsymbol{S} = \int_v \rho_v dv$	Gauss' law of Electrostatics
4.	$\nabla \cdot \boldsymbol{B} = 0$	$\oint_S \boldsymbol{B} \cdot d\boldsymbol{S} = 0$	Gauss' law of Magnetostatic (non-existence of magnetic mono-pole)

6.6.3 Maxwell's Equations in Phasor Form

In a time-varying field, the field quantities $\boldsymbol{E}(x,y,z,t)$, $\boldsymbol{D}(x,y,z,t)$, $\boldsymbol{B}(x,y,z,t)$, $\boldsymbol{H}(x,y,z,t)$, $\boldsymbol{J}(x,y,z,t)$ and $\rho_v(x,y,z,t)$ can be represented in their respective phasor forms as below:

$$\boldsymbol{E} = \mathrm{Re}\{\boldsymbol{E}_s e^{j\omega t}\} \qquad ...(6.8a)$$

$$\boldsymbol{D} = \mathrm{Re}\{\boldsymbol{D}_s e^{j\omega t}\} \qquad ...(6.8b)$$

$$\boldsymbol{B} = \mathrm{Re}\{\boldsymbol{B}_s e^{j\omega t}\} \qquad ...(6.8c)$$

$$\boldsymbol{H} = \mathrm{Re}\{\boldsymbol{H}_s e^{j\omega t}\} \qquad ...(6.8d)$$

$$\boldsymbol{J} = \mathrm{Re}\{\boldsymbol{J}_s e^{j\omega t}\} \qquad ...(6.8e)$$

and $$\rho_v = \mathrm{Re}\{\rho_{vs} e^{j\omega t}\} \qquad ...(6.8f)$$

where $\boldsymbol{E}_s, \boldsymbol{D}_s, \boldsymbol{B}_s, \boldsymbol{H}_s, \boldsymbol{J}_s$ and ρ_{vs} are the phasor forms of respective field quantities. Using these relations, we can directly obtain the phasor form of Maxwell's equations as described below.

Maxwell's First Equation: Faraday's Law

In time varying field, first Maxwell's equation is written as

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \qquad ...(6.9)$$

Now, from equation (6.8a) we can obtain

$$\nabla \times \boldsymbol{E} = \nabla \times \mathrm{Re}\{\boldsymbol{E}_s e^{j\omega t}\} = \mathrm{Re}\{\nabla \times \boldsymbol{E}_s e^{j\omega t}\}$$

and using equation (6.8c) we get

$$\frac{\partial \boldsymbol{B}}{\partial t} = \frac{\partial}{\partial t}\mathrm{Re}\{\boldsymbol{B}_s e^{j\omega t}\} = \mathrm{Re}\{j\omega \boldsymbol{B}_s e^{j\omega t}\}$$

Substituting the two results in equation (6.9) we get

$$\mathrm{Re}\{\nabla \times \boldsymbol{E}_s e^{j\omega t}\} = -\mathrm{Re}\{j\omega \boldsymbol{B}_s e^{j\omega t}\}$$

Hence, $$\nabla \times \boldsymbol{E}_s = -j\omega \boldsymbol{B}_s \qquad \text{(Differential form)}$$

or, $$\oint_L \boldsymbol{E}_s \cdot d\boldsymbol{L} = -j\omega \int_S \boldsymbol{B}_s \cdot d\boldsymbol{S} \qquad \text{(Integral form)}$$

Maxwell's Second Equation: Modified Ampere's Circuital Law

In time varying field, second Maxwell's equation is written as

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t} \qquad ...(6.10)$$

From equation (6.8d) we can obtain

$$\nabla \times \boldsymbol{H} = \nabla \times \mathrm{Re}\{\boldsymbol{H}_s e^{j\omega t}\} = \mathrm{Re}\{\nabla \times \boldsymbol{H}_s e^{j\omega t}\}$$

From equation (6.8e), we have

$$\boldsymbol{J} = \text{Re}\{\boldsymbol{J}_s e^{j\omega t}\}$$

and using equation (6.8b) we get

$$\frac{\partial \boldsymbol{D}}{\partial t} = \frac{\partial}{\partial t}\text{Re}\{\boldsymbol{D}_s e^{j\omega t}\} = \text{Re}\{j\omega \boldsymbol{D}_s e^{j\omega t}\}$$

Substituting these results in equation (6.10) we get

$$\text{Re}\{\nabla \times \boldsymbol{H}_s e^{j\omega t}\} = \text{Re}\{\boldsymbol{J}_s e^{j\omega t} + j\omega \boldsymbol{D}_s e^{j\omega t}\}$$

Hence, $\nabla \times \boldsymbol{H}_s = \boldsymbol{J}_s + j\omega \boldsymbol{D}_s$ (Differential form)

or, $\oint_L \boldsymbol{H}_s \cdot d\boldsymbol{L} = \int_S (\boldsymbol{J}_s + j\omega \boldsymbol{D}_s) \cdot d\boldsymbol{S}$ (Integral form)

Maxwell's Third Equation : Gauss's Law for Electric Field

In time varying field, third Maxwell's equation is written as

$$\nabla \cdot \boldsymbol{D} = \rho_v \qquad \text{...(6.11)}$$

From equation (6.8b) we can obtain

$$\nabla \cdot \boldsymbol{D} = \nabla \cdot \text{Re}\{\boldsymbol{D}_s e^{j\omega t}\} = \text{Re}\{\nabla \cdot \boldsymbol{D}_s e^{j\omega t}\}$$

and from equation (6.8f) we have

$$\rho_v = \text{Re}\{\rho_{vs} e^{j\omega t}\}$$

Substituting these two results in equation (6.11) we get

$$\text{Re}\{\nabla \cdot \boldsymbol{D}_s e^{j\omega t}\} = \text{Re}\{\rho_{vs} e^{j\omega t}\}$$

Hence, $\nabla \cdot \boldsymbol{D}_s = \rho_{vs}$ (Differential form)

or, $\oint_S \boldsymbol{D}_s \cdot d\boldsymbol{S} = \int_v \rho_{vs}\, dv$ (Integral form)

Maxwell's Fourth Equation : Gauss's Law for Magnetic Field

$$\nabla \cdot \boldsymbol{B} = 0 \qquad \text{...(6.12)}$$

From equation (6.8c) we can obtain

$$\nabla \cdot \boldsymbol{B} = \nabla \cdot \text{Re}\{\boldsymbol{B}_s e^{j\omega t}\} = \text{Re}\{\nabla \cdot \boldsymbol{B}_s e^{j\omega t}\}$$

Substituting it in equation (6.12) we get

$$\text{Re}\{\nabla \cdot \boldsymbol{B}_s e^{j\omega t}\} = 0$$

Hence, $\nabla \cdot \boldsymbol{B}_s = 0$ (Differential form)

or, $\oint_S \boldsymbol{B}_s \cdot d\boldsymbol{S} = 0$ (Integral form)

Table 6.3 summarizes the Maxwell's equations in phasor form.

Table 6.3: Maxwell's Equations in Phasor Form

S.N.	Differential form	Integral form	Name
1.	$\nabla \times \boldsymbol{E}_s = -j\omega \boldsymbol{B}_s$	$\oint_L \boldsymbol{E}_s \cdot d\boldsymbol{L} = -j\omega \int_S \boldsymbol{B}_s \cdot d\boldsymbol{S}$	Faraday's law of electromagnetic induction
2.	$\nabla \times \boldsymbol{H}_s = \boldsymbol{J}_s + j\omega \boldsymbol{D}_s$	$\oint_L \boldsymbol{H}_s \cdot d\boldsymbol{L} = \int_S (\boldsymbol{J}_s + j\omega \boldsymbol{D}_s) \cdot d\boldsymbol{S}$	Modified Ampere's circuital law
3.	$\nabla \cdot \boldsymbol{D}_s = \rho_{vs}$	$\oint_S \boldsymbol{D}_s \cdot d\boldsymbol{S} = \int_v \rho_{vs}\, dv$	Gauss' law of Electrostatics
4.	$\nabla \cdot \boldsymbol{B}_s = 0$	$\oint_S \boldsymbol{B}_s \cdot d\boldsymbol{S} = 0$	Gauss' law of Magnetostatic (non-existence of magnetic mono-pole)

6.7 MAXWELL'S EQUATIONS IN FREE SPACE

For electromagnetic fields, free space is characterised by the following parameters:

1. Relative permittivity, $\varepsilon_r = 1$
2. Relative permeability, $\mu_r = 1$
3. Conductivity, $\sigma = 0$
4. Conduction current density, $\boldsymbol{J} = 0$
5. Volume charge density, $\rho_v = 0$

As we have already obtained the four Maxwell's equations for time-varying fields, static fields, and harmonic fields; these equations can be easily written for the free space by just replacing the variables to their respective values in free space.

