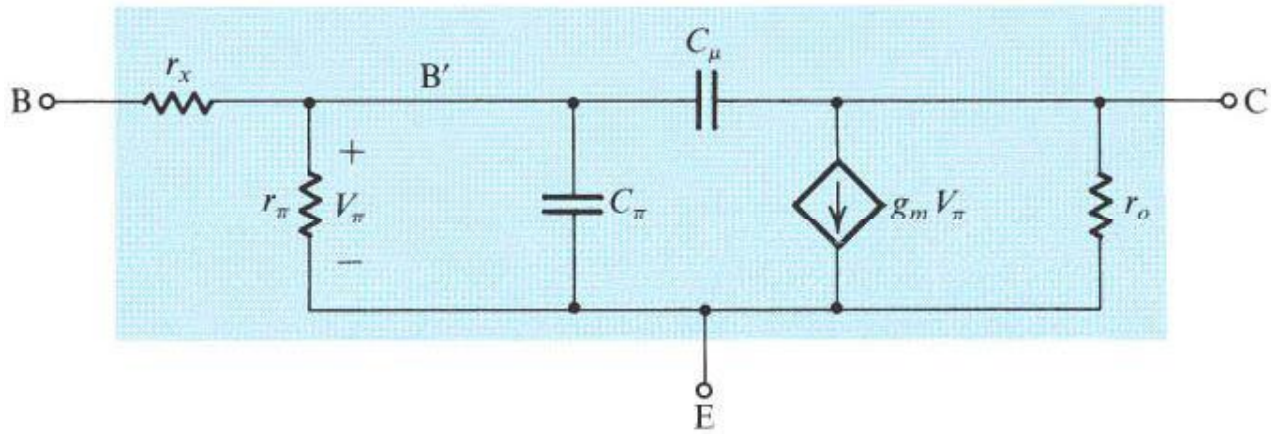
Frequency Response of BJT Amplifiers

The High-frequency Hybrid-π Model

The figure given below shows the hybrid-π model of the BJT, including capacitive effects. Specifically there are two capacitances: the emitter-base capacitance C_π, and the collector-base capacitance C_μ. Typically, C_π is in the range of a few picofarads to a few tens of picofarads, and C_μ is in the range of a fraction of a picofarad to a few picofarads. Note that we have also added a resistor r_x to model the resistance of the silicon material of the base region between the base terminal B and a fictitious internal, or intrinsic, base terminal B' that is right under the emitter region.

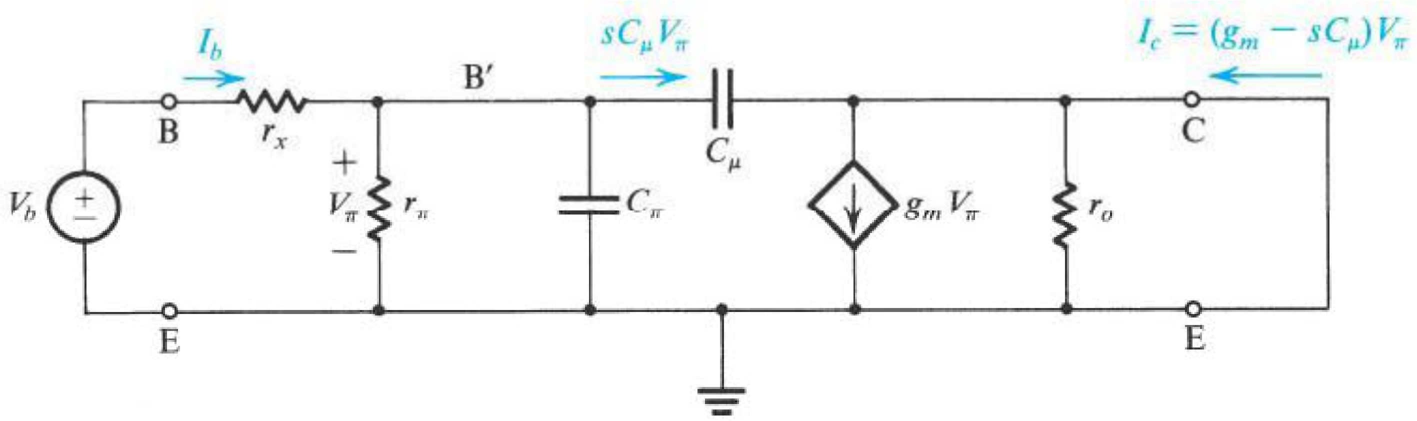


Typically r_x is in the range of few tens of ohms, and its value depends on the current level in a rather complicated manner. Since (usually) $r_x \ll r_\pi$, its effect is negligible at low frequencies. Its presence is felt, however, at high frequencies.

The values of the hybrid-π equivalent circuit parameters can be determined at a given bias point using the formulas given in this unit.

The Cutoff Frequency

In order to determine C_π and C_μ ewe shall derive the expression for h_{fe}, the CE short-circuit current gain, as a function of frequency in terms of the hybrid-π components. For this purpose consider the circuit shown in the figure below, in which the collector is shorted to the emitter.



A node equation C provides the short-circuit collector current I_c as

$$I_c = (g_m - sC_\mu)V_\pi$$

A relationship between the V_π and I_b can be established by multiplying the impedance seen between B' and E:

$$V_\pi = I_b\left(r_\pi || C_\pi || C_\mu\right) = \frac{I_b}{1/r_\pi + sC_\mu + sC_\pi}$$

Thus hfe can be obtained by combining above two equations as:

$$h_{fe} = \frac{I_c}{I_b} = \frac{g_m - sC_\mu}{1/r_\pi + s\left(C_\pi + C_\mu\right)}$$

At the frequencies for which this model is valid, $g_m \gg \omega C_\mu$; thus we can neglect the sC_μ term in the numerator and write

$$h_{fe} \cong \frac{g_m r_\pi}{1 + s\left(C_\pi + C_\mu\right) r_\pi}$$

Thus,

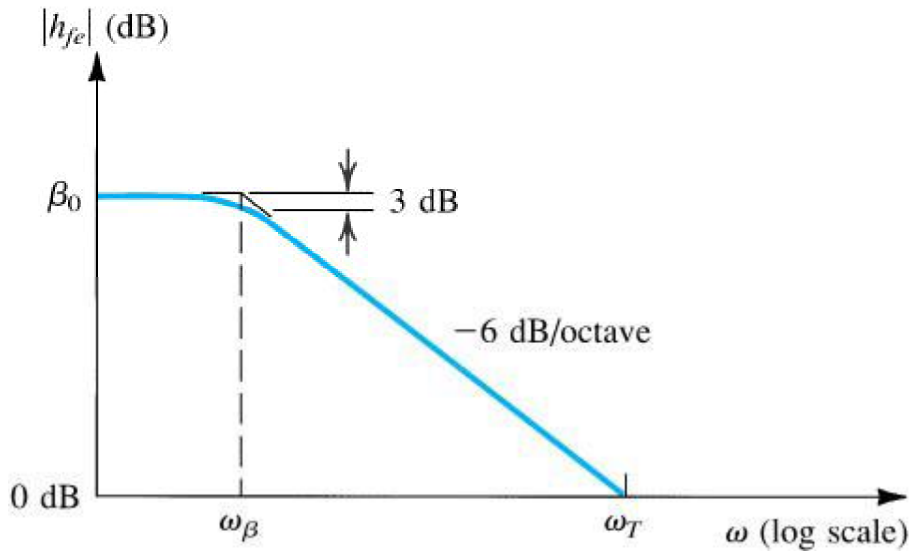
$$h_{fe} = \frac{\beta_o}{1 + s\left(C_\pi + C_\mu\right) r_\pi}$$

Where βo is the low-frequency value of β. Thus h_{fe} has the single-pole (or STC) response with a 3-dB frequency at $\omega = \omega_\beta$, where

$$\omega_\beta = \frac{1}{(C_\pi + C_\mu) r_\pi}$$

The Figure given below shows a Bode plot for $|h_{fe}|$. From the -6 dB/octave slope it follows that the frequency at which $|h_{fe}|$ drops to unity, which is called unity-gain bandwidth ω_T is given by

$$\omega_T = \beta_o \omega_\beta$$



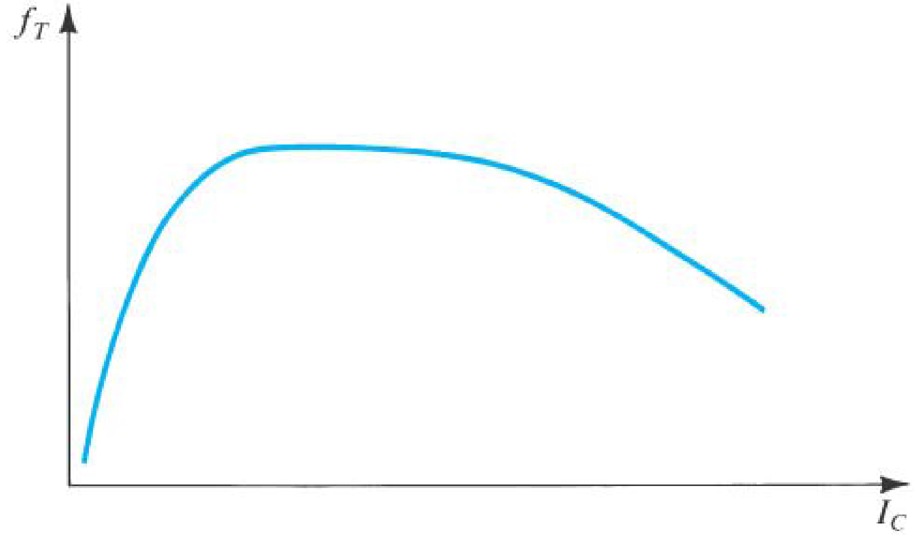
Thus,

$$\omega_T = \frac{g_m}{C_\pi + C_\mu}$$

And

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

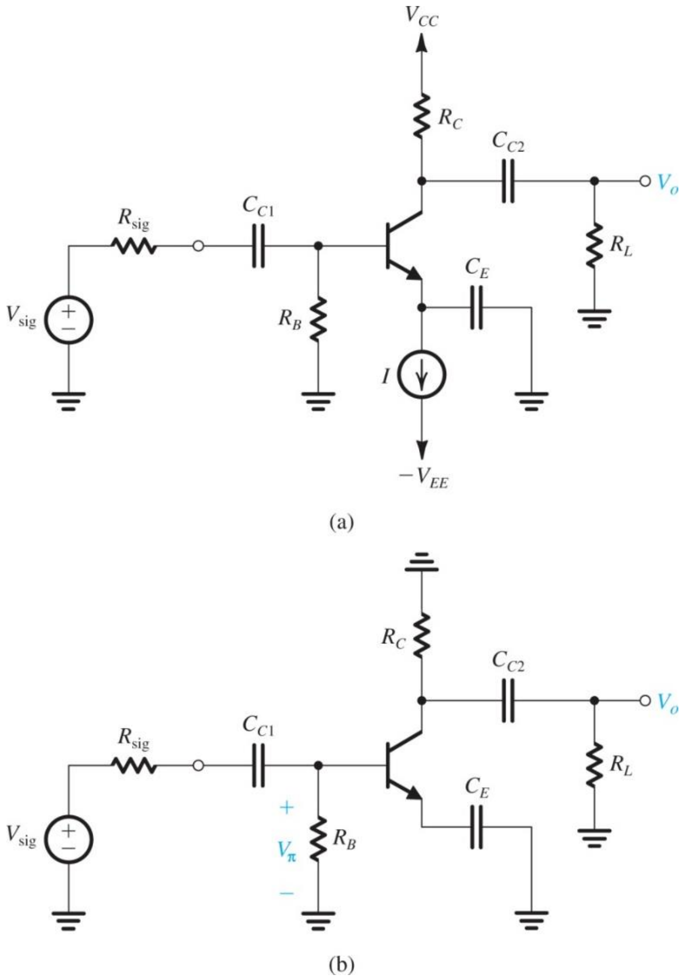
The unity gain bandwidth f_T is usually specified in the data sheets of the transistor. In the same cases f_T is given as a function of I_c and V_{CE}. Typically, f_T is in the range of 100 MHz to tens of GHz. The value of f_T can be used in the equations above to determine the value of $C_\pi + C_\mu$. The decrease in f_T at high currents is shown in figure below:



Frequency Response of The Common-Emitter Amplifier

The Three Frequency Bands

The circuit of CE amplifier with coupling capacitors is given below:



(a) A capacitively coupled common-emitter amplifier.
(b) The circuit prepared for small-signal analysis.

