Measures Of Central Tendency

central value

middle

Measures of Central Tendency

- One of the important objectives of statistical analysis is to get one single value that describes the characteristic of entire mass of selected data
- Such value is called as “Central Value” or “Average” or expected value of the variable
- Average
  - Average is an attempt to find one single figure to describe the whole of figures
  - Average is a single value selected from a group of values to represent them in some way
  - Average is sometimes described as a number which is typical of the whole group
- Objectives of averaging
  - To get single value that describes the characteristics of the entire group
  - To facilitate comparison

Requisites of good average

- Easy to understand
- Simple to compute
- Based on all the items *
- Not be unduly affected by extreme observations

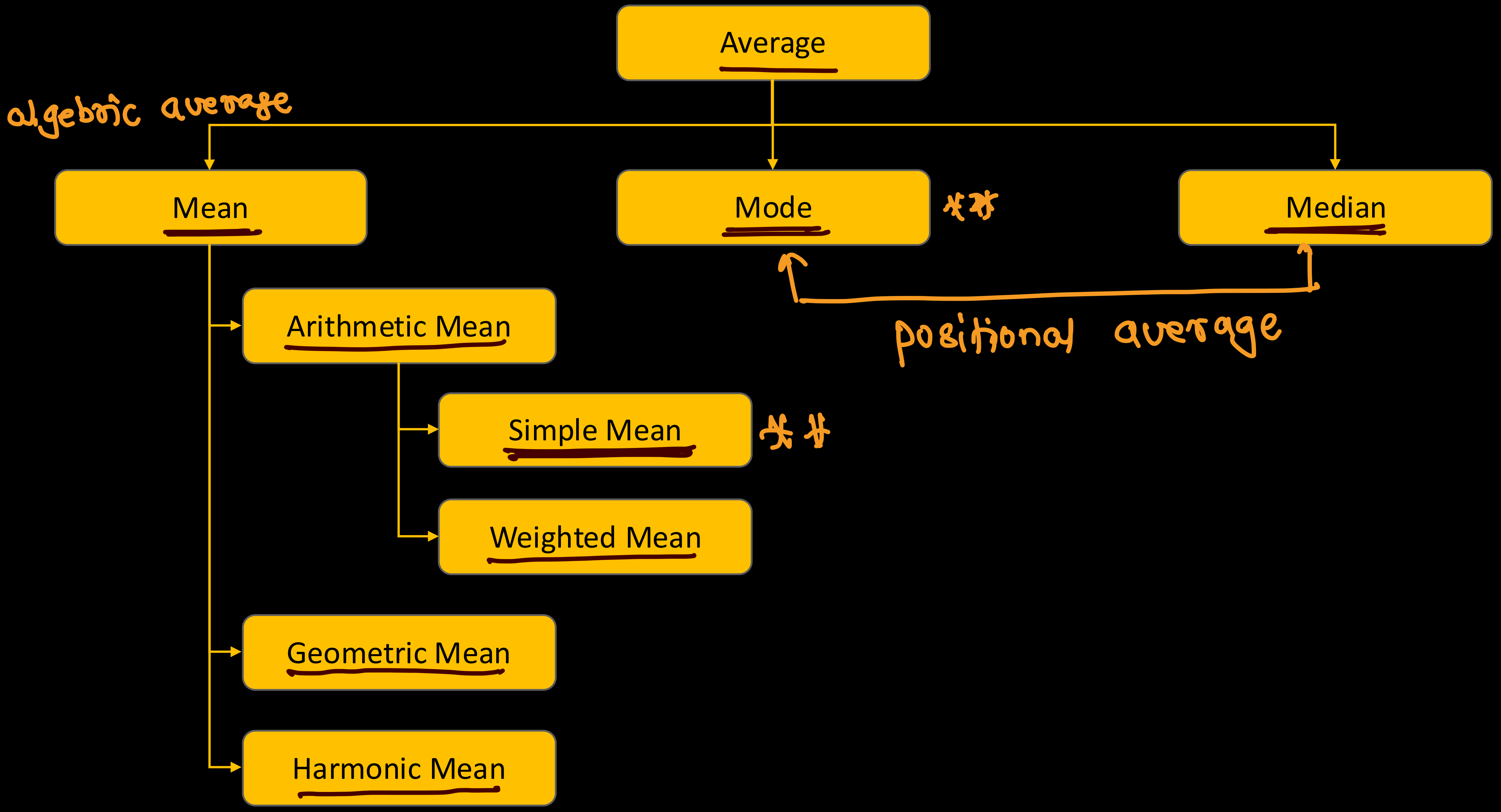
outliers

- Rigidly defined
- Capable of further algebraic treatment
- Sampling stability

25, 28, 27, 25, 26, 0

outlier

Types of Averages



Mean

algebric average

Simple Arithmetic Mean – Individual Series

- Direct method
- Steps
  - Add all the observations together and obtain the total $\sum X$
  - Divide the total by number of observations