6.7.1 Maxwell's Equations for Time Varying Fields in Free Space

By substituting the parameters, $\boldsymbol{J} = 0$ and $\rho_v = 0$ in the Maxwell's equations given in Table 6.1, we get the Maxwell's equation for time-varying fields in free space as summarized below:

Table 6.4: Maxwell's Equations for Time Varying Fields in Free Space

S.N.	Differential form	Integral form	Name
1.	$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$	$\oint_L \boldsymbol{E} \cdot d\boldsymbol{L} = -\frac{\partial}{\partial t}\int_S \boldsymbol{B} \cdot d\boldsymbol{S}$	Faraday's law of electromagnetic induction
2.	$\nabla \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t}$	$\oint_L \boldsymbol{H} \cdot d\boldsymbol{L} = \int_S \frac{\partial \boldsymbol{D}}{\partial t} \cdot d\boldsymbol{S}$	Modified Ampere's circuital law
3.	$\nabla \cdot \boldsymbol{D} = 0$	$\oint_S \boldsymbol{D} \cdot d\boldsymbol{S} = 0$	Gauss' law of Electrostatics
4.	$\nabla \cdot \boldsymbol{B} = 0$	$\oint_S \boldsymbol{B} \cdot d\boldsymbol{S} = 0$	Gauss' law of Magnetostatic (non-existence of magnetic mono-pole)

6.7.2 Maxwell's Equations for Static Fields in Free Space

Substituting the parameters, $\boldsymbol{J} = 0$ and $\rho_v = 0$ in the Maxwell's equation given in Table 6.2, we get the Maxwell's equation for static fields in free space as summarized below.

Table 6.5: Maxwell's Equations for Static Fields in Free Space

S.N.	Differential Form	Integral Form	Name
1.	$\nabla \times \boldsymbol{E} = 0$	$\oint_L \boldsymbol{E} \cdot d\boldsymbol{L} = 0$	Faraday's law of electromagnetic induction
2.	$\nabla \times \boldsymbol{H} = 0$	$\oint_L \boldsymbol{H} \cdot d\boldsymbol{L} = 0$	Modified Ampere's circuital law
3.	$\nabla \cdot \boldsymbol{D} = 0$	$\oint_S \boldsymbol{D} \cdot d\boldsymbol{S} = 0$	Gauss' law of Electrostatics
4.	$\nabla \cdot \boldsymbol{B} = 0$	$\oint_S \boldsymbol{B} \cdot d\boldsymbol{S} = 0$	Gauss' law of Magnetostatic (non-existence of magnetic mono-pole)

Thus, all the four Maxwell's equation vanishes for static fields in free space.

6.7.3 Maxwell's Equations for Time Harmonic Fields in Free Space

Again, substituting the parameters, $J = 0$ and $\rho_v = 0$ in the Maxwell's equations given in Table 6.3, we get the Maxwell's equation for time harmonic fields in free space as summarized below.

Table 6.6 : Maxwell's Equations for Time-Harmonic Fields in Free Space

S.N.	Differential form	Integral form	Name
1.	$\nabla \times \boldsymbol{E}_s = -j\omega \boldsymbol{B}_s$	$\oint_L \boldsymbol{E}_s \cdot d\boldsymbol{L} = -j\omega \int_S \boldsymbol{B}_s \cdot d\boldsymbol{S}$	Faraday's law of electromagnetic induction
2.	$\nabla \times \boldsymbol{H}_s = j\omega \boldsymbol{D}_s$	$\oint_L \boldsymbol{H}_s \cdot d\boldsymbol{L} = \int_S j\omega \boldsymbol{D}_s \cdot d\boldsymbol{S}$	Modified Ampere's circuital law
3.	$\nabla \cdot \boldsymbol{D}_s = 0$	$\oint_S \boldsymbol{D}_s \cdot d\boldsymbol{S} = 0$	Gauss' law of Electrostatics
4.	$\nabla \cdot \boldsymbol{B}_s = 0$	$\oint_S \boldsymbol{B}_s \cdot d\boldsymbol{S} = 0$	Gauss' law of Magnetostatic (non-existence of magnetic mono-pole)

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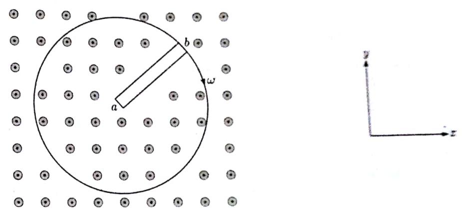
EXERCISE 6.1

MCQ 6.1.1 A perfect conducting sphere of radius r is such that it's net charge resides on the surface. At any time t, magnetic field $B(r, t)$ inside the sphere will be
(A) 0
(B) uniform, independent of r
(C) uniform, independent of t
(D) uniform, independent of both r and t

MCQ 6.1.2 A straight conductor ab of length l lying in the xy plane is rotating about the centre a at an angular velocity ω as shown in the figure.

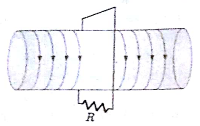


If a magnetic field B is present in the space directed along $\boldsymbol{a}_z$ then which of the following statement is correct ?
(A) V_{ab} is positive
(B) V_{ab} is negative
(C) V_{ba} is positive
(D) V_{ba} is zero

MCQ 6.1.3 **Assertion (A) :** A small piece of bar magnet takes several seconds to emerge at bottom when it is dropped down a vertical aluminum pipe where as an identical unmagnetized piece takes a fraction of second to reach the bottom.
Reason (R) : When the bar magnet is dropped inside a conducting pipe, force exerted on the magnet by induced eddy current is in upward direction.
(A) Both A and R are true and R is correct explanation of A.
(B) Both A and R are true but R is not the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

MCQ 6.1.4 Self inductance of a long solenoid having n turns per unit length will be proportional to
(A) n (B) $1/n$
(C) n^2 (D) $1/n^2$

MCQ 6.1.5 A wire with resistance R is looped on a solenoid as shown in figure.

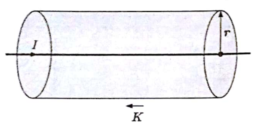


If a constant current is flowing in the solenoid then the induced current flowing in the loop with resistance R will be

(A) non uniform (B) constant

(C) zero (D) none of these

MCQ 6.1.6 A long straight wire carries a current $I = I_0 \cos(\omega t)$. If the current returns along a coaxial conducting tube of radius r as shown in figure then magnetic field and electric field inside the tube will be respectively.

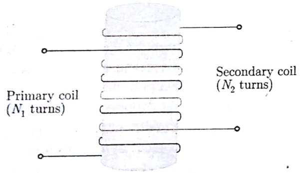


(A) radial, longitudinal (B) circumferential, longitudinal

(C) circumferential, radial (D) longitudinal, circumferential

MCQ 6.1.7 Assertion (A) : Two coils are wound around a cylindrical core such that the primary coil has N_1 turns and the secondary coils has N_2 turns as shown in figure. If the same flux passes through every turn of both coils then the ratio of emf induced in the two coils is

$$\frac{V_{emf2}}{V_{emf1}} = \frac{N_2}{N_1}$$



Reason (R) : In a primitive transformer, by choosing the appropriate no. of turns, any desired secondary emf can be obtained.

(A) Both A and R are true and R is correct explanation of A.

