Contents

Section outline

KMap technique

- Aim is to have an optimal 2-level SOP (or POS) form
- Algebraic operation used repeatedly on FPs pz and $p\overline{z}$ where p is contained in FPs pz and $p\overline{z}$
- $pz + p\overline{z} = p(z+\overline{z}) = p$
- FPs pz and $p\overline{z}$ are *adjacent*
- By absorbtion [$p = p + p$], FPs are not exclusive
- For convenience minterms are placed on a Karnaugh map where adjacent minterms get placed in adjacent cells
- Enables easier identification of adjacent FPs for simplification

f

a,b \ c,d	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

$$\underbrace{\overline{a}\,\overline{b}\,\overline{c}\,\overline{d}}_{0000\leftrightarrow 0} + \underbrace{\overline{a}\,b\,c\,d}_{0111\leftrightarrow 7} + \underbrace{a\,\overline{b}\,\overline{c}\,d}_{1001\leftrightarrow 9} +$$

$$\underbrace{a\,b\,\overline{c}\,\overline{d}}_{1100\leftrightarrow 12} + \underbrace{a\,b\,c\,d}_{1111\leftrightarrow 15}$$

$$f = \sum_m (0, 7, 9, 12, 15)$$

$$f = \underbrace{\bar{a}\,\bar{b}\,\bar{c}\,\bar{d}}_{0000\leftrightarrow 0} + \underbrace{\bar{a}\,b\,c\,d}_{0111\leftrightarrow 7} + \underbrace{a\,\bar{b}\,\bar{c}\,d}_{1001\leftrightarrow 9} + \underbrace{a\,b\,\bar{c}\,\bar{d}}_{1100\leftrightarrow 12} + \underbrace{a\,b\,c\,d}_{1111\leftrightarrow 15}$$

f — a,b \ c,d	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

$$f = \underline{b\,c\,d} + \underline{a\,b\,\bar{c}\,\bar{d}} + \underline{a\,\bar{b}\,\bar{c}\,d} + \underline{\bar{a}\,\bar{b}\,\bar{c}\,\bar{d}}$$

$$f(a,b,c,d) = \sum_m (0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$$

f

a,b \ c,d	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

$$f = \bar{c} + \bar{a}\,\bar{d} + b\,\bar{d}$$

$$f(a,b,c,d) = \sum_m (0,5,7,8,11,13,14,15)$$

f

a,b \ c,d	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

$$f = b\,d + a\,b\,c + a\,c\,d + \bar{b}\,\bar{c}\,\bar{d}$$

$$f(a,b,c,d,e) = \sum_m (0,1,2,7,8,9,10,15,16,17,18,24,25,26,28,30)$$

f

a,b \ c,d,e	000	001	011	010	110	111	101	100
00	0	1	3	2	6	7	5	4
01	8	9	11	10	14	15	13	12
11	24	25	27	26	30	31	29	28
10	16	17	19	18	22	23	21	20

$$f = \underline{\bar{c}\,\bar{d}} + \underline{\bar{e}\,\bar{c}} + \underline{a\,b\,\bar{e}} + \underline{\bar{a}\,c\,d\,e}$$

$$f(a,b,c,d,e) = \sum_m (3,6,7,8,10,12,14,17,20,21,24,25,27,28) + \sum_d (0,1,18,19,31)$$

f

a,b \ c,d,e	000	001	011	010	110	111	101	100
00	0	1	3	2	6	7	5	4
01	8	9	11	10	14	15	13	12
11	24	25	27	26	30	31	29	28
10	16	17	19	18	22	23	21	20

$$f = \bar{a}\,b\,\bar{e} + a\,\bar{c}\,e + \bar{b}\,\bar{c}\,e + b\,\bar{d}\,\bar{e} + a\,\bar{b}\,c\,\bar{d} + \bar{a}\,\bar{b}\,c\,d$$

$$f(a,b,c,d,e) = \sum_m (0,2,3,4,5,6,7,11,15,16,18,19,23,27,31) + \sum_d (1,9,24,30)$$

f

a,b \ c,d,e	000	001	011	010	110	111	101	100
00	0	1	3	2	6	7	5	4
01	8	9	11	10	14	15	13	12
11	24	25	27	26	30	31	29	28
10	16	17	19	18	22	23	21	20

