Probabilistic logic and statistical inference

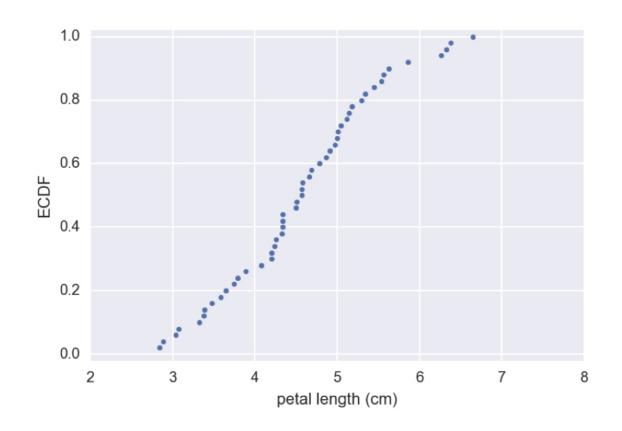
STATISTICAL THINKING IN PYTHON (PART 1)

Justin Bois

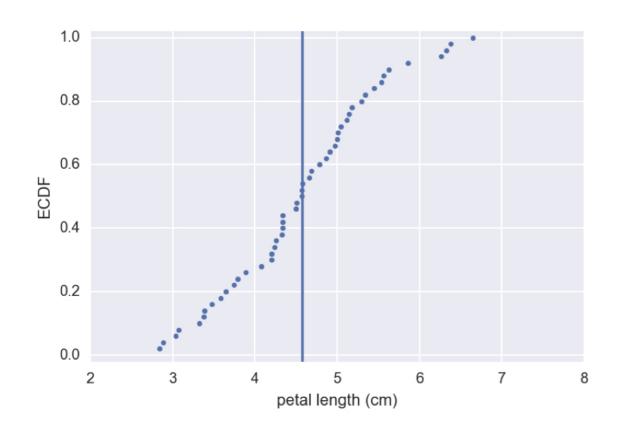
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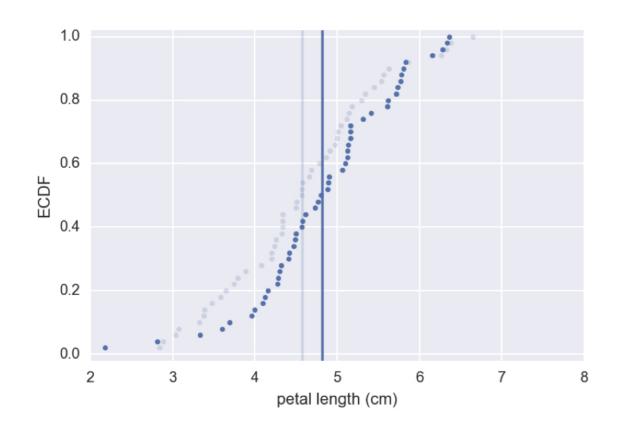




