

Bandits

CS 486 / 686

University of Waterloo


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Exploration/Exploitation Tradeoff

- Fundamental problem of RL due to the active nature of the learning process
- Consider one-state RL problems known as **bandits**



Stochastic Bandits


- Formal definition:
 - Single state: $S = \{s\}$
 - A : set of actions (also known as **arms**)
 - Space of rewards (typically assumed to be $[0,1]$)
- **No transition function to be learned** since there is a single state
- We simply need to **learn the  stochastic reward function**

Origin

- The term bandit comes from gambling where **slot machines can be thought as one-armed bandits.**
- Problem: which slot machine should we play at each turn when their payoffs are not necessarily the same and initially unknown?



Examples

- Design of experiments (Clinical Trials)
- Online ad selection 
- Games
- Networks (packet routing)

Online Ad Optimization

The screenshot displays the homepage of The Globe and Mail website. At the top, there is a navigation bar with the site's logo and a search bar. Below this, a large banner advertisement for IBM is visible, featuring the text "Can your business anticipate shifts in the marketplace?" and "Learn how to use Big Data and Analytics to get better business outcomes". The main content area is divided into several sections. On the left, there are two news articles: "Six Ontarians charged in alleged \$200-million investment fraud" and "122 'potential objects' spotted in ocean offer fresh jet lead". In the center, there is a video player showing a debate between candidates. On the right, there is a large yellow advertisement for Porter's plans, titled "ASK your Toronto City Councillor TO VOTE YES" and "on April 1 for Porter's plans". The bottom of the page features a navigation bar with links to various sections like News, Opinion, Business, etc.

Can your business anticipate shifts in the marketplace? Learn how to use Big Data and Analytics to get better business outcomes

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GLOBE UNLIMITED FLASH SALE SAVE 50% ON THE FIRST 6 MONTHS | OFFER ENDS MARCH 31ST ACCESS EVERYTHING GLOBEANDMAIL.COM HAS TO OFFER SEE MY OPTIONS

Six Ontarians charged in alleged \$200-million investment fraud

- WATCH Video: How to protect your bank account from fraud

122 'potential objects' spotted in ocean offer fresh jet lead

- WATCH Sailing the waters where Flight 370 went down

TORONTO Chow presses Ford to 'take down the circus tent' as candidates hammer each other in mayoral debate

ASK your Toronto City Councillor TO VOTE YES on April 1 for Porter's plans

porterporterplans.com Take Action

Online Ad Optimization

- Problem: **which ad should be presented?**
- Answer: present ad with highest payoff

$$\text{payoff} = \text{clickThroughRate} \times \text{payment}$$

- Click through rate: probability that user clicks on ad
- Payment: \$\$ paid by advertiser
 - Amount determined by an auction

Simplified Problem

- Assume payment is 1 unit for all ads
- Need to estimate click through rate
- Formulate as a bandit problem:
 - Arms: the set of possible ads
 - Rewards: 0 (no click) or 1 (click)
- In what order should ads be presented to maximize revenue?
 - How should we balance exploitation and exploration?

Simple yet difficult problem

- Simple: description of the problem is short
- Difficult: no known tractable optimal solution

Simple heuristics


- **Greedy strategy**: select the arm with the highest average so far
 - May get stuck in local optimum due to lack of exploration
- **ϵ -greedy**: select an arm at random with probability ϵ and otherwise do a greedy selection
 - Convergence rate depends on choice of ϵ

Regret

- Let $R(a)$ be the unknown average reward of a
- Let $r^* = \max_a R(a)$ and $a^* = \operatorname{argmax}_a R(a)$
- Denote by $loss(a)$ the **expected regret** of a
$$loss(a) = r^* - R(a)$$
- Denote by $Loss_n$ the **expected cumulative regret** for n time steps

$$Loss_n = \sum_{t=1}^n loss(a_t)$$

Theoretical Guarantees

- When ϵ is constant, then
 - For large enough t : $\Pr(a_t \neq a^*) \approx \epsilon$
 - Expected cumulative regret: $Loss_n = O(n)$
 - Linear regret
- When $\epsilon_t \propto 1/t$
 - For large enough t : $\Pr(a_t \neq a^*) \approx \epsilon_t = O\left(\frac{1}{t}\right)$
 - Expected cumulative regret: $Loss_n = O(\log n)$
 - Logarithmic regret 

$$Pr(a_k \neq a_*) \rightarrow \epsilon$$

$$\Delta = r^* - r_{min}$$

Loss in the constant ϵ .

