Electromagnetic Engineering (EC21006)

Tutorial III
Flux, Divergence

1. Given $\vec{A} = 2xy\,\vec{a}_x + z\,\vec{a}_y + yz^2\vec{a}_z$

   a) Find $\int A \cdot ds$ along all the faces of the differential volume centered at (2,-1, 3)

   b) $\vec{A} = (10/r^2)\,\vec{a}_r + 5e^{-2z}\,\vec{a}_z$ at (2, ϕ, 1)

   c) $\vec{A} = [10\sin^2\theta/r]\,\vec{a}_r$ at (2, π/4, π/2)

   d) Find ∇.A in the three cases by shrinking the differential volume.

   e) Find ∇.A using the formula in the three cases and cross check with the results obtained in (d).

2. If the flux density $\vec{D}$ in a region is given as $\vec{D} = (2 + 16\,\rho^2)\,\vec{a}_z$, determine the total flux $\int \vec{D} \cdot \overrightarrow{ds}$ passing through a circular surface of radius ρ=2 in the xy plane.

3. Verify the divergence theorem for each of the following cases :

$$\oint_s A \cdot ds = \int_v \nabla \cdot A\,dv$$

   a) $A = xy^2a_x + y^3a_y + y^2z\,a_z$ and S is the surface of the cuboid defined by 0<x<1, 0<y<1, 0<z<1.

   b) $A = 2\,\rho z a_\rho + 32\sin\phi\,a_\phi - 4\rho\cos\phi\,a_z$ and S is the surface of the wedge $0 < \rho < 2$, $0 < \phi < 45^0$, 0<z<5.

   c) $A = \gamma^2\,a_r + \gamma\sin\theta\cos\phi\,a_\theta$ and S is the surface of a quarter of a sphere defined by $0 < \gamma < 3$, $0 < \phi < \pi/2$, $0 < \theta < \pi/2$.

4. In free space, let $D = 8xyz^4\boldsymbol{a}_x + 4x^2z^4\boldsymbol{a}_y + 16x^2yz^3\boldsymbol{a}_z\,pC/m^2$

   a) Find the total electric flux passing through the rectangular surface $z = 2, 0 < x < 2, 1 < y < 3$, in the $\boldsymbol{a}_z$ direction.

   b) Find **E** at *P*(2,-1,3).

   c) Find an approximate value for the total charge contained in an incremental sphere located at *P*(2,-1,3) and having a volume of $10^{-12}\,m^3$.

5. Let $\boldsymbol{D} = 5.00\,r^2\boldsymbol{a}_r\,mC/m^2$ for $r \le 0.08\,m$ and $D = 0.205\,\boldsymbol{a}_r/r^2\,\mu C/m^2$ for $r \ge 0.08\,m$

   a) Find ρ_v for r=0.06m.

   b) Find ρ_v for r=0.1m

c) What surface charge density could be located at r=0.08m to cause **D**=0 for r>0.08m?

6. In the region of free space that includes the volume, 2 < x, y, z < 3, $\boldsymbol{D} = \frac{2}{z^2}\left(yz\,\boldsymbol{a}_x + xz\,\boldsymbol{a}_y - 2xy\,\boldsymbol{a}_z\right) C/m^2$

   a) Evaluate the volume integral side of the divergence theorem for the volume defined here.

   b) Evaluate the surface integral side for the corresponding closed surface.

7. Let $\boldsymbol{D} = 20\rho^2\,\boldsymbol{a}_\rho\; nC/m^2$

   a) What is the volume charge density at the point $P(0.5, 60^0,\ 2)$?

   b) Use two different methods to find the amount of charge lying within the closed surface bounded by $\rho = 3, 0 \le z \le 2$.

8. Given the flux density $\boldsymbol{D} = \frac{16}{r}\cos(2\theta)\,\boldsymbol{a}_\theta\; C/m^2$, use two different methods to find the total charge within the region 1 < r < 2m, 1 < θ < 2 rad, 1 < ϕ < 2 rad.

9. Determine the net flux of the vector field **F**=2 $\mathbf{a}_r$ +r $\mathbf{a}_\theta$ -$\mathbf{a}_\phi$ out of a unit radius sphere that is centered on the origin of a spherical coordinate system.

10. Verify the Divergence theorem for the vector field **F** = r $\mathbf{a}_r$ over a closed surface that is a quadrant of a sphere defined by r =1, 0 ≤ θ ≤ π /2.