Test 1: MA20104 Probability and Statistics

Time: 45 minutes   **Marks: 40**

Instructions: There are ten questions. You have to answer all. They carry equal marks.

Q.1: A fair dice is rolled twice independently and the observed face values X and Y respectively. What is the probability that $X^2 + Y^2 + XY$ is an even number? (answer should be correct up to three decimal places)

ANS 1: 0.25 (error range: 0.01)

Q.2: Ten cards (numbered 1 to 10) are randomly arranged in a row. Find the probability that cards numbered "3", "6" and "9" are placed together. (answer should be correct up to three decimal places)

ANS 2: 0.067 (error range: 0.005)

Q.3: Consider the set of all one-one and onto functions (bijections) from the set of English alphabets $\{A, B, ..., Z\}$ to the set of integers $\{1, 2, ..., 26\}$. If a function is chosen at random, then the probability that all the vowels will be mapped exactly to digits which are divisible by 5 is $\frac{1}{N}$, where N is an integer. Find the value of N.

ANS 3: 65780 (error range: 0)

Q.4: A fair dice is rolled once. If the upper face value is X, then a coin (which has probability $\frac{1}{X}$ of getting head) is tossed once. If a head is observed as a final outcome then what is the probability that '3' was observed when the dice was rolled? (answer should be correct up to three decimal places)

ANS 4: 0.136 (error range: 0.005)

Q.5: Suppose 50% of the emails that we receive in Gmail are spam emails. A software can detect 99% of the spam emails correctly, however it also incorrectly identifies 5% of non-spam emails as spam emails. Assume that an email is detected as spam by the software, then calculate the probability that it is not a spam email. (answer should be correct up to three decimal places)

ANS 5: 0.048 (error range: 0.005)

Q.6: Let A and B be two independent events of a random experiment such that $P(A) = 2P(B)$ and $P(A \cap B) = 0.15$. Find the value of $P(A^C \cap B^C)$. (answer should be correct up to two decimal places)

ANS 6: 0.33 (error range: 0.005)

Q.7: A machine is to be assembled by randomly selecting three components from a store having 100 components. However, it is known that there are 10 defective components in that collection. The machine will work if at least 2 of the components used in the assembly are non-defective. Determine the probability of assembling a working machine. (answer should be correct up to three decimal places)

ANS 7: 0.768 (error range: 0.005)

Q8. Three distinct integers are chosen at random from the first 13 positive integers. Compute the probability that the product ~~of~~ is divisible by 3. (answer should be correct up to three decimal places)

ANS 8: 0.706 (error range: 0.005)

Q.9: A drawer contains 5 different pairs of socks. If 4 pieces of socks are chosen at random without replacement, compute the probability that there is at least one complete pair among these 4 socks. (answer should be correct up to three decimal places)

ANS 9: 0.619 (error range: 0.005)

Q.10: A random experiment has two mutually exclusive and exhaustive possible outcomes, the first occurs with probability $\left(p^2 + \frac{p}{4}\right)$ and the second occurs with probability $\left(\frac{3-p}{4}\right)$, where $0 < p < 1$. Find the value of p.

ANS 10: 0.5 (error range: 0)

Q.11: An integer is chosen at random from the first 500 positive integers. Find the probability that the number chosen is divisible by 7 or 11. (answer should be correct up to two decimal places)

ANS 11: 0.22 (error range: 0.01)

Q.12: Let A, B, C be independent events with $P(A) = P(B) = \frac{1}{3}$ and $P(C) = \frac{1}{4}$. The value of $P((A - B) - C)$ is equal to $\frac{1}{N}$, where N is an integer. Find the value of N.

ANS 12: 6

Q.13: Let A and B be two events with $P(A \cup B) = 0.6, P(A) = 0.5$ and $P(B) = 0.3$. The value of $P(A^C|B)$ equals $\frac{1}{N}$, where N is an integer. Find the value of N.

ANS 13: 3

Q.14: Each of the coefficients a, b, c are determined by throwing a fair dice three times independently. The probability that the system of linear equations $ax + by = 0, bx + cy = 0$ has non-zero solutions equals $\frac{1}{N}$, where N is an integer. Find the value of N.

ANS 14: 27

Q.15: Three shooters S_1, S_2, S_3 hit a target successfully with respective probabilities $\frac{2}{5}, \frac{1}{3}, \frac{3}{7}$. They shoot simultaneously and independently of each other. Find the conditional probability that S_2 has missed the target given that there were exactly two hits. (answer should be correct up to three decimal places)

ANS 15: 0.414 (error range: 0.005)

Q.16: A missile has probability 0.3 of destroying its target and probability 0.7 of missing it. Assume that n missiles are fired simultaneously and independently of each other at the same target. Find the least value of n which gives the probability of destroying the target at least 0.9.

ANS 16: 7

Q17: Two Indians are to be seated in a row along with eight other persons (foreigners) randomly. Then find the probability that an Indian occupies the second position but not the first position. (answer should be correct up to three decimal places)

ANS 17: 0.178 (error range: 0.005)

Q.18: A class has ten students. Assuming none have them have birthday on 29th February, find the probability that exactly two of them have the same birthday and all others have distinct birthdays (answer should be correct up to three decimal places).

ANS 18: 0.112 (error range: 0.005)

Q.19: Let A, B, C be disjoint events with $P(A) = P(B) = \frac{1}{4}$ and $P(C) = \frac{1}{5}$. Find the value of $P((A \cup B^C) \cup (B^C \cup C^C)^C)$. (answer should be correct up to two decimal places)

ANS 19: 0.75 (error range: 0.001)

Q.20: From a set of 10 elements, a nonempty subset is chosen at random in the sense that all of the nonempty subsets are equally likely to be selected. Find the probability that the number of elements is divisible by 4. (answer should be correct up to three decimal places)

ANS 20: 0.249 (error range: 0.005)