Assignment 3

19CS10060
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1. (a) <u>Given</u> $\mathcal{E}_0 \cup y : bool$

$y := false$

$\Rightarrow$ Assuming $false : Bool \in \mathcal{E}C$

Given $\mathcal{E}_0 \cup \{y : Ref\ Bool\}$

$\mathcal{E}_0 \cup \{y : Ref\ Bool\} \vdash y : Ref\ Bool$ --- (1)

and, [By Identifier Rule]

$\mathcal{E}_0 \cup \{y : Ref\ Bool\} \vdash false : Bool$ --- (2)

[By constant Rule]

$\mathcal{E}_0 \cup \{y : Ref\ Bool\} \vdash y := false : Command$ -- (3)

[By Assignment Rule]

$$\therefore \frac{\overline{\mathcal{E}_0 \cup \{y : Ref\ Bool\} \vdash y : Ref\ Bool}\ (1) \quad \overline{\mathcal{E}_0 \cup \{y : Ref\ Bool\} \vdash false : Bool}\ (2)}{\mathcal{E}_0 \cup \{y : Ref\ Bool\} \vdash y := false : Command}\ (3)$$

$$\left(\text{Assingment} : \frac{\mathcal{E} \vdash N : Ref\ T,\ \mathcal{E} \vdash M : T}{\mathcal{E} \vdash N := M : Command} \right)$$

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1.(5) Given:

$func1 : monad \rightarrow \Psi$

$func2 : \varphi \rightarrow \Psi$

$\mathcal{E}_0 \cup \{x : monad\} \vdash func1 : monad \rightarrow \Psi$ -- (1) [Constant Rule]

$\mathcal{E}_0 \cup \{x : monad\} \vdash x : monad$ -- (2) [Indentifier Rule]

$\mathcal{E}_0 \cup \{x : monad\} \vdash func1\ x : \Psi$ -- (3) [App. Rule]

$\left(\because \text{Application} : \dfrac{\mathcal{E} \vdash M : S \rightarrow T;\ \mathcal{E} \vdash N : S}{\mathcal{E} \vdash MN : T} \right)$

$\Rightarrow \mathcal{E}_0 \cup \{x : monad\} \vdash (func1\ x) : \Psi$ [Paren.. Rule] -- (4)

$\mathcal{E}_0 \vdash \lambda(x : monad).(func1\ x) : monad \rightarrow \Psi$ [Function Rule] -- (5)

Let, $\mathcal{E}_2 = \mathcal{E}_0 \cup \{q : \varphi\}$

$\mathcal{E}_2 \vdash func2 : \varphi \rightarrow \Psi$ [Const. Rule] -- (6)

$\mathcal{E}_2 \vdash q : \varphi$ [Indentifier Rule] -- (7)

$\mathcal{E}_2 \vdash func2\ q : \Psi$ [App. Rule] -- (8)

$\mathcal{E}_2 \vdash (func2\ q) : \Psi$ [ParenRule] -- (9)

$\mathcal{E}_0 \vdash \lambda(q : \varphi).(func2\ q) : \varphi \rightarrow \Psi$ [Sequence Rule] -- (10)

$\mathcal{E}_0 \vdash \lambda(x : monad)(func1\ x);\ \lambda(q : \varphi).(func2\ q) : \varphi \rightarrow \Psi$ [Seq. Rule] -- (11)

Let, $\mathcal{E}_1 = \mathcal{E}_0 \cup \{x : monad\}$

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$$\dfrac{\dfrac{\dfrac{\dfrac{\mathcal{E}_1 \vdash func1 : monad \to \Psi \;\; (1) \qquad \mathcal{E}_1 \vdash x : monad \;\; (2)}{\mathcal{E}_1 \vdash func1\ x : \Psi} (3)}{\mathcal{E}_1 \vdash (func1\ x) : \Psi} (4)}{\mathcal{E}_0 \vdash \lambda(x : monad).(func1\ x) : monad \to \Psi} (5) \qquad \dfrac{\dfrac{\dfrac{\mathcal{E}_2 \vdash func2 : \varphi \to \Psi \;\; (6) \qquad \mathcal{E}_2 \vdash q : \varphi \;\; (7)}{\mathcal{E}_2 \vdash func2\ q : \Psi} (8)}{\mathcal{E}_2 \vdash (func2\ q) : \Psi} (9)}{\mathcal{E}_0 \vdash \lambda(q : \varphi).(func2\ q) : \varphi \to \Psi} (10)}{\mathcal{E}_0 \vdash \lambda(x : monad).(func1\ x);\ \lambda(q : \varphi).func2\ q) : \varphi \to \Psi} (11)$$

