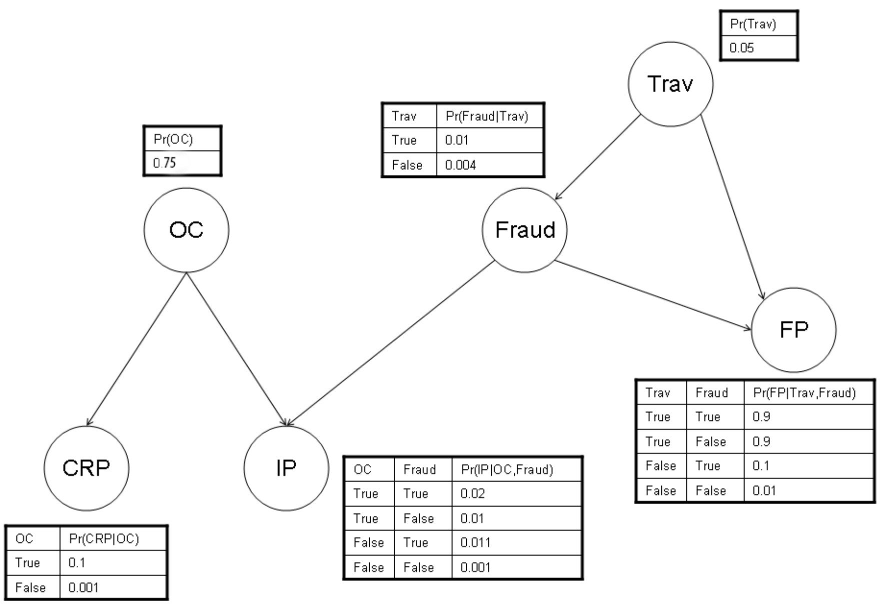
# CS 486/686 Spring 2015
# Assignment 2 Solutions

June 15, 2015

## Question 2a




| Pr(Trav) |
|---|
| 0.05 |

| Trav | Pr(Fraud\|Trav) |
|---|---|
| True | 0.01 |
| False | 0.004 |

| Pr(OC) |
|---|
| 0.75 |

| Trav | Fraud | Pr(FP\|Trav,Fraud) |
|---|---|---|
| True | True | 0.9 |
| True | False | 0.9 |
| False | True | 0.1 |
| False | False | 0.01 |

| OC | Fraud | Pr(IP\|OC,Fraud) |
|---|---|---|
| True | True | 0.02 |
| True | False | 0.01 |
| False | True | 0.011 |
| False | False | 0.001 |

| OC | Pr(CRP\|OC) |
|---|---|
| True | 0.1 |
| False | 0.001 |

## Question 2b

### Part I - Prior Probability

$$\begin{aligned} Pr(fraud) &= Pr(fraud|trav)Pr(trav) + Pr(fraud|\neg trav)Pr(\neg trav) \\ &= 0.01 * 0.05 + 0.004 * 0.95 \\ &= 0.0043 \end{aligned}$$

### Part II - Posterior Probability

| $OC$ | $f_1(OC) = Pr(OC)$ |
|---|---|
| t | 0.75 |
| f | 0.25 |

| $Fraud$ | $Trav$ | $f_2(Fraud, Trav) = Pr(Fraud|Trav)$ |
|---|---|---|
| t | t | 0.01 |
| t | f | 0.004 |
| f | t | 0.99 |
| f | f | 0.996 |

| $Trav$ | $f_3(Trav) = Pr(Trav)$ |
|---|---|
| t | 0.05 |
| f | 0.95 |

| $OC$ | $f_4(OC) = Pr(crp|OC)$ |
|---|---|
| t | 0.1 |
| f | 0.001 |

| $OC$ | $Fraud$ | $f_5(OC, Fraud) = Pr(\neg ip|OC, Fraud)$ |
|---|---|---|
| t | t | 0.98 |
| t | f | 0.99 |
| f | t | 0.989 |
| f | f | 0.999 |

| $Trav$ | $Fraud$ | $f_6(Trav, Fraud) = Pr(fp|Trav, Fraud)$ |
|---|---|---|
| t | t | 0.9 |
| t | f | 0.9 |
| f | t | 0.1 |
| f | f | 0.01 |

