SOLUTIONS 6.1

SOL 6.1.1 Option (C) is correct.
The z-transform is

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\,z^{-n} = \sum_{n=-\infty}^{-1} -\left(\frac{1}{2}\right)^n z^{-n}$$
$$= -\sum_{n=-\infty}^{-1}\left(\frac{1}{2}z^{-1}\right)^n = -\sum_{n=-\infty}^{-1}(2z)^{-n}$$
$$= -\sum_{m=1}^{\infty}(2z)^m \qquad \text{Let } -n = m \text{ so,}$$

The above series converges if $|2z| < 1$ or $|z| < \frac{1}{2}$

$$X(z) = -\frac{2z}{1-2z} = \frac{2z}{2z-1}, \quad |z| < \frac{1}{2}$$

SOL 6.1.2 Option (A) is correct.

We have $\quad x[n] = \left(\frac{1}{2}\right)^{|n|} = \left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{2}\right)^{-n} u[-n-1]$

z-transform is $\quad X(z) = \sum_{n=-\infty}^{\infty} x[n]\,z^{-n}$

$$= \sum_{n=-\infty}^{\infty}\left(\frac{1}{2}\right)^{-n} z^{-n} u[-n-1] + \sum_{n=-\infty}^{\infty}\left(\frac{1}{2}\right)^n z^{-n} u[n]$$
$$= \sum_{n=-\infty}^{-1}\left(\frac{1}{2}\right)^{-n} z^{-n} + \sum_{n=0}^{\infty}\left(\frac{1}{2z}\right)^n$$
$$= \underbrace{\sum_{n=1}^{\infty}\left(\frac{1}{2}z\right)^n}_{\text{II}} + \underbrace{\sum_{n=0}^{\infty}\left(\frac{1}{2z}\right)^n}_{\text{I}}$$
$$= \frac{1}{1-\frac{1}{2z}} + \frac{\frac{1}{2}z}{1-\frac{1}{2}z} = \frac{z}{z-\frac{1}{2}} - \frac{z}{z-2}$$

Series I converges, if $\left|\frac{1}{2}z\right| < 1$ or $|z| < 2$

Series II converges, if $\left|\frac{1}{2z}\right| < 1$ or $|z| > \frac{1}{2}$

ROC is intersection of both, therefore ROC : $\frac{1}{2} < |z| < 2$

SOL 6.1.3 Option (D) is correct.

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\,z^{-n}$$
$$= \sum_{n=-\infty}^{\infty}\left(\frac{1}{2}\right)^n z^{-n} u[-n-1] + \sum_{n=-\infty}^{\infty}\left(\frac{1}{3}\right)^n z^{-n} u[n]$$
$$= \sum_{n=-\infty}^{-1}\left(\frac{1}{2}\right)^n z^{-n} + \sum_{n=0}^{\infty}\left(\frac{1}{3z}\right)^n$$
$$= \sum_{n=1}^{\infty}(2z)^n + \sum_{n=0}^{\infty}\left(\frac{1}{3z}\right)^n = \underbrace{\frac{2z}{1-2z}}_{\text{I}} + \underbrace{\frac{1}{1-\frac{1}{3}z^{-1}}}_{\text{II}}$$

Series I converges, when $|2z| < 1$ or $|z| < \frac{1}{2}$

Series II converges, when $\left|\frac{1}{3z}\right| < 1$ or $|z| > \frac{1}{3}$

So ROC of $X(z)$ is intersection of both ROC: $\frac{1}{3} < |z| < \frac{1}{2}$

SOL 6.1.4 Option (C) is correct.
z-transform of $x[n]$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\,z^{-n}$$
$$= \sum_{n=-\infty}^{\infty} \alpha^n z^{-n} u[n] + \sum_{n=-\infty}^{\infty} \alpha^{-n} z^{-n} u[n]$$
$$= \sum_{n=0}^{\infty}(\alpha z^{-1})^n + \sum_{n=0}^{\infty}(\alpha z)^{-n} = \underbrace{\frac{1}{1-\alpha z^{-1}}}_{\text{I}} + \underbrace{\frac{1}{1-(\alpha z)^{-1}}}_{\text{II}}$$

Series I converges, if $\alpha z^{-1} < 1$ or $|z| > |\alpha|$

Series II converges, if $(\alpha z)^{-1} < 1$ or $\alpha z > 1$ or $|z| > \frac{1}{|\alpha|}$

So ROC is interaction of both

$$\text{ROC} : |z| > \max\left(|\alpha|, \frac{1}{|\alpha|}\right)$$

SOL 6.1.5 Option (C) is correct.

(P → 4) $\quad x_1[n] = u[n-2]$

$$X_1(z) = \sum_{n=-\infty}^{\infty} u[n-2]\,z^{-n} = \sum_{n=2}^{\infty} z^{-n} = \frac{z^{-2}}{1-z^{-1}}, \quad |z| > 1$$

(Q → 1) $\quad x_2[n] = -u[-n-3]$

$$X_2[z] = -\sum_{n=-\infty}^{\infty} u[-n-3]\,z^{-n}$$
$$= -\sum_{n=-\infty}^{-3} z^{-n} = -\sum_{m=3}^{\infty} z^m \qquad \text{Let, } n = -m$$
$$= \frac{-z^3}{1-z} = \frac{1}{z^{-2}(1-z^{-1})}, \quad |z| < 1$$

(R → 3) $\quad x_3[n] = (1)^n u[n+4]$

$$X_3(z) = \sum_{n=-\infty}^{\infty} u[n+4]\,z^{-n} = \sum_{n=-4}^{\infty} z^{-n}$$
$$= \frac{z^4}{1-z^{-1}} = \frac{1}{z^{-4}(1-z^{-1})}, \quad |z| > 1$$

(S → 2) $\quad x_4[n] = (1)^n u[-n]$

$$= \sum_{n=-\infty}^{\infty} u[-n]\,z^{-n}$$
$$= \sum_{n=-\infty}^{0} z^{-n} = \sum_{m=0}^{\infty} z^m = \frac{1}{1-z} = \frac{-z^{-1}}{1-z^{-1}}, \quad |z| < 1$$

SOL 6.1.6 Option (A) is correct.

$$X(z) = \sum_{n=-\infty}^{\infty} e^{jn\pi} z^{-n} u[n] = \sum_{n=0}^{\infty}(e^{j\pi} z^{-1})^n$$
$$= \frac{1}{1-e^{j\pi} z^{-1}}, \quad |z| > 1$$
$$= \frac{z}{z-e^{j\pi}} = \frac{z}{z+1} \qquad \because e^{j\pi} = -1$$

SOL 6.1.7 Option (D) is correct.
We can write, transfer function

$$H(z) = \frac{Az^2}{(z-2)(z-3)}$$

$$H(1) = \frac{A}{(-1)(-2)} = 1 \text{ or } A = 2$$

so,
$$H(z) = \frac{2z^2}{(z-2)(z-3)}$$

$$\frac{H(z)}{z} = \frac{2z}{(z-2)(z-3)}$$

$$H(z) = \frac{-4z}{(z-2)} + \frac{6z}{(z-3)} \qquad \text{From partial fraction}$$

We can see that for ROC: $|z| > 3$, the system is causal and unstable because ROC is exterior of the circle passing through outermost pole and does not include unit circle.

so,
$$h[n] = [(-4)2^n + (6)(3)^n]u[n], \ |z| > 3 \qquad (P \to 2)$$

For ROC $2 < |z| < 3$, The sequence corresponding to pole at $z = 2$ corresponds to right-sided sequence while the sequence corresponds to pole at $z = 3$ corresponds to left sided sequence

$$h[n] = (-4)2^n u[n] + (-6)3^n u[-n-1] \qquad (Q \to 4)$$

For ROC: $|z| < 2$, ROC is interior to circle passing through inner most pole, hence the system is non causal.

$$h[n] = (4)2^n u[-n-1] + (-6)3^n u[-n-1] \qquad (R \to 3)$$

For the response

$$h[n] = 4(2)^n u[-n-1] + (-6)3^n u[n]$$

ROC: $|z| < 2$ and $|z| > 3$ which does not exist $\qquad (S \to 1)$

SOL 6.1.8 Option (A) is correct.

$$X(z) = e^z + e^{1/z}$$

$$X(z) = \left(1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \ldots..\right) + \left(1 + \frac{1}{z} + \frac{1}{2!}\frac{1}{z^2} + \ldots..\right)$$

$$= \left(1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \ldots.\right) + \left(1 + z^{-1} + \frac{z^{-2}}{2!} + \ldots.\right)$$

Taking inverse z-transform

$$x[n] = \delta[n] + \frac{1}{n!}$$

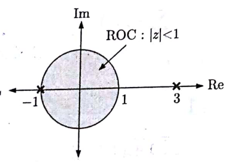
SOL 6.1.9 Option (A) is correct.

$$X(z) = \frac{z^2 + 5z}{z^2 - 2z - 3} = \frac{z(z+5)}{(z-3)(z+1)}$$

$$\frac{X(z)}{z} = \frac{z+5}{(z-3)(z+1)} = \frac{2}{z-3} - \frac{1}{z+1} \qquad \text{By partial fraction}$$

Thus
$$X(z) = \frac{2z}{z-3} - \frac{z}{z+1}$$

Poles are at $z = 3$ and $z = -1$



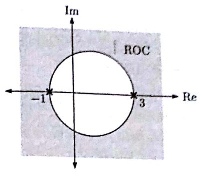
ROC : $|z| < 1$, which is not exterior of circle outside the outermost pole $z = 3$. So, $x[n]$ is anticausal given as

$$x[n] = [-2(3)^n + (-1)^n]u[-n-1]$$

SOL 6.1.10 Option (A) is correct.

$$X(z) = \frac{2z}{z-3} - \frac{z}{z+1}$$

If $|z| > 3$, ROC is exterior of a circle outside the outer most pole, x causal.

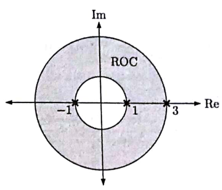


$$x[n] = [2(3)^n - (-1)^n]u[n]$$

SOL 6.1.11 Option (C) is correct.

$$X(z) = \frac{2z}{z-3} - \frac{z}{z+1}$$

If ROC is $1 < |z| < 3$, $x[n]$ is two sided with anticausal part $\frac{2z}{z-3}$, $|z| < 3$ and causal part $\frac{-z}{z+1}$, $|z| > 1$



$$x[n] = -2(3)^n u[-n-1] - (-1)^n u[n]$$

SOL 6.1.12 Option (D) is correct.

$$X_1(z) = \sum_{n=-\infty}^{\infty}(0.7)^n z^{-n} u[n-1] = \sum_{n=1}^{\infty}(0.7z^{-1})^n$$

$$= \frac{0.7z^{-1}}{1 - 0.7z^{-1}}$$

ROC : $|0.7z^{-1}| < 1$ or $|z| > 0.7$

$$X_2(z) = \sum_{n=-\infty}^{\infty}(-0.4)^n z^{-n} u[-n-2] = \sum_{n=-\infty}^{-2}(-0.4)^n z^{-n}$$

$$= \sum_{m=2}^{\infty}(-0.4)^{-m} z^m \qquad \text{Let } n = -m$$

$$= \sum_{m=2}^{\infty}[(-0.4)^{-1}z]^m = \frac{-(0.4)^{-1}z}{1 + (0.4)^{-1}z}$$

ROC : $|(0.4)^{-1}z| < 1$ or $|z| < 0.4$

The ROC of z-transform of $x[n]$ is intersection of both which does not exist.

SOL 6.1.13 Option (D) is correct.

If $x[n] \xleftrightarrow{z} X(z)$

From time shifting property

$$x[n-n_0] \xleftrightarrow{z} z^{-n_0}X(z)$$

So $z(x[n-4]) = z^{-4}X(z) = \dfrac{1}{8z^5 - 2z^4 - z^3}$

SOL 6.1.14 Option (A) is correct.

We can see that

$$X_1(z) = z^1X_2(z) = z^2X_3(z)$$

or $z^{-2}X_1(z) = z^{-1}X_2(z) = X_3(z)$

So $x_1[n-2] = x_2[n-1] = x_3[n]$

SOL 6.1.15 Option (C) is correct.

$x[n]$ can be written in terms of unit sequence as

$$x[n] = u[n] - u[n-k]$$

so $X(z) = \dfrac{z}{z-1} - z^{-k}\dfrac{z}{z-1} = \dfrac{1-z^{-k}}{1-z^{-1}}$

SOL 6.1.16 Option (C) is correct.

For positive shift

If, $x[n] \xleftrightarrow{z} X(z)$

then, $x[n-n_0] \xleftrightarrow{z} z^{-n_0}X(z),\ n_0 \geq 0$

So $x[n-1] \xleftrightarrow{z} z^{-1}\left(\dfrac{z}{z-1}\right) = \dfrac{1}{z-1}$

For negative shift

$$x[n+n_0] \xleftrightarrow{z} z^{n_0}\left(X(z) - \sum_{m=0}^{n_0-1} x[n]z^{-m}\right),\ n_0 > 0$$

$$x[n+1] \xleftrightarrow{z} z(X(z) - x[0])$$

We know that $x[n] = u[n]$ so $x[0] = 1$

and $x[n+1] \xleftrightarrow{z} z(X(z)-1) = z\left(\dfrac{z}{z-1} - 1\right) = \dfrac{z}{z-1}$

SOL 6.1.17 Option (B) is correct.

Even part of $x[n]$, $x_e[n] = \frac{1}{2}(x[n] + x[-n])$

z- transform of $x_e[n]$, $X_e(z) = \frac{1}{2}\left[X(z) + X\left(\frac{1}{z}\right)\right] \qquad \because\ x[-n] \xleftrightarrow{z} X\left(\frac{1}{z}\right)$

$$= \underbrace{\frac{1}{2}\left(\frac{z}{z-0.4}\right)}_{\text{I}} + \underbrace{\frac{1}{2}\left(\frac{1/z}{1/z-0.4}\right)}_{\text{II}}$$

Region of convergence for I series is $|z| > 0.4$ and for II series it is $|z| < 2.5$. Therefore, $X_e(z)$ has ROC $0.4 < |z| < 2.5$

SOL 6.1.18 Option (B) is correct.

$(P \to 4)$ $y[n] = n(-1)^n u[n]$

We know that

$$(-1)^n u[n] \xleftrightarrow{z} \frac{1}{1+z^{-1}},\ |z| > 1$$

If $x[n] \xleftrightarrow{z} X(z)$

then, $nx[n] \xleftrightarrow{z} -z\dfrac{dX(z)}{dz}$ (z-domain differentiation)

so, $n(-1)^n u[n] \xleftrightarrow{z} -z\dfrac{d}{dz}\left[\dfrac{1}{1+z^{-1}}\right]$, ROC: $|z| > 1$

$$Y(z) = \frac{-z^{-1}}{(1+z^{-1})^2},\ \text{ROC}: |z| > 1$$

$(Q \to 3)$ $y[n] = -nu[-n-1]$

We know that,

$$u[-n-1] \xleftrightarrow{z} \frac{-1}{(1-z^{-1})},\ \text{ROC}: |z| < 1$$

Again applying z-domain differentiation property

$$-nu[-n-1] \xleftrightarrow{z} z\frac{d}{dz}\left[\frac{-1}{1-z^{-1}}\right],\ \text{ROC}: |z| < 1$$

$$Y(z) = \frac{z^{-1}}{(1-z^{-1})^2},\ \text{ROC}: |z| < 1$$

$(R \to 2)$ $y[n] = (-1)^n u[n]$

$$Y(z) = \sum_{n=-\infty}^{\infty}(-1)^n z^{-n}u[n] = \sum_{n=0}^{\infty}(-z^{-1})^n$$

$$= \frac{1}{1+z^{-1}},\ \text{ROC}: |z| > 1$$

$(S \to 1)$ $y[n] = nu[n]$

We know that $u[n] \xleftrightarrow{z} \dfrac{1}{1-z^{-1}}$, ROC: $|z| > 1$

so, $nu[n] \xleftrightarrow{z} z\dfrac{d}{dz}\left(\dfrac{1}{1-z^{-1}}\right)$, ROC: $|z| > 1$

$$Y(z) = \frac{z^{-1}}{(1-z^{-1})^2},\ \text{ROC}: |z| > 1$$

SOL 6.1.19 Option (A) is correct.

Its difficult to obtain z-transform of $x[n]$ directly due to the term $1/n$.

Let $y[n] = nx[n] = (-2)^{-n}u[-n-1]$

So z-transform of $y[n]$

$$Y(z) = \frac{-z}{z+\frac{1}{2}},\ \text{ROC}: |z| < \frac{1}{2}$$

Since $y[n] = nx[n]$

so, $Y(z) = -z\dfrac{dX(z)}{dz}$ (Differentiation in z-domain)

$$-z\frac{dX(z)}{dz} = \frac{-z}{z+\frac{1}{2}}$$

$$\frac{dX(z)}{dz} = \frac{1}{z+\frac{1}{2}}$$

or $X(z) = \log\left(z + \frac{1}{2}\right)$, ROC: $|z| < \frac{1}{2}$

SOL 6.1.20 Option (A) is correct.

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n},\ \text{ROC}: R_x$$

$$Y(z) = \sum_{n=-\infty}^{\infty} y[n]z^{-n} = \sum_{n=-\infty}^{\infty} a^n x[n]z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} x[n]\left(\frac{z}{a}\right)^{-n} = X\left(\frac{z}{a}\right),\ \text{ROC}: aR_x$$

SOL 6.1.21 Option (B) is correct.

Using time shifting property of z-transform

If, $x[n] \xleftrightarrow{z} X(z)$, ROC: R_x

then, $$x[n-n_0] \xleftrightarrow{\mathcal{Z}} z^{-n_0}X(z)$$

with same ROC except the possible deletion or addition of $z=0$ or $z=\infty$.