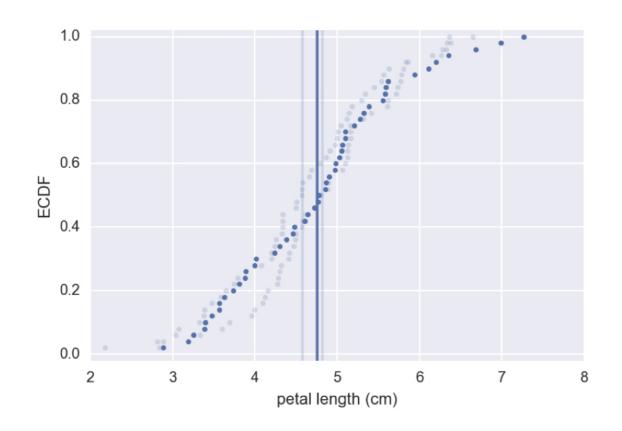




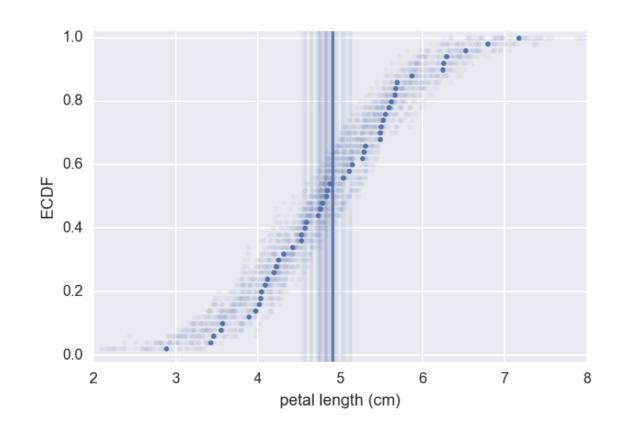




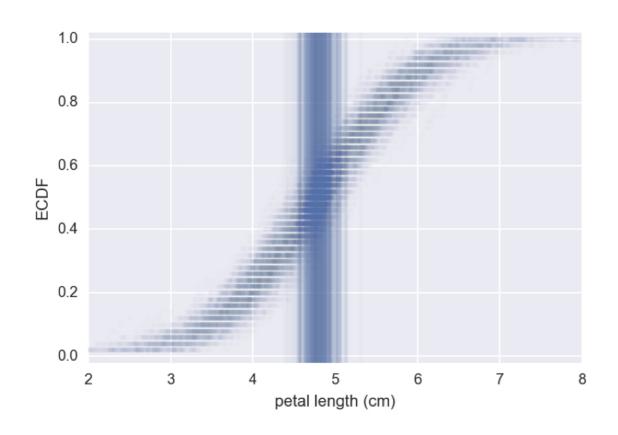








Repeats of 50 measurements of petal length



Let's practice!

STATISTICAL THINKING IN PYTHON (PART 1)



Random number generators and hacker statistics

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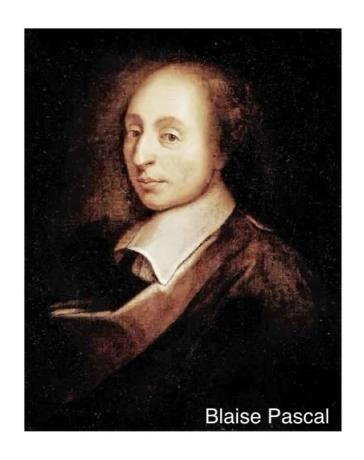
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Hacker statistics

• Uses simulated repeated measurements to compute probabilities.



¹ Image: artist unknown











¹ Image: Heritage Auction

Simulating coin flips



Bernoulli trials

• An experiment that has two options, "success" (True) and "failure" (False).

The np.random module

```
import numpy as np
rng = np.random.default_rng()
rng
```

Generator(PCG64) at 0x7F9433D38120

Random number seed

- Integer fed into random number generating algorithm
- Manually seed random number generator (only) if you need reproducibility
- Specified using rng = np.random.default_rng(seed)

Simulating 4 coin flips

```
rng = np.random.default_rng(42)
random_numbers = rng.random(size=4)
random_numbers
array([0.77395605, 0.43887844, 0.85859792, 0.69736803])
heads = random_numbers < 0.5
heads
array([False, True, False, False])
np.sum(heads)
```



Simulating 4 coin flips

```
n_all_heads = 0 # Initialize number of 4-heads trials
for _ in range(10000):
    heads = np.random.random(size=4) < 0.5
    n_heads = np.sum(heads)
    if n_heads == 4:
        n_all_heads += 1</pre>
n_all_heads / 10000
```

0.0607

Hacker stats probabilities

- Determine how to simulate data
- Simulate many many times
- Probability is approximately fraction of trials with the outcome of interest

Let's practice!

STATISTICAL THINKING IN PYTHON (PART 1)



Probability distributions and stories: The Binomial distribution

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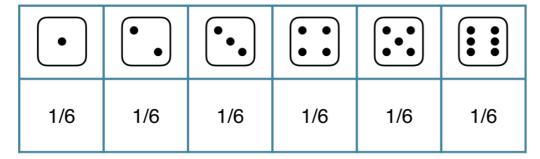
Probability mass function (PMF)

• The set of probabilities of discrete outcomes



Discrete Uniform PMF

Tabular



Graphical



Probability distribution

• A mathematical description of outcomes



Discrete Uniform distribution: the story

The outcome of rolling a single fair die is

- Discrete
- Uniformly distributed.