Eliminate variable $Trav$:

$$f_7(Fraud) = sumout_{Trav}[f_2(Fraud, Trav) * f_3(Trav) * f_6(Trav, Fraud)]$$

| $Fraud$ | $f_7(Fraud)$ |
|---|---|
| t | 0.0008 |
| f | 0.0540 |

Eliminate variable $OC$:

$$f_8(Fraud) = sumout_{OC}[f_1(OC) * f_4(OC) * f_5(OC, Fraud)]$$

| $Fraud$ | $f_8(Fraud)$ |
|---|---|
| t | 0.0737 |
| f | 0.0745 |

$$Pr(fraud|fp, \neg ip, crp) = k * f_7(fraud) * f_8(fraud) = 0.0150$$

where $k$ is a normalizing constant:

$$k = \frac{1}{f_7(fraud) * f_8(fraud) + f_7(\neg fraud) * f_8(\neg fraud)}$$

## Question 2c

| $OC$ | $f_1(OC) = Pr(OC)$ |
|---|---|
| t | 0.75 |
| f | 0.25 |

| $Fraud$ | $f_2(Fraud) = Pr(Fraud|trav)$ |
|---|---|
| t | 0.01 |
| f | 0.99 |

| $Trav$ | $f_3() = Pr(trav)$ |
|---|---|
| | 0.05 |

| $OC$ | $f_4(OC) = Pr(crp|OC)$ |
|---|---|
| t | 0.1 |
| f | 0.001 |

| $OC$ | $Fraud$ | $f_5(OC, Fraud) = Pr(\neg ip|OC, Fraud)$ |
|---|---|---|
| t | t | 0.98 |
| t | f | 0.99 |
| f | t | 0.989 |
| f | f | 0.999 |

| $Fraud$ | $f_6(Fraud) = Pr(fp|trav, Fraud)$ |
|---|---|
| t | 0.9 |
| f | 0.9 |

Eliminate variable $OC$:

$$f_7(Fraud) = sumout_{OC}[f_1(OC) * f_4(OC) * f_5(OC, Fraud)]$$

| $Fraud$ | $f_7(Fraud)$ |
| --- | --- |
| t | 0.0737 |
| f | 0.0745 |

$$Pr(fraud|fp, \neg ip, crp, trav) = k * f_2(fraud) * f_3() * f_6(fraud) * f_7(fraud) = 0.0099$$

where $k$ is a normalizing constant:

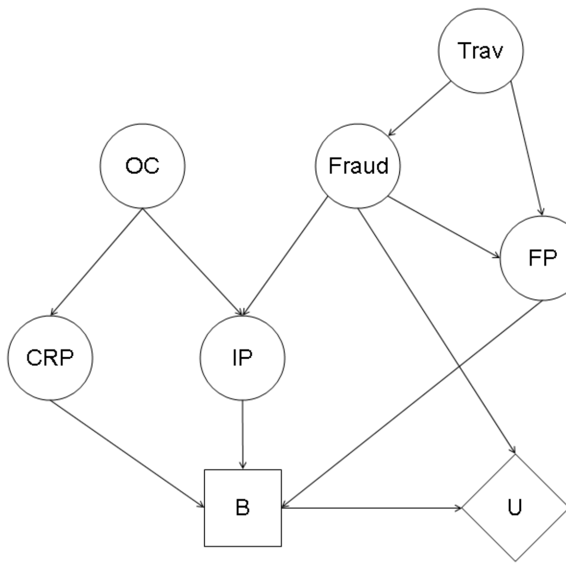
$$k = \frac{1}{\sum_{Fraud} f_2(Fraud) * f_3() * f_6(Fraud) * f_7(Fraud)}$$

# Question 2d

When an internet purchase is made, the fraud detection system is likely to believe that the transaction is fraudulent unless it has reasons to believe that the card holder owns a computer. Therefore, an ingenious thief could simply make a computer related purchase first to fool the fraud detection system into believing the card holder owns a computer. After that, the thief can make the intended internet purchase with a reduced risk of being rejected.

One can verify that the probability of a fraudulent transaction decreases when a computer related purchase is observed since $Pr(fraud|ip) = 0.0098$ whereas $Pr(fraud|ip, crp) = 0.0086$.

