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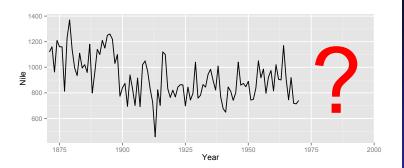
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Dynamic Linear Models And Kalman Filtering

Dr. Holger Zien

12th December 2014

One of The Most Common Problems Is To Forecast Time Series



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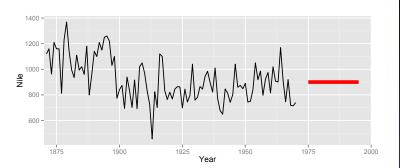
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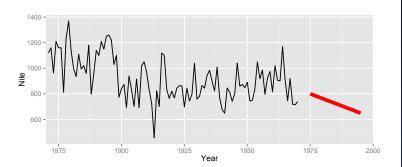
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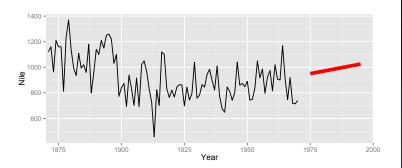
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Typical Time Series In Classical Textbooks

- many samples
- more or less stationary
- typical for physical measurements
- the method of choice is ARMA

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In Practice We Are Facing A Different Kind of Time Series

- ridiculous short
- often non-stationary
- ARMA models do not work
- Sometimes Bayesian modeling of Dynamic Linear Models/Kalman Filtering will help

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Autoregressive Moving Average Model (ARMA)

$$y_t = \varepsilon_t + \underbrace{\sum_{i=1}^p \phi_i y_{t-i}}_{\text{autoregressive part}} + \underbrace{\sum_{j=1}^q \psi_j \varepsilon_{t-j}}_{\text{moving average part}}$$

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Dynamic Linear Model

Simplest Version

observation eq. $y_t = F\theta_t + \nu_t$ system eq. $\theta_t = G\theta_{t-1} + \omega$

 ν_t, ω_t : mutually independent random variables

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Dynamic Linear Model

Vector-Valued State/Observation, Time-Dependent Coefficents

observation eq. $m{y}_t = m{F}_t^{ op} \cdot m{ heta}_t + m{
u}_t$ system eq. $m{ heta}_t = m{G}_t \cdot m{ heta}_{t-1} + m{\omega}_t$

 θ , ω : *n*-dimensional random vectors

y, ν : r-dimensional random vectors

G: $n \times n$ dimensional state evolution matrix

 \mathbf{F} : $n \times r$ dimensional dynamic regression matrix

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$$egin{aligned} oldsymbol{y}_t &= oldsymbol{F}_t^T \cdot oldsymbol{ heta}_t + oldsymbol{
u}_t \ oldsymbol{ heta}_t &= oldsymbol{G}_t \cdot oldsymbol{ heta}_{t-1} + oldsymbol{\omega}_t \end{aligned}$$

post. distr.
$$oldsymbol{ heta}_t$$
: $oldsymbol{ heta}_t | D_t \sim N(oldsymbol{m}_t, oldsymbol{\mathcal{C}}_t)$

forecast:
$$\mathbf{y}_{t}|D_{t-1} \sim N(\mathbf{f}_{t}, \mathbf{Q}_{t})$$

prior distr.
$$heta_t$$
: $heta_t | D_{t-1} \sim N(extbf{\emph{a}}_t, extbf{\emph{R}}_t)$

$$egin{aligned} \omega_t &\sim \textit{N}(\mathbf{0}, oldsymbol{W}_t) \
u_t &\sim \textit{N}(\mathbf{0}, oldsymbol{V}_t) \
heta_0 | D_0 &\sim \textit{N}(oldsymbol{m}_0, oldsymbol{\mathcal{C}}_0) \end{aligned}$$

