6.1 INTRODUCTION

As we studied in previous chapter, the Laplace transform is an important tool for analysis of continuous time signals and systems. Similarly, z-transforms enables us to analyze discrete time signals and systems in the z-domain.

Like, the Laplace transform, it is also classified as bilateral z-transform and unilateral z-transform.

The bilateral or two-sided z-transform is used to analyze both causal and non-causal LTI discrete systems, while the unilateral z-transform is defined only for causal signals.

NOTE :
The properties of z-transform are similar to those of the Laplace transform.

6.1.1 The Bilateral or Two-Sided z-transform

The z-transform of a discrete-time sequence $x[n]$, is defined as

$$X(z) = \mathcal{Z}\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n} \tag{6.1.1}$$

Where, $X(z)$ is the transformed signal and $\mathcal{Z}$ represents the z -transformation. z is a complex variable. In polar form, z can be expressed as

$$z = re^{j\Omega}$$

where r is the magnitude of z and Ω is the angle of z. This corresponds to a circle in z plane with radius r as shown in figure 6.1.1 below

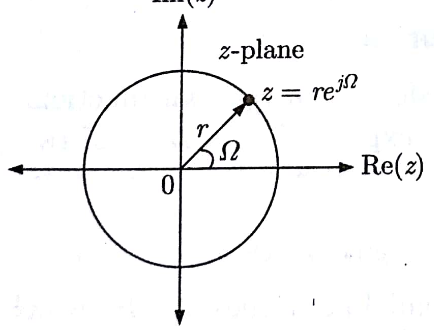


Figure 6.1.1 z-plane

NOTE :
The signal $x[n]$ and its z-transform $X(z)$ are said to form a z-transform pair de:

$$x[n] \xleftrightarrow{\mathcal{Z}} X(z)$$

6.1.2 The Unilateral or One-sided z-transform

The z-transform for causal signals and systems is referred to as the unilateral z-transform. For a causal sequence

$$x[n] = 0, \text{ for } n < 0$$

Therefore, the unilateral z-transform is defined as

$$X(z) = \sum_{n=0}^{\infty} x[n]\, z^{-n} \tag{6.1.2}$$

NOTE :
For causal signals and systems, the unilateral and bilateral z-transform are the same.

6.2 EXISTENCE OF Z-TRANSFORM

Consider the bilateral z-transform given by equation (6.1.1)

$$X[z] = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}$$

The z-transform exists when the infinite sum in above equation converges. For this summation to be converged $|x[n]\, z^{-n}|$ must be absolutely summable.

Substituting $z = re^{j\Omega}$

$$X[z] = \sum_{n=-\infty}^{\infty} x[n]\,(re^{j\Omega})^{-n}$$

or,

$$X[z] = \sum_{n=-\infty}^{\infty} \{x[n]\, r^{-n}\}\, e^{-j\Omega n}$$

Thus for existence of z-transform

$$|X(z)| < \infty$$

$$\sum_{n=-\infty}^{\infty} x[n]\, r^{-n} < \infty \tag{6.2.1}$$

6.3 REGION OF CONVERGENCE

The existence of z-transform is given from equation (6.2.1). The values of r for which $x[n]\, r^{-n}$ is absolutely summable is referred to as region of convergence. Since, $z = re^{j\Omega}$ so $r = |z|$. Therefore we conclude that the range of values of the variable $|z|$ for which the sum in equation (6.1.1) converges is called the region of convergence. This can be explained through the following examples.

6.3.1 Poles and Zeros of Rational z-transforms

The most common form of z-transform is a rational function. Let $X(z)$ be the z-transform of sequence $x[n]$, expressed as a ratio of two polynomials $N(z)$ and $D(z)$.

$$X(z) = \frac{N(z)}{D(z)}$$

The roots of numerator polynomial i.e. values of z for which $X(z) = 0$ is referred to as zeros of $X(z)$. The roots of denominator polynomial for which $X(z) = \infty$ is referred to as poles of $X(z)$. The representation of $X(z)$ through its poles and zeros in the z-plane is called pole-zero plot of $X(z)$.
For example consider a rational transfer function $X(z)$ given as

$$H(z) = \frac{z}{z^2 - 5z + 6}$$

$$= \frac{z}{(z-2)(z-3)}$$

Now, the zeros of $X(z)$ are roots of numerator that is $z = 0$ and poles are roots of equation $(z-2)(z-3) = 0$ which are given as $z = 2$ and $z = 3$. The poles and zeros of $X(z)$ are shown in pole-zero plot of figure 6.3.1.

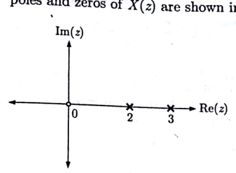


Figure 6.3.1 Pole-zero plot of $X(z)$

NOTE :
In pole-zero plot poles are marked by a small cross '×' and zeros are marked by a small dot 'o' as shown in figure 6.3.1.

6.3.2 Properties of ROC

The various properties of ROC are summarized as follows. These properties can be proved by taking appropriate examples of different DT signals.

PROPERTY 1

The ROC is a concentric ring in the z-plane centered about the origin.

PROOF :
The z-transform is defined as

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}$$

Put $z = re^{j\Omega}$

$$X(z) = X(re^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n]\, r^{-n}\, e^{-j\Omega n}$$

$X(z)$ converges for those values of z for which $x[n]\, r^{-n}$ is absolutely summbable that is

$$\sum_{n=-\infty}^{\infty} x[n]\, r^{-n} < \infty$$

Thus, convergence is dependent only on r, where, $r = |z|$
The equation $z = re^{j\Omega}$, describes a circle in z-plane. Hence the ROC will consists of concentric rings centered at zero.

PROPERTY 2

The ROC cannot contain any poles.

PROOF :
ROC is defined as the values of z for which z-transform $X(z)$ converges. We know that $X(z)$ will be infinite at pole, and, therefore $X(z)$ does not converge at poles. Hence the region of convergence does not include any pole.

PROPERTY 3

If $x[n]$ is a finite duration two-sided sequence then the ROC is entire z-plane except at $z = 0$ and $z = \infty$.

PROOF :

A sequence which is zero outside a finite interval of time is called 'finite duration sequence'. Consider a finite duration sequence $x[n]$ shown in figure 6.3.2a; $x[n]$ is non-zero only for some interval $N_1 \le n \le N_2$.

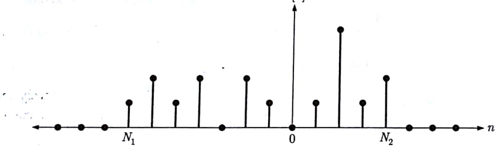


Figure 6.3.2a A Finite Duration Sequence

The z-transform of $x[n]$ is defined as

$$X(z) = \sum_{n=N_1}^{N_2} x[n] z^{-n}$$

This summation converges for all finite values of z. If N_1 is negative and N_2 is positive, then $X(z)$ will have both positive and negative powers of z. The negative powers of z becomes unbounded (infinity) if $|z| \to 0$. Similarly positive powers of z becomes unbounded (infinity) if $|z| \to \infty$. So ROC of $X(z)$ is entire z-plane except possible $z = 0$ and/or $z = \infty$.

NOTE :

Both N_1 and N_2 can be either positive or negative.

PROPERTY 4

If $x[n]$ is a right-sided sequence, and if the circle $|z| = r_0$ is in the ROC, then all values of z for which $|z| > r_0$ will also be in the ROC.

PROOF :

A sequence which is zero prior to some finite time is called the *right-sided sequence.* Consider a right-sided sequence $x[n]$ shown in figure 6.3.2b; that is;

$$x[n] = 0 \text{ for } n < N_1.$$

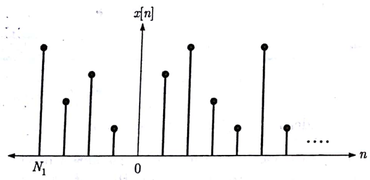


Figure 6.3.2b A Right - Sided Sequence

Let the z-transform of $x[n]$ converges for some value of $|z|$ (i.e. $|z| = r_0$). From the condition of convergence we can write

$$\left|\sum_{n=-\infty}^{\infty} x[n] z^{-n}\right| < \infty$$

$$\sum_{n=-\infty}^{\infty} |x[n]| r_0^{-n} < \infty$$

The sequence is right sided, so limits of above summation changes as

$$\sum_{n=N_1}^{\infty} |x[n]| r_0^{-n} < \infty \qquad (6.3.1)$$

Now if we take another value of z as $|z| = r_1$ with $r_1 < r_0$, then $x[n] r_1^{-n}$ decays faster than $x[n] r_0^{-n}$ for increasing n. Thus we can write

$$\sum_{n=N_1}^{\infty} |x[n]| z^{-n} = \sum_{n=N_1}^{\infty} |x[n]| z^{-n} r_0^{-n} r_0^{n}$$

$$= \sum_{n=N_1}^{\infty} |x[n]| r_0^{-n} \left(\frac{z}{r_0}\right)^{-n} \qquad (6.3.2)$$

From equation (6.3.1) we know that $x[n] r_0^{-n}$ is absolutely summable. Let, it is bounded by some value M_x, then equation (6.3.2) becomes as

$$\sum_{n=N_1}^{\infty} |x[n]| z^{-n} \le M_x \sum_{n=N_1}^{\infty} \left(\frac{z}{r_0}\right)^{-n} \qquad (6.3.3)$$

The right hand side of above equation converges only if

$$\left|\frac{z}{r_0}\right| > 1 \text{ or } |z| > r_0$$

Thus, we conclude that if the circle $|z| = r_0$ is in the ROC, then all values of z for which $|z| > r_0$ will also be in the ROC. The ROC of a right-sided sequence is illustrated in figure 6.3.2c.

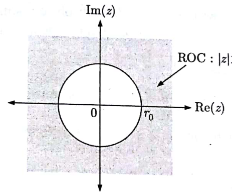


Figure 6.3.2c ROC of a right-sided sequence

PROPERTY 5

If $x[n]$ is a left-sided sequence, and if the circle $|z| = r_0$ is in the ROC, then all values of z for which $|z| < r_0$ will also be in the ROC.

PROOF :

A sequence which is zero after some finite time interval is called a 'left-sided signal'. Consider a left-sided signal $x[n]$ shown in figure 6.3.2d; that is $x[n] = 0$ for $n > N_2$.

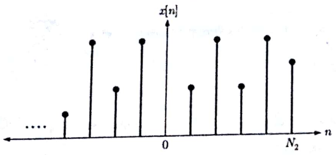


Figure 6.3.2d A left-sided sequence

Let z-transform of $x[n]$ converges for some values of $|z|$ (i.e. $|z| = r_0$). From the condition of convergence we write

$$\left|\sum_{n=-\infty}^{\infty} x[n]\, z^{-n}\right| < \infty$$

or
$$\sum_{n=-\infty}^{\infty} |x[n]|\, r_0^{-n} < \infty \qquad (6.3.4)$$

The sequence is left sided, so the limits of summation changes as

$$\sum_{n=-\infty}^{N_2} |x[n]|\, r_0^{-n} < \infty \qquad (6.3.5)$$

Now if take another value of z as $|z| = r_1$, then we can write

$$\sum_{n=-\infty}^{N_2} |x[n]|\, z^{-n} = \sum_{n=-\infty}^{N_2} |x[n]|\, z^{-n} r_0^{-n} r_0^{n}$$

$$= \sum_{n=-\infty}^{N_2} |x[n]|\, r_0^{-n} \left(\frac{r_0}{z}\right)^n \qquad (6.3.6)$$

From equation (6.3.4), we know that $x[n]\, r_0^{-n}$ is absolutely summable. Let it is bounded by some value M_x, then equation (6.3.6) becomes as

$$\sum_{n=-\infty}^{N_2} |x[n]|\, z^{-n} \le M_x \sum_{n=-\infty}^{N_2} \left(\frac{r_0}{z}\right)^n$$

The above summation converges if $\left|\frac{r_0}{z}\right| > 1$ (because n is increasing negatively), so $|z| < r_0$ will be in ROC.

The ROC of a left-sided sequence is illustrated in figure 6.3.2e.

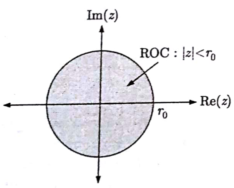


Figure 6.3.2e ROC of a Left - Sided Sequence

PROPERTY 6

If $x[n]$ is a two-sided signal, and if the circle $|z| = r_0$ is in the ROC, then the ROC consists of a ring in the z-plane that includes the circle $|z| = r_0$

PROOF :

A sequence which is defined for infinite extent for both $n > 0$ and $n < 0$ is called 'two-sided sequence'. A two-sided signal $x[n]$ is shown in figure 6.3.2f.

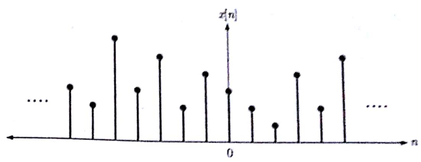


Figure 6.3.2f A Two - Sided Sequence

For any time N_0, a two-sided sequence can be divided into sum of left-sided and right-sided sequences as shown in figure 6.3.2g.

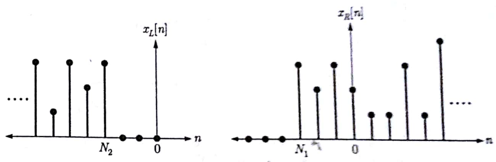
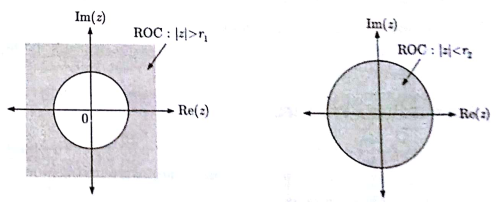


Figure 6.3.2g A Two Sided Sequence Divided into Sum of a Left - Sided and Right - Sided Sequence

The z-transform of $x[n]$ converges for the values of z for which the transform of both $x_R[n]$ and $x_L[n]$ converges. From property 4, the ROC of a right-sided sequence is a region which is bounded on the inside by a circle and extending outward to infinity i.e. $|z| > r_1$. From property 5, the ROC of a left sided sequence is bounded on the outside by a circle and extending inward to zero i.e. $|z| < r_2$. So the ROC of combined signal includes intersection of both ROCs which is ring in the z-plane.

The ROC for the right-sided sequence $x_R[n]$, the left-sequence $x_L[n]$ and their combination which is a two sided sequence $x[n]$ are shown in figure 6.3.2h.



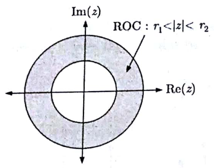


Figure 6.3.2h ROC of a left-sided sequence, a right-sided sequence and two sided sequence

PROPERTY 7

If the z-transform $X(z)$ of $x[n]$ is rational, then its ROC is bounded by poles or extends to infinity.

PROOF :

The exponential DT signals also have rational z-transform and the poles of $X(z)$ determines the boundaries of ROC.

PROPERTY 8

If the z-transform $X(z)$ of $x[n]$ is rational and $x[n]$ is a right-sided sequence then the ROC is the region in the z-plane outside the outermost pole i.e. ROC is the region outside a circle with a radius greater than the magnitude of largest pole of $X(z)$.

PROOF :

This property can be be proved by taking property 4 and 7 together.

PROPERTY 9

If the z-transform $X(z)$ of $x[n]$ is rational and $x[n]$ is a left-sided sequence then the ROC is the region in the z-plane inside the innermost pole i.e. ROC is the region inside a circle with a radius equal to the smallest magnitude of poles of $X(z)$.

PROOF :

This property can be be proved by taking property 5 and 7 together.

Z-Transform of Some Basic Functions

Z-transform of basic functions are summarized in the Table 6.1 with their respective ROCs.

6.4 THE INVERSE Z-TRANSFORM

Let $X(z)$ be the z-transform of a sequence $x[n]$. To obtain the sequence $x[n]$ from its z-transform is called the inverse z-transform. The inverse z -transform is given as

TABLE 6.1 : z-Transform of Basic Discrete Time Signals

	DT sequence $x[n]$	z-transform	ROC
1.	$\delta[n]$	1	entire z-plane
2.	$\delta[n-n_0]$	z^{-n_0}	entire z-plane, except $z=0$
3.	$u[n]$	$\frac{1}{1-z^{-1}}=\frac{z}{z-1}$	$\lvert z\rvert>1$
4.	$\alpha^n u[n]$	$\frac{1}{1-\alpha z^{-1}}=\frac{z}{z-\alpha}$	$\lvert z\rvert>\lvert\alpha\rvert$
5.	$\alpha^{n-1}u[n-1]$	$\frac{z^{-1}}{1-\alpha z^{-1}}=\frac{1}{z-\alpha}$	$\lvert z\rvert>\lvert\alpha\rvert$
6.	$nu[n]$	$\frac{z^{-1}}{(1-z^{-1})^2}=\frac{z}{(z-1)^2}$	$\lvert z\rvert>1$
7.	$n\alpha^n u[n]$	$\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}=\frac{\alpha z}{(z-\alpha)^2}$	$\lvert z\rvert>\alpha$
8.	$\cos(\Omega_0 n)u[n]$	$\frac{1-z^{-1}\cos\Omega_0}{1-2z^{-1}\cos\Omega_0+z^{-2}}$ or $\frac{z[z-\cos\Omega_0]}{z^2-2z\cos\Omega_0+1}$	$\lvert z\rvert>1$
9.	$\sin(\Omega_0 n)u[n]$	$\frac{z^{-1}\sin\Omega_0}{1-2z^{-1}\cos\Omega_0+z^{-2}}$ or $\frac{z\sin\Omega_0}{z^2-2z\cos\Omega_0+1}$	$\lvert z\rvert>1$
10.	$\alpha^n\cos(\Omega_0 n)u[n]$	$\frac{1-\alpha z^{-1}\cos\Omega_0}{1-2\alpha z^{-1}\cos\Omega_0+\alpha^2 z^{-2}}$ or $\frac{z[z-\alpha\cos\Omega_0]}{z^2-2\alpha z\cos\Omega_0+\alpha^2}$	$\lvert z\rvert>\lvert\alpha\rvert$
11.	$\alpha^n\sin(\Omega_0 n)u[n]$	$\frac{\alpha z^{-1}\sin\Omega_0}{1-2\alpha z^{-1}\cos\Omega_0+\alpha^2 z^{-2}}$ or $\frac{\alpha z\sin\Omega_0}{z^2-2\alpha z\cos\Omega_0+\alpha^2}$	$\lvert z\rvert>\alpha$
12.	$r\alpha^n\sin(\Omega_0 n+\theta)u[n]$ with $\alpha\in R$	$\frac{A+Bz^{-1}}{1+2\gamma z^{-1}+\alpha^2 z^{-2}}$ or $\frac{z(Az+B)}{z^2+2\gamma z+\gamma^2}$	$\lvert z\rvert\le\lvert\alpha\rvert^{(a)}$

$$x[n] = \frac{1}{2\pi j}\oint X(z)\,z^{n-1}dz$$

This method involves the contour integration, so difficult to solve. There are other commonly used methods to evaluate the inverse z-transform given as follows

1. Partial fraction method
2. Power series expansion

6.4.1 Partial Fraction Method

If $X(z)$ is a rational function of z then it can be expressed as follows.

$$X(z) = \frac{N(z)}{D(z)}$$

It is convenient if we consider $X(z)/z$ rather than $X(z)$ to obtain the inverse z-transform by partial fraction method.

