CHAPTER 2

ELECTROSTATIC FIELDS

2.1 INTRODUCTION

Electrostatic fields are produced by charges at rest. The main objective of this chapter is to provide detailed concepts of electrostatics. They include:

- Fundamental concept of electric charges and charge distributions
- Coulomb's law and its applications
- Electric field intensity due to various charge distributions
- Gauss's law and concept of Gaussian surface
- Potential functions and potential gradient
- Energy stored in electrostatic fields
- Electric Dipoles and dipole moment

2.2 ELECTRIC CHARGE

Electric charge is a fundamental conserved property of some subatomic particles, which determines their electromagnetic interaction. The SI unit of electric charge is Coulomb. In electrostatic formulations charges have four types of idealisations, as described below.

2.2.1 Point Charge

Point charges are very small charges assumed to be of infinitesimally small volume, although they have finite volume considered as a single charge. For example, an electron is considered to be a point charge and has a charge of 1.6×10^{-19} Coulombs (C). The idealisation here is that the whole charge Q is concentrated at a point. In general, the point charges are denoted by Q or q.

2.2.2 Line Charge

This is a charge distribution in which the charge is distributed along a line like a filament, as shown in Figure 2.1(a). The charge per unit length along the line charge is called line charge density. It is denoted by ρ_L and defined as

$$\rho_L = \lim_{\Delta L \to 0} \frac{\Delta Q}{\Delta L} = \frac{dQ}{dL}$$

where ΔQ is small charge, and ΔL is small length.

2.2.3 Surface Charge

When a charge is confined to the surface of a conductor, it is said to be surface charge distribution, as shown in Figure 2.1(b). The charge per unit area over the surface is called the surface charge density. It is denoted by ρ_S and defined as

$$\rho_S = \lim_{\Delta s \to 0} \frac{\Delta Q}{\Delta S} = \frac{dQ}{dS}$$

where ΔQ is small charge, and ΔS is small area.

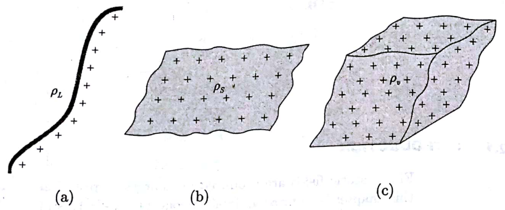

Figure 2.1: Various Charge Distributions (a) Line Charge Distribution, (b) Surface Charge Distribution, (c) Volume Charge Distribution

2.2.4 Volume Charge

When the charge is being distributed within a certain defined region, it is said to be the volume charge distribution, as shown in Figure 2.1(c). The charge per unit volume in the region is called volume charge density. It is denoted by ρ_v and defined as

$$\rho_v \equiv \lim_{\Delta v \to 0} \frac{\Delta Q}{\Delta v} = \frac{dQ}{dv}$$

where ΔQ is small charge, and Δv is small volume.

2.3 COULOMB'S LAW

According to Coulomb's law, the electrostatic force F between the two point charges Q_1 and Q_2 separated by a distance R, is given by

$$F = \frac{1}{4\pi\varepsilon} \frac{Q_1 Q_2}{R^2} = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \frac{Q_1 Q_2}{R^2}$$

where ε is the absolute permittivity of medium, ε_r is the relative permittivity of medium, and $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m is the permittivity in free space. In free space,

$$F = \frac{1}{4\pi\varepsilon_o} \frac{Q_1 Q_2}{R^2} = (9 \times 10^9)\left(\frac{Q_1 Q_2}{R^2}\right)$$

2.3.1 Vector Form of Coulomb's Law

Consider the two point charges Q_1 and Q_2 with separation distance R as shown in Figure 2.2(a). The force exerted by Q_1 on Q_2 is

$$\boldsymbol{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{R^2} \boldsymbol{a}_{R12}$$

where $\boldsymbol{a}_{R12}$ is a unit vector directed from Q_1 to Q_2. If two charges have the position vectors $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$, respectively, as shown in Figure 2.2(b); then the force acting on charge Q_2 due to charge Q_1 is

$$\boldsymbol{F}_{12} = \frac{Q_1 Q_2 (\boldsymbol{r}_2 - \boldsymbol{r}_1)}{4\pi\varepsilon_0 |\boldsymbol{r}_2 - \boldsymbol{r}_1|^3}$$

Similarly, force exerted by Q_2 on Q_1 is

$$\boldsymbol{F}_{21} = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{R^2} \boldsymbol{a}_{R21} = \frac{Q_1 Q_2 (\boldsymbol{r}_1 - \boldsymbol{r}_2)}{4\pi\varepsilon_0 |\boldsymbol{r}_1 - \boldsymbol{r}_2|^3} = -\boldsymbol{F}_{12}$$

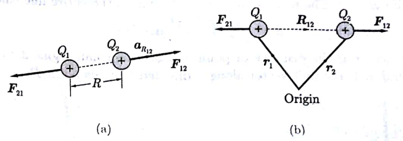

Figure 2.2: Illustration of Coulomb's Law: (a) Force Representation in Vector Form, (b) Electrostatic Force in terms of Position Vectors

2.3.2 Principle of Superposition

If there is a number of charges $Q_1, Q_2, ..., Q_n$ placed at points with position vectors $\boldsymbol{r}_1, \boldsymbol{r}_2, ..., \boldsymbol{r}_n$, respectively, then the resultant force $\boldsymbol{F}$ on a charge Q located at point $\boldsymbol{r}$ is the vector sum of the forces exerted on Q by each of the charges $Q_1, Q_2, ..., Q_n$. i.e.,

$$\boldsymbol{F} = \frac{QQ_1(\boldsymbol{r} - \boldsymbol{r}_1)}{4\pi\varepsilon_0 |\boldsymbol{r} - \boldsymbol{r}_1|^3} + \frac{QQ_2(\boldsymbol{r} - \boldsymbol{r}_2)}{4\pi\varepsilon_0 |\boldsymbol{r} - \boldsymbol{r}_2|^3} + ... + \frac{QQ_n(\boldsymbol{r} - \boldsymbol{r}_n)}{4\pi\varepsilon_0 |\boldsymbol{r} - \boldsymbol{r}_n|^3}$$

or

$$\boldsymbol{F} = \frac{Q}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{Q_i(\boldsymbol{r} - \boldsymbol{r}_i)}{|\boldsymbol{r} - \boldsymbol{r}_i|^3}$$

2.4 ELECTRIC FIELD INTENSITY

Electric field intensity is defined as the force per unit charge when placed in the field. If a point charge q placed in a field experiences a force $\boldsymbol{F}$ then the electric field intensity in the region is defined as

$$\boldsymbol{E} = \lim_{q \to 0} \frac{\boldsymbol{F}}{q}$$

or simply

$$\boldsymbol{E} = \frac{\boldsymbol{F}}{q}$$

It is seen that the electric field intensity is in the same direction as the force and is expressed in Newton per Coulomb (N/C) or volt per meter (V/m).

2.4.1 Electric Field Intensity due to a Point Charge

The electric field intensity due to a point charge Q at a distance R from the charge is given by

$$\boldsymbol{E} = \frac{Q}{4\pi\varepsilon_0 R^2} \boldsymbol{a}_R$$

If n point charges $Q_1, Q_2, ..., Q_n$ be located at points $\boldsymbol{r}_1, \boldsymbol{r}_2, ...\boldsymbol{r}_n$ then, using the principal of superposition the electric field intensity at point $\boldsymbol{r}$ is given as

$$\boldsymbol{E} = \frac{Q_1(\boldsymbol{r} - \boldsymbol{r}_1)}{4\pi\varepsilon_0 |\boldsymbol{r} - \boldsymbol{r}_1|^3} + \frac{Q_2(\boldsymbol{r} - \boldsymbol{r}_2)}{4\pi\varepsilon_0 |\boldsymbol{r} - \boldsymbol{r}_2|^3} + ... + \frac{Q_3(\boldsymbol{r} - \boldsymbol{r}_n)}{4\pi\varepsilon_0 |\boldsymbol{r} - \boldsymbol{r}_n|^3}$$

$$= \frac{1}{4\pi\varepsilon_0} \sum_{k=1}^{n} \frac{Q_k(\boldsymbol{r} - \boldsymbol{r}_k)}{|\boldsymbol{r} - \boldsymbol{r}_k|^3}$$

2.4.2 Electric Field Intensity due to a Line Charge Distribution

Consider a line charge distribution with charge density ρ_L as shown in Figure 2.3. The electric field intensity due to the entire line charge is

$$E = \frac{1}{4\pi\varepsilon_0}\int_L \frac{\rho_L dL}{r^2} a_r \text{ (V/m)} \qquad \text{...(2.1)}$$

where r is the distance of point P from the small segment of line charge, and a_r is the unit vector along r directed towards point P.

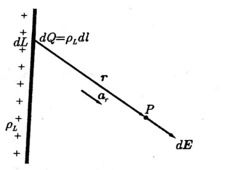

Figure 2.3 : Electric Field Intensity due to a Small Segment of Line Charge

Let us generalize equation (2.1) for some special cases.

