MA 20104 Probability and Statistics
Assignment No. 7

1. Let (X, Y) be discrete with the joint pmf

$Y \setminus X$	-1	0	1
-2	1/6	1/12	1/6
1	1/6	1/12	1/6
2	1/12	0	1/12

   Find the joint pmf of (U, V) where $U = |X|, V = Y^2$.

2. Projectiles are fired at the origin of an XY – coordinate system. Assume that the point which is hit, say (X, Y), consists of a pair of independent standard normal r.v.'s. For two projectiles fired independently of one another, let (X_1, Y_1) and (X_2, Y_2) represent the points which are hit and Z be the distance between them. What is the distribution of Z^2?

3. Let X_1 and X_2 be independent r.v.'s each with negative exponential distribution with pdf $\lambda\, e^{-\lambda\, x}, x > 0$. Find the joint and marginal distributions of $Y_1 = X_1/X_2$ and $Y_2 = X_1 + X_2$.

4. Let X_1, X_2 be i.i.d. $N(0, 1)$ and $Y_1 = X_1^2 + X_2^2, Y_2 = X_1/X_2$. Find the joint and marginal distributions of Y_1 and Y_2. Are Y_1, Y_2 independent?

5. Let X_1 and X_2 have independent gamma distributions with parameters (n_1, λ) and (n_2, λ). Find the distributions of $Y = \frac{X_1}{X_1+X_2}$ and $Z = X_1 + X_2$. Is Y independent of Z? Is Z independent of $U = X_1/X_2$?

6. Let $X_1, X_2, \dots, X_n$ be independent exponential random variables with the probability density $f(x) = e^{-x}, x > 0$. Define random variables $Y_1, Y_2, \dots, Y_n$ as

$$Y_1 = X_1 + X_2 + \cdots + X_n, Y_2 = \frac{X_1 + X_2 + \cdots + X_{n-1}}{X_1 + X_2 + \cdots + X_n},$$

$$Y_3 = \frac{X_1 + X_2 + \cdots + X_{n-2}}{X_1 + X_2 + \cdots + X_{n-1}}, \dots, Y_{n-1} = \frac{X_1 + X_2}{X_1 + X_2 + X_3},$$

$$Y_n = \frac{X_1}{X_1 + X_2}.$$

Find the joint and marginal densities of $Y_1, Y_2, \dots, Y_n$. Are they independent?

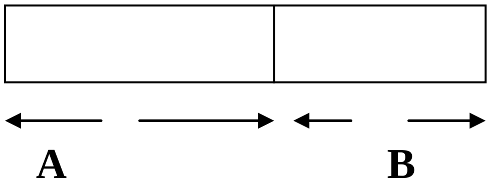
7. Suppose independent random variables Y_1, Y_2, Y_3 are such that $Y_1 = \ln X_1 \sim N(4, 1), Y_2 = \ln X_2 \sim N(3, 1)$ and $Y_3 = \ln X_3 \sim N(2, 0.5)$. Find the distribution and the median of $W = e^2 X_1^2 X_2^4 X_3^4$. Determine L and R such that $P(L \le W \le R) = 0.90$.

8. Let (X, Y) have bivariate normal distribution with density function

$$f(x, y) = \frac{1}{\pi\sqrt{3}} Exp\left\{-\frac{2}{3}(x^2 - xy + y^2)\right\}, \quad -\infty < x, y < \infty$$

Find the correlation coefficient between X and Y, $V(X - Y)$ and $P(-1 < X + Y < 2)$.

9. A straight rod consists of two sections **A** and **B**, each of which is manufactured independently on a different machine. The length (in inches) of section **A** is normally distributed with mean **20** and variance **0.03** and the length of section **B** is normally distributed with mean **14** and variance **0.01**. The rod is formed by joining the two sections together as shown below:



Suspect that the rod can be used in the construction of an airplane wing if its total length is between **33.6** to **34.4** inches. What is the probability that the rod can be used in the construction?

10. Let (X, Y) be a continuous bivariate random variable with the joint pdf

$$f(x,y) = \frac{1}{x^2y^2} \quad ,x > 1, y > 1$$

Find the joint and marginal distributions of $U = XY, V = \frac{X}{Y}$.