ASSIGNMENT 1

19CS10060
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1.a) $\lambda x.\ x\ z\ \lambda y.\ x\ y$

paranthesized:

$$(\lambda x.((\underbrace{x\ z}_{z})(\lambda y.\ x\ y)))$$

z in this part is free variable

b) $(\lambda x.\ x\ z)\ \lambda y.\ w\ \lambda w.\ w\ y\ z\ x$

$$((\lambda x.(\underbrace{x\ z}_{z}))(\lambda y.(\underbrace{w}_{w}\ (\lambda w.\underbrace{(((w\ y)z)\ x}_{z,\ x})))))$$

These are the free variables in corresponding marked part.

c) $\lambda x.\ x\ y\ \lambda x.\ y\ x$

$$(\lambda x.((\underbrace{x\ y}_{y})(\lambda x.(\underbrace{y}_{y}\ x))))$$

These are free variables in corresponding marked parts.

19CS10060

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2.(a) $NOT = \lambda x. ((x\ TRUE)\ FALSE)$

$TRUE = \lambda x. \lambda y. x$

$FALSE = \lambda x. \lambda y. y$

Now,

$NOT\ (NOT\ FALSE)$

$= \lambda x. ((x\ TRUE)\ FALSE)(NOT\ FALSE)$

$= (((NOT\ FALSE)\ TRUE)\ FALSE)$

$= (((\lambda x. ((x\ TRUE)\ FALSE)\ FALSE)\ TRUE)\ FALSE)$

$= ((((FALSE\ TRUE)\ FALSE)\ \cancel{TRUE})\ FALSE)$

$= (((((\lambda x. \lambda y. y)\ TRUE)\ FALSE)\ TRUE)\ FALSE)$

$= ((((\lambda y. y)\ FALSE)\ TRUE)\ FALSE)$

$= (((FALSE)\ TRUE)\ FALSE)$

$= ((\lambda x. \lambda y. y)\ TRUE)\ FALSE$

$= (\lambda y. y)\ FALSE = FALSE$ (Proved)

(b) $OR\ FALSE\ TRUE$

$= ((\lambda x. \lambda y. ((x\ TRUE)\ y))\ FALSE)\ TRUE$

$= \lambda y. ((FALSE\ TRUE)\ y)\ TRUE$

$= ((FALSE\ TRUE)\ TRUE)$

$= ((\lambda x. \lambda y. y)\ TRUE)\ TRUE$

$= (\lambda y. y)\ TRUE = TRUE$ (Proved)

19CS10060

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2.(c) $\text{add } \bar{5}\ \bar{1}$

$= \left(\left(\lambda n.\ \lambda m.\ \lambda f.\ \lambda x.\left((n\ f)(m\ f\ x)\right)\right)\bar{5}\right)\bar{1}$

$= \left(\lambda m.\ \text{[illegible]}\right)\bar{1}$

$= \lambda f.\ \lambda x.\left((\bar{5} f)\left((\bar{1} f)\ x\right)\right)$

$[\bar{n} = \lambda f.\ \lambda x.\ f^{n} x]$

$= \lambda f.\ \lambda x.\ \left(\left((\lambda f.\ \lambda x.\ f^5 x) f\right)\left(\left((\lambda f.\ \lambda x. f x) f\right) x\right)\right)$

$= \lambda f.\ \lambda x.\ \left((\lambda x.\ f^5 x)\left((\lambda x.\ f\ x)\ x\right)\right)$

$= \lambda f.\ \lambda x.\left((\lambda x.\ f^5\ x)\ (f\ x)\right)$

$= \lambda f.\ \lambda x.\ \left(f^5 (f(x))\right)$

$= \lambda f.\ \lambda x.\ f^6 x = \bar{6}$ [Solved]

2.(d) IF TRUE THEN x ELSE y

$=$ TRUE x y [$\because$ IF a THEN b ELSE c = a b c]

$= \left((\lambda x.\ \lambda y.\ x)\ x\right) y$ [$\because$ TRUE $= \lambda x.\ \lambda y.\ x$]

$= (\lambda y.\ x)\ y$

$= x$ [Proved]

