Contents

2-level optimisation by Quine-McCluskey method



Section outline

- 2-level optimisation by Quine-McCluskey method
 - QM method

- QM ex-1
- QM ex-2
- QM ex-3
- QM ex-4



QM method

- Partition cubes into groups \(\lambda m : 1, n : D \) of m on-set and n DC-set minterms
- Combine adjacent terms between groups (matching in the position of 1's and don't cares) to get prime implicants
- Cover on-set minterms using prime implicants
 - reduce table by row dominance
 C₁ dominates C₂ if C₁ covers every on-set minterm covered by C₂
 - reduce table by column dominance
 m₂ is dominated m₁ if any P covering m₁ also covers m₂
 however, if C₂ is a bigger cube (due to DCs) it may still be retained
 - reduce table by dropping essential PMs
 - finally apply branch and bound (recursive application of covering needed)
 arbitrarily decide to keep or not to keep a cube bound exploration if the cost matches or exceeds the cost of an explored solution
 - Petrick's method may be used to generate all possible covers (especially for the cyclic core)



$$f(a, b, c, d) = \sum_{m} (0, 5, 8, 9, 10, 11, 14, 15)$$

$\langle 0:1,0:D \rangle$						
m	а	b	С	d		
0	0	0	0	0		
⟨1 : 1, 0 : D⟩						
\ ,	-	/				
		b	С	d		



$$f(a, b, c, d) = \sum_{m} (0, 5, 8, 9, 10, 11, 14, 15)$$

⟨0 : 1, 0 : D⟩						
m abcd						
0	0	0	0	0		
⟨1 : 1,	0 : [\overline{O}				
⟨1 : 1, m) b	С	d		

⟨0 : 1, 1 : D⟩						
cube	а	b	С	d		
0, 8	_	0	0	0		



$$f(a,b,c,d) = \sum_{m} (0,5,8,9,10,11,14,15)$$

$\langle 1:1,0:D \rangle$						
m	а	b	С	d		
8	1	0	0	0		
⟨2 : 1, 0 : D⟩						
\ \ /						
m	а		С	d		
			c	d		
m	а	b		d 1		



$$f(a,b,c,d) = \sum_{m} (0,5,8,9,10,11,14,15)$$

$\langle 1:1,0:D \rangle$					
m	а	b	С	d	
8	1	0	0	0	
⟨2 : 1,0 : D⟩					
,		,			
m	а	b	С	d	
			c	d	
m	а			d 1 1	

⟨1 : 1, 1 : D⟩					
cube	а	b	С	d	
8, 9	1	0	0	-	



$$f(a,b,c,d) = \sum_{m} (0,5,8,9,10,11,14,15)$$

$\langle 1:1,0:D \rangle$						
m	а	b	С	d		
8	1	0	0	0		
⟨2 : 1,0 : D⟩						
		,				
m	а	b	С	d		
m 5		b	c	d		
m	а	b 1 0		d 1		

⟨1 : 1, 1 : D⟩					
cube	а	b	С	d	
8, 9	1	0	0	-	
8, 10	1	0	-	0	



$$f(a,b,c,d) = \sum_{m} (0,5,8,9,10,11,14,15)$$

$\langle 2:1,0:D \rangle$						
m	а	b	С	d		
5	0	1	0	1		
9	1	0	0	1		
10	1	0	1	0		
⟨3:1,	0 : [\rangle				
m	а	b	С	d		
11	1	0	1	1		
14	1	1	1	0		



$$f(a,b,c,d) = \sum_{m} (0,5,8,9,10,11,14,15)$$

$\langle 2:1,0:D \rangle$						
m	а	b	С	d		
5	0	1	0	1		
9	1	0	0	1		
10	1	0	1	0		
⟨3:1,	0 : [O				
m	а	b	С	d		
11	1	0	1	1		
14	1	1	1	0		

⟨2 : 1,1 : D⟩					
cube	а	b	С	d	
9, 11	1	0	-	1	



$$f(a,b,c,d) = \sum_{m} (0,5,8,9,10,11,14,15)$$

⟨2 : 1, 0 : D⟩						
m	а	b	С	d		
5	0	1	0	1		
9	1	0	0	1		
10	1	0	1	0		
⟨3:1,	0 : [O				
m	а	b	С	d		
11	1	0	1	1		
14	1	1	1	0		

⟨2 : 1, 1 : D⟩					
cube	а	b	С	d	
9, 11	1	0	-	1	
10, 11	1	0	1	-	



$$f(a,b,c,d) = \sum_{m} (0,5,8,9,10,11,14,15)$$

$\langle 2:1,0:D\rangle$						
m	а	b	С	d		
5	0	1	0	1		
9	1	0	0	1		
10	1	0	1	0		
⟨3:1,	0 : [O				
m	а	b	С	d		
11	1	0	1	1		
14	1	1	1	0		

$\langle 2:1,1:D \rangle$					
cube	а	b	С	d	
9, 11	1	0	-	1	
10, 11	1	0	1	-	
10, 14	1	-	1	0	



$$f(a,b,c,d) = \sum_{m} (0,5,8,9,10,11,14,15)$$

⟨3 : 1, 0 : D⟩						
m	а	b	С	d		
11	1	0	1	1		
14	1	1	1	0		
⟨4:1,0:D⟩						
m	а	b	С	d		
15	1	1	1	1		



$$f(a, b, c, d) = \sum_{m} (0, 5, 8, 9, 10, 11, 14, 15)$$

⟨3 : 1, 0 : D⟩						
m	а	b	С	d		
11	1	0	1	1		
14	1	1	1	0		
⟨4:1,	0 : [\overline{O}				
m	а	b	С	d		
15	1	1	1	1		

⟨3 : 1, 1 : D⟩					
cube	а	b	С	d	
11, 15	1	-	1	1	



$$f(a, b, c, d) = \sum_{m} (0, 5, 8, 9, 10, 11, 14, 15)$$

⟨3 : 1, 0 : D⟩						
m	а	b	С	d		
11	1	0	1	1		
14	1	1	1	0		
⟨4:1,						
m	а	b	С	d		
15	1	1	1	1		

⟨3 : 1, 1 : D⟩						
cube	а	b	С	d		
11, 15	1	-	1	1		
14, 15	1	1	1	-		



$$f(a,b,c,d) = \sum_{m} (0,5,8,9,10,11,14,15)$$

$\langle 1:1,1:D \rangle$					
cube	а	b	С	d	
8, 9	1	0	0	-	
8, 10	1	0	-	0	
⟨2 : 1, 1 : D⟩					
\Z.I,	1 . L	<i>)</i>			
cube	a		С	d	
•		b	C	d	
cube	а	b	c -	d 1	



$$f(a,b,c,d) = \sum_{m} (0,5,8,9,10,11,14,15)$$

$\langle 1:1,1:D\rangle$					
cube	а	b	С	d	
8, 9	1	0	0	-	
8, 10	1	0	-	0	
⟨2:1,	1 : C	\rangle			
cube	а	b	С	d	
9, 11	1	0	-	1	
10, 11	1	0	1	-	
10, 14	1		1	Λ	

⟨1 : 1,2 : D⟩				
cube	a b c d			
8, 9, 10, 11	10			



$$f(a,b,c,d) = \sum_{m} (0,5,8,9,10,11,14,15)$$

$\langle 1 : 1, 1 : D \rangle$					
cube	а		С	d	
8, 9	1		0	-	
8, 10	1	0	-	0	
⟨2:1,	1 : C	\rangle			
cube	а	b	С	d	
9, 11	1	0	-	1	
10, 11	1	0	1	-	
10, 14	1	-	1	0	

⟨1 : 1,2 : D⟩						
cube	a b c d					
8, 9, 10, 11	10					
8. 10. 9. 11	10					



$$f(a,b,c,d) = \sum_{m} (0,5,8,9,10,11,14,15)$$

⟨2 : 1, 1 : D⟩								
cube	а	b	С	d				
9, 11	1	0	-	1				
10, 11	1	0	1	-				
10, 14	1	-	1	0				
⟨3:1,	1:	$D \rangle$						
cube	а	b	С	d				
11, 15	1	-	1	1				



$$f(a,b,c,d) = \sum_{m} (0,5,8,9,10,11,14,15)$$

$\langle 2:1,1:D\rangle$								
cube	а		С	d				
9, 11	1		-	1				
10, 11	1	0	1	-				
10, 14	1	-	1	0				
⟨3:1,	1 : [$O\rangle$						
cube	а	b	С	d				
11, 15	1	-	1	1				
14, 15	1	1	1	-				

⟨2 : 1,2 : D⟩							
cube	a b c d						
10,11,14,15	1 - 1 -						



$$f(a,b,c,d) = \sum_{m} (0,5,8,9,10,11,14,15)$$

⟨2 : 1, 1 : D⟩								
cube	а		С	d				
9, 11	1	0	-	1				
10, 11	1	0	1	-				
10, 14	1	_	1	0				
⟨3:1,	1 : [\rangle						
cube	а	b	С	d				
11, 15	1	_	1	1				
14, 15	1	1	1	-				

