MCQ 8.4.94 A loss-less transmission line with characteristic impedance of 600 ohms is terminated in a purely resistive load of 900 ohms. The reflection coefficient is

(A) 0.2 (B) 0.5
(C) 0.667 (D) 1.5

MCQ 8.4.95 A transmission line has R, L, G and C distributed parameters per unit length of the line, γ is the propagation constant of the lines. Which expression gives the characteristic impedance of the line ?

(A) $\frac{\gamma}{R+j\omega L}$ (B) $\frac{R+j\omega L}{\gamma}$
(C) $\frac{G+j\omega C}{\gamma}$ (D) $\sqrt{\frac{G+j\omega C}{R+j\omega L}}$

MCQ 8.4.96 The open circuit impedance of a certain length of a loss-less line is 100 Ω. The short circuit impedance of the same line is also 100 Ω. The characteristic impedance of the line is

(A) $100\sqrt{2}\ \Omega$ (B) 50 Ω
(C) $100/\sqrt{2}\ \Omega$ (D) 100 Ω

MCQ 8.4.97 In the relations $S = \frac{1+|\Gamma|}{1-|\Gamma|}$; the values of S and Γ (where S stands for wave ratio and Γ is reflection coefficient), respectively, vary as

(A) 0 to 1 and −1 to 0 (B) 1 to ∞ and −1 to +1
(C) −1 to +1 and 1 to ∞ (D) −1 to 0 and 0 to 1

MCQ 8.4.98 Consider the following statements :
The characteristic impedance of a transmission line can increase with the increase in

1. resistance per unit length
2. conductance per unit length
3. capacitance per unit length
4. inductance per unit length

Which of these statements are correct ?

(A) 1 and 2 (B) 2 and 3
(C) 1 and 4 (D) 3 and 4

****

SOLUTIONS 8.1

SOL 8.1.1 Option (A) is correct.
Given
the input voltage, $v_i = V_0 \cos(4 \times 10^4 \pi t)$
and length of transmission line, $l = 20\text{ cm} = 20 \times 10^{-2}\text{ m}$
So, the angular frequency of the applied voltage is

$$\omega = 4 \times 10^4 \pi$$

and the wavelength of the voltage wave is

$$\lambda = \frac{v_p}{f} = \frac{2\pi v_p}{\omega} \qquad \left(f = \frac{\omega}{2\pi}\right)$$

Therefore,
$$\frac{l}{\lambda} = \frac{\omega(20 \times 10^{-2})}{2\pi v_p}$$
$$= \frac{(4 \times 10^4 \pi) \times (20 \times 10^{-2})}{2\pi \times (3 \times 10^8)} \quad (\text{in free space } v_p = 3 \times 10^8\text{ m/s})$$
$$= 1.33 \times 10^{-5}$$

Since, $\frac{l}{\lambda} \leq 0.01$

So the effect of transmission line on the voltage wave is negligible i.e. the output voltage will be in the same phase to the input voltage.
Thus, A and R both are true and R is correct explanation of A.

SOL 8.1.2 Option (C) is correct.
The width of strips, $w = 9.6\text{ cm} = 9.6 \times 10^{-2}\text{ m}$
Separation between the strips, $d = 0.6\text{ cm} = 0.6 \times 10^{-2}\text{ m}$
Relative permittivity of dielectric, $\varepsilon_r = 1.3$
Conductivity of dielectric, $\sigma \approx 0$
So, the conductance per unit length of line is given as

$$G' = \frac{\sigma w}{d} = 0 \qquad \sigma \approx 0$$

and the capacitance per unit length of the line is given as

$$C' = \frac{\varepsilon w}{d} = \varepsilon_0 \varepsilon_r \frac{w}{d} = (8.85 \times 10^{-12}) \times 1.3 \times \frac{9.6 \times 10^{-2}}{0.6 \times 10^{-2}}$$
$$= 1.84 \times 10^{-10}\text{ F/m} = 0.18\text{ nF/m}$$

SOL 8.1.3 Option (C) is correct.
Inductance per unit length, $L' = 250\text{ nH/m} = 250 \times 10^{-9}\text{ H/m}$
Capacitance per unit length $C' = 0.1\text{ nF/m} = 0.1 \times 10^{-9}\text{ F/m}$
So, the velocity of wave propagation along the lossless transmission line is given as

$$v_p = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{(250 \times 10^{-9})(0.1 \times 10^{-9})}}$$
$$= 2 \times 10^8\text{ m/s}$$

The characteristic impedance of the lossless transmission line is given as

$$Z_0 = \sqrt{\frac{L'}{C'}} = \sqrt{\frac{250 \times 10^{-9}}{0.1 \times 10^{-9}}} \qquad \text{(for lossless line, } R' = G' = 0\text{)}$$
$$= 50\,\Omega$$

SOL 8.1.4 Option (B) is correct.

Given the operating angular frequency of the transmission line is
$$\omega = 6 \times 10^8 \text{ rad/s}$$
and the parameters of transmission line are
$$R' = 0.2\,\text{k}\Omega/\text{m} = 200\,\Omega/\text{m}$$
$$L' = 4\,\mu\text{H/m} = 4 \times 10^{-6}\,\text{H/m}$$
$$G' = 8\,\mu\text{S/m} = 8 \times 10^{-6}\,\text{S/m}$$
$$C' = 4\,\text{pF/m} = 4 \times 10^{-12}\,\text{F/m}$$
So, the propagation constant of the transmission line is given as
$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$$
$$= \sqrt{[200 + j(6 \times 10^8)(4 \times 10^{-6})][(8 \times 10^{-6}) + j(6 \times 10^8)(4 \times 10^{-12})]}$$
$$= \sqrt{[200 + j(24 \times 10^2)][(8 \times 10^{-6}) + j(24 \times 10^{-4})]}$$
$$= (0.10 + j2.4) \text{ per meter}$$

SOL 8.1.5 Option (C) is correct.

Given the Operating angular frequency of the transmission line,
$$\omega = 1.2 \times 10^9 \text{ rad/s}$$
and the parameters of transmission line are
$$R' = 10\,\Omega/\text{m}$$
$$L' = 0.4\,\mu\text{H/m} = 0.1 \times 10^{-6}\,\text{H/m}$$
$$C' = 10\,\text{pF/m} = 10 \times 10^{-12}\,\text{F/m}$$
$$G' = 40\,\mu\text{S/m} = 40 \times 10^{-6}\,\text{S/m}$$
So, the characteristic impedance of the line is given as
$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} = \sqrt{\frac{10 + j(1.2 \times 10^9)(0.1 \times 10^{-6})}{40 \times 10^{-6} + j(1.2 \times 10^9)(10 \times 10^{-12})}}$$
$$= 100 - j4\,\Omega$$

SOL 8.1.6 Option (C) is correct.

The width of strips $= w$

Separation between strips $= d$

So, the characteristic impedance of lossless transmission line is given as
$$Z_0 = \frac{d}{w}\sqrt{\frac{\mu}{\varepsilon}}$$
When d and W is doubled, the characteristic impedance of the transmission line will be given as
$$Z_0' = \frac{2d}{2W}\sqrt{\frac{\mu}{\varepsilon}} = Z_0$$
Therefore, the characteristic impedance will remain same.

SOL 8.1.7 Option (D) is correct.

Attenuation constant, $\alpha = 10\,\text{mNP/m} = 10^{-2}\,\text{NP/m}$

Characteristic impedance, $Z_0 = 0.1\,\text{k}\Omega = 100\,\Omega$

Phase velocity, $v_p = 0.5 \times 10^8\,\text{m/s}$

Since the transmission line is distortion less so, the resistance per unit length of the transmission line is given as
$$R' = \alpha Z_0 = (10^{-2})(100) = 1\,\Omega/\text{m}$$
and the inductance per unit length of the lossless transmission line is given as
$$L' = \frac{Z_0}{v_p} = \frac{100}{0.5 \times 10^8} = 2\,\mu\text{H/m}$$

SOL 8.1.8 Option (C) is correct.

Given the parameters of distortionless transmission line are
$$R' = 4\,\Omega/\text{m}$$
and
$$G' = 4 \times 10^{-4}\,\text{S/m}$$
So, the attenuation constant of the distortion less transmission line is given as
$$\alpha = \sqrt{R'G'} = \sqrt{4 \times 4 \times 10^{-4}} = 4 \times 10^{-2}\,\text{NP/m}$$
and the characteristic impedance of the distortionless transmission line is given as
$$Z_0 = \sqrt{\frac{R'}{G'}} = \sqrt{\frac{4}{4 \times 10^{-4}}} = 100\,\Omega \qquad \text{distortionless line}$$

SOL 8.1.9 Option (D) is correct.

Load impedance, $Z_L = 300\,\Omega$

Characteristic impedance, $Z_0 = 150\,\Omega$

So, the reflection coefficient at the load terminal is given as
$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{300 - 150}{300 + 150} = \frac{1}{3}$$
and the reflection coefficient at generator end is given as
$$\Gamma_g = \frac{Z_g - Z_0}{Z_g + Z_0}$$
where Z_g is internal impedance of the generator. Since, it is given that the internal resistance of the generator is zero (i.e., $Z_g = 0$) so, we get
$$\Gamma_g = \frac{0 - 150}{0 + 150} = -1$$

SOL 8.1.10 Option (C) is correct.

Characteristic impedance, $Z_0 = 50\,\Omega$

Voltage standing wave ratio, $S = 3$

Since, the load connected to the lossless transmission line is purely resistive so, phase angle of the reflection coefficient of the line will be
$$\theta_\Gamma = 0 \text{ or } \pi$$
Now, the magnitude of the reflection coefficient is given as
$$|\Gamma| = \frac{S-1}{S+1} = \frac{3-1}{3+1} = 0.5$$
So, reflection coefficient of the transmission line is
$$\Gamma = |\Gamma| e^{j\theta_\Gamma}$$
$$= 0.5e^{j0} \text{ or } 0.5e^{j\pi}$$
$$= 0.5 \text{ or } -0.5$$
For $\Gamma = 0.5$ the load impedance of the transmission line is given as
$$Z_L = Z_0\left[\frac{1+\Gamma}{1-\Gamma}\right] = 150\left[\frac{1+0.5}{1-0.5}\right] = 450\,\Omega$$
and for $\Gamma = -0.5$ the load impedance of the transmission line is given as

$$Z_L = Z_0\left[\frac{1+\Gamma}{1-\Gamma}\right] = 150\left[\frac{1-0.5}{1+0.5}\right] = 50\,\Omega$$

Therefore, the possible values of load impedance connected to the transmission line are

$$Z_L = 50\,\Omega \text{ or } 450\,\Omega$$

SOL 8.1.11 Option (D) is correct.

Load impedance, $Z_L = (200 - j200)\,\Omega$

Characteristic impedance, $Z_0 = 100\,\Omega$

Length of transmission line, $l = 10\text{ cm} = 10 \times 10^{-2} = 0.1\text{ m}$

Generator voltage, $v_g(t) = 3\cos(\pi \times 10^9 t)$ volt

So, we get the angular frequency

$$\omega = \pi \times 10^9$$

and the phase constant of the wave on the transmission line is

$$\beta = \frac{\omega}{v_p} = \frac{\pi \times 10^9}{3 \times 10^8} = \frac{10\pi}{3} \qquad \text{(in air } v_p = 3 \times 10^8\text{ m/s)}$$

or

$$\beta l = \frac{10\pi}{3} \times 0.1 = \frac{\pi}{3}$$

Therefore, the input impedance of the lossless transmission line is given as

$$Z_{in} = Z_0\left(\frac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l}\right)$$

$$= 100\left(\frac{200 - j200 + j100\tan(\pi/3)}{100 + j(200 - j200)\tan(\pi/3)}\right)$$

$$= (25 - j25.4)\,\Omega$$

SOL 8.1.12 Option (C) is correct.

The natural frequency of oscillation of a wave in a transmission line of length l which is open circuited at one end and short circuited at other end is given as

$$f_n = \frac{(2n+1)v_p}{4l}, \quad n = 1,2,3,\ldots\infty$$

where v_p is phase velocity of the wave.

SOL 8.1.13 Option (C) is correct.

The dimension of the transmission line will remain same at all frequencies i.e. l will be constant but as it is defined in terms of wavelength which changes with the frequency so, the expression for length will vary in terms of wavelength λ. The wavelength of a wave is defined in terms of frequency f as

$$\lambda = \frac{c}{f}$$

where, c is the velocity of wave in free space so, at $f = 500$ Hz we have

$$\lambda = \frac{c}{500}$$

Therefore, the length of transmission line is

$$l = \frac{\lambda}{4} = \frac{c}{2000} \qquad (1)$$

Now, the wavelength at frequency, $f = 1\text{ kHz} = 1000$ Hz is given as

$$\lambda = \frac{c}{1000} \qquad (2)$$

Since, the length of the transmission line will be same as determined in equation (1). So, we get

$$l = \frac{c}{2000} = \frac{(c/1000)}{2} = \frac{\lambda}{2} \qquad \text{(from eq. (2))}$$

SOL 8.1.14 Option (A) is correct.

Given, the distance between successive maxima and minima is 10 cm.

i.e. $\lambda/2 = 10$ cm

$\lambda = 20$ cm

Now, the distance between first minima and load is

$$l_{min} = 7.5\text{ cm}$$

$$l_{min} > \frac{\lambda}{4}$$

So, the distance between first maxima and load will be

$$l_{max} = l_{min} - \frac{\lambda}{4} = 7.5 - \frac{\lambda}{4} = \frac{7.5}{20} \times \lambda - \frac{\lambda}{4} = \frac{\lambda}{8}$$

SOL 8.1.15 Option (C) is correct.

Since, the TEM wave is z-polarized i.e. the electric field of the wave is directed along $+\boldsymbol{a}_z$.

i.e. $\boldsymbol{a}_E = \boldsymbol{a}_z$

and the direction of wave propagation is along $\boldsymbol{a}_x$

i.e. $\boldsymbol{a}_k = \boldsymbol{a}_x$

So, the direction of magnetic field intensity will be

$$\boldsymbol{a}_H = \boldsymbol{a}_k \times \boldsymbol{a}_E = \boldsymbol{a}_x \times \boldsymbol{a}_z = -\boldsymbol{a}_y$$

As $\boldsymbol{E}$ is in $+\boldsymbol{a}_z$ direction and $\boldsymbol{H}$ is in $-\boldsymbol{a}_y$ direction so, we can consider the two vectors as

$$\boldsymbol{E} = E_z\boldsymbol{a}_z \qquad (1)$$

and

$$\boldsymbol{H} = -H_y\boldsymbol{a}_y \qquad (2)$$

Now, from the maxwell's equation in phasor form we have

$$\nabla \times \boldsymbol{H} = j\omega\varepsilon\boldsymbol{E} \qquad \text{(for perfect dielectric } \sigma = 0)$$

$$\begin{vmatrix} \boldsymbol{a}_x & \boldsymbol{a}_y & \boldsymbol{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & -H_y & 0 \end{vmatrix} = j\omega\varepsilon E_z\boldsymbol{a}_z \qquad \text{Using equation (1) and (2)}$$

$$\frac{\partial H_y}{\partial z}\boldsymbol{a}_x - \frac{\partial H_y}{\partial x}\boldsymbol{a}_z = j\omega\varepsilon E_z\boldsymbol{a}_z$$

It gives the result as

$$\frac{\partial H_y}{\partial z} = 0$$

Again from Maxwell's equation we have

$$\nabla \times \boldsymbol{E} = -j\omega\mu\boldsymbol{H}$$

$$\begin{vmatrix} \boldsymbol{a}_x & \boldsymbol{a}_y & \boldsymbol{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & E_z \end{vmatrix} = +j\omega\mu H_y\boldsymbol{a}_y \qquad \text{Using equation (1) and (2)}$$

$$\frac{\partial E_z}{\partial y}\boldsymbol{a}_x - \frac{\partial E_z}{\partial x}\boldsymbol{a}_y = j\omega\mu H_y\boldsymbol{a}_y$$

So, it gives the result as

$$\frac{\partial E_z}{\partial y} = 0$$

Thus, Both (A) and (B) are correct.

SOL 8.1.16 Option (C) is correct.

Given,

The position of first voltage maximum, $l_{max} = 4.5\text{ cm}$

Position of first current maximum(voltage minima), $l_{min} = 1.5\text{ cm}$

Standing wave ratio, $S = 3$

Characteristic impedance, $Z_0 = 50\,\Omega$

Since, the distance between a maximum and an adjacent minimum is $\lambda/4$ as discussed in previous question.

i.e.
$$l_{max} - l_{min} = \lambda/4$$
$$4.5 - 1.5 = \lambda/4$$

So,
$$\lambda = 12\text{ cm}$$

Again the distance of first voltage maximum from the load is given as

$$l_{max} = \frac{\theta_\Gamma \lambda}{4\pi} + \frac{n\lambda}{2}$$

$$4.5 = \frac{\theta_\Gamma (12)}{4\pi} + 0 \qquad \text{(For } n = 0)$$

$$\theta_\Gamma = \frac{3\pi}{2}$$

Now, the magnitude of reflection coefficient is given as

$$|\Gamma| = \frac{S-1}{S+1} = \frac{3-1}{3+1} = \frac{2}{4} = 0.5$$

So, the reflection coefficient of the transmission line is

$$\Gamma = |\Gamma| \underline{/\theta_\Gamma}$$
$$= 0.5 < 3\pi/2 = 0.5e^{j3\pi/2} = -j0.5$$

Therefore, the load impedance of the transmission line is given as

$$Z_L = Z_0\left[\frac{1+\Gamma}{1-\Gamma}\right] = 50\left[\frac{1-j0.5}{1+j0.5}\right]$$
$$= (30 - j40)\,\Omega$$

SOL 8.1.17 Option (D) is correct.