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(e) add

For add to be commutative:

$$\text{add } \bar{a}\ \bar{b} = \text{add } \bar{b}\ \bar{a}$$

$$LHS = \text{add } \bar{a}\ \bar{b}$$

$$= ((\lambda n.\ \lambda m.\ \lambda f.\ \lambda x.\ ((n\ f)\ ((m\ f)\ x)))\ \bar{a})\ \bar{b}$$

$$= (\lambda m.\ \lambda f.\ \lambda x.\ ((\bar{a} f)\ ((m\ f)\ x)))\ \bar{b}$$

$$= \lambda f.\ \lambda x.\ ((\bar{a} f)\ ((\bar{b} f)\ x))$$

$$= \lambda f.\ \lambda x.\ (((\lambda f.\lambda x.\ f^{a} x) f)\ (((\lambda f.\lambda x.\ f^{b} x)\ f)\ x))$$

$$= \lambda f.\ \lambda x.\ ((\lambda x.\ f^{a} x)\ ((\lambda x.\ f^{b} x)\ x))$$

$$= \lambda f.\ \lambda x.\ ((\lambda x.\ f^{a} x)(f^{b} x))$$

$$= \lambda f.\ \lambda x.\ (f^{a}(f^{b} x))$$

$$= \lambda f.\ \lambda x.\ f^{a+b} x$$

$$= \overline{a+b}$$

$$RHS = \text{add } \bar{b}\ \bar{a}$$

$$= ((\lambda n.\lambda m.\ \lambda f.\ \lambda x.\ ((n\ f)\ ((m\ f)\ x)))\ \bar{b})\ \bar{a}$$

$$= (\lambda m.\ \lambda f.\ \lambda x.\ ((\bar{b} f)\ ((m\ f)\ x)))\ \bar{a}$$

$$= \lambda f.\ \lambda x.\ ((\bar{b} f)\ ((\bar{a} f)\ x))$$

$$= \lambda f.\ \lambda x.\ ((\lambda x.\ f^{b} x)\ ((\lambda x.\ f^{a} x) x))$$

$$= \lambda f.\ \lambda x.\ ((\lambda x.\ f^{b} x)\ (f^{a} x))$$

$$= \lambda f.\ \lambda x.\ (f^{b}(f^{a} x))$$

$$= \lambda f.\ \lambda x.\ f^{a+b} x$$

$$= \overline{a+b} = LHS$$

Hence Proved: add is commutative.

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19CS10060

2(e) $\underline{\text{mul}}$

Need to prove: $\text{mul}\ \bar{a}\ \bar{b} = \text{mul}\ \bar{b}\ \bar{a}$

LHS $= \text{mul}\ \bar{a}\ \bar{b}$

$= (\lambda n.\lambda m.\lambda x.(n(m\ x))\ \bar{a})\ \bar{b}$

$= (\lambda m.\lambda x.(\bar{a}\ (m\ x)))\ \bar{b}$

$= \lambda x.(\bar{a}\ (\bar{b}\ x))$

$= \lambda x.(\bar{a}\ ((\lambda g.\lambda y.g^b\ y)\ x))$

$= \lambda x.(\bar{a}\ (\lambda y.x^b\ y))$

$= \lambda x.(\lambda g.\lambda z.g^a\ z)\ (\lambda y.x^b\ y)$

$= \lambda x.\lambda z.(\lambda y.x^b\ y)^a\ z$

$= \lambda x.\lambda z.(\lambda y.x^b y)^{a-1}\ (\lambda y.x^b y)\ z$

$= \lambda x.\lambda z.(\lambda y.x^b y)^{a-2}\ (\lambda y.x^b y)\ (x^b z)$

$= \lambda x.\lambda z.(\lambda y.x^b y)^{a-2}\ (x^{2b}\ z)$

$\vdots$ In this way after a-2 step

$= \lambda x.\lambda z.\ x^{ab}\ z$

$= \overline{ab}$

In similar way for

RHS $= \text{mul}\ \bar{b}\ \bar{a}$

$= \lambda x.(\bar{b}\ (\bar{a}\ x))$

$= \lambda x.(\bar{b}\ (\lambda y.x^a\ y))$

$= \lambda x.\lambda z.(\lambda y.x^a y)^b\ z$

$= \lambda x.\lambda z.\ x^{ba}\ z$

$= \overline{ab} =$ LHS

Hence proved: mul is commutative.

19CS10060

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3. (a)

True $$! True = True

True $$! False = False

False $$! True = False

False $$! False = False.

(b) [n is the input, l is the list]

~~(defun rot-left (n l)~~
~~(mod (n (length l)))~~
~~(append (nthcdr n l)~~
~~(butlast l~~

```
(defun rot-left (n l)
    (mod n (length l))
    (append (nthcdr n l)
            (butlast l (- (length l) n)))
    )
)
```

(c)

```
( define (ack m n)
    (cond ((fx=? m 0)
           (fx+ n 1))
          ((fx=? n 0)
           (ack (fx- m 1) 1))
          (else
            (ack (fx- m 1) (ack m (fx- n 1)))
    )    )
)
```

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3 (d)

```
SumofOddSq :: Integer
SumOddSq
  = sum (takeWhile (<10,000) [ x * x | x <- [1..], odd x])
```

3 (e)

```
Sm.Divisor :: Integer -> Integer
Sm Divisor n
  | n < 0     = error " In valid Input"
  | n == 0    = 0
  | n == 1    = 1
  | otherwise = head [x | x <- [2...n], n 'mod' x == 0]
```