So, ROC for $x[n-2]$ is R_x (S_1, R_1)

Similarly for $x[n+2]$, ROC : R_x (S_2, R_1)

Using time-reversal property of z-transform

If, $$x[n] \xleftrightarrow{\mathcal{Z}} X(z),\quad \text{ROC}: R_x$$

then, $$x[-n] \xleftrightarrow{\mathcal{Z}} X\left(\frac{1}{z}\right),\quad \text{ROC}: \frac{1}{R_x}$$

For S_3, $$x[-n] \xleftrightarrow{\mathcal{Z}} X\left(\frac{1}{z}\right),$$

Because z is replaced by $1/z$, so ROC would be $|z| < \frac{1}{a}$ (S_3, R_3)

$S_4 : (-1)^n x[n]$

Using the property of scaling in z-domain, we have

If, $$x[n] \xleftrightarrow{\mathcal{Z}} X(z),\quad \text{ROC}: R_x$$

then, $$\alpha^n x[n] \xleftrightarrow{\mathcal{Z}} X\left(\frac{z}{\alpha}\right)$$

z is replaced by z/α so ROC will be $\frac{R_x}{|\alpha|}$

Here $$(-1)^n x[n] \xleftrightarrow{\mathcal{Z}} X\left(\frac{z}{1}\right),\quad = 1$$

so, $$\text{ROC}: |z| > a \qquad (S_4, R_1)$$

SOL 6.1.22 Option (D) is correct.

Time scaling property :

If, $$x[n] \xleftrightarrow{\mathcal{Z}} X(z)$$

then, $$x[n/2] \xleftrightarrow{\mathcal{Z}} X(z^2) \qquad (P \to 2)$$

Time shifting property :

If, $$x[n] \xleftrightarrow{\mathcal{Z}} X(z)$$

then, $$x[n-2]\,u[n-2] \xleftrightarrow{\mathcal{Z}} z^{-2}X(z) \qquad (Q \to 1)$$

For $x[n+2]\,u[n]$ we can not apply time shifting property directly.

Let, $$y[n] = x[n+2]\,u[n]$$
$$= \alpha^{n+2}u[n+2]\,u[n] = \alpha^{n+2}u[n]$$

so, $$Y(z) = \sum_{n=-\infty}^{\infty} y[n]\,z^{-n} = \sum_{n=0}^{\infty}\alpha^{n+2}z^{-n}$$
$$= \alpha^2\sum_{n=0}^{\infty}(\alpha z^{-1})^n = \alpha^2 X(z) \qquad (R \to 4)$$

Let, $$g[n] = \beta^{2n}x[n]$$
$$G(z) = \sum_{n=-\infty}^{\infty} g[n]\,z^{-n} = \sum_{n=-\infty}^{\infty}\beta^{2n}\alpha^n z^{-n}u[n]$$
$$= \sum_{n=0}^{\infty}\alpha^n\left(\frac{z}{\beta^2}\right)^n = X\left(\frac{z}{\beta^2}\right) \qquad (S \to 3)$$

SOL 6.1.23 Option (C) is correct.

Let, $$y[n] = x[2n]$$
$$Y(z) = \sum_{n=-\infty}^{\infty} x[2n]\,z^{-n}$$
$$= \sum_{k=-\infty}^{\infty} x[k]\,z^{-k/2} \qquad \text{Put } 2n = k \text{ or } n = \frac{k}{2},\ k \text{ is even}$$

Since k is even, so we can write

$$Y(z) = \sum_{k=-\infty}^{\infty}\left[\frac{x[k] + (-1)^k x[k]}{2}\right]z^{-k/2}$$
$$= \frac{1}{2}\sum_{k=-\infty}^{\infty} x[k]\,z^{-k/2} + \frac{1}{2}\sum_{k=-\infty}^{\infty} x[k]\,(-z^{1/2})^k$$
$$= \frac{1}{2}\left[X(\sqrt{z}) \cdot\ X(-\sqrt{z})\right]$$

SOL 6.1.24 Option (A) is correct.

From the accumulation property we know that

If, $$x[n] \xleftrightarrow{\mathcal{Z}} X(z)$$

then, $$\sum_{k=-\infty}^{n} x[k] \xleftrightarrow{\mathcal{Z}} \frac{z}{(z-1)}X(z)$$

Here, $$y[n] = \sum_{k=0}^{n} x[k]$$
$$Y(z) = \frac{z}{(z-1)}X(z) = \frac{4z^2}{(z-1)(8z^2-2z-1)}$$

SOL 6.1.25 Option (B) is correct.

By taking z-transform of both the sequences
$$X(z) = (-1 + 2z^{-1} + 0 + 3z^{-3})$$
$$H(z) = 2z^2 + 3$$

Convolution of sequences $x[n]$ and $h[n]$ is given as
$$y[n]\ x[n] * h[n]$$

Applying convolution property of z-transform, we have
$$Y(z) = X(z)\,H(z)$$
$$= (-1 + 2z^{-1} + 3z^{-3})(2z^2 + 3) = -2z^2 + 4z - 3 + 12z^{-1} + 9z^{-3}$$

or, $$y[n] = \{-2, 4, \underset{\uparrow}{-3}, 12, 0, 9\}$$

SOL 6.1.26 Option (C) is correct.

The z-transform of signal $x^*[n]$ is given as follows
$$\mathcal{Z}\{x^*[n]\} = \sum_{n=-\infty}^{\infty} x^*[n]\,z^{-n} = \sum_{n=-\infty}^{\infty}\left[x[n]\,(z^*)^{-n}\right]^* \qquad ...(1)$$

Let z-transform of $x[n]$ is $X(z)$
$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\,z^{-n}$$

Taking complex conjugate on both sides of above equation
$$X^*(z) = \sum_{n=-\infty}^{\infty}\left[x[n]\,z^{-n}\right]^*$$

Replacing $z \to z^*$, we will get
$$X^*(z^*) = \sum_{n=-\infty}^{\infty}\left[x[n]\,(z^*)^{-n}\right]^* \qquad ...(2)$$

Comparing equation (1) and (2)
$$\mathcal{Z}\{x^*[n]\} = X^*(z^*)$$

SOL 6.1.27 Option (B) is correct.

By taking z-transform on both sides of given difference equation
$$Y(z) - \frac{1}{2}z^{-1}[Y(z) + y[-1]\,z] = X(z)$$

Let impulse response is $H(z)$, so the impulse input is $X(z) = 1$
$$H(z) - \frac{1}{2}z^{-1}[H(z) + 3z] = 1$$

$$H(z)[1-\tfrac{1}{2}z^{-1}] = \frac{5}{2}$$

$$H(z) = \frac{5/2}{1-\frac{1}{2}z^{-1}} = \frac{5}{2}\left(\frac{z}{z-\frac{1}{2}}\right)$$

$$h[n] = \frac{5}{2}\left(\frac{1}{2}\right)^n, \quad n \geq 0$$

SOL 6.1.28 Option (B) is correct.

$$h[n] = (2)^n u[n]$$

Taking z-transform

$$H(z) = \frac{z}{z-2} = \frac{Y(z)}{X(z)}$$

so, $\quad (z-2)\,Y(z) = zX(z)$

or, $\quad (1-2z^{-1})\,Y(z) = X(z)$

Taking inverse z-transform

$$y[n] - 2y[n-1] = x[n]$$

SOL 6.1.29 Option (B) is correct.

$$h[n] = \delta[n] - \left(\frac{-1}{2}\right)^n u[n]$$

z-transform of $h[n]$

$$H(z) = 1 - \frac{z}{z+\frac{1}{2}} = \frac{\frac{1}{2}}{z+\frac{1}{2}} = \frac{Y(z)}{X(z)}$$

$$\left(z+\frac{1}{2}\right)Y(z) = \frac{1}{2}X(z)$$

$$\left(1+\frac{1}{2}z^{-1}\right)Y(z) = \frac{1}{2}z^{-1}X(z)$$

Taking inverse z-transform

$$y[n] + \frac{1}{2}y[n-1] = \frac{1}{2}x[n-1]$$

$$y[n] + 0.5y[n-1] = 0.5x[n-1]$$

SOL 6.1.30 Option (C) is correct.

We have $\quad y[n] - 0.4y[n-1] = (0.4)^n u[n]$

Zero state response refers to the response of system with zero initial condition.

So, by taking z-transform

$$Y(z) - 0.4z^{-1}Y(z) = \frac{z}{z-0.4}$$

$$Y(z) = \frac{z^2}{(z-0.4)^2}$$

Taking inverse z-transform

$$y[n] = (n+1)(0.4)^n u[n]$$

SOL 6.1.31 Option (B) is correct.

Zero state response refers to response of the system with zero initial conditions.

Taking z-transform

$$Y(z) - \frac{1}{2}z^{-1}Y(z) = X(z)$$

$$Y(z) = \left(\frac{z}{z-0.5}\right)X(z)$$

For an input $\quad x[n] = u[n],\ X(z) = \dfrac{z}{z-1}$

so, $\quad Y(z) = \dfrac{z}{(z-0.5)}\dfrac{z}{(z-1)} = \dfrac{z^2}{(z-1)(z-0.5)}$

$$\frac{Y(z)}{z} = \frac{z}{(z-1)(z-0.5)}$$

$$= \frac{2}{z-1} - \frac{1}{z-0.5} \qquad \text{By partial fraction}$$

Thus $\quad Y(z) = \dfrac{2z}{z-1} - \dfrac{z}{z-0.5}$

Taking inverse z-transform

$$y[n] = 2u[n] - (0.5)^n u[n]$$

SOL 6.1.32 Option (C) is correct.

Input, $\quad x[n] = 2\delta[n] + \delta[n+1]$

By taking z-transform

$$X(z) = 2 + z$$

$$\frac{Y(z)}{X(z)} = H(z), \qquad Y(z) \text{ is } z\text{-transform of output } y[n]$$

$$Y(z) = H(z)\,X(z)$$

$$= \frac{2z(z-1)}{(z+2)^2}(z+2)$$

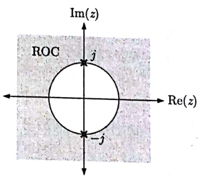
$$= \frac{2z(z-1)}{(z+2)} = 2z - \frac{6z}{z+2}$$

Taking inverse z-transform

$$y[n] = 2\delta[n+1] - 6(-2)^n u[n]$$

SOL 6.1.33 Option (B) is correct.

Poles of the system function are at $z = \pm j$ ROC is shown in the figure.



Causality :

We know that a discrete time LTI system with transfer function $H(z)$ is causal if and only if ROC is the exterior of a circle outside the outer most pole.

For the given system ROC is exterior to the circle outside the outer most pole ($z = \pm j$). The system is causal.

Stability :

A discrete time LTI system is stable if and only if ROC of its transfer function $H(z)$ includes the unit circle $|z| = 1$.

The given system is unstable because ROC does not include the unit circle.

Impulse Response :

$$H(z) = \frac{z}{z^2+1}$$

We know that

$$\sin(\Omega_0 n)\,u[n] \xleftrightarrow{Z} \frac{z\sin\Omega_0}{z^2-2z\cos\Omega_0+1},\ |z|>1$$

Here $\quad z^2+1 = z^2-2z\cos\Omega_0+1$

So $\quad 2z\cos\Omega_0 = 0$ or $\Omega_0 = \frac{\pi}{2}$

Taking the inverse Laplace transform of $H(z)$

$$h[n] = \sin\left(\frac{\pi}{2}n\right)u[n]$$

SOL 6.1.34 Option (D) is correct.
Statement (A), (B) and (C) are true.

SOL 6.1.35 Option (D) is correct.
First we obtain transfer function (z-transform of $h[n]$) for all the systems and then check for stability

(A) $\quad H(z) = \dfrac{\frac{1}{3}z}{(z-\frac{1}{3})^2}$

Stable because all poles lies inside unit circle.

(B) $\quad h[n] = \frac{1}{3}\delta[n]$

$$\sum_n |h[n]| = \frac{1}{3} \qquad \text{(absolutely summable)}$$

Thus this is also stable.

(C) $\quad h[n] = \delta[n] - \left(\frac{-1}{3}\right)^n u[n]$

$$H(z) = 1 - \frac{z}{z+\frac{1}{3}}$$

Pole is inside the unit circle, so the system is stable.

(D) $\quad h[n] = [(2)^n - (3)^n]\,u[n]$

$$H(z) = \frac{z}{z-2} - \frac{z}{z-3}$$

Poles are outside the unit circle, so it is unstable.

SOL 6.1.36 Option (B) is correct.
By taking z-transform

$$(1+3z^{-1}+2z^{-2})\,Y(z) = (2+3z^{-1})\,X(z)$$

So, transfer function

$$H(z) = \frac{Y(z)}{X(z)} = \frac{(2+3z^{-1})}{(1+3z^{-1}+2z^{-2})} = \frac{2z^2+3z}{z^2+3z+2}$$

or $\quad \dfrac{H(z)}{z} = \dfrac{2z+3}{z^2+3z+2} = \dfrac{1}{z+2}+\dfrac{1}{z+1}$ $\quad$ By partial fraction

Thus $\quad H(z) = \dfrac{z}{z+2}+\dfrac{z}{z+1}$

Both the poles lie outside the unit circle, so the system is unstable.

SOL 6.1.37 Option (B) is correct.

$$y[n] = x[n] + y[n-1]$$

Put $x[n] = \delta[n]$ to obtain impulse response $h[n]$

$$h[n] = \delta[n] + h[n-1]$$

For $n=0$, $\quad h[0] = \delta[0] + h[-1]$

$h[0] = 1 \qquad (\,h[-1] = 0$, for causal system)

$n=1$, $\quad h[1] = \delta[1] + h[0]$

$h[1] = 1$

$n=2$, $\quad h[2] = \delta[2] + h[1]$

$h[2] = 1$

In general form

$h[n] = u[n]$ $\qquad$ Thus, statement 1 is true.

Let $\quad x[n] \xleftrightarrow{Z} X(z)$

$$X(z) = \frac{z}{z-0.5}$$

$$h[n] \xleftrightarrow{Z} H(z)$$

$$H(z) = \frac{z}{z-1}$$

Output $\quad Y(z) = H(z)\,X(z)$

$$= \left(\frac{z}{z-1}\right)\left(\frac{z}{z-0.5}\right) = \frac{2z}{z-1} - \frac{z}{z-0.5} \quad \text{By partial fraction}$$

Inverse z-transform

$y[n] = 2u[n] - (0.5)^n u[n]$ $\qquad$ Statement 3 is also true.

$$H(z) = \frac{z}{z-1}$$

System pole lies at unit circle $|z| = 1$, so the system is not BIBO stable.

SOL 6.1.38 Option (C) is correct.
(P → 3) ROC is exterior to the circle passing through outer most pole at $z = 1.2$, so it is causal. ROC does not include unit circle, therefore it is unstable.
(Q → 1) ROC is not exterior to the circle passing through outer most pole at $z = 1.2$, so it is non causal. But ROC includes unit circle, so it is stable.
(R → 2), ROC is not exterior to circle passing through outermost pole $z = 0.8$, so it is not causal. ROC does not include the unit circle, so it is unstable also.
(S → 4), ROC contains unit circle and is exterior to circle passing through outermost pole, so it is both causal and stable.

SOL 6.1.39 Option (D) is correct.

$$H(z) = \frac{P(z-0.9)}{z-0.9+Pz}$$

$$= \frac{P(z-0.9)}{(1+P)z-0.9} = \frac{P}{1+P}\left(\frac{z-0.9}{z-\frac{0.9}{1+P}}\right)$$

Pole at $\quad z = \dfrac{0.9}{1+P}$

For stability pole lies inside the unit circle, so

$$|z| < 1$$

or $\quad \left|\dfrac{0.9}{1+P}\right| < 1$

$$0.9 < |1+P|$$

$$P > -0.1 \text{ or } P < -1.9$$

SOL 6.1.40 Option (A) is correct.
For a system to be causal and stable, $H(z)$ must not have any pole outside the unit circle $|z| = 1$.

S_1 : $\quad H(z) = \dfrac{z-\frac{1}{2}}{z^2+\frac{1}{2}z-\frac{3}{16}} = \dfrac{z-\frac{1}{2}}{(z-\frac{1}{4})(z+\frac{3}{4})}$

Poles are at $z = 1/4$ and $z = -3/4$, so it is causal.

S_2 : $$H(z) = \frac{z+1}{(z+\frac{4}{3})(1-\frac{1}{2}z^{-3})}$$

one pole is at $z = -4/3$, which is outside the unit circle, so it is not causal.

S_3 : one pole is at $z = \infty$, so it is also non-causal.

SOL 6.1.41 Option (D) is correct.

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\,z^{-n} = \sum_{n=-\infty}^{\infty} \delta[n-k]\,z^{-n} = z^{-k},\ z \neq 0$$

ROC : We can find that $X(z)$ converges for all values of z except $z = 0$, because at $z = 0$ $X(z) \to \infty$.

SOL 6.1.42 Option (D) is correct.

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\,z^{-n} = \sum_{n=-\infty}^{\infty} \delta[n+k]\,z^{-n} = z^{k},\ \text{all } z$$

ROC : We can see that above summation converges for all values of z.

SOL 6.1.43 Option (A) is correct.

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\,z^{-n} = \sum_{n=-\infty}^{\infty} u[n]\,z^{-n}$$
$$= \underbrace{\sum_{n=0}^{\infty} z^{-n}}_{\text{I}} = \frac{1}{1-z^{-1}}$$

ROC : Summation I converges if $|z| > 1$, because when $|z| < 1$, then $\sum_{n=0}^{\infty} z^{-n} \to \infty$.

SOL 6.1.44 Option (D) is correct.

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\,z^{-n}$$
$$= \sum_{n=-\infty}^{\infty} \left(\frac{1}{4}\right)^n (u[n]-u[n-5])\,z^{-n}$$
$$= \underbrace{\sum_{n=0}^{4} \left(\frac{1}{4}z^{-1}\right)^n}_{\text{I}} \qquad u[n]-u[n-5] = 1,\ \text{for } 0 \le n \le 4$$
$$= \frac{1-(\frac{1}{4}z^{-1})^5}{1-(\frac{1}{4}z^{-1})} = \frac{z^5-(0.25)^5}{z^4(z-0.5)},\quad \text{all } z$$

ROC : Summation I converges for all values of z because n has only four value.