⟨2 : 1,2 : D⟩							
cube	a b c d						
10,11,14,15	1 - 1 -						
10,14,11,15	1 - 1 -						



$$f(a, b, c, d) = \sum_{m} (0, 5, 8, 9, 10, 11, 14, 15)$$

Pls	0	5	8	9	10	11	14	15
8, 9, 10, 11			Х	$ \mathbf{x} $	Х	X		
10, 11, 14, 15					Х	X	X	X
0, 8	X		Х					
5		(\mathbf{x})						

$$f = a\bar{b} + ac + \bar{b}\bar{c}\bar{d} + \bar{a}b\bar{c}d$$



$$f(a,b,c,d) = \sum_{m} (1,2,3,7,8,9,10,11,14,15)$$

$\langle 1:1,0:D\rangle$								
m								
1	0	0	0	1				
2	0	0	1	0				
8	1	0	0	0				
⟨2:1,0:D⟩								
⟨2 : 1,	0 : C	\rangle						
⟨2 : 1, m	0 : E a) b	С	d				
m			c	d				
m	а	b						



$$f(a,b,c,d) = \sum_{m} (1,2,3,7,8,9,10,11,14,15)$$

$\langle 1:1,0:D\rangle$								
m								
1	0	0	0	1				
2	0	0	1	0				
8	1	0	0	0				
⟨2:1,0:D⟩								
⟨2:1,	0 : C	\rangle						
⟨2:1, m	0 : C) b	С	d				
			C	d				
m	а	b		d 1				

⟨1 : 1, 1 : D⟩									
cube	а	b	С	d					
1, 3	0	0	-	1					



$$f(a,b,c,d) = \sum_{m} (1,2,3,7,8,9,10,11,14,15)$$

⟨1 : 1,0 : D⟩									
m	а	b	С	d					
1	0	0	0	1					
2	0	0	1	0					
8	1	0	0	0					
⟨2:1,	0 : C	\rangle							
⟨2 : 1, m	0 : E) b	С	d					
•			C	d					
m	а	b							

$\langle 1:1,1:D \rangle$								
cube	(а	b	С	d			
1, 3		0	0	-	1			
1, 9		-	0	0	1			



$$f(a,b,c,d) = \sum_{m} (1,2,3,7,8,9,10,11,14,15)$$

⟨1 : 1,0 : D⟩					
m	а	b	С	d	
1	0	0	0	1	
2	0	0	1	0	
8	1	0	0	0	
⟨2:1,0:D⟩					
⟨2:1,	0 : C	\rangle			
⟨2:1, m	0 : C) b	С	d	
•			C	d	
m	а	b		d 1	

⟨1 : 1, 1 : D⟩				
cube	а	b	С	d
1, 3	0	0	-	1
1, 9	-	0	0	1
2, 3	0	0	1	-



$$f(a,b,c,d) = \sum_{m} (1,2,3,7,8,9,10,11,14,15)$$

⟨1 : 1,0 : D⟩					
m	а	b	С	d	
1	0	0	0	1	
2	0	0	1	0	
8	1	0	0	0	
⟨2:1,	0 : C	\rangle			
⟨2 : 1, m	0 : E) b	С	d	
•			C	d	
m	а	b		d 1	

$\langle 1:1,1:D\rangle$					
cube	а	b	С	d	
1, 3	0	0	-	1	
1, 9	-	0	0	1	
2, 3	0	0	1	-	
2, 10	-	0	1	0	



$$f(a,b,c,d) = \sum_{m} (1,2,3,7,8,9,10,11,14,15)$$

⟨1 : 1,0 : D⟩					
m	а	b	С	d	
1	0	0	0	1	
2	0	0	1	0	
8	1	0	0	0	
⟨2:1,	0 : C	\rangle			
m	а	b	С	d	
3	0	0	1	1	
9	1	0	0	1	

\left(1:1,	1 : C	\rangle		
cube	а	b	С	d
1, 3	0	0	-	1
1, 9	-	0	0	1
2, 3	0	0	1	-
2, 10	-	0	1	0
8, 9	1	0	0	-



$$f(a,b,c,d) = \sum_{m} (1,2,3,7,8,9,10,11,14,15)$$

⟨1 : 1,0 : D⟩					
m	а	b	С	d	
1	0	0	0	1	
2	0	0	1	0	
8	1	0	0	0	
⟨2:1,	0 : C	\rangle			
m	а	b	С	d	
3	0	0	1	1	
9	1	0	0	1	
9		_	_		

⟨1 : 1, 1 : D⟩					
cube	а	b	С	d	
1, 3	0	0	-	1	
1, 9	-	0	0	1	
2, 3	0	0	1	-	
2, 10	-	0	1	0	
8, 9	1	0	0	-	
8, 10	1	0	-	0	



$$f(a,b,c,d) = \sum_{m} (1,2,3,7,8,9,10,11,14,15)$$

⟨2:1,0:D⟩					
m	а	b	С	d	
3	0	0	1	1	
9	1	0	0	1	
10	1	0	1	0	
⟨3:1,0:D⟩					
⟨3:1,	0 : E	\rangle			
⟨3:1, m	0 : C a) b	С	d	
•			c	d	
m	а	b		d 1	



$$f(a, b, c, d) = \sum_{m} (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

⟨2 : 1,0 : D⟩						
m	а	b	С	d		
3	0	0	1	1		
9	1	0	0	1		
10	1	0	1	0		
⟨3:1,	0 : [\rangle				
m	а	b	С	d		
7	0	1	1	1		
11	1	0	1	1		
14	1	1	1	0		

⟨2 : 1,1 : D⟩					
cube	а	b	С	d	
3, 7	0	-	1	1	



$$f(a, b, c, d) = \sum_{m} (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

⟨2 : 1,0 : D⟩					
m	а	b	С	d	
3	0	0	1	1	
9	1	0	0	1	
10	1	0	1	0	
⟨3:1,	0 : C	\rangle			
m	а	b	С	d	
7	0	1	1	1	
11	1	0	1	1	

⟨2 : 1,1 : D⟩						
cube	а	b	С	d		
3, 7	0	-	1	1		
3, 11	-	0	1	1		



$$f(a, b, c, d) = \sum_{m} (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

$\langle 2:1,0:D \rangle$					
m	а	b	С	d	
3	0	0	1	1	
9	1	0	0	1	
10	1	0	1	0	
⟨3 : 1, 0 : D⟩					
m	а	b	С	d	
7	0	1	1	1	
11	1	0	1	1	
14	1	1	1	0	

⟨2 : 1, 1 : D⟩					
cube	а	b	С	d	
3, 7	0	-	1	1	
3, 11	-	0	1	1	
9, 11	1	0	-	1	



$$f(a,b,c,d) = \sum_{m} (1,2,3,7,8,9,10,11,14,15)$$

⟨2 : 1, 0 : D⟩						
m	а	b	С	d		
3	0	0	1	1		
9	1	0	0	1		
10	1	0	1	0		
⟨3:1,	⟨3 : 1, 0 : D⟩					
m	а	b	С	d		
7	0	1	1	1		
11	1	0	1	1		
14	1	1	1	0		

⟨2 : 1, 1 : D⟩					
cube	а	b	С	d	
3, 7	0	-	1	1	
3, 11	-	0	1	1	
9, 11	1	0	-	1	
10, 11	1	0	1	-	



$$f(a, b, c, d) = \sum_{m} (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

⟨2 : 1,0 : D⟩					
m	а	b	С	d	
3	0	0	1	1	
9	1	0	0	1	
10	1	0	1	0	
⟨3 : 1, 0 : D⟩					
(0.1,	U . L	•/			
m	a	b	С	d	
,			c	d	
m	а		C 1	d 1	

⟨2 : 1,1 : D⟩					
cube	а	b	С	d	
3, 7	0	-	1	1	
3, 11	-	0	1	1	
9, 11	1	0	-	1	
10, 11	1	0	1	-	
10, 14	1	-	1	0	



$$f(a,b,c,d) = \sum_{m} (1,2,3,7,8,9,10,11,14,15)$$

⟨3 : 1, 0 : D⟩					
m	а	b	С	d	
7	0	1	1	1	
11	1	0	1	1	
14	1	1	1	0	
⟨4 : 1,0 : D⟩					
m	а	b	С	d	
15	1	1	1	1	



$$f(a,b,c,d) = \sum_{m} (1,2,3,7,8,9,10,11,14,15)$$

⟨3:1,	0 : E	\rangle		
m	а	b	С	d
7	0	1	1	1
11	1	0	1	1
14	1	1	1	0
(4:1,	0 : E			
m	а	b	С	d
15	1	1	1	1

⟨3:1,	1:	D〉		
cube	а	b	С	d
7, 15	-	1	1	1



$$f(a, b, c, d) = \sum_{m} (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