marks = 40, 41, 42, 45,

mean = $\frac{40 + 41 + 42 + 45}{4} = \frac{168}{4} = 42$

$\boxed{\text{mean } (\bar{x}) = 42}$

$$\bar{X} = \frac{X1 + X2 + X3 \ \ldots + Xn}{N}$$

OR

$$\bar{X} = \frac{\sum X}{N}$$

Simple Arithmetic Mean – Individual Series

- Shortcut method (Using Assumed Mean)
- Steps
  - Take an assumed mean and denote it as A
  - Take the deviations of items from assumed mean and denote them by d
  - Obtain the sum of these deviations i.e. $\sum d$
  - Apply the formula

$$\bar{X} = A + \frac{\sum d}{N}$$

Simple Arithmetic Mean – Individual Series

- Following are the monthly income of 10 employees in an office
  - 14780, 15760, 26690, 27750, 24840, 24920, 16100, 17810, 27050, 16950
- Calculate arithmetic mean of income

Simple Arithmetic Mean – Discrete Series

- Direct method
- Steps
  - Multiply the frequency of each row with the variable and obtain the total $\sum fX$
  - Divide the total by number of observation that is the total frequency

$$\bar{X} = \frac{\sum fX}{N}$$

- Where
  - f = frequency
  - X = observations
  - N = total frequency

Simple Arithmetic Mean – Discrete Series

- Shortcut method - Using Assumed mean
- Steps
  - Take an assumed mean and denote it by A
  - Take the deviations of the variable X from the assumed mean and denote the deviations by d
  - Multiply this deviation by respective frequency and take the total $\sum fd$
  - Apply the formula

$$\bar{X} = A + \frac{\sum fd}{N}$$

- Where
  - f = frequency
  - d = deviation from Assumed mean
  - A = assumed mean
  - N = total frequency

Simple Arithmetic Mean – Discrete Series

- From the following data of marks obtained by students, calculate arithmetic mean

Marks	20	30	40	50	60	70
# students	8	12	20	10	6	4

Simple Arithmetic Mean – Continuous Series

- Direct method
- Steps
  - Obtain the mid point of each class and denote it by m
  - Multiply these mid points by the respective frequency of each class and obtain $\sum fm$
  - Divide the total obtained by the sum of frequency (N)

$$\bar{X} = \frac{\sum fm}{N}$$

- Where
  - f = frequency
  - m = mid point of each class
  - N = total frequency

Simple Arithmetic Mean – Continuous Series

- Shortcut method - Using Assumed mean
- Steps
  - Take an assumed mean and denote it by A
  - From the mid point of each class deduct the assumed mean
  - Multiply the respective frequencies of each class by the deviations and obtain $\sum fd$
  - Apply formula

$$\bar{X} = A + \frac{\sum fd}{N}$$

- Where
  - f = frequency
  - d = deviation of class mid point from assumed mean
  - A = assumed mean
  - N = total frequency

Simple Arithmetic Mean – Continuous Series

- From the following data of marks obtained by students, calculate arithmetic mean

Marks	0-10	10-20	20-30	30-40	40-50	50-60
# students	5	10	25	30	20	10

Mathematical Properties of Arithmetic Mean

- Sum of the deviations of the items from the arithmetic mean (taking sign into account) is always zero
- Sum of the squared deviations of the items from arithmetic mean is minimum, that is, less than the sum of squared deviations of the items from any other value
- Including the mean value in the series multiple times wont change the mean
- If we have arithmetic mean and number of items of two or more than two related groups, we can compute combined me… formula

$$\overline{X_{12}} = \frac{N_1\overline{X_1} + N_2\overline{X_2}}{N_1 + N_2}$$

X	D	D^2
1	-2	4
2	-1	1
3	0	0
4	1	1
5	2	4
	0	10

$\bar{x} = \frac{1+2+3+4+5}{5} = \frac{15}{5} = 3$

X	X-4	$(X-4)^2$
1	-3	9
2	-2	4
3	-1	1
4	0	0
5	1	1
		15

X	X-1	$(X-1)^2$
1	0	0
2	1	1
3	2	4
4	3	9
5	4	16
		30

$\bar{x} = \frac{41+42+43+44+45}{5}$

$\bar{x} = \frac{215}{5} = 43$

X	(D) $X - \bar{X}$
41	-2
42	-1
43	0
44	1
45	2
	0

$\Sigma d = 0$

$X_1 = \underline{1, 2, 3, 4, 5},$

$\bar{X}_1 = \underline{\underline{3}}$

new addition

$X_2 = \underline{1, 2, 3, 4, 5, 6}$

$\bar{X}_2 = 3.5 \leftarrow$ mean is changed

new addition (mean)