Electric Field Intensity due to a Finite Straight Line Charge

Consider the finite straight line charge AB shown in Figure 2.4. The net electric field intensity at point P due to the finite straight line charge is given by

$$E = \frac{\rho_L}{4\pi\varepsilon_0 R}(\sin\alpha_1 - \sin\alpha_2)a_z + \frac{\rho_L}{4\pi\varepsilon_0 R}(\cos\alpha_2 - \cos\alpha_1)a_R \qquad \text{...(2.2)}$$

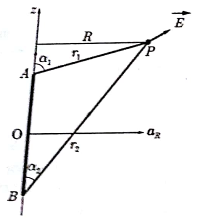

Figure 2.4 : Electric Field Intensity due to a Finite Straight Line Charge

Electric Field Intensity due to an Infinite Straight Line Charge

As a special case of the above expression, for an infinite line charge, point A is located at $(z = \infty)$ and B at $(z = -\infty)$. So,

$$\alpha_1 = \pi \quad \text{and} \quad \alpha_2 = 0$$

Substituting these values in Eq (2.2), we get

$$E = \frac{\rho_L}{2\pi\varepsilon_0 R}a_R$$

Electric Field Intensity due to a Charged Circular Ring

Consider the charged circular ring shown in Figure 2.5. The circular ring of radius a carries a uniform charge ρ_L C/m and is placed on the xy-plane with axis the same as the z-axis. The net electric field intensity at point P due to the charged circular ring is given by

$$E = \frac{\rho_L a h a_z}{2\varepsilon_0[h^2 + a^2]^{3/2}}$$

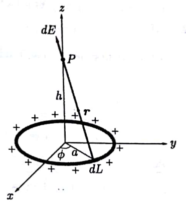

Figure 2.5: Electric Field Intensity due to a Charged Circular Ring

2.4.3 Electric Field Intensity due to Surface Charge Distribution

Figure 2.6 shows a body with surface charges. Let ρ_S be the surface charge density in C/m^2. Then, the electric field intensity due to the entire surface charge is given by

$$E = \frac{1}{4\pi\varepsilon_0}\int_S \frac{\rho_S dS}{r^2} a_r \qquad \text{...(2.3)}$$

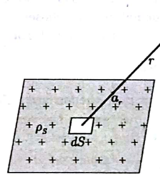

Figure 2.6: Electric Field Intensity due to a Surface Charge Distribution

Electric Field Intensity due to an Infinite Sheet Charge

For an infinite sheet of charge, the electric field intensity at any point P is defined as

$$E = \frac{\rho_S}{2\varepsilon_0} a_n \qquad \text{...(2.4)}$$

where a_n is a unit vector normal to the sheet and directed toward point P.

2.5 ELECTRIC FLUX DENSITY

As the electric field intensity is dependent on the medium, we define a new vector field D which is independent of the medium. This vector field is called electric flux density and given by

$$D = \varepsilon_0 E$$

Electric flux density is also called electric displacement vector. It is measured in C/m^2.

2.5.1 Electric Flux

The total electric flux passing through a surface S is given by

$$\psi = \int_S D \cdot dS$$

where dS is the surface area vector directed normal to the surface.

2.6 GAUSS'S LAW

In Electrostatics, the Gauss's law states that the electric flux passing through any closed surface is equal to the total charge enclosed by that surface, i.e.

$$\psi = \oint_S D \cdot dS = Q_{enclosed}$$

This expression can be generalized in the following two forms:

1. **Integral Form:** The Gauss's Law can be expressed in integral form as

$$\oint_S D \cdot ds = \int_v \rho_v dv$$

where ρ_v is the volume charge density.

2. **Differential Form:** In differential form, the Gauss's Law is defined as

$$\nabla \cdot D = \rho_v$$

2.6.1 Gaussian Surface

The closed surface to which Gauss' law is applied is known as Gaussian surface. The Gaussian surface must satisfy the following two conditions :

1. **The D and E field lines are normal to the chosen surface :** This condition removes the dot product from the integral, leaving

$$\int_S DdS = Q_{enclosed}$$

2. **The D and E field lines are constant over the surface :** This condition means that D is independent of the position on the surface and hence can be removed from the integral, leaving

$$D\int_S dS = Q_{enclosed}$$

Following table provides some examples of systems in which Gauss' law is applicable for electric field computation, with the corresponding Gaussian surface.

Table 2.1: Gaussian surfaces for various charge configurations

System	Symmetry	Gaussian surface
Point charge	Spherical	Concentric sphere
Infinite rod	Cylindrical	Coaxial cylinder
Infinite plane	Planner	Gaussian "Pillbox"
Sphere, spherical shell	Spherical	Concentric sphere

2.7 ELECTRIC POTENTIAL

The electric potential at a point is defined as the work done to bring a unit positive charge from infinity to that point. The unit of electric potential is Joule per Coulomb (J/C) or Volt.

2.7.1 Potential Difference :

The potential difference between two points A and B is the work done to bring a unit positive charge from point B to point A. It is defined as

$$V_{AB} = -\int_B^A E \cdot dL$$

where V_{AB} represents the potential difference between the two points such that B is the initial point and A is the final point. Following are some important points related to potential difference.

POINTS TO REMEMBER

1. If V_{AB} is positive, there is a gain in the potential energy in the movement and hence an external agent performs the work.
2. Conversely, if V_{AB} is negative, there is a loss in the potential energy in moving the charge from B to A. This means that the work is done by the field.

2.7.2 Potential Gradient

The rate of change of potential with respect to the distance is called the potential gradient. Electric field is equal to negative of potential gradient, i.e.

$$E = -\nabla V$$

This relation can be generalised for the three coordinate systems as

1. Cartesian coordinate system:

$$E = -\left[\frac{\partial V}{\partial x} a_x + \frac{\partial V}{\partial y} a_y + \frac{\partial V}{\partial z} a_z\right]$$

2. Cylindrical coordinate system:

$$E = -\left[\frac{\partial V}{\partial \rho} a_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi} a_\phi + \frac{\partial V}{\partial z} a_z\right]$$

3. Spherical coordinate system:

$$E = -\left[\frac{\partial V}{\partial r} a_r + \frac{1}{r}\frac{\partial V}{\partial \theta} a_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi} a_\phi\right]$$

2.7.3 Equipotential Surfaces

Equipotential surface is a surface with equal value of potential at every point on the surface. Following are some important properties of equipotential surfaces:

PROPERTIES OF EQUIPOTENTIAL SURFACE

1. The electric field lines are perpendicular to the equipotential surfaces and are directed from higher to lower potentials.
2. The tangential component of the electric field along the equipotential surface is zero; otherwise non-vanishing work would be done to move a charge from one point on the surface to the other.

3. No work is required to move a particle along an equipotential surface.
4. The equipotential surfaces for a point charge or a sphere with uniform charge distribution are spheres concentric with the charge.
5. The equipotential surfaces for a line charge or a cylinder with uniform charge distribution are concentric cylinders centred on the axis of the charge distribution.
6. The equipotential surfaces for a flat surface with uniform charge distribution are planes parallel to the surface.

2.8 ENERGY STORED IN ELECTROSTATIC FIELD

Electrostatic energy is defined as the energy required to establish the given charge distribution in space. The electrostatic energy in different charge distributions are determined below.

2.8.1 Energy Stored in a Region with Discrete Charges

Consider a region with n point charges $Q_1, Q_2, \ldots\ldots, Q_n$ located at points $P_1, P_2, \ldots\ldots, P_n$ respectively. If the total electrostatic potential at the points $P_1, P_2, \ldots\ldots, P_n$ be respectively $V_1, V_2, \ldots.., V_n$. Then, the energy stored in the charge system is given by

$$W = \frac{1}{2}[Q_1 V_1 + Q_2 V_2 + \ldots\ldots\ldots\ldots + Q_N V_N]$$

or

$$W = \frac{1}{2}\sum_{i=1}^{N} Q_i V_i$$

2.8.2 Energy Stored in a Region with Continuous Charge Distribution

If, instead of point charges, the region has a continuous charge distribution, the summation becomes integration. The electrostatic energy for line, surface and volume charge distributions are given below :

1. Line charge distribution

$$W = \frac{1}{2}\int_L \rho_L V dl$$

2. Surface charge distribution

$$W = \frac{1}{2}\int_S \rho_S V dS$$

3. Volume charge distribution

$$W = \frac{1}{2}\int_v \rho_v V dv.$$

where ρ_L, ρ_S, ρ_v are the charge density and V is the electric potential in the region.

2.8.3 Electrostatic Energy in terms of Electric Field Intensity

In a certain region with continuous charge distribution, assume that the electric field intensity is $\boldsymbol{E}$, and the electric flux density is $\boldsymbol{D}$. Then, the electrostatic energy stored in the system is given by

$$W = \frac{1}{2}\int_v \boldsymbol{D} \cdot \boldsymbol{E}\, dv$$

From the above expression, we define the electrostatic energy density in the region as

$$w = \frac{dW}{dv} = \frac{d}{dv}\left(\frac{1}{2}\int_v \boldsymbol{D} \cdot \boldsymbol{E} dv\right)$$

$$= \frac{1}{2}(\boldsymbol{D} \cdot \boldsymbol{E}) = \frac{1}{2}\varepsilon_0 E^2$$

2.9 ELECTRIC DIPOLE

An electric dipole is formed when two point charges of equal magnitude but opposite sign are separated by a small distance. Figure 2.7 shows an electric dipole with charges $+q$ and $-q$ separated by a small distance $\boldsymbol{d}$.

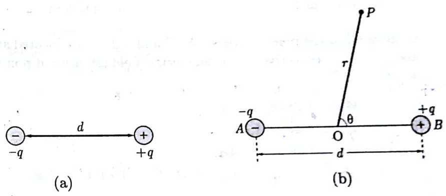

Figure 2.7 : (a) Electric Dipole, (b) Illustration of Electric Field due to a dipole

2.9.1 Electric Dipole Moment

The dipole moment is defined as the product of the small charge q and the distance d between the charges. It is a vector quantity and denoted by $\boldsymbol{p}$, i.e.

$$\boldsymbol{p} = q\boldsymbol{d}$$

where $\boldsymbol{d}$ is the vector joining the negative charge to the positive charge. The line along the direction of dipole moment is called the axis of the dipole.

2.9.2 Electric Potential due to a Dipole

Consider a point P located at a distance r from the dipole as shown in Figure 2.7(b). The electrostatic potential at point P due to the dipole is given by

$$V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$$

where p is the dipole moment.

