**MCQ 2.4.47** A charge is uniformly distributed throughout the sphere of radius $a$. Taking the potential at infinity as zero, the potential at $r = b < a$ is

(A) $-\int_{\infty}^{b} \frac{Qr}{4\pi\varepsilon_0 a^3} dr$

(B) $-\int_{\infty}^{b} \frac{Q}{4\pi\varepsilon_0 r^2} dr$

(C) $-\int_{\infty}^{a} \frac{Q}{4\pi\varepsilon_0 r^2} dr - \int_{a}^{b} \frac{Qr}{4\pi\varepsilon_0 a^3} dr$

(D) $-\int_{\infty}^{a} \frac{Q}{4\pi\varepsilon_0 r^2} dr$

**MCQ 2.4.48** A potential field is given by $V = 3x^2y - yz$. Which of the following is not true ?

(A) At the point $(1,0,-1)$, V and the electric field $\boldsymbol{E}$ vanish

(B) $x^2y = 1$ is an equipotential plane in the $xy$-plane

(C) The equipotential surface $V = -8$ passes through the point $P(2,-1,4)$

(D) A unit vector normal to the equipotential surface $V = -8$ at P is $(-0.83\boldsymbol{x} + 0.55\boldsymbol{y} + 0.07\boldsymbol{z})$

**MCQ 2.4.49** The relation between electric intensity $E$, voltage applied $V$ and the distance $d$ between the plates of a parallel plate condenser is

(A) $E = V/d$

(B) $E = V \times d$

(C) $E = V/(d)^2$

(D) $E = V \times (d)^2$

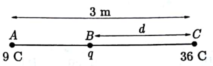
***********

# SOLUTIONS 2.1



**SOL 2.1.1** Option (B) is correct.

Since the two point charges are positive so the introduced third point charge must be negative as to make the entire system in equilibrium as shown below




as the system must be in equilibrium so the force between all the pair of charges will be equal

i.e. $$F_{AB} = F_{CB} = F_{AC}$$

$$\frac{(9)q}{(3-d)^2} = \frac{(36)q}{d^2} = \frac{(36)(9)}{(3)^2}$$

Solving the equation we get, $q = -4\,\text{C}$ and $d = 2\,\text{m}$

**SOL 2.1.2** Option (A) is correct.

Electric field intensity at any point $P$ due to the two point charges $Q_1$ and $Q_2$ is defined as

$$\boldsymbol{E} = k\left(\frac{Q_1}{(R_1)^3}\boldsymbol{R}_1 + \frac{Q_2}{(R_2)^3}\boldsymbol{R}_2\right)$$

where, $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ is the vector distance of the point $P$ from the two point charges.

So the net electric field due to the two given point charges is

$$\boldsymbol{E} = \frac{9\times10^9\times(-5)\times10^{-9}[(-7+4)\,\boldsymbol{a}_x + (3-0)\,\boldsymbol{a}_y + (-1+2)\,\boldsymbol{a}_z]}{\sqrt{(-7+4)^2+(3-0)^2+(-1+2)^2}}$$
$$+\frac{9\times10^9\times2\times10^{-9}[(-7+5)\,\boldsymbol{a}_x + (3-0)\,\boldsymbol{a}_y + (-1-3)\,\boldsymbol{a}_z]}{\sqrt{(-7+5)^2+(3-0)^2+(-1-3)^2}}$$
$$= \frac{-45[-3\boldsymbol{a}_x + 3\boldsymbol{a}_y + \boldsymbol{a}_z]}{19^{3/2}} + \frac{18[-2\boldsymbol{a}_x + 3\boldsymbol{a}_y - 4\boldsymbol{a}_z]}{29^{3/2}}$$
$$= 1.4\boldsymbol{a}_x - 1.284\boldsymbol{a}_y - 1.004\boldsymbol{a}_z$$

**SOL 2.1.3** Option (D) is correct.

For an electric field to exist, the its curl must be zero. So, we check the existence of the given field vector first.

Given the electric field intensity

$$\boldsymbol{E} = 2xy\boldsymbol{a}_x + 4yz\boldsymbol{a}_y + 6xz\boldsymbol{a}_z \text{ V/m}$$

So, $$\nabla\times\boldsymbol{E} = 2\begin{vmatrix} \boldsymbol{a}_x & \boldsymbol{a}_y & \boldsymbol{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & 2yz & 3xz \end{vmatrix}$$

$$= 2[-2y\boldsymbol{a}_x - 3z\boldsymbol{a}_y - x\boldsymbol{a}_z] \neq 0$$

Therefore, as the curl of the given electric field is not equal to zero so, the field does not exist.

**SOL 2.1.4** Option (C) is correct.
Electric field intensity in free space at a distance $\boldsymbol{R}$ from an infinite line charge with charge density $\rho_L$ is defined as

$$\boldsymbol{E} = \frac{\rho_L}{2\pi\varepsilon_0}\frac{\boldsymbol{R}}{R^2}$$

Given $\rho_L = 1\,\mu\text{C/m} = 1\times 10^{-6}\,\text{C/m}$

$$\boldsymbol{R} = -2\boldsymbol{a}_x - \boldsymbol{a}_y$$

So,

$$\boldsymbol{E} = \frac{(1\times 10^{-6})}{2\pi\varepsilon_0}\left(\frac{-2\boldsymbol{a}_x - \boldsymbol{a}_y}{5}\right)$$

$$= -7.2\boldsymbol{a}_x - 3.6\boldsymbol{a}_y\ \text{kV/m}$$

**SOL 2.1.5** Option (B) is correct.
Electric flux density in a certain region for the given electric field intensity is defined as

$$\boldsymbol{D} = \varepsilon_0\boldsymbol{E} = \varepsilon_0(x^2\boldsymbol{a}_x + 2xy\boldsymbol{a}_y)$$

So at the point $(-1,0,1)$

$$\boldsymbol{D} = \varepsilon_0(\boldsymbol{a}_x)$$

**SOL 2.1.6** Option (A) is correct.
According to Gauss law net outward electric flux from any closed surface is equal to the total charge enclosed by the volume

i.e.

$$\psi = Q_{enc}$$

or,

$$\psi = \int \rho_v\,dv$$

$$= \int_{r=0}^{1}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\left(\frac{1}{r^2}\right)(r^2\sin\theta\,dr\,d\theta\,d\phi)$$

$$= 1\times 2\times 2\pi = 4\pi\ \text{C}$$

**SOL 2.1.7** Option (B) is correct.
As we have already determined the total electric flux crossing the surface $r = 1\,\text{m}$ So, electric flux density $\boldsymbol{D}$ at $r = 1\,\text{m}$ is evaluated as below:

Total electric flux

$$\psi = \oint \boldsymbol{D}\cdot d\boldsymbol{S}$$

So we have

$$\oint \boldsymbol{D}\cdot d\boldsymbol{S} = 4\pi \qquad (\psi = 4\pi)$$

$$D(4\pi r^2) = 4\pi$$

$$D = \frac{1}{r^2} = 1\,\text{C/m}^2$$

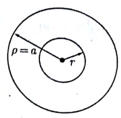
Thus

$$\boldsymbol{D} = \boldsymbol{a}_r\ \text{C/m}^2$$

**SOL 2.1.8** Option (A) is correct.
As the point charge is located at origin. So flux due to it will be emanating from all the eight quadrants symmetrically.
So the flux through the portion of plane $x + y = 2\,\text{m}$ lying in first octant is 1/8 of the total flux emanating from the charge located at origin.
and from Gauss law, total flux $= Q_{enc} = 8\,\text{C}$
So, flux through the surface $x + y = 2\,\text{m}$ is $\frac{Q_{enc}}{8} = \frac{8}{8} = 1\,\text{C}$

**SOL 2.1.9** Option (A) is correct.
We construct a Gaussian surface at $\rho = r$ as shown in figure.






So, according to Gauss law the total outward flux through the surface $\rho = r$ will be equal to the charge enclosed by it.

i.e.

$$D(2\pi rh) = \rho_v(\pi r^2 h) \qquad \text{(assume the height of the cylinder is } h\text{)}$$

So,

$$D = \rho_v\frac{r}{2}$$

Therefore the electric field intensity at a distance $r$ from the cylindrical axis is

$$E = \frac{D}{\varepsilon_0} = \frac{\rho_v}{\varepsilon_0}\left(\frac{r}{2}\right)$$

Thus

$$E \propto r$$

**SOL 2.1.10** Option (C) is correct.
According to Gauss law the surface integral of the electric flux density over a closed surface is equal to the total charge enclosed inside the region defined by closed surface.

i.e.

$$\oint \boldsymbol{D}\cdot d\boldsymbol{S} = Q_{enc}$$

or

$$\oint \boldsymbol{E}\cdot d\boldsymbol{S} = \frac{1}{\varepsilon_0}Q_{enc} \qquad \left(\text{since } \boldsymbol{E} = \frac{\boldsymbol{D}}{\varepsilon_0}\right)$$

As we have to evaluate $\boldsymbol{E}$ for $r \le 2$ and since the charge density is zero for $r \le 2$ so

$$Q_{enc} = 0 \qquad (\text{for } r \le 2)$$

Therefore,

$$\oint \boldsymbol{E}\cdot d\boldsymbol{S} = \frac{1}{\varepsilon_0}\times 0$$

$$\boldsymbol{E} = 0$$

**SOL 2.1.11** Option (A) is correct.
Again from Gauss law, we have the surface integral of electric field intensity over the Gaussian surface at $r = 3$ as

$$\oint \boldsymbol{E}\cdot d\boldsymbol{S} = \frac{1}{\varepsilon_0}Q_{enc}$$

$$\oint \boldsymbol{E}\cdot d\boldsymbol{S} = \frac{1}{\varepsilon_0}\int \rho_v\,dv = \underbrace{\frac{1}{\varepsilon_0}\int 0\,dv}_{r\le 2} + \underbrace{\frac{1}{\varepsilon_0}\int (4/r^2)\,dv}_{2<r\le 3}$$

$$E(4\pi\times(3)^2) = \frac{1}{\varepsilon_0}\int_{r=2}^{3}\int_0^{\pi}\int_0^{2\pi}\left(\frac{4}{r^2}\right)(r^2\sin\theta\,dr\,d\theta\,d\phi)$$

$$E(4\pi\times 9) = \frac{4\pi\times 4}{\varepsilon_0}(3-2)$$

$$\boldsymbol{E} = \frac{4}{9\varepsilon_0}\boldsymbol{a}_r$$

**SOL 2.1.12** Option (D) is correct.
As calculated in the previous question, we have the surface integral of the electric field intensity over the Gaussian surface $r = 5$ as

$$\oint \boldsymbol{E}\cdot d\boldsymbol{S} = \frac{1}{\varepsilon_0}\int \rho_v\,dv = \underbrace{\frac{1}{\varepsilon_0}\int 0\,dv}_{r\le 2} + \underbrace{\frac{1}{\varepsilon_0}\int (4/r^2)\,dv}_{2<r\le 4} + \underbrace{\frac{1}{\varepsilon_0}\int 0\,dv}_{4<r\le 5}$$

$$E(4\pi \times (5)^2) = \frac{1}{\varepsilon_0}\int_{r=2}^{4}\int_0^{\pi}\int_0^{2\pi}\left(\frac{4}{r^2}\right)(r^2 \sin\theta dr d\theta d\phi)$$

$$E(100\pi) = \frac{4\pi \times 4}{\varepsilon_0}\int_2^4 dr$$

$$\boldsymbol{E} = \frac{8}{25\varepsilon_0}\boldsymbol{a}_r$$

**SOL 2.1.13** Option (A) is correct.
According to Gauss law the volume Charge density in a certain region is equal to the divergence of electric flux density in that region

i.e.
$$\rho_v = \nabla \cdot \boldsymbol{D}$$
$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\cos\theta}{r^3}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\frac{\sin\theta\sin\theta}{2r^3}\right)$$
$$= -\frac{1}{r^4}\cos\theta + \frac{1}{r^4}\cos\theta$$
$$= 0$$

**SOL 2.1.14** Option (B) is correct.
Electric field at any point is equal to the negative gradient of potential

i.e.
$$\boldsymbol{E} = -\nabla V = -\left(\frac{\partial}{\partial x}V + \frac{\partial}{\partial y}V + \frac{\partial}{\partial z}V\right)$$
$$= -\left[\left(y^2z^3 + \frac{3x}{x^2+2y^2+3z^2}\right)\boldsymbol{a}_x + \left(2xyz^3 + \frac{6y}{x^2+2y^2+3z^2}\right)\boldsymbol{a}_y + \left(3xy^2z^2 + \frac{9z}{x^2+2y^2+3z^2}\right)\boldsymbol{a}_z\right]$$

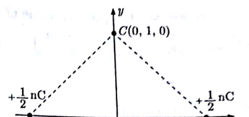
So, at the point $P(x = 3, y = 2, z = -1)$
$$\boldsymbol{E} = 3.6\boldsymbol{a}_x + 11.4\boldsymbol{a}_y - 35.6\boldsymbol{a}_z \text{ V/m}$$

**SOL 2.1.15** Option (A) is correct.
Electric flux density in terms of field intensity is defined as
$$\boldsymbol{D} = \varepsilon_0\boldsymbol{E}$$
So, at point $P(3,2,-1)$,
$$\boldsymbol{D} = \varepsilon_0(3.6\boldsymbol{a}_x + 11.4\boldsymbol{a}_y - 35.6\boldsymbol{a}_z)$$
$$= 31.4\boldsymbol{a}_x + 101\boldsymbol{a}_y - 314.5\boldsymbol{a}_z \text{ pC/m}^2$$

**SOL 2.1.16** Option (A) is correct.
Electric force experienced by a point charge $q$ located in the field $\boldsymbol{E}$ is defined as
$$\boldsymbol{F} = q\boldsymbol{E}$$
So, the force applied at the point charge $+1$ C located at $(0, y, 0)$ is
$$\boldsymbol{F} = (1)\frac{Qd}{4\pi\varepsilon_0 r^3}[2\cos^2\theta a_r + \sin\theta a_\theta] \qquad (q = +1\text{C})$$
$$= \frac{Qd}{4\pi\varepsilon_0 r^3}[\sin 90^\circ(-\boldsymbol{a}_z)] \qquad (\theta = 90^\circ,\ \boldsymbol{a}_\theta = -\boldsymbol{a}_z,\ r = y)$$
$$= \frac{-Qd}{4\pi\varepsilon_0 y^3}\boldsymbol{a}_z$$

**SOL 2.1.17** Option (B) is correct.
For determining the position of the third charge, first of all we evaluate the total electric field at the given point $C(0,1,0)$ due to the two point charges located at points $A(1,0,0)$ and $B(-1,0,0)$ respectively as shown in figure.





