Test 3: MA20104 Probability and Statistics

Time: 45 minutes **Marks: 40**

Instructions: There are ten questions. You have to answer all. They carry equal marks.

Q. 1: Suppose that the amount of time one spends in a bank is exponentially distributed with mean 14 minutes. Find the probability that a customer will spend more than 17 minutes in the bank given that she is still in the bank after 10 minutes? (answer should be correct up to three decimal places)

ANS 1: 0.607 (error range: 0.005)

Q. 2: A defected spot can be detected as in a Poisson process at the rate of 0.02 per square feet of the sheet. The cost to repair a defected spot in a metal sheet is Rs. 5. Find the probability that more than Rs. 10 will be required to repair defects in a 100 square feet metal sheet. (answer should be correct up to three decimal places)

ANS 2: 0.323 (error range: 0.005)

Q.3: Suppose the arrival of emails in your inbox follows a Poisson process with a fixed rate. Given that no mail arrived in first 10 minutes, the probability that no mail arrived in next five minutes is 0.1. Find the probability that at least one email is received in next 2 minutes? (answer should be correct up to three decimal places)

ANS 3: 0.602 (error range: 0.005)

Q. 4: There are three connections in a series circuit between two nodes of current. Lifetimes of connections are independently and exponentially distributed with means 3, 4 and 6 months respectively. Find the probability that the circuit has a failure within a month? (answer should be correct up to three decimal places)

ANS 4: 0.528 (error range: 0.005)

Q. 5: The maximum height (in meters) that a particular firecracker can achieve (when shot vertically) is a normally distributed random variable with mean 28 and standard deviation 2. Find the probability that out of 5 such firecrackers at most one achieves a height of 30. (answer should be correct up to three decimal places)
(Given $\Phi(0.5)= 0.6915, \Phi(1) = 0.8413, \Phi(1.96) = 0.975, \Phi(2) = 0.9772,$
$\Phi(2.32) = 0.99, \Phi(1.65) = 0.95, \Phi(1.28) = 0.9, \Phi(3) = 0.9987$)

ANS 5: 0.819 (error range: 0.005)

Q. 6: Cars pass through an out of the way village according to a Poisson process with rate λ per hour. The probability of one or more cars passing through the village between 8:00 a.m. to 9:00 a.m. is 0.51. Find the probability that at least one car will pass through the village during the next half an hour. (answer should be correct up to three decimal places)

ANS 6: 0.300 (error range: 0.005)

Q. 7: Let θ be a randomly chosen angle uniformly on the interval $(0,\pi/4)$. The random variable Y is defined as the y-coordinate of the point at which the ray through the origin at the angle θ intersects the line $x = 1$ in the plane. Find the expected value of the area of the triangle with vertices (0,0), (1,0) and $(1,Y)$. (answer should be correct up to three decimal places)

ANS 7: 0.221 (error range: 0.005)

Q. 8: Let $t(p)$ be a real number such that $\Phi\left(t(p)\right) = \frac{p}{2}, 0 < p < 1$. Let X be a normal random variable with mean 10 and standard deviation 3. Find $P\left(10 + 3t(0.35) \le X \le 10 + 3t(0.55)\right)$.
(answer should be correct up to three decimal places)

ANS 8: 0.1 (error range :0.005)

Q. 9: The life (in years) of an equipment follows a gamma distribution with mean 4 and variance 8. Find the probability that the equipment will fail within 2 years. (answer should be correct up to three decimal places)

ANS 9: 0.264 (error range: 0.005)

Q. 10: The proportion of persons surviving a certain disease has a beta distribution with parameters 4 and 2. Find the probability that in a particular region this proportion is less than one third. (answer should be correct up to three decimal places)

ANS 10: 0.045 (error range: 0.005)

Q. 11: The lives (in years) of a battery have a Weibull distribution with parameters $\alpha = 2, \beta = 3$. Find the probability that the battery is operational after one year given that it was working after 6 months. CDF of Weibull (α, β) is $F(x) = 1 - \exp(-\alpha x^{\beta})$ for nonnegative x. (answer should be correct up to three decimal places)

ANS 11: 0.174 (error range: 0.005)

Q. 12: A parallel system has 2 independent components each having lifetimes (in years) distributed as Weibull $(\alpha = 2, \beta = 2)$ and Weibull $(\alpha = 1, \beta = 3)$. Find the probability that the system is working after 1 year. CDF of Weibull (α, β) is $F(x) = 1 - \exp(-\alpha x^{\beta})$ for nonnegative x. (answer should be correct up to three decimal places)

ANS 12: 0.453 (error range: 0.005)

Q. 13: Let a random variable X have a uniform distribution on the interval (0,3). Find the probability that the equation $4t^2 + 4tX + X + 2 = 0$ has real roots in t. (answer should be correct up to three decimal places)

ANS 13: 0.333 (error range: 0.005)

Q. 14: The diameter X of a bolt (in cm) is normally distributed with mean 7 cm and standard deviation 2 cm. If X lies in the specification limits of 6 to 8 cm, a profit of Rs.100 is gained on sale. However, in case $X < 6$ or $X > 8$, there is a loss of Rs. 40 or Rs. 20 respectively on sale. Find the expected profit (in Rs.) on sale of 100 bolts. (answer should be correct up to three decimal places)
(Given $\Phi(0.5) = 0.6915, \Phi(1) = 0.8413, \Phi(1.96) = 0.975, \Phi(2) = 0.9772,$
$\Phi(2.32) = 0.99, \Phi(1.65) = 0.95, \Phi(1.28) = 0.9, \Phi(3) = 0.9987$)

ANS 14: 1979.00 (error range: 0.005)

Q. 15: A computer lab has two printers. Printer I handles 60% of all jobs and its printing time (in minutes) follows a gamma distribution with mean time 2 and variance 2. Printer II handles remaining 40% of all the jobs and its printing time follows a uniform distribution between 0 and 4. Find the probability that a randomly selected job will be printed in more than one minute. (answer should be correct up to three decimal places)

ANS 15: 0.741 (error range: 0.005)

Q. 16: Let X be a random variable with $\log_e M_X(t) = 2t(1+t), t \in R$. Find the fourth central moment. (answer should be correct up to three decimal places)

ANS 16: 48 (error range: 0.005)

Q. 17: Let X be a random variable with $M_X(t) = \frac{2}{2-t}$, $t < 2$. Find the probability

$$P(0.5 < X < 1).$$

(answer should be correct up to three decimal places)

ANS: 0.233 (error range: 0.005)

Q. 18: A series system has three components connected independently with lifetimes (in years) exponentially distributed with means $\frac{1}{2\lambda}, \frac{1}{2\lambda}, \frac{1}{\lambda}$ respectively. If the reliability (i.e. the probability that the system is functioning beyond time 't') of the system after one year is 0.5, find the reliability after six months. (answer should be correct up to three decimal places)

ANS 18: 0.707 (error range: 0.005)

Q. 19: Decay times (in thousands of years) of radioactive particles are denoted by a random variable X having a Cauchy distribution with the pdf

$$f(x) = \frac{1}{\pi(1 + (x - 10)^2)}, x \in R$$

Find the probability that out of two such particles, only one will have decay time between 11 to $10 + \sqrt{3}$ thousand years. (answer should be correct up to three decimal places)

ANS 19: 0.153 (error range: 0.005)

Q. 20: The income (in thousands of Rs.) distribution of a country is represented by the Laplace (double exponential) pdf

$$f(x) = e^{-2|x-1|}, x \in R$$

Find the variance of the income distribution. (answer should be correct up to three decimal places)

ANS 20: 0.5 (error range: 0.005)