SOL 6.1.45 Option (D) is correct.

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\,z^{-n} = \sum_{n=-\infty}^{\infty} \left(\frac{1}{4}\right)^n u[-n]\,z^{-n}$$
$$= \sum_{n=-\infty}^{0} \left(\frac{1}{4}z^{-1}\right)^n = \sum_{n=-\infty}^{0} (4z)^{-n}$$
$$= \underbrace{\sum_{m=0}^{\infty} (4z)^m}_{\text{I}} = \frac{1}{1-4z},\quad |z| < \frac{1}{4} \qquad \text{Taking } n \to -m,$$

ROC : Summation I converges if $|4z| < 1$ or $|z| < \frac{1}{4}$.

SOL 6.1.46 Option (B) is correct.

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\,z^{-n} = \sum_{n=-\infty}^{\infty} 3^n u[-n-1]\,z^{-n}$$
$$= \sum_{n=-\infty}^{-1} (3z^{-1})^n = \underbrace{\sum_{n=1}^{\infty} \left(\frac{1}{3}z\right)^n}_{\text{I}} \qquad u[-n-1] = 1,\ n \le -1$$
$$= \frac{\frac{1}{3}z}{1-\frac{1}{3}z} = \frac{z}{3-z},\quad |z| < 3$$

ROC : Summation I converges when $|\frac{1}{3}z| < 1$ or $|z| < 3$

SOL 6.1.47 Option (B) is correct.

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\,z^{-n} = \sum_{n=-\infty}^{\infty} \left(\frac{2}{3}\right)^{|n|} z^{-n}$$
$$= \sum_{n=-\infty}^{-1} \left(\frac{2}{3}\right)^{-n} z^{-n} + \sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n z^{-n}$$

In first summation taking $n = -m$,

$$X(z) = \sum_{m=1}^{\infty} \left(\frac{2}{3}\right)^m z^m + \sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n z^{-n}$$
$$= \underbrace{\sum_{m=1}^{\infty} \left(\frac{2}{3}z\right)^m}_{\text{I}} + \underbrace{\sum_{n=0}^{\infty} \left(\frac{2}{3}z^{-1}\right)^n}_{\text{II}}$$
$$= \frac{\frac{2}{3}z}{(1-\frac{2}{3}z)} + \frac{1}{(1-\frac{2}{3}z^{-1})}$$
$$= \frac{-1}{(1-\frac{3}{2}z^{-1})} + \frac{1}{(1-\frac{2}{3}z^{-1})}$$

ROC : Summation I converges if $|\frac{2}{3}z| < 1$ or $|z| < \frac{3}{2}$ and summation II converges if $|\frac{2}{3}z^{-1}| < 1$ or $|z| > \frac{2}{3}$. ROC of $X(z)$ would be intersection of both, that is $\frac{2}{3} < |z| < \frac{3}{2}$

SOL 6.1.48 Option (B) is correct.

$$x[n] = \cos\left(\frac{\pi}{3}n\right)u[n] = \frac{e^{j(\frac{\pi}{3})n} + e^{-j(\frac{\pi}{3})n}}{2}u[n]$$
$$= \frac{1}{2}e^{j(\frac{\pi}{3})n}u[n] + \frac{1}{2}e^{-j(\frac{\pi}{3})n}u[n]$$
$$X[z] = \frac{1}{2}\left[\underbrace{\frac{1}{1-e^{\frac{\pi}{3}}z^{-1}}}_{\text{I}} + \underbrace{\frac{1}{1-e^{-\frac{\pi}{3}}z^{-1}}}_{\text{II}}\right]$$
$$a^n u[n] \xleftrightarrow{\mathcal{Z}} \frac{1}{1-az^{-1}},\quad |z| > |a|$$
$$= \frac{1}{2}\left[\frac{2-z^{-1}[e^{-\frac{\pi}{3}}+e^{\frac{\pi}{3}}]}{1-z^{-1}(e^{\frac{\pi}{3}}+e^{-\frac{\pi}{3}})+z^{-2}}\right],\quad |z| > 1$$
$$= \frac{z}{2}\frac{(2z-1)}{(z^2-z+1)},\quad |z| > 1$$

ROC : First term in $X(z)$ converges for $|z| > |e^{\frac{\pi}{3}}| \Rightarrow |z| > 1$. Similarly II term also converges for $|z| > |e^{-\frac{\pi}{3}}| \Rightarrow |z| > 1$, so ROC would be simply $|z| > 1$.

SOL 6.1.49 Option (B) is correct.

$$x[n] = 3\delta[n+5] + 6\delta[n] + \delta[n-1] - 4\delta[n-2]$$
$$X[z] = 3z^5 + 6 + z^{-1} - 4z^{-2},\qquad 0 < |z| < \infty$$

$$\delta[n \pm n_0] \xleftrightarrow{\mathcal{Z}} z^{\pm n_0}$$

ROC : $X(z)$ is finite over entire z plane except $z = 0$ and $z = \infty$ because when $z = 0$ negative power of z becomes infinite and when $z \to \infty$ the positive powers of z tends to becomes infinite.

SOL 6.1.50 Option (D) is correct.

$$x[n] = 2\delta[n+2] + 4\delta[n+1] + 5\delta[n] + 7\delta[n-1] + \delta[n-3]$$
$$X(z) = 2z^2 + 4z + 5 + 7z^{-1} + z^{-3},\quad 0 < |z| < \infty \quad \delta[n \pm n_0] \xleftrightarrow{\mathcal{Z}} z^{\pm n_0}$$

ROC is same as explained in previous question.

SOL 6.1.51 Option (B) is correct.

$$x[n] = \delta[n] - \delta[n-2] + \delta[n-4] - \delta[n-5]$$

$$X(z) = 1 - z^{-2} + z^{-4} - z^{-5}, \quad z \neq 0 \qquad \delta[n \pm n_0] \xleftrightarrow{\mathcal{Z}} z^{\pm n_0}$$

ROC : $X(z)$ has only negative powers of z, therefore transform $X(z)$ does not converges for $z = 0$.

SOL 6.1.52 Option (A) is correct.

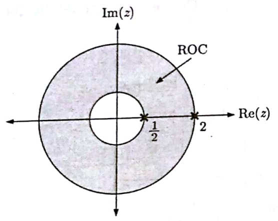
Using partial fraction expansion, $X(z)$ can be simplified as

$$X(z) = \frac{z^2 - 3z}{z^2 + \frac{3}{2}z - 1} = \frac{1 - 3z^{-1}}{1 + \frac{3}{2}z^{-1} - z^{-2}}$$

$$\frac{(1 - 3z^{-1})}{(1 + 2z^{-1})(1 - \frac{1}{2}z^{-1})}$$

$$= \underbrace{\frac{2}{1 + 2z^{-1}}}_{\mathrm{I}} - \underbrace{\frac{1}{1 - \frac{1}{2}z^{-1}}}_{\mathrm{II}}$$

Poles are at $z = -2$ and $z = \frac{1}{2}$. We obtain the inverse z-transform using relationship between the location of poles and region of convergence as shown in the figure.



ROC : $\frac{1}{2} < |z| < 2$ has a radius less than the pole at $z = -2$ therefore the I term of $X(z)$ corresponds to a left sided signal

$$\frac{2}{1 + 2z^{-1}} \xleftrightarrow{\mathcal{Z}^{-1}} -2(2)^n u[-n-1] \qquad \text{(left-sided signal)}$$

While, the ROC has a greater radius than the pole at $z = \frac{1}{2}$, so the second term of $X(z)$ corresponds to a right sided sequence.

$$\frac{1}{1 - \frac{1}{2}z^{-1}} \xleftrightarrow{\mathcal{Z}^{-1}} \frac{1}{2^n} u[n] \qquad \text{(right-sided signal)}$$

So, the inverse z-transform of $X(z)$ is

$$x[n] = -2(2)^n u[-n-1] - \frac{1}{2^n} u[n]$$

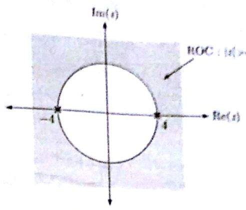
SOL 6.1.53 Option (A) is correct.

Using partial fraction expansion $X(z)$ can be simplified as follows

$$X(z) = \frac{3z^2 - \frac{1}{4}z}{z^2 - 16} = \frac{3 - \frac{1}{4}z^{-1}}{1 - 16z^{-2}}$$

$$= \frac{\frac{49}{32}}{1 + 4z^{-1}} + \frac{\frac{47}{32}}{1 - 4z^{-1}}$$

Poles are at $z = -4$ and $z = 4$. Location of poles and ROC is shown in the figure below



ROC : $|z| > 4$ has a radius greater that the pole at $z = -4$ and $z = 4$, therefore both the terms of $X(z)$ corresponds to right sided sequences. Taking inverse z- transform we have

$$x[n] = \left[\frac{49}{32}(-4)^n + \frac{47}{32}4^n\right]u[n]$$

SOL 6.1.54 Option (C) is correct.

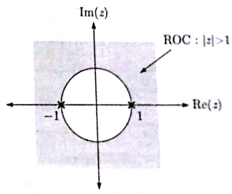
Using partial fraction expansion $X(z)$ can be simplified as

$$X(z) = \frac{2z^4 - 2z^3 - 2z^2}{z^2 - 1}$$

$$= \left[\frac{2 - 2z^{-1} - 2z^{-2}}{1 - z^{-2}}\right]z^2$$

$$= \left[2 + \frac{1}{1 + z^{-1}} + \frac{-1}{1 - z^{-1}}\right]z^2$$

Poles are at $z = -1$ and $z = 1$. Location of poles and ROC is shown in the following figure



ROC : $|z| > 1$ has radius grater than both the poles at $z = -1$ and $z = 1$, therefore both the terms in $X(z)$ corresponds to right sided sequences.

$$\frac{1}{1 + z^{-1}} \xleftrightarrow{\mathcal{Z}} (-1)^n u[n] \qquad \text{(right-sided)}$$

$$\frac{1}{1 - z^{-1}} \xleftrightarrow{\mathcal{Z}} u[n] \qquad \text{(right-sided)}$$

Now, using time shifting property the complete inverse z-transform of $X(z)$ is

$$x[n] = 2\delta[n+2] + ((-1)^n - 1)u[n+2]$$

SOL 6.1.55 Option (A) is correct.

We have, $X(z) = 1 + 2z^{-6} + 4z^{-8}, \quad |z| > 0$

Taking inverse z-transform we get

$$x[n] = \delta[n] + 2\delta[n-6] + 4\delta[n-8] \qquad z^{-n_0} \xleftrightarrow{\mathcal{Z}^{-1}} \delta[n-n_0]$$

SOL 6.1.56 Option (B) is correct.

Since $x[n]$ is right sided,

$$x[n] = \sum_{k=5}^{10} \frac{1}{k}\delta[n-k] \qquad z^{-n_0} \xleftrightarrow{\mathcal{Z}^{-1}} \delta[n-n_0]$$

SOL 6.1.57 Option (D) is correct.

We have , $X(z) = (1+z^{-1})^3 = 1 + 3z^{-1} + 3z^{-2} + z^{-3}, \quad |z| > 0$

Since $x[n]$ is right sided signal, taking inverse z-transform we have

$$x[n] = \delta[n] + 3\delta[n-1] + 3\delta[n-2] + \delta[n-3] \qquad z^{-n_0} \xleftrightarrow{\mathcal{Z}^{-1}} \delta[n-n_0]$$

SOL 6.1.58 Option (A) is correct.

We have, $X(z) = z^6 + z^2 + 3 + 2z^{-3} + z^{-4}, \quad |z| > 0$

Taking inverse z transform we get

$$x[n] = \delta[n+6] + \delta[n+2] + 3\delta[n] + 2\delta[n-3] + \delta[n-4]$$

SOL 6.1.59 Option (A) is correct.

$$X(z) = \frac{1}{1-\frac{1}{4}z^{-2}}, \quad |z| > \frac{1}{2}$$

The power series expansion of $X(z)$ with $|z| > \frac{1}{2}$ or $|\frac{1}{4}z^{-2}| < 1$ is written as

$$X(z) = 1 + \frac{z^{-2}}{4} + \left(\frac{z^{-2}}{4}\right)^2 + \left(\frac{z^{-2}}{4}\right)^3 + \dots\dots$$

$$= \sum_{k=0}^{\infty}\left(\frac{1}{4}z^{-2}\right)^k = \sum_{k=0}^{\infty}\left(\frac{1}{4}\right)^k z^{-2k}$$

Series converges for $|\frac{1}{4}z^{-2}| < 1$ or $|z| > \frac{1}{2}$. Taking inverse z-transform we get

$$x[n] = \sum_{k=0}^{\infty}\left(\frac{1}{4}\right)^k \delta[n-2k] \qquad z^{-2k} \xleftrightarrow{\mathcal{Z}^{-1}} \delta[n-2k]$$

$$= \begin{cases} \left(\frac{1}{4}\right)^{\frac{n}{2}}, & n \text{ even and } n \geq 0 \\ 0, & n \text{ odd} \end{cases}$$

$$= \begin{cases} 2^{-n}, & n \text{ even and } n \geq 0 \\ 0, & n \text{ odd} \end{cases}$$

SOL 6.1.60 Option (C) is correct.

$$X(z) = \frac{1}{1-\frac{1}{4}z^{-2}}, \quad |z| < \frac{1}{2}$$

Since ROC is left sided so power series expansion of $X(z)$ will have positive powers of z, we can simplify above expression for positive powers of z as

$$X(z) = \frac{-4z^2}{1-(2z)^2}, \quad |z| < \frac{1}{2}$$

The power series expansion of $X(z)$ with $|z| < \frac{1}{2}$ or $|4z^2| < 1$ is written as

$$X(z) = -4z^2[1 + (2z)^2 + (2z)^4 + (2z)^6 + \dots\dots]$$

$$X(z) = -4z^2\sum_{k=0}^{\infty}(2z)^{2k} = -\sum_{k=0}^{\infty}2^{2(k+1)}z^{2(k+1)}$$

Taking inverse z-transform, we get

$$x[n] = -\sum_{k=0}^{\infty}2^{2(k+1)}\delta[n+2(k+1)] \qquad z^{2(k+1)} \xleftrightarrow{\mathcal{Z}^{-1}} \delta[n+2(k+1)]$$

SOL 6.1.61 Option (A) is correct.

Using Taylor's series expansion for a right-sided signal, we have

$$\ln(1+\alpha) = \alpha - \frac{\alpha^2}{2} + \frac{\alpha^3}{3} - \frac{\alpha^4}{4} + \dots\dots = \sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k}(\alpha)^k$$

$$X(z) = \ln(1+z^{-1}) = \sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k}(z^{-1})^k$$

Taking inverse z-transform we get

$$x[n] = \sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k}\delta[n-k] \qquad z^{-k} \xleftrightarrow{\mathcal{Z}^{-1}} \delta[n-k]$$

SOL 6.1.62 Option (D) is correct.

From the given pole-zero pattern

$$X(z) = \frac{Az}{(z-\frac{1}{3})(z-2)}, \qquad A \to \text{Some constant}$$

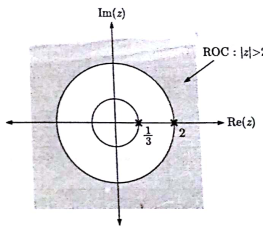
Using partial fraction expansion, we write

$$\frac{X(z)}{z} = \frac{\alpha}{z-\frac{1}{3}} + \frac{\beta}{z-2}, \quad \alpha \text{ and } \beta \text{ are constants.}$$

$$X(z) = \underbrace{\frac{\alpha}{(1-\frac{1}{3}z^{-1})}}_{\mathrm{I}} + \underbrace{\frac{\beta}{(1-2z^{-1})}}_{\mathrm{II}} \qquad \dots(1)$$

Poles are at $z = \frac{1}{3}$ and $z = 2$. We obtain the inverse z-transform using relationship between the location of poles and region of convergence as shown in following figures.

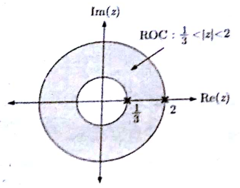
ROC : $|z| > 2$



ROC is exterior to the circle passing through right most pole so both the term in equation (1) corresponds to right sided sequences

$$x_1[n] = \alpha\left(\frac{1}{3}\right)^n u[n] + \beta(2)^n u[n]$$

ROC : $\frac{1}{3} < |z| < 2$



Since ROC has greater radius than the pole at $z = \frac{1}{3}$, so first term in equation (1) corresponds to right-sided sequence

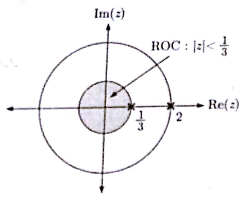
$$\frac{\alpha}{(1-\frac{1}{3}z^{-1})} \xleftrightarrow{z^{-1}} \alpha\left(\frac{1}{3}\right)^n u[n] \qquad \text{(right-sided)}$$

ROC $|z| < 2$ has radius less than the pole at $z = 2$, so the second term in equation (1) corresponds to left sided sequence.

$$\frac{\beta}{(1-2z^{-1})} \xleftrightarrow{z^{-1}} \beta(2)^n u[n-1] \qquad \text{(left-sided)}$$

So,
$$x_2[n] = \alpha\left(\frac{1}{3}\right)^n u[n] + \beta(2)^n u[n-1]$$

ROC : $|z| < \frac{1}{3}$



ROC is left side to both the poles of $X(z)$, so they corresponds to left sided signals.

$$x_3[n] = \alpha\left(\frac{1}{3}\right)^n u[-n-1] + \beta(2)^n u[-n-1]$$

All gives the same z-transform with different ROC so, all are the solution.

SOL 6.1.63 Option (C) is correct.

