Generative learning algorithms

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Discriminant learning algorithms and generative learning algorithm

- So far, we build a model

$$p(y|x)$$

- Generative learning algorithm

$$p(x|y)$$

Bayes rule

- The posterior distribution:

$$p(y|x) = \frac{p(x|y)p(y)}{p(x)}$$

  - (class) prior: $p(y)$
  - total

$$p(x) = p(x|y=0)p(y=0) + p(x|y=1)p(y=1)$$

Probability basics

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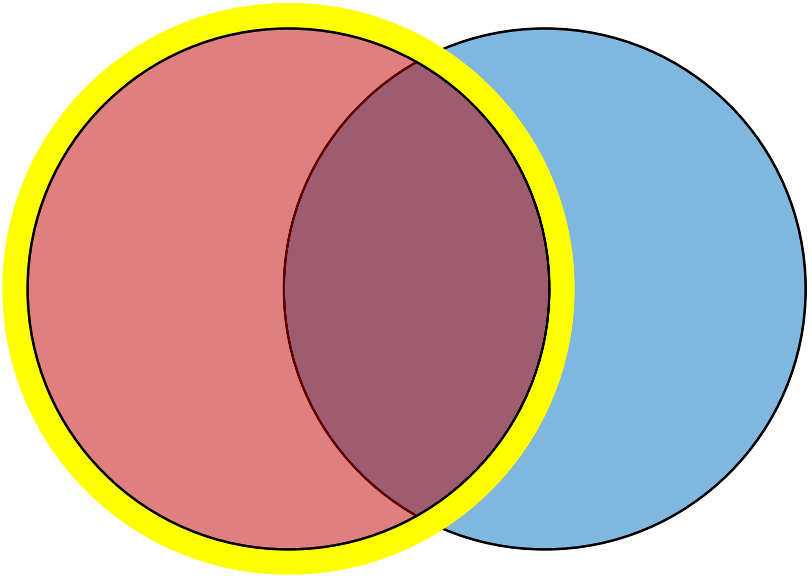

$P(X)$

- Single event probability:

$$P(X)$$

Probability basics (continued)

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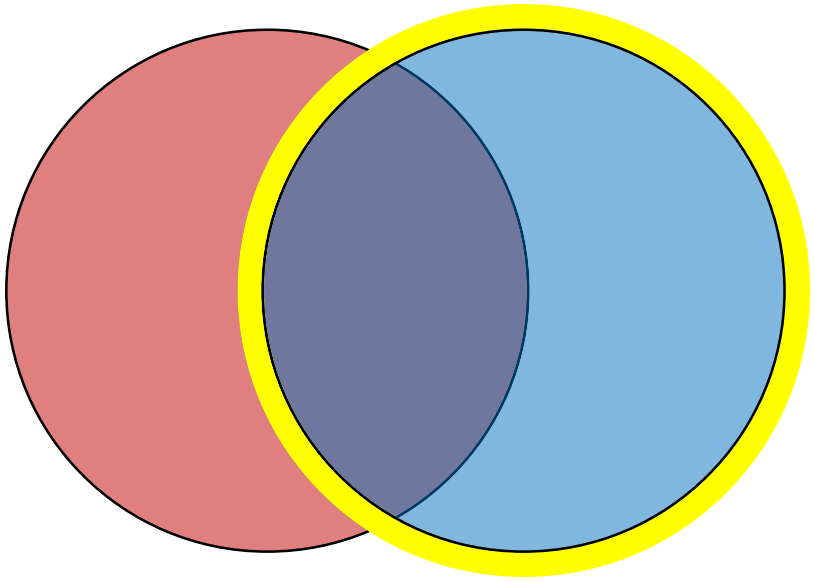

- Single event probability:

  $P(X), P(Y)$

Probability basics (continued)

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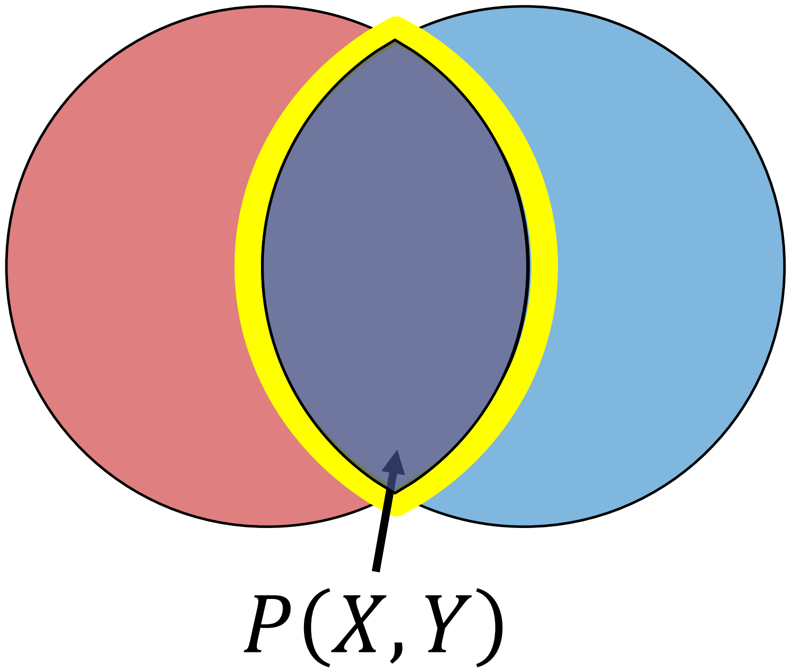