2.9.3 Electric Field Intensity due to a Dipole

Electric field intensity at point P due to the dipole is obtained as

$$\boldsymbol{E} = -\nabla V$$

$$= \frac{p}{4\pi\varepsilon_0 r^3}[2\cos\theta \boldsymbol{a}_r + \sin\theta \boldsymbol{a}_\theta]$$

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EXERCISE 2.1

MCQ 2.1.1 Two point charges of 9 C and 36 C are located on x-axis at a separation of 3 m. A third point charge q is placed on the x-axis at a distance d from the 36 C charge which makes the entire system in equilibrium. The value of q and d are

(A) 4 C and 1 m
(B) -4 C and 2 m
(C) 4 C and 2 m
(D) -4 C and 1 m

MCQ 2.1.2 Consider that the point charges -5 nC and $+2$ nC are located at $(-4,0,-2)$ and $(-5,0,3)$ respectively The net electric field intensity at point $(-7,3,-1)$ will be

(A) $-1.004\boldsymbol{a}_z - 1.284\boldsymbol{a}_y + 1.4\boldsymbol{a}_x$
(B) $1.004\boldsymbol{a}_z - 1.284\boldsymbol{a}_y + 1.4\boldsymbol{a}_x$
(C) $-1.004\boldsymbol{a}_x - 1.284\boldsymbol{a}_y + 1.4\boldsymbol{a}_z$
(D) $+1.004\boldsymbol{a}_x + 1.284\boldsymbol{a}_y + 1.4\boldsymbol{a}_z$

MCQ 2.1.3 Which of the following charge distribution produces the electric field intensity ?

$$\boldsymbol{E} = 2xy\boldsymbol{a}_x + 4yz\boldsymbol{a}_y + 6xz\boldsymbol{a}_z \text{ V/m ?}$$

(A) infinite line charge of 2 nC/m along x-axis
(B) spherical shell of charge density 3 nC/m^3
(C) plane sheet of charge density 3 nC/m^2 at x-y plane
(D) field doesn't exist

MCQ 2.1.4 An infinite line charge of 1 μC/m is located on the z-axis. Electric field due to the line charge at point $(-2,-1,5)$ will be

(A) $2.4\boldsymbol{a}_x + 1.8\boldsymbol{a}_y$
(B) $7.2\boldsymbol{a}_x + 14.4\boldsymbol{a}_y$
(C) $-7.2\boldsymbol{a}_x - 3.6\boldsymbol{a}_y$
(D) $-2\boldsymbol{a}_x - \boldsymbol{a}_y$

MCQ 2.1.5 Electric field intensity at any point (x,y,z) in free space is $\boldsymbol{E} = x^2\boldsymbol{a}_x + 2xy\boldsymbol{a}_y$. The electric flux density at the point $(-1,0,1)$ will be

(A) 0
(B) $\varepsilon_0\boldsymbol{a}_x$
(C) $-\varepsilon_0\boldsymbol{a}_x$
(D) $4\pi\varepsilon_0\boldsymbol{a}_y$

Common Data For Q. 6 and 7 :

Volume charge density in the free space in spherical coordinate system is given by

$$\rho_v = \begin{cases} \dfrac{1}{r^2}\text{ C/m}^3 & 0 < r < 3\text{ m} \\ 0 & r > 3\text{ m} \end{cases}$$

MCQ 2.1.6 Net electric flux crossing the surface $r = 1$ m is

(A) 4π C
(B) $-\pi$ C
(C) 2π C
(D) 0

MCQ 2.1.7 Electric flux density at $r = 1$ m is

(A) $\frac{\pi}{2}\boldsymbol{a}_r$ C/m^2
(B) $\boldsymbol{a}_r$ C/m^2
(C) $4\pi\boldsymbol{a}_r$ C/m^2
(D) $\boldsymbol{a}_r + \boldsymbol{a}_\theta$ C/m^2

MCQ 2.1.8 A point charge 8 C is located at the origin. The total electric flux crossing the portion of plane $x + y = 2$ m lying in the first octant is

(A) 1 C
(B) 4 C
(C) 1 C/m
(D) 4 C/m

MCQ 2.1.9 A uniform volume charge density ρ_v C/m^3 is distributed inside the region defined by a cylindrical surface of cross sectional radius a. The electric field intensity at a distance $r (< a)$ from the cylindrical axis is proportional to

(A) r
(B) $\frac{a}{r}$
(C) $\frac{1}{r^2}$
(D) ar^2

Common Data For Q. 10 to 12 :

Charge density inside a hollow spherical shell of radius $r = 4$m centered at origin is defined as

$$\rho_v = \begin{cases} 0 & \text{for } r \le 2 \\ \dfrac{4}{r^2}\text{ c/m}^3 & \text{for } 2 < r \le 4 \end{cases}$$

MCQ 2.1.10 The Electric field intensity at any point in the region $r \le 2$ will be

(A) -1 V/m
(B) -4 V/m
(C) 0
(D) 2 V/m

MCQ 2.1.11 Electric field intensity at $r = 3$ will be

(A) $\frac{4}{9\varepsilon_0}\boldsymbol{a}_r$
(B) $\frac{4}{\varepsilon_0}\boldsymbol{a}_r$
(C) $\frac{20}{9\varepsilon_0}\boldsymbol{a}_r$
(D) $\frac{9}{\varepsilon_0}\boldsymbol{a}_r$

MCQ 2.1.12 If the region outside the spherical shell is charge free then what will be the electric field intensity at $r = 5$?

(A) $\frac{1}{3\varepsilon_0}\boldsymbol{a}_r$
(B) $\frac{16\pi}{\varepsilon_0}\boldsymbol{a}_r$
(C) $\frac{25}{8}\varepsilon_0\boldsymbol{a}_r$
(D) $\frac{8}{25}\varepsilon_0\boldsymbol{a}_r$

MCQ 2.1.13 In a certain region the electric flux density is $\boldsymbol{D} = \frac{\cos\theta}{r^3}\boldsymbol{a}_r + \frac{\sin\theta}{2r^3}\boldsymbol{a}_\theta$ C/m^2. Volume charge density in the region will be

(A) 0 C/m^3
(B) $\frac{2\cos\theta}{r^4}$ C/m^3
(C) $\frac{\sin\theta}{r^3}$ C/m^3
(D) $\frac{4}{r^3}$ C/m^3

Common Data For Q. 14 and 15 :

In the entire free space electric potential is given by

$$V = xy^2z^3 + \frac{3}{2}\ln(x^2 + 2y^2 + 3z^2)$$

MCQ 2.1.14 Electric field at point $P(3,2,-1)$ will be

(A) $7.1\boldsymbol{a}_x + 22.8\boldsymbol{a}_y - 71.1\boldsymbol{a}_z$
(B) $3.6\boldsymbol{a}_x + 11.4\boldsymbol{a}_y - 35.6\boldsymbol{a}_z$
(C) $3.6\boldsymbol{a}_x - 11.4\boldsymbol{a}_y + 35.6\boldsymbol{a}_z$
(D) $2.2\boldsymbol{a}_x - 11.4\boldsymbol{a}_y + 35.6\boldsymbol{a}_z$

MCQ 2.1.15 Electric flux density at point P will be

(A) $31.4\boldsymbol{a}_x + 101\boldsymbol{a}_y - 314.5\boldsymbol{a}_z\ \text{pC/m}^2$
(B) $62.8\boldsymbol{a}_x + 202\boldsymbol{a}_y - 629\boldsymbol{a}_z\ \text{nC/m}^2$
(C) $-0.095\boldsymbol{a}_x - 0.304\boldsymbol{a}_y + 0.948\boldsymbol{a}_z\ \text{nC/m}^2$
(D) $7.1\boldsymbol{a}_x + 22.8\boldsymbol{a}_y - 71.1\boldsymbol{a}_z\ \text{pC/m}^2$

MCQ 2.1.16 An electric dipole consists of two point charges of equal and opposite magnitude$\pm Q$ is lying along x-axis such that $+Q$ is at $x = d/2$ and $-Q$ is at $x = -d/2$. Electric field due to the dipole at any point (r,θ,ϕ) in spherical coordinate system is given by

$$\boldsymbol{E} = \frac{Qd}{4\pi\varepsilon_0 r^3}[2\cos^2\theta\boldsymbol{a}_r + \sin\theta\boldsymbol{a}_\theta] \qquad \text{where } r >> d$$

The force applied by the dipole on a charge of $+1$ C located at point $(0,y,0)$ is

(A) $\frac{-Qd}{4\pi\varepsilon_0 y^3}\boldsymbol{a}_z$
(B) $\frac{+Qd}{4\pi\varepsilon_0 y^3}\boldsymbol{a}_z$
(C) $\frac{-Qd}{4\pi\varepsilon_0 y^3}\boldsymbol{a}_z + \frac{2Qd}{4\pi\varepsilon_0 r^3}\boldsymbol{a}_y$
(D) $\frac{-Qd}{4\pi\varepsilon_0 y^3}\boldsymbol{a}_x$

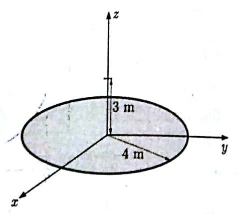
MCQ 2.1.17 Two equal point charges of $+\frac{1}{2}$ nC each are located at points $(-1,0,0)$ and $(1,0,0)$ respectively. What will be the position of third point charge of $+\sqrt{2}$ nC such that the net electric field $\boldsymbol{E} = 0$ at $(0,1,0)$?