Let $p_1, p_2, p_3 .... p_n$ are the roots of denominator polynomial, also the poles of $X(z)$. Then, using partial fraction method $X(z)/z$ can be expressed as

$$\frac{X(z)}{z} = \frac{A_1}{z-p_1} + \frac{A_2}{z-p_2} + \frac{A_3}{z-p_3} + ... + \frac{A_n}{z-p_n}$$

$$X(z) = A_1\frac{z}{z-p_1} + A_2\frac{z}{z-p_2} + ... + \frac{z}{z-p_n}$$

Now, the inverse z-transform of above equation can be obtained by comparing each term with the standard z-tranform pair given in table 6.1. The values of coefficients $A_1, A_2, A_3 .... A_n$ depends on whether the poles are real & distinct or repeated or complex. Three cases are given as follows

Case I : Poles are Simple and Real

$X(z)/z$ can be expanded in partial fraction as

$$\frac{X(z)}{z} = \frac{A_1}{z-p_1} + \frac{A_2}{z-p_2} + \frac{A_3}{z-p_3} + ... + \frac{A_n}{z-p_n} \quad (6.4.1)$$

where $A_1, A_2, ... A_n$ are calculated as follows

$$A_1 = (z-p_1)\frac{X(z)}{z}\Big|_{z=p_1}$$

$$A_2 = (z-p_2)\frac{X(z)}{z}\Big|_{z=p_2}$$

In general,

$$A_i = (z-p_i)X(z)\big|_{z=p_i} \quad (6.4.2)$$

Case II : If Poles are Repeated

In this case $X(z)/z$ has a different form. Let p_k be the root which repeats l times, then the expansion of equation must include terms

$$\frac{X(z)}{z} = \frac{A_{1k}}{z-p_k} + \frac{A_{2k}}{(z-p_k)^2} + ...$$

$$+\frac{A_{ik}}{(z-p_k)^i} + ... + \frac{A_{lk}}{(z-p_k)^l} \quad (6.4.3)$$

The coefficient A_{ik} are evaluated by multiplying both sides of equation (6.4.3) by $(z-p_k)^l$, differentiating $(l-i)$ times and then evaluating the resultant equation at $z = p_k$.

Thus,

$$C_{ik} = \frac{1}{\boxed{(l-i)}}\frac{d^{l-i}}{dz^{l-i}}\left[(z-p_k)^l\frac{X(z)}{z}\right]\Big|_{z=p_k} \quad (6.4.4)$$

Case III : Complex Poles

If $X(z)$ has complex poles then partial fraction of the $X(z)/z$ can be expressed as

$$\frac{X(z)}{z} = \frac{A_1}{z-p_1} + \frac{A_1^*}{z-p_1^*} \quad (6.4.5)$$

where A_1^* is complex conjugate of A_1 and p_1^* is complex conjugate of z_1. The coefficients are obtained by equation (6.4.2)

6.4.2 Power Series Expansion Method

Power series method is also convenient in calculating the inverse z-transform. The z-transform of sequence $x[n]$ is given as

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\,z^{-n}$$

Now, $X(z)$ is expanded in the following form

$$X(z) = .. + x[-2]\,z^2 + x[-1]\,z^1 + x[0] + x[1]\,z^{-1} + x[2]\,z^{-2} + ...$$

To obtain inverse z-transform(i.e.$x[n]$), represent the given $X(z)$ in the form of above power series. Then by comparing we can get

$$x[n] = \{...x[-2],\ x[-1],\ x[0],\ x[1],\ x[2],...\}$$

6.5 PROPERTIES OF Z-TRANSFORM

The unilateral and bilateral z-transforms possess a set of properties, which are useful in the analysis of DT signals and systems. The proofs of properties are given for bilateral transform only and can be obtained in a similar way for the unilateral transform.

6.5.1 Linearity

Like Laplace transform, the linearity property of z transform states that, the linear combination of DT sequences in the time domain is equivalent to linear combination of their z transform.

Let $\quad x_1[n] \xleftrightarrow{\mathcal{Z}} X_1(z), \quad$ with ROC: R_1

and $\quad x_2[n] \xleftrightarrow{\mathcal{Z}} X_2(z), \quad$ with ROC: R_2

then, $\quad ax_1[n] + bx_2[n] \xleftrightarrow{\mathcal{Z}} aX_1(z) + bX_2(z),$

with ROC: at least $R_1 \cap R_2$

for both unilateral and bilateral z-transform.

PROOF :

The z-transform of signal $\{ax_1[n] + bx_2[n]\}$ is given by equation (6.1.1) as follows

$$\mathcal{Z}\{ax_1[n] + bx_2[n]\} = \sum_{n=-\infty}^{\infty}\{ax_1[n] + bx_2[n]\}\,z^{-n}$$

$$= a\sum_{n=-\infty}^{\infty}x_1[n]\,z^{-n} + b\sum_{n=-\infty}^{\infty}x_2[n]\,z^{-n}$$

$$= aX_1(z) + bX_2(z)$$

Hence, $ax_1[n] + bx_2[n] \xleftrightarrow{\mathcal{Z}} aX_1(z) + bX_2(z)$

ROC : Since, the z-transform $X_1(z)$ is finite within the specified ROC, R_1.

Similarly, $X_2(z)$ is finite within its ROC, R_2. Therefore, the linear combination $aX_1(z)+bX_2(z)$ should be finite at least within region $R_1 \cap R_2$.

NOTE :
In certain cases, due to the interaction between $x_1[n]$ and $x_2[n]$, which may lead to cancellation of certain terms, the overall ROC may be larger than the intersection of the two regions. On the other hand, if there is no common region between R_1 and R_2, the z-transform of $ax_1[n]+bx_2[n]$ does not exist.

6.5.2 Time Shifting

For the bilateral z-transform

If $\quad x[n] \xleftrightarrow{\mathcal{Z}} X(z), \quad$ with ROC R_x

then $\quad x[n-n_0] \xleftrightarrow{\mathcal{Z}} z^{-n_0}X(z),$

and $\quad x[n+n_0] \xleftrightarrow{\mathcal{Z}} z^{n_0}X(z),$

with ROC : R_x except for the possible deletion or addition of $z=0$ or $z=\infty$.

PROOF :
The bilateral z-transform of signal $x[n-n_0]$ is given by equation (6.1.1) as follows

$$\mathcal{Z}\{x[n-n_0]\} = \sum_{n=-\infty}^{\infty} x[n-n_0]\,z^{-n}$$

Substituting $n-n_0=\alpha$ on RHS, we get

$$\mathcal{Z}\{x[n-n_0]\} = \sum_{\alpha=-\infty}^{\infty} x[\alpha]\,z^{-(n_0+\alpha)}$$

$$= \sum_{\alpha=-\infty}^{\infty} x[\alpha]\,z^{-n_0}z^{-\alpha} = z^{-n_0}\sum_{\alpha=-\infty}^{\infty} x[\alpha]\,z^{-\alpha}$$

$$\mathcal{Z}\{x[n-n_0]\} = z^{-n_0}X[z]$$

Similarly we can prove

$$\mathcal{Z}\{x[n+n_0]\} = z^{n_0}X[z]$$

ROC : The ROC of shifted signals is altered because of the terms z^{n_0} or z^{-n_0}, which affects the roots of the denominator in $X(z)$.

TIME SHIFTING FOR UNILATERAL z-TRANSFORM

For the unilateral z-transform

If $\quad x[n] \xleftrightarrow{\mathcal{Z}} X(z), \quad$ with ROC R_x

then $\quad x[n-n_0] \xleftrightarrow{\mathcal{Z}} z^{-n_0}\left(X(z)+\sum_{m=1}^{n_0} x[-m]\,z^{m}\right),$

and $\quad x[n+n_0] \xleftrightarrow{\mathcal{Z}} z^{n_0}\left(X(z)-\sum_{m=0}^{n_0-1} x[m]\,z^{-m}\right),$

with ROC : R_x except for the possible deletion or addition of $z=0$ or $z=\infty$.

PROOF :
The unilateral z-transform of signal $x[n-n_0]$ is given by equation (6.1.2) as follows

$$\mathcal{Z}\{x[n-n_0]\} = \sum_{n=0}^{\infty} x[n-n_0]\,z^{-n}$$

Multiplying RHS by z^{n_0} and z^{-n_0}

$$\mathcal{Z}\{x[n-n_0]\} = \sum_{n=0}^{\infty} x[n-n_0]\,z^{-n}z^{n_0}z^{-n_0}$$

$$= z^{-n_0}\sum_{n=0}^{\infty} x[n-n_0]\,z^{-(n-n_0)}$$

Substituting $n-n_0=\alpha$

Limits; when $n\to 0$, $\alpha\to -n_0$

when $n\to+\infty$, $\alpha\to+\infty$

Now, $\quad \mathcal{Z}\{x[n-n_0]\} = z^{-n_0}\sum_{\alpha=-n_0}^{\infty} x[\alpha]\,z^{-\alpha}$

$$= z^{-n_0}\sum_{\alpha=-n_0}^{-1} x[\alpha]\,z^{-\alpha} + z^{-n_0}\sum_{\alpha=0}^{\infty} x[\alpha]\,z^{-\alpha}$$

or, $\quad \mathcal{Z}\{x[n-n_0]\} = z^{-n_0}\sum_{\alpha=0}^{\infty} x[\alpha]\,z^{-\alpha} + z^{-n_0}\sum_{\alpha=-n_0}^{-1} x[\alpha]\,z^{-\alpha}$

or, $\quad \mathcal{Z}\{x[n-n_0]\} = z^{-n_0}\sum_{\alpha=0}^{\infty} x[\alpha]\,z^{-\alpha} + z^{-n_0}\sum_{\alpha=1}^{n_0} x[-\alpha]\,z^{\alpha}$

Changing the variables as $\alpha\to n$ and $\alpha\to m$ in first and second summation respectively

$$\mathcal{Z}\{x[n-n_0]\} = z^{-n_0}\sum_{n=0}^{\infty} x[n]\,z^{-n} + z^{-n_0}\sum_{m=1}^{n_0} x[-m]\,z^{m}$$

$$= z^{-n_0}X[z] + z^{-n_0}\sum_{m=1}^{n_0} x[-m]\,z^{m}$$

In similar way, we can also prove that

$$x[n+n_0] \xleftrightarrow{\mathcal{Z}} z^{n_0}\left(X(z)-\sum_{m=0}^{n_0-1} x[m]\,z^{-m}\right)$$

6.5.3 Time Reversal

Time reversal property states that time reflection of a DT sequence in time domain is equivalent to replacing z by $1/z$ in its z-transform.

If $\quad x[n] \xleftrightarrow{\mathcal{Z}} X(z), \quad$ with ROC : R_x

then $\quad x[-n] \xleftrightarrow{\mathcal{Z}} X\left(\frac{1}{z}\right), \quad$ with ROC : $1/R_x$

for bilateral z-transform.

PROOF :
The bilateral z-transform of signal $x[-n]$ is given by equation (6.1.1) as follows

$$\mathcal{Z}\{x[-n]\} = \sum_{n=-\infty}^{\infty} x[-n]\,z^{-n}$$

Substituting $-n=k$ on the RHS, we get

$$\mathcal{Z}\{x[-n]\} = \sum_{k=\infty}^{-\infty} x[k]\,z^{k} = \sum_{k=-\infty}^{\infty} x[k]\,(z^{-1})^{-k} = X\left(\frac{1}{z}\right)$$

Hence, $\quad x[-n] \xleftrightarrow{\mathcal{Z}} X\left(\frac{1}{z}\right)$

ROC : $z^{-1}\in R_x$ or $z\in 1/R_x$

47d.

6.5.4 Differentiation in the z-domain

This property states that multiplication of time sequence $x[n]$ with n corresponds to differentiation with respect to z and multiplication of result by $-z$ in the z-domain.

If	$x[n] \xleftrightarrow{\mathcal{Z}} X(z)$,	with ROC : R_x
then	$nx[n] \xleftrightarrow{\mathcal{Z}} -z\frac{dX(z)}{dz}$,	with ROC : R_x

For both unilateral and bilateral z-transforms.

PROOF :

The bilateral z-transform of signal $x[n]$ is given by equation (6.1.1) as follows

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

Differentiating both sides with respect to z gives

$$\frac{dX(z)}{dz} = \sum_{n=-\infty}^{\infty} x[n]\frac{dz^{-n}}{dz} = \sum_{n=-\infty}^{\infty} x[n](-nz^{-n-1})$$

Multiplying both sides by $-z$, we obtain

$$-z\frac{dX(z)}{dz} = \sum_{n=-\infty}^{\infty} nx[n]z^{-n}$$

Hence, $\quad nx[n] \xleftrightarrow{\mathcal{Z}} -z\frac{dX(z)}{dz}$

ROC : This operation does not affect the ROC.

6.5.5 Scaling in z-Domain

Multiplication of a time sequence with an exponential sequence a^n corresponds to scaling in z-domain by a factor of a.

If	$x[n] \xleftrightarrow{\mathcal{Z}} X(z)$,	with ROC : R_x
then	$a^n x[n] \xleftrightarrow{\mathcal{Z}} X\left(\frac{z}{a}\right)$,	with ROC : $\lvert a \rvert R_x$

for both unilateral and bilateral transform.

PROOF :

The bilateral z-transform of signal $x[n]$ is given by equation (6.1.1) as

$$\mathcal{Z}\{a^n x[n]\} = \sum_{n=-\infty}^{\infty} a^n x[n]z^{-n} = \sum_{n=-\infty}^{\infty} x[n][a^{-1}z]^{-n}$$

$$a^n x[n] \xleftrightarrow{\mathcal{Z}} X\left(\frac{z}{a}\right)$$

ROC : If z is a point in the ROC of $X(z)$ then the point $\lvert a \rvert z$ is in the ROC of $X(z/a)$.

6.5.6 Time Scaling

As we discussed in Chapter 2, there are two types of scaling in the DT domain decimation(compression) and interpolation(expansion).

Time Compression

Since the decimation (compression) of DT signals is an irreversible process (because some data may lost), therefore the z-transform of $x[n]$ and its decimated sequence $y[n] = x[an]$ not be related to each other.

Time Expansion

In the discrete time domain, time expansion of sequence $x[n]$ is defined as

$$x_k[n] = \begin{cases} x[n/k] & \text{if } n \text{ is a multiple of integer } k \\ 0 & \text{otherwise} \end{cases} \qquad (6.5.1)$$

Time-scaling property of z-transform is derived only for time expansion which is given as

If	$x[n] \xleftrightarrow{\mathcal{Z}} X(z)$,	with ROC : R_x
then	$x_k[n] \xleftrightarrow{\mathcal{Z}} X_k(z) = X(z^k)$,	with ROC : $(R_x)^{1/k}$

for both the unilateral and bilateral z-transform.

PROOF :

The unilateral z-transform of expanded sequence $x_k[n]$ is given by

$$\mathcal{Z}\{x_k[n]\} = \sum_{n=0}^{\infty} x_k[n]z^{-n}$$
$$= x_k[0] + x_k[1]z^{-1} + ... + x_k[k]z^{-k}$$
$$+ x_k[k+1]z^{-(k+1)} + ... x_k[2k]z^{-2k} + ...$$

Since the expanded sequence $x_k[n]$ is zero everywhere except when n is a multiple of k. This reduces the above transform as follows

$$\mathcal{Z}\{x_k[n]\} = x_k[0] + x_k[k]z^{-k} + x_k[2k]z^{-2k} + x_k[3k]z^{-3k} + ...$$

As defined in equation 6.5.1, interpolated sequence is

$$x_k[n] = x[n/k]$$

$n = 0 \qquad x_k[0] = x[0],$

$n = k \qquad x_k[k] = x[1]$

$n = 2k \qquad x_k[2k] = x[2]$

Thus, we can write

$$\mathcal{Z}\{x_k[n]\} = x[0] + x[1]z^{-k} + x[2]z^{-2k} + x[3]z^{-3k} + ...$$
$$= \sum_{n=0}^{\infty} x[n](z^k)^{-n} = X(z^k)$$

NOTE :

Time expansion of a DT sequence by a factor of k corresponds to replacing z as z^k in its z-transform.

6.5.7 Time Differencing

If	$x[n] \xleftrightarrow{\mathcal{Z}} X(z)$,	with ROC : R_x
then	$x[n] - x[n-1] \xleftrightarrow{\mathcal{Z}} (1 - z^{-1})X(z)$,	with the ROC

: R_x except for the possible deletion of $z = 0$, for both unilateral and bilateral transform.

PROOF :

The z-transform of $x[n] - x[n-1]$ is given by equation (6.1.1) as follows

$$\mathcal{Z}\{x[n] - x[n-1]\} = \sum_{n=-\infty}^{\infty}\{x[n] - x[n-1]\}z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} x[n]z^{-n} - \sum_{n=-\infty}^{\infty} x[n-1]z^{-n}$$

In the second summation, substituting $n-1=r$

$$\mathcal{Z}\{x[n]-x[n-1]\} = \sum_{n=-\infty}^{\infty} x[n]z^{-n} - \sum_{r=-\infty}^{\infty} x[r]z^{-(r+1)}$$

$$= \sum_{n=-\infty}^{\infty} x[n]z^{-n} - z^{-1}\sum_{r=-\infty}^{\infty} x[r]z^{-r}$$

$$= X(z) - z^{-1}X(z)$$

Hence, $x[n]-x[n-1] \xleftrightarrow{\mathcal{Z}} (1-z^{-1})X(z)$

6.5.8 Time Convolution

Time convolution property states that convolution of two sequence in time domain corresponds to multiplication in z-domain.

Let	$x_1[n] \xleftrightarrow{\mathcal{Z}} X_1(z)$,	ROC : R_1
and	$x_2[n] \xleftrightarrow{\mathcal{Z}} X_2(z)$,	ROC : R_2
then the convolution property states that		
$x_1[n] * x_2[n] \xleftrightarrow{\mathcal{Z}} X_1(z)X_2(z)$,		ROC : at least $R_1 \cap R_2$
for both unilateral and bilateral z-transforms.		

PROOF :

As discussed in chapter 4, the convolution of two sequences is given by

$$x_1[n] * x_2[n] = \sum_{k=-\infty}^{\infty} x_1[k]x_2[n-k]$$

Taking the z-transform of both sides gives

$$x_1[n] * x_2[n] \xleftrightarrow{\mathcal{Z}} \sum_{n=-\infty}^{\infty}\sum_{k=-\infty}^{\infty} x_1[k]x_2[n-k]z^{-n}$$

Interchanging the order of the two summations, we get

$$x_1[n] * x_2[n] \xleftrightarrow{\mathcal{Z}} \sum_{k=-\infty}^{\infty} x_1[k]\sum_{n=-\infty}^{\infty} x_2[n-k]z^{-n}$$

Substituting $n-k=\alpha$ in the second summation

$$x[n] * x_2[n] \xleftrightarrow{\mathcal{Z}} \sum_{k=-\infty}^{\infty} x_1[k]\sum_{\alpha=-\infty}^{\infty} x_2[\alpha]z^{-(\alpha+k)}$$

or
$$x[n] * x_2[n] \xleftrightarrow{\mathcal{Z}} \left(\sum_{k=-\infty}^{\infty} x_1[k]z^{-k}\right)\left(\sum_{\alpha=-\infty}^{\infty} x_2[\alpha]z^{-\alpha}\right)$$

$$x_1[n] * x_2[n] \xleftrightarrow{\mathcal{Z}} X_1(z)X_2(z)$$

6.5.9 Conjugation Property

If	$x[n] \xleftrightarrow{\mathcal{Z}} X(z)$,	with ROC : R_x
then	$x^*[n] \xleftrightarrow{\mathcal{Z}} X^*(z^*)$,	with ROC : R_x
If $x[n]$ is real, then	$X(z) = X^*(z^*)$	

PROOF :

The z-transform of signal $x^*[n]$ is given by equation (6.1.1) as follows

$$\mathcal{Z}\{x^*[n]\} = \sum_{n=-\infty}^{\infty} x^*[n]z^{-n} = \sum_{n=-\infty}^{\infty}[x[n](z^*)^{-n}]^* \quad (6.5.2)$$

Let z-transform of $x[n]$ is $X(z)$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

by taking complex conjugate on both sides of above equation

$$X^*(z) = \sum_{n=-\infty}^{\infty}[x[n]z^{-n}]^*$$

Replacing $z \to z^*$, we will get

$$X^*(z^*) = \sum_{n=-\infty}^{\infty}[x[n](z^*)^{-n}]^* \quad (6.5.3)$$

Comparing equation (6.5.2) and (6.5.3)

$$\mathcal{Z}\{x^*[n]\} = X^*(z^*) \quad (6.5.4)$$

For real $x[n]$, $x^*[n] = x[n]$, so

$$\mathcal{Z}\{x^*[n]\} = \sum_{n=-\infty}^{\infty} x[n]z^{-n} = X(z) \quad (6.5.5)$$

Comparing equation (6.5.4) and (6.5.5)

$$X(z) = X^*(z^*)$$

6.5.10 Initial Value Theorem

If	$x[n] \xleftrightarrow{\mathcal{Z}} X(z)$,	with ROC : R_x
then initial-value theorem states that,		
	$x[0] = \lim_{z\to\infty} X(z)$	
The initial-value theorem is valid only for the unilateral Lapalce transform		

PROOF :

For a causal signal $x[n]$

$$X(z) = \sum_{n=0}^{\infty} x[n]z^{-n}$$

$$= x[0] + x[1]z^{-1} + x[2]z^{-2} + \dots$$

Taking limit as $z\to\infty$ on both sides we get

$$\lim_{z\to\infty} X(z) = \lim_{z\to\infty}(x[0] + x[1]z^{-1} + x[2]z^{-2} + \dots) = x[0]$$

$$x[0] = \lim_{z\to\infty} X(z)$$

6.5.11 Final Value Theorem

If	$x[n] \xleftrightarrow{\mathcal{Z}} X(z)$,	with ROC : R_x
then final-value theorem states that		
	$x[\infty] = \lim_{z\to 1}(z-1)X(z)$	
Final value theorem is applicable if $X(z)$ has no poles outside the unit circle. This theorem can be applied to either the unilateral or bilateral z-transform.		