When the CE amplifier of the figure above was studied previously, it was assumed that the coupling capacitors C_{C1} and C_{C2} and the bypass capacitor C_E were acting as a perfect short circuits at all the signal frequencies of interest. We also neglected the internal capacitances of the BJT. That is C_π and C_μ of the BJT high-frequency model were assumed to be sufficiently small to act as open circuits at all the signal frequencies of interest. As a result, ignoring all the capacitive effects, the gain expression derived in the previous sections were independent of frequency. In reality, however, this situation only appears over a limited , though wide, band of frequencies. This is illustrated in the figure below, which shows a sketch of the magnitude of the overall gain , $|G_v|$, of the common-emitter amplifier versus frequency. We observe that the gain is almost constant over a wide frequency band called the **midband**. The value of the midband gain A_M corresponds to the overall voltage gain G_v that we derived earlier namely,

$$A_M = \frac{V_o}{V_{sig}} = -\frac{(R_B||r_\pi)}{(R_B||r_\pi) + R_{sig}} g_m(r_o||R_C||R_L)$$

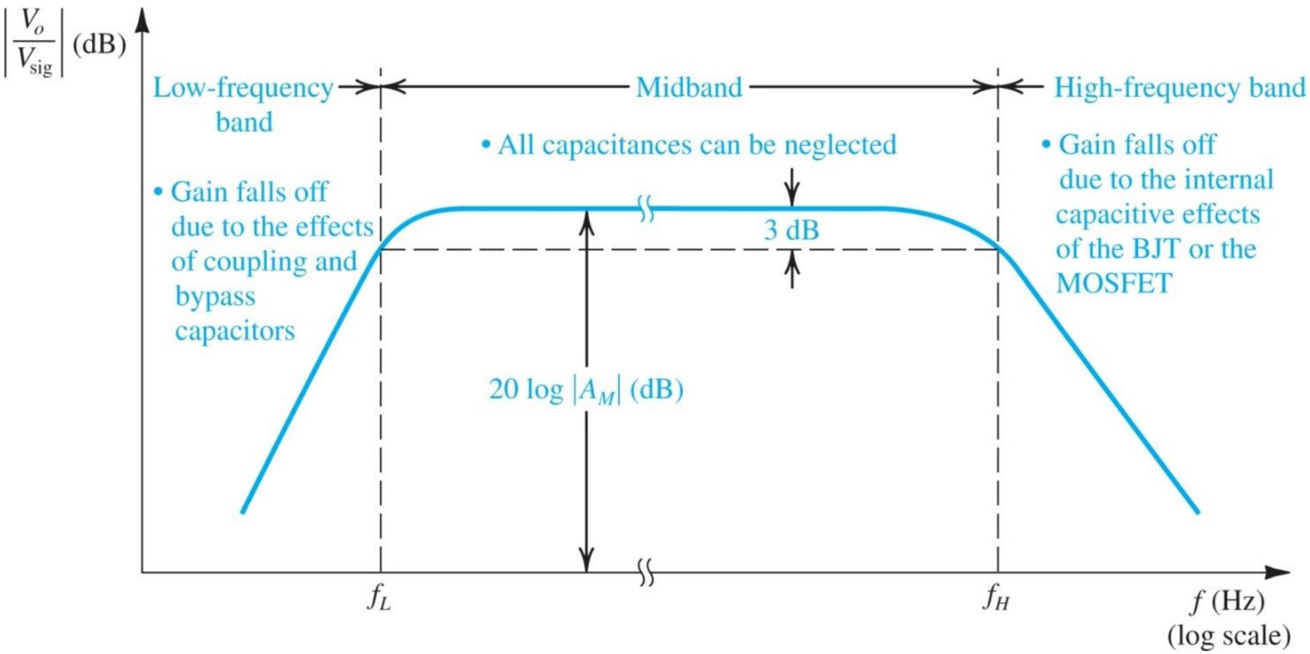


Figure given above shows that the gain falls off at a signal frequencies below and above the midband. The gain fall off in the **low-frequency band** is due to the fact that the coupling capacitors have high impedances and they no longer behave as short circuits. On the other hand, the gain falls off in the **high-frequency band** as result of the Cμ and Cπ, which though very small, their impedances at sufficiently large frequencies decrease; thus they can no longer be considered as open circuits.

The midband is obviously the useful frequency band of the amplifier. Usually, f_L and f_H are the frequencies at which the gain drops by 3 dB below its value at the midband; that is, at f_L and f_H, $|\text{gain}| = |A_M|/\sqrt{2}$. The amplifier bandwidth or 3-dB bandwidth is defined as the difference between the lower (f_L) and the upper (f_H) 3-dB frequencies.

$$BW \equiv f_H - f_L$$

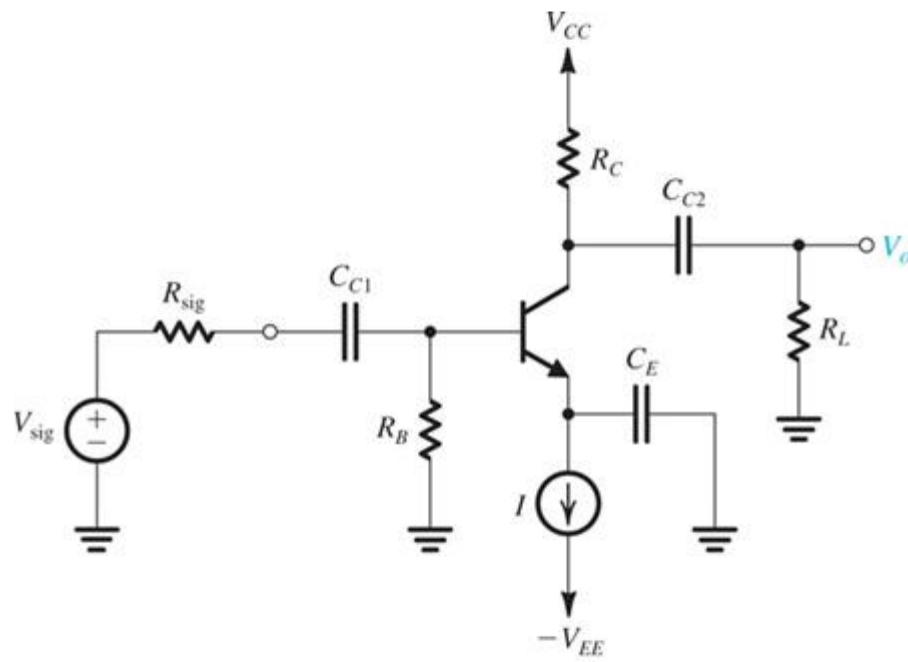
A figure of merit for the amplifier is its gain-bandwidth product, defined as

$$GB = |A_M| \text{ BW}$$

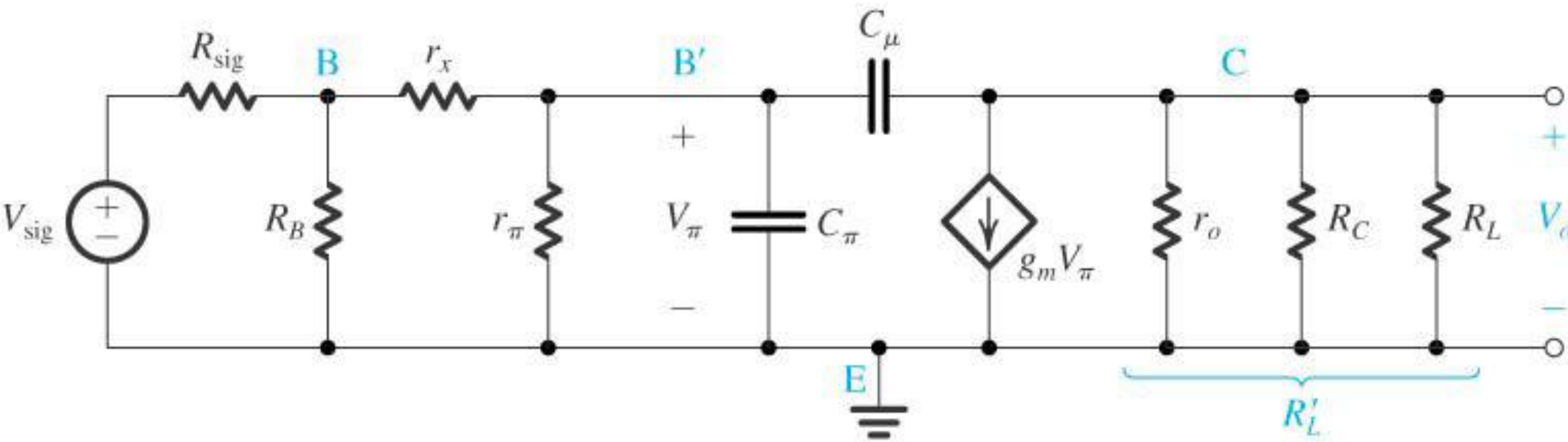
It will be shown at a later stage that in amplifier design, it is usually possible to trade off gain for bandwidth.

The High-Frequency Response

To determine the gain, or the transfer function, of the amplifier of figure below, we replace the BJT with the high frequency model as seen earlier in Unit II.

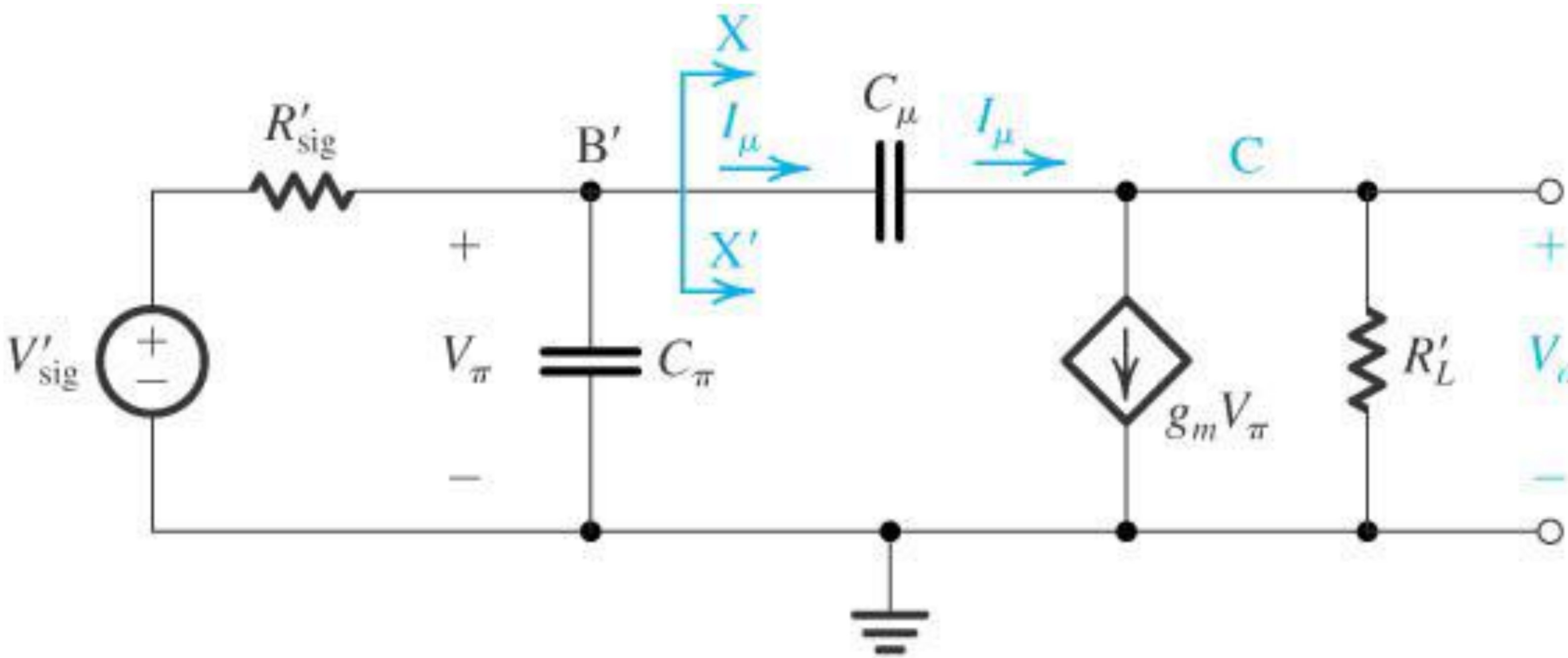


(a)



(a)

(a) Determining the high-frequency response of the CE amplifier : Equivalent circuit



