# Maxwell’s Equation

# Electromagnetic Induction

- In the middle part of the nineteenth century Michael Faraday formulated his law of induction.
- It had been known for some time that a current could be produced in a wire by a changing magnetic field.
- Faraday showed that the induced electromotive force is directly related to the rate at which the magnetic field lines cut across the path.

# Faraday’s Law

- Faraday's law of induction can be expressed as:
- The emf is equal to minus the change of the magnetic flux with time.

$$\xi = -\frac{d\Phi_B}{dt}$$

# Magnetic Flux

- The magnetic flux is given by:

$$\Phi_B \equiv -\int \vec{B} \cdot d\vec{A}$$

# Faraday's Law

- Therefore the induced emf can be expressed as:

$$\xi = -\frac{d}{dt}\int \vec{B} \cdot d\vec{A}$$

# Faraday's Law for Simple Cases

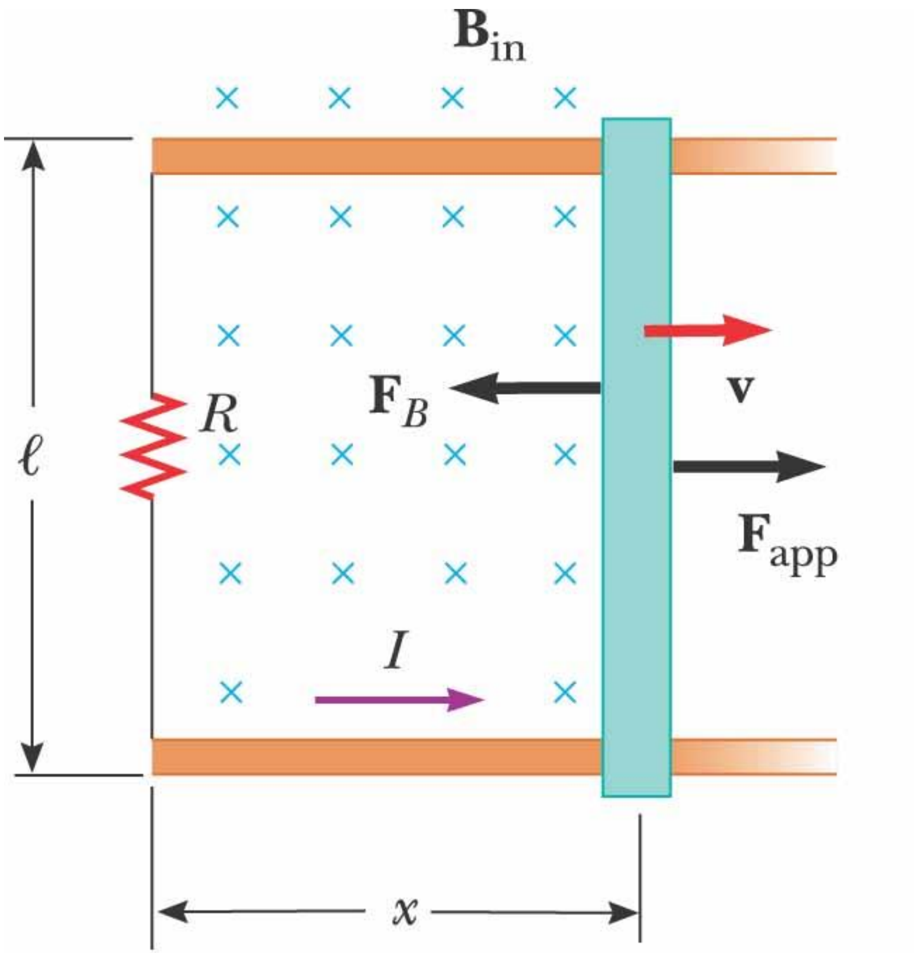
- If the magnetic field is spatially uniform and the area is simple enough then the induced emf can be expressed as:

$$\xi = B\frac{dA}{dt}$$

# Example

- A rectangular loop of wire has an area equal to its width (x) times its length (L).
- Suppose that the length of the conducting wire loop can be arbitrarily shortened by sliding a conducting rod of length L along its width.
- Furthermore, suppose that a constant magnetic field is moving perpendicular to the rectangular loop.

# A sliding rod of length L in a magnetic field.



# Solution

- If we move the conductor along the conducting wires the area of the loop that encloses the magnetic field is changing with time.
- The amount of change is proportional to the velocity that which we move the rod.