The z-transform of all the signal is same given as

$$X(z) = \frac{1}{1-2z^{-1}} - \frac{1}{1-\frac{1}{2}z^{-1}}$$

Poles are at $z = 2$ and $z = \frac{1}{2}$. Now consider the following relationship between ROC and location of poles.

1. Since $x_1[n]$ is right-sided signal, so ROC is region in z-plane having radius greater than the magnitude of largest pole. So, $|z| > 2$ and $|z| > \frac{1}{2}$ gives $R_1 : |z| > 2$
2. Since $x_2[n]$ is left-sided signal, so ROC is the region inside a circle having radius equal to magnitude of smallest pole. So, $|z| < 2$ and $|z| < \frac{1}{2}$ gives $R_2 : |z| < \frac{1}{2}$
3. Since $x_3[n]$ is double sided signal. So ROC is the region in z-plane such as $|z| > \frac{1}{2}$ and $|z| < 2$ which gives $R_3 : \frac{1}{2} < |z| < 2$

SOL 6.1.64 Option (B) is correct.

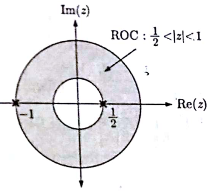
We have
$$X(z) = \frac{1+\frac{7}{6}z^{-1}}{(1-\frac{1}{2}z^{-1})(1+\frac{1}{3}z^{-1})} = \frac{2}{1-\frac{1}{2}z^{-1}} + \frac{-1}{1+\frac{1}{3}z^{-1}}$$

$X(z)$ has poles at $z = \frac{1}{2}$ and $z = -\frac{1}{3}$, we consider the different ROC's and location of poles to obtain the inverse z-transform.

1. ROC $|z| > \frac{1}{2}$ is exterior to the cicle which passes through outtermost pole, so both the terms in equation (1) contributes to right sided sequences.
$$x[n] = \frac{2}{2^n}u[n] - \left(\frac{-1}{3}\right)^n u[n]$$
2. ROC $|z| < \frac{1}{3}$ is interior to the circle passing through left most poles, so both the terms in equation (1) corresponds to left sided sequences.
$$x[n] = \left[\frac{-2}{2^n} + \left(\frac{-1}{3}\right)^n\right]u[-n-1]$$
3. ROC $\frac{1}{3} < |z| < \frac{1}{2}$ is interior to the circle passing through pole at $z = \frac{1}{2}$ so the first term in equation (1) corresponds to a right sided sequence, while the ROC is exterior to the circle passing through pole at $z = -\frac{1}{3}$, so the second term corresponds to a left sided sequence. Therefore, inverse z-transform is
$$x[n] = -\frac{2}{2^n}u[-n-1] - \left(\frac{-1}{3}\right)^n u[n]$$

SOL 6.1.65 Option (A) is correct.

The location of poles and the ROC is shown in the figure. Since the ROC includes the point $z = \frac{3}{4}$, ROC is $\frac{1}{2} < |z| < 1$



$$X(z) = \frac{A}{1-\frac{1}{2}z^{-1}} + \frac{B}{1+z^{-1}}$$

ROC is exterior to the pole at $z = \frac{1}{2}$, so this term corresponds to a right-sided sequence, while ROC is interior to the pole at $z = -1$ so the second term corresponds to a left sided sequence. Taking inverse z-transform we get

$$x[n] = \frac{A}{2^n}u[n] + B(-1)^n u[-n-1]$$

For $n = 1$, $\quad x[1] = \frac{A}{2}(1) + B \times 0 = 1 \Rightarrow \frac{A}{2} = 1 \Rightarrow A = 2$

For $n = -1$, $\quad x[-1] = A \times 0 + B(-1) = 1 \Rightarrow B = -1$

So, $\quad x[n] = \frac{1}{2^{n-1}}u[n] - (-1)^n u[-n-1]$

SOL 6.1.66 Option (B) is correct.

Let, $x[n] = Cp^n u[n]$ (right-sided sequence having a single pole)

$$x[0] = 2 = C$$

$$x[2] = \frac{1}{2} = 2p^2 \Rightarrow p = \frac{1}{2},$$

So, $x[n] = 2\left(\frac{1}{2}\right)^n u(n)$

SOL 6.1.67 Option (D) is correct.

$$X(z) = \sum_{n=0}^{\infty}\left(\frac{1}{2}z^{-1}\right)^n + \sum_{n=-\infty}^{-1}\left(\frac{1}{4}z^{-1}\right)^n$$

$$= \underbrace{\sum_{n=0}^{\infty}\left(\frac{1}{2}z^{-1}\right)^n}_{\text{I}} + \underbrace{\sum_{m=1}^{\infty}(4z)^m}_{\text{II}} = \frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-\frac{1}{4}z^{-1}}$$

ROC : Summation I converges if $\left|\frac{1}{2}z^{-1}\right| < 1$ or $|z| > \frac{1}{2}$ and summation II converges if $|4z| < 1$ or $|z| < \frac{1}{4}$. ROC would be intersection of both which does not exist.

SOL 6.1.68 Option (C) is correct.

$$x[n] \xleftrightarrow{Z} \frac{z^2}{z^2-16}, \quad \text{ROC } |z| < 4$$

$$x[n-2] \xleftrightarrow{Z} z^{-2}\left(\frac{z^2}{z^2-16}\right) = \frac{1}{z^2-16} \quad \text{(Time shifting property)}$$

SOL 6.1.69 Option (B) is correct.

$$x[n] \xleftrightarrow{Z} \frac{z^2}{z^2-16}, \quad \text{ROC } |z| < 4$$

$$\frac{1}{2^n}x[n] \xleftrightarrow{Z} \frac{(2z)^2}{(2z)^2-16} = \frac{z^2}{z^2-4} \quad \text{(Scaling in } z\text{-domain)}$$

SOL 6.1.70 Option (C) is correct.

$$x[n] \xleftrightarrow{Z} \frac{z^2}{z^2-16}, \quad \text{ROC } |z| < 4$$

$$x[-n] \xleftrightarrow{Z} \frac{\left(\frac{1}{z}\right)^2}{\left(\frac{1}{z}\right)^2-16} \quad \text{(Time reversal property)}$$

$$x[-n] * x[n] \xleftrightarrow{Z} \left[\frac{\left(\frac{1}{z}\right)^2}{\left(\frac{1}{z}\right)^2-16}\right]\left[\frac{z^2}{z^2-16}\right] \quad \text{(Time convolution property)}$$

$$\xleftrightarrow{Z} \frac{z^2}{257z^2-16z^4-16}$$

SOL 6.1.71 Option (A) is correct.

$$x[n] \xleftrightarrow{Z} \frac{z^2}{z^2-16}, \quad \text{ROC } |z| < 4$$

$$nx[n] \xleftrightarrow{Z} -z\frac{d}{dz}\frac{z^2}{z^2-16} \quad \text{(Differentiation in } z\text{-domain)}$$

$$\xleftrightarrow{Z} \frac{32z^2}{(z^2-16)^2}$$

SOL 6.1.72 Option (B) is correct.

$$x[n] \xleftrightarrow{Z} \frac{z^2}{z^2-16}, \quad \text{ROC } |z| < 4$$

$$x[n+1] \xleftrightarrow{Z} zX(z) \quad \text{(Time shifting)}$$

$$x[n-1] \xleftrightarrow{Z} z^{-1}X(z) \quad \text{(Time shifting)}$$

$$x[n+1] + x[n-1] \xleftrightarrow{Z} (z+z^{-1})X(z) \quad \text{(Linearity)}$$

$$\xleftrightarrow{Z} \frac{z(z^2)+z^{-1}(z^2)}{z^2-16} = \frac{z(z^2+1)}{z^2-16}$$

SOL 6.1.73 Option (D) is correct.

$$x[n] \xleftrightarrow{Z} \frac{z^2}{z^2-16}, \quad \text{ROC } |z| < 4$$

$$x[n-3] \xleftrightarrow{Z} z^{-3}\left(\frac{z^2}{z^2-16}\right) = \frac{z^{-1}}{z^2-16} \quad \text{(Time shifting property)}$$

$$x[n] * x[n-3] \xleftrightarrow{Z} \left(\frac{z^2}{z^2-16}\right)\left(\frac{z^{-1}}{z^2-16}\right) \quad \text{(Time convolution property)}$$

$$\xleftrightarrow{Z} \frac{z}{(z^2-16)^2}$$

SOL 6.1.74 Option (C) is correct.

We have, $X(z) \xleftrightarrow{Z^{-1}} 3^n n^2 u[n]$

$$X(2z) \xleftrightarrow{Z^{-1}} \frac{1}{2^n}\{3^n n^2 u[n]\} \quad \text{(Scaling in } z\text{-domain)}$$

SOL 6.1.75 Option (B) is correct.

$$X(z) \xleftrightarrow{Z^{-1}} 3^n n^2 u[n]$$

$$X\left(\frac{1}{z}\right) \xleftrightarrow{Z^{-1}} 3^{(-n)}(-n)^2 u[-n] \quad \text{(Time reversal)}$$

$$\xleftrightarrow{Z^{-1}} 3^{-n} n^2 u[-n]$$

SOL 6.1.76 Option (C) is correct.

$$X(z) \xleftrightarrow{Z^{-1}} 3^n n^2 u[n]$$

$$-z\frac{d}{dz}X(z) \xleftrightarrow{Z^{-1}} nx[n] \quad \text{(Differentiation in } z\text{-domain)}$$

$$z^{-1}\left[-z\frac{d}{dz}X(z)\right] \xleftrightarrow{Z^{-1}} (n-1)x[n-1] \quad \text{(Time shifting)}$$

So, $\frac{dX(z)}{dz} \xleftrightarrow{Z^{-1}} -(n-1)x[n-1] \qquad -z^{-1}\left[-z\frac{d}{dz}X(z)\right] = \frac{dX(z)}{dz}$

$$\xleftrightarrow{Z^{-1}} -(n-1)3^{n-1}(n-1)^2 u[n-1]$$

$$\xleftrightarrow{Z^{-1}} -(n-1)^3 3^{n-1} u[n-1]$$

SOL 6.1.77 Option (A) is correct.

$$\frac{1}{2}z^2 X(z) \xleftrightarrow{Z^{-1}} \frac{1}{2}x[n+2] \quad \text{(Time shifting)}$$

$$\frac{1}{2}z^{-2} X(z) \xleftrightarrow{Z^{-1}} \frac{1}{2}x[n-2] \quad \text{(Time shifting)}$$

$$\frac{z^2-z^{-2}}{2}X(z) \xleftrightarrow{Z^{-1}} \frac{1}{2}(x[n+2]-x[n-2]) \quad \text{(Linearity)}$$

SOL 6.1.78 Option (B) is correct.

$$X(z) \xleftrightarrow{Z^{-1}} 3^n n^2 u[n]$$

$$X(z)X(z) \xleftrightarrow{Z^{-1}} x[n] * x[n] \quad \text{(Time convolution)}$$

SOL 6.1.79 Option (A) is correct.

$$X(z) = 1 + \frac{z^{-1}}{4} - \frac{z^{-2}}{8}, \quad Y(z) = 1 - \frac{3z^{-1}}{4}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\frac{5}{3}}{1+\frac{1}{2}z^{-1}} + \frac{-\frac{2}{3}}{1-\frac{1}{4}z^{-1}}$$

For a causal system impulse response is obtained by taking right-sided inverse

z-transform of transfer function $H(z)$. Therefore,

$$h[n] = \frac{1}{3}\left[5\left(\frac{-1}{2}\right)^n - 2\left(\frac{1}{4}\right)^n\right]u[n]$$

SOL 6.1.80 Option (D) is correct.

We have $\quad x[n] = (-3)^n u[n]$

and $\quad y[n] = \left[4(2)^n - \left(\frac{1}{2}\right)^n\right]u[n]$

Taking z transform of above we get

$$X(z) = \frac{1}{1+3z^{-1}}$$

and $\quad Y(z) = \frac{4}{1-2z^{-1}} - \frac{1}{1-\frac{1}{2}z^{-1}} = \frac{3}{(1-2z^{-1})(1-\frac{1}{2}z^{-1})}$

Thus transfer function is

$$H(z) = \frac{Y(z)}{X(z)} = \frac{10}{1-2z^{-1}} + \frac{-7}{1-\frac{1}{2}z^{-1}}$$

For a causal system impulse response is obtained by taking right-sided inverse z-transform of transfer function $H(z)$. Therefore,

$$h[n] = \left[10(2)^n - 7\left(\frac{1}{2}\right)^n\right]u[n]$$

SOL 6.1.81 Option (D) is correct.

We have $\quad h[n] = (\frac{1}{2})^n u[n]$

and $\quad y[n] = 2\delta[n-4]$

Taking z-transform of above we get

$$H(z) = \frac{1}{1-\frac{1}{2}z^{-1}}$$

and $\quad Y(z) = 2z^{-4}$

Now $\quad X(z) = \frac{Y(z)}{H(z)} = 2z^{-4} - z^{-5}$

Taking inverse z-transform we have

$$x[n] = 2\delta[n-4] - \delta[n-5]$$

SOL 6.1.82 Option (C) is correct.

We have, $\quad y[n] = x[n] - x[n-2] + x[n-4] - x[n-6]$

Taking z-transform we get

$$Y(z) = X(z) - z^{-2}X(z) + z^{-4}X(z) - z^{-6}X(z)$$

or $\quad H(z) = \frac{Y(z)}{X(z)} = (1 - z^{-2} + z^{-4} - z^{-6})$

Taking inverse z-transform we have

$$h[n] = \delta[n] - \delta[n-2] + \delta[n-4] - \delta[n-6]$$

SOL 6.1.83 Option (A) is correct.

We have $\quad h[n] = \frac{3}{4}\left(\frac{1}{4}\right)^{n-1}u[n-1]$

Taking z-transform we get

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\frac{3}{4}z^{-1}}{1-\frac{1}{4}z^{-1}} \qquad \left(\frac{1}{4}\right)^{n-1}u[n-1] \xleftrightarrow{z} z^{-1}\left(\frac{1}{1-\frac{1}{4}z^{-1}}\right)$$

or, $\quad Y(z) - \frac{1}{4}z^{-1}Y(z) = \frac{3}{4}z^{-1}X(z)$

Taking inverse z-transform we have

$$y[n] - \frac{1}{4}y[n-1] = \frac{3}{4}x[n-1]$$

SOL 6.1.84 Option (A) is correct.

We have, $\quad h[n] = \delta[n] - \delta[n-5]$

Taking z-transform we get

$$H(z) = \frac{Y(z)}{X(z)} = 1 - z^{-5}$$

or $\quad Y(z) = X(z) - z^{-5}X(z)$

Taking inverse z-transform we get

$$y[n] = x[n] - x[n-5]$$

SOL 6.1.85 Option (A) is correct.

Taking z transform of all system we get

$$Y_1(z) = 0.2z^{-1}Y(z) + X(z) - 0.3z^{-1}X(z) + 0.02z^{-2}X(z)$$

$$H_1(z) = \frac{Y_1(z)}{X(z)} = \frac{1-0.3z^{-1}+0.02z^{-2}}{1-0.2z^{-1}}$$

$$= \frac{(1-0.2z^{-1})(1-0.1z^{-1})}{(1-0.2z^{-1})} = (1-0.1z^{-1})$$

$$Y_2(z) = X(z) - 0.1z^{-1}X(z)$$

$$H_2(z) = \frac{Y_2(z)}{X(z)} = (1-0.1z^{-1})$$

$$Y_3(z) = 0.5z^{-1}Y(z) + 0.4X(z) - 0.3z^{-1}X(z)$$

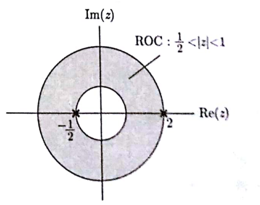
$$H_3(z) = \frac{Y_3(z)}{X(z)} = \frac{0.4-0.3z^{-1}}{1-0.5z^{-1}}$$

$H_1(z) = H_2(z)$, so y_1 and y_2 are equivalent.

SOL 6.1.86 Option (B) is correct.

We have $\quad H(z) = \frac{1}{1-2z^{-1}} + \frac{1}{1+\frac{1}{2}z^{-1}}$

Poles of $H(z)$ are at $z = 2$ and $z = -\frac{1}{2}$. Since $h[n]$ is stable, so ROC includes unit circle $|z| = 1$ and for the given function it must be $\frac{1}{2} < |z| < 2$. The location of poles and ROC is shown in the figure below



Consider the following two cases :

1. ROC is interior to the circle passing through pole at $z = 2$, so this term corresponds to a left-sided signal.

$$\frac{1}{1-2z^{-1}} \xleftrightarrow{z^{-1}} -(2)^n u[-n-1] \qquad \text{(left-sided)}$$

2. ROC is exterior to the circle passing through pole at $z = -\frac{1}{2}$, so this term corresponds to a right-sided signal.

$$\frac{1}{1+\frac{1}{2}z^{-1}} \xleftrightarrow{z^{-1}} \left(\frac{-1}{2}\right)^n u[n] \qquad \text{(right-sided)}$$

Impulse response,

$$h[n] = -(2)^n u[-n-1] + \left(\frac{-1}{2}\right)^n u[n]$$

SOL 6.1.87 Option (B) is correct.

We have
$$H(z) = \frac{5z^2}{z^2 - z - 6} = \frac{5z^2}{(z-3)(z+2)}$$
$$= \frac{5}{(1-3z^{-1})(1+2z^{-1})} = \frac{3}{1-3z^{-1}} + \frac{2}{1+2z^{-1}}$$

Since $h[n]$ is causal, therefore impulse response is obtained by taking right-sided inverse z-transform of the transfer function $X(z)$

$$h[n] = [3^{n+1} + 2(-2)^n] u[n]$$

SOL 6.1.88 Option (D) is correct.

Zero at : $z = 0, \frac{2}{3}$, poles at $z = \frac{1 \pm \sqrt{2}}{2}$

(1) For a causal system all the poles of transfer function lies inside the unit circle $|z| = 1$. But, for the given system one of the pole does not lie inside the unit circle, so the system is not causal and stable.

