Frequency-Response-Based Controller Design

1) A Long Prelab

This is a long prelab, intended to prepare you for the maglev design problem next week. It also introduces you to the matlab suite of computational tools for system analysis, tools which we will be using extensively for the remainder of the semester. We had intended to switch over to python exclusively this term, but that transition will have to wait until next term. We've had less time for development than expected, and we did not realize the diversity of python enviroments we would need to support. Our apologies for this, and we will try to make this change in plans as painless as possible.

You can use matlab on-line or install it on your computer, information on the alternatives for MIT students is available here. If you are new to matlab, we suggest using the on-line version, and there is a tutorial video here, but we will describe most of the commands you will need.

For this Friday's lab, if would help to skim over the matlab tools for creating CT transfer functions and computing step responses. But before beginning the design problem next week, you should finish this prelab.

2) Phase Margin and Leads

In much of the literature on linear systems analysis, the term "frequency domain analysis" is a very general catagory, and encompasses nearly every use of transforms to replace difference or differential equations involving functions of n or t, with rational functions (typically) of z or s. But when WE use the term "frequency response", we are referring to a system's eventual response to sinusoidal inputs as a function of the sinusoid frequency (the sinusoidal steady-state response). And we usually characterize the frequency response by its magnitude and angle (or phase) with respect to the input sinusoid.

When one uses frequency domain analysis to