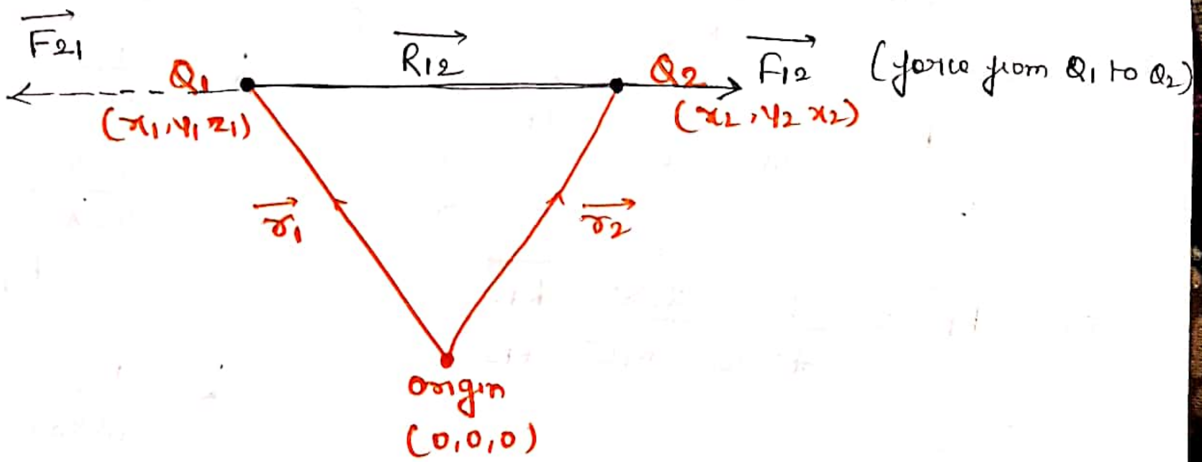
Chapter - 01

Electrostatics & Magnetostatics

* Coulomb's law :- It states that " force exerted between two point charges is - d

1) Along the line joining between them
2) Directly proportional to product of point charges
3) Inversely proportional to square of distance between them



$$\vec{r_1} = x_1 \hat{a_x} + y_1 \hat{a_y} + z_1 \hat{a_z}$$

$$\vec{r_2} = x_2 \hat{a_x} + y_2 \hat{a_y} + z_2 \hat{a_z}$$

$\vec{F_{12}}$ = Force on Q_2

$\vec{R_{12}}$ = Displacement Vector between Q_2 & Q_2

Displacement Vector ⟶ Displacement from one point to another point

Position Vector ⟶ Displacement from origin to a point.

$$|\vec{R_{12}}| = |\vec{r_2} - \vec{r_1}| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

$$\vec{R_{12}} = \vec{r_2} - \vec{r_1} = (x_2-x_1)\hat{a_x} + (y_2-y_1)\hat{a_y} + (z_2-z_1)\hat{a_z}$$

Force on charge Q_2 due to Q_1

$$\vec{F}_{12} \propto Q_1 Q_2$$

$$\vec{F}_{12} \propto \frac{1}{|\vec{R}_{12}|^2}$$

So,

$$\vec{F}_{12} \propto \frac{Q_1 Q_2}{R_{12}^2} \hat{a}_{R12}$$

$$\vec{F}_{12} = k \frac{Q_1 Q_2}{R_{12}^2} \hat{a}_{R12}$$

$$\boxed{\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{R_{12}^2} \hat{a}_{R12}}$$

$$\left\{ \begin{array}{l} k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \\ \epsilon_0 = \frac{10^{-9}}{36\pi} \text{ or } 8.854 \times 10^{-12} \text{ F/m} \end{array} \right.$$

or $$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{R_{12}^2} \frac{\vec{R}_{12}}{R_{12}}$$

$$\boxed{\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2 \vec{R}_{12}}{R_{12}^3}}$$

$$\left\{ \begin{array}{l} k = \frac{1}{4\pi\epsilon_0 \epsilon_r} \\ \text{for dielectric Medium} \end{array} \right.$$

or $$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2 \left[(x_2 - x_1)\hat{a}_x + (y_2 - y_1)\hat{a}_y + (z_2 - z_1)\hat{a}_z\right]}{\left[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2\right]^{3/2}}$$

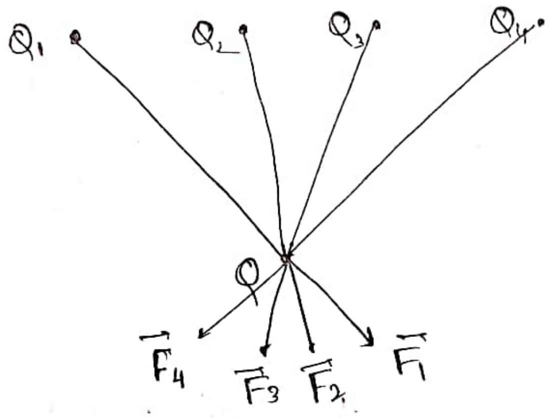
Force on charge Q_1 due to Q_2

$$\vec{F}_{21} = |\vec{F}_{21}| \hat{a}_{R21}$$

$$= |\vec{F}_{12}| (-\hat{a}_{R12})$$

$$= -|\vec{F}_{12}| \hat{a}_{R12}$$

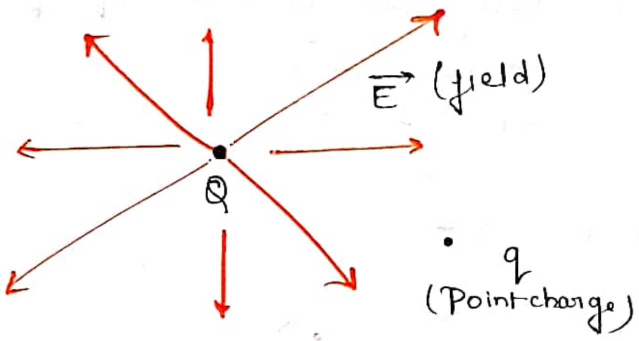
$$\vec{F}_{21} = -\vec{F}_{12}$$



Coulomb's Law obeys Superposition theorem

$$\vec{F_{net}} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4}$$

<u>Electric field Intensity</u>



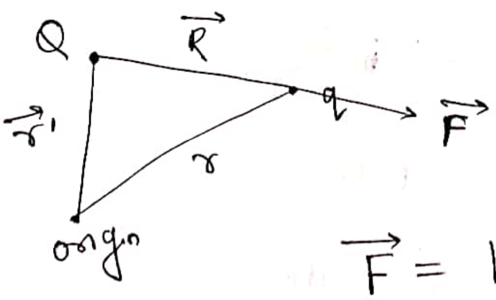
Force exerted on point charge q because of field.

$$\vec{F} = q\vec{E}$$

Electric field Intensity

$$\vec{E} = \frac{\vec{F}}{q} = \text{force per unit charge}$$

$$\frac{\text{Newton}}{\text{Coulomb}} = \frac{\text{Volt}}{\text{m}}$$



$$\vec{F} = k\frac{Qq}{R^2}\hat{a}_R$$

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$$\boxed{\vec{E} = \frac{\vec{F}}{q} = \frac{kQ\hat{a}_R}{R^2}}$$

or $$\vec{E} = \frac{kQ}{R^3}\vec{R}$$

$$\boxed{\vec{E} = \frac{kQ}{R^3}(\vec{r} - \vec{r}')}$$ (force direction is also field direction)

Electric field Intensity also follows superposition theorem

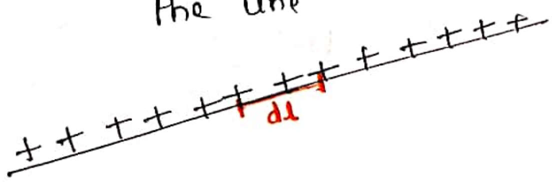
$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4$$

* Charge Distribution :-

1) point charge $\cdot Q$

$$\boxed{\vec{E} = \frac{kQ}{R^2}\hat{a}_R}$$

ii) line charge :- Charge distributed uniformly along the line



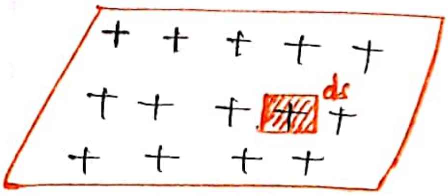
line charge density (ρ_L) in (C/m)

$$dQ = \rho_L\, dl$$

$$Q = \int_l \rho_l\, dl$$

$$\boxed{\vec{E} = \int_l \frac{k \rho_L dl}{R^2} \hat{a}_R}$$

III) Surface charge



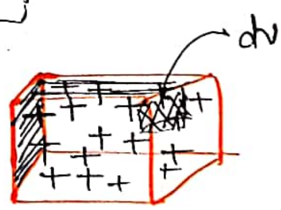
Surface charge density ρ_s in C/m^2

$$dQ = \rho_s \, ds$$

$$Q = \int_s \rho_s \, ds$$

$$\boxed{\vec{E} = \int_s \frac{k \rho_s ds}{R^2} \hat{a}_R}$$

IV) Volume charge :



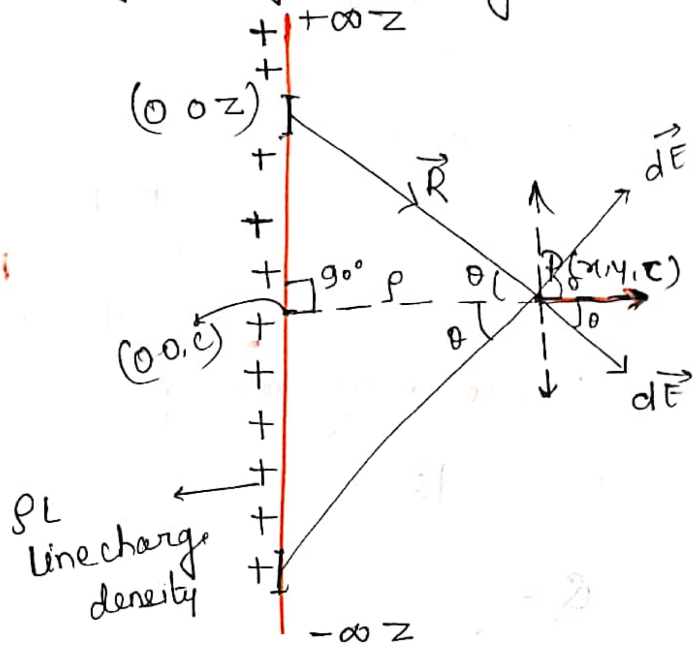
Volume charge density ρ_v in C/m^3

$$dQ = \rho_v \, dv$$

$$Q = \int_V \rho_v \, dv$$

$$\boxed{\vec{E} = \int_V \frac{k \rho_v dv}{R^2} \hat{a}_R}$$

* Electric field Intensity due to Infinite Long line charge



$$\vec{dE} = \frac{k\rho_L \, dl \, \hat{a_R}}{R^2}$$

$$\vec{E} = \int_l \frac{k\rho_L \, dl \, \hat{a_R}}{R^2}$$

$$\Rightarrow \vec{E} = \int_l \frac{k\rho_L \, dl}{R^3} \vec{R}$$

$$\vec{R} = x\hat{a_x} + y\hat{a_y} + (c - z)\hat{a_z}$$

$$R = |\vec{R}| = \sqrt{x^2 + y^2 + (c-z)^2}$$

$$\vec{E} = \int_l \frac{k\rho_L \, dz \, [x\hat{a_x} + y\hat{a_y} + (c-z)\hat{a_z}]}{(x^2 + y^2 + (c-z)^2)^{3/2}}$$

Due to Symmetry, z Component will Cancel each other

$$E = \int_{-\infty}^{\infty} \frac{k\rho_L \, dz \, [x\hat{a_x} + y\hat{a_y}]}{[x^2 + y^2 + (c-z)^2]^{3/2}}$$

let $(c - z) = \sqrt{x^2+y^2} \tan\theta \qquad \Rightarrow -dz = \sqrt{x^2+y^2} \sec^2\theta \, d\theta$

$$E = -\int_{+\pi/2}^{-\pi/2} \frac{k\rho_L [x\hat{a_x} + y\hat{a_y}]}{[x^2+y^2+(x^2+y^2)\tan^2\theta]^{3/2}} \sec^2\theta \sqrt{x^2+y^2} \, d\theta$$

$$\Rightarrow -\int_{-\pi/2}^{\pi/2} \frac{k\,\rho_L\,[x\hat{a_x} + y\hat{a_y}]}{\sec\theta\,(x^2+y^2)}\,d\theta$$

$$\Rightarrow -k\,\rho_L\,\frac{x\hat{a_x} + y\hat{a_y}}{x^2+y^2}\,(\sin\theta)_{-\pi/2}^{\pi/2}$$

$$\Rightarrow 2k\,\rho_L\,\frac{x\hat{a_x} + y\hat{a_y}}{x^2+y^2}$$

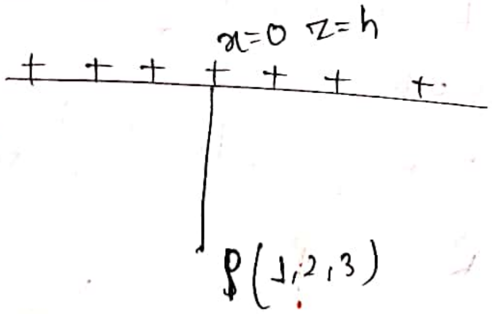
$$\Rightarrow \frac{1}{4\pi\epsilon_0} \times 2 \times \rho_L\,\frac{(x\hat{a_x} + y\hat{a_y})}{x^2+y^2}$$

$$= \frac{\rho_L}{2\pi\epsilon_0}\,\frac{x\hat{a_x} + y\hat{a_y}}{(x^2+y^2)}$$

$$\vec{\rho} = x\hat{a_x} + y\hat{a_y} + (c-c)\hat{a_z}$$
$$= x\hat{a_x} + y\hat{a_y}$$
$$|\vec{\rho}| = \rho = \sqrt{x^2+y^2}$$

$$\boxed{\vec{E} = \frac{\rho_L}{2\pi\epsilon_0\,\rho}\,\hat{a_\rho}}$$

eg. ρ_L



$$\vec{\rho} = (1-0)\hat{a_x} + (2-2)\hat{a_y} + (3-h)\hat{a_z}$$

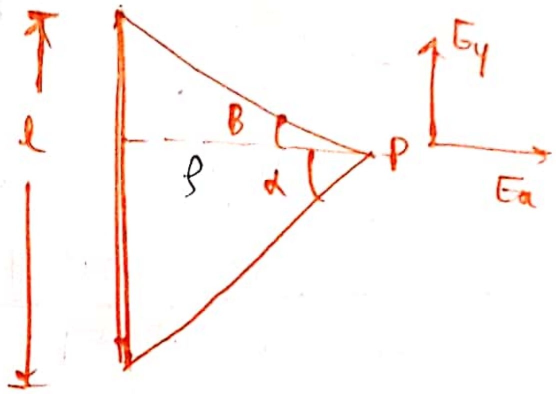
$$\vec{\rho} = \hat{a_x} + (3-h)\hat{a_z}$$

* For finite length

$$E_x = \frac{k\rho_L}{\rho}(\sin\beta + \sin\alpha)$$

$$E_y = \frac{k\rho_L}{\rho}(\cos\beta - \cos\alpha)$$

$$E = \sqrt{E_x^2 + E_y^2}$$

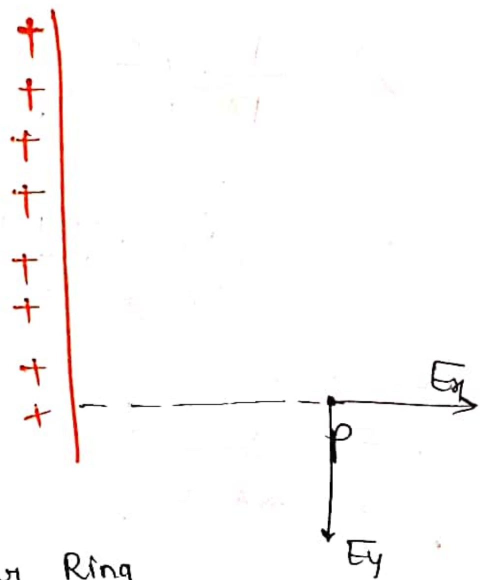


* for Infinite long line with point at $-\infty$

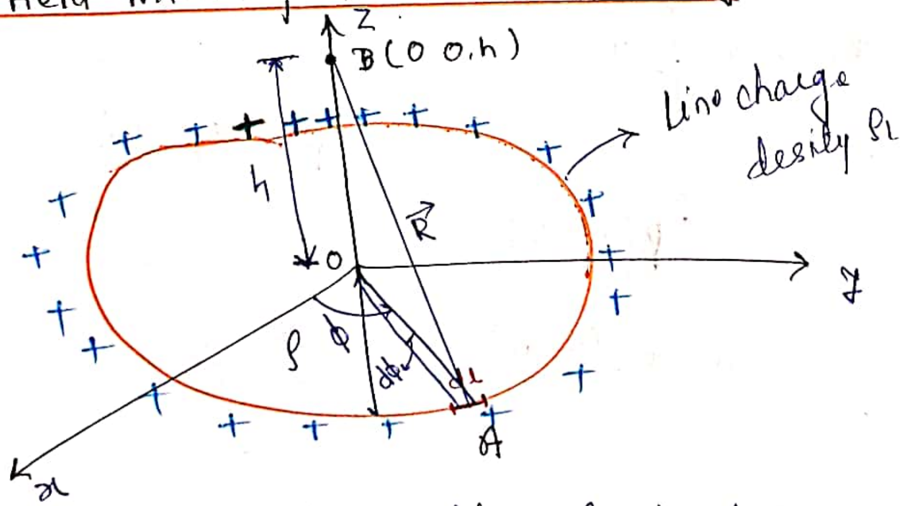
$$|E_x| = |E_y| = \frac{k\rho_L}{\rho}$$

$$\vec{E} = \sqrt{E_x^2 + E_y^2}$$

$$\boxed{E = \frac{\sqrt{2}\,k\rho_L}{\rho}}$$



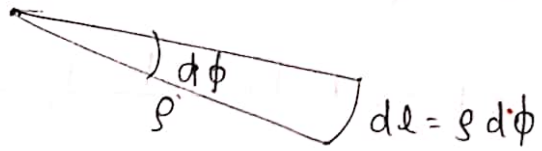
* Electric Field Intensity due to Circular Ring



ρ = radius of Circular loop

$$\vec{E} = \oint_l \frac{k\,\rho_L\,dl}{R^2}\,\hat{a}_R$$

$$\vec{E} = \oint_l \frac{k\,\rho_L\,dl\,\vec{R}}{R^3}$$



$dl = \rho\, d\phi$

$\vec{AB} = \vec{AO} + \vec{OB}$

$\vec{AO} = -\rho\,\hat{a}_\rho$

$\vec{OB} = h\,\hat{a}_z$.

