Electromagnetic Engineering (EC 21006)
T U T O R I A L - VII

MAXWELL'S EQUATIONS

1. Given $\boldsymbol{H} = \ 300\cos(3\times10^8\mathrm{t}-\mathrm{y})\,\hat{a}_z$ A/m in free space, find the emf developed in the general $a_\emptyset$ direction about the closed path having corners at: (a) (0,0,0),(1,0,0,),(1,1,0), and (0,1,0); (b) (0,0,0),(2π,0,0),(2π, 2π,0), and (0, 2π,0),
2. The location of the sliding bar in figure 1 is given by $x = 5t + 2t^3$, and the separation of the two rails is 20 cm. Let $\boldsymbol{B} = 0.8x^2\hat{a}_z$ T. Find the voltmeter reading at : (a) $t = 0.4\ s$; (b) $x = 0.6\ m$.

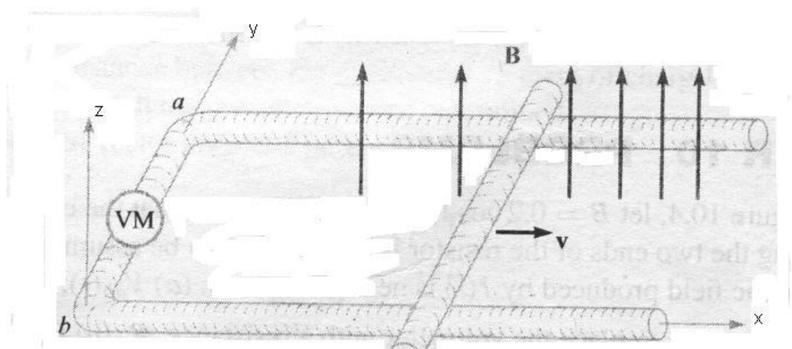
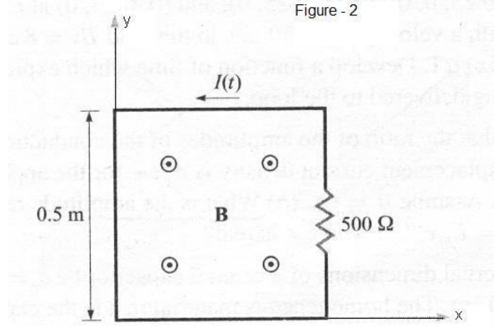


Figure - 1

3. A perfectly conducting filament containing a small 500 Ω resister is formed into a square, as illustrated by figure 2. Find I(t) if $\boldsymbol{B} =: (a)\ 0.3\cos(120\pi t - 30^0)\,\hat{a}_z$ T; $(b)0.4\cos[\pi(ct-y)]\hat{a}_z\ \mu$T, where $c = 3\times10^8\ m/s$.

Figure - 2



4. A square filamentary loop of wire is 25 cm on a side and has a resistance of 125 Ω per meter length. The loop lies in the $z = 0$ plane with its corners at (0,0,0), (0.25,0,0), (0.25,0.25,0), and (0,0.25,0) at $t = 0$. The loop is moving with a velocity of $v_y = 50\ m/s$ in the field $\boldsymbol{B}_z$ =8 cos $(1.5\times10^8\mathrm{t}-0.5x)\ \mu$T. Develop a function of time which expresses the ohmic power being delivered to the loop.
5. Consider the region defined by $|x|, |y|, |z| < 1$. Let $\boldsymbol{\varepsilon}_r$ =5, μ_r=4, and σ =0. If $\mathrm{J_d}=20\cos(1.5\times10^8\mathrm{t}-bx)\hat{a}_y\ \mu A/\mathrm{m}^2$: (a)find **D** and **E**;(b) use the point form of Faraday's law and an integration with respect to time to find **B** and **H**;(c)use $\nabla\times\boldsymbol{H} = \boldsymbol{J}_d + \boldsymbol{J}$ to find $\boldsymbol{J}_{d;}$(d)what is the numerical value of b?

6. In a source free dielectric medium the electric field intensity is given as $E = C\,sin\alpha x\cos(\omega t - \beta z)\,\hat{a}_y$ V/m where C is the amplitude and α and β are constant quantities. Determine (a)the magnetic field intensity and (b)the electric flux density.
7. Let $\mu = 3 \times 10^{-5}\ H/m, \varepsilon = 1.2 \times 10^{-10}\ F/m$ and σ =0 everywhere. If **H**=2 cos (10^{10}t $-\beta$x) $\hat{a}_z\ A/$m, use Maxwell's equations to obtain expressions for **B**,**D**,**E** and β.
8. The electric field intensity in the region $0 < x < 5, 0 < y < \pi/12, 0 < z < 0.06$ m in free space is given by $\boldsymbol{E}$ =C $sin\ 12y\ sin\ az\ cos\ 2 \times 10^{10} t\,\hat{a}_x$ V/m. Beginning with the $\nabla \times \boldsymbol{E}$ relationship**,** use Maxwell's equations to find a numerical value for a, if it is known that a is greater than zero.
9. In region 1, z<0, $\varepsilon_1 = 2 \times 10^{-11}\ F/m$, $\mu_1 = 2 \times 10^{-6}\ H/m$, and $\sigma_1 = 4 \times 10^{-3}\ S/m$; in region 2, z >0, $\varepsilon_2 = \varepsilon_1/2$, $\mu_2 = 2\mu_1$, and $\sigma_2 = \sigma_1/4$. It is known that $\boldsymbol{E}_1 = (30\,\hat{a}_x + 20\,\hat{a}_y + 10\,\hat{a}_z)\cos 10^9 t$ V/m at P(0,0,0$^-$).
   (a) find $\mathbf{E}_{N1}$, $\mathbf{E}_{t1}$, $\mathbf{D}_{N1}$, and $\mathbf{D}_{t1}$ at P_1;
   (b) find $\boldsymbol{J}_{N1}$ and $\boldsymbol{J}_{t1}$ at P_1;
   (c) find $\mathbf{E}_{t2}$, $\mathbf{D}_{t2}$, and $\boldsymbol{J}_{t2}$ at P_2(0,0,0$^+$);
   (d) use the continuity equation to help show that $\boldsymbol{J}_{N1} - \boldsymbol{J}_{N2} = \partial \boldsymbol{D}_{N2}/\partial t - \partial \boldsymbol{D}_{N1}/\partial t$, and then determine $\mathbf{D}_{N2}$, $\boldsymbol{J}_{N2}$, and $\mathbf{E}_{N2}$.
10. The parallel plate transmission line shown in figure 3 has dimensions $b = 4cm$ and $d = 8mm$, while the medium between the plates is characterized by $\boldsymbol{\varepsilon}_r$ =20, μ_r=1,and $\sigma = 0$. Neglect fields outside the dielectric. Given $\boldsymbol{H} = 5\cos(10^9$t $-\beta$x$)\,\hat{a}_y$ A/m, use Maxwell's equations to help find; (a) β, if $\beta > 0$; (b) the displacement current density at $z = 0$; (c) the total displacement current crossing the surface $x = 0.5d, 0 < y < b,\ 0 < z < 0.1$ m in the $\hat{a}_x$ direction.