⟨3:1,	0 : E	\rangle		
m	а	b	С	d
7	0	1	1	1
11	1	0	1	1
14	1	1	1	0
⟨ 4 : 1 ,	0 : E			
m	а	b	С	d
15	1	1	1	1

⟨3:1,	1:[$O\rangle$		
cube	а	b	С	d
7, 15	-	1	1	1
11, 15	1	-	1	1



$$f(a,b,c,d) = \sum_{m} (1,2,3,7,8,9,10,11,14,15)$$

⟨3 : 1,	0 : E	\rangle		
m	а	b	С	d
7	0	1	1	1
11	1	0	1	1
14	1	1	1	0
⟨4:1,	0 : E	\rangle		
m	а	b	С	d
15	1	1	1	1

⟨3:1,	1 : [\rangle		
cube	а	b	С	d
7, 15	-	1	1	1
11, 15	1	-	1	1
14, 15	1	1	1	-



$$f(a,b,c,d) = \sum_{m} (1,2,3,7,8,9,10,11,14,15)$$

⟨1 : 1,	1 : C	\rangle		
cube	а	b	С	d
1, 3	0	0	-	1
1, 9	-	0	0	1
2, 3	0	0	1	-
2, 10	-	0	1	0
8, 9	1	0	0	-
8, 10	1	0	_	0
, , ,				U
⟨2 : 1,				
·			С	d
⟨2:1,	1 : C	\rangle	c	
⟨2 : 1, cube	1 : C	\rangle		d
⟨2 : 1, cube 3, 7	1 : C	b -	1	d
〈2:1, cube 3,7 3,11	1 : C a 0	b - 0	1	d 1

$$f(a,b,c,d) = \sum_{m} (1,2,3,7,8,9,10,11,14,15)$$

⟨1 : 1,	1 : C	\rangle		
cube	а	b	С	d
1, 3	0	0	-	1
1, 9	-	0	0	1
2, 3	0	0	1	-
2, 10	-	0	1	0
8, 9	1	0	0	-
8, 10	1	0		0
0, 10	1	U		U
(2:1,				
			C	d
⟨2:1,	1 : C	\rangle	c	
⟨2 : 1, cube	1 : C	\rangle		d
(2:1, cube 3,7	1 : C) b -	1	d
(2:1, cube 3,7 3,11	1 : C a 0	b - 0	1	d 1

(1 : 1,	2 : D>	
cube	a b	c d
1, 3, 9 ,11	- 0	- 1

$$f(a,b,c,d) = \sum_{m} (1,2,3,7,8,9,10,11,14,15)$$

⟨ 1 : 1 ,	1 : C	\rangle		
cube	а	b	С	d
1, 3	0	0	-	1
1, 9	-	0	0	1
2, 3	0	0	1	-
2, 10	-	0	1	0
8, 9	1	0	0	-
8, 10	1	0	_	0
0, 10				U
(2:1,				
			С	d
⟨2:1,	1 : C	\rangle	c	
⟨2 : 1, cube	1 : C	\rangle		d
(2:1, cube 3,7	1 : C) b -	1	d
(2:1, cube 3,7 3,11	1 : C a 0	b - 0	1	d 1

⟨1 : 1,	2 : [\rangle		
cube	а	b	С	d
1, 3, 9 ,11	-	0	-	1
2, 3, 10 ,11	-	0	1	-

$$f(a,b,c,d) = \sum_{m} (1,2,3,7,8,9,10,11,14,15)$$

	1 : C	\rangle		
cube	а	b	С	d
1, 3	0	0	-	1
1, 9	-	0	0	1
2, 3	0	0	1	-
2, 10	-	0	1	0
8, 9	1	0	0	-
8, 10	1	0	-	0
⟨2 : 1,	1 · Γ) \		
	٠. ٢	'		
cube	а	b	С	d
•		•	C	d
cube	а	•		
cube 3, 7	а	b -	1	1
3, 7 3, 11	a 0	b - 0	1	1

⟨1 : 1,2 : D⟩								
cube	а	b	С	d				
1, 3, 9 ,11	-	0	-	1				
2, 3, 10 ,11	-	0	1	-				
8, 9, 10 ,11	1	0	-	-				

$$f(a, b, c, d) = \sum_{m} (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

⟨2 : 1, 1 : D⟩								
cube	а	b	С	d				
3, 7	0	-	1	1				
3, 11	-	0	1	1				
9, 11	1	0	-	1				
10, 11	1	0	1	-				
10, 14	1	-	1	0				
10, 14 \(\langle 3 : 1,			1	0				
-				0 d				
⟨3 : 1,	1 : C)						
(3 : 1, cube	1 : C)	C					



$$f(a, b, c, d) = \sum_{m} (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

$\langle 2:1,1:D \rangle$								
cube	а	b	С	d				
3, 7	0	-	1	1				
3, 11	-	0	1	1				
9, 11	1	0	-	1				
10, 11	1	0	1	-				
10, 14	1	-	1	0				
⟨3:1,	1 : [$O\rangle$						
cube	а	b		d				
7, 15	_	1	1	1				
11, 15	1	-	1	1				

⟨2 : 1,2 : D⟩							
cube	а	b	С	d			
3, 7, 11, 15	-	-	1	1			



$$f(a, b, c, d) = \sum_{m} (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

$\langle 2:1,1:D \rangle$								
cube	а	b	С	d				
3, 7	0	-	1	1				
3, 11	-	0	1	1				
9, 11	1	0	-	1				
10, 11	1	0	1	-				
10, 14	1	-	1	0				
⟨3:1,	1 : C	\rangle						
cube	а	b		d				
7, 15	-	1	1	1				
11, 15	1	-	1	1				
14, 15	1	1	1					

⟨2 : 1,2 : D⟩								
cube	а	b	С	d				
3, 7, 11, 15	-	-	1	1				
10, 11, 14,	1	-	1	-				
15								



$$f(a, b, c, d) = \sum_{m} (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

Pls	1	2	3	7	8	9	10	11	14	15
1, 3, 9, 11	\mathbf{x}		X			X		Х		
2, 3, 10, 11		\mathbf{X}	X				X	X		
8, 9, 10, 11					\mathbf{X}	X	X	X		
3, 7, 11, 15			X	\mathbf{x}				Х		Х
10, 11, 14, 15							X	X	X	X

$$f = \bar{B}D + \bar{B}R + A\bar{B} + RD + AR$$



$$f(a,b,c,d) = \sum_{m} (4,6,9,10,11,13) + \sum_{d} (2,12,15)$$

$\langle 1:1,0:D \rangle$								
m	а	b	С	d				
2	0	0	1	0				
4	0	1	0	0				
⟨2 : 1, 0 : D⟩								
m	а	b	С	d				
m 6	a	b	c	d				
6	0	1	1	0				



$$f(a,b,c,d) = \sum_{m} (4,6,9,10,11,13) + \sum_{d} (2,12,15)$$

$\langle 1:1,0:D\rangle$									
m	а	b	С	d					
2	0	0	1	0					
4	0	1	0	0					
⟨2 : 1, 0 : D⟩									
m	а	b	С	d					
6	0	1	1	0					
9	1	0	0	1					
10	1	0	1	0					

⟨1 : 1,1 : D⟩									
cube	а	b	С	d					
2, 6	0	-	1	0					



$$f(a,b,c,d) = \sum_{m} (4,6,9,10,11,13) + \sum_{d} (2,12,15)$$

$\langle 1:1,0:D\rangle$									
m	а	b	С	d					
2	0	0	1	0					
4	0	1	0	0					
⟨2 : 1, 0 : D⟩									
m	а	b	С	d					
6	0	1	1	0					
9	1	0	0	1					
10	1	0	1	0					
			0	0					

⟨1 : 1, 1 : D⟩									
cube	а	b	С	d					
2, 6	0	-	1	0					
2, 10	-	0	1	0					



$$f(a,b,c,d) = \sum_{m} (4,6,9,10,11,13) + \sum_{d} (2,12,15)$$

⟨1 : 1,0 : D⟩						
m	а	b	С	d		
2	0	0	1	0		
4	0	1	0	0		
⟨2 : 1, 0 : D⟩						
m	а	b	С	d		
6	0	1	1	0		
9	1	0	0	1		
10	1	0	1	0		
12	1	1	0	0		

⟨1 : 1,1 : D⟩					
cube	а	b	С	d	
2, 6	0	-	1	0	
2, 10	-	0	1	0	
4, 6	0	1	-	0	



$$f(a, b, c, d) = \sum_{m} (4, 6, 9, 10, 11, 13) + \sum_{d} (2, 12, 15)$$

$\langle 1:1,0:D\rangle$						
m	а	b	С	d		
2	0	0	1	0		
4	0	1	0	0		
⟨2:1,	⟨2 : 1, 0 : D⟩					
m	а	b	С	d		
6	0	1	1	0		
9	1	0	0	1		
10	1	0	1	0		
12	4	4	0	0		