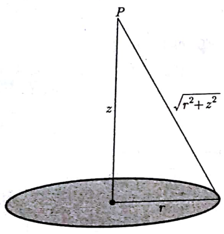

Electric field due to the charge located at point $A$ is
$$\boldsymbol{E}_1 = kQ\frac{\boldsymbol{AC}}{|\boldsymbol{AC}|^3} = 9 \times 10^9\left(\frac{1}{2}\right) \times 10^{-9} \times \frac{(\boldsymbol{a}_x + \boldsymbol{a}_y)}{(\sqrt{1+1})^3}$$
$$= \frac{9}{4\sqrt{2}}(\boldsymbol{a}_x + \boldsymbol{a}_y)$$
and the electric field due to charge at point $B$ is
$$\boldsymbol{E}_2 = kQ\frac{(\boldsymbol{BC})}{|\boldsymbol{BC}|^3} = 9 \times 10^9 \times \left(\frac{1}{2}\right) \times 10^{-9} \times \frac{(-\boldsymbol{a}_x + \boldsymbol{a}_y)}{(\sqrt{1+1})^3}$$
$$= \frac{9}{4\sqrt{2}}(-\boldsymbol{a}_x + \boldsymbol{a}_y)$$
So,
$$\boldsymbol{E}_1 + \boldsymbol{E}_2 = \frac{9}{4\sqrt{2}}(\boldsymbol{a}_x + \boldsymbol{a}_y) + \frac{9}{4\sqrt{2}}(-\boldsymbol{a}_x + \boldsymbol{a}_y)$$
$$= \frac{9}{2\sqrt{2}}\boldsymbol{a}_y$$
As the field is directed in $\boldsymbol{a}_y$ direction so for making $\boldsymbol{E} = 0$ the third charge of $+\sqrt{2}$ nC must be placed on $y$-axis at any point $y > 1$. Consider the position of the third charge is $(0, y, 0)$. So, electric field at point $C$ due to the third charge is.
$$\boldsymbol{E}_3 = \frac{9 \times 10^9 \times (\sqrt{2}) \times 10^{-9}}{(y-1)^2}(-\boldsymbol{a}_y) = -\frac{9\sqrt{2}}{(y-1)^2}\boldsymbol{a}_y$$
and since the total electric field must be zero
So, we have
$$\boldsymbol{E}_1 + \boldsymbol{E}_2 + \boldsymbol{E}_3 = 0$$
$$\frac{9}{2\sqrt{2}}\boldsymbol{a}_y - \frac{9\sqrt{2}}{(y-1)^2}\boldsymbol{a}_y = 0$$
$$(y-1)^2 = 4 \text{ or } y = 3, -1$$
as discussed above $y > 1$, so the point will be located at $y = 3$
i.e. Point $P$ will have the coordinate $(0,3,0)$

**SOL 2.1.18** Option (B) is correct.
Electric field intensity at any point $P$ due to the uniformly charged plane with charge density $\rho_S$ is defined as
$$\boldsymbol{E} = \frac{\rho_s}{2\varepsilon_0}\boldsymbol{a}_n$$
where $\boldsymbol{a}_n$ is the unit vector normal to the plane directed toward point $P$
Since the unit vector normal to any plane $f = 0$ is defined as
$$\boldsymbol{a}_n = \pm\frac{\nabla f}{|\nabla f|}$$
So, we have the unit vector normal to the given charged plane $3x + 4y = 0$ as
$$\boldsymbol{a}_n = \pm\frac{3\boldsymbol{a}_x + 4\boldsymbol{a}_y}{\sqrt{3^2+4^2}} = \pm\frac{3\boldsymbol{a}_x + 4\boldsymbol{a}_y}{5} \qquad (f = 3x + 4y)$$

Since at point (1,0,3) $f > 0$, so, we take the positive value of $\boldsymbol{a}_n$.

Therefore, $$\boldsymbol{E} = \frac{\rho_s}{2\varepsilon_0}\boldsymbol{a}_n = \frac{(2\times 10^{-9})}{2(10^{-9}/36\pi)}\left(\frac{3\boldsymbol{a}_z + 4\boldsymbol{a}_y}{5}\right) \qquad (\rho_S = 2\,\text{nC/m}^2)$$
$$= \frac{36\pi}{5}(3\boldsymbol{a}_z + 4\boldsymbol{a}_y)$$
$$= 67.85\boldsymbol{a}_z + 90.48\boldsymbol{a}_y \text{ V/m}$$

SOL 2.1.19 Option (B) is correct.
Horizontal component of the electric field intensity will be cancelled due to the uniform distribution of charge in the circular loop. So the net electric field will have only the component in $\boldsymbol{a}_z$ direction and defined as below :




$$\boldsymbol{E} = \frac{1}{4\pi\varepsilon_0}\rho_L(2\pi r)\frac{z}{(r^2+z^2)^{3/2}}\boldsymbol{a}_z$$
$$= (9\times 10^9)\times(2\times 10^{-9})\times(2\pi\times 4)\frac{3}{(4^2+3^2)^{3/2}}\boldsymbol{a}_z$$
$$= 9\times 2\times\frac{2\pi\times 4\times 3}{125}\boldsymbol{a}_z = 10.86\boldsymbol{a}_z \text{ V/m}$$

SOL 2.1.20 Option (D) is correct.
Electric flux density produced at a distance $r$ from a point charge $Q$ located at origin is defined as
$$\boldsymbol{D} = \frac{Q}{4\pi r^2}\boldsymbol{a}_r$$
So, the divergence of the electric flux density is
$$\nabla\cdot\boldsymbol{D} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{Q}{4\pi r^2}\right) = 0$$
So it is 0 for all the points but at origin ($r = 0$) its divergence can't be defined.

SOL 2.1.21 Option (C) is correct.
Given the moment $\qquad \boldsymbol{p} = 4\pi\varepsilon_0\boldsymbol{a}_z$ C-m
The electric field intensity at any point $(r,\theta,\phi)$ produced due to an electric dipole lying along $z$-axis and having the dipole moment $\boldsymbol{p}$ in $\boldsymbol{a}_z$ direction is defined as
$$\boldsymbol{E} = \frac{p}{4\pi\varepsilon_0 r^3}(2\cos\theta\boldsymbol{a}_r + \sin\theta\boldsymbol{a}_\theta)$$
$$\boldsymbol{E} = \frac{1}{r^3}(2\cos\theta\boldsymbol{a}_r + \sin\theta\boldsymbol{a}_\theta) \qquad (\boldsymbol{p} = 4\pi\varepsilon_0\boldsymbol{a}_z \text{ C-m})$$
Now, given that the $z$-component of electric field is zero



i.e. $$E_z = \boldsymbol{E}\cdot\boldsymbol{a}_z = 0$$
$$\frac{1}{r^3}[2\cos\theta(\boldsymbol{a}_r\cdot\boldsymbol{a}_z) + \sin\theta(\boldsymbol{a}_\theta\cdot\boldsymbol{a}_z)] = 0$$
$$\frac{1}{r^3}[2\cos^2\theta - \sin^2\theta] = 0$$
$$2\cos^2\theta - \sin^2\theta = 0 \qquad E_z = 0$$
$$\frac{1}{2}[1 + 3\cos 2\theta] = 0$$
Thus $\qquad \theta = 54.7^\circ$ or $\theta = 125.3^\circ$
Therefore the conical surface of angle $\theta = 54.7^\circ$ or $125.3^\circ$ will have the electric field component $E_z = 0$.

SOL 2.1.22 Option (B) is correct.
Electric field intensity produced at a distance $\rho$ from an infinite line charge with charge density $\rho_L$ is defined as
$$\boldsymbol{E} = \frac{\rho_L}{2\pi\varepsilon_0\rho}$$
and since the electric potential at point $(1,\pi/2,2)$ is zero so, the electric potential at point $(\rho,\phi,z)$ will be equal to the integral of the electric field from point $(1,\pi/2,2)$ to the point to $(\rho,\phi,z)$.

i.e. $$V = -\int_{(1,\pi/2,2)}^{(\rho,\phi,z)}\boldsymbol{E}\cdot d\boldsymbol{l} = -\int_1^\rho\left(\frac{\rho_L}{2\pi\varepsilon_0\rho}\right)d\rho = \left[-\frac{\rho_L}{2\pi\varepsilon_0}\ln(\rho)\right]_1^\rho$$
$$V = 2\times 10^{-9}\times 9\times 10^9\ln\left(\frac{1}{\rho}\right) = 18\ln\left(\frac{1}{\rho}\right) \quad (\rho_L = +1\,\text{nC})$$

**NOTE :**
Since the infinite line charge has the equipotential cylindrical surface so for taking the line integral, $\phi$ and $z$ has not been considered.

SOL 2.1.23 Option (B) is correct.
Electric potential at any point for a given electric field $\boldsymbol{E}$ is defined as
i.e. $$V = -\int\boldsymbol{E}\cdot d\boldsymbol{l} + C$$
Now given the electric field intensity in spherical coordinate system
$$\boldsymbol{E} = \frac{2r}{(r^2+4)^2}\boldsymbol{a}_r$$
and since the differential displacement in the spherical system is given as
$$d\boldsymbol{l} = dr\boldsymbol{a}_r + rd\theta\boldsymbol{a}_\theta + r\sin\theta d\phi\boldsymbol{a}_\phi$$
So we have the electric potential
$$V = -\int\frac{2r}{(r^2+4)^2}dr + C = \frac{1}{r^2+4} + C$$
At maxima, $$\frac{dV}{dr} = 0$$
$$\frac{-1}{(r^2+4)^2}\times 2r = 0$$
Solving the equation we get, $r = 0$ and $r = \infty$
At $r = 0$ $$\frac{d^2V}{dr^2} = -\text{ve}$$
So the electric potential will be maximum at origin.

SOL 2.1.24 Option (C) is correct.
As calculated in the previous question, the electric potential at point $(r,\theta,\phi)$ is

$$V = \frac{1}{r^2+4} + C$$

So at $r = 0$, electric potential is $\qquad V_1 = \frac{1}{4} + C$

and at $r = 2$ electric potential is $\qquad V_2 = \frac{1}{8} + C$

So potential difference between the two surfaces is :

$$V_{12} = \left(\frac{1}{4} + C\right) - \left(\frac{1}{8} + C\right) = \frac{1}{8} \text{ volt}$$

**SOL 2.1.25** Option (B) is correct.

The charged sphere will be treated as a point charge for the field at any point outside the sphere. So, the electric field at distance $r$ from the centre of the sphere will be :

$$\boldsymbol{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \qquad \text{(For } r > R\text{)}$$

So the electric potential at the point will be :

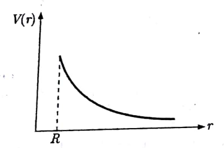
$$V(r) = -\int_{\infty}^{r} \boldsymbol{E} \cdot \boldsymbol{dl} \qquad \text{(Taking } \infty \text{ as a reference point)}$$

$$= -\frac{1}{4\pi\varepsilon_0}\int_{\infty}^{r} \frac{Q}{r^2}\,dr = -\frac{1}{4\pi\varepsilon_0}\left[-\frac{Q}{r}\right]_{\infty}^{r}$$

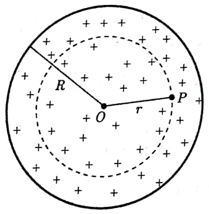
$$= \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{r}$$

So, $\qquad V(r) \propto \frac{1}{r}$

The graph of $V(r)$ will be as :




**SOL 2.1.26** Option (B) is correct.




For determining the electric field inside the spherical region at distance $r (\leq R)$ from the centre of sphere we construct a Gaussian surface as shown in the figure. So the surface integral of the electric field over the Gaussian surface is given as



$$E(4\pi r^2) = \frac{1}{\varepsilon_0}Q_{enc} = \frac{1}{\varepsilon_0}\left[Q\left(\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3}\right)\right]$$

So, the electric field at a distance $r$ from the center is

$$\boldsymbol{E} = \frac{1}{4\pi r^2}\left(\frac{Q}{\varepsilon_0}\frac{r^3}{R^3}\right)\boldsymbol{a}_r = \frac{Q}{4\pi\varepsilon_0}\frac{r}{R^3}\boldsymbol{a}_r \qquad \text{(for } r \leq R\text{)}$$

Therefore the electric potential at the point $P$ will be the line integral of the field intensity from infinity to the point $P$

i.e. $\qquad V(r) = -\left[\int_{\infty}^{R} \boldsymbol{E}_1 \cdot \boldsymbol{dr} + \int_{R}^{r} \boldsymbol{E}_2 \cdot \boldsymbol{dr}\right]$

where $\qquad E_1 \rightarrow$ electric field outside the sphere as calculated in previous question.

$E_2 \rightarrow$ electric field inside the sphere

$$V(r) = -\left[\int_{\infty}^{R} \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\,dr + \int_{R}^{r}\left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}r\right)dr\right]$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{R^3}\left(\frac{r^2 - R^2}{2}\right)\right]$$

So, $V(r)$ decreases with increase in $r$.

