SOLUTIONS 6.1

SOL 6.1.1 Option (C) is correct.

From Faraday's law, the relation between electric field and magnetic field is

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

Since the electric field inside a conducting sphere is zero.

i.e. $E = 0$

So the rate of change in magnetic flux density will be

$$\frac{\partial B}{\partial t} = -(\nabla \times E) = 0$$

Therefore $B(r, t)$ will be uniform inside the sphere and independent of time.

SOL 6.1.2 Option (A) is correct.

Electric field intensity experienced by the moving conductor ab in the presence of magnetic field B is given as

$$E = v \times B \quad \text{where } v \text{ is the velocity of the conductor.}$$

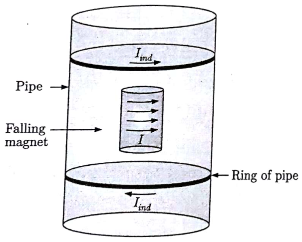
So, electric field will be directed from b to a as determined by right hand rule for the cross vector. Therefore, the voltage difference between the two ends of the conductor is given as

$$V_{ab} = -\int_a^b E \cdot dl$$

Thus, the positive terminal of voltage will be a and V_{ab} will be positive.

SOL 6.1.3 Option (A) is correct.

Consider a magnet bar being dropped inside a pipe as shown in figure.



Suppose the current I in the magnet flows counter clockwise (viewed from above) as shown in figure. So near the ends of pipe, it's field points upward. A ring of pipe below the magnet experiences an increasing upward flux as the magnet approaches and hence by Lenz's law a current will be induced in it such as to produce downward flux.

Thus, I_{ind} must flow clockwise which is opposite to the current in the magnet.

Since opposite currents repel each other so, the force exerted on the magnet due to the induced current is directed upward. Meanwhile a ring above the magnet experiences a decreasing upward flux; so it's induced current parallel to I and it attracts magnet upward. And flux through the rings next to the magnet bar is constant. So no current is induced in them.

Thus, for all we can say that the force exerted by the eddy current (induced current according to Lenz's law) on the magnet is in upward direction which causes the delay to reach the bottom. Whereas in the cases of unmagnetized bar no induced current is formed. So it reaches in fraction of time.

Thus, A and R both true and R is correct explanation of A.

SOL 6.1.4 Option (C) is correct.

The magnetic flux density inside a solenoid of n turns per unit length carrying current I is defined as

$$B = \mu_0 n I$$

Let the length of solenoid be l and its cross sectional radius be r. So, the total magnetic flux through the solenoid is

$$\Phi = (\mu_0 n I)(\pi r^2)(nl) \qquad (1)$$

Since the total magnetic flux through a coil having inductance L and carrying current I is given as

$$\Phi = LI$$

So comparing it with equation (1) we get,

$$L = \mu_0 n^2 I \pi^2 l$$

and as for a given solenoid, radius r and length l is constant therefore

$$L \propto n^2$$

SOL 6.1.5 Option (C) is correct.

The magnetic flux density inside the solenoid is defined as

$$B = \mu_0 n I$$

where $n \rightarrow$ no. of turns per unit length

$I \rightarrow$ current flowing in it.

So the total magnetic flux through the solenoid is

$$\Phi = \int B \cdot dS = (\mu_0 n I)(\pi a^2)$$

where $a \rightarrow$ radius of solenoid

Induced emf in a loop placed in a magnetic field is defined as

$$V_{emf} = -\frac{d\Phi}{dt}$$

where Φ is the total magnetic flux passing through the loop. Since the resistance R is looped over the solenoid so total flux through the loop will be equal to the total flux through the solenoid and therefore the induced emf in the loop of resistance will be

$$V_{emf} = -\pi a^2 \mu_0 n \frac{dI}{dt}$$

Since current I flowing in the solenoid is constant so, the induced emf is

$$V_{emf} = 0$$

and therefore the induced current in the loop will be zero.

SOL 6.1.6 Option (B) is correct.

It will be similar to the current in a solenoid.
So, the magnetic field will be in circumferential while the electric field is longitudinal.

SOL 6.1.7 Option (B) is correct.
In Assertion (A) the magnetic flux through each turn of both coils are equal
So, the net magnetic flux through the two coils are respectively

$$\Phi_1 = N_1\Phi$$

and
$$\Phi_2 = N_2\Phi$$

where Φ is the magnetic flux through a single loop of either coil and N_1, N_2 are the total no. of turns of the two coils respectively.
Therefore the induced emf in the two coils are

$$V_{emf1} = -\frac{d\Phi_1}{dt} = -N_1\frac{d\Phi}{dt}$$

$$V_{emf2} = -\frac{d\Phi_2}{dt} = -N_2\frac{d\Phi}{dt}$$

Thus, the ratio of the induced emf in the two loops are

$$\frac{V_{emf2}}{V_{emf1}} = \frac{N_2}{N_1}$$

Now, in Reason (R) : a primitive transformer is similar to the cylinder core carrying wound coils. It is the device in which by choosing the appropriate no. of turns, any desired secondary emf can be obtained.
So, both the statements are correct but R is not the explanation of A.

SOL 6.1.8 Option (B) is correct.
Electric flux density in the medium is given as

$$D = \varepsilon E = \varepsilon E_0 \cos\omega t \qquad (E = E_0\cos\omega t)$$

Therefore the displacement current density in the medium is

$$J_d = \frac{\partial D}{\partial t} = -\omega\varepsilon E_0 \sin\omega t$$

and the conduction current density in the medium is

$$J_c = \sigma E = \sigma E_0 \cos\omega t$$

So, the ratio of amplitudes of conduction current density and displacement current density is

$$\frac{|J_c|}{|J_d|} = \frac{\sigma}{\omega\varepsilon}$$

SOL 6.1.9 Option (C) is correct.
Given the volume charge density, $\rho_v = 0$
So, from Maxwell's equation we have

$$\nabla\cdot \boldsymbol{D} = \rho_v$$

$$\nabla\cdot \boldsymbol{D} = 0 \qquad (1)$$

Now, the electric flux density in a medium is defined as

$$\boldsymbol{D} = \varepsilon\boldsymbol{E} \qquad \text{(where } \varepsilon \text{ is the permittivity of the medium)}$$

So, putting it in equation (1) we get,

$$\nabla\cdot(\varepsilon\boldsymbol{E}) = 0$$

or,
$$\boldsymbol{E}\cdot(\nabla\varepsilon) + \varepsilon(\nabla\cdot\boldsymbol{E}) = 0$$

and since
$$\frac{\nabla\varepsilon}{\varepsilon} \approx 0 \Rightarrow \nabla\varepsilon \approx 0 \qquad \text{(given)}$$

Therefore,
$$\nabla\cdot\boldsymbol{E} \approx 0$$

SOL 6.1.10 Option (A) is correct.
Given the electric field intensity in time domain as

$$\boldsymbol{E} = \frac{\sin\theta\cos(\omega t - kr)}{r}\boldsymbol{a}_\theta$$

So, the electric field intensity in phasor form is given as

$$\boldsymbol{E}_s = \frac{\sin\theta}{r}e^{-jkr}\,\boldsymbol{a}_\theta$$

and
$$\nabla\times\boldsymbol{E}_s = \frac{1}{r}\frac{\partial}{kdr}(rE_{\theta s})\boldsymbol{a}_\phi = (-jk)\frac{\sin\theta}{r}e^{-jkr}\,\boldsymbol{a}_\phi$$

Therefore, from Maxwell's equation we get the magnetic field intensity as

$$\boldsymbol{H}_s = -\frac{\nabla\times\boldsymbol{E}_s}{j\omega r_0} = \frac{k}{\omega r_0}\frac{\sin\theta}{r}e^{-jkr}\,\boldsymbol{a}_\phi$$

SOL 6.1.11 Option (B) is correct.
Magnetic flux density produced at a distance ρ from a long straight wire carrying current I is defined as

$$\boldsymbol{B} = \frac{\mu_0 I}{2\pi\rho}\boldsymbol{a}_\phi$$

where $\boldsymbol{a}_\phi$ is the direction of flux density as determined by right hand rule.
So, the magnetic flux density produced by the straight conducting wire linking through the loop is normal to the surface of the loop.
Now consider a strip of width $d\rho$ of the square loop at distance ρ from the wire for which the total magnetic flux linking through the square loop is given as

$$\Phi = \int_S \boldsymbol{B}\cdot d\boldsymbol{S}$$

$$= \frac{\mu_0 I}{2\pi}\int_\rho^{\rho+a}\frac{1}{\rho}(ad\rho) \qquad \text{(area of the square loop is } dS = ad\rho)$$

$$= \frac{\mu_0 Ia}{2\pi}\ln\left(\frac{\rho+a}{\rho}\right)$$

The induced emf due to the change in flux (when pulled away) is given as

$$V_{emf} = -\frac{d\Phi}{dt} = -\frac{\mu_0 Ia}{2\pi}\frac{d}{dt}\left[\ln\left(\frac{\rho+a}{\rho}\right)\right]$$

Therefore,
$$V_{emf} = -\frac{\mu_0 Ia}{2\pi}\left(\frac{1}{\rho+a}\frac{d\rho}{dt} - \frac{1}{\rho}\frac{d\rho}{dt}\right)$$

Given
$$\frac{d\rho}{dt} = \text{velocity of loop} = 5\,\text{m/s}$$

and since the loop is currently located at 3 m distance from the straight wire, so after 0.6 sec it will be at

$$\rho = 3 + (0.6)\times v \qquad (v \rightarrow \text{velocity of the loop})$$

$$= 3 + 0.6\times 5 = 6\,\text{m}$$

So,
$$V_{emf} = -\frac{\mu_0\times(30)\times 2}{2\pi}\left[\frac{1}{8}(5) - \frac{1}{6}(5)\right] \qquad (a = 2\,\text{m}, I = 30\,\text{A})$$

$$= 25\times 10^{-7}\,\text{volt} = 2.5\,\mu\text{volt}$$

SOL 6.1.12 Option (B) is correct.
Since total magnetic flux through the loop depends on the distance from the straight wire and the distance is constant. So the flux linking through the loop will be constant, if it is pulled parallel to the straight wire. Therefore the induced emf in the loop is

$$V_{emf} = -\frac{d\Phi}{dt} = 0 \qquad (\Phi \text{ is constant})$$

SOL 6.1.13 Option (D) is correct.

Total magnetic flux through the solenoid is given as

$$\Phi = \mu_0 nI$$

where n is the no. of turns per unit length of solenoid and I is the current flowing in the solenoid.

Since the solenoid carries current that is increasing linearly with time

i.e. $I \propto t$

So the net magnetic flux through the solenoid will be

$$\Phi \propto t$$

or, $\Phi = kt$ where k is a constant.