$$m_{t} = a_{t} + A_{t} \cdot (y_{t} - f_{t})$$

$$C_{t} = R_{t} - A_{t} \cdot Q_{t} \cdot A_{t}^{T}$$

$$A_{t} = R_{t} \cdot F_{t} \cdot Q_{t}^{-1}$$

$$f_{t} = F_{t}^{T} \cdot a_{t}$$

$$egin{aligned} oldsymbol{Q}_t &= oldsymbol{F}_t^T \cdot oldsymbol{R}_t \cdot oldsymbol{F}_t + oldsymbol{V}_t \ oldsymbol{a}_t &= oldsymbol{G}_t \cdot oldsymbol{m}_{t-1} \end{aligned}$$

$$\mathbf{R}_t = \mathbf{G}_t \cdot \mathbf{C}_{t-1} \cdot \mathbf{G}_t^\mathsf{T} + \mathbf{W}_t$$

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$$egin{aligned} oldsymbol{y}_t &= oldsymbol{F}_t^{ op} \cdot oldsymbol{ heta}_t + oldsymbol{
u}_t \ oldsymbol{ heta}_t &= oldsymbol{G}_t \cdot oldsymbol{ heta}_{t-1} + oldsymbol{\omega}_t \end{aligned}$$

post. distr.
$$\theta_t$$
: $\theta_t | D_t \sim N(\boldsymbol{m}_t, \boldsymbol{C}_t)$

forecast:
$$\mathbf{v}_{t}|D_{t-1} \sim N(\mathbf{f}_{t}, \mathbf{Q}_{t})$$

prior distr.
$$\theta_t$$
: $\theta_t | D_{t-1} \sim N(\boldsymbol{a}_t, \boldsymbol{R}_t)$

$$egin{aligned} oldsymbol{\omega}_t &\sim \mathcal{N}(oldsymbol{0}, oldsymbol{W}_t) \ oldsymbol{
u}_t &\sim \mathcal{N}(oldsymbol{0}, oldsymbol{V}_t) \ _0 | D_0 &\sim \mathcal{N}(oldsymbol{m}_0, oldsymbol{\mathcal{C}}_0) \end{aligned}$$

$$oldsymbol{
u}_t \sim oldsymbol{\mathcal{N}}(oldsymbol{0}, oldsymbol{V}_t) \ oldsymbol{ heta}_0 | D_0 \sim oldsymbol{\mathcal{N}}(oldsymbol{m}_0, oldsymbol{\mathcal{C}}_0)$$

$$egin{aligned} m_t &= a_t + A_t \cdot (y_t - f_t) \ C_t &= R_t - A_t \cdot Q_t \cdot A_t^{\mathsf{T}} \ A_t &= R_t \cdot F_t \cdot Q_t^{-1} \ f_t &= F_t^{\mathsf{T}} \cdot a_t \ Q_t &= F_t^{\mathsf{T}} \cdot R_t \cdot F_t + V_t \end{aligned}$$

Dynamic Linear Model

Kalman Filtering

$$egin{aligned} oldsymbol{y}_t &= oldsymbol{F}_t^T \cdot oldsymbol{ heta}_t + oldsymbol{
u}_t \ oldsymbol{ heta}_t &= oldsymbol{G}_t \cdot oldsymbol{ heta}_{t-1} + oldsymbol{\omega}_t \end{aligned}$$

post. distr.
$$\theta_t$$
: $\theta_t | D_t \sim N(\boldsymbol{m}_t, \boldsymbol{C}_t)$

forecast:
$$\mathbf{v}_t | D_{t-1} \sim N(\mathbf{f}_t, \mathbf{Q}_t)$$

prior distr.
$$\theta_t$$
: $\theta_t | D_{t-1} \sim N(\boldsymbol{a}_t, \boldsymbol{R}_t)$

