Individual Round

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1 Individual — Division A

This is an **individual** round. You will have 30 minutes to solve these 20 questions. **Only pencils, pen, and paper are allowed.** You may not use help from any outside source.

Problem 1. Atticus has a sock drawer which contains twenty red socks, twenty blue socks, and twenty green socks. Without looking, Atticus selects socks from his drawer at random. What is the least number of socks Atticus needs to take to guarantee that he takes a pair of some color?

Problem 2. Sunay rolls two fair standard dice. What is the probability that both dice show a prime number on the top face?

Problem 3. Equilateral $\triangle ABC$ has a perimeter of 12. What is its area?

Problem 4. Let $f(x) = 2x + 5$ and $g(x) = x^2$. Find $g(f(1)) - f(g(1))$.

Problem 5. A frog is climbing up a well with height 10 feet. Each day, he jumps up 4 feet, and each night, he slips down 3 feet. If he starts climbing on Monday, on what day will he reach the top?

Problem 6. How many ways are there to walk from $(0, 0)$ to $(3, 3)$ if, at each step, you only go up or to the right?

Problem 7. Two real numbers x and y are randomly selected from the interval $[0, 2]$. What is the probability that $x + y > 1$?

Problem 8. How many positive cubes are factors of $3! \cdot 5! \cdot 7!$?

Problem 9. Eric rolls a standard fair die. If he rolls an x, what is the expected value of x^2?

Problem 10. Triangle $\triangle ABC$ on the coordinate plane satisfies $A = (0, 0)$,

$B = (0, 2\sqrt{3})$, and $C = (2, 0)$. Triangle $\triangle ABC$ is then reflected about the vertical line $x = a$, where $a > 1$. If the area of intersection between the old and new triangle is $\frac{\sqrt{3}}{36}$, find a. Express your answer as a common fraction.

Problem 11. Let p be a prime. Find the sum of all possible values of $\gcd(p, 1001)$, where $\gcd(a, b)$ is the greatest positive integer d such that a and b are both multiples of d.

Problem 12. Suppose that $a + b = 6$ and $a^2 - b^2 = 24$. Find a.

Problem 13. Katherine is listening to Papa Roach again. Her favorite song is "Last Resort". In how many ways can the letters in Katherine's favorite song be ordered such that the L is not next to an R?

Problem 14. If $10^{\frac{3}{x}} = x$, find $x \log_{10} x$.

Problem 15. A standard deck of cards contains 13 cards in each of 4 suits. Find the probability that a randomly drawn card is a 2 or a club.

Problem 16. If

$$\begin{aligned} \frac{5xy}{x+y} &= 1, \\ \frac{yz}{y+z} &= \frac{1}{7}, \text{ and} \\ \frac{2zx}{z+z} &= \frac{1}{3}, \end{aligned}$$

find $x + y + z$. Express your answer as a common fraction.

Problem 17. How many zeroes are at the end of the number

$$115 \cdot 116 \cdot 117 \ldots 201?$$

Problem 18. Triangle $\triangle ABC$ has sides $AB = 13$, $BC = 14$, and $AC = 15$. Point D is on $\overline{BC}$ such that AD bisects $\angle BAC$. If $DC = x$ and the area of $\triangle ABC$ is K, compute $\frac{x}{k}$. Express your answer as a common fraction.

Problem 19. In triangle ABC, $AB = 4$, $BC = 5$, and $AC = 3$. Point D is on BC such that $BD = BC$, and point E is on AC such that $\frac{AE}{AC} = \frac{1}{3}$. If AD meets BE at X, and CX intersects AB at F, find BF.

Problem 20. Suppose that a, b, c, and d are real numbers that satisfy

$$\begin{aligned} a + 2b + 3c + 4d &= 25, \text{ and} \\ 2a^2 + 4b^2 + 6c^2 + 8d^2 &= 125. \end{aligned}$$

Compute $a^2 + 4b^2 + 9c^2 + 16d^2$.

Tiebreaker Question. What is the average value of all answers to this question?