Therefore the emf induced in the loop consisting resistances R_A, R_B is

$$V_{emf} = -\frac{d\Phi}{dt}$$

$$V_{emf} = -k$$

and the current through R_1 and R_2 will be

$$I_{ind} = -\frac{k}{R_1 + R_2}$$

Now according to Lenz's law the induced current I in a loop flows such as to produce a magnetic field that opposes the change in $\boldsymbol{B}(t)$.

i.e. the induced current in the loop will be opposite to the direction of current in solenoid (in anticlockwise direction).

So, $V_A = I_{ind} R_A = -\frac{kR_A}{R_A + R_B}$

and $V_B = -I_{ind} R_B = \left(\frac{kR_B}{R_A + R_B}\right)$

Thus, the ratio of voltmeter readings is

$$\frac{V_A}{V_B} = -\frac{R_A}{R_B}$$

SOL 6.1.14 Option (D) is correct.

Induced emf in the conducting loop formed by rail, bar and the resistor is given by

$$V_{emf} = -\frac{d\Phi}{dt}$$

where Φ is total magnetic flux passing through the loop.

The bar is located at a distance x from the resistor at time t. So the total magnetic flux passing through the loop at time t is

$\Phi = \int \boldsymbol{B} \cdot d\boldsymbol{S} = Blx$ where l is separation between the rails

Now the induced emf in a loop placed in magnetic field is defined as

$$V_{emf} = -\frac{d\Phi}{dt}$$

where Φ is the total magnetic flux passing through the loop. Therefore the induced emf in the square loop is

$$V_{emf} = -\frac{d}{dt}(Blx) = -Bl\frac{dx}{dt} \qquad (\Phi = Blx)$$

Since from the given figure, we have

$$l = 5\text{ m}$$

$$B = 2\text{ T}$$

and $dx/dt \to$ velocity of bar $= 4\text{ m/s}$

So, induced emf is

$$V_{emf} = -(2)(5)(4) = -40\text{ volt}$$

Therefore the current in the bar loop will be

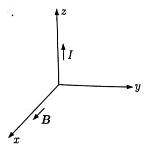
$$I = \frac{V_{emf}}{R} = -\frac{40}{10} = -4\text{ A}$$

SOL 6.1.15 Option (B) is correct.

As obtained in the previous question the current flowing in the sliding bar is

$$I = -4\text{ A}$$

Now we consider magnetic field acts in $\boldsymbol{a}_x$ direction and current in the sliding bar is flowing in $+\boldsymbol{a}_z$ direction as shown in the figure.



Therefore, the force exerted on the bar is

$$\boldsymbol{F} = \int Id\boldsymbol{l} \times \boldsymbol{B} = \int_0^5 (-4dz\boldsymbol{a}_z) \times (2\boldsymbol{a}_x)$$

$$= -8\boldsymbol{a}_y[z]_0^5 = -40\boldsymbol{a}_y\text{ N}$$

i.e. The force exerted on the sliding bar is in opposite direction to the motion of the sliding bar.

SOL 6.1.16 Option (C) is correct.

Given the magnetic flux density through the square loop is

$$\boldsymbol{B} = 7.5\cos(120\pi t - 30°)\,\boldsymbol{a}_z$$

So the total magnetic flux passing through the loop will be

$$\Phi = \oint_S \boldsymbol{B} \cdot d\boldsymbol{S}$$

$$= [-7.5\cos(120\pi t - 30°)\,\boldsymbol{a}_z](1 \times 1)(-\boldsymbol{a}_z)$$

$$= 7.5\cos(120\pi t - 30°)$$

Now, the induced emf in the square loop is given by

$$V_{emf} = -\frac{d\Phi}{dt} = 7.5 \times 120\pi\sin(120\pi t - 30°)$$

The polarity of induced emf (according to Lenz's law) will be such that induced current in the loop will be in opposite direction to the current $I(t)$ shown in the figure. So we have

$$I(t) = -\frac{V_{emf}}{R}$$

$$= -\frac{7.5 \times 120\pi}{500}\sin(120\pi t - 30°) \qquad (R = 250 + 250 = 500\,\Omega)$$

$$= -5.7\sin(120\pi t - 30°)$$

SOL 6.1.17 Option (A) is correct.

Consider the mutual inductance between the rectangular loop and straight wire be M. So applying KVL in the rectangular loop we get,

$$M\frac{di_1}{dt} = L\frac{di_2}{dt} + Ri_2 \qquad \dots(1)$$

Now from the shown figure (b), the current flowing in the straight wire is given as

$$i_1 = I_1 u(t) - I_1 u(t-T) \quad (I_1 \text{ is amplitude of the current})$$

or,
$$\frac{di_1}{dt} = I_1\delta(t) - I_1\delta(t-T) \qquad (2)$$

So, at $t = 0$
$$\frac{di_1}{dt} = I_1$$

and
$$MI_1 = L\frac{di_2}{dt} + Ri_2 \qquad \text{(from equation (1))}$$

Solving it we get
$$i_2 = \frac{M}{L}I_1 e^{-(R/L)t} \qquad \text{for } 0 < t < T$$

Again in equation (2) at $t = T$ we have
$$\frac{di_1}{dt} = -I_1$$

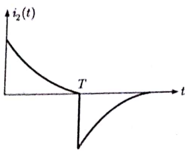
and
$$-MI_1 = L\frac{di_2}{dt} + Ri_2 \qquad \text{(from equation (1))}$$

Solving it we get
$$i_2 = -\frac{M}{L}I_1 e^{-(R/L)(t-T)} \qquad \text{for } t > T$$

Thus, the current in the rectangular loop is

$$i_2 = \begin{cases} \frac{M}{L}I_1 e^{-(R/L)t} & 0 < t < T \\ -\frac{M}{L}I_1 e^{-(R/L)(t-T)} & t > T \end{cases}$$

Plotting i_2 versus t we get



SOL 6.1.18 Option (A) is correct.

Total magnetic flux passing through the loop formed by the resistance, bar and the rails is given as:

$$\Phi = \int_S \boldsymbol{B} \cdot d\boldsymbol{S}$$
$$= \boldsymbol{B} \cdot \boldsymbol{S} = [0.2\cos\omega t\boldsymbol{a}_z] \cdot [0.5(1-y)\boldsymbol{a}_z]$$
$$= 0.1[1 - 0.5(1-\cos\omega t)]\cos\omega t \quad (y = 0.5(1-\cos\omega t)\,\text{m})$$
$$= 0.05\cos\omega t(1+\cos\omega t) = 0.05(\cos\omega t + \cos^2\omega t)$$

So, the induced emf in the loop is

$$V_{emf} = -\frac{d\Phi}{dt}$$

and as determined by Lenz's law, the induced current will be flowing in

opposite direction to the current i. So the current i in the loop will be

$$i = -\frac{V_{emf}}{R} = -\frac{1}{R}\left(-\frac{d\Phi}{dt}\right)$$
$$= \frac{0.05}{5}[-\omega\sin\omega t - 2\omega\cos\omega t\sin\omega t]$$
$$= -0.01\omega\sin\omega t(1 + 2\cos\omega t)$$

SOL 6.1.19 Option (D) is correct.

Given the electric flux density in the medium is

$$\boldsymbol{D} = 1.33\sin(3\times10^8 t - 0.2x)\boldsymbol{a}_y\,\mu\text{C/m}^2$$

So, the electric field intensity in the medium is given as

$$\boldsymbol{E} = \frac{\boldsymbol{D}}{\varepsilon} \qquad \text{where } \varepsilon \text{ is the permittivity of the medium}$$

or,
$$\boldsymbol{E} = \frac{\boldsymbol{D}}{\varepsilon_r\varepsilon_0} = \frac{1.33\times10^{-6}\sin(3\times10^8 t - 0.2x)}{10\times8.85\times10^{-12}}\boldsymbol{a}_y \quad (\varepsilon_r = 10)$$
$$= 1.5\times10^4\sin(3\times10^8 t - 0.2x)\boldsymbol{a}_y$$

Now, from maxwell's equation we have

$$\nabla\times\boldsymbol{E} = -\frac{\partial\boldsymbol{B}}{\partial t}$$

or,
$$\frac{\partial\boldsymbol{B}}{\partial t} = -\nabla\times\boldsymbol{E}$$
$$= -\frac{\partial E_y}{\partial x}\boldsymbol{a}_z$$
$$= -(-0.2)\times(1.5\times10^4)\cos(3\times10^8 t - 0.2x)\boldsymbol{a}_y$$
$$= 3\times10^3\cos(3\times10^8 t - 0.2x)\boldsymbol{a}_y$$

Integrating both sides, we get the magnetic flux density in the medium as

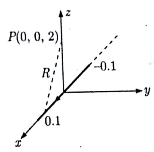
$$\boldsymbol{B} = \int 3\times10^3\cos(3\times10^8 t - 0.2x)\boldsymbol{a}_y$$
$$= \frac{3\times10^3}{3\times10^8}\sin(3\times10^8 t - 0.2x)\boldsymbol{a}_y$$
$$= 10^{-5}\sin(3\times10^8 t - 0.2x)\boldsymbol{a}_y \text{ Tesla}$$

Therefore the magnetic field intensity in the medium is

$$\boldsymbol{H} = \frac{\boldsymbol{B}}{\mu} = \frac{\boldsymbol{B}}{\mu_r\mu_0} = \frac{10^{-5}\sin(3\times10^8 t - 0.2x)}{2\times4\pi\times10^{-7}} \qquad \mu_r = 2$$

Thus
$$\boldsymbol{H} = 4\sin(3\times10^8 t - 0.2x)\boldsymbol{a}_y\,\text{A/m}$$

SOL 6.1.20 Option (B) is correct.



