

**Computer Science & Engineering Department**  
**I. I. T. Kharagpur**  
**Principles of Programming Languages: CS40032**

Elective

**Assignment – 2:  $\lambda$ -Calculus**

Marks: 20

Assign Date: 25th February, 2022

Submit Date: 23:55, 4th March, 2022

1. Reduce the following  $\lambda$ -expressions. Show every step of  $\alpha$ -,  $\beta$ -,  $\eta$ - and  $\delta$ -reductions.

**[2\*6 = 12]**

- (a)  $(\lambda z. z) (\lambda y. y y) (\lambda x. x a)$
- (b)  $(\lambda z. z) (\lambda z. z z) (\lambda z. z y)$
- (c)  $(\lambda x. \lambda y. x y y) (\lambda a. a) b$
- (d)  $(\lambda x. \lambda y. x y y) (\lambda y. y) y$
- (e)  $(\lambda x. (\lambda y. (x y)) y) z$
- (f)  $(\lambda x. x x) (\lambda y. y x) z$

2. (a) Define a recursive version of this function that takes the recursion point as a parameter.

**[ 1 Mark]**

```
def add x y =  
  if iszero y  
  then x  
  else add (succ x) (pred y)
```

- (b) Feed this revised *add* function to the Y combinator and evaluate using the regular rules of lambda calculus.

**[4 marks]**

3. Solve:

**[3 marks]**

$(((((\lambda f. (\lambda g. (\lambda x. ((fx)(gx)))))(\lambda m. (\lambda n. (nm)))))(\lambda n. z))p)$