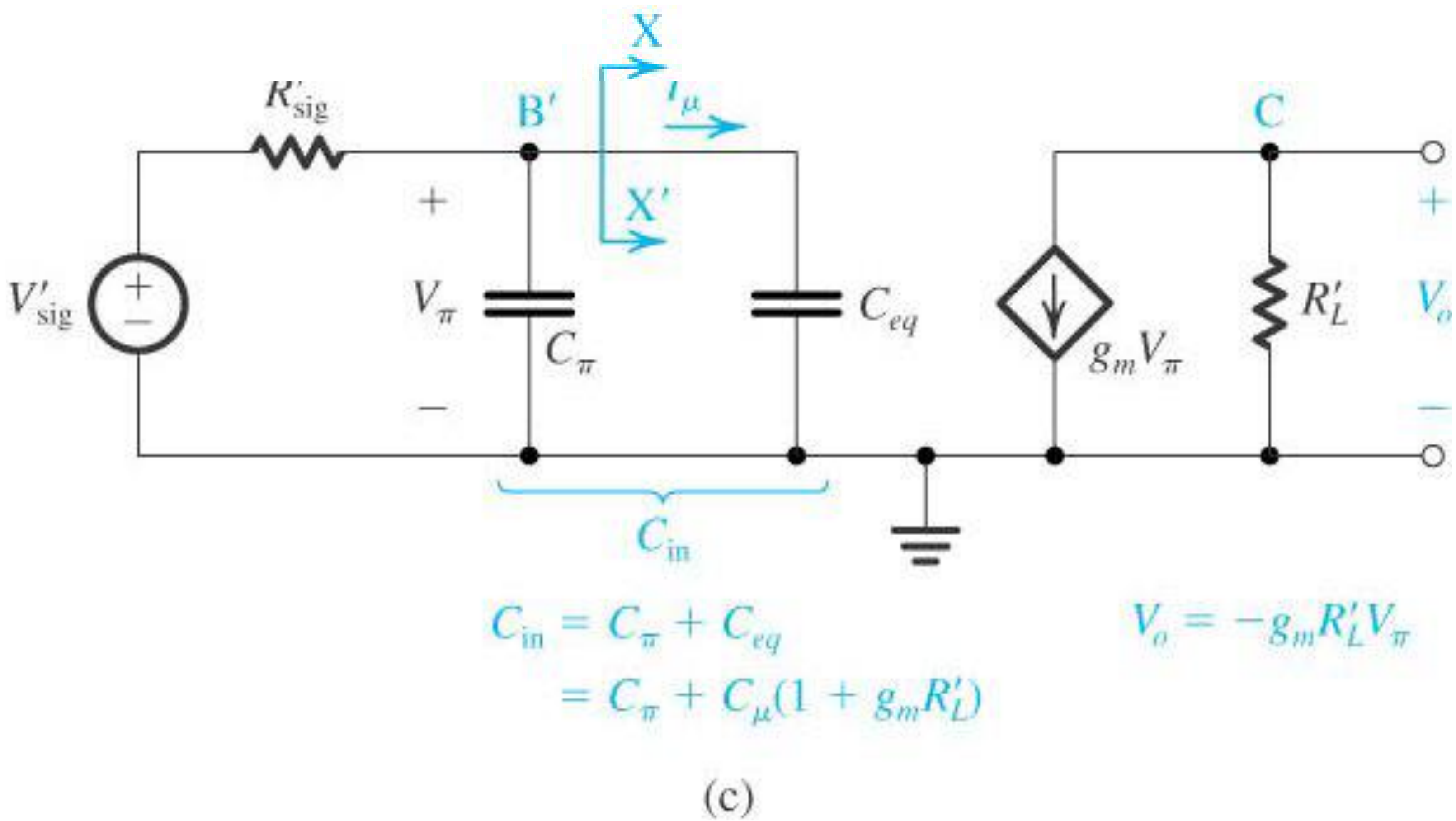
$$V'_{sig} = V_{sig} \frac{R_B}{R_B + R_{sig}} \frac{r_\pi}{r_\pi + r_x + (R_{sig} /\!/ R_B)}$$

$$R'_L = r_o /\!/ R_C /\!/ R_L$$

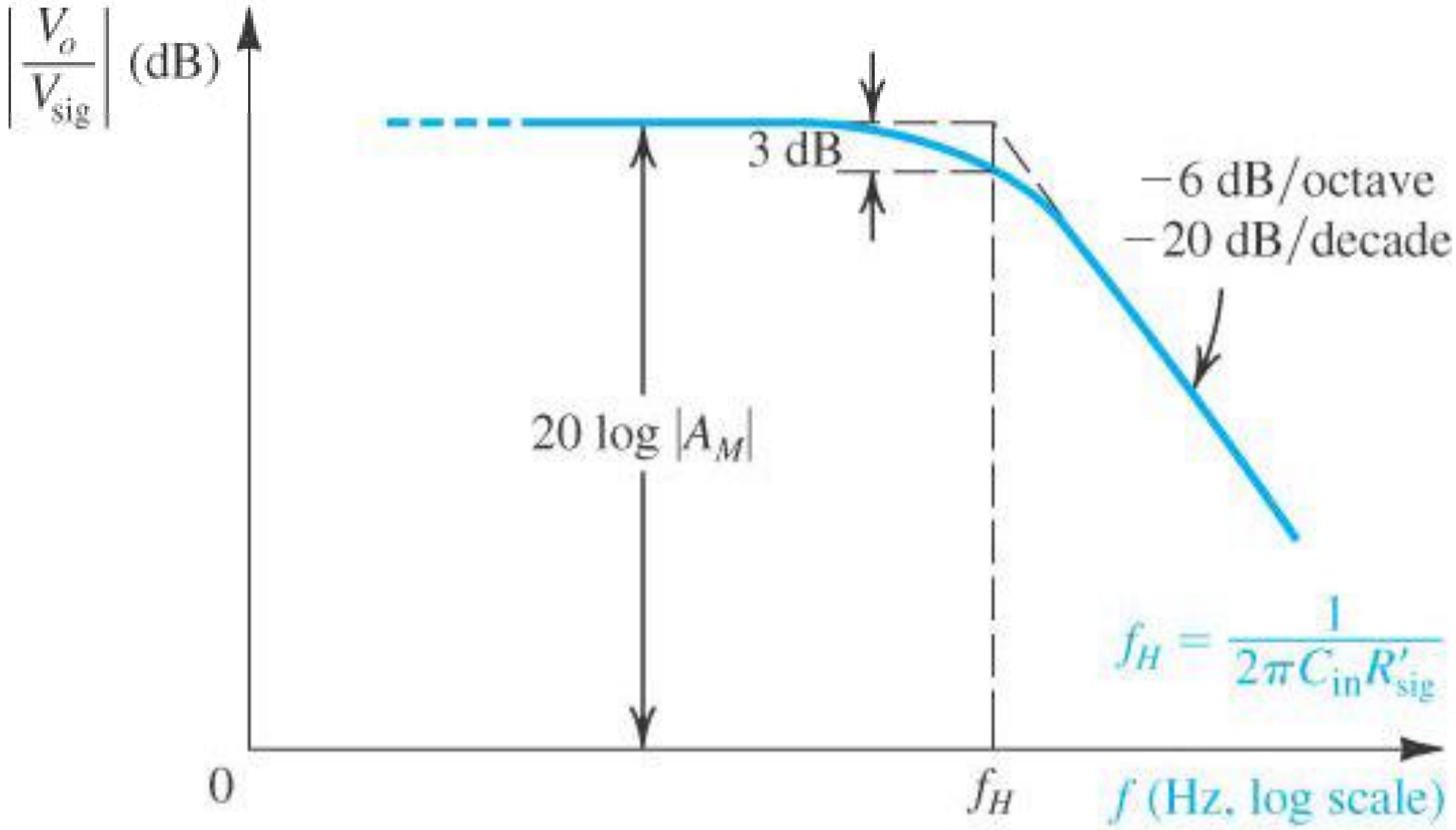
$$R'_{sig} = r_\pi /\!/ [r_x + (R_B /\!/ R_{sig})]$$

(b)

The circuit of (a) simplified at both the input and the output side.



Equivalent circuit with C_μ replaced at the input side with the equivalent capacitance C_{eq}



(d)

Sketch of the frequency-response plot, which is that of a low pass STC circuit.

Since $V_o = V_{ce}$, equation above indicates that the gain from B' to C is $-g_m R_L'$, the same value as in the midband. The current I_μ can now be found from:

$$I_\mu = sC_\mu(V_\pi - V_o)$$

$$= sC_\mu[V_\pi - (-g_m R_L' V_\pi)] = sC_\mu(1 + g_m R_L')V_\pi$$

Now in the figure given above, the left-hand side of the circuit, at XX', knows of the existence of C_μ only through the current I_μ. Therefore, we can replace C_μ by an equivalent capacitance C_{eq} between B' and the ground as long as C_{eq} draws a current equal to I_μ. That is

$$sC_{eq}V_\pi = I_\mu = sC_\mu(1 + g_m R_L')V_\pi$$

Which results in $$C_{eq} = C_\mu(1 + g_m R_L')$$

Thus we can express V_π in terms of V_{sig}' as $$V_\pi = V_{sig}' \frac{1}{1 + s/\omega_o}$$

where $$\omega_o = 1/C_{in}R_{sig}' \qquad C_{in} = C_\pi + C_{eq} = C_\pi + C_\mu(1 + g_m R_L')$$

$$\frac{V_o}{V_{sig}} = -\left[\frac{R_B}{R_B + R_{sig}} \frac{r_\pi g_m R_L'}{r_\pi + r_x + \left(R_{sig} || R_B\right)}\right]\left(\frac{1}{1 + \frac{s}{\omega_o}}\right)$$