Characteristic impedance, $Z_0 = 50\,\Omega$

Load impedance, $Z_L = (30 + j15)\,\Omega$

Length of transmission line, $l = 7\lambda/20$

Since, the transmission line is lossless so, the attenuation constant is zero

i.e.
$$\alpha = 0$$

or,
$$\gamma = \alpha + j\beta = j\beta$$

Therefore, the input impedance of the lossless transmission line is given as

$$Z_{in} = Z_0\left(\frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}\right) = Z_0\left[\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}\right] \qquad (\gamma = j\beta)$$

$$= 50\left[\frac{(30+j15) + j50\tan\left(\frac{2\pi}{\lambda}\frac{7\lambda}{20}\right)}{50 + j(30+j15)\tan\left(\frac{2\pi}{\lambda}\frac{7\lambda}{20}\right)}\right] \qquad \left(\beta = \frac{2\pi}{\lambda}\right)$$

$$= 50\left[\frac{(30+j15) + j50\tan\left(\frac{7\pi}{10}\right)}{50 + j(30+j15)\tan\left(\frac{7\pi}{10}\right)}\right]$$

$$= (32.4 - j19.2)\,\Omega$$

SOL 8.1.18 Option (B) is correct.

In the assertion (A) given,

Length of the transmission line, $l = \lambda/4$

Load impedance, $Z_L = 0$

So, we get
$$\beta l = \left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{4}\right) = \frac{\pi}{2} \qquad (\beta = 2\pi/\lambda)$$

The input impedance of the lossless transmission line is given as

$$Z_{in} = Z_0\left(\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}\right)$$
$$= Z_0\left(\frac{jZ_0 \tan\frac{\pi}{2}}{Z_0}\right) = j\infty$$

Now, we consider the reason part,

Distance of the maxima from load is given as

$$l_{max} = (\theta_\Gamma + 2n\pi)/2\beta$$

where, θ_Γ is the phase angle of reflection coefficient

β is the phase constant of the voltage wave

and $n = 0, 1, 2, ...$

Therefore, the input impedance at the point of maxima is given as

$$Z_{in} = Z_0\left(\frac{1 + \Gamma e^{-j2\beta l_{max}}}{1 - \Gamma e^{-j2\beta l_{max}}}\right) = Z_0\left(\frac{1 + |\Gamma| e^{j\theta_\Gamma} e^{-j(\theta_\Gamma + 2n\pi)}}{1 - |\Gamma| e^{j\theta_\Gamma} e^{-j(\theta_\Gamma + 2n\pi)}}\right) \qquad (\Gamma = |\Gamma| e^{j\theta_\Gamma})$$

$$= Z_0\left(\frac{1+|\Gamma|}{1-|\Gamma|}\right)$$

So, Z_{in} is real if Z_0 is real and since, Z_0 is always real for a distortionless line. Thus, Z_{in} will be purely real at the position of voltage maxima in a distortionless line.

i.e. A and R both are true but R is not the explanation of A.

SOL 8.1.19 Option (D) is correct.

Length of transmission line, $l = 6\text{ m}$

Characteristic impedance, $Z_0 = 30\,\Omega$

Relative permittivity, $\varepsilon_r = 2.25$

Load impedance, $Z_L = (30 - j10)\,\Omega$

So, we get the angular frequency,

$$\omega = 8\pi \times 10^7$$

and the phase constant of the volatge wave along the transmission line is

$$\beta = \frac{\omega}{v_p} = \frac{8\pi \times 10^7}{c/\sqrt{\varepsilon_r}} \qquad \left(v_p = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{\varepsilon_r}}\right)$$

$$= \frac{8\pi \times 10^7 \times \sqrt{2.25}}{3 \times 10^8} = \frac{8\pi \times 10^7 \times 1.5}{3 \times 10^8}$$

$$= \frac{2\pi}{5}$$

or,
$$\beta l = \frac{2\pi}{5} \times 6 = 2.4\pi\text{ rad}$$

Therefore, the input impedance of the lossless transmission line is given as

$$Z_{in} = Z_0\left(\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}\right) = 30\left(\frac{30 - j10 + j30\tan(2.4\pi)}{30 + j(30 - j10)\tan(2.4\pi)}\right)$$
$$= (23.14 + j5.48)\,\Omega$$

SOL 8.1.20 Option (C) is correct.

Given the generator voltage to the transmission line,

$$V_g(t) = 10\cos(8\pi \times 10^7 t - 30°)$$

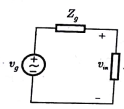
So, in phasor form the generator voltage is

$$V_{sg} = 10e^{-j30°}$$

and as determined in previous question, the input impedance of transmission line is

$$Z_{in} = (23.14 + j5.48)\,\Omega$$

So, for determining the input voltage, we draw the equivalent circuit for the transmission line as shown in figure below :



Using voltage division, we get the input voltage to the transmission line as

$$V_{in} = V_g \times \left(\frac{Z_{in}}{Z_{in} + Z_g}\right)$$

or,

$$V_{s,in} = V_{sg}\left(\frac{Z_{in}}{Z_{in} + Z_g}\right) \quad \text{(in phasor form)}$$

$$= 10e^{-j30}\left(\frac{23.14 + j5.48}{30 + 23.14 + j5.48}\right)$$

$$= (10e^{-j30})(0.44e^{7.44°})$$

$$= 4.4e^{-j22.56°}$$

Thus, the instantaneous input voltage of the transmission line is

$$v_{in}(t) = \mathrm{Re}[V_{s,in}\,e^{j\omega t}]$$

$$= 4.4\cos(8\pi \times 10^7 t - 22.56°)\ \text{volt}$$

SOL 8.1.21 Option (C) is correct.

Given,

Load impedances to the line 1 and 2, $\quad Z_{L1} = Z_{L2} = 150\,\Omega$

Length of the transmission lines 1 and 2, $\quad l_1 = l_2 = \lambda/5$

Now, we consider the input impedance of line 1 and line 2 be Z_{in1} and Z_{in2} respectively. Since, the transmission line are identical so, the input impedances of the transmission lines 1 and 2 will be equal and given as

$$Z_{in1} = Z_{in2} = Z_0\left(\frac{Z_{L1} + jZ_0\tan\beta l_1}{Z_0 + jZ_L\tan\beta l_1}\right) \quad \text{(lossless transmission line)}$$

$$= 100\left(\frac{150 + j100\tan\left(\frac{2\pi}{\lambda}\frac{\lambda}{5}\right)}{100 + j150\tan\left(\frac{2\pi}{\lambda}\frac{\lambda}{5}\right)}\right) \quad (\beta = 2\pi/\lambda)$$

$$= 100\left(\frac{150 + j100\tan\left(\frac{2\pi}{5}\right)}{100 + j150\tan\left(\frac{2\pi}{5}\right)}\right)$$

$$= (70.4 - j17.24)\,\Omega$$

Therefore, the effective load impedance of the feedline will be equal to the equivalent input impedance of the parallel combination of the line 1 and 2.

i.e.

$$Z_L' = Z_{in1} \,||\, Z_{in2}$$

$$= \frac{Z_{in1}}{2} = \frac{(70.4 - j17.24)}{2} \qquad Z_{in1} = Z_{in2}$$

$$= (35.20 - j8.62)\,\Omega$$

SOL 8.1.22 Option (B) is correct.

Given the length of the feed line,

$$l = 0.3\lambda$$

and as calculated in above question, the effective load impedance of the feedline is

$$Z_L' = (35.20 - j8.62)\,\Omega$$

So,

$$\beta l = \left(\frac{2\pi}{\lambda}\right)(0.3\lambda) = 0.6\lambda$$

Therefore, input impedance of the feedline (lossless transmission line) is given as

$$Z_{in} = Z_0\left(\frac{Z_L' + jZ_0\tan\beta l}{Z_0 + jZ_L'\tan\beta l}\right)$$

$$= 100\left(\frac{35.20 - j8.62 + j100\tan(0.6\pi)}{100 + j(35.20 - j8.62)\tan(0.6\pi)}\right)$$

$$= (215.14 - j113.4)\,\Omega$$

SOL 8.1.23 Option (B) is correct.

Operating frequency $\quad f = 0.3\,\text{GHz} = 0.3 \times 10^9\,\text{Hz}$

Load impedance, $\quad Z_L = (100 - j100)\,\Omega$

Characteristic impedance $\quad Z_0 = 100\,\Omega$

Generator voltage in phasor form, $\quad V_{sg} = 150$ volt

Internal resistance of generator $\quad Z_g = 100\,\Omega$

Length of the transmission line, $\quad l = 0.375\lambda$

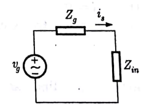
So, the input impedance of the lossless transmission line is given as

$$Z_{in} = Z_0\left(\frac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l}\right)$$

$$= 100\left(\frac{100 - j100 + j100\tan\left(\frac{2\pi}{\lambda}0.375\lambda\right)}{100 + j(100 - j100)\tan\left(\frac{2\pi}{\lambda}0.375\lambda\right)}\right)$$

$$= (200 + j100)\,\Omega$$

Now, for determining the load current, we draw the equivalent circuit for the transmission line as shown in the figure below :



Therefore, the voltage across the input terminal of the transmission line is given as

$$V_{s,in} = V_{sg}\left(\frac{Z_{in}}{Z_g + Z_{in}}\right)$$

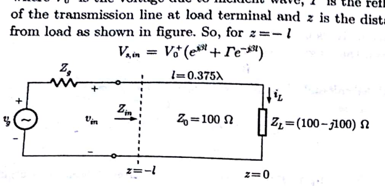
$$= 150\left(\frac{200 + j100}{100 + 200 + j100}\right) = 106.1e^{j8.13^\circ}$$

Since, at any point, on the transmission line voltage is given as

$$V_s(z) = V_0^+(e^{-j\beta z} + \Gamma e^{j\beta z})$$

where V_0^+ is the voltage due to incident wave, Γ is the reflection coefficient of the transmission line at load terminal and z is the distance of the point from load as shown in figure. So, for $z = -l$

$$V_{s,in} = V_0^+(e^{j\beta l} + \Gamma e^{-j\beta l}) \qquad (1)$$



Now, the reflection coefficient of the transmission line at load terminal is

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100 - j100 - 100}{100 - j100 + 100} = 0.45e^{-j63.43^\circ}$$

Putting the value of Γ and $V_{s,in}$ in equation (1), we get

$$106.1e^{j8.13^\circ} = V_0^+\left(e^{j\left(\frac{2\pi}{\lambda}\right)(0.375\lambda)} + 0.45e^{-j6343^\circ}e^{-j\left(\frac{2\pi}{\lambda}\right)(0.375\lambda)}\right)$$

$$V_0^+ = \frac{106.1e^{j8.13^\circ}}{e^{j135^\circ} + 0.45e^{-j63.43^\circ}e^{-j135^\circ}}$$

$$= 75e^{-j135^\circ}$$

The current at any point on the transmission line is given as

$$I_s(z) = \frac{V_0^+}{Z_0}(e^{-j\beta z} - \Gamma e^{j\beta z})$$

So, the current flowing in the load (at $z = 0$) is

$$I_{sL} = \frac{V_0^+}{Z_0}(1 - \Gamma)$$

$$= \frac{75e^{-j135^\circ}}{100}(1 - 0.45e^{-j63.43^\circ})$$

$$= 0.67e^{-j108.4^\circ}$$

Therefore, the instantaneous current at the load terminal will be

$$i_L(t) = \mathrm{Re}\{I_{sL}e^{j\omega t}\}$$

$$= 0.67\cos(2\pi \times 0.3 \times 10^9 t - 108.4^\circ)$$

$$= 0.67\cos(6\pi \times 10^8 t - 108.4^\circ)$$

SOL 8.1.24 Option (A) is correct.

Given, transmission line is of infinite length i.e. $l = \infty$.

and input impedance of the transmission line is equal to its characteristic impedance

i.e. $\quad Z_{in} = Z_0$

Since, the input impedance of a transmission line is defined as

$$Z_{in} = Z_0\left(\frac{Z_L + Z_0\tanh\gamma l}{Z_0 + Z_L\tanh\gamma l}\right)$$

So, $$Z_0 = Z_0\left(\frac{Z_L + Z_0\tanh\gamma l}{Z_0 + Z_L\tanh\gamma l}\right)$$

Solving the equation, we get

$$\tanh\gamma l = 1$$

$$\frac{e^{\gamma l} - e^{-\gamma l}}{e^{\gamma l} + e^{-\gamma l}} = 1$$

$$e^{-\gamma l} = 0$$

Since, $l = \infty$. So for satisfying the above condition propagation constant γ must have a real part.

i.e. $\quad$ real part of $\gamma \neq 0 \qquad (\gamma = \alpha + j\beta)$

or, $\quad \alpha \neq 0$

As the attenuation constant of the voltage wave along the transmission line is not equal to zero therefore, it is a lossy transmission line.

SOL 8.1.25 Option (B) is correct.

Observing the waveform we conclude that at the sending end voltage changes at $t = t_1$. The changed voltage at the sending is given as

$$v(t_1) = V_0^+ + \Gamma_L V_0^+ + \Gamma_g\Gamma_L V_0^+ \qquad (1)$$

where V_0^+ is voltage at sending end at $t = 0$, Γ_L and Γ_g are the reflection coefficients at the load terminal and the source terminal respectively. So, we get

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = -1 \qquad (Z_L = 0)$$

and $$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0} \qquad (Z_g = R_g)$$

Putting these values in equation (1), we get

$$v(t_1) = V_1^+ - V_1^+ - \Gamma_g V_1^+$$

$$v(t_1) = -\Gamma_g V_1^+ \qquad (2)$$

From the shown wave form of the voltage at sending end, we have

$$v(t_1) = 6 \text{ volt}$$

$$V_0^+ = 24 \text{ volt}$$

Putting these values in equation (2), we get

$$6 = -\Gamma_g(24)$$

or, $$\Gamma_g = -4$$

$$\frac{R_g - Z_0}{R_g + Z_0} = -4$$

$$R_g = 60\,\Omega \qquad (Z_0 = 100\,\Omega)$$

At $t = 0$ as the voltage just applied to transmission line, the input impedance is independent of Z_L and equals to Z_0 (i.e. $Z_{in} = Z_0$ at $t = 0$). Therefore, using voltage division the voltage at the sending end is given as

$$V_0^+ = V_g\left(\frac{Z_0}{R_g + Z_0}\right)$$

$$24 = V_g\left(\frac{100}{60 + 100}\right) \qquad (V_0^+ = 24 \text{ volt})$$

$$V_g = \frac{24 \times 160}{100} = 38.4 \text{ volt}$$

SOL 8.1.26 Option (A) is correct.

Length of the transmission line, $l = 1.5$ m	
Internal resistance of generator,	$R_g = 200\,\Omega$
Characteristic impedance,	$Z_0 = 100\,\Omega$
Generator voltage,	$V_g = 30$ volt
Load impedance,	$Z_L = 50\,\Omega$

So, the reflection coefficient at the load terminal is

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50-100}{50+100} = -\frac{1}{3}$$

and the reflection coefficient at the source terminal is

$$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0} = \frac{200-100}{200+100} = \frac{1}{3}$$

Again as discussed in previous question at time, $t = 0$ as the voltage is just applied to the transmission line, the input impedance is independent of Z_L and equals to Z_0 (i.e. $Z_{in} = Z_0$ at $t = 0$). Therefore, using voltage division the input voltage at the sending end is given as

$$V_0^+ = V_g\left(\frac{Z_0}{R_g + Z_0}\right) = 30 \times \left(\frac{100}{200+100}\right) = 10 \text{ volt}$$

Now, the time taken by the wave to travel from source terminal to the load terminal (or load terminal to source terminal) is given as

$$T = \frac{l}{c}$$

where, l is length of transmission line and c is the velocity of the voltage wave along the transmission line. So, we get

$$T = \frac{1.5}{3 \times 10^8} = 5 \text{ ns}$$

Therefore, for the interval $0 \le t < 5$ ns, the incident wave will be travelling from source to load and will have the voltage

$$V_1^+ = 10 \text{ volt}$$

For the interval $5 \text{ ns} \le t < 10$ ns an additional reflected wave will be travelling from load to source and will have the voltage

$$V_1^- = \Gamma_L V_1^+ = -\frac{10}{3} = -3.33 \text{ volt}$$

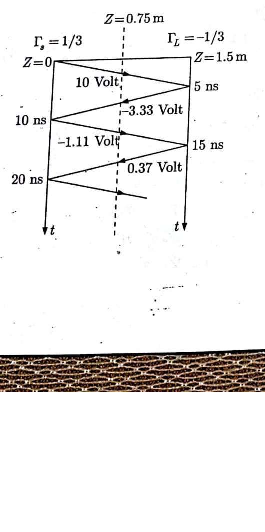
For $10 \text{ ns} \le t \le 15$ ns the wave reflected by source resistance travelling from source to load will be added to that has the voltage

$$V_2^+ = \Gamma_g V_1^- = -\frac{3.33}{3} = -1.11 \text{ volt}$$

For $15 \text{ ns} \le t \le 20$ ns again the wave reflected by load travelling from load to source will be added that has the voltage

$$V_2^- = \Gamma_L V_2^+ = \frac{1.11}{3} = 0.37 \text{ volt}$$

This will be continuous and the bounce diagram obtained between source (at $z = 0$) and load (at $z = 1.5$ m) will be as shown in figure below :



SOL 8.1.27 Option (C) is correct.