**SOL 2.1.27** Option (D) is correct.

The total stored energy inside a region having charge density $\rho_v$ and potential $V$ is defined as

$$W_E = \frac{1}{2}\int \rho_v V dv$$

As calculated in previous question the electric potential at any point inside the sphere is

$$V(r) = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{R^3}\left(\frac{r^2 - R^2}{2}\right)\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{1}{2}(3 - r^2)\right] \qquad (R = 1\,\text{m},\ Q = 1\,\text{C})$$

Therefore the total energy stored inside the sphere is

$$W_E = \frac{1}{2}\int_0^1\left(\frac{Q}{\frac{4}{3}\pi R^3}\right)\left[\frac{1}{4\pi\varepsilon_0} \times \frac{1}{2}(3 - r^2)\right](4\pi r^2\,dr)$$

$$= \frac{3}{8\pi} \times \frac{1}{4\pi\varepsilon_0} \times \frac{4\pi}{2}\int_0^1 (3r^2 - r^4)\,dr \qquad (R = 1\,\text{m},\ Q = 1\,\text{C})$$

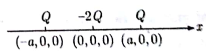
$$= \frac{3}{16\pi\varepsilon_0}\left[r^3 - \frac{r^5}{5}\right]_0^1$$

$$= \frac{3}{16\pi\varepsilon_0} \times \frac{4}{5} = \frac{3 \times 9 \times 10^9 \times 4}{4 \times 5}$$

$$= \frac{27}{5} \times 10^9 = 5.4 \times 10^9\ \text{J}$$

**SOL 2.1.28** Option (A) is correct.

The point charges can be represented as shown below.




So the electric field at point $(x,0,0)$ will be directed along $x$-axis. Taking only magnitude we have the net electric field intensity at $(x,0,0)$ as

$$E = \frac{Q}{4\pi\varepsilon_0(x-a)^2} - \frac{2Q}{4\pi\varepsilon_0 x^2} + \frac{Q}{4\pi\varepsilon_0(x+a)^2}$$

$$= \frac{Q}{4\pi\varepsilon_0 x^2}\left[1+\frac{2a}{x}+3\left(\frac{a}{x}\right)^2+\dots\right]-\frac{2Q}{4\pi\varepsilon_0 x^2}+\frac{Q}{4\pi\varepsilon_0 x^2}\left[1-\frac{2a}{x}+3\left(\frac{a}{x}\right)^2-\dots\right]$$

Since $x >> a$, neglecting higher powers of $\left(\frac{a}{x}\right)$ we get

$$E = \frac{Q}{4\pi\varepsilon_0 x^2}\left[1+\frac{2a}{x}+3\left(\frac{a}{x}\right)^2\right]-\frac{2Q}{4\pi\varepsilon_0 x^2}+\frac{Q}{4\pi\varepsilon_0 x^2}\left[1-\frac{2a}{x}+3\left(\frac{a}{x}\right)^2\right]$$

$$= \frac{6Qa^2}{4\pi\varepsilon_0 x^4} = K\left(\frac{6Qa^2}{x^4}\right)$$

**SOL 2.1.29** Option (B) is correct.

According to Gauss law the surface integral of electric field intensity over a Gaussian surface is defined as

$$\oint \boldsymbol{E}\cdot d\boldsymbol{S} = \frac{1}{\varepsilon_0}Q_{enc}$$

So for the Gaussian surface outside the sphere at a distance $r\,(> R)$ from the centre of the sphere we have

$$E(4\pi r^2) = \frac{\rho_v\left(\frac{4}{3}\pi R^3\right)}{\varepsilon_0} \qquad \text{(there is no charge outside the sphere)}$$

Therefore at any point outside the sphere $r\,(> R)$ the electric field intensity will be

$$\boldsymbol{E} = \frac{\rho_v\left(\frac{4}{3}\pi R^3\right)}{4\pi\varepsilon_0 r^2}\boldsymbol{a}_r = \frac{\rho_v}{\varepsilon_0}\left(\frac{R^3}{3r^2}\right)\boldsymbol{a}_r$$

and for the Gaussian surface inside the sphere at a distance $r\,(\le R)$ from the center of the sphere we have

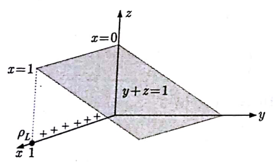
$$E(4\pi r^2) = \frac{\rho_v\left(\frac{4}{3}\pi r^3\right)}{\varepsilon_0}$$

Therefore at any point inside the sphere, the electric field intensity will be

$$\boldsymbol{E} = \frac{1}{\varepsilon_0}\frac{\rho_v\left(\frac{4}{3}\pi r^3\right)}{4\pi r^2}\boldsymbol{a}_r = \frac{\rho_v}{\varepsilon_0}\left(\frac{r}{3}\right)\boldsymbol{a}_r$$

**SOL 2.1.30** Option (C) is correct.

The portion of the plane $y+z=1\,\text{m}$ lying in the first octant bounded by the planes $x=0$ and $x=1\,\text{m}$ has been shown in the figure through which we have to determine the total electric field flux.




According to Gauss law the total outward flux through a closed surface is equal to the charge enclosed by it.

i.e. $$\psi = \oint \boldsymbol{D}\cdot d\boldsymbol{S} = Q_{enc}$$

So the total electric field flux emanating flux from the line charge between $x=0$ and $x=1\,\text{m}$ is

$$\oint \boldsymbol{E}\cdot d\boldsymbol{S} = \frac{Q_{enc}}{\varepsilon_0} = \frac{\rho_L(1)}{\varepsilon_0} = \frac{\rho_L}{\varepsilon_0}$$



and by symmetry, flux through the defined surface will be one fourth of the total electric field flux emanating from the defined portion .

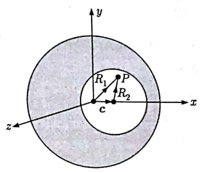
i.e. the electric flux crossing the surface $= \dfrac{\oint \boldsymbol{E}\cdot d\boldsymbol{S}}{4} = \dfrac{\rho_L}{4\varepsilon_0}$

**NOTE :**

It must be kept in mind that the total electric flux is $\int \boldsymbol{D}\cdot d\boldsymbol{S}$ while the total electric field flux is $\int \boldsymbol{E}\cdot d\boldsymbol{S}$

**SOL 2.1.31** Option (A) is correct.

Consider a point $P$ inside the cylindrical surface of 2 m as shown in figure.




Now we make the use of superposition to evaluate the electric field at point $P$ by considering the given charge distribution as the sum of two uniformly distributed cylindrical charges, one of radius 5 m and the other of radius 2 m, and such that the total charge in the hole is zero. Thus we obtain the net electric field at point $P$ as

$$\boldsymbol{E}_{net} = \boldsymbol{E}_1 + \boldsymbol{E}_2$$

where $\boldsymbol{E}_1$ is the electric field intensity at point $P$ due to the uniformly charged cylinder of radius 5 m that has the charge density $(5\,\text{nC/m}^3)$, while $\boldsymbol{E}_2$ is the electric field intensity at point $P$ due to charged cylinder of radius 2 m that has the charge density $(-5\,\text{nC/m}^3)$

As calculated in MCQ.61 the electric field intensity at a distance $\boldsymbol{r}$ from the cylindrical axes having uniform charge density $\rho_v$ is

$$\boldsymbol{E} = \frac{\rho_v \boldsymbol{r}}{\varepsilon_0\,2}$$

So we have $$\boldsymbol{E}_1 = \frac{\rho_v}{2\varepsilon_0}\boldsymbol{R}_1 = \frac{5\times10^{-9}}{2\varepsilon_0}\boldsymbol{R}_1$$

and $$\boldsymbol{E}_2 = \frac{\rho_v}{2\varepsilon_0}\boldsymbol{R}_2 = \frac{-5\times10^{-9}}{2\varepsilon_0}\boldsymbol{R}_2$$

So the net electric field at point $P$ is

$$\boldsymbol{E}_{net} = \frac{5\times10^{-9}}{2\varepsilon_0}(\boldsymbol{R}_1-\boldsymbol{R}_2)$$

By the triangle law of vector

$$\boldsymbol{R}_1-\boldsymbol{R}_2 = \boldsymbol{C} = \boldsymbol{a}_x \qquad \text{(separation} = 1\,\text{m)}$$

So, $$\boldsymbol{E}_{net} = \frac{5\times10^{-9}}{2\varepsilon_0}(\boldsymbol{a}_x)$$

$$= 282.5\boldsymbol{a}_x\ \text{V/m}$$

**SOL 2.1.32** Option (B) is correct.
As we have calculated the electric field for the same distribution in Q.55. So we evaluate the electric potential by taking the line integral of the field intensity.

i.e.
$$V = -\int \boldsymbol{E}\cdot d\boldsymbol{l}$$

$$\boldsymbol{E} = \begin{cases} \frac{\rho_v}{\varepsilon_0}\left(\frac{r}{3}\right)\mathbf{a}_r & \text{for } r \le R \\ \frac{\rho_v}{\varepsilon_0}\left(\frac{R^3}{3r^2}\right)\mathbf{a}_r & \text{for } r > R \end{cases}$$

The electric potential at any point outside the sphere $(r > R)$ is

$$V = -\int_{\infty}^{r}\boldsymbol{E}\cdot d\boldsymbol{l} = -\int_{\infty}^{r}\frac{\rho_v}{\varepsilon_0}\left(\frac{R^3}{3l^2}\right)dl$$

$$= -\frac{\rho_v R^3}{3\varepsilon_0}\left[-\frac{1}{l}\right]_{\infty}^{r} = \frac{\rho_v R^3}{3\varepsilon_0 r}$$

and the electric potential at any point inside the sphere $(r \le R)$ is

$$V = -\left[\int_{\infty}^{R}\boldsymbol{E}\cdot d\boldsymbol{l} + \int_{R}^{r}\boldsymbol{E}\cdot d\boldsymbol{l}\right]$$

$$= -\int_{\infty}^{R}\frac{\rho_v}{\varepsilon_0}\frac{R^3}{3r^2}dr - \int_{R}^{r}\frac{\rho_v}{\varepsilon_0}\left(\frac{l}{3}\right)dl$$

$$= -\frac{\rho_v R^3}{3\varepsilon_0}\left[-\frac{1}{r}\right]_{\infty}^{R} - \frac{\rho_v}{3\varepsilon_0}\left[\frac{l^2}{2}\right]_{R}^{r}$$

$$= -\frac{\rho_v R^3}{3\varepsilon_0}\left[-\frac{1}{R}\right] - \frac{\rho_v}{3\varepsilon_0}\left[\frac{r^2}{2} - \frac{R^2}{2}\right]$$

$$= \frac{\rho_v}{3\varepsilon_0}\left(\frac{3R^2}{2} - \frac{r^2}{2}\right) = \frac{\rho_v}{2\varepsilon_0}\left(R^2 - \frac{r^2}{3}\right)$$

**SOL 2.1.33** Option (A) is correct.
Electric field at any point due to infinite surface charge distribution is defined as

$$\boldsymbol{E} = \frac{\rho_s}{2\varepsilon_0}\boldsymbol{a}_n$$

where $\rho_s \rightarrow$ surface charge density
$\boldsymbol{a}_n \rightarrow$ unit vector normal to the sheet directed toward the point where field is to be determined.

At origin electric field intensity due to sheet at $y = +1$ is

$$\boldsymbol{E}_1 = \frac{\rho_s}{2\varepsilon_0}(-\boldsymbol{a}_y) = -\frac{5}{2\varepsilon_0}\boldsymbol{a}_y \qquad (\boldsymbol{a}_n = -\boldsymbol{a}_y)$$

and electric field intensity at origin due to sheet at $y = -1$ is

$$\boldsymbol{E}_{-1} = \frac{\rho_s}{2\varepsilon_0}(\boldsymbol{a}_y) = \frac{5}{2\varepsilon_0}\boldsymbol{a}_y \qquad (\boldsymbol{a}_n = \boldsymbol{a}_y)$$

So net field intensity at origin is

$$\boldsymbol{E} = \boldsymbol{E}_{+1} + \boldsymbol{E}_{-1} = -\frac{5}{2\varepsilon_0}\boldsymbol{a}_y + \frac{5}{2\varepsilon_0}\boldsymbol{a}_y = 0$$

**SOL 2.1.34** Option (A) is correct.
As the test charge is placed at point (2,5,4). So it will be in the region $y > +1$ for which electric field is given as

$$\boldsymbol{E} = \boldsymbol{E}_{+1} + \boldsymbol{E}_{-1}$$

$$= \frac{\rho_s}{2\varepsilon_0}(\boldsymbol{a}_y) + \frac{\rho_s}{2\varepsilon_0}(\boldsymbol{a}_y) \qquad \text{(for both the sheet } \boldsymbol{a}_n = \boldsymbol{a}_y)$$

$$= \frac{2\times(5\times10^{-9})}{2\varepsilon_0}\boldsymbol{a}_y = \frac{5\times10^{-9}}{\varepsilon_0}\boldsymbol{a}_y$$



Therefore the net force on the charge will be

$$F = q\boldsymbol{E} = (5\times10^{-6})\left(\frac{5\times10^{-9}}{\varepsilon_0}\right)\boldsymbol{a}_y = 2.83\times10^{-3}\ \text{N}$$

**SOL 2.1.35** Option (C) is correct.
Since the electric field intensity due to a sheet charge is defined as

$$\boldsymbol{E} = \frac{\rho_s}{2\varepsilon_0}\boldsymbol{a}_n$$

So it doesn't depend on the distance from the sheet and given as

$$\boldsymbol{E} = \boldsymbol{E}_{+1} + \boldsymbol{E}_{-1}$$

$$= \frac{\rho_s}{2\varepsilon_0}(-\boldsymbol{a}_y) + \frac{\rho_s}{2\varepsilon_0}(-\boldsymbol{a}_y)$$

$$= -\frac{\rho_s}{2\varepsilon_0}\boldsymbol{a}_y = -\frac{5\times10^{-9}}{2\varepsilon_0}\boldsymbol{a}_y$$

So, it will be constant as we move away from the sheet.