$X3 = 1, 2, 3, 4, 5, 3, 3, 3$

$\bar{X}3 = 3$

Merits

- It is simplest average to understand and easiest to compute
- It is affected by value of every item in the series
- It is defined by rigid mathematical formula with the result that everyone who computes the average gets the same answer
- It lends itself to subsequent algebraic treatment better than median or mode
- The mean is typical in the sense that it is the center of gravity, balancing the values on the either sides of it
- It is calculated values and not based on the positions

Geometric Mean

- Steps
  - Multiply all the values and get the result
  - Get the square root to the Nth power to find the geometric mean

$$\bar{X} = \sqrt[N]{x_1 * x_2 * \ldots .* x_n}$$

Harmonic Mean

- Steps
  - Get reciprocal of each number and add together
  - Divide the number of values by the total calculated eariler

$$\bar{X} = \frac{N}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}}$$

Weighted Mean

- Steps
  - Multiply every value with corresponding weight
  - Add the values together
  - Divide the total by sum of all the weights

$$\bar{X} = \frac{\sum WiXi}{W1 + W2 \ldots. + Wn}$$

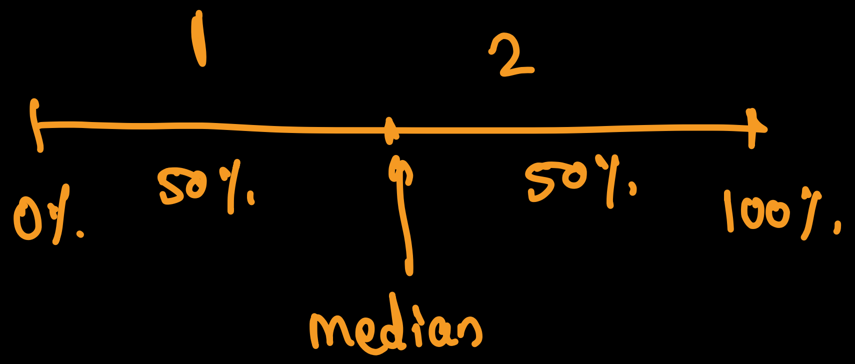
Median

middlemost value

textual / numeric

Median

- By definition, it refers to the middle value in a distribution
- The median is just 50th percentile value below which 50% of the values in the sample fall
- It splits the observations into two halves
- Unlike the mean, median is calculated by position (which refers to the place of the value in the series)



Median – Individual Series

- Steps
  - Arrange the data in the ascending or descending order of magnitude
  - In a group composed of an odd number of values such as 7, add 1 to the total number of values and divide it by 2. Thus 7 + 1 would be 8 which divided by 2 gives 4 – the position used to calculate the ~~mean~~ median
  - In a group composed of even number of values such as 10, use the average of middle two values. Thus 10 / 2 gives 5 – which will produce a median by taking average of 5th and 6th position values

$$median = \frac{N+1}{2}$$

Example 1 → odd no of values

2 3 5 9 12 14 30 32 35

median

middle position = $\frac{N+1}{2} = \frac{9+1}{2} = 5$

Example 2 → even no of values

2 3 5 9 $\boxed{12 \quad 14}$ 30 35 45 48

middle positions = 5 & 6 , median = $\frac{12+14}{2} = \frac{26}{2} = \underline{13}$

Countries = bhutan india japan usa uk

median

Median – Individual Series

- E.g. 1:
  - find median of 14100, 14150, 16080, 17120, 15200, 16160, 17400
  - Arrange them in ascending order
    - 14100, 14150, 15200, 16080, 16160, 17120, 17400
  - Median = (N + 1) / 2th item
  - Median = 7 + 1 / 2 = 4th item => 16080

- E.g. 2:
  - Find median of 19, 28, 40, 10, 29, 50, 37, 89, 90, 60
  - Arrange them in ascending order
    - 10, 19, 28, 29, 37, 40, 50, 60, 89, 90
  - Median = (N + 1)/ 2 the item
  - Median = average of 5th and 6th items => Average(37, 40) =>38.50