$$\frac{V_o}{V_{sig}} = \frac{A_M}{1 + \frac{s}{\omega_o}}$$

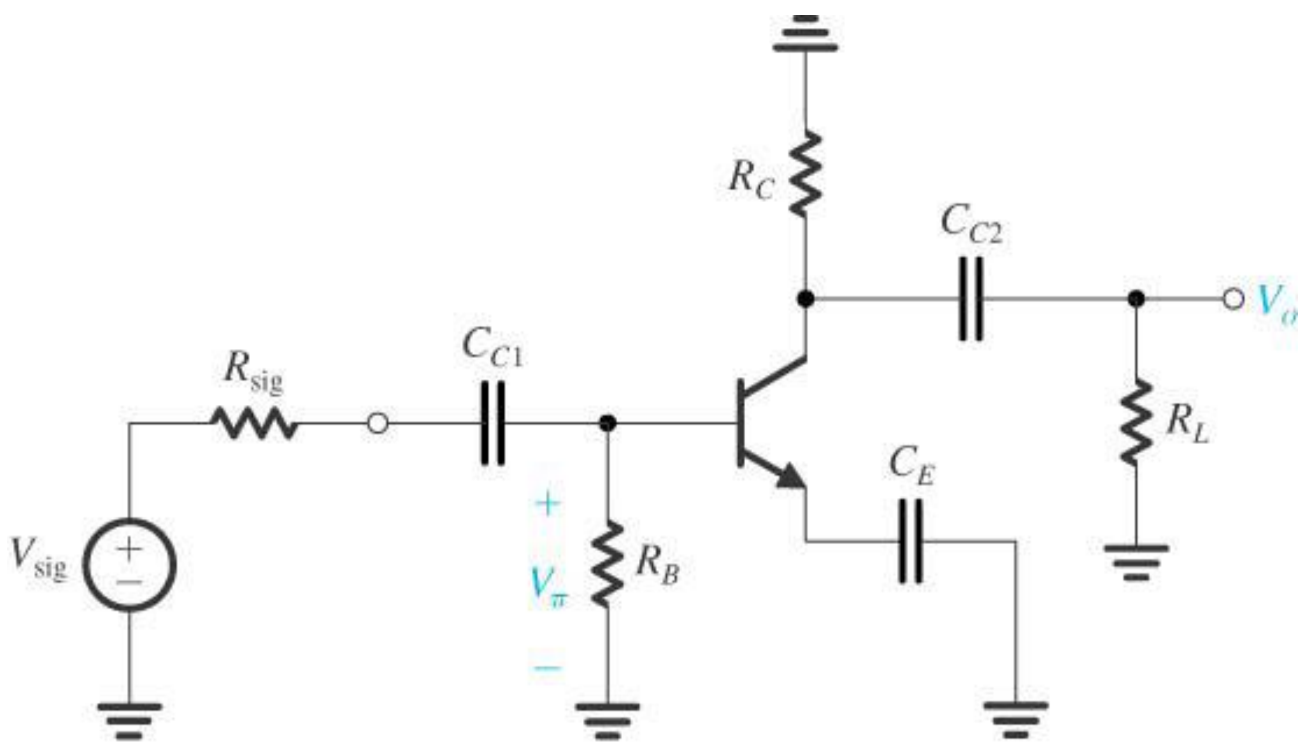
Where A_M is the midband gain

From which we deduce that the upper 3-dB frequency f_H must be

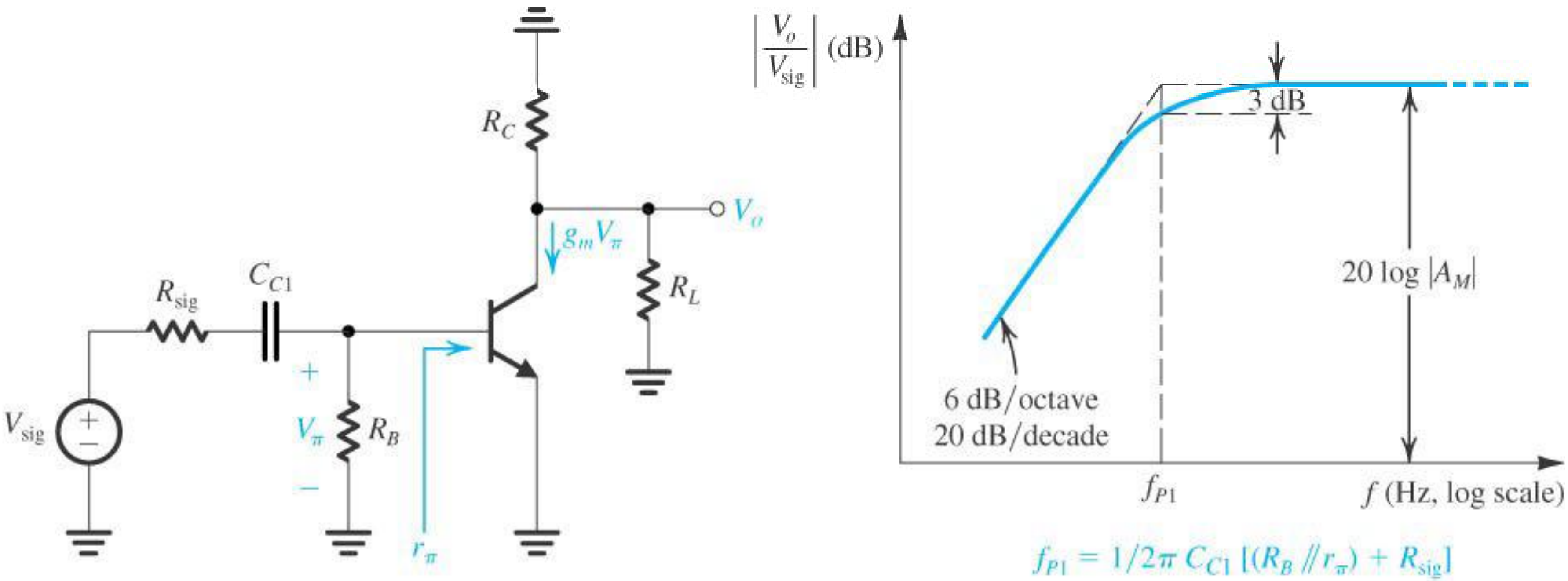
$$f_H = \frac{\omega_o}{2\pi} = \frac{1}{2\pi C_{in} R_{sig}'}$$

Low Frequency Response

To show the low frequency gain (or transfer function) of the CE amplifier circuit, we show the figure as (a) given below with the dc sources eliminated (current source I open circuited and voltage source V_{cc} short circuited). We perform the small circuit analysis directly on this circuit. We will ignore C_π and C_μ since at low frequencies their impedance will be very high and thus can be considered as open circuits. Also we will neglect r_o . Finally, we also neglect r_x which is usually much smaller than r_π with which it appears in series.

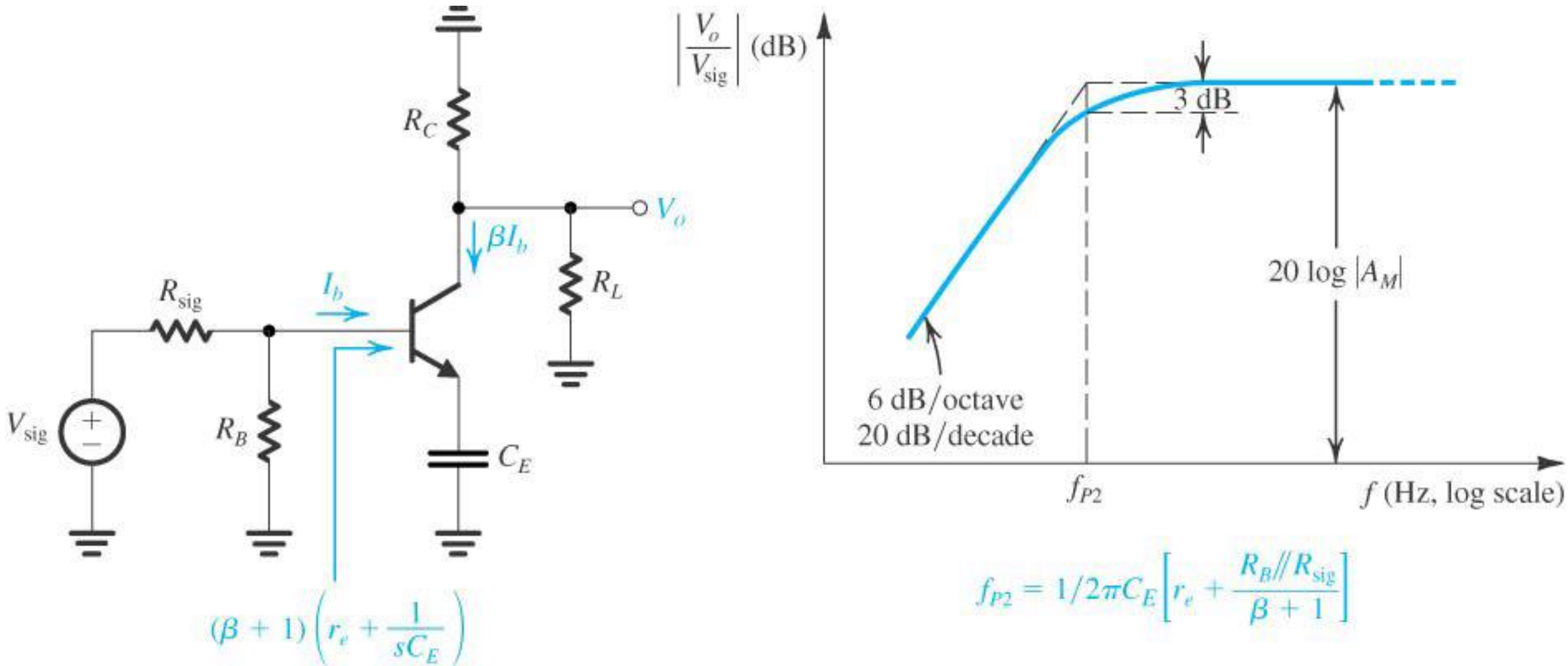


(a)



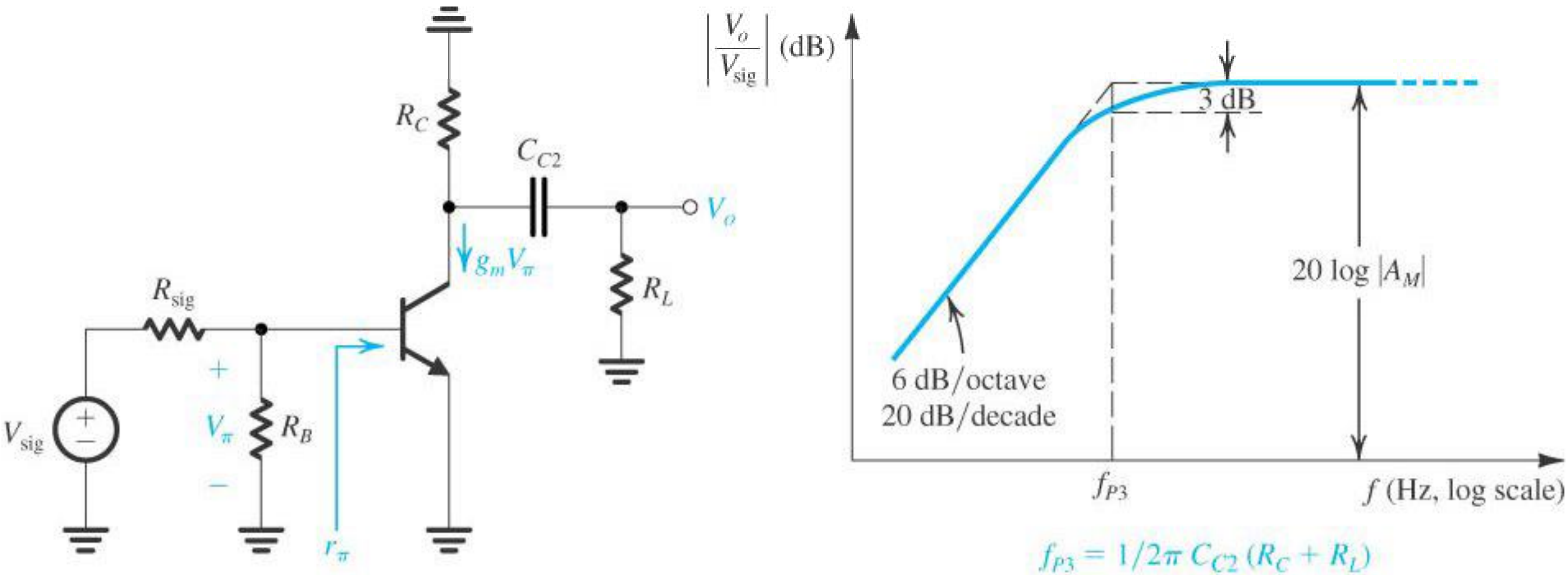
R_C
V_o
$g_m V_\pi$
R_{sig}
C_{C1}
R_L
+
V_{sig}
V_π
R_B
−
r_π
$\left|\frac{V_o}{V_{sig}}\right|$ (dB)
3 dB
20 log $|A_M|$
6 dB/octave
20 dB/decade
f_{P1}
f (Hz, log scale)
$f_{P1} = 1/2\pi C_{C1}\,[(R_B \,/\!/\, r_\pi) + R_{sig}]$

(b)



$\left|\frac{V_o}{V_{sig}}\right|$ (dB)
R_C
V_o
βI_b
I_b
R_{sig}
R_L
V_{sig}
R_B
C_E
$(\beta + 1)\left(r_e + \frac{1}{sC_E}\right)$
3 dB
20 log $|A_M|$
6 dB/octave
20 dB/decade
f_{P2}
f (Hz, log scale)
$f_{P2} = 1/2\pi C_E\left[r_e + \frac{R_B /\!/ R_{sig}}{\beta + 1}\right]$

(c)



R_C
C_{C2}
V_o
$g_m V_\pi$
R_L
R_{sig}
V_{sig}
V_π
R_B
r_π
$\left|\frac{V_o}{V_{sig}}\right|$ (dB)
3 dB
20 log $|A_M|$
6 dB/octave
20 dB/decade
f_{P3}
f (Hz, log scale)
$f_{P3} = 1/2\pi\, C_{C2}\, (R_C + R_L)$

(d)

Figure (a) above is to consider the effect of the three capacitors C_{C1}, C_E, and C_{C2} one at a time, considering the other two as a perfect short-circuit.
Consider the Figure (b) above, shows that C_{C2} and C_E are replaced by short circuits. The voltage V_π at the base is of the transistor can be written as

$$V_\pi = V_{sig} \frac{R_B||r_\pi}{(R_B||r_\pi) + R_{sig} + \frac{1}{sC_{C1}}}$$

$$V_o = -g_m V_\pi (R_C||R_L)$$

$$\frac{V_o}{V_{sig}} = -\frac{(R_B||r_\pi)}{(R_B||r_\pi) + R_{sig}} g_m (R_C||R_L) \left[\frac{s}{s + \frac{1}{C_{C1}\left[(R_B||r_\pi) + R_{sig}\right]}}\right]$$

This factor is recognized as the transfer function of a single time constant (STC) network of the high pass type with a -3 dB frequency ω_{P1} which is written as

$$\omega_{P1} = \frac{1}{C_{C1}\left[(R_B||r_\pi) + R_{sig}\right]}$$

Next we consider the effect of C_E. For this purpose we assume that C_{C1} and C_{C2} are acting as a perfect short circuits and thus obtain the circuit in the Figure (c) above. Reflecting r_π and C_E into the base and utilizing Thevenin theorem we obtain the base current as

$$V_{BB} = V_{sig}\frac{R_B}{R_B + R_{sig}} \qquad R_{BB} = R_B||R_{sig} \qquad V_{BB} = R_{BB}I_b + (1+\beta)\left(r_e + \frac{1}{sC_E}\right)I_b$$

$$I_b = V_{sig}\frac{R_B}{R_B + R_{sig}}\frac{1}{\left(R_B||R_{sig}\right) + (\beta + 1)\left(r_e + \frac{1}{sC_E}\right)}$$

$$V_o = -\beta I_b(R_C||R_L) = -\frac{R_B}{R_B + R_{sig}} \frac{\beta(R_C||R_L)}{\left(R_B||R_{sig}\right) + (\beta + 1)\left(r_e + \frac{1}{sC_E}\right)} V_{sig}$$

$$\frac{V_o}{V_{sig}} = -\frac{R_B}{R_B + R_{sig}} \frac{\beta(R_C||R_L)}{\left(R_B||R_{sig}\right) + (\beta + 1)r_e} \frac{s}{s + \left[1/C_E\left(r_e + \frac{R_B||R_{sig}}{(\beta + 1)}\right)\right]}$$