From the bounce diagram that obtained between source terminal ($z = 0$) and load terminal ($z = 1.5$ m) in previous question, we can determine the voltage $v(t)$ at any instant by just summing all the voltage waves existing at any time t.

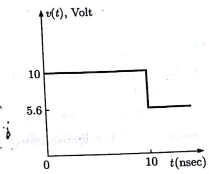
Since, for interval $0 \le t \le 10$ ns only a single voltage waye with $V_1^+ = 10$ volt exists at sending end so, the voltage at the sending end ($z = 0$) for the interval is

$$v(t) = V_1^+ = 10 \text{ volt}, \quad \text{for } 0 \le t < 10 \text{ ns}$$

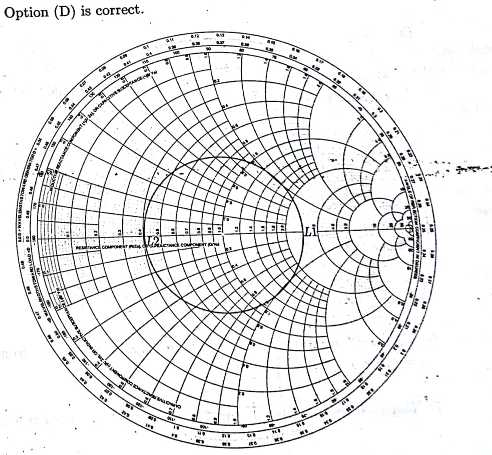
again for the interval $10 \text{ ns} \le t < 20$ ns, three voltage waves with $V_1^+ = 10$ volt, $V_1^- = -3.33$ volt and $V_2^+ = -1.11$ volt exists at the sending end so, the voltage at the sending end for the interval is

$$v(t) = V_1^+ + V_1^- + V_2^+ = 10 - 3.33 - 1.11 = 5.6 \text{ volt} \quad \text{for } 10 \text{ ns} \le t < 20 \text{ ns}$$

Thus, the obtained voltage wave form is plotted in the figure below



SOL 8.1.28 Option (D) is correct.



As shown in the smith chart, SWR circle meets the Γ_r axis (real part of reflection coefficient) at L_1 and L_2 respectively. So, We have the two possible values of normalised impedance (real values of z_L).

$$z_{L1} = 2.5 \quad \text{at } L_1$$

$$z_{L2} = 0.4 \quad \text{at } L_2$$

Since, the normalised impedance is defined as

$$z_L = \frac{\text{Load impedance}}{\text{Characteristic impedance}}$$

So, we have $\quad z_{L1} = \frac{Z_L}{Z_{01}} = 2.5$

or, $\quad Z_{01} = \frac{Z_L}{2.5}$

$$= \frac{50}{2.5} = 20\,\Omega$$

Similarly, $\quad z_{L2} = \frac{Z_L}{Z_{02}} = 0.4$

or, $\quad Z_{02} = \frac{Z_L}{0.4}$

$$= \frac{50}{0.4} = 125\,\Omega$$

Therefore, the two possible values of the characteristic impedance of the lossless transmission line are $20\,\Omega$ and $125\,\Omega$.

SOL 8.1.29 Option (B) is correct.

We can determine the reflection coefficient of the transmission line using smith chart as explained below :

(1) First we determine the normalized load impedance of the transmission line as

$$z_L = \frac{Z_L}{Z_0} = \frac{100 + j50}{100} = 1 + j0.5$$

(2) Comparing the normalized impedance to its general form

$$z_L = r + jx$$

where r is the normalized resistance (real component) and x is the normalized reactance (imaginary component). we get

$$r = 1 \text{ and } x = 0.5$$

(3) Now, we determine the intersection point of $r = 1$ circle and $x = 0.5$ circle on the smith charge and denote it by point P as shown in the smith chart. It gives the position of normalized load impedance.

(4) We join the point P and the centre O to form the line OP

(5) Extend the line OP to meet the $r = 0$ circle at Q. The magnitude of the reflection coefficient of the transmission line is given as

$$|\Gamma| = \frac{OP}{OQ}$$

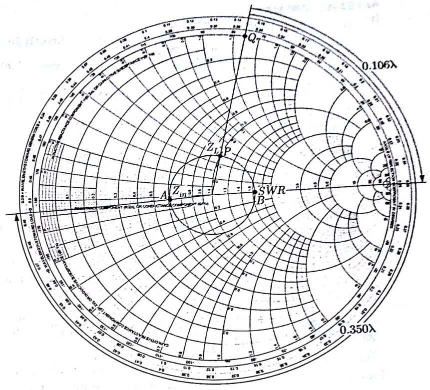
$$= \frac{2.1\text{ cm}}{9.4\text{ cm}} = 0.22$$

(6) Angle of the reflection coefficient in degrees is read out from the scale at point Q as

$$\theta_\Gamma = 76.0^\circ$$

(7) Thus, we get the reflection coefficient of the transmission line as

$$\Gamma = |\Gamma| \underline{/\theta_\Gamma} = 0.22e^{j76^\circ}$$



ALTERNATIVE METHOD :

Reflection coefficient of the transmission line is defined as

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100 + j50 - 100}{100 + j50 + 100}$$

$$= 0.24\underline{/76^\circ} = 0.24e^{j76^\circ}$$

which is same as calculated from smith chart.

SOL 8.1.30 Option (C) is correct.

As shown in the smith chart in previous question normalized load impedance is located at point P. So, for determining the input impedance at a distance of 0.35λ from the load we follow the steps as explained below :

(1) First we draw a SWR circle (circle centered at origin with radius OP)

(2) For finding input impedance at a distance of 0.35λ from load we move a distance of 0.35λ on WTG scale (wave length toward generator) along the SWR circle.

(3) Since, the line OP corresponds to the reading of 0.144λ on WTG scale so, after moving a distance of 0.35λ on WTG scale we reach at $0.144\lambda + 0.35\lambda = 0.494\lambda$ on WTG scale. The reading corresponds to the point A on the SWR circle.

(4) Taking the values of r and x-circle at point A we find out normalized input impedance as

$$z_{in} = r + jx = 0.61 + j(-0.022) = 0.61 - j0.022$$

(5) Therefore, the input impedance at a distance of 0.35λ from load is given as

$$Z_{in} = z_{in} Z_0$$

$$= 100(0.61 - j0.022) = (61 - j2.2)\,\Omega$$

ALTERNATIVE METHOD :

We can conclude the input impedance at $l = 0.35\lambda$ directly by using formula

$$Z_{in} = Z_0\left(\frac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l}\right) \qquad \text{lossless transmission line}$$

$$= 100\left[\frac{100 + j50 + j100\tan\left(\frac{2\pi}{\lambda}0.35\lambda\right)}{100 + j(100 + j50)\tan\left(\frac{2\pi}{\lambda}0.35\lambda\right)}\right]$$

$$= (61 - j2.2)\,\Omega$$

as calculated above using with chart.

SOL 8.1.31 Option (C) is correct.

For determining the shortest length of the transmission line for which the input impedance appears to be purely resistive, we follow the steps as explained below :

(1) First we determine the WTG reading of the point denoting the normalized load impedance on the smith chart. From the above question, we have the reading of point P as 0.144λ on WTG circle.

(2) Since, the resistive load lies on the real axis of reflection coefficient (Γ_r-axis). So, we move along the SWR circle to reach the Γ_r-axis and denote the points as A and B.

(4) Since, point B is nearer to the point P so, it will give the shortest length of the transmission line for which the input impedance appears to be purely resistive.

(5) Now, we have the reading of point B on WTG scale as 0.25λ. So, the shortest length for the input impedance to be purely resistive is given as the difference between the readings at point B and P. i.e.,

$$l = 0.25\lambda - 0.144\lambda$$
$$= 0.106\lambda$$

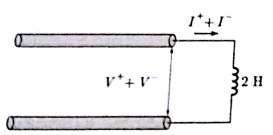
SOL 8.1.32 Option (A) correct.

The voltage maximum occurs at the point where the SWR circle intersects the positive Γ_r axis on smith chart. The SWR circle of the load impedance intersects the positive Γ_r axis at point B as shown in the Smith chart. So, the point B gives the position of first voltage maxima.

As calculated in previous question the distance between point B and point A on the WTG scale is 0.106λ. Therefore, the 1[st] voltage maximum occurs at a distance of 0.106λ from load.

SOL 8.1.33 Option (B) is correct.

At any time t, the currents of positive and negative waves are respectively I^+ and I^- and the voltages of positive and negative waves are respectively V^+ and V^- as shown in the figure.



$$I^+ = \frac{V^+}{Z_0} = \frac{1}{Z_0} \qquad (V^+ = 1\,\text{Volt})$$

and $\quad I^- = -\dfrac{V^-}{Z_0}$

Now, the voltage and current across an inductor are related as

$$v = L\frac{di}{dt}$$

$$V^+ + V^- = 2\frac{d}{dt}(I^+ + I^-)$$

$$1 + V^- = 2\frac{d}{dt}\left[\frac{1 - V^-}{50}\right] \qquad (V^+ = 1, Z_0 = 50)$$

$$1 + V^- = -\frac{1}{25}\frac{dV^-}{dt}$$

$$-25\,dt = \frac{dV^-}{1 + V^-}$$

Taking integration both sides we get

$$\ln(1 + V^-) = -25t + C_1 \qquad \text{where } C_1 \text{ is a constant}$$

$$(1 + V^-) = Ae^{-25t} \qquad (1)$$

Since, the voltage (V^+) wave is incident at $t = 0$ so, at $t = 0^+$ the current through inductor is zero and therefore, from the property of an inductor at $t = 0^+$ the current through inductor will be also zero.

i.e. $\quad (I^+ + I^-)_{\text{at } t=0^+} = 0$

$$\left[\frac{V^+}{Z_0} - \frac{V^-}{Z_0}\right]_{\text{at } t=0^+} = 0$$

$$\left[\frac{1}{Z_0} - \frac{V_0^-}{Z_0}\right]_{\text{at } t=0^+} = 0 \qquad (V^+ = 1\,\text{Volt})$$

So at $t = 0^+$, $\quad V_0^- = 1$ volt

Putting it in equation (1), we get

$$(1 + 1) = A$$
$$A = 2$$

Thus, the voltage of the reflected wave is

$$V^- = (2e^{-25t} - 1)\,\text{Volt}$$

SOL 8.1.34 Option (A) is correct.

The voltage of positive wave in transmission line is V_0^+. So, at the voltage maxima, magnitude of the voltage is given as

$$|V_s|_{max} = |V_0^+|[1 + \Gamma]$$

and at the point of voltage maxima the current will be minimum and given as

$$|I_s|_{min} = \frac{|V_0^+|}{Z_0}[1 - \Gamma]$$

So, the line impedance at the point of voltage maxima will be

$$Z_{max} = \frac{|V_s|_{max}}{|I_s|_{min}} = Z_0\left(\frac{1 + \Gamma}{1 - \Gamma}\right)$$

$$= Z_0 S \qquad \left(S = \frac{1 + \Gamma}{1 - \Gamma}\right)$$

Now, at the voltage minimum the voltage magnitude is

$$|V_s|_{min} = |V_0^+|[1 - \Gamma]$$

and at the point of voltage minimum current will be maximum and given as,

$$|I_s|_{max} = \frac{|V_0^+|}{Z_0}[1 + \Gamma]$$

and the line impedance at the point will be

$$Z_{min} = \frac{|V_s|_{min}}{|I_s|_{max}} = Z_0\left(\frac{1-\Gamma}{1+\Gamma}\right) = \frac{Z_0}{S} \qquad \left(S = \frac{1+\Gamma}{1-\Gamma}\right)$$

SOL 8.1.35 Option (A) is correct

To determine the required quantity, we note that for a particular line of characteristic impedance Z_0, the product of the line impedances at two positions (two values of d) separated by an odd multiple of $\lambda/4$ is given by

$$\{Z[d]\}\left\{Z\left[d+(2n-1)\frac{\lambda}{4}\right]\right\} = \left\{Z_0\left(\frac{1+\Gamma(d)}{1-\Gamma(d)}\right)\right\}\left\{Z_0\frac{1+\Gamma\left(d+(2n-1)\frac{\lambda}{4}\right)}{1-\Gamma(d+(2n-1)\lambda/4)}\right\}$$

$$= Z_0^2\left[\frac{1+\Gamma(d)}{1-\Gamma(d)}\right]\left[\frac{1+\Gamma(d)e^{-j2\beta(2n-1)\frac{\lambda}{4}}}{1-\Gamma(d)e^{-j2\beta(2n-1)\frac{\lambda}{4}}}\right]$$

$$= Z_0^2\left[\frac{1+\Gamma(d)}{1-\Gamma(d)}\right]\left[\frac{1+\Gamma(d)e^{-j(2n-1)\pi}}{1-\Gamma(d)e^{-j(2n-1)\pi}}\right]$$

$$= Z_0^2\left[\frac{1+\Gamma(d)}{1-\Gamma(d)}\right]\left[\frac{1-\Gamma(d)}{1+\Gamma(d)}\right]$$

$$= Z_0^2$$

As the intrinsic impedance of medium 1 is η_1 and that of medium 3 is η_3 so, for required match, thickness t is $\lambda/4$ and the intrinsic impedance (η_2) of the medium 2 is given as

$$\eta_1\eta_3 = \eta_2^2 \quad \text{or } \eta_2 = \sqrt{\eta_1\eta_3}$$

SOL 8.1.36 Option (B) is correct.

Distance between load and first voltage maxima, $l_{max} = 0.125\lambda$

Characteristics impedance, $Z_0 = 100\,\Omega$

Standing wave ratio, $S = 3$

Position of voltage maxima (l_{max}) in terms of reflection coefficient $|\Gamma|\underline{/\theta_\Gamma}$ is

$$l_{max} = \frac{\theta_\Gamma\lambda}{4\pi} + \frac{n\lambda}{2} \qquad \text{where } n = 0, 1, 2, ...$$

So, for 1st voltage maxima we have $n = 0$ and so, we get the position of first voltage maxima as

$$l_{max} = \frac{\theta_\Gamma\lambda}{4\pi}$$

$$0.125\lambda = \frac{\theta_\Gamma\lambda}{4\pi} \Rightarrow \theta_\Gamma = \frac{\pi}{2}$$

The magnitude of reflection coefficient is defined in terms of SWR as

$$|\Gamma| = \frac{S-1}{S+1} = \frac{3-1}{3+1} = \frac{1}{2}$$

So, the reflection coefficient of the transmission line is

$$\Gamma = |\Gamma|\underline{/\theta_\Gamma} = \frac{1}{2}e^{j\pi/2} = \frac{j}{2}$$

Therefore, the load impedance of the transmission line is given as

$$Z_L = Z_0\left(\frac{1+\Gamma}{1-\Gamma}\right)$$

$$= 100\left(\frac{1+\frac{j}{2}}{1-\frac{j}{2}}\right)$$

$$= (60 + j80)\,\Omega$$

SOL 8.1.37 Option (D) is correct.

Given, the transmission line is terminated by its characteristic impedance i.e.,

$$Z_L = Z_0$$

So, there will be no reflected wave and therefore, the height of the voltage pulse will be given as

$$V_1^+ = \frac{Z_0 V_g}{Z_0 + Z_g} \qquad (Z_g \to \text{internal resistance of generator})$$

$$= \frac{100 \times 15}{100 + 50} = 10 \text{ Volt}$$

As the wave travels in the $+Z$ direction along transmission line at velocity

$$v_p = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{(0.25 \times 10^{-6}) \times (100 \times 10^{-12})}}$$

$$= 2 \times 10^8 \text{ m/s}$$

So, the voltage pulse will reach at $l = 5$ m at time,

$$t_0 = \frac{5}{2 \times 10^8} = 25 \text{ ns}$$

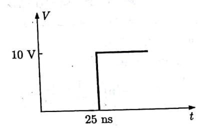
So, at $l = 5$ m for $0 < t < 25$ ns,

$$V = 0$$

and for $t \geq 25$ ns

$$V = V_1^+ = 10 \text{ Volt}$$

Therefore the plot of voltage against time at a distance 5 m from the source is as shown in graph below.



SOL 8.1.38 Option (D) is correct.

As the first forward voltage pulse is V_1^+ so, the first reflected pulse voltage is

$$V_1^- = \Gamma_L V_1^+$$

The 2nd forward pulse voltage is given as

$$V_2^+ = \Gamma_g V_1^- = \Gamma_g \Gamma_L V_1^+$$

The 2nd reflected pulse voltage is given as

$$V_2^- = \Gamma_L V_2^+ = \Gamma_g \Gamma_L^2 V_1^+$$

So, summing up all the pulses at load end for steady state ($t \to \infty$) we get the load voltage as

$$V_L = V_1^+ + V_1^- + V_2^+ + V_2^- + ...$$

$$= V_1^+[1 + \Gamma_L + \Gamma_g\Gamma_L + \Gamma_g\Gamma_L^2 + ...]$$

$$= V_1^+[(1 + \Gamma_g\Gamma_L + \Gamma_g^2\Gamma_L^2 + ...) + \Gamma_L(1 + \Gamma_g\Gamma_L + ...)]$$

$$= V_1^+\left[\left(\frac{1}{1-\Gamma_g\Gamma_L}\right) + \left(\frac{\Gamma_L}{1-\Gamma_g\Gamma_L}\right)\right]$$

$$= V_1^+\left(\frac{1+\Gamma_L}{1-\Gamma_g\Gamma_L}\right)$$

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SOLUTIONS 8.2

SOL 8.2.1 Correct answer is 0.788 .