(A) $(-1,0,0)$
(B) $(0,-1,0)$
(C) $(3,0,0)$
(D) $(0,3,0)$

MCQ 2.1.18 Plane $3x + 4y = 0$ carries a uniform charge distribution with $\rho_S = 2\ \text{nC/m}^2$. The electric field intensity at point $(1,0,3)$ will be

(A) $-67.8\boldsymbol{a}_x - 90.48\boldsymbol{a}_y$ V/m
(B) $67.85\boldsymbol{a}_x + 90.48\boldsymbol{a}_y$ V/m
(C) $3\boldsymbol{a}_x + 4\boldsymbol{a}_y$ V/m
(D) $-3\boldsymbol{a}_x - 4\boldsymbol{a}_y$ V/m

MCQ 2.1.19 Electric field intensity at a distance 3 m above the center of a circular loop of radius 4 m lying in the xy-plane and carrying a uniform line charge $+2$ nC/m as shown in the figure is



(A) $21.72\boldsymbol{a}_z + 10.86\boldsymbol{a}_z$ V/m
(B) $10.86\boldsymbol{a}_z$ V/m
(C) $10.86\boldsymbol{a}_z + 21.72\boldsymbol{a}_z$ V/m
(D) $72\boldsymbol{a}_z$ V/m

MCQ 2.1.20 Consider a point charge Q is located at the origin. Divergence of the electric flux density produced by the charge is

(A) 0, at all points
(B) +1, at all points
(C) +1, at all points except origin
(D) 0, at all points except origin

MCQ 2.1.21 A dipole having a moment $\boldsymbol{p} = 4\pi\varepsilon_0\boldsymbol{a}_z$ C-m is located at origin in free space. If the electric field produce due to the dipole is given by $\boldsymbol{E} = E_x\boldsymbol{a}_x + E_y\boldsymbol{a}_y + E_z\boldsymbol{a}_z$ then surface on which $E_z = 0$ but $E_x,\ E_y \neq 0$ will be

(A) a cone of angle 54.7°
(B) a cone of angle 125.3°
(C) (a) and (b) both
(D) none of these

MCQ 2.1.22 An infinite line charge $+1$ nC/m is lying along entire z-axis. If the electric potential at the point $(1,\pi/2,2)$ due to the line charge is zero then the electric potential at any point (ρ,ϕ,z) will be

(A) $\frac{18}{\rho}$ volt
(B) $18\left(\ln\left(\frac{1}{\rho}\right)\right)$
(C) $\frac{10^{-9}}{2}\ln\left(\frac{1}{\rho}\right)$
(D) $\frac{9\times10^9}{2}\ln\left(\frac{1}{\rho}\right)$

Common Data For Q. 23 and 24 :

Electric field at any point (r,θ,ϕ) in free space is given by

$$\boldsymbol{E} = \frac{2r}{(r^2+4)^2}\boldsymbol{a}_r$$

MCQ 2.1.23 The electric potential will be maximum at

(A) infinity
(B) origin
(C) at $r = -2$
(D) $r = +2$

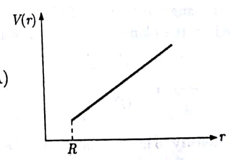
MCQ 2.1.24 Potential difference between the spherical surfaces $r = 0$ and $r = 2$ will be

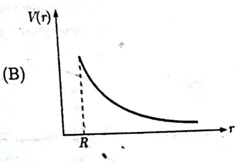
(A) 1/2 volt
(B) 1 volt
(C) 1/8 volt
(D) 1/4 volt

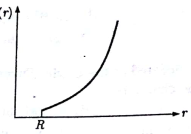
Common Data For Q. 25 to 27 :

A uniformly charged solid sphere of radius R has the total charge Q. Consider the electric potential at a distance r from the centre of the sphere is $V(r)$.

MCQ 2.1.25 For $r > R$, plot of $V(r)$ versus r will be

(A)


(B)


(C)


(D) None of these

MCQ 2.1.26 With the increase in r potential $V(r)$ inside the charged sphere will
(A) increase
(B) decrease
(C) remain constant
(D) be zero always

MCQ 2.1.27 If $R = 1\,\text{m}$ and $Q = 1\,\text{C}$ then the total stored energy inside the sphere will be
(A) $4.34 \times 10^9\,\text{J}$
(B) $6.75 \times 10^9\,\text{J}$
(C) 4.5×10^9
(D) $5.4 \times 10^9\,\text{J}$

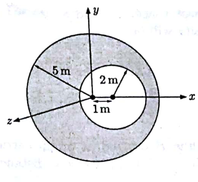
MCQ 2.1.28 Three point charges Q, $-2Q$ and Q are located at $(a,0,0)$, $(0,0,0)$ and $(-a,0,0)$ respectively. The electric field intensity at any point $(x,0,0)$ for $x >> a$ is
(A) $K\left(\frac{6Qa^2}{x^4}\right)$ (B) $K\left(\frac{3Qa^2}{x^4}\right)$
(C) $K\left(\frac{Qa^2}{6x^4}\right)$ (D) zero

MCQ 2.1.29 A volume charge is distributed throughout a sphere of radius R, and centered at the origin, with uniform density $\rho_v\,\text{C/m}^3$. The electric field intensity at a distance r from the origin is

	inside the sphere($r \le R$)	outside the sphere($r > R$)
(A)	$\frac{\rho_v}{\varepsilon_0}\left(\frac{r^2}{3R}\right)\boldsymbol{a}_r$	$\frac{\rho_v}{\varepsilon_0}\left(\frac{R^3}{3r^2}\right)\boldsymbol{a}_r$
(B)	$\frac{\rho_v}{\varepsilon_0}\left(\frac{r}{3}\right)\boldsymbol{a}_r$	$\frac{\rho_v}{\varepsilon_0}\left(\frac{R^3}{3r^2}\right)\boldsymbol{a}_r$
(C)	$\frac{\rho_v}{\varepsilon_0}\left(\frac{r^2}{3R}\right)\boldsymbol{a}_r$	$\frac{\rho_v}{\varepsilon_0}\left(\frac{r^3}{3R^2}\right)\boldsymbol{a}_r$
(D)	$\frac{\rho_v}{\varepsilon_0}\left(\frac{r}{3}\right)\boldsymbol{a}_r$	$\frac{\rho_v}{\varepsilon_0}\left(\frac{r^3}{3R^2}\right)\boldsymbol{a}_r$

MCQ 2.1.30 An infinite line charge of uniform density ρ_L is situated along the x-axis. The total electric field flux crossing the portion of plane $y + z = 1\,\text{m}$ lying in the first octant and bounded by the planes $x = 0$ and $x = 1\,\text{m}$
(A) $\frac{\rho_L}{2\varepsilon_0}$ (B) $\frac{\rho_L}{8\varepsilon_0}$
(C) $\frac{\rho_L}{4\varepsilon_0}$ (D) $\frac{4\rho_L}{\varepsilon_0}$

MCQ 2.1.31 Volume charge of uniform density $5\,\text{nC/m}^3$ is distributed in the region between two infinitely long, parallel cylindrical surfaces of radii $5\,\text{m}$ and $2\,\text{m}$ and with their axes separated by distance of $1\,\text{m}$ as shown in the figure.



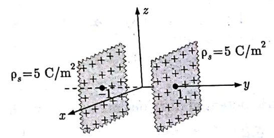
The electric field intensity in the charge-free region inside the cylindrical surface of radius $2\,\text{m}$ is
(A) $282.5\boldsymbol{a}_x\,\text{V/m}$ (B) $5.65 \times 10^{11}\,\text{V/m}$
(C) $3.54\boldsymbol{a}_x\,\text{mV/m}$ (D) $1.77 \times 10^{-12}\,\text{V/m}$

MCQ 2.1.32 A volume charge is distributed throughout a sphere of radius R and centered at the origin with uniform density $\rho_v\,\text{C/m}^3$. The electric potential at a distance r from the origin is

	inside the sphere($r \le R$)	outside the sphere($r > R$)
(A)	$\frac{2\rho_v}{\varepsilon_0}\left(r^2 - \frac{R^2}{3}\right)$	$\frac{\rho_v R^3}{3\varepsilon_0 r}$
(B)	$\frac{\rho_v}{2\varepsilon_0}\left(R^2 - \frac{r^2}{3}\right)$	$\frac{\rho_v R^3}{3\varepsilon_0 r}$
(C)	$\frac{2\rho_v}{\varepsilon_0}\left(R^2 - \frac{r^2}{3}\right)$	$\frac{\rho_v R^3}{3\varepsilon_0 r}$
(D)	$\frac{\rho_v}{2\varepsilon_0}\left(R^2 - \frac{r^2}{3}\right)$	$\frac{3\rho_v R^3}{\varepsilon_0 R}$

Common Data For Q. 33 and 34:

Two infinite uniform sheets of charge, each with density $5\,\text{C/m}^2$, are located at $y = +1$ and $y = -1$ as shown in figure.



MCQ 2.1.33 Electric field intensity at the origin will be
(A) 0 (B) $\frac{10}{2\varepsilon_0}\boldsymbol{a}_y\,\text{V/m}$
(C) $-\frac{5}{2\varepsilon_0}\boldsymbol{a}_y\,\text{V/m}$ (D) $\frac{5}{2\varepsilon_0}\boldsymbol{a}_y\,\text{V/m}$

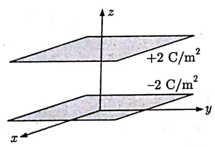
MCQ 2.1.34 If a test charge of $5\,\mu\text{C}$ is placed at point $(2,5,4)$ then the force applied by the sheets on test charge is
(A) $2.83\,\text{mN}$ (B) $2.5 \times 10^{-14}\,\text{N}$
(C) $2.83\,\text{N}$ (D) $5.65 \times 10^2\,\text{N}$

MCQ 2.1.35 As we move away from the sheet charge located at $y = -1$ in the region $y < -1$, the electric field intensity will be
(A) linearly increasing
(B) linearly decreasing
(C) constant
(D) zero

MCQ 2.1.36 Consider a hollow sphere of radius R centred at origin carries a uniform surface charge density ρ_S. The electric field intensity at distance r from the center of the sphere is