We observe that C_E introduces the STC high-pass factor on the extreme right hand side. Thus C_E cause the gain to fall off at low frequencies, with the -3 dB frequency equal to the frequency of the high-pass STC factor, that is

$$\omega_{P2} = \frac{1}{C_E\left[r_e + \frac{R_B||R_{sig}}{\beta + 1}\right]}$$

Finally, we consider the effect of C_{C2}. The circuit with C_{C1} and C_E assumed to be acting as perfect short circuits is shown in the Figure (d) above, for which we can write

$$V_\pi = V_{sig}\frac{R_B||r_\pi}{(R_B||r_\pi) + R_{sig}}$$

$$V_o = -g_m V_\pi \frac{R_C}{R_C + \frac{1}{sC_{C2}} + R_L} R_L$$

$$\frac{V_o}{V_{sig}} = -\frac{R_B||r_\pi}{(R_B||r_\pi) + R_{sig}} g_m (R_C||R_L)\left[\frac{s}{s + \frac{1}{C_{C2}(R_C + R_L)}}\right]$$

We observe that C_{C2} introduces the frequency-dependent factor which we recognize as the transfer function of a high-pass STC network with a break frequency ω_{P3} given as

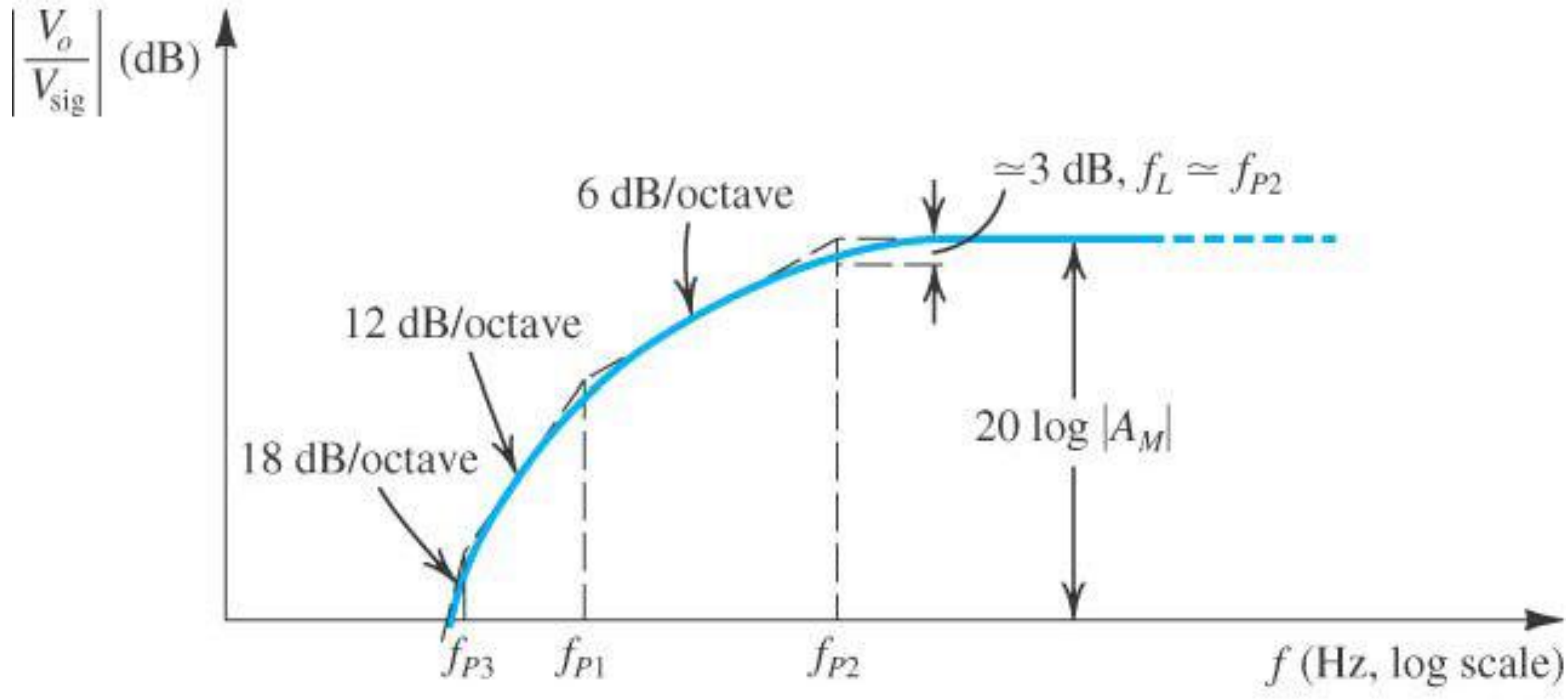
$$\omega_{P3} = \frac{1}{C_{C2}(R_C + R_L)}$$

Now that we have determined the effects of each of C_{C1}, C_E, C_{C2} acting alone, the amplifier low frequency gain can be expressed as

$$\frac{V_o}{V_{sig}} = -A_M\left(\frac{s}{s+\omega_{P1}}\right)\left(\frac{s}{s+\omega_{P2}}\right)\left(\frac{s}{s+\omega_{P3}}\right)$$

From which we see that it acquires three break frequencies at f_{P1}, f_{P2}, f_{P3}, all in the low frequency band. If the three frequencies are widely separated, their effects will be distinct, as indicated by the Figure below. The important point is that the -3 dB frequency f_L is determined by the highest of these break frequencies. This is usually the break frequency caused by the bypass capacitor C_E, simply because the resistance that it sees is usually quite small. Thus, even if one uses a large value of C_E, f_{P2} is usually the highest of the three break frequencies.

If f_{P1}, f_{P2}, and f_{P3} are close together, none of the three dominates, and to determine f_L, we have to evaluate $\left[V_o/V_{sig}\right]$ in the equation given above. We can obtain a reasonably good estimate for f_L using the following formula.



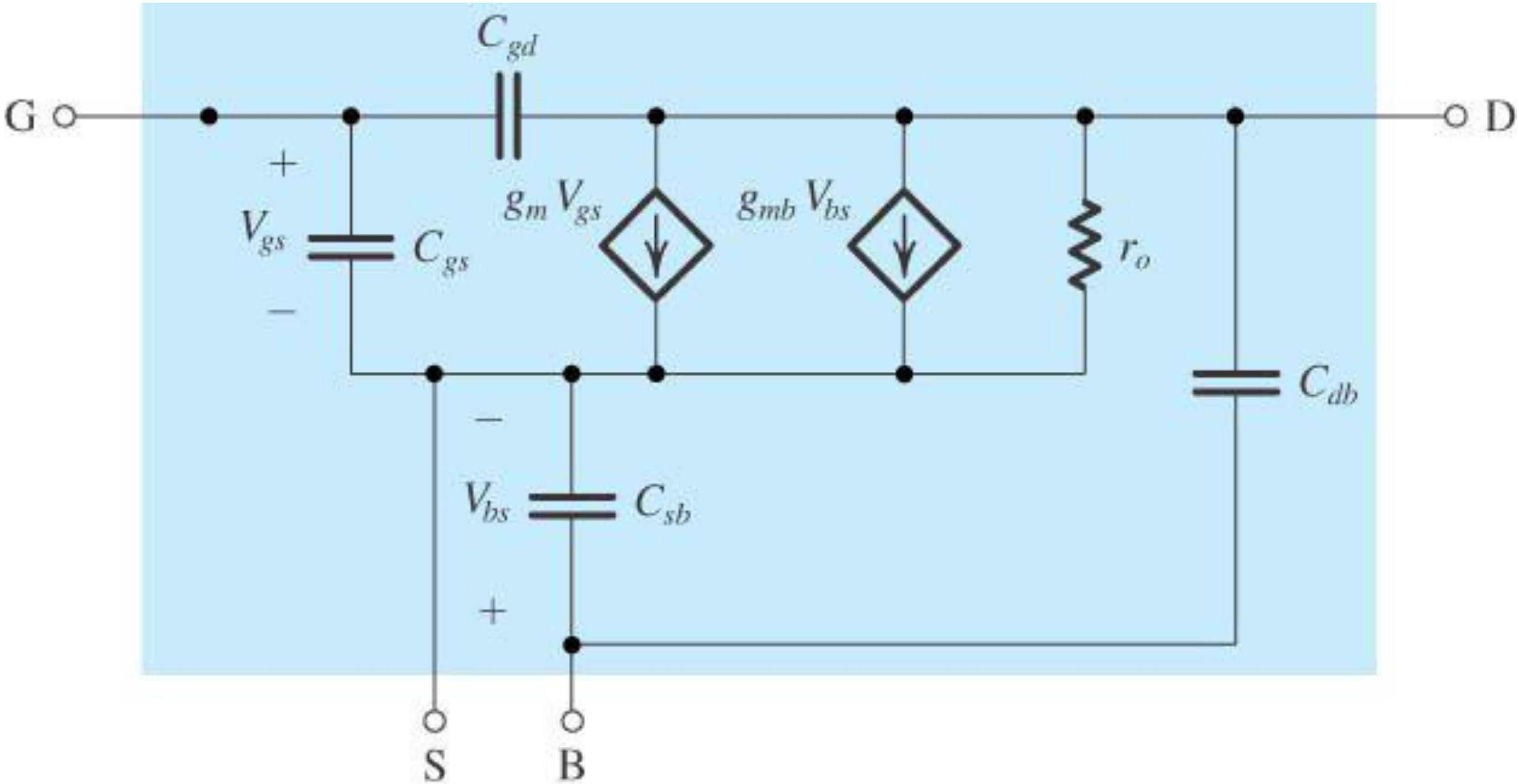
(e)

$$f_L \equiv \frac{1}{2\pi}\left[\frac{1}{C_{C1}R_{C1}} + \frac{1}{C_E R_E} + \frac{1}{C_{C2}R_{C2}}\right] \quad \text{or} \quad f_L = f_{P1} + f_{P2} + f_{P3}$$

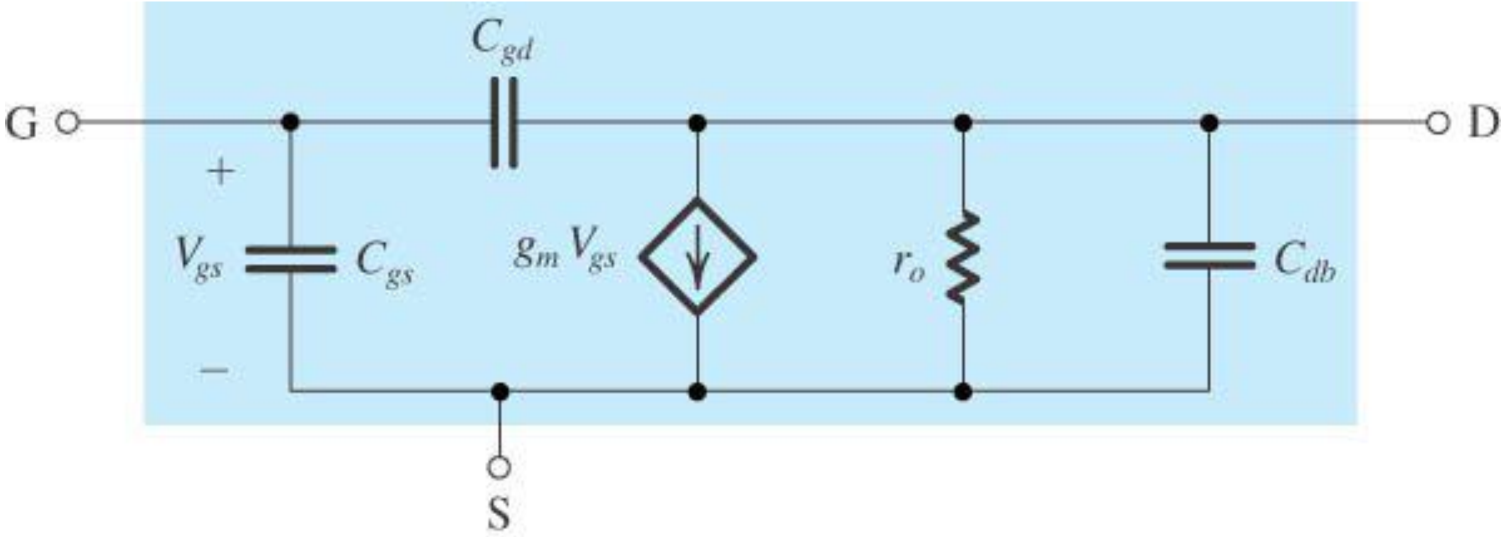
Where R_{C1}, R_E, and R_{C2} are the resistances seen by C_{C1}, C_E, C_{C2}, respectively, when V_{sig} is set to zero and the other two capacitances are replaced with short circuits.