PROOF :

$$\mathcal{Z}\{x[n+1]\} - \mathcal{Z}\{x[n]\} = \lim_{k\to\infty}\sum_{n=0}^{k}\{x[n+1] - x[n]\}\, z^{-n} \quad (6.5.6)$$

From the time shifting property of unilateral z-transform discussed in section 6.5.2

$$x[n+n_0] \xleftrightarrow{\mathcal{Z}} z^{n_0}\left(X(z) - \sum_{m=0}^{n_0-1} x[m]\, z^{-m}\right)$$

For $n_0 = 1$
$$x[n+1] \xleftrightarrow{\mathcal{Z}} z\left(X(z) - \sum_{m=0}^{0} x[m]\, z^{-m}\right)$$

$$x[n+1] \xleftrightarrow{\mathcal{Z}} z(X(z) - x[0])$$

Put above transformation in the equation (6.5.6)

$$zX[z] - zx[0] - X[z] = \lim_{k\to\infty}\sum_{n=0}^{k}(x[n+1] - x[n])\, z^{-n}$$

$$(z-1)X[z] - zx[0] = \lim_{k\to\infty}\sum_{n=0}^{k}(x[n+1] - x[n])\, z^{-n}$$

Taking limit as $z \to 1$ on both sides we get

$$\lim_{z\to 1}(z-1)X[z] - x[0] = \lim_{k\to\infty}\sum_{n=0}^{k} x[n+1] - x[n]$$

$$\lim_{z\to 1}(z-1)X[z] - x[0] = \lim_{k\to\infty}\{(x[1] - x[0]) + (x[2] - x[1]) + (x[3] - x[2]) + \dots$$
$$\dots + (x[k+1] - x[k])$$

$$\lim_{z\to 1}(z-1)X[z] - x[0] = x[\infty] - x[0]$$

Hence, $$x[\infty] = \lim_{z\to 1}(z-1)X(z)$$

Summary of Properties

Let, $x[n] \xleftrightarrow{\mathcal{Z}} X(z)$, with ROC R_x

$x_1[n] \xleftrightarrow{\mathcal{Z}} X_1(z)$, with ROC R_1

$x_2[n] \xleftrightarrow{\mathcal{Z}} X_2(z)$, with ROC R_2

The properties of z-transforms are summarized in the following table.

TABLE 6.2 Properties of z-transform

Properties	Time domain	z-transform	ROC
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	at least $R_1 \cap R_2$
Time shifting (bilateral or non-causal)	$x[n-n_0]$	$z^{-n_0}X(z)$	R_x except for the possible deletion or addition of $z = 0$ or $z = \infty$
	$x[n+n_0]$	$z^{n_0}X(z)$	
Time shifting (unilateral or causal)	$x[n-n_0]$	$z^{-n_0}\left(X(z) + \sum_{m=1}^{n_0} x[-m]\, z^{m}\right)$	R_x except for the possible deletion or addition of $z = 0$ or $z = \infty$
	$x[n+n_0]$	$z^{n_0}\left(X(z) - \sum_{m=0}^{n_0-1} x[m]\, z^{-m}\right)$	
Time reversal	$x[-n]$	$X\left(\frac{1}{z}\right)$	$1/R_x$
Differentiation in z domain	$nx[n]$	$-z\frac{dX(z)}{dz}$	R_x
Scaling in z domain	$a^n x[n]$	$X\left(\frac{z}{a}\right)$	$\lvert a\rvert R_x$
Time scaling (expansion)	$x_k[n] = x[n/k]$	$X(z^k)$	$(R_x)^{1/k}$
Time differencing	$x[n] - x[n-1]$	$(1 - z^{-1})X(z)$	R_x, except for the possible deletion of the origin
Time convolution	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	at least $R_1 \cap R_2$
Conjugations	$x^*[n]$	$X^*(z^*)$	R_x
Initial-value theorem		$x[0] = \lim_{z\to\infty} X(z)$	provided $x[n] = 0$ for $n < 0$
Final-value theorem		$x[\infty] = \lim_{n\to\infty} x[n] = \lim_{z\to 1}(z-1)X(z)$	provided $x[\infty]$ exists

6.6 ANALYSIS OF DISCRETE LTI SYSTEMS USING Z-TRANSFORM

The z-transform is very useful tool in the analysis of discrete LTI system. As the Laplace transform is used in solving differential equations which describe continuous LTI systems, the z-transform is used to solve difference equation which describe the discrete LTI systems.

Similar to Laplace transform, for CT domain, the z-transform gives transfer function of the LTI discrete systems which is the ratio of the z-transform of the output variable to the z-transform of the input variable. These applications are discussed as follows

6.6.1 Response of LTI Continuous Time System

As discussed in chapter 4 (section 4.8), a discrete-time LTI system is always described by a linear constant coefficient difference equation given as follows

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

$$a_N y[n-N] + a_{N-1} y[n-(N-1)] + \dots\dots + a_1 y[n-1] + a_0 y[n]$$
$$= b_M x[n-M] + b_{M-1} x[n-(M-1)] + \dots\dots + b_1 x[n-1] + b_0 x[n] \quad (6.6.1)$$

where, N is order of the system.

The time-shift property of z-transform $x[n-n_0] \xleftrightarrow{\mathcal{Z}} z^{-n_0}X(z)$, is used to solve the above difference equation which converts it into an algebraic equation. By taking z-transform of above equation

$$a_N z^{-N} Y(z) + a_{N-1} z^{-(N-1)} Y(z)] + \ldots\ldots + a_1 z^{-1} + a_0 Y(z)$$
$$= b_M z^{-M} X(z) + b_{M-1} z^{-(M-1)} X(z) + \ldots.. + b_1 z^{-1} X(x) + b_0 X(z)$$
$$\frac{Y(z)}{X(z)} = \frac{b_M z^{-N} + b_{M-1} z^{M-1} + \ldots.. + b_1 + b_0}{a_N z^N + a_{N-1} z^{N-1} + \ldots.. + a_1 + a_0}$$

this equation can be solved for $Y(z)$ to find the response $y[n]$.The solution or total response $y[n]$ consists of two parts as discussed below.

1. Zero-input Response or Free Response or Natural Response

The zero input response $y_{zi}[n]$ is mainly due to initial output in the system. The zero-input response is obtained from system equation (6.6.1) when input $x[n] = 0$.

By substituting $x[n] = 0$ and $y[n] = y_{zi}[n]$ in equation (6.6.1), we get

$$a_N y[n-M] + a_{N-1} y[n-(N-1)] + \ldots\ldots + a_1 y[n-1] + a_0 y[n] = 0$$

On taking z-transform of the above equation with given initial conditions, we can form an equation for $Y_{zi}(z)$. The zero-input response $y_{zi}[n]$ is given by inverse z-transform of $Y_{zi}(z)$.

NOTE :
The zero input response is also called the natural response of the system and it is denoted as $y_N[n]$.

2. Zero-State Response or Forced Response

The zero-state response $y_{zs}[n]$ is the response of the system due to input signal and with zero initial conditions. The zero-state response is obtained from the difference equation (6.6.1) governing the system for specific input signal $x[n]$ for $n \geq 0$ and with zero initial conditions.

Substituting $y[n] = y_{zs}[n]$ in equation (6.6.1) we get,

$$a_N y_{zs}[n-M] + a_{N-1} y_{zs}[n-(N-1)] + \ldots\ldots + a_1 y_{zs}[n-1] + a_0 y_{zs}[n]$$
$$= b_M x[n-M] + b_{M-1} x[n-(M-1)] + \ldots.. + b_1 x[n-1] + b_0 x[n]$$

Taking z-transform of the above equation with zero initial conditions for output (i.e., $y[-1] = y[-2] \ldots = 0$ we can form an equation for $Y_{zs}(z)$.

The zero-state response $y_{zs}[n]$ is given by inverse z-transform of $Y_{zs}(z)$.

NOTE :
The zero state response is also called the forced response of the system and it is denoted as $y_F[n]$.

Total Response

The total response $y[n]$ is the response of the system due to input signal and initial output. The total response can be obtained in following two ways :

Taking z-transform of equation (6.6.1) with non-zero initial conditions for both input and output, and then substituting for $X(z)$ we can form an equation for $Y(z)$. The total response $y[n]$ is given by inverse Laplace transform of $Y(s)$.

Alternatively, that total response $y[n]$ is given by sum of zero-input response $y_{zi}[n]$ and zero-state response $y_{zs}[n]$.

Therefore total response,

$$y[n] = y_{zi}[n] + y_{zs}[n]$$

6.6.2 Impulse Response and Transfer Function

System function or transfer function is defined as the ratio of the z-transform of the output $y[n]$ and the input $x[n]$ with zero initial conditions.

Let $x[n] \xleftrightarrow{\mathcal{Z}} X(z)$ is the input and $y[n] \xleftrightarrow{\mathcal{L}} Y(z)$ is the output of an LTI discrete time system having impulse response $h(n) \xleftrightarrow{\mathcal{L}} H(z)$. The response $y[n]$ of the discrete time system is given by convolution sum of input and impulse response as

$$y[n] = x[n] * h[n]$$

By applying convolution property of z-transform we obtain

$$Y(z) = X(z)\, H(z)$$
$$H(z) = \frac{Y(z)}{X(z)}$$

where, $H(z)$ is defined as the transfer function of the system. It is the z-transform of the impulse response.

Alternatively we can say that the inverse z-transform of transfer function is the impulse response of the system.

Impulse response

$$h[n] = \mathcal{Z}^{-1}\{H(z)\} = \mathcal{Z}^{-1}\left\{\frac{Y(z)}{X(z)}\right\}$$

6.7 STABILITY AND CAUSALITY OF LTI DISCRETE SYSTEMS USING Z -TRANSFORM

z-transform is also used in characterization of LTI discrete systems. In this section, we derive a z-domain condition to check the stability and causality of a system directly from its z-transfer function.

6.7.1 Causality

A linear time-invariant discrete time system is said to be causal if the impulse response $h[n] = 0$, for $n < 0$ and it is therefore right-sided. The ROC of such a system $H(z)$ is the exterior of a circle. If $H(z)$ is rational then the system is said to be causal if

1. The ROC is the exterior of a circle outside the outermost pole ; and
2. The degree of the numerator polynomial of $H(z)$ should be less than or equal to the degree of the denominator polynomial.

6.7.2 Stability

An LTI discrete-time system is said to be BIBO stable if the impulse response $h[n]$ is summable. That is

$$\sum_{n=-\infty}^{\infty} |h[n]| < \infty$$

z-transform of $h[n]$ is given as

$$H(z) = \sum_{n=-\infty}^{\infty} h[n] z^{-n}$$

Let $z = e^{j\Omega}$ (which describes a unit circle in the z-plane), then

$$|H[e^{j\Omega}]| = \left|\sum_{n=-\infty}^{\infty} h[n] e^{-j\Omega n}\right|$$
$$\leq \sum_{n=-\infty}^{\infty} |h[n] e^{-j\Omega n}|$$
$$= \sum_{n=-\infty}^{\infty} |h[n]| < \infty$$

which is the condition for the stability. Thus we can conclude that

STABILITY OF LTI DISCRETE SYSTEM

An LTI system is stable if the ROC of its system function $H(z)$ contains the unit circle $|z| = 1$

6.7.3 Stability and Causality

As we discussed previously, for a causal system with rational transfer function $H(z)$, the ROC is outside the outermost pole. For the BIBO stability the ROC should include the unit circle $|z| = 1$. Thus, for the system to be causal and stable theses two conditions are satisfied if all the poles are within the unit circle in the z-plane.

STABILITY AND CAUSALITY OF LTI DISCRETE SYSTEM

An LTI discrete time system with the rational system function $H(z)$ is said to be both causal and stable if all the poles of $H(z)$ lies inside the unit circle.

6.8 BLOCK DIAGRAM REPRESENTATION

In z-domain, the input-output relation of an LTI discrete time system is represented by the transfer function $H(z)$,which is a rational function of z, as shown in equation

$$H(z) = \frac{Y(z)}{X(z)}$$
$$= \frac{b_0 z^M + b_1 z^{M-1} + b_2 z^{M-2} + \dots + b_{M-1} z + b_M}{a_0 z^N + a_1 z^{N-1} + a_2 z^{N-2} + \dots + a_{N-1} z + a_N}$$

where, $N =$ Order of the system, $M \le N$ and $a_0 = 1$

The above transfer function is realized using unit delay elements, unit advance elements, adders and multipliers. Basic elements of block diagram with their z-domain representation is shown in table 6.3.

TABLE 6.3 : Basic Elements of Block Diagram

Elements of Block diagram	Time Domain Representation	s-domain Representation
Adder	$x_1[n] \to (+) \to x_1[n]+x_2[n]$, $x_2[n]$	$X_1(z) \to (+) \to X_1(z)+X_2(z)$, $X_2(z)$
Constant multiplier	$x[n] \to a \to ax[n]$	$X(z) \to a \to aX(z)$
Unit delay element	$x[n] \to \boxed{z^{-1}} \to x[n-1]$	$X(z) \to \boxed{z^{-1}} \to z^{-1}X(z)$
Unit advance element	$x[n] \to \boxed{z} \to x[n+1]$	$X(z) \to \boxed{z} \to zX(z)$

The different types of structures for realizing discrete time systems are same as we discussed for the continuous-time system in the previous chapter.

6.8.1 Direct Form I Realization

Consider the difference equation governing the discrete time system with $a_0 = 1$,

$$y[n] + a_1 y[n-1] + a_2 y[n-2] + \dots + a_N y[n-N]$$
$$= b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + \dots + b_M x[n-M]$$

Taking $\mathcal{Z}$ transform of the above equation we get,

$$Y(z) = -a_1 z^{-1} Y(z) - a_2 z^{-2} Y(z) - \dots - a_N z^{-N} Y(z) +$$
$$b_0 X(z) + b_1 z^{-1} X(z) + b_2 z^{-2} X(z) + \dots + b_M z^{-M} X(z) \qquad (6.8.1)$$

The above equation of $Y(z)$ can be directly represented by a block diagram as shown in figure 6.8.1a. This structure is called direct form-I structure. This structure uses separate delay elements for both input and output of the system. So, this realization uses more memory.

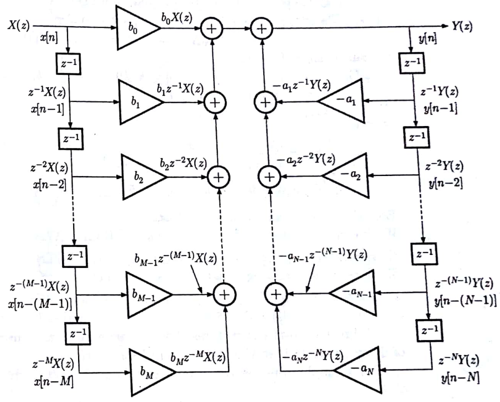


Figure 6.8.1a General structure of direct form-realization

For example consider a discrete LTI system which has the following impulse response

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + 2z^{-1} + 2z^{-2}}{1 + 4z^{-1} + 3z^{-2}}$$

$$Y(z) + 4z^{-1} Y(z) + 3z^{-2} Y(z) = 1X(z) + 2z^{-1} X(z) + 2z^{-2} X(z)$$

Comparing with standard form of equation (6.8.1), we get $a_1 = 4$, $a_2 = 3$ and $b_0 = 1$, $b_1 = 2$, $b_2 = 2$. Now put these values in general structure of Direct form-I realization we get

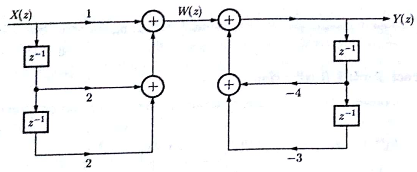


Figure 6.8.1b

6.8.2 Direct Form II Realization

Consider the general difference equation governing a discrete LTI system

$$y[n] + a_1 y[n-1] + a_2 y[n-2] + ... + a_N y[n-N]$$
$$= b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + ... + b_M x[n-M]$$

Taking $\mathcal{Z}$ transform of the above equation we get,

$$Y(z) = -a_1 z^{-1} Y(z) - a_2 z^{-2} Y(z) - ... - a_N z^{-N} Y(z) +$$
$$b_0 X(z) + b_1 z^{-1} X(z) + b_2 z^{-2} X(z) + ... + b_M z^{-M} X(z)$$

It can be simplified as,

$$Y(z)[1 + a_1 z^{-1} + a_2 z^{-2} + ... + a_N z^{-N}] = X(z)[b_0 + b_1 z^{-1} + b_2 z^{-2} + ... + b_M z^{-M}]$$

Let, $\dfrac{Y(z)}{X(z)} = \dfrac{W(z)}{X(z)} \times \dfrac{Y(z)}{W(z)}$

where,

$$\frac{W(z)}{X(z)} = \frac{1}{1 + a_1 z^{-1} + a_2 z^{-2} + ... + a_N z^{-N}} \qquad (6.8.2)$$

$$\frac{Y(z)}{W(z)} = b_0 + b_1 z^{-1} + b_2 z^{-2} + ... + b_M z^{-M} \qquad (6.8.3)$$

Equation (6.8.2) can be simplified as,

$$W(z) + a_1 z^{-1} W(z) + a_2 z^{-2} W(z) + ... + a_N z^{-N} W(z) = X(z)$$

$$W(z) = X(z) - a_1 z^{-1} W(z) - a_2 z^{-2} W(z) - ... - a_N z^{-N} W(z) \qquad (6.8.4)$$

Similarly by simplifying equation (6.8.3), we get

$$Y(z) = b_0 W(z) + b_1 z^{-1} W(z) + b_2 z^{-2} W(z) + ... + b_M z^{-M} W(z) \qquad (6.8.5)$$

Equation (6.8.4) and (6.8.5) can be realized together by a direct structure called direct form-II structure as shown in figure 6.8.2a. It uses less number of delay elements then the Direct Form I structure.

