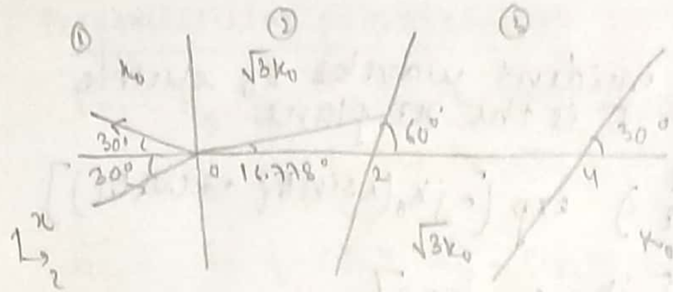
5.



Region ① and ④ is free space

$n_2 = \frac{c}{v} = \frac{\sqrt{\mu\epsilon}}{\sqrt{\mu_0\epsilon_0}} = \sqrt{\mu_r\epsilon_r} = \sqrt{3}$

$n_3 = \sqrt{3}$

i) If the frequency of the wave is assumed to be ω

$k_0^2 = \omega^2\mu_0\epsilon_0$ since magnitude of k is not known (or ω is not known) we would take k_0 as the magnitude of the propagation vector

or $k_0 = \frac{\omega}{c}$

$\vec{k}_{inci} = k_0(\cos 30°\ \hat{a}_z + \sin 30°\ \hat{a}_x)$

$= \frac{k_0}{2}(\sqrt{3}\,\hat{a}_z + \hat{a}_x)$

$= \frac{k_0}{2}(\hat{a}_x + \sqrt{3}\,\hat{a}_z)$

$\vec{k}_{refl} = k_0(\sin 30°\ \hat{a}_x - \cos 30°\ \hat{a}_z)$

$= \frac{k_0}{2}(\hat{a}_x - \sqrt{3}\,\hat{a}_z)$

We can find angle of refraction by Snell's Law

$\sin\theta_i = n_2 \sin\theta_r$

or $\frac{1}{2} = \sqrt{3}\sin\theta_r$

or $\theta_r = \sin^{-1}\left(\frac{1}{2\sqrt{3}}\right) = 16.778°$

$\vec{k}_{tr2} = \sqrt{3}k_0(\sin\theta_r\ \hat{a}_x + \cos\theta_r\ \hat{a}_z)$

$= \sqrt{3}k_0\left(\frac{1}{2\sqrt{3}}\hat{a}_x + \frac{\sqrt{11}}{2\sqrt{3}}\hat{a}_z\right)$

$= \frac{\sqrt{3}k_0}{2\sqrt{3}}(\hat{a}_x + \sqrt{11}\,\hat{a}_z)$

$= \frac{\cancel{\sqrt{3}}}{2}k_0(\hat{a}_x + \sqrt{11}\,\hat{a}_z)$

$= k_0\left(0.5\,\hat{a}_x + \frac{\sqrt{11}}{2}\,\hat{a}_z\right)$

Parallel Polarization

we can assume that the incident wave has E_0 electric field and H_0 magnetic field. $\vec{E}$ is the xz plane

$$\vec{E}_{i1} = E_0 (\cos\theta_i \hat{a}_x - \sin\theta_i \hat{a}_z) \exp[-jk_0(x\sin\theta_i + z\cos\theta_i)]$$

$$= \frac{E_0}{2}(\sqrt{3}\hat{a}_x - \hat{a}_z) \exp\left[-\frac{jk_0}{2}(x+\sqrt{3}z)\right]$$

$$\vec{H}_{i1} = \frac{E_0}{\eta_0} \exp\left[-\frac{jk_0}{2}(x+\sqrt{3}z)\right]$$

$$= E_0 \exp\left[-\frac{jk_0}{2}(x+\sqrt{3}z)\right]$$

The reflected wave is

$$\vec{E}_{r1} = E_0 R_{12} (\cos\theta_i \hat{a}_x + \sin\theta_i \hat{a}_z) \exp[-jk_0(x\sin\theta_i + z\cos\theta_i)]$$