**SOL 2.1.36** Option (A) is correct.
For any point inside the sphere when we draw a symmetrical spherical surface (Gaussian surface) then the charge enclosed is zero as all the charge is concentrated on the surface of the hollow sphere.
So according to Gauss's law

$$\varepsilon_0\int\boldsymbol{E}\cdot d\boldsymbol{S} = \int\rho_v\,dv = 0$$

Therefore $\boldsymbol{E} = 0$ at any point inside the hollow sphere.
now at any point outside the sphere at a distance $r$ from the center when we draw a symmetrical closed surface(Gaussian surface) then the charge enclosed is

$$Q_{enc} = \rho_s(4\pi R^2)$$

and according to Gauss's law

$$\varepsilon_0\int\boldsymbol{E}\cdot d\boldsymbol{S} = Q_{enc}$$

$$\varepsilon_0 E(4\pi R^2) = \rho_s(4\pi R^2)$$

$$\boldsymbol{E} = \frac{\rho_s}{\varepsilon_0}\left(\frac{R^2}{r^2}\right)\boldsymbol{a}_r$$

**SOL 2.1.37** Option (A) is correct.
Electric field intensity at any point due to uniform surface charge distribution is defined as

$$\boldsymbol{E} = \frac{\rho_s}{2\varepsilon_0}\boldsymbol{a}_n$$

where $\rho_s \rightarrow$ surface charge density
$\boldsymbol{a}_n \rightarrow$ unit vector normal to the sheet directed toward the point where field is to be determined.

The electric field intensity due to the upper plate will be

$$\boldsymbol{E}_U = \frac{2}{2\varepsilon_0}(-\boldsymbol{a}_z) \qquad (\boldsymbol{a}_n = -\boldsymbol{a}_z)$$

and the field intensity due to lower plate will be

$$\boldsymbol{E}_l = -\frac{2}{2\varepsilon_0}(\boldsymbol{a}_z) \qquad (\boldsymbol{a}_n = \boldsymbol{a}_z)$$

So the net field between the plates is

$$\boldsymbol{E} = \boldsymbol{E}_U + \boldsymbol{E}_l$$
$$= \frac{2}{2\varepsilon_0}(-\boldsymbol{a}_z) + \left[-\frac{2}{2\varepsilon_0}(\boldsymbol{a}_z)\right] = -\frac{4}{2\varepsilon_0}\boldsymbol{a}_z = -\frac{2}{\varepsilon_0}\boldsymbol{a}_z$$

**SOL 2.1.38** Option (A) is correct.

Electric field intensity at any point is equal to the negative gradient of electric potential at the point

i.e. $\boldsymbol{E} = -\nabla V$

So, the $y$-component of the field is

$$E_y = -\frac{\partial V}{\partial y}$$

Now, for the interval $-3 \le y \le -2$, $V = 20(t+3)$

$$E_y = -\frac{\partial V}{\partial y} = -20 \text{ V/m}$$

For the interval $-2 \le y \le -1$, $V = 20$

So, $E_y = -\dfrac{\partial V}{\partial y} = 0$

For the interval $-1 \le y \le +1$, $V = -20t$

So, $E_y = -\dfrac{\partial V}{\partial y} = 20 \text{ V/m}$

For the interval $1 \le y \le 2$, $V = -20$

So $E_y = 0$

For the interval $2 \le y \le 3$, $V = 20(t-3)$

So, $E_y = -\dfrac{\partial V}{\partial y} = -20 \text{ V/m}$

Therefore, the plot field component $E_y$ with respect to $y$ for the defined intervals will be same as in option (A).

**SOL 2.1.39** Option (B) is correct.

Since the electrons are moving with equal but opposite velocities so assume that their velocities are $+v_0\boldsymbol{a}_x$ and $-v_0\boldsymbol{a}_x$.

Now let the electric field is applied in $\boldsymbol{a}_x$ direction

i.e. $\boldsymbol{E} = E_0\boldsymbol{a}_x$

So the force applied on the electrons will be

$$F = e\boldsymbol{E} = -(1.6 \times 10^{-19})\boldsymbol{E}$$
$$m\frac{dv}{dt} = -(1.6 \times 10^{-19})\boldsymbol{E}$$

therefore, change in the velocity

$$dv = -\frac{(1.6 \times 10^{-19})\boldsymbol{E}}{m}(dt) = -\frac{(1.6 \times 10^{-19})E_0\,dt}{m}\boldsymbol{a}_x$$

So, the velocity of electron moving in $+\boldsymbol{a}_x$ direction will change to

$$v_1 = v_0\boldsymbol{a}_x - \frac{(1.6 \times 10^{-19})E_0\,dt}{m}\boldsymbol{a}_x$$
$$= \left[v_0 - \frac{(1.6 \times 10^{-19})E_0\,dt}{m}\right]\boldsymbol{a}_x$$

Since velocity deceases so loss in K.E. is

$$K.E_{Loss} = \frac{1}{2}mv_0^2 - \frac{1}{2}mv_1^2$$
$$= (1.6 \times 10^{-19})E_0\,dt - \frac{1}{2}\frac{(1.6 \times 10^{-19})^2E_0^2(dt)^2}{m} \qquad ...(1)$$

Again the velocity of electron moving in $-\boldsymbol{a}_x$ direction will change to



$$v_2 = -v_0\boldsymbol{a}_x - \frac{(1.6 \times 10^{-19})E_0\,dt}{m}\boldsymbol{a}_x$$
$$= -\left[v_0 + \frac{(1.6 \times 10^{-19})E_0\,dt}{m}\right]\boldsymbol{a}_x$$

Since velocity increases, so Gain in K.E. is

$$K.E_{Gain} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_0^2$$
$$= (1.6 \times 10^{-19})E_0\,dt + \frac{1}{2}\frac{(1.6 \times 10^{-19})^2E_0^2(dt)^2}{m} \qquad ...(2)$$

Comparing eq (1) and eq (2) we get

$$K.E_{Gain} > K.E_{Loss}$$

***********

# SOLUTIONS 2.2

**SOL 2.2.1** Correct answer is 0.
Since all the charges are exactly equal and at same distance from the centre. So, the forces get cancelled by the diagonally opposite charges and so the net force on the charge located at centre is $F_{net} = 0\,\mathrm{N}$

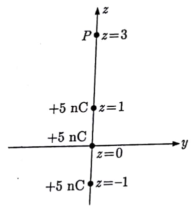
**SOL 2.2.2** Correct answer is 18.
Since one of the four charges has been removed so, it will be treated as an additional $-2\,\mathrm{C}$ charge has been put on the corner, so the force due to the additional charge will be :

$$F = \left| k\frac{(-2)\times(+1)\times 10^{-9}}{(1)^2} \right|$$
$$= |-9\times 10^{9}\times 2\times 10^{-9}| = 18\,\mathrm{N}$$

and so the net force experienced by the charge located at center is

$$F_{net} = 18 + 0 = 18\,\mathrm{N}$$

**SOL 2.2.3** Correct answer is 19.0625 .
From the positions of the three point charges as shown in the figure below, we conclude that the electric field intensity due to all the point charges will be directed along $\boldsymbol{a}_z$.

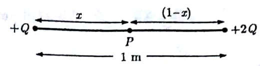



So the net electric field intensity produced at the point $P$ due to the three point charges is

$$\boldsymbol{E} = \sum \frac{Q}{4\pi\varepsilon_0 R}\boldsymbol{a}_R \quad \text{(where } R \text{ is the distance of point } P \text{ from the charge } Q)$$
$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{(3+1)^2}+\frac{1}{(3)^2}+\frac{1}{(3-1)^2}\right]\boldsymbol{a}_z \qquad (\boldsymbol{a}_R = \boldsymbol{a}_z)$$
$$= 5\times 10^{-9}\times 9\times 10^{9}\times\left[\frac{1}{16}+\frac{1}{9}+\frac{1}{4}\right]\boldsymbol{a}_z$$
$$= 19.0625\boldsymbol{a}_z$$

**SOL 2.2.4** Correct answer is 0.414 .
Since, both the point charges are positive, so the point $P$ must be located on the line joining the two charges as shown in figure.






Given the net electric field intensity at point $P$ is zero
i.e. $\quad \Sigma E = 0$
Since, the direction of electric field intensity due to the two charges will be opposite

So, $$\left[\frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}\right]-\left[\frac{1}{4\pi\varepsilon_0}\frac{2q}{(1-x)^2}\right] = 0$$
$$2x^2 = (1-x)^2$$
$$x^2 + 2x - 1 = 0$$
$$x = \frac{-2\pm\sqrt{4+4}}{2} = -1\pm\sqrt{2}$$
$$x = 0.414 \text{ and } x = -2.414$$

As discussed above the point $P$ must be located between the two charges, so we have the distance of point $P$ from charge $+Q$ as: $x = 0.414\,\mathrm{m}$

**SOL 2.2.5** Correct answer is 160.
Given the volume charge density, $\rho_v = 2\,\mu\mathrm{C} = 2\times 10^{-6}\,\mathrm{C}$
So the total charge present throughout the shell is defined as the volume integral of the charge density inside the region:

i.e. $$Q = \oint \rho_v dv$$
$$= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{r=0.02}^{0.03}(2\times 10^{-6})(r^2\sin\theta dr d\theta d\phi)$$
$$= \left[4\pi(2\times 10^{-6})\times\frac{r^3}{3}\right]_{0.02}^{0.03}$$
$$= 1.6\times 10^{-10} = 160\,\mathrm{pC}$$

**SOL 2.2.6** Correct answer is 2.6 .
The charge located in the region $2\,\mathrm{cm} < r < a$ is

$$q = \frac{Q}{2} = \frac{1}{2}\times 160 = 80\,\mathrm{pC}$$

Similarly as calculated in previous question we have

$$q = \oint \rho_v dv\backslash$$

or $$80\,\mathrm{pc} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{r=0.02}^{0.03}(2\times 10^{-6})(r^2\sin\theta dr d\theta d\phi)$$

or $$80\times 10^{-12} = \left[4\pi\times 2\times 10^{6}\times\frac{r^3}{3}\right]_{0.02}^{a}$$

Therefore, $$a = \left[\frac{3\times 80\times 10^{-12}}{4\pi\times 2\times 10^{-6}}+(0.02)^3\right]^{1/3} = 2.59\,\mathrm{cm} = 2.6\,\mathrm{cm}$$

**SOL 2.2.7** Correct answer is $-48$.
Charge density in a certain region is defined as the charge per unit volume.
Since the net charge in the subregion= 30% of the electronic charge

So the $$\text{charge density} = \frac{\text{net charge}}{\text{volume}}$$
$$= \frac{\frac{30}{100}\times(-1.6\times 10^{-19})}{10^{-12}}$$

$$= -4.8 \times 10^{-8} = -48\,\text{nC/m}^3$$

**SOL 2.2.8** Correct answer is 25.1 .

Given the surface charge density $\rho_S = \rho^2 z$

So the total charge distributed over the cylindrical surface is

$$Q = \int \rho_S\, dS$$

$$= \int_{z=0}^{1}\int_{\phi=0}^{2\pi} (\rho^2 z)(\rho d\phi dz) \qquad (dS = \rho d\phi dz)$$

$$= 8 \times \left[\frac{z^2}{2}\right]_0^1 \times [\phi]_0^{2\pi} \qquad \text{at } \rho = 2$$

$$= 8 \times \frac{1}{2} \times 2\pi = 8\pi = 25.1\,\mu\text{C}$$

**SOL 2.2.9** Correct answer is 6.5 .

Given the surface charge density

$$\rho_S = 3xy\ \text{C/m}^2$$

So, total stored charge on the triangular surface is

$$Q = \int \rho_S\, dS = \int_{x=1}^{2}\int_{y=1}^{-2x+5} (3xy)\, dx dy = 6.5\,\text{C}$$

**SOL 2.2.10** Correct answer is 785.398 .

Total stored charge on the disk is evaluated by taking surface integral of the charge density.

i.e. $$Q = \int \rho_s dS = \int_0^5 (3r)(2\pi r dr)$$

$$= 6\pi\left[\frac{r^3}{3}\right]_0^5 = 250\pi = 785.398\,\text{C}$$

**SOL 2.2.11** Correct answer is 5.3 .

$$\boldsymbol{E} = 3r^2 \boldsymbol{a}_r$$

According to Gauss's law the total charge stored in a closed surface is equal to the surface integral of its flux density over the closed surface.

i.e. $$Q_{enc} = \oint \boldsymbol{D} \cdot d\boldsymbol{S} = \varepsilon_0 \oint \boldsymbol{E} \cdot d\boldsymbol{S}$$

$$= \varepsilon_0 \oint (3r^2 \boldsymbol{a}_r)\, dS$$

$$= \varepsilon_0 (3r^2)(4\pi r^2) \qquad \left(\oint dS = 4\pi r^2 \boldsymbol{a}_r\right)$$

$$= \varepsilon_0 \times 3 \times 4\pi \times 2^4 \qquad r = 2\,\text{m}$$

$$= 5.3 \times 10^{-9} = 5.3\,\text{nC}$$

**SOL 2.2.12** Correct answer is 16.

According to Gauss law the volume Charge density in a certain region is equal to the divergence of electric flux density in that region.

i.e. $$\rho_v = \nabla \cdot \boldsymbol{D} = 2x$$

So total charge enclosed by the cube is

$$Q = \int \rho_v dv = \int_0^2\int_0^2\int_{-1}^{1} (2x)\,(dx dy dz)$$

$$= 4 \times 2 \times 2 = 16\,\text{C}$$

**SOL 2.2.13** Correct answer is $-578.9$ .

Net electric potential due to two or more point charges is defined as :

$$V = \sum \frac{Q}{4\pi\varepsilon_0 R}$$



So, the electric potential at point $P$ due to the two point charges is

$$V = \frac{Q_1}{4\pi\varepsilon_0 R_1} + \frac{Q_2}{4\pi\varepsilon_0 R_2}$$

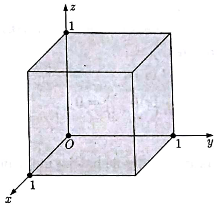
where $Q_1 = +1\,\mu\text{C}$, $Q_2 = -1\,\mu\text{C}$ and $R_1, R_2$ are the distance of the point $P$ from the two point charges respectively.

