MA 20104 Probability and Statistics
Assignment No. 4

1. Under a certain complicated birth situation, the mortality rate of a new born child is given by $Z(t) = 0.5+2t$, $t > 0$. If the baby survives to age one, find the probability that he/she will survive to age 2.

2. In an examination, a student is considered to have failed, secured second class, first class and distinction, according as he scores less than 45%, between 45% and 60%, between 60% and 75% and above 75% respectively. In a particular year 10% of students failed in the examination and 5% students got distinction. Assuming the marks to be normally distributed, find the percentage of students who get first class and second class respectively.

3. In an industrial process the diameter of a ball bearing is an important component. The buyer sets specifications on the diameter to be 3.0 ± 0.01 cm. The diameter has a normal distribution with mean 3 cm. and s.d. 0.005 cm. On the average how many manufactured balls will be scrapped?

4. The width of a duralumin forging is (in inches) normally distributed with $\mu = 0.9$ and $\sigma = 0.003$. The specification limits were given as 0.9 ± 0.005. What percentage of forgings will be defective? What is the maximum allowable value of σ that will permit no more than 1 defective in 100 when the widths are $N(0.9, \sigma^2)$?

5. The height a university high jumper will clear, each time he jumps, is a normal r.v. with mean 2 meters and s.d. 10 cm. What is the greatest height that he will jump with probability 0.95? What is the height that he will clear only 10% of the time?

6. If a set of marks on a Statistics exam is approximately $N(74, 62.41)$, find
   a) the lowest passing grade if the lowest 10% of the students are given F's;
   b) the highest B if the top 5% of the students are given A's;
   c) the lowest B if the top 10% of the students are given A's and the next 25% are given B's.

7. The diameters X of a ball-bearing are distributed normally with mean μ and standard deviation unity, If X lies in the specification limits of 6 to 8 inches, a profit of Rs. C_0 is gained. However, in case $X < 6$ or $X > 8$, there is a loss of Rs. C_1 or Rs. C_2 respectively. Find the value of μ that maximizes the expected profit.

8. IQ levels of the candidates for a particular job selection are normally distributed with mean 90 and standard deviation 5. Find the approximate probability that a randomly selected candidate has IQ level between 85 and 95. Suppose four

candidates are randomly selected. Find the probability that at least two of them have IQ levels between 85 and 95.

9. A production company has 200 machines and they operate independently. The time to failure for each machine is given by a random variable X, measured in years with pdf given as

$$f(x) = \begin{cases} \dfrac{2}{x^3}, & \text{if } x > 1 \\ 0, & \text{if } x \leq 1 \end{cases}$$

   Use binomial approximation to normal to find the probability that at least 60 machines will be working for more than two years. (Use continuity corrections).

10. In a probability and statistics class, the total number of students is 200. A professor gives a question to the students as a surprise test. The probability that a randomly selected student can solve the question is 0.5. What is the probability that at least 110 students in the class cannot solve the Question? Use normal approximation to binomial. (Use continuity corrections).

11. Trains arrive and depart at Kanpur railway station according to a Poisson process at a rate one per three minutes. What is the probability that between 2:00 p.m. to 3:00 p.m. the number of trains arriving or departing is at least 17 and not more than 25? Use normal approximation with continuity corrections.

12. Let Y denote the diameter in mm. of certain type of nuts. Assume that Y has a log-normal distribution with parameters $\mu = 0.8$ and $\sigma = 0.1$. Find the probability that a randomly selected nut has diameter more than 2.7 mm. Between what two values will Y fall with probability 0.95?

13. A random variable X has a beta distribution with mean $\frac{2}{3}$ and the variance is $\frac{1}{18}$. Find $P(0.2 < X < 0.5)$.

14. Let X follow a zero truncated Poisson distribution with the probability mass function given by

$$P(X = x) = \frac{1}{1 - e^{-\lambda}} \frac{e^{-\lambda}\lambda^x}{x!} \quad , x = 1, 2, 3, \ldots$$

   Find $E\left(\frac{1}{1+X}\right)$.