## Question 3a




| Fraud | B | U |
|---|---|---|
| True | True | 0 |
| True | False | -1000 |
| False | True | -10 |
| False | False | 5 |

## Question 3b

Probabilities taken from 2b:

$$\begin{aligned}
EU(b|crp, fp, \neg ip) &= Pr(fraud|crp, fp, \neg ip) * U(fraud, b) \\
&\quad + Pr(\neg fraud|crp, fp, \neg ip) * U(\neg fraud, b) \\
&= 0.015 * 0 + 0.985 * -10 \\
&= -9.85
\end{aligned}$$

$$\begin{aligned}
EU(\neg b|crp, fp, \neg ip) &= Pr(fraud|crp, fp, \neg ip) * U(fraud, \neg b) \\
&\quad + Pr(\neg fraud|crp, fp, \neg ip) * U(\neg fraud, \neg b) \\
&= 0.015 * -1000 + 0.985 * 5 \\
&= -10.075
\end{aligned}$$

The optimal policy is therefore to block the transaction.

## Question 3c

Let $evid$ be the evidence $crp, fp, \neg ip$.

The value of information gained is the difference between the EU of the optimal policy when we do not have any information about $Trav$, and the EU of the optimal policy when we do have some information about $Trav$.

The EU of the optimal policy with no information about $Trav$ is:

$$\max_B EU(B|evid) = -9.85 \qquad \text{(from 3b)}$$

If we call to verify, the card holder would either be travelling with probability $Pr(trav|evid)$, or not with probability $Pr(\neg trav|evid)$. The EU of the optimal policy when we do call is therefore given by:

$$\sum_{Trav} \max_B EU(B|evid, Trav) Pr(Trav|evid)$$

The expected value of information gained is therefore given by:

$$\begin{aligned}
&\sum_{Trav} \max_B EU(B|evid, Trav) Pr(Trav|evid) - \max_B EU(B|evid) \\
&\quad = \sum_{Trav} \max_B EU(B|evid, Trav) Pr(Trav|evid) - -9.85 \\
&\quad = -5.787 - -9.85 \qquad \text{(see eq 2 below)} \\
&\quad = 4.063
\end{aligned} \tag{1}$$

$$
\begin{aligned}
\sum_{Trav} & \max_B EU(B|evid, Trav) * Pr(Trav|evid) \\
&= \max_B EU(B|evid, trav) * Pr(trav|evid) \\
&\quad + \max_B EU(B|evid, \neg trav) * Pr(\neg trav|evid) \\
&= -4.062 + -1.725 \qquad \text{(see eqs 3-6 below)} \\
&= -5.787
\end{aligned} \tag{2}
$$

$$
\begin{aligned}
& EU(b|evid, trav) * Pr(trav|evid) \\
&\quad = [Pr(fraud|evid, trav) * U(fraud, b) \\
&\quad\quad + \Pr(\neg fraud|evid, trav) * U(\neg fraud, b)] \\
&\quad\quad * Pr(trav|evid) \\
&\quad = [0.0099 * 0 + 0.9901 * -10] * 0.8206 \qquad \text{(computations not shown)} \\
&\quad = -8.125
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
& EU(\neg b|evid, trav) * Pr(trav|evid) \\
&\quad = [Pr(fraud|evid, trav) * U(fraud, \neg b) \\
&\quad\quad + \Pr(\neg fraud|evid, trav) * U(\neg fraud, \neg b)] \\
&\quad\quad * Pr(trav|evid) \\
&\quad = [0.0099 * -1000 + 0.9901 * 5] * 0.8206 \qquad \text{(computations not shown)} \\
&\quad = -4.062
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
& EU(b|evid, \neg trav) * Pr(\neg trav|evid) \\
&\quad = [Pr(fraud|evid, \neg trav) * U(fraud, b) \\
&\quad\quad + \Pr(\neg fraud|evid, \neg trav) * U(\neg fraud, b)] \\
&\quad\quad * Pr(\neg trav|evid) \\
&\quad = [0.0382 * 0 + 0.9618 * -10] * 0.1794 \qquad \text{(computations not shown)} \\
&\quad = -1.725
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
& EU(\neg b|evid, \neg trav) * Pr(\neg trav|evid) \\
&\quad = [Pr(fraud|evid, \neg trav) * U(fraud, \neg b) \\
&\quad\quad + \Pr(\neg fraud|evid, \neg trav) * U(\neg fraud, \neg b)] \\
&\quad\quad * Pr(\neg trav|evid) \\
&\quad = [0.0382 * -1000 + 0.9618 * 5] * 0.1794 \qquad \text{(computations not shown)} \\
&\quad = -5.990
\end{aligned} \tag{6}
$$