Given,

Inner diameter of coaxial line, $2a = 1\,\text{cm} \Rightarrow a = 0.5 \times 10^{-2}\,\text{m}$

and outer diameter of coaxial line, $2b = 2\,\text{cm} \Rightarrow b = 10^{-2}\,\text{m}$

Permeability of conductor, $\mu_c = 2\mu_0$

Conductivity of conductor, $\sigma_c = 11.6 \times 10^7\,\text{S/m}$

Operating frequency, $f = 4\,\text{GHz} = 4 \times 10^9\,\text{Hz}$

So, the resistance per unit length of transmission line is given as :

$$R' = \frac{1}{2\pi}\sqrt{\frac{\pi f \mu_c}{\sigma_c}}\left(\frac{1}{a} + \frac{1}{b}\right)$$

$$= \frac{1}{2\pi}\sqrt{\frac{\pi \times (4 \times 10^9) \times (2 \times 4\pi \times 10^{-7})}{11.6 \times 10^7}}\left(\frac{1}{0.5 \times 10^{-2}} + \frac{1}{10^{-2}}\right)$$

$$= 0.788\,\Omega/\text{m}$$

SOL 8.2.2 Correct answer is 277.

Inner diameter of coaxial line, $2a = 1.5\,\text{cm} \Rightarrow a = 0.75 \times 10^{-2}\,\text{m}$

Outer diameter of coaxial line, $2b = 3\,\text{cm} \Rightarrow b = 1.5 \times 10^{-2}\,\text{m}$

Permeability of the filled dielectric, $\mu = 2\mu_0$

So, it's inductance per unit length is given as

$$L' = \frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right) = \frac{2 \times (4\pi \times 10^{-7})}{2\pi}\ln\left(\frac{1.5 \times 10^{-2}}{0.75 \times 10^{-2}}\right)$$

$$= 2.77 \times 10^{-7}\,\text{H/m} = 277\,\text{nH/m}$$

SOL 8.2.3 Correct answer is 9.1 .

Given,

Inner radius of the coaxial line, $a = 1/8\,\text{cm} = 1.25 \times 10^{-3}\,\text{m}$

Outer radius of the coaxial line, $b = 1/2\,\text{cm} = 5 \times 10^{-3}\,\text{m}$

Conductivity of dielectric, $\sigma = 2 \times 10^{-3}\,\text{S/m}$

So, the conductance per unit length of the transmission line is given as

$$G' = \frac{2\pi\sigma}{\ln\frac{b}{a}} = \frac{2\pi \times (2 \times 10^{-3})}{\ln\left(\frac{5 \times 10^{-3}}{1.25 \times 10^{-3}}\right)} = 9.1\,\text{mS/m}$$

SOL 8.2.4 Correct answer is 361.

Given,

Inner diameter of coaxial line, $2a = 1\,\text{cm} \Rightarrow a = 0.5 \times 10^{-2}\,\text{m}$

Outer diameter of coaxial line, $2b = 4\,\text{cm} \Rightarrow b = 2 \times 10^{-2}\,\text{m}$

Permittivity of the dielectric, $\varepsilon = 9\varepsilon_0$

So, the capacitance per unit length of the line is given as

$$C' = \frac{2\pi\varepsilon}{\ln\frac{b}{a}} = \frac{2\pi \times 9 \times 8.85 \times 10^{-12}}{\ln\left(\frac{2 \times 10^{-2}}{0.5 \times 10^{-2}}\right)}$$

$$= 3.61 \times 10^{-10}\,\text{F/m} = 361\,\text{pF/m}$$

SOL 8.2.5 Correct answer is 0.9722 .

Given,

Width of strips, $w = 2.4 \times 10^{-2}\,\text{m}$

Conductivity of strips, $\sigma = 1.16 \times 10^8\,\text{S/m}$

Permeability of strips, $\mu = \mu_0$

Operating frequency, $f = 4\,\text{GHz} = 4 \times 10^9\,\text{Hz}$

So, the parameter R' is given as

$$R' = \frac{2}{w}\sqrt{\frac{\pi f \mu}{\sigma}}$$

$$= \frac{2}{2.4 \times 10^{-2}}\sqrt{\frac{\pi \times 4 \times 10^9 \times 4\pi \times 10^{-7}}{1.16 \times 10^8}}$$

$$= 0.9722\,\Omega/\text{m}$$

SOL 8.2.6 Correct answer is 157.

Strips width, $w = 4.8\,\text{cm} = 4.8 \times 10^{-2}\,\text{m}$

Separation between the plates, $d = 0.3\,\text{cm} = 0.3 \times 10^{-2}\,\text{m}$

Permittivity of dielectric, $\mu = 2\mu_0$

So, the inductance per unit length is given as

$$L' = \frac{\mu d}{w} = \frac{2 \times 4\pi \times 10^{-7} \times 0.3 \times 10^{-2}}{4.8 \times 10^{-2}}$$

$$= 1.57 \times 10^{-7}\,\text{H/m} = 157\,\text{nH/m}$$

SOL 8.2.7 Correct answer is 7.22 .

Operating frequency, $f = 1\,\text{GHz} = 10^9\,\text{Hz}$

Conductivity, $\sigma = 6.4 \times 10^7\,\text{S/m}$

Permittivity, $\varepsilon = 6\varepsilon_0$

Axial component of electric field $= E_z$

Transverse component of electric field $= E_y$

So, the ratio of the two components for the transmission line is

$$\frac{E_z}{E_y} = \sqrt{\frac{\omega\varepsilon}{\sigma}} = \sqrt{\frac{2\pi \times 10^9 \times 6\varepsilon_0}{6.4 \times 10^7}} \qquad (\omega = 2\pi f)$$

$$= 7.22 \times 10^{-5}$$

SOL 8.2.8 Correct answer is 0.10 .

The amplitude of voltage wave after travelling a distance l along a transmission line is given as

$$V_1 = V_0 e^{-\alpha l}$$

where V_0 is the amplitude of the source voltage wave

Now, in the given problem, after travelling 20 m distance along the transmission line the voltage wave remains 13% of it's source amplitude. So, we get

$$V_1 = V_0 e^{-\alpha l} = 13\% \text{ of } V_0$$

$$e^{-\alpha(20)} = 0.13 \qquad (l = 20\,\text{m})$$

$$\alpha = 0.10\,\text{NP/m}$$

SOL 8.2.9 Correct answer is 561.

Given the propagation constant of the voltage wave

$$\gamma = \alpha + j\beta = 0.5 + j2.4$$

So, we get the attenuation constant of the wave

$$\alpha = 0.5$$

and phase constant of the wave along the transmission line is

$$\beta = 2.4$$

Since, the amplitude of voltage wave after travelling a distance l along a transmission line is given as

$$V_1 = V_0 e^{-\alpha l}$$

where V_0 is the amplitude of the source voltage wave. Since the amplitude of a voltage wave after travelling a certain distance down a transmission line is reduced by 87% so, for the given transmission line we have

$$V_1 = V_0 e^{-\alpha l} = \left(1 - \frac{87}{100}\right) V_0$$

$$e^{-\alpha l} = 0.13$$

$$l = \frac{1}{\alpha} \ln\left(\frac{1}{0.13}\right) = 4.08 \text{ m}$$

Therefore, the shift in phase angle for the travelled distance is given as

$$\phi = \beta l \left(\frac{360^\circ}{2\pi}\right)$$

$$= (2.4)(4.08)\left(\frac{360^\circ}{2\pi}\right) = 561^\circ$$

SOL 8.2.10 Correct answer is 3.81 .

Operating frequency, $f = 5\text{ GHz} = 5 \times 10^9 \text{ Hz}$

Characteristic impedance, $Z_0 = 80\,\Omega$

Phase constant, $\beta = 1.5 \text{ rad/m}$

So, the inductance per unit length of the transmission line is given as

$$L' = \frac{\beta Z_0}{\omega} = \frac{1.5 \times 80}{2\pi \times 5 \times 10^9} \qquad (\omega = 2\pi f)$$

$$= 3.81 \text{ nH/m}$$

SOL 8.2.11 Correct answer is 0.43 .

The maximum magnitude of voltage wave, $V_{max} = 6$ volt

The minimum magnitude of voltage wave, $V_{min} = 2.4$ volt

So, the standing wave ratio on the transmission line is given as

$$S = \frac{V_{max}}{V_{min}} = \frac{6}{2.4} = 2.5$$

Therefore, the reflection coefficient of the transmission line is evaluated as

$$\Gamma = \frac{S-1}{S+1} = \frac{2.5-1}{2.5+1} = 0.43$$

SOL 8.2.12 Correct answer is 2.1 .

Characteristic impedance, $Z_0 = 25\,\Omega$

Inner radius of the coaxial line, $a = 0.6 \text{ mm} = 0.6 \times 10^{-3}$

Permittivity of insulated material, $\varepsilon = 9\varepsilon_0 \Rightarrow \varepsilon_r = 9$

Now, the characteristic impedance of a lossless coaxial line is given as

$$Z_0 = \frac{60}{\sqrt{\varepsilon_r}} \ln\left(\frac{b}{a}\right)$$

where b is the outer radius of the coaxial line. So, we get

$$25 = \frac{60}{\sqrt{9}} \ln\left(\frac{b}{0.6 \times 10^{-3}}\right)$$

or, $$b = (0.6 \times 10^{-3}) e^{25\sqrt{9}/60}$$

$$= 0.0021 \text{ m} = 2.1 \text{ mm}$$

SOL 8.2.13 Correct answer is 3.65 .

Load impedance, $Z_L = (15 - j25)\,\Omega$

Characteristic impedance $Z_0 = 25\,\Omega$

So, the reflection coefficient of the transmission line is given as

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(15 - j25) - 25}{(15 - j25) + 25}$$

$$= 0.57 e^{-j79.8^\circ}$$

Therefore, the standing wave ratio of the transmission line is determined as

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.57}{1 - 0.57} = 3.65$$

SOL 8.2.14 Correct answer is 40.

Given,

Operating frequency, $f = 2\text{ MHz} = 2 \times 10^6 \text{ Hz}$

So, the angular frequency of voltage wave is

$$\omega = 2\pi f = 4\pi \times 10^6 \text{ rad/sec}$$

When the line is short circuited, input impedance is

$$Z_{in}^{sc} = j\omega L \qquad \text{(Equivalent to 32 nH inductance)}$$

$$= j(4\pi \times 10^6)(32 \times 10^{-9}) = j0.4\,\Omega$$

When the line is open circuited, input impedance is

$$Z_{in}^{oc} = \frac{1}{j\omega C} \qquad \text{(Equivalent to 20 pH capacitance)}$$

$$= \frac{1}{j(4\pi \times 10^6)(20 \times 10^{-12})} = -j3979.9\,\Omega$$

Therefore, the characteristic impedance of the transmission line is given as

$$Z_0 = \sqrt{Z_{in}^{oc} Z_{in}^{sc}}$$

$$= \sqrt{j(0.4)(-j3978.9)}$$

$$= 40\,\Omega$$

SOL 8.2.15 Correct answer is 600.

Given, the length of the transmission lines 1 and 2

$$l_1 = l_2 = \lambda/4$$

So, the input impedance for line 1 is given as :

$$Z_{in1} = \frac{Z_{01}^2}{Z_L} = \frac{(100)^2}{150} = \frac{200}{3}\,\Omega$$

From the shown arrangement of the transmission line it is clear that the effective load for line 2 will be equal to the input impedance of line 1.

i.e. $$Z_L' = Z_{in1} = \frac{200}{3}\,\Omega$$

Therefore, the input impedance for the whole combination is

$$Z_{in} = \frac{Z_{02}^2}{Z_L'} = \frac{(200)^2}{(200)/3} = 600\,\Omega$$

SOL 8.2.16 Correct answer is 1.56 .

The voltage maximum exists at the point where the incident and the reflected voltage wave both are in same phase and the distance of voltage maximum from the load is given as

$$l_{max} = \frac{\theta_\Gamma \lambda}{4\pi} + \frac{n\lambda}{2} \qquad (1)$$

where θ_Γ is phase angle of reflection coefficient, λ is the wavelength of the voltage wave and $n = 0, 1, 2, ...$

Now, the reflection coefficient of a transmission line is given as

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(0.3 - j0.5) - 0.5}{(0.3 - j0.5) + 0.5}$$

$$= \frac{-0.2 - j0.5}{0.8 - j0.5} = 0.57e^{-79.8^\circ}$$

i.e. $\theta_\Gamma = -79.8^\circ$

So, from equation (1) for $n = 0$ we have

$$l_{max} = \frac{\theta_\Gamma \lambda}{4\pi} = \frac{-79.8^\circ \times 4 \times 10^{-2}}{4\pi} \times \frac{\pi}{180^\circ}$$

$$= -0.44 \times 10^{-2}\,\text{m}$$

which is negative (i.e. the point doesn't exist). Therefore, the 1st maximum voltage will exist for $n = 1$ and the distance of the 1st maximum from the load is

i.e. $$l_{max} = \frac{\theta_\Gamma \lambda}{4\pi} + \frac{\lambda}{2} \qquad (n = 1)$$

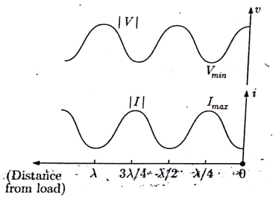
$$= -0.44 \times 10^{-2} + 2 \times 10^{-2}$$

$$= 1.56 \times 10^{-2}\,\text{m}$$

$$= 1.56\,\text{cm}$$

SOL 8.2.17 Correct answer is 0.56 .

In a lossless transmission line, the current maximum lies at the same point where the voltage minima lies and similarly, the current minima lies at the same point where the voltage maxima lies as shown in the figure below :



Now, it is clear from the figure that the distance between two adjacent maxima and minima is $\lambda/4$

i.e. $$l_{max} - l_{min} = \frac{\lambda}{4}$$

Since the maximum voltage wave lies at a distance

$$l_{max} = 1.56\,\text{cm}$$

So, the distance of 1st voltage minimum (the distance of 1st current maxima) from the load will be

$$l_{min} = l_{max} - \frac{\lambda}{4} = 1.56 - \frac{4}{4} = 0.56\,\text{cm}$$

Thus, the distance of 1st current maximum from the load is 0.56 cm.

SOL 8.2.18 Correct answer is 27.

Generator voltage in phasor form, $V_{sg} = 150\,\text{V}$

Internal impedance of generator, $Z_g = 100\,\Omega$

Load impedance $Z_L = 150\,\Omega$

Length of transmission line, $l = 0.15\lambda$

Characteristic impedance $Z_0 = 100\,\Omega$

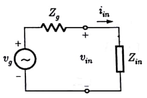
So, the input impedance of the lossless transmission line is given as

$$Z_{in} = Z_0\left(\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right)$$

$$= 100\left[\frac{150 + j100\tan(\frac{2\pi}{\lambda}15\lambda)}{100 + j150\tan(\frac{2\pi}{\lambda}0.15\lambda)}\right]$$

$$= 100\left(\frac{150 + j100\tan 54^\circ}{100 + j50\tan 54^\circ}\right) = (80.5 - j32.7)\,\Omega$$

Now, for determining the power delivered, we draw the equivalent circuit for the transmission line as shown in figure below :



Using voltage division, we get the input voltage as

$$V_{s,in} = V_g\left(\frac{Z_{in}}{Z_{in} + Z_g}\right)$$

$$= 150\left(\frac{82.5 - j32.7}{82.5 - j32.7 + 100}\right) = 71.8e^{-j11.46^\circ}$$

So, the current at the input current is

$$I_{s,in} = \frac{V_{s,in}}{Z_{in}} = \frac{71.8e^{-j11.46^\circ}}{82.5 - j32.7} = 0.81e^{j10.16^\circ}$$

Therefore, the average input power delivered to the transmission line is given as

$$P_{in} = \frac{1}{2}\text{Re}[V_{s,in} I^*_{s,in}]$$

$$= \frac{1}{2}\text{Re}[(71.8e^{-j11.46^\circ})(0.81e^{-j10.16^\circ})]$$

$$= 27\,\text{Watt}$$

SOL 8.2.19 Correct answer is 153.1 .

