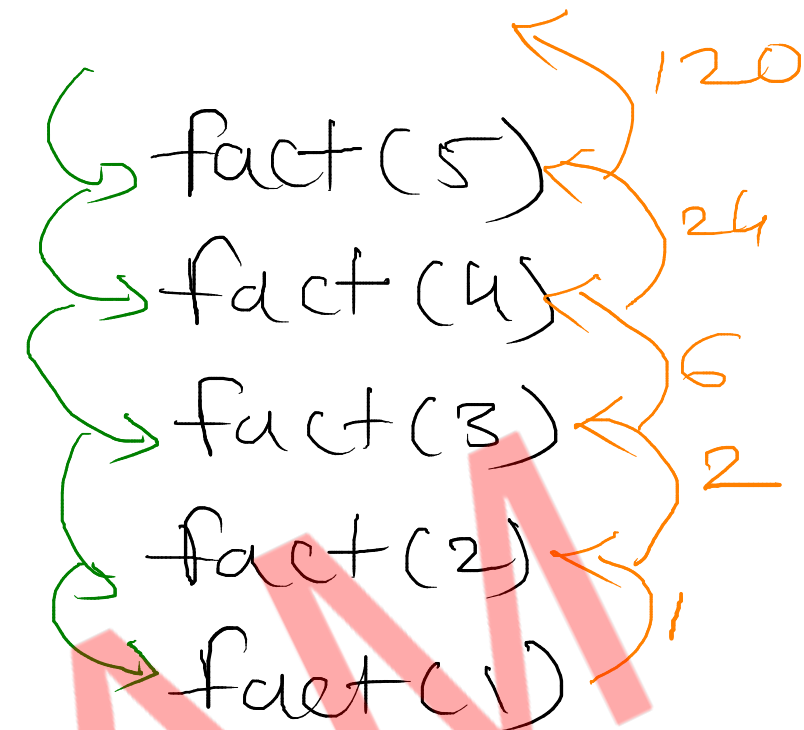


Analysis of Recursion

```
int fact(int n)
{
    if(n == 1)
        return 1;
    return n * fact(n-1);
}
```



$n=5$

recursive calls = 5

Time \propto no. of recursive calls

Time $\propto n$

$$T(n) = O(n)$$

Sorting Algorithms

- arrangement of data/values in either ascending or descending order of their values
- basic algorithms
 1. Selection sort
 2. Bubble sort
 3. Insertion sort
- advanced algorithms
 4. Merge sort
 5. Quick sort
 6. Heap sort

Selection sort

- //1. select one position of the array (generally start from 0)
- //2. compare selected position element with all other elements one by one
- //3. if selected position element is greater than other position element, then swap both elements
- //4. repeat above three steps till array is not sorted

no. of elements = n

no. of passes = $n-1$

$n=6$
pass 1 $\rightarrow 5$
pass 2 $\rightarrow 4$
pass 3 $\rightarrow 3$
pass 4 $\rightarrow 2$
pass 5 $\rightarrow 1$

15

$n-1$
 $n-2$
 $n-3$
 \vdots
 1

8

Total comps = $1 + 2 + 3 + \dots + (n-2) + (n-1)$

$$= \frac{n(n+1)}{2}$$

$$= \frac{n^2 + n}{2}$$

Time \propto comps

$$\text{Time} \propto \frac{n^2 + n}{2}$$

$$T(n) = O(n^2)$$

Best
Avg
Worst

$$f(n) = n^2 + n$$

↓
mathematical polynomial
↳ Degree of polynomial

↓
highest power of var
eg. degree = 2

↑ while writing time complexity
only consider degree term
because it is highest growing
term

n	n ²
1	1
10	100
100	10000
1000	1000000

Auxiliary
space

$$S(n) = O(1)$$

Bubble sort

//1. compare all pairs of consecutive elements

//2. if left element is greater than right element then swap both elements

//3. repeat above two steps till array is not sorted

$n=6$

$i < n$	$j < n-1$	$j < n-i$
i	j	j
1	0, 1, 2, 3, 4	0, 1, 2, 3, 4
2	0, 1, 2, 3, 4	0, 1, 2, 3
3	0, 1, 2, 3, 4	0, 1, 2
4	0, 1, 2, 3, 4	0, 1
5	0, 1, 2, 3, 4	0

10	20	30	40	50	60
10	20	30	40	50	60
10	20	30	40	50	60
10	20	30	40	50	60
10	20	30	40	50	60

Auxiliary
space

$$S(n) = O(1)$$

$$T(n) = O(n^2)$$

} Avg
Worst

$$T(n) = O(n)$$

- best

Insertion sort

- //1. pick one element of array (start from 1 index)
- //2. compare picked element with all its left neighbours one by one
- //3. if left neighbour is greater than picked element
 //then move it by one position ahead
- //4. insert picked element at its appropriate position
- //5. repeat above steps till array is not sorted

```
public static void insertionSort(int arr[], int N) {  
    for(int i = 1 ; i < N ; i++) {  
        int temp = arr[i];  
        int j = i - 1;  
        while(j >= 0 && arr[j] > temp) {  
            arr[j+1] = arr[j];  
            j--;  
        }  
        arr[j+1] = temp;  
    }  
}
```

N = 6					
20	40	50	60	10	30
0	1	2	3	4	5
i	i < 6	j	temp	j >= 0 && arr[j] > temp	
1	T	0, -1	40	T, F	
2	T	1, 0, -1	20	T, T, F	
3	T	2	60	F	
4	T				
5	T				

No. of element = n

No. of passes = $n-1$ (pick elements)

$$n=6$$

pass 1	- 1	1
pass 2	- 2	2
pass 3	- 3	3
pass 4	- 4	4
pass 5	- 5	$n-1$

$$\text{Total Comps} = 1 + 2 + 3 + \dots + n-1$$

$$= \frac{n(n+1)}{2}$$

$$\text{Time} \propto \frac{n^2 + n}{2}$$

$$S(n) = O(1)$$

$$T(n) = O(n^2) \quad \left. \vphantom{T(n)} \right\} \begin{array}{l} \text{Avg} \\ \text{Worst} \end{array}$$

$$T(n) = O(n) \quad \left. \vphantom{T(n)} \right\} \text{best}$$

10, 20, 30, 40, 50, 60