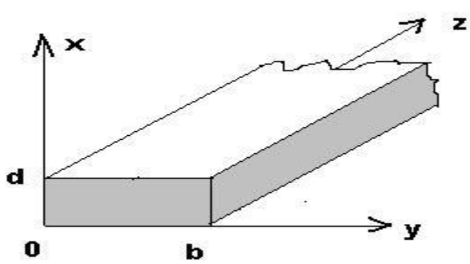


Figure 3

11. A right circularly polarized plane wave in air is incident at Brewster's angle onto a semi-infinite slab of Plexiglas ($\epsilon_r' = 3.45, \epsilon_r'' = 0$). (a)Determine the fractions of the incident power that are reflected and transmitted. (b)Describe the polarizations of the reflected and transmitted waves.
12. The conductivity of seawater is approximately 0.4 mS/m and its dielectric constant is 81. Determine the frequency at which the magnitude of the displacement current density is equal to the magnitude of the conduction current density. Comment on the electrical behavior of seawater at very low and very high frequencies.
13. In free space $\boldsymbol{E}(z,t) = 150\sin(\omega t - \beta z)\,\hat{a}_x$ (V/m). Find the total power passing through a rectangular area, of sides 30mm and 15mm, in the $z = 0$ plane.

14. In a dielectric medium ($\varepsilon = 4\varepsilon_0,\ \mu = \mu_0$) the **E** and **H** fields are given as follows: $\boldsymbol{E}_z = 1000\cos\left(\omega t - \frac{\pi}{3}x\right) V/m$ and $\boldsymbol{H}_y = -\frac{1000}{\eta}\cos\left(\omega t - \frac{\pi}{3}x\right) A/m$. Using phasor analysis, determine (a) ω and η , (b)the direction of power flow, and (c) the average power crossing the surface area bounded by the corners of a triangle at (2,0,0)m, (2,4,0)m, and (2,4,2)m.
15. A plane wave in free space ($z \leq 0$)is incident normally on a large block of material with $\boldsymbol{\varepsilon}_r$ =12, μ_r=3, $\sigma = 0$ which occupies $z \geq 0$. If the incident electric field is $\boldsymbol{E} = 30\cos(\omega t - z)\ \hat{a}_y$ V/m, find (a) ω, (b) the standing wave ratio, (c) the reflected magnetic field, and (d) the average power density of the transmitted wave.
16. A 30 MHz uniform plane wave with $\boldsymbol{H} = 10\sin(\omega t + \beta x)\ \hat{a}_z$ mA/m exists in region $x \geq 0$ having $\sigma = 0$, $\varepsilon = 9\varepsilon_0$, $\mu = 4\mu_0$. At $x = 0$, the wave encounters free space. Determine
    a. the polarization of the wave,
    b. the phase constant β,
    c. the displacement current density in region $x \geq 0$,
    d. the reflected and transmitted magnetic fields, and
    e. the average power density in each region.
17. In a dielectric medium ($\varepsilon = 9\varepsilon_0$, $\mu = \mu_0$), a plane wave with $\boldsymbol{H} = 0.2\cos(10^9 t - kx - k\sqrt{8}z)\ \hat{a}_y$ A/m is incident on an air boundary at $z = 0$, find
    a. θ_r and θ_t,
    b. k,
    c. the wavelength in the dielectric and air,
    d. the incident **E**,
    e. the transmitted and reflected **E**, and
    f. the Brewster's angle.
18. The unit vector $0.64\hat{a}_x + 0.6\hat{a}_y - 0.48\hat{a}_z$ is directed from region 2 ($\boldsymbol{\varepsilon}_r$ =2, μ_r=3, $\sigma_2 = 0$) toward region 1 ($\boldsymbol{\varepsilon}_{r1}$ =4, μ_{r1}=2, $\sigma_1 = 0$). If $\boldsymbol{B}_1 = \hat{a}_x - 2\hat{a}_y + 3\hat{a}_z)\sin 300t$ (T) at point P in region 1 adjacent to the boundary, find the amplitude at P of $\boldsymbol{B}_{N1}$; $\boldsymbol{B}_{t1}$; $\boldsymbol{B}_{N2}$; and $\boldsymbol{B}_2$.