The High-Frequency MOSFET Model

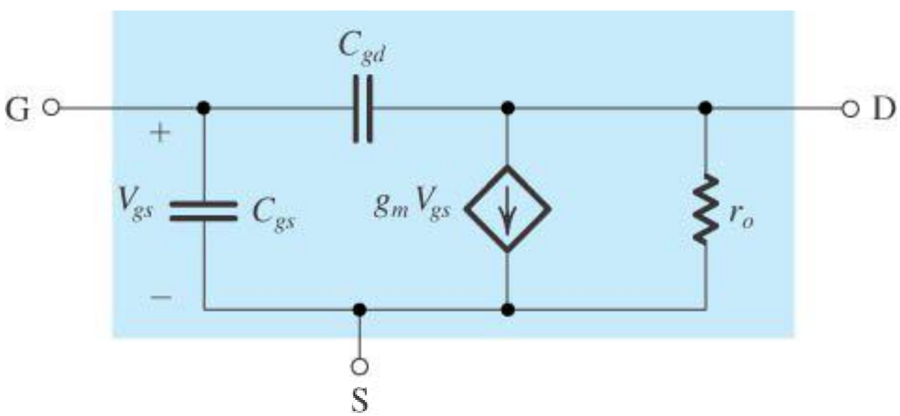
Figure (a) given below shows the small-signal model of the MOSFET, including the four capacitances C_{gs}, C_{gd}, C_{sb}, and C_{db}. This model can be used to predict high-frequency response of the MOSFET amplifier.



(a)



(b)



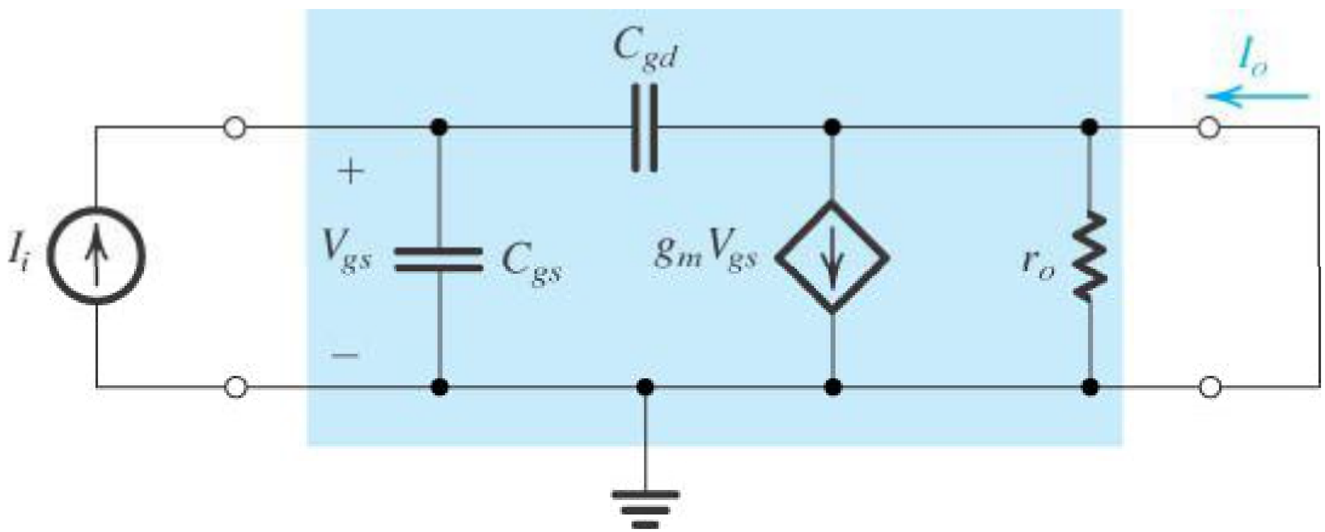
(c)

(b) The equivalent circuit for the case in which the source is connected to the substrate (body).

(c) The equivalent circuit model of (b) with C_{db} neglected (to simplify analysis)

The MOSFET Unity-Gain Frequency (f_T)

A figure of merit for the high frequency operation of the MOSFET as an amplifier is the unity-gain frequency, f_T. This is defined as the frequency at which the short-circuit current gain of common-source configuration becomes unity. Figure given below shows the MOSFET hybrid-π model with the source as the common terminal between the input and the output ports.



To determine the short-circuit current gain, the input is fed with the current source signal I_i and the output terminals are short circuited. It is easy to see that the current in the short circuit is given by

$$I_o = g_m V_{gs} - sC_{gd}V_{gs}$$

Since C_{gd} is small, at the frequencies of interest, the second term in the Equation above can be neglected,

$$I_o \cong g_m V_{gs}$$

From the Figure above we can express V_{gs} in terms of input current Ii as

$$V_{gs} = I_i/s\left(C_{gs} + C_{gd}\right)$$

Equations above can be combined to obtain the short circuit current gain,

$$\frac{I_o}{I_i} = \frac{g_m}{s\left(C_{gs}+C_{gd}\right)}$$

For the physical frequencies s = jω, it can be seen that the magnitude of the current gain becomes unity at the frequency

$$\omega_T = g_m/\left(C_{gs} + C_{gd}\right)$$

Thus the unity-gain frequency $f_T = \omega_T/2\pi$ is

$$f_T = \frac{g_m}{2\pi(C_{gs}+C_{gd})}$$

Since f_T is proportional to g_m and inversely proportional to the FET internal capacitances, the higher the value of f_T, the more effective the FET becomes as an amplifier. Substituting for g_m using the Equation given below

$$g_m = \sqrt{2k_n'}\sqrt{W/L}\sqrt{I_D}$$

We can express f_T in terms of the bias current I_D. Alternatively, we can substitute for g_m from the equation given below to express f_T in terms of the overdrive voltage V_{ov}.

$$g_m = k_n'(W/L)(V_{GS} - V_t) = k_n'(W/L)V_{OV}$$

Both the expressions yield additional insight into the high-frequency operation of the MOSFET.

Typically, f_T ranges from about 100 MHz for the older technologies to many GHz for newer high speed technologies.

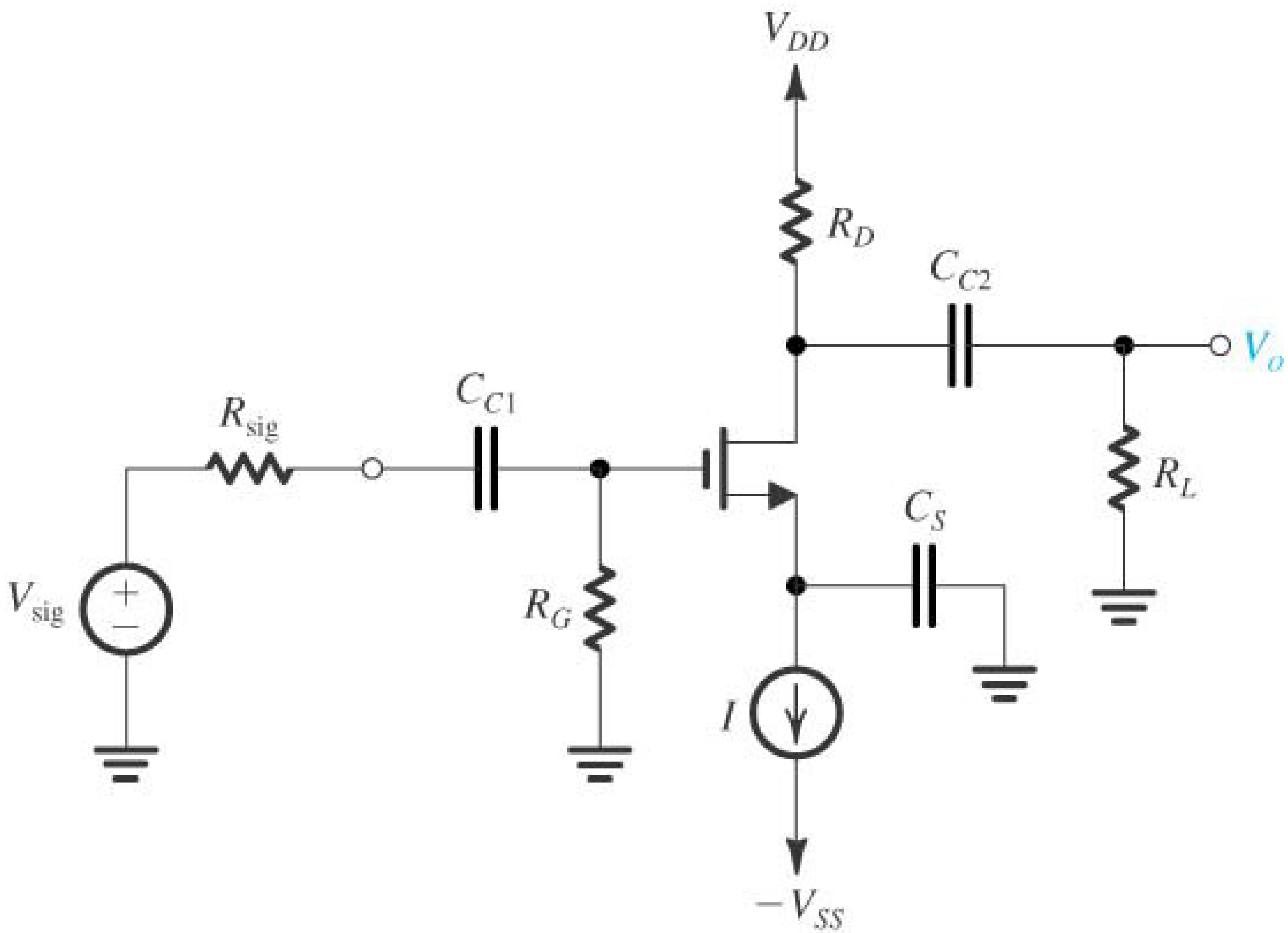
Frequency Response of the Common Source Amplifier

Three Frequency Bands

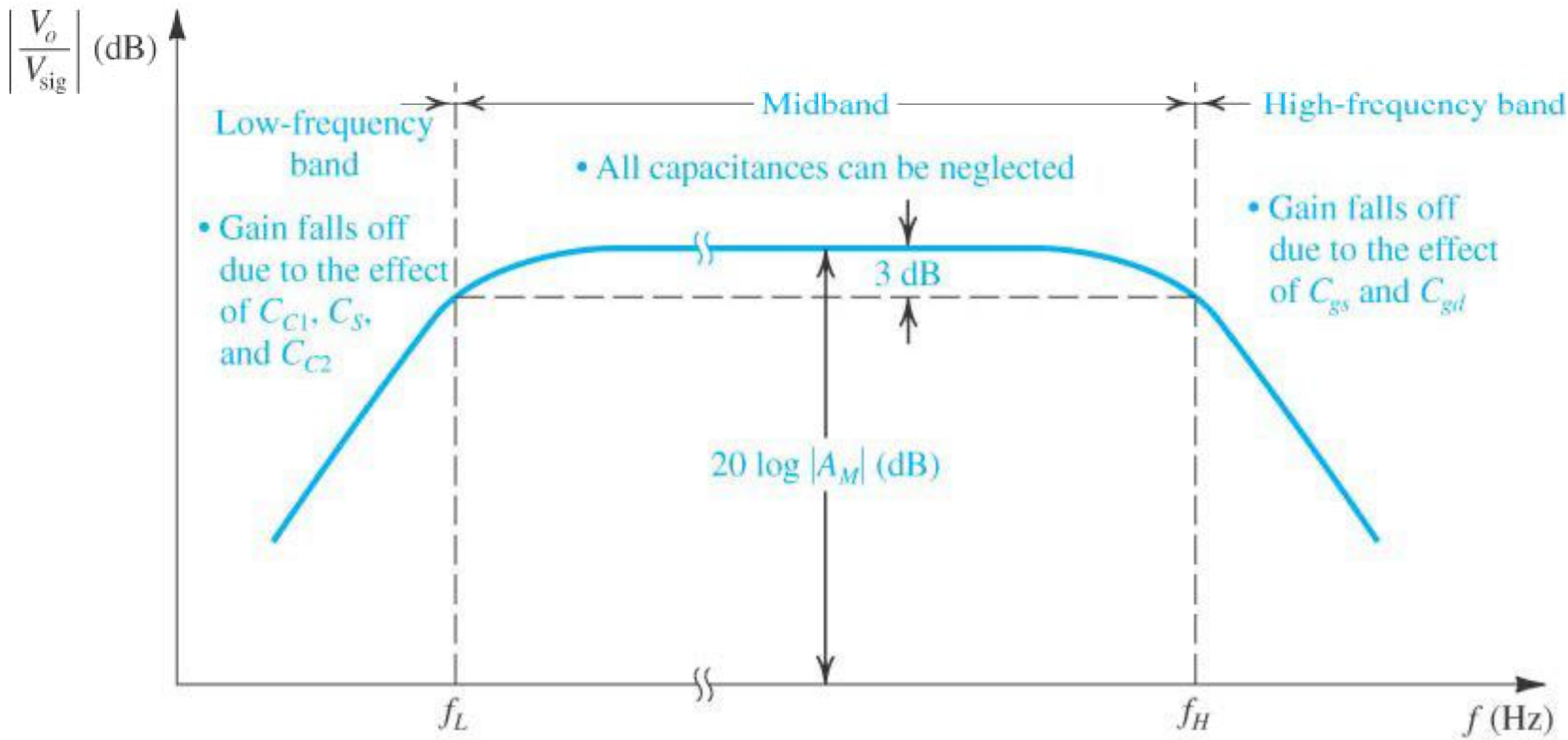
When the circuit of Figure (a) below was studied earlier, it was assumed that the coupling capacitors C_{C1} and C_{C2} and the bypass capacitor C_S were acting as perfect short circuit at all signal frequencies of interest. We also neglected the internal capacitances of the MOSFET, that is C_{gd} and C_{gs}. We observe that the gain of the amplifier is almost constant over a wide frequency band, called the **midband**. The value of the midband gain A_M corresponds to the overall voltage gain G_v namely,

$$A_M \equiv \frac{V_o}{V_{sig}} = -\frac{R_G}{R_G + R_{sig}} g_m(r_o||R_D||R_L)$$