$$= \frac{E_0 R_{12}}{2}(\sqrt{3}\hat{a}_x + \hat{a}_z) \exp\left[-\frac{jk_0}{2}(x-\sqrt{3}z)\right]$$

$$\vec{H}_{r1} = -\frac{E_0 R_{12}}{\eta_0}\cancel{\sqrt{3}} \exp\left[-\frac{jk_0}{2}(x-\sqrt{3}z)\right]\hat{a}_y$$

$$= -E_0 R_{12} \exp\left[-\frac{jk_0}{2}(x-\sqrt{3}z)\right]\hat{a}_y$$

$$R_{12} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} = \frac{\frac{\eta_2}{\eta_1}\frac{\cos\theta_t}{\cos\theta_i} - 1}{\frac{\eta_2}{\eta_1}\frac{\cos\theta_t}{\cos\theta_i} + 1} = 0.3138$$

$$\cancel{T_{12} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}}$$

The transmitted wave is

$$\vec{E}_{t2} = E_0 T_{12} (\cos\theta_t \hat{a}_x - \sin\theta_t \hat{a}_z) \exp -j\sqrt{3}k_0 (x\sin\theta_t + z\cos\theta_t)$$

$$= \frac{E_0 T_{12}}{2\sqrt{3}}(\sqrt{11}\hat{a}_x - 1) \exp\left[-\frac{jk_0}{2}(x+\sqrt{11}z)\right]$$

$$\vec{H}_{t2} = \frac{E_0 T_{12}}{\eta_2} \exp\left[-j\sqrt{3}k_0(x\sin\theta_t + z\cos\theta_t)\right]$$

$$= \frac{E_0 T_{12}}{\sqrt{3}} \exp\left[-\frac{jk_0}{2}(x+\sqrt{11}z)\right]$$

$$T_{12} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} = \frac{\frac{2\eta_2}{\eta_1}\cos\theta_i}{\frac{\eta_2}{\eta_1}\cos\theta_t + \cos\theta_i} = 1.1884$$

Perpendicular polarization

$$\vec{E_{i1}} = E_0 \exp\left[-jk_0(x\sin\theta_i + z\cos\theta_i)\right]\hat{a_y}$$

$$= E_0 \exp\left[-j\frac{k_0}{2}(x+\sqrt{3}z)\right]\hat{a_y}$$

$$\vec{H_{i1}} = \frac{E_0}{\eta_1}(-\cos\theta_i\hat{a_x} + \sin\theta_i\hat{a_z})\exp\left[-jk_0(x\sin\theta_i + z\cos\theta_i)\right]$$

$$= \frac{E_0}{2}(-\sqrt{3}\hat{a_x} + \hat{a_z})\exp\left[-j\frac{k_0}{2}(x+\sqrt{3}z)\right]$$

The reflected wave is

$$\vec{E_{r1}} = E_0R_{12}\exp\left[-jk_0(x\sin\theta_i - z\cos\theta_i)\right]\hat{a_y}$$

$$= E_0R_{12}\exp\left[-j\frac{k_0}{2}(x-\sqrt{3}z)\right]\hat{a_y}$$

$$\vec{H_{r1}} = \frac{E_0R_{12}}{\eta_1}\cancel{\exp}(\cos\theta_i\hat{a_x} + \sin\theta_i\hat{a_z})\exp\left[-j\frac{k_0}{2}(x-\sqrt{3}z)\right]$$

$$= \frac{E_0R_{12}}{2}(\sqrt{3}\hat{a_x} + \hat{a_z})\exp\left[-j\frac{k_0}{2}(x-\sqrt{3}z)\right]$$

$$R_{12} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} = \frac{\frac{\eta_2}{\eta_1}\frac{\cos\theta_i}{\cos\theta_t} - 1}{\frac{\eta_2}{\eta_1}\frac{\cos\theta_i}{\cos\theta_t} + 1} = 0.22$$