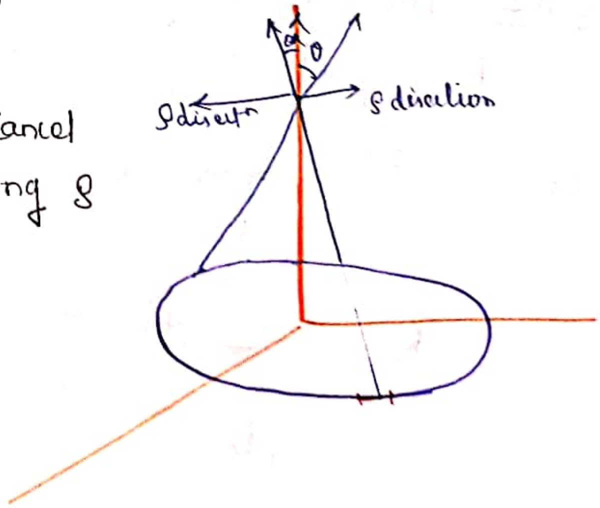
$\left\{ AO = \int_\rho^0 d\rho\,\hat{a}_\rho = -\rho\,\hat{a}_\rho \right.$

$\vec{AB} = -\rho\,\hat{a}_\rho + h\,\hat{a}_z$

$\left\{ \vec{OB} = \int_0^h dz\,\hat{a}_z = (z)_0^h\,\hat{a}_z = h\,\hat{a}_z \right.$

$\Rightarrow \vec{R} = -\rho\,\hat{a}_\rho + h\,\hat{a}_z$

$$\Rightarrow \vec{E} = \oint_l \frac{k\,\rho_L\,\rho\,d\phi\,(-\rho\,\hat{a}_\rho + h\,\hat{a}_z)}{(\rho^2+h^2)^{3/2}}$$



$\Rightarrow$ ρ component of electric field cancel each other so after neglecting ρ

$$\vec{E} = \oint_l \frac{k\,\rho_L\,\rho\,d\phi\,h\,\hat{a}_z}{(\rho^2+h^2)^{3/2}}$$

$$\vec{E} = \int_0^{2\pi} \frac{k\,\rho_L\,\rho\,h\,\hat{a}_z}{(\rho^2+h^2)^{3/2}}\,d\phi$$

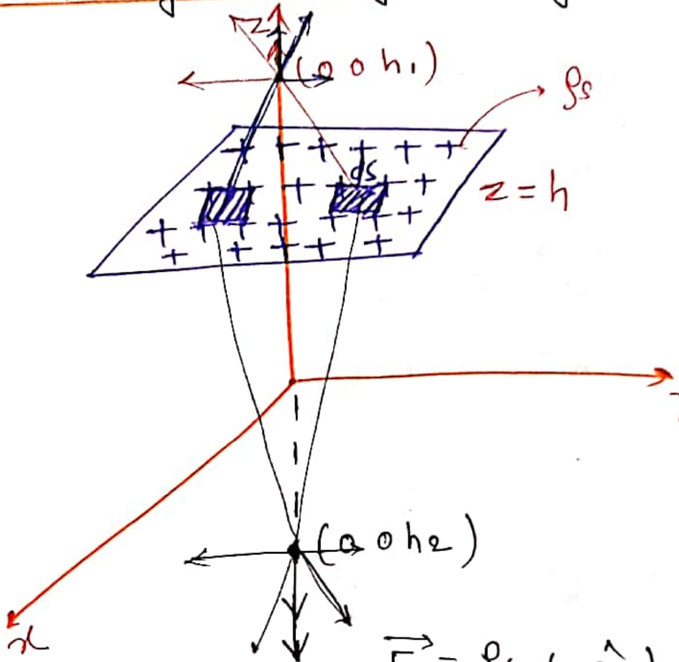
$$\vec{E} = \frac{k\,\rho_L\,\rho\,h\,\hat{a}_z}{(\rho^2+h^2)^{3/2}}\,2\pi$$

$$\boxed{\vec{E} = \frac{\rho_L \rho h \hat{a}_z}{2\varepsilon_0 (\rho^2+h^2)^{3/2}}}$$

Electric field exist only perpendicular (z dirxn) to the circularstrip (placed in xy)

* Electric Field Intensity due to Infinite Surface charge

$$\boxed{\vec{E} = \frac{\rho_s}{2\varepsilon_0} \hat{a}_z}$$



$$\vec{E} = \frac{\rho_s}{2\varepsilon_0}(-\hat{a}_z)$$

Que1. The Electric field at point (1,1,0) due to a point charge of 1 μC located at (-1,1,1) is

Sol
$$\vec{E} = \frac{kQ}{R^2}\hat{a}_R$$

P (1, 1 0)

(0 0 0)

$$\vec{R} = 2\hat{a}_x + 0\hat{a}_y + (0-1)\hat{a}_z$$

Q (-1 1 1)

$$\vec{R} = 2\hat{a}_x - \hat{a}_z$$

$$R = \sqrt{2^2 + ((-1))^2}$$

$$= \sqrt{5}$$

$$\hat{a}_R = \frac{\vec{R}}{|\vec{R}|}$$

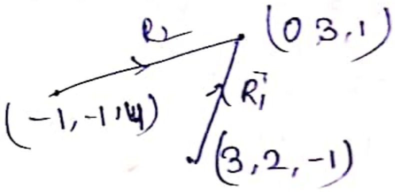
$$\vec{E} = \frac{kQ}{5^{3/2}}(2\hat{a}_x - \hat{a}_z)$$

$$\vec{E} = \frac{10^{-6}}{4\pi\epsilon_0 (\sqrt{5})^{3/0}} (2\hat{a_x} - \hat{a_z})$$

$$= \frac{10^{-6}}{20\pi\epsilon_0 \sqrt{5}} (2\hat{a_x} - \hat{a_z}) \quad //.$$

<u>Que 2</u> point charges of 1mC and -2mC are located at (3, 2, -1) and (-1, -1, 4) respectively. Calculate force on 10nC charge located at (0, 3, 1) and $\vec{E}$ at this point

<u>Sol</u> $$\vec{F} = k \frac{Q_1 Q_2}{R^2} \vec{a_R}$$



$$E_1 = \frac{kQ}{R^3} \vec{a_R}$$

$$= \frac{k\,1\,(-3a_x - \hat{a_y} - 2\hat{a_z})}{[\sqrt{9+1+4}]^3}$$

$$E_1 = \frac{k(-3a_x + \hat{a_y} + 2\hat{a_z})}{(14)^{3/2}} \quad \Big\{ \text{due to } 1mC$$

$$E_2 = \frac{k \times (-2)(+\hat{a_x} + 4\hat{a_y} + 3\hat{a_z})}{(26)^{3/2}} = k(-0.015\hat{a_x} + 0.03\hat{a_y} - 0.022\hat{a_z})$$

$$\vec{E} = E_1 + E_2 = \frac{k\, 1\times10^{-3}(-3a_x + a_y + 2a_z)}{(14)^{3/2}} + \frac{k(-2\times10^{-3})(+a_x + 4a_y + 3\hat{a_z})}{(26)^{3/2}}$$

$$\Rightarrow \cancel{k\times10^{-3}}\Big[-15\cancel{7}\cancel{4}49$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$\vec{E} = 10^{-5} k \left(-6.8\, \hat{a}_x - 1.2\, \hat{a}_y + 6.4\, \hat{a}_z \right)$$

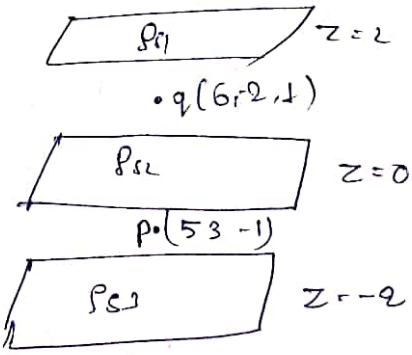
Que. Infinite plane sheet of charge lie in $z = -2$, $z = 0$, $z = 2$ with uniform surface charge density $\rho_{s1}, \rho_{s2}\ \rho_{s3}$. Given that the resultant electric field at $(5, 3, -1)$ $(6, -2, 1)$ and $(4, -7, 8)$ are $0, -2\hat{a}_z, +\hat{a}_z$ respectively. The value of $\rho_{s1}\ \rho_{s2}\ \rho_{s3}$ in term of ϵ_0.

Sol

$$E = \frac{\rho_s}{2\epsilon_0} \hat{a}_z$$

$\cdot (4\ {-7}\ 8)$

$$E_1 = \frac{\rho_{s1}}{2\epsilon_0} \hat{a}_z$$



$\Rightarrow$ At $(5, 3, -1)$

$$E_1 + E_2 + E_3 = 0$$

$$\Rightarrow \cancel{\frac{\rho_{s1}}{2\epsilon_0}} \cancel{\frac{-3a_z}{3}} + \frac{\cancel{\rho_{s2}}}{2\epsilon_0} \cancel{\frac{\hat{a}_z}{1}} + \cancel{\frac{\rho_{s3}}{2\epsilon_0}} \cancel{\frac{-2\hat{a}_z}{2}}$$

$$\Rightarrow \frac{\rho_{s1}}{2\epsilon_0}(-\hat{a}_z) + \frac{\rho_{s2}}{2\epsilon_0}(-\hat{a}_z) + \frac{\rho_{s3}}{2\epsilon_0}\hat{a}_z = 0 \quad \text{——— (1)}$$

At (6, -2, 1)

$E_1 + E_2 + E_3 = -2\hat{a_z}$

$\Rightarrow \frac{\rho_{s1}}{2\varepsilon_0}(-\hat{a_z}) + \frac{\rho_{s2}}{2\varepsilon_0}(\hat{a_z}) + \frac{\rho_{s3}}{2\varepsilon_0}(\hat{a_z}) = -2\hat{a_z}$ —(2)

At (4, -7, 8)

$\frac{\rho_{s1}}{2\varepsilon_0}(\hat{a_z}) + \frac{\rho_{s2}}{2\varepsilon_0}(\hat{a_z}) + \frac{\rho_{s3}}{2\varepsilon_0}(\hat{a_z}) = \hat{a_z}$ —(3)

from (1) $-\rho_{s1} \cancel{+} \rho_{s2} + \rho_{s3} = 0$ —(4)

& $-\rho_{s1} + \rho_{s2} + \rho_{s3} = -4\varepsilon_0$ —(5)

$\rho_{s1} + \rho_{s2} + \rho_{s3} = 2\varepsilon_0$ —(6)

(5)-(6)

$-2\rho_{s1} = -6\varepsilon_0$

$\rho_{s1} = 3\varepsilon_0$

(4)-(5)

$-2\rho_2 = 4\varepsilon_0$

$\rho_2 = -2\varepsilon_0$

(4)+(6)

$2\rho_3 = 2\varepsilon_0$

$\rho_3 = \varepsilon_0$ //

Que3 Plane $x=2$, $y=-3$ carrying charges $10\,nC/m^2$ and $15\,nC/m^2$. If line $x=0, z=2$ carries charge $10\pi\,nC/m$. Calculate electric field at (1, 1, -1) due to three charge distribution

Sol $E_1 = \frac{10}{2\varepsilon_0}(-\hat{a_x})$

$E_2 = \frac{15}{2\varepsilon_0}\hat{a_y}$

} Surface charge

line charge

$\cancel{E} = \int_l \frac{\cancel{k\ \rho_L\ dl}}{\cancel{R^2}}\ \cancel{\hat{a_e}}$

$$E_3 = \frac{\rho_L}{2\pi\epsilon_0 \rho}\hat{a}_\rho$$

$$= \frac{\cancel{10\pi} \quad \cancel{-3\hat{a}_z}}{\cancel{2\pi\epsilon_0} \; \cancel{3}}$$

$$\epsilon_0 = \frac{10^{-9}}{36\pi}$$

$$E_1 = \frac{10\times 10^{-9}}{2\epsilon_0}(-\hat{a}_x) = \frac{10\times \cancel{10^{-9}}}{2\times \frac{\cancel{10^{-9}}}{36\pi}}(-\hat{a}_x)$$

$$= -180\pi\,\hat{a}_x$$

$$E_2 = \frac{15\times 10^{-9}}{2\times \frac{10^{-9}}{36\pi}}\hat{a}_y = 15\times 18\pi\,\hat{a}_y$$

$$= 270\pi\,\hat{a}_y$$

$$E_3 = \frac{\rho_L\,\hat{a}_\rho}{2\pi\epsilon_0\rho} = \frac{\rho_L\,\vec{\rho}}{2\pi\epsilon_0\rho^2}$$

$$\rho = (1-0)\hat{a}_x + (-1-2)\hat{a}_z$$

$$= \hat{a}_x - 3\hat{a}_z$$

$$|\rho| = \sqrt{1+9} = \sqrt{10}$$

$$E_3 = \frac{10\pi\times \cancel{10^{-9}}}{\cancel{2\pi}\,\frac{\cancel{10^{-9}}}{\cancel{36\pi}_{18}}\times 10}(\hat{a}_x - 3\hat{a}_z)$$

$$E_3 \Rightarrow 18\pi\,\hat{a}_x - 54\pi\,\hat{a}_z \quad V/m$$

Total

$E = E_1 + E_2 + E_3$

$= -180\pi \hat{a_x} + 270\pi \hat{a_y} + 18\pi \hat{a_x} - 54\pi \hat{a_z}$

$\Rightarrow -162\pi \hat{a_x} + 270\pi \hat{a_y} - 54\pi \hat{a_z}$ //.

Que 4. A line charge density of 24 nC/m is located in free space on the line y = 1 & z = 2

i) find $\vec{E}$ at (6, -1, 3)

ii) what point charge Q should be located at (-3, 4, 1) to cause y component of Electric field at (6, -1, 3) to be equal to zero. ?

Sol $E = \frac{\rho_L \hat{a_\rho}}{2\pi\epsilon_0 \rho} \Rightarrow \frac{\rho_L \vec{\rho}}{2\pi\epsilon_0 \rho^2}$

$\Rightarrow \frac{24 \times 10^{-9} (-2\hat{a_y} + \hat{a_z})}{2\pi\epsilon_0 \times 5}$

$\Rightarrow \frac{24 \times 36\pi}{10\pi} (-2\hat{a_y} + \hat{a_z})$

$\Rightarrow 86.4(-2\hat{a_y} + \hat{a_z})$

ii) Q (-3 4 1) —— P (6, -1, 3)

$\Rightarrow \vec{E} = \frac{kQ}{R^3}(\vec{R})$

$\cancel{kQ(9\hat{a_x} - 5\hat{a_y} + 2\hat{a_z})}$ $\Rightarrow 0$

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$$\vec{E} = \frac{kQ\,\hat{a}_R}{R^2}$$

$\cdot P(6,-1,3)$

$$\vec{E} = \frac{kQ(9\hat{a_x} - 5\hat{a_y} + 2\hat{a_z})}{(\sqrt{81+25+4})^3}$$

$\cdot Q(-3\ 4\ 1)$

$$\Rightarrow \frac{kQ(9\hat{a_x} - 5\hat{a_y} + 2\hat{a_z})}{(\sqrt{110})^3}$$

$$\vec{E_{y_1}}\ \frac{-5kQ\,\hat{a_y}}{(\sqrt{110})^3}$$

y Comp. due to line charge

$$\vec{E_{y_2}} = -86.4 \times 2\,\hat{a_y}$$

$$E_y = \vec{E_{y_1}} + \vec{E_{y_2}} = \frac{-5\times10^{-9}\times 9 \times Q\,\hat{a_y}}{(\sqrt{110})^3} - 86.4\times2\,\hat{a_y} = 0$$

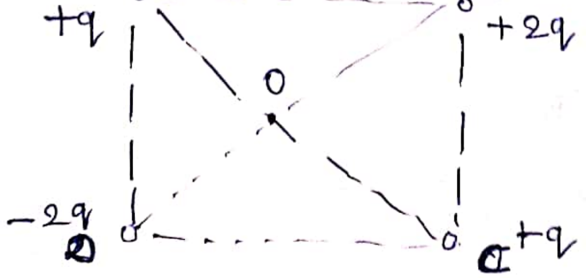
$$\Rightarrow \quad -0.039\,Q = 86.4\times2$$

$$Q = -4430.76\ nC$$

$$= -4.43\ \mu C \quad //$$

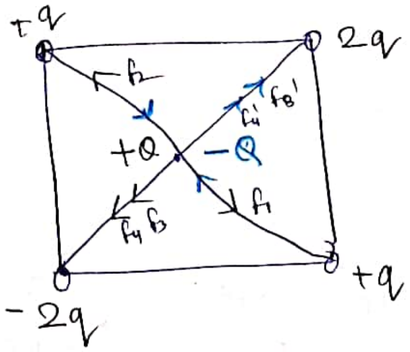
$\{ Q = -4.43\ \mu C$

<u>Que.</u> 4 charges are arranged at the corner of a square ABCD as shown in figure



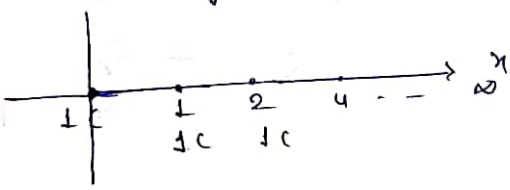
The force on the charge kept at the Centre O is
a) zero b) along the Diagonal AC
c) along the diagonal BD d) Perpendicular to side AB

Sol



Que. An Infinite no. of charges each of charge 1 μC are placed on the x axis with coordinates x = 1, 2, 4, 8 ... ∞
If a charge of 1C is kept at origin the net force (in kN) acting on 1C charge is

Sol



$f = \frac{kq^2}{r^2}$

$= k\left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{8^2} \cdots\right] \times 10^{-6}$

$= k \frac{1}{1 - \frac{1}{2^2}} = k\frac{4}{3} = 1.33 \times 10^{-6}\ k$

$= 1.33 \times 10^{-6} \times 9 \times 10^{9}$

$\Rightarrow 1.33 \times 9 \times 10^{3}$

$\Rightarrow 11.97 \times 10^{3}$

$\approx$ 12 kN //

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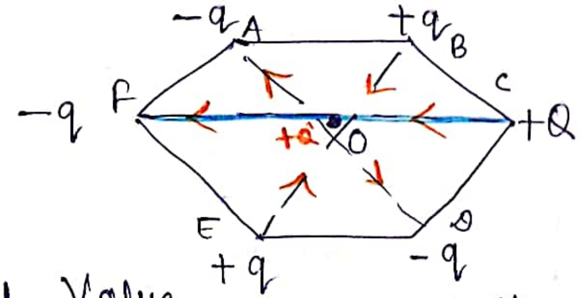
Que. 3 negative point charges $-q$ each and 3 positive point charges $+q$ ~~each~~, $+q$ & $+Q$ are placed at vertices of a Regular Hexagon as shown in fig.



for what value of Q will the electric field at O due to five charges at A B D E & F be twice the electric field at centre O due to charge Q at C alone.