(B) Both A and R are true but R is not the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

MCQ 6.1.8 In a non magnetic medium electric field $E = E_0 \cos \omega t$ is applied. If the permittivity of medium is ε and the conductivity is σ then the ratio of the amplitudes of the conduction current density and displacement current density will be

(A) $\mu_0/\omega\varepsilon$ (B) $\sigma/\omega\varepsilon$

(C) $\sigma\mu_0/\omega\varepsilon$ (D) $\omega\varepsilon/\sigma$

MCQ 6.1.9 In a medium, the permittivity is a function of position such that $\frac{\nabla\varepsilon}{\varepsilon} \approx 0$. If the volume charge density inside the medium is zero then $\nabla \cdot \boldsymbol{E}$ is roughly equal to

(A) $\varepsilon \boldsymbol{E}$ (B) $-\varepsilon \boldsymbol{E}$

(C) 0 (D) $-\nabla\varepsilon \cdot \boldsymbol{E}$

MCQ 6.1.10 In free space, the electric field intensity at any point (r, θ, ϕ) in spherical coordinate system is given by

$$\boldsymbol{E} = \frac{\sin\theta \cos(\omega t - kr)}{r} \boldsymbol{a}_\theta$$

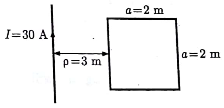
The phasor form of magnetic field intensity in the free space will be

(A) $\frac{k \sin\theta}{\omega\mu_0 r} e^{-jkr} \boldsymbol{a}_\phi$ (B) $-\frac{k \sin\theta}{\omega\mu_0 r} e^{-jkr} \boldsymbol{a}_\phi$

(C) $\frac{k\omega\mu_0}{r} e^{-jkr} \boldsymbol{a}_\phi$ (D) $\frac{k \sin\theta}{r} e^{-jkr} \boldsymbol{a}_\phi$

Common Data For Q. 11 and 12 :

A conducting wire is formed into a square loop of side 2 m. A very long straight wire carrying a current $I = 30$ A is located at a distance 3 m from the square loop as shown in figure.



MCQ 6.1.11 If the loop is pulled away from the straight wire at a velocity of 5 m/s then the induced e.m.f. in the loop after 0.6 sec will be

(A) 5 μvolt (B) 2.5 μvolt

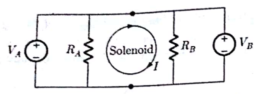
(C) 25 μvolt (D) 5 mvolt

MCQ 6.1.12 If the loop is pulled downward in the parallel direction to the straight wire, such that distance between the loop and wire is always 3 m then the induced e.m.f. in the loop at any time t will be

(A) linearly increasing with t (B) always 0

(C) linearly decreasing with t (D) always constant but not zero.

MCQ 6.1.13 Two voltmeters A and B with internal resistances R_A and R_B respectively is connected to the diametrically opposite points of a long solenoid as shown in figure. Current in the solenoid is increasing linearly with time. The correct relation between the voltmeter's reading V_A and V_B will be

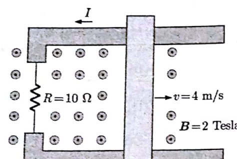


(A) $V_A = V_B$ (B) $V_A = -V_B$

(C) $\frac{V_A}{V_B} = \frac{R_A}{R_B}$ (D) $\frac{V_A}{V_B} = -\frac{R_A}{R_B}$

Common Data For Q. 14 and 15 :

Two parallel conducting rails are being placed at a separation of 5 m with a resistance $R = 10\,\Omega$ connected across it's one end. A conducting bar slides frictionlessly on the rails with a velocity of 4 m/s away from the resistance as shown in the figure.



MCQ 6.1.14 If a uniform magnetic field $B = 2$ Tesla pointing out of the page fills entire region then the current I flowing in the bar will be

(A) 0 A (B) -40 A

(C) 4 A (D) -4 A

MCQ 6.1.15 The force exerted by magnetic field on the sliding bar will be

(A) 4 N, opposes it's motion

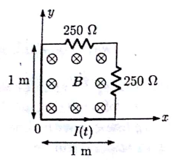
(B) 40 N, opposes it's motion

(C) 40 N, in the direction of it's motion

(D) 0

MCQ 6.1.16 Two small resistor of 250 Ω each is connected through a perfectly conducting filament such that it forms a square loop lying in x-y plane as shown in the figure. Magnetic flux density passing through the loop is given as

$$\boldsymbol{B} = -7.5\cos(120\pi t - 30°)\,\boldsymbol{a}_z$$

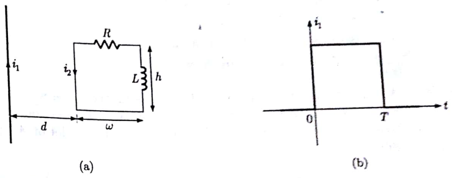


The induced current $I(t)$ in the loop will be

(A) $0.02\sin(120\pi t - 30°)$ (B) $2.8 \times 10^3\sin(120\pi t - 30°)$

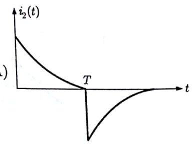
(C) $-5.7\sin(120\pi t - 30°)$ (D) $5.7\sin(120\pi t - 30°)$

MCQ 6.1.17 A rectangular loop of self inductance L is placed near a very long wire carrying current i_1 as shown in figure (a). If i_1 be the rectangular pulse of current as shown in figure (b) then the plot of the induced current i_2 in the loop versus time t will be (assume the time constant of the loop, $\tau \gg L/R$)

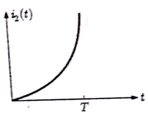


(a) (b)

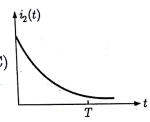
(A)



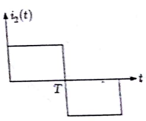
(B)



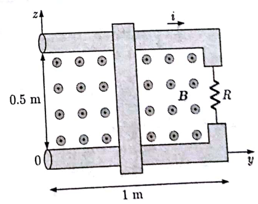
(C)



(D)



MCQ 6.1.18 Two parallel conducting rails is placed in a varying magnetic field $B = 0.2\cos\omega t\,\boldsymbol{a}_x$. A conducting bar oscillates on the rails such that it's position is given by $y = 0.5(1 - \cos\omega t)$ m. If one end of the rails are terminated in a resistance $R = 5\,\Omega$, then the current i flowing in the rails will be



(A) $0.01\omega\sin\omega t(1+2\cos\omega t)$
(B) $-0.01\omega\sin\omega t(1+2\cos\omega t)$
(C) $0.01\omega\cos\omega t(1+2\sin\omega t)$
(D) $0.05\omega\sin\omega t(1+2\sin\omega t)$

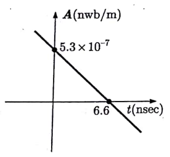
MCQ 6.1.19 Electric flux density in a medium ($\varepsilon_r = 10$, $\mu_r = 2$) is given as

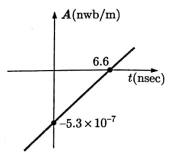
$$\boldsymbol{D} = 1.33\sin(3\times10^8 t - 0.2x)\boldsymbol{a}_y\ \mu\text{C/m}^2$$

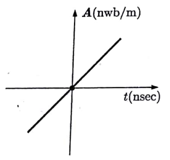
Magnetic field intensity in the medium will be

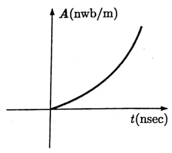
(A) $10^{-5}\sin(3\times10^8 t - 0.2x)\boldsymbol{a}_y$ A/m
(B) $2\sin(3\times10^8 t - 0.2x)\boldsymbol{a}_y$ A/m
(C) $-4\sin(3\times10^8 t - 0.2x)\boldsymbol{a}_y$ A/m
(D) $4\sin(3\times10^8 t - 0.2x)\boldsymbol{a}_y$ A/m

MCQ 6.1.20 A current filament located on the x-axis in free space with in the interval $-0.1 < x < 0.1$ m carries current $I(t) = 8t$ A in $\boldsymbol{a}_x$ direction. If the retarded vector potential at point $P(0,0,2)$ be $\boldsymbol{A}(t)$ then the plot of $\boldsymbol{A}(t)$ versus time will be

(A)


(B)


(C)


(D)


Common Data For Q. 21 and 22 :

In a region of electric and magnetic fields $\boldsymbol{E}$ and $\boldsymbol{B}$, respectively, the force experienced by a test charge qC are given as follows for three different velocities.

Velocity m/sec	Force, N
$\boldsymbol{a}_x$	$q(\boldsymbol{a}_y + \boldsymbol{a}_z)$
$\boldsymbol{a}_y$	$q\boldsymbol{a}_y$
$\boldsymbol{a}_z$	$q(2\boldsymbol{a}_y + \boldsymbol{a}_z)$

MCQ 6.1.21 What will be the magnetic field $\boldsymbol{B}$ in the region ?