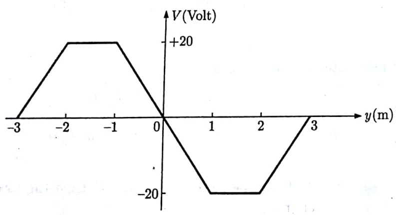
	inside the sphere($r \le R$)	outside the sphere($r > R$)
(A)	0	$\frac{\rho_s}{\varepsilon_0}\left(\frac{R}{r}\right)^2 \boldsymbol{a}_r$
(B)	$\frac{\rho_s}{\varepsilon_0}\boldsymbol{a}_r$	$\frac{\rho_s}{\varepsilon_0}\left(\frac{R}{r}\right)^2 \boldsymbol{a}_r$
(C)	$\frac{\rho_s}{\varepsilon_0}\boldsymbol{a}_r$	0
(D)	0	$\frac{\rho_s}{\varepsilon_0}\boldsymbol{a}_r$

MCQ 2.1.37 An air filled parallel plate capacitor is arranged such that the lower side of upper plate carries surface charge density $2\,\text{C/m}^2$ and upper side of lower plate carries surface charge density $-2\,\text{C/m}^2$ as shown in figure. The electric field intensity between the plates will be

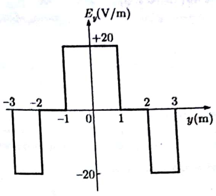


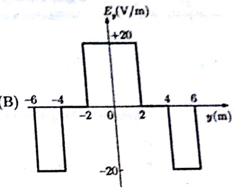
(A) $-\frac{2}{\varepsilon_0}\boldsymbol{a}_z$ (B) $\frac{2}{\varepsilon_0}\boldsymbol{a}_z$
(C) $-\frac{4}{\varepsilon_0}\boldsymbol{a}_z$ (D) $\frac{4}{\varepsilon_0}\boldsymbol{a}_z$

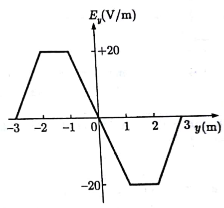
MCQ 2.1.38 In a certain region electric potential distribution is as shown in the figure.

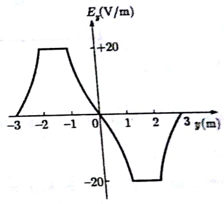


The corresponding plot of electric field component E_y will be

(A)


(B)


(C)


(D)


MCQ 2.1.39 Two electrons are moving with equal velocities in opposite directions. A uniform electric field is applied along the direction of the motion of one of the electrons, so the electron gets accelerated while the electron moving in opposite direction gets decelerated. If the gain in the kinetic energy of accelerating electron is $K.E_{Gain}$ and the loss in Kinetic energy of decelerating electron is $K.E_{Loss}$ then the correct relation between them is
(A) $K.E_{Gain} = K.E_{Loss}$
(B) $K.E_{Gain} > K.E_{Loss}$
(C) $K.E_{Gain} < K.E_{Loss}$
(D) Can't be determined as initial velocities are not given

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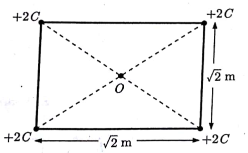
EXERCISE 2.2

Common Data For Q. 1 and 2 :

Four equal charges of $+2\,\text{C}$ are being placed at the corners of the square of side $\sqrt{2}$ m in free space as shown in figure.



QUES 2.2.1 The net force on a test charge $+1\,\text{nC}$ at the centre O of the square will be ____ N.

QUES 2.2.2 If one of the four charges is being removed, what will be the magnitude of the net force (in Newton) on the test charge $+1\,\text{nC}$ placed at the centre ?

QUES 2.2.3 The three point charges, each $+5\,\text{nC}$, are located on the z-axis at $z = -1, 0, 1$ in free space. The electric field intensity at point $P(0,0,3)$ will be ____ N/m in $\boldsymbol{a}_z$ direction.

QUES 2.2.4 Charges $+Q$ and $+2Q$ are separated by a distance 1 m. What will be the distance (in meter) of point P form $+Q$ charge such that the net electric field intensity at P is zero. ?

Common Data For Q. 5 and 6 :

A uniform volume charge density of $2\,\mu\text{C/m}^3$ is present throughout the spherical shell extending from $r = 2$ cm to $r = 3$ cm.

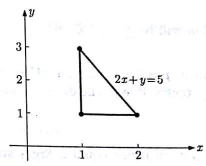
QUES 2.2.5 The total charge present throughout the spherical shell will be ____ pC.

QUES 2.2.6 For what value of a (in cm) half of the total charge will be located in the region $2\text{ cm} < r < a$?

QUES 2.2.7 Electrons are moving randomly in a fixed region in free space. During a time interval T the probability of finding an electron in a subregion of volume $10^{-12}\,\text{m}^3$ is 30%.The volume charge density in the subregion for the time interval will be ____ nC/m^3.

QUES 2.2.8 Total stored charge on the cylindrical surface $\rho = 2$, $0 < z < 1$ m having surface charge density $\rho^2 z\,\mu\text{C/m}^2$ is ____ μC.

QUES 2.2.9 Consider a triangular surface in the plane $z = 0$ as shown in the figure.



If the triangular surface has charge density $\rho_S = 3xy\,\text{C/m}^2$ then the total charge on it will be ____ Coulomb.

QUES 2.2.10 A circular disk of radius 5 m has surface charge density $\rho_S = 3r$, where r (≤ 5 m) is the distance of any point on the disk from its centre. The total charge stored on the disk is ____ Coulomb.

QUES 2.2.11 Consider the electric field intensity in some region is found to be $\boldsymbol{E} = 3r^2\boldsymbol{a}_r\,\text{V/m}$, in spherical coordinate system. What will be the total charge stored (nC) in a sphere of radius 2 m, centered at origin ?

QUES 2.2.12 If electric flux density in a certain region is

$$\boldsymbol{D} = (3y^2 + 4z)\,\boldsymbol{a}_x + 2xy\boldsymbol{a}_y + 4x\boldsymbol{a}_z\,\text{C/m}^2$$

The total charge enclosed by the cube $0 \leq x \leq 2, 0 \leq y \leq 2, -1 \leq z \leq 1$ is ____ Coulomb.

QUES 2.2.13 Two point charges $+1\,\mu\text{C}$ and $-1\,\mu\text{C}$ are being located at points (0,0,1) and (0,0,−1) respectively. The net electric potential at point $P(-3,0,-4)$ due to the two charges will be ____ Volt.

Common Data For Q. 14 and 15 :

In the region of free space that includes the cubical volume $0 < x, y, z < 1$, electric flux density is given by

$$\boldsymbol{D} = x^2 y\boldsymbol{a}_x + y^2 x^2\,\boldsymbol{a}_y\,\text{C/m}^2$$

QUES 2.2.14 What is the total flux (in Coulomb) leaving the closed surface of the cube ?

QUES 2.2.15 At center of the cube, div $\boldsymbol{D} =$ ____ C/m^3.

Common Data For Q. 16 and 17 :

In free space, flux charge density is given by

$$D = \begin{cases} 5r^2 a_r \text{ nC/m}^2 & r < 0.5\text{ m} \\ 2/r^2 a_r \text{ nC/m}^2 & r \geq 0.5\text{ m} \end{cases}$$

QUES 2.2.16 Volume charge density at $r = 0.2$ m will be ____ C/m^3.

QUES 2.2.17 Volume charge density at $r = 1$ m will be ____ C/m^3.

QUES 2.2.18 An electric dipole having moment $p = \frac{3}{5}a_x - a_y + 2a_z$ nC-m is located at point $B(0,1,-6)$. The electric potential due to the dipole at point $A(1,2,2)$ will be ____ Volt.

QUES 2.2.19 A total charge 20 nC is being split into four equal charges spaced at 90° intervals around a circular loop of radius 5 m. What will be the electric potential (in Volt) at the center of the loop ?

QUES 2.2.20 The work done in carrying a 2 C charge from point $A(1,1/2,3)$ to the point $B(4,1,0)$ in the field $E = 2ya_x + 2xa_y$ V/m along the curve $y = \sqrt{x/2}$ will be ____ Joule.

QUES 2.2.21 In a certain region, the electric field intensity is given as $E = xa_x - ya_y$ V/m. The amount of work done in moving a $+2$ C charge along a circular arc centred at origin from $x = 1$ m to $x = y = \frac{1}{\sqrt{2}}$ m in the region will be ____ Joule.

Common Data For Q. 22 and 23 :

Four equal charges of $+1$ nC is being carried from infinity and placed at different corners of a square. Consider the side of the square is 1 m and the charges are being carried as one at a time.

QUES 2.2.22 How much energy (in nJ) does it require to bring in the last charge from infinity and place it in the fourth corner ?

QUES 2.2.23 Total work done for assembling the whole configuration of four charges will be ____ nJ.

QUES 2.2.24 The electric field in a certain region is given by

$$E = \sin\phi\,a_\rho + (z+1)\rho\cos\phi\,a_\phi + \rho\sin\phi\,a_z \text{ V/m}$$

Work done in moving a 2 C charge from $A(2,0°,1)$ to $B(2,30°,1)$ in the field is ____ Joule.

QUES 2.2.25 Total work done in transferring two point charges $+1\,\mu$C and $+2$ mC from infinity to the points $A(-3,6,0)$ and $B(2,-4,-1)$ respectively is ____ Joule.

QUES 2.2.26 Four point charges of 8 nC are placed at the corners of a square of side 1 cm. The total potential energy stored in the system of charges is ____ mJ.

Common Data For Q. 27 and 28 :

The potential field in free space is expressed as

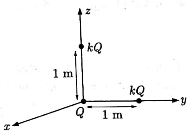
$$V = \frac{1}{xyz}\text{ V}$$

QUES 2.2.27 The total energy stored within the cube $1 < x,\ y,\ z < 2$ will be ____ $\times 10^{-13}$ J

QUES 2.2.28 The energy density at the centre of the cube will be ____$\times 10^{-13}$ J.

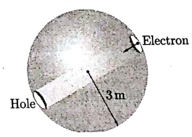
QUES 2.2.29 The electric field intensity required to counter act the earth's gravitational force on an electron is ____ $\times 10^{-11}$ V/m.