Since, the lengths of line 1 and line 2 are

$$l_1 = l_2 = \lambda/2$$

So, the input impedance of the line 1 is given as

$$Z_{in1} = Z_0\left(\frac{Z_{L1} + jZ_0\tan\beta l}{Z_0 + jZ_{L1}\tan\beta l}\right)$$

$$= Z_0\left[\frac{Z_{L1} + jZ_0\tan(\frac{2\pi}{\lambda}\frac{\lambda}{2})}{Z_0 + jZ_{L1}\tan(\frac{2\pi}{\lambda}\frac{\lambda}{2})}\right] \qquad (\beta = 2\pi/\lambda)$$

$$= Z_0\left(\frac{Z_{L1} + 0}{Z_0 + 0}\right) = Z_{L1}$$

$$= 150\,\Omega$$

Similarly, the input impedance of line 2 is given as

$$Z_{in2} = Z_{L2} = 150\,\Omega$$

The effective load for line 3 will be equal to the equivalent impedance of the parallel combination of input impedances of line 1 and line 2.

i.e. $$Z_L' = Z_{in1} \,||\, Z_{in2}$$
$$= \frac{150}{2} = 75\,\Omega$$

So, the input impedance for line 3 is given as

$$Z_{in} = Z_L' = 75\,\Omega \qquad \text{(Length of line 3, } l = \lambda/2\text{)}$$

Therefore, the input voltage of line 3 is

$$V_{s,in} = V_{sg}\left(\frac{Z_{in}}{Z_{in} + Z_g}\right) = 500\left(\frac{75}{75 + 100}\right)$$
$$= 214.28 \text{ volt}$$

and so the current at the input terminal of line 3 is

$$I_{s,in} = \frac{V_{s,in}}{Z_{in}} = 2.86 \text{ A}$$

Thus, the average power delivered to the lossless transmission line 3 is given as

$$P_{in} = \text{Re}[V_{s,in} I_{s,in}^*]$$
$$= \frac{1}{2} \times (214.28) \times (2.86)$$
$$= 306.11 \text{ Watt}$$

Since, the transmission line is lossless so, the power delivered to each load will be same and given as

$$P_1 = P_2 = \frac{P_{in}}{2} = \frac{1}{2} \times 306.11 = 153.1 \text{ Watt}$$

SOL 8.2.20 Correct answer is 17.8 .

As discussed in previous question the input impedance of infinitely long lossy transmission line is equal to it's characteristic impedance. So, the input impedance to line 1 will be

$$Z_{in1} = Z_{01} = 200\,\Omega$$

From the shown arrangement of the transmission line it is clear that the effective load impedance for line 2 will be equal to the input impedance of line 1.

i.e. $$Z_{L2} = Z_{in1} = 200\,\Omega$$

Since the length of the line 2 is $\lambda/2$ so, the input impedance of line 2 will be equal to its load

i.e. $$Z_{in2} = Z_{L2} = 200\,\Omega \qquad (l = \lambda/2)$$

Therefore, the reflection coefficient at the load terminal of line 2 is given as

$$\Gamma = \frac{Z_{L2} - Z_{02}}{Z_{L2} + Z_{02}} = \frac{200 - 100}{200 + 100} = \frac{1}{3}$$

Now, the input voltage of line 2 is determined by using voltage division rule as

$$V_{s,in} = V_{s,g}\left(\frac{Z_{in2}}{Z_{in2} + Z_g}\right)$$
$$= 4\left(\frac{200}{200 + 100}\right) = \frac{8}{3} \text{ volt}$$

Again, the voltage at any point on line 2 is given as

$$V_s(z) = V_0^+(e^{j\beta z} + \Gamma e^{-j\beta z}) \qquad \text{(lossless line)}$$

where V_0^+ is voltage of incident wave β is phase constant of the voltage wave and z is distance from load. So, for $z = -\lambda/2$

$$V_s(z) = V_0^+\left(e^{j\frac{2\pi}{\lambda}\left(-\frac{\lambda}{2}\right)} + \Gamma e^{-j\frac{2\pi}{\lambda}\left(-\frac{\lambda}{2}\right)}\right)$$
$$\frac{8}{3} = V_0^+(e^{-j\pi} + \Gamma e^{j\pi}) \qquad (V_s(z) = V_{s,in} \text{ at } z = -\lambda/2)$$
$$V_0^+ = \frac{8}{3} \times \frac{1}{\left(-1 - \frac{1}{3}\right)} \qquad \left(\Gamma = \frac{1}{3}\right)$$
$$= -2 \text{ volt}$$

Therefore, the incident average power to the line 2 is given as

$$P_{av}^i = \frac{|V_0^+|^2}{2Z_{02}} = \frac{4}{2 \times 100} = 20 \text{ mWatt}$$

So, the reflected average power at the input terminal of line 1 (load terminal of line 2) is

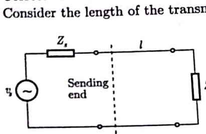
$$P_{av}^r = |\Gamma|^2 P_{av}^i = \left(\frac{1}{3}\right)^2 \times 20 = 2.2 \text{ mWatt}$$

Thus, we get the transmitted power to the line 1 as

$$P_{av}^t = P_{av}^i - P_{av}^r = 20 - 2.2 = 17.8 \text{ mWatt}$$

SOL 8.2.21 Correct answer is 600.

Consider the length of the transmission line l as shown in figure below.



The generator voltage is applied to the transmission line at time $t = 0$ for which the voltage at the sending end is

$$v(0) = 10 \text{ volt} \qquad (\text{at } t = 0)$$

After time $\Delta t = 4\,\mu\text{s}$ the voltage $v(t)$ at the sending end changes to 6 V. This change in the voltage will be caused only if the reflected voltage wave from the load comes to the sending end. So, the time duration for the change in voltage at sending end can be given as

Δt = (time taken by incident wave to reach the load)
+ (time taken by reflected wave to reach sending end from the load)

or, $$\Delta t = \frac{l}{v_p} + \frac{l}{v_p} = \frac{2l}{v_p} \qquad (1)$$

where l is the length of the transmission line (distance between load and sending terminal) and v_p is phase velocity of the wave along the transmission line. Since, the line is air spaced so,

$$v_p = c = 3 \times 10^8 \text{ m/s}$$

Putting it in equation (1) we get

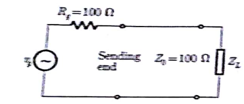
$$4\,\mu\text{s} = \frac{2l}{3 \times 10^8} \qquad (\Delta t = 4\,\mu\text{s})$$

Thus, length of the transmission line is

$$l = \frac{3 \times 10^8 \times 4 \times 10^{-6}}{2} = 600 \text{ m}$$

SOL 8.2.22 Correct answer is 42.86 .

Let the load impedance connected to the transmission line is Z_L so the equivalent circuit for the transmission line will be as shown in figure below :



Since, the internal resistance of the generator is equal to the characteristic impedance of the line

i.e. $$R_g = Z_0 = 100\,\Omega$$

So, the reflection coefficient due to source resistance will be zero and therefore, the change in voltage at sending will be caused only due to the reflection coefficient at load terminal given as

$$\Delta v(t) = \Gamma V_0^+$$

where, V_0^+ is amplitude of the incident voltage wave and Γ is the reflection coefficient at the load terminal. Since, the change in voltage at $t = 4\,\mu s$ is

$$\Delta v(t) = 6 - 10 = -4$$

So, we get $$-4 = 10\Gamma \qquad (V_0^+ = 10\,\text{V})$$

$$\Gamma = -\frac{4}{10} = -0.4$$

$$\left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) = -0.4$$

$$\left(\frac{Z_L - 100}{Z_L + 100}\right) = -0.4 \qquad (Z_0 = 100\,\Omega)$$

$$Z_L - 100 = -0.4Z_L - 40$$

$$Z_L = 42.86\,\Omega$$

SOL 8.2.23 Correct answer is 4.

As determined in previous question, for a wave travelling through the three mediums of intrinsic impedances η_1, η_2 and η_3, the condition for matching dielectric (the intrinsic impedance of medium 2 that eliminates the reflected wave in medium 1) is

$$\eta_2 = \sqrt{\eta_1 \eta_3}$$

Since, all the media have $\mu = \mu_0$ so, for the dielectrics ($\sigma = 0$) the above equation can be rewritten as

$$\sqrt{\frac{\mu_0}{\varepsilon_2}} = \sqrt{\left(\sqrt{\frac{\mu_0}{\varepsilon_0}}\right)\left(\sqrt{\frac{\mu_0}{16\varepsilon_0}}\right)} \qquad \left(\eta = \sqrt{\frac{\mu}{\varepsilon}}\right)$$

where ε_2 is the permittivity of the medium 2.

$$\varepsilon = 4\varepsilon_0$$

So, the relative permittivity of the medium 2 is

$$\varepsilon_r = 4$$

SOL 8.2.24 Correct answer is 2.5 .

The thickness 't' of the dielectric coating for the perfect matching (the condition for eliminating reflection) is given as

$$t = \frac{\lambda}{4} \qquad \text{(quarter wave)}$$

where λ is the wavelength of plane wave. The wavelength in terms of frequency is

$$\lambda = \frac{v_p}{f}$$

where v_p is the phase velocity of the wave in the propagation medium which is given as

$$v_p = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0 4\varepsilon_0}} = \frac{3 \times 10^8}{2} = 1.5 \times 10^8$$

So, at frequency $f = 1.5\,\text{GHz}$ the thickness of the dielectric coating is given as

So, $$t = \frac{v_p}{4f} = \frac{1.5 \times 10^8}{4(1.5 \times 10^9)} = 0.25\,\text{m} = 2.5\,\text{cm}$$

SOL 8.2.25 Correct answer is 60.

Characteristic impedance, $Z_0 = 60\,\Omega$

Load impedance, $Z_L = 180\,\Omega$

Voltage generator, $V_g = 100\,\text{V}$

Internal resistance, $Z_g = 120\,\Omega$

So, the first forward voltage pulse will be

$$V_1^+ = \left(\frac{Z_0}{Z_0 + Z_g}\right)V_g = \left(\frac{60}{60 + 120}\right)100 = \frac{100}{3}\,\text{Volt}$$

The reflection coefficient at load terminal is given as

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{180 - 60}{180 + 60} = \frac{1}{2}$$

The reflection coefficient at source terminal is given as

$$\Gamma_g = \frac{Z_g - Z_0}{Z_g + Z_0} = \frac{120 - 60}{120 + 60} = \frac{1}{3}$$

Therefore, the voltage across the load at steady state is given by the expression as determined in previous question

i.e. $$V_L = V_1^+\left(\frac{1 + \Gamma_L}{1 - \Gamma_g\Gamma_L}\right)$$

$$= \frac{100}{3}\left(\frac{1 + \frac{1}{2}}{1 - (\frac{1}{3})(\frac{1}{2})}\right)$$

$$= \frac{100}{3} \times \frac{3}{2} \times \frac{6}{5} = 60\,\text{Volt}$$

SOL 8.2.26 Correct answer is 0.67 .

Voltage generator, $V_g = 50\,\text{Volt}$

Internal impedance, $Z_g = 30\,\Omega$

Characteristic impedance, $Z_0 = 15\,\Omega$

Load impedance, $Z_L = 45\,\Omega$

So, first forward voltage pulse is

$$V_1^+ = \left(\frac{Z_0}{Z_0 + Z_g}\right)V_g = \left(\frac{15}{15 + 30}\right)50 = \frac{50}{3}$$

Now, the reflection coefficient at source terminal is

$$\Gamma_g = \frac{Z_g - Z_0}{Z_g + Z_0} = \frac{30 - 15}{30 + 15} = \frac{1}{3}$$

and the reflection coefficient at load terminal is

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{45 - 15}{45 + 15} = \frac{1}{2}$$

So, at steady state ($t = \infty$) voltage across load is given as

$$V_L = V_1^+\left(\frac{1+\Gamma_L}{1-\Gamma_g\Gamma_L}\right)$$

$$= \frac{50}{3}\left(\frac{1-\frac{1}{2}}{1-(\frac{1}{2})(\frac{1}{3})}\right)$$

$$= \frac{50}{3} \times \frac{6}{5} \times \frac{3}{2} = 30 \text{ Volt}$$

Therefore, the current through load at steady state is given as

$$I_L = \frac{V_L}{Z_L} = \frac{30}{45} = \frac{2}{3} = 0.67 \text{ A}$$

SOL 8.2.27 Correct answer is 80.

Since, the internal resistance of the battery is zero so, the 1[st] forward voltage pulse is

$$V_1^+ = V_g = 6 \text{ Volt}$$

and from the plot we get the first forward pulse current as

$$I_1^+ = 75 \text{ mA}$$

Therefore, the characteristic impedance of the transmission line is given as

$$Z_0 = \frac{V_1^+}{I_1^+} = \frac{6}{75 \times 10^{-3}} = 80\,\Omega$$

SOL 8.2.28 Correct answer is 262.85 .

Reflection coefficient at source and load end are given as

$$\Gamma_g = \frac{Z_g - Z_0}{Z_g + Z_0} = -1$$

and $$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Now, from the plot of input current (current at generator end) we get,

$$V_1^+ = 75 \text{ mA} \qquad (1)$$

and $$V_1^+ - V_1^- + V_2^+ = -5 \text{ mA} \qquad (2)$$

where, V_1^+ is the first forward voltage pulse, V_1^- is the first reflected voltage pulse and V_2^+ is the second forward voltage pulse. So, putting the values of these voltages in terms of reflection coefficients we get

$$V_1^+ - \Gamma_L V_1^+ + \Gamma_L \Gamma_g V_1^+ = -5 \text{ mA}$$

$$V_1^+(1 - \Gamma_L - \Gamma_L) = -5 \text{ mA} \qquad (\Gamma_g = -1)$$

$$1 - 2\Gamma_L = -\frac{5}{75} \qquad (V_1^+ = 75 \text{ mA})$$

or, $$\Gamma_L = \frac{8}{15}$$

For determining load resistance of the line the reflection coefficient is written in the terms of impedances as

$$\frac{Z_L - Z_0}{Z_L + Z_0} = \frac{8}{15}$$

$$\frac{Z_L - 80}{Z_L + 80} = \frac{8}{15} \qquad (Z_0 = 80\,\Omega \text{ as calculated in previous question})$$

$$Z_L(15 - 8) = 80 \times 8 + 15 \times 80$$

Thus, $$Z_L = 262.85\,\Omega$$

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SOLUTIONS 8.3

SOL 8.3.1 Option (C) is correct.

Characteristic impedance of a transmission line is defined as

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

and the propagation constant of the transmission line is defined as

$$\gamma = \alpha + j\beta = \sqrt{(R' + j\omega C')(G' + j\omega C')}$$

where, α is attenuation constant

β is phase constant

R' is resistance per unit length of the line

G' is conductance per unit length of the line

L' is inductance per unit length of the line

C' is capacitance per unit length of the line

Now, for lossless line, $R' = G' = 0$

So, the characteristic impedance of lossless transmission line is

$$Z_0 = \sqrt{\frac{L'}{C'}}$$

and the propagation constant of lossless transmission line is

$$\gamma = \alpha + j\beta = j\omega\sqrt{L'C'}$$

or $$\alpha = 0$$

Therefore, the attenuation constant of lossless line is always zero (real).

i.e. statement (A) is correct.

Again for distortionless line,

$$\frac{R'}{L'} = \frac{G'}{C'}$$

So, the characteristic impedance of distortionless line is

$$Z_0 = \sqrt{\frac{L'}{C'}} = \sqrt{\frac{R'}{G'}}$$

and the propagation constant of the distortionless line is

$$\gamma = \alpha + j\beta = \sqrt{R'G'} + j\omega\sqrt{L'C'}$$

or, $$\alpha = \sqrt{R'G'} \neq 0$$

Therefore, the attenuation constant of distortion less line is not zero but it is real.

Thus, (A) and (B) is correct statement while (C) is not a correct statement.

SOL 8.3.2 Option (D) is correct.

Load impedance, $Z_L = 0$ (Short circuit)

Input impedance, $Z_{in} = \infty$ (Open circuit)

and, wave length $= \lambda$

Now, the input impedance of lossless transmission line is defined as

$$Z_{in} = Z_0\left(\frac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l}\right)$$

where, l is the length of the transmission line and β is the phase constant of the voltage wave along the transmission line. So, we get

$$\infty = \frac{Z_0(0 + jZ_0\tan\beta l)}{(Z_0 + j0\tan\beta l)}$$

or, $$\infty = jZ_0\tan\beta l$$

Since, for a practical transmission line, $Z_0 \neq \infty$ so, we have

$$\tan\beta l = \infty$$

or, $$\beta l = \pi/2 \qquad \text{(for minimum length)}$$

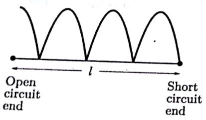
Therefore, the minimum required length of the transmission line is

$$l = \frac{\pi}{2}\times\frac{1}{\beta} = \frac{\pi}{2}\times\frac{\lambda}{2\pi} \qquad \left(\beta = \frac{2\pi}{\lambda}\right)$$

$$= \lambda/4$$

SOL 8.3.3 Option (C) is correct.

Since the transmission line has one short circuited and one open circuited end so at the short circuit end voltage must be zero while at open circuit end voltage must be maximum. So the voltage standing wave pattern will be half sinusoids with zeros at short circuited end and maxima at the open circuited end.



SOL 8.3.4 Option (A) is correct.

Given, the transmission line is terminated in short circuit i.e., $Z_L = 0$ and line should be short circuited at its input terminal i.e. $Z_{in} = 0$.