Sol

$$E_{Q0} = \frac{F}{+Q}$$

$$E_0 = \frac{F}{-q}$$

$$\cancel{2}\frac{\cancel{2}F}{\cancel{Q}} = \frac{2\cancel{2}f}{-q}$$

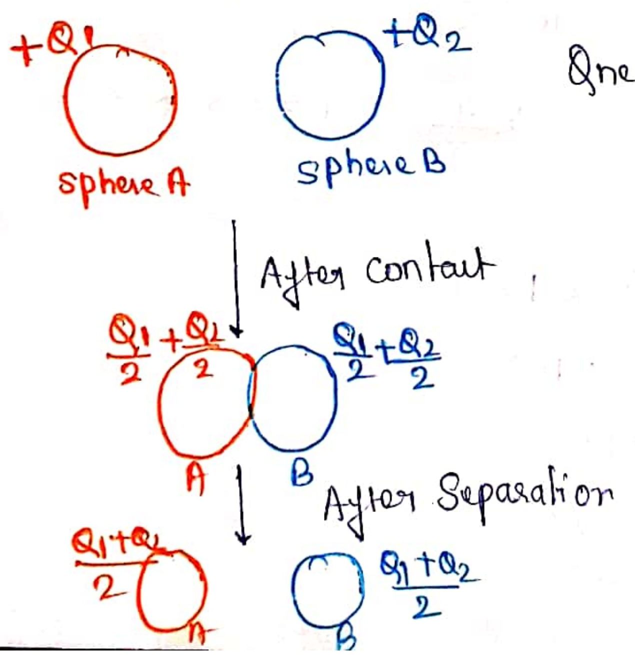
$$\cancel{Q = \frac{q/2}{2}}$$

$2E_{ABDEF}$ = ~~$2Q$~~ Electric field due to Q

$$2\frac{f}{-q} = \frac{\cancel{2}f}{Q}$$

$$Q = -\frac{q}{2} \quad //.$$

Que.



$Q_{net} = Q_1 + Q_2$

Law of Conservation of charge

Que. Two Identical Conducting spheres Carrying different charges attract each other with a force F at a distance 'd' apart. The spheres are brought into contact and then taken to their original position. Now the two sphere Repel each other with a force whose Magnitude is equal to that of the initial attractive force. The Ratio between initial charges on the sphere is

a) $-(3+\sqrt{8})$ b) $-3+\sqrt{8}$ c) $-3 \pm \sqrt{8}$ d) $3 \pm \sqrt{8}$

Sol

$Q_1 \underset{d}{\text{———}} Q_2$

$\cancel{f_1} = \cancel{-f_2}$

$$f_1 = \frac{k Q_1 Q_2}{d^2}$$

$\cancel{E}$

$$f_2 = \cancel{\frac{k\, Q_1+Q_2}{d^2}} \quad k\frac{\left(\frac{Q_1 - Q_2}{2}\right)^2}{d^2}$$

$F \neq$

$$\frac{f_1}{f_2} = \frac{Q_1 Q_1}{\left(\frac{Q_1 - Q_2}{2}\right)^2}$$

$$\Rightarrow Q_1^2 + Q_2^2 - 2Q_1Q_2 = 4Q_1Q_2$$

$$\Rightarrow Q_1^2 + Q_2^2 - 6Q_1Q_2 = 0$$

$$\Rightarrow \left(\frac{Q_1}{Q_2}\right)^2 - \left(6\frac{Q_1}{Q_2}\right) + 1 = 0$$

$$x^2 - 6x + 1 = 0$$

$$\Rightarrow \cancel{x^2} - \cancel{6x} + \quad \frac{6 \pm \sqrt{36-4}}{2}$$

$$\Rightarrow 3 \pm \sqrt{8} \; //$$

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Que. Two Small Conducting sphere of equal radii have charges $+10\mu C$ and $-20\mu C$ resp. and placed at distance 'R' from each other experience force F_1. If they are brought in Contact and Separated to the Same distance they experience force F_2. find F_1/F_2.

a) 1 : 8
b) −8 : 1
c) 1 : 2
d) −2 : 1

Sol

$$F_1 = -k\frac{20 \times 10}{R^2}$$

after Contact

A: $\frac{-20+10}{2}$ B: $\frac{-20+10}{2}$

$$F_2 = k\frac{+5 \times 5}{R^2}$$

$$\frac{F_1}{F_2} = \frac{-200}{25} = -8 : 1 \text{ //.}$$

Que. Two spherical Conductors B & C having equal radius and Carrying equal charges in them repel each other with a force F when kept apart at Some distance. A third spherical Conductor having Same radius as that of B but uncharged is brought in Contact with B then brought in Contact with C and finally removed away from both. The net force of Repulsion betn B & C is

a) F/4   b) 3F/4   c) F/8   d) 3F/8

Sol

B: Q   C: Q

After contact

B: $\frac{Q}{2}$ $\xleftrightarrow{F'}$ C: $\frac{3Q}{4}$

$Q/4$

$Q + \frac{Q}{2}$

$\frac{3Q}{4}$

$$F = \frac{kQ^2}{r^2}$$

$$F' = k\frac{3Q^2}{8}$$

$$F' = \frac{3}{8}F \text{ //.}$$

Que. A point charge is located at origin. At point (a,a) Electric field is E_1. At point $(-a,a)$ Electric field is E_2 and at $(-a,-a)$ E_3. Which is correct

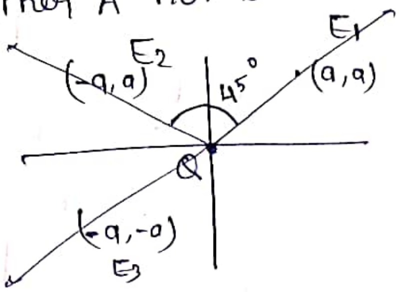
1) $E_1 \cdot E_2 = 0$

II) $E_1 \times E_3 = 0$

III) Both $E_1 \cdot E_2$ & $E_1 \cdot E_3 = 0$

IV) Neither A nor B.

Sol



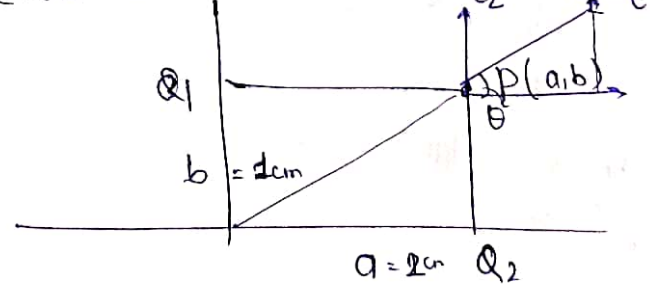
$$\vec{E_1} \cdot \vec{E_2} = |\vec{E_1}||\vec{E_2}| \cos 90^\circ = 0$$

$$\vec{E_1} \times \vec{E_3} = |\vec{E_1}||\vec{E_3}| \sin 180^\circ = 0$$

opt. c //

Que. Two point charges $q_1 = 1\,\mu C$ & $q_2 = 2\,\mu C$ are placed at distance $b = 1\,cm$ and $a = 2\,cm$ from origin on y and x axis as shown in fig. The Electric field vector at point p with angle θ with x axis is

So



$E_1 = \frac{kQ_1}{2^2}$

$\theta = \frac{E_2}{E_1}$

~~$r = \sqrt{4+1}$~~

$= \sqrt{5}$

~~$r = 5$~~

~~$E_1 =$~~

~~$\theta = \tan^{-1} 1/2$~~

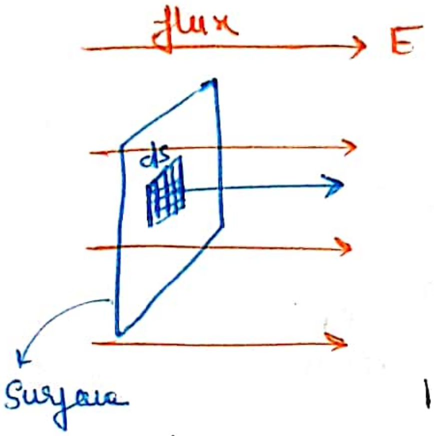
$$\tan\theta = \frac{kQ_2}{\frac{kQ_1}{2^2}}$$

$$= \frac{2^2}{2}$$

$$\tan\theta = 2 \,//$$

* Electric Flux

- It is a measure of field lines passing through any Surface



i) In this Case

$\vec{E} \cdot \vec{ds} = E\,ds \cos 0°$

$= E\,ds$

$\boxed{\psi = \int E\,ds}$ Max

ii) If Surface is place such that 90° betn E & ds

then

$\psi = \int \vec{E} \cdot \vec{ds}$

$= \int E\,ds \cos 90$

$\boxed{\psi = 0}$ min

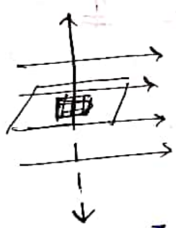
$d\psi = \vec{E} \cdot \vec{ds}$

$$\boxed{\psi = \int_S \vec{E} \cdot \vec{ds}} = \text{Coulombs}$$

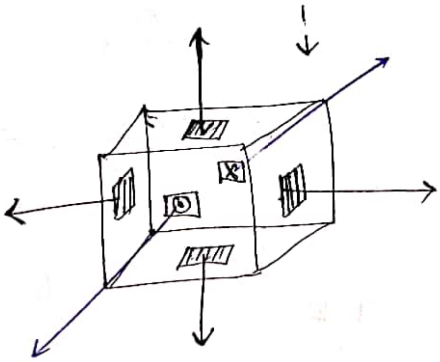
iii) If angle θ betn $\vec{E}$ & ds

$\boxed{\psi = \int E\,ds \cos\theta}$

(I) open Surface



(II) Cube



③

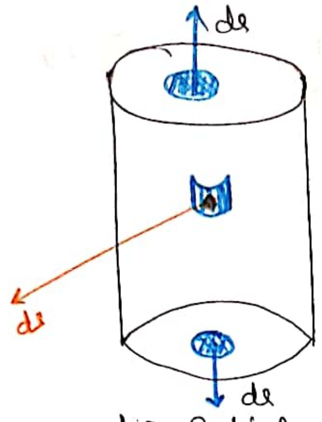
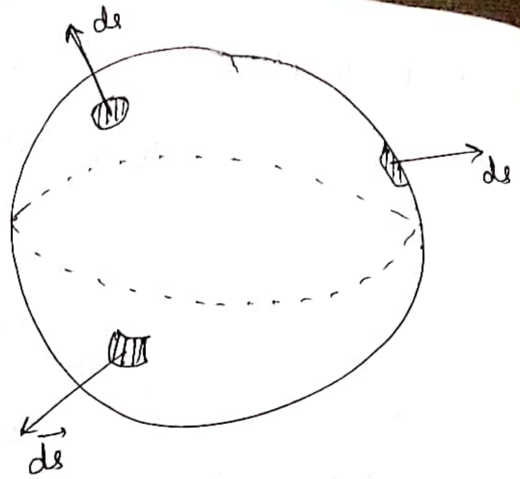


fig. Cylinder

④



* Gauss law for Electrostatic

It states that " Total Electric flux passing through any closed surface is equal to charge enclosed by that surface divided by ϵ_0. "

$$\oint_s \vec{E} \cdot \vec{ds} = \frac{Q_{enc}}{\epsilon_0}$$

$$\Rightarrow \quad \epsilon_0 \oint_s \vec{E} \cdot \vec{ds} = Q_{enc}$$

$$\Rightarrow \quad \oint \epsilon_0 \vec{E} \cdot \vec{ds} = Q_{enc.}$$

$$\Rightarrow \quad \oint_s \vec{D} \cdot \vec{ds} = Q_{enc.} \quad \text{—— (1)}$$

$\vec{D}$ = Electric flux density $= \epsilon_0 \vec{E}$ (C/m^2)

If volume charge density ρ_v C/m^3

$$Q = \int_v \rho_v \, dv \quad \text{—— (2)}$$

from (1) & (2)

$$\boxed{\oint \vec{D} \cdot \vec{ds} = \int_v \rho_v \, dv}$$

Maxwell first eqn in Integral form.

Divergence theorem. { when $\oint$ surface $\longrightarrow \int_V$ }

$$\oint_S \vec{D}\cdot\vec{ds} = \int_V (\nabla\cdot\vec{D})\, dv$$

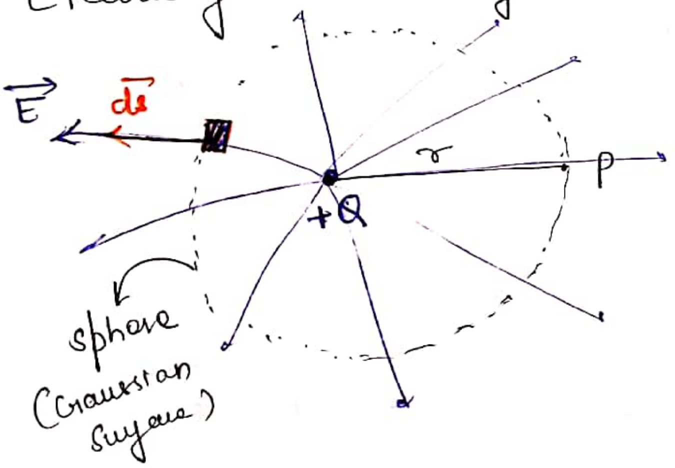
$$\int_V (\nabla\cdot\vec{D})\, dv = \int_V \rho_v\, dv$$

$$\boxed{\nabla\cdot\vec{D} = \rho_v}$$ Differential form of Maxwell eqⁿ

for Source free region $\rho_v = 0$

$$\nabla\cdot\vec{D} = 0$$

* Electric field Intensity due to point charge :-



Electric field at any pt in Gaussian surface is equal at any

$$\oint \vec{E}\cdot\vec{ds} = \oint E\, ds \cos 0$$

$$= \oint E\, ds = \frac{Q}{\epsilon_0}$$

$$= E\oint ds = \frac{Q}{\epsilon_0}$$

$$= ES = \frac{Q}{\epsilon_0}$$

$$\Rightarrow \boxed{E = \frac{Q}{4\pi\epsilon_0 r^2}}$$

$S = 4\pi r^2$ = Surface area of sphere.

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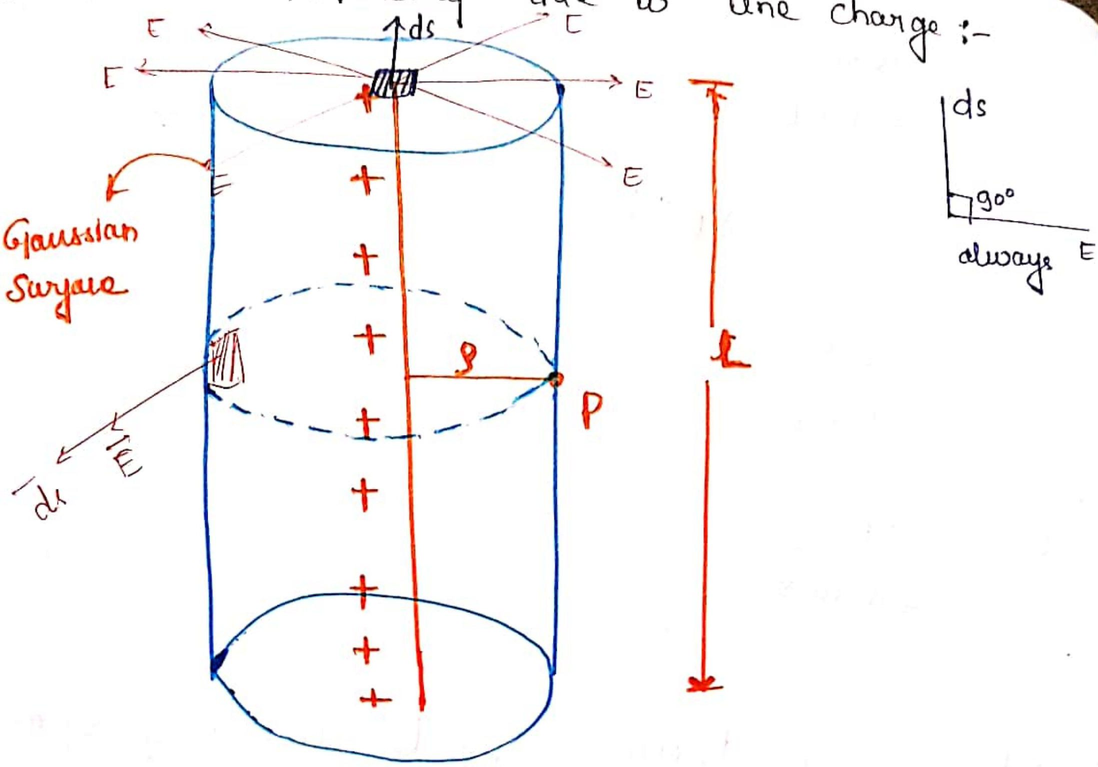
* Electric Field Intensity due to line charge :-



$$\oint_S \vec{E} \cdot \vec{ds} = \frac{Q_{enc}}{\epsilon_0}$$

or $$\left(\int_{Top\,surface} + \int_{Bottom\,surface} + \int_{curved\,surface} \right) \vec{E} \cdot \vec{ds} = \frac{Q_{enc}}{\epsilon_0}$$

$$\int_{Top\,surface} \vec{E} \cdot \vec{ds} = \int_{Top} E\,ds \cos 90 = 0$$

also, $$\int_{Bottom} \vec{E} \cdot \vec{ds} = 0$$

now $$\int_{Curved} \vec{E} \cdot \vec{ds} = \frac{Q_{enc}}{\epsilon_0}$$

$$\int_{Curv} \vec{E} \cdot \vec{ds} = \int E\,ds \cos 0 = \int E\,ds = \frac{Q_{en}}{\epsilon_0}$$

$$\Rightarrow E\, 2\pi \rho L = \frac{Q_{enc}}{\epsilon_0}$$

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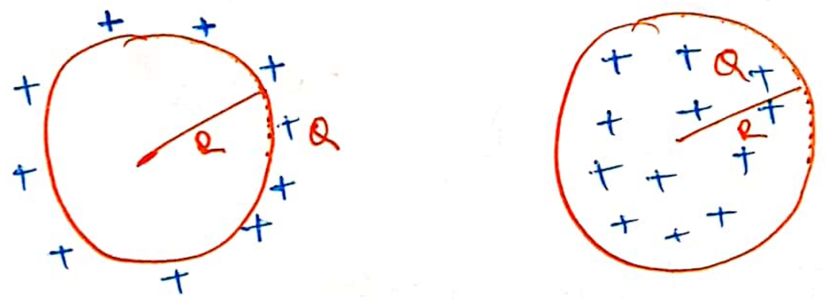
$$E = \frac{Q_{enc}}{2\pi \rho L \epsilon_0}$$

where $Q_{enc} = \rho_L \times L$

$$E = \frac{\rho_L L}{2\pi \rho L \epsilon_0}$$

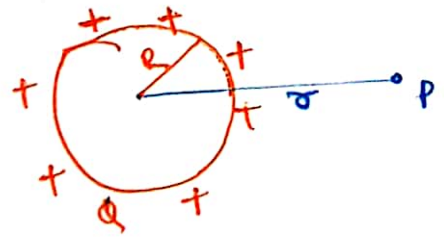
$$\boxed{E = \frac{\rho_L}{2\pi \epsilon_0 \rho}}$$

* Electric field Intensity due to Conducting sphere of Radius 'R'.