- Single event probability:

$$P(X), P(Y)$$

- Joint event probability:

$$P(X,Y)$$

Probability basics (continued)

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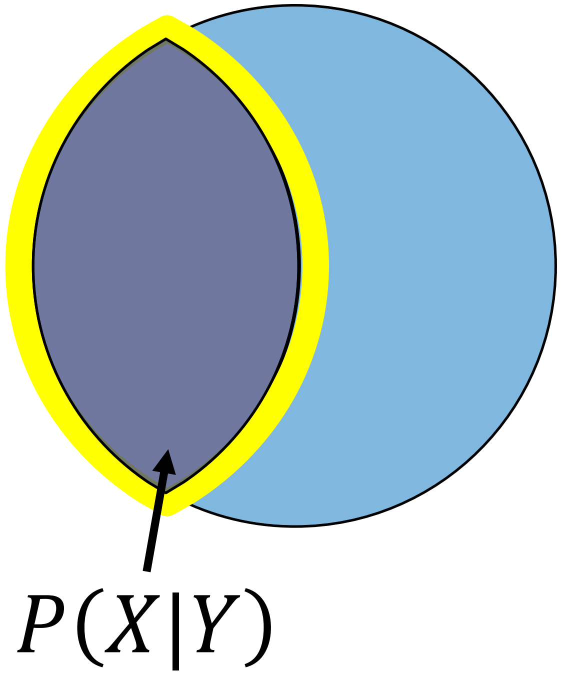

- Single event probability:

$$P(X), P(Y)$$

- Joint event probability:

$$P(X, Y)$$

- Conditional probability:

$$P(X|Y)$$

Probability basics (continued)

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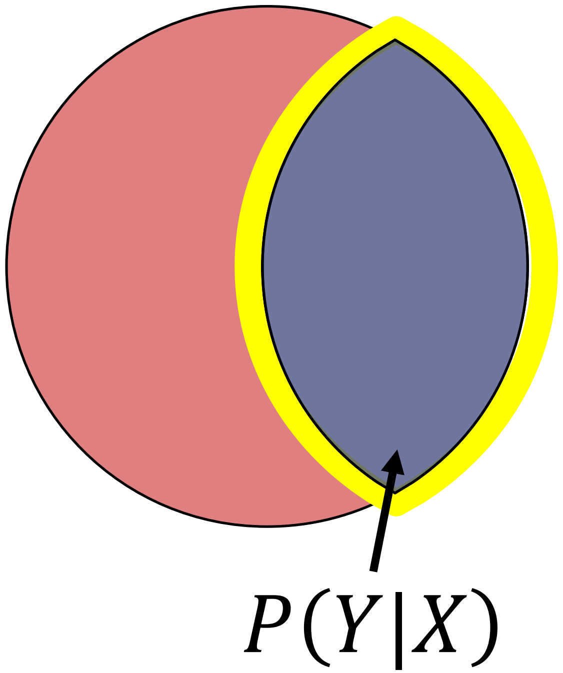

- Single event probability:

$$P(X), P(Y)$$

- Joint event probability:

$$P(X, Y)$$

- Conditional probability:

$$P(X|Y), P(Y|X)$$

Probability basics (continued)

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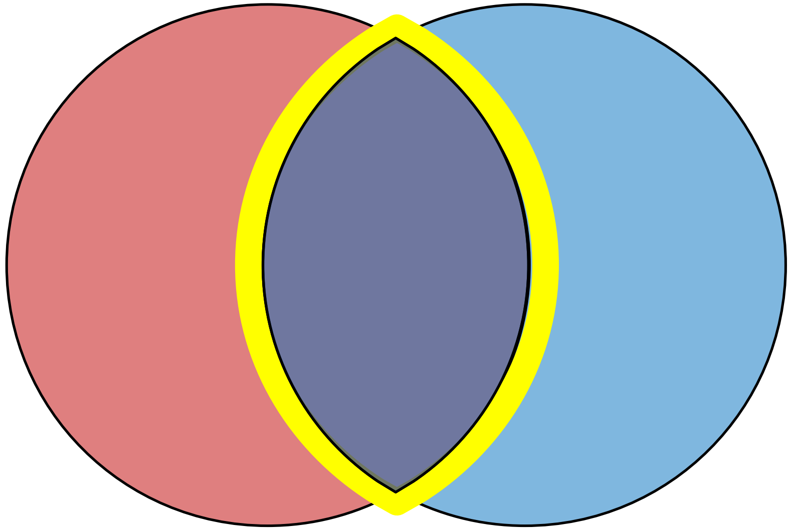