# Solution cont.

- The induced emf depends on the magnitude of the magnetic field and the change of the area of the loop with time.

$$\xi = B\frac{dA}{dt} = BL\frac{dx}{dt}$$

# Solution cont.

- Since the length L remains constant the width changes as:

$$\frac{dx}{dt} = \mathrm{v}$$

- Then the induced emf is:

$$BL\mathrm{v}$$

# Example

- Suppose the rod in the previous figure is moving at a speed of 5.0 m/s in a direction perpendicular to a 0.80 T magnetic field.
- The conducting rod has a length of 1.6 meters.
- A light bulb replaces the resistor in the figure.
- The resistance of the bulb is 96 ohms.

# Example cont.

- Find the emf produced by the circuit,
- the induced current in the circuit,
- the electrical power delivered to the bulb,
- and the energy used by the bulb in 60.0 s.

# Solution

- The induced emf is given by Faraday's law:

$$\xi = \mathrm{v}BL = \left(5.0\,m/s\right)\left(0.8T\right)\left(1.6m\right) = 6.4V$$

# Solution cont.

- We can obtain the induced current in the circuit by using Ohm's law.

$$I = \frac{\xi}{R} = \frac{6.4V}{96\Omega} = 0.067\,A$$

# Solution cont.

- The power can now be determined.

$$P = I\xi = (0.067\,A)(6.4V) = 0.43W$$

# Solution cont.

- Since the power is not changing with time, then the energy is the product of the power and the time.

$$E = Pt = (0.43W)(60.0\,s) = 26\,J$$

# The Emf Induced by a rotating Coil

- Faraday's law of induction states that an emf is induced when the magnetic flux changes over time.
- This can be accomplished
- by changing the magnitude of the magnetic field,
- by changing the cross-sectional area that the flux passes through, or
- by changing the angle between the magnetic field and the area with which it passes.

# The Emf Induced by a rotating Coil

- If a coil of *N* turns is made to rotate in a magnetic field then the angle between the B-field and the area of the loop will be changing.
- Faraday's law then becomes:

$$emf = -\frac{d\Phi_B}{dt} = -NAB\left(\frac{d\cos\theta}{dt}\right) = NAB\sin\theta\frac{d\theta}{dt}$$

# Angular Speed

- The angular speed can be defined as:

$$\omega = \frac{d\theta}{dt}$$

- Then integrating we get that:

$$\theta = \omega t$$

# The Emf Induced by a rotating Coil

- Substitution of the angular speed into our relation for the emf for a rotating coil gives the following:

$$\xi = NAB\omega \sin \omega t$$

# Example

- The armature of a 60-Hz ac generator rotates in a 0.15-T magnetic field.
- If the area of the coil is $2 \times 10^{-2}$ $m^2$ , how many loops must the coil contain if the peak output is 170 V?

# Solution

- The maximum emf occurs when the sin ωt equals one. Therefore:

$$\xi_{\max} = NBA\omega$$

- Furthermore, we can calculate the angular speed by noting that the angular frequency is:

$$\omega = 2\pi f = 2\pi(60\,Hz) = 377\,s^{-1}$$

# Solution cont.

- The number of turns is then:

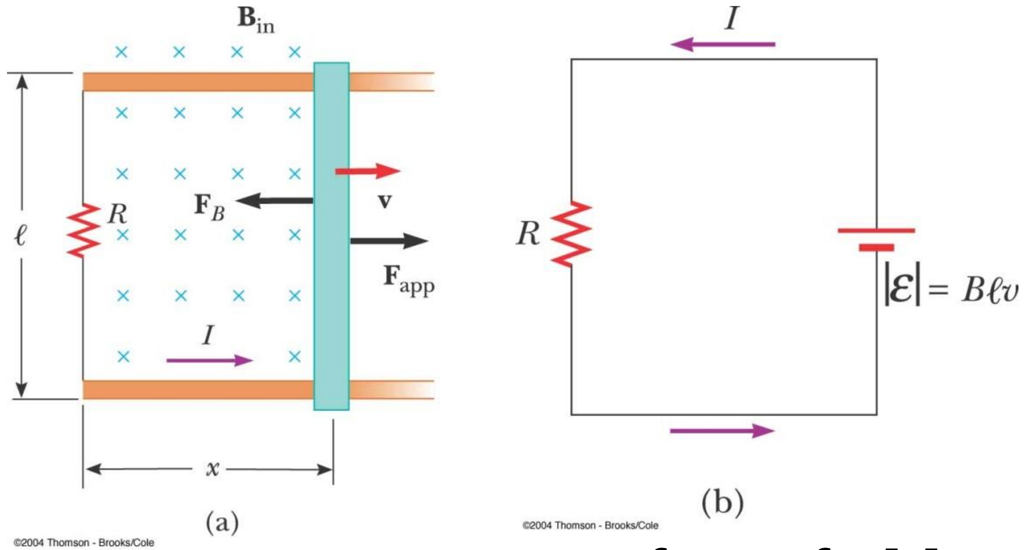
$$N = \frac{\xi_{\max}}{BA\omega} = \frac{170V}{(0.15T)\left(2.0\times10^{-2}m^2\right)\left(377s^{-1}\right)} = 150$$