The magnetic vector potential for a direct current flowing in a filament is given as

$$\boldsymbol{A} = \int \frac{\mu_0 I}{4\pi R}\boldsymbol{a}_z\,dx$$

Here current $I(t)$ flowing in the filament shown in figure is varying with time as

$$I(t) = 8t \text{ A}$$

So, the retarded vector potential at the point P will be given as

$$\mathbf{A} = \int \frac{\mu_0 I(t - R/c)}{4\pi R}\mathbf{a}_x\,dx$$

where R is the distance of any point on the filamentary current from P as shown in the figure and c is the velocity of waves in free space. So, we have

$$R = \sqrt{x^2+4} \text{ and } c = 3 \times 10^8 \text{ m/s}$$

Therefore,

$$\mathbf{A} = \int_{x=-0.1}^{0.1} \frac{\mu_0 8(t - R/c)}{4\pi R}\mathbf{a}_x\,dx$$

$$= \frac{8\mu_0}{4\pi}\left[\int_{-0.1}^{0.1}\frac{t}{\sqrt{x^2+4}}dx - \int_{-0.1}^{0.1}\frac{1}{c}dx\right]$$

$$= 8 \times 10^{-7} t\left[\ln(x+\sqrt{x^2+4})\right]_{-0.1}^{0.1} - \frac{8 \times 10^{-7}}{3 \times 10^8}[x]_{-0.1}^{0.1}$$

$$= 8 \times 10^{-7} t\ln\left(\frac{0.1+\sqrt{4.01}}{-0.1+\sqrt{4.01}}\right) - 0.53 \times 10^{-15}$$

$$= 8 \times 10^{-8} t - 0.53 \times 10^{-15}$$

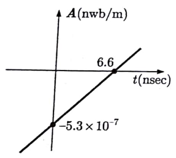
or,

$$\mathbf{A} = (80t - 5.3 \times 10^{-7})\mathbf{a}_x \text{ nWb/m} \tag{1}$$

So, when $\mathbf{A} = 0$ $\qquad t = 6.6 \times 10^{-9} = 6.6 \text{ nsec}$

and when $t = 0$ $\qquad A = -5.3 \times 10^{-7} \text{ nWb/m}$

From equation (1) it is clear that $\mathbf{A}$ will be linearly increasing with respect to time. Therefore the plot of $\mathbf{A}$ versus t is



NOTE :

Time varying potential is usually called the retarded potential.

SOL 6.1.21 Option (A) is correct.

The force experienced by a test charge q in presence of both electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$ in the region will be evaluated by using Lorentz force equation as

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

So, putting the given three forces and their corresponding velocities in above equation we get the following relations

$$q(\mathbf{a}_y + \mathbf{a}_z) = q(\mathbf{E} + \mathbf{a}_x \times \mathbf{B}) \tag{1}$$

$$q\mathbf{a}_y = q(\mathbf{E} + \mathbf{a}_y \times \mathbf{B}) \tag{2}$$

$$q(2\mathbf{a}_y + \mathbf{a}_z) = q(\mathbf{E} + \mathbf{a}_z \times \mathbf{B}) \tag{3}$$

Subtracting equation (2) from (1) we get

$$\mathbf{a}_z = (\mathbf{a}_x - \mathbf{a}_y) \times \mathbf{B} \tag{4}$$

and subtracting equation (1) from (3) we get

$$\mathbf{a}_y = (\mathbf{a}_z - \mathbf{a}_x) \times \mathbf{B} \tag{5}$$

Now we substitute $\mathbf{B} = B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z$ in eq (4) to get

$$\mathbf{a}_z = B_y\mathbf{a}_z - B_z\mathbf{a}_y + B_x\mathbf{a}_z - B_z\mathbf{a}_x$$

So, comparing the x, y and z components of the two sides we get

$$B_x + B_y = 1$$

and

$$B_z = 0$$

Again by substituting $\mathbf{B} = B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z$ in eq (5), we get

$$\mathbf{a}_y = B_x\mathbf{a}_y - B_y\mathbf{a}_x - B_y\mathbf{a}_z + B_z\mathbf{a}_y$$

So, comparing the x, y and z components of the two sides we get

$$B_x + B_z = 1$$

and

$$B_y = 0$$

as calculated above $B_z = 0$, therefore $B_x = 1$

Thus, the magnetic flux density in the region is

$$\mathbf{B} = \mathbf{a}_x \text{ Wb/m}^2 \qquad (B_x = 1,\ B_y = B_z = 0)$$

SOL 6.1.22 Option (C) is correct.

As calculated in previous question the magnetic flux density in the region is

$$\mathbf{B} = \mathbf{a}_x \text{ Wb/m}^2$$

So, putting it in Lorentz force equation we get

$$\mathbf{F} = q(\mathbf{E} + \mathbf{V} \times \mathbf{B})$$

or,

$$q(\mathbf{a}_y + \mathbf{a}_z) = q(\mathbf{E} + \mathbf{a}_x \times \mathbf{a}_x)$$

Therefore, the electric field intensity in the medium is

$$\mathbf{E} = \mathbf{a}_y + \mathbf{a}_z \text{ V/m}$$

SOL 6.1.23 Option (C) is correct.

Given

Retarded scalar potential, $\qquad V = y(x - ct) \text{ volt}$

and retarded vector potential, $\qquad \mathbf{A} = y\left(\frac{x}{c} - t\right)\mathbf{a}_z \text{ Wb/m}$

Now the magnetic flux density in the medium is given as

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$= -\frac{\partial A_y}{\partial y}\mathbf{a}_z = \left(t - \frac{x}{c}\right)\mathbf{a}_z \text{ Tesla} \tag{1}$$

So, the magnetic field intensity in the medium is

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} \qquad (\mu_0 \text{ is the permittivity of the medium})$$

$$= \frac{1}{\mu_0}\left(t - \frac{x}{c}\right)\mathbf{a}_z \text{ A/m} \tag{2}$$

and the electric field intensity in the medium is given as

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}$$

$$= -(x - ct)\mathbf{a}_y - y\mathbf{a}_x + y\mathbf{a}_x = (ct - x)\mathbf{a}_y \tag{3}$$

So, the electric flux density in the medium is

$$\mathbf{D} = \varepsilon_0\mathbf{E} \qquad (\varepsilon_0 \text{ is the permittivity of the medium})$$

$$= \varepsilon_0(ct - x)\mathbf{a}_y \text{ C/m}^2 \tag{4}$$

Now we determine the condition for the field to satisfy all the four Maxwell's equation.

(a) $\qquad \nabla \cdot \mathbf{D} = \rho_v$

or, $\qquad \rho_v = \nabla \cdot [\varepsilon_0(ct - x)\mathbf{a}_y] \qquad$ (from equation (4))

$$= 0$$

It means the field satisfies Maxwell's equation if $\rho_v = 0$.

(b) $\nabla \cdot \boldsymbol{B} = 0$

Now, $\nabla \cdot \boldsymbol{B} = \nabla \cdot \left[\left(t - \frac{x}{c}\right)\boldsymbol{a}_z\right] = 0$ (from equation (1))

So, it already, satisfies Maxwell's equation

(c) $\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$

Now, $\nabla \times \boldsymbol{H} = -\frac{\partial H_z}{\partial x}\boldsymbol{a}_y = \frac{1}{\mu_0 c}\boldsymbol{a}_y = \sqrt{\frac{\varepsilon_0}{\mu_0}}\boldsymbol{a}_y$ (from equation (2))

and from equation (4) we have

$$\frac{\partial \boldsymbol{D}}{\partial t} = \varepsilon_0 c \boldsymbol{a}_y = \sqrt{\frac{\varepsilon_0}{\mu_0}}\boldsymbol{a}_y \qquad \left(\text{Since in free space } c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}\right)$$

Putting the two results in Maxwell's equation, we get the condition

$$\boldsymbol{J} = 0$$

(d) $\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$

Now $\nabla \times \boldsymbol{E} = \frac{\partial E_y}{\partial x}\boldsymbol{a}_z = -\boldsymbol{a}_z$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{a}_z$$

So, it already satisfies Maxwell's equation. Thus, by combining all the results we get the two required conditions as $\boldsymbol{J} = 0$ and $\rho_v = 0$ for the field to satisfy Maxwell's equation.

SOL 6.1.24 Option (A) is correct.

Given the magnetic flux density through the loop is

$$\boldsymbol{B} = -2/x\, \boldsymbol{a}_z$$

So the total magnetic flux passing through the loop is given as

$$\Phi = \int \boldsymbol{B} \cdot d\boldsymbol{S} = \int_x^{x+2}\int_y^{y+2}\left(-\frac{2}{x}\boldsymbol{a}_z\right) \cdot (-dxdy\boldsymbol{a}_z)$$

$$= \left(2\ln\frac{x+2}{x}\right)(2) = 4\ln\left(\frac{x+2}{x}\right)$$

Therefore, the circulation of induced electric field in the loop is

$$\oint_C \boldsymbol{E} \cdot d\boldsymbol{l} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\left[4\ln\left(\frac{x+2}{x}\right)\right]$$

$$= -\frac{4}{\left(\frac{x+2}{x}\right)}\frac{d}{dt}\left(\frac{x+2}{x}\right)$$

$$= -\frac{4x}{x+2}\left(-\frac{2}{x^2}\frac{dx}{dt}\right)$$

$$= \frac{8}{x(x+2)}(2) = \frac{16}{x(x+2)} \qquad \left(\frac{dx}{dt} = v = 2\boldsymbol{a}_x\right)$$

SOL 6.1.25 Option (A) is correct.

As the magnetic flux density for $\rho < 4$ is $\boldsymbol{B} = 0$ so, the total flux passing through the closed loop defined by $\rho = 4$ m is

$$\Phi = \int \boldsymbol{B} \cdot d\boldsymbol{S} = 0$$

So, the induced electric field circulation for the region $\rho < 4$ m is given as

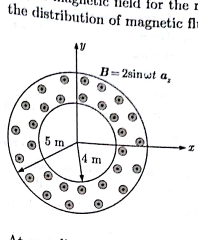
$$\oint_C \boldsymbol{E} \cdot d\boldsymbol{l} = -\frac{d\Phi}{dt} = 0$$

or,

$$\boldsymbol{E} = 0 \qquad \text{for } \rho < 4\text{ m}$$

SOL 6.1.26 Option (B) is correct.

As the magnetic field for the region $\rho < 4$ m and $\rho > 5$ m is zero so we get the distribution of magnetic flux density as shown in figure below.



At any distance ρ from origin in the region $4 < \rho < 5$ m, the circulation of induced electric field is given as

$$\oint_C \boldsymbol{E} \cdot d\boldsymbol{l} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\left(\int \boldsymbol{B} \cdot d\boldsymbol{S}\right)$$

$$= -\frac{d}{dt}[2\sin\omega t(\pi\rho^2 - \pi 4^2)]$$

$$= -2\omega\cos\omega t(\pi\rho^2 - 16\pi)$$

or,

$$E(2\pi\rho) = -2\omega\cos\omega t(\pi\rho^2 - 16\pi)$$

$$E = -\frac{2(\rho^2 - 16)\omega\cos\omega t}{2\rho}$$

So, the induced electric field intensity at $\rho = 4.5$ m is

$$\boldsymbol{E} = -\frac{2}{4.5}((4.5)^2 - 16)\omega\cos\omega t$$

$$= -\frac{17}{18}\omega\cos\omega t$$

SOL 6.1.27 Option (B) is correct.