(2) Not all poles and zero are inside unit circle $|z| = 1$, the system is not minimum phase.

SOL 6.1.89 Option (A) is correct.

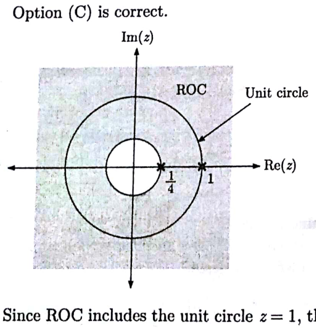
$$X(z) = \frac{-\frac{3}{8}}{1-\frac{1}{3}z^{-1}} + \frac{\frac{27}{8}}{1-3z^{-1}}$$

Poles are at $z = \frac{1}{3}$ and $z = 3$. Since $X(z)$ converges on $|z| = 1$, so ROC must include this circle. Thus for the given signal ROC : $\frac{1}{3} < |z| < 3$

ROC is exterior to the circle passing through the pole at $z = \frac{1}{3}$ so this term will have a right sided inverse z-transform. On the other hand ROC is interior to the circle passing through the pole at $z = 3$ so this term will have a left sided inverse z-transform.

$$x[n] = -\frac{1}{3^{n-1}8}u[n] - \frac{3^{n+3}}{8}u[-n-1]$$

SOL 6.1.90 Option (C) is correct.



Since ROC includes the unit circle $z = 1$, therefore the system is both stable and causal.

SOL 6.1.91 Option (C) is correct.

(1) Pole of system $z = -\frac{1}{2}, \frac{1}{3}$ lies inside the unit circle $|z| = 1$, so the system is causal and stable.

(2) Zero of system $H(z)$ is $z = -\frac{1}{2}$, therefore pole of the inverse system is at $z = -\frac{1}{2}$ which lies inside the unit circle, therefore the inverse system is also causal and stable.

SOL 6.1.92 Option (C) is correct.

Writing the equation from given block diagram we have

$$[2Y(z) + X(z)]z^{-2} = Y(z)$$

or
$$H(z) = \frac{z^{-2}}{1-2z^{-2}} = -\frac{1}{2} + \frac{\frac{1}{4}}{1-\sqrt{2}z^{-1}} + \frac{\frac{1}{4}}{1+\sqrt{2}z^{-1}}$$

Taking inverse laplace transform we have

$$h[n] = -\frac{1}{2}\delta[n] + \frac{1}{4}\{(\sqrt{2})^n + (-\sqrt{2})^n\}u[n]$$

SOL 6.1.93 Option (D) is correct.

$$Y(z) = X(z)z^{-1} - \{Y(z)z^{-1} + Y(z)z^{-2}\}$$
$$\frac{Y(z)}{X(z)} = \frac{z^{-1}}{1+z^{-1}+z^{-2}} = \frac{z}{z^2+z+1}$$

So this is a solution but not unique. Many other correct diagrams can be drawn.

****

SOLUTIONS 6.2

SOL 6.2.1 Correct answer is 2.

z-transform is given as

$$X(z) = \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n z^{-n} = \sum_{n=0}^{\infty}\left(\frac{1}{2z}\right)^n$$

$$= \frac{1}{1-\frac{1}{2z}} = \frac{2z}{2z-1} \qquad \text{...(1)}$$

From the given question, we have

$$X(z) = \frac{az}{az-1} \qquad \text{...(2)}$$

So, by comparing equations (1) and (2), we get $a = 2$

SOL 6.2.2 Correct answer is -1.125.

The z-transform of given sequence is

$$X(z) = z^3 + z^2 - z^1 - z^0$$

$$= z^3 + z^2 - z - 1$$

Now $\quad X\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) - 1 = -1.125$

SOL 6.2.3 Correct answer is 3.

$$X(z) = \frac{z+1}{z(z-1)} = -\frac{1}{z} + \frac{2}{z-1}$$

$$= -\frac{1}{z} + 2z^{-1}\left(\frac{z}{z-1}\right) \qquad \text{By partial fraction}$$

Taking inverse z-transform

$$x[n] = -\delta[n-1] + 2u[n-1]$$

$$x[0] = -0 + 0 = 0$$

$$x[1] = -1 + 2 = 1$$

$$x[2] = -0 + 2 = 2$$

Thus, we obtain

$$x[0] + x[1] + x[2] = 3$$

SOL 6.2.4 Correct answer is 3.

$$x[n] = \alpha^n u[n]$$

Let, $\quad y[n] = x[n+3]\,u[n] = \alpha^{n+3}u[n+3]\,u[n]$

$$= \alpha^{n+3}u[n] \qquad u[n+3]\,u[n] = u[n]$$

$$Y(z) = \sum_{n=-\infty}^{\infty} y[n]z^{-n} = \sum_{n=-\infty}^{\infty}\alpha^{n+3}z^{-n}u[n] = \sum_{n=0}^{\infty}\alpha^{n+3}z^{-n}$$

$$= \alpha^3\sum_{n=0}^{\infty}(\alpha z^{-1})^n = \alpha^3\frac{1}{1-\alpha z^{-1}} = \alpha^3\left(\frac{z}{z-\alpha}\right) \qquad \text{...(1)}$$

From the given question, we have

$$Y(z) = \alpha^k\left(\frac{z}{z-\alpha}\right) \qquad \text{...(2)}$$

So, by comparing equations (1) and (2), we get $k = 3$

NOTE :
Do not apply time shifting property directly because $x[n]$ is a causal signal.

SOL 6.2.5 Correct answer is 10.

We know that

$$\alpha^n u[n] \xleftrightarrow{Z} \frac{z}{z-\alpha}$$

$$\alpha^{n-10}u[n-10] \xleftrightarrow{Z} \frac{z^{-10}z}{z-\alpha} \qquad \text{(time shifting property)}$$

So, $\quad x[n] = \alpha^{n-10}u[n-10] \qquad \text{...(1)}$

From the given question, we have

$$x[n] = \alpha^{n-k}u[n-k] \qquad \text{...(2)}$$

So, by comparing equations (1) and (2), we get

$$k = 10$$

SOL 6.2.6 Correct answer is 9.

We know that $\quad a^n u[n] \xleftrightarrow{Z} \frac{z}{z-a}$

or $\quad 3^n u[n] \xleftrightarrow{Z} \frac{z}{z-3}$

$$3^{n-3}u[n-3] \xleftrightarrow{Z} z^{-3}\left(\frac{z}{z-3}\right)$$

So $\quad x[n] = 3^{n-3}u[n-3]$

$$x[5] = 3^2u[2] = 9$$

SOL 6.2.7 Correct answer is 2.

$$y[n] = n[n+1]\,u[n]$$

$$y[n] = n^2u[n] + nu[n]$$

We know that $\quad u[n] \xleftrightarrow{Z} \frac{z}{z-1}$

Applying the property of differentiation in z-domain

If, $\quad x[n] \xleftrightarrow{Z} X(z)$

then, $\quad nx[n] \xleftrightarrow{Z} -z\frac{d}{dz}X(z)$

so, $\quad nu[n] \xleftrightarrow{Z} -z\frac{d}{dz}\left(\frac{z}{z-1}\right)$

or, $\quad nu[n] \xleftrightarrow{Z} \frac{z}{(z-1)^2}$

Again by applying the above property

$$n(nu[n]) \xleftrightarrow{Z} -z\frac{d}{dz}\left[\frac{z}{(z-1)^2}\right]$$

$$n^2u[n] \xleftrightarrow{Z} \frac{z(z+1)}{(z-1)^3}$$

So $\quad Y(z) = \frac{z}{(z-1)^2} + \frac{z(z+1)}{(z-1)^3} = \frac{2z^2}{(z-1)^3} \qquad \text{...(1)}$

From the given question, we have

$$x[n] = \frac{kz^k}{(z-1)^{k+1}} \qquad \text{...(2)}$$

So, by comparing equations (1) and (2), we get

$$k = 2$$

SOL 6.2.8 Correct answer is -1.

Given that $\quad X(z) = \log(1-2z),\ |z| < \frac{1}{2}$

Differentiating

$$\frac{dX(z)}{dz} = \frac{-2}{1-2z} = \frac{z^{-1}}{1-\frac{1}{2}z^{-1}}$$

or,
$$\frac{zdX(z)}{dz} = \frac{1}{1-\frac{1}{2}z^{-1}}$$

From z-domain differentiation property
$$nx[n] \overset{z}{\longleftrightarrow} -z\frac{dX(z)}{dz}$$

so,
$$nx[n] \overset{z}{\longleftrightarrow} \frac{-1}{1-\frac{1}{2}z^{-1}}$$

From standard z-transform pair, we have
$$\left(\frac{1}{2}\right)^n u[-n-1] \overset{z}{\longleftrightarrow} \frac{-1}{1-\frac{1}{2}z^{-1}}$$

Thus
$$nx[n] = \left(\frac{1}{2}\right)^n u[-n-1]$$

or,
$$x[n] = \frac{1}{n}\left(\frac{1}{2}\right)^n u[-n-1] \qquad ...(1)$$

From the given question, we have
$$x[n] = \frac{1}{n}\left(\frac{1}{2}\right)^n u[a-n] \qquad ...(2)$$

So, by comparing equations (1) and (2), we get $a = -1$

SOL 6.2.9 Correct answer is 2.

$$Y(z) = \sum_{n=-\infty}^{\infty} x\left[\frac{n}{2}\right]z^{-n}$$
$$= \sum_{k=-\infty}^{\infty} x[k]\,z^{-2k} \qquad \text{Put } \frac{n}{2} = k \text{ or } n = 2k$$
$$= X(z^2) \qquad ...(1)$$

From the given question, we have
$$Y(z) = X(z^k) \qquad ...(2)$$

So, by comparing equations (1) and (2), we get $k = 2$

SOL 6.2.10 Correct answer is 0.

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\,z^{-n}$$
$$Y(z) = X(z^3) = \sum_{n=-\infty}^{\infty} x[n]\,(z^3)^{-n} = \sum_{n=-\infty}^{\infty} x[n]\,z^{-3n}$$
$$= \sum_{k=-\infty}^{\infty} x[k/3]\,z^{-k} \qquad \text{Put } 3n = k \text{ or } n = k/3$$

Thus
$$y[n] = x[n/3]$$
$$y[n] = \begin{cases} (-0.5)^{n/3}, & n = 0, 3, 6, ... \\ 0, & \text{otherwise} \end{cases}$$

Thus
$$y[4] = 0$$

SOL 6.2.11 Correct answer is -6.

By taking z-transform of $x[n]$ and $h[n]$
$$H(z) = 1 + 2z^{-1} - z^{-3} + z^{-4}$$
$$X(z) = 1 + 3z^{-1} - z^{-2} - 2z^{-3}$$

From the convolution property of z-transform
$$Y(z) = H(z)\,X(z)$$
$$Y(z) = 1 + 5z^{-1} + 5z^{-2} - 5z^{-3} - 6z^{-4} + 4z^{-5} + z^{-6} - 2z^{-7}$$

Sequence is
$$y[n] = \{1, 5, 5, -5, -6, 4, 1, -2\}$$

$$y[4] = -6$$

SOL 6.2.12 Correct answer is 0.5 .

$x[n]$ can be written as
$$x[n] = \frac{1}{2}[u[n] + (-1)^n u[n]]$$

z-transform of $x[n]$
$$X(z) = \frac{1}{2}\left[\frac{1}{1-z^{-1}} + \frac{1}{1+z^{-1}}\right]$$

From final value theorem
$$x(\infty) = \lim_{z \to 1}(z-1)\,X(z)$$
$$= \frac{1}{2}\lim_{z \to 1}(z-1)\left[\frac{z}{z-1} + \frac{z}{z+1}\right]$$
$$= \frac{1}{2}\lim_{z \to 1}\left[z + \frac{z(z-1)}{(z+1)}\right]$$
$$= \frac{1}{2}(1) = 0.5$$

SOL 6.2.13 Correct answer is 0.5 .

From initial value theorem
$$x[0] = \lim_{z \to \infty} X(z)$$
$$= \lim_{z \to \infty} \frac{0.5z^2}{(z-1)(z-0.5)}$$
$$= \lim_{z \to \infty} \frac{0.5}{\left(1-\frac{1}{z}\right)\left(1-\frac{0.5}{z}\right)} = 0.5$$

SOL 6.2.14 Correct answer is -2.5 .

Taking z transform of input and output
$$X(z) = \frac{z}{z-0.5}$$
$$Y(z) = 1 - 2z^{-1} = \frac{z-2}{z}$$

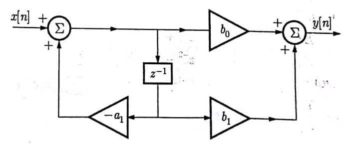
Transfer function of the filter
$$H(z) = Y(z)/X(z)$$
$$= \left(\frac{z-2}{z}\right)\left(\frac{z-0.5}{z}\right) = \frac{z^2 - 2.5z + 1}{z^2}$$
$$= 1 - 2.5z^{-1} + z^{-2}$$

Taking inverse z-transform
$$h[n] = \{\underset{\uparrow}{1}, -2.5, 1\}$$

Therefore
$$h[1] = -2.5$$

SOL 6.2.15 Correct answer is 4.

Comparing the given system realization with the generic first order direct form II realization



Difference equation for above realization is

$$y[n] + a_1 y[n-1] = b_0 x[n] + b_1 x[n-1]$$

Here $a_1 = -2, b_1 = 3, b_0 = 4$

So $\quad y[n] - 2y[n-1] = 4x[n] + 3x[n-1]$

Taking z-transform on both sides

$$Y(z) - 2z^{-1}Y(z) = 4X(z) + 3z^{-1}X(z)$$

Transfer function

$$H(z) = \frac{Y(z)}{X(z)} = \frac{4+3z^{-1}}{1-2z^{-1}} = \frac{4z+3}{z-2} \qquad ...(1)$$

From the given question, we have

$$H(z) = \frac{Y(z)}{X(z)} = \frac{k(z+1)-1}{z-2} \qquad ...(2)$$

So, by comparing equations (1) and (2), we get

$$k = 4$$

SOL 6.2.16 Correct answer is 0.3333 .

The z-transform of each system response

$$H_1(z) = 1 + \frac{1}{2}z^{-1}, \quad H_2(z) = \frac{z}{z-\frac{1}{2}}$$

The overall system function

$$H(z) = H_1(z)H_2(z)$$

$$= \left(1+\frac{1}{2}z^{-1}\right)\left(\frac{z}{z-\frac{1}{2}}\right) = \frac{(z+\frac{1}{2})}{(z-\frac{1}{2})}$$

Input, $\quad x[n] = \cos(n\pi)$

So, $z = -1$ and $H(z=-1) = \frac{-1+\frac{1}{2}}{-1-\frac{1}{2}} = \frac{1}{3}$

Output of system

$$y[n] = H(z=-1)x[n] = \frac{1}{3}\cos n\pi \qquad ...(1)$$

From the given question, we have

$$y[n] = k\cos n\pi \qquad ...(2)$$

So, by comparing equations (1) and (2), we get

$$k = \frac{1}{3} = 0.3333$$

SOL 6.2.17 Correct answer is 1.

From the given block diagram

$$Y(z) = \alpha z^{-1}X(z) + \alpha z^{-1}Y(z)$$

$$Y(z)(1-\alpha z^{-1}) = \alpha z^{-1}X(z)$$

Transfer function

$$\frac{Y(z)}{X(z)} = \frac{\alpha z^{-1}}{1-\alpha z^{-1}}$$

For stability poles at $z = 1$ must be inside the unit circle.

So $\quad |\alpha| < 1$

SOL 6.2.18 Correct answer is -2.

$$X^+(z) = \sum_{n=0}^{\infty} x[n]z^{-n} = \sum_{n=0}^{\infty}\delta[n-2]z^{-n} = z^{-2} \qquad ...(1)$$

$$\sum_{n=-\infty}^{\infty} f[n]\delta[n-n_0] = f[n_0]$$

From the given question, we have

$$X^+(z) = z^{-2} \qquad ...(2)$$

So, by comparing equations (1) and (2), we get

$$k = -2$$

SOL 6.2.19 Correct answer is -1.

$$X^+(z) = \sum_{n=0}^{\infty} x[n]z^{-n} = \sum_{n=0}^{\infty} z^{-n} = \frac{1}{1-z^{-1}} \qquad ...(1)$$

From the given question, we have

$$X^+(z) = \frac{1}{1+a/z} \qquad ...(2)$$

So, by comparing equations (1) and (2), we get

$$k = -1$$

SOL 6.2.20 Correct answer is 0.0417 .

We know that,

$$\cos\alpha = 1 - \frac{\alpha^2}{2!} + \frac{\alpha^4}{4!} - \frac{\alpha^6}{6!} + \frac{\alpha^8}{8!} - ....$$

$$= \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!}\alpha^{2k}$$

Thus, $\quad X(z) = \cos(z^{-3}) = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!}(z^{-3})^{2k}, \quad |z| > 0$

Taking inverse z-transform we get

$$x[n] = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!}\delta[n-6k]$$

Now for $n = 12$ we get, $12 - 6k = 0 \Rightarrow k = 2$

Thus, $\quad x[12] = \frac{(-1)^2}{4!} = \frac{1}{24} = 0.0417$

SOL 6.2.21 Correct answer is 4.

For anticausal signal initial value theorem is given as,

$$x[0] = \lim_{z\to\infty} X(z) = \lim_{z\to\infty}\frac{12-21z}{3-7z+12z^2} = \frac{12}{3} = 4$$

SOL 6.2.22 Correct answer is 1.12 .

Taking z-transform on both sides

$$Y(z) = cz^{-1}Y(z) - 0.12z^{-2}Y(z) + z^{-1}X(z) + z^{-2}X(z)$$

Transfer function,

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1}+z^{-2}}{1-cz^{-1}+0.12z^{-2}} = \frac{z+1}{z^2-cz+0.12}$$

Poles of the system are $z = \frac{c \pm \sqrt{c^2-0.48}}{2}$.