For example, consider the same transfer function $H(z)$ which is discussed above

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + 2z^{-1} + 2z^{-2}}{1 + 4z^{-1} + 3z^{-2}}$$

Let $\dfrac{Y(z)}{X(z)} = \dfrac{Y(z)}{W(z)} \times \dfrac{W(z)}{X(z)}$

where, $\dfrac{W(z)}{X(z)} = \dfrac{1}{1 + 4z^{-1} + 3z^{-2}}$,

$$\frac{Y(z)}{W(z)} = 1 + 2z^{-1} + 2z^{-2}$$

so, $W(z) = X(z) - 4z^{-1} W(z) - 3z^{-2} W(z)$

and $Y(z) = 1 W(z) + 2z^{-1} W(z) + 2z^{-2} W(z)$

Comparing these equations with standard form of equation (6.8.4) and (6.8.5), we have $a_1 = 4$, $a_2 = 3$ and $b_0 = 1, b_1 = 2, b_2 = 2$. Substitute these values in general structure of Direct form II , we get as shown in figure 6.8.2b

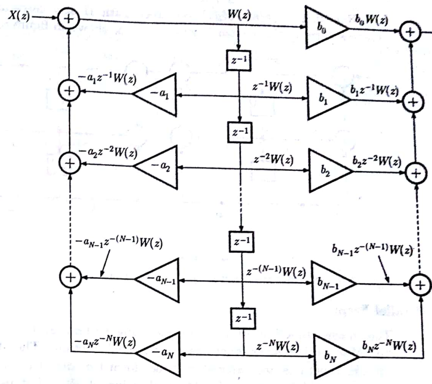


Figure 6.8.2a General structure of direct form-II realization

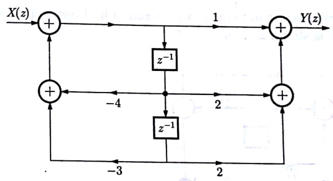


Figure 6.8.2b

6.8.3 Cascade Form

The transfer function $H(z)$ of a discrete time system can be expressed as a product of several transfer functions. Each of these transfer functions is realized in direct form-I or direct form II realization and then they are cascaded.

Consider a system with transfer function

$$H(z) = \frac{(b_{k0} + b_{k1} z^{-1} + b_{k2} z^{-2})(b_{m0} + b_{m1} z^{-1} + b_{m2} z^{-2})}{(1 + a_{k1} z^{-1} + a_{k2} z^{-2})(1 + a_{m1} z^{-1} + a_{m2} z^{-2})}$$
$$= H_1(z) H_2(z)$$

where $H_1(z) = \dfrac{b_{k0} + b_{k1} z^{-1} + b_{k2} z^{-2}}{1 + a_{k1} z^{-1} + a_{k2} z^{-2}}$

$$H_2(z) = \frac{b_{m0} + b_{m1}z^{-1} + b_{m2}z^{-2}}{1 + a_{m1}z^{-1} + a_{m2}z^{-2}}$$

Realizing $H_1(z)$ and $H_2(z)$ in direct form II and cascading we obtain cascade form of the system function $H(z)$ as shown in figure 6.8.3.

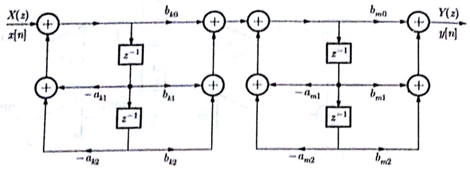


Figure 6.8.3 Cascaded form realization of discrete LTI system

6.8.4 Parallel Form

The transfer function $H(z)$ of a discrete time system can be expressed as the sum of several transfer functions using partial fractions. Then the individual transfer functions are realized in direct form I or direct form II realization and connected in parallel for the realization of $H(z)$. Let us consider the transfer function

$$H(z) = c + \frac{c_1}{1 - p_1 z^{-1}} + \frac{c_2}{1 - p_2 z^{-1}} + \ldots\ldots \frac{c_N}{1 - p_N z^{-1}}$$

Now each factor in the system is realized in direct form II and connected in parallel as shown in figure 6.8.4.

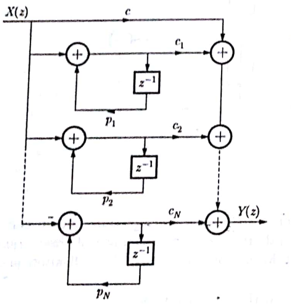


Figure 6.8.4 Parallel form realization of discrete LTI system

6.9 RELATIONSHIP BETWEEN S-PLANE & Z-PLANE

There exists a close relationship between the Laplace and z-transforms. We know that a DT sequence $x[n]$ is obtained by sampling a CT signal $x(t)$ with a sampling interval T, the CT sampled signal $x_s(t)$ is written as follows

$$x_s(t) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t - nT)$$

where $x(nT)$ are sampled value of $x(t)$ which equals the DT sequence $x[n]$. Taking the Laplace transform of $x_s(t)$, we have

$$X(s) = L\{x_s(t)\} = \sum_{n=-\infty}^{\infty} x(nT)\, L\{\delta(t - nT)\}$$

$$= \sum_{n=-\infty}^{\infty} X(nT)\, e^{-nTs} \qquad (6.9.1)$$

The z-transform of $x[n]$ is given by

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n} \qquad (6.9.2)$$

Comparing equation (6.9.1) and (6.9.2)

$$X(s) = X(z)\big|_{z=e^{sT}} \qquad x[n] = x(nT)$$

****

EXERCISE 6.1

MCQ 6.1.1 The z-transform and its ROC of a discrete time sequence

$$x[n] = \begin{cases} -\left(\frac{1}{2}\right)^n, & n < 0 \\ 0, & n \geq 0 \end{cases}$$

will be

(A) $\frac{2z}{2z-1}$, $|z| > \frac{1}{2}$ (B) $\frac{z}{z-2}$, $|z| < \frac{1}{2}$

(C) $\frac{2z}{2z-1}$, $|z| < \frac{1}{2}$ (D) $\frac{2z^{-1}}{z-1}$, $|z| > \frac{1}{2}$

MCQ 6.1.2 The ROC of z-transform of the discrete time sequence $x[n] = \left(\frac{1}{2}\right)^{|n|}$ is

(A) $\frac{1}{2} < |z| < 2$ (B) $|z| > 2$

(C) $-2 < |z| < 2$ (D) $|z| < \frac{1}{2}$

MCQ 6.1.3 Consider a discrete-time signal $x[n] = \left(\frac{1}{3}\right)^n u[n] + \left(\frac{1}{2}\right)^n u[-n-1]$. The ROC of its z-transform is

(A) $3 < |z| < 2$ (B) $|z| < \frac{1}{2}$

(C) $|z| > \frac{1}{3}$ (D) $\frac{1}{3} < |z| < \frac{1}{2}$

MCQ 6.1.4 For a signal $x[n] = [\alpha^n + \alpha^{-n}]\,u[n]$, the ROC of its z-transform would be

(A) $|z| > \min\left(|\alpha|, \frac{1}{|\alpha|}\right)$ (B) $|z| > |\alpha|$

(C) $|z| > \max\left(|\alpha|, \frac{1}{|\alpha|}\right)$ (D) $|z| < |\alpha|$

MCQ 6.1.5 Match List I (discrete time sequence) with List II (z-transform) and choose the correct answer using the codes given below the lists:

List-I (Discrete Time Sequence)		List-II (z-Transform)	
P.	$u[n-2]$	**1.**	$\frac{1}{z^{-2}(1-z^{-1})}$, $\lvert z\rvert < 1$
Q.	$-u[-n-3]$	**2.**	$\frac{-z^{-1}}{1-z^{-1}}$, $\lvert z\rvert < 1$
R.	$u[n+4]$	**3.**	$\frac{1}{z^{-4}(1-z^{-1})}$, $\lvert z\rvert > 1$
S.	$u[-n]$	**4.**	$\frac{z^{-2}}{1-z^{-1}}$, $\lvert z\rvert > 1$

Codes :

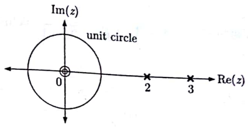
	P	Q	R	S
(A)	1	4	2	3
(B)	2	4	1	3
(C)	4	1	3	2
(D)	4	2	3	1

MCQ 6.1.6 The z-transform of signal $x[n] = e^{jn\pi}u[n]$ is

(A) $\frac{z}{z+1}$, ROC: $|z| > 1$ (B) $\frac{z}{z-j}$, ROC: $|z| > 1$

(C) $\frac{z}{z^2+1}$, ROC: $|z| < 1$ (D) $\frac{1}{z+1}$, ROC: $|z| < 1$

MCQ 6.1.7 Consider the pole zero diagram of an LTI system shown in the figure which corresponds to transfer function $H(z)$.



Match List I (The impulse response) with List II (ROC which corresponds to above diagram) and choose the correct answer using the codes given below: {Given that $H(1) = 1$}

List-I (Impulse Response)		List-II (ROC)	
P.	$[(-4)2^n + 6(3)^n]\,u[n]$	**1.**	does not exist
Q.	$(-4)2^n u[n] + (-6)3^n u[-n-1]$	**2.**	$\lvert z\rvert > 3$
R.	$(4)2^n u[-n-1] + (-6)3^n u[-n-1]$	**3.**	$\lvert z\rvert < 2$
S.	$4(2)^n u[-n-1] + (-6)3^n u[n]$	**4.**	$2 < \lvert z\rvert < 3$

Codes :

	P	Q	R	S
(A)	4	1	3	2
(B)	2	1	3	4
(C)	1	4	2	3
(D)	2	4	3	1

MCQ 6.1.8 The z-transform of a signal $x[n]$ is $X(z) = e^z + e^{1/z}$, $|z| \neq 0$. $x[n]$ would be

(A) $\delta[n] + \frac{1}{n!}$ (B) $u[n] + \frac{1}{n!}$

(C) $u[n-1] + n!$ (D) $\delta[n] + (n-1)!$

Common Data For Q. 9 to 11:

Consider a discrete time signal $x[n]$ and its z-transform $X(z)$ given as

$$X(z) = \frac{z^2 + 5z}{z^2 - 2z - 3}$$

MCQ 6.1.9 If ROC of $X(z)$ is $|z| < 1$, then signal $x[n]$ would be

(A) $[-2(3)^n + (-1)^n]\,u[-n-1]$ (B) $[2(3)^n - (-1)^n]\,u[n]$

(C) $-2(3)^n u[-n-1] - (-1)^n u[n]$ (D) $[2(3)^n + 1]\,u[n]$

MCQ 6.1.10 If ROC of $X(z)$ is $|z| > 3$, then signal $x[n]$ would be

(A) $[2(3)^n - (-1)^n u[n]$ (B) $[-2(3)^n + (-1)^n]\,u[-n-1]$

(C) $-2(3)^n u[-n-1] - (-1)^n u[n]$ (D) $[2(3)^n + 1]\,u[n]$

MCQ 6.1.11 If ROC of $X(z)$ is $1 < |z| < 3$, the signal $x[n]$ would be

(A) $[2(3)^n - (-1)^n]u[n]$
(B) $[-2(3)^n + (-1)^n]u[-n-1]$
(C) $-2(3)^n u[-n-1] - (-1)^n u[n]$
(D) $[2(3)^n + (-1)^n]u[-n-1]$

MCQ 6.1.12 Consider a DT sequence $x[n] = x_1[n] + x_2[n]$ where, $x_1[n] = (0.7)^n u[n-1]$ and $x_2[n] = (-0.4)^n u[n-2]$. The region of convergence of z-transform of $x[n]$ is

(A) $0.4 < |z| < 0.7$
(B) $|z| > 0.7$
(C) $|z| < 0.4$
(D) none of these

MCQ 6.1.13 The z-transform of a DT signal $x[n]$ is $X(z) = \frac{z}{8z^2 - 2z - 1}$. What will be the z-transform of $x[n-4]$?

(A) $\frac{(z+4)}{8(z+4)^2 - 2(z+4) - 1}$
(B) $\frac{z^5}{8z^2 - 2z - 1}$
(C) $\frac{4z}{128z^2 - 8z - 1}$
(D) $\frac{1}{8z^5 - 2z^4 - z^3}$

MCQ 6.1.14 Let $x_1[n]$, $x_2[n]$ and $x_3[n]$ be three discrete time signals and $X_1(z)$, $X_2(z)$ and $X_3(z)$ are their z-transform respectively given as

$$X_1(z) = \frac{z^2}{(z-1)(z-0.5)},$$

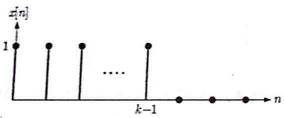
$$X_2(z) = \frac{z}{(z-1)(z-0.5)}$$

and $$X_3(z) = \frac{1}{(z-1)(z-0.5)}$$

Then $x_1[n]$, $x_2[n]$ and $x_3[n]$ are related as

(A) $x_1[n-2] = x_2[n-1] = x_3[n]$
(B) $x_1[n+2] = x_2[n+1] = x_3[n]$
(C) $x_1[n] = x_2[n-1] = x_3[n-2]$
(D) $x_1[n+1] = x_2[n-1] = x_3[n]$

MCQ 6.1.15 The z-transform of the discrete time signal $x[n]$ shown in the figure is



(A) $\frac{z^{-k}}{1 - z^{-1}}$
(B) $\frac{z^{-k}}{1 + z^{-1}}$
(C) $\frac{1 - z^{-k}}{1 - z^{-1}}$
(D) $\frac{1 + z^{-k}}{1 - z^{-1}}$

MCQ 6.1.16 Consider the unilateral z-transform pair $x[n] \xleftrightarrow{z} X(z) = \frac{z}{z-1}$. The z-transform of $x[n-1]$ and $x[n+1]$ are respectively

(A) $\frac{z^2}{z-1}, \frac{1}{z-1}$
(B) $\frac{1}{z-1}, \frac{z^2}{z-1}$
(C) $\frac{1}{z-1}, \frac{z}{z-1}$
(D) $\frac{z}{z-1}, \frac{z^2}{z-1}$

MCQ 6.1.17 A discrete time causal signal $x[n]$ has the z-transform

$$X(z) = \frac{z}{z-0.4}, \quad \text{ROC}: |z| > 0.4$$

The ROC for z-transform of the even part of $x[n]$ will be

(A) same as ROC of $X(z)$
(B) $0.4 < |z| < 2.5$
(C) $|z| > 0.2$
(D) $|z| > 0.8$

MCQ 6.1.18 Match List I (Discrete time sequence) with List II (z-transform) and select the correct answer using the codes given below the lists.

List-I (Discrete time sequence)

P. $n(-1)^n u[n]$

Q. $-nu[-n-1]$

R. $(-1)^n u[n]$

S. $nu[n]$

List-II (z-transform)

1. $\frac{z^{-1}}{(1-z^{-1})^2}$, ROC: $|z| > 1$

2. $\frac{1}{(1+z^{-1})}$, ROC: $|z| > 1$

3. $\frac{z^{-1}}{(1-z^{-1})^2}$, ROC: $|z| < 1$

4. $-\frac{z^{-1}}{(1+z^{-1})^2}$, ROC: $|z| > 1$

Codes :

	P	Q	R	S
(A)	4	1	2	3
(B)	4	3	2	1
(C)	3	1	4	2
(D)	2	4	1	3

MCQ 6.1.19 A discrete time sequence is defined as $x[n] = \frac{1}{n}(-2)^{-n} u[-n-1]$. The z-transform of $x[n]$ is

(A) $\log\left(z + \frac{1}{2}\right)$, ROC: $|z| < \frac{1}{2}$
(B) $\log\left(z - \frac{1}{2}\right)$, ROC: $|z| < \frac{1}{2}$
(C) $\log(z-2)$, ROC: $|z| > 2$
(D) $\log(z+2)$, ROC: $|z| < 2$

MCQ 6.1.20 Consider a z-transform pair $x[n] \xleftrightarrow{z} X(z)$ with ROC R_x. The z transform and its ROC for $y[n] = a^n x[n]$ will be

(A) $X\left(\frac{z}{a}\right)$, ROC: $|a| R_x$
(B) $X(z+a)$, ROC: R_x
(C) $z^{-a} X(z)$, ROC: R_x
(D) $X(az)$, ROC: $|a| R_x$

MCQ 6.1.21 Let $X(z)$ be the z-transform of a causal signal $x[n] = a^n u[n]$ with ROC: $|z| > a$. Match the discrete sequences S_1, S_2, S_3 and S_4 with ROC of their z-transforms R_1, R_2 and R_3.

Sequences

S_1: $x[n-2]$

S_2: $x[n+2]$

S_3: $x[-n]$

S_4: $(-1)^n x[n]$

ROC

R_1: $|z| > a$

R_2: $|z| < a$

R_3: $|z| < \frac{1}{a}$

(A) $(S_1, R_1), (S_2, R_2), (S_3, R_3), (S_4, R_3)$
(B) $(S_1, R_1), (S_2, R_1), (S_3, R_3), (S_4, R_1)$
(C) $(S_1, R_2), (S_2, R_1), (S_3, R_2), (S_4, R_3)$
(D) $(S_1, R_1), (S_2, R_2), (S_3, R_3), (S_4, R_3)$

MCQ 6.1.22 Consider a discrete time signal $x[n] = \alpha^n u[n]$ and its z-transform $X(z)$. Match List I (discrete signals) with List II (z-transform) and select the correct answer using the codes given below:

	List-I (Discrete time signal)		List-II (z-transform)
P.	$x[n/2]$	1.	$z^{-2}X(z)$
Q.	$x[n-2]\,u[n-2]$	2.	$X(z^2)$
R.	$x[n+2]\,u[n]$	3.	$X(z/\beta^2)$
S.	$\beta^{2n}x[n]$	4.	$\alpha^2 X(z)$

Codes :

	P	Q	R	S
(A)	1	2	4	3
(B)	2	4	1	3
(C)	1	4	2	3
(D)	2	1	4	3

9 **MCQ 6.1.23** The z-transform of a discrete sequence $x[n]$ is $X(z)$, then the z-transform of $x[2n]$ will be

(A) $X(2z)$ (B) $X\left(\frac{z}{2}\right)$

(C) $\frac{1}{2}\left[X(\sqrt{z}) + X(-\sqrt{z})\right]$ (D) $X(\sqrt{z})$

MCQ 6.1.24 Consider a signal $x[n]$ and its z transform $X(z)$ given as

$$X(z) = \frac{4z}{8z^2 - 2z - 1}$$

The z-transform of the sequence $y[n] = x[0] + x[1] + x[2] + \ldots.. + x[n]$ will be

(A) $\dfrac{4z^2}{(z-1)(8z^2-2z-1)}$ (B) $\dfrac{4z(z-1)}{8z^2-2z-1}$

(C) $\dfrac{4z^2}{(z+1)(8z^2-2z-1)}$ (D) $\dfrac{4z(z+1)}{8z^2-2z-1}$

MCQ 6.1.25 What is the convolution of two DT sequence $x[n] = \{-\underset{\uparrow}{1}, 2, 0, 3\}$ and $h[n] = \{2, 0, \underset{\uparrow}{3}\}$

(A) $\{-2, -4, \underset{\uparrow}{3}, 6, 9\}$ (B) $\{-2, 4, -\underset{\uparrow}{3}, 12, 0, 9\}$

(C) $\{9, 6, \underset{\uparrow}{3}, -4, -2\}$ (D) $\{-\underset{\uparrow}{3}, 6, 7, 4, 6\}$

MCQ 6.1.26 If $x[n] \xleftrightarrow{z} X(z)$ be a z-transform pair, then which of the following is true?

(A) $x^*[n] \xleftrightarrow{z} X^*(-z)$ (B) $x^*[n] \xleftrightarrow{z} -X^*(z)$

(C) $x^*[n] \xleftrightarrow{z} X^*(z^*)$ (D) $x^*[n] \xleftrightarrow{z} X^*(-z^*)$

MCQ 6.1.27 A discrete-time system with input $x[n]$ and output $y[n]$ is governed by following difference equation

$$y[n] - \frac{1}{2}y[n-1] = x[n], \text{ with initial condition } y[-1] = 3$$

The impulse response of the system

(A) $\frac{5}{2}\left(\frac{n}{2} - 1\right),\ n \geq 0$ (B) $\frac{5}{2}\left(\frac{1}{2}\right)^n,\ n \geq 0$

(C) $\frac{5}{2}\left(\frac{1}{2}\right)^{n-1},\ n \geq 0$ (D) $\frac{5}{2}\left(\frac{1}{2}\right)^{n+1},\ n \geq 0$

MCQ 6.1.28 Consider a causal system with impulse response $h[n] = (2)^n u[n]$. If $x[n]$ is the input and $y[n]$ is the output to this system, then which of the following difference equation describes the system ?