$$f = \bar{a}\,\bar{b} + d\,e + \bar{b}\,\bar{c}\,\bar{e}$$

$$f(a,b,c,d,e,f) = \sum_m \left(\begin{array}{l} 0,2,4,8,10,13,15,16,18,20,23,24,26,32,34,40,41,42,45,47,48, \\ 50,56,57,58,60,61 \end{array} \right)$$

g

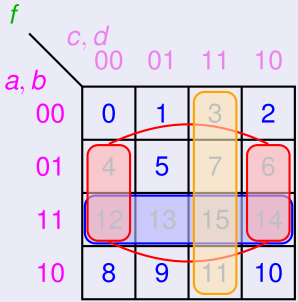
a,b,c \ d,e,f	000	001	011	010	110	111	101	100
000	0	1	3	2	6	7	5	4
001	8	9	11	10	14	15	13	12
011	24	25	27	26	30	31	29	28
010	16	17	19	18	22	23	21	20
110	48	49	51	50	54	55	53	52
111	56	57	59	58	62	63	61	60
101	40	41	43	42	46	47	45	44
100	32	33	35	34	38	39	37	36

$g = \bar{e}\,\bar{f} + a\,b\,\bar{c}\,f + \bar{a}\,\bar{b}\,\bar{c}\,\bar{f} + \bar{a}\,\bar{b}\,c\,d\,e\,f + a\,b\,\bar{c}\,d + b\,\bar{d}\,e\,f$

$$f = \begin{cases} \underbrace{(a+b+\bar{c}+\bar{d})}_{0011\leftrightarrow 3} \cdot \underbrace{(a+\bar{b}+c+d)}_{0100\leftrightarrow 4} \cdot \underbrace{(a+\bar{b}+\bar{c}+d)}_{0110\leftrightarrow 6} \cdot \underbrace{(a+\bar{b}+\bar{c}+\bar{d})}_{0111\leftrightarrow 7} \cdot \underbrace{(\bar{a}+b+\bar{c}+\bar{d})}_{1011\leftrightarrow 11} \cdot \\ \underbrace{(\bar{a}+\bar{b}+c+d)}_{1100\leftrightarrow 12} \cdot \underbrace{(\bar{a}+\bar{b}+c+\bar{d})}_{1101\leftrightarrow 13} \cdot \underbrace{(\bar{a}+\bar{b}+\bar{c}+d)}_{1110\leftrightarrow 14} \cdot \underbrace{(\bar{a}+\bar{b}+\bar{c}+\bar{d})}_{1111\leftrightarrow 15} \end{cases}$$

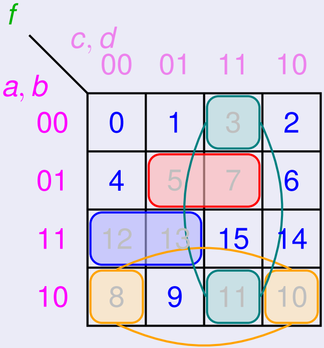
- Minterm accepts iff maxterm rejects
- Cover is obtained where f is false
- Core step: $(s+x)(s+\overline{x}) = s$
- $f = \begin{cases} M_3 \cdot M_4 \cdot M_6 \cdot M_7 \cdot M_{11} \cdot \\ M_{12} \cdot M_{13} \cdot M_{14} \cdot M_{15} \end{cases}$
- $\overline{f} = \begin{cases} \overline{M_3} + \overline{M_4} + \overline{M_6} + \overline{M_7} + \overline{M_{11}} + \\ \overline{M_{12}} + \overline{M_{13}} + \overline{M_{14}} + \overline{M_{15}} \end{cases}$
- $\overline{f} = \begin{cases} m_3 + m_4 + m_6 + m_7 + m_{11} + \\ m_{12} + m_{13} + m_{14} + m_{15} \end{cases}$
- $f = m_0 + m_1 + m_2 + m_5 + m_8 + m_9 + m_{10}$

NB Literals in a minterm and the corresponding maxterm are complemented



$f = (\bar{a}+\bar{b})\ (\bar{b}+d)\ (\bar{c}+\bar{d})$

$$f(a,b,c,d) = \prod_M (3,5,7,8,10,11,12,13)$$



$$f = (\bar{a}+\bar{b}+c)\ (a+\bar{b}+\bar{d})\ (\bar{a}+b+d)\ (b+\bar{c}+\bar{d})$$