CS21003 ALGORITHMS-1
(WorkSheet 4: Binary Trees, Binary Search Trees)
Date: Oct 3 2020

1 Draw the AVL Trees

1.1 Only ODD Roll Numbered Students Must attempt this

Consider this set of 7 elements $\{1, 2, 3, 4, 5, 6, 7\}$ and an AVL tree T which is **empty** initially. Insert these elements one by one into this AVL tree T starting from **1**. You need to draw the tree every time you insert an element into it. (You don't have to mention any intermediate steps while inserting an element. Only the final tree after inserting every element is sufficient)

After inserting all the 7 numbers into the AVL tree T, Delete the node containing the element **4** and draw the resultant tree.

After deleting the element 4, write the Inorder, Preorder and Postorder traversal of T.

1.2 Only EVEN Roll Numbered Students Must attempt this

Consider this set of 7 elements $\{9, 8, 7, 6, 5, 4, 3\}$ and an AVL tree T which is **empty** initially. Insert these elements one by one into this AVL tree T starting from **9**. You need to draw the tree every time you insert an element into it. (You don't have to mention any intermediate steps while inserting an element. Only the final tree after inserting every element is sufficient)

After inserting all the 7 numbers into the AVL tree T, Delete the node containing the element **6** and draw the resultant tree.

After deleting the element 6, write the Inorder, Preorder and Postorder traversal of T.

1.3 BONUS QUESTION: All students can attempt. Will be checked only when you are at borderline of grades at the end of the semester.

Suppose that we insert n distinct keys into an initially empty binary search tree. Assuming that the $n!$ permutations are equally likely to occur, what is the average height of the tree?
Pointers to solving the problem:
$n =$ number of keys
$X =$ height of the tree of n keys
$Y_n = 2^{X_n}$
First, we need to estimate an upper bound on $E[Y_n]$.
Show that $E[Y_n] \leqslant \frac{1}{4}\left({}^{n+3}C_3\right)$ for $n \geqslant 1$.
Further assuming $f(E[X]) \leqslant E[f(X)]$ and $X = X_n$ and $f(X) = 2^X$ find an estimate of $E[X_n]$.

Further hint: Note that if x is the first key randomly inserted in the tree, then all keys smaller than it goes to the left subtree and all those larger go to the right subtree. The height of the tree thus constructed is one plus the larger of the height of the left and the right subtree.