Figure (b) below shows that the gain falls off at signal frequencies below and above the midband. The gain falloff in the **low-frequency band** is due to the fact that even though C_{C1}, C_{C2}, and C_S are large capacitors, as the signal frequency is reduced, their impedances increase, and they no longer behave as short circuits. On the other hand, the gain falls off in the **high-frequency band** as a result of C_{gs} and C_{gd}, which though very small, their impedances at high frequencies decrease and thus no longer be considered as open circuits.



(a)

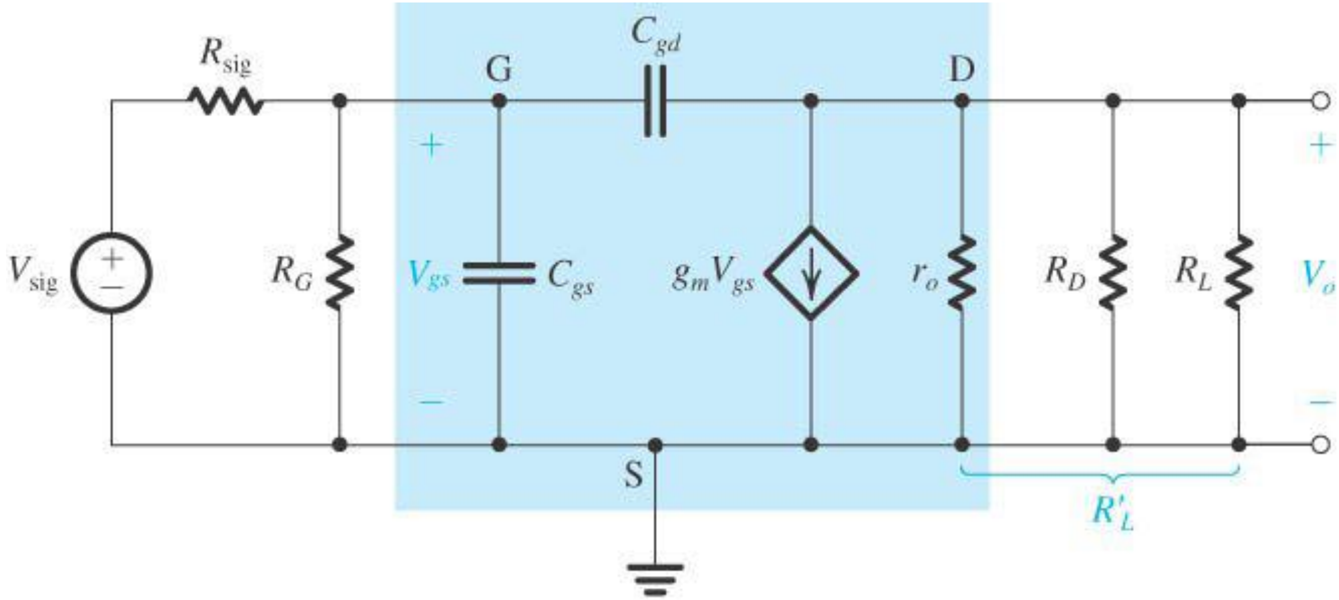


$\left|\frac{V_o}{V_{sig}}\right|$ (dB)
Low-frequency band
Midband
High-frequency band
• All capacitances can be neglected
• Gain falls off due to the effect of C_{C1}, C_S, and C_{C2}
• Gain falls off due to the effect of C_{gs} and C_{gd}
3 dB
20 log $|A_M|$ (dB)
f_L
f_H
f (Hz)

(b)

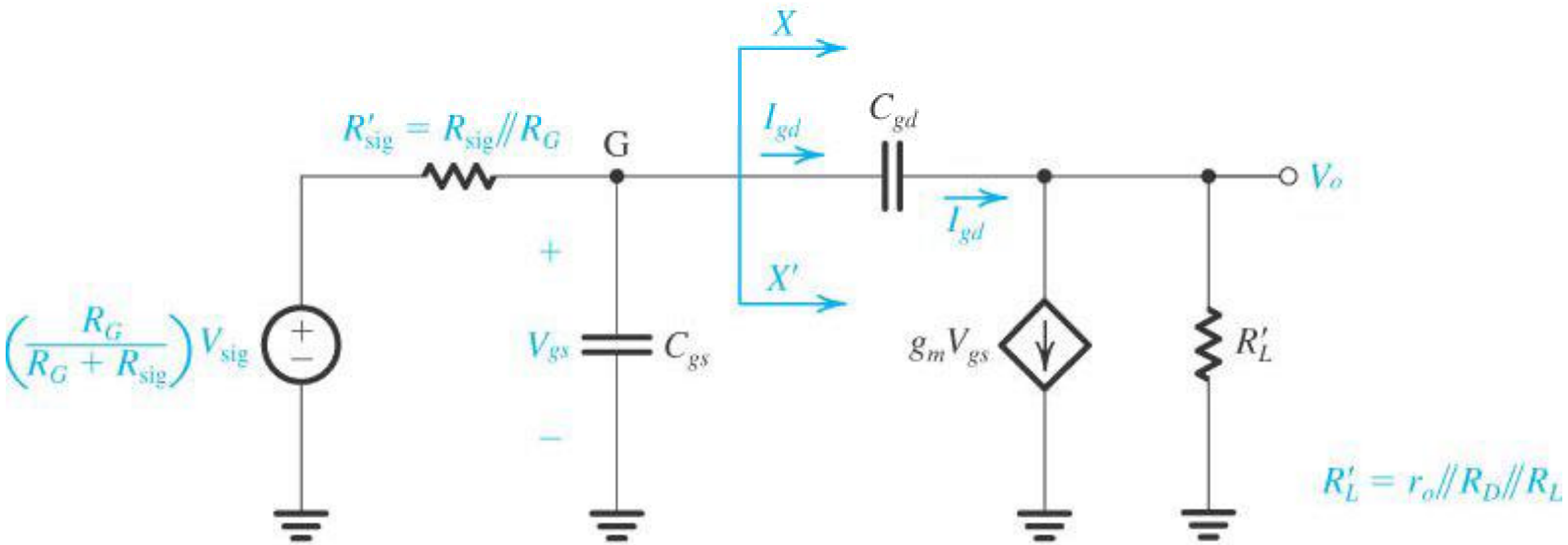
The High-Frequency Response

To determine the gain, or transfer function of the amplifier given in Figure (a) above at high frequencies, we replace the MOSFET with its high-frequency model. At these frequencies C_{C1}, C_{C2}, and C_S will behave as perfect short circuits. The result is the high frequency amplifier equivalent circuit shown in Figure (a) below.



(a)

The equivalent circuit of Figure (a) above can be simplified by utilizing the Thevenin theorem at the input side by combining the three parallel resistance at the output side. The resulting simplified circuit is shown in Figure (b) below.



(b)

$$R_L' = r_o||R_D||R_L \qquad \textit{and since} \qquad V_o = V_{ds}$$

$$I_{gd} = sC_{gd}\left(V_{gs} - V_o\right) = sC_{gd}\left[V_{gs} - \left(-g_m R_L' V_{gs}\right)\right] = sC_{gd}(1 + g_m R_L')V_{gs}$$

$$sC_{eq}V_{gs} = sC_{gd}(1 + g_m R_L')V_{gs}$$

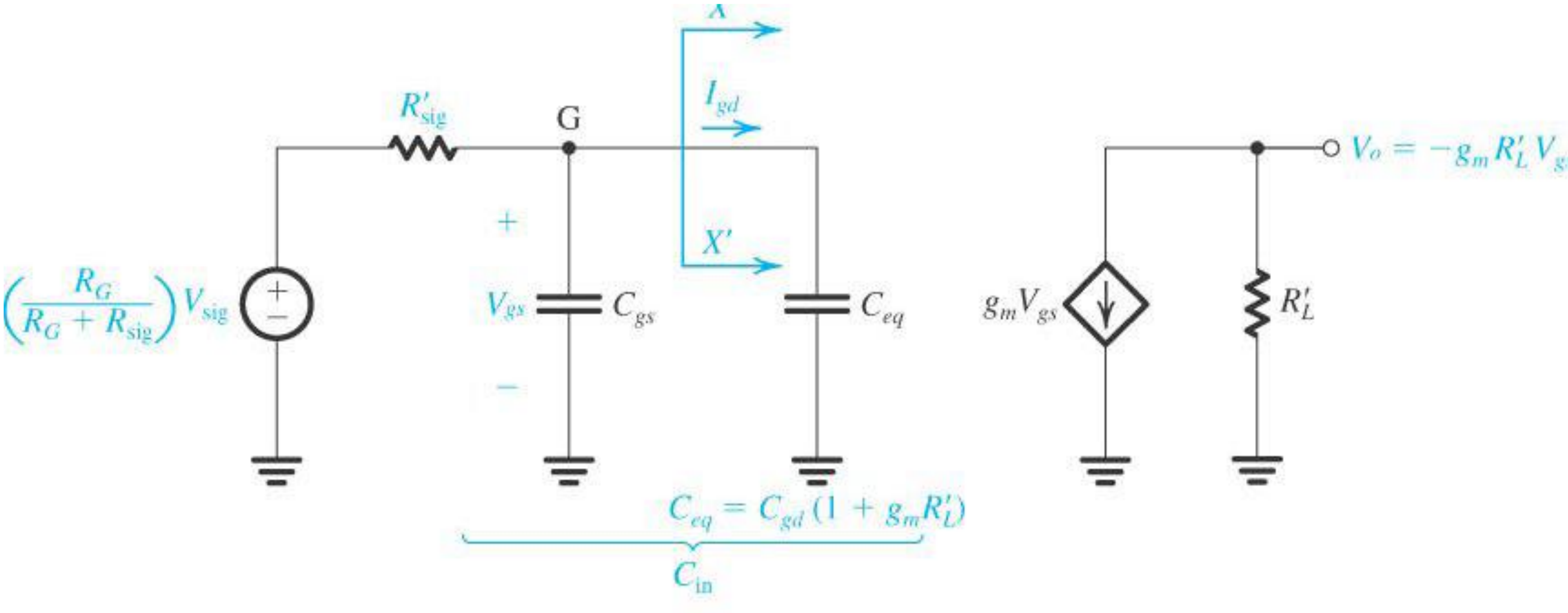
$$C_{eq} = C_{gd}(1 + g_m R_L')$$

V_{gs} of the STC circuit can be written as

$$V_{gs} = \left(\frac{R_G}{R_G + R_{sig}} V_{sig}\right)\frac{1}{1 + \frac{s}{\omega_o}}$$

Where ω_o is the corner frequency or the break frequency of the STC circuit

$$\omega_o = 1/C_{in}R_{sig}' \qquad \text{with} \qquad C_{in} = C_{gs} + C_{eq} = C_{gs} + C_{gd}(1 + g_m R_L')$$



(c)

$$R'_{sig} = R_{sig} || R_G$$

Combining the above equations we get

$$\frac{V_o}{V_{sig}} = -\left(\frac{R_G}{R_G + R_{sig}}\right)(g_m R'_L)\frac{1}{1 + \frac{s}{\omega_o}}$$