1(c) Let, $\mathcal{E}_1 = \mathcal{E}_0 \cup \{\omega : \Psi \to \Pi\}$

$\mathcal{E}_1 \cup \{x : \Psi\} \vdash x : \Psi$	[Id. Rule] (1)
$\mathcal{E}_1 \cup \{x : \Psi\} \vdash 1 : \Psi \to \Psi \to \Psi \to \Psi$	[Const. Rule] (2)
$\mathcal{E}_1 \cup \{x : \Psi\} \vdash 1x : \Psi \to \Psi \to \Psi$	[App. Rule] (3)
$\mathcal{E}_1 \cup \{x : \Psi\} \vdash t : \Psi$	[const. Rule] (4)
$\mathcal{E}_1 \cup \{x : \Psi\} \vdash 1xt : \Psi \to \Psi$	[App. "] (5)
$\mathcal{E}_1 \cup \{x : \Psi\} \vdash \omega : \Psi \to \Pi$	[Id "] (6)
$\mathcal{E}_1 \cup \{x : \Psi\} \vdash (1xt) : \Psi \to \Psi$	[Pare. from 5] (7)
$\mathcal{E}_1 \cup \{x : \Psi\} \vdash (1xt)x : \Psi$	[App. Rule from 7 and 1] (8)
$\mathcal{E}_1 \cup \{x : \Psi\} \vdash ((1xt)x) : \Psi$	[Paren.] (9)
$\mathcal{E}_1 \cup \{x : \Psi\} \vdash \omega((1xt)x) : \Pi$	[App. from 9 and 6] (10)
$\mathcal{E}_1 \cup \{x : \Psi\} \vdash (\omega((1xt)x)) : \Pi$	[Paren Rule] (11)

$\therefore \mathcal{E}_1 \vdash \lambda(x : \Psi).(\omega((1xt)x)) : \Psi \to \Pi$

[Func Rule]

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$\therefore\ \mathcal{E}_0 \vdash \lambda(\omega : \psi \to \pi).\ \lambda(x:\psi).\ (\omega\ ((1\ x\ t)\ x)) :$

$(\psi \to \pi) \to (\psi \to \pi)$

[Func. Rule]

1. (d) let, $\mathcal{E}_1 = \mathcal{E}_0 \cup \{f : X \to Y\}$

let, $\mathcal{E}_2 = \mathcal{E}_1 \cup \{x : X\}$

$\mathcal{E}_2 \vdash + : X \to X$ [Const] (1)

$\mathcal{E}_2 \vdash x : X$ [Id.] (2)

$\mathcal{E}_2 \vdash +x : X$ [App.] (3)

$\mathcal{E}_2 \vdash (+x) : X$ [Paren] (4)

$\mathcal{E}_2 \vdash f : X \to Y$ [Id] (5)

$\mathcal{E}_2 \vdash f(+x) : Y$ [App from 5 & 3] (6)

$\Rightarrow \mathcal{E}_1 \vdash \lambda(x:X).\ f(+x) : X \to Y$ [Func.] (7)

$\Rightarrow \mathcal{E}_0 \vdash \lambda(f:X\to Y).\ \lambda(x:X).\ f(+x):$

$(X \to Y) \to (X \to Y)$ [Func.] (8)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathcal{E}_2 \vdash + : X \to X}(1) \quad \dfrac{}{\mathcal{E}_2 \vdash x : X}(2)}{\mathcal{E}_2 \vdash +x : X}(3)}{\mathcal{E}_2 \vdash (+x) : X}(4) \quad \dfrac{}{\mathcal{E}_0 \vdash f : X \to Y}(5)}{\mathcal{E}_2 \vdash f(+x) : Y}(6)$$

$$\dfrac{\dfrac{\mathcal{E}_2 \vdash f(+x) : Y}{\mathcal{E}_1 \vdash \lambda(x:X).\ f(+x) : X \to Y}(7)}{\mathcal{E}_0 \vdash \lambda(f : X \to Y).\ \lambda(x:X).\ f(+x) : X \to Y}(8)$$

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1. (e)

$\mathcal{E}_0 = \{ x : \text{Ref Bool}, y : \text{Bool} \}$

$\mathcal{E}_0 \vdash succ : Int \to Int$ [Const] (1)