(A) $\boldsymbol{a}_x$
(B) $\boldsymbol{a}_x - \boldsymbol{a}_y$
(C) $\boldsymbol{a}_z$
(D) $\boldsymbol{a}_y - \boldsymbol{a}_z$

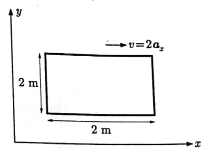
MCQ 6.1.22 What will be electric field $\boldsymbol{E}$ in the region ?

(A) $\boldsymbol{a}_x - \boldsymbol{a}_z$
(B) $\boldsymbol{a}_y - \boldsymbol{a}_z$
(C) $\boldsymbol{a}_y + \boldsymbol{a}_z$
(D) $\boldsymbol{a}_y + \boldsymbol{a}_z - \boldsymbol{a}_x$

MCQ 6.1.23 In a non-conducting medium ($\sigma = 0$, $\mu_r = \varepsilon_r = 1$), the retarded potentials are given as $V = y(x - ct)$ volt and $\boldsymbol{A} = y(\frac{x}{c} - t)\boldsymbol{a}_x$ Wb/m where c is velocity of waves in free space. The field (electric and magnetic) inside the medium satisfies Maxwell's equation if

(A) $\boldsymbol{J} = 0$ only
(B) $\rho_v = 0$ only
(C) $\boldsymbol{J} = \rho_v = 0$
(D) Can't be possible

MCQ 6.1.24 In Cartesian coordinates magnetic field is given by $\boldsymbol{B} = -2/x\,\boldsymbol{a}_z$. A square loop of side 2 m is lying in xy plane and parallel to the y-axis. Now, the loop is moving in that plane with a velocity $v = 2\boldsymbol{a}_x$ as shown in the figure.



What will be the circulation of the induced electric field around the loop ?

(A) $\dfrac{16}{x(x+2)}$
(B) $\dfrac{8}{x}$
(C) $\dfrac{8}{x(x+2)}$
(D) $\dfrac{x(x+2)}{16}$

Common Data For Q. 25 to 27 :

In a cylindrical coordinate system, magnetic field is given by

$$\boldsymbol{B} = \begin{cases} 0 & \text{for } \rho < 4\text{ m} \\ 2\sin\omega t\,\boldsymbol{a}_z & \text{for } 4 < \rho < 5\text{ m} \\ 0 & \text{for } \rho > 5\text{ m} \end{cases}$$

MCQ 6.1.25 The induced electric field in the region $\rho < 4$ m will be

(A) 0
(B) $\dfrac{2\omega\cos\omega t}{\rho}\boldsymbol{a}_\phi$
(C) $-2\cos\omega t\boldsymbol{a}_\phi$
(D) $\dfrac{1}{2\sin\omega t}\boldsymbol{a}_\phi$

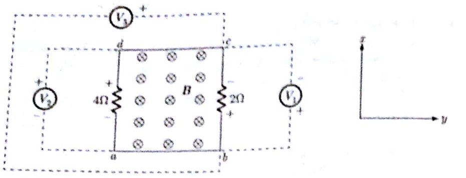
MCQ 6.1.26 The induced electric field at $\rho = 4.5$ m is

(A) 0
(B) $-\dfrac{17\omega\cos\omega t}{18}$
(C) $\dfrac{4\omega\cos\omega t}{9}$
(D) $-\dfrac{17\omega\cos\omega t}{4}$

MCQ 6.1.27 The induced electric field in the region $\rho > 5$ m is

(A) $-\dfrac{18}{\rho}\omega\cos\omega t\boldsymbol{a}_\phi$
(B) $\dfrac{-9\omega\cos\omega t}{\rho}\boldsymbol{a}_\phi$
(C) $-9\rho\cos\omega t\boldsymbol{a}_\phi$
(D) $\dfrac{9\omega\cos\omega t}{\rho}\boldsymbol{a}_\phi$

MCQ 6.1.28 Magnetic flux density, $B = 0.1t\,a_z$ Tesla threads only the loop $abcd$ lying in the plane xy as shown in the figure.

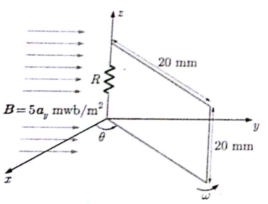


Consider the three voltmeters V_1, V_2 and V_3, connected across the resistance in the same xy plane. If the area of the loop $abcd$ is 1 m^2 then the voltmeter readings are

	V_1	V_2	V_3
(A)	66.7 mV	33.3 mV	66.7 mV
(B)	33.3 mV	66.7 mV	33.3 mV
(C)	66.7 mV	66.7 mV	33.3 mV
(D)	33.3 mV	66.7 mV	66.7 mV

Common Data For Q. 29 and 30 :

A square wire loop of resistance R rotated at an angular velocity ω in the uniform magnetic field $B = 5a_y$ mWb/m^2 as shown in the figure.



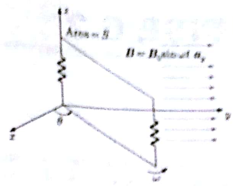
MCQ 6.1.29 If the angular velocity, $\omega = 2$ rad/sec then the induced e.m.f. in the loop will be

(A) $2\sin\theta\,\mu$V/m (B) $2\cos\theta\,\mu$V/m
(C) $4\cos\theta\,\mu$V/m (D) $4\sin\theta\,\mu$V/m

MCQ 6.1.30 If resistance, $R = 40$ mΩ then the current flowing in the square loop will be

(A) $0.2\sin\theta$ mA (B) $0.1\sin\theta$ mA
(C) $0.1\cos\theta$ mA (D) $0.5\sin\theta$ mA

MCQ 6.1.31 In a certain region magnetic flux density is given as $B = B_0\sin\omega t\,a_y$. A rectangular loop of wire is defined in the region with it's one corner at origin and one side along z-axis as shown in the figure.

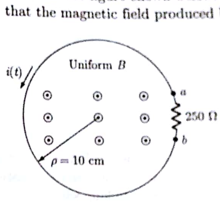


If the loop rotates at an angular velocity ω (same as the angular frequency of magnetic field) then the maximum value of induced e.m.f in the loop will be

(A) $\frac{1}{2}B_0S\omega$ (B) $2B_0S\omega$
(C) $B_0S\omega$ (D) $4B_0S\omega$

Common Data For Q. 32 and 33 :

Consider the figure shown below. Let $B = 10\cos 120\pi t$ Wb/m^2 and assume that the magnetic field produced by $i(t)$ is negligible



MCQ 6.1.32 The value of v_{ab} is

(A) $-118.43\cos 120\pi t$ V (B) $118.43\cos 120\pi t$ V
(C) $-118.43\sin 120\pi t$ V (D) $118.43\sin 120\pi t$ V

MCQ 6.1.33 The value of $i(t)$ is

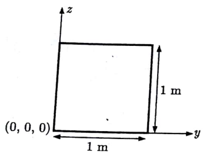
(A) $-0.47\cos 120\pi t$ A (B) $0.47\cos 120\pi t$ A
(C) $-0.47\sin 120\pi t$ A (D) $0.47\sin 120\pi t$ A

****

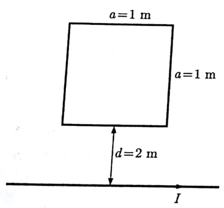
EXERCISE 6.2

QUES 6.2.1 A small conducting loop is released from rest with in a vertical evacuated cylinder. What is the voltage induced (in mV) in the falling loop ? (Assume earth magnetic field $= 10^{-6}\,\text{T}$ at a constant angle of $10°$ below the horizontal)

QUES 6.2.2 A square loop of side 1 m is located in the plane $x = 0$ as shown in figure. A non-uniform magnetic flux density through it is given as $\boldsymbol{B} = 4z^3t^2\boldsymbol{a}_x$, The emf induced in the loop at time $t = 2\,\text{sec}$ will be ____ Volt.



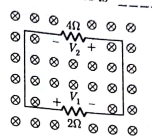
QUES 6.2.3 A very long straight wire carrying a current $I = 5\,\text{A}$ is placed at a distance of 2 m from a square loop as shown. If the side of the square loop is 1 m then the total flux passing through the square loop will be ____ $\times 10^{-7}$ wb



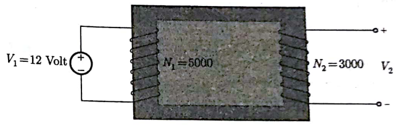
QUES 6.2.4 In a medium where no D.C. field is present, the conduction current density at any point is given as $\boldsymbol{J}_d = 20\cos(1.5 \times 10^8 t)\boldsymbol{a}_y\,\text{A/m}^2$. Electric flux density in the medium will be $D_0\sin(1.5 \times 10^8 t)\,\boldsymbol{a}_y\,\text{nC/m}^2$ such that $D_0 =$ ____

QUES 6.2.5 A conducting medium has permittivity, $\varepsilon = 4\varepsilon_0$ and conductivity, $\sigma = 1.14 \times 10^8\,\text{s/m}$. The ratio of magnitude of displacement current and conduction current in the medium at 50 GHz will be _____ $\times 10^{-8}$.