So, we have $$R_1 = \sqrt{(-3-0)^2 + (0-0)^2 + (-4-1)^2} = 5.83$$

$$R_2 = \sqrt{(-3-0)^2 + (0-0)^2 + (-4+1)^2} = 4.24$$

Thus $$V = \frac{10^{-6}}{4\pi\varepsilon_0}\left[\frac{1}{5.83} - \frac{1}{4.24}\right] = -578.9\,\text{V}$$

**SOL 2.2.14** Correct answer is $-0.1667$ .




The total flux leaving the closed surface is

$$\psi = \oint \boldsymbol{D} \cdot d\boldsymbol{S} \qquad (\ d\boldsymbol{S} \text{ is normal vector to surface})$$

The closed cube has total eight surfaces but as the vector field has no component in $\boldsymbol{a}_z$ direction so we have the integrals only through the four separate surfaces as shown in the figure

So, $$\psi = \underbrace{\int_0^1\int_0^1 x^2 y\, dy dz}_{\text{at } x=0,\ \text{front}} + \underbrace{\int_0^1\int_0^1 -x^2 y\, dy dz}_{\text{at } x=1,\ \text{back}} + \underbrace{\int_0^1\int_0^1 -x^2 y^2\, dx dz}_{\text{at } y=0,\ \text{left}} + \underbrace{\int_0^1\int_0^1 x^2 y^2\, dx dz}_{\text{at } y=1,\ \text{right}}$$

$$= -\int_0^1\int_0^1 y dy dz + \int_0^1\int_0^1 x^2\, dx dz$$

$$= -\left[\frac{y^2}{2}\right]_0^1 [z]_0^1 + \left[\frac{x^3}{3}\right]_0^1 [z]_0^1$$

$$= -\frac{1}{2} \times 1 + \frac{1}{3} \times 1 = -\frac{1}{6} = -0.1667$$

**SOL 2.2.15** Correct answer is 0.75 .

Given the electric flux density

$$\boldsymbol{D} = x^2 y \boldsymbol{a}_x + y^2 x^2 \boldsymbol{a}_y\ \text{C/m}^2$$

So, $$\text{div}\,\boldsymbol{D} = \nabla \cdot \boldsymbol{D}$$

$$= \left(\frac{\partial}{\partial x}\boldsymbol{a}_x + \frac{\partial}{\partial y}\boldsymbol{a}_y + \frac{\partial}{\partial z}\boldsymbol{a}_z\right) \cdot (x^2 y \boldsymbol{a}_x + y^2 x^2 \boldsymbol{a}_y)$$

$$\nabla \cdot \boldsymbol{D} = [2xy + 2x^2 y]$$

$$= \frac{1}{2} + \frac{1}{4} = 0.75 \qquad \left(\text{center of the cube is located at } \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)\right)$$

**SOL 2.2.16** Correct answer is 4.

From the given data we have the electric flux density at $r = 0.2\,\text{m}$ as

$$\boldsymbol{D} = 5r^2\mathbf{a}_r\,\text{nC/m}^2$$

According to Gauss law the volume charge density at any point is equal to the divergence of the flux density at that point, so we have the volume charge density at $r = 0.2\,\text{m}$ as

$$\rho_v = \nabla \cdot \boldsymbol{D}$$
$$= \frac{1}{r^2}\frac{\partial}{\partial r}(r^2(5r^2)) = \frac{1}{r^2} \times 5 \times 4r^3$$
$$= 20r = 4\,\text{nC/m}^3 \qquad (r = 0.2\,\text{m})$$

**SOL 2.2.17** Correct answer is 0.

Again from the given data we have the electric flux density at $r = 1\,\text{m}$ as

$$\boldsymbol{D} = 2/r^2\mathbf{a}_r\,\text{nC/m}^2$$

So, the volume charge density at $r = 1\,\text{m}$ is

$$\rho_v = \nabla \cdot \boldsymbol{D} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\left(\frac{2}{r^2}\right)\right) = 0$$

**SOL 2.2.18** Correct answer is 0.6 .

Electric potential at a distance $\boldsymbol{R}$ from a dipole having moment $\boldsymbol{p}$ is defined as

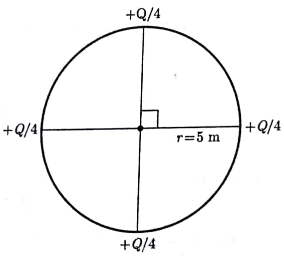
$$V = \frac{\boldsymbol{p} \cdot \boldsymbol{R}}{4\pi\varepsilon_0 R^3}$$

So we have the potential at point $A$ due to the dipole located at point $B$ as:

$$V = \frac{\boldsymbol{p} \cdot \boldsymbol{AB}}{4\pi\varepsilon_0|\boldsymbol{AB}|^3}$$
$$= \frac{(\frac{3}{5}\boldsymbol{a}_x - \boldsymbol{a}_y + 2\boldsymbol{a}_z) \cdot (\boldsymbol{a}_x + \boldsymbol{a}_y + 8\boldsymbol{a}_z) \times 10^{-9}}{4\pi\varepsilon_0(\sqrt{1^2 + 1^2 + 8^2})^3}$$
$$= 0.6\,\text{V}$$

**SOL 2.2.19** Correct answer is 36.

Since the charge is being split and placed on a circular loop so the distance of all the newly formed point charges from the center of the loop will be equal as shown in the figure.




Therefore, the potential at the center of the loop will be

$$V = 4\left(\frac{Q/4}{4\pi\varepsilon_0 r}\right) = (9 \times 10^9) \times \frac{(20 \times 10^{-9})}{5} \qquad (Q = 20\,\text{nC})$$

$$= 36\,\text{V}$$



**SOL 2.2.20** Correct answer is $-15.5$ .

The work done in carrying a charge $q$ from point $A$ to point $B$ in the field $\boldsymbol{E}$ is defined as

$$W = -q\int_A^B \boldsymbol{E} \cdot \boldsymbol{dl}$$

Given the electric field intensity in the cartesian system as

$$\boldsymbol{E} = 2y\boldsymbol{a}_x + 2x\boldsymbol{a}_y$$

and since the differential displacement in cartesian system is given as

$$\boldsymbol{dl} = dx\boldsymbol{a}_x + dy\boldsymbol{a}_y + dz\boldsymbol{a}_z$$

So, the work done in carrying charge $q = +2\,\text{C}$ from point $A(1,1/2,3)$ to the point $B(4,1,0)$ is

$$W = -2\left[\int_{x=1}^{4} 2y\,dx + \int_{y=1/2}^{1} 2x\,dy\right]$$

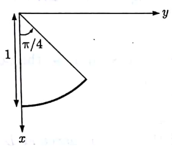
The curve along which the charge is being carried is given as

$$y = \sqrt{\frac{x}{2}} \qquad \Rightarrow x = 2y^2$$

Therefore, we have $\quad W = -2\left[\int_1^4 2\left(\sqrt{x/2}\right)dx + \int_{1/2}^{1} 2(2y^2)\,dy\right]$

$$= -4\left[\frac{\sqrt{2}}{3}[x^{3/2}]_1^4 + \frac{2}{3}[y^3]_{1/2}^1\right]$$
$$= -4\left[\frac{7\sqrt{2}}{3} + \frac{7}{12}\right]$$
$$= -15.5\,\text{J}$$

**SOL 2.2.21** Correct answer is 1.




The work done in carrying a charge $q$ from one point to other point in the field $\boldsymbol{E}$ is defined as

$$W = -q\int \boldsymbol{E} \cdot \boldsymbol{dl}$$

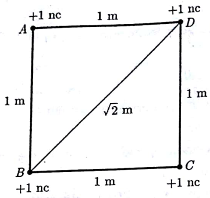
and since the differential displacement for the defined circular arc is $\boldsymbol{dl} = \rho d\phi\boldsymbol{a}_\phi$ as obtained from the figure

So, the work done is $\quad W = -2\int_{\phi=0}^{\pi/4}(x\boldsymbol{a}_x - y\boldsymbol{a}_y) \cdot (\rho d\phi\boldsymbol{a}_\phi)$

now we put $x = \rho\cos\phi$, $y = \rho\sin\phi$ and $\boldsymbol{a}_x \cdot \boldsymbol{a}_\phi = -\sin\phi$, $\boldsymbol{a}_y \cdot \boldsymbol{a}_\phi = \cos\phi$ in the expression to get

$$W = -2\int_0^{\pi/4} -2\rho^2\sin\phi\cos\phi\,d\phi$$
$$= -2 \times 1\int_0^{\pi/4} -\sin(2\phi)\,d\phi \qquad (\rho = 1)$$
$$= +1\,\text{J}$$

**SOL 2.2.22** Correct answer is 24.36 .




Consider the last charge is being placed at corner $D$ so the potential at $D$ due to the charges placed at the corners $A, B, C$ is

$$V = \frac{1}{4\pi\varepsilon_0}\sum\frac{q}{r} = \frac{10^{-9}}{4\pi\varepsilon_0}\left[\frac{1}{1} + \frac{1}{\sqrt{2}} + \frac{1}{1}\right]$$
$$= (9 \times 10^9) \times 10^{-9} \times \left(2 + \frac{1}{\sqrt{2}}\right)$$
$$= 24.36 \text{ volt}$$

As the potential at infinity is zero so the work done in carrying the last charge from infinity to the fourth corner is

$$W = qV = 10^{-9} \times 24.36 \qquad (q = 1\,\text{nC})$$
$$= 24.36 \text{ nJ}$$

**SOL 2.2.23** Correct answer is 48.72 .

Consider the first charge is being placed at $A$ so the potential at $A$ will be zero as there is no any charge present at any of the corner and therefore the work done in carrying the first charge is

$$W_1 = 0$$

now consider the second charge is being placed at $B$ so the potential at $B$ will be only due to the charge at corner $A$

i.e. $$V_2 = \frac{q}{4\pi\varepsilon_0 a}$$

and therefore the work done in placing the second charge at $B$ is

$$W_2 = qV_2 = q\left(\frac{q}{4\pi\varepsilon_0 a}\right)$$
$$= \frac{1}{4\pi\varepsilon_0} \times \frac{10^{-18}}{1} = 9 \text{ nJ}$$

and similarly the potential at the corner $C$ will be due to the charges at corners $A$ and $B$

i.e. $$V_3 = \frac{1}{4\pi\varepsilon_0}\sum\frac{q}{r} = \frac{10^{-9}}{4\pi\varepsilon_0}\left(\frac{1}{1} + \frac{1}{\sqrt{2}}\right)$$

therefore the work done in placing the third charge at $C$ is

$$W_3 = qV_3 = q\left[\frac{1}{4\pi\varepsilon_0}\left(1 + \frac{1}{\sqrt{2}}\right)\right]$$
$$= (9 \times 10^9) \times 10^{-18}\left(\frac{1}{\sqrt{2}} + 1\right)$$

and the work done in placing the last charge at $D$ has already been calculated in previous question

i.e. $$W_4 = 24.36 \text{ nJ}$$



So the total work done in assembling the whole configuration of four charges is

$$W = W_1 + W_2 + W_3 + W_4$$
$$= 0 + 9 + 15.36 + 24.36 = 48.72 \text{ nJ}$$

**SOL 2.2.24** Correct answer is 8.

The work done in carrying a charge $q$ from point $A$ to point $B$ in the field $\boldsymbol{E}$ is defined as

$$W = -q\int_A^B \boldsymbol{E}\cdot d\boldsymbol{l}$$

Given that $$q = 2\,\text{C}$$
$$\boldsymbol{E} = \sin\phi\,\boldsymbol{a}_\rho + (z+1)\rho\cos\phi\,\boldsymbol{a}_\phi + \rho\sin\phi\,\boldsymbol{a}_z$$

and since the given points $A$ and $B$ have $\rho_1 = \rho_2 = 2$ and $z_1 = z_2 = 1$ so the differential displacement in the cylindrical coordinate system from $A$ to $B$ may be given as

$$d\boldsymbol{l} = \rho d\phi\,\boldsymbol{a}_\phi \qquad \text{for } 0 < \phi < 30^\circ.$$

Therefore the work done is, $$W = -2\int_{\phi=0^\circ}^{30^\circ}((z+1)\rho\cos\phi)(\rho d\phi)$$
$$= -2 \times (1+1) \times (2)^2 \times [\sin\phi]_{0^\circ}^{30^\circ}$$
$$= -16 \times \frac{1}{2} = -8\,\text{J}$$

**SOL 2.2.25** Correct answer is 1.604 .

Consider the $+1\,\mu$C charge is transferred first, from infinity to the given point $A(-3,6,0)$ so the work done for transferring the charge will be zero as there is no charge initially present.