$\langle 1:1,1:D \rangle$						
cube	а	b	С	d		
2, 6	0	-	1	0		
2, 10	-	0	1	0		
4, 6	0	1	-	0		
4, 12	-	1	0	0		



$$f(a,b,c,d) = \sum_{m} (4,6,9,10,11,13) + \sum_{d} (2,12,15)$$

⟨2 : 1, 0 : D⟩					
m	а	b	С	d	
6	0	1	1	0	
9	1	0	0	1	
10	1	0	1	0	
12	1	1	0	0	
⟨3:1,	0 : [O			
m	а	b	С	d	
11	1	0	1	1	
13	1	1	0	1	



$$f(a,b,c,d) = \sum_{m} (4,6,9,10,11,13) + \sum_{d} (2,12,15)$$

⟨2 : 1,0 : D⟩					
m	а	b	С	d	
6	0	1	1	0	
9	1	0	0	1	
10	1	0	1	0	
12	1	1	0	0	
⟨3:1,	0 : [O			
m	а	b	С	d	
11	1	0	1	1	
13	1	1	0	1	

⟨2:1,1:D⟩					
cube	а	b	С	d	
9, 11	-	1 0	-	1	



$$f(a,b,c,d) = \sum_{m} (4,6,9,10,11,13) + \sum_{d} (2,12,15)$$

⟨2 : 1,0 : D⟩					
m	а	b	С	d	
6	0	1	1	0	
9	1	0	0	1	
10	1	0	1	0	
12	1	1	0	0	
⟨3:1,	0 : [O			
m	а	b	С	d	
11	1	0	1	1	
13	1	1	0	1	

⟨2:1,1:D⟩					
cube	а	b	С	d	
9, 11	1	0	-	1	
9, 13	1	-	0	1	



$$f(a,b,c,d) = \sum_{m} (4,6,9,10,11,13) + \sum_{d} (2,12,15)$$

⟨2 : 1, 0 : D⟩						
m	а	b	С	d		
6	0	1	1	0		
9	1	0	0	1		
10	1	0	1	0		
12	1	1	0	0		
⟨3:1,	0 : [\overline{O}				
m	а	b	С	d		
11	1	0	1	1		
13	1	1	0	1		

/-				
$\langle 2:1,1:D\rangle$				
cube	а	b	С	d
9, 11	1	0	-	1
9, 13	1	-	0	1
10, 11	1	0	1	-



$$f(a,b,c,d) = \sum_{m} (4,6,9,10,11,13) + \sum_{d} (2,12,15)$$

⟨2 : 1,0 : D⟩					
m	а	b	С	d	
6	0	1	1	0	
9	1	0	0	1	
10	1	0	1	0	
12	1	1	0	0	
⟨3:1,	0 : [O			
m	а	b	С	d	
11	1	0	1	1	
13	1	1	0	1	

⟨2 : 1,1 : D⟩								
cube	а	b	С	d				
9, 11	1	0	-	1				
9, 13	1	-	0	1				
10, 11	1	0	1	-				
12, 13	1	1	0	-				



$$f(a,b,c,d) = \sum_{m} (4,6,9,10,11,13) + \sum_{d} (2,12,15)$$

⟨3 : 1, 0 : D⟩								
m a b c d								
11 1 0 1 1								
13	1	1	0	1				
⟨4:1,1:D⟩								
m	а	b	С	d				
15	1	1	1	1				



$$f(a,b,c,d) = \sum_{m} (4,6,9,10,11,13) + \sum_{d} (2,12,15)$$

⟨3 : 1, 0 : D⟩								
m abc								
11 1 0 1 1								
13	1	1	0	1				
⟨4:1,1:D⟩								
m	а	b	С	d				
15	1	1	1	1				

⟨3 : 1, 1 : D⟩						
cube	а	b	С	d		
11, 15	1	-	1	1		



$$f(a,b,c,d) = \sum_{m} (4,6,9,10,11,13) + \sum_{d} (2,12,15)$$

⟨3 : 1, 0 : D⟩								
m a b c d								
11 1 0 1 1								
13	1	1	0	1				
⟨4:1 ,								
m	а	b	С	d				
15	1	1	1	1				

⟨3 : 1, 1 : D⟩							
cube	а	b	С	d			
11, 15	1	-	1	1			
13, 15	1	1	-	1			



$$f(a,b,c,d) = \sum_{m} (4,6,9,10,11,13) + \sum_{d} (2,12,15)$$

⟨2 : 1, 1 : D⟩								
cube	а		С	d				
9, 11	1	0	-	1				
9, 13	1		0	1				
10, 11	1		1	-				
12, 13	1	1	0	-				
⟨3:1,	1 : [\rangle						
cube	а	b	С	d				
11, 15	1	-	1	1				
13, 15	1	1	-	1				



$$f(a,b,c,d) = \sum_{m} (4,6,9,10,11,13) + \sum_{d} (2,12,15)$$

⟨2:1,1:D⟩							
cube	а	b	С	d			
9, 11	1	0	-	1			
9, 13	1	-	0	1			
10, 11	1	0	1	-			
12, 13	1	1	0	-			
⟨3:1,	1:[O					
cube	а	b	С	d			
11, 15	1	-	1	1			
13, 15	1	1	-	1			

⟨2 : 1,2 : D⟩						
cube	а		b	С	d	
9,11,13,15		1	-	-	1	



$$f(a,b,c,d) = \sum_{m} (4,6,9,10,11,13) + \sum_{d} (2,12,15)$$

⟨2 : 1, 1 : D⟩							
cube	а	b	С	d			
9, 11	1	0	-	1			
9, 13	1	-	0	1			
10, 11	1	0	1	-			
12, 13	1	1	0	-			
⟨3:1,	1 : [\rangle					
cube	а	b	С	d			
11, 15	1	-	1	1			
13, 15	1	1	-	1			

⟨2 : 1,2 : D⟩						
cube	a b c (d				
9,11,13,15	1	1				
9,13,11,15	1	1				



$$f(a,b,c,d) = \sum_{m} (4,6,9,10,11,13) + \sum_{d} (2,12,15)$$

Pls	4	6	9	10	11	13
2, 6		X				
2, 10				X		
4, 6	X	X				
4, 12	X					
9, 11, 13, 15			\mathbf{x}		Х	X
10, 11				X	Х	
12, 13						X

- $\langle 9, 11, 13, 15 \rangle$ dominates $\langle 12, 13 \rangle$
- $\langle 10, 11 \rangle$ dominates $\langle 2, 10 \rangle$
- $\langle 4,6 \rangle$ dominates $\langle 4,12 \rangle$ and $\langle 2,6 \rangle$

$$f = ad + \bar{a}b\bar{d}$$



$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 0:1,0:D\rangle$								
m	а	b	С	d	е			
0	0	0	0	0	0			
⟨1 :	: 1,0) : C	\rangle					
		-						
m	а	b	С	d	е			
	a	b	c	d	e			
1 2					e 1 0			



$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

⟨0 : 1, 0 : D⟩								
m	а	b	С	d	е			
0	0	0	0	0	0			
⟨1 :	: 1,0) : C	\rangle					
m	_	b	_	d	_			
111	а	U	С	u	е			
1	0	0	0	0	1			
1 2					1 0			

⟨0 : 1, 1 : D⟩								
cube	а	b	С	d	е			
0, 1	0	0	0	0	-			



$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

⟨0 : 1, 0 : D⟩										
m	а	b	С	d	е					
0	0	0	0	0	0					
⟨1 :	: 1,0	⟨1 : 1,0 : D⟩								
m	а	b	С	d	е					
	a	b	c	d	e 1					
1 2										

⟨0 : 1, 1 : D⟩							
cube	а	b	С	d	е		
0, 1	0	0	0	0	-		
0, 2	0	0	0	-	0		



$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

⟨0 : 1, 0 : D⟩								
m	а	b	С	d	е			
0	0	0	0	0	0			
⟨1 : 1,0 : D⟩								
m	а	b	С	d	е			
m 1	a	b	c	d	e			
1 2								

⟨0 : 1, 1 : D⟩							
cube	а	b	С	d	е		
0, 1	0	0	0	0	-		
0, 2	0	0	0	-	0		
0, 8	0	-	0	0	0		



⟨1 : 1, 0 : D⟩								
m	а	b	С	d	е			
1	0	0	0	0	1			
2	0	0	0	1	0			
8	0	1	0	0	0			
⟨2	: 1,0) : C	\rangle					
m	а	b	С	d	е			
3	0	0	0	1	1			
6	0	0	1	1	$\overline{}$			
	"	U		ı	0			
9	0	1	0	0	1			
9	0	1	0	0	1			