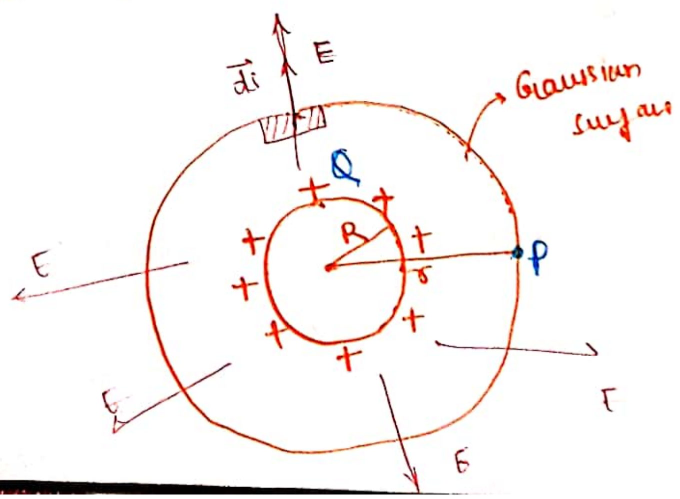


Conducting sphere

Nonconducting sphere



Case 1 $r > R$



$$\oint E\,ds\cos 0^\circ = \frac{Q_{enc}}{\epsilon_0}$$

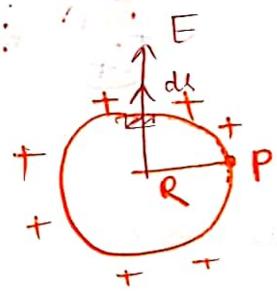
$$\Rightarrow \oint E\,ds = \frac{Q_{enc}}{\epsilon_0}$$

$$\Rightarrow E\,S = \frac{Q_{enc}}{\epsilon_0}$$

$$\Rightarrow E = \frac{Q_{enc}}{S\,\epsilon_0} \qquad \left\{ \begin{array}{l} Q_{enc} = Q \\ S = 4\pi r^2 \end{array} \right.$$

$$\Rightarrow E = \frac{Q}{4\pi r^2 \epsilon_0}$$

or $$\boxed{E = \frac{Q}{4\pi\epsilon_0 r^2}}$$

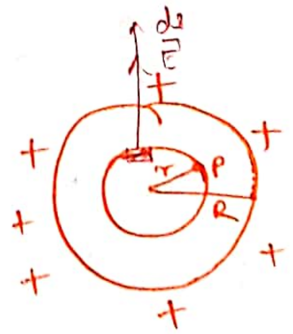


<u>Case II</u> $r = R$

$$\oint E\,ds\cos 0 = \frac{Q_{enc}}{\epsilon_0}$$

$$\Rightarrow E\int ds = \frac{Q}{\epsilon_0}$$

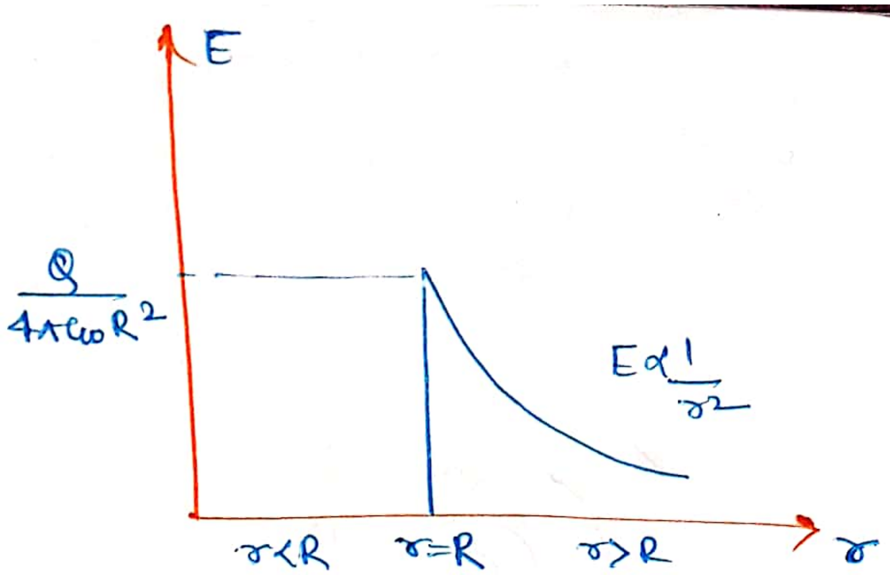
$$\boxed{E = \frac{Q}{4\pi\epsilon_0 R^2}}$$



<u>Case III</u> $r < R$

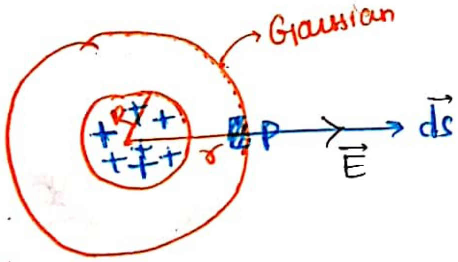
$$\int E\,ds\cos 0 = \frac{Q_{enc}}{\epsilon_0}$$

$$\boxed{E = 0} \qquad \{ Q_{enc} = 0$$



* Electric Field Intensity due to Non Conducting sphere.

<u>Case I</u> $r > R$



$$\oint \vec{E} \cdot \vec{ds} \cos 0 = \frac{Q_{enc}}{\epsilon_0}$$

$$\oint E\, ds = \frac{Q}{\epsilon_0}$$

$$\boxed{E = \frac{Q}{4\pi\epsilon_0 r^2}}$$

<u>Case II</u> :- $r = R$

$$\boxed{E = \frac{Q}{4\pi\epsilon_0 R^2}}$$

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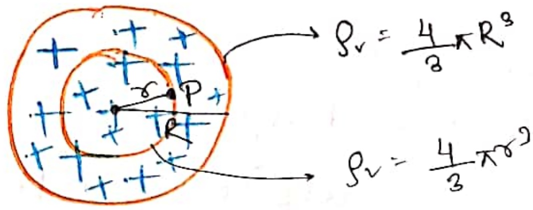
Case III



$$\oint \vec{E}\cdot\vec{ds} = \frac{Q_{enc}}{\epsilon_0}$$

$$\frac{4}{3}\pi R^3 \longrightarrow Q$$

$$\oint E\,ds\cos 0 = \frac{Q_{enc}}{\epsilon_0}$$

$$1 \longrightarrow \frac{Q}{\frac{4}{3}\pi R^3}$$

$$\Rightarrow \oint E\,ds = \frac{Q_{enc}}{\epsilon_0}$$

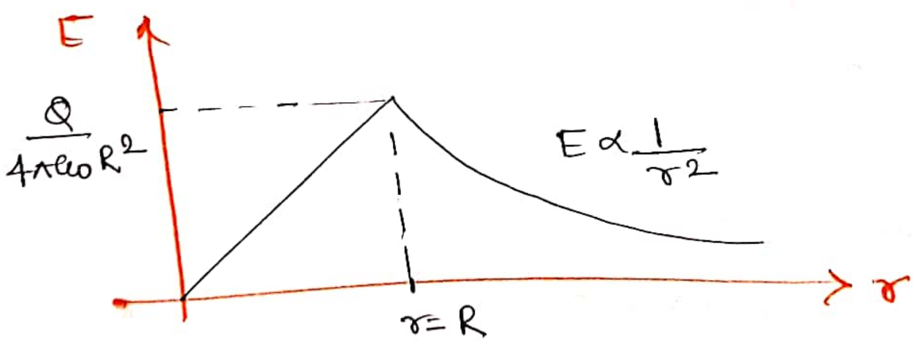
$$\frac{4}{3}\pi r^3 \longrightarrow \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3}Q$$

$$\Rightarrow E = \frac{Q_{enc}}{4\pi\epsilon_0 r^2}$$

where $Q_{enc} = \dfrac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3}Q$

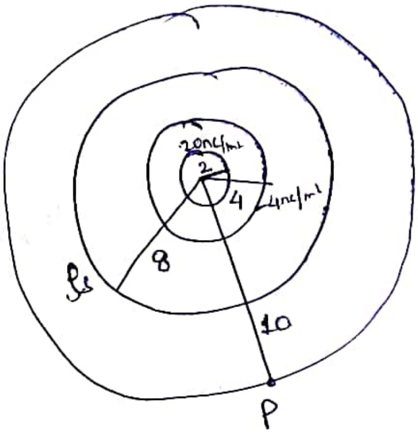
$$\frac{\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3}Q}{\cancel{4\pi\epsilon_0 r^2}}$$

$$\boxed{E = \frac{Qr}{4\pi\epsilon_0 R^3}}$$



Que1 Cocentric spherical shell of radius 2m, 4m & 8m Carry uniform surface charge density of 20nC/m2, -4 nC/m2 & ρ_s respectively. The value of ρ_s in nC/m2 required to ensure that the Electric flux density $D=0$ at radius 10m is—

Sol



Electric field at P

$\left\{ \begin{array}{l} D = \epsilon_0 E \quad \text{at } 10m \\ D = 0 \end{array} \right.$

$\therefore \quad E = 0$

$$\int E \cdot ds = \frac{Q_{enc}}{\epsilon_0}$$

$$\Rightarrow \quad E = \frac{Q_{enc}}{4\pi\epsilon_0 \times 10^2}$$

$$Q_{enc} = 0$$

charge in 2m radius spher

$$Q = 20 \times 4\pi \times 2^2$$

charge in 4m rad

$$Q = -4 \times 4\pi \times 4^2$$

charge in 8m rad.

$$Q = \rho_s \times 4\pi (8^2)$$

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$$Q_{enc} = 20 \times 4\pi \times 2^2 + \rho_s \, 4\pi 8^2 - 4 \times 4\pi \times 4^2 = 0$$

$$\Rightarrow 80 + \rho_s 64 - 64 = 0$$

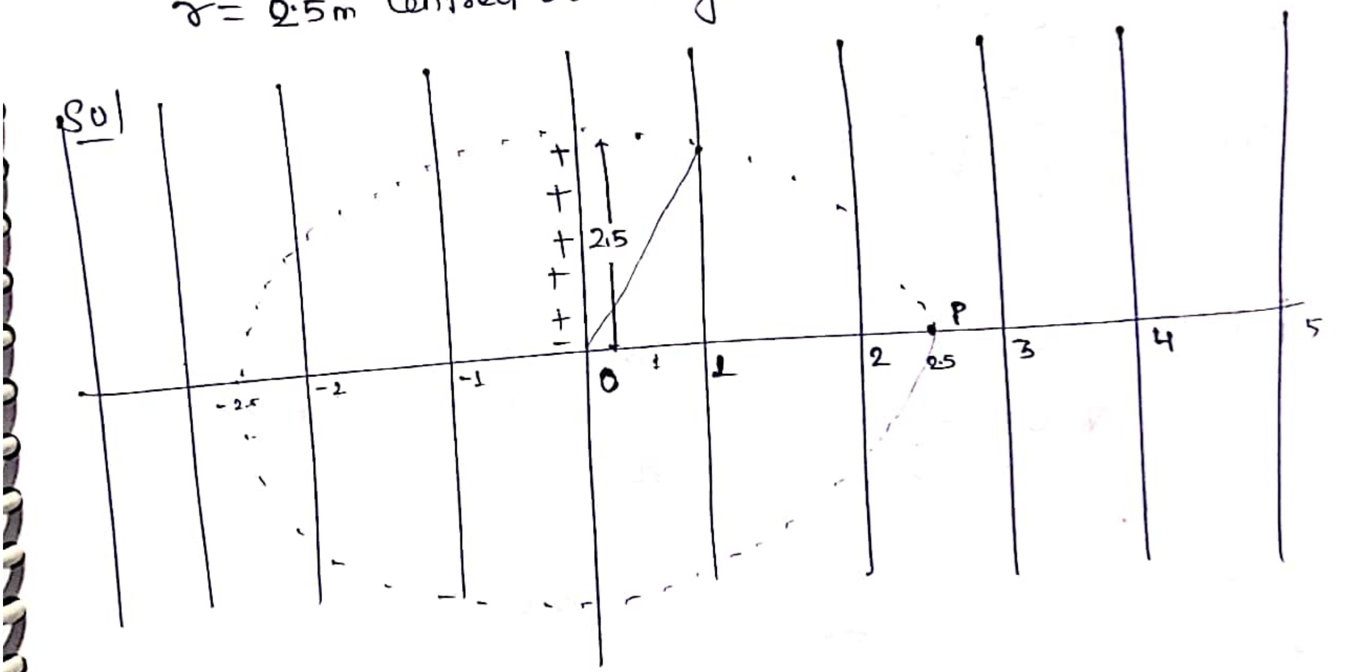
$$\rho_s = \frac{64-80}{64}$$

$$= 1 - \frac{10}{8}$$

$$\Rightarrow -0.25 \ nC/m^2$$

Que2 Uniform line charges of 20nC/m each lie in $z=0$ plane at $x = 0, \pm 1, \pm 2 \ldots \pm 5$. Find total electric flux leaving the spherical surface $r = 2.5m$ centred at origin.

Sol



$$Q = \rho_L L$$

$$= 2\left[20 \times 2.5 \times 10^{-9}\right] + 4\left[20 \times 10^{-9} \times \sqrt{2.5^2 - 1^2}\right]$$

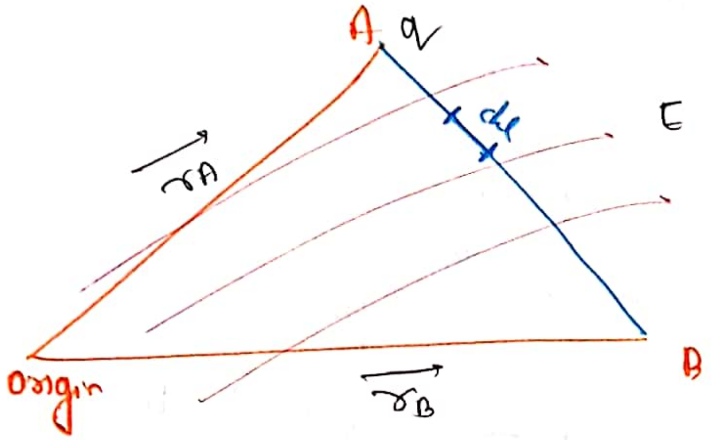
$$+ 4\left[20 \times 10^{-9} \times \sqrt{2.5^2 - 2^2}\right]$$

$$\Rightarrow 20 \times 10^{-9}\left[4 \times 2.5 + 4 \times 2.29 + 4 \times 1.5\right]$$

$$\Rightarrow \cancel{20} \times 10^{-9}\left[2303.2\right] \qquad \Big\} \ 448 \ \text{nC}$$

$$\cancel{\psi} = \cancel{\frac{Q}{4\pi\epsilon_0 2.5^2}} \qquad \psi = \frac{Q}{\epsilon_0}$$

* Potential difference



$$dW = -\vec{F}\cdot\vec{dl}$$

$$dW = -q\vec{E}\cdot\vec{dl}$$

$$W = -\int_A^B q\vec{E}\cdot dl$$

Potential difference

$$\boxed{V_{AB} = \frac{W}{q} = -\int_A^B \vec{E}\cdot\vec{dl}} \quad \text{Volt}$$

$$\underset{\text{electric field Intensity}}{\vec{E}} = \frac{kQ}{r^2}\,\hat{a_r} \qquad dl = dr\,\hat{a_r}$$

$$\boxed{V_{AB} = -\int_A^B \frac{kQ}{r^2}\,dr}$$

$$= -\int_{r_A}^{r_B} \frac{kQ}{r^2}\,dr = -kQ\left[\frac{-1}{r}\right]_{r_A}^{r_B}$$

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$$V_{AB} = kQ\left[\frac{1}{r_B} - \frac{1}{r_A}\right] = \frac{kQ}{r_B} - \frac{kQ}{r_A} = V_B - V_A$$

where $r_A = \infty$ $r_B = r$

$$\boxed{V_{AB} = \frac{kQ}{r}}$$ Absolute potential at r

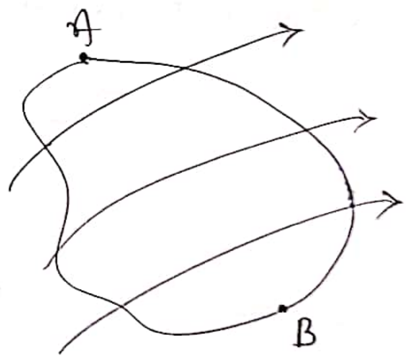
* Potential difference

$$V_{AB} = -\int_A^B \vec{E}\cdot\vec{dl}$$

$$-V_{AB} = \int_A^B \vec{E}\cdot\vec{dl}$$

$$-V_{AB} = -\int_B^A \vec{E}\cdot\vec{dl}$$

$$-V_{AB} = V_{BA}$$



$\Rightarrow$ $$V_{AB} + V_{BA} = 0$$

$$\int_A^B \vec{E}\cdot\vec{dl} + \int_B^A \vec{E}\cdot\vec{dl} = 0$$

$$\boxed{\oint_l \vec{E}\cdot\vec{dl} = 0}$$ Maxwell 2^{nd} eq^n Integral form

"Total Workdone to move a charge in a closed loop in presence of Electric field is 0".
Zero