QUES 6.2.6 In a certain region magnetic flux density is given as $\boldsymbol{B} = 0.1t\boldsymbol{a}_z\,\text{Wb/m}^2$. An electric loop with resistance $2\,\Omega$ and $4\,\Omega$ is lying in x-y plane as shown in the figure. If the area of the loop is $1\,\text{m}^2$ then, the voltage drop V_1 across the $2\,\Omega$ resistance is ____ mV.



QUES 6.2.7 A magnetic core of uniform cross section having two coils (Primary and secondary) wound on it as shown in figure. The no. of turns of primary coil is 5000 and no. of turns of secondary coil is 3000. If a voltage source of 12 volt is connected across the primary coil then what will be the voltage (in Volt) across the secondary coil ?

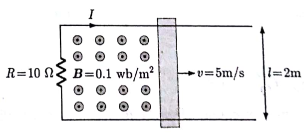


QUES 6.2.8 Magnetic field intensity in free space is given as

$$\boldsymbol{H} = 0.1\cos(15\pi y)\sin(6\pi \times 10^9 t - bx)\boldsymbol{a}_z\,\text{A/m}$$

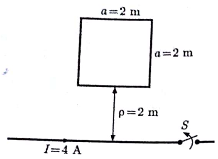
It satisfies Maxwell's equation when $|b| =$ ____

QUES 6.2.9 Two parallel conducting rails are being placed at a separation of 2 m as shown in figure. One end of the rail is being connected through a resistor $R = 10\,\Omega$ and the other end is kept open. A metal bar slides frictionlessly on the rails at a speed of 5 m/s away from the resistor. If the magnetic flux density $\boldsymbol{B} = 0.1\,\text{Wb/m}^2$ pointing out of the page fills entire region then the current I flowing in the resistor will be ____ Ampere.

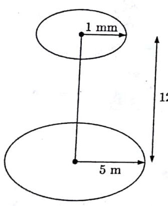


QUES 6.2.10 An infinitely long straight wire with a closed switch S carries a uniform current $I = 4\,\text{A}$ as shown in figure. A square loop of side $a = 2\,\text{m}$ and resistance

$R = 4\,\Omega$ is located at a distance 2 m from the wire. Now at any time $t = t_0$ the switch is open so the current I drops to zero. What will be the total charge (in nC) that passes through a corner of the square loop after $t = t_0$?



QUES 6.2.11 A circular loop of radius 5 m carries a current $I = 2$ A. If another small circular loop of radius 1 mm lies a distance 12 m above the large circular loop such that the planes of the two loops are parallel and perpendicular to the common axis as shown in figure then total flux through the small loop will be ____ fermi-weber.



QUES 6.2.12 A non magnetic medium at frequency $f = 1.6 \times 10^8$ Hz has permittivity $\varepsilon = 54\varepsilon_0$ and resistivity $\rho = 0.77\,\Omega$ - m. What will be the ratio of amplitudes of conduction current to the displacement current ?

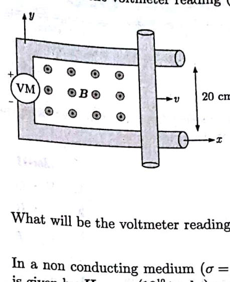
QUES 6.2.13 In a certain region a test charge is moving with an angular velocity 2 rad/sec along a circular path of radius 2 m centred at origin in the x-y plane. If the magnetic flux density in the region is $\boldsymbol{B} = 2\boldsymbol{a}_z$ Wb/m^2 then the electric field viewed by an observer moving with the test charge is ____ V/m in $\boldsymbol{a}_\rho$ direction.

Common Data For Q. 13 and 14 :

In a non uniform magnetic field $\boldsymbol{B} = 8x^2\boldsymbol{a}_z$ Tesla, two parallel rails with a separation of 20 cm and connected with a voltmeter at it's one end is located in x-y plane as shown in figure. The Position of the bar which is sliding on the rails is given as

$$x = t(1 + 0.4t^2)$$

QUES 6.2.14 What will be the voltmeter reading (in volt) at $t = 0.4$ sec ?



QUES 6.2.15 What will be the voltmeter reading (in volt) at $x = 12$ cm ?

QUES 6.2.16 In a non conducting medium ($\sigma = 0$) magnetic field intensity at any point is given by $\boldsymbol{H} = \cos(10^{10}t - bx)\boldsymbol{a}_z$ A/m . The permittivity of the medium is $\varepsilon = 0.12$ nF/m and permeability of the medium is $\mu = 3 \times 10^{-5}$ H/m. D.C. field is not present in medium. Field satisfies Maxwell's equation, if $|b| =$ ____

QUES 6.2.17 Electric field in free space in given as

$$\boldsymbol{E} = 5\sin(10\pi y)\cos(6\pi \times 10^9 - bx)\boldsymbol{a}_z$$

It satisfies Maxwell's equation for $|b| =$?

QUES 6.2.18 8 A current is flowing along a straight wire from a point charge situated at the origin to infinity and passing through the point (2,2,2). The circulation of the magnetic field intensity around the closed path formed by the triangle having the vertices (2,0,0), (0,2,0) and (0,0,2) is equal to ____ Ampere.

QUES 6.2.19 A 50 turn rectangular loop of area 64 cm^2 rotates at 60 revolution per seconds in a magnetic field $\boldsymbol{B} = 0.25\sin 377t$ Wb/m^2 directed normal to the axis of rotation. What is the rms value of the induced voltage (in volt) ?

****

EXERCISE 6.3

MCQ 6.3.1 Match List I with List II and select the correct answer using the codes given below (Notations have their usual meaning)

	List-I		List-II
a	Ampere's circuital law	1.	$\nabla \cdot \boldsymbol{D} = \rho_v$
b	Faraday's law	2.	$\nabla \cdot \boldsymbol{B} = 0$
c	Gauss's law	3.	$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$
d	Non existence of isolated magneticharge	4.	$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$

Codes :

	a	b	c	d
(A)	4	3	2	1
(B)	4	1	3	2
(C)	2	3	1	4
(D)	4	3	1	2

MCQ 6.3.2 Magneto static fields is caused by
(A) stationary charges
(B) steady currents
(C) time varying currents
(D) none of these

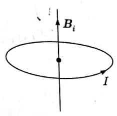
MCQ 6.3.3 Let $\boldsymbol{A}$ be magnetic vector potential and $\boldsymbol{E}$ be electric field intensity at certain time in a time varying *EM* field. The correct relation between $\boldsymbol{E}$ and $\boldsymbol{A}$ is
(A) $\boldsymbol{E} = -\frac{\partial \boldsymbol{A}}{\partial t}$
(B) $\boldsymbol{A} = -\frac{\partial \boldsymbol{E}}{\partial t}$
(C) $\boldsymbol{E} = \frac{\partial \boldsymbol{A}}{\partial t}$
(D) $\boldsymbol{A} = \frac{\partial \boldsymbol{E}}{\partial t}$

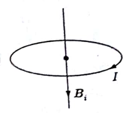
MCQ 6.3.4 A closed surface S defines the boundary line of magnetic medium such that the field intensity inside it is $\boldsymbol{B}$. Total outward magnetic flux through the closed surface will be
(A) $\boldsymbol{B} \cdot \boldsymbol{S}$
(B) 0
(C) $\boldsymbol{B} \times \boldsymbol{S}$
(D) none of these

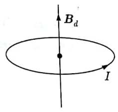
MCQ 6.3.5 The total magnetic flux through a conducting loop having electric field $\boldsymbol{E} = 0$ inside it will be
(A) 0
(B) constant
(C) varying with time only
(D) varying with time and area of the surface both

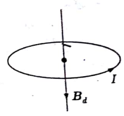
MCQ 6.3.6 A cylindrical wire of a large cross section made of super conductor carries a current I. The current in the superconductor will be confined.
(A) inside the wire
(B) to the axis of cylindrical wire
(C) to the surface of the wire
(D) none of these

MCQ 6.3.7 If $\boldsymbol{B}_i$ denotes the magnetic flux density increasing with time and $\boldsymbol{B}_d$ denotes the magnetic flux density decreasing with time then which of the configuration is correct for the induced current I in the stationary loop ?