⟨1 : 1, 0 : D⟩								
m	а	b	С	d	е			
1	0	0	0	0	1			
2	0	0	0	1	0			
8	0	1	0	0	0			
⟨2	: 1,0) : [\rangle					
m	а	b	С	d	е			
3	0	0	0	1	1			
6	0	0	0	1	1			
6	0	0	1	1	0			
6	0	0	1	1	0			

⟨1 : 1, 1 : D⟩								
cube	а	b	С	d	е			
1, 3	0	0	0	-	1			

⟨1 : 1,0 : D⟩								
m	а	b	С	d	е			
1	0	0	0	0	1			
2	0	0	0	1	0			
8	0	1	0	0	0			
⟨2	: 1,0) : C	\rangle					
m	а	b	С	d	е			
3	$\overline{}$	$\overline{}$	_	-				
S	0	0	0	1	1			
6	0	0	1	1	0			
6	0	0	1	1	0			
6	0	0	1	1	0			

$\langle 1:1,1:D \rangle$								
cube	а	b	С	d	е			
1, 3	0	0	0	-	1			
1, 9	0	-	0	0	1			

⟨1 : 1, 0 : D⟩									
m	а	b	С	d	е				
1	0	0	0	0	1				
2	0	0	0	1	0				
8	0	1	0	0	0				
⟨2 : 1, 0 : D⟩									
m	а	b	С	d	е				
3	0	0	0	1	1				
6	0	0	1	1	0				
9	0	0	1	1	0				
9	0	1	0	0	1				

⟨1 : 1, 1 : D⟩							
cube	а	b	С	d	е		
1, 3	0	0	0	-	1		
1, 9	0	-	0	0	1		
1, 17	-	0	0	0	1		

⟨1 : 1, 0 : D⟩								
m	а	b	С	d	е			
1	0	0	0	0	1			
2	0	0	0	1	0			
8	0	1	0	0	0			
⟨2	: 1,0) : C	\rangle					
m	а	b	С	d	е			
3	0	0	0	1	1			
6	0	0	0	1	0			
6	0	0	1	1	0			
6	0	0	1	1	0			

$\langle 1:1,1:D \rangle$							
cube	а	b	С	d	е		
1, 3	0	0	0	-	1		
1, 9	0	-	0	0	1		
1, 17	-	0	0	0	1		
2, 3	0	0	0	1	-		

$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

⟨1 : 1,0 : D⟩								
m	а	b	С	d	е			
1	0	0	0	0	1			
2	0	0	0	1	0			
8	0	1	0	0	0			
⟨2	: 1,0) : C	\rangle					
m	а	b	С	d	е			
3	0	0	0	1	1			
6	0	0	1	1	0			
9	0	1	0	0	1			
10	0	1	0	1	0			
17	1	0	0	0	1			
20	1	0	1	0	0			

⟨1 : 1, 1 : D⟩							
cube	а	b	С	d	е		
1, 3	0	0	0	-	1		
1, 9	0	-	0	0	1		
1, 17	-	0	0	0	1		
2, 3	0	0	0	1	-		
2, 6	0	0	-	1	0		

⟨1 :									
m	а	b	С	d	е				
1	0	0	0	0	1				
2	0	0	0	1	0				
8	0	1	0	0	0				
⟨2	: 1,0) : C	\rangle						
m	а	b	С	d	е				
3	0	0	0	1	1				
6	0	0	1	1	0				
9	0	1	0	0	1				
					-				
10	0	1	0	1	0				
10 17	1	0	0	0	1				

⟨1 : 1, 1 : D⟩								
cube	а	b	С	d	е			
1, 3	0	0	0	-	1			
1, 9	0	-	0	0	1			
1, 17	-	0	0	0	1			
2, 3	0	0	0	1	-			
2, 6	0	0	-	1	0			
2, 10	0	-	0	1	0			
	•							



⟨1 : 1,0 : D⟩								
m	а	b	С	d	е			
1	0	0	0	0	1			
2	0	0	0	1	0			
8	0	1	0	0	0			
⟨2	: 1, C) : C	\rangle					
m	а	b	С	d	е			
3	0	0	0	1	1			
6	0	0	1	1	0			
	0	0	1	1	0			
6								
6	0	1	0	0	1			

(1 :	$\langle 1 : 1, 1 : D \rangle$							
cube	а	b	С	d	е			
1, 3	0	0	0	-	1			
1, 9	0	-	0	0	1			
1, 17	-	0	0	0	1			
2, 3	0	0	0	1	-			
2, 6	0	0	-	1	0			
2, 10	0	-	0	1	0			
8, 9	0	1	0	0	-			

⟨1 : 1,0 : D⟩								
m	а	b	С	d	е			
1	0	0	0	0	1			
2	0	0	0	1	0			
8	0	1	0	0	0			
⟨2	: 1,0) : C	\rangle					
m	а	b	С	d	е			
•	_	_	_	-				
3	0	0	0	1	1			
6	0	0	1	1	0			
6	0	0	1	1	0			
6 9	0	0	1	1	0			

$\langle 1:1,1:D\rangle$							
cube	а	b	С	d	е		
1, 3	0	0	0	-	1		
1, 9	0	-	0	0	1		
1, 17	-	0	0	0	1		
2, 3	0	0	0	1	-		
2, 6	0	0	-	1	0		
2, 10	0	-	0	1	0		
8, 9	0	1	0	0	-		
8, 10	0	1	0	-	0		

$$f(a,b,c,d,e) = \sum_{m} (0,1,2,3,6,8,9,10,11,17,20,21,23,25,28,30,31)$$

⟨2 : 1, 0 : D⟩								
m	а	b	С	d	е			
3	0	0	0	1	1			
6	0	0	1	1	0			
9	0	1	0	0	1			
10	0	1	0	1	0			
17	1	0	0	0	1			
20	1	0	1	0	0			
⟨3 ∣	: 1,0) : C	\rangle					
m	а	b	С	d	е			
11	0	1	0	1	1			
21	1	0	1	0	1			
25	1	1	0	0	1			
28	1	1	1	0	0			

$$f(a,b,c,d,e) = \sum_{m} (0,1,2,3,6,8,9,10,11,17,20,21,23,25,28,30,31)$$

⟨2 : 1, 0 : D⟩								
m	а	b	С	d	е			
3	0	0	0	1	1			
6	0	0	1	1	0			
9	0	1	0	0	1			
10	0	1	0	1	0			
17	1	0	0	0	1			
20	1	0	1	0	0			
⟨3	: 1,0) : C	\rangle					
m	а	b	С	d	е			
11	0	1	0	1	1			
21	1	0	1	0	1			
25	1	1	0	0	1			
28	1	1	1	0	0			

⟨2 : 1, 1 : D⟩									
cube	а	b	С	d	е				
3, 11	0	-	0	1	1				

$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 2:1,0:D \rangle$									
m	а	b	С	d	е				
3	0	0	0	1	1				
6	0	0	1	1	0				
9	0	1	0	0	1				
10	0	1	0	1	0				
17	1	0	0	0	1				
20	1	0	1	0	0				
⟨3	: 1,0) : C	\rangle						
m	а	b	С	d	е				
11	0	1	0	1	1				
21	1	0	1	0	1				
25	1	1	0	0	1				
28	1	1	1	0	0				

⟨2 : 1, 1 : D⟩									
cube	а	b	С	d	е				
3, 11	0	-	0	1	1				
9, 11	0	1	0	-	1				

$$f(a,b,c,d,e) = \sum_{m} (0,1,2,3,6,8,9,10,11,17,20,21,23,25,28,30,31)$$

⟨2 : 1, 0 : D⟩									
m	а	b	С	d	е				
3	0	0	0	1	1				
6	0	0	1	1	0				
9	0	1	0	0	1				
10	0	1	0	1	0				
17	1	0	0	0	1				
20	1	0	1	0	0				
⟨3 ∣	: 1,0) : [\rangle						
m	а	b	С	d	е				
11	0	1	0	1	1				
21	1	0	1	0	1				
25	1	1	0	0	1				
28	1	1	1	0	0				

⟨2 : 1, 1 : D⟩									
cube	а	b	С	d	е				
3, 11	0	-	0	1	1				
9, 11	0	1	0	-	1				
9, 25	-	1	0	0	1				

$$f(a,b,c,d,e) = \sum_{m} (0,1,2,3,6,8,9,10,11,17,20,21,23,25,28,30,31)$$

$\langle 2:1,0:D\rangle$									
m	а	b	С	d	е				
3	0	0	0	1	1				
6	0	0	1	1	0				
9	0	1	0	0	1				
10	0	1	0	1	0				
17	1	0	0	0	1				
20	1	0	1	0	0				
⟨3 ∣	: 1,0) : [\rangle						
m	а	b	С	d	е				
11	0	1	0	1	1				
21	1	0	1	0	1				
25	1	1	0	0	1				
28	1	1	1	0	0				

⟨2 : 1, 1 : D⟩								
cube	а	b	С	d	е			
3, 11	0	-	0	1	1			
9, 11	0	1	0	-	1			
9, 25	-	1	0	0	1			
10, 11	0	1	0	1	-			