Binomial distribution: the story

- The number r of successes in n Bernoulli trials with probability p of success, is Binomially distributed
- The number r of heads in 4 coin flips with probability 0.5 of heads, is Binomially distributed

Sampling from the Binomial distribution

```
rng.binomial(4, 0.5)
```

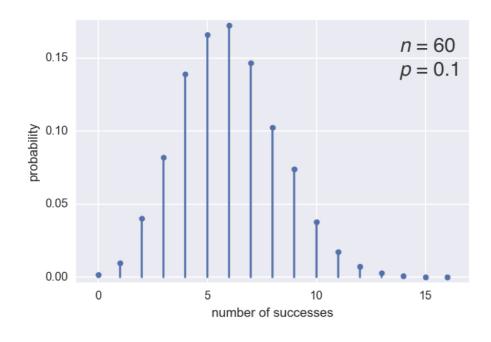
2

```
rng.binomial(4, 0.5, size=10)
```

```
array([4, 3, 2, 1, 1, 0, 3, 2, 3, 0])
```

The Binomial PMF

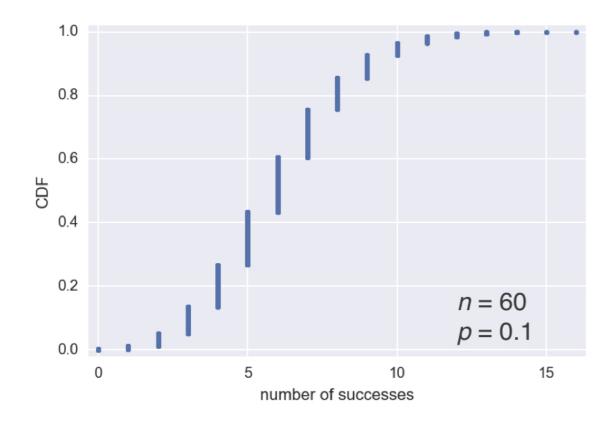
```
samples = rng.binomial(60, 0.1, size=10000)
n = 60
p = 0.1
```



The Binomial CDF

```
import matplotlib.pyplot as plt
import seaborn as sns
sns.set()
x, y = ecdf(samples)
_ = plt.plot(x, y, marker='.', linestyle='none')
plt.margins(0.02)
  = plt.xlabel('number of successes')
_ = plt.ylabel('CDF')
plt.show()
```

The Binomial CDF



Let's practice!

STATISTICAL THINKING IN PYTHON (PART 1)



Poisson processes and the Poisson distribution

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Poisson process

 The timing of the next event is completely independent of when the previous event happened

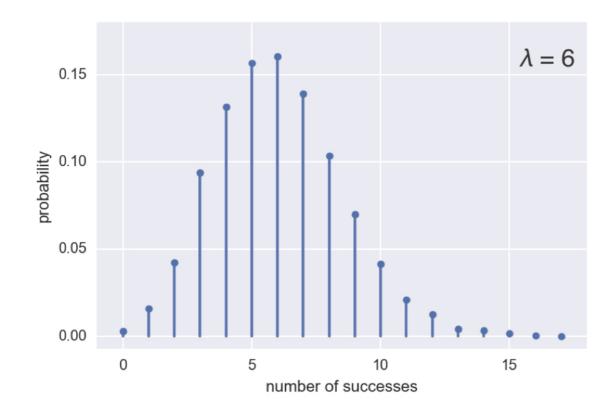
Examples of Poisson processes

- Natural births in a given hospital
- Hits on a website during a given hour
- Meteor strikes
- Molecular collisions in a gas
- Aviation incidents
- Buses in Poissonville

Poisson distribution

- The number r of arrivals of a Poisson process in a given time interval with average rate of lambda arrivals per interval is Poisson distributed.
- The number r of hits on a website in one hour with an average hit rate of 6 hits per hour is Poisson distributed.

Poisson PMF

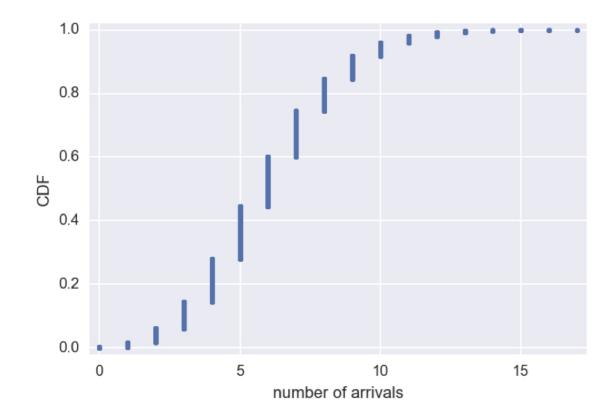


Poisson Distribution

- Limit of the Binomial distribution for low probability of success and large number of trials.
- That is, for rare events.

The Poisson CDF

The Poisson CDF



Let's practice!

STATISTICAL THINKING IN PYTHON (PART 1)