The input impedance of a lossless transmission line is defined as

$$Z_{in} = Z_0\left(\frac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l}\right)$$

So, $$0 = Z_0\left(\frac{0 + jZ_0\tan\beta l}{Z_0 + 0}\right) \qquad (Z_L = 0, Z_{in} = 0)$$

$$j\tan\beta l = 0$$

$$\beta l = 0,\ \pi,\ 2\pi, \dots\dots\dots$$

Since, length of transmission line cant be zero i.e., $l \neq 0$ so, we get

$$l = \frac{\pi}{\beta} \Rightarrow l = \frac{\pi}{(2\pi/\lambda)} \Rightarrow l = \frac{\lambda}{2}$$

SOL 8.3.5 Option (A) is correct.

SOL 8.3.6 Option (A) is correct.

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SOLUTIONS 8.4

SOL 8.4.1 Option (B) is correct.

Characteristic impedance of a coaxial cable is defined as

$$Z_0 = \sqrt{\frac{\mu}{\varepsilon}}\ln\left(\frac{b}{a}\right)$$

where, $b \rightarrow$ outer cross sectional diameter

$a \rightarrow$ inner cross sectional diameter

So, $$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0\varepsilon_r}}\ln\left(\frac{b}{a}\right)$$

$$= \sqrt{\frac{4\pi\times10^{-7}\times36\pi}{10^{-9}\times10.89}}\ln\left(\frac{2.4}{1}\right)$$

$$= 100\,\Omega$$

SOL 8.4.2 Option (C) is correct.

Since, $$Z_0 = \sqrt{Z_1Z_2}$$

$$100 = \sqrt{50\times200}$$

As this is quarter wave matching so, the length of the transmission line would be odd multiple of $\lambda/4$.

Now, $$l = (2m+1)\frac{\lambda}{4}$$

For $f_1 = 429\,\text{MHz}$, $$l_1 = \frac{c}{f_1\times4} = \frac{3\times10^8}{429\times10^6\times4} = 0.174\,\text{m}$$

For $f_2 = 1\,\text{GHz}$, $$l_2 = \frac{c}{f_2\times4} = \frac{3\times10^8}{1\times10^9\times4} = 0.075\,\text{m}$$

Now, only the length of the line given in option (C) is the odd multiple of both l_1 and l_2 as :

$$(2m+1) = \frac{1.58}{l_1} = 9$$

$$(2m+1) = \frac{1.58}{l_2} \simeq 21$$

Therefore, the length of the line can be approximately 1.58 cm.

SOL 8.4.3 Option (C) is correct.

Length on the transmission line, $d = 2\,\text{mm}$

Operating frequency, $f = 10$ GHz

Phase difference, $\theta = \pi/4$

Since the phase difference between the two points on the line is defined as

$$\theta = \frac{2\pi}{\lambda}d$$

where λ is operating wavelength and d is the distance between the two points. So, we get

$$\frac{\pi}{4} = \frac{2\pi}{\lambda}d$$

or $$\lambda = 8d = 8\times2\,\text{mm} = 16\,\text{mm}$$

Therefore, the phase velocity of the wave is given as

$$v_p = f\lambda = 10 \times 10^9 \times 16 \times 10^{-3}$$
$$= 1.6 \times 10^8 \text{ m/sec}$$

SOL 8.4.4 Option (A) is correct.

Since, voltage maxima is observed at a distance of $\lambda/4$ from the load and we know that the separation between one maxima and minima equals to $\lambda/4$ so voltage minima will be observed at the load.

Now, the input impedance at the point of voltage minima on the line is defined as

$$[Z_{in}]_{min} = \frac{Z_0}{S}$$

where, Z_0 is characteristic impedance and S is the standing wave ratio on the line. Therefore, the load impedance of the transmission line (equal to the input impedance at load) is given as

$$Z_L = [Z_{in}]_{min} = \frac{Z_0}{S} = \frac{50}{5} = 10\,\Omega \qquad (Z_0 = 50\,\Omega,\ S = 5)$$

SOL 8.4.5 Option (C) is correct.

For a lossless network,

$$|S_{11}|^2 + |S_{21}|^2 = 1$$

Since, from the given scattering matrix we have

$$S_{11} = 0.2\angle 0^\circ,\ S_{12} = 0.9\angle 90^\circ$$
$$S_{21} = 0.9\angle 90^\circ,\ S_{22} = 0.1\angle 90^\circ$$

So, we get $\quad (0.2)^2 + (0.9)^2 \neq 1$

Therefore, the two port is not lossless.

Now, for a reciprocal network, $\quad S_{12} = S_{21}$

As for the given scattering matrix we have

$$S_{12} = S_{21} = 0.9\angle 90^\circ$$

Therefore , the two port is reciprocal.

SOL 8.4.6 Option (D) is correct.

For a distortion less transmission line characteristics impedance

$$Z_0 = \sqrt{\frac{R}{G}} \qquad (1)$$

Attenuation constant for distortionless line is

$$\alpha = \sqrt{RG} \qquad (2)$$

So, using equation (1) and (2) we get

$$\alpha = \frac{R}{Z_0} = \frac{0.1}{50} = 0.002$$

SOL 8.4.7 Option (B) is correct.

For a lossless transmission line, the input impedance is defined as

$$Z_{in} = Z_o\left[\frac{Z_L + jZ_o\tan\beta l}{Z_o + jZ_L\tan\beta l}\right]$$

Now, for the quarter wave ($\lambda/4$) line we have

Load impedance, $\quad Z_L = 30\,\Omega$

Characteristic impedance, $\quad Z_o = 30\,\Omega$

Length of the line, $\quad l = \frac{\lambda}{4}$

So, $\quad \tan\beta l = \tan\left(\frac{2\pi}{\lambda}\frac{\lambda}{4}\right) = \infty \qquad \left(\beta = \frac{2\pi}{\lambda}\right)$

Therefore, the input impedance of the quarter wave line is

$$Z_{in1} = Z_o\left[\frac{\frac{Z_L}{\tan\beta l} + jZ_o}{\frac{Z_o}{\tan\beta l} + jZ_L}\right] = \frac{Z_0^2}{Z_L} = 60\,\Omega$$

Now, for $\lambda/8$ transmission line we have

Load impedance, $\quad Z_L = 0\,\Omega \qquad$ (Short Circuit)

Characteristic impedance, $\quad Z_o = 30\,\Omega$

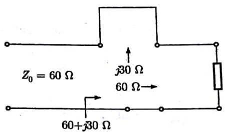
Length of the line, $\quad l = \frac{\lambda}{8}$

So, we get $\quad \tan\beta l = \tan\left(\frac{2\pi}{\lambda}\frac{\lambda}{8}\right) = 1$

Therefore, the input impedance of the $\lambda/8$ transmission line is given as

$$Z_{in2} = jZ_o\tan\beta l = j30$$

The equivalent circuit is shown below :



The effective load impedance of the $60\,\Omega$ transmission line is

$$Z_L = 60 + j30$$

So, the reflection coefficient at the load terminal is

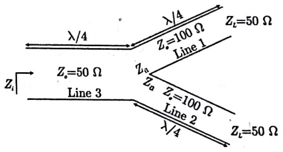
$$\Gamma = \left|\frac{Z_L - Z_o}{Z_L + Z_o}\right| = \left|\frac{60 + j3 - 60}{60 + j3 + 60}\right| = \frac{1}{\sqrt{17}}$$

Therefore, the voltage standing wave ratio of the line is given as

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + \sqrt{17}}{1 - \sqrt{17}} = 1.64$$

SOL 8.4.8 Option (D) is correct.

The transmission line are as shown below. Length of all line is $\frac{\lambda}{4}$



The input impedance of a quarter wave ($\lambda/4$) lossless transmission line is defined as

$$Z_{in} = \frac{Z_0^2}{Z_L}$$

where, Z_o is the characteristic impedance of the line and Z_L is the load impedance of the line. So, for line 1 we have the input impedance as

$$Z_{i1} = \frac{Z_{01}^2}{Z_{L1}} = \frac{100^2}{50} = 200\,\Omega$$

Similarly, for line 2, the input impedance is

$$Z_{i2} = \frac{Z_{02}^2}{Z_{L2}} = \frac{100^2}{50} = 200\,\Omega$$

So, the effective load impedance of line 3 is given as

$$Z_{L3} = Z_{i1} \,||\, Z_{i2} = 200\,\Omega \,||\, 200\,\Omega = 100\,\Omega$$

Therefore, the input impedance of line 3 is

$$Z_i = \frac{Z_0^2}{Z_{L3}} = \frac{50^2}{100} = 25\,\Omega$$

SOL 8.4.9 Option (D) is correct.

The input impedance of the lossless transmission line is defined as

$$Z_{in} = Z_o\left(\frac{Z_L + jZ_o\tan(\beta l)}{Z_o + jZ_L\tan(\beta l)}\right)$$

Since, the given transmission line of characteristic impedance $Z_0 = 75\,\Omega$ is short circuited ($Z_L = 0$) at its one end. Therefore, the input impedance of the line is

$$Z_{in} = jZ_o\tan(\beta l)$$

Now, the operating wavelength of the line is

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{3\times10^9} = 0.1\text{ m or }10\text{ cm} \qquad (f = 3\,\text{GHz})$$

So, $$\beta l = \frac{2\pi}{\lambda}l = \frac{2\pi}{10}\times 1 = \frac{\pi}{5} \qquad (l = 1\,\text{cm})$$

Therefore, $$Z_{in} = jZ_o\tan\frac{\pi}{5}$$

Since, $Z_o\tan(\pi/5)$ is positive so, Z_{in} is inductive.

SOL 8.4.10 Option (C) is correct.

The 2-port scattering parameter matrix is

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22}\end{bmatrix}$$

$$S_{11} = \frac{(Z_L\,||\,Z_0) - Z_o}{(Z_L\,||\,Z_0) + Z_o} = \frac{(50\,||\,50) - 50}{(50\,||\,50) + 50} = -\frac{1}{3}$$

$$S_{12} = S_{21} = \frac{2(Z_L\,||\,Z_o)}{(Z_L\,||\,Z_o) + Z_o} = \frac{2(50\,||\,50)}{(50\,||\,50) + 50} = \frac{2}{3}$$

$$S_{22} = \frac{(Z_L\,||\,Z_o) - Z_o}{(Z_L\,||\,Z_o) + Z_o} = \frac{(50\,||\,50) - 50}{(50\,||\,50) + 50} = -\frac{1}{3}$$

SOL 8.4.11 Option (D) is correct.

The input impedance of a quarter wave ($l = \lambda/4$) lossless transmission line is defined as

$$Z_{in} = \frac{Z_o^2}{Z_L}$$

where, Z_0 is characteristic impedance and Z_L is the load impedance of the line. So, we have the input impedance of line 1 as

$$Z_{in1} = \frac{Z_{01}^2}{Z_{L1}} = \frac{50^2}{100} = 25$$

Similarly, the input impedance of line 2 is

$$Z_{in2} = \frac{Z_{02}^2}{Z_{L2}} = \frac{50^2}{200} = 12.5$$

The effective load impedance of the line 3 is given as

$$Z_L = Z_{in1}\,||\,Z_{in2}$$
$$= 25\,||\,12.5 = \frac{25}{3}$$

So, the input impedance of the $50\,\Omega$ transmission line is

$$Z_S = \frac{(50)^2}{\frac{25}{3}} = 300$$

Therefore, the reflection coefficient at the input terminal is given as

$$\Gamma = \frac{Z_S - Z_0}{Z_S + Z_0} = \frac{300 - 50}{300 + 50} = \frac{5}{7}$$

SOL 8.4.12 Option (A) is correct.

We have $\quad 10\log G_p = 10\text{ dB}$

or $\quad G_p = 10$

The power gain of the antenna is defined as

$$G_p = \frac{P_{rad}}{P_{in}}$$

where P_{rad} is the radiated power of the antenna and P_{in} is the input power feed to the antenna. So, putting all the values we get

$$10 = \frac{P_{rad}}{1\text{W}}$$

or $\quad P_{rad} = 10$ Watts

SOL 8.4.13 Option (D) is correct.

The characteristic impedance of a transmission line is defined as

$$Z_0^2 = Z_{oc}\,Z_{sc}$$

where Z_{oc} and Z_{sc} are input impedance of the open circuited and is short circuited line. So, we get

$$Z_{sc} = \frac{Z_0^2}{Z_{oc}} = \frac{50\times 50}{100 + j150} = \frac{50}{2+3j}$$
$$= \frac{50(2-3j)}{13} = 7.69 - 11.54j$$

SOL 8.4.14 Option (C) is correct.

From the diagram, VSWR is given as

$$S = \frac{V_{max}}{V_{min}} = \frac{4}{1} = 4$$

Since, voltage minima is located at the load terminal so, the load impedance of the transmission line is given as

$$Z_L = [Z_{in}]_{min} = \frac{Z_o}{S} = \frac{50}{4} = 12.5\,\Omega \qquad (Z_0 = 50\,\Omega, S = 4)$$

SOL 8.4.15 Option (A) is correct.

The reflection coefficient at the load terminal is given as

$$\Gamma = \frac{Z_L - Z_O}{Z_L + Z_O} = \frac{12.5 - 50}{125. + 50} = -0.6$$

SOL 8.4.16 Option (C) is correct.

The given circles represent constant reactance circle.

SOL 8.4.17 Option (C) is correct.

The ratio of the load impedance to the input impedance of the transmission line is given as

$$\frac{V_L}{V_{in}} = \frac{Z_0}{Z_{in}}$$

or $$V_L = \frac{Z_0}{Z_{in}} V_{in}$$

$$= \frac{10 \times 300}{50} = 60 \text{ V}$$

SOL 8.4.18 Option (A) is correct.

Suppose at point P impedance is

$$Z = r + j(-1)$$

If we move in constant resistance circle from point P in clockwise direction by an angle $45°$, the reactance magnitude increase. Let us consider a point Q at $45°$ from point P in clockwise direction. It's impedance is

$$Z_1 = r - 0.5j$$

or $$Z_1 = Z + 0.5j$$

Thus movement on constant r - circle by an $\underline{/45°}$ in CW direction is the addition of inductance in series with Z.

SOL 8.4.19 Option (D) is correct.

The VSWR of a transmission line is defined as

$$S = \frac{1 - |\Gamma|}{1 + |\Gamma|}$$

where Γ is the reflection coefficient of the transmission line. So, we get

$$2 = \frac{1 - |\Gamma|}{1 + |\Gamma|} \qquad (S = 2)$$

or $$|\Gamma| = \frac{1}{3}$$

Thus, the ratio of the reflected and incident wave is given as

$$\frac{P_r}{P_i} = |\Gamma|^2 = \frac{1}{9}$$

or $$P_r = \frac{P_i}{9}$$

i.e. 11.11% of incident power is reflected.

SOL 8.4.20 Option (A) is correct.

The input impedance of a lossless transmission line is defined as

$$Z_{in} = Z_o \frac{[Z_L + jZ_o \tan \beta l]}{[Z_o + jZ_L \tan \beta l]}$$

Now, for $\lambda/2$ transmission line we have

$$l = \lambda/2$$

and $$Z_{L1} = 100\,\Omega$$

So, the input impedance of the $\lambda/2$ transmission line is

$$Z_{in1} = Z_o \frac{[Z_{L1} + jZ_o \tan \pi]}{[Z_o + jZ_{L1} \tan \pi]} = Z_{L1} = 100\,\Omega \qquad \left(\beta = \frac{2\pi}{\lambda}\right)$$

For $\lambda/8$ transmission line, we have

$$l = \lambda/8$$

and $$Z_{L2} = 0 \qquad \text{(short circuit)}$$

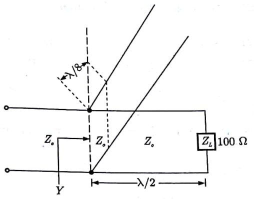
So, the input impedance of $\lambda/8$ line is

$$Z_{in2} = Z_o \frac{[0 + jZ_o \tan \frac{\pi}{4}]}{[Z_o + 0]} = jZ_o = j50\,\Omega \qquad \left(\beta = \frac{2\pi}{\lambda}\right)$$

Thus, the net admittance at the junction of the stub is given as

$$Y = \frac{1}{Z_{in1}} + \frac{1}{Z_{in2}}$$

$$= \frac{1}{100} + \frac{1}{j50} = 0.01 - j0.02$$



SOL 8.4.21 Option (D) is correct.

VSWR (voltage standing wave ratio) of a transmission line is defined as

$$S = \frac{1 + \Gamma}{1 - \Gamma}$$

where Γ is the reflection coefficient of the transmission line that varies from 0 to 1. Therefore, S varies from 1 to ∞.

SOL 8.4.22 Option (B) is correct.

Reactance increases, if we move along clockwise direction in the constant resistance circle.

SOL 8.4.23 Option (C) is correct.

A transmission line is distortion less if $LG = RC$

SOL 8.4.24 Option (B) is correct.

$$Z_o = \sqrt{Z_{OC} Z_{SC}} = \sqrt{100 \times 25} = 10 \times 5 = 50\,\Omega$$

SOL 8.4.25 Option (B) is correct.

We know that distance between two adjacent voltage maxima is equal to $\lambda/2$, where λ is wavelength. So, we get

$$\frac{\lambda}{2} = 27.5 - 12.5$$

or, $$\lambda = 2 \times 15 = 30 \text{ cm}$$

Therefore, the operating frequency of the transmission line is

$$f = \frac{c}{\lambda} = \frac{3 \times 10^{10}}{30} = 1 \text{ GHz} \qquad (c = 3 \times 10^{10} \text{ cm/s})$$

SOL 8.4.26 Option (C) is correct.

