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19EC10088
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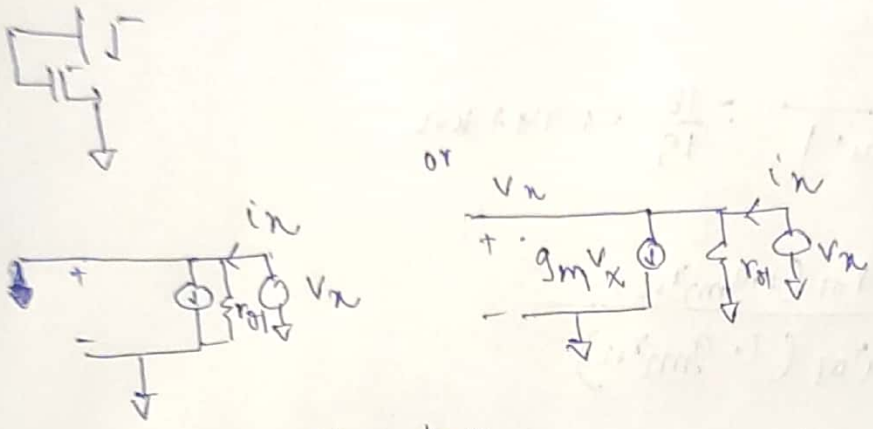
3. $\Delta_1 = 88 \bmod 20 = 8$
$\Delta_2 = 88 \bmod 15 = 13$

i)



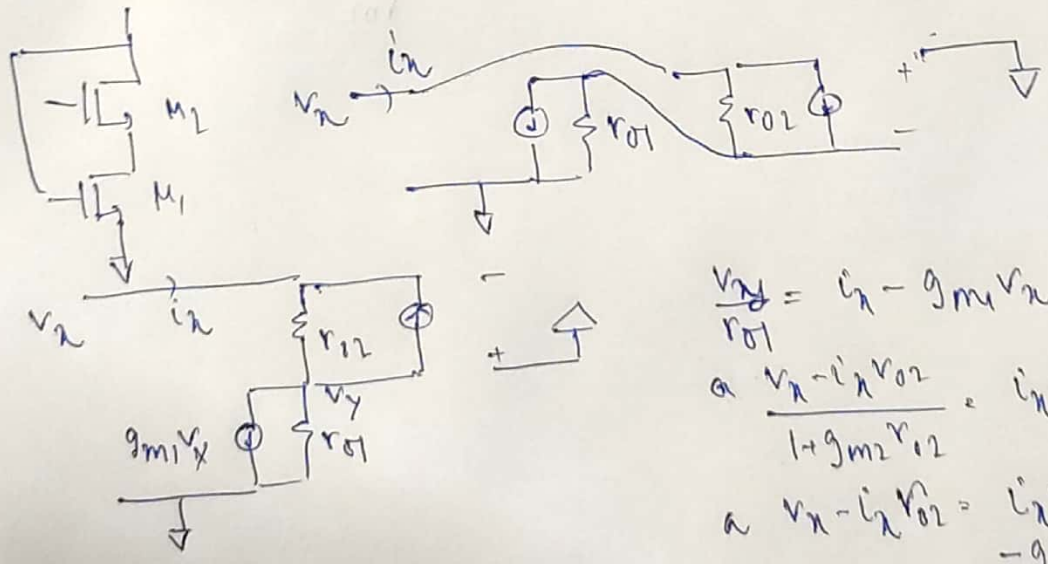
Applying KCL at node x

$$i_x = \frac{v_x}{r_{o1}} + g_{m1} v_x$$

$$\text{or } \frac{i_x}{v_x} = \frac{1}{r_{o1}} + g_{m1} = \frac{1 + g_{m1} r_{o1}}{r_{o1}}$$

$$\text{or } \frac{v_x}{i_x} = \frac{r_{o1}}{1 + g_{m1} r_{o1}} = r_{o1} \| \frac{1}{g_{m1}}$$

This diode connected device is 2 terminal. Alternatively we can see it from source to obtain the same impedence



$$\frac{v_x - v_y}{r_{o1}} = i_x - g_{m1} v_x$$

$$\text{or } \frac{v_x - i_x r_{o2}}{1 + g_{m2} r_{o2}} = i_x r_{o1} - g_{m1} v_x r_{o1}$$

$$\text{or } v_x - i_x r_{o2} = i_x r_{o1} (1 + g_{m2} r_{o2}) - g_{m1} r_{o1} (1 + g_{m2} r_{o2}) v_x$$

$$\text{or } v_x \left[1 + g_{m1} r_{o1} (1 + g_{m2} r_{o2})\right] = i_x \left[r_{o2} + r_{o1} (1 + g_{m2} r_{o2})\right]$$

$$\text{or } R_{out} = \frac{r_{o2} + r_{o1} (1 + g_{m2} r_{o2})}{1 + g_{m1} r_{o1} (1 + g_{m2} r_{o2})}$$

$$v_y = v_x - (i_x + g_{m2} v_y) r_{o2}$$

$$\text{or } v_y = v_x - i_x r_{o2} - g_{m2} v_y r_{o2}$$

$$\text{or } v_y (1 + g_{m2} r_{o2}) = v_x - i_x r_{o2}$$

$$\text{or } v_y = \frac{v_x - i_x r_{o2}}{1 + g_{m2} r_{o2}}$$

ii) $r_{o1} = 10 + \Delta_1 = 18\ k\Omega$

$r_{o2} = 9 + \Delta_2 = 22\ k\Omega$

$g_{m1} = g_{m2} = 1\ (k\Omega)^{-1}$

$$R_{out1} = \frac{r_{o1}}{r_{o1} g_{m1} + 1} = \frac{18}{19} = 0.947\ k\Omega$$

$$R_{out2} = \frac{r_{o2} + r_{o1}(1 + g_{m2} r_{o2})}{1 + g_{m1} r_{o1}(1 + g_{m2} r_{o2})}$$

$$= \frac{22 + 18 \times 23}{1 + 18 \times 23}$$

$$= \frac{436}{415} = 1.05\ k\Omega$$

Intuitively

$$R_{out2} = \frac{\frac{r_{o2}}{1 + g_{m2} r_{o2}} + r_{o1}}{\frac{1}{g_{m2} r_{o2} + 1} + r_{o1}} = \frac{r_{o2} \| \frac{1}{g_{m2}} + r_{o1}}{\frac{r_{o2} \| \frac{1}{g_{m2}}}{r_{o2}} + r_{o1}}$$