The transmitted wave is

$$\vec{E_{t2}} = E_0T_{12}\exp\left[-j\sqrt{3}k_0(x\sin\theta_t + z\cos\theta_t)\right]\hat{a_y}$$

$$= E_0T_{12}\exp\left[-j\frac{k_0}{2}(x+\sqrt{11}z)\right]\hat{a_y}$$

$$\vec{H_{t2}} = \frac{E_0T_{12}}{\eta_2}(-\cos\theta_t\hat{a_x} + \sin\theta_t\hat{a_z})\exp\left[-j\sqrt{3}k_0(x\sin\theta_t + z\cos\theta_t)\right]$$

$$= \frac{E_0T_{12}}{\sqrt{3}\times 2\sqrt{3}}(-\sqrt{11}\hat{a_x} + \hat{a_z})\exp\left[-j\frac{k_0}{2}(x+\sqrt{11}z)\right]$$

ii> since the refractive indices of the two medium are same, there can't be any reflection here.

$$\overrightarrow{K_{inc2}} = \overrightarrow{K_{tr2}}$$
$$= k_0\left(\tfrac{1}{2}\hat{a}_x + \tfrac{\sqrt{11}}{2}\hat{a}_z\right)$$

$$\overrightarrow{K_{r2}} = 0$$

$$\overrightarrow{K_{t3}} = \overrightarrow{K_{inc2}}$$
$$= k_0\left(\tfrac{1}{2}\hat{a}_x + \tfrac{\sqrt{11}}{2}\hat{a}_z\right)$$

The incident field may be both parallel or perpendicular polarization. The field incident is same as that of the transmitted field in regi

$$\overrightarrow{E_{i2}} = \overrightarrow{E_{t2}}$$
$$\overrightarrow{H_{i2}} = \overrightarrow{H_{t2}}$$

Irrespective of the nature of incident wave the ~~tra~~ reflected wave is 0

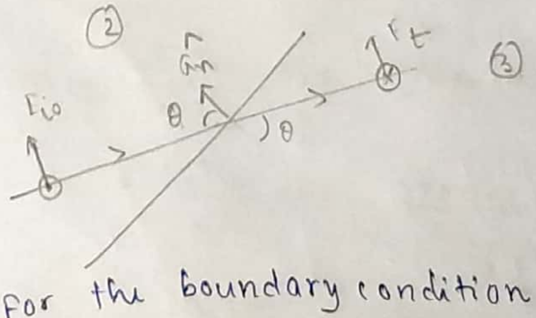
$$\overrightarrow{E_{r2}} = 0$$
$$\overrightarrow{H_{r2}} = 0$$

The transmitted wave would depend on whether the incident wave is parallel or perpendicularly polarized.

Parallel polarization

$$E_{i0} = E_0 T_{12p} = 1.1884 E_0$$
$$H_{i0} = \frac{E_{i0}}{\sqrt{3}} = 0.686 E_0$$

$$T_{23} = \frac{2n_1\cos\theta_i}{n_2\cos\theta_t + n_1\cos\theta_i} = 1$$



For the boundary condition
$(\vec{E}_{i0} - \vec{E}_{t3}) \times \hat{a}_n = 0$ to be true
we must have
$$\overrightarrow{E_{t3}} = \overrightarrow{E_{i2}} = \frac{1.1884 E_0}{2\sqrt{3}} \exp\left[-j\frac{k_0}{2}(x + \sqrt{11}z)\right]\left[-\hat{a}_z + \sqrt{11}\hat{a}_x\right]$$
Thus the electric field gets transmitted without any change

The bother boundary condition to be satisfied is

$(\vec{B_3} - \vec{B_2}) \cdot \hat{a_n} = 0$

or $B_3 - B_2 \perp \hat{a_n}$

$$\vec{H_{t3}} = \vec{H_{i2}} \sqrt{\frac{\mu_2 \epsilon_2}{\mu_3 \epsilon_3}}$$

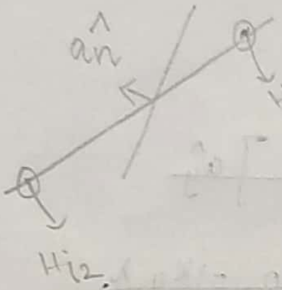
$$\vec{H_{t3}} = \frac{1.188 E_0}{\sqrt{3}} \exp\left[-j\frac{k_0}{2}(x - \sqrt{11}z)\right]$$