(A)


(B)


(C)


(D)


MCQ 6.3.8 A circular loop is rotating about z-axis in a magnetic field $\boldsymbol{B} = B_0 \cos \omega t \boldsymbol{a}_y$. The total induced voltage in the loop is caused by
(A) Transformer emf
(B) motion emf.
(C) Combination of (A) and (B)
(D) none of these

MCQ 6.3.9 For static magnetic field,
(A) $\nabla \times \boldsymbol{B} = \rho$
(B) $\nabla \times \boldsymbol{B} = \mu \boldsymbol{J}$
(C) $\nabla \cdot \boldsymbol{B} = \mu_0 J$
(D) $\nabla \times \boldsymbol{B} = 0$

MCQ 6.3.10 Displacement current density is
(A) $\boldsymbol{D}$
(B) $\boldsymbol{J}$
(C) $\partial \boldsymbol{D}/\partial t$
(D) $\partial \boldsymbol{J}/\partial t$

MCQ 6.3.11 The time varying electric field is
(A) $\boldsymbol{E} = -\nabla V$
(B) $\boldsymbol{E} = -\nabla V - \dot{\boldsymbol{A}}$
(C) $\boldsymbol{E} = -\nabla V - \boldsymbol{B}$
(D) $\boldsymbol{E} = -\nabla V - \boldsymbol{D}$

MCQ 6.3.12 A field can exist if it satisfies
(A) Gauss's law
(B) Faraday's law
(C) Coulomb's law
(D) All Maxwell's equations

MCQ 6.3.13 Maxwell's equations give the relations between
(A) different fields
(B) different sources
(C) different boundary conditions
(D) none of these

MCQ 6.3.14 If $\boldsymbol{E}$ is a vector, then $\nabla \cdot \nabla \times E$ is
(A) 0
(B) 1
(C) does not exist
(D) none of these

MCQ 6.3.15 The Maxwell's equation, $\nabla \cdot B = 0$ is due to
(A) $\boldsymbol{B} = \mu \boldsymbol{H}$
(B) $\boldsymbol{B} = \frac{H}{\mu}$
(C) non-existence of a mono pole
(D) none of these

MCQ 6.3.16 For free space,
(A) $\sigma = \infty$
(B) $\sigma = 0$
(C) $J \neq 0$
(D) none of these

MCQ 6.3.17 For time varying EM fields
(A) $\nabla \times \boldsymbol{H} = \boldsymbol{J}$
(B) $\nabla \times \boldsymbol{H} = \dot{\boldsymbol{D}} + \boldsymbol{J}$
(C) $\nabla \times \boldsymbol{E} = 0$
(D) none of these

****

EXERCISE 6.4

MCQ 6.4.1 A magnetic field in air is measured to be $\boldsymbol{B} = B_0\left(\frac{x}{x^2+y^2}\boldsymbol{a}_y - \frac{y}{x^2+y^2}\boldsymbol{a}_x\right)$
What current distribution leads to this field ?
[Hint : The algebra is trivial in cylindrical coordinates.]
(A) $\boldsymbol{J} = \frac{B_0 \boldsymbol{z}}{\mu_0}\left(\frac{1}{x^2+y^2}\right), r \neq 0$
(B) $\boldsymbol{J} = -\frac{B_0 \boldsymbol{z}}{\mu_0}\left(\frac{2}{x^2+y^2}\right), r \neq 0$
(C) $\boldsymbol{J} = 0, r \neq 0$
(D) $\boldsymbol{J} = \frac{B_0 \boldsymbol{z}}{\mu_0}\left(\frac{1}{x^2+y^2}\right), r \neq 0$

MCQ 6.4.2 For static electric and magnetic fields in an inhomogeneous source-free medium, which of the following represents the correct form of Maxwell's equations ?
(A) $\nabla \cdot E = 0, \nabla \times B = 0$
(B) $\nabla \cdot E = 0, \nabla \cdot B = 0$
(C) $\nabla \times E = 0, \nabla \times B = 0$
(D) $\nabla \times E = 0, \nabla \cdot B = 0$

MCQ 6.4.3 If C is closed curve enclosing a surface S, then magnetic field intensity $\boldsymbol{H}$, the current density $\boldsymbol{J}$ and the electric flux density $\boldsymbol{D}$ are related by
(A) $\iint_S \boldsymbol{H} \cdot d\boldsymbol{S} = \oiint_C \left(\boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}\right) \cdot d\boldsymbol{l}$
(B) $\int_S \boldsymbol{H} \cdot d\boldsymbol{l} = \oiint_S \left(\boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}\right) \cdot d\boldsymbol{S}$
(C) $\oiint_S \boldsymbol{H} \cdot d\boldsymbol{S} = \int_C \left(\boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}\right) \cdot d\boldsymbol{l}$
(D) $\oint_C \boldsymbol{H} \cdot d\boldsymbol{l} = \iint_S \left(\boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}\right) \cdot d\boldsymbol{S}$

MCQ 6.4.4 The unit of $\nabla \times \boldsymbol{H}$ is
(A) Ampere
(B) Ampere/meter
(C) Ampere/meter^2
(D) Ampere-meter

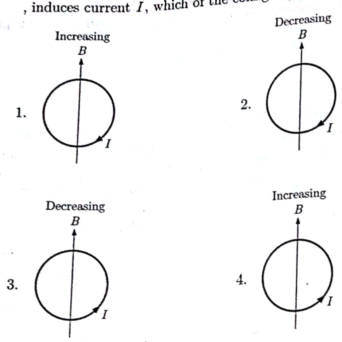
MCQ 6.4.5 The Maxwell equation $\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$ is based on
(A) Ampere's law
(B) Gauss' law
(C) Faraday's law
(D) Coulomb's law

MCQ 6.4.6 A loop is rotating about they y-axis in a magnetic field $\boldsymbol{B} = B_0 \cos(\omega t + \phi)\,\boldsymbol{a}_x$ T. The voltage in the loop is
(A) zero
(B) due to rotation only
(C) due to transformer action only
(D) due to both rotation and transformer action

MCQ 6.4.7 The credit of defining the following current is due to Maxwell
(A) Conduction current
(B) Drift current
(C) Displacement current
(D) Diffusion current

MCQ 6.4.8 A varying magnetic flux linking a coil is given by $\Phi = 1/3\lambda t^3$. If at time $t = 3$ s, the emf induced is 9 V, then the value of λ is.
(A) zero
(B) 1 Wb/s^2
(C) -1 Wb/s^2
(D) 9 Wb/s^2

MCQ 6.4.9 Assuming that each loop is stationary and time varying magnetic field B, induces current I, which of the configurations in the figures are correct ?



(A) 1, 2, 3 and 4
(B) 1 and 3 only
(C) 2 and 4 only
(D) 3 and 4 only

MCQ 6.4.10 **Assertion (A) :** For time varying field the relation $E = -\nabla V$ is inadequate.
Reason (R) : Faraday's law states that for time varying field $\nabla \times E = 0$
(A) Both **Assertion (A)** and **Reason (R)** are individually true and **Reason (R)** is the correct explanation of **Assertion (A)**
(B) Both **Assertion (A)** and **Reason (R)** are individually true but **Reason (R)** is not the correct explanation of **Assertion (A)**
(C) **Assertion (A)** is true but **Reason (R)** is false
(D) **Assertion (A)** is false but **Reason (R)** is true

MCQ 6.4.11 Who developed the concept of time varying electric field producing a magnetic field ?
(A) Gauss
(B) Faraday
(C) Hertz
(D) Maxwell

MCQ 6.4.12 A single turn loop is situated in air, with a uniform magnetic field normal to its plane. The area of the loop is $5\,\text{m}^2$ and the rate of charge of flux density is $2\,\text{Wb/m}^2/\text{s}$. What is the emf appearing at the terminals of the loop ?
(A) -5 V
(B) -2 V
(C) -0.4 V
(D) -10 V

MCQ 6.4.13 Which of the following equations results from the circuital form of Ampere's law ?
(A) $\nabla \times E = -\frac{\partial B}{\partial t}$
(B) $\nabla \cdot B = 0$
(C) $\nabla \cdot D = \rho$
(D) $\nabla \times H = J + \frac{\partial D}{\partial t}$

MCQ 6.4.14 **Assertion (A) :** Capacitance of a solid conducting spherical body of radius a is given by $4\pi\varepsilon_0 a$ in free space.