Using stokes theorem $\{ \text{when } \oint_L \longrightarrow \int_s \}$

$$\oint_l \vec{E}\cdot\vec{dl} = \int_s (\nabla \times \vec{E})\, ds$$

$$\int_s (\nabla \times \vec{E})\, \vec{ds} = 0$$

$$\boxed{\nabla \times \vec{E} = 0}$$

Maxwell eqn in differential or point form

This is for static field

for Time Varying field

$$\boxed{\nabla \times E = -\frac{\partial \vec{B}}{\partial t}}$$

* Relationship between Electric field Intensity & potential

$$\boxed{\vec{E} = -\nabla V}$$

$E = -$(Gradient of V)

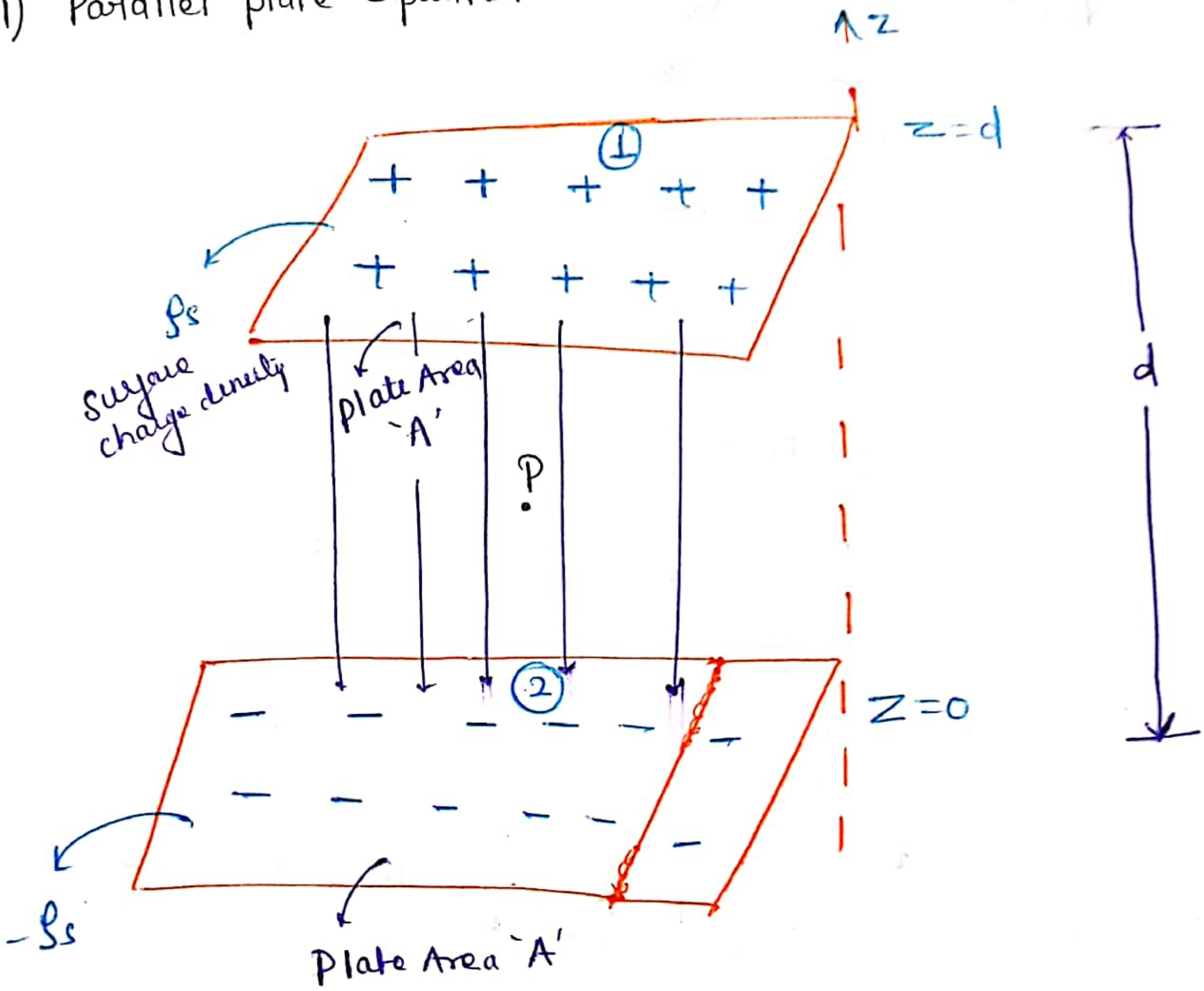
* Capacitance :- When there are two Conductors having equal and opposite charged and separated by some distance forms a Capacitive System

i) Parallel plate Capacitor

ii) Co axial Capacitor

iii) spherical Capacitor

1) Parallel plate Capacitor



$$V_{21} = -\int_2^1 \vec{E} \cdot \vec{dl}$$

$$\vec{E} = \frac{\rho_s}{2\varepsilon_0}(-\hat{a}_z) + \frac{-\rho_s}{2\varepsilon_0}(+a_z)$$

given by direction of Movement found at point p { point of intrest

$$= \frac{-\rho_s \hat{a}_z}{\varepsilon_0}$$

Total charge = $Q = \rho_s A$

$$\rho_s = Q/A$$

$$\vec{E} = \frac{-Q}{A\varepsilon_0}\hat{a}_z$$

$$V_{21} = -\int_2^1 \left(-\frac{Q}{A\epsilon_0}\hat{a_z}\right)\cdot(dz\,\hat{a_z}) \qquad \left\{ dl = dz\hat{a_z} \right.$$

$$= \int_2^1 \frac{Q}{A\epsilon_0}\,dz$$

$$\Rightarrow \int_0^d \frac{Q}{A\epsilon_0}\,dz$$

$$\Rightarrow \frac{Q}{A\epsilon_0}(d-0)$$

$$V_{21} \Rightarrow \frac{Qd}{A\epsilon_0}$$

$$\Rightarrow \boxed{C = \frac{Q}{V_{21}} = \frac{\epsilon_0 A}{d}}$$ for air between plates

Energy stored $\boxed{W_E = \frac{1}{2}\int_v (\vec{D}\cdot\vec{E})\,dv}$

$$W_E = \frac{1}{2}\int_v \left(\epsilon_0\vec{E}\cdot\vec{E}\right) dv$$

$$= \frac{1}{2}\int \epsilon_0 E^2\,dv$$

$$= \frac{1}{2}\epsilon_0 E^2 \int dv$$

$$= \frac{1}{2}\epsilon_0 E^2 A\cdot d \qquad \left\{ V = A\cdot d \right.$$

$$= \frac{1}{2}\epsilon_0 \frac{Q^2}{(A\epsilon_0)^2} A d$$

$$= \frac{1}{2} \frac{\varepsilon_0}{\varepsilon_0^2} \frac{Q^2}{A} d$$

$$= \frac{1}{2} \frac{Q^2 d}{\varepsilon_0 A}$$

$$= \frac{1}{2} \frac{Q^2}{C} \qquad \left\{ Q = CV \right.$$

$$= \frac{1}{2} \frac{C^2 V^2}{C}$$

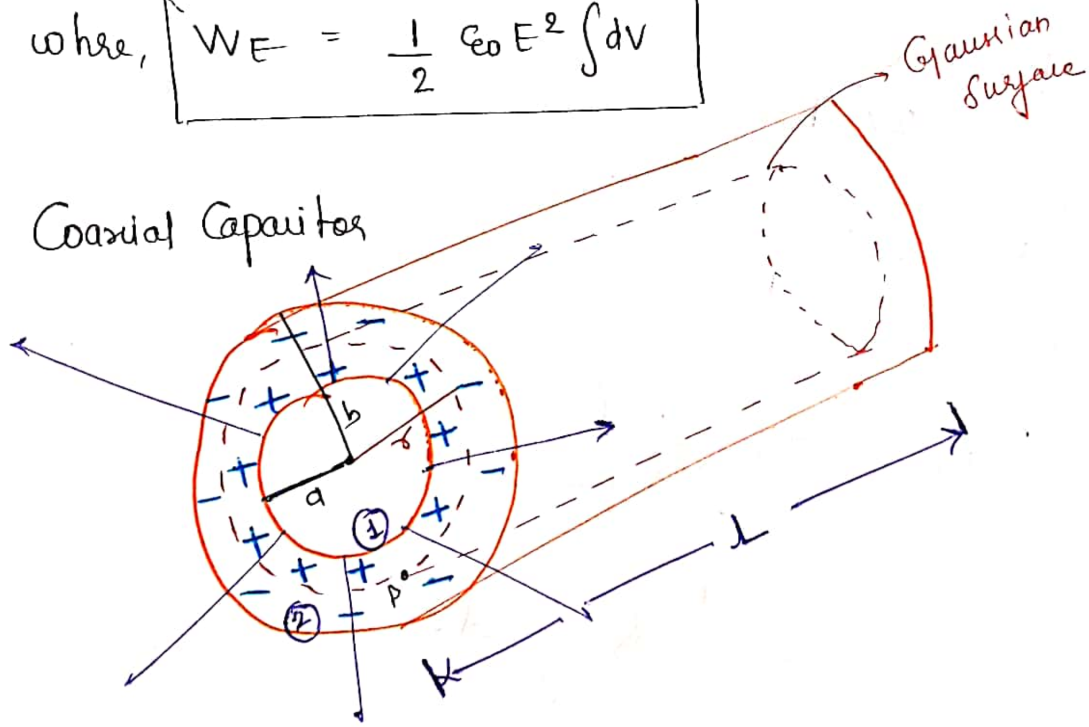
$$\boxed{W_E = \frac{1}{2} C V^2}$$

$$\text{Energy density} = \frac{\text{Energy}}{\text{Volume}} = \frac{W_E}{\int_v dv}$$

$$\boxed{W_d = \frac{1}{2} \varepsilon_0 E^2}$$

where, $\boxed{W_E = \frac{1}{2} \varepsilon_0 E^2 \int dv}$

2) Coaxial Capacitor



$$C = \frac{Q}{V_{21}}$$

$$V_{21} = -\int_2^1 \vec{E}\cdot\vec{dl}$$

for top & bottom Surface of Cylinder

$$\vec{E}\cdot\vec{dS} = E\,dS\,\cos 90^\circ = 0$$

$$\int_{\text{Curved surface}} E\,dl\,\cos 0^\circ = \frac{Q_{enc}}{\epsilon_0}$$

$$E\int dS = \frac{Q}{\epsilon_0}$$

$$E\,2\pi r L = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{2\pi r L \epsilon_0}$$

$$\boxed{E = \frac{Q}{2\pi\epsilon_0 L}}$$

$$V_{21} = -\int_2^1 \frac{Q}{2\pi\epsilon_0 r L}\,\hat{a_r}\cdot dr\,\hat{a_r}$$

$$= -\int_2^1 \frac{Q}{2\pi\epsilon_0 r L}\,dr$$

$$= -\frac{Q}{2\pi\epsilon_0 L}\left(\log r\right)_b^a$$

$$V_{21} = \frac{-Q}{2\pi\epsilon_0 L} \ln\frac{a}{b}$$

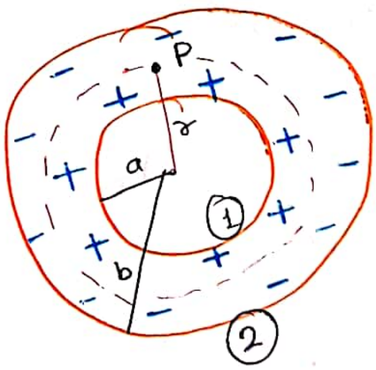
$$V_{21} = \frac{Q}{2\pi\epsilon_0 L} \ln\frac{b}{a}$$

$$C = \frac{Q}{V_{21}} = \frac{Q}{\frac{Q}{2\pi\epsilon_0 L}\ln\left(\frac{b}{a}\right)}$$

$$C = +\, 2\pi\epsilon_0 L / \ln\left(\frac{b}{a}\right)$$

$$\boxed{C = \frac{2\pi\epsilon_0 L}{\ln(b/a)}}$$

* Spherical Capacitor



$$C = \frac{Q}{V_{21}}$$

$$V_{21} = -\int_2^1 \vec{E} \cdot \vec{dl}$$

$$\int \vec{E}\cdot d\vec{l} = \frac{Q_{enc}}{\epsilon_0}$$

$$\int \vec{E}\cdot \vec{dl} = \frac{Q_{en}}{\epsilon_0}$$

$$\int E\,dl = \frac{Q}{\epsilon_0}$$

$$\left\{ \begin{aligned} \vec{E}\cdot\vec{dl} &= E\,dl \cos 0^\circ \\ &= E\,dl \end{aligned} \right.$$

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{a_r}$$

$$V_{21} = -\int_2^1 \frac{Q}{4\pi\epsilon_0 r^2}\hat{a_r}\cdot\vec{dl}$$

$$= -\int_2^1 \frac{Q}{4\pi\epsilon_0 r^2}\hat{a_r}\, dr\, \hat{a_r}$$

$$= -\int_b^a \frac{Q}{4\pi\epsilon_0 r^2} dr$$

$$= +\frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r}\right]_b^a = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{a}-\frac{1}{b}\right]$$

$$V_{21} > \frac{Q}{4\pi\epsilon_0 \left(\frac{1}{a}-\frac{1}{b}\right)}$$

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$$C = \frac{4\pi\epsilon_0 (a \times b)}{b - a}$$

$$\boxed{C = \frac{4\pi\epsilon_0 ab}{b - a}}$$

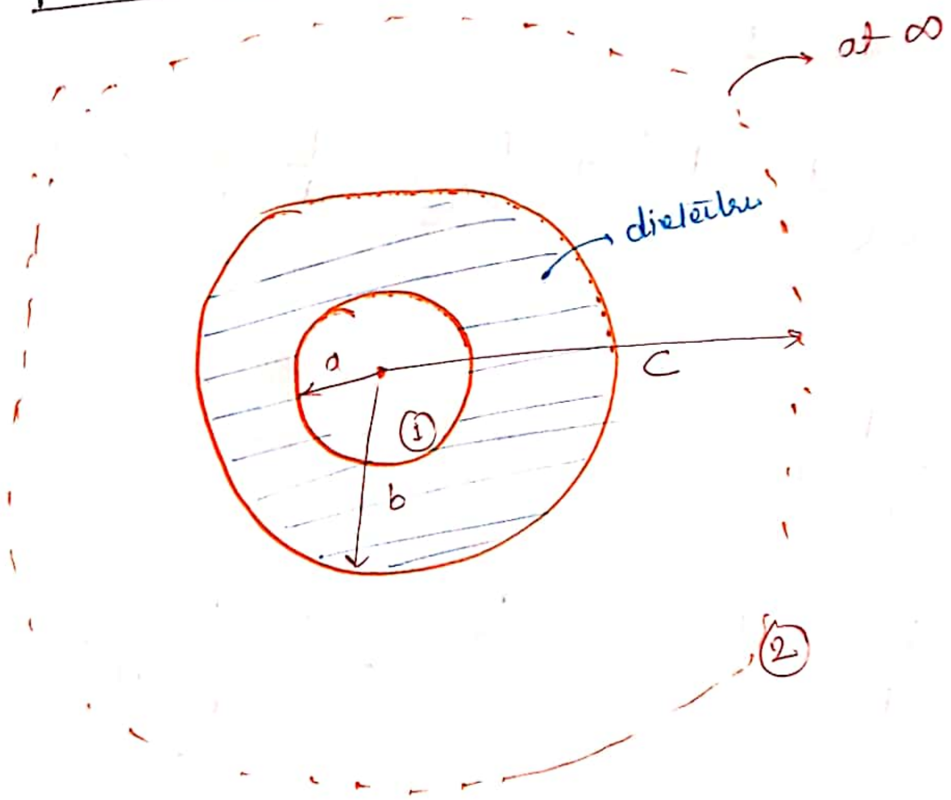
or

$$\boxed{C = \frac{4\pi\epsilon_0}{\left(\frac{1}{a} - \frac{1}{b}\right)}} = \frac{1}{\frac{1}{4\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)}$$

If $b = \infty$ thenit is Called Isolated spere

$$\boxed{C_{iso\,sp.} = 4\pi\epsilon_0 a}$$

*



$$C = \frac{Q}{V_{21}}$$

$$V_{21} = -\int_2^1 \vec{E}\cdot\vec{dl} = -\left[\int_c^b \frac{Q}{4\pi\epsilon_0 r^2}\,dr + \int_b^a \frac{Q}{4\pi\epsilon_0 \epsilon_r r^2}\,dr\right.$$

$$V_{21} = \frac{-Q}{4\pi\epsilon_0}\left[-\left(\frac{1}{r}\right)_c^b - \left(\frac{1}{\epsilon_r r}\right)_b^a\right]$$

$$= \frac{-Q}{4\pi\epsilon_0}\left[-\frac{1}{b}+\frac{1}{c}-\frac{1}{\epsilon_r a}+\frac{1}{\epsilon_r b}\right]$$

$$= \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{b}-\frac{1}{c}\right] + \frac{Q}{4\pi\epsilon_0\epsilon_r}\left[\frac{1}{a}-\frac{1}{b}\right]$$

$$C = \frac{Q}{V_{21}}$$

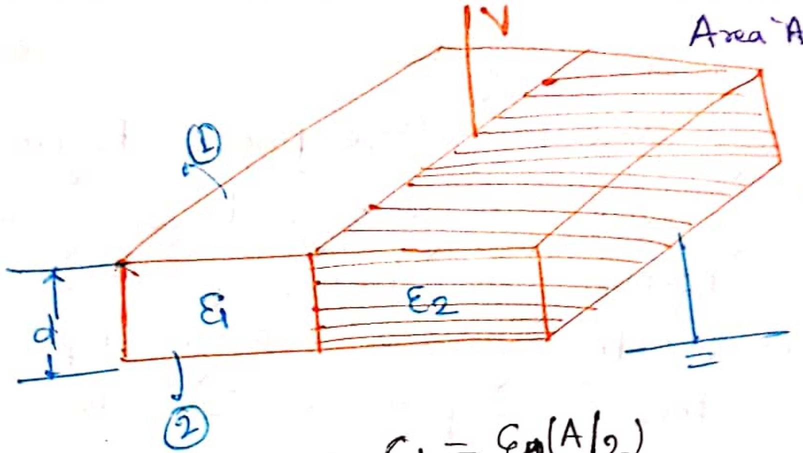
$$\boxed{C = \frac{1}{\frac{1}{4\pi\epsilon_0}\left(\frac{1}{b}-\frac{1}{c}\right) + \frac{1}{4\pi\epsilon_0\epsilon_r}\left(\frac{1}{a}-\frac{1}{b}\right)}}$$