$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

⟨2 : 1, 0 : D⟩									
m	а	b	С	d	е				
3	0	0	0	1	1				
6	0	0	1	1	0				
9	0	1	0	0	1				
10	0	1	0	1	0				
17	1	0	0	0	1				
20	1	0	1	0	0				
⟨3	: 1,0) : [O						
m	а	b	С	d	е				
11	0	1	0	1	1				
21	1	0	1	0	1				
25	1	1	0	0	1				
28	1	1	1	0	0				

⟨2 : 1, 1 : D⟩								
cube	а	b	С	d	е			
3, 11	0	-	_	1	1			
9, 11	0	1	0	-	1			
9, 25	-	1	0	0	1			
10, 11	0	1	0	1	-			
17, 21	1	0	-	0	1			

$$f(a,b,c,d,e) = \sum_{m} (0,1,2,3,6,8,9,10,11,17,20,21,23,25,28,30,31)$$

$\langle 2:1,0:D \rangle$									
m	а	b	С	d	е				
3	0	0	0	1	1				
6	0	0	1	1	0				
9	0	1	0	0	1				
10	0	1	0	1	0				
17	1	0	0	0	1				
20	1	0	1	0	0				
⟨3	: 1,0) : [\rangle						
m	а	b	С	d	е				
11	0	1	0	1	1				
21	1	0	1	0	1				
25	1	1	0	0	1				
28	1	1	1	0	0				

⟨2 : 1, 1 : D⟩								
cube	а	b	С	d	е			
3, 11	0	-	0	1	1			
9, 11	0	1	0	-	1			
9, 25	-	1	0	0	1			
10, 11	0	1	0	1	-			
17, 21	1	0	-	0	1			
17, 25	1	-	0	0	1			

$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 2:1,0:D \rangle$									
m	а	b	С	d	е				
3	0	0	0	1	1				
6	0	0	1	1	0				
9	0	1	0	0	1				
10	0	1	0	1	0				
17	1	0	0	0	1				
20	1	0	1	0	0				
⟨3	: 1,0) : [\rangle						
m	а	b	С	d	е				
11	0	1	0	1	1				
21	1	0	1	0	1				
25	1	1	0	0	1				
28	1	1	1	0	0				

⟨2 : 1, 1 : D⟩									
cube	а	b	С	d	е				
3, 11	0	-	0	1	1				
9, 11	0	1	0	-	1				
9, 25	-	1	0	0	1				
10, 11	0	1	0	1	-				
17, 21	1	0	-	0	1				
17, 25	1	-	0	0	1				
20, 21	1	0	1	0	-				

$$f(a,b,c,d,e) = \sum_{m} (0,1,2,3,6,8,9,10,11,17,20,21,23,25,28,30,31)$$

⟨2 : 1, 0 : D⟩									
m	а	b	С	d	е				
3	0	0	0	1	1				
6	0	0	1	1	0				
9	0	1	0	0	1				
10	0	1	0	1	0				
17	1	0	0	0	1				
20	1	0	1	0	0				
⟨3	: 1,0) : [\rangle						
m	а	b	С	d	е				
11	0	1	0	1	1				
21	1	0	1	0	1				
25	1	1	0	0	1				
28	1	1	1	0	0				

⟨2 : 1, 1 : D⟩									
cube	а	b	С	d	е				
3, 11	0	-	0	1	1				
9, 11	0	1	0	-	1				
9, 25	-	1	0	0	1				
10, 11	0	1	0	1	-				
17, 21	1	0	-	0	1				
17, 25	1	-	0	0	1				
20, 21	1	0	1	0	-				
20, 28	1	-	1	0	0				

 $f(a,b,c,d,e) = \sum_{m} (0,1,2,3,6,8,9,10,11,17,20,21,23,25,28,30,31)$

$\langle 3:1,0:D \rangle$									
m	а	b	С	d	е				
11	0	1	0	1	1				
21	1	0	1	0	1				
25	1	1	0	0	1				
28	1	1	1	0	0				
⟨4 ∣	: 1,0) : C	\rangle						
m	а	b	С	d	е				
23	1	0	1	1	1				
30	1	1	1	1	0				



$$f(a,b,c,d,e) = \sum_{m} (0,1,2,3,6,8,9,10,11,17,20,21,23,25,28,30,31)$$

$\langle 3:1,0:D \rangle$								
m	а	b	С	d	е			
11	0	1	0	1	1			
21	1	0	1	0	1			
25	1	1	0	0	1			
28	1	1	1	0	0			
⟨4 ∣	: 1,0) : C	\rangle					
m	а	b	С	d	е			
23	1	0	1	1	1			
30	1	1	1	1	0			

⟨3 : 1,1 : D⟩									
cube	а	b	С	d	е				
21, 23	1	0	1	-	1				



$$f(a,b,c,d,e) = \sum_{m} (0,1,2,3,6,8,9,10,11,17,20,21,23,25,28,30,31)$$

$\langle 3:1,0:D \rangle$									
m	а	b	С	d	е				
11	0	1	0	1	1				
21	1	0	1	0	1				
25	1	1	0	0	1				
28	1	1	1	0	0				
⟨4	: 1,0) : C	\rangle						
m	а	b	С	d	е				
23	1	0	1	1	1				
30	1	1	1	1	0				

⟨3 : 1, 1 : D⟩								
cube	а	b	С	d	е			
21, 23	1	0	1	-	1			
28, 30	1	1	1	-	0			



$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

⟨4:1,0:D⟩									
m	а	b	С	d	е				
23	1	0	1	1	1				
30	1	1	1	1	0				
⟨5 ∣	: 1,0) : [\rangle						
m	а	b	С	d	е				
31	1	1	1	1	1				



$$f(a,b,c,d,e) = \sum_{m} (0,1,2,3,6,8,9,10,11,17,20,21,23,25,28,30,31)$$

⟨4:1,0:D⟩									
m	а	b	С	d	е				
23	1	0	1	1	1				
30	1	1	1	1	0				
⟨5	: 1,0) : [\overline{O}						
m	а	b	С	d	е				
31	1	1	1	1	1				

⟨4:1,1:D⟩								
cube	а	b	С	d	е			
23, 31	1	-	1	1	1			



$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 4:1,0:D \rangle$									
m	а	b	С	d	е				
23	1	0	1	1	1				
30	1	1	1	1	0				
⟨5 :	: 1,0) : C	\rangle						
m	а	b	С	d	е				
31	1	1	1	1	1				

⟨4:1,1:D⟩								
cube	а	b	С	d	е			
23, 31	1	-	1	1	1			
30, 31	1	1	1	1	_			



$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

⟨0:	1, 1 : [$O\rangle$								
cube	а	b	С	d	е					
0, 1	0	0	0	0	-					
0, 2	0	0	0	-	0					
0, 8	0	-	0	0	0					
(1 : '	⟨1 : 1, 1 : D⟩									
cube	а	b	С	d	е					
1, 3	0	0	0	-	1					
1, 9	0	-	0	0	1					
1, 17	-	0	0	0	1					
2, 6	0	0	-	1	0					
2, 10	0	-	0	1	0					
8, 9	0	1	0	0	-					
0, 0										
2, 3	0	0	0	1	-					

$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

⟨0 : ¹	1,1:	$D\rangle$			
cube	а	b	С	d	е
0, 1	0	0	0	0	-
0, 2	0	0	0	-	0
0, 8	0	-	0	0	0
(1 : ·	1,1:	D angle			
cube	а	b	С	d	е
1, 3	0	0	0	-	1
1, 9	_		0	0	1
1, 9	0	-	U	U	•
1, 17	-	0	0	0	1
	- 0	0			
1, 17	-			0	1
1, 17 2, 6	- 0		0	0	1
1, 17 2, 6 2, 10	- 0 0	0	0 - 0	0 1 1	1

⟨0 : 1, 2 : D⟩								
cube	a b c d e							
0, 1, 8, 9	0 - 0 0 -							

$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

⟨0 : ⁻	1,1:	$D \rangle$			
cube	а	b	С	d	е
0, 1	0	0	0	0	-
0, 2	0	0	0	-	0
0, 8	0	-	0	0	0
(1 : ·	1,1:	$D \rangle$			
cube	а	b	С	d	е
1, 3	0	0	0	-	1
1, 9	0	-	0	0	1
1, 9 1, 17	-	0	0	0	1
· ·	- 0	- 0 0			
1, 17	-			0	1
1, 17 2, 6	- 0		0	0	1
1, 17 2, 6 2, 10	- 0 0	0 -	0 -	0 1 1	1

⟨0 : 1,2 : D⟩								
cube	а	b	С	d	е			
0, 1, 8, 9	0	-	0	0	-			
0, 1, 2, 3	0	0	0	-	-			