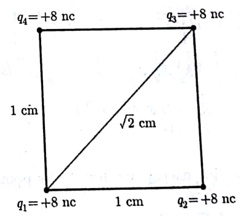
now the potential at point $B$ due to the charge at $A$ is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_A}{|\boldsymbol{AB}|}$$
$$= 9 \times 10^9 \frac{10^{-6}}{\sqrt{5^2 + 10^2 + 1^2}} = \frac{9 \times 10^3}{\sqrt{126}} \qquad (q_A = 1\,\mu\text{C})$$

So the work done in transferring the charge $+2$ mC at point $B$ is

$$W = q_B V$$
$$= (2 \times 10^{-3}) \times \left(\frac{9 \times 10^3}{\sqrt{126}}\right) \qquad (q_B = 2\,\text{mC})$$
$$= 1.604\,\text{J}$$

**SOL 2.2.26** Correct answer is 0.312 .



The total potential energy stored in the system is given by

$$W = \frac{1}{2}\sum_{n=1}^{4} q_n V_n$$

where $q_n$ is the charges at the four corners and $V_n$ is the total electric potential at the corresponding corners.
For the 1st corner :

Charge, $\quad q_1 = 8\,\text{nC}$

and potential, $\quad V_1 = V_{21} + V_{31} + V_{41}$

where $V_{21}$, $V_{31}$ and $V_{41}$ are the potential at the 1st corner due to the charges $q_2$, $q_3$ and $q_4$ respectively

So,
$$V_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_2}{0.01} + \frac{q_3}{0.01\sqrt{2}} + \frac{q_4}{0.01}\right] \qquad (q_2 = q_3 = q_4 = 8\,\text{nC})$$
$$= \frac{8\times 10^{-9}}{4\pi\varepsilon_0}\left[\frac{1}{0.01} + \frac{1}{0.01\sqrt{2}} + \frac{1}{0.01}\right]$$
$$= 1.944 \times 10^4\ \text{V}$$

Since all the charges are equal so the potential will be same at all the corners and therefore the total potential energy stored in the system of the charges is

$$W = \frac{1}{2}\times 4(q_1 V_1)$$
$$= 2 \times (8\times 10^{-9}) \times (1.944\times 10^4) = 0.312\ \text{mJ}$$

**SOL 2.2.27** Correct answer is 9.68 .

Energy density in a certain region in free space having electric field intensity $\boldsymbol{E}$ is defined as

$$w_E = \frac{1}{2}\varepsilon_0 \boldsymbol{E}\cdot\boldsymbol{E}$$

and since the electric field is equal to the negative gradient of the potential so we have

$$\boldsymbol{E} = -\nabla V$$
$$= -\left[\frac{\partial V}{\partial x}\boldsymbol{a}_x + \frac{\partial V}{\partial y}\boldsymbol{a}_y + \frac{\partial V}{\partial z}\boldsymbol{a}_z\right]$$
$$= \left[\frac{1}{x^2yz}\boldsymbol{a}_x + \frac{1}{xy^2z}\boldsymbol{a}_y + \frac{1}{xyz^2}\boldsymbol{a}_z\right]\text{V/m}$$

So the energy density inside the cube will be

$$w_E = \frac{1}{2}\varepsilon_0(\boldsymbol{E}\cdot\boldsymbol{E}) = \frac{1}{2}\varepsilon_0\left[\frac{1}{x^4y^2z^2} + \frac{1}{x^2y^4z^2} + \frac{1}{x^2y^2z^4}\right]$$

Therefore the total energy stored in the cube is

$$W_E = \int w_E\,dv$$
$$W_E = \frac{1}{2}\varepsilon_0\int_1^2\int_1^2\int_1^2\left[\frac{1}{x^4y^2z^2} + \frac{1}{x^2y^4z^2} + \frac{1}{x^2y^2z^4}\right]dx\,dy\,dz$$
$$= \frac{\varepsilon_0}{2}\int_1^2\int_1^2\left[-\left(\frac{1}{3}\right)\frac{1}{x^3y^2z^2} - \frac{1}{xy^4z^2} - \frac{1}{xy^2z^4}\right]_1^2 dy\,dz$$
$$= \frac{\varepsilon_0}{2}\times 3\times\frac{7}{96} = 9.68\times 10^{-13}\ \text{J}$$

**SOL 2.2.28** Correct answer is 5.18 .

As calculated in the above question energy density at any point inside the cube is

$$w_E = \frac{1}{2}\varepsilon_0\left[\frac{1}{x^4y^2z^2} + \frac{1}{x^2y^4z^2} + \frac{1}{x^2y^2z^4}\right]$$

So, at the centre of the cube (1.5, 1.5, 1.5) the energy density is

$$w_E = \frac{1}{2}\varepsilon_0\left[\frac{3}{(1.5)^4(1.5)^2(1.5)^2}\right] = 5.18\times 10^{-12}\ \text{J}$$

**SOL 2.2.29** Correct answer is 5.57 .

The electric field to counter act the gravitational force must produce the same force as applied by gravity but in opposite direction.

i.e. $\quad e(\boldsymbol{E}) = m_e g(-\boldsymbol{a}_r)$

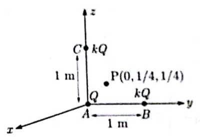
where $e$ is the charge of an electron, $m_e$ is the mass of electron, $g$ is acceleration due to gravity and $\boldsymbol{a}_r$ is radial direction of earth.

So, taking the magnitude only we have the required field intensity,

$$E = \frac{m_e g}{e} = \frac{(9.1\times 10^{-31})\times 9.8}{1.6\times 10^{-19}} = 5.57\times 10^{-11}\ \text{V/m}$$

**SOL 2.2.30** Correct answer is 5.59 .

Consider the electric field intensity produced at point $P\left(0,\frac{1}{4},\frac{1}{4}\right)$ due to the charges located at points $A$, $B$ and $C$ respectively as shown in figure is $\boldsymbol{E}_A$, $\boldsymbol{E}_B$ and $\boldsymbol{E}_C$ respectively.




So the net electric field at point $P$ is

$$\boldsymbol{E}_{net} = \boldsymbol{E}_A + \boldsymbol{E}_B + \boldsymbol{E}_C$$

and since the electric field intensity at any distance $\boldsymbol{R}$ from a point charge $Q$ is defined as $\quad \boldsymbol{E} = \frac{Q}{4\pi\varepsilon_0}\frac{\boldsymbol{R}}{|\boldsymbol{R}|^3}$

So
$$\boldsymbol{E}_{net} = \frac{1}{4\pi\varepsilon_0}\left[Q\frac{\boldsymbol{PA}}{|\boldsymbol{PA}|^3} + kQ\frac{\boldsymbol{PB}}{|\boldsymbol{PB}|^3} + kQ\frac{\boldsymbol{PC}}{|\boldsymbol{PC}|^3}\right]$$
$$= \frac{1}{4\pi\varepsilon_0}Q\frac{\left(\frac{1}{4}\boldsymbol{a}_y + \frac{1}{4}\boldsymbol{a}_z\right)}{\left[\left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^2\right]^{3/2}} + kQ\frac{\left(-\frac{3}{4}\boldsymbol{a}_y + \frac{1}{4}\boldsymbol{a}_z\right)}{\left[\left(\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2\right]^{3/2}} + kQ\frac{\left(\frac{1}{4}\boldsymbol{a}_y - \frac{3}{4}\boldsymbol{a}_z\right)}{\left[\left(\frac{1}{4}\right)^2 + \left(\frac{3}{4}\right)^2\right]^{3/2}}$$

and since $\boldsymbol{E}_{net} = 0$ so we have

$$\frac{\left(\frac{1}{4}\right)\times(16)^{3/2}}{(2)^{3/2}} - \frac{\frac{3}{4}k}{\left[\left(\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2\right]^{3/2}} + \frac{\frac{1}{4}k}{\left[\left(\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2\right]^{3/2}} = 0$$

Solving the equation we get $\qquad k = 5.59$

**SOL 2.2.31** Correct answer is 2.83 .

As discussed earlier, the electric field at any point inside a charged solid sphere is

$$\boldsymbol{E} = \frac{\rho_v}{\varepsilon_0}\left(\frac{r}{3}\right)\boldsymbol{a}_r$$

where $r$ is the distance from center of the sphere and $\rho_v$ is the volume charge density given as

$$\rho_v = \frac{Q}{\frac{4}{3}\pi R^3} = \frac{2 \times 10^{-9}}{\frac{4}{3}\pi(3)^3} \qquad (Q = 2\,\text{nC}, R = 3\,\text{m})$$
$$= 1.77 \times 10^{-11}\,\text{C/m}^3$$

So the force acting on electron when it is at a distance $r$ from the center of the sphere is

$$F = eE \qquad (e \text{ is the charge of an electron})$$
$$m_e \frac{d^2 r}{dt^2} = e\frac{\rho_v}{\varepsilon_0}\left(\frac{r}{3}\right) \qquad (m_e \text{ is mass of an electron})$$
$$\frac{d^2 r}{dt^2} = \frac{(-1.6 \times 10^{-19})(1.77 \times 10^{-11})}{(9.1 \times 10^{-31})(8.85 \times 10^{-12})} \times \frac{r}{3}$$
$$\frac{d^2 r}{dt^2} = -(1.17 \times 10^{11})r$$
$$\frac{d^2 r}{dt^2} + (1.17 \times 10^{11})r = 0$$

Solving the differential equation we have

$$r = A_1\cos(\sqrt{1.17 \times 10^{11}}\,t) + A_2\sin(\sqrt{1.17 \times 10^{11}}\,t) \quad ....(1)$$

where $A_1$ and $A_2$ are constants.

Now, at $t = 0$, $r = 3\,\text{m}$ as the electron is located at one end of the hole.

So putting it in equation (1) we get, $\quad A_1 = 3$

again at $t = 0$, $\frac{dr}{dr} = 0$ as the electron is released from rest.

So putting it in equation (1) we get $\quad A_2 = 0$

Thus the position of electron at any time $t$ is

$$r = 3\cos(\sqrt{1.17 \times 10^{11}}\,t)$$

At $t = 1\,\mu\text{sec}$ $\quad r = 2.83\,\text{m}$

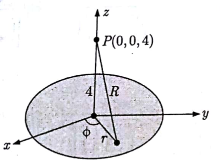
**SOL 2.2.32** Correct answer is 54.4 .

As calculated in above question the position of the electron at any time $t$ is

$$r = 3\cos(\sqrt{1.17 \times 10^{11}}\,t)$$

So,
$$2\pi f = \sqrt{1.17 \times 10^{11}}$$
$$f = \frac{\sqrt{1.17 \times 10^{11}}}{2\pi} = 5.44 \times 10^4\,\text{Hz} = 54.4\,\text{KHz}$$

**SOL 2.2.33** Correct answer is 9.44 .




Given the total charge on the disk is $\quad Q = 900\pi\,\mu\text{C} = 900\pi \times 10^{-6}\,\text{C}$

radius of the disk is $\quad a = 6\,\text{m}$

and since the charge has been distributed uniformly over the surface so the small charge element $dQ$ on the disk at a distance $r$ from the center as shown in figure is given as

$$dQ = \left(\frac{Q}{S}\right)dS = \frac{900\pi \times 10^{-6}}{\pi(6)^2}(rdrd\phi)$$
$$= 25 \times 10^{-6}\,rdrd\phi$$



The force applied by the charge element $dQ$ on the $150\,\mu\text{C}$ charge located at point $P$ is

$$dF = \frac{(150 \times 10^{-6})\,dQ}{R^2} = \frac{(150 \times 10^{-6})\,dQ}{(r^2 + 16)}$$

As the disk has uniformly distributed charge so the horizontal component of the field is get cancelled and the net force will have the only component in $\boldsymbol{a}_z$ direction and the net force by projection on $z$-axis is given as

$$F = \int_{\phi=0}^{2\pi}\int_{r=0}^{6} \frac{(150 \times 10^{-6})(25 \times 10^{-6}\,rdrd\phi)}{4\pi\varepsilon_0(r^2 + 16)} \times \left(\frac{4}{\sqrt{r^2 + 16}}\right)$$
$$F = 270\pi\left[-\frac{1}{\sqrt{r^2 + 16}}\right]_0^6 = 9.44\,\text{N}$$

**SOL 2.2.34** Correct answer is 2.5 .

As the charge is redistributed so the total charge will remain same on the sphere.

Total charge before redistribution.

$$Q_1 = \int \rho_v\,dv = (6\,\text{C/m}^3)\left(\frac{4}{3}\pi(1)^3\right) \qquad (\rho_v = 6\,\text{C/m}^2)$$
$$= 8\pi \text{ Coulomb}$$

and total charge after redistribution

$$Q_2 = \int \rho_v\,dv = \int_{r=0}^{1} k(3 - r^2)4\pi r^2\,dr$$

Since $\quad Q_1 = Q_2$

So, we have
$$8\pi = \int_0^1 k(4\pi)(3r^2 - r^4)\,dr$$
$$= 4\pi k\left[r^3 - \frac{r^5}{5}\right]_0^1 = 4\pi k\left[1 - \frac{1}{5}\right]$$

or $\quad k = 2.5$

**SOL 2.2.35** Correct answer is 25.

According to Gauss's law the total electric flux through any closed surface is equal to the total charge enclosed by the volume.

Now consider the complete spherical surface defined by $r = 48\,\text{m}$ through which the total flux is equal to the point charge.

So the total flux passing through the hemispherical surface will be half of the point charge.

i.e.
$$\psi = \frac{Q}{2} = \frac{50\,\mu\text{C}}{2} = 25\,\mu\text{C}$$

**SOL 2.2.36** Correct answer is 19.2 .

The electric field intensity produced at a distance $\rho$ from a line charge of density $\rho_L$ is defined as

$$\boldsymbol{E} = \frac{\rho_L}{2\pi\varepsilon_0\rho}\boldsymbol{a}_\rho$$

where $\boldsymbol{a}_\rho$ is unit vector directed toward point $P$ along $\rho$. So, the electric field acting on the line charge at $y = 3\,\text{m}$ due to the line charge located at $y = -3\,\text{m}$ is

$$\boldsymbol{E} = \frac{80 \times 10^{-9}}{2\pi\varepsilon_0(6)}\boldsymbol{a}_y \qquad (\rho_L = 80\,\text{nC}, \boldsymbol{a}_\rho = \boldsymbol{a}_y, \rho = 6\,\text{m})$$
$$= 240\boldsymbol{a}_y\,\text{V/m}$$

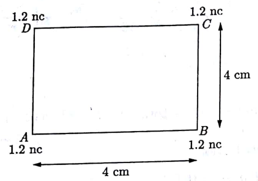
Therefore, the force per unit length exerted on the line charge located at

$y = 3\text{ m}$ is

$$F = \int_{z=0}^{1} (\rho_L dz)(E)$$
$$= (80 \times 10^{-9})(240 a_y) = 19.2 a_y\ \mu\text{N}$$

**SOL 2.2.37** Correct answer is 1.75 .

The four charges located at the corners of square 4 cm has been shown in figure below :




The net potential at the charge located at $A$ due to the other three charges is

$$V_A = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_B}{AB} + \frac{q_C}{AC} + \frac{q_D}{AD}\right)$$
$$= 9 \times 10^9 \times 1.2 \times 10^{-9}\left(\frac{1}{4 \times 10^{-2}} + \frac{1}{4\sqrt{2} \times 10^{-2}} + \frac{1}{4 \times 10^{-2}}\right)$$
$$= \frac{10.8 \times 10^2}{4}\left(2 + \frac{1}{\sqrt{2}}\right)$$
$$= 730.92\text{ Volt}$$

Similarly, the electric potential at all the corners will be

$$V_B = V_C = V_D = V_A = 730.92\text{ Volt}$$

Therefore, the net potential energy stored in the system is given as

$$W = \sum \frac{1}{2} qV = \frac{1}{2}(q_A V_A + q_B V_B + q_C V_C + q_D V_D)$$
$$= \frac{1}{2} \times 4 \times (1.2 \times 10^{-9}) \times (730.92)$$
$$= 1.75\ \mu\text{J}$$

***********

# SOLUTIONS 2.3



**SOL 2.3.1** Option (C) is correct.

According to Gauss law the total outward electric flux from a closed surface is equal to the charge enclosed by it

i.e. $$\psi = \oint D \cdot dS = Q_{enc}$$

So when the charge enclosed by the volume is zero then the net outward flux is zero, or in other words, the net electric field flux emanating from an arbitrary surface not enclosing a point charge is zero.