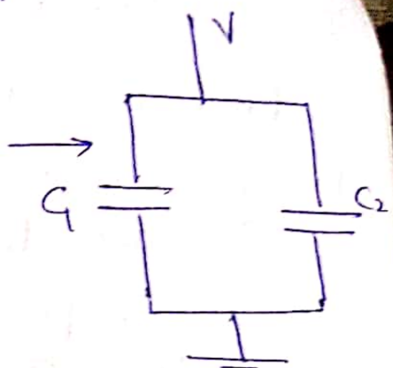
$$C = \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}}$$

$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}}$$

Series Connection of Capacitor

*





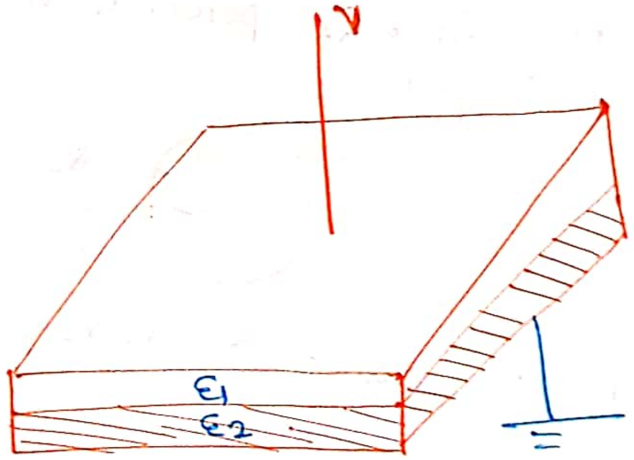
$$C_1 = \frac{\varepsilon_1 (A/2)}{d}$$

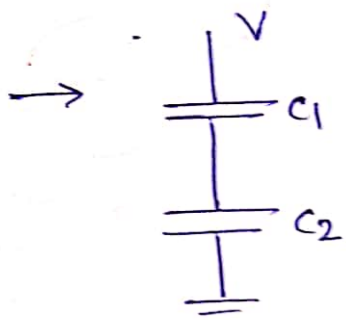
$$C_2 = \frac{\varepsilon_2 A/2}{d}$$

$$C_{eq} = \cancel{\frac{C_1 C_2}{C_1 + C_2}}$$

$$C_{eq} = C_1 + C_2$$

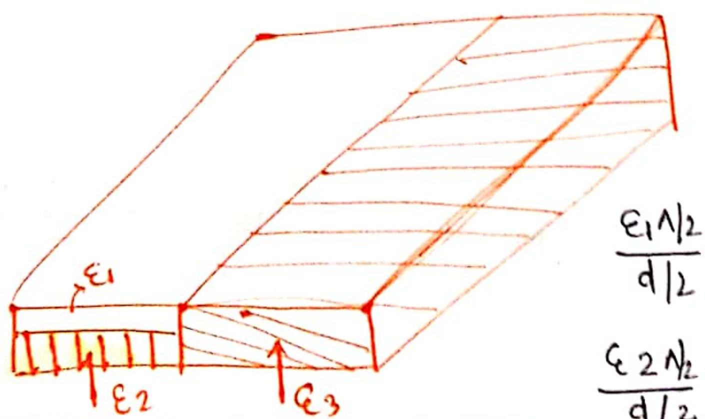
$$\boxed{C_{eq} = \frac{\varepsilon_1 A}{2d} + \frac{\varepsilon_2 A}{2d}}$$





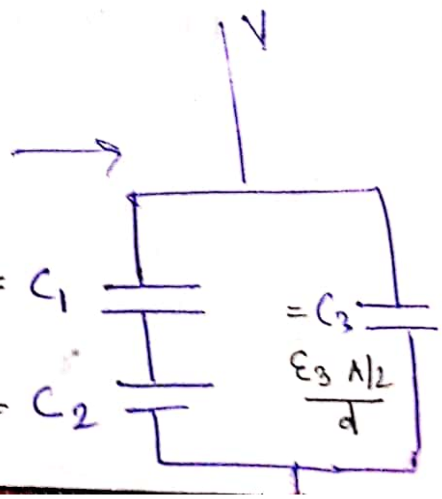
$$C_1 = \frac{\varepsilon_1 A}{d/2}, \quad C_2 = \frac{\varepsilon_2 A}{d/2}$$

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$



$$\frac{\varepsilon_1 A/2}{d/2} = C_1$$

$$\frac{\varepsilon_2 A/2}{d/2} = C_2$$



$$= C_3 = \frac{\varepsilon_3 A/2}{d}$$

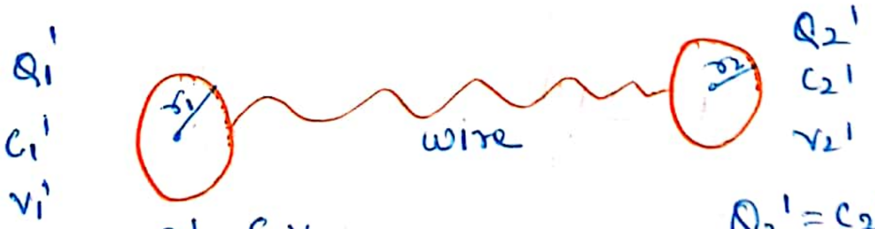
* Re-Distribution of charges

When two charge Conductors are joined together through a Conducting wire, charge begins to flow from one Conductor to another from higher potential to lower potential. This flow of charge stops when they attain the Same potential. Due to flow of charge Loss of energy also take place in the form of heat through the Connecting wire.

Suppose there are two spherical Conductor of radius r_1 & r_2 having charge Q_1 & Q_2 potential V_1 & V_2 and Capacitance C_1 & C_2 respectively

$Q_1 = C_1 V_1$ $\qquad$ $Q_2 = C_2 V_2$

Net charge = $Q_1 + Q_2$



$Q_1' = C_1 V$ $\qquad$ $Q_2' = C_2 V$

Net charge $= Q_1' + Q_2'$

$$V_1 = \frac{Q_1}{C_1}$$

$$\boxed{V_1 = \frac{Q_1}{4\pi\varepsilon_0 r_1} \quad \text{for spherical Conductor}} = \frac{kQ}{\text{radius}}$$

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After Connecting both Conductor through wire

- $Q_1 + Q_2 = Q_1' + Q_2' = Q$ { Net charge is equal

- potential in both conductor $= V$ i.e equal

$$\boxed{\begin{aligned} Q_2' &= \frac{Q r_2}{r_1 + r_2} \\ Q_1' &= \frac{Q r_1}{r_1 + r_1} \end{aligned}}$$

{ like Voltage division rule.

$$V = \frac{Q_1' + Q_2'}{C_1 + C_2}$$

$$= \frac{Q_1 + Q_2}{C_1 + C_2}$$

$$V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

Que. Two Metallic sphere of radius 1cm & 3cm are given charges of -1×10^{-2} C & 5×10^{-2} C resp. If these are Connected by Conducting wire. The final charge on bigger sphere is —

Sol

$C_1 = -4\pi \epsilon_0 \times 1 \times \cancel{10^{-2}}\ 10^{-2}$

$C_2 = 4\pi \epsilon_0 \times 5 \times 10^{-2}$

$Q_{net} = -1 \times 10^{-2} + 5 \times 10^{-2}$

$= 4 \times 10^{-2}$

$$Q_2' = \frac{4 \times 10^{-2} \times 3 \times 10^{-2}}{(3+1)\, 10^{-2}}$$

$\Rightarrow 3 \times 10^{-2}\, C$ //

Que Two charge sphere of radius r_1 & r_2 having equal surface charge density. The ratio of their potential is

Sol $V_1 \propto \frac{Q_1}{r_1}$

Surface charge density

$Q = \rho 4\pi r^2$

$V_2 \propto \frac{Q_2}{r_2}$

$$\frac{V_1}{V_2} = \frac{\cancel{r_2^2}}{\cancel{r_1^2}} \cdot \frac{Q_1}{r_1} \times \frac{r_2}{Q_2} \Rightarrow \frac{Q_1}{Q_2} \times \frac{r_2}{r_1}$$

$$= \Rightarrow \frac{r_1^2}{r_2^2} \times \frac{r_2}{r_1}$$

$$\boxed{\frac{V_1}{V_2} = \frac{r_2}{r_1}}$$ //

Que. The Electric potential at a point (x, y, z) is given by $V = -x^2y - xz^3 + 4$ The electric field at that point is

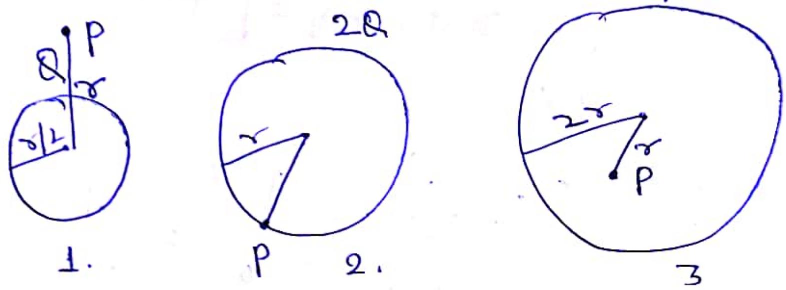
Sol $\vec{E} = -\nabla V$

$$= -\left[(-2xy - z^3)\hat{a_x} - x^2\hat{a_y} - 3z^2x\hat{a_z}\right]$$

$$= (2xy + z^3)\hat{a_x} + x^2\hat{a_y} + 3z^2x\hat{a_z} \quad //.$$

Que. Charges Q, 2Q & 4Q are Uniformly distributed in 3 dielectric Solid spheres 1, 2 & 3 of radius $r/2$, r, $2r$

$2r$ — 4Q
r — 4Q
4Q → $4\pi r^2$



1) Magnitudes of Electric field at point p at a distance r from Centre of sphere 1, 2 & 3 are E_1 E_2 & E_3 resp. then (non Conducting sphere Case)

a) $E_1 > E_2 \quad E_2 > E_3$

b) $E_3 > E_1 > E_2$

c) $E_2 > E_1 > E_3$

d) $E_3 > E_2 > E_1$

$E_1 \propto \frac{Q}{r^2}$

$E_2 \propto \frac{2Q}{r^2}$

$E_3 \propto \frac{4Q r'}{r^3}$ $\{r' = r$

$\propto \frac{4Q}{2r^2}$

page 1-75

Q 10

Sol $\vec{E} = x\hat{a_x} + y\hat{a_y} + z\vec{a_z}$

$X(2, 0, 0)$ $Y(1, 2, 3)$

$$V_{xy} = -\int_X^Y \vec{E}\cdot d\vec{l}$$

$$\Rightarrow -\int_X^Y (x\hat{a_x} + y\hat{a_y} + z\hat{a_z})\cdot(\hat{a_x}\,dx + dy\,a_y + a_z\,dz)$$

$$\Rightarrow -\int_X^Y x\,dx + y\,dy + z\,dz$$

$$\Rightarrow -\left[\left(\frac{x^2}{2}\right)_2^1 + \left(\frac{y^2}{2}\right)_0^2 + \left(\frac{z^2}{2}\right)_0^3\right]$$

$$= -\left[\frac{1}{2} - 2 + 2 + \frac{9}{2}\right]$$

$$= -[5]\ V$$

$$= -5V$$

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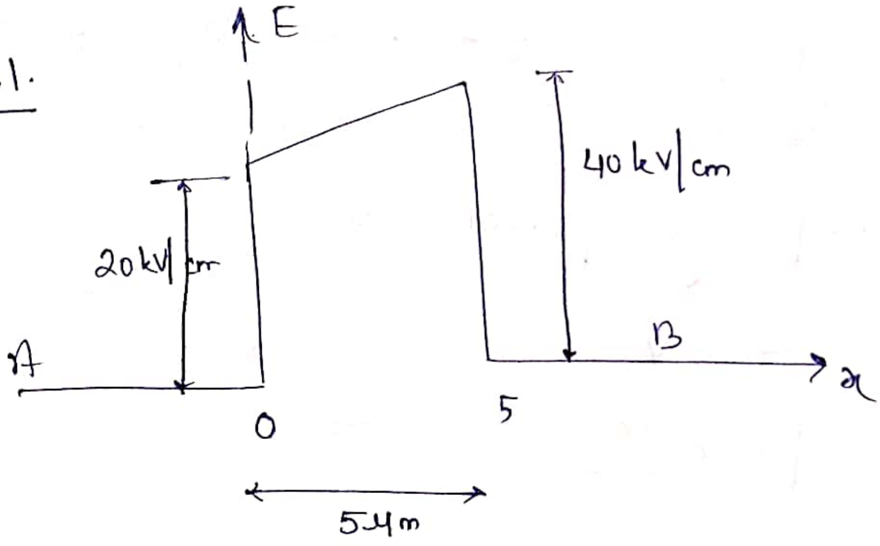
Q21.



$V_B - V_A = V_{AB}$

$= -\int_B^A \vec{E}\cdot d\vec{l}$

$= -\int_0^5 \vec{E}\cdot dx$

20 kV/cm = 2 V/μm

$\Rightarrow -\int_0^5 \left(\frac{40-20}{5}\cancel{x}\right) + 20 \; dx$

$\Rightarrow -\int_0^5 (4x+20)\, dx$

$\Rightarrow -\left[\frac{4}{2}(5^2) + 20\times5\right]$

$\Rightarrow -\left[2\times25 + 100\right]$

$= -150$

$-\int_0^5 \left(\frac{2x}{5} + 2\right) dx$

$\Rightarrow -\int_0^5 \left(\frac{2x^2}{5\times2}\right) + 2(x$

$\Rightarrow -\left[5 + 10\right]$

$= -15V$

Q22. $E = -(2y^3 - 3yz^2)\hat{a_x} - (6xy^2 - 3xz^2)a_y + 6xyz\, a_z$

Correction

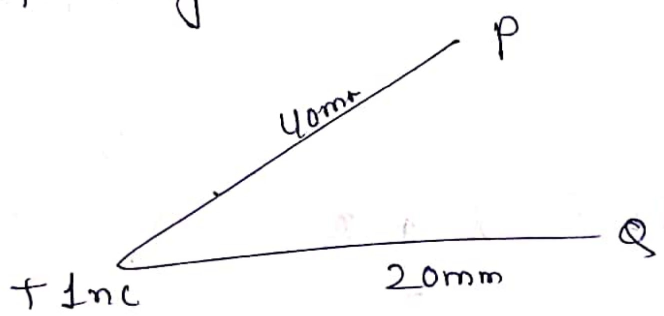
Valid exp for potential is —

Sol $\vec{E} = -\nabla V$

check with option D

Que A point charge of 1nC is placed in a space with permittivity ϵ_0 as shown in fig - The potential difference V_{pq} b/w two point P & Q at distance of 20mm & 40mm respectively from the point charge is-

Sol



$V_{pq} = V_q - V_p$

$V_q = \frac{kQ}{20mm} = \frac{k\, 1\times10^{-9}}{20\times10^{-3}} = \frac{k10^{-5}}{2}$

$V_p = \frac{kQ}{40mm} = \frac{k\, 1\times10^{-9}}{40\times10^{-3}} = \frac{k10^{-5}}{4}$

$V_{pq} = k10^{-5}\left[\frac{1}{2} - \frac{1}{4}\right]$

$k10^{-5}\left[\frac{1}{4}\right] \Rightarrow \frac{9\times10^{9}\times10^{-5}}{4}$

$\Rightarrow 4.5\times10^{4} \Rightarrow 225$ //

Que Given the potential in free space $V = 50x^2 + 50y^2 + 50z^2$

The Magnitude & direction of ~~Magnetic~~ electric field at point $(1,-1,1)$ and its direction also.