For the region $\rho > 5$ m the magnetic flux density is 0 and so the total magnetic flux passing through the closed loop defined by $\rho = 5$ m is

$$\Phi = \int_0^5 \boldsymbol{B} \cdot d\boldsymbol{S} = \int_0^4 \boldsymbol{B} \cdot d\boldsymbol{S} + \int_4^5 \boldsymbol{B} \cdot d\boldsymbol{S}$$

$$= 0 + \int_4^5 (2\sin\omega t)\boldsymbol{a}_z \cdot d\boldsymbol{S}$$

$$= (2\sin\omega t)[\pi(5)^2 - \pi(4)^2] = 18\pi\sin\omega t$$

So, the circulation of magnetic flux density for any loop in the region $\rho > 5$ m is

$$\oint \boldsymbol{E} \cdot d\boldsymbol{l} = -\frac{d\psi}{dt}$$

$$E(2\pi\rho) = -\frac{d}{dt}(18\pi\sin\omega t)$$

$$= -18\pi\omega\cos\omega t$$

So, the induced electric field intensity in the region $\rho > 5$ m is

$$\boldsymbol{E} = \frac{-18\pi\omega\cos\omega t}{2\pi\rho}\boldsymbol{a}_\phi$$

$$= -\frac{9}{\rho}\omega\cos\omega t\, a_\phi$$

SOL 6.1.28 Option (D) is correct.
The distribution of magnetic flux density and the resistance in the circuit are same as given in section A (Q. 31) so, as calculated in the question, the two voltage drops in the loop due to magnetic flux density $B = 0.1t\, a_z$ are

$$V_1 = 33.3\text{ mV}$$

and $$V_2 = 66.67\text{ mV} = 66.7\text{ mV}$$

Now V_3 (voltmeter) which is directly connected to terminal cd is in parallel to both V_2 and V_1. It must be kept in mind that the loop formed by voltmeter V_3 and resistance $2\,\Omega$ also carries the magnetic flux density crossing through it. So, in this loop the induced emf will be produced which will be same as the field produced in loop $abcd$ at the enclosed fluxes will be same.
Therefore as calculated above induced emf in the loop of V_3 is

$$V_{emf} = 100\text{ mV}$$

According to lenz's law it's polarity will be opposite to V_3 and so

$$-V_{emf} = V_1 + V_3$$

or, $$V_3 = 100 - 33.3 = 66.7\text{ mV}$$

SOL 6.1.29 Option (D) is correct.
The induced emf in a closed loop is defined as

$$V_{emf} = -\frac{d\Phi}{dt}$$

where Φ is the total magnetic flux passing through the square loop
At any time t, angle between B and dS is θ since B is in a_y direction so the total magnetic flux passing through the square loop is

$$\Phi = \int B \cdot dS$$
$$= (B)(S)\cos\theta$$
$$= (5 \times 10^{-3})(20 \times 10^{-3} \times 20 \times 10^{-3})\cos\theta$$
$$= 2 \times 10^{-6}\cos\theta$$

Therefore the induced emf in the loop is

$$V_{emf} = -\frac{d\Phi}{dt}$$
$$= -2 \times 10^{-6}\frac{d}{dt}(\cos\theta)$$
$$= 2 \times 10^{-6}\sin\theta\frac{d\theta}{dt}$$

and as $$\frac{d\theta}{dt} = \text{angular velocity} = 2\text{ rad/sec}$$

So, $$V_{emf} = (2 \times 10^{-6})\sin\theta(2)$$
$$= 4 \times 10^{-6}\sin\theta\text{ V/m} = 4\sin\theta\,\mu\text{V/m}$$

SOL 6.1.30 Option (B) is correct.
As calculated in previous question the induced emf in the closed square loop is

$$V_{emf} = 4\sin\theta\,\mu\text{V/m}$$

So the induced current in the loop is

$$I = \frac{V_{emf}}{R}$$ where R is the resistance in the loop.

$$= \frac{4\sin\theta \times 10^{-6}}{40 \times 10^{-3}} \qquad (R = 40\text{ m}\Omega)$$
$$= 0.1\sin\theta\text{ mA}$$

SOL 6.1.31 Option (C) is correct.
The total magnetic flux through the square loop is given as

$$\Phi = \oint B \cdot dS = (B_0\sin\omega t)(S)\cos\theta$$

So, the induced emf in the loop is

$$V_{emf} = -\frac{d\Phi}{dt} = -\frac{d}{dt}[(B_0\sin\omega t)(S)\cos\theta]$$
$$= -B_0 S\frac{d}{dt}[\sin\omega t\cos\omega t] \qquad (\theta = \omega t)$$
$$= -B_0 S\cos 2\omega t$$

Thus, the maximum value of induced emf is

$$|V_{emf}| = B_0 S\omega$$

SOL 6.1.32 Option (C) is correct.
e.m.f. induced in the loop due to the magnetic flux density is given as

$$V_{emf} = -\frac{\partial\Phi}{\partial t} = -\frac{\partial}{\partial t}(10\cos 120\pi t)(\pi\rho^2)$$
$$= -\pi(10 \times 10^{-2})^2 \times (120\pi)(-10\sin 120\pi t)$$
$$= 12\pi^2\sin 120\pi t$$

As determined by Lenz's law the polarity of induced e.m.f will be such that b is at positive terminal with respect to a.

i.e. $$V_{ba} = V_{emf} = 12\pi^2\sin 120\pi t$$

or $$V_{ab} = -12\pi^2\sin 120\pi t$$
$$= -118.43\sin 120\pi t\text{ Volt}$$

SOL 6.1.33 Option (D) is correct.
As calculated in previous question, the voltage induced in the loop is

$$V_{ab} = -12\pi^2\sin 120\pi t$$

Therefore, the current flowing in the loop is given as

$$I(t) = -\frac{V_{ab}}{250} = \frac{12\pi^2\sin 120\pi t}{250}$$
$$= 0.47\sin 120\pi t$$

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SOLUTIONS 6.2

SOL 6.2.1 Correct answer is 0.

As the conducting loop is falling freely So, the flux through loop will remain constant. Therefore, the voltage induced in the loop will be zero.

SOL 6.2.2 Correct answer is -4.

The magnetic flux density passing through the loop is given as

$$B = 4z^3t^2 a_x$$

Since the flux density is directed normal to the plane $x = 0$ so the total magnetic flux passing through the square loop located in the plane $x = 0$ is

$$\Phi = \int B \cdot dS = \int_{y=0}^{1}\int_{z=0}^{1}(4z^3t^2)\,dydz = t^2 \qquad (dS = (dydz)\,a_x)$$

Induced emf in a loop placed in magnetic field is defined as

$$V_{emf} = -\frac{d\Phi}{dt}$$

where Φ is the total magnetic flux passing through the loop. So the induced emf in the square loop is

$$V_{emf} = -\frac{d(t^2)}{dt} = -2t \qquad (\Phi = t^2)$$

Therefore at time $t = 2$ sec the induced emf is

$$V_{emf} = -4 \text{ volt}$$

SOL 6.2.3 Correct answer is 4.05 .

Magnetic flux density produced at a distance ρ from a long straight wire carrying current I is defined as

$$B = \frac{\mu_0 I}{2\pi\rho}a_\phi$$

where a_ϕ is the direction of flux density as determined by right hand rule. So the flux density produced by straight wire at a distance ρ from it is

$$B = \frac{\mu_0 I}{2\pi\rho}a_n \qquad (a_n \text{ is unit vector normal to the loop})$$

Therefore the total magnet flux passing through the loop is

$$\Phi = \int B \cdot dS = \int_d^{d+a}\frac{\mu_0 I}{2\pi\rho}a\,d\rho \qquad (dS = a\,d\rho\,a_n)$$

where $d\rho$ is width of the strip of loop at a distance ρ from the straight wire. Thus,

$$\Phi = \int_2^3\left(\frac{\mu_0 I}{2\pi}\right)\frac{d\rho}{\rho} = \frac{\mu_0 I}{2\pi}\ln\left(\frac{3}{2}\right) = \frac{\mu_0(5)}{2\pi}\ln(1.5)$$

$$= (2\times10^{-7})(5)\ln(1.5) = 4.05\times10^{-7}\text{ Wb}$$

SOL 6.2.4 Correct answer is 133.3 .

The displacement current density in a medium is equal to the rate of change in electric flux density in the medium.

$$J_d = \frac{\partial D}{\partial t}$$

Since the displacement current density in the medium is given as

$$J_d = 20\cos(1.5\times10^8 t)a_y \text{ A/m}^2$$

So, the electric flux density in the medium is

$$D = \int J_d dt + C \qquad (C \rightarrow \text{constant})$$

$$= \int 20\cos(1.5\times10^8 t)a_y dt + C$$

As there is no D.C. field present in the medium so, we get $C = 0$ and thus,

$$D = \frac{20\sin(1.5\times10^8 t)}{1.5\times10^8}a_y = 1.33\times10^{-7}\sin(1.5\times10^8 t)a_y$$

$$= 133.3\sin(1.5\times10^8 t)a_y \text{ nC/m}^2$$

Since, from the given problem we have the flux density

$$D = D_0\sin(1.5\times10^8 t)a_y \text{ nC/m}^2$$

So, we get

$$D_0 = 133.3$$

SOL 6.2.5 Correct answer is 9.75 .

The ratio of magnitudes of displacement current to conduction current in any medium having permittivity ε and conductivity σ is given as

$$\left|\frac{\text{Displacement current}}{\text{Conduction current}}\right| = \frac{\omega\varepsilon}{\sigma}$$

where ω is the angular frequency of the current in the medium.

Given frequency, $f = 50$ GHz

Permittivity, $\varepsilon = 4\varepsilon_0 = 4\times8.85\times10^{-12}$

Conductivity, $\sigma = 1.14\times10^8$ s/m

So, $\omega = 2\pi f = 2\pi\times50\times10^9 = 100\pi\times10^9$

Therefore, the ratio of magnitudes of displacement current to the conduction current is

$$\left|\frac{I_d}{I_c}\right| = \frac{100\pi\times10^9\times4\times8.85\times10^{-12}}{1.14\times10^8} = 9.75\times10^{-8}$$

SOL 6.2.6 Correct answer is 33.3 .

Given magnetic flux density through the square loop is

$$B = 0.1t a_z \text{ Wb/m}^2$$

So, total magnetic flux passing through the loop is

$$\Phi = B\cdot dS = (0.1t)(1) = 0.1t$$

The induced emf (voltage) in the loop is given as

$$V_{emf} = -\frac{d\phi}{dt} = -0.1\text{ Volt}$$

As determined by Lenz's law the polarity of induced emf will be such that

$$V_1 + V_2 = -V_{emf}$$

Therefore, the voltage drop in the $2\,\Omega$ resistance is

$$V_1 = \left(\frac{2}{2+4}\right)(-V_{emf}) = \frac{0.1}{3} = 33.3\text{ mV}$$

SOL 6.2.7 Correct answer is 7.2 .

Voltage, $V_1 = -N_1\frac{d\Phi}{dt}$

where Φ is total magnetic flux passing through it.