$\mathcal{E}_0 \vdash 4 : Int$ [Const] (2)

$\mathcal{E}_0 \vdash succ\ 4 : Int$ [App.] (3)

$\mathcal{E}_0 \vdash x : \text{Ref Bool}$ [Id] (4)

$\mathcal{E}_0 \vdash true : Bool$ [Const] (5)

$\mathcal{E}_0 \vdash x := true : Command$ [Assingment Rule] (6)

$\Rightarrow \mathcal{E}_0 \vdash succ\ 4;\ x := true : Command.$ [Seq. Rule]

$$\cfrac{\cfrac{\mathcal{E}_0 : succ : Int \to Int \ (1) \qquad \mathcal{E}_0 \vdash 4 : Int \ (2)}{\mathcal{E}_0 : succ\ 4 : Int} (4) \qquad \cfrac{\mathcal{E}_0 \vdash x : \text{Ref Bool} \ (4) \qquad \mathcal{E}_0 \vdash true : Bool \ (5)}{\mathcal{E}_0 \vdash x := true : Command} (6)}{\mathcal{E}_0 \vdash succ\ 4;\ x := true : Command} (7)$$

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2.(a) Let, $\mathcal{E}_1 = \mathcal{E}_0 \cup \{z : \xi \to \xi\}$

$\mathcal{E}_2 = \mathcal{E}_1 \cup \{+ : \xi \to \xi\}$

$\mathcal{E}_3 = \mathcal{E}_2 \cup \{n : \xi \to \xi\}$

$\mathcal{E}_4 = \mathcal{E}_3 \cup \{\mu : \xi\}$

$\mathcal{E}_4 \vdash n : \xi \to \xi$ [Id.] (1)

$\mathcal{E}_4 \vdash \mu : \xi$ [Id] (2)

$\mathcal{E}_4 \vdash n\mu : \xi$ [App.] (3)

$\mathcal{E}_4 \vdash (n\mu) : \xi$ [Paren Rule] (4)

$\mathcal{E}_4 \vdash n(n\mu) : \xi$ [App from 1 & 4] (5)

$\mathcal{E}_4 \vdash (n(n\mu)) : \xi$ [Paren Rule] (6)

$\cancel{\mathcal{E}_5 \vdash +(n(n\mu)) : \mathcal{E}}$

$\mathcal{E}_4 \vdash + : \xi \to \xi$ [Id Rule] (7)

$\mathcal{E}_4 \vdash +(n(n\mu)) : \xi$ [App. from 6 & 7] (8)

$\mathcal{E}_4 \vdash (+(n(n\mu))) : \xi$ [Paren.] (9)

$\mathcal{E}_4 \vdash z : \xi \to \xi$ [Id] (10)

$\mathcal{E}_4 \vdash z(+(n(n\mu))) : \xi$ [App.] (11)

$\Rightarrow \mathcal{E}_3 \vdash \lambda(\mu : \xi).\, z(+(n(n\mu))) : \xi \to \xi$ [Func.] (12)

$\Rightarrow \mathcal{E}_2 \vdash \lambda(n : \xi \to \xi).\, \lambda(\mu : \xi).\, z(+(n(n\mu))) : (\xi \to \xi) \to (\xi \to \xi)$ [Func.] (13)

$\mathcal{E}_1 \vdash \lambda(+ : \xi \to \xi).\, \lambda(n : \xi \to \xi).\, \lambda(\mu : \xi).\, z(+(n(n\mu))) :$

$(\xi \to \xi) \to (\xi \to \xi) \to (\xi \to \xi)$ [Func] (14)

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$$\xi_0 \vdash \lambda(z:\xi \to \xi).\ \lambda(+:\xi \to \xi).\ \lambda(n:\xi \to \xi).\ \lambda(\mu:\xi).$$
$$z\,(+\,(n(n\mu))) : (\xi \to \xi) \to (\xi \to \xi) \to (\xi \to \xi) \to \xi \to \xi$$
[Func.] (15)

$$\xi_0 \vdash (\lambda(z:\xi \to \xi).\ \lambda(+:\xi \to \xi).\ \lambda(n:\xi \to \xi).\ \lambda(\mu:\xi).$$
$$z(+(n(n\mu)))) : \xi \to \xi \to \xi \to \cancel{\xi} \to \xi \to \xi \to \xi \to \xi$$
[Paren.] (16)

$$\xi_0 \vdash \phi : \xi \to \xi$$ [Const Rule] (17)