Electrical path length $= \beta l$

where $$\beta = \frac{2\pi}{\lambda}, \; l = 50 \text{ cm}$$

Now, the operating wavelength λ of the transmission line is given as

$$\lambda = \frac{v}{f} = \frac{1}{f} \times \frac{1}{\sqrt{LC}} \qquad v = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{25 \times 10^{6}} \times \frac{1}{\sqrt{10 \times 10^{-6} \times 40 \times 10^{-12}}}$$

$$= \frac{5 \times 10^{7}}{25 \times 10^{6}} = 2\text{ m}$$

So, the electric path length is

$$\beta l = \frac{2\pi}{2} \times 50 \times 10^{-2} = \frac{\pi}{2} \text{ radian}$$

SOL 8.4.27 Option (B) is correct.

The input impedance at the voltage minima on the transmission line is defined as

$$[Z_{in}]_{\min} = \frac{Z_0}{S}$$

where S is standing wave ratio along the transmission line. Since, the reflection coefficient Γ_L of the transmission line is given as

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100 - 50}{100 + 50} = \frac{50}{150} = \frac{1}{3}$$

So, the standing wave ratio of the line is

$$S = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} = \frac{1 + \frac{1}{3}}{1 - \frac{1}{3}} = 2$$

Therefore, the minimum input impedance measured on the line is equal to

$$|Z_{in}|_{\min} = \frac{50}{2} = 25\,\Omega$$

SOL 8.4.28 Option (A) is correct.

For a lossy transmission line the input impedance is given as

$$Z_{in} = Z_0\left[\frac{Z_L + jZ_0 \tanh \gamma l}{Z_0 + jZ_L \tanh \gamma l}\right]$$

Load impedance, $Z_L = \infty$ (open circuited at load end)

Length of line, $l = \lambda/4$

So,
$$Z_{in} = Z_0 \lim_{Z_L \to \infty}\left[\frac{1 + \frac{jZ_0 \tanh l\gamma}{Z_L}}{\frac{Z_0}{Z_L} + j\tanh l\gamma}\right]$$

$$= \frac{Z_0}{j\tanh l\gamma} = 0 \qquad \left(\tanh\frac{\gamma\pi}{4} \to \infty\right)$$

SOL 8.4.29 Option (A) is correct.

Input impedance of a lossless transmission line is given by

$$Z_{in} = Z_0\left[\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}\right]$$

where $Z_0 \to$ Charateristic impedance of line

$Z_L \to$ Load impedance

$l \to$ Length of transmission line

and $\beta = 2\pi/\lambda$

So, we have $\beta l = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2}$

$Z_L = 0$ (Short circuited at load)

and $Z_0 = 50\,\Omega$

Therefore, the input impedance of the transmission line is

$$Z_{in} = 50\left[\frac{0 + j50 \tan\frac{\pi}{2}}{50 + j0 \tan\frac{\pi}{2}}\right] = \infty$$

i.e. infinite input impedance and thus, the current drawn from the voltage source will be zero.

SOL 8.4.30 Option (B) is correct.

For lossless transmission line, the phase velocity is defined as

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} \qquad \text{...(1)}$$

Characteristics impedance for a lossless transmission line is given as

$$Z_0 = \sqrt{\frac{L}{C}} \qquad \text{...(2)}$$

So, from equation (1) and (2) we get

$$v_p = \frac{1}{\sqrt{C}\,(Z_0\sqrt{C})} = \frac{1}{Z_0 C}$$

SOL 8.4.31 Option (C) is correct.

Input impedance of a $(\lambda/4)$ transmission line is defined as

$$Z_{in} = \frac{Z_0^2}{Z_L}$$

where Z_0 is characteristic impedance of the line and Z_L is load impedance of the line. Since, the $\lambda/4$ line is shorted at one end (i.e. $Z_L = 0$) So, we get,

$$Z_{in} = \lim_{Z_L \to 0}\frac{Z_0^2}{Z_L} = \infty$$

SOL 8.4.32 Option (A) is correct.

Voltage minima of a short circuited transmission line is located at it's load. As the location of minima is same for the load R_L (i.e. the minima located at R_L) so, the first voltage maxima will be located at $\lambda/4$ distance from load.

Now,
$$l_{\max} = \frac{\theta_\Gamma \lambda}{4\pi} \qquad \text{...(1)}$$

where $l_{\max}$ is the distance of point of maxima from the load, θ_Γ is phase angle of reflection coefficient and λ is operating wavelength of line. So, putting the value of $l_{\max}$ is equation (1), we get

$$\frac{\lambda}{4} = \frac{\theta_\Gamma \lambda}{4\pi}$$

$$\theta_\Gamma = \pi$$

Now, the standing wave ratio of the line is given as

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

or,
$$3 = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} \qquad (S = 3)$$

$$|\Gamma_L| = 1/2$$

i.e.
$$\Gamma_L = |\Gamma_L|\angle\theta_\Gamma = \frac{1}{2}\angle\pi = -\frac{1}{2}$$

The reflection coefficient at the load terminal is given as

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$-\frac{1}{2} = \frac{R_L - 75}{R_L + 75} \qquad (Z_L = R_L),\ (Z_0 = 75\,\Omega)$$

$$-R_L - 75 = 2R_L - 150$$
$$3R_L = 75 \quad \Rightarrow \quad R_L = 25\,\Omega$$

SOL 8.4.33 Option (B) is correct.

The VSWR (voltage standing wave ratio) in terms of maxima and minima voltage is defined as

$$S = \frac{|V_{max}|}{|V_{min}|} = \frac{4}{2} = 2$$

SOL 8.4.34 Option (A) is correct.

Characteristic impedance, $Z_0 = 60\,\Omega$

SWR $S = 4$

So, we have $\frac{1+\Gamma_L}{1-\Gamma_L} = S = 4$

$$\Gamma_L = \frac{3}{5} = 0.6$$

The reflection coefficient at load is defined as

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

So, $$0.6 = \frac{Z_L - 60}{Z_L + 60}$$

$$Z_L = \frac{1.6}{0.4} \times 60 = 240\,\Omega$$

SOL 8.4.35 Option (D) is correct.

Loading of a cable is done to increase the inductance as well as to achieve the distortionless condition.

i.e. statement (1) and (4) are correct.

SOL 8.4.36 Option (C) is correct.

Single stub with adjustable position is the best method for transmission line load matching for a given frequency range.

SOL 8.4.37 Option (B) is correct.

The reflection coefficient at load terminal is defined as

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{+j50 - 50}{j50 + 50}$$
$$= j$$

Therefore, the standing wave, ratio is

$$\text{VSWR} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = \frac{1+1}{1-1} = \infty$$

SOL 8.4.38 Option (A) is correct.

Given, the load impedance is short circuit

i.e. $Z_L = 0$

So, input impedance for lossless line is given as

$$Z_{in} = Z_0\left(\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right) = jZ_0 \tan\beta l$$

Now, for $l < \lambda/4 \Rightarrow \beta l < \frac{\pi}{2}$

So, $\tan\beta l$ is positive and therefore, Z_{in} is inductive a → 2

For $\frac{\lambda}{4} < l < \frac{\lambda}{2} \Rightarrow \frac{\pi}{2} < \beta l < \pi$

$\tan\beta l$ is $-$ve and therefore, Z_{in} is capacitive b → 1

For $l = \frac{\lambda}{4} \Rightarrow \beta l = \frac{\pi}{2}$

$\tan\beta l = \infty$ and therefore $Z_{in} = \infty$ c → 4

For $l = \frac{\lambda}{2} \Rightarrow \beta l = \pi$

$\tan\beta l = 0$ and therefore, $Z_{in} = 0$ d → 3

SOL 8.4.39 Option (C) is correct.

For distortionless transmission line,

$$\alpha = \sqrt{RG}, \quad \beta = \omega\sqrt{LC}$$

and for lossless transmission line,

$$\alpha = 0, \qquad \beta = \omega\sqrt{LC}$$

So, for both the type of transmission line attenuation is constant and is independent of frequency. Where as the phase shift β varies linearly with frequency ω.

i.e. statement 1 and 3 are correct.

SOL 8.4.40 Option (D) is correct.

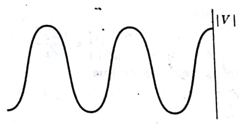
Given,

Length of transmission line, $l = 500$ m

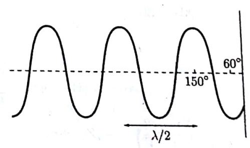
Phase angle, $\underline{/\theta_\Gamma} = -150°$

Operating wavelength, $\lambda = 150$ m

Consider the reflected voltage wave for the lossless transmission line terminated in resistive load as shown in figure.



Since, the reflection coefficient has a phase angle $-150°$ So, the wave lags by 150° angle.



The voltage wave has the successive maxima at each $\lambda/2$ distance,

So, the total no. of maxima $= \frac{\text{Total length}}{\text{Distance between two maxima}}$

$$= \frac{500}{(150/2)} = 6\frac{2}{3}$$

i.e. 6 maxima and remaining phase angle $= \frac{2}{3} \times 360° = 240°$

From the wave pattern shown above we conclude that the remaining phase (240°) will include one more maxima and therefore the total no. of maxima is 7.

SOL 8.4.41 Option (D) is correct.

Reflection coefficient at load terminal is defined as

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

For a matched transmission line we have

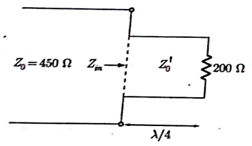
$$Z_L = Z_0$$

So, $\Gamma_L = 0$

i.e. matching eliminated the reflected wave between the source and the matching device location.

SOL 8.4.42 Option (B) is correct.

Consider the quarter wave transformer connected to load has the characteristic impedance Z'_0 as shown in the figure.



So, we have the input impedance,

$$Z_{in} = Z'_0\left[\frac{Z_L + jZ'_0\tan\left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{4}\right)}{Z_0 + jZ_L\tan\left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{4}\right)}\right] = \frac{(Z'_0)^2}{Z_L}$$

This will be the load to 450 Ω transmission line

i.e. $$Z'_L = \frac{(Z'_0)^2}{Z_L} = \frac{(Z'_0)^2}{200}$$

and for matching $Z_0 = Z'_L$

$$450 = \frac{(Z'_0)^2}{200}$$

$$Z'_0 = \sqrt{(450)(200)} = 300\ \Omega$$

SOL 8.4.43 Option (A) is correct.

Given $Z_L = \infty$ (open circuit)

and $l = \frac{\lambda}{4}$ (quarter wave)

$$Z_{in} = Z_0\left(\frac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l}\right) = -jZ_0\cot\beta l \qquad (Z_L \to \infty)$$

$$= -jZ_0\cot\left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{4}\right) = 0$$

SOL 8.4.44 Option (D) is correct.

Length of transmission line $l < \lambda/4$ (open circuit)

Load impedance, $Z_L = \infty$

So, the input impedance of the transmission line is given as

$$Z_{in} = Z_0\left(\frac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l}\right) \qquad \text{(lossless line)}$$

$$= Z_0\left(\frac{1}{j\tan\beta l}\right) \qquad (Z_L = \infty)$$

$$= -jZ_0\cot\beta l$$

SOL 8.4.45 Option (C) is correct.

Given,

Load impedance, $Z_L = 0$ (short circuit)

Line parameters, $R = G = 0$ (loss free line)

Attenuation constant, $\alpha = 0$ (loss free line)

So, the input impedance of the line is given as

$$Z_{in} = jZ_0\tan\beta l$$

i.e. pure reactance

Statement (A) is correct.

Since $\tan\beta l$ can be either positive or negative So Z_{in} can be either capacitive or inductive.

Statement (B) is correct.

The reflection coefficient at load is

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = -1 \neq 0$$

So, the reflection exists.

Statement (C) is incorrect.

and since the standing waves of voltage and current are set up along length of the lines so, statement (D) is also correct.

SOL 8.4.46 Option (C) is correct.

(a) short circuit ($Z_L = 0$)

So $$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = -1 \qquad (a \to 2)$$

(b) Open circuit ($Z_L = \infty$)

So, $$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 1 \qquad (b \to 3)$$

(c) Line characteristic impedance ($Z_L = Z_0$)

$$\Gamma_L = \frac{Z_0 - Z_0}{Z_0 + Z_0} = 0 \qquad (c \to 1)$$

(d) 2 × line characteristic impedance ($Z_L = 2Z_0$)

$$\Gamma_L = \frac{2Z_0 - Z_0}{2Z_0 + Z_0} = \frac{1}{3} \qquad (d \to 4)$$

SOL 8.4.47 Option (D) is correct.

Given, reflection coefficient,

$$\Gamma_L = 1\angle 0°$$

So, $$\text{VSWR} = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+1}{1-1} = \infty$$

SOL 8.4.48 Option (A) is correct.

Given, reflection coefficient, $\Gamma = \frac{1}{5}$

So, $$\text{VSWR} = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{6}{4} = \frac{3}{2}$$

SOL 8.4.49 Option (C) is correct.

Characteristic impedance of transmission line is defined as

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

So, for lossless transmission line ($R = G = 0$)

$$Z_0 = \sqrt{\frac{L}{C}}$$

SOL 8.4.50 Option (C) is correct.

Input impedance has the range from 0 to ∞. (a $\rightarrow$ 3)
VSWR has the range from 1 to ∞ (c $\rightarrow$ 2)
Reflection coefficient (Γ) ranges from -1 to $+1$. (b $\rightarrow$ 1)

SOL 8.4.51 Option (C) is correct.

Input impedance of a quarter wave transformer is defined as

$$Z_{in} = \frac{Z_0^2}{Z_L}$$

where Z_0 is the characteristic impedance of the line and Z_L is the load impedance. Since the quarter wave transformer is terminated by a short circuit ($Z_L = 0$) so, we get the input impedance of the transformer as

$$Z_{in} = \infty$$

SOL 8.4.52 Option (A) is correct.

The scattering parameters linearly relate the reflected wave to incident wave and it is frequency invariant so the scattering parameters are more suited than impedance parameters.

SOL 8.4.53 Option (C) is correct.

Given, the reflection coefficient as

$$\Gamma_L = 0.3e^{-j30^\circ}$$

At any point on the transmission line the reflection coefficient is defined as

$$\Gamma(z) = \Gamma_L e^{-2\gamma z}$$

where z is the distance of point from load.

$$z = 0.1\lambda \qquad \text{(Given)}$$

So,

$$\Gamma(z) = \Gamma_L e^{-2\gamma(0.1\lambda)} = (0.3e^{-j30^\circ})(e^{-2j\beta(0.1\lambda)}) \qquad (\text{Assume } \alpha = 0)$$

$$= 0.3e^{-j30^\circ}(e^{-j0.4\pi})$$

$$= 0.3e^{-j102^\circ} = 0.3e^{j258^\circ}$$

SOL 8.4.54 Option (A) is correct.

Balun is used to couple a coaxial line to a parallel wire.

SOL 8.4.55 Option (B) is correct.

The reflection coefficient of the conducting sheet is $\Gamma = -1$ where as the transmission coefficient is $\Gamma = 0$. So, there will be x-directed surface current on the sheet.

SOL 8.4.56 Option (B) is correct.

Given,

Operating frequency, $f = 25$ kHz
Conductivity, $\sigma = 5$ mho/m
Relative permittivity, $\varepsilon_r = 80$

The attenuation constant for the medium is defined as

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}} \qquad (\sigma >> \omega\varepsilon)$$

$$= \sqrt{\frac{(2\pi \times 25 \times 10^3) \times (4\pi \times 10^{-7})(5)}{2}} \qquad (\mu = \mu_0)$$

$$= 0.7025$$

The attenuated voltage at any point is given as

$$V = V_0 e^{-\alpha l} \qquad (1)$$

where V_0 is source voltage and l is the distance travelled by wave

Since, the radio signal is to be transmitted with 90% attenuation so, the voltage of the signal after 90% attenuation is

$$V = V_0 - 90\% \text{ of } V_0 = 0.1V_0$$

Comparing it with equation (1) we get

$$(0.1) = e^{-\alpha l}$$

or,

$$l = -\frac{\ln(0.1)}{0.7025} = 3.27 \text{ m}$$

SOL 8.4.57 Option (D) is correct.

In Smith chart, the distance towards the load is always measured in anticlockwise direction. So, statement 3 is incorrect while statement 1 and 2 are correct.

SOL 8.4.58 Option (D) is correct.

Given,

Characteristic impedance, $Z_0 = 75\,\Omega$
Load impedance, $Z_L = (100 - j75)\,\Omega$

The condition for matching is

$$Z'_L = Z_0$$

where Z'_L is the equivalent load impedance of the transmission line after connecting an additional circuit. So, the best matching will be obtained by a short circuited stub at some specific distance from load.

SOL 8.4.59 Option (B) is correct.

Given, the voltage standing wave ratio in decibels is

$$VSWR \text{ in decibels} = 6 \text{ dB}$$

or,

$$20\log_{10}S = 6$$

$$S = (10)^{6/20} = 2$$

So, the reflection coefficient at the load terminal is given as

$$\Gamma = \frac{S-1}{S+1} = \frac{2-1}{2+1} = 0.33$$

SOL 8.4.60 Option (A) is correct.

Input impedance of a quarter wave transformer (lossless transmission line) is defined as

$$Z_{in} = \frac{Z_0^2}{Z_L}$$

where Z_0 is the characteristic impedance of the line and Z_L is the load impedance of the line. So, we get

$$Z_0 = \sqrt{Z_{in}Z_L} = \sqrt{(50)(200)} = 100\,\Omega$$

SOL 8.4.61 Option (D) is correct.