Reason (R) : $\nabla \times H = j\omega\varepsilon E + J$
(A) Both A and R are individually true and R is the correct explanation of A.
(B) Both A and R are individually true but R is not the correct explanation of A.
(C) A is true but R is false
(D) A is false but R is true

MCQ 6.4.15 Two conducting thin coils X and Y (identical except for a thin cut in coil Y) are placed in a uniform magnetic field which is decreasing at a constant rate. If the plane of the coils is perpendicular to the field lines, which of the following statement is correct ? As a result, emf is induced in
(A) both the coils
(B) coil Y only
(C) coil X only
(D) none of the two coils

MCQ 6.4.16 **Assertion (A) :** Time varying electric field produces magnetic fields.
Reason (R) : Time varying magnetic field produces electric fields.
(A) Both A and R are true and R is the correct explanation of A
(B) Both A and R are true but R is NOT the correct explanation of A
(C) A is true but R is false
(D) A is false but R is true

MCQ 6.4.17 Match List I (Electromagnetic Law) with List II (Different Form) and select the correct answer using the code given below the lists :

List-I		List-II	
a.	Ampere's law	1.	$\nabla \cdot D = \rho_v$
b.	Faraday's law	2.	$\nabla \cdot J = -\frac{\partial h}{\partial t}$
c.	Gauss law	3.	$\nabla \times H = J + \frac{\partial D}{\partial t}$
d.	Current	4.	$\nabla \times E = -\frac{\partial B}{\partial t}$

Codes :

	a	b	c	d
(A)	1	2	3	4
(B)	3	4	1	2
(C)	1	4	3	2
(D)	3	2	1	4

MCQ 6.4.18 Two metal rings 1 and 2 are placed in a uniform magnetic field which is decreasing with time with their planes perpendicular to the field. If the rings are identical except that ring 2 has a thin air gap in it, which one of the following statements is correct ?
(A) No e.m.f is induced in ring 1
(B) An e.m.f is induced in both the rings
(C) Equal Joule heating occurs in both the rings
(D) Joule heating does not occur in either ring.

MCQ 6.4.19 Which one of the following Maxwell's equations gives the basic idea of radiation ?

(A) $\left.\begin{matrix}\nabla \times \boldsymbol{H} = \partial \boldsymbol{D}/\partial t \\ \nabla \times \boldsymbol{E} = \partial \boldsymbol{B}/\partial t\end{matrix}\right\}$

(B) $\left.\begin{matrix}\nabla \times \boldsymbol{E} = -\partial \boldsymbol{B}/\partial t \\ \nabla \cdot \boldsymbol{D} = -\partial \boldsymbol{B}/\partial t\end{matrix}\right\}$

(C) $\left.\begin{matrix}\nabla \cdot \boldsymbol{D} = \rho \\ \nabla \cdot \boldsymbol{D} = 0\end{matrix}\right\}$

(D) $\left.\begin{matrix}\nabla \cdot \boldsymbol{B} = \rho \\ \nabla \times \boldsymbol{H} = (\partial \boldsymbol{D}/\partial t)\end{matrix}\right\}$

MCQ 6.4.20 Which one of the following is NOT a correct Maxwell equation ?

(A) $\nabla \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t} + \boldsymbol{J}$

(B) $\nabla \times \boldsymbol{E} = \frac{\partial \boldsymbol{H}}{\partial t}$

(C) $\nabla \cdot \boldsymbol{D} = \rho$

(D) $\nabla \cdot \boldsymbol{B} = 0$

MCQ 6.4.21 Match List I (Maxwell equation) with List II (Description) and select the correct answer :

List I

a. $\oint \boldsymbol{B} \cdot d\boldsymbol{S} = 0$

b. $\oint \boldsymbol{D} \cdot d\boldsymbol{S} = \int_v \rho_v dv$

c. $\oint \boldsymbol{E} \cdot d\boldsymbol{l} = -\int \frac{\partial \boldsymbol{B}}{\partial t} \cdot d\boldsymbol{S}$

d. $\oint \boldsymbol{H} \cdot d\boldsymbol{l} = \int \frac{\partial(\boldsymbol{D} + \boldsymbol{J})}{\partial t} \cdot d\boldsymbol{S}$

List II

1. The mmf around a closed path is equal to the conduction current plus the time derivative of the electric displacement current through any surface bounded by the path.
2. The emf around a closed path is equal to the time derivative is equal to the time derivative of the magnetic displacement through any surface bounded by the path.
3. The total electric displacement through the surface enclosing a volume is equal to total charge within the volume
4. The net magnetic flux emerging through any closed surface is zero.

Codes :

	a	b	c	d
(A)	1	3	2	4
(B)	4	3	2	1
(C)	4	2	3	1
(D)	1	2	3	4

MCQ 6.4.22 The equation of continuity defines the relation between

(A) electric field and magnetic field

(B) electric field and charge density

(C) flux density and charge density

(D) current density and charge density

MCQ 6.4.23 What is the generalized Maxwell's equation $\nabla \times \boldsymbol{H} = \boldsymbol{J}_c + \frac{\partial \boldsymbol{D}}{\partial t}$ for the free space ?

(A) $\nabla \times \boldsymbol{H} = 0$

(B) $\nabla \times \boldsymbol{H} = \boldsymbol{J}_c$

(C) $\nabla \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t}$

(D) $\nabla \times \boldsymbol{H} = \boldsymbol{D}$

MCQ 6.4.24 Magnetic field intensity is $\boldsymbol{H} = 3\boldsymbol{a}_z + 7y\boldsymbol{a}_y + 2x\boldsymbol{a}_z$ A/m. What is the current density $\boldsymbol{J}$ A/m^2 ?

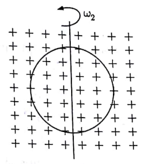
(A) $-2\boldsymbol{a}_y$

(B) $-7\boldsymbol{a}_z$

(C) $3\boldsymbol{a}_z$

(D) $12\boldsymbol{a}_y$

MCQ 6.4.25 A circular loop placed perpendicular to a uniform sinusoidal magnetic field of frequency ω_1 is revolved about an axis through its diameter at an angular velocity ω_2 rad/sec ($\omega_2 < \omega_1$) as shown in the figure below. What are the frequencies for the e.m.f induced in the loop ?



(A) ω_1 and ω_2

(B) $\omega_1, \omega_2 + \omega_2$ and ω_2

(C) $\omega_2, \omega_1 - \omega_2$ and ω_2

(D) $\omega_1 - \omega_2$ and $\omega_1 + \omega_2$

MCQ 6.4.26 Which one of the following is not a Maxwell's equation ?

(A) $\nabla \times \boldsymbol{H} = (\sigma + j\omega\varepsilon)\boldsymbol{E}$

(B) $\boldsymbol{F} = Q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$

(C) $\oint_c \boldsymbol{H} \cdot d\boldsymbol{l} = \oint_s \boldsymbol{J} \cdot d\boldsymbol{S} + \oint_s \frac{\partial \boldsymbol{D}}{\partial t} \cdot d\boldsymbol{S}$

(D) $\oint_S \boldsymbol{B} \cdot d\boldsymbol{S} = 0$

MCQ 6.4.27 Consider the following three equations :

1. $\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$

2. $\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$

3. $\nabla \cdot \boldsymbol{B} = 0$

Which of the above appear in Maxwell's equations ?

(A) 1, 2 and 3

(B) 1 and 2

(C) 2 and 3

(D) 1 and 3

MCQ 6.4.28 In free space, if $\rho_v = 0$, the Poisson's equation becomes

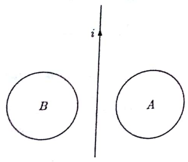
(A) Maxwell's divergence equation $\nabla \cdot \boldsymbol{B} = 0$

(B) Laplacian equation $\nabla^2 V = 0$

(C) Kirchhoff's voltage equation $\Sigma V = 0$

(D) None of the above

MCQ 6.4.29 A straight current carrying conductor and two conducting loops A and B are shown in the figure given below. What are the induced current in the two loops ?