Again $\quad V_2 = -N_2 \dfrac{d\Phi}{dt}$

Since both the coil are in same magnetic field so, change in flux will be same for both the coil.

Comparing the equations (1) and (2) we get

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

$$V_2 = V_1 \frac{N_2}{N_1} = (12)\frac{3000}{5000} = 7.2 \text{ volt}$$

SOL 6.2.8 Correct answer is 41.6 .

In phasor form the magnetic field intensity can be written as

$$\boldsymbol{H}_s = 0.1\cos(15\pi y)e^{-jbx}\boldsymbol{a}_z \text{ A/m}$$

Similar as determined in MCQ 42 using Maxwell's equation we get the relation

$$(15\pi)^2 + b^2 = \omega^2 \mu_0 \varepsilon_0$$

Here $\quad \omega = 6\pi \times 10^9$

So, $$(15\pi)^2 + b^2 = \left(\frac{6\pi \times 10^9}{3 \times 10^8}\right)^2$$

$$(15\pi)^2 + b^2 = 400\pi^2$$

$$b^2 = 175\pi^2 \quad \Rightarrow \quad b = \pm 41.6 \text{ rad/m}$$

So, $$|b| = 41.6 \text{ rad/m}$$

SOL 6.2.9 Correct answer is 0.01 .

Induced emf. in the conducting loop formed by rail, bar and the resistor is given by

$$V_{emf} = -\frac{d\Phi}{dt}$$

where Φ is total magnetic flux passing through the loop.

Consider the bar be located at a distance x from the resistor at time t. So the total magnetic flux passing through the loop at time t is

$$\Phi = \int \boldsymbol{B} \cdot d\boldsymbol{S} = Blx \qquad \text{(area of the loop is } S = lx\text{)}$$

Now the induced emf in a loop placed in magnetic field is defined as

$$V_{emf} = -\frac{d\Phi}{dt}$$

where Φ is the total magnetic flux passing through the loop. Therefore the induced emf in the square loop is

$$V_{emf} = -\frac{d}{dt}(Blx) = -Bl\frac{dx}{dt} \qquad (\Phi = Blx)$$

Since from the given figure, we have

$$l = 2 \text{ m} \quad \text{and} \quad B = 0.1 \text{ Wb/m}^2$$

and $\quad dx/dt = \text{velocity of bar} = 5 \text{ m/s}$

So, induced emf is

$$V_{emf} = -(0.1)(2)(5) = -1 \text{ volt}$$

According to Lenz's law the induced current I in a loop flows such as to produce magnetic field that opposes the change in $\boldsymbol{B}(t)$. As the bar moves away from the resistor the change in magnetic field will be out of the page so the induced current will be in the same direction of I shown in figure. Thus, the current in the loop is

$$I = -\frac{V_{emf}}{R} = -\frac{(-1)}{10} = 0.01 \text{ A} \qquad (R = 10\Omega)$$

SOL 6.2.10 Correct answer is 277.

Magnetic flux density produced at a distance ρ from a long straight wire carrying current I is defined as

$$\boldsymbol{B} = \frac{\mu_0 I}{2\pi\rho}\boldsymbol{a}_\phi$$

where $\boldsymbol{a}_\phi$ is the direction of flux density as determined by right hand rule. Since the direction of magnetic flux density produced at the loop is normal to the surface of the loop So, total flux passing through the loop is given by

$$\Phi = \int_S \boldsymbol{B} \cdot d\boldsymbol{S} = \int_{\rho=2}^{4}\left(\frac{\mu_0 I}{2\pi\rho}\right)(a\,d\rho) \qquad (dS = a\,d\rho)$$

$$= \frac{\mu_0 I a}{2\pi}\int_2^4 \frac{d\rho}{\rho}$$

$$= \frac{\mu_0 I2}{2\pi}\ln 2 = \frac{\mu_0 I}{\pi}\ln(2)$$

The current flowing in the loop is I_{loop} and induced e.m.f. is V_{emf}.

So, $$V_{emf} = I_{loop}R = -\frac{d\Phi}{dt}$$

$$\frac{dQ}{dt}(R) = -\frac{\mu_0}{\pi}\ln(2)\frac{dI}{dt}$$

where Q is the total charge passing through a corner of square loop.

$$\frac{dQ}{dt} = -\frac{\mu_0}{4\pi}\ln(2)\frac{dI}{dt} \qquad (R = 4\,\Omega)$$

$$dQ = -\frac{\mu_0}{4\pi}\ln(2)\,dI$$

Therefore the total charge passing through a corner of square loop is

$$Q = -\frac{\mu_0}{4\pi}\ln(2)\int_4^0 dI$$

$$= -\frac{\mu_0}{4\pi}\ln(2)(0-4)$$

$$= \frac{4 \times 4\pi \times 10^{-7}}{4\pi}\ln(2)$$

$$= 2.77 \times 10^{-7}\text{ C} = 277 \text{ nC}$$

SOL 6.2.11 Correct answer is 44.9 .

Since the radius of small circular loop is negligible in comparison to the radius of the large loop. So, the flux density through the small loop will be constant and equal to the flux on the axis of the loops.

So, $$\boldsymbol{B} = \frac{\mu_0 I}{2}\frac{R^2}{(z^2 + R^2)^{3/2}}\boldsymbol{a}_z$$

where $\quad R \rightarrow$ radius of large loop $= 5$ m

$\quad z \rightarrow$ distance between the loops $= 12$ m

$$\boldsymbol{B} = \frac{\mu_0 \times 2}{2} \times \frac{(5)^2}{[(12)^2 + (5)^2]^{3/2}}\boldsymbol{a}_z = \frac{25\mu_0}{(13)^3}\boldsymbol{a}_z$$

Therefore, the total flux passing through the small loop is

$$\Phi = \int \boldsymbol{B} \cdot d\boldsymbol{S} = \frac{25\mu_0}{(13)^3} \times \pi r^2 \qquad \text{wherer is radius of small circular loop.}$$

$$= \frac{25 \times 4\pi \times 10^{-7}}{(13)^3} \times \pi(10^{-3})^2 = 44.9 \text{ fWb}$$

SOL 6.2.12 Correct answer is 2.7 .

Electric field in any medium is equal to the voltage drop per unit length.

i.e.
$$E = \frac{V}{d}$$

where $V \rightarrow$ potential difference between two points.

$d \rightarrow$ distance between the two points.

The voltage difference between any two points in the medium is
$$V = V_0 \cos 2\pi ft$$

So the conduction current density in the medium is given as
$$J_c = \sigma E \qquad (\sigma \rightarrow \text{conductivity of the medium})$$
$$= \frac{E}{\rho} \qquad (\rho \rightarrow \text{resistivity of the medium})$$
$$= \frac{V}{\rho d} = \frac{V_0 \cos 2\pi ft}{\rho d} \qquad (V = V_0 \cos 2\pi ft)$$

or,
$$|J_c| = \frac{V_0}{\rho d}$$

and displacement current density in the medium is given as
$$J_d = \frac{\partial D}{\partial t} = \varepsilon \frac{\partial E}{\partial t} = \varepsilon \frac{\partial}{\partial t}\left[\frac{V_0 \cos(2\pi ft)}{d}\right] \qquad (V = V_0 \cos 2\pi ft)$$
$$= \frac{\varepsilon V_0}{d}[-2\pi f t \sin(2\pi ft)]$$

or,
$$|J_d| = \frac{2\pi f \varepsilon V_0}{d}$$

Therefore, the ratio of amplitudes of conduction current and displacement current in the medium is
$$\frac{|I_c|}{|I_d|} = \frac{|J_C|}{|J_d|} = \frac{(V_0)/(\rho d)}{(d)/(2\pi f \varepsilon V_0)} = \frac{1}{2\pi f \varepsilon \rho}$$
$$= \frac{1}{2\pi \times (1.6 \times 10^8) \times (54 \times 8.85 \times 10^{-12}) \times 0.77}$$
$$= 2.7$$

SOL 6.2.13 Correct answer is 8.

Let the test charge be q coulomb So the force presence of experienced by the test charge in the presence of magnetic field is
$$\boldsymbol{F} = q(\boldsymbol{v} \times \boldsymbol{B})$$

and the force experienced can be written in terms of the electric field intensity as
$$\boldsymbol{F} = q\boldsymbol{E}$$

Where $\boldsymbol{E}$ is field viewed by observer moving with test charge.

Putting it in Eq. (i)
$$q\boldsymbol{E} = q(\boldsymbol{v} \times \boldsymbol{B})$$
$$\boldsymbol{E} = (\omega \rho \boldsymbol{a}_\phi) \times (2\boldsymbol{a}_z)$$

where ω is angular velocity and ρ is radius of circular loop.
$$= (2)(2)(2)\boldsymbol{a}_\rho = 8\boldsymbol{a}_\rho \text{ V/m}$$

SOL 6.2.14 Correct answer is -0.35 .

As shown in figure the bar is sliding away from origin.

Now when the bar is located at a distance dx from the voltmeter, then, the vector area of the loop formed by rail and the bar is
$$d\boldsymbol{S} = (20 \times 10^{-2})(dx)\,\boldsymbol{a}_z$$

So, the total magnetic flux passing through the loop is
$$\Phi = \int_S \boldsymbol{B} \cdot d\boldsymbol{S}$$
$$= \int_0^x (8x^2 \boldsymbol{a}_z)(20 \times 10^{-2} dx \boldsymbol{a}_z)$$
$$= \frac{1.6[t(1+0.4t^2)]^3}{3}$$

Therefore, the induced e.m.f. in the loop is given as
$$V_{emf} = -\frac{d\Phi}{dt} = -\frac{1.6}{3} \times 3(t + 0.4t^3)^2 \times (1 + 1.2t^2)$$
$$V_{emf} = -1.6[(0.4)+(0.4)^4]^2 \times [1 + (1.2)(0.4)^2] \qquad (t = 0.4 \text{ sec})$$
$$= -0.35 \text{ volt}$$

Since the voltmeter is connected in same manner as the direction of induced emf (determined by Lenz's law).

So the voltmeter reading will be
$$V = V_{emf} = -0.35 \text{ volt}$$

SOL 6.2.15 Correct answer is -23.4 .

Since the position of bar is give as
$$x = t(1 + 0.4t^2)$$

So for the position $x = 12$ cm we have
$$0.12 = t(1 + 0.4t^2)$$

or,
$$t = 0.1193 \text{ sec}$$

As calculated in previous question, the induced emf in the loop at a particular time t is
$$V_{emf} = -(1.6)[t + 0.4t^3]^2(1 + 1.2t^2)$$

So, at $t = 0.1193$ sec,
$$V_{emf} = -1.6[(0.1193) + 0.4(0.1193)^3]^2[1 + (1.2)(0.1193)^2]$$
$$= -0.02344 = -23.4 \text{ mV}$$

Since the voltmeter is connected in same manner as the direction of induced emf as determined by Lenz's law. Therefore, the voltmeter reading at $x = 12$ cm will be
$$V = V_{emf} = -23.4 \text{ mvolt}$$

SOL 6.2.16 Correct answer is ± 600.