Sol $\vec{E} = -\nabla E$

$$= -\left[100x\hat{i} + 100y\hat{i} + 100z\hat{k}\right]$$

$$-\left[100\hat{i} - 100\hat{j} + 100\hat{k}\right]$$

$$\Rightarrow -100\hat{i} + 100\hat{j} - 100\hat{k}$$

$$|E| = \sqrt{100^2 + 100^2 + 100^2}$$

$$= 100\sqrt{3}$$

$$\text{directio} = \frac{\vec{E}}{|E|}$$

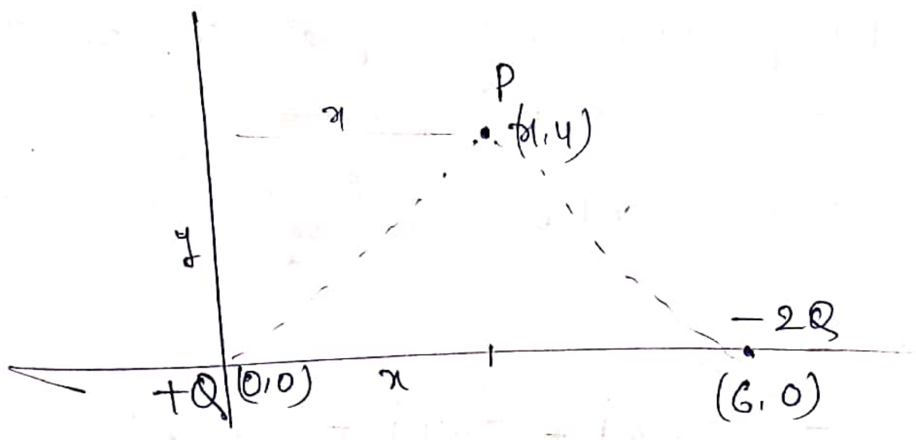
$$= \frac{-100\hat{i} + 100\hat{j} - 100\hat{k}}{100\sqrt{3}}$$

$$= \frac{-\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}} \quad //.$$

Que. Two Electric charges $+Q$ & $-2Q$ are placed at $(0,0)$ & $(6,0)$. The eqn of Zero equipotential Curve in xy plane is

$+Q$ $(0,0)$ $\qquad$ $-2Q$ $(6,0)$

Sol



$$V_{PO} = \frac{kQ}{\sqrt{(x^2+y^2)}}$$

$$V_{P6} = \frac{-k2Q}{\sqrt{(6-x)^2+y^2}}$$

$$V_{PO} + V_{P6} = 0$$

$$\frac{kQ}{\sqrt{x^2+y^2}} - \frac{2Qk}{\sqrt{(6-x)^2+y^2}} = 0$$

$$\Rightarrow \sqrt{(6-x)^2+y^2} = 2\sqrt{x^2+y^2} \quad —①$$

$$\Rightarrow (6-x)^2+y^2 = 4(x^2+y^2) \quad —②$$

$$\Rightarrow 36+x^2-12x+y^2 = 4x^2+4y^2$$

$$3x^2+3y^2+12x-36 = 0$$

$$\Rightarrow x^2+y^2+4x-12 = 0$$

$$\Rightarrow (x+2)^2+y^2 = 16 \quad //.$$

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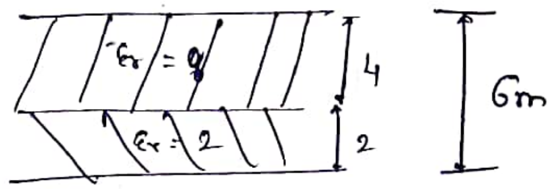
Que. A Capacitor Consist of two Metal plates $500 \times 500\ mm^2$ and spaced 6mm apart. The space beth metal plate is filled with a glass plate 4mm thickness and permittivity 8 and a layer of paper 2mm thickness and permittivity 2. The Capacitance in pF is

Sol $A = 500 \times 500\ mm^2$

$d = 6\ mm$



$$C = \cancel{C_1 + C_2}$$

$$= \cancel{\frac{\varepsilon_0 A}{d_1}} + \cancel{\frac{\varepsilon_0 A}{d_2}}$$

$$C = C_1 + C_2$$

$$= \cancel{\frac{\varepsilon_0 \varepsilon A}{4}} + \cancel{\varepsilon_0 A \varepsilon_{r2}}$$

$$\frac{1}{C_1} = \frac{1}{\frac{\varepsilon_0 \varepsilon_{r1} A}{4}} + \frac{1}{\frac{\varepsilon_0 \varepsilon_{r2} A}{2}}$$

$$= \frac{1}{\frac{8 \varepsilon_0 \times 500 \times 500 \times 10^{-6}}{4 \times 10^{-3}}} + \frac{1}{\frac{2 \varepsilon_0\ 500 \times 500 \times 10^{-6}}{2 \times 10^{-3}}}$$

$$C \Rightarrow \frac{C_1 C_2}{C_1 + C_2} = \frac{\frac{8 \varepsilon_0\ 500^2 \times 10^{-6}}{4 \times 10^{-3}} \times \frac{2 \varepsilon_0\ 500 \times 10^{-6}}{2 \times 10^{-3}}}{\frac{8 \varepsilon_0\ 500^2 \times 10^{-6}}{4 \times 10^{-3}} + \frac{2 \varepsilon_0\ 500 \times 10^{-6}}{2 \times 10^{-3}}}$$

$$\Rightarrow \frac{8 \varepsilon_0 \times 5 \times 10^{-1}}{4} \times \frac{2 \varepsilon_0 \times 5 \times 10^{-1}}{2}$$

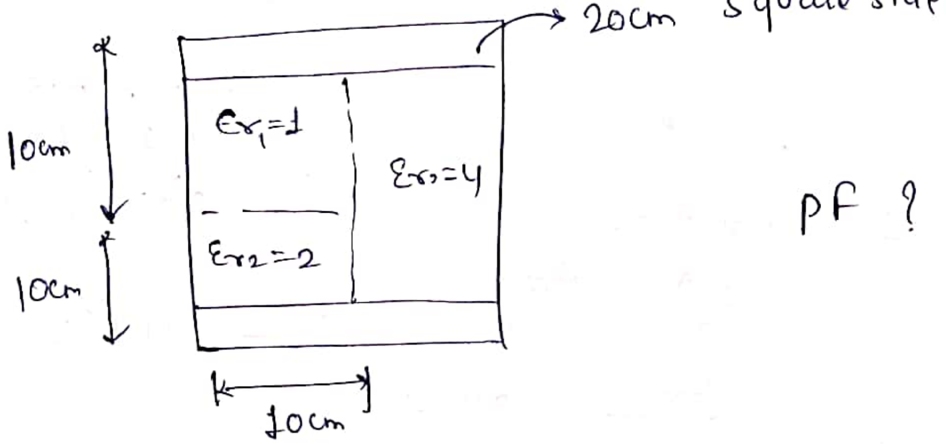
$= 147567$ pF //

Que. The capacitance of an isolated sphere of radius 10cm in air in pF is ——

Sol $C = 4\pi\epsilon_0 a$

$= 11.12\,pF$

Que. The capacitance of arrangement shown in fig is



pF ?

Sol $C_1 = \dfrac{\epsilon_0\, \cancel{20 \times 10^{-2}} \times \cancel{10^{-12}} \times 10^{-2}}{\cancel{10 \times 10^{-2}} \times \cancel{10^{-12}}}$

$C_1 \Rightarrow \cancel{2\epsilon_0 \times 10^{-2}}$

$= \dfrac{\epsilon_0 A}{d} = \dfrac{\epsilon_0\, 20 \times 10^{-4} \times 10}{10 \times 10^{-2}}$

$= 20\epsilon_0 \times 10^{-2}$

$C_2 = \dfrac{\epsilon_0\, 20 \times 10^{-4} \times 2 \times 10}{10 \times 10^{-2}}$

$C_2 \Rightarrow 40\epsilon_0 \times 10^{-2}$

$$C_3 = \frac{\epsilon_0 \epsilon_r A}{d} = \frac{\epsilon_0 \times 4 \times 20 \times 10^{-4} \times 10}{20 \times 10^{-2}}$$

$$= 40\epsilon_0 \times 10^{-2}$$

$$C_{eq} = C' + C_3$$

$$C' = \frac{C_1 C_2}{C_1 + C_2} = \frac{20\epsilon_0 \times 10^{-2} \times 40\epsilon_0 \times 10^{-2}}{20\epsilon_0 \times 10^{-2} + 40\epsilon_0 \times 10^{-2}}$$

$$= \frac{800\epsilon_0^2 \times 10^{-4}}{60\epsilon_0 \; 10^{-2}}$$

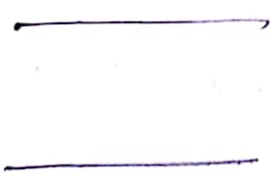
$$C' \Rightarrow \frac{40}{3} \times 10^{-2}$$

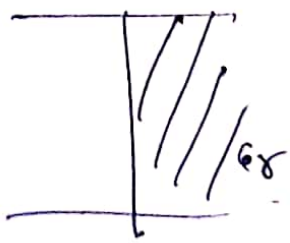
$$C_{eq} = \frac{40}{3} \times 10^{-2}\epsilon_0 + 40\epsilon_0 \times 10^{-2}$$

$$\Rightarrow \cancel{4\times10^{-2}} \Big[\quad + \quad 40 \times \epsilon_0 \times 10^{-2} \left[\frac{4}{3} \right]$$

$$= 4.7 \text{ pF} \; //.$$

Que. C_0 is Capacitance of parallel plate capacitor with air as dielectric. If half of entire gap is filled with a dielectric of permittivity ϵ_r. The expression for Modified capacitance is —

Sol a) 

b) 

$$C_0' = \frac{\varepsilon_0 A/2}{d} + \frac{\varepsilon_0 \varepsilon_r A/2}{d}$$

$$C_0 = \frac{\varepsilon_0 A}{d}$$

$$\Rightarrow \frac{C_0}{2}(1+\varepsilon_r) \;//.$$

<u>Que.</u> A capacitance is made of dielectric of Dielectric Const. of 2.26 and a dielectric breakdown strength of 50kV/cm. If the rectangular plate of capacitor have width of 20cm and length of 40cm. The max. electric charge in μC in capacitor is

<u>Sol</u>:- Dielectric Breakdown = Maxm electric field that a dielectric can tolerate

Relative permittivity also called Dielectric Const

$\varepsilon_r = 2.26$

$A = 20 \times 40 \text{ cm}^2$

$Q = CV$

$$Q = \frac{\varepsilon A}{d} V$$

$$= \frac{2.26 \times 800 \times 10^{-4}}{\cancel{d}} \; \frac{50 \times 10^3}{10^{-2}} \cancel{d} \times \varepsilon_0$$

$$= 2.26 \times 8 \times 5 \times 10^{4} \times \varepsilon_0$$

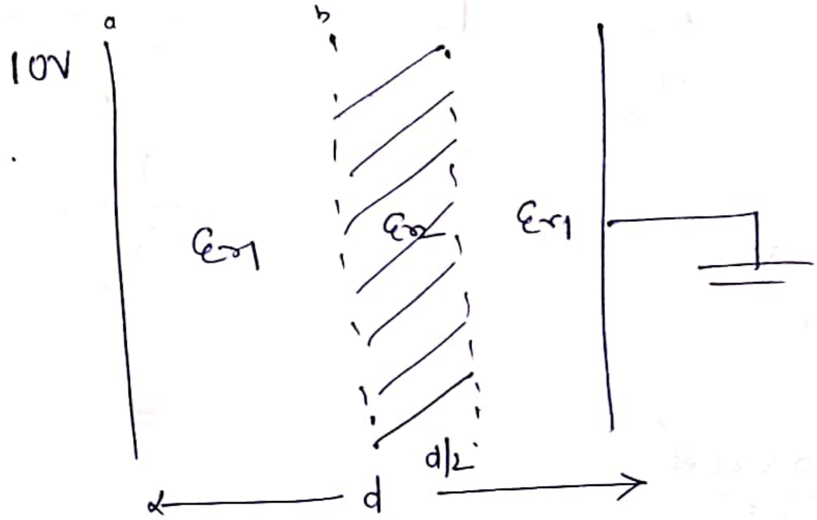
$$\Rightarrow 90.4 \times 10^4 \times \varepsilon_0$$

$$\Rightarrow 800 \times 10^4 \times 10^{-12}$$

$$\Rightarrow 8 \times 10^{-6}\ C = 8\ \mu C //.$$

Que. A parallel plate Capacitor Consisting two dielectric Material as shown in fig. The middle dielectric slab is placed symmetrically wrt plates. If the potential difference betn one of the plate and nearest surface of dielectric interface is 2V then ratio of $\frac{\epsilon_{r1}}{\epsilon_{r2}}$.

Sol



$Q = CV$

$C_1 = \frac{\epsilon_0 \epsilon_{r1} A}{d/4}$ $\qquad$ $C_3 = \frac{\epsilon_0 \epsilon_{r1} A}{d/4}$

$C_2 = \frac{\epsilon_0 \epsilon_{r2} A}{d/2}$

$C_{eq1} = \frac{C_1 C_2}{C_1 + C_2}$

$= \dfrac{\frac{\epsilon_0 \epsilon_{r1} A}{d/4} \frac{\epsilon_0 \epsilon_{r2} A}{d/2}}{\frac{\epsilon_0 A \epsilon_{r1}}{\frac{d}{4}} + \frac{\epsilon_0 \epsilon_{r2} A}{\frac{d}{2}}}$

$\Rightarrow \frac{\epsilon_0 \epsilon_{r1} \epsilon_{r2}}{\frac{d}{2}\epsilon_{r1} + \frac{d}{4}\epsilon_{r2}}$

$V_{ab} = V_b - V_a$

$= 2 - 10$

$= -8V$

$C = Q/V$

$C_1 = \frac{Q}{-8}$ $\Rightarrow$

$$C_{eq1} = \frac{2\epsilon_0 \epsilon_{r1} \epsilon_{r2} A}{(\epsilon_{r1} + \epsilon_{r2}) d}$$

$$C_1 = C_3$$

$$C \propto \frac{1}{V}$$

$$\frac{C_1}{C_{eq}} = \frac{V}{V_1} = \frac{10}{2}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C_{eq}} = \frac{2}{C_1} + \frac{1}{C_2} \quad (1)$$

$$\frac{C_1}{C_{eq}} = 5$$

$$\Rightarrow \frac{\frac{2\cancel{A}\cancel{\epsilon_0}\epsilon_{r1}\cancel{A}}{\cancel{d}}}{\frac{\cancel{2}\cancel{\epsilon_0}\epsilon_{r1}\epsilon_{r2}\cancel{A}}{(\epsilon_{r1}+\epsilon_{r2})\cancel{d}}} = 5$$

$$\Rightarrow \frac{2\epsilon_{r1}(\epsilon_{r1}+\epsilon_{r2})}{\epsilon_{r1}\epsilon_{r2}} = 5$$

$$\Rightarrow 2\epsilon_{r1} + 2\epsilon_{r2} = 5\epsilon_{r2}$$

$$2\epsilon_{r1} = 3\epsilon_{r2}$$

$$\frac{\epsilon_{r1}}{\epsilon_{r2}} = 3/2 \quad //.$$

or $\quad CV = \text{Const}$

$$C_1V_1 = C_2V_2$$

drop across Middle

$$\left(\frac{4\epsilon_0\epsilon_{r1}A}{d}\right)2 = \left(\frac{2\epsilon_0\epsilon_{r2}A}{d}\right)6$$

$$\frac{\epsilon_{r1}}{\epsilon_{r2}} = \frac{3}{2} \;//$$

Que 30

Isolated Electrically = charge stored remain Same

sol

$$Q = CV$$

$$E = \frac{1}{2}CV^2 = \frac{1}{2}\frac{\epsilon_0 A}{d}V^2$$

$$= \frac{1}{2}QV$$

$$= \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q^2 d}{\epsilon_0 A} = k\frac{d}{2} \checkmark$$

$$E' = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2\,2d}{\epsilon_0 A} = kd \checkmark$$

$$E' = d = 2E \;//$$

OR

$$CV = \text{Constant}$$

$$C_1V_1 = C_2V_2$$

$$V_1 = \frac{V_2}{2}$$

$$E_2 = \frac{1}{2}C_2(2V_1)^2 = \frac{1}{2}\frac{4}{2}V_1^2C_1$$

$$= 2E_1 \;//.$$

QII.

Sol

$A = 10^{-4} m^2$

$d = 10^{-3} m$

$V = 0.5 V$

$f = 3.6 GHz$

$\varepsilon_0 = \frac{10^{-9}}{36\pi}$

$\vec{J_D} = \frac{\partial \vec{D}}{\partial t}$

$|\vec{J_D}| = |j\omega\varepsilon_0 \vec{E}|$

$= \omega \varepsilon_0 E$

$= 2\pi \times 3.6 \times \cancel{10^{9}} \times \frac{\cancel{10^{-9}}}{36\pi} \times \frac{0.5}{10^{-3}}$

$\Rightarrow 2 \times \frac{3.6}{36} \times \frac{1}{2} \times 10^{3}$

$= + 10^2 \times 1$

$= 100$

$I_D = J_D A$

$= 100 \times 10^{-4}$

$= \frac{1\cancel{0}\cancel{0}}{100\cancel{0}\cancel{0}}$

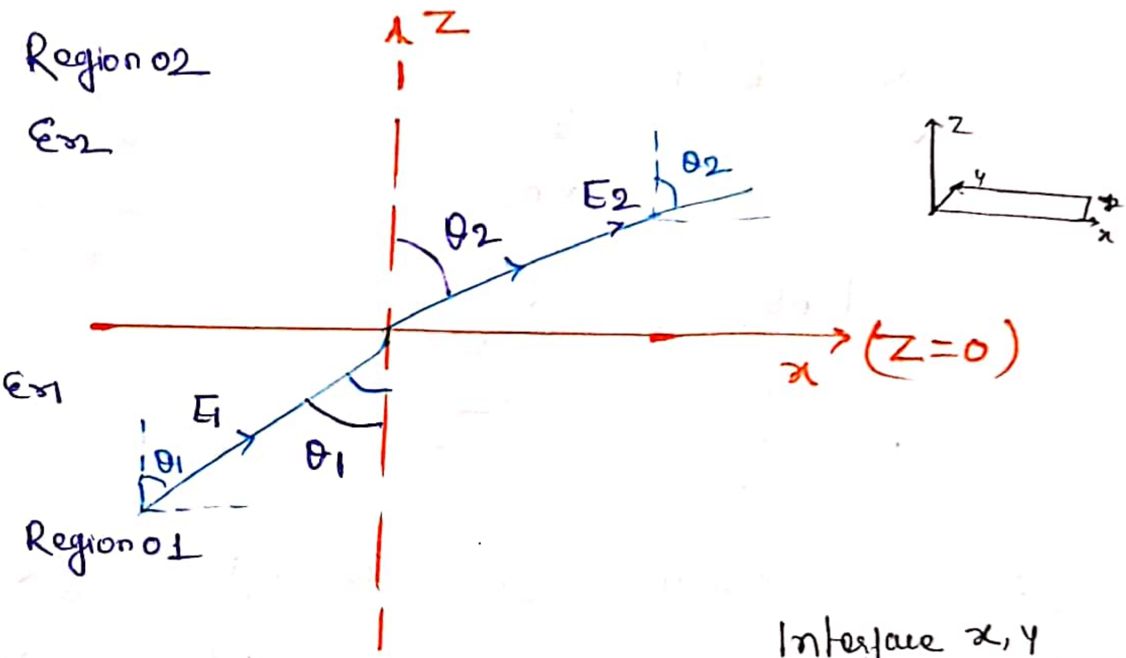
$\Rightarrow 10 mA$ //

* Electric Boundary Condition

① $E_{tan1} = E_{tan2}$ { tangential Comp. of Electric field

② $D_{n1} = D_{n2}$ { Normal Component of flux

③ $(D_{n2} - D_{n1}) = \rho_s$ { final Reg. − Initial transmit − Incident Reg.



Interface x, y
tangential Comp. - xy Parallel to Parallel
Normal comp. - z

$E_{n1} = E_1 \cos\theta_1$

$E_{t1} = E_1 \sin\theta_1$

$$\tan\theta_1 = \frac{E_{t1}}{E_{n1}} \quad \text{—— ①}$$

$E_{n2} = E_2 \cos\theta_2$

$E_{t2} = E_2 \sin\theta_2$

$$\tan\theta_2 = \frac{E_{t2}}{E_{n2}} \quad \text{—— ②}$$

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{E_{t1}}{E_{n1}} \times \frac{E_{n2}}{E_{t2}}$$

from boundary Condition

$$E_{t1} = E_{t2}$$

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{E_{n2}}{E_{n1}}$$

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{D_{n2}\,\epsilon_{r1}}{D_{n1}\,\epsilon_{r2}}$$

from Boundary Condⁿ

$$D_{n1} = D_{n2}$$

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_{r1}}{\epsilon_{r2}}}$$

Que 9. Medium 1 $\epsilon_1 = 15\epsilon_0$ f/m left of x=0 plane

Medium 2 $\epsilon_2 = 25\epsilon_0$ f/m right of x=0 plane

E_1 in Medium 1 $2.0\,\vec{a}_x - 3.0\,\vec{a}_y + 1\cdot\hat{a}_z$

E_2 in Med 2 = ?