For stability poles should lie inside the unit circle, so $|z| < 1$

$$\left|\frac{c \pm \sqrt{c^2-0.48}}{2}\right| < 1$$

Solving this inequality, we get $|c| < 1.12$.

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SOLUTIONS 6.3

Answers						
1. (B)	5. (C)	9. (A)	13. (D)	17. (C)	21. (C)	25. (B)
2. (D)	6. (C)	10. (C)	14. (C)	18. (A)	22. (D)	26. (B)
3. (C)	7. (D)	11. (C)	15. (B)	19. (A)	23. (C)	
4. (D)	8. (A)	12. (A)	16. (D)	20. (B)	24. (C)	

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SOLUTIONS 6.4

SOL 6.4.1 Option (C) is correct.

z-transform of $x[n]$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = \sum_{n=-\infty}^{\infty} \alpha^n u[n] z^{-n}$$

$$= \sum_{n=0}^{\infty} (\alpha z^{-1})^n = \frac{1}{1-\alpha z^{-1}} = \frac{z}{z-\alpha}$$

SOL 6.4.2 Option (B) is correct.

We have $$x[n] = \sum_{k=0}^{\infty} \delta[n-k]$$

$$X(z) = \sum_{k=0}^{\infty} x[n] z^{-n} = \sum_{n=-\infty}^{\infty}\left[\sum_{k=0}^{\infty} \delta[n-k] z^{-n}\right]$$

Since $\delta[n-k]$ defined only for $n = k$ so

$$X(z) = \sum_{k=0}^{\infty} z^{-k} = \frac{1}{(1-1/z)} = \frac{z}{(z-1)}$$

SOL 6.4.3 Option (A) is correct.

We have $$f(nT) = a^{nT}$$

Taking z-transform we get

$$F(z) = \sum_{n=-\infty}^{\infty} a^{nT} z^{-n} = \sum_{n=-\infty}^{\infty} (a^T)^n z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{a^T}{z}\right)^n = \frac{z}{z-a^T}$$

SOL 6.4.4 Option ( ) is correct.

SOL 6.4.5 Option (A) is correct.

$$x[n] = b^n u[n] + b^{-n} u[-n-1]$$

z-transform of $x[n]$ is given as

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} b^n u[n] z^{-n} + \sum_{n=-\infty}^{\infty} b^{-n} u[-n-1] z^{-n}$$

$$= \sum_{n=0}^{\infty} b^n z^{-n} + \sum_{n=-\infty}^{-1} b^{-n} z^{-n}$$

In second summation, Let $n = -m$

$$X(z) = \sum_{n=0}^{\infty} b^n z^{-n} + \sum_{m=1}^{\infty} b^m z^m$$

$$= \underbrace{\sum_{n=0}^{\infty} (bz^{-1})^n}_{\text{I}} + \underbrace{\sum_{m=1}^{\infty} (bz)^m}_{\text{II}}$$

Summation I converges, if $|bz^{-1}| < 1$ or $|z| > |b|$

Summation II converges, if $|bz| < 1$ or $|z| < \frac{1}{|b|}$

since $|b| < 1$ so from the above two conditions ROC : $|z| < 1$.

SOL 6.4.6 Option (B) is correct.

z-transform of signal $a^n u[n]$ is

$$X(z) = \sum_{n=-\infty}^{\infty} a^n u[n] z^{-n} = \underbrace{\sum_{n=0}^{\infty} (az^{-1})^n}_{\mathrm{I}} \qquad u[n] = 1,\ n \geq 0$$

$$= \frac{1}{1 - az^{-1}} = \frac{z}{z-a}$$

Similarly, z-transform of signal $a^n u[-n-1]$ is

$$X(z) = \sum_{n=-\infty}^{\infty} - a^n u[-n-1] z^{-n}$$

$$= -\sum_{n=-\infty}^{-1} a^n z^{-n} \qquad \because u[-n-1] = 1,\ n \leq -1$$

Let $n = -m$, then

$$X(z) = -\sum_{m=1}^{\infty} a^{-m} z^m = -\underbrace{\sum_{m=1}^{\infty} (a^{-1}z)^m}_{\mathrm{II}}$$

$$= -\frac{a^{-1}z}{1 - a^{-1}z} = \frac{z}{z-a}$$

z-transform of both the signal is same.
(A) is true
ROC : To obtain ROC we find the condition for convergences of $X(z)$ for both the transform.
Summation I converges if $|a^{-1}z| < 1$ or $|z| > a$, so ROC for $a^n u[n]$ is $|z| > |a|$
Summation II converges if $|a^{-1}z| < 1$ or $|z| < |a|$, so ROC for $-a^n u[-n-1]$ is $|z| < |a|$.
(R) is true, but (R) is NOT the correct explanation of (A).

SOL 6.4.7 Option (B) is correct.
z- transform of $x[n]$ is defined as

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} x[n] r^{-n} e^{-j\Omega n} \qquad \text{Putting } z = re^{j\Omega}$$

z-transform exists if $|X(z)| < \infty$

$$\sum_{n=-\infty}^{\infty} |x[n] r^{-n} e^{-j\Omega n}| < \infty$$

or

$$\sum_{n=-\infty}^{\infty} |x[n] r^{-n}| < \infty$$

Thus, z-transform exists if $x[n] r^{-n}$ is absolutely summable.

SOL 6.4.8 Option (A) is correct.

$$x[n] = \left(\frac{1}{3}\right)^n u[n] - \left(\frac{1}{2}\right)^n u[-n-1]$$

Taking z transform we have

$$X(z) = \sum_{n=0}^{n=\infty} \left(\frac{1}{3}\right)^n z^{-n} - \sum_{n=-\infty}^{n=-1} \left(\frac{1}{2}\right)^n z^{-n}$$

$$= \sum_{n=0}^{n=\infty} \left(\frac{1}{3} z^{-1}\right)^n - \sum_{n=-\infty}^{n=-1} \left(\frac{1}{2} z^{-1}\right)^n$$

First term gives

$$\frac{1}{3} z^{-1} < 1 \rightarrow \frac{1}{3} < |z|$$

Second term gives

$$\frac{1}{2} z^{-1} > 1 \rightarrow \frac{1}{2} > |z|$$

Thus its ROC is the common ROC of both terms. that is

$$\frac{1}{3} < |z| < \frac{1}{2}$$

SOL 6.4.9 Option (C) is correct.
Here

$$x_1[n] = \left(\frac{5}{6}\right)^n u[n]$$

$$X_1(z) = \frac{1}{1 - (\frac{5}{6} z^{-1})} \qquad \text{ROC} : R_1 \rightarrow |z| > \frac{5}{6}$$

$$x_2[n] = -\left(\frac{6}{5}\right)^n u[-n-1]$$

$$X_1(z) = 1 - \frac{1}{1 - (\frac{6}{5} z^{-1})} \qquad \text{ROC} : R_2 \rightarrow |z| < \frac{6}{5}$$

Thus ROC of $x_1[n] + x_2[n]$ is $R_1 \cap R_2$ which is $\frac{5}{6} < |z| < \frac{6}{5}$

SOL 6.4.10 Option (A) is correct.

$$x[n] = 2^n u[n]$$

z-transform of $x[n]$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = \sum_{n=-\infty}^{\infty} 2^n u[n] z^{-n}$$

$$= \sum_{n=0}^{\infty} (2z^{-1})^n = 1 + 2z^{-1} + (2z^{-1})^2 + \dots$$

$$= \frac{1}{1 - 2z^{-1}}$$

the above series converges if $|2z^{-1}| < 1$ or $|z| > 2$

SOL 6.4.11 Option (A) is correct.
We have

$$h[n] = u[n]$$

$$H(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = \sum_{n=0}^{\infty} 1 z^{-n} = \sum_{n=0}^{\infty} (z^{-1})^n$$

$H(z)$ is convergent if

$$\sum_{n=0}^{\infty} (z^{-1})^n < \infty$$

and this is possible when $|z^{-1}| < 1$. Thus ROC is $|z^{-1}| < 1$ or $|z| > 1$

SOL 6.4.12 Option (B) is correct.
(Please refer to table 6.1 of the book **Gate Guide signals & Systems** by same authors)

(A) $\quad u[n] \xleftrightarrow{z} \dfrac{z}{z-1}$ (A → 3)

(B) $\quad \delta[n] \xleftrightarrow{z} 1$ (B → 1)

(C) $\quad \sin \omega t\big|_{t=nT} \xleftrightarrow{z} \dfrac{z \sin \omega T}{z^2 - 2z\cos \omega T + 1}$ (C → 4)

(D) $\quad \cos \omega t\big|_{t=nT} \xleftrightarrow{z} \dfrac{z - \cos \omega T}{z^2 - 2z\cos \omega t + 1}$ (D → 2)

SOL 6.4.13 Option (A) is correct.
Inverse z-transform of $X(z)$ is given as

$$x[n] = \frac{1}{2\pi j} \oint X(z) z^{n-1} dz$$

SOL 6.4.14 Option (B) is correct.

$$H(z) = z^{-m}$$

so

$$h[n] = \delta[n-m]$$

SOL 6.4.15 Option (C) is correct.

We know that

$$\alpha^n u[n] \xleftrightarrow{z} \frac{1}{1-\alpha z^{-1}}$$

For $\alpha = 1$, $$u[n] \xleftrightarrow{z} \frac{1}{1-z^{-1}}$$

SOL 6.4.16 Option (B) is correct.

$$X(z) = \frac{0.5}{1-2z^{-1}}$$

Since ROC includes unit circle, it is left handed system

$$x[n] = -(0.5)(2)^{-n} u[-n-1]$$

$$x(0) = 0$$

If we apply initial value theorem

$$x(0) = \lim_{z \to \infty} X(z) = \lim_{z \to \infty} \frac{0.5}{1-2z^{-1}} = 0.5$$

That is wrong because here initial value theorem is not applicable because signal $x[n]$ is defined for $n < 0$.

SOL 6.4.17 Option (B) is correct.

z-transform $$F(z) = \frac{1}{z+1} = 1 - \frac{z}{z+1} = 1 - \frac{1}{1+z^{-1}}$$

so, $$f(k) = \delta(k) - (-1)^k$$

Thus $$(-1)^k \xleftrightarrow{z} \frac{1}{1+z^{-1}}$$

SOL 6.4.18 Option (C) is correct.

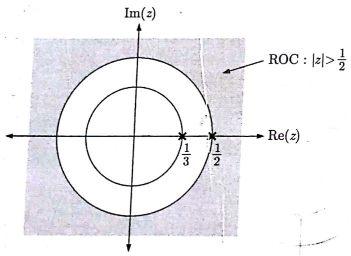
$$X(z) = \frac{z(2z-\frac{5}{6})}{(z-\frac{1}{2})(z-\frac{1}{3})}$$

$$\frac{X(z)}{z} = \frac{(2z-\frac{5}{6})}{(z-\frac{1}{2})(z-\frac{1}{3})} = \frac{1}{(z-\frac{1}{2})} + \frac{1}{(z-\frac{1}{3})}$$

or $$X(z) = \underbrace{\frac{z}{(z-\frac{1}{2})}}_{\text{term I}} + \underbrace{\frac{z}{(z-\frac{1}{3})}}_{\text{term II}} \qquad ...(1)$$

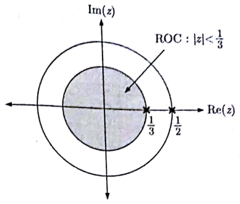
Poles of $X(z)$ are at $z = \frac{1}{2}$ and $z = \frac{1}{3}$

ROC : $|z| > \frac{1}{2}$ Since ROC is outside to the outer most pole so both the terms in equation (1) corresponds to right sided sequence.



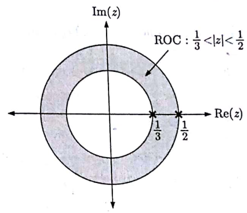
So, $$x[n] = \left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{3}\right)^n u[n] \qquad (A \to 4)$$

ROC : $|z| < \frac{1}{3}$:Since ROC is inside to the innermost pole so both the terms in equation (1) corresponds to left sided signals.



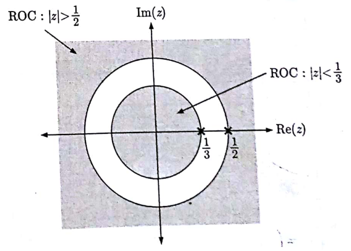
So, $$x[n] = -\left(\frac{1}{2}\right)^n u[-n-1] - \left(\frac{1}{3}\right)^n u[-n-1] \qquad (D \to 2)$$

ROC : $\frac{1}{3} < |z| < \frac{1}{2}$: ROC is outside to the pole $z = \frac{1}{3}$, so the second term of equation (1) corresponds to a causal signal. ROC is inside to the pole at $z = \frac{1}{2}$, so First term of equation (1) corresponds to anticausal signal.



So, $$x[n] = -\left(\frac{1}{2}\right)^n u[-n-1] + \left(\frac{1}{3}\right)^n u[n] \qquad (C \to 1)$$

ROC : $|z| < \frac{1}{3}$ & $|z| > \frac{1}{2}$: ROC : $|z| < \frac{1}{3}$ is inside the pole at $z = \frac{1}{3}$ so second term of equation (1) corresponds to anticausal signal. On the other hand, ROC : $|z| > \frac{1}{2}$ is outside to the pole at $z = \frac{1}{2}$, so the first term in equation (1) corresponds to a causal signal.



So, $$x[n] = \left(\frac{1}{2}\right)^n u[n] - \left(\frac{1}{3}\right)^n u[-n-1] \qquad (B \to 3)$$

SOL 6.4.19 Option (A) is correct.

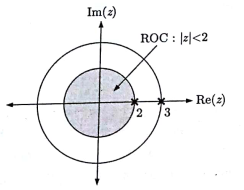
Given, $$X(z) = \frac{z}{(z-2)(z-3)},\ |z| < 2$$

$$\frac{X(z)}{z} = \frac{1}{(z-2)(z-3)} = \frac{1}{z-3} - \frac{1}{z-2} \quad \text{By partial fraction}$$

or, $$X(z) = \frac{z}{z-3} - \frac{z}{z-2} \qquad ...(1)$$

Poles of $X(z)$ are $z = 2$ and $z = 3$

ROC : $|z| < 2$



Since ROC is inside the innermost pole of $X(z)$, both the terms in equation (1) corresponds to anticausal signals.

$$x[n] = -3^n u[-n-1] + 2^n u[-n-n] = (2^n - 3^n)\, u[-n-1]$$

SOL 6.4.20 Option (D) is correct.

Given that $$X(z) = \frac{z}{(z-a)^2},\ |z| > a$$

Residue of $X(z)\, z^{n-1}$ at $z = a$ is

$$= \frac{d}{dz}(z-a)^2 X(z)\, z^{n-1}\Big|_{z=a}$$

$$= \frac{d}{dz}(z-a)^2 \frac{z}{(z-a)^2} z^{n-1}\Big|_{z=a}$$

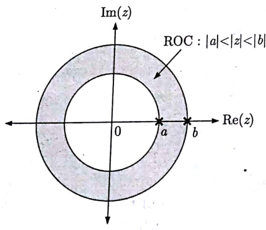
$$= \frac{d}{dz} z^n\Big|_{z=a} = n z^{n-1}\Big|_{z=a} = n a^{n-1}$$

SOL 6.4.21 Option (C) is correct.

$$X(z) = \frac{\frac{1}{2}}{1 - az^{-1}} + \frac{\frac{1}{3}}{1 - bz^{-1}}, \qquad \text{ROC} : |a| < |z| < |b|$$

Poles of the system are $z = a,\ z = b$

ROC : $|a| < |z| < |b|$



Since ROC is outside to the pole at $z = a$, therefore the first term in $X(z)$ corresponds to a causal signal.

$$\frac{\frac{1}{2}}{1 - az^{-1}} \xleftrightarrow{Z^{-1}} \frac{1}{2}(a)^n u[n]$$

ROC is inside to the pole at $z = b$, so the second term in $X(z)$ corresponds to a anticausal signal.

$$\frac{\frac{1}{3}}{1 - bz^{-1}} \xleftrightarrow{Z^{-1}} -\frac{1}{3}(b)^n u[-n-1]$$

$$x[n] = \frac{1}{2}(a)^n u[n] - \frac{1}{3}(b)^n u[-n-1]$$

$$x[0] = \frac{1}{2}u[0] - \frac{1}{3}u[-1] = \frac{1}{2}$$

SOL 6.4.22 Option (A) is correct.

$$X(z) = \frac{(z + z^{-3})}{(z + z^{-1})} = \frac{z(1 + z^{-4})}{z(1 + z^{-2})}$$
$$= (1 + z^{-4})(1 + z^{-2})^{-1}$$

Writing binomial expansion of $(1 + z^{-2})^{-1}$, we have

$$X(z) = (1 + z^{-4})(1 - z^{-2} + z^{-4} - z^{-6} + ...)$$
$$= 1 - z^{-2} + 2z^{-4} - 2z^{-6} + ...$$

For a sequence $x[n]$, its z-transform is

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}$$

Comparing above two

$$x[n] = \delta[n] - \delta[n-2] + 2\delta[n-4] - 2\delta[n-6] + ...$$
$$= \{1, 0, -1, 0, 2, 0, -2, ...\}$$

$x[n]$ has alternate zeros.

SOL 6.4.23 Option (A) is correct.