(A) $y[n] + 2y[n+1] = x[n]$ (B) $y[n] - 2y[n-1] = x[n]$

(C) $y[n] + 2y[n-1] = x[n]$ (D) $y[n] - \frac{1}{2}y[n-1] = x[n]$

MCQ 6.1.29 The impulse response of a system is given as $h[n] = \delta[n] - (-\frac{1}{2})^n u[n]$. For an input $x[n]$ and output $y[n]$, the difference equation that describes the system is

(A) $y[n] + 2y[n-1] = 2x[n]$ (B) $y[n] + 0.5y[n-1] = 0.5x[n-1]$

(C) $y[n] + 2ny[n-1] = x[n]$ (D) $y[n] - 0.5y[n-1] = 0.5x[n-1]$

MCQ 6.1.30 The input-output relationship of a system is given as $y[n] - 0.4y[n-1] = x[n]$ where, $x[n]$ and $y[n]$ are the input and output respectively. The zero state response of the system for an input $x[n] = (0.4)^n u[n]$ is

(A) $n(0.4)^n u[n]$ (B) $n^2(0.4)^n u[n]$

(C) $(n+1)(0.4)^n u[n]$ (D) $\frac{1}{n}(0.4)^n u[n]$

MCQ 6.1.31 A discrete time system has the following input-output relationship $y[n] - \frac{1}{2}y[n-1] = x[n]$. If an input $x[n] = u[n]$ is applied to the system, then its zero state response will be

(A) $\left[\frac{1}{2} - (2)^n\right]u[n]$ (B) $\left[2 - \left(\frac{1}{2}\right)^n\right]u[n]$

(C) $\left[\frac{1}{2} - \left(\frac{1}{2}\right)^n\right]u[n]$ (D) $[2 - (2)^n]\,u[n]$

MCQ 6.1.32 Consider the transfer function of a system

$$H(z) = \frac{2z(z-1)}{z^2 + 4z + 4}$$

For an input $x[n] = 2\delta[n] + \delta[n+1]$, the system output is

(A) $2\delta[n+1] + 6(2)^n u[n]$ (B) $2\delta[n] - 6(-2)^n u[n]$

(C) $2\delta[n+1] - 6(-2)^n u[n]$ (D) $2\delta[n+1] + 6\left(\frac{1}{2}\right)^n u[n]$

MCQ 6.1.33 The transfer function of a discrete time LTI system is given as

$$H(z) = \frac{z}{z^2+1}, \quad \text{ROC}: |z| > 1$$

Consider the following statements

1. The system is causal and BIBO stable.
2. The system is causal but BIBO unstable.
3. The system is non-causal and BIBO unstable.
4. Impulse response $h[n] = \sin\left(\frac{\pi}{2}n\right)u[n]$

Which of the above statements are true ?

(A) 1 and 4 (B) 2 and 4

(C) 1 only (D) 3 and 4

MCQ 6.1.34 Which of the following statement is not true?
An LTI system with rational transfer function $H(z)$ is

(A) causal if the ROC is the exterior of a circle outside the outermost pole.

(B) stable if the ROC of $H(z)$ includes the unit circle $|z| = 1$.

(C) causal and stable if all the poles of $H(z)$ lie inside unit circle.

(D) none of above

MCQ 6.1.35 If $h[n]$ denotes the impulse response of a causal system, then which of the following system is not stable?

(A) $h[n] = n\left(\frac{1}{3}\right)^n u[n]$
(B) $h[n] = \frac{1}{3}\delta[n]$
(C) $h[n] = \delta[n] - \left(-\frac{1}{3}\right)^n u[n]$
(D) $h[n] = [(2)^n - (3)^n]\,u[n]$

MCQ 6.1.36 A causal system with input $x[n]$ and output $y[n]$ has the following relationship

$$y[n] + 3y[n-1] + 2y[n-2] = 2x[n] + 3x[n-1]$$

The system is

(A) stable
(B) unstable
(C) marginally stable
(D) none of these

MCQ 6.1.37 A causal LTI system is described by the difference equation $y[n] = x[n] + y[n-1]$

Consider the following statement

1. Impulse response of the system is $h[n] = u[n]$
2. The system is BIBO stable
3. For an input $x[n] = (0.5)^n u[n]$, system output is $y[n] = 2u[n] - (0.5)^n u[n]$

Which of the above statements is/are true?

(A) 1 and 2
(B) 1 and 3
(C) 2 and 3
(D) 1, 2 and 3

MCQ 6.1.38 Match List I (system transfer function) with List II (property of system) and choose the correct answer using the codes given below

	List-I (System transfer function)		List-II (Property of system)
P.	$H(z) = \frac{z^3}{(z-1.2)^3}$, ROC: $\lvert z\rvert > 1.2$	1.	Non causal but stable
Q.	$H(z) = \frac{z^2}{(z-1.2)^3}$, ROC: $\lvert z\rvert < 1.2$	2.	Neither causal nor stable
R.	$H(z) = \frac{z^4}{(z-0.8)^3}$, ROC: $\lvert z\rvert < 0.8$	3.	Causal but not stable
S.	$H(z) = \frac{z^3}{(z-0.8)^3}$, ROC: $\lvert z\rvert > 0.8$	4.	Both causal and stable

Codes :

	P	Q	R	S
(A)	4	2	1	3
(B)	1	4	2	3
(C)	3	1	2	4
(D)	3	2	1	4

• **MCQ 6.1.39** The transfer function of a DT feedback system is

$$H(z) = \frac{P}{1 + P\left(\frac{z}{z-0.9}\right)}$$

The range of P, for which the system is stable will be

(A) $-1.9 < P < -0.1$
(B) $P < 0$
(C) $P > -1$
(D) $P > -0.1$ or $P < -1.9$

•• **MCQ 6.1.40** Consider three stable LTI systems S_1, S_2 and S_3 whose transfer functions are

$$S_1 : H(z) = \frac{z - \frac{1}{2}}{2z^2 + \frac{1}{2}z - \frac{3}{16}}$$

$$S_2 : H(z) = \frac{z+1}{-\frac{2}{3}z^{-3} - \frac{1}{2}z^{-2} + \frac{4}{3} + z}$$

$$S_3 : H(z) = \frac{1 + \frac{1}{2}z^{-2} - \frac{4}{3}z^{-1}}{z^{-1}(1 - \frac{1}{3}z^{-1})(1 - \frac{1}{2}z^{-1})}$$

Which of the above systems is/are causal?

(A) S_1 only
(B) S_1 and S_2
(C) S_1 and S_3
(D) S_1, S_2 and S_3

MCQ 6.1.41 The z-transform of $\delta[n-k]$, $k > 0$ is

(A) z^k, $z > 0$
(B) z^{-k}, $z > 0$
(C) z^k, $z \neq 0$
(D) z^{-k}, $z \neq 0$

MCQ 6.1.42 The z-transform of $\delta[n+k]$, $k > 0$ is

(A) z^{-k}, $z \neq 0$
(B) z^k, $z \neq 0$
(C) z^{-k}, all z
(D) z^k, all z

MCQ 6.1.43 The z-transform of $u[n]$ is

(A) $\frac{1}{1-z^{-1}}$, $\lvert z\rvert > 1$
(B) $\frac{1}{1-z^{-1}}$, $\lvert z\rvert < 1$
(C) $\frac{z}{1-z^{-1}}$, $\lvert z\rvert < 1$
(D) $\frac{z}{1-z^{-1}}$, $\lvert z\rvert > 1$

MCQ 6.1.44 The z-transform of $\left(\frac{1}{4}\right)^n (u[n] - u[n-5])$ is

(A) $\frac{z^5 - (0.25)^5}{z^4(z-0.25)}$, $\lvert z\rvert > 0.25$
(B) $\frac{z^5 - (0.25)^5}{z^4(z-0.25)}$, $\lvert z\rvert > 0.5$
(C) $\frac{z^5 - (0.25)^5}{z^3(z-0.25)}$, $\lvert z\rvert < 0.25$
(D) $\frac{z^5 - (0.25)^5}{z^4(z-0.25)}$, all z

MCQ 6.1.45 The z-transform of $\left(\frac{1}{4}\right)^n u[-n]$ is

(A) $\frac{4z}{4z-1}$, $\lvert z\rvert > \frac{1}{4}$
(B) $\frac{4z}{4z-1}$, $\lvert z\rvert < \frac{1}{4}$
(C) $\frac{1}{1-4z}$, $\lvert z\rvert > \frac{1}{4}$
(D) $\frac{1}{1-4z}$, $\lvert z\rvert < \frac{1}{4}$

MCQ 6.1.46 The z-transform of $3^n u[-n-1]$ is

(A) $\frac{z}{3-z}$, $\lvert z\rvert > 3$
(B) $\frac{z}{3-z}$, $\lvert z\rvert < 3$
(C) $\frac{3}{3-z}$, $\lvert z\rvert > 3$
(D) $\frac{3}{3-z}$, $\lvert z\rvert < 3$

• **MCQ 6.1.47** The z-transform of $\left(\frac{2}{3}\right)^{\lvert n\rvert}$ is

(A) $\frac{-5z}{(2z-3)(3z-2)}$, $-\frac{3}{2} < \lvert z\rvert < -\frac{2}{3}$
(B) $\frac{-5z}{(2z-3)(3z-2)}$, $\frac{2}{3} < \lvert z\rvert < \frac{3}{2}$
(C) $\frac{5z}{(2z-3)(3z-2)}$, $\frac{2}{3} < \lvert z\rvert < \frac{2}{3}$
(D) $\frac{5z}{(2z-3)(3z-2)}$, $-\frac{3}{2} < \lvert z\rvert < -\frac{2}{3}$

MCQ 6.1.48 The z-transform of $\cos\left(\frac{\pi}{3}n\right)u[n]$ is

(A) $\frac{z(2z-1)}{2(z^2-z+1)},\ 0<|z|<1$ (B) $\frac{z(2z-1)}{2(z^2-z+1)},\ |z|>1$

(C) $\frac{z(1-2z)}{2(z^2-z+1)},\ 0<|z|<1$ (D) $\frac{z(1-2z)}{2(z^2-z+1)},\ |z|>1$

MCQ 6.1.49 The z-transform of $\{3, 0, 0, 0, 0, \underset{\uparrow}{6}, 1, -4\}$

(A) $3z^5+6+z^{-1}-4z^{-2},\quad 0\le|z|<\infty$
(B) $3z^5+6+z^{-1}-4z^{-2},\quad 0<|z|<\infty$
(C) $3z^{-5}+6+z-4z^2,\quad 0<|z|<\infty$
(D) $3z^{-5}+6+z-4z^2,\quad 0\le|z|<\infty$

MCQ 6.1.50 The z-transform of $x[n]=\{2, 4, \underset{\uparrow}{5}, 7, 0, 1\}$

(A) $2z^2+4z+5+7z+z^3,\qquad z\ne\infty$
(B) $2z^{-2}+4z^{-1}+5+7z+z^3,\qquad z\ne\infty$
(C) $2z^{-2}+4z^{-1}+5+7z+z^3,\quad 0<|z|<\infty$
(D) $2z^2+4z+5+7z^{-1}+z^{-3},\quad 0<|z|<\infty$

MCQ 6.1.51 The z-transform of $x[n]=\{\underset{\uparrow}{1}, 0, -1, 0, 1, -1\}$ is

(A) $1+2z^{-2}-4z^{-4}+5z^{-5},\quad z\ne0$
(B) $1-z^{-2}+z^{-4}-z^{-5},\quad z\ne0$
(C) $1-2z^2+4z^4-5z^5,\quad z\ne0$
(D) $1-z^2+z^4-z^5,\quad z\ne0$

MCQ 6.1.52 The time signal corresponding to $\frac{z^2-3z}{z^2+\frac{3}{2}z-1},\ \frac{1}{2}<|z|<2$ is

(A) $-\frac{1}{2^n}u[n]-2^{n+1}u[-n-1]$ (B) $-\frac{1}{2^n}u[n]-2^{n+1}u[n+1]$

(C) $\frac{1}{2^n}u[n]+2^{n+1}u[n+1]$ (D) $\frac{1}{2^n}u[n]-2^{-n-1}u[-n-1]$

MCQ 6.1.53 The time signal corresponding to $\frac{3z^2-\frac{1}{4}z}{z^2-16},\ |z|>4$ is

(A) $\left[\frac{49}{32}(-4)^n+\frac{47}{32}4^n\right]u[n]$ (B) $\left[\frac{49}{32}4^n+\frac{47}{32}4^n\right]u[n]$

(C) $\frac{49}{32}(-4)^n u[-n]+\frac{47}{32}4^n u[n]$ (D) $\frac{49}{32}4^n u[n]+\frac{47}{32}(-4)^n u[-n]$

MCQ 6.1.54 The time signal corresponding to $\frac{2z^4-2z^3-2z^2}{z^2-1},\ |z|>1$ is

(A) $2\delta[n-2]+[1-(-1)^n]u[n-2]$
(B) $2\delta[n+2]+[1-(-1)^n]u[n+2]$
(C) $2\delta[n+2]+[(-1)^n-1]u[n+2]$
(D) $2\delta[n-2]+[(-1)^n-1]u[n-2]$

MCQ 6.1.55 The time signal corresponding to $1+2z^{-6}+4z^{-8},\ |z|>0$ is

(A) $\delta[n]+2\delta[n-6]+4\delta[n-8]$
(B) $\delta[n]+2\delta[n+6]+4\delta[n+8]$
(C) $\delta[-n]+2\delta[-n+6]+4\delta[-n+8]$
(D) $\delta[-n]+2\delta[-n-6]+4\delta[-n-8]$

MCQ 6.1.56 The time signal corresponding to $\sum_{k=5}^{10}\frac{1}{k}z^{-k},\ |z|>0$ is

(A) $\sum_{k=5}^{10}\frac{1}{k}\delta[n+k]$ (B) $\sum_{k=5}^{10}\frac{1}{k}\delta[n-k]$

(C) $\sum_{k=5}^{10}\frac{1}{k}\delta[-n+k]$ (D) $\sum_{k=5}^{10}\frac{1}{k}\delta[-n-k]$

MCQ 6.1.57 The time signal corresponding to $(1+z^{-1})^3,\ |z|>0$ is

(A) $\delta[-n]+3\delta[-n-1]+3\delta[-n-2]+\delta[-n-3]$
(B) $\delta[-n]+3\delta[-n+1]+3\delta[-n+2]+\delta[-n+3]$
(C) $\delta[n]+3\delta[n+1]+3\delta[n+2]+\delta[n+3]$
(D) $\delta[n]+3\delta[n-1]+3\delta[n-2]+\delta[n-3]$

MCQ 6.1.58 The time signal corresponding to $z^6+z^2+3+2z^{-3}+z^{-4},\ |z|>0$ is

(A) $\delta[n+6]+\delta[n+2]+3\delta[n]+2\delta[n-3]+\delta[n-4]$
(B) $\delta[n-6]+\delta[n-2]+3\delta[n]+2\delta[n+3]+\delta[n+4]$
(C) $\delta[-n+6]+\delta[-n+2]+3\delta[-n]+2\delta[-n+3]+\delta[-n+4]$
(D) $\delta[-n-6]+\delta[-n-2]+3\delta[-n]+2\delta[-n-3]+\delta[-n-4]$

MCQ 6.1.59 The time signal corresponding to $\frac{1}{1-\frac{1}{4}z^{-2}},\ |z|>\frac{1}{2}$ is

(A) $\begin{cases}2^{-n}, & n \text{ even and } n\ge0\\ 0, & \text{otherwise}\end{cases}$ (B) $\left(\frac{1}{4}\right)^{2n}u[n]$

(C) $\begin{cases}2^{-n}, & n \text{ odd}, n>0\\ 0, & n \text{ even}\end{cases}$ (D) $2^{-n}u[n]$

MCQ 6.1.60 The time signal corresponding to $\frac{1}{1-\frac{1}{4}z^{-2}},\ |z|<\frac{1}{2}$ is

(A) $-\sum_{k=0}^{\infty}2^{2(k+1)}\delta[-n-2(k+1)]$

(B) $-\sum_{k=0}^{\infty}2^{2(k+1)}\delta[-n+2(k+1)]$

(C) $-\sum_{k=0}^{\infty}2^{2(k+1)}\delta[n+2(k+1)]$

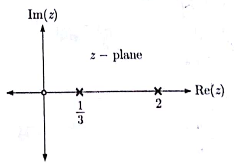
(D) $-\sum_{k=0}^{\infty}2^{2(k+1)}\delta[n-2(k+1)]$

MCQ 6.1.61 The time signal corresponding to $\ln(1+z^{-1}),\ |z|>0$ is

(A) $\frac{(-1)^{k-1}}{k}\delta[n-k]$ (B) $\frac{(-1)^{k-1}}{k}\delta[n+k]$

(C) $\frac{(-1)^k}{k}\delta[n-k]$ (D) $\frac{(-1)^k}{k}\delta[n+k]$

MCQ 6.1.62 $X[z]$ of a system is specified by a pole zero pattern as following :



Consider three different solution of $x[n]$

$$x_1[n] = \left[2^n - \left(\frac{1}{3}\right)^n\right]u[n]$$

$$x_2[n] = -2^n u[n-1] - \frac{1}{3^n}u[n]$$

$$x_3[n] = -2^n u[n-1] + \frac{1}{3^n}u[-n-1]$$

Correct solution is

(A) $x_1[n]$ (B) $x_2[n]$

(C) $x_3[n]$ (D) All three

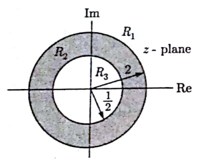
MCQ 6.1.63 Consider three different signal

$$x_1[n] = \left[2^n - \left(\frac{1}{2}\right)^n\right]u[n]$$

$$x_2[n] = -2^n u[-n-1] + \frac{1}{2^n}u[-n-1]$$

$$x_3[n] = -2^n u[-n-1] - \frac{1}{2^n}u[n]$$

Following figure shows the three different region. Choose the correct for the ROC of signal



	R_1	R_2	R_3
(A)	$x_1[n]$	$x_2[n]$	$x_3[n]$
(B)	$x_2[n]$	$x_3[n]$	$x_1[n]$
(C)	$x_1[n]$	$x_3[n]$	$x_2[n]$
(D)	$x_3[n]$	$x_2[n]$	$x_1[n]$

MCQ 6.1.64 Given the z-transform

$$X(z) = \frac{1 + \frac{7}{6}z^{-1}}{(1 - \frac{1}{2}z^{-1})(1 + \frac{1}{3}z^{-1})}$$

For three different ROC consider there different solution of signal $x[n]$:

(a) $|z| > \frac{1}{2}$, $x[n] = \left[\frac{1}{2^{n-1}} - \left(\frac{-1}{3}\right)^n\right]u[n]$

(b) $|z| < \frac{1}{3}$, $x[n] = \left[\frac{-1}{2^{n-1}} + \left(\frac{-1}{3}\right)^n\right]u[-n+1]$

(c) $\frac{1}{3} < |z| < \frac{1}{2}$, $x[n] = -\frac{1}{2^{n-1}}u[-n-1] - \left(\frac{-1}{3}\right)^n u[n]$

Correct solution are

(A) (a) and (b)

(B) (a) and (c)

(C) (b) and (c)

(D) (a), (b), (c)

MCQ 6.1.65 The $X(z)$ has poles at $z = \frac{1}{2}$ and $z = -1$. If $x[1] = 1$, $x[-1] = 1$, and the ROC includes the point $z = \frac{3}{4}$. The time signal $x[n]$ is

(A) $\frac{1}{2^{n-1}}u[n] - (-1)^n u[-n-1]$ (B) $\frac{1}{2^n}u[n] - (-1)^n u[-n-1]$