- Single event probability:

$$P(X), P(Y)$$

- Joint event probability:

$$P(X, Y)$$

- Conditional probability:

$$P(X|Y), P(Y|X)$$

- Joint and conditional relationship:

$$P(X, Y) = P(Y|X) * P(X) = P(X|Y) * P(Y)$$

Bayes theorem derivation

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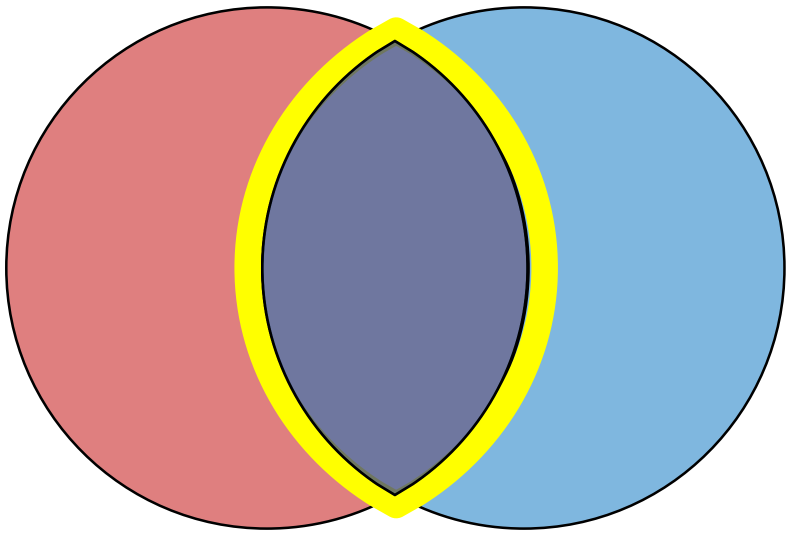

- By conditional and joint relationship:

$$P(Y|X) * P(X) = P(X|Y) * P(Y)$$

Bayes theorem derivation (continued)

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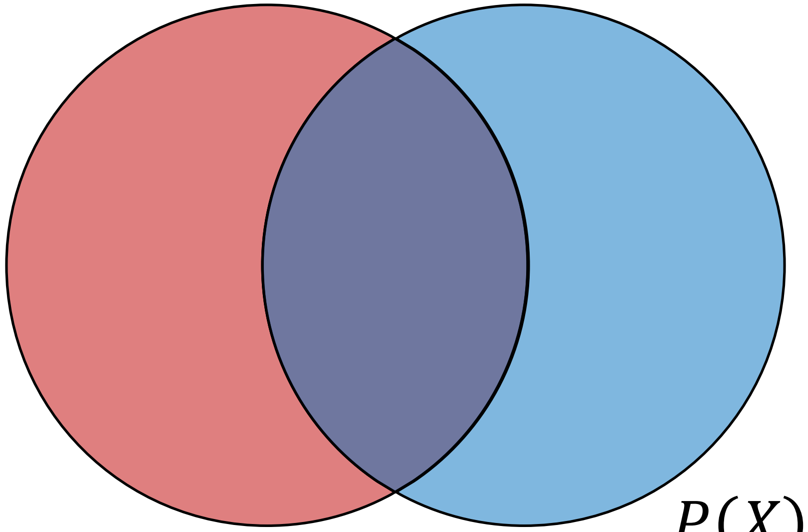


- Use conditional and joint relationship:

$$P(Y|X) * P(X) = P(X|Y) * P(Y)$$

- To invert conditional probability:

$$P(Y|X) = \frac{P(X|Y) * P(Y)}{\boxed{P(X)}}$$

$$P(X) = \sum_{Z} P(X,Z) = \sum_{Z} P(X|Z) * P(Z)$$

Your first generative learning algorithm

- Assume $p(x|y)$ is distributed according to a multivariate normal distribution
  - Multivariate normal distribution
    - Notebook

Gaussian discriminant analysis model

- Binary classification
  - Class prior

$$y \sim \text{Bernoulli}(\pi)$$

$$p(y) = \pi^y (1-\pi)^{(1-y)}$$

  - Likelihood

$$p(x|y=0) = \frac{1}{(2\pi)^{\frac{d}{2}} |\Sigma|^{\frac{1}{2}}} e^{\left(-\frac{1}{2}(x-\mu_0)^\intercal \Sigma^{-1} (x-\mu_0)\right)}$$

$$p(x|y=1) = \frac{1}{(2\pi)^{\frac{d}{2}} |\Sigma|^{\frac{1}{2}}} e^{\left(-\frac{1}{2}(x-\mu_1)^\intercal \Sigma^{-1} (x-\mu_1)\right)}$$