$$egin{aligned} oldsymbol{\omega}_t &\sim \mathcal{N}(oldsymbol{0}, oldsymbol{W}_t) \ oldsymbol{
u}_t &\sim \mathcal{N}(oldsymbol{0}, oldsymbol{V}_t) \ oldsymbol{ heta}_0 | D_0 &\sim \mathcal{N}(oldsymbol{m}_0, oldsymbol{\mathcal{C}}_0) \end{aligned}$$

$$egin{aligned} oldsymbol{f}_t &= oldsymbol{F}_t^T \cdot oldsymbol{a}_t \ oldsymbol{Q}_t &= oldsymbol{F}_t^T \cdot oldsymbol{R}_t \cdot oldsymbol{F}_t + oldsymbol{V}_t \end{aligned}$$

$$a_t = G_t \cdot m_{t-1}$$

$$\mathbf{R}_t = \mathbf{G}_t \cdot \mathbf{C}_{t-1} \cdot \mathbf{G}_t^T + \mathbf{W}_t$$

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Dynamic Linear Model

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$$egin{aligned} oldsymbol{y}_t &= oldsymbol{F}_t^{\mathsf{T}} \cdot oldsymbol{ heta}_t + oldsymbol{
u}_t \ oldsymbol{ heta}_t &= oldsymbol{G}_t \cdot oldsymbol{ heta}_{t-1} + oldsymbol{\omega}_t \end{aligned}$$

post. distr.
$$m{ heta}_t$$
: $m{ heta}_t | D_t \sim N(m{m}_t, m{C}_t)$

forecast:
$$\mathbf{y}_t | D_{t-1} \sim N(\mathbf{f}_t, \mathbf{Q}_t)$$

prior distr.
$$\theta_t$$
: $\theta_t | D_{t-1} \sim N(\boldsymbol{a}_t, \boldsymbol{R}_t)$

$$egin{aligned} oldsymbol{\omega}_t &\sim \mathcal{N}(oldsymbol{0}, oldsymbol{W}_t) \ oldsymbol{
u}_t &\sim \mathcal{N}(oldsymbol{0}, oldsymbol{V}_t) \ 0 | D_0 &\sim \mathcal{N}(oldsymbol{m}_0, oldsymbol{\mathcal{C}}_0) \end{aligned}$$

$$egin{aligned} oldsymbol{
u}_t &\sim oldsymbol{\mathcal{N}}(oldsymbol{0}, oldsymbol{V}_t) \ oldsymbol{ heta}_0 | D_0 &\sim oldsymbol{\mathcal{N}}(oldsymbol{m}_0, oldsymbol{\mathcal{C}}_0) \end{aligned}$$

$$egin{aligned} oldsymbol{
u}_t &\sim oldsymbol{\mathcal{N}}(oldsymbol{0}, oldsymbol{V}_t) \ oldsymbol{ heta}_0 | D_0 &\sim oldsymbol{\mathcal{N}}(oldsymbol{m}_0, oldsymbol{\mathcal{C}}_0) \end{aligned}$$

$$m_t = a_t + A_t \cdot (y_t - f_t)$$

 $C_t = R_t - A_t \cdot Q_t \cdot A_t^T$

$$oldsymbol{A}_t = oldsymbol{R}_t \cdot oldsymbol{F}_t \cdot oldsymbol{Q}_t^{-1}$$

$$f_t = F_t^T \cdot a_t$$

$$Q_t = F_t^T \cdot R_t \cdot F_t + V_t$$

$$\mathbf{Q}_t = \mathbf{F}_t \cdot \mathbf{R}_t \cdot \mathbf{F}_t + \mathbf{V}_t$$

 $\mathbf{a}_t = \mathbf{G}_t \cdot \mathbf{m}_{t-1}$

$$\mathbf{R}_t = \mathbf{G}_t \cdot \mathbf{C}_{t-1} \cdot \mathbf{G}_t^T + \mathbf{W}_t$$