(C) $\frac{1}{2^{n-1}}u[n] + u[-n+1]$ (D) $\frac{1}{2^n}u[n] + u[-n+1]$

MCQ 6.1.66 If $x[n]$ is right-sided, $X(z)$ has a signal pole and $x[0] = 2$, $x[2] = \frac{1}{2}$, then $x[n]$ is

(A) $\frac{u[-n]}{2^{n-1}}$ (B) $\frac{u[n]}{2^{n-1}}$

(C) $\frac{u[-n]}{2^{n+1}}$ (D) $a\frac{u[-n]}{2^{n+1}}$

MCQ 6.1.67 The z-transform of $\left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{4}\right)^n u[-n-1]$ is

(A) $\frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-\frac{1}{4}z^{-1}}$, $\frac{1}{4} < |z| < \frac{1}{2}$

(B) $\frac{1}{1-\frac{1}{2}z^{-1}} + \frac{1}{1-\frac{1}{4}z^{-1}}$, $\frac{1}{4} < |z| < \frac{1}{2}$

(C) $\frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-\frac{1}{4}z^{-1}}$, $|z| > \frac{1}{2}$

(D) None of the above

Common Data For Q. 68 - 73:

Given the z-transform pair $x[n] \overset{Z}{\longleftrightarrow} \frac{z^2}{z^2 - 16}$, $|z| < 4$

MCQ 6.1.68 The z-transform of the signal $x[n-2]$ is

(A) $\frac{z^4}{z^2 - 16}$ (B) $\frac{(z+2)^2}{(z+2)^2 - 16}$

(C) $\frac{1}{z^2 - 16}$ (D) $\frac{(z-2)^2}{(z-2)^2 - 16}$

MCQ 6.1.69 The z-transform of the signal $y[n] = \frac{1}{2^n}x[n]$ is

(A) $\frac{(z+2)^2}{(z+2)^2 - 16}$ (B) $\frac{z^2}{z^2 - 4}$

(C) $\frac{(z-2)^2}{(z-2)^2 - 16}$ (D) $\frac{z^2}{z^2 - 64}$

MCQ 6.1.70 The z-transform of the signal $x[-n] * x[n]$ is

(A) $\frac{z^2}{16z^2 - 257z^4 - 16}$ (B) $\frac{-16z^2}{(z^2 - 16)^2}$

(C) $\frac{z^2}{257z^2 - 16z^4 - 16}$ (D) $\frac{16z^2}{(z^2 - 16)^2}$

MCQ 6.1.71 The z-transform of the signal $nx[n]$ is

(A) $\frac{32z^2}{(z^2 - 16)^2}$ (B) $\frac{-32z^2}{(z^2 - 16)^2}$

(C) $\frac{32z}{(z^2 - 16)^2}$ (D) $\frac{-32z}{(z^2 - 16)^2}$

MCQ 6.1.72 The z-transform of the signal $x[n+1]+x[n-1]$ is

(A) $\frac{(z+1)^2}{(z+1)^2-16}+\frac{(z-1)^2}{(z-1)^2-16}$ (B) $\frac{z(z^2+1)}{z^2-16}$

(C) $\frac{z^2(-1+z)}{z^2-16}$ (D) None of the above

MCQ 6.1.73 The z-transform of the signal $x[n]*x[n-3]$ is

(A) $\frac{z^{-3}}{(z^2-16)^2}$ (B) $\frac{z^7}{(z^2-16)^2}$

(C) $\frac{z^5}{(z^2-16)^2}$ (D) $\frac{z}{(z^2-16)^2}$

Common Data For Q. 74 - 78:-

Given the z-transform pair $3^n n^2 u[n] \xleftrightarrow{z} X(z)$

MCQ 6.1.74 The time signal corresponding to $X(2z)$ is

(A) $n^2 3^n u[2n]$ (B) $\left(-\frac{3}{2}\right)^n n^2 u[n]$

(C) $\left(\frac{3}{2}\right)^n n^2 u[n]$ (D) $6^n n^2 u[n]$

MCQ 6.1.75 The time signal corresponding to $X(z^{-1})$ is

(A) $n^2 3^{-n} u[-n]$ (B) $n^2 3^{-n} u[-n]$

(C) $\frac{1}{n^2} 3^{\frac{1}{n}} u[n]$ (D) $\frac{1}{n^2} 3^{\frac{1}{n}} u[-n]$

MCQ 6.1.76 The time signal corresponding to $\frac{d}{dz}X(z)$ is

(A) $(n-1)^3 3^{n-1} u[n-1]$

(B) $n^3 3^n u[n-1]$

(C) $(1-n)^3 3^{n-1} u[n-1]$

(D) $(n-1)^3 3^{n-1} u[n]$

MCQ 6.1.77 The time signal corresponding to $\left(\frac{z^2-z^{-2}}{2}\right)X(z)$ is

(A) $\frac{1}{2}(x[n+2]-x[n-2])$ (B) $x[n+2]-x[n-2]$

(C) $\frac{1}{2}x[n-2]-x[n+2])$ (D) $x[n-2]-x[n+2]$

MCQ 6.1.78 The time signal corresponding to $\{X(z)\}^2$ is

(A) $[x[n]]^2$ (B) $x[n]*x[n]$

(C) $x(n)*x[-n]$ (D) $x[-n]*x[-n]$

MCQ 6.1.79 A causal system has

Input, $x[n]=\delta[n]+\frac{1}{4}\delta[n-1]-\frac{1}{8}\delta[n-2]$ and

Output, $y[n]=\delta[n]-\frac{3}{4}\delta[n-1]$

The impulse response of this system is

(A) $\frac{1}{3}\left[5\left(\frac{-1}{2}\right)^n-2\left(\frac{1}{4}\right)^n\right]u[n]$ (B) $\frac{1}{3}\left[5\left(\frac{1}{2}\right)^n+2\left(\frac{-1}{4}\right)^n\right]u[n]$

(C) $\frac{1}{3}\left[5\left(\frac{1}{2}\right)^n-2\left(\frac{-1}{4}\right)^n\right]u[n]$ (D) $\frac{1}{3}\left[5\left(\frac{1}{2}\right)^n+2\left(\frac{1}{4}\right)^n\right]u[n]$

MCQ 6.1.80 A causal system has

Input, $x[n]=(-3)^n u[n]$

Output, $y[n]=[4(2)^n-(\frac{1}{2})^n]u[n]$

The impulse response of this system is

(A) $\left[7\left(\frac{1}{2}\right)^n-10\left(\frac{1}{2}\right)^n\right]u[n]$ (B) $\left[7(2^n)-10\left(\frac{1}{2}\right)^n\right]u[n]$

(C) $\left[10\left(\frac{1}{2}\right)^2-7(2)^n\right]u[n]$ (D) $\left[10(2^n)-7\left(\frac{1}{2}\right)^n\right]u[n]$

MCQ 6.1.81 A system has impulse response $h[n]=(\frac{1}{2})^n u[n]$. The output $y[n]$ to the input $x[n]$ is given by $y[n]=2\delta[n-4]$. The input $x[n]$ is

(A) $2\delta[-n-4]-\delta[-n-5]$

(B) $2\delta[n+4]-\delta[n+5]$

(C) $2\delta[-n+4]-\delta[-n+5]$

(D) $2\delta[n-4]-\delta[n-5]$

MCQ 6.1.82 A system is described by the difference equation

$$y[n]=x[n]-x[n-2]+x[n-4]-x[n-6]$$

The impulse response of system is

(A) $\delta[n]-2\delta[n+2]+4\delta[n+4]-6\delta[n+6]$

(B) $\delta[n]+2\delta[n-2]-4\delta[n-4]+6\delta[n-6]$

(C) $\delta[n]-\delta[n-2]+\delta[n-4]-\delta[n-6]$

(D) $\delta[n]-\delta[n+2]+\delta[n+4]-\delta[n+6]$

MCQ 6.1.83 The impulse response of a system is given by $h[n]=\frac{3}{4^n}u[n-1]$. The difference equation representation for this system is

(A) $4y[n]-y[n-1]=3x[n-1]$

(B) $4y[n]-y[n+1]=3x[n+1]$

(C) $4y[n]+y[n-1]=-3x[n-1]$

(D) $4y[n]+y[n+1]=3x[n+1]$

MCQ 6.1.84 The impulse response of a system is given by $h[n]=\delta[n]-\delta[n-5]$. The difference equation representation for this system is

(A) $y[n]=x[n]-x[n-5]$

(B) $y[n]=x[n]-x[n+5]$

(C) $y[n]=x[n]+5x[n-5]$

(D) $y[n]=x[n]-5x[n+5]$

MCQ 6.1.85 Consider the following three systems

$$y_1[n]=0.2y[n-1]+x[n]-0.3x[n-1]+0.02x[n-2]$$
$$y_2[n]=x[n]-0.1x[n-1]$$
$$y_3[n]=0.5y[n-1]+0.4x[n]-0.3x[n-1]$$

The equivalent system are

(A) $y_1[n]$ and $y_2[n]$

(B) $y_2[n]$ and $y_3[n]$

(C) $y_3[n]$ and $y_1[n]$

(D) all

MCQ 6.1.86 The z-transform function of a stable system is $H(z) = \frac{2-\frac{3}{2}z^{-1}}{(1-2z^{-1})(1+\frac{1}{2}z^{-1})}$. The impulse response $h[n]$ is

(A) $2^n u[-n+1] - \left(\frac{1}{2}\right)^n u[n]$ (B) $-2^n u[-n-1] + \left(\frac{-1}{2}\right)^n u[n]$

(C) $-2^n u[-n-1] - \left(\frac{-1}{2}\right)^n u[n]$ (D) $2^n u[n] - \left(\frac{1}{2}\right)^n u[n]$

MCQ 6.1.87 The transfer function of a causal system is $H(z) = \frac{5z^2}{z^2-z-6}$. The impulse response is

(A) $(3^n + (-1)^n 2^{n+1})\,u[n]$
(B) $(3^{n+1} + 2(-2)^n)\,u[n]$
(C) $(3^{n-1} + (-1)^n 2^{n+1})\,u[n]$
(D) $(3^{n-1} - (-2)^{n+1})\,u[n]$

MCQ 6.1.88 The transfer function of a system is given by $H(z) = \frac{z(3z-2)}{z^2-z-\frac{1}{4}}$. The system is

(A) causal and stable
(B) causal, stable and minimum phase
(C) minimum phase
(D) none of the above

MCQ 6.1.89 The z-transform of a signal $x[n]$ is $X(z) = \frac{3}{1-\frac{10}{3}z^{-1}+z^{-2}}$. If $X(z)$ converges on the unit circle, $x[n]$ is

(A) $-\frac{1}{3^{n-1}(8)}u[n] - \frac{3^{n+3}}{8}u[-n-1]$ (B) $\frac{1}{3^{n-1}(8)}u[n] - \frac{3^{n+3}}{(8)}u[-n]$

(C) $\frac{1}{3^{n-1}(8)}u[n] - \frac{3^{n+3}}{(8)}u[-n]$ (D) $-\frac{1}{3^{n-1}(8)}u[n] - \frac{3^{n+3}}{(8)}u[-n]$

MCQ 6.1.90 The transfer function of a system is $H(z) = \frac{4z^{-1}}{(1-\frac{1}{4}z^{-1})^2}$, $|z| > \frac{1}{4}$. The $h[n]$ is

(A) stable
(B) causal
(C) stable and causal
(D) none of the above

MCQ 6.1.91 The transfer function of a system is given as

$$H(z) = \frac{2(z+\frac{1}{2})}{(z-\frac{1}{2})(z-\frac{1}{3})}$$

Consider the two statements

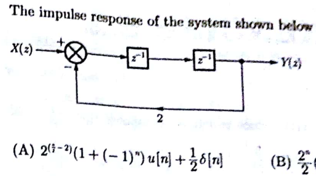
Statement (1) : System is causal and stable.

Statement (2) : Inverse system is causal and stable.

The correct option is

(A) (1) is true
(B) (2) is true
(C) Both (1) and (2) are true
(D) Both are false

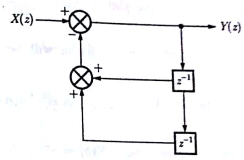
MCQ 6.1.92 The impulse response of the system shown below is



(A) $2^{(\frac{n}{2}-2)}(1+(-1)^n)\,u[n] + \frac{1}{2}\delta[n]$ (B) $\frac{2^n}{2}(1+(-1)^n)\,u[n] + \frac{1}{2}\delta[n]$

(C) $2^{(\frac{n}{2}-2)}(1+(-1)^n)\,u[n] - \frac{1}{2}\delta[n]$ (D) $\frac{2^n}{2}[1+(-1)^n]\,u[n] - \frac{1}{2}\delta[n]$

MCQ 6.1.93 The system diagram for the transfer function $H(z) = \frac{z}{z^2+z+1}$. is shown below.



The system diagram is a
(A) Correct solution
(B) Not correct solution
(C) Correct and unique solution
(D) Correct but not unique solution

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EXERCISE 6.2

QUES 6.2.1 Consider a DT signal which is defined as follows

$$x[n] = \begin{cases} \left(\frac{1}{2}\right)^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

The z-transform of $x[n]$ will be $\frac{az}{az-1}$ such that the value of a is _______

QUES 6.2.2 If the z-transform of a sequence $x[n] = \{1, 1, -1, -\underset{\uparrow}{1}\}$ is $X(z)$, then what is the value of $X(1/2)$?

QUES 6.2.3 The z-transform of a discrete time signal $x[n]$ is $X(z) = \frac{z+1}{z(z-1)}$. Then, $x[0] + x[1] + x[2] =$ _______

QUES 6.2.4 If $x[n] = \alpha^n u[n]$, then the z-transform of $x[n+3]u[n]$ will be $\alpha^k\left(\frac{z}{z-k}\right)$, where $k =$ _______

QUES 6.2.5 The inverse z-transform of a function $X(z) = \frac{z^{-9}}{z-\alpha}$ is $\alpha^{n-k}u[n-k]$ where the value of k is _______

QUES 6.2.6 Let $x[n] \xleftrightarrow{Z} X(z)$ be a z-transform pair, where $X(z) = \frac{z^{-2}}{z-3}$. What will be the value of $x[5]$?

QUES 6.2.7 The z-transform of a discrete time sequence $y[n] = n[n+1]u[n]$ is $\frac{kz^k}{(z-1)^{k+1}}$ such that the value of k is _______

QUES 6.2.8 A signal $x[n]$ has the following z-transform $X(z) = \log(1-2z)$, ROC: $|z| < \frac{1}{2}$. Let the signal be

$$x[n] \frac{1}{n}\left(\frac{1}{2}\right)^n u[a-n]$$

what is the value of a in the expression ?

QUES 6.2.9 Let $x[n] \xleftrightarrow{Z} X(z)$ be a z-transform pair. Consider another signal $y[n]$ defined as

$$y[n] = \begin{cases} x[n/2], & \text{if } n \text{ is even} \\ 0, & \text{if } n \text{ is odd} \end{cases}$$

The z-transform of $y[n]$ is $X(z^k)$ such that the value of k is _______

QUES 6.2.10 Let $X(z)$ be z-transform of a discrete time sequence $x[n] = (-\frac{1}{2})^n u[n]$. Consider another signal $y[n]$ and its z-transform $Y(z)$ given as $Y(z) = X(z^3)$. What is the value of $y[n]$ at $n = 4$?

QUES 6.2.11 Let $h[n] = \{1, 2, 0, -1, 1\}$ and $x[n] = \{1, 3, -1, -2\}$ be two discrete time sequences. What is the value of convolution $y[n] = h[n] * x[n]$ at $n = 4$?

QUES 6.2.12 A discrete time sequence is defined as follows

$$x[n] = \begin{cases} 1, & n \text{ is even} \\ 0, & \text{otherwise} \end{cases}$$

What is the final value of $x[n]$?

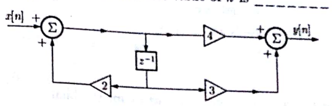
QUES 6.2.13 Let $X(z)$ be the z-transform of a DT signal $x[n]$ given as

$$X(z) = \frac{0.5z^2}{(z-1)(z-0.5)}$$

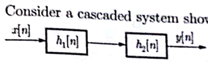
The initial value of $x[n]$ is _______

QUES 6.2.14 The signal $x[n] = (0.5)^n u[n]$ is when applied to a digital filter, it yields the following output $y[n] = \delta[n] - 2\delta[n-1]$. If impulse response of the filter is $h[n]$, then what will be the value of sample $h[1]$?

QUES 6.2.15 The transfer function for the system realization shown in the figure will be $\frac{k(z+1)-1}{z-2}$ such that the value of k is _______



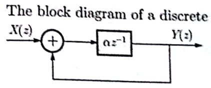
QUES 6.2.16 Consider a cascaded system shown in the figure



where, $h_1[n] = \delta[n] + \frac{1}{2}\delta[n-1]$ and $h_2[n] = \left(\frac{1}{2}\right)^n u[n]$

If an input $x[n] = \cos(n\pi)$ is applied, then output $y[n] = k\cos n\pi$ where the constant k is _______

QUES 6.2.17 The block diagram of a discrete time system is shown in the figure below



The system is BIBO stable for $|\alpha| <$ _______

QUES 6.2.18 Let $x[n] = \delta[n-1] + \delta[n+2]$. If unilateral z-transform of the signal $x[n]$ be

$$X(z) = z^k$$

then, the value of constant k is _______

QUES 6.2.19 The unilateral z-transform of signal $x[n] = u[n+4]$ is $\frac{1}{1+a/z}$ such that the value of a is _______

QUES 6.2.20 If z-transform is given by $X(z) = \cos(z^{-3})$, $|z| > 0$, then what will be the value of $x[12]$?

QUES 6.2.21 The z-transform of an anticausal system is $X(z) = \frac{12-21z}{3-7z+12z^2}$. What will be the yalue of $x[0]$?

