

FIRST SEMESTER B.Tech EXAMINATION, JANUARY - 2024

CALCULUS AND MATRIX ALGEBRA

Time: 3 Hours

Maximum Marks: 100

Instructions: i. Missing data may be suitably assumed.

ANSWER ALL QUESTIONS

PART - A			05 × 02=10		
1.	a.	Evaluate $\int_0^{\pi/2} \sin^7 x \cos^5 x dx$.	L-2	CO1	02
	b.	Calculate total differential du for the function $u = x^3 + xy^2 + x^2y + y^3$.	L-2	CO2	02
	c.	Find $\text{curl } \vec{v}$, where $\vec{v} = yz\hat{i} + 3xz\hat{j} + z\hat{k}$.	L-2	CO1	02
	d.	Identify the eigen values of matrix $A = \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$	L-2	CO1	02
	e.	Solve the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$	L-2	CO3	02
PART - B			06 × 05=30		
2.	a.	Evaluate $\int_0^{\pi} \frac{1-\cos\theta}{1+\cos\theta} \sin^2 \theta d\theta$	L-3	CO1	05
		OR			
	b.	Evaluate $\int_0^{\pi} x \sin^7 x dx$	L-3	CO1	05
3.	a.	Prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$, where $u = f(x-y, y-z, z-x)$.	L-3	CO2	05
		OR			
	b.	If $z = e^{ax+by} f(ax-by)$ then prove that $b \frac{\partial z}{\partial x} + a \frac{\partial z}{\partial y} = 2abz$	L-3	CO2	05
4.	a.	A particle moves along a curve whose parametric equations are $x = e^{-t}$, $y = 2\cos 3t$, $z = 2\sin 3t$ where t is the time. Find the velocity and acceleration at any time t and also their magnitudes at $t = 0$.	L-3	CO6	05
		OR			
	b.	Show that the vector point function $\vec{F} = (3x^2 - 2yz)\hat{i} + (3y^2 - 2zx)\hat{j} + (3z^2 - 2xy)\hat{k}$ is irrotational.	L-3	CO6	05
5.	a.	Solve the system of equations by Gauss-Seidel method $10x + y + z = 12, x + 10y + z = 12, \text{ and } x + y + 10z = 12$	L-3	CO1	05
		OR			

	b.	Find rank of the following matrix by reducing it to the row echelon form.	L-3	CO1	05
		$A = \begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$			
6.	a.	Solve the differential equation $4 \frac{d^4 y}{dx^4} - 4 \frac{d^3 y}{dx^3} - 23 \frac{d^2 y}{dx^2} + 12 \frac{dy}{dx} + 36y = 0$	L-3	CO5	05
		OR			
	b.	Solve the differential equation $(D^2 - 5D + 6)y = \sin 3x$	L-3	CO5	05
7.	a.	Write the MATLAB code to find $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial y^2}$ and $\frac{\partial^2 f}{\partial x \partial y}, \frac{\partial^2 f}{\partial y \partial x}$ where $f(x, y) = x^3 + 3xy - 15x^2 - 15y^2 + 72x - 4y$	L-3	CO1	05
		OR			
	b.	List out at least three Vector Built-in Functions in MATLAB and also explain its functionalities.	L-3	CO1	05
PART - C			06 × 10 = 60		
8.	a.	Using Beta Gamma function show that $\int_0^\infty x e^{-x^8} dx \times \int_0^\infty x^2 e^{-x^4} dx = \frac{\pi}{16\sqrt{2}}$	L-3	CO4	10
		OR			
	b.	Prove that $\int_0^{\pi/2} \sqrt{\sin \theta} d\theta \times \int_0^{\pi/2} \frac{1}{\sqrt{\sin \theta}} d\theta = \pi$.	L-3	CO4	10
9.	a.	Temperature T at any point (x, y, z) in the space is given by $T = 400xyz^2$. Find the highest temperature at the surface of unit sphere $x^2 + y^2 + z^2 = 1$.	L-3	CO6	10
		OR			
	b.	Investigate the extreme value of the function $f(x, y, z) = \cos x \cos y \cos z$, where x, y, z are the angles of the triangle.	L-3	CO6	10
10.	a.	Find unit tangent and unit normal vectors to the curve $\vec{r}(t) = \cos 2t \hat{i} + \sin 2t \hat{j} + t \hat{k}$ at $x = \frac{1}{\sqrt{2}}$.	L-3	CO6	10
		OR			

	b.	Prove that the vector point function $\vec{F} = (2xy^2 + yz)\hat{i} + (2x^2y + xz + 2yz^2)\hat{j} + (2y^2z + xy)\hat{k}$ is a conservative force field. Find its scalar potential ϕ such that $\nabla\phi = \vec{F}$.	L-3	CO6	10
11.	a.	Investigate the values of λ and μ such that the system of equations $x + y + z = 6; x + 2y + 3z = 10; x + 2y + \lambda z = \mu$ will have i. Unique solution ii. Infinite Solution iii. No solution.	L-3	CO3	10
		OR			
	b.	Find Eigen values and the corresponding Eigen vectors of the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$	L-3	CO3	10
12.	a.	Solve $X' = \begin{bmatrix} -5 & 1 \\ 4 & -2 \end{bmatrix} X$ by diagonalization and hence discuss the stability of the system	L-3	CO3	10
		OR			
	b.	Solve the differential equation $(D^2 - 4D + 1)y = e^{2x} \sin 2x$	L-3	CO5	10
13.	a.	Explain the significance of MATLAB also discuss its advantage and disadvantage. Write the MATLAB code to evaluate the integral $\int_0^\pi (\sin y - 5y^3) dy$	L-3	CO1	10
		OR			
	b.	Write the MATLAB code to create the matrix A and perform following operations for $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 3 \\ 2 & 1 & -1 \end{bmatrix}$ i. Determinant of A ii. Rank of A iii. Size of matrix A iv. Characteristic polynomial of the square matrix A v. Eigen values of A	L-3	CO3	10