ASSIGNMENT 2

19CS10060
Sunanda Mandal

1 (b) $(\lambda z.\, z)\,(\lambda z.\, z\,z)\,(\lambda z.\, z\,y)$

$= (\lambda x.\, x)\,(\lambda z.\, z\,z)\,(\lambda z.\, z\,y)$ [α reduction: rename z with x]

$= (\lambda z.\, z\,z)\,(\lambda z.\, z\,y)$ [β reduction: replace x with $(\lambda z.\, z\,z)$]

$= (\lambda x.\, x\,x)\,(\lambda z.\, z\,y)$ [α reduction rename z with x]

$= (\lambda z.\, z\,y)\,(\lambda z.\, z\,y)$ [β reduction: replace x with $(\lambda z.\, z\,y)$]

$= (\lambda x.\, x\,y)\,(\lambda z.\, z\,y)$ [α reduction: rename z with x]

$= (\lambda z.\, z\,y)\,y$ [β reduction: replace x with $(\lambda z.\, z\,y)$]

$= y\,y$ [β reduction: replace z with y]

[NOTE: α-reductions above are not necessary, used for clarity]

(d) $(\lambda x.\, \lambda y.\, x\,y\,y)\,(\lambda y.\, y)\,y$

$= (\lambda x.\, \lambda z.\, x\,z\,z)\,(\lambda y.\, y)\,y$ [α reduction: rename y with z]

$= (\lambda z.\, ((\lambda y.\, y)\,z\,z))\,y$ [β reduction: replace x with $(\lambda y.\, y)$]

$= (\lambda y.\, y)\,y\,y$ [β reduction: replace z with y]

$= (\lambda y.\, y)\,y\,y$ [β reduction: replace y with y]

$= y\,y$

(f) $(\lambda x.\, x\,x)\,(\lambda y.\, y\,x)\,z$

$= (\lambda y.\, y\,x)\,(\lambda y.\, y\,x)\,z$ [β reduction: replacing x by $(\lambda y.\, y\,x)$]

$= (\lambda y.\, y\,x)\,x\,z$ [β reduction: replacing y by $(\lambda y.\, y\,x)$]

$= x\,x\,z$ [β reduction: replacing y by x]

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1. (a) ~~$(\lambda z.z)(\lambda z.zz)(\lambda z.zy)$~~

$= (\lambda z.z)(\lambda z.zz)(\lambda z.zy)$ [β reduction: replace z with y

$= (\lambda z.z)(\lambda z.zz)\,y$ [β reduction: replace z with y]

$= (\lambda z.z)\,y\,y$

1. (a) $(\lambda z.z)(\lambda y.yy)(\lambda x.xa)$

$= (\lambda y.yy)(\lambda x.xa)$ [β reduction: replacing z with $(\lambda y.yy)$]

$= (\lambda x.xa)(\lambda x.xa)$ [β reduction: replacing y with $(\lambda x.xa)$]

$= (\lambda x.xa)a$ [β reduction: replacing x with $(\lambda x.xa)$]

$= a\,a$ [β reduction replacing x with a]

(c) $(\lambda x.\lambda y.x\,y\,y)(\lambda a.a)\,b$

$= (\lambda y.(\lambda a.a)\,y\,y)\,b$ [β reduction: replacing x with $(\lambda a.a)$]

$= (\lambda a.a)\,\underline{b}\,b$ [β reduction: replacing y with $\underline{b}$]

$= b\,b$ [β reduction: replacing a with $\underline{b}$]

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1. (e) $(\lambda x.(\lambda y.(x\ y))y)\ z$

$= (\lambda x.(\lambda a.(x\ a))\ y)\ z$ [α-reduction: rename y with a]

$= (\lambda a.(z\ a)\ y)$ [β reduction: replace x with z]

$= z\ y$ [β reduction: replace a with y]

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2. (a) Revised add function:

$$add = \lambda f.\ \lambda x.\ \lambda y.\ (\text{if iszero } y \text{ then } x \text{ else } f\ (succ\ x)\ (pred\ y))$$

(b)

Y add

$= add\ (Y\ add)$

$= \lambda f.\ \lambda x.\ \lambda y\ (\text{if iszero } y \text{ then } x \text{ else } f\ (succ\ x)\ (pred\ y))\ (Y\ add)$

$= \lambda x.\ \lambda y\ (\text{if iszero } y \text{ then } x \text{ else } (Y\ add)\ (succ\ x)\ (pred\ y))$ [β reduction] — (i)

For some value $x = 1,\ y = 1$

$= (Y\ add)\ 1\ 1$

$= \lambda x.\ \lambda y.\ (\text{if iszero } y \text{ then } x \text{ else } (Y\ add)\ (succ\ x)\ (pred\ y))\ 1\ 1$

$= (\text{if iszero } 1 \text{ then } 1 \text{ else } (Y\ add)\ (succ\ 1)\ (pred\ 1))$ [from prev evaluation]

$= (Y\ add)\ (succ\ 1)\ (pred\ 1)$ [β reduction] [else branch]

$= (Y\ add)\ 2\ 0$ [evaluating succ & pred]

$= \lambda x.\ \lambda y.\ (\text{if iszero } y \text{ then } x \text{ else } (Y\ add)\ (succ\ x)\ (pred\ y))\ 2\ 0$ [from (i)]

$= (\text{if iszero } 0 \text{ then } 2 \text{ else } (Y\ add)\ (succ\ 2)\ (pred\ 0))$

$= 2$ [if branch]

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3. $(((((\lambda f.(\lambda g.(\lambda x.((fx)(gx))))) (\lambda m.(\lambda n.(nm)))) (\lambda n.z)) p)$

[β reduction: replacing f by $(\lambda m.(\lambda n.(nm)))$]

$= (((\lambda g.(\lambda x.(((\lambda m.(\lambda n.(nm)))x)(gx)))) (\lambda n.z)) p)$

[β reduction: replacing g by $(\lambda n.z)$]

$= ((\lambda x.(((\lambda m.(\lambda n.(nm)))x)((\lambda n.z)x))) p)$

[β reduction: replacing x by p]

$= (((\lambda m.(\lambda n.(nm)))p)((\lambda n.z)p))$

[β reduction: replacing m by p]

$= ((\lambda n.(np))((\lambda n.z)p))$

[β reduction: replacing n by $((\lambda n.z)p)$]

$= ((\lambda n.z)p)p$

[β reduction: replacing n by p]

$= z\ p$