Computer Science & Engineering Department
I. I. T. Kharagpur

Principles of Programming Languages: CS40032

Elective

Assignment – 3: Typed λ-Calculus *Marks: 25*

Assign Date: *09 March, 2022* Submit Date: *23:55, 16 March, 2022*

Instructions: Please solve the questions using pen and paper and scan the images. Every image should contain your roll number and name.

1. Derive the type of each lambda expression. Show all derivation steps and mark them like constant rule, application rule, etc. No marks will be awarded without the rule markings **[3 * 5 = 15]**

   (a) Given $\mathcal{E}_0 \cup y : bool$:

   $$y \ := \ false$$

   (b) Given type constants $func1 : monad \rightarrow \Psi$ and $func2 : (\varphi \rightarrow \Psi)$:

   $$\lambda(x \ : \ monad).(func1 \ x); \lambda(q \ : \ \varphi).(func2 \ q)$$

   (c) Given | be a constant of type $\psi \ \rightarrow \ \psi \ \rightarrow \ \psi \rightarrow \ \psi$ and type of t is ψ

   $$\lambda(\omega \ : \ \psi \ \rightarrow \ \pi). \ \lambda(x \ : \ \psi). \ (\omega \ ((x \mid t)x))$$

   (d) Given + is type constant with the type $X \ \rightarrow \ X$.

   $$\lambda(f \ : \ X \ \rightarrow \ Y). \ \lambda(x \ : \ X). \ f(+x)$$

   (e) Given $\mathcal{E}_0 = \{x \ : \ Ref \ Bool, \ y \ : \ Bool\}$ and the constants $succ \ : \ Int \ \rightarrow \ Int$, $true \ : \ Bool$, $4 \ : \ Int$:

   $$succ \ 4; \ x \ := \ true$$

2. Derive the type of each of the following expression. Any assumptions are welcome but must be clearly stated. **[5 * 2 = 10]**

   (a) Given the type of ϕ and Φ is $\xi \ \rightarrow \ \xi$

   $$((\lambda(z : \ \xi \ \rightarrow \ \xi).\lambda(+ : \ \xi \ \rightarrow \ \xi). \ \lambda(\eta : \ \xi \ \rightarrow \ \xi). \ \lambda(\mu : \ \xi). \ z \ (+ \ (\eta \ (\eta \ \mu)))) \ \phi)\Phi$$

   (b) Given ϕ be the constant with the type $\theta \ \rightarrow \ \theta \ \rightarrow \ \theta$ and $true$ with the type θ

   $$\lambda(func1 : \ \theta \ \rightarrow \ Char). \ \lambda(\tau : \ \theta). \ func1 \ (\tau \ \phi \ true)$$