Computer Science and Engineering

Course work portal

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Principles of

Question 2

Not yet answered

Marked out of 10.00

Flag question

A program in the language is a sequence of commands. Stores belong to the domain Store and serve as arguments to the valuation function:

$$\mathbf{C} : Command \to Store_{\perp} \to Store_{\perp}$$

Example Language with Assignment:

Abstract Syntax:

- Consider the entities as:
  $P \in Program$
  $C \in Command$
  $E \in Expression$
  $B \in Boolean\_expr$
  $I \in Identifier$
  $N \in Numeral$

  $P ::= C.$
  $C ::= C_1;\ C_2 \mid \textbf{if } B \textbf{ then } C \mid \textbf{if } B \textbf{ then } C_1 \textbf{ else } C_2 \mid$
  $I := E \mid \textbf{diverge}$
  $E ::= E_1 + E_2 \mid I \mid N$
  $B ::= E_1 = E_2 \mid \neg B$
  diverge is a non-terminating command

Semantic Algebras:

- *Truth values*
  Domain: $t \in Tr = B$
  Operations:
  $true, false : Tr$
  $not : Tr \to Tr$

- *Identifiers*
  Domain: $i \in Id = Identifier$
- *Natural Numbers*
  Domain: $n \in Nat = \mathcal{N}$
  Operations:
  $zero, one, ... : Nat$
  $plus : Nat \times Nat \to Nat$
  $equals : Nat \times Nat \to Tr$
- *Store*
  Domain: $s \in Store = Id \to Nat$
  Operations:
  $newstore : Store$
  $newstore = \lambda i.zero$
  $access : Id \to Store \to Nat$
  $access = \lambda i.\lambda s.s(i)$
  $update : Id \to Nat \to Store \to Store$
  $update = \lambda i.\lambda n.\lambda s.[i \mapsto n]s$

Valuation Functions:

- $\mathbf{P} : Program \to Nat \to Nat_\perp$
  $\mathbf{P}[[C.]] = \lambda n.let\ s = (update\ [[A]]\ n\ newstore)\ in$
  $let\ s' = \mathbf{C}[[C]]s\ in\ (access\ [[Z]]\ s')$
  where the input number n is associated with identifier $[[A]]$ in a new store. As the program body is evaluated, and the answer is extracted from the store at $[[Z]]$

- $\mathbf{C} : Command \to Store_\perp \to Store_\perp$
  $\mathbf{C}[[C_1;\ C_2]] = \underline{\lambda} s.\mathbf{C}[[C_2]](\mathbf{C}[[C_1]]s)$
  $\mathbf{C}[[\textbf{if } B \textbf{ then } C]] = \underline{\lambda} s.\mathbf{B}[[B]]s \to \mathbf{C}[[C]]s \ [] \ s$
  $\mathbf{C}[[\textbf{if } B \textbf{ then } C_1 \textbf{ else } C_2]] =$
  $\underline{\lambda} s.\mathbf{B}[[B]]s \to \mathbf{C}[[C_1]]s \ [] \ \mathbf{C}[[C_2]]s$
  $\mathbf{C}[[I := E]] = \underline{\lambda} s.update[[I]]\ (\mathbf{E}[[E]]s)\ s$
  $\mathbf{C}[[\textbf{diverge}]] = \underline{\lambda} s.\perp$

- $\mathbf{E} : Expression \to Store \to Nat$
  $\mathbf{E}[[E_1 + E_2]] = \lambda s.\mathbf{E}[[E_1]]s\ plus\ \mathbf{E}[[E_2]]s$
  $\mathbf{E}[[I]] = \lambda s.access\ [[I]]\ s$
  $\mathbf{E}[[N]] = \lambda s.\mathbf{N}[[N]]$

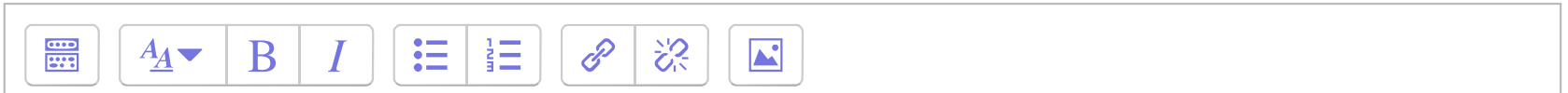
- $\mathbf{B} : Boolean\_expr \to Store \to Tr$
  $\mathbf{B}[[E_1 = E_2]] = \lambda s.\mathbf{E}[[E_1]]s\ equals\ \mathbf{E}[[E_2]]s$
  $\mathbf{B}[[\neg B]] = \lambda s.not(\mathbf{B}[[B]]s)$

- $\mathbf{N} : Numeral \to Nat$ *(omitted)*

i. What is a Valuation Function? **[2]**

ii. Based on the above definition solve the following: **[8]**

$$\mathbf{P}[[X := 5; Y := X\ +\ 1; \textbf{if } (A = 5) \textbf{ then diverge};\ Z := Y\ +\ X.]](one)$$



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QUIZ NAVIGATION

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Finish attempt ...