We know that $\alpha Z^{\pm a} \xleftrightarrow{Z^{-1}} \alpha\delta[n \pm a]$

Given that $X(z) = 5z^2 + 4z^{-1} + 3$

Inverse z-transform $x[n] = 5\delta[n+2] + 4\delta[n-1] + 3\delta[n]$

SOL 6.4.24 Option (A) is correct.

$$X(z) = e^{1/z}$$

$$X(z) = e^{1/z} = 1 + \frac{1}{z} + \frac{1}{2!\, z^2} + \frac{1}{3!\, z^3} + ...$$

z-transform of $x[n]$ is given by

$$X(z) = \sum_{n=0}^{\infty} x[n]\, z^{-n} = x[0] + \frac{x[1]}{z} + \frac{x[2]}{z^2} + \frac{x[3]}{z^3} + ...$$

Comparing above two

$$\{x[0], x[1], x[2], x[3] ....\} = \left\{1, \frac{1}{1!}, \frac{1}{2!}, \frac{1}{3!}, ...\right\}$$

$$x[n] = \frac{1}{n!} u[n]$$

SOL 6.4.25 Option (D) is correct.

The ROC of addition or subtraction of two functions $x_1[n]$ and $x_2[n]$ is $R_1 \cap R_2$. We have been given ROC of addition of two function and has been asked ROC of subtraction of two function. It will be same.

SOL 6.4.26 Option (D) is correct.

(A)
$$x[n] = \alpha^n u[n]$$
$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = \sum_{n=0}^{\infty} \alpha^n z^{-n} \qquad u[n] = 1,\ n \geq 0$$
$$= \sum_{n=0}^{\infty} (\alpha z^{-1})^n$$
$$= \frac{1}{1-\alpha z^{-1}}, \quad |\alpha z^{-1}| < 1 \text{ or } |z| > |\alpha| \qquad (A \rightarrow 2)$$

(B)
$$x[n] = -\alpha^n u[-n-1]$$
$$X(z) = -\sum_{n=-\infty}^{\infty} \alpha^n u[-n-1] z^{-n}$$
$$= -\sum_{n=-\infty}^{-1} \alpha^n z^{-n} \qquad u[-n-1] = 1,\ n \leq -1$$
Let $n = -m$,
$$X(z) = -\sum_{m=1}^{\infty} \alpha^{-m} z^m = -\sum_{m=1}^{\infty} (\alpha^{-1} z)^m$$
$$= \frac{-\alpha^{-1} z}{1-\alpha^{-1} z}, \quad |\alpha^{-1} z| < 1 \text{ or } |z| < |\alpha|$$
$$= \frac{1}{(1-\alpha z^{-1})}, \quad |z| < |\alpha| \qquad (B \rightarrow 3)$$

(C)
$$x[n] = -n\alpha^n u[-n-1]$$
We have,
$$-\alpha^n u[-n-1] \xleftrightarrow{z} \frac{1}{(1-\alpha z^{-1})}, \quad |z| < |\alpha|$$
From the property of differentiation in z-domain
$$-n\alpha^n u[-n-1] \xleftrightarrow{z} -z\frac{d}{dz}\left[\frac{1}{1-\alpha z^{-1}}\right], \quad |z| < |\alpha|$$
$$\xleftrightarrow{z} \frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}, \quad |z| < |\alpha| \qquad (C \rightarrow 4)$$

(D)
$$x[n] = n\alpha^n u[n]$$
We have,
$$\alpha^n u[n] \xleftrightarrow{z} \frac{1}{(1-\alpha z^{-1})}, \quad |z| > |\alpha|$$
From the property of differentiation in z-domain
$$n\alpha^n u[n] \xleftrightarrow{z} -z\frac{d}{dz}\left[\frac{1}{(1-\alpha z^{-1})}\right], \quad |z| > |\alpha|$$
$$\xleftrightarrow{z} \frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}, \quad |z| > |\alpha| \qquad (D \rightarrow 1)$$

SOL 6.4.27 Given that, z transform of $x[n]$ is
$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$
z-transform of $\{x[n] e^{j\omega_0 n}\}$
$$Y(z) = \sum_{n=-\infty}^{\infty} x[n] e^{j\omega_0 n} z^{-n} = \sum_{n=-\infty}^{\infty} x[n] (ze^{-j\omega_0})^{-n} = X(z')\big|_{z' = ze^{-j\omega_0}}$$
so,
$$Y(z) = X(ze^{-j\omega_0})$$

SOL 6.4.28 Option (A) is correct.

SOL 6.4.29 Option (C) is correct.

We know that,
$$a^n u[n] \xleftrightarrow{z} \frac{z}{z-a} \qquad (A \rightarrow 3)$$
From time shifting property
$$a^{n-2} u[n-2] \xleftrightarrow{z} z^{-2}\frac{z}{z-a} \qquad (B \rightarrow 4)$$
From the property of scaling in z-domain

If,
$$x[n] \xleftrightarrow{z} X(z)$$
then,
$$\alpha^n x[n] \xleftrightarrow{z} X\left(\frac{z}{\alpha}\right)$$
so
$$(e^j)^n a^n \xleftrightarrow{z} \frac{\left(\frac{z}{e^j}\right)}{\left(\frac{z}{e^j} - a\right)} = \frac{ze^{-j}}{ze^{-j} - a} \qquad (C \rightarrow 2)$$
From the property of differentiation in z-domain

If,
$$a^n u[n] \xleftrightarrow{z} \frac{z}{z-a}$$
then,
$$na^n u[n] \xleftrightarrow{z} \frac{d}{dz}\left(\frac{z}{z-a}\right) = \frac{az}{(z-a)^2} \qquad (D \rightarrow 1)$$

SOL 6.4.30

SOL 6.4.31 Option (C) is correct.

The convolution of a signal $x[n]$ with unit step function $u[n]$ is given by
$$y[n] = x[n] * u[n] = \sum_{k=0}^{\infty} x[k]$$
Taking z-transform
$$Y(z) = X(z)\frac{1}{1-z^{-1}}$$

SOL 6.4.32 Option (B) is correct.

From the property of z-transform.
$$x_1[n] * x_2[n] \xleftrightarrow{z} X_1(z) X_2(z)$$

SOL 6.4.33 Option (C) is correct.

Given z transform
$$C(z) = \frac{z^{-1}(1-z^{-4})}{4(1-z^{-1})^2}$$
Applying final value theorem
$$\lim_{n\to\infty} f(n) = \lim_{z\to 1} (z-1) f(z)$$
$$\lim_{z\to 1} (z-1) F(z) = \lim_{z\to 1} (z-1)\frac{z^{-1}(1-z^{-4})}{4(1-z^{-1})^2}$$
$$= \lim_{z\to 1} \frac{z^{-1}(1-z^{-4})(z-1)}{4(1-z^{-1})^2}$$
$$= \lim_{z\to 1} \frac{z^{-1}z^{-4}(z^4-1)(z-1)}{4z^{-2}(z-1)^2}$$
$$= \lim_{z\to 1} \frac{z^{-3}}{4}\frac{(z-1)(z+1)(z^2+1)(z-1)}{(z-1)^2}$$
$$= \lim_{z\to 1} \frac{z^{-3}}{4}(z+1)(z^2+1) = 1$$

SOL 6.4.34 Option (C) is correct.
$$H_1(z) = 1 + 1.5z^{-1} - z^{-2}$$
$$= 1 + \frac{3}{2z} - \frac{1}{z^2} = \frac{2z^2+3z-2}{2z^2}$$
Poles $z^2 = 0 \Rightarrow z = 0$

zeros $(2z^2+3z-2) = 0 \Rightarrow \left(z-\frac{1}{2}\right)(z+2) = 0 \Rightarrow z = \frac{1}{2}, z = -2$

zeros of the two systems are identical.

SOL 6.4.35 Option (D) is correct.

Taking z-transform on both sides of given equation.

$$z^3 Y(z) + 6z^2 Y(z) + 11zY(z) + 6Y(z) = z^2 R(z) + 9zR(z) + 20R(z)$$

Transfer function

$$\frac{Y(z)}{R(z)} = \frac{z^2 + 9z + 20}{z^3 + 6z^2 + 11z + 6}$$

SOL 6.4.36 Option (A) is correct.

Characteristic equation of the system

$$|zI - A| = 0$$

$$zI - A = \begin{bmatrix} z & 0 \\ 0 & z \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ \beta & -\alpha \end{bmatrix} = \begin{bmatrix} z & -1 \\ \beta & z+\alpha \end{bmatrix}$$

$$|zI - A| = z(z+\alpha) + \beta = 0$$

$$z^2 + z\alpha + \beta = 0$$

In the given options, only option (A) satisfies this characteristic equation.

$$c[k+2] + \alpha c[k+1] + \beta c[k] = u[k]$$

$$z^2 + z\alpha + \beta = 0$$

SOL 6.4.37 Option (B) is correct.

We can see that the given impulse response is decaying exponential, i.e.

$$h[n] = a^n u[n], \quad 0 < a < 1$$

z-transform of $h[n]$ $\quad H(z) = \dfrac{z}{z-a}$

Pole of the transfer function is at $z = a$, which is on real axis between $0 < a < 1$.

SOL 6.4.38 Option (A) is correct.

$$y[n] + y[n-1] = x[n] - x[n-1]$$

Taking z-transform

$$Y(z) + z^{-1} Y(z) = X(z) - z^{-1} X(z)$$

$$Y(z) = \frac{(1 - z^{-1})}{(1 + z^{-1})} \quad \text{which has a linear phase response.}$$

SOL 6.4.39 Option (A) is correct.

For the linear phase response output is the delayed version of input multiplied by a constant.

$$y[n] = kx[n - n_0]$$

$$Y(z) = kz^{-n_0} X(z) = \frac{kX(z)}{z^{n_0}}$$

Pole lies at $z = 0$

SOL 6.4.40 Option (B) is correct.

Given impulse response can be expressed in mathematical form as

$$h[n] = \delta[n] - \delta[n-1] + \delta[n-2] - \delta[n-3] + \dots$$

By taking z-transform

$$H(z) = 1 - z^{-1} + z^{-2} - z^{-3} + z^{-4} - z^{-5} + \dots$$

$$= (1 + z^{-2} + z^{-4} + \dots) - (z^{-1} + z^{-3} + z^{-5} + \dots)$$

$$= \frac{1}{1 - z^{-2}} - \frac{z^{-1}}{1 - z^{-2}} = \frac{z^2}{z^2 - 1} - \frac{z}{z^2 - 1}$$

$$= \frac{(z^2 - z)}{(z^2 - 1)} = \frac{z(z-1)}{(z-1)(z+1)} = \frac{z}{z+1} \quad \text{Pole at } z = -1$$

SOL 6.4.41 Option (B) is correct.

Let Impulse response of system $h[n] \xleftrightarrow{z} H(z)$

First consider the case when input is unit step.

Input, $\quad x_1[n] = u[n]$ or $X_1(z) = \dfrac{z}{(z-1)}$

Output, $\quad y_1[n] = \delta[n]$ or $Y_1(z) = 1$

so,

$$Y_1(z) = X_1(z) H(z)$$

$$1 = \frac{z}{(z-1)} H(z)$$

Transfer function, $\quad H(z) = \dfrac{(z-1)}{z}$

Now input is ramp function

$$x_2[n] = nu[n]$$

$$X_2(z) = \frac{z}{(z-1)^2}$$

Output,

$$Y_2(z) = X_2(z) H(z)$$

$$= \left[\frac{z}{(z-1)^2}\right]\left[\frac{(z-1)}{(z)}\right] = \frac{1}{(z-1)}$$

$$Y_2(z) \xleftrightarrow{Z^{-1}} y_2[n]$$

$$\frac{1}{(z-1)} \xleftrightarrow{Z^{-1}} u[n-1]$$

SOL 6.4.42 Option (C) is correct.

Given state equations

$$s[n+1] = As[n] + Bx[n] \quad \text{...(1)}$$

$$y[n] = Cs[n] + Dx[n] \quad \text{...(2)}$$

Taking z-transform of equation (1)

$$zS(z) = AS(z) + BX(z)$$

$$S(z)[zI - A] = BX(z) \quad I \to \text{unit matrix}$$

$$S(z) = (zI - A)^{-1} BX(z) \quad \text{...(3)}$$

Now, taking z-transform of equation (2)

$$Y(z) = CS(z) + DX(z)$$

Substituting $S(z)$ from equation (3), we get

$$Y(z) = C(zI - A)^{-1} BX(z) + DX(z)$$

Transfer function

$$H(z) = \frac{Y(z)}{X(z)} = C(zI - A)^{-1} B + D$$

SOL 6.4.43 Option (B) is correct.

$$F(z) = 4z^3 - 8z^2 - z + 2$$

$$F(z) = 4z^2(z-2) - z(z-2)$$

$$= (4z^2 - 2)(z-2)$$

$$4z^2 - 2 = 0 \text{ and } (z-2) = 0$$

$$z = \pm\frac{1}{2} \text{ and } z = 2$$

Only one root lies outside the unit circle.

SOL 6.4.44 Option (A) is correct.

We know that convolution of $x[n]$ with unit step function $u[n]$ is given by

$$x[n] * u[n] = \sum_{k=-\infty}^{n} x[k]$$

so $$y[n] = x[n] * u[n]$$

Taking z-transform on both sides

$$Y(z) = X(z)\frac{z}{(z-1)} = X(z)\frac{1}{(1-z^{-1})}$$

Transfer function,

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{(1-z^{-1})}$$

Now consider the inverse system of $H(z)$, let impulse response of the inverse system is given by $H_1(z)$, then we can write

$$H(z)\,H_1(z) = 1$$

$$H_1(z) = \frac{X(z)}{Y(z)} = 1 - z^{-1}$$

$$(1-z^{-1})\,Y(z) = X(z)$$

$$Y(z) - z^{-1}\,Y(z) = X(z)$$

Taking inverse z-transform

$$y[n] - y[n-1] = x[n]$$

SOL 6.4.45 Option (B) is correct.

$$y[n] - \frac{1}{2}y[n-1] = x[n]$$

Taking z-transform on both sides

$$Y(z) - \frac{1}{2}z^{-1}\,Y(z) = X(z)$$

Transfer function $$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1-\frac{1}{2}z^{-1}}$$

Now, for input $x[n] = k\delta[n]$Output is

$$Y(z) = H(z)\,X(z)$$

$$= \frac{k}{(1-\frac{1}{2}z^{-1})} \qquad X(z) = k$$

Taking inverse z-transform

$$y[n] = k\left(\frac{1}{2}\right)^n u[n] = k\left(\frac{1}{2}\right)^n, \quad n \geq 0$$

SOL 6.4.46 Option (A) is correct.

$$y[n] + y[n-1] = x[n]$$

For unit step response, $x[n] = u[n]$

$$y[n] + y[n-1] = u[n]$$

Taking z-transform $$Y(z) + z^{-1}\,Y(z) = \frac{z}{z-1}$$

$$(1+z^{-1})\,Y(z) = \frac{z}{(z-1)}$$

$$\frac{(1+z)}{z}\,Y(z) = \frac{z}{(z-1)}$$

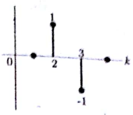
$$Y(z) = \frac{z^2}{(z+1)(z-1)}$$

SOL 6.4.47 Option (A) is correct.

SOL 6.4.48 Option (A) is correct.

SOL 6.4.49 Option (A) is correct.

We have $h(2) = 1$, $h(3) = -1$ otherwise $h[k] = 0$. The diagram of response is as follows :



It has the finite magnitude values. So it is a finite impulse response filter. Thus S_2 is true but it is not a low pass filter. So S_1 is false.

SOL 6.4.50 Option (D) is correct.

$$H(z) = \frac{z}{z-0.2} \qquad |z| < 0.2$$

We know that

$$-a^n u[-n-1] \longleftrightarrow \frac{1}{1-az^{-1}} \qquad |z| < a$$

Thus $$h[n] = -(0.2)^n u[-n-1]$$

SOL 6.4.51 Option (B) is correct.

We have $$h[n] = 3\delta[n-3]$$

or $$H(z) = 2z^{-3} \qquad \text{Taking } z \text{ transform}$$

$$X(z) = z^4 + z^2 - 2z + 2 - 3z^{-4}$$

Now $$Y(z) = H(z)\,X(z)$$

$$= 2z^{-3}(z^4 + z^2 - 2z + 2 - 3z^{-4})$$

$$= 2(z + z^{-1} - 2z^{-2} + 2z^{-3} - 3z^{-7})$$

Taking inverse z transform we have

$$y[n] = 2[\delta[n+1] + \delta[n-1] - 2\delta[n-2] + 2\delta[n-3] - 3\delta[n-7]]$$

At $n = 4$,

$$y[4] = 0$$

SOL 6.4.52 Option (A) is correct.

z-transform of $x[n]$ is

$$X(z) = 4z^{-3} + 3z^{-1} + 2 - 6z^2 + 2z^3$$

Transfer function of the system

$$H(z) = 3z^{-1} - 2$$

Output, $$Y(z) = H(z)\,X(z)$$

$$= (3z^{-1} - 2)(4z^{-3} + 3z^{-1} + 2 - 6z^2 + 2z^3)$$

$$= 12z^{-4} + 9z^{-2} + 6z^{-1} - 18z + 6z^2 - 8z^{-3} - 6z^{-1} - 4 + 12z^2 - 4z^3$$

$$= 12z^{-4} - 8z^{-3} + 9z^{-2} - 4 - 18z + 18z^2 - 4z^3$$

Or sequence $y[n]$ is

$$y[n] = 12\delta[n-4] - 8\delta[n-3] + 9\delta[n-2] - 4\delta[n]$$
$$- 18\delta[n+1] + 18\delta[n+2] - 4\delta[n+3]$$

$$y[n] \neq 0, \; n < 0$$

So $y[n]$ is non-causal with finite support.

SOL 6.4.53 Option (C) is correct.