QUES 6.2.22 The system $y[n] = cy[n-1] - 0.12y[n-2] + x[n-1] + x[n-2]$ is stable if $|c| <$ _______

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EXERCISE 6.3

MCQ 6.3.1 The z-transform is used to analyze
(A) discrete time signals and system
(B) continuous time signals and system
(C) both (A) and (B)
(D) none

MCQ 6.3.2 Which of the following expression is correct for the bilateral z-transform of $x[n]$?
(A) $\sum_{n=0}^{\infty} x[n] z^n$ (B) $\sum_{n=0}^{\infty} x[n] z^{-n}$
(C) $\sum_{n=-\infty}^{\infty} x[n] z^n$ (D) $\sum_{n=-\infty}^{\infty} x[n] z^{-n}$

MCQ 6.3.3 The unilateral z-transform of sequence $x[n]$ is defined as
(A) $\sum_{n=0}^{\infty} x[n] z^n$ (B) $\sum_{n=-\infty}^{\infty} x[n] z^n$
(C) $\sum_{n=0}^{\infty} x[n] z^{-n}$ (D) $\sum_{n=-\infty}^{\infty} x[n] z^{-n}$

MCQ 6.3.4 The z-transform of a causal signal $x[n]$ is given by
(A) $\sum_{n=-\infty}^{\infty} x[n] z^n$ (B) $\sum_{n=0}^{\infty} x[n] z^n$
(C) $\sum_{n=-\infty}^{\infty} x[n] z^{-n}$ (D) $\sum_{n=0}^{\infty} x[n] z^{-n}$

MCQ 6.3.5 For a signal $x[n]$, its unilateral z-transform is equivalent to the bilateral z-transform of
(A) $x[n]\, r[n]$ (B) $x[n]\, \delta[n]$
(C) $x[n]\, u[n]$ (D) none of these

MCQ 6.3.6 The ROC of z-transform $X(z)$ is defined as the range of values of z for which $X(z)$
(A) zero (B) diverges
(C) converges (D) none

MCQ 6.3.7 In the z-plane the ROC of z-transform $X(z)$ consists of a
(A) strip (B) parabola
(C) rectangle (D) ring

MCQ 6.3.8 If $x[n]$ is a right-sided sequence, and if the circle $z = r_0$ is in the ROC, then
(A) the values of z for which $z > r_0$ will also be in the ROC
(B) the values of z for which $z < r_0$ will also be in the ROC
(C) both (A) & (B)
(D) none of these

MCQ 6.3.9 The ROC does not contain any
(A) poles (B) 1's
(C) zeros (D) none

MCQ 6.3.10 Let $x[n] \xleftrightarrow{z} X(z)$ be a z-transform pair. If $x[n] = \delta[n]$, then the ROC of $X(z)$ is
(A) $|z| < 1$ (B) $|z| > 1$
(C) entire z-plane (D) none of the above

MCQ 6.3.11 The ROC of z-transform of unit-step sequence $u[n]$, is
(A) entire z-plane (B) $|z| < 1$
(C) $|z| > 1$ (D) none of the above

MCQ 6.3.12 The ROC of the unilateral z-transform of α^n is
(A) $|z| > |\alpha|$
(B) $|z| < |\alpha|$
(C) $|z| < 1$
(D) $|z| > 1$

MCQ 6.3.13 Which of the following statement about ROC is not true ?
(A) ROC never lies exactly at the boundary of a circle
(B) ROC consists of a circle in the z-plane centred at the origin
(C) ROC of a right handed finite sequence is the entire z-plane except $z = 0$
(D) ROC contains both poles and zeroes

MCQ 6.3.14 The z-transform of unit step sequence is
(A) 1 (B) $\frac{1}{z-1}$
(C) $\frac{z}{z-1}$ (D) 0

MCQ 6.3.15 The ROC for the z-transform of the sequence $x[n] = u[-n]$ is
(A) $|z| > 0$ (B) $|z| < 1$
(C) $|z| > 1$ (D) does not exist

MCQ 6.3.16 Let $x[n] \xleftrightarrow{z} X(z)$, then unilateral z-transform of sequence $x_1[n] = x[n-1]$ will be
(A) $X_1(z) = z^{-1}X(z) + x[0]$
(B) $X_1(z) = z^{-1}X(z) - x[1]$
(C) $X_1(z) = z^{-1}X(z) - x[-1]$
(D) $X_1(z) = z^{-1}X[z] + x[-1]$

MCQ 6.3.17 Let $x[n] \xleftrightarrow{z} X(z)$, the bilateral z-transform of $x[n-n_0]$ is given by
(A) $zX(z)$ (B) $z^{n_0}X(z)$
(C) $z^{-n_0}X(z)$ (D) $\frac{1}{z}X(z)$

MCQ 6.3.18 If the ROC of z-transform of $x[n]$ is R_x then the ROC of z-transform of $x[-n]$ is
(A) R_x
(B) $-R_x$
(C) $1/R_x$
(D) none of these

MCQ 6.3.19 If $X(z) = \mathcal{Z}\{x[n]\}$, then $X(z) = \mathcal{Z}\{a^{-n}x[n]\}$ will be
(A) $X(az)$ (B) $X\left(\frac{z}{a}\right)$
(C) $X\left(\frac{a}{z}\right)$ (D) $X\left(\frac{1}{az}\right)$

MCQ 6.3.20 If $x[n]$ and $y[n]$ are two discrete time sequences, then the z-transform of correlation of the sequences $x[n]$ and $y[n]$ is
(A) $X(z^{-1})\,Y(z^{-1})$
(B) $X(z)\,Y(z^{-1})$
(C) $X(z) * Y(z)$
(D) $X^*(z)\,Y^*(z^{-1})$

MCQ 6.3.21 If $X(z) = \mathcal{Z}\{x[n]\}$, then, value of $x[0]$ is equal to
(A) $\lim\limits_{z\to 0} zX(z)$
(B) $\lim\limits_{z\to 1}(z-1)X(z)$
(C) $\lim\limits_{z\to\infty} X(z)$
(D) $\lim\limits_{z\to 0} X(z)$

MCQ 6.3.22 The choice of realization of structure depends on
(A) computational complexity
(B) memory requirements
(C) parallel processing and pipelining
(D) all the above

MCQ 6.3.23 Which of the following schemes of system realization uses separate delays for input and output samples ?
(A) parallel form
(B) cascade form
(C) direct form-I
(D) direct form-II

MCQ 6.3.24 The direct form-I and II structures of IIR system will be identical in
(A) all pole system
(B) all zero system
(C) both (A) and (B)
(D) first order and second order systems

MCQ 6.3.25 The number of memory locations required to realize the system,
$$H(z) = \frac{1+3z^{-2}+2z^{-3}}{1+2z^{-2}+z^{-4}} \text{ is}$$
(A) 5
(B) 7
(C) 2
(D) 10

MCQ 6.3.26 The mapping $z = e^{sT}$ from s-plane to z-plane, is
(A) one to one
(B) many to one
(C) one to many
(D) many to many

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EXERCISE 6.4

MCQ 6.4.1 **IES EC 2007**
What is the z-transform of the signal $x[n] = \alpha^n u[n]$?
(A) $X(z) = \frac{1}{z-1}$
(B) $X(z) = \frac{1}{1-z}$
(C) $X(z) = \frac{z}{z-\alpha}$
(D) $X(z) = \frac{1}{z-\alpha}$

MCQ 6.4.2 **GATE EC 1998**
The z-transform of the time function $\sum\limits_{k=0}^{\infty}\delta[n-k]$ is
(A) $\frac{z-1}{z}$
(B) $\frac{z}{z-1}$
(C) $\frac{z}{(z-1)^2}$
(D) $\frac{(z-1)^2}{z}$

MCQ 6.4.3 **GATE EC 1999**
The z-transform $F(z)$ of the function $f(nT) = a^{nT}$ is
(A) $\frac{z}{z-a^T}$
(B) $\frac{z}{z+a^T}$
(C) $\frac{z}{z-a^{-T}}$
(D) $\frac{z}{z+a^{-T}}$

MCQ 6.4.4 **GATE EE 2006**
The discrete-time signal $x[n] \xleftrightarrow{\mathcal{Z}} X(z) = \sum_{n=0}^{\infty}\frac{3^n}{2+n}z^{2n}$, where $\longleftrightarrow$ denotes a transform-pair relationship, is orthogonal to the signal
(A) $y_1[n] \leftrightarrow Y_1(z) = \sum_{n=0}^{\infty}\left(\frac{2}{3}\right)^n z^{-n}$
(B) $y_2[n] \leftrightarrow Y_2(z) = \sum_{n=0}^{\infty}(5^n - n)\,z^{-(2n+1)}$
(C) $y_3[n] \leftrightarrow Y_3(z) = \sum_{n=-\infty}^{\infty} 2^{-|n|}z^{-n}$
(D) $y_4[n] \leftrightarrow Y_4(z) = 2z^{-4}+3z^{-2}+1$

MCQ 6.4.5 **IES E & T 19[illegible]**
Which [illegible] of the following [illegible] ion of convergence (ROC) for the [illegible]quence $x[n] = b^n u[n] +$ [illegible]
(A) Region $|z| < 1$
(B) Annular strip in the region $b > |z|$ [illegible]
(C) Region $|z| > 1$
(D) Annular str[illegible] in the region $b < |z| < \frac{1}{b}$

MCQ 6.4.6 **IES EC 2002**
[illegible] signals $a^n u[n]$ and $-a^n u[-n-1]$ have the same z [illegible]
[illegible] son (R[illegible] son of Convergence (ROC) for $a^n u[n]$ is $|z| > |a|$, whereas the [illegible] $u[-n-1]$ is $|z| < |a|$.
(A) Both A and [illegible] true and R is the correct explanation of A
(B) Both A and [illegible] e true but R is NOT the correct explanation of A
(C) A is true [illegible] is false
(D) A is false but R is true

MCQ 6.4.7 **IES EC 2006**
Which one [illegible] the following is the correct statement ?
The region [illegible] convergence of z-transform of $x[n]$ consists of the values of z for which $x[n]$ [illegible]$^{-n}$ is
(A) absolutely integrable
(B) [illegible] summable
(C) unity
(D) $<$ 1

MCQ 6.4.8
GATE EC 2009

The ROC of z-transform of the sequence $x[n] = \left(\frac{1}{3}\right)^n u[n] - \left(\frac{1}{2}\right)^n u[-n-1]$ is

(A) $\frac{1}{3} < |z| < \frac{1}{2}$ (B) $|z| > \frac{1}{2}$

(C) $|z| < \frac{1}{3}$ (D) $2 < |z| < 3$

MCQ 6.4.9
GATE EC 2005

The region of convergence of $z-$ transform of the sequence $\left(\frac{5}{6}\right)^n u[n] - \left(\frac{6}{5}\right)^n u[-n-1]$ must be

(A) $|z| < \frac{5}{6}$ (B) $|z| > \frac{5}{6}$

(C) $\frac{5}{6} < |z| < \frac{6}{5}$ (D) $\frac{6}{5} < |z| < \infty$

MCQ 6.4.10
GATE IN 2008

The region of convergence of the z-transform of the discrete-time signal $x[n] = 2^n u[n]$ will be

(A) $|z| > 2$ (B) $|z| < 2$

(C) $|z| > \frac{1}{2}$ (D) $|z| < \frac{1}{2}$

MCQ 6.4.11
GATE EC 2001

The region of convergence of the $z-$ transform of a unit step function is

(A) $|z| > 1$ (B) $|z| < 1$

(C) (Real part of z) > 0 (D) (Real part of z) < 0

MCQ 6.4.12
IES EC 2005

Match List I (Discrete Time signal) with List II (Transform) and select the correct answer using the codes given below the lists :

List I	List II
A. Unit step function	**1.** 1
B. Unit impulse function	**2.** $\frac{z-\cos\omega T}{z^2-2z\cos\omega T+1}$
C. $\sin\omega t,\ t = 0, T, 2T$	**3.** $\frac{z}{z-1}$
D. $\cos\omega t,\ t = 0, T, 2T, \ldots$	**4.** $\frac{z\sin\omega T}{z^2-2z\cos\omega T+1}$

Codes :

	A	B	C	D
(A)	2	4	1	3
(B)	3	1	4	2
(C)	2	1	4	3
(D)	3	4	1	2

MCQ 6.4.13
IES EC 2006

What is the inverse z-transform of $X(z)$

(A) $\frac{1}{2\pi j}\oint X(z)\, z^{n-1}\, dz$ (B) $2\pi j \oint X(z)\, z^{n+1}\, dz$

(C) $\frac{1}{2\pi j}\oint X(z)\, z^{1-n}\, dz$ (D) $2\pi j \oint X(z)\, z^{-(n+1)}\, dz$

MCQ 6.4.14
IES E&T 1997

Which one of the following represents the impulse response of a system defined by $H(z) = z^{-m}$?

(A) $u[n-m]$ (B) $\delta[n-m]$

(C) $\delta[m]$ (D) $\delta[m-n]$

MCQ 6.4.15
IES EC 2008

If $X(z)$ is $\frac{1}{|1-z^{-1}|}$ with $|z| > 1$, then what is the corresponding $x[n]$?

(A) e^{-n} (B) e^{n}

(C) $u[n]$ (D) $\delta(n)$

MCQ 6.4.16
GATE EC 2007

The $z-$transform $X(z)$ of a sequence $x[n]$ is given by $X[z] = \frac{0.5}{1-2z^{-1}}$. It is given that the region of convergence of $X(z)$ includes the unit circle. The value of $x[0]$ is

(A) -0.5 (B) 0

(C) 0.25 (D) 05

MCQ 6.4.17
GATE EE 2005

If $u(t)$ is the unit step and $\delta(t)$ is the unit impulse function, the inverse z-transform of $F(z) = \frac{1}{z+1}$ for $k > 0$ is

(A) $(-1)^k \delta(k)$ (B) $\delta(k) - (-1)^k$

(C) $(-1)^k u(k)$ (D) $u(k) - (-1)^k$

MCQ 6.4.18
IES EC 2002

For a z-transform $X(z) = \frac{(2z-\frac{5}{6})}{(z-\frac{1}{2})(z-\frac{1}{3})}$

Match List I (The sequences) with List II (The region of convergence) and select the correct answer using the codes given below the lists :

List I	List II
A. $[(1/2)^n + (1/3)^n]\, u[n]$	**1.** $(1/3) < \lvert z\rvert < (1/2)$
B. $(1/2)^n u[n] - (1/3)^n u[-n-1]$	**2.** $\lvert z\rvert < (1/3)$
C. $-(1/2)^n u[-n-1] + (1/3)^n u[n]$	**3.** $\lvert z\rvert < 1/3\ \&\ \lvert z\rvert > 1/2$
D. $-[(1/2)^n + (1/3)^n]\, u[-n-1]$	**4.** $\lvert z\rvert > 1/2$

Codes :

	A	B	C	D
(A)	4	2	1	3
(B)	1	3	4	2
(C)	4	3	1	2
(D)	1	2	4	3

MCQ 6.4.19
IES EC 2005

Which one of the following is the inverse z-transform of

$$X(z) = \frac{z}{(z-2)(z-3)},\quad |z| < 2 \ ?$$

(A) $[2^n - 3^n]\, u[-n-1]$ (B) $[3^n - 2^n]\, u[-n-1]$

(C) $[2^n - 3^n]\, u[n+1]$ (D) $[2^n - 3^n]\, u[n]$

MCQ 6.4.20
GATE EE 2008

Given $X(z) = \frac{z}{(z-a)^2}$ with $|z| > a$, the residue of $X(z) z^{n-1}$ at $z = a$ for $n \geq 0$ will be

(A) a^{n-1} (B) a^n

(C) na^n (D) na^{n-1}

MCQ 6.4.21
GATE IN 2004

Given $X(z) = \frac{\frac{1}{2}}{1-az^{-1}} + \frac{\frac{1}{3}}{1-bz^{-1}}$, $|a|$ and $|b| < 1$ with the ROC specified as $|a| < |z| < |b|$, then $x[0]$ of the corresponding sequence is given by

(A) $\frac{1}{3}$ (B) $\frac{5}{6}$

(C) $\frac{1}{2}$ (D) $\frac{1}{6}$

MCQ 6.4.22 IES EC 2002

If $X(z) = \frac{z+z^{-3}}{z+z^{-1}}$ then $x[n]$ series has

(A) alternate 0's
(B) alternate 1's
(C) alternate 2's
(D) alternate -1's

MCQ 6.4.23 GATE EC 2010

Consider the z-transform $x(z) = 5z^2 + 4z^{-1} + 3;\ 0 < |z| < \infty$. The inverse z - transform $x[n]$ is

(A) $5\delta[n+2] + 3\delta[n] + 4\delta[n-1]$
(B) $5\delta[n-2] + 3\delta[n] + 4\delta[n+1]$
(C) $5u[n+2] + 3u[n] + 4u[n-1]$
(D) $5u[n-2] + 3u[n] + 4u[n+1]$

MCQ 6.4.24 GATE IN 2003

The sequence $x[n]$ whose z-transform is $X(z) = e^{1/z}$ is

(A) $\frac{1}{n!}u[n]$
(B) $\frac{1}{-n!}u[-n]$
(C) $(-1)^n \frac{1}{n!}u[n]$
(D) $\frac{1}{-(n+1)!}u[-n-1]$

MCQ 6.4.25 GATE EC 2006

If the region of convergence of $x_1[n] + x_2[n]$ is $\frac{1}{3} < |z| < \frac{2}{3}$ then the region of convergence of $x_1[n] - x_2[n]$ includes

(A) $\frac{1}{3} < |z| < 3$
(B) $\frac{2}{3} < |z| < 3$
(C) $\frac{3}{2} < |z| < 3$
(D) $\frac{1}{3} < |z| < \frac{2}{3}$

MCQ 6.4.26 IES EC 2000

Match List I with List II and select the correct answer using the codes given below the lists :

List I		List II	
A.	$\alpha^n u[n]$	1.	$\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$, ROC : $\lvert z\rvert > \lvert\alpha\rvert$
B.	$-\alpha^n u[-n-1]$	2.	$\frac{1}{(1-\alpha z^{-1})}$, ROC : $\lvert z\rvert > \lvert\alpha\rvert$
C.	$-n\alpha^n u[-n-1]$	3.	$\frac{1}{(1-\alpha z^{-1})}$, ROC : $\lvert z\rvert < \lvert\alpha$
D.	$n\alpha^n u[n]$	4.	$\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$, ROC : $\lvert z\rvert < \lvert\alpha$

Codes :

	A	B	C	D
(A)	2	4	3	1
(B)	1	3	4	2
(C)	1	4	3	2
(D)	2	3	4	1

MCQ 6.4.27 IES EC 2007

Algebraic expression for z-transform of $x[n]$ is $X[z]$. What is the algebraic expression for z-transform of $\{e^{j\omega_0 n}x[n]\}$?

(A) $X(z - z_0)$
(B) $X(e^{-j\omega_0}z)$
(C) $X(e^{j\omega_0}z)$
(D) $X(z)e^{j\omega_0 z}$

MCQ 6.4.28 IES E&T 1997

Given that $F(z)$ and $G(z)$ are the one-sided z-transforms of discrete time functions $f(nT)$ and $g(nT)$, the z-transform of $\sum f(kT)g(nT-kT)$ is given by

(A) $\sum f(nT)g(nT)z^{-n}$
(B) $\sum f(nT)z^{-n}\sum g(nT)z^{-n}$
(C) $\sum f(kT)g(nT-kT)z^{-n}$
(D) $\sum f(nT-kT)g(nT)z^{-n}$

MCQ 6.4.29 IES E&T 1997

Match List-I ($x[n]$) with List-II ($X(z)$) and select the correct answer using the codes given below the Lists:

List-I		List-II	
A.	$a^n u[n]$	1.	$\frac{az}{(z-a)^2}$
B.	$a^{n-2}u[n-2]$	2.	$\frac{ze^{-j}}{ze^{-j}-a}$
C.	$e^{jn}a^n$	3.	$\frac{z}{z-a}$
D.	$na^n u[n]$	4.	$\frac{z^{-1}}{z-a}$

Codes :

	A	B	C	D
(A)	3	2	4	1
(B)	2	3	4	1
(C)	3	4	2	1
(D)	1	4	2	3

MCQ 6.4.30 IES EC 2005

The output $y[n]$ of a discrete time LTI system is related to the input $x[n]$ as given below :

$$y[n] = \sum_{k=0}^{\infty} x[k]$$

Which one of the following correctly relates the z-transform of the input and output, denoted by $X(z)$ and $Y(z)$, respectively ?