Given the magnetic field intensity in the medium is
$$\boldsymbol{H} = \cos(10^{10} t - bx)\boldsymbol{a}_z \text{ A/m}$$

Now from the Maxwell's equation, we have
$$\nabla \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t}$$

or,
$$\frac{\partial \boldsymbol{D}}{\partial t} = -\frac{\partial H_z}{\partial x}\boldsymbol{a}_y = -b\sin(10^{10}t - bx)\boldsymbol{a}_y$$
$$\boldsymbol{D} = \int -b\sin(10^{10}t - bx)dt + C \qquad \text{where } C \text{ is a constant.}$$

Since no D.C. field is present in the medium so, we get $C = 0$ and therefore.
$$\boldsymbol{D} = \frac{b}{10^{10}}\cos(10^{10}t - bx)\boldsymbol{a}_y \text{ C/m}^2$$

and the electric field intensity in the medium is given as
$$\boldsymbol{E} = \frac{\boldsymbol{D}}{\varepsilon} = \frac{b}{0.12 \times 10^{-9} \times 10^{10}}\cos(10^{10t} - bx)\boldsymbol{a}_y \qquad (\varepsilon = 0.12 \text{ nF/m})$$

Again From the Maxwell's equation

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

or,
$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \left[\frac{b}{1.2}\cos(10^{10t} - bx)\boldsymbol{a}_y\right]$$

$$= -\frac{b^2}{1.2}\sin(10^{10t} - bx)\boldsymbol{a}_z$$

So, the magnetic flux density in the medium is

$$\boldsymbol{B} = -\int \frac{b^2}{1.2}\sin(10^{10t} - bx)\boldsymbol{a}_z\,dt$$

$$= \frac{b^2}{(1.2)\times 10^{10}}\cos(10^{10}t - bx)\boldsymbol{a}_z \qquad (1)$$

We can also determine the value of magnetic flux density as :

$$\boldsymbol{B} = \mu \boldsymbol{H}$$

$$= (3 \times 10^{-5})\cos(10^{10}t - bx)\boldsymbol{a}_z \qquad (2)$$

Comparing the results of equation (1) and (2) we get,

$$\frac{b^2}{(1.2)\times 10^{10}} = 3 \times 10^{-5}$$

$$b^2 = 3.6 \times 10^5$$

$$b = \pm\, 600 \text{ rad/m}$$

SOL 6.2.17 Correct answer is 54.414 .

Given the electric field in time domain as

$$\boldsymbol{E} = 5\sin(10\pi y)\cos(6\pi \times 10^9 - bx)\boldsymbol{a}_z$$

Comparing it with the general equation for electric field intensity given as

$$\boldsymbol{E} = E_0\cos(\omega t - \beta x)\boldsymbol{a}_z$$

We get, $\omega = 6\pi \times 10^9$

Now in phasor form, the electric field intensity is

$$\boldsymbol{E}_s = 5\sin(10\pi y)e^{-jbx}\,\boldsymbol{a}_z \qquad (1)$$

From Maxwell's equation we get the magnetic field intensity as

$$\boldsymbol{H}_s = -\frac{1}{j\omega\mu_0}(\nabla \times \boldsymbol{E}_s) = \frac{j}{\omega\mu_0}\left[\frac{\partial E_{sz}}{\partial y}\boldsymbol{a}_x - \frac{\partial E_{sz}}{\partial x}\boldsymbol{a}_y\right]$$

$$= \frac{j}{\omega\mu_0}\left[50\pi\cos(10\pi y)e^{-jbx}\,\boldsymbol{a}_x + j5b\sin(10\pi y)\boldsymbol{a}_y\right]e^{-jbx}$$

Again from Maxwell's equation we have the electric field intensity as

$$\boldsymbol{E}_s = \frac{1}{j\omega\varepsilon_0}(\nabla \times \boldsymbol{H}_s) = \frac{1}{j\omega\varepsilon_0}\left[\frac{\partial H_{sy}}{\partial x} - \frac{\partial H_{sx}}{\partial y}\right]\boldsymbol{a}_z$$

$$= \frac{1}{\omega^2\mu_0\varepsilon_0}\left[(j5b)(-jb)\sin(10\pi y)\,e^{-jbx} + (50\pi)(10\pi)\sin(10\pi y)\,e^{-jbx}\right]\boldsymbol{a}_z$$

$$= \frac{1}{\omega^2\mu_0\varepsilon_0}\left[5b^2 + 500\pi^2\right]\sin 10\pi y e^{-jbx}\,\boldsymbol{a}_z$$

Comparing this result with equation (1) we get

$$\frac{1}{\omega^2\mu_0\varepsilon_0}(5b^2 + 500\pi^2) = 5$$

or,
$$b^2 + 100\pi^2 = \omega^2\mu_0\varepsilon_0$$

$$b^2 + 100\pi^2 = (6\pi \times 10^9)^2 \times \frac{1}{(3\times 10^8)^2} \quad \left(\omega = 6\pi \times 10^9, \sqrt{\mu_0\varepsilon_0} = \frac{1}{c}\right)$$

$$b^2 + 100\pi^2 = 400\pi^2$$

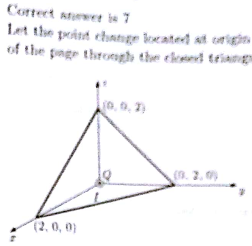
$$b^2 = 300\pi^2$$

So,
$$b = \pm\sqrt{300}\,\pi \text{ rad/m}$$

$$|b| = \sqrt{300}\,\pi = 54.414 \text{ rad/m}$$

SOL 6.2.18 Correct answer is 7

Let the point charge located at origin be Q and the current I is flowing out of the page through the closed triangular path as shown in the figure.



As the current I flows away from the point charge along the wire, the net charge at origin will change with increasing time and given as

$$\frac{dQ}{dt} = -I$$

So the electric field intensity will also vary through the surface and for the varying field circulation of magnetic field intensity around the triangular loop is defined as

$$\oint \boldsymbol{H}\cdot d\boldsymbol{l} = [I_d]_{enc} + [I_c]_{enc}$$

where $[I_c]_{enc}$ is the actual flow of charge called enclosed conduction current and $[I_d]_{enc}$ is the current due to the varying field called enclosed displacement current which is given as

$$[I_d]_{enc} = \frac{d}{dt}\int_S(\varepsilon_0\boldsymbol{E})\cdot d\boldsymbol{S} = \frac{d}{dt}\int_S \boldsymbol{D}\cdot d\boldsymbol{S} \qquad (1)$$

From symmetry the total electric flux passing through the triangular surface is

$$\int_S \boldsymbol{D}\cdot d\boldsymbol{S} = \frac{Q}{8}$$

So,
$$[I_d]_{enc} = \frac{d}{dt}\left(\frac{Q}{8}\right) = \frac{1}{8}\frac{dQ}{dt} = -\frac{I}{8} \qquad \text{(from equation (1))}$$

whereas
$$[I_c]_{enc} = I$$

So, the net circulation of the magnetic field intensity around the closed triangular loop is

$$\oint_C \boldsymbol{H}\cdot d\boldsymbol{l} = [I_d]_{enc} + [I_c]_{enc}$$

$$= -\frac{I}{8} + I = \frac{7}{8}(8) = 7\text{ A} \qquad (I = 8\text{ A})$$

SOL 6.2.19 Correct answer is 21.33 .

As calculated in previous question the maximum induced voltage in the rotating loop is given as

$$|V_{emf}| = B_0 S\omega$$

From the given data, we have

$$B_0 = 0.25 \text{ Wb/m}^2$$

$$S = 64 \text{ cm}^2 = 64 \times 10^{-4} \text{ m}^2$$

and $\omega = 60 \times 2\pi = 377$ rad/sec (In one revolution 2π radian is covered)

So, the r.m.s. value of the induced voltage is

$$[V_{emf}]_{r.m.s} = \frac{1}{\sqrt{2}}|V_{emf}| = \frac{1}{\sqrt{2}}B_0 S\omega$$

$$= \frac{1}{\sqrt{2}}(0.25 \times 64 \times 10^{-4} \times 377)$$

$$= 0.4265$$

Since the loop has 50 turns so net induced voltage will be 50 times the calculated value.

i.e. $$[V_{emf}]_{r.m.s} = 50 \times (0.4265)$$

$$= 21.33 \text{ volt}$$

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SOLUTIONS 6.3

SOL 6.3.1 Option (D) is correct.

SOL 6.3.2 Option (B) is correct.

The line integral of magnetic field intensity along a closed loop is equal to the current enclosed by it.

i.e. $$\int \boldsymbol{H} \cdot d\boldsymbol{l} = I_{enc}$$

So, for the constant current, magnetic field intensity will be constant i.e. magnetostatic field is caused by steady currents.

SOL 6.3.3 Option (A) is correct.

From Faraday's law the electric field intensity in a time varying field is defined as

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$ where $\boldsymbol{B}$ is magnetic flux density in the EM field.

and since the magnetic flux density is equal to the curl of magnetic vector potential

i.e. $$\boldsymbol{B} = \nabla \times \boldsymbol{A}$$

So, putting it in equation (1), we get

$$\nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t}(\nabla \times \boldsymbol{A})$$

or $$\nabla \times \boldsymbol{E} = \nabla \times \left(-\frac{\partial}{\partial t}\boldsymbol{A}\right)$$

Therefore, $$\boldsymbol{E} = -\frac{\partial \boldsymbol{A}}{\partial t}$$

SOL 6.3.4 Option (B) is correct.

Since total magnetic flux through a surface S is defined as

$$\Phi = \int_S \boldsymbol{B} \cdot d\boldsymbol{S}$$

From Maxwell's equation it is known that ~~curl~~ div. of magnetic flux density is zero

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\int_S \boldsymbol{B} \cdot d\boldsymbol{S} = \int_v (\nabla \cdot \boldsymbol{B})\, dv = 0$$ (Stokes Theorem)

Thus, net outwards flux will be zero for a closed surface.

SOL 6.3.5 Option (B) is correct.

From the integral form of Faraday's law we have the relation between the electric field intensity and net magnetic flux through a closed loop as

$$\oint \boldsymbol{E} \cdot d\boldsymbol{l} = -\frac{d\Phi}{dt}$$

Since electric field intensity is zero ($E = 0$) inside the conducting loop. So, the rate of change in net magnetic flux through the closed loop is

$$\frac{d\Phi}{dt} = 0$$

i.e. Φ is constant and doesn't vary with time.

