Probability and Stochastic Process (MA20106)
Assignment –Random variable

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1. The probability mass function of a random variable X is given by
$$P(X = x) = k\binom{n}{x}, x = 0, 1, \ldots, n,$$
where k is a constant. Then calculate the moment generating function $M_X(t)$.

2. Let the probability density function of a random variable X be given by
$$f(x) = \alpha e^{-x^2-\beta x}, \quad -\infty < x < \infty.$$
If $E(X) = -\frac{1}{2}$, then find α and β.

3. Let X be a $Geom(0.4)$ random variable. Then find $P(X = 5|X \geq 2)$.

4. X is a random variable with density $f(x) = \dfrac{1}{4}e^{-|x|/2}, \quad -\infty < x < \infty$. Then find $E(|X|)$.

5. Let X be a normal random variable with mean 2 and variance 4, and $g(a) = P(a \leq X \leq a + 2)$. Then calculate the value of a that maximizes $g(a)$.

6. The probability density function of a random variable X is given by
$$f(x) = \begin{cases} \dfrac{1}{4}, & \text{if } |x| < 1, \\ \dfrac{1}{4x^2}, & \text{otherwise.} \end{cases}$$
Then calculate $P(-\frac{1}{2} \leq X \leq 2)$.

7. The cumulative distribution function of a random variable X is given by
$$F(x) = \begin{cases} 0, & \text{if } x < 0, \\ \dfrac{1}{4} + \dfrac{1}{6}(4x - x^2), & \text{if } 0 \leq x < 1, \\ 1, & \text{if } x \geq 1. \end{cases}$$
Then calculate $P(X = 0|0 \leq X < 1)$.

8. The probability density function $f(x)$ of a random variable X is symmetric about 0. Then find
$$\int_{-2}^{2}\int_{-\infty}^{x} f(u)dudx.$$

9. A circle of random radius R (in cm) is constructed, where the random variable R has $U[0, 1]$ distribution. Then find the probability that the area of circle is less than 1 cm^2.

10. Let the random variable X have moment generating function

$$M_X(t) = e^{2t(1+t)}, \ t \ \in \ \mathbb{R}.$$

Then find the $P(X \leq 2)$.

11. The distribution function of a random variable X is given by

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{4}, & 0 \leq x < \frac{1}{4} \\ \frac{1}{2}, & \frac{1}{4} \leq x < \frac{1}{2} \\ \frac{3}{4}, & \frac{1}{2} \leq x < \frac{3}{4} \\ \frac{x+3}{5}, & \frac{3}{4} \leq x < 2 \\ 1, & x \geq 2 \end{cases}$$

Then calculate $P\Big(\frac{1}{4} \leq X \leq 1\Big)$.

12. Let X be a discrete random variable with the moment generating function

$$M_X(t) = e^{0.5(e^t-1)}, t \in \mathcal{R}.$$

Then find $P(X \leq 1)$.

13. Let X be a continuous random variable with the probability density function

$$f(x) = \frac{1}{(2+x^2)^{3/2}}, x \in \mathcal{R}.$$

Then find $E(X^2)$.
Answer: does not exist

14. The probability density function of a random variable X is given by

$$f(x) = \begin{cases} \alpha x^{\alpha-1}, 0 < x < 1, \\ 0 \text{ otherwise} \end{cases}, \alpha > 0.$$

Then find the distribution of the random variable $Y = log_e X^{-2\alpha}$.

15. Let X be a random variable with the cumulative distribution function

$$F(x) = \begin{cases} 0, x < 0, \\ \frac{x}{8}, 0 \leq < 2, \\ \frac{x^2}{16}, 2 \leq < 4, \\ 1, x \geq 4. \end{cases}$$

Then calculate $E(X)$.

16. An institute purchases laptops from either vendor V_1 or vendor V_2 with equal probability. The lifetimes (in years) of laptops from vendor V_1 have a $U(0,4)$ distribution, and the lifetimes (in years) of laptops from vendor V_2 have an $Exp(1/2)$ distribution. If a randomly selected laptop in the institute has lifetime more than two years, then find the probability that it was supplied by vendor V_2.

17. Let X be a continuous random variable with the probability density function

$$f(x) = \begin{cases} \frac{2x}{9}, 0 < x < 3, \\ 0, \text{ otherwise.} \end{cases}$$

Then, using Chebyshev's inequality, the find the upper bound of $P(|X - 2| > 1)$.

18. Let Y be a $Bin(72, \frac{1}{3})$ random variable. Using normal approximation to binomial distribution, find an approximate value of $P(22 \leq Y \leq 28)$.

19. Let X be a $Bin(2,p)$ random variable and Y be a $Bin(4,p)$ random variable, $0 < p < 1$. If $P(X \geq 1) = \frac{5}{9}$, then find $P(Y \geq 1)$.

20. Let X be a continuous random variable with the probability density function

$$f(x) = \begin{cases} \frac{x+1}{2}, -1 < x < 1, \\ 0, \text{ otherwise.} \end{cases}$$

Then calculate $P(\frac{1}{4} < X^2 < \frac{1}{2})$.

21. If X is a $U(0,1)$ random variable, then find the $P(\min(X, 1-X) \leq \frac{1}{4})$.

22. Let X be a continuous random variable with the probability density function

$$f(x) = \begin{cases} 0, \text{ if } x \leq 0 \\ x^3, \text{ if } 0 < x \leq 1 \\ \frac{3}{x^5}, \text{ if } x > 1 \end{cases} .$$

Then find the $P(1/2 < X < 2)$.

23. Let X be a random variable with the moment generating function

$$M_X(t) = \frac{1}{216}(5 + e^t)^3, t \in \mathcal{R}.$$

Then find the $P(X > 1)$.

24. Let X be a discrete random variable with the probability mass function

$$p(x) = k(1 + |x|)^2, x = -2, -1, 0, 1, 2,$$

where k is a real constant. Then calculate the $P(X = 0)$.

25. Let the random variable X have uniform distribution on the interval $\left(\frac{\pi}{6}, \frac{\pi}{2}\right)$.Then find the $P(cosX > sinX)$.

26. Let the random variable X have uniform distribution on the interval $(0, 1)$ and $Y = -2log_e X$. Then find $E(Y)$.

27. If $Y = \log_{10} X$ has $N(\mu, \sigma^2)$ distribution with moment generating function $M_y(t) = e^{5t+2t^2}$, $t \in (-\infty, \infty)$, then find the $P(X < 1000)$.

28. Let X be a continuous random variable with the probability density function

$$f(x) = \begin{cases} \frac{x}{8}, \text{ if } 0 < x < 2 \\ \frac{k}{8}, \text{ if } 2 \leq x \leq 4 \\ \frac{6-x}{8}, \text{ if } 4 < x < 6 \\ 0, \text{ otherwise} \end{cases}$$

where k is a real constant. Then calculate $P(1 < X < 5)$.

29. Let X be a random variable with the probability density function

$$f(x|r, \lambda) = \frac{\lambda^r}{(r-1)!} x^{r-1} e^{-\lambda x}, x > 0, \lambda > 0, r > 0.$$

If $E(X) = 2$ and $Var(X) = 2$, then find the $P(X < 1)$.