# Another Form

- Suppose the have a stationary loop in a changing magnetic field.
- Then, since the path of integration around the loop is stationary, we can rewrite Faraday's law.

$$\xi = \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

# Sliding Conducting Bar




- A bar moving through a uniform field and the equivalent circuit diagram
- Assume the bar has zero resistance
- The work done by the applied force appears as internal energy in the resistor *R*

# Lenz's Law

- Faraday's law indicates that the induced emf and the change in flux have opposite algebraic signs
- This has a physical interpretation that has come to be known as **Lenz's law**
- Developed by German physicist Heinrich Lenz

# Lenz's Law, cont.

- **Lenz's law**: *the induced current in a loop is in the direction that creates a magnetic field that opposes the change in magnetic flux through the area enclosed by the loop*
- The induced current tends to keep the original magnetic flux through the circuit from changing

# Induced emf and Electric Fields

- An electric field is created in the conductor as a result of the changing magnetic flux
- Even in the absence of a conducting loop, a changing magnetic field will generate an electric field in empty space
- This induced electric field is nonconservative
  - Unlike the electric field produced by stationary charges

# Induced emf and Electric Fields

- The induced electric field is a nonconservative field that is generated by a changing magnetic field
- The field cannot be an electrostatic field because if the field were electrostatic, and hence conservative, the line integral of $\mathbf{E} \cdot d\mathbf{s}$ would be zero and it isn't

# Another Look at Ampere's Law

- Ampere’s Law states the following:

$$\oint \vec{B} \cdot d\vec{s} = \mu_o I$$

# Another Look at Ampere’s Law

- Thus we can determine the magnetic field around a current carrying wire by integrating around a closed loop that surrounds the wire and the result should be proportional to the current enclosed by the loop.

# Another Look at Ampere’s Law

- What if however, we place a capacitor in the circuit?
- If we use Ampere's law we see that it fails when we place our loop in between the plates of the capacitor.
- The current in between the plates is zero sense the flow of electrons is zero.

- What do we do now?

# Maxwell’s Solution

- In 1873 James Clerk Maxwell altered ampere's law so that it could account for the problem of the capacitor.

# Maxwell's Solution cont.

- The solution to the problem can be seen by recognizing that even though there is no current passing through the capacitor there is an electric flux passing through it.
- As the charge is building up on the capacitor, or if it is oscillating in the case of an ac circuit, the flux is changing with time.

# Maxwell’s Solution cont.

- Therefore, the expression for the magnetic flux around a capacitor is:

$$\oint \vec{B} \cdot d\vec{s} = \mu_o \varepsilon_o \frac{d\Phi_E}{dt}$$

# Maxwell’s Solution cont.

- If there is a dielectric between the plates of a capacitor that has a small conductivity then there will be a small current moving through the capacitor thus:

$$\oint \vec{B} \cdot d\vec{s} = \mu_o I + \mu_o \varepsilon_o \frac{d\Phi_E}{dt}$$

# Maxwell’s Solution cont.

- Maxwell proposed that this equation is valid for any arbitrary system of electric fields, currents, and magnetic fields.
- It is now known as the Ampere-Maxwell Law.
- The last term in the previous equation is known as the displacement current.

# What Have We Learned So Far?

- Gauss's Law for Electricity.

$$\oint_S \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_o}$$

- Gauss's Law for Magnetism.

$$\oint_S \vec{B} \cdot d\vec{A} = 0$$

# What Have We Learned So Far?

- Faraday's Law. $$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

- Maxwell-Ampere's Law. $$\oint \vec{B} \cdot d\vec{s} = \mu_o I + \mu_o \varepsilon_o \frac{d\Phi_E}{dt}$$

- The four previous equations are known as Maxwell's equations.

# Maxwell’s Equations

- The differential form of Maxwell's when dielectric and magnetic materials are present are as follows:

$$\nabla \cdot \vec{D} = \rho_f$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{H} = J_f + \frac{\partial \vec{D}}{\partial t}$$