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Kalman Filtering

$$egin{aligned} oldsymbol{y}_t &= oldsymbol{F}_t^T \cdot oldsymbol{ heta}_t +
u_t \ oldsymbol{ heta}_t &= oldsymbol{G}_t \cdot oldsymbol{ heta}_{t-1} + \omega_t \end{aligned}$$

post. distr.
$$\theta_t$$
: $\theta_t | D_t \sim N(\boldsymbol{m}_t, \boldsymbol{C}_t)$

$$\mathsf{Bayes}^t \mathsf{LaW}$$

forecast:
$$m{y}_t | D_{t-1} \sim N(m{f}_t, m{Q}_t)$$

prior distr.
$$m{ heta}_t$$
: $m{ heta}_t | D_{t-1} \sim N(m{a}_t, m{R}_t)$

$$egin{aligned} oldsymbol{\omega}_t &\sim extstyle extstyle N(oldsymbol{0}, oldsymbol{W}_t) \ oldsymbol{
u}_t &\sim extstyle N(oldsymbol{0}, oldsymbol{V}_t) \ oldsymbol{ heta}_0 | D_0 &\sim extstyle N(oldsymbol{m}_0, oldsymbol{\mathcal{C}}_0) \end{aligned}$$

$$m_t = a_t + A_t \cdot (y_t - f_t)$$
 $C_t = R_t - A_t \cdot Q_t \cdot A_t^T$

$$m{A}_t = m{R}_t \cdot m{F}_t \cdot m{Q}_t^{-1}$$

$$oldsymbol{f}_t = oldsymbol{F}_t^T \cdot oldsymbol{a}_t$$

$$oldsymbol{Q}_t = oldsymbol{F}_t^T \cdot oldsymbol{R}_t \cdot oldsymbol{F}_t + oldsymbol{V}_t$$

$$a_t = G_t \cdot m_{t-1}$$

$$\mathbf{R}_t = \mathbf{G}_t \cdot \mathbf{C}_{t-1} \cdot \mathbf{G}_t^T + \mathbf{W}_t$$

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Glossary Applications

Estimate of the current value of the Filtering:

state/system variable.

Smoothing: Estimate of past values of the state/system

variable, i.e., estimating at time t given

measurements up to time t' > t.

Forecasting: Forecasting future observations or values of the

state/system variable.

- Models composed of different components (trends, seasonality, ARMA)
- ► Non-stationary models
- Models with interventions
- ▶ Regression model with time-dependent coefficients

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Regression Model

Regression model:
$$y_t = \alpha_t + \beta_{1,t} x_{1,t} + \beta_{2,t} x_{2,t} + \varepsilon_t$$

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Regression

Regression Model

Regression model:
$$y_t = \alpha_t + \beta_{1,t} x_{1,t} + \beta_{2,t} x_{2,t} + \varepsilon_t$$

DLM:
$$y_t = \boldsymbol{F}_t^T \cdot \boldsymbol{\theta}_t + \varepsilon_t$$
$$\boldsymbol{\theta}_t = \boldsymbol{\theta}_{t-1} + \boldsymbol{\omega}_t$$

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Applications Regression

Regression model:
$$y_t = \alpha_t + \beta_{1,t} x_{1,t} + \beta_{2,t} x_{2,t} + \varepsilon_t$$

DLM:
$$y_t = \boldsymbol{F}_t^T \cdot \boldsymbol{\theta}_t + \varepsilon_t$$

$$\boldsymbol{\theta}_t = \boldsymbol{\theta}_{t-1} + \boldsymbol{\omega}_t$$

with:
$$\mathbf{F}_{t} = (1, x_{1,t}, x_{2,t})^{T}$$

$$\boldsymbol{\theta}_t = (\alpha_t, \beta_{1,t}, \beta_{2,t})^T$$

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Applications Regression

- ▶ Modeling of short time series (Bayes model).
- Models composed of different components (trends, seasonality, ARMA)
- ► Non-stationary models
- Models with interventions
- Regression model with time-dependent coefficients
- ► ARMA models with known or unknown coefficients