Median – Discrete Series

- Steps
  - Arrange the data in ascending or descending order of magnitude
  - Find out cumulative frequencies
  - Apply the formula (N + 1) / 2 the item
  - Now look at the cumulative frequency and find the total which is either equal to (N + 1) /2 or next higher to that and determine the value of variable corresponding to it
  - This gives the value of median

Median – Discrete Series

Marks	20	30	40	50	60	70
# students	8	12	20	10	6	4

Marks	#students	Cumulative frequency
20	8	8
30	12	20
40	20	40
50	10	50
60	6	56
70	4	60

- Median is (N + 1) / 2 th item => (60 + 1) / 2 = 30.5 th item
- Since the value at 30.5th (or just higher than it) is 40
- Median = 40

Median – Continuous Series

- Steps
  - Determine the particular class in which the value of median lies, consider this as median class
  - Calculate the cumulative frequencies
  - Use N/2 as the rank of the median
  - Use the formula

$$median = L + \frac{\frac{N}{2} - cf}{f} * i$$

- Where
  - L = Lower limit of the median class (the class in which middle item of the distribution lies)
  - cf = cumulative frequency of the class preceding the median class
  - f = frequency of the median class
  - i = class interval of the median class

Median – Continuous Series

Marks	0-10	10-20	20-30	30-40	40-50	50-60
# students	5	10	25	30	20	10

- The median class is => 100 /2 => 50 lies in (30-40)
- Median = 30 + ((100/2 – 40) / 30) * 10
- Median = 30 + (10/30) * 10 = 33.33

Marks	#students	cf
0-10	5	5
10-20	10	15
20-30	25	40
30-40	30	70
40-50	20	90
50-60	10	100

Merits

- It is useful in case of open-end classes since only the position and not the values of the items must be known
- Median is recommended if the distribution has unequal classes
- Extreme values do not affect the median as strongly as they do the mean
- It is most appropriate average in dealing with qualitative data
- Value of median can be calculated graphically
- It represents clear-cut the middle value in the distribution

Limitations

- For calculating median, it is necessary to arrange the data in a specific order
- Since it is a middle value, its value is not determined by each and every observation
- It is not capable of algebraic treatment
- The value of median is affected more by fluctuations than the value of the arithmetic mean
- It is erratic if the number of observations is very small

Mode

value which has
highest frequency

one which repeats
max no of times

Mode

- The mode or modal value is that value in a series which occurs most frequently
- That is the mode always will have the highest frequency in the data
- There are many situations where mean and median fails to reveal the true middle value, in such scenarios mode is used to find the central value

↳ textual / qualitative data

Mode – Individual Series

- Steps
  - Count the number of times the various values repeate themselves and the value occurring maximum number of times is the modal value

- E.g. 10, 28, 39, 40, 10, 20, 40, 50, 10 => mode = [10] → single mode
- E.g. 10, 20, 40, 50, 10, 20, 30, 40, 50 => mode = [10, 20, 50, 40] → multi mode
- E.g. 10, 20, 30, 40, 50, 60, 70, 80, 90 => mode = [] → No mode / all mode

Mode – Discrete Series

- Steps
  - Mode can be determined just be inspection
  - i.e. by looking to that value of the variable around which the items are most heavily concentrated

- E.g.

Marks	20	30	40	50	60	70
# students	8	12	20	10	6	4

- The mode here is 40

Mode – Continuous Series

- Steps
  - Find the modal class by finding the largest value
  - Determine the value of mode by applying the following formula

$$mode = L + \frac{\Delta_1}{\Delta_1 - \Delta_2} * 1$$

- Where
  - L = Lower limit of modal class
  - Δ_1 = difference between the frequency of modal class and frequency of pre-modal class
  - Δ_2 = difference between the frequency of modal class and frequency of post-modal class

Mode – Continuous Series

Marks	0-10	10-20	20-30	30-40	40-50	50-60
# students	5	10	25	30	20	10

- Modal class here is: 30-40
- Using the formula
  - Mode = 30 + ((30-25) / ((30-25) + (30-20))) * 10
  - Mode = 30 + (5 / (5 + 10)) * 10
  - Mode = 30 + 3.33 = 33.33

Merits

- Mode is the most typical or representative value of the distribution
- Like median, mode is not unduly affected by extreme values
- It can be used to describe the qualitative phenomenon
- The value of mode can be calculated graphically

Limitations

- The value of mode can not always be determined
- It is not capable of algebraic manipulation
- The value of mode is not based on each and every value of distribution