Impulse response of given LTI system.

$$h[n] = x[n-1] * y[n]$$

Taking z-transform on both sides.

$$H(z) = z^{-1}X(z)\,Y(z) \qquad x[n-1] \xleftrightarrow{z} z^{-1}x(z)$$

We have $X(z) = 1 - 3z^{-1}$ and $Y(z) = 1 + 2z^{-2}$

So $\quad H(z) = z^{-1}(1-3z^{-1})(1+2z^{-2})$

Output of the system for input $u[n] = \delta[n-1]$ is,

$$y(z) = H(z)\,U(z) \qquad U[n] \xleftrightarrow{z} U(z) = z^{-1}$$

So

$$\begin{aligned} Y(z) &= z^{-1}(1-3z^{-1})(1+2z^{-2})z^{-1} \\ &= z^{-2}(1-3z^{-1}+2z^{-2}-6z^{-3}) \\ &= z^{-2}-3z^{-3}+2z^{-4}-6z^{-5} \end{aligned}$$

Taking inverse z-transform on both sides we have output.

$$y[n] = \delta[n-2] - 3\delta[n-3] + 2\delta[n-4] - 6\delta[n-5]$$

SOL 6.4.54 Option (D) is correct.

$$H(z) = (1 - az^{-1})$$

We have to obtain inverse system of $H(z)$. Let inverse system has response $H_1(z)$.

$$H(z)\,H_1(z) = 1$$

$$H_1(z) = \frac{1}{H(z)} = \frac{1}{1-az^{-1}}$$

For stability $H(z) = (1 - az^{-1})$, $|z| > a$ but in the inverse system $|z| < a$, for stability of $H_1(z)$.

so $\quad h_1[n] = -a^n u[-n-1]$

SOL 6.4.55 Option (C) is correct.

$$H(z) = \frac{z}{z+\frac{1}{2}}$$

Pole, $\quad z = -\frac{1}{2}$

The system is stable if pole lies inside the unit circle. Thus (A) is true, (R) is false.

SOL 6.4.56 Option (A) is correct.

Difference equation of the system.

$$y[n+2] - 5y[n+1] + 6y[n] = x[n]$$

Taking z-transform on both sides of above equation.

$$z^2Y(z) - 5zY(z) + 6Y(z) = X(z)$$

$$(z^2 - 5z + 6)\,y(z) = X(z)$$

Transfer function,

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{(z^2-5z+6)} = \frac{1}{(z-3)(z-2)}$$

Roots of the characteristic equation are $z = 2$ and $z = 3$

We know that an LTI system is unstable if poles of its transfer function (roots of characteristic equation) lies outside the unit circle. Since, for the given system the roots of characteristic equation lies outside the unit circle $(z = 2, z = 3)$ so the system is unstable.

SOL 6.4.57 Option (C) is correct.

System function, $\quad H(z) = \dfrac{z^2+1}{(z+0.5)(z-0.5)}$

Poles of the system lies at $z = 0.5$, $z = -0.5$. Since, poles are within the unit circle, therefore the system is stable.

From the initial value theorem

$$h[0] = \lim_{z\to\infty} H(z) = \lim_{z\to\infty}\frac{(z^2+1)}{(z+0.5)(z-0.5)}$$

$$= \lim_{z\to\infty}\frac{\left(1+\frac{1}{z^2}\right)}{\left(1+\frac{0.5}{z}\right)\left(1-\frac{0.5}{z}\right)} = 1$$

SOL 6.4.58 Option (D) is correct.

$$y[n] = 2x[n] + 4x[n-1]$$

Taking z-transform on both sides

$$Y(z) = 2X(z) + 4z^{-1}X(z)$$

Transfer Function,

$$H(z) = \frac{Y(z)}{X(z)} = 2 + 4z^{-1} = \frac{2z+4}{z}$$

Pole of $H(z)$, $z = 0$

Since Pole of $H(z)$ lies inside the unit circle so the system is stable.

(A) is not True.

$$H(z) = 2 + 4z^{-1}$$

Taking inverse z-transform

$$h[n] = 2\delta[n] + 4\delta[n-1] = \{2, 4\}$$

Impulse response has finite number of non-zero samples.

(R) is true.

SOL 6.4.59 Option (B) is correct.

For left sided sequence we have

$$-a^n u[-n-1] \xleftrightarrow{z} \frac{1}{1-az^{-1}} \qquad \text{where } |z| < a$$

Thus

$$-5^n u[-n-1] \xleftrightarrow{z} \frac{1}{1-5z^{-1}} \qquad \text{where } |z| < 5$$

or

$$-5^n u[-n-1] \xleftrightarrow{z} \frac{z}{z-5} \qquad \text{where } |z| < 5$$

Since ROC is $|z| < 5$ and it include unit circle, system is stable.

ALTERNATIVE METHOD :

$$h[n] = -5^n u[-n-1]$$

$$H(z) = \sum_{n=-\infty}^{\infty} h[n]z^{-n} = \sum_{n=-\infty}^{-1} -5^n z^{-n} = -\sum_{n=-\infty}^{-1}(5z^{-1})^n$$

Let $n = -m$, then

$$H(z) = -\sum_{m=1}^{\infty}(5z^{-1})^{-m} = 1 - \sum_{m=0}^{\infty}(5^{-1}z)^{m}$$

$$= 1 - \frac{1}{1-5^{-1}z}, \qquad |5^{-1}z| < 1 \text{ or } |z| < 5$$

$$= 1 - \frac{5}{5-z} = \frac{z}{z-5}$$

SOL 6.4.60 Option (B) is correct.

For a system to be stable poles of its transfer function $H(z)$ must be inside the unit circle. In inverse system poles will appear as zeros, so zeros must be inside the unit circle.

SOL 6.4.61 Option (C) is correct.

An LTI discrete system is said to be BIBO stable if its impulse response $h[n]$

is summable, that is

$$\sum_{n=-\infty}^{\infty} h[n] < \infty$$

z-transform of $h[n]$ is given as

$$H(z) = \sum_{n=-\infty}^{\infty} h[n] z^{-n}$$

Let $z = e^{j\Omega}$ (which describes a unit circle in the z-plane), then

$$|H(z)| = \left| \sum_{n=-\infty}^{\infty} h[n] e^{-j\Omega n} \right|$$

$$= \sum_{n=-\infty}^{\infty} |h[n] e^{-j\Omega n}|$$

$$= \sum_{n=-\infty}^{\infty} |h[n]| < \infty$$

which is the condition of stability. So LTI system is stable if ROC of its system function includes the unit circle $|z| = 1$.
(A) is true.
We know that for a causal system, the ROC is outside the outermost pole. For the system to be stable ROC should include the unit circle $|z| = 1$. Thus, for a system to be causal & stable these two conditions are satisfied if all the poles are within the unit circle in z-plane.
(R) is false.

SOL 6.4.62 Option (B) is correct.
We know that for a causal system, the ROC is outside the outermost pole. For the system to be stable ROC should include the unit circle $|z| = 1$. Thus, for a system to be causal & stable these two conditions are satisfied if all the poles are within the unit circle in z-plane.
(A) is true.
If the z-transform $X(z)$ of $x[n]$ is rational then its ROC is bounded by poles because at poles $X(z)$ tends to infinity.
(R) is true but (R) is not correct explanation of (A).

SOL 6.4.63 Option (C) is correct.
We have,

$$H(z) = \frac{2 - \frac{3}{4}z^{-1}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

$$= \frac{1}{(1 - \frac{1}{2}z^{-1})} + \frac{1}{(1 - \frac{1}{4}z^{-1})} \qquad \text{By partial fraction}$$

For ROC : $|z| > 1/2$

$$h[n] = \left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{4}\right)^n u[n],\ n > 0 \qquad \frac{1}{1 - az^{-1}} = a^n u[n], |z| > a$$

Thus, system is causal. Since ROC of $H(z)$ includes unit circle, so it is stable also. Hence S_1 is True

For ROC : $|z| < \frac{1}{4}$

$$h[n] = -\left(\frac{1}{2}\right)^n u[-n-1] + \left(\frac{1}{4}\right)^n u[n], \qquad |z| > \frac{1}{4}, |z| < \frac{1}{2}$$

System is not causal. ROC of $H(z)$ does not include unity circle, so it is not stable and S_3 is True.

SOL 6.4.64 Option (C) is correct.
We have $\quad 2y[n] = \alpha y[n-2] - 2x[n] + \beta x[n-1]$

Taking z transform we get

$$2Y(z) = \alpha Y(z) z^{-2} - 2X(z) + \beta X(z) z^{-1}$$

or $$\frac{Y(z)}{X(z)} = \left(\frac{\beta z^{-1} - 2}{2 - \alpha z^{-2}}\right) \qquad ...(1)$$

or $$H(z) = \frac{z(\frac{\beta}{2} - z)}{(z^2 - \frac{\alpha}{2})}$$

It has poles at $\pm\sqrt{\alpha/2}$ and zero at 0 and $\beta/2$. For a stable system poles must lie inside the unit circle of z plane. Thus

$$\left|\sqrt{\frac{\alpha}{2}}\right| < 1$$

or $$|\alpha| < 2$$

But zero can lie anywhere in plane. Thus, β can be of any value.

SOL 6.4.65 Option (D) is correct.
Let $H_1(z)$ and $H_2(z)$ are the transfer functions of systems s_1 and s_2 respectively. For the second order system, transfer function has the following form

$$H_1(z) = az^{-2} + bz^{-1} + c$$

$$H_2(z) = pz^{-2} + qz^{-1} + r$$

Transfer function of the cascaded system

$$H(z) = H_1(z) H_2(z)$$

$$= (az^{-2} + bz^{-1} + c)(pz^{-2} + qz^{-1} + r)$$

$$= apz^{-4} + (aq + bp) z^{-3} + (ar + cp) z^{-2} + (br + qc) z^{-1} + cr$$

So, impulse response $h[n]$ will be of order 4.

SOL 6.4.66 Option (B) is correct.
Output is equal to input with a delay of two units, that is

$$y(t) = x(t-2)$$

$$Y(z) = z^{-2} X(z)$$

Transfer function,

$$H(z) = \frac{Y(z)}{X(z)} = z^{-2}$$

For the cascaded system, transfer function

$$H(z) = H_1(z) H_2(z)$$

$$z^{-2} = \frac{(z-0.5)}{(z-0.8)} H_2(z)$$

$$H_2(z) = \frac{z^{-1} - 0.8z^{-2}}{z - 0.5} = \frac{z^{-2} - 0.8z^{-3}}{1 - 0.5z^{-1}}$$

SOL 6.4.67 Option (B) is correct.

$$y[n] = x[n-1]$$

or $$Y(z) = z^{-1} X(z)$$

or $$\frac{Y(z)}{X(z)} = H(z) = z^{-1}$$

Now $$H_1(z) H_2(z) = z^{-1}$$

$$\left(\frac{1 - 0.4z^{-1}}{1 - 0.6z^{-1}}\right) H_2(z) = z^{-1}$$

$$H_2(z) = \frac{z^{-1}(1 - 0.6z^{-1})}{(1 - 0.4z^{-1})}$$

SOL 6.4.68 Option (C) is correct.

We have $h_1[n] = \delta[n-1]$ or $H_1[Z] = Z^{-1}$

and $h_2[n] = \delta[n-2]$ or $H_2(Z) = Z^{-2}$

Response of cascaded system

$$H(z) = H_1(z) \cdot H_2(z) = z^{-1} \cdot z^{-2} = z^{-3}$$

or, $$h[n] = \delta[n-3]$$

SOL 6.4.69 Option (B) is correct.

Let three LTI systems having response $H_1(z), H_2(z)$ and $H_3(z)$ are Cascaded as showing below

I/P → $\boxed{H_1(z)}$ → $\boxed{H_2(z)}$ → $\boxed{H_3(z)}$ → $H(z)$

Assume $H_1(z) = z^2 + z^1 + 1$ (non-causal)

$H_2(z) = z^3 + z^2 + 1$ (non-causal)

Overall response of the system

$$H(z) = H_1(z)\,H_2(z)\,H_3(z)$$
$$= (z^2 + z^1 + 1)(z^3 + z^2 + 1)\,H_3(z)$$

To make $H(z)$ causal we have to take $H_3(z)$ also causal.

Let $H_3(z) = z^{-6} + z^{-4} + 1$

$$H(z) = (z^2 + z^1 + 1)(z^3 + z^2 + 1)(z^{-6} + z^{-4} + 1)$$
$$H(z) \rightarrow \text{causal}$$

Similarly to make $H(z)$ unstable atleast one of the system should be unstable.

SOL 6.4.70 Option (B) is correct.

$$H(z) = \frac{1 + az^{-1} + bz^{-2}}{1 + cz^{-1} + dz^{-2} + ez^{-3}}$$

We know that number of minimum delay elements is equal to the highest power of z^{-1} present in the denominator of $H(z)$.

No. of delay elements $= 3$

SOL 6.4.71 Option (A) is correct.

From the given system realization, we can write

$$(X(z) + Y(z)\,z^{-2}a_2 + Y(z)\,a_1 z^{-1}) \times a_0 = Y(z)$$

System Function

$$H(z) = \frac{Y(z)}{X(z)} = \frac{a_0}{1 - a_1 z^{-1} - a_2 z^{-2}}$$
$$= \frac{1}{\frac{1}{a_0} - \frac{a_1}{a_0} z^{-1} - \frac{a_2}{a_0} z^{-2}}$$

Comparing with given $H(z)$

$$\frac{1}{a_0} = 1 \Rightarrow a_0 = 1$$
$$-\frac{a_1}{a_0} = -0.7 \Rightarrow a_1 = 0.7$$
$$-\frac{a_2}{a_0} = 0.13 \Rightarrow a_2 = -0.13$$

SOL 6.4.72 Option (B) is correct.

Let, $M \rightarrow$ highest power of z^{-1} in numerator.

$N \rightarrow$ highest power of z^{-1} in denominator

Number of delay elements in direct form-I realization equals to $M + N$

Number of delay elements in direct form-II realization equal to N.

Here, $M = 3$, $N = 3$

So delay element in direct form-I realization will be 6 and in direct form realization will be 3.

SOL 6.4.73 Option (A) is correct.

System response is given as

$$H(z) = \frac{G(z)}{1 - KG(z)}$$
$$g[n] = \delta[n-1] + \delta[n-2]$$
$$G(z) = z^{-1} + z^{-2}$$

So $$H(z) = \frac{(z^{-1} + z^{-2})}{1 - K(z^{-1} + z^{-2})} = \frac{z+1}{z^2 - Kz - K}$$

For system to be stable poles should lie inside unit circle.

$$|z| \le 1$$
$$z = \frac{K \pm \sqrt{K^2 + 4K}}{2} \le 1$$
$$K \pm \sqrt{K^2 + 4K} \le 2$$
$$\sqrt{K^2 + 4K} \le 2 - K$$
$$K^2 + 4K \le 4 - 4K + K^2$$
$$8K \le 4$$
$$K \le 1/2$$

SOL 6.4.74 Option (B) is correct.

Input-output relationship of the system

$$y[n] = x[n] + ay[n-1]$$

Taking z-transform

$$Y(z) = X(z) + z^{-1}aY(z)$$

Transform Function,

$$\frac{Y(z)}{X(z)} = \frac{1}{1 - z^{-1}a}$$

Pole of the system $(1 - z^{-1}a) = 0 \Rightarrow z = a$

For stability poles should lie inside the unit circle $|z| < 1$ so $|a| < 1$.

SOL 6.4.75 Option (D) is correct.

The relation ship between Laplace transform and z-transform is given as

$$X(s) = X(z)\big|_{z = e^{sT}}$$
$$z = e^{sT} \quad \text{...(1)}$$

We know that $z = re^{j\Omega}$ ...(2)

and $s = \sigma + j\omega$ (3)

From equation (1), (2) and (3), we can write

$$z = re^{j\Omega} = e^{(\sigma + j\omega)T} = e^{\sigma T} e^{j\omega T}$$

From above relation we can find that $|z| = e^{\sigma T}$ and $\Omega = \omega T$. It is concluded that,

- If $\sigma = 0$ then $|z| = 1$, the $j\omega$-axis of s-plane maps into unit circle.
- If $\sigma < 0$, $|z| < 1$, it implies that left half of s-plane maps into inside of unit circle $(|z| < 1)$.
- Similarly, if $\sigma > 0$, $|z| > 1$ which implies that right half of s-plane maps into outside of unit circle.

SOL 6.4.76 Option (D) is correct.

The relation ship between Laplace transform and z-transform is given as

$$X(s) = X(z)\big|_{z=e^{st}} \quad ...(1)$$

$$z = e^{sT} \quad ...(2)$$

We know that $z = re^{j\Omega}$ (3)

and $s = \sigma + j\omega$

From equation (1), (2) and (3), we can write

$$z = re^{j\Omega} = e^{(\sigma+j\omega)T}$$

$$= e^{\sigma T}e^{j\omega T}$$

$$z = re^{j\Omega} = e^{\sigma T}e^{j\omega T}$$

From above relation we can find that $|z| = e^{\sigma T}$ and $\Omega = \omega T$. It is concluded that,

- If $\sigma = 0$ then $|z| = 1$, the $j\omega$-axis of s-plane maps into unit circle.
- If $\sigma < 0$, $|z| < 1$, it implies that left half of s-plane maps into inside of unit circle $(|z| < 1)$.
- Similarly, if $\sigma > 0$, $|z| > 1$ which implies that right half of s-plane maps into outside of unit circle.

SOL 6.4.77 Option (C) is correct.

Ideal sampler output is given by

$$f(t) = \sum_{n=0}^{\infty} K_n \delta[t - nT_s]$$

where $T_s \rightarrow$ sampling period

$n \rightarrow$ integer

$$f(t) = K_0\delta[n] + K_1\delta[n-1] + K_2\delta[n-2] + ...$$

$$\mathcal{Z}[f(t)] = K_0 + K_1 z^{-1} + K_2 z^{-2} + ... + K_n z^{-n}$$

SOL 6.4.78 Option (B) is correct.

We know that $X(s) = X(z)\big|_{z=e^{sT}}$

so,

$$z = e^{sT}$$

$$\ln z = sT$$

$$s = \frac{\ln z}{T}$$

SOL 6.4.79 Option (D) is correct.

SOL 6.4.80 Option (C) is correct.

$$H(s) = \frac{a}{s^2 + a^2}$$

Poles in s-domain are at $s = \pm ja$. In z-domain poles will be at $z = e^{sT}$, so

$$z_1 = e^{-jaT} \text{ and } z_2 = e^{jaT}$$

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