QUES 2.2.30 Three point charges Q, kQ and kQ are arranged as shown in figure. What will be the value of k for which the net electric field intensity at the point $P(0, \frac{1}{4}, \frac{1}{4})$ is zero ?



Common Data For Q. 31 and 32 :

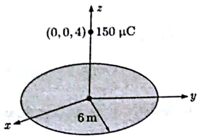
Consider a total charge of 2 nC is distributed throughout a spherical volume of radius 3 m. A small hole is drilled through the center of the spherical volume charge as shown in figure. The size of the hole is negligible compared to the size of the sphere.



QUES 2.2.31 If an electron is placed at one end of the hole and released from rest at $t = 0$, what will be the distance (in meter) of the electron from center of sphere at $t = 1\,\mu$sec. ?

QUES 2.2.32 The frequency of the oscillation of point charge is ____ kHz.

QUES 2.2.33 A total charge of $900\pi\ \mu C$ is uniformly distributed over a circular disk of radius 6 m The applied force on a $150\ \mu C$ charge located on the axis of disk and 4 m from it's center as shown in figure is ____ N.



QUES 2.2.34 A charged sphere of radius 1 m carries a uniform charge density of $6\ C/m^3$. A redistribution of the charge results in the density function given by

$$\rho_v = k(3 - r^2)\ C/m^3$$

where r is distance of the point from center of the sphere. What is the value of k ?

QUES 2.2.35 A $50\ \mu C$ point charge is located at the origin. The total electric flux passing through the hemispherical surface defined by $r = 48$ m, $0 \le \theta \le \pi/2$ is ____ μC.

QUES 2.2.36 Two identical uniform charges with $\rho_L = 80$ nC/m are located in free space at $x = 0$, $y = \pm 3$ m. The force per unit length acting on the line at positive y arising from the charge at negative y is ____ μN in a_y direction.

QUES 2.2.37 Four 1.2 nC point charge are located in free space at the corners of a square 4 cm on a side. The total potential energy stored is ____ μJ.

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EXERCISE 2.3

MCQ 2.3.1 **Assertion (A) :** Net electric field flux emanating from an arbitrary surface not enclosing a point charge is zero.
Reason (R) : Electric field intensity at any point outside the uniformly charged sphere is always zero.
(A) A and R both are true and R is correct explanation of A.
(B) A and R both are true but R is not the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

MCQ 2.3.2 **Assertion (A) :** No charge can be present in a uniform electric field.
Reason (R) : According to Gauss's law volume charge density in a region having electric field intensity E is given by $\rho_v = \varepsilon \nabla E$.
(A) A and R both are true and R is correct explanation of A.
(B) A and R both are true but R is not the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

MCQ 2.3.3 A potential function V satisfies Laplace's equation inside a certain region. In this region the potential function will have
(A) a maxima only
(B) a minima only
(C) a maxima and a minima both
(D) neither a maxima nor a minima

MCQ 2.3.4 Coulomb's force is proportional to
(A) r
(B) r^2
(C) $\frac{1}{r}$
(D) $\frac{1}{r^2}$

MCQ 2.3.5 The proportionality constant in Coulomb's law has unit of
(A) Farads
(B) Farads/metre
(C) Newton
(D) metre/Farad

MCQ 2.3.6 The value of proportionality constant in Coulomb's law is
(A) 9×10^9
(B) 9×10^{-9}
(C) 8.854×10^{-12}
(D) $\frac{1}{36\pi} \times 10^9$

MCQ 2.3.7 The unit of electric field is
(A) Newton
(B) Coulomb/Newton
(C) Newton/Coulomb
(D) Coulomb/metre

MCQ 2.3.8 If the direction of Coulomb's force on a unit charge is $\mathbf{a}_x$, the direction of electric field is
(A) $-\mathbf{a}_x$
(B) $\mathbf{a}_y$
(C) $\mathbf{a}_x$
(D) $\mathbf{a}_z$

MCQ 2.3.9 The unit of electric flux is
(A) Coulomb
(B) Coulomb/metre
(C) Weber
(D) Weber/m^2

MCQ 2.3.10 The electric field on x-axis due to a line charge extending from $-\infty$ to ∞ is
(A) $\frac{\rho_l}{2\pi\varepsilon_0\rho}$
(B) $\frac{\rho_l}{2\varepsilon_0\rho}$
(C) $\frac{\rho_l}{2\rho}$
(D) $\frac{\rho_l}{\varepsilon_0\rho}$

MCQ 2.3.11 Potential at all the points on the surface of a conductor is
(A) the same
(B) not the same
(C) zero
(D) infinity

MCQ 2.3.12 Gradient of the potential and an equipotential surface
(A) have the same direction
(B) have opposite directions
(C) are orthogonal to each other
(D) have no directional relation

MCQ 2.3.13 The unit of electric moment is
(A) C/m
(B) C-m
(C) C/m^2
(D) C-m^2

MCQ 2.3.14 Point form of Gauss's law is
(A) $\nabla \cdot \boldsymbol{D} = \rho_v$
(B) $\nabla \times \boldsymbol{D} = \rho_s$
(C) $\nabla \times \boldsymbol{D} = \rho_v/\varepsilon_0$
(D) $\nabla \times \boldsymbol{D} = Q$

MCQ 2.3.15 The unit of electric flux is
(A) Coulomb
(B) Coulomb/m
(C) Weber
(D) Tesla

MCQ 2.3.16 Gauss's law is
(A) $\int_s \boldsymbol{D}.d\boldsymbol{S} = Q$
(B) $\oint_s \boldsymbol{D}.d\boldsymbol{S} = Q$
(C) $\oint D\, dS = Q$
(D) $\int_s \boldsymbol{D}\, dS = Q$

MCQ 2.3.17 Potential has the unit of
(A) Joules/Coulomb
(B) Joules
(C) Joules/m^3
(D) Joules/m^2

MCQ 2.3.18 Electric flux lines
(A) originate at (+)ve charge
(B) originate at (−) ve charge
(C) are closed loops
(D) originate at (+)ve charge and also terminate at (+)ve charge

MCQ 2.3.19 The electric field in free space
(A) $\frac{\boldsymbol{D}}{\varepsilon_0}$
(B) $\frac{\boldsymbol{D}}{\mu_0}$
(C) $\varepsilon_0\boldsymbol{D}$
(D) $\frac{\sigma}{\varepsilon_0}$

****

EXERCISE 2.4

MCQ 2.4.1 If the electric field intensity is given by $\boldsymbol{E} = (x\boldsymbol{a}_x + y\boldsymbol{a}_y + z\boldsymbol{a}_z)$ volt/m, the potential difference between $X(2,0,0)$ and $Y(1,2,3)$ is
(A) +1 volt
(B) −1 volt
(C) +5 volt
(D) +6 volt

MCQ 2.4.2 There are three charges, which are given by $Q_1 = 1\,\mu$C, $Q_2 = 2\,\mu$C and $Q_3 = 3\,\mu$C. The field due to each charge at a point P in free space is $(\boldsymbol{a}_x + 2\boldsymbol{a}_y - \boldsymbol{a}_z)$, $(\boldsymbol{a}_y + 3\boldsymbol{a}_z)$ and $(2\boldsymbol{a}_x - \boldsymbol{a}_y)$ newtons/coulomb. The total field at the point P due to all three charges is given by
(A) $1.6\boldsymbol{a}_x + 2.2\boldsymbol{a}_y + 2.5\boldsymbol{a}_z$ newtons/coulomb
(B) $0.3\boldsymbol{a}_x + 0.2\boldsymbol{a}_y + 0.2\boldsymbol{a}_z$ newtons/coulomb
(C) $3\boldsymbol{a}_x + 2\boldsymbol{a}_y + 2\boldsymbol{a}_z$ newtons/coulomb
(D) $0.6\boldsymbol{a}_x + 0.2\boldsymbol{a}_y + 0.5\boldsymbol{a}_z$ newtons/coulomb

MCQ 2.4.3 Given that the electric flux density $\boldsymbol{D} = z\rho(\cos^2\Phi)\,\boldsymbol{a}_z$ C/m^2. The charge density at point $(1, \pi/4, 3)$ is
(A) 3
(B) 1
(C) 0.5
(D) 0.5 $\boldsymbol{a}_z$

MCQ 2.4.4 An electric charge of Q coulombs is located at the origin. Consider electric potential V and electric field intensity E at any point (x, y, z). Then
(A) $\boldsymbol{E}$ and V are both scalars
(B) $\boldsymbol{E}$ and V are both vectors
(C) $\boldsymbol{E}$ is a scalar and V is a vector
(D) $\boldsymbol{E}$ is a vector and V is a scalar

MCQ 2.4.5 **Assertion (A) :** Capacitance between two parallel plates of area 'A' each and distance of separation 'd' is $\varepsilon A/d$ for large A/d ratio.
Reason (R) : Fringing electric field can be neglected for large A/d ratio.
(A) Both A and R are individually true and R is the correct explanation of A
(B) Both A and R are individually true but R is not the correct explanation of A
(C) A is true but R is false
(D) A is false but R is true

MCQ 2.4.6 **Assertion (A) :** In solving boundary value problems, the method of images is used.
Reason (R) : By this technique, conducting surfaces can be removed from the solution domain.

(A) Both A and R are individually true and R is the correct explanation of A
(B) Both A and R are individually true but R is not the correct explanation of A
(C) A is true but R is false
(D) A is false but R is true

MCQ 2.4.7 What will be the equipotential surfaces for a pair of equal and opposite line charges ?

(A) Spheres | (B) Concentric cylinders
(C) Non-concentric cylinders | (D) None of the above

MCQ 2.4.8 If the potential functions V_1 and V_2 satisfy Laplace's equation within a closed region and assume the same values on its surface, then which of the following is correct ?