Sol

M1 | x=0 (yz plan) | M2

$\epsilon_1 = 15\epsilon_0$ | | $\epsilon_2 = 25\epsilon_0$

$E_1 = 2.0\,\vec{a}_x - 3.0\,\hat{a}_y + 1\,\hat{a}_z$

~~Etan~~ $E = E_{tan} + E_{nor}$

$= (-3.0\,\hat{a}_y + \hat{a}_z) + 2.0\,\hat{a}_x$

By Boundary condn

$$E_{tan1} = E_{tan2}$$

$$E_{tan2} = -3\hat{a_y} + \hat{a_z}$$

also, $D_{n1} = D_{n2}$

$$\epsilon_0 15\, E_{n1} = D_{n2}$$

$$2\hat{a_x}\, \epsilon_0 15 = \epsilon_0 25\, E_{n2}$$

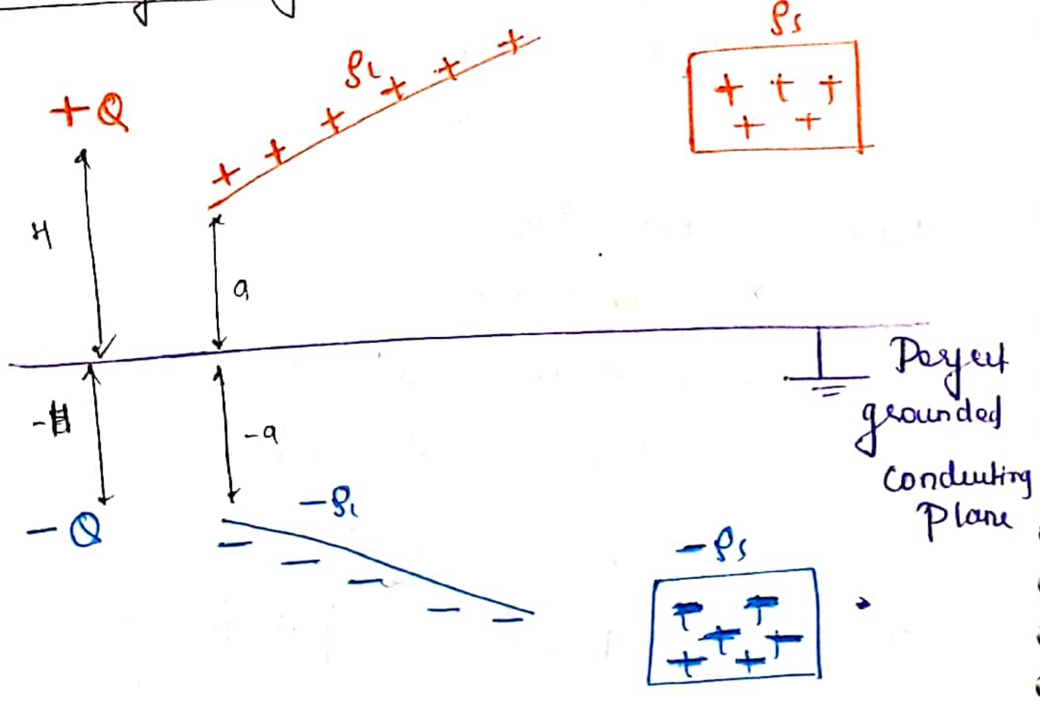
$$E_{n2} = \frac{6}{5}\hat{a_x}$$

$$E_{n2} = 1.2\hat{a_x}$$

Total $E_2 = 1.2\hat{a_x} - 3\hat{a_y} + \hat{a_z}$ //.

Q12.

* Method of Images



1) Image theory state that " the given charge configuration above ~~an~~ infinite grounded perfect conductor may be replaced by a charge configuration itself. its image and equipotential surface in place of conducting plane.

11) Condition that must be satisfied in the method of image.

a) The image charges must be located in conducting region.

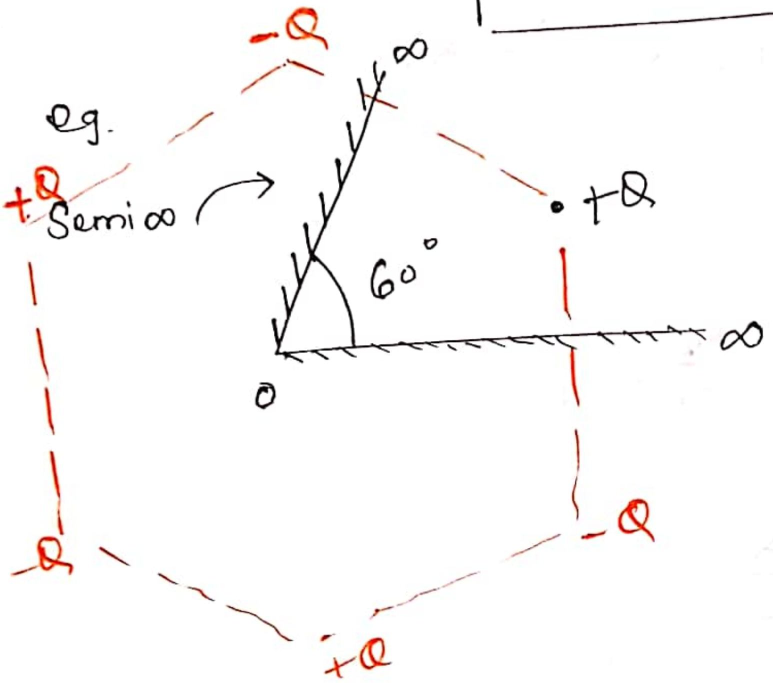
b) The image charges must be located on the conducting surface, the potential is zero or constant

c) In General. when Method of Image is used

For a system consisting of a point charge between two semi infinite conducting plane inclined at an angle of ϕ°. The no. of image is given by -

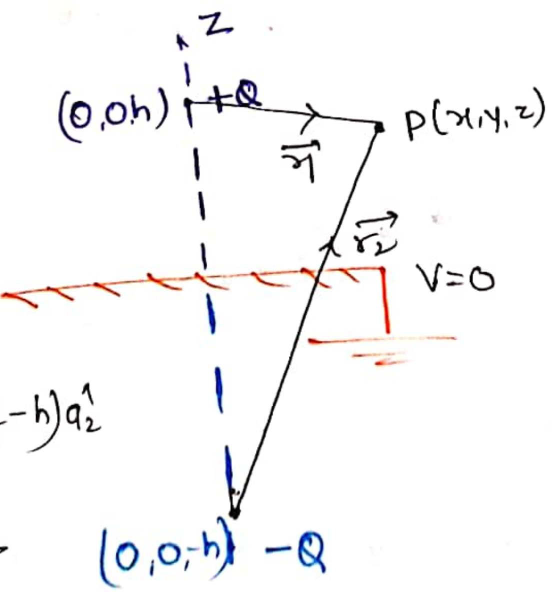
$$\boxed{N = \frac{360^\circ}{\phi^\circ} - 1}$$

eg.



$$N = \frac{360^\circ}{60} - 1$$
$$= 5$$

* Electric field Intensity due to point charge placed above a perfect grounded plane:-



$$\vec{r}_1 = x\hat{a_x} + y\hat{a_y} + (z-h)\hat{a_z}$$

$$|\vec{r}_1| = \sqrt{x^2+y^2+(z-h)^2}$$

$$|\vec{r}_2| = \sqrt{x^2+y^2+(z+h)^2}$$

$$\vec{r}_2 = x\hat{a_x} + y\hat{a_y} + (z+h)\hat{a_z}$$

$$E = \frac{kQ}{R^2}\hat{a}_R$$

$$E_\phi = \frac{kQ}{R^3}\vec{R}$$

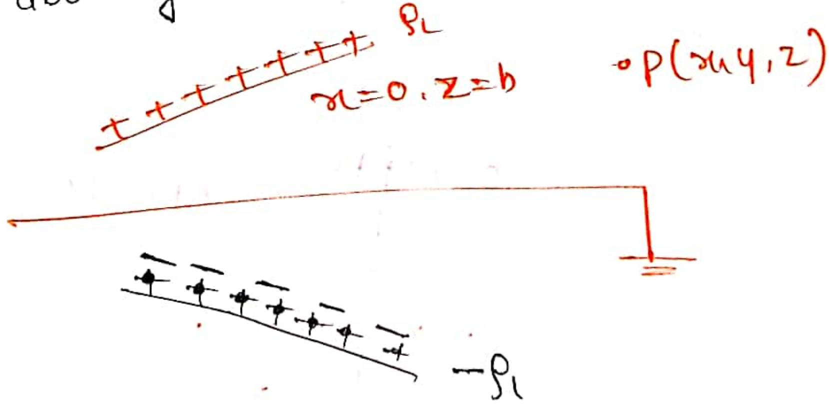
$$E_1 = \frac{kQ\vec{r}_1}{r_1^3} \quad \text{and} \quad E_2 = \frac{-kQ\vec{r}_2}{r_2^3}$$

$$E = E_1 + E_2$$

$$\boxed{E = kQ\left[\frac{\vec{r}_1}{r_1^3} - \frac{\vec{r}_2}{r_2^3}\right]}$$

$$\boxed{V = \frac{kQ}{r_1} - \frac{kQ}{r_2}}$$

* Electric field intensity due to infinite line charge placed above grounded conducting plane.



E at P due to ρ_L

$$\vec{E} = \frac{\rho_L}{2\pi\epsilon_0 \rho}\hat{a}_\rho = \frac{\rho_L}{2\pi\epsilon_0 \rho_1^2}\vec{\rho_1}$$

$$= \frac{\rho_L\left((x-0)\hat{a}_x + (y-y)\hat{a}_y + (z-h)\hat{a}_z\right)}{2\pi\epsilon_0\left(x^2+(z-h)^2\right)}$$

$$\rho_2 = x\,\hat{a_x} + (z+h)\,\hat{a_z}$$

$$|\rho_2| = \sqrt{x^2 + (z+h)^2}$$

$$\vec{E} = \vec{E_1} + \vec{E_2}$$

$$\vec{E} = \frac{\rho_L}{2\pi\epsilon_0 \rho_1^2}\vec{\rho_1} - \frac{\rho_L}{2\pi\epsilon_0 \rho_2^2}\vec{\rho_2}$$

Que. force on a point charge $+Q$ kept at a distance d from the surface of infinite grounded metal plate in a medium of permittivity ϵ is

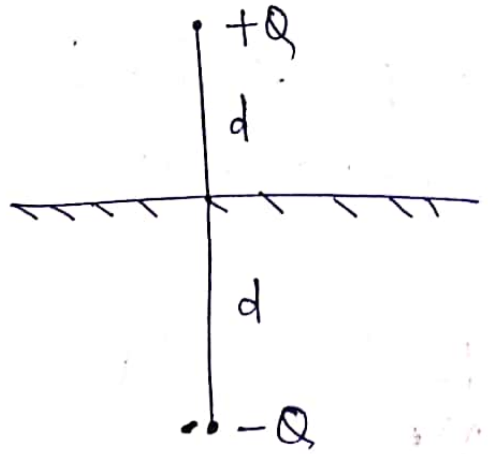
a) 0

b) $\frac{Q^2}{16\pi\epsilon d^2}$ away from plate

c) $\frac{Q^2}{16\pi\epsilon d^2}$ Toward plane

d) $\frac{Q^2}{4\pi\epsilon d^2}$ toward the plate

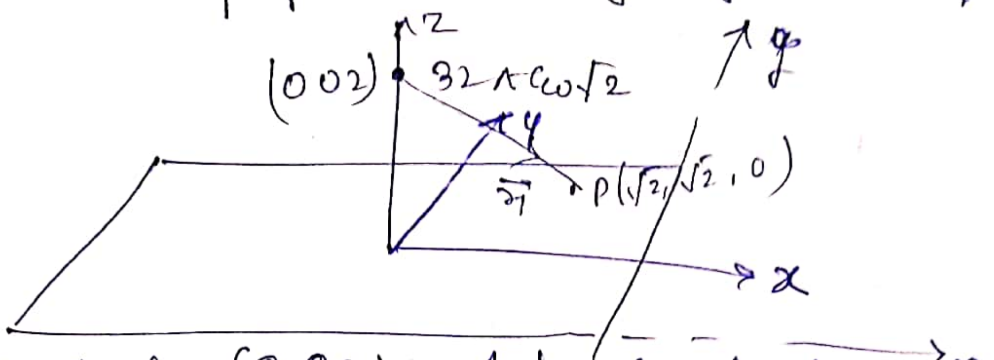
Sol)



$$|F| = \frac{Q^2}{4\pi\epsilon_0 (2d)^2}$$

$$= \frac{Q^2}{16\pi\epsilon d^2} \quad //$$

Que A perfectly Conducting Metal plate is placed in xy plane. A charge of $32\pi\epsilon_0\sqrt{2}$



at Coordinate (0 0 2). At Coordinate $(\sqrt{2}, \sqrt{2}, 0)$ The Electric field Vector will be

Sol

a) $2\sqrt{2}\,\hat{k}$

b) $2\hat{k}$

c) $-2\hat{k}$

d) $-2\sqrt{2}\,\hat{k}$

$$E_1 = \frac{1}{4\pi\epsilon_0}\,\frac{32\pi\epsilon_0\sqrt{2}\,(\sqrt{2}\,\hat{a_x} + \sqrt{2}\,\hat{a_y} - 2\hat{a_z})}{(\sqrt{2}^2 + \sqrt{2}^2 + 2^2)^{3/2}}$$

$$E_2 = \frac{-1}{4\pi\epsilon_0}\,\frac{32\pi\epsilon_0\sqrt{2}\,(\sqrt{2}\,\hat{a_x} + \sqrt{2}\,\hat{a_y} + 2\hat{a_z})}{((\sqrt{2})^2 + \sqrt{2}^2 + 2^2)^{3/2}}$$

$$= -\left(\frac{2 \cdot 8\sqrt{2} + 16\sqrt{2}}{(2+2+4)^{3/2}}\right) \qquad (E = E_1 + E_2)$$

$$\Rightarrow \frac{\cancel{-2\sqrt{2}}}{\cancel{8\sqrt{8}}}\,\hat{k} \;// \qquad -2\hat{k} \;//$$

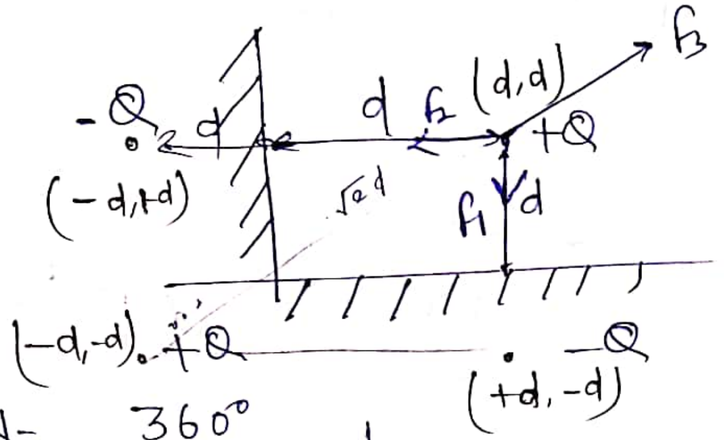
$\Rightarrow$

Que. Two Semi Infinite Conducting sheets are placed at right angle to each other a Point charge of $+Q$ is placed at a distance d from both the sheet. The Net force on the charge is

$\frac{Q^2}{4\pi\epsilon_0 d^2}k$ where k is ____

a) 0 b) $-\frac{\hat{i}}{4}-\frac{\hat{j}}{4}$ c) $-\frac{\hat{i}}{8}-\frac{\hat{j}}{8}$ d) $\frac{1-2\sqrt{2}}{8\sqrt{2}}\hat{i}+\frac{1-2\sqrt{2}}{8\sqrt{2}}\hat{j}$

Sol



$$N = \frac{360^\circ}{90^\circ} - 1$$

$$N = 3$$

$$\cancel{F_{net}} = \cancel{\frac{Q^2}{4\pi\epsilon_0 d^2}}\left[\cancel{-\frac{1}{4}} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{(2\sqrt{2})^2}}\right]$$

$$\Rightarrow \cancel{\frac{Q^2}{4\pi\epsilon_0 d^2}}\left[\cancel{\frac{-2-2+1}{8}}\right]$$

$$F_1 = \frac{1}{4\pi\epsilon_0}\left[\frac{-Q^2\left[(d-d)\hat{a_x} + d-(-d)\hat{a_y}\right]}{\left[(d-d)^2+(d+d)^2\right]^{3/2}}\right]$$

$$F_2 = \frac{1}{4\pi\epsilon_0}\left[\frac{-Q^2\left[(d+d)\hat{a_x} + (d-d)\hat{a_y}\right]}{\left[(d-d)^2 + (d+d)^2\right]^{3/2}}\right]$$

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$$\vec{F_3} = \frac{1}{4\pi\epsilon_0} Q^2 \frac{\left[(d+d)\hat{a_x} + (d+d)\hat{a_y}\right]}{\left((d+d)^2 + (d+d)^2\right)^{3/2}}$$

$$F_{net} = F_1 + F_2 + F_3$$

$$= \frac{Q^2}{4\pi\epsilon_0}\left[\frac{-2d\hat{a_y}}{(4d^2)^{3/2}} - \frac{2d\hat{a_x}}{(4d^2)^{3/2}} + \frac{2d\hat{a_x} + 2d\hat{a_y}}{[4d^2+4d^2]^{3/2}}\right]$$

$$\Rightarrow \frac{Q^2}{4\pi\epsilon_0}\left[\frac{-2d\hat{a_y}}{8d^3} - \frac{2d\hat{a_x}}{8d^3} + \frac{2d\hat{a_x} + 2d\hat{a_y}}{(8d^2)^{3/2}}\right]$$

$$\Rightarrow \frac{Q^2}{4\pi\epsilon_0 d^2}\left[\frac{1 - 2\sqrt{2}\,\hat{a_x}}{8\sqrt{2}} + \frac{1 - 2\sqrt{2}\,\hat{a_y}}{8\sqrt{2}}\right]$$