$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

⟨0 : ⁻	1,1:	$D \rangle$			
cube	а	b	С	d	е
0, 1	0	0	0	0	-
0, 2	0	0	0	-	0
0, 8	0	-	0	0	0
(1 : ·	1,1:	$D \rangle$			
cube	а	b	С	d	е
1, 3	0	0	0	-	1
1, 9	0	_	0	0	1
1, 17	-	0	0	0	1
1, 17 2, 6	- 0	0	0	0	1
	- 0 0		0 - 0		
2, 6			-	1	0
2, 6 2, 10	0	0	- 0	1	0

$\langle 0:1,2:D \rangle$									
cube	а	b	С	d	е				
0, 1, 8, 9	0	-	0	0	-				
0, 1, 2, 3	0	0	0	-	-				
0, 2, 8, 10	0	-	0	-	0				

$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

(1 : '	1, 1	: C	\rangle			
cube	a	3	b	С	d	е
1, 3		0	0	0	-	1
1, 9		0	-	0	0	1
1, 17		-	0	0	0	1
2, 6		0	0	-	1	0
2, 10		0	-	0	1	0
8, 9		0	1	0	0	-
2, 3		0	0	0	1	-
8, 10		0	1	0	-	0

⟨2 : 1, 1 : D⟩									
cube		а	b	С	d	е			
3, 11		0	-	0	1	1			
9, 11		0	1	0	-	1			
9, 25		-	1	0	0	1			
10, 11		0	1	0	1	-			
17, 21		1	0	-	0	1			
17, 25		1	-	0	0	1			
20, 21		1	0	1	0	-			
20, 28		1	-	1	0	0			



$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

⟨1:	1, 1 : [\overline{O}			
cube	а	b	С	d	е
1, 3	0	0	0	-	1
1, 9	0	-	0	0	1
1, 17	-	0	0	0	1
2, 6	0	0	-	1	0
2, 10	0	-	0	1	0
8, 9	0	1	0	0	-
2, 3	0	0	0	1	-
8, 10	0	1	0	-	0
⟨1 :	1,2: [\overline{O}			
cube	а	b	С	d	е
1, 3, 9, 11	С) -	0	-	1

⟨2:1,1:D⟩									
cube		1		С	d	е			
3, 11		0	-	0	1	1			
9, 11		0	1	0	-	1			
9, 25		-	1	0	0	1			
10, 11		0	1	0	1	-			
17, 21		1	0	-	0	1			
17, 25		1	-	0	0	1			
20, 21		1	0	1	0	-			
20, 28		1	-	1	0	0			

$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

⟨1:	1, 1	: E	\rangle			
cube		а	b	С	d	е
1, 3		0	0	0	-	1
1, 9		0	-	0	0	1
1, 17		-	0	0	0	1
2, 6		0	0	-	1	0
2, 10		0	-	0	1	0
8, 9		0	1	0	0	-
2, 3		0	0	0	1	-
8, 10		0	1	0	-	0
⟨1:	1,2	2 : C	\rangle			
cube		а	b	С	d	е
1, 3, 9, 11		0	-	0	-	1
1, 9, 17, 25		-	-	0	0	1

⟨2 : 1, 1 : D⟩												
cube a b c d												
3, 11		0	-	0	1	1						
9, 11		0	1	0	-	1						
9, 25		-	1	0	0	1						
10, 11		0	1	0	1	-						
17, 21		1	0	-	0	1						
17, 25		1	-	0	0	1						
20, 21		1	0	1	0	-						
20, 28		1	-	1	0	0						

$$f(a,b,c,d,e) = \sum_{m} (0,1,2,3,6,8,9,10,11,17,20,21,23,25,28,30,31)$$

⟨1:	1, 1	: [\rangle			
cube		а	b	С	d	е
1, 3		0	0	0	-	1
1, 9		0	-	0	0	1
1, 17		-	0	0	0	1
2, 6		0	0	-	1	0
2, 10		0	-	0	1	0
8, 9		0	1	0	0	-
2, 3		0	0	0	1	-
8, 10		0	1	0	-	0
⟨1:	1,2	: E	\rangle			
cube		а	b	С	d	е
1, 3, 9, 11		0	-	0	-	1
1, 9, 17, 25		-	-	0	0	1
2, 10, 3, 11		0	-	0	1	-

⟨2:1,1:D⟩												
cube												
3, 11	0	-	0	1	1							
9, 11	0	1	0	-	1							
9, 25	-	1	0	0	1							
10, 11	0	1	0	1	-							
17, 21	1	0	-	0	1							
17, 25	1	-	0	0	1							
20, 21	1	0	1	0	-							
20, 28	1	-	1	0	0							

$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

⟨1:	1,1:[O			
cube	а	b	С	d	е
1, 3	0	0	0	-	1
1, 9	0	-	0	0	1
1, 17	-	0	0	0	1
2, 6	0	0	-	1	0
2, 10	0	-	0	1	0
8, 9	0	1	0	0	-
2, 3	0	0	0	1	-
8, 10	0	1	0	-	0
⟨1 :	1,2:[\overline{O}			
cube	а	b	С	d	е
1, 3, 9, 11	0	-	0	-	1
1, 9, 17, 25	-	-	0	0	1
2, 10, 3, 11	0	-	0	1	-
8, 9, 10, 11	0	1	0	-	-

⟨2 : 1, 1 : D⟩												
cube												
3, 11		0	-	0	1	1						
9, 11		0	1	0	-	1						
9, 25		-	1	0	0	1						
10, 11		0	1	0	1	-						
17, 21		1	0	-	0	1						
17, 25		1	-	0	0	1						
20, 21		1	0	1	0	-						
20, 28		1	-	1	0	0						

$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 0:1,2:D \rangle$											
cube	a b c d e										
0, 1, 8, 9	0 - 0 0 -										
0, 1, 2, 3	0 0 0										
0, 2, 8, 10	0 - 0 - 0										
√1 : ¹	1,2 : D>										
cube	a b c d e										
1, 3, 9, 11	0 - 0 - 1										
1, 9, 17, 25	0 0 1										
2, 10, 3, 11	0 - 0 1 -										
8, 9, 10, 11	0 1 0										



$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 0 : 1, 2 : D \rangle$												
cube	а	b	С	d	е							
0, 1, 8, 9	0	-	0	0	-							
0, 1, 2, 3	0	0	0	-	-							
0, 2, 8, 10	0	-	0	-	0							
⟨1 : 1,2 : D⟩												
\'.	· , <u>~</u> .	D /										
cube	a		С	d	е							
· · · · · · · · · · · · · · · · · · ·		b	c	d -	e							
cube	а	b		d - 0	e 1							
cube 1, 3, 9, 11	а	b	0	-	e 1 1							

$\langle 0:1,3:D \rangle$										
cube	a b c d e									
0, 1, 8, 9,	0 - 0									
2, 10, 3, 11										



2, 6	P ₁
17, 21	P ₂
20, 21	P ₃
20, 28	P ₄
21, 23	P ₅
28, 30	P ₆
23, 31	P ₇
30, 31	P ₈
1, 9, 17, 25	P ₉
0, 1, 8, 9, 2, 10, 3, 11	P ₁₀



Pls	0	1	2	3	6	8	9	10	11	17	20	21	23	25	28	30	31
P ₁			х		х												
P ₂										х		х					
P ₃											х	х					
P ₄											Х				Х		
P ₅												х	х				
P ₆															х	х	
P ₇													х				х
P ₈																х	х
P ₉		х					х			х				х			
P ₁₀	х	х	х	х		х	х	х	х								



Pls	0	1	2	3	6	8	9	10	11	17	20	21	23	25	28	30	31
P ₁			х		х												
P ₂										х		х					
P ₃											х	х					
P ₄											х				Х		
P ₅												х	х				
P ₆															х	х	
P ₇													х				х
P ₈																х	х
P ₉		х					х			х				х			
P ₁₀	X	х	х	х		х	х	х	х								



$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

Pls	0	1	2	3	6	8	9	10	11	17	20	21	23	25	28	30	31
P ₁			X		х												
P ₂										х		х					
P ₃											х	х					
P ₄											х				Х		
P ₅												х	х				
P ₆															X	х	
P ₇													х				х
P ₈																х	х
P ₉		х					х			х				х			
P ₁₀	x	х	x	х		х	х	X	х								