Now, the electric field intensity outside a charged sphere having total charge $Q$ is determined by treating the sphere as a point charge

i.e. $$E = \frac{Q}{4\pi\varepsilon_0 r^2} a_r$$

where $r$ is distance of the point form center of sphere and $a_r$ is it's radial direction.

So the electric field intensity at any point outside the charged sphere is not zero.

Therefore, Assertion(A) is true but Reason(R) is false.

**SOL 2.3.2** Option (A) is correct.

According to Gauss's law

$$\rho_v = \varepsilon \nabla E$$

So when the field intensity is uniform

$$\nabla E = 0$$

and $$\rho_v = \varepsilon \nabla E = 0$$

So no charge can be present in a uniform electric field.

**SOL 2.3.3** Option (D) is correct.

Laplace's equation for a scalar function $V$ is defined as

$$\nabla^2 V = 0$$

but at the point of maxima $\nabla^2 V$ must have a negative value while at the point of minima $\nabla^2 V$ must have a positive value. So the condition of maxima/minima doesn't satisfy the Laplace's equation, therefore the potential function will have neither a maxima nor a minima inside the defined region.

**SOL 2.3.4** Option (D) is correct.

**SOL 2.3.5** Option (D) is correct.

**SOL 2.3.6** Option (A) is correct.

**SOL 2.3.7** Option (C) is correct.

**SOL 2.3.8** Option (C) is correct.

**SOL 2.3.9** Option (A) is correct.

**SOL 2.3.10** Option (A) is correct.

**SOL 2.3.11** Option (A) is correct.

**SOL 2.3.12** Option (C) is correct.

**SOL 2.3.13** Option (B) is correct.

**SOL 2.3.14** Option (A) is correct.

**SOL 2.3.15** Option (A) is correct.

**SOL 2.3.16** Option (B) is correct.

**SOL 2.3.17** Option (A) is correct.

**SOL 2.3.18** Option (A) is correct.

**SOL 2.3.19** Option (A) is correct.

***********

# SOLUTIONS 2.4



**SOL 2.4.1** Option (C) is correct.

Given, the electric field intensity,

$$\boldsymbol{E} = x\boldsymbol{a}_x + y\boldsymbol{a}_y + z\boldsymbol{a}_z$$
$$d\boldsymbol{l} = \boldsymbol{a}_x dx + \boldsymbol{a}_y dy + \boldsymbol{a}_z dz$$

So, the potential difference between point $X$ and $Y$ is

$$V_{XY} = -\int_X^Y \boldsymbol{E}\cdot d\boldsymbol{l} = \int_1^2 x dx + \int_2^0 y dy + \int_3^0 z dz$$
$$= -\left[\frac{x^2}{2}\Big|_1^2 + \frac{y^2}{2}\Big|_2^0 + \frac{z^2}{2}\Big|_3^0\right]$$
$$= -\frac{1}{2}[2^2 - 1^2 + 0^2 - 2^2 + 0^2 - 3^2] = 5$$

**SOL 2.4.2** Option (C) is correct.

Given the electric field vector at point $P$ due to the three charges $Q_1$, $Q_2$ and $Q_3$ are respectively.

$$\boldsymbol{E}_1 = \boldsymbol{a}_x + 2\boldsymbol{a}_y - \boldsymbol{a}_z$$
$$\boldsymbol{E}_2 = \boldsymbol{a}_y + 3\boldsymbol{a}_z$$
$$\boldsymbol{E}_3 = 2\boldsymbol{a}_x - \boldsymbol{a}_y$$

So, the net field intensity at point $P$ is

$$\boldsymbol{E} = \boldsymbol{E}_1 + \boldsymbol{E}_2 + \boldsymbol{E}_3$$
$$= 3\boldsymbol{a}_x + 2\boldsymbol{a}_y + 2\boldsymbol{a}_z$$

**SOL 2.4.3** Option (C) is correct.

Charge density at any point in terms of electric flux density $\boldsymbol{D}$ is defined as

$$\rho_v = \nabla\cdot\boldsymbol{D}$$

Since, $\boldsymbol{D} = z\rho(\cos^2\phi)\boldsymbol{a}_z\ \text{C/m}^2$

So, we get $\rho_v = \nabla\cdot\boldsymbol{D}$

$$= \frac{\partial}{\partial z}[z\rho(\cos^2\phi)\boldsymbol{a}_z]$$
$$= \rho\cos^2\phi\ \text{C/m}^3$$

At point $\left(1, \frac{\pi}{4}, 3\right)$, $\rho_v = (1)\cos^2\left(\frac{\pi}{4}\right)$

$$= \frac{1}{2} = 0.5\ \text{C/m}^3$$

**SOL 2.4.4** Option (D) is correct.

Electric field intensity $\boldsymbol{E}$ is a vector quantity while the electric potential $V$ is a scalar quantity.

**SOL 2.4.5** Option (A) is correct.

For an ideal capacitance the area of plates, $A$ is assumed very high in comparison to the separation $d$ between the plates.

i.e. $\frac{A}{d} \approx \infty$

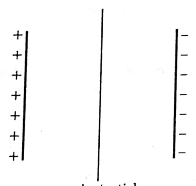
So, the fringing effect at the plates edges can be neglected and therefore, we get the capacitance between the parallel plates as

$$C = \frac{\varepsilon A}{d}$$

So A and R both true and R is correct explanation of A.

**SOL 2.4.6** Option (A) is correct.
Using method of images, the conducting surfaces are being replaced by the image of charge distribution which gives a system of charge distribution. So, in solving boundary value problems we can avoid solving Laplace's or Poisson's equation and directly apply the method of images to solve it.
Thus both A and R are true and R is correct explanation of A.

**SOL 2.4.7** Option (D) is correct.
For a pair of line charges equipotential surface exists where the normal distance from both the line charges are same. So, the plane surface between the two line charges will be equipotential.




This is the similar case to method of images.

**SOL 2.4.8** Option (A) is correct.
According to uniqueness theorem : If a solution to Laplace's equation (a) be found that satisfies the boundary condition then the solution is unique. Here it is given that the potential functions $V_1$ and $V_2$ satisfy Laplace's equation within a closed region and has the same value at its boundary so both the functions are identical.

**SOL 2.4.9** Option (A) is correct.
From Maxwell's equation we have

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t}(\nabla \times \boldsymbol{A}) \qquad (\boldsymbol{B} = \nabla \times \boldsymbol{A})$$

$$\nabla \times \left(\boldsymbol{E} + \frac{\partial}{\partial t}\boldsymbol{A}\right) = 0$$

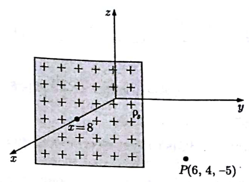
Since, the curl of a gradient of a scalar field is identically zero. So, we get

$$\boldsymbol{E} + \frac{\partial}{\partial t}\boldsymbol{A} = -\nabla V$$

i.e. $\boldsymbol{E} \neq -\nabla V$ in time varying field therefore A and R both are true and R is the correct explanation of A.



**SOL 2.4.10** Option (A) is correct.
The surface charge density at plane $x = 8$ is shown in the figure.




The point $P$ is located at $(6, 4, -5)$. So, the normal vector to the plane $x = 8$ pointing toward $P$ is

$$\boldsymbol{a}_n = -\boldsymbol{a}_x$$

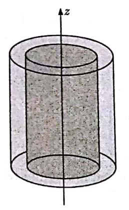
Therefore, the electric flux density produced at point $P$ is

$$\boldsymbol{D} = \frac{\rho_s}{2}\boldsymbol{a}_n$$

$$= \frac{60}{2}(-\boldsymbol{a}_x) = -30\boldsymbol{a}_x$$

**SOL 2.4.11** Option (C) is correct.
Consider the coaxial cylinder is located along $z$-axis. So at any point between the two surfaces the electric field is given as

$$\boldsymbol{E} = -\nabla V = -\frac{\partial}{\partial \rho}V\boldsymbol{a}_\rho \quad \text{(Since all other derivatives will be zero)}$$




Given that the inner surface is at potential $V_0$ while the outer one is grounded so the region between the two surfaces will have a gradually decreasing potential and so, $\boldsymbol{E}$ will not be uniform and it is radially directed as calculated above (in $\boldsymbol{a}_\rho$ direction).

**SOL 2.4.12** Option (C) is correct.
The Poisson's equation is defined as

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon}$$

where $V$ is electric potential and $\rho_v$ is charge density. So, in charge free space ($\rho_v = 0$) we get the Poisson's equation as

$$\nabla^2 V = 0 \qquad \text{which is Laplace equation.}$$

**SOL 2.4.13** Option (C) is correct.

Consider the three equal charges of $Q$ C is placed at a separation of 0.5 m as shown in figure below :


The net stored charge in the system of $n$ charges is defined as

$$W = \frac{1}{2}\sum_{k=1}^{n} Q_k V_k$$

where $Q_k$ is one point charge and $V_k$ is the net electric potential at the point charge due to the other charges.

Now, we have the net electric potential at any of the point charge $Q$ located in the system as

$$V_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{0.5} + \frac{Q}{0.5}\right)$$

$$= \frac{Q}{\pi\varepsilon_0}$$

So, total energy stored in the system of charges is given as

$$W_1 = 3\left(\frac{1}{2} Q V_1\right) = \frac{3Q^2}{2\pi\varepsilon_0} \quad (1)$$

Now, when the charges are separated by 1 m then the electric potential at any of the charge $Q$ due to the other two charges is

$$V_2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{1} + \frac{Q}{1}\right) = \frac{Q}{2\pi\varepsilon_0}$$

So, the stored energy in the new system is

$$W_2 = 3\left(\frac{1}{2} Q V_2\right) = \frac{3Q^2}{4\pi\varepsilon_0} \quad (2)$$

From equation (1) and (2) we have

$$W_2 = 0.5\, W_1 \quad \text{or} \quad W_1 = 2W_2$$

**SOL 2.4.14** Option (A) is correct.

Electric potential due to point charge is defined as

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

So, for the equal distance $r$ potential will be same i.e. equipotential surface about a point charge is sphere.

**SOL 2.4.15** Option (D) is correct.

An electrostatic field has its curl always equals to zero. So electric field is irrotational.

Statement 1 is correct.

Electric field divergence is not zero and so it is not solenoidal.

Statement 2 is correct.

Electric field is static only from a macroscopic view point.

Statement 3 is correct.

Work done in moving a charge in the electric field from one point to other is independent of the path.

Statement 4 is correct.



**SOL 2.4.16** Option (D) is correct.

Given electric potential,

$$V = 10y^4 + 20x^3$$

From Poisson's equation we have

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon_0}$$

where, $V \rightarrow$ Electric potential

$\rho_v \rightarrow$ Charge density

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)(10y^4 + 20x^3) = -\frac{\rho_v}{\varepsilon_0}$$

$$120x + 120y = -\frac{\rho_v}{\varepsilon_0}$$

$$\rho_v = \varepsilon_0(120 \times 2 + 120 \times 0) \qquad (x = 2,\ y = 0)$$

$$\rho_v = -240\varepsilon_0$$

**SOL 2.4.17** Option (C) is correct.

Given, the wave equation in space for a propagating wave in $z$-direction is

$$\nabla^2 E_x + k^2 E_x = 0$$

Now, from option (C) we have the electric field component as

$$E_x = E_0 e^{-jkz}$$

The Lapalacian of electric field is

$$\nabla^2 E_x = (-jk)^2 E_0 e^{-jkz}$$

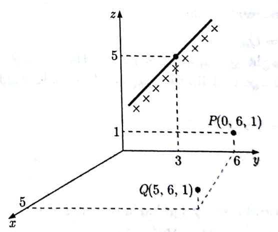
$$\nabla^2 E_x = -k^2 E_0 e^{-jkz} = -k^2 E_x$$

or, $\nabla^2 E_x + k^2 E_x = 0$ So, it satisfies the wave equation.

**SOL 2.4.18** Option (A) is correct.

Consider the infinitely long uniform charge density shown in the figure. The electric field intensity produced at a distance $\rho$ from an infinite line charge with density $\rho_L$ is defined as

$$\boldsymbol{E} = \frac{\rho_L}{2\pi\varepsilon_0\rho}$$

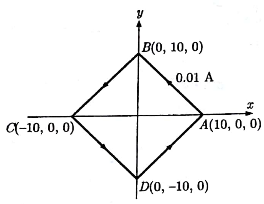



Since, the normal distance vector of points $P$ (0, 6, 1) and $Q$ (5,6,1) from the line charge will be same so, the field intensity produced due to the infinite line at both the points $P$ and $Q$ will be same.

Therefore, the field intensity at (5,6,1) is $\boldsymbol{E}$.

**SOL 2.4.19** Option (A) is correct.

Consider the square loop $ABCD$ carrying current 0.1 A as shown in figure.




The magnetic dipole moment is

$$m = IS$$

where $I$ is current in the loop and $S$ is the area enclosed by loop.