or $\vec{H_{t3}} = \frac{\vec{H_{i2}}}{\sqrt{3}}$ The magnetic field ~~gets scaled remains same~~ gets scaled

perpendicular polarization

$E_{i0} = 1.22 E_0$

$H_{i0} = \frac{E_{i0}}{\sqrt{3}}$



$(\vec{B_{i2}} - \vec{B_{t3}}) \cdot \hat{a_n} = 0$

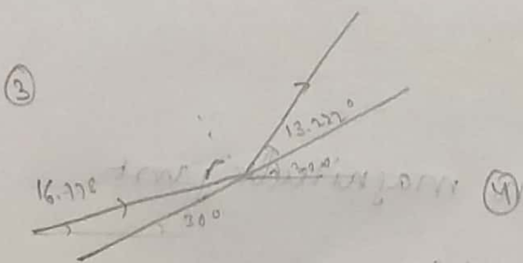
Here also the B incident and transmitted are parallel. So to satisfy boundary conditions

$\vec{H_{t3}} = \frac{\vec{H_{i2}}}{\sqrt{3}} = \frac{\vec{H_{t2}}}{3}$

$\vec{E_{t3}} = \vec{E_{i2}}$

~~Thus irrespective of the polarization, magnetic field is scaled and electric field remains same~~.

iii) The angle of incidence is 76.778°
The critical angle is 35.26°
Thus there is total internal reflection

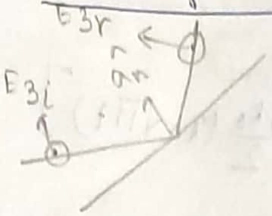


$\vec{k_{inc3}} = \vec{k_{r2}}$

$\vec{k_{r3}} = k_0[(\cos 43.222°)\hat{a_z} + \sin 43.222° \, \hat{a_x}] = 0.728\hat{a_z} + 0.684\hat{a_x}$

$\vec{k_{t4}} = 0$

Parallel polarization



$(\vec{E}_{3i} - \vec{E}_{3r}) \times \hat{a}_n = 0$

This implies that both should have equal magnitude

$|\vec{E}_{r3}| = |\vec{E}_{i3}|$

$(\vec{B}_{3i} - \vec{B}_{r3}) \cdot \hat{a}_n = 0$

The B fields are perpendicular to the vector. so this is satisfied

$$\vec{H}_{r3} = \frac{|E_{r3}|}{n_1} \exp[-jk_0(x\sin\theta_i - z\cos\theta_i)]\ \hat{a}_y$$

$$\cancel{\vec{H}_{r3} = \frac{1.1884}{\sqrt{3}} \exp\left[jk_0(0.684x - 0.728z)\right]\hat{a}_y}$$

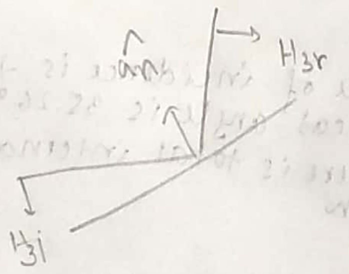
$$\cancel{\vec{E}_{r3} = \frac{1.1884E_0}{2\sqrt{3}}(-0.684\hat{a}_z + 0.728\hat{a}_x)\exp -jk_0 x}$$

$\theta_i = 76.778^\circ$

$$\vec{H}_{r3} = \frac{1.1884E_0}{\sqrt{3}} \exp\left[jk_0(0.684x - 0.728z)\right]\hat{a}_y$$

$$\vec{E}_{r3} = 1.1884E_0(-0.684\hat{a}_z + 0.728\hat{a}_x)\exp[-jk_0(0.728x - 0.684$$

Perpendicular polarization



Here H must be same in magnitude just rotated

$\vec{H}_{3r} = \vec{H}_{3i}\, e^{-j0.46}$ rotated by 26.44° = 0.46 radians

$\vec{E}_{3r} = 1.22E_0 \exp\left[-jk_0(0.684x + 0.728z)\right]$

(v) The following assumptions are made

a. The wave impedance is small.

b. There is no loss in medium

c. Boundaries are smooth and abrupt.