(1) Given,

Length of line, $l = \lambda/8$

Load impedance, $Z_L = 0$

So, $\beta l = \left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{8}\right) = \frac{\pi}{4}$

Therefore the input impedance of the lossless transmission line is

$$Z_{in} = Z_0\left(\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right) = Z_0\left(\frac{jZ_0 \tan\frac{\pi}{4}}{Z_0}\right)$$

$$= jZ_0 \qquad \text{(i.e., incorrect statement)}$$

(2) Given,

Length of line, $l = \lambda/4$

Load impedance, $Z_L = 0$

So, $\beta l = \left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{4}\right) = \frac{\pi}{2}$

Therefore the input impedance of the lossless transmission line is

$$Z_{in} = Z_0\left(\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right)$$

$$= Z_0\left(\frac{jZ_0 \tan\frac{\pi}{2}}{Z_0}\right) = j\infty \qquad \text{(i.e., correct statement)}$$

(3) Given,

Length of line, $l = \lambda/2$

Load impedance, $Z_L = \infty$

So, $\beta l = \left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{2}\right) = \pi$

Therefore, the input impedance of the lossless transmission line is

$$Z_{in} = Z_0\left(\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right)$$

$$= Z_0\left(\frac{1}{j\tan\pi}\right) = -j\infty \qquad \text{(i.e., incorrect statement)}$$

(4) Matched line have the load impedance equal to its characteristic impedance

i.e. $Z_L = Z_0$

So, for the matched line the input impedance is

$$Z_{in} = Z_0\left(\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right) = Z_0 \qquad \text{(i.e., correct statement)}$$

SOL 8.4.62 Option (C) is correct.

Given,

Length of line, $l = \lambda/8$

Load impedance, $Z_L = 0$

So, $\beta l = \left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{8}\right) = \frac{\pi}{4}$

Therefore, the input impedance of the transmission line is

$$Z_{in} = Z_0\left(\frac{Z_L + Z_0 \tanh\gamma l}{Z_0 + Z_L \tanh\gamma l}\right) = Z_0 \tanh\gamma l$$

If the line is distortion less (i.e. $\alpha = 0$) then, the input impedance of the line is

$$Z_{in} = jZ_0 \tan\beta l = jZ_0$$

So, it will depend on characteristic impedance as the line is resistive or reactive.

SOL 8.4.63 Option (B) is correct.

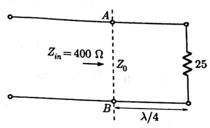
Since, a transmission line of output impedance 400 Ω is to be matched to a load of 25 Ω through a quarter wavelength line. So, for the quarter wave line we have

Input impedance, $Z_{in} = 400\,\Omega$ (same as the o/p impedance of the matched line)

Load impedance, $Z_L = 25\,\Omega$

Length of line, $l = \lambda/4$

The characteristic impedance of quarter wave transmission line is Z_0 that connected between the load and the transmission line of output impedance 400 Ω as shown in figure.



So,, the input impedance at AB is given as

$$Z_{in} = Z_0\left(\frac{Z_L + jZ_0 \tan\frac{\pi}{2}}{Z_0 + jZ_L \tan\frac{\pi}{2}}\right) = \frac{(Z_0)^2}{Z_L}$$

Therefore, $Z_0 = \sqrt{Z_{in} Z_L} = \sqrt{400 \times 25} = 100\,\Omega$

SOL 8.4.64 Option (B) is correct.

Given,

Length of transmission line, $l = \lambda/8$

Load impedance, $Z_L = 0$ (Short circuited line)

So, we get $\beta l = \left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{8}\right) = \frac{\pi}{4}$

Therefore, the input impedance of lossless transmission line is given as

$$Z_{in} = Z_0\left(\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right) = Z_0\left(\frac{jZ_0}{Z_0}\right) = jZ_0 \qquad \text{which is inductive}$$

So, the input impedance of $\lambda/8$ long short-circuited section of a lossless transmission line is inductive.

SOL 8.4.65 Option (C) is correct.

In list I

(a) Characteristic impedance of a transmission line is defined as

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{Z}{Y}} \qquad (a \rightarrow 2)$$

(b) Propagation constant of the line is given as

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{ZY} \qquad (b \rightarrow 1)$$

(c) Sending end input impedance is

$$Z_{in} = Z_0\left(\frac{Z_L + Z_0 \tan\gamma l}{Z_0 + Z_L \tan\gamma l}\right)$$

Given, $Z_L = Z_0$ (terminated in characteristic impedance, Z_0)

So, we get the input impedance as

$$Z_{in} = Z_0 = \sqrt{\frac{Z}{Y}}$$

(c→ 2)

SOL 8.4.66 Option (C) is correct.

For a distortionless transmission line, the attenuation constant (α) must be independent of frequency (ω) and the phase constant (β) should be linear function of ω.

(a) $$R = G = 0$$

For this condition propagation constant is given as

$$\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$$

i.e. $$\alpha = 0 \text{ and } \beta = \omega\sqrt{LC}$$

As the attenuation constant is independent of frequency and the phase constant is linear function of ω so, it is a distortionless transmission line.

(b) $$RC = GL$$

$$\frac{R}{L} = \frac{G}{C}$$

This is the general condition for distortionless line for which

$$\alpha = \sqrt{RG} \text{ and } \beta = \omega\sqrt{LC}$$

(c) $$R >> \omega L,\ G >> \omega C$$

$$\gamma = \alpha + j\beta = \sqrt{RG}$$

i.e. $$\alpha = \sqrt{RG}, \text{ and } \beta = 0$$

Since, β is not function of ω so, it is not the distortionless line.

(d) $$R << \omega L,\ G << \omega C$$

$$\gamma = \alpha + j\beta = \sqrt{(j\omega L)(j\omega C)}$$

i.e. $$\alpha = 0 \text{ and } \beta = \omega\sqrt{LC}$$

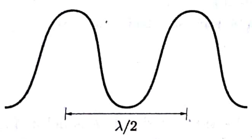
SOL 8.4.67 Option (B) is correct.

Distance between adjacent maxima of an EM wave propagating along a transmission line is $\lambda/2$. So, we get

$$\lambda/2 = (37.5 - 12.5)$$

$$\lambda/2 = 25 \text{ cm}$$

i.e. $$\lambda = 50 \text{ cm}$$



Therefore, the operating frequency of the line is

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{50 \times 10^{-2}}$$

$$= 600 \text{ MHz}$$

SOL 8.4.68 Option (C) is correct.

Forward voltage wave along the transmission line is given as

$$V_0^+ = \frac{Z_0}{Z_0 + Z_0} E = \frac{E}{2}$$

As the transmission line is open circuited at its load terminal ($Z_L = \infty$) so, the reflection coefficient at the load terminal is

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 1$$

Therefore, the voltage travelling in reverse direction is

$$V_0^- = \Gamma_L V_0^+ = \frac{E}{2}$$

The time taken by the wave to travel the distance between source and load terminal is given as

$$t_1 = \frac{l}{c}$$

where l is the length of transmission line and c is velocity of propagating wave. Now, from the plot we observe that at $z = 0$, voltage of the line is $E/2$ where as at $z = l$, voltage is E therefore, it is clear that the voltage wave has been reflected from the load but not reached yet to the generator.

i.e. $$\frac{l}{c} < t < \frac{2l}{c}$$

SOL 8.4.69 Option (C) is correct.

SOL 8.4.70 Option (D) is correct.

The characteristic impedance for a lossy transmission line does not depend on the length of the line.

SOL 8.4.71 Option (C) is correct.

SOL 8.4.72 Option (B) is correct.

A distortionless transmission line has it's parameters related as

$$\frac{R}{L} = \frac{G}{C}$$

or $$RC = GL$$

SOL 8.4.73 Option (B) is correct.

Given the reflection coefficient of the line is

$$\Gamma = 0.6$$

So, the voltage standing wave ratio is defined as

$$SWR = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.6}{1-0.6} = 4$$

SOL 8.4.74 Option (B) is correct.

Characteristic impedance, $Z_0 = 50\,\Omega$

Load impedance, $Z_L = 100\,\Omega$

Forward voltage $V^+ = 10$ V

So, the reflection coefficient of the line is given as

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100 - 50}{100 + 50} = \frac{1}{3}$$

SOL 8.4.75 Option (C) is correct.

Characteristic impedance, $Z_0 = 50\,\Omega$

Load impedance, $Z_L = 15 - j20\,\Omega$

So, the normalized load impedance is given as

$$z_L = \frac{Z_L}{Z_0} = \frac{15}{50} - j\frac{20}{50} = 0.3 - j0.4$$

SOL 8.4.76 Option (A) is correct.

Since both the transmission lines are identical except that the loads connected to them are $2Z$ and $Z/2$ respectively. Let the maximum voltage across the loads be V_m So, the power transmitted to the loads are

$$P_A = \frac{V_m^2}{2Z}$$

and $$P_B = \frac{V_m^2}{Z/2}$$

Given, $$P_A = W_1$$

So, $$V_m^2 = (2Z)\,W_1$$

and $$P_B = \frac{V_m^2}{(Z/2)} = \frac{2ZW_1}{Z/2} = 4\,W_1$$

SOL 8.4.77 Option (D) is correct.

Given, the short circuited and open circuited input impedance as

$$Z_{s.c.} = 36\,\Omega, \quad Z_{o.c.} = 64\,\Omega$$

So, the characteristic impedance of the transmission line is defined as

$$Z_0 = \sqrt{Z_{o.c.}Z_{s.c.}} = \sqrt{36 \times 64} = 48\,\Omega$$

SOL 8.4.78 Option (A) is correct.

(1) $Z_L = Z_0$ (line terminated by its characteristic impedance)

So, reflection coefficient

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = 0$$

i.e. no any reflected wave.

(2) $$Z_L = Z_0$$
$$\Gamma = 0$$

and so, there will be no reflected wave and the wave will have only forward voltage and current wave which will be equal at all the points on the line.

(3) For a lossless half wave transmission line

$$Z_{in} = Z_L$$

So, statement 3 is incorrect while statements 1 and 2 are correct.

SOL 8.4.79 Option (C) is correct.

Since, the standing wave ratio of the wave is 1.

i.e. $$SWR = 1$$

So, expressing it in terms of reflection coefficient, we get

$$\frac{1+|\Gamma|}{1-|\Gamma|} = 1$$

$$|\Gamma| = 0$$

$$\frac{Z_L - Z_0}{Z_L + Z_0} = 0$$

$$Z_L = Z_0$$

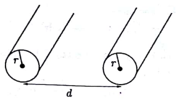
i.e. characteristic impedance is equal to load impedance.

SOL 8.4.80 Option (A) is correct.

Given, the two wire transmission line has

$$\text{Half center to center spacing} = h = \frac{d}{2}$$

$$\text{Conductor radius} = r$$



So, the capacitance per unit length of the line is defined as

$$C = \frac{\pi\varepsilon}{\log_e\left[\left(\frac{d}{2r}\right) + \left(\sqrt{\left(\frac{d}{2r}\right)^2 - 1}\right)\right]}$$

$$= \frac{\pi\varepsilon}{\log_e\left[\frac{h}{r} + \sqrt{\left(\frac{h}{r}\right)^2 - 1}\right]}$$

SOL 8.4.81 Option (D) is correct.

Reflection coefficient, $$\Gamma = \frac{Z_R - Z_0}{Z_R + Z_0}$$

$$= \frac{\frac{Z_0}{3} - Z_0}{\frac{Z_0}{3} + Z_0} = -1/2$$

SOL 8.4.82 Option (B) is correct.

Propagation constant, $$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

The characteristic impedance of the transmission line is given as

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$Z_0 = \frac{R + j\omega L}{\gamma}$$

SOL 8.4.83 Option (D) is correct.

Given the reflection coefficient,

$$\Gamma = -\frac{1}{3}$$

So, the standing wave ratio

$$S = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{4}{2} = 2$$

SOL 8.4.84 Option (B) is correct.

For distortionless transmission line

$$\frac{R}{G} = \frac{L}{C}$$

and so, the attenuation constant,

$$\alpha = \sqrt{RG} = \sqrt{R\left(\frac{RC}{L}\right)} = R\sqrt{\frac{C}{L}}$$

SOL 8.4.85 Option (A) is correct.

Capacitance per unit length, $C = 10^{-10}\,\mathrm{F/m}$

Characteristic impedance, $Z_0 = 50\,\Omega$

Now, for distortionless line the characteristic impedance is given as

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$50 = \sqrt{\frac{L}{10^{-10}}}$$

So, the inductance per unit length is

$$L = (50)^2 \times (10^{-10}) = 0.25\,\mu\text{H/m}$$

SOL 8.4.86 Option (B) is correct.

The characteristic impedance Z_0 in terms of open circuit impedance Z_{oc} and short circuit impedance Z_{sc} is defined as

$$Z_0 = \sqrt{Z_{oc}Z_{sc}} = \sqrt{(100)(100)} \qquad (\text{Given } Z_{oc} = Z_{sc} = 100)$$
$$= 100\,\Omega$$

SOL 8.4.87 Option (B) is correct.

Given,

Load impedance, $Z_L = (75 - j50)\,\Omega$

Characteristic impedance, $Z_0 = 75\,\Omega$

Since, for matching the load impedance is equal to the characteristic impedance (i.e., $Z_L = Z_0$) so, we have to produce an additional impedance of $+j50$ at load to match it with transmission line. Therefore, for matching the transmission line a short circuit stub is connected at some specific distance from load.

SOL 8.4.88 Option (C) is correct.

Given,

The load impedance = Surge impedance

i.e. $Z_L = Z_0$

So, reflection coefficient of the line is given as

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$
$$= 0$$

SOL 8.4.89 Option (C) is correct.

Given,

Length of transmission line, $l = 50\,\text{cm} = 0.5\,\text{m}$

Operating frequency, $f = 30\,\text{MHz} = 30 \times 10^6\,\text{Hz}$

Line parameters, $L = 10\,\mu\text{H/m} = 10 \times 10^{-6}\,\text{H/m}$

and $C = 40\,\text{pF/m} = 40 \times 10^{-12}\,\text{F/m}$

So, the phase constant of the wave along the transmission line is

$$\beta = \omega\sqrt{LC}$$
$$= 2\pi \times 30 \times 10^6 \sqrt{(10 \times 10^{-6})(40 \times 10^{-12})}$$
$$= \frac{6\pi}{5}$$

Therefore, $\beta l = \frac{6\pi}{5} \times 0.5 = 0.6\pi = 108^\circ$

SOL 8.4.90 Option (D) is correct.

Propagation constant in a transmission line is defined as.

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

SOL 8.4.91 Option (D) is correct.

For a series resonant circuit the required conditions are

(1) The angular frequency is

$$\omega = \frac{1}{\sqrt{LC}}$$

(2) The total equivalent impedance is pure resistive

i.e. $Z = R$

Now, the input impedance at a distance $\lambda/4$ from the load is defined as

$$Z_{in} = \frac{Z_0^2}{Z_L}$$

And since the transmission line is open ($Z_L = \infty$)

So, $Z_{in} = 0$ which is purely resistive

i.e. R is correct statement.

In a lossless line voltage or current along the line are not constant.

i.e. A is not a correct statement.

SOL 8.4.92 Option (A) is correct.

The characteristic impedance of a transmission line can be defined as below.

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$
$$Z_0 = \sqrt{Z_{oc}Z_{sc}}$$
$$Z_0 = \frac{V^+}{I^+}$$

So, all the three statements are correct.

SOL 8.4.93 Option (C) is correct.

Given, load impedance of the transmission line is

$$Z_L = 0 \qquad \text{(Short circuit)}$$

So, the input impedance of the lossless line is given as

$$Z_{in} = Z_0\left(\frac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l}\right)$$
$$= Z_0\left(\frac{jZ_0\tan\beta l}{Z_0}\right)$$
$$= jZ_0\tan\beta l$$

SOL 8.4.94 Option (A) is correct.

Given,

Characteristic impedance, $Z_0 = 600\,\Omega$

Load impedance, $Z_L = 900\,\Omega$

So, the reflection coefficient of the transmission line is given as

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$
$$= \frac{900 - 600}{900 + 600} = 0.2$$

SOL 8.4.95 Option (B) is correct.

The propagation constant of a transmission line is defined as

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

and the characteristic impedance is defined as

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$
$$Z_0 = \frac{\gamma}{G + j\omega C} = \frac{R + j\omega L}{\gamma}$$

SOL 8.4.96 Option (D) is correct.

Characteristic impedance of a lossless transmission line is defined as

$$Z_0 = \sqrt{Z_{oc}Z_{sc}}$$

where Z_{oc} is open circuit impedance and Z_{sc} is short circuit impedance.

So, $$Z_0 = \sqrt{(100)(100)} = 100\,\Omega$$

SOL 8.4.97 Option (B) is correct.

The range of standing wave ratio S and reflection coefficient Γ is defined as

$$|\Gamma| \leq 1$$

or $$-1 \leq \Gamma \leq 1 \text{ and } 1 \leq S \leq \infty$$

SOL 8.4.98 Option (C) is correct.

Characteristic impedance of a transmission line is defined as

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

So, Z_0 can increase with increase in resistance or inductance per unit length.

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