P₁₀ is an essential cube

$$f = \bar{a}\bar{c} +$$

H

$$f(a,b,c,d,e) = \sum_{m} (0,1,2,3,6,8,9,10,11,17,20,21,23,25,28,30,31)$$

Pls	6	17	20	21	23	25	28	30	31
P ₁	X								
P ₂		х		х					
P ₃			х	х					
P ₄			х				х		
P ₅				х	х				
P ₆							Х	х	
P ₇					х				х
P ₈								х	х
P ₉		х				х			

$$f = \bar{a}\bar{c} +$$



$$f(a,b,c,d,e) = \sum_{m} (0,1,2,3,6,8,9,10,11,17,20,21,23,25,28,30,31)$$

Pls	6	17	20	21	23	25	28	30	31
P ₁	X								
P ₂		х		х					
P ₃			х	х					
P ₄			х				х		
P ₅				х	х				
P ₆							х	х	
P ₇					х				х
P ₈								Х	х
P ₉		Х				Х			

$$f = \bar{a}\bar{c} +$$



QM ex-4

$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

Pls	6	17	20	21	23	25	28	30	31
P ₁	X								
P ₂		х		х					
P ₃			х	Х					
P ₄			х				Х		
P ₅				Х	х				
P ₆							х	X	
P ₇					х				х
P ₈								X	х
P ₉		х				х			

P₁ is an essential cube

$$f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} +$$



$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

Pls	17	20	21	23	25	28	30	31
P ₂	х		х					
P ₃		х	х					
P ₄		х				х		
P ₅			х	х				
P ₆						х	Х	
P ₇				Х				х
P ₈							х	х
P ₉	х				х			

$$f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} +$$



$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

Pls	17	20	21	23	25	28	30	31
P ₂	X		х					
P ₃		х	х					
P ₄		х				х		
P ₅			х	Х				
P ₆						Х	Х	
P ₇				Х				х
P ₈							х	х
P ₉	Х				X			

$$f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} +$$



QM ex-4

$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

Pls	17	20	21	23	25	28	30	31
P ₂	х		х					
P ₃		х	х					
P ₄		х				х		
P ₅			х	х				
P ₆						х	Х	
P ₇				х				х
P ₈							Х	х
P ₉	Х				x			

P9 is an essential cube

$$f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e +$$



$$f(a,b,c,d,e) = \sum_{m} (0,1,2,3,6,8,9,10,11,17,20,21,23,25,28,30,31)$$

Pls	20	21	23	28	30	31
P ₂		х				
P ₃	х	х				
P ₄	Х			Х		
P ₅		х	х			
P ₆				х	х	
P ₇			х			х
P ₈					х	х

$$f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e +$$



$$f(a,b,c,d,e) = \sum_{m} (0,1,2,3,6,8,9,10,11,17,20,21,23,25,28,30,31)$$

Pls	20	21	23	28	30	31
P ₂		х				
P ₃	х	х				
P ₄	х			Х		
P ₅		х	х			
P ₆				Х	Х	
P ₇			х			х
P ₈					х	х

P₃ dominates P₂

$$f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e +$$



$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

Pls	20	21	23	28	30	31
P ₃	Х	Х				
P ₄	Х			х		
P ₅		Х	х			
P ₆				х	Х	
P ₇			х			Х
P ₈					Х	Х

$$f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e +$$



$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

Pls	20	21	23	28	30	31
P ₃	х	X				
P ₄	х			Х		
P ₅		Х	х			
P ₆				Х	х	
P ₇			х			х
P ₈					х	х

$$\mathbf{P}_3: f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + a\bar{b}c\bar{d} +$$



$$f(a,b,c,d,e) = \sum_{m} (0,1,2,3,6,8,9,10,11,17,20,21,23,25,28,30,31)$$

Pls	23	28	30	31
P ₄		х		
P ₅	х			
P ₆		х	х	
P ₇	х			Х
P ₈			х	х

$$\mathbf{P}_3: f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + a\bar{b}c\bar{d} +$$



$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

Pls	23	28	30	31
P ₄		х		
P ₅	х			
P ₆		х	х	
P ₇	х			Х
P ₈			х	Х

P₆ dominates P₄

$$\mathbf{P}_3: f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + a\bar{b}c\bar{d} +$$



$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

Pls	23	28	30	31
P ₅	X			
P ₆		х	х	
P ₇	X			X
P ₈			х	Х

$$\mathbf{P}_3: f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + a\bar{b}c\bar{d} +$$



$$f(a,b,c,d,e) = \sum_{m} (0,1,2,3,6,8,9,10,11,17,20,21,23,25,28,30,31)$$

Pls	23	28	30	31
P ₅	х			
P ₆		х	х	
P ₇	х			х
P ₈			х	х

P₇ dominates P₅

$$\mathbf{P}_3: f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + a\bar{b}c\bar{d} +$$



$$f(a,b,c,d,e) = \sum_{m} (0,1,2,3,6,8,9,10,11,17,20,21,23,25,28,30,31)$$

Pls	23	28	30	31
P ₆		х	х	
P ₇	X			Х
P ₈			х	Х

$$\mathbf{P}_3: f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + a\bar{b}c\bar{d} +$$



$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

Pls	23	28	30	31
P ₆		х	х	
P ₇	Х			Х
P ₈			х	Х

23 dominates 31

$$\mathbf{P}_3: f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + a\bar{b}c\bar{d} +$$



$$f(a,b,c,d,e) = \sum_{m} (0,1,2,3,6,8,9,10,11,17,20,21,23,25,28,30,31)$$

Pls	23	28	30
P ₆		X	Х
P ₇	Х		
P ₈			Х

$$\mathbf{P}_3: f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + a\bar{b}c\bar{d} +$$



$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

Pls	23	28	30
P ₆		х	х
P ₇	х		
P ₈			х

P₆ dominates P₈

$$\mathbf{P}_3: f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + a\bar{b}c\bar{d} +$$



$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

Pls	23	28	30
P ₆		Х	х
P ₇	Х		

$$\mathbf{P}_3: f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + a\bar{b}c\bar{d} +$$



$$f(a,b,c,d,e) = \sum_{m} (0,1,2,3,6,8,9,10,11,17,20,21,23,25,28,30,31)$$

Pls	23	28	30
P ₆		X	X
P ₇	X		

P₆ and P₇ are essential cubes

$$\mathbf{P}_3: f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + a\bar{b}c\bar{d} + abc\bar{e} + acde$$



$$f(a,b,c,d,e) = \sum_{m} (0,1,2,3,6,8,9,10,11,17,20,21,23,25,28,30,31)$$

Pls	20	21	23	28	30	31
P ₄	х			Х		
P ₅		Х	х			
P ₆				Х	Х	
P ₇			х			х
P ₈					Х	х

$$ar{\mathbf{P}}_3: f = ar{a}\,ar{c} + ar{a}\,ar{b}\,d\,ar{e} + ar{c}\,ar{d}\,e +$$



$$f(a,b,c,d,e) = \sum_{m} (0,1,2,3,6,8,9,10,11,17,20,21,23,25,28,30,31)$$

Pls	20	21	23	28	30	31
P ₄	X			X		
P ₅		х	х			
P ₆				X	х	
P ₇			х			х
P ₈					х	х

$$ar{\mathbf{P}}_3: f = ar{a}\,ar{c} + ar{a}\,ar{b}\,d\,ar{e} + ar{c}\,ar{d}\,e +$$



$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

Pls	20	21	23	28	30	31
P ₄	(x)			X		
P ₅		х	Х			
P ₆				х	X	
P ₇			х			х
P ₈					х	х

P4 is an essential cube

$$\mathbf{P}_{3}: f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + ac\bar{d}\bar{e} +$$



$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

Pls	21	23	30	31
P ₅	Х	X		
P ₆			х	
P ₇		Х		х
P ₈			х	Х

$$\bar{\mathbf{P}}_3: f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + ac\bar{d}\bar{e} +$$



$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

Pls	21	23	30	31
P ₅	X	х		
P ₆			Х	
P ₇		х		X
P ₈			х	Х

$$\bar{\mathbf{P}}_3: f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + ac\bar{d}\bar{e} +$$



$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

Pls	21	23	30	31
P ₅	X	х		
P ₆			х	
P ₇		х		Х
P ₈			Х	Х

P₅ is an essential cube

$$\mathbf{P}_{3}: f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + ac\bar{d}\bar{e} + a\bar{b}ce +$$



$$f(a,b,c,d,e) = \sum_{m} (0,1,2,3,6,8,9,10,11,17,20,21,23,25,28,30,31)$$

Pls	30	31
P ₆	Х	
P ₇		х
P ₈	Х	х

$$\mathbf{P}_{3}: f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + ac\bar{d}\bar{e} + a\bar{b}ce +$$



$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

Pls	30	31
P ₆	х	
P ₇		х
P ₈	Х	х

P₈ dominates P₆ and P₇

$$\bar{\mathbf{P}}_3: f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + ac\bar{d}\bar{e} +$$



$$f(a, b, c, d, e) = \sum_{m} (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

Pls	30	31
P ₈	(x)	X

P₈ is an essential cube

$$oldsymbol{ar{P}}_3: f = ar{a}ar{c} + ar{a}ar{b}dar{e} + ar{c}ar{d}e + acar{d}ar{e} + aar{b}ce + abcd$$

$$\mathbf{P}_3: f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + a\bar{b}c\bar{d} + abc\bar{e} + acde$$

Both decisions lead to (distinct) solutions of the same cost