SOL 6.3.6 Option (C) is correct.
A superconductor material carries zero magnetic field and zero electric field inside it.

i.e. $B = 0$ and $E = 0$

Now from Ampere-Maxwell equation we have the relation between the magnetic flux density and electric field intensity as

$$\nabla \times B = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

So, $J = 0$ $\qquad (B = 0,\ E = 0)$

Since the net current density inside the superconductor is zero so all the current must be confined at the surface of the wire.

SOL 6.3.7 Option (C) is correct.
According to Lenz's law the induced current I in a loop flows such as to produce a magnetic field that opposes the change in $B(t)$.
Now the configuration shown in option (A) and (B) for increasing magnetic flux B_i, the change in flux is in same direction to B_i as well as the current I flowing in the loop produces magnetic field in the same direction so it does not follow the Lenz's law.
For the configuration shown in option (D), as the flux B_d is decreasing with time so the change in flux is in opposite direction to B_d as well as the current I flowing in the loop produces the magnetic field in opposite direction so it also does not follow the Lenz's law.
For the configuration shown in option (C), the flux density B_d is decreasing with time so the change in flux is in opposite direction to B_d but the current I flowing in the loop produces magnetic field in the same direction to B_d (opposite to the direction of change in flux density). Therefore this is the correct configuration.

SOL 6.3.8 Option (C) is correct.
Induced emf in a conducting loop is given by

$$V_{emf} = -\frac{d\Phi}{dt} \qquad \text{where } \Phi \text{ is total magnetic flux passing}$$

through the loop.

Since, the magnetic field is non-uniform so the change in flux will be caused by it and the induced emf due to it is called transformer emf.
Again the field is in a_y direction and the loop is rotating about z-axis so flux through the loop will also vary due to the motion of the loop. This causes the emf which is called motion emf. Thus, total induced voltage in the rotating loop is caused by the combination of both the transformer and motion emf.

SOL 6.3.9 Option (B) is correct.

SOL 6.3.10 Option (C) is correct.

SOL 6.3.11 Option (B) is correct.

SOL 6.3.12 Option (D) is correct.

SOL 6.3.13 Option (A) is correct.

SOL 6.3.14 Option (A) is correct.

SOL 6.3.15 Option (C) is correct.

SOL 6.3.16 Option (B) is correct.

SOL 6.3.17 Option (B) is correct.

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SOLUTIONS 6.4

SOL 6.4.1 Option (C) is correct.
Given, the magnetic flux density in air as

$$\boldsymbol{B} = B_0\left(\frac{x}{x^2+y^2}a_y - \frac{y}{x^2+y^2}a_x\right) \qquad \ldots(1)$$

Now, we transform the expression in cylindrical system, substituting

$$x = r\cos\phi \quad \text{and} \quad y = r\sin\phi$$

$$a_x = \cos\phi a_r - \sin\phi a_\phi$$

and $$a_y = \sin\phi a_r + \cos\phi a_\phi$$

So, we get $$\boldsymbol{B} = B_0 \boldsymbol{a}_\phi$$

Therefore, the magnetic field intensity in air is given as

$$\boldsymbol{H} = \frac{\boldsymbol{B}}{\mu_0} = \frac{B_0 \boldsymbol{a}_\phi}{\mu_0}, \text{ which is constant}$$

So, the current density of the field is

$$\boldsymbol{J} = \nabla \times \boldsymbol{H} = 0 \qquad \text{(since } H \text{ is constant)}$$

SOL 6.4.2 Option (D) is correct.
Maxwell equations for an EM wave is given as

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \cdot \boldsymbol{E} = \frac{\rho_v}{\varepsilon}$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t} + \boldsymbol{J}$$

So, for static electric magnetic fields

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \cdot \boldsymbol{E} = \rho_v/\varepsilon$$

$$\nabla \times \boldsymbol{E} = 0 \qquad \left(\frac{\partial \boldsymbol{B}}{\partial t} = 0\right)$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} \qquad \left(\frac{\partial \boldsymbol{D}}{\partial t} = 0\right)$$

SOL 6.4.3 Option (D) is correct.

$$\nabla \times \boldsymbol{H} = J + \frac{\partial \boldsymbol{D}}{\partial t} \qquad \text{Maxwell Equations}$$

$$\iint_S (\nabla \times \boldsymbol{H}) \cdot d\boldsymbol{S} = \iint_S \left(J + \frac{\partial \boldsymbol{D}}{\partial t}\right) \cdot d\boldsymbol{S} \qquad \text{Integral form}$$

$$\oint \boldsymbol{H} \cdot d\boldsymbol{l} = \iint_S \left(J + \frac{\partial \boldsymbol{D}}{\partial t}\right) \cdot d\boldsymbol{S} \qquad \text{Stokes Theorem}$$

SOL 6.4.4 Option (C) is correct.
From Maxwells equations we have

$$\nabla \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t} + \boldsymbol{J}$$

Thus, $\nabla \times \boldsymbol{H}$ has unit of current density $\boldsymbol{J}$ (i.e., A/m^2)

SOL 6.4.5 Option (A) is correct.
This equation is based on Ampere's law as from Ampere's circuital law we have

$$\oint_l \boldsymbol{H} \cdot d\boldsymbol{l} = I_{enclosed}$$

or, $$\oint_l \boldsymbol{H} \cdot d\boldsymbol{l} = \int_S \boldsymbol{J} \cdot d\boldsymbol{S}$$

Applying Stoke's theorem we get

$$\int_S (\nabla \times \boldsymbol{H}) \cdot d\boldsymbol{S} = \int_S \boldsymbol{J} \cdot d\boldsymbol{S}$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{J}$$

Then, it is modified using continuity equation as

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$$

SOL 6.4.6 Option (D) is correct.
When a moving circuit is put in a time varying magnetic field induced emf have two components. One due to time variation of magnetic flux density $\boldsymbol{B}$ and other due to the motion of circuit in the field.

SOL 6.4.7 Option (C) is correct.
From maxwell equation we have

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$$

The term $\frac{\partial \boldsymbol{D}}{\partial t}$ defines displacement current.

SOL 6.4.8 Option (C) is correct.
Emf induced in a loop carrying a time varying magnetic flux Φ is defined as

$$V_{emf} = -\frac{d\Phi}{dt}$$

$$9 = -\frac{d}{dt}\left(\frac{1}{3}\lambda t^3\right)$$

$$9 = -\lambda t^2$$

at time, $t = 3$ s, we have

$$9 = -\lambda(3)^2$$

$$\lambda = -1 \text{ Wb/s}^2$$

SOL 6.4.9 Option (B) is correct.
According to Lenz's law the induced emf (or induced current) in a loop flows such as to produce a magnetic field that opposed the change in $\boldsymbol{B}$. The direction of the magnetic field produced by the current is determined by right hand rule.
Now, in figure (1), $\boldsymbol{B}$ directed upwarded increases with time where as the field produced by current I is downward so, it obey's the Lenz's law.
In figure (2), $\boldsymbol{B}$ directed upward is decreasing with time whereas the field produced by current I is downwards (i.e. additive to the change in $\boldsymbol{B}$) so, it doesn't obey Lenz's law.
In figure (3), $\boldsymbol{B}$ directed upward is decreasing with time where as current I produces the field directed upwards (i.e. opposite to the change in $\boldsymbol{B}$) So, it also obeys Lenz's law.
In figure (4), $\boldsymbol{B}$ directed upward is increasing with time whereas current I produces field directed upward (i.e. additive to the change in $\boldsymbol{B}$) So, it

doesn't obey Lenz's law.
Thus, the configuration 1 and 3 are correct.

SOL 6.4.10 Option (C) is correct.
Faraday's law states that for time varying field,

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

Since, the curl of gradient of a scalar function is always zero
i.e. $\nabla \times (\nabla V) = 0$
So, the expression for the field, $\boldsymbol{E} = -\nabla V$ must include some other terms is

$$\boldsymbol{E} = -\nabla V - \frac{\partial \boldsymbol{A}}{\partial t}$$

i.e. A is true but R is false.

SOL 6.4.11 Option (B) is correct.
Faraday develops the concept of time varying electric field producing a magnetic field. The law he gave related to the theory is known as Faraday's law.

SOL 6.4.12 Option (D) is correct.
Given, the area of loop

$$S = 5\,\mathrm{m}^2$$

Rate of change of flux density,

$$\frac{\partial B}{\partial t} = 2\,\mathrm{Wb/m^2/S}$$

So, the emf in the loop is

$$V_{emf} = -\frac{\partial}{\partial t}\int \boldsymbol{B}\cdot d\boldsymbol{S}$$
$$= (5)(-2) = -10\,\mathrm{V}$$

SOL 6.4.13 Option (D) is correct.
The modified Maxwell's differential equation.

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$$

This equation is derived from Ampere's circuital law which is given as

$$\oint \boldsymbol{H}\cdot dl = I_{enc}$$
$$\oint (\nabla \times \boldsymbol{H})\cdot d\boldsymbol{S} = \int \boldsymbol{J} d\boldsymbol{S}$$
$$\nabla \times \boldsymbol{H} = \boldsymbol{J}$$

SOL 6.4.14 Option (B) is correct.
Electric potential of an isolated sphere is defined as

$$C = 4\pi\varepsilon_0 a \qquad \text{(free space)}$$

The Maxwell's equation in phasor form is written as

$$\nabla \times \boldsymbol{H} = j\omega\varepsilon \boldsymbol{E} + \sigma \boldsymbol{E} = j\omega\varepsilon \boldsymbol{E} + \boldsymbol{J} \qquad (\boldsymbol{J} = \sigma \boldsymbol{E})$$

So A and R both are true individually but R is not the correct explanation of A.

SOL 6.4.15 Option (A) is correct.
If a coil is placed in a time varying magnetic field then the e.m.f. will induce in coil. So here in both the coil e.m.f. will be induced.

SOL 6.4.16 Option (B) is correct.
Both the statements are individually correct but R is not explanation of A.

SOL 6.4.17 Option (B) is correct.

Ampere's law	$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$	(a → 3)
Faraday' law	$\nabla \times \boldsymbol{E} = \frac{\partial \boldsymbol{B}}{\partial t}$	(b → 4)
Gauss law	$\nabla \cdot \boldsymbol{D} = \rho_v$	(c → 1)
Current continuity	$\nabla \cdot \boldsymbol{J} = -\frac{\partial \rho}{\partial t}$	(d → 2)

SOL 6.4.18 Option (B) is correct.
Since, the magnetic field perpendicular to the plane of the ring is decreasing with time so, according to Faraday's law emf induced in both the ring is

$$V_{emf} = -\frac{\partial}{\partial t}\int \boldsymbol{B}\cdot d\boldsymbol{S}$$

Therefore, emf will be induced in both the rings.