(A) $Y(z) = (1-z^{-1})X(z)$
(B) $Y(z) = z^{-1}X(z)$
(C) $Y(z) = \frac{X(z)}{1-z^{-1}}$
(D) $Y(z) = \frac{dX(z)}{dz}$

MCQ 6.4.31 IES EC 2010

Convolution of two sequence $x_1[n]$ and $x_2[n]$ is represented as

(A) $X_1(z) * X_2(z)$
(B) $X_1(z)X_2(z)$
(C) $X_1(z) + X_2(z)$
(D) $X_1(z)/X_2(z)$

MCQ 6.4.32 GATE EC 1999

The z-transform of a signal is given by $C(z) = \frac{1z^{-1}(1-z^{-4})}{4(1-z^{-1})^2}$. Its final value is

(A) 1/4
(B) zero
(C) 1.0
(D) infinity

MCQ 6.4.33 IES E&T 1996

Consider a system described by the following difference equation:
$y(n+3) + 6y(n+2) + 11y(n+1) + 6y(n) = r(n+2) + 9r(n+1) + 20r(n)$
where y is the output and r is the input. The transfer function of the system is

(A) $\frac{2z^2+z+20}{3z^3+2z^2+z+6}$
(B) $\frac{z^2+9z+20}{z^3+6z^2+6z+11}$
(C) $\frac{z^3+6z^2+6z+11}{z^2+9z+20}$
(D) none of the above

MCQ 6.4.34 IES E&T 1998

If the function $H_1(z) = (1 + 1.5z^{-1} - z^{-2})$ and $H_2(z) = z^2 + 1.5z - 1$, then

(A) the poles and zeros of the functions will be the same
(B) the poles of the functions will be identical but not zeros
(C) the zeros of the functions will be identical but not the poles
(D) neither the poles nor the zeros of the two functions will be identical

MCQ 6.4.35
IES EC 1999

The state model

$$x[k+1] = \begin{bmatrix} 0 & 1 \\ -\beta & -\alpha \end{bmatrix} x[k] + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u[k]$$

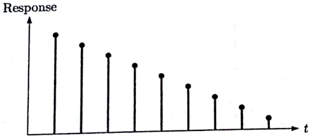
$$y[k] = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1[k] \\ x_2[k] \end{bmatrix}$$

is represented in the difference equation as

(A) $c[k+2] + \alpha c[k+1] + \beta c[k] = u[k]$
(B) $c[k+1] + \alpha c[k] + \beta c[k-1] = u[k-1]$
(C) $c[k-2] + \alpha c[k-1] + \beta c[k] = u[k]$
(D) $c[k-1] + \alpha c[k] + \beta c[k+1] = u[k+1]$

MCQ 6.4.36
IES EC 2000

The impulse response of a discrete system with a simple pole shown in the figure below. The pole of the system must be located on the



(A) real axis at $z = -1$
(B) real axis between $z = 0$ and $z = 1$
(C) imaginary axis at $z = j$
(D) imaginary axis between $z = 0$ and $z = j$

MCQ 6.4.37
IES EC 2001

Which one of the following digital filters does have a linear phase response ?

(A) $y[n] + y[n-1] = x[n] - x[n-1]$
(B) $y[n] = 1/6(3x[n] + 2x[n-1] + x[n-2])$
(C) $y[n] = 1/6(x[n] + 2x[n-1] + 3x[n-2])$
(D) $y[n] = 1/4(x[n] + 2x[n-1] + x[n-2])$

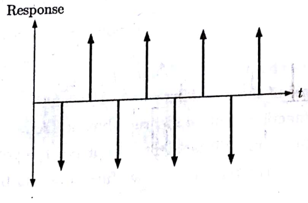
MCQ 6.4.38
IES EC 2001

The poles of a digital filter with linear phase response can lie

(A) only at $z = 0$
(B) only on the unit circle
(C) only inside the unit circle but not at $z = 0$
(D) on the left side of $\text{Real}(z) = 0$ line

MCQ 6.4.39
IES EC 2001

The impulse response of a discrete system with a simple pole is shown in the given figure



The pole must be located

(A) on the real axis at $z = 1$
(B) on the real axis at $z = -1$
(C) at the origin of the z-plane
(D) at $z = \infty$

MCQ 6.4.40
IES EC 2002

The response of a linear, time-invariant discrete-time system to a unit step input $u[n]$ is the unit impulse $\delta[n]$. The system response to a ramp input $nu[n]$ would be

(A) $u[n]$
(B) $u[n-1]$
(C) $n\delta[n]$
(D) $\sum_{k=0}^{\infty} k\delta[n-k]$

MCQ 6.4.41
IES EC 2002

A system can be represented in the form of state equations as

$$s[n+1] = As[n] + Bx[n]$$
$$y[n] = Cs[n] + Dx[n]$$

where A, B, C and D are matrices, $s[n]$ is the state vector. $x[n]$ is the input and $y[n]$ is the output. The transfer function of the system $H(z) = Y(z)/X(z)$ is given by

(A) $A(zI - B)^{-1}C + D$
(B) $B(zI - C)^{-1}D + A$
(C) $C(zI - A)^{-1}B + D$
(D) $D(zI - A)^{-1}C + B$

MCQ 6.4.42
IES EC 2004

What is the number of roots of the polynomial $F(z) = 4z^3 - 8z^2 - z + 2$, lying outside the unit circle ?

(A) 0
(B) 1
(C) 2
(D) 3

MCQ 6.4.43
IES EC 2004

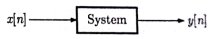
$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

Which one of the following systems is inverse of the system given above ?

(A) $x[n] = y[n] - y[n-1]$
(B) $x[n] = y[n]$
(C) $x[n] = y[n+4]$
(D) $x[n] = ny[n]$

MCQ 6.4.44
IES EC 2006

For the system shown, $x[n] = k\delta[n]$, and $y[n]$ is related to $x[n]$ as $y[n] - \frac{1}{2}y[n-1] = x[n]$



What is $y[n]$ equal to ?

(A) k
(B) $(1/2)^n k$
(C) nk
(D) 2^n

MCQ 6.4.45
IES EC 2010

Unit step response of the system described by the equation $y[n] + y[n-1] = x[n]$ is

(A) $\frac{z^2}{(z+1)(z-1)}$
(B) $\frac{z}{(z+1)(z-1)}$
(C) $\frac{z+1}{z-1}$
(D) $\frac{z(z-1)}{(z+1)}$

MCQ 6.4.46
IES EC 2011

Unit step response of the system described by the equation $y[n] + y[n-1] = x[n]$ is

(A) $\frac{z^2}{(z+1)(z-1)}$
(B) $\frac{z}{(z+1)(z-1)}$
(C) $\frac{(z+1)}{(z-1)}$
(D) $\frac{z(z-1)}{(z+1)}$

MCQ 6.4.47 IES EC 2011

System transformation function $H(z)$ for a discrete time LTI system expressed in state variable form with zero initial conditions is

(A) $c(zI-A)^{-1}b+d$ (B) $c(zI-A)^{-1}$

(C) $(zI-A)^{-1}z$ (D) $(zI-A)^{-1}$

MCQ 6.4.48 GATE EC 2009

A system with transfer function $H(z)$ has impulse response $h(.)$ defined as $h(2)=1, h(3)=-1$ and $h(k)=0$ otherwise. Consider the following statements.

S1 : $H(z)$ is a low-pass filter.
S2 : $H(z)$ is an FIR filter.

Which of the following is correct?

(A) Only S2 is true

(B) Both S1 and S2 are false

(C) Both S1 and S2 are true, and S2 is a reason for S1

(D) Both S1 and S2 are true, but S2 is not a reason for S1

MCQ 6.4.49 GATE EC 2004

The z-transform of a system is $H(z)=\frac{z}{z-0.2}$. If the ROC is $|z|<0.2$, then the impulse response of the system is

(A) $(0.2)^n u[n]$

(B) $(0.2)^n u[-n-1]$

(C) $-(0.2)^n u[n]$

(D) $-(0.2)^n u[-n-1]$

MCQ 6.4.50 GATE EC 2003

A sequence $x(n)$ with the z-transform $X(z)=z^4+z^2-2z+2-3z^{-4}$ is applied as an input to a linear, time-invariant system with the impulse response $h[n]=2\delta[n-3]$ where

$$\delta[n]=\begin{cases}1, & n=0\\ 0, & \text{otherwise}\end{cases}$$

The output at $n=4$ is

(A) -6 (B) zero

(C) 2 (D) -4

MCQ 6.4.51 GATE EE 2009

The z-transform of a signal $x[n]$ is given by $4z^{-3}+3z^{-1}+2-6z^2+2z^3$

It is applied to a system, with a transfer function $H(z)=3z^{-1}-2$

Let the output be $y[n]$. Which of the following is true ?

(A) $y[n]$ is non causal with finite support

(B) $y[n]$ is causal with infinite support

(C) $y[n]=0; |n|>3$

(D) $\mathrm{Re}[Y(z)]_{z=e^{j\theta}}=-\mathrm{Re}[Y(z)]_{z=e^{-j\theta}}$

$\mathrm{Im}[Y(z)]_{z=e^{j\theta}}=\mathrm{Im}[Y(z)]_{z=e^{-j\theta}};\ -\pi\le\theta<\pi$

MCQ 6.4.52 GATE EE 2008

$H(z)$ is a transfer function of a real system. When a signal $x[n]=(1+j)^n$ is the input to such a system, the output is zero. Further, the Region of convergence (ROC) of $(1-\frac{1}{2}z^{-1})$ H(z) is the entire Z-plane (except $z=0$). It can then be

inferred that $H(z)$ can have a minimum of

(A) one pole and one zero

(B) one pole and two zeros

(C) two poles and one zero

(D) two poles and two zeros

MCQ 6.4.53 GATE IN 2004

A discrete-time signal, $x[n]$, suffered a distortion modeled by an LTI system with $H(z)=(1-az^{-1})$, a is real and $|a|>1$. The impulse response of a stable system that exactly compensates the magnitude of the distortion is

(A) $\left(\frac{1}{a}\right)^n u[n]$

(B) $-\left(\frac{1}{a}\right)^n u[-n-1]$

(C) $a^n u[n]$

(D) $a^n u[-n-1]$

MCQ 6.4.54 IES E&T 1998

Assertion (A) : A linear time-invariant discrete-time system having the system function

$$H(z)=\frac{z}{z+\frac{1}{2}} \text{ is a stable system.}$$

Reason (R) : The pole of $H(z)$ is in the left-half plane for a stable system.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT a correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

MCQ 6.4.55 IES EC 1999

Assertion (A) : An LTI discrete system represented by the difference equation $y[n+2]-5y[n+1]+6y[n]=x[n]$ is unstable.

Reason (R) : A system is unstable if the roots of the characteristic equation lie outside the unit circle.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

MCQ 6.4.56 IES EC 1999

Consider the following statements regarding a linear discrete-time system

$$H(z)=\frac{z^2+1}{(z+0.5)(z-0.5)}$$

1. The system is stable
2. The initial value $h(0)$ of the impulse response is -4
3. The steady-state output is zero for a sinusoidal discrete time input of frequency equal to one-fourth the sampling frequency.

Which of these statements are correct ?

(A) 1, 2 and 3

(B) 1 and 2

(C) 1 and 3

(D) 2 and 3

MCQ 6.4.57 IES EC 2005

Assertion (A) : The discrete time system described by $y[n]=2x[n]+4x[n-1]$ is unstable, (here $y[n]$ is the output and $x[n]$ the input)

Reason (R) : It has an impulse response with a finite number of non-zero samples.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

122) d 47) d.

MCQ 6.4.58 GATE EC 2002

If the impulse response of discrete - time system is $h[n] = -5^n u[-n-1]$, then the system function $H(z)$ is equal to

(A) $\frac{-z}{z-5}$ and the system is stable

(B) $\frac{z}{z-5}$ and the system is stable

(C) $\frac{-z}{z-5}$ and the system is unstable

(D) $\frac{z}{z-5}$ and the system is unstable

MCQ 6.4.59 GATE IN 2010

$H(z)$ is a discrete rational transfer function. To ensure that both $H(z)$ and its inverse are stable its

(A) poles must be inside the unit circle and zeros must be outside the unit circle.

(B) poles and zeros must be inside the unit circle.

(C) poles and zeros must be outside the unit circle

(D) poles must be outside the unit circle and zeros should be inside the unit circle

MCQ 6.4.60 IES EC 2002

Assertion (A) : The stability of the system is assured if the Region of Convergence (ROC) includes the unit circle in the z-plane.

Reason (R) : For a causal stable system all the poles should be outside the unit circle in the z-plane.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false

(D) A is false but R is true

MCQ 6.4.61 IES EC 2002

Assertion (A) : For a rational transfer function $H(z)$ to be causal, stable and causally invertible, both the zeros and the poles should lie within the unit circle in the z-plane.

Reason (R) : For a rational system, ROC bounded by poles.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

MCQ 6.4.62 GATE EC 2010

The transfer function of a discrete time LTI system is $H(z) = \frac{2 - \frac{3}{4}z^{-1}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$

Consider the following statements:

S1: The system is stable and causal for ROC: $|z| > 1/2$

S2: The system is stable but not causal for ROC: $|z| < 1/4$

S3: The system is neither stable nor causal for ROC: $1/4 < |z| < 1/2$

Which one of the following statements is valid ?

(A) Both S1 and S2 are true (B) Both S2 and S3 are true

(C) Both S1 and S3 are true (D) S1, S2 and S3 are all true

MCQ 6.4.63 GATE EC 2004

A causal LTI system is described by the difference equation

$$2y[n] = \alpha y[n-2] - 2x[n] + \beta x[n-1]$$

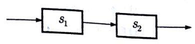
The system is stable only if

(A) $|\alpha| = 2, |\beta| < 2$ (B) $|\alpha| > 2, |\beta| > 2$

(C) $|\alpha| < 2$, any value of β (D) $|\beta| < 2$, any value of α

MCQ 6.4.64 IES EC 2000

Two linear time-invariant discrete time systems s_1 and s_2 are cascaded as shown in the figure below. Each system is modelled by a second order difference equation. The difference equation of the overall cascaded system can be of the order of



(A) 0, 1, 2, 3 or 4

(B) either 2 or 4

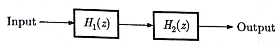
(C) 2

(D) 4

MCQ 6.4.65 IES EC 2000

Consider the compound system shown in the figure below. Its output is equal to the input with a delay of two units. If the transfer function of the first system is given by

$$H_1(z) = \frac{z-0.5}{z-0.8},$$



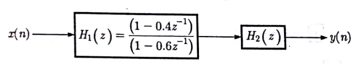
then the transfer function of the second system would be

(A) $H_2(z) = \frac{z^{-2} - 0.2z^{-3}}{1 - 0.4z^{-1}}$ (B) $H_2(z) = \frac{z^{-2} - 0.8z^{-3}}{1 - 0.5z^{-1}}$

(C) $H_2(z) = \frac{z^{-1} - 0.2z^{-3}}{1 - 0.4z^{-1}}$ (D) $H_2(z) = \frac{z^{-2} + 0.8z^{-3}}{1 + 0.5z^{-1}}$

MCQ 6.4.66 GATE EC 2011

Two systems $H_1(z)$ and $H_2(z)$ are connected in cascade as shown below. The overall output $y[n]$ is the same as the input $x[n]$ with a one unit delay. The transfer function of the second system $H_2(z)$ is



(A) $\frac{1 - 0.6z^{-1}}{z^{-1}(1 - 0.4z^{-1})}$ (B) $\frac{z^{-1}(1 - 0.6z^{-1})}{(1 - 0.4z^{-1})}$

(C) $\frac{z^{-1}(1 - 0.4z^{-1})}{(1 - 0.6z^{-1})}$ (D) $\frac{1 - 0.4z^{-1}}{z^{-1}(1 - 0.6z^{-1})}$

MCQ 6.4.67 GATE EC 2010

Two discrete time system with impulse response $h_1[n] = \delta[n-1]$ and $h_2[n] = \delta[n-2]$ are connected in cascade. The overall impulse response of the cascaded system is

(A) $\delta[n-1] + \delta[n-2]$ (B) $\delta[n-4]$

(C) $\delta[n-3]$ (D) $\delta[n-1]\delta[n-2]$

MCQ 6.4.68 GATE EE 2009

A cascade of three Linear Time Invariant systems is causal and unstable. From this, we conclude that

(A) each system in the cascade is individually causal and unstable

(B) at least on system is unstable and at least one system is causal

(C) at least one system is causal and all systems are unstable

(D) the majority are unstable and the majority are causal

MCQ 6.4.69 IES EC 2001

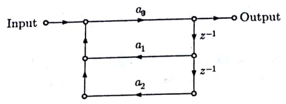
The minimum number of delay elements required in realizing a digital filter with the transfer function

$$H(z) = \frac{1 + az^{-1} + bz^{-2}}{1 + cz^{-1} + dz^{-2} + ez^{-3}}$$

(A) 2 (B) 3
(C) 4 (D) 5

MCQ 6.4.70 GATE IN 2004

A direct form implementation of an LTI system with $H(z) = \frac{1}{1 - 0.7z^{-1} + 0.13z^{-2}}$ is shown in figure. The value of a_0, a_1 and a_2 are respectively



(A) 1.0, 0.7 and -0.13 (B) -0.13, 0.7 and 1.0
(C) 1.0, -0.7 and 0.13 (D) 0.13, -0.7 and 1.0

MCQ 6.4.71 GATE IN 2010

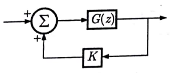
A digital filter having a transfer function $H(z) = \frac{p_0 + p_1 z^{-1} + p_3 z^{-3}}{1 + d_3 z^{-3}}$ is implemented

using Direct Form-I and Direct Form-II realizations of IIR structure. The number of delay units required in Direct Form-I and Direct Form-II realizations are, respectively

(A) 6 and 6 (B) 6 and 3
(C) 3 and 3 (D) 3 and 2

MCQ 6.4.72 GATE EE 2007

Consider the discrete-time system shown in the figure where the impulse response of $G(z)$ is $g(0) = 0, g(1) = g(2) = 1, g(3) = g(4) = \cdots = 0$

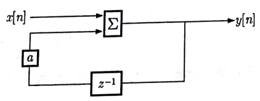


This system is stable for range of values of K

(A) $[-1, \frac{1}{2}]$ (B) $[-1, 1]$
(C) $[-\frac{1}{2}, 1]$ (D) $[-\frac{1}{2}, 2]$

MCQ 6.4.73 GATE IN 2006

In the IIR filter shown below, a is a variable gain. For which of the following cases, the system will transit from stable to unstable condition ?



(A) $0.1 < a < 0.5$
(B) $0.5 < a < 1.5$
(C) $1.5 < a < 2.5$
(D) $2 < a < \infty$

MCQ 6.4.74 IES EC 2002

The poles of an analog system are related to the corresponding poles of the digital system by the relation $z = e^{sT}$. Consider the following statements.

1. Analog system poles in the left half of s-plane map onto digital system poles inside the circle $|z| = 1$.
2. Analog system zeros in the left half of s-plane map onto digital system zeros inside the circle $|z| = 1$.
3. Analog system poles on the imaginary axis of s-plane map onto digital system zeros on the unit circle $|z| = 1$.
4. Analog system zeros on the imaginary axis of s-plane map onto digital system zeros on the unit circle $|z| = 1$.

Which of these statements are correct ?

(A) 1 and 2 (B) 1 and 3
(C) 3 and 4 (D) 2 and 4

MCQ 6.4.75 IES EC 2007

Which one of the following rules determines the mapping of s-plane to z-plane ?

(A) Right half of the s-plane maps into outside of the unit circle in z-plane
(B) Left half of the s-plane maps into inside of the unit circle
(C) Imaginary axis in s-plane maps into the circumference of the unit circle
(D) All of the above

MCQ 6.4.76 IES EC 2008

Assertion (A) : The z-transform of the output of an ideal sampler is given by

$$\mathcal{Z}[f(t)] = K_0 + \frac{K_1}{z} + \frac{K_2}{z^2} + ... + \frac{K_n}{z^n}$$

Reason (R) : The relationship is the result of application of $z = e^{-sT}$, where T stands for the time gap between the samples.

(A) Both A and R are true and R is the correct explanation of A
(B) Both A and R are true but R is NOT the correct explanation of A
(C) A is true but R is false
(D) A is false but R is true

MCQ 6.4.77 IES EC 2010

z and Laplace transform are related by

(A) $s = \ln z$ (B) $s = \frac{\ln z}{T}$
(C) $s = z$ (D) $\frac{T}{\ln z}$

MCQ 6.4.78 IES EC 2010

Frequency scaling [relationship between discrete time frequency (Ω) and continuous time frequency (ω)] is defined as

(A) $\omega = 2\Omega$ (B) $\omega = 2T_S/\Omega$
(C) $\Omega = 2\omega/T_S$ (D) $\Omega = \omega T_S$

MCQ 6.4.79 GATE IN 2004

A casual, analog system has a transfer function $H(s) = \frac{a}{s^2 + a^2}$. Assuming a sampling

time of T seconds, the poles of the transfer function $H(z)$ for an equivalent digital system obtained using impulse in variance method are at

(A) (e^{aT}, e^{-aT}) (B) $\left(j\frac{a}{T}, -j\frac{a}{T}\right)$
(C) (e^{jaT}, e^{-jaT}) (D) $(e^{aT/2}, e^{-aT/2})$

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