$$\xi_0 \vdash (\lambda(z:\xi \to \xi).\ \lambda(+:\xi \to \xi).\ \lambda(n:\xi \to \xi).\ \lambda(\mu:\xi).\ z(+(n(n\mu)))$$
$$)\ \phi : (\xi \to \xi) \to (\xi \to \xi) \to \xi \to \xi$$ [App] (18)

$$\xi_0 \vdash ((\lambda(z:\xi \to \xi).\ \lambda(+:\xi \to \xi).\ \lambda(\xi \to \xi).\ \lambda(\mu:\xi).$$
$$z(+(n(n\mu))))\ \phi) : (\xi \to \xi) \to (\xi \to \xi) \to \xi \to \xi$$ [Paren] (19)

$$\xi_0 \vdash \Phi : \xi \to \xi$$ [Const] (20)

$$\xi_0 \vdash ((\lambda(z:\xi \to \xi).\ \lambda(+:\xi \to \xi).\ \lambda(n:\xi \to \xi).\ \lambda(\mu:\xi).$$
$$z(+(n(n\mu))))\ \phi)\ \Phi : (\xi \to \xi) \to \xi \to \xi$$ [App.] (21)

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2.(b)

$$\left.\begin{array}{l} \phi : \theta \to \theta \to \theta \\ true : \theta \end{array}\right\} \mathcal{E}C,\ \mathcal{E}I$$

Let, $\mathcal{E}_1 = \mathcal{E}_0 \cup \{ func1 : \theta \to Char \}$

$\mathcal{E}_2 = \mathcal{E}_1 \cup \{ \tau : \theta \}$

$\mathcal{E}_2 \vdash \tau : \theta$ [Id] (1)

$\mathcal{E}_2 \vdash \phi : \theta \to \theta \to \theta$ [Const] (2)

$\mathcal{E}_2 \vdash \phi\, \tau : \theta \to \theta$ [App.] (3)

$\mathcal{E}_2 \vdash true : \theta$ [Const] (4)

$\mathcal{E}_2 \vdash \phi\, \tau\, true : \theta$ [App.] (5)

$\mathcal{E}_2 \vdash (\phi\, \tau\, true) : \theta$ [Paren.] (6)

$\mathcal{E}_2 \vdash func1 : \theta \to Char$ [Id] (7)

$\mathcal{E}_2 \vdash func1\,(\phi\, \tau\, true) : Char$ [App.] (8)

$\mathcal{E}_1 \vdash \lambda(\tau : \theta).\, func1\,(\phi\, \tau\, true) : \theta \to Char$

[Func.] (9)

$\mathcal{E}_0 \vdash \lambda(func1 : \theta \to Char).\, \lambda(\tau : \theta).\, func1\,(\phi\, \tau\, true) :$

$(\theta \to Char) \to (\theta \to Char)$

[Func.] (10)

∴ ~~Hence~~

$$\frac{\mathcal{E}_2 \vdash \phi : \theta \to \theta \to \theta \ \ (2) \qquad \mathcal{E}_2 \vdash \tau : \theta \ \ (1)}{\mathcal{E}_2 \vdash \phi\, \tau : \theta \to \theta \ \ (3) \qquad \mathcal{E}_2 \vdash true : \theta \ \ (4)}$$

$$\frac{\mathcal{E}_2 \vdash \phi\, \tau : \theta \to \theta \qquad \mathcal{E}_2 \vdash true : \theta}{\mathcal{E}_2 \vdash (\phi\, \tau\, true) : \theta}\ (5)\ \&\ (6)$$

$$\frac{\mathcal{E}_2 \vdash (\phi\, \tau\, true) : \theta \qquad \mathcal{E}_2 \vdash func1 : \theta \to Char \ \ (7)}{\mathcal{E}_2 \vdash func1(\phi\, \tau\, true) : Char}\ (8)$$

$$\frac{\mathcal{E}_2 \vdash func1(\phi\, \tau\, true) : Char}{\mathcal{E}_1 \vdash \lambda(\tau : \theta).\, func1(\phi\, \tau\, true) : \theta \to Char}\ (9)$$

$$\frac{\mathcal{E}_1 \vdash \lambda(\tau : \theta).\, func1(\phi\, \tau\, true) : \theta \to Char}{\mathcal{E}_0 \vdash \lambda(func1 : \theta \to Char).\, \lambda(\tau : \theta).\, func1\,(\phi\, \tau\, true) : (\theta \to Char) \to (\theta \to Char)}\ (10)$$