(A) V_1 and V_2 are identical
(B) V_1 is inversely proportional as V_2
(C) V_1 has the same direction as V_2
(D) V_1 has the same magnitude as V_2 but has different direction

MCQ 2.4.9 **Assertion (A) :** The expression $\boldsymbol{E} = -\nabla V$, where $\boldsymbol{E}$ is the electric field and V is the potential is not valid for time varying fields.
Reason (R) : The curl of a gradient is identically zero.

(A) Both A and R are individually true and R is the correct explanation of A.
(B) Both A and R are individually true but R is not the correct explanation of A.
(C) A is true but R is false
(D) A is false but R is true

MCQ 2.4.10 What is the electric flux density (in $\mu C/m^2$) at a point (6, 4, – 5) caused by a uniform surface charge density of $60\,\mu C/m^2$ at a plane $x = 8$?

(A) $-30\boldsymbol{a}_x$ | (B) $-60\boldsymbol{a}_x$
(C) $30\boldsymbol{a}_x$ | (D) $60\boldsymbol{a}_x$

MCQ 2.4.11 Of two concentric long conducting cylinders, the inner one is kept at a constant positive potential $+V_0$ and the outer one is grounded. What is the electric field in the space between the cylinders?

(A) Uniform and directed radially outwards
(B) Uniform and directed radially inwards
(C) Non-uniform and directed radially outwards
(D) Non-uniform and directed parallel to the axis of the cylinders

MCQ 2.4.12 In a charge free space, the Poisson's equation results in which one of the following?

(A) Continuity equation | (B) Maxwell's equation
(C) Laplace equation | (D) None of the above

MCQ 2.4.13 W_1 is the electrostatic energy stored in a system of three equal point charges arranged in a line with 0.5 m separation between them. If W_2 is the energy stored with 1 m separation between them, then which one of the following is correct ?

(A) $W_1 = 0.5\,W_2$ | (B) $W_1 = W_2$
(C) $W_1 = 2\,W_2$ | (D) $W_1 = 4\,W_2$

MCQ 2.4.14 Equivalent surface about a point charge are in which one of the following forms ?

(A) Spheres | (B) Planes
(C) Cylinders | (D) Cubes

MCQ 2.4.15 Consider the following statements regarding an electrostatic field :

1. It is irrotational
2. It is solensoidal
3. It is static only form a macroscopic view point.
4. Work done in moving a charge in the field form one point to another is independent of the path of movement.

Which of the statements given above are correct ?

(A) 1, 2 and 3 | (B) 1, 2 and 4
(C) Only 2 and 4 | (D) 1, 3 and 4

MCQ 2.4.16 The potential (scalar) distribution is given as $V = 10y^4 + 20x^3$. If ε_0 is the permittivity of free space, what is the volume charge density ρ_v at the point (2, 0) ?

(A) $-200\varepsilon_0$ | (B) $-200/\varepsilon_0$
(C) $200\varepsilon_0$ | (D) $-240\varepsilon_0$

MCQ 2.4.17 The x-directed electric field E_x having sinusoidal time variation $e^{j\omega t}$ and space variation in z-direction satisfies the equation $\nabla^2 E_x = 0$ under source free condition in a lossless medium. What is the solution representing propagation in positive z-direction ?

(A) $E_x = E_0 e^{-kz}$ | (B) $E_x = E_0 e^{+jkz}$
(C) $E_x = E_0 e^{-jkz}$ | (D) $E_x = E_0 e^{+kz}$

MCQ 2.4.18 An infinitely long uniform charge of density 30 nC/m is located at $y = 3, z = 5$. The field intensity at (0, 6, 1) is $\boldsymbol{E} = 64.7\boldsymbol{a}_y - 86.3\boldsymbol{a}_z$ V/m. What is the field intensity at (5,6,1) ?

(A) $\boldsymbol{E}$ | (B) $\left(\frac{6^2+1^2}{5^2+6^2+1^2}\right)\boldsymbol{E}$
(C) $\left(\frac{6^2+1^2}{5^2+6^2+1^2}\right)^{1/2}\boldsymbol{E}$ | (D) $\left(\frac{5^2+6^2+1^2}{6^2+1^2}\right)^{1/2}\boldsymbol{E}$

MCQ 2.4.19 What is the magnetic dipole moment in Am^2 for a square current loop having the vertices at the point $A(10,0,0)$, $B(0,10,0)$, $C(-10,0,0)$ and $D(0,-10,0)$ and with current 0.01 A flowing in the sense $ABCDA$?

(A) $2\boldsymbol{a}_z$ | (B) $-2\boldsymbol{a}_z$
(C) $4\boldsymbol{a}_z$ | (D) $4(\boldsymbol{a}_x + \boldsymbol{a}_y)$

MCQ 2.4.20 An electric charge Q is placed in a dielectric medium. Which of the following quantities are independent of the dielectric constant ε of the medium ?

(A) Electric potential V and Electric field intensity E
(B) Displacement density D and Displacement ψ
(C) Electric field intensity E and Displacement density D
(D) Electric potential V and Displacement ψ

MCQ 2.4.21 Two coaxial cylindrical sheets of charge are present in free space, $\rho_S = 5\,\text{C/m}^2$ at $r = 2\,\text{m}$ and $\rho_S = -2\,\text{C/m}^2$ at $r = 4\,\text{m}$. The displacement flux density $\boldsymbol{D}$ at $r = 3\,\text{m}$ is

(A) $\boldsymbol{D} = 5\boldsymbol{a}_r\,\text{C/m}^2$
(B) $\boldsymbol{D} = 2/3\boldsymbol{a}_r\,\text{C/m}^2$
(C) $\boldsymbol{D} = 10/3\boldsymbol{a}_r\,\text{C/m}^2$
(D) $\boldsymbol{D} = 18/3\boldsymbol{a}_r\,\text{C/m}^2$

MCQ 2.4.22 An electric potential field is produced in air by point charge $1\,\mu\text{C}$ and $4\,\mu\text{C}$ located at $(-2,1,5)$ and $(1,3,-1)$ respectively. The energy stored in the field is

(A) 2.57 mJ
(B) 5.14 mJ
(C) 10.28 mJ
(D) 12.50 mJ

MCQ 2.4.23 A dipole produces an electric field intensity of 1 mV/m at a distance of 2 km. The field intensity at a distance of 4 km will be

(A) 1 mV/m
(B) 0.75 mV/m
(C) 0.50 mV/m
(D) 0.25 mV/m

MCQ 2.4.24 The energy stored per unit volume in an electric field (with usual notations) is given by

(A) $1/2\varepsilon H^2$
(B) $1/2\varepsilon E$
(C) $1/2\varepsilon E^2$
(D) εE^2

MCQ 2.4.25 A positive charge of Q coulomb is located at point $A(0,0,3)$ and a negative charge of magnitude Q coulombs is located at point $B(0,0,-3)$. The electric field intensity at point $C(4,0,0)$ is in the

(A) negative x-direction
(B) negative z-direction
(C) positive x-direction
(D) positive z-direction

MCQ 2.4.26 The force between two points charges of 1 nC each with a 1 mm separation in air is

(A) $9 \times 10^{-3}\,\text{N}$
(B) $9 \times 10^{-6}\,\text{N}$
(C) $9 \times 10^{-9}\,\text{N}$
(D) $9 \times 10^{-12}\,\text{N}$

MCQ 2.4.27 Gauss law relates the electric field intensity $\boldsymbol{E}$ with the volume charge density ρ_v at a point as

(A) $\nabla \times \boldsymbol{E} = \varepsilon_0 \rho_v$
(B) $\nabla \cdot \boldsymbol{E} = \varepsilon_0 \rho_v$
(C) $\nabla \times \boldsymbol{E} = \rho_v/\varepsilon_0$
(D) $\nabla \cdot \boldsymbol{E} = \rho_v/\varepsilon_0$

MCQ 2.4.28 The electric field strength at any point at a distance r from the point charge q located in a homogeneous isotropic medium with dielectric constant ε, is given by

(A) $\boldsymbol{E} = \dfrac{q\varepsilon^{-1}}{4\pi r^2}\boldsymbol{a}_r$
(B) $\boldsymbol{E} = \oint \boldsymbol{D}\,d\boldsymbol{S}\cos\theta$
(C) $\boldsymbol{E} = \dfrac{q\varepsilon}{4\pi r^2}\boldsymbol{a}_r$
(D) $\boldsymbol{E} = \dfrac{q^2}{4\pi\varepsilon r^2}\boldsymbol{a}_r$

MCQ 2.4.29 The vector statement of Gauss's a law is

(A) $\oint_v \boldsymbol{D}\cdot d\boldsymbol{S} = \int_s \rho_v dv$
(B) $\int_v \boldsymbol{D}\cdot d\boldsymbol{S} = \oint_s \rho_v dv$
(C) $\iint_s \boldsymbol{D}\cdot d\boldsymbol{S} = \int_v \rho_v^2 dv$
(D) $\oint_s \boldsymbol{D}\cdot d\boldsymbol{S} = \int_v \rho_v dv$

MCQ 2.4.30 Two charges are placed at a distance apart. Now, if a glass slab is inserted between them, then the force between the charge will

(A) reduce to zero
(B) increase
(C) decrease
(D) not change

MCQ 2.4.31 The following point charges are located in air :

$+0.008\,\mu\text{C}$ at $(0,0)\,\text{m}$
$+0.05\,\mu\text{C}$ at $(3,0)\,\text{m}$
$-0.009\,\mu\text{C}$ at $(0,4)\,\text{m}$

The total electric flux over a sphere of 5 m radius with centre (0,0) is

(A) $0.058\,\mu\text{C}$
(B) $0.049\,\mu\text{C}$
(C) $0.029\,\mu\text{C}$
(D) $0.016\,\mu\text{C}$

MCQ 2.4.32 Electric flux through a surface area is the integral of the

(A) normal component of the electric field over the area
(B) parallel component of the electric field over the area
(C) normal component of the magnetic field over the area
(D) parallel component of the magnetic field over the area

MCQ 2.4.33 **Assertion (A) :** The electric field around a positive charge is outward.
Reason (R) : Gauss's law states that the differential of the normal component of the outward electric flux density over a closed surface yields the positive charge enclosed.