$$Loss = \sum_{t=1}^n \epsilon \Delta = n\epsilon \Delta = O(n)$$

Loss in the decreasing ϵ .

$$Loss = \sum_{t=1}^n \epsilon_t \Delta = \sum_{t=1}^n \frac{1}{t} \Delta = \log \Delta = O(\log n)$$

Empirical mean

- Problem: how far is the empirical mean $\tilde{R}(a)$ from the true mean $R(a)$?
- If we knew that $|R(a) - \tilde{R}(a)| \leq bound$
 - Then we would know that $R(a) < \tilde{R}(a) + bound$
 - And we could select the arm with best $\tilde{R}(a) + bound$
- Overtime, additional data will allow us to refine $\tilde{R}(a)$ and compute a tighter *bound*.

Positivism in the Face of Uncertainty

- Suppose that we have an oracle that returns an **upper bound** $UB_n(a)$ on $R(a)$ for each arm based on n trials of arm a .
- Suppose the upper bound returned by this oracle converges to $R(a)$ in the limit:
 - i.e. $\lim_{n \rightarrow \infty} UB_n(a) = R(a)$
- **Optimistic algorithm**
 - At each step, **select** $\operatorname{argmax}_a UB_n(a)$

Convergence

- **Theorem:** An optimistic strategy that always selects $\operatorname{argmax}_a UB_n(a)$ will converge to a^*
- Proof by contradiction:
 - Suppose that we converge to suboptimal arm a after infinitely many trials.
 - Then $R(a) = UB_\infty(a) \geq UB_\infty(a') = R(a') \forall a'$
 - But $R(a) \geq R(a') \forall a'$ contradicts our assumption that a is suboptimal.

Probabilistic Upper Bound

- Problem: We can't compute an upper bound with certainty since we are sampling
- However we can obtain measures f that are upper bounds most of the time
 - i.e., $\Pr(R(a) \leq f(a)) \geq 1 - \delta$
 - Example: Hoeffding's inequality

$$\Pr\left(R(a) \leq \tilde{R}(a) + \sqrt{\frac{\log\left(\frac{1}{\delta}\right)}{2n_a}}\right) \geq 1 - \delta$$

where n_a is the number of trials for arm a

Upper Confidence Bound (UCB)

- Set $\delta_n = 1/n^4$ in Hoeffding's bound
- Choose a with highest Hoeffding bound

UCB(h)

$V \leftarrow 0, n \leftarrow 0, n_a \leftarrow 0 \quad \forall a$

Repeat until $n = h$

Execute $\operatorname{argmax}_a \tilde{R}(a) + \sqrt{\frac{2 \log n}{n_a}}$

Receive r

$V \leftarrow V + r$

$\tilde{R}(a) \leftarrow \frac{n_a \tilde{R}(a) + r}{n_a + 1}$

$n \leftarrow n + 1, n_a \leftarrow n_a + 1$

Return V

UCB Convergence

- Theorem: Although Hoeffding's bound is probabilistic, **UCB converges**
- Proof: As n increases, the term $\sqrt{\frac{2 \log n}{n_a}}$ increases, ensuring that all arms are tried infinitely often
- Expected cumulative regret: **$Loss_n = O(\log n)$**
 - **Logarithmic regret**

Summary

- Stochastic bandits
 - Exploration/exploitation tradeoff
- ϵ -greedy and UCB
 - Theory: logarithmic expected cumulative regret
- In practice:
 - UCB often performs better than ϵ -greedy
 - Many variants of UCB improve performance