(A) Anticlockwise in A and clockwise in B
(B) Clockwise in A and anticlockwise in B
(C) Clockwise both in A and B
(D) Anticlockwise both in A and B

MCQ 6.4.30 Which one of the following equations is not Maxwell's equation for a static electromagnetic field in a linear homogeneous medium ?

(A) $\nabla \cdot B = 0$ (B) $\nabla \times D = \bar{0}$
(C) $\oint_c B \cdot dl = \mu_0 I$ (D) $\nabla^2 A = \mu_0 J$

MCQ 6.4.31 Match **List I** with **List II** and select the correct answer using the codes given below :

	List I		List II
a	Continuity equation	1.	$\nabla \times H = J + \frac{\partial D}{\partial t}$
b	Ampere's law	2.	$J = \frac{\partial D}{\partial t}$
c	Displacement current	3.	$\nabla \times E = -\frac{\partial B}{\partial t}$
d	Faraday's law	4.	$\nabla \cdot J = -\frac{\partial \rho_v}{\partial t}$

Codes :

	a	b	c	d
(A)	4	3	2	1
(B)	4	1	2	3
(C)	2	3	4	1
(D)	2	1	4	3

MCQ 6.4.32 The magnetic flux through each turn of a 100 turn coil is $(t^3 - 2t)$ milli-Webers where t is in seconds. The induced e.m.f at $t = 2$ s is

(A) 1 V (B) −1 V
(C) 0.4 V (D) −0.4 V

MCQ 6.4.33 Match **List I** (Type of field denoted by A) with **List II** (Behaviour) and select the correct answer using the codes given below :

	List I		List II
a	A static electric field in a charge free region	1.	$\nabla \cdot A = 0$, $\nabla \times A \neq 0$
b	A static electric field in a charged region	2.	$\nabla \cdot A \neq 0$, $\nabla \times A = 0$
c	A steady magnetic field in a current carrying conductor	3.	$\nabla \cdot A \neq 0$, $\nabla \times A \neq 0$
d	A time-varying electric field in a charged medium with time-varying magnetic field	4.	$\nabla \cdot A = 0$, $\nabla \times A = 0$

Codes :

	a	b	c	d
(A)	4	2	3	1
(B)	4	2	1	3
(C)	2	4	3	1
(D)	2	4	1	3

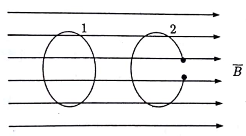
MCQ 6.4.34 Which one of the following pairs is not correctly matched ?

(A) Gauss Theorem : $\oint_s D \cdot ds = \oint_v \nabla \cdot D dv$
(B) Gauss's Law : $\oint_s D \cdot ds = \oint_v \rho dv$
(C) Coulomb's Law : $V = -\frac{d\phi_m}{dt}$
(D) Stoke's Theorem : $\oint_l \xi \cdot dl = \oint_s (\nabla \times \xi) \cdot ds$

MCQ 6.4.35 Maxwell equation $\nabla \times E = -(\partial B/\partial t)$ is represented in integral form as

(A) $\oint E \cdot dl = -\frac{\partial}{\partial t}\oint B \cdot dl$ (B) $\oint E \cdot dl = -\frac{\partial}{\partial t}\int_s B \cdot ds$
(C) $\oint E \times dl = -\frac{\partial}{\partial t}\oint B \cdot dl$ (D) $\oint E \times dl = -\frac{\partial}{\partial t}\int_s B \cdot dl$

MCQ 6.4.36 Two conducting coils 1 and 2 (identical except that 2 is split) are placed in a uniform magnetic field which decreases at a constant rate as in the figure. If the planes of the coils are perpendicular to the field lines, the following statements are made :



1. an e.m.f is induced in the split coil 2
2. e.m.fs are induced in both coils

3. equal Joule heating occurs in both coils
4. Joule heating does not occur in any coil

Which of the above statements is/are true ?

(A) 1 and 4
(B) 2 and 4
(C) 3 only
(D) 2 only

MCQ 6.4.37 For linear isotropic materials, both $\boldsymbol{E}$ and $\boldsymbol{H}$ have the time dependence $e^{j\omega t}$ and regions of interest are free of charge. The value of $\nabla \times \boldsymbol{H}$ is given by

(A) $\sigma \boldsymbol{E}$
(B) $j\omega\varepsilon \boldsymbol{E}$
(C) $\sigma \boldsymbol{E} + j\omega\varepsilon \boldsymbol{E}$
(D) $\sigma \boldsymbol{E} - j\omega\varepsilon \boldsymbol{E}$

MCQ 6.4.38 Which of the following equations is/are not Maxwell's equations(s) ?

(A) $\nabla \cdot \boldsymbol{J} = -\frac{\partial \rho_v}{\partial t}$
(B) $\nabla \cdot \boldsymbol{D} = \rho_v$
(C) $\nabla \cdot \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$
(D) $\oint \boldsymbol{H} \cdot dl = \int_s \left(\sigma \boldsymbol{E} + \varepsilon \frac{\partial \boldsymbol{E}}{\partial t}\right) \cdot ds$

Select the correct answer using the codes given below :

(A) 2 and 4
(B) 1 alone
(C) 1 and 3
(D) 1 and 4

MCQ 6.4.39 **Assertion (A) :** The relationship between Magnetic Vector potential $\boldsymbol{A}$ and the current density J in free space is

$$\nabla \times (\nabla \times \boldsymbol{A}) = \mu_0 \boldsymbol{J}$$

For a magnetic field in free space due to a dc or slowly varying current is $\nabla^2 \boldsymbol{A} = -\mu_0 \boldsymbol{J}$

Reason (R) : For magnetic field due to dc or slowly varying current $\nabla \cdot \boldsymbol{A} = 0$.

(A) Both A and R are true and R is the correct explanation of A
(B) Both A and R are true but R is NOT the correct explanation of A
(C) A is true but R is false
(D) A is false but R is true

MCQ 6.4.40 Given that $\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$

Assertion (A) : In the equation, the additional term $\frac{\partial \boldsymbol{D}}{\partial t}$ is necessary.

Reason (R) : The equation will be consistent with the principle of conservation of charge.

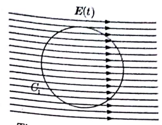
(A) Both A and R are true and R is the correct explanation of A
(B) Both A and R are true but R is NOT the correct explanation of A
(C) A is true but R is false
(D) A is false but R is true

MCQ 6.4.41 A circular loop is rotating about the y-axis as a diameter in a magnetic field $\boldsymbol{B} = B_0 \sin\omega t \boldsymbol{a}_x$ Wb/m^2. The induced emf in the loop is

(A) due to transformer emf only
(B) due to motional emf only
(C) due to a combination of transformer and motional emf
(D) zero

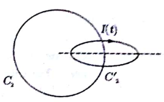
MCQ 6.4.42 Consider coils C_1, C_2, C_3 and C_4 (shown in the given figures) which are placed in the time-varying electric field $\boldsymbol{E}(t)$ and electric field produced by the coils C'_2, C'_3 and C'_4 carrying time varying current $I(t)$ respectively :

1.



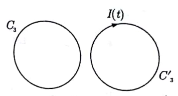
Time varying electric field $E(t)$ parallel to the plane of coil C_1

2.



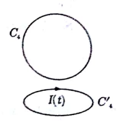
Coil planes are orthogonal

3.



Co-planer coils

4.



Coil planes are orthogonal

The electric field will induce an emf in the coils

(A) C_1 and C_2
(B) C_2 and C_3
(C) C_1 and C_3
(D) C_2 and C_4

MCQ 6.4.43 Match **List I** (Law/quantity) with **List II** (Mathematical expression) and select the correct answer :

	List I		**List II**
a.	Gauss's law	**1.**	$\nabla \cdot \boldsymbol{D} = \rho$
b.	Ampere's law	**2.**	$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$
c.	Faraday's law	**3.**	$\boldsymbol{\mathcal{P}} = \boldsymbol{E} \times \boldsymbol{H}$
d.	Poynting vector	**4.**	$\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$
		5.	$\nabla \times \boldsymbol{H} = \boldsymbol{J}_c + \frac{\partial \boldsymbol{D}}{\partial t}$

Codes :

	a	b	c	d
(A)	1	2	4	3
(B)	3	5	2	1
(C)	1	5	2	3
(D)	3	2	4	1

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