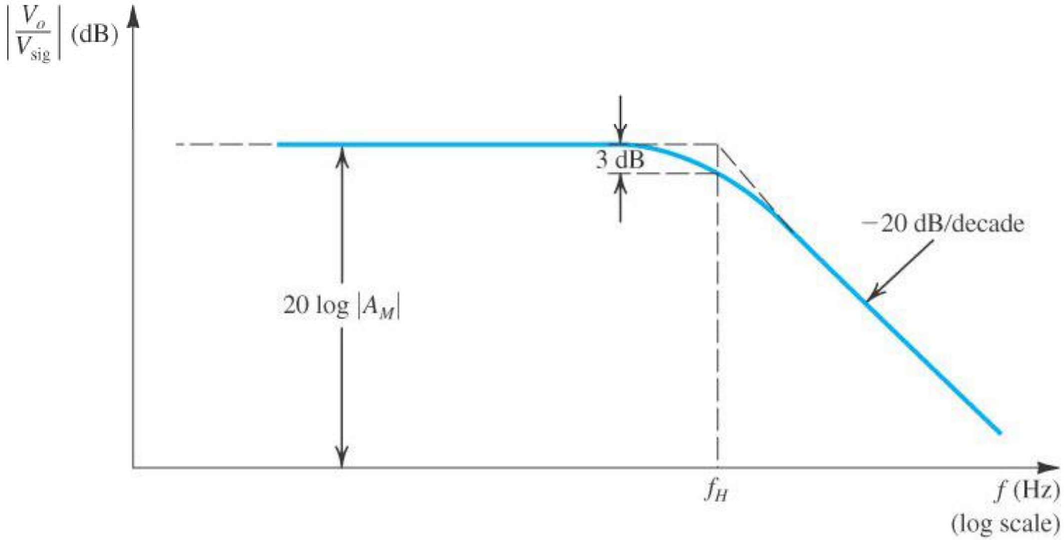
$$\frac{V_o}{V_{sig}} = \frac{A_M}{1 + \frac{s}{\omega_H}}$$

Where A_M is the midband gain and ω_H is the upper 3-dB frequency

$$A_M = -\frac{R_G}{R_G + R_{sig}} g_m R_L'$$

$$\omega_H = \omega_o = \frac{1}{C_{in} R_{sig}'}$$

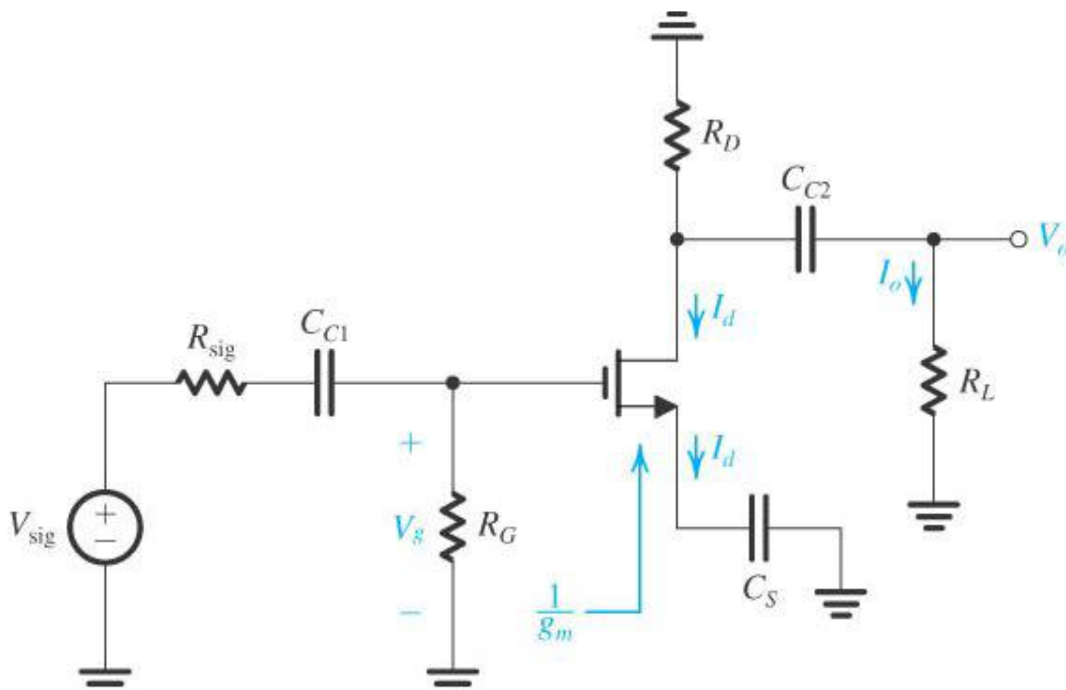
$$f_H = \frac{\omega_H}{2\pi} = \frac{1}{2\pi C_{in} R_{sig}'}$$



(d)

The Low-Frequency Response

To determine the low-frequency gain or transfer function of the common-source amplifier, we show the Figure below where the dc sources are eliminated (current source I open-circuited and voltage source V_{DD} short-circuited). We shall perform the small-signal analysis directly on this circuit. However, we shall ignore ro. This is done in order to keep the analysis simple and thus focus attention on significant issues.



The analysis begins at the signal generator by finding the fraction of V_{sig} that appears at the transistor gate,

$$V_g = V_{sig}\frac{R_G}{R_G+\frac{1}{sC_{C1}}+R_{sig}}$$

Which can be written in the alternate form

$$V_g = V_{sig}\frac{R_G}{R_G+R_{sig}}\frac{s}{s+\frac{1}{C_{C1}\left(R_G+R_{sig}\right)}}$$

Thus we see that the expression for the signal transmission from signal generator to amplifier input has acquired a frequency-dependent factor. We recognize this factor as the transfer function of an STC network of the high-pass type with a break or corner frequency $\omega_o = 1/C_{C1}\left(R_G + R_{sig}\right)$. Thus the effect of the coupling capacitor C_{C1} is to introduce a high-pass STC response with a break frequency that we shall denote ω_{P1},

$$\omega_{P1} = \omega_o = \frac{1}{C_{C1}\left(R_G+R_{sig}\right)}$$

Continuing with the analysis we next determine the drain current I_d by dividing V_g by the total impedance in the source circuit which is $[(1/g_m) + (1/sC_S)]$ to obtain

$$I_d = \frac{V_g}{\frac{1}{g_m}+\frac{1}{sC_S}}$$

Which can be written in the alternate form

$$I_d = g_m V_g \frac{s}{s+\frac{g_m}{C_S}}$$

We observe that C_S introduces a frequency-dependent factor, which is also of the STC high-pass type. Thus the amplifier acquires another break frequency,

$$\omega_{P2} = \frac{g_m}{C_S}$$

To complete the analysis, we find V_o by first using the current divider rule to determine the fraction of I_d that flows through R_L,

$$I_o = -I_d \frac{R_D}{R_D+\frac{1}{sC_{C2}}+R_L}$$

And then multiplying I_o by R_L to obtain

$$V_o = I_o R_L = -I_d \frac{R_D R_L}{R_D+R_L} \frac{s}{s+\frac{1}{C_{C2}(R_D+R_L)}}$$

From which we see that C_{C2} introduces a third STC high-pass factor, giving the amplifier a third break frequency at

$$\omega_{P3} = \frac{1}{C_{C2}(R_D+R_L)}$$

The overall low-frequency transfer function of the amplifier can be found out by combining the above equations and replacing the break frequencies by their symbols from the above equations,

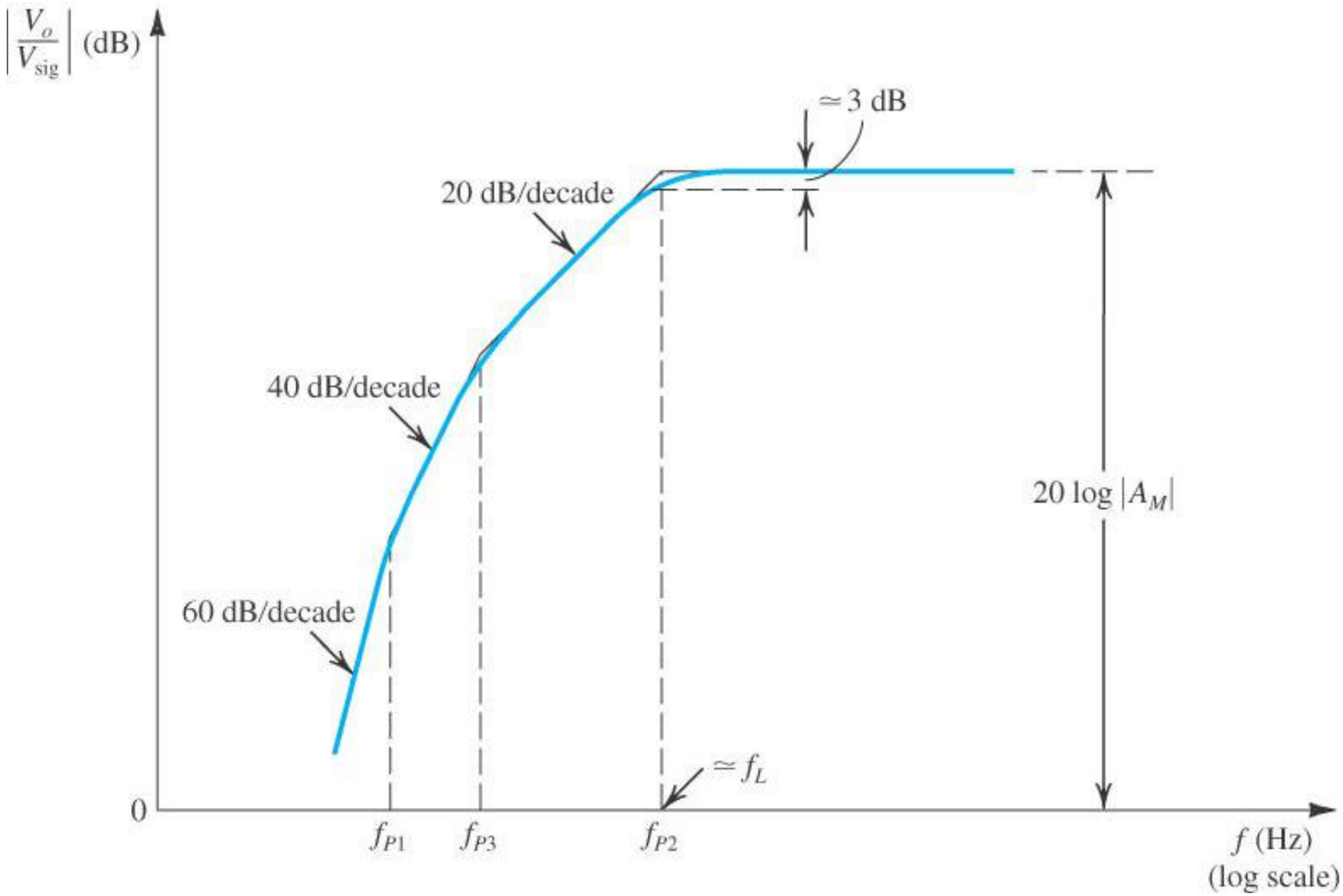
$$\frac{V_o}{V_{sig}} = -\left(\frac{R_G}{R_G+R_{sig}}\right)[g_m(R_D||R_L)]\left(\frac{s}{s+\omega_{P1}}\right)\left(\frac{s}{s+\omega_{P2}}\right)\left(\frac{s}{s+\omega_{P3}}\right)$$

The low frequency magnitude response can be obtained from the above equation by replacing s by jω and finding $|V_o/V_{sig}|$. In many cases, however, one of the three break frequencies can be much higher than the other two, say by a factor greater than 4. in such case, it is this highest frequency break point that will determine the lower 3-dB frequency f_L. For instance, because the expression for ω_{P2} includes g_m, ω_{P2} is usually higher than ω_{P1} and ω_{P3}. If ω_{P2} is sufficiently separated from ω_{P1} and ω_{P3} then

$$f_L \cong f_{P2}$$

Which means that in such case, the bypass capacitor determines the low end of the mid-band.

Figure given below shows a sketch of the low-frequency gain of the CS amplifier in which the three break frequencies are sufficiently separated so that their effects appear distinct.



Selecting values for the Coupling and Bypass Capacitors

The design objective is to place the lower 3-dB frequency f_L at a specified value while minimizing the capacitor values. Since as mentioned above C_S results in the highest of the three break frequencies, the total capacitance is minimized by selecting C_S so that its break frequency $f_{P2} = f_L$. We then decide on the location of the other two break frequencies, say 5 to 10 times lower than the frequency of the dominant one, f_{P2}. However, the values selected for f_{P1} and f_{P3} should not be too low, for that would require larger values for C_{C1} and C_{C2} than may be necessary.