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ARMA Model With Known Coefficients

ARMA(2,2) model:
$$y_t = \varepsilon_t + \sum_{i=1}^2 \phi_i y_{t-i} + \sum_{j=1}^2 \psi_j \varepsilon_{t-j}$$

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ARMA Model With Known Coefficients

$$\mathsf{ARMA(2,2)} \; \mathsf{model:} \qquad y_t = \varepsilon_t + \sum_{i=1}^2 \phi_i y_{t-i} + \sum_{j=1}^2 \psi_j \varepsilon_{t-j}$$

DLM:
$$y_t = \boldsymbol{F}_t^T \cdot \boldsymbol{\theta}_t$$
 $\boldsymbol{\theta}_t = \boldsymbol{G} \cdot \boldsymbol{\theta}_{t-1} + \boldsymbol{\omega}_t$

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ARMA(2,2) model:
$$y_t = \varepsilon_t + \sum_{i=1}^2 \phi_i y_{t-i} + \sum_{j=1}^2 \psi_j \varepsilon_{t-j}$$

DLM:
$$y_t = \boldsymbol{F}_t^T \cdot \boldsymbol{\theta}_t$$
 $\boldsymbol{\theta}_t = \boldsymbol{G} \cdot \boldsymbol{\theta}_{t-1} + \boldsymbol{\omega}_t$

with:
$$m{F}_t = egin{pmatrix} \left(1,0,0
ight)^T \\ m{G} = egin{pmatrix} \phi_1 & 1 & 0 \\ \phi_2 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\boldsymbol{\omega}_t = \left(1, \psi_1, \psi_2\right)^T \varepsilon_t$$

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ARMA Model With Unknown Coefficients

$$y_t = \varepsilon_t + \sum_{i=1}^2 \phi_i y_{t-i}$$

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ARMA Model With Unknown Coefficients

ARMA(2,0) model:
$$y_t = \varepsilon_t + \sum_{i=1}^{2} \phi_i y_{t-i}$$

DLM:
$$y_t = \boldsymbol{F}_t^T \cdot \boldsymbol{\theta}_t + \varepsilon_t$$
$$\boldsymbol{\theta}_t = \boldsymbol{\theta}_{t-1} + \boldsymbol{\omega}_t$$

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ARMA Model With Unknown Coefficients

ARMA(2,0) model:
$$y_t = \varepsilon_t + \sum_{i=1}^{2} \phi_i y_{t-i}$$

DLM:
$$y_t = \boldsymbol{F}_t^T \cdot \boldsymbol{\theta}_t + \varepsilon_t$$

$$oldsymbol{ heta}_t = oldsymbol{ heta}_{t-1} + oldsymbol{\omega}_t$$

with:
$$\boldsymbol{F}_t = \left(y_{t-1}, y_{t-2}\right)^T$$

$$\boldsymbol{\theta}_t = \left(\phi_1, \phi_2\right)^T$$

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Pros:

- Applicable to short time series
- Not restricted to stationary data
- Large DLM models can be build through composing small DLM models
- May be extended to non-normal distributions
- ► Results are easy to interpret

Cons:

- Finding size of noise terms ν_t , ω_t difficult
 - MLE yields unreasonable results frequently.
 - A solution might be Bayesian estimation of ν_t , ω_t which is not part of the R-libraries
- ▶ Incomplete R libraries, no "standard" library
- ▶ Difficult numerical implementation

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dlm Personally preferred library, see Petris (2010)

FKF Performance–optimized Kalman filtering, no functions for model building

sspir Library used by Cowpertwait u. Metcalfe (2009), no longer maintained

dlmodeler Common interface to libraries dlm, FKF, KFAS

... Several other libraries, see CRAN Task View: "Time Series Analysis", for comparison see Tusell (2011) and Commandeur u. a. (2011)...

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Finally

Questions?

Thank You! ©