So, $$m = (0.01)(10\sqrt{2})^2 = 2\ \text{A-m}^2$$

The direction of the magnetic dipole moment is determined by right hand rule.

i.e. $$\boldsymbol{m} = 2\boldsymbol{a}_z\ \text{A-m}^2$$

**SOL 2.4.20** Option (B) is correct.

Electric flux density at a distance $r$ from a point charge $Q$ is defined as

$$\boldsymbol{D} = \frac{Q}{4\pi r^2}\boldsymbol{a}_r$$

and the total flux through any defined surface is

$$\psi = \int \boldsymbol{D} \cdot d\boldsymbol{S}$$

So, both the quantities has not the permittivity $\varepsilon$ in their expression. Therefore, $\boldsymbol{D}$ and $\psi$ are independent of permittivity $\varepsilon$ of the medium.

**SOL 2.4.21** Option (C) is correct.

According to Gauss's law, the total outward flux through a closed surface is equal to the charge enclosed inside it.

i.e. $$\oint \boldsymbol{D} \cdot d\boldsymbol{S} = Q_{enc}$$

Now, consider the height of cylinder is $h$. So, the cylindrical surface at $\rho = 3$ encloses the charge distribution ($\rho_S = 5\,\text{C/m}^2$) located at $\rho = 2\,\text{m}$. Therefore, we get

$$D(2\pi(3)h) = 5 \times 2\pi(2)h$$

or, $$\boldsymbol{D} = \frac{10}{3}\boldsymbol{a}_\rho$$

**SOL 2.4.22** Option (B) is correct.

The electric potential produced by $1\,\mu\text{C}$ at a distance $r$ is

$$V = 9 \times 10^9\frac{(1 \times 10^{-6})}{r} = \frac{9000}{r}$$

So, the potential energy stored in the field will be the energy of the charges as,

i.e. $$W = qV$$

$$= (4 \times 10^{-6})\frac{9000}{r} = \frac{36 \times 10^{-3}}{r}$$

where $r$ is the distance between the charges given as

$$r = \sqrt{(-2-1)^2 + (1-3)^2 + (5+1)^2} = 7$$

So, $$W = \frac{36 \times 10^{-3}}{7} = 5.15 \times 10^{-3}\ \text{Joule}$$

**SOL 2.4.23** Option (D) is correct.

Electric field intensity due to a dipole having moment $P$ at a distance $r$ from it is

$$E \propto \frac{1}{r^3}$$

$$\frac{E_2}{E_1} = \frac{r_1^3}{r_2^3}$$

$$\frac{E_2}{1} = \frac{(2)^3}{(4)^3}$$

$$E_2 = \frac{1}{8}\ \text{mV/m}$$

**SOL 2.4.24** Option (C) is correct.

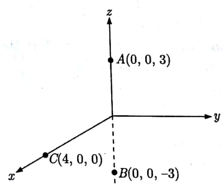
Energy density (energy stored per unit volume) in an electric field is defined as

$$w_e = \frac{1}{2}\boldsymbol{D} \cdot \boldsymbol{E}$$

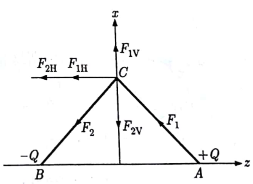
$$= \frac{1}{2}\varepsilon_0\boldsymbol{E} \cdot \boldsymbol{E} = \frac{1}{2}\varepsilon_0 E^2$$

**SOL 2.4.25** Option (B) is correct.

The position of points $A$, $B$ and $C$ are shown below




Since, position charge is placed at $A$ and negative charge at $B$ so, their resultant field intensity at $C$ is as shown below :




Since, the forces $F_1 = F_2$ so the vertical component $F_{1V}$ and $F_{2V}$ are get cancelled while $F_{2H}$ and $F_{1H}$ are get summed to provide the resultant field in $-\boldsymbol{a}_z$ direction.

**SOL 2.4.26** Option (A) is correct.

Given,

Charges, $Q_1 = Q_2 = 1\,\text{nC} = 10^{-9}\,\text{C}$

Separation between charges, $r = 1\,\text{mm} = 10^{-3}\,\text{m}$

So, the force acting between the charges is

$$F = \frac{kQ_1Q_2}{r^2} = \frac{9\times10^9(10^{-9})^2}{(10^{-3})^2}$$

$$= 9\times10^{-3}\,\text{N}$$

**SOL 2.4.27** Option (B) is correct.

According to Gauss's law, the surface integral of flux density through a closed surface is equal to the charge enclosed inside the closed surface (volume integral of charge density)

i.e. $$\oint \boldsymbol{D}\cdot d\boldsymbol{S} = \int \rho_v\,dv$$

In differential form, the Gauss's law can be written as

$$\nabla\cdot\boldsymbol{D} = \rho_v$$

$$\nabla\cdot\boldsymbol{E} = \frac{\rho_v}{\varepsilon_0} \qquad (\boldsymbol{D} = \varepsilon_0\boldsymbol{E})$$

**SOL 2.4.28** Option (A) is correct.

The electric field at a distance $r$ from the point charge $q$ located in a medium with permittivity $\varepsilon$ is defined as

$$\boldsymbol{E} = \frac{q}{4\pi\varepsilon r^2}\boldsymbol{a}_r = \frac{q\varepsilon^{-1}}{4\pi r^2}\boldsymbol{a}_r$$

**SOL 2.4.29** Option (D) is correct.

For according to Gauss's law the total outward electric flux through a closed surface is equal to the charge enclosed by the surface.

i.e. $$\oint_S \boldsymbol{D}\cdot d\boldsymbol{S} = Q_{enc}$$

or, $$\int_S \boldsymbol{D}\cdot d\boldsymbol{S} = \int_v \rho_v\,dv$$

**SOL 2.4.30** Option (C) is correct.

The force between the two charges $q_1$ and $q_2$ placed in a medium with permittivity $\varepsilon$ located at a distance $r$ apart is defined as

$$F = \frac{1}{4\pi\varepsilon}\frac{q_1q_2}{r^2}$$

or $$F \propto \frac{1}{\varepsilon}$$

i.e. force is inversely proportional to permittivity of the medium.

Since, glass has the permittivity greater than 1 (i.e. permittivity of free space) So, the force between the two charges will decreases as the glass is placed between the two charges.

**SOL 2.4.31** Option (B) is correct.

According to Gauss's law the total electric flux through a closed surface is equal to the charge enclosed by it. Since, the sphere centred at origin and of radius 5 m encloses all the charges therefore, the total electric flux over the sphere is given as

$$\psi_E = Q_1 + Q_2 + Q_3$$

$$= 0.008 + 0.05 - 0.009$$

$$= 0.049\,\mu\text{C}$$



**SOL 2.4.32** Option (A) is correct.

Electric flux through a surface area is the integral of the normal component of electric field over the area.

**SOL 2.4.33** Option (C) is correct.

The electric field due to a positive charge is directed away from it (i.e. outwards.)

According to Gauss's law the surface integral of normal component of flux density over a closed surface is equal to the charge enclosed inside it.

So, A is true but R is false.

**SOL 2.4.34** Option (B) is correct.

Force between the two charges $Q_1$ and $Q_2$ is defined as

$$\boldsymbol{F} = \frac{Q_1Q_2}{4\pi\varepsilon_0R^2}\boldsymbol{a}_R$$

When the charges are of same polarity then the force between them is repulsive. The electric force on both the charges will have same magnitude. As the expression of Force includes the term $\varepsilon$ (permittivity of the medium) so it depends on the medium in which the charges are placed.

So the statements (a), (c) and (d) are correct while (b) is incorrect.

**SOL 2.4.35** Option (C) is correct.

Since the electric field is negative gradient of the electric potential so the field lines will be orthogonal to the equipotential lines (surface).

**SOL 2.4.36** Option (C) is correct.

Electric potential at $-10\,\text{nC}$ due to $10\,\text{nC}$ charge is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

$$= 9\times10^9\times\frac{10\times10^{-9}}{\sqrt{2^2+0+0}}$$

$$= 45\,\text{Volt}$$

and so the energy stored is

$$W_e = QV$$

$$= (-10\times10^{-9})\times45$$

$$= -450\,\text{nJ}$$

**SOL 2.4.37** Option (D) is correct.

According to Gauss's the outward electric flux density through any closed surface is equal to the charge enclosed by it. So electric field out side the spherical balloon doesn't change with the change in its radius and so the energy density at point $P$ is $w_E$ for the inflated radius $b$ of the balloon.

**SOL 2.4.38** Option (B) is correct.

The curl of $\boldsymbol{E}$ is identically zero.

i.e. $$\nabla\times\boldsymbol{E} = 0$$

So, it is conservative.

The electrostatic field is a gradient of a scalar potential.

i.e. $E = -\nabla V$

So, $\nabla \times E = 0$ (Conservative)

Work done in a closed path inside the field is zero

i.e. $\int E \cdot dl = 0$

$\nabla \times E = 0$ (Conservative)

So, (a), (c) and (d) satisfies that the field is conservative.
As the potential difference between two points is not zero inside a field so, the statement (b) is incorrect.

**SOL 2.4.39** Option (A) is correct.
Net outward electric flux through the spherical surface, $r = a$ is

$$\oint D \cdot dS = \psi = \rho_v\left(\frac{4}{3}\pi a^3\right)$$

$$D(4\pi a^2) = \frac{\rho_v 4}{3}\pi a^3$$

$$D = \frac{\rho_v a}{3} a_r \text{ C/m}^2$$

**SOL 2.4.40** Option (D) is correct.
For a pair of equal and opposite linear chargers the electric potential is defined as

$$V = \frac{Q}{4\pi\varepsilon_0 r_1} - \frac{Q}{4\pi\varepsilon_0 r_2}$$

where $r_1$ and $r_2$ are the distances from the charges respectively. For the same value of $V$ (equipotential surface) a plane can be defined exactly at the centre point between them.

**SOL 2.4.41** Option (A) is correct.
In a charge free region $(\rho_v = 0)$ electrostatic field has the following characteristic

$$\nabla \cdot E = \frac{\rho_v}{\varepsilon} = 0$$

and $\nabla \times E = 0$ (for static field)

**SOL 2.4.42** Option (D) is correct.
Consider the force experienced by $Q$ is $F_1$. Since, there is no any external applied field (or force) so, sum of all the forces in the system of charges will be zero.

i.e. $\Sigma F = 0$

or, $3F + 2F + F_1 = 0$

$F_1 = -5F$

**SOL 2.4.43** Option (B) is correct.
Poissions law is derived from Gauss's law as

$$\nabla \cdot D = \rho$$

For inhomogeneous medium $\varepsilon$ is variable and so,

$$\nabla \cdot (\varepsilon E) = \rho$$

$$\nabla \cdot [\varepsilon(-\nabla V)] = \rho$$

$$\nabla \cdot (\varepsilon \nabla V) = -\rho$$

This is the Poissions law for inhomogenous medium.



**SOL 2.4.44** Option (D) is correct.
Electric field intensity due to a infinite charged surface is defined as

$$E = \frac{\rho_S}{2\varepsilon_0} a_n$$

where $\rho_S$ is surface charge density and $a_n$ is the unit vector normal to the surface directed towards the point of interest.

Given that, $\rho_S = 20 \text{ nC/m}^2 = 20 \times 10^{-9} \text{ C/m}^2$

and $a_n = -a_z$ (Since the surface $z = 10$ m is above the origin).

So we have,

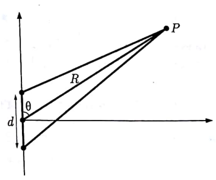
$$E = \frac{20 \times 10^{-9}}{2\varepsilon_0}(-a_z)$$

$$= \frac{20 \times 10^{-9} \times 9 \times 10^9 \times 4\pi}{2} a_z$$

$$= -360\pi a_z \text{ V/m}$$

**SOL 2.4.45** Option (C) is correct.
Electric field intensity due to a short dipole having a very small separation $d$, $a+$ a distance $R$ from it is defined as

$$E = \frac{Qd}{4\pi\varepsilon_0 R^3}(2\cos\theta a_r + \sin\theta a_\theta) \quad \text{(for } d << R)$$




So, for the given dipole, $\theta = 90°$

and $R = \sqrt{r^2 - d^2} \approx r$ $(r >> d)$

Therefore, $E = \frac{Qd}{4\pi\varepsilon_0 r^3}(0 + a_\theta)$

i.e. $E \propto \frac{1}{r^3}$

**SOL 2.4.46** Option (C) is correct.
According to Gauss' law the total outward flux from a closed surface is equal to the total charge enclosed by the surface.

**SOL 2.4.47** Option (C) is correct.
Electric field intensity at any point $r$ outside the sphere is defined as

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} a_r \quad \text{for } r > a$$

and the field intensity inside the sphere is

$$E = \frac{Q}{\left(\frac{4}{3}\pi a^3\right)} \frac{\frac{4}{3}\pi r^3}{4\pi\varepsilon_0 r^2} a_r \quad \text{for } r \le a$$

$$= \frac{Qr}{4\pi\varepsilon_0 a^3} a_r$$

So the electric potential at any point $r = b < a$ is

$$V = -\int \boldsymbol{E} \cdot d\boldsymbol{l}$$
$$= -\int_{r=\infty}^{a} \boldsymbol{E} \cdot (dr\boldsymbol{a}_r) - \int_{a}^{b} E \cdot (dr\boldsymbol{a}_r)$$
$$= -\int_{\infty}^{a} \frac{Q}{4\pi\varepsilon_0 r^2} dr - \int_{a}^{b} \frac{Qr}{4\pi\varepsilon_0 a^3} dr$$

**SOL 2.4.48** Option (A) is correct.

Given, the electric potential,

$$V = 3x^2 y - yz$$

Electric field intensity at any point is equal to the negative gradient of the potential.

i.e.
$$\boldsymbol{E} = -\nabla V$$
$$= -(6xy)\boldsymbol{a}_x - (3x^2 - z)\boldsymbol{a}_y - (-y)\boldsymbol{a}_z \text{ at } (x = 1,\ y = 0,\ z = -1)$$
$$\boldsymbol{E} = -4\boldsymbol{a}_y \neq 0$$

So, electric field does not vanish at given point.

**SOL 2.4.49** Option (A) is correct.

Consider two parallel plates separated by a distance $d$ is connected to a voltage source $V$. So, the field intensity between the plates is defined as

$$E = \frac{V}{d}$$

***********