Gaussian discriminant analysis model (continued)

- Log-likelihood:

$$\ell(\pi, \mu_0, \mu_1, \Sigma) = \log \prod_{i=1}^{n_{\mathrm{train}}} p(x^{(i)}, y^{(i)}; \pi, \mu_0, \mu_1, \Sigma)$$

$$= \log \prod_{i=1}^{n_{\mathrm{train}}} p(x^{(i)} | y^{(i)}; \mu_0, \mu_1, \Sigma) p(y^{(i)}; \pi)$$

Gaussian discriminant analysis model (continued)

- Parameter estimate:
  - Parameter for prior

$$\pi = \frac{\sum_{i=1}^{n_{\text{train}}} \mathbb{1}[y^{(1)} = 1]}{n_{\text{train}}}$$

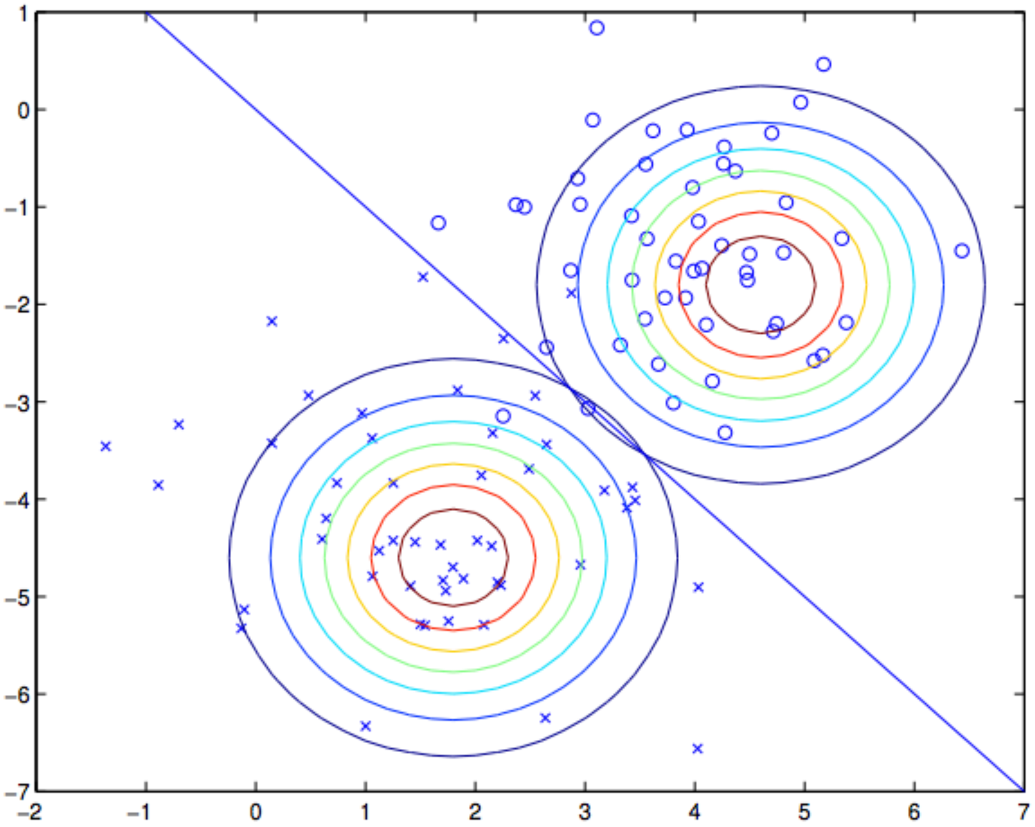
  - Parameters for posterior

$$\mu_0 = \frac{\sum_{i=1}^{n_{\text{train}}} \mathbb{1}[y^{(1)} = 0]x^{(i)}}{\sum_{i=1}^{n_{\text{train}}} \mathbb{1}[y^{(1)} = 0]} \qquad \mu_1 = \frac{\sum_{i=1}^{n_{\text{train}}} \mathbb{1}[y^{(1)} = 1]x^{(i)}}{\sum_{i=1}^{n_{\text{train}}} \mathbb{1}[y^{(1)} = 1]}$$

$$\Sigma = \frac{1}{n_{\text{train}}} \sum_{i=1}^{n_{\text{train}}} (x^{(i)} - \mu_{y^{(i)}})(x^{(i)} - \mu_{y^{(i)}})^{\mathsf{T}}$$

GDA model

- Decision boundary – linear discriminant analysis



GDA model – quadratic discriminant analysis

- LDA and QDA