SOL 6.4.19 Option (A) is correct.
The Basic idea of radiation is given by the two Maxwell's equation

$$\nabla \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t}$$
$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

SOL 6.4.20 Option (B) is correct.
The correct Maxwell's equation are

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t} \qquad \nabla \cdot \boldsymbol{D} = \rho$$
$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \qquad \nabla \cdot \boldsymbol{B} = 0$$

SOL 6.4.21 Option (B) is correct.
In List I

a. $\oint \boldsymbol{B}\cdot d\boldsymbol{S} = 0$

The surface integral of magnetic flux density over the closed surface is zero or in other words, net outward magnetic flux through any closed surface is zero. (a → 4)

b. $\oint \boldsymbol{D}\cdot d\boldsymbol{S} = \int_v \rho_v dv$

Total outward electric flux through any closed surface is equal to the charge enclosed in the region. (b → 3)

c. $\int \boldsymbol{E}\cdot dl = -\int \frac{\partial \boldsymbol{B}}{\partial t} d\boldsymbol{S}$

i.e. The line integral of the electric field intensity around a closed path is equal to the surface integral of the time derivative of magnetic flux density (c → 2)

d. $\oint \boldsymbol{H}\cdot d\boldsymbol{S} = \int \left(\frac{\partial \boldsymbol{D}}{\partial t} + \boldsymbol{J}\right) da$

i.e. The line integral of magnetic field intensity around a closed path is equal to the surface integral of sum of the current density and time derivative of electric flux density. (d → 1)

SOL 6.4.22 Option (D) is correct.

The continuity equation is given as

$$\nabla \cdot \boldsymbol{J} = -\rho_v$$

i.e. it relates current density ($\boldsymbol{J}$) and charge density ρ_v.

SOL 6.4.23 Option (C) is correct.

Given Maxwell's equation is

$$\nabla \times \boldsymbol{H} = J_c + \frac{\partial \boldsymbol{D}}{\partial t}$$

For free space, conductivity, $\sigma = 0$ and so,

$$J_c = \sigma \boldsymbol{E} = 0$$

Therefore, we have the generalized equation

$$\nabla \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t}$$

SOL 6.4.24 Option (A) is correct.

Given the magnetic field intensity,

$$\boldsymbol{H} = 3\boldsymbol{a}_x + 7y\boldsymbol{a}_y + 2x\boldsymbol{a}_z$$

So from Ampere's circuital law we have

$$\boldsymbol{J} = \nabla \times \boldsymbol{H}$$

$$= \begin{vmatrix} \boldsymbol{a}_x & \boldsymbol{a}_y & \boldsymbol{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3 & 7y & 2x \end{vmatrix}$$

$$= \boldsymbol{a}_x(0) - \boldsymbol{a}_y(2-0) + \boldsymbol{a}_z(0) = -2\boldsymbol{a}_y$$

SOL 6.4.25 Option (A) is correct.

The emf in the loop will be induced due to motion of the loop as well as the variation in magnetic field given as

$$V_{emf} = -\int \frac{\partial \boldsymbol{B}}{\partial t} d\boldsymbol{S} + \oint (\boldsymbol{v} \times \boldsymbol{B}) d\boldsymbol{l}$$

So, the frequencies for the induced e.m.f. in the loop is ω_1 and ω_2.

SOL 6.4.26 Option (B) is correct.

$\boldsymbol{F} = Q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$ is Lorentz force equation.

SOL 6.4.27 Option (A) is correct.

All of the given expressions are Maxwell's equation.

SOL 6.4.28 Option (B) is correct.

Poission's equation for an electric field is given as

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon}$$

where, V is the electric potential at the point and ρ_v is the volume charge density in the region. So, for $\rho_v = 0$ we get,

$$\nabla^2 V = 0$$

Which is Laplacian equation.

SOL 6.4.29 Option (A) is correct.

The direction of magnetic flux due to the current 'i' in the conductor is determined by right hand rule. So, we get the flux through A is pointing into the paper while the flux through B is pointing out of the paper.

According to Lenz's law the induced e.m.f. opposes the flux that causes it.

So again by using right hand rule we get the direction of induced e.m.f. is anticlockwise in A and clockwise in B.

SOL 6.4.30 Option (D) is correct.

$$\nabla^2 \boldsymbol{A} = -\mu_0 \boldsymbol{J}$$

This is the wave equation for static electromagnetic field.

i.e. It is not Maxwell's equation.

SOL 6.4.31 Option (B) is correct.

Continuity equation	$\nabla \times \boldsymbol{J} = -\frac{\partial \rho_v}{\partial t}$	(a → 4)
Ampere's law	$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$	(b → 1)
Displacement current	$\boldsymbol{J} = \frac{\partial \boldsymbol{D}}{\partial t}$	(c → 2)
Faraday' law	$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$	(d → 3)

SOL 6.4.32 Option (B) is correct.

Induced emf in a coil of N turns is defined as

$$V_{emf} = -N\frac{d\Phi}{dt}$$

where Φ is flux linking the coil. So, we get

$$V_{emf} = -100\frac{d}{dt}(t^3 - 2t)$$

$$= -100(3t^2 - 2)$$

$$= -100(3(2)^2 - 2) = -1000\,\text{mV} \qquad \text{(at } t = 2\,\text{s)}$$

$$= -1\,\text{V}$$

SOL 6.4.33 Option (B) is correct.

A static electric field in a charge free region is defined as

$$\nabla \cdot \boldsymbol{E} = 0 \qquad (a \to 4)$$

and $\nabla \times \boldsymbol{E} = 0$

A static electric field in a charged region have

$$\nabla \cdot \boldsymbol{E} = \frac{\rho_v}{\varepsilon} \neq 0 \qquad (b \to 2)$$

and $\nabla \times \boldsymbol{E} = 0$

A steady magnetic field in a current carrying conductor have

$$\nabla \cdot \boldsymbol{B} = 0 \qquad (c \to 1)$$

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} \neq 0$$

A time varying electric field in a charged medium with time varying magnetic field have

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \neq 0 \qquad (d \to 3)$$

$$\nabla \cdot \boldsymbol{E} = \frac{\rho_v}{\varepsilon} \neq 0$$

SOL 6.4.34 Option (C) is correct.

$$V = -\frac{d\Phi_m}{dt}$$

It is Faraday's law that states that the change in flux through any loop induces e.m.f. in the loop.

SOL 6.4.35 Option (B) is correct.

From stokes theorem, we have

$$\int(\nabla \times \boldsymbol{E}) \cdot d\boldsymbol{S} = \oint \boldsymbol{E} \cdot d\boldsymbol{l} \qquad (1)$$

Given, the Maxwell's equation

$$\nabla \times \boldsymbol{E} = -(\partial \boldsymbol{B}/\partial t)$$

Putting this expression in equation (1) we get,

$$\oint \boldsymbol{E} \cdot d\boldsymbol{l} = -\frac{\partial}{\partial t}\int_s \boldsymbol{B} \cdot d\boldsymbol{S}$$

SOL 6.4.36 Option (D) is correct.

Since, the flux linking through both the coil is varying with time so, emf are induced in both the coils.

Since, the loop 2 is split so, no current flows in it and so joule heating does not occur in coil 2 while the joule heating occurs in closed loop 1 as current flows in it.

Therefore, only statement 2 is correct.

SOL 6.4.37 Option (C) is correct.

The electric field intensity is

$$\boldsymbol{E} = \boldsymbol{E}_0 e^{j\omega t} \qquad \text{where } \boldsymbol{E}_0 \text{ is independent of time}$$

So, from Maxwell's equation we have

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\varepsilon\,\partial \boldsymbol{E}}{\partial t}$$

$$= \sigma \boldsymbol{E} + \varepsilon(j\omega)\boldsymbol{E}_0 e^{j\omega t} = \sigma \boldsymbol{E} + j\omega\varepsilon \boldsymbol{E}$$

SOL 6.4.38 Option (C) is correct.

Equation (1) and (3) are not the Maxwell's equation.

SOL 6.4.39 Option (A) is correct.

From the Maxwell's equation for a static field (DC) we have

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J}$$

$$\nabla \times (\nabla \times \boldsymbol{A}) = \mu_0 \boldsymbol{J}$$

$$\nabla(\nabla \cdot \boldsymbol{A}) - \nabla^2 \boldsymbol{A} = \mu_0 \boldsymbol{J}$$

For static field (DC), $\nabla \cdot \boldsymbol{A} = 0$

therefore we have, $\nabla^2 \boldsymbol{A} = -\mu_0 \boldsymbol{J}$

So, both A and R are true and R is correct explanation of A.

SOL 6.4.40 Option (A) is correct.

For a static field, Maxwells equation is defined as

$$\nabla \times \boldsymbol{H} = \boldsymbol{J}$$

and since divergence of the curl is zero

i.e. $\nabla \cdot (\nabla \times \boldsymbol{H}) = 0$

$$\nabla \cdot \boldsymbol{J} = 0$$

but in the time varying field, from continuity equation (conservation of charges)

$$\nabla \cdot \boldsymbol{J} = -\frac{\partial \rho_v}{\partial t} \neq 0$$

So, an additional term is included in the Maxwell's equation.

i.e. $\nabla \times \boldsymbol{H} = \boldsymbol{J} + \dfrac{\partial \boldsymbol{D}}{\partial t}$

where $\dfrac{\partial \boldsymbol{D}}{\partial t}$ is displacement current density which is a necessary term.

Therefore A and R both are true and R is correct explanation of A.

SOL 6.4.41 Option (C) is correct.

Since, the circular loop is rotating about the y-axis as a diameter and the flux lines is directed in $\boldsymbol{a}_x$ direction. So, due to rotation magnetic flux changes and as the flux density is function of time so, the magnetic flux also varies w.r.t time and therefore the induced e.m.f. in the loop is due to a combination of transformer and motional e.m.f. both.

SOL 6.4.42 Option (A) is correct.

For any loop to have an induced e.m.f., magnetic flux lines must link with the coil.

Observing all the given figures we conclude that loop C_1 and C_2 carries the flux lines through it and so both the loop will have an induced e.m.f.

SOL 6.4.43 Option (C) is correct.

Gauss's law	$\nabla \cdot \boldsymbol{D} = \rho$	(a → 1)
Ampere's law	$\nabla \times \boldsymbol{H} = \boldsymbol{J}_c + \dfrac{\partial \boldsymbol{D}}{\partial t}$	(b → 5)
Faraday's law	$\nabla \times \boldsymbol{E} = -\dfrac{\partial \boldsymbol{B}}{\partial t}$	(c → 2)
Poynting vector	$\boldsymbol{\mathcal{P}} = \boldsymbol{E} \times \boldsymbol{H}$	(d → 3)

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