(A) Both **Assertion (A)** and **Reason (R)** are individually true and **Reason (R)** is the correct explanation of **Assertion (A)**
(B) Both **Assertion (A)** and **Reason (R)** are individually true but **Reason (R)** is not the correct explanation of **Assertion (A)**
(C) **Assertion (A)** is true but **Reason (R)** is false
(D) **Assertion (A)** is false but **Reason (R)** is true

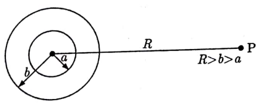
MCQ 2.4.34 Point charges of $Q_1 = 2nC$ and $Q_2 = 3nC$ are located at a distance apart. With regard to this situation, which one of the following statements is not correct ?

(A) The force on the 3 nC charge is repulsive.
(B) A charge of $-5\,nC$ placed midway between Q_1 and Q_2 will experience no force.
(C) The forces Q_1 and Q_2 are same in magnitude.
(D) The forces on Q_1 and Q_2 will depend on the medium in which they are placed.

MCQ 2.4.35 Which one of the following is the correct statement ?
Equi-potential lines and field lines
(A) are parallel
(B) are anti-parallel
(C) are orthogonal
(D) bear no definite relationship

MCQ 2.4.36 Point charges of -10 nC and 10 nC are located in free space at $(-1,0,0)$ m and $(1,0,0)$ m respectively. What is the energy stored in the field ?
(A) Zero
(B) 450 nJ
(C) -450 nJ
(D) -900 nJ

MCQ 2.4.37 A spherical balloon of radius a is charged. The energy density in the electric field at point P shown in the figure given below is w. If the balloon is inflated to a radius b without altering its charge, what is the energy density at P ?



(A) $w\left(\frac{b}{a}\right)^3$
(B) $w\left(\frac{b}{a}\right)^2$
(C) $w\left(\frac{b}{a}\right)$
(D) w

MCQ 2.4.38 Which one of the following statements does not state that electrostatic field is conservative ?
(A) The curl of $\boldsymbol{E}$ is identically zero
(B) The potential difference between two points is zero
(C) The electrostatic field is a gradient of a scalar potential
(D) The work done in a closed path inside the field is zero

MCQ 2.4.39 Sphere of radius a with a uniform charge density ρ_v C/m^3 shall have electric flux density at $r=a$, equal to
(A) $\frac{a}{3}\rho_v \boldsymbol{i}_r$ C/m^2
(B) $\frac{1}{3}\rho_v \boldsymbol{i}_r$ C/m^2
(C) $a\rho_v \boldsymbol{i}_r$ C/m^2
(D) $\frac{a}{4}\rho_v \boldsymbol{i}_r$ C/m^2

MCQ 2.4.40 Equipotential surfaces about a pair of equal and opposite linear charges exist in what form ?
(A) Concentric spheres
(B) Concentric cylinders
(C) Non-concentric cylinders
(D) Planes

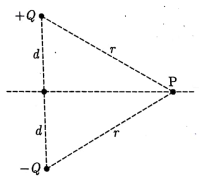
MCQ 2.4.41 For electrostatic fields in charge free atmosphere, which one of the following is correct ?
(A) $\nabla \times \boldsymbol{E} = 0$ and $\nabla \cdot \boldsymbol{E} = 0$
(B) $\nabla \times \boldsymbol{E} \neq 0$ and $\nabla \cdot \boldsymbol{E} = 0$
(C) $\nabla \times \boldsymbol{E} = 0$ and $\nabla \cdot \boldsymbol{E} \neq 0$
(D) $\nabla \times \boldsymbol{E} \neq 0$ and $\nabla \cdot \boldsymbol{E} \neq 0$

MCQ 2.4.42 If the electric field established by three point charge Q, $2Q$ and $3Q$ exerts a force $3\boldsymbol{F}$ on $3Q$ and $2\boldsymbol{F}$ on $2Q$, then what is the force exerted on the point charge Q ?
(A) $\boldsymbol{F}$
(B) $-\boldsymbol{F}$
(C) $5\boldsymbol{F}$
(D) $-5\boldsymbol{F}$

MCQ 2.4.43 Which one of the following is the Poission's equation for a linear and isotropic but inhomogeneous medium ?
(A) $\nabla^2 E = -\frac{\rho}{\varepsilon}$
(B) $\nabla \cdot (\varepsilon \nabla V) = -\rho$
(C) $\nabla \cdot \nabla(\varepsilon V) = -\rho$
(D) $\nabla^2 V = -\frac{\rho}{\varepsilon}$

MCQ 2.4.44 Plane $z = 10$ m carries surface charge density 20 nc/m^2. What is the electric field at the origin ?
(A) $-10\boldsymbol{a}_z$ v/m
(B) $-18\pi\boldsymbol{a}_z$ v/m
(C) $72\pi\boldsymbol{a}_z$ v/m
(D) $-360\pi\boldsymbol{a}_z$ v/m

MCQ 2.4.45 Consider the following diagram :



The electric field E at a point P due to the presence of dipole as shown in the above diagram (considering distance $r >>$ distance d) is proportional to
(A) $1/r$
(B) $1/r^2$
(C) $1/r^3$
(D) $1/r^4$

MCQ 2.4.46 What is the value of total electric flux coming out of a closed surface ?
(A) Zero
(B) Equal to volume charge density
(C) Equal to the total charge enclosed by the surface
(D) Equal to the surface charge density

MCQ 2.4.47 A charge is uniformly distributed throughout the sphere of radius a. Taking the potential at infinity as zero, the potential at $r = b < a$ is

(A) $-\int_{\infty}^{b} \frac{Qr}{4\pi\varepsilon_0 a^3} dr$

(B) $-\int_{\infty}^{b} \frac{Q}{4\pi\varepsilon_0 r^2} dr$

(C) $-\int_{\infty}^{a} \frac{Q}{4\pi\varepsilon_0 r^2} dr - \int_{a}^{b} \frac{Qr}{4\pi\varepsilon_0 a^3} dr$

(D) $-\int_{\infty}^{a} \frac{Q}{4\pi\varepsilon_0 r^2} dr$

MCQ 2.4.48 A potential field is given by $V = 3x^2y - yz$. Which of the following is not true ?

(A) At the point $(1, 0, -1)$, V and the electric field $\boldsymbol{E}$ vanish

(B) $x^2y = 1$ is an equipotential plane in the xy-plane

(C) The equipotential surface $V = -8$ passes through the point $P(2, -1, 4)$

(D) A unit vector normal to the equipotential surface $V = -8$ at P is $(-0.83\boldsymbol{x} + 0.55\boldsymbol{y} + 0.07\boldsymbol{z})$

MCQ 2.4.49 The relation between electric intensity E, voltage applied V and the distance d between the plates of a parallel plate condenser is

(A) $E = V/d$

(B) $E = V \times d$

(C) $E = V/(d)^2$

(D) $E = V \times (d)^2$

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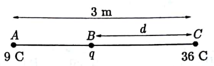
SOLUTIONS 2.1

SOL 2.1.1 Option (B) is correct.

Since the two point charges are positive so the introduced third point charge must be negative as to make the entire system in equilibrium as shown below



as the system must be in equilibrium so the force between all the pair of charges will be equal

i.e. $F_{AB} = F_{CB} = F_{AC}$

$$\frac{(9)q}{(3-d)^2} = \frac{(36)q}{d^2} = \frac{(36)(9)}{(3)^2}$$

Solving the equation we get, $q = -4\,\text{C}$ and $d = 2\,\text{m}$

SOL 2.1.2 Option (A) is correct.

Electric field intensity at any point P due to the two point charges Q_1 and Q_2 is defined as

$$\boldsymbol{E} = k\left(\frac{Q_1}{(R_1)^3}\boldsymbol{R}_1 + \frac{Q_2}{(R_2)^3}\boldsymbol{R}_2\right)$$

where, $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ is the vector distance of the point P from the two point charges.

So the net electric field due to the two given point charges is

$$\boldsymbol{E} = \frac{9\times10^9\times(-5)\times10^{-9}[(-7+4)\boldsymbol{a}_x + (3-0)\boldsymbol{a}_y + (-1+2)\boldsymbol{a}_z]}{\sqrt{(-7+4)^2 + (3-0)^2 + (-1+2)^2}}$$
$$+ \frac{9\times10^9\times2\times10^{-9}[(-7+5)\boldsymbol{a}_x + (3-0)\boldsymbol{a}_y + (-1-3)\boldsymbol{a}_z]}{\sqrt{(-7+5)^2 + (3-0)^2 + (-1-3)^2}}$$
$$= \frac{-45[-3\boldsymbol{a}_x + 3\boldsymbol{a}_y + \boldsymbol{a}_z]}{19^{3/2}} + \frac{18[-2\boldsymbol{a}_x + 3\boldsymbol{a}_y - 4\boldsymbol{a}_z]}{29^{3/2}}$$
$$= 1.4\boldsymbol{a}_x - 1.284\boldsymbol{a}_y - 1.004\boldsymbol{a}_z$$

SOL 2.1.3 Option (D) is correct.

For an electric field to exist, the its curl must be zero. So, we check the existence of the given field vector first.

Given the electric field intensity

$$\boldsymbol{E} = 2xy\boldsymbol{a}_x + 4yz\boldsymbol{a}_y + 6xz\boldsymbol{a}_z \text{ V/m}$$

So, $$\nabla \times \boldsymbol{E} = 2\begin{vmatrix} \boldsymbol{a}_x & \boldsymbol{a}_y & \boldsymbol{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & 2yz & 3xz \end{vmatrix}$$
$$= 2[-2y\boldsymbol{a}_x - 3z\boldsymbol{a}_y - x\boldsymbol{a}_z] \neq 0$$

Therefore, as the curl of the given electric field is not equal to zero so, the field does not exist.