CS21003 ALGORITHMS-1
(Tutorial 5 : Dynamic Programming – Solutions)
Date: Oct 3 2020

1 Contiguous Subsequence of Maximum Sum

Solution -
The idea is to maintain two values at every point (index i). The global maximum sum that we have seen (MX), and the maximum sum of contiguous elements including the element i (max_i).
Subproblems: (max_i) for $i = 1$ to n, MX
Recurrence:
Suppose you are at the $i+1$th element in the array.
$max_{i+1} = max(a_i, a_i + max_i)$
$MX = max(max_{i+1}, MX)$
Final answer is MX and you may make small changes to be able to come up with the actual contiguous sequence.

2 Minimize the Rental Cost

Solution -
Let $m[i]$ be the rental cost for the best solution to go from post i to post n for $1 \leqslant i < j \leqslant n$. The final answer is in $m[1]$, and $m[n] = 0$.
Recurrence: $m[i] = min_{j:i<j\leqslant n}(m[j] + f_{i,j})$
The subproblems can be solved in the order $m[n], m[n-1], \ldots, m[1]$. The time complexity if $O(n^2)$.

3 Parenthesization

Solution-
We need to record both the maximum and minimum value of each subproblem to take care of both the signs.
Subproblems: Let $M[i,j]$ denote the max value of the expression $a_i \circ_i \ldots x_j$ and $m[i,j]$ denote its minimum value.
Recurrence: Consider $M[i,j]$. You will first parenthesize at $[i,k]$ and $[k+1,j]$ for some $i \leqslant k \leqslant j-1$.
If $\circ_k =$ '$+$', $M[i,j]$ can be written as:
$M[i,j] = M[i,k] + M[k+1,j]$
On the other hand, if $\circ_k =$ '$-$', it can be written as:
$M[i,j] = M[i,k] - m[k+1,j]$
Since $M[i,j]$ denotes the max value,
$M[i,j] = max_{i\leqslant k\leqslant j-1} M[i,k] + M[k+1,j],\ \text{if } \circ_k = \text{'}+\text{'}, M[i,k] - m[k+1,j] \text{ if } \circ_k = \text{'}-\text{'}$
Analogously, you can define recurrence for $m[i,j]$.
The complexity will be $O(n^3)$. The table of subproblems should be filled in a very similar way as we did for matrix chain multiplication problem.

4 Longest Good Subsequence

Solution-
Note that similar to the last problem, you will need to maintain the length of longest sequences ending either with $<$ or $>$.
$maxL_i$: Length of the longest good sequence of the form $a_1 < a_2 > a_3 < ... < a_i$ ending at i
$maxR_i$: Length of the longest good sequence of the form $a_1 < a_2 > a_3 < ... > a_i$ ending at i
Recurrence:
$maxL_i = max(1, max\{maxR_j + 1 | j < i, A_j < A_i\})$
$maxR_i = max(1, max\{maxL_j + 1 | j < i, A_j > A_i\})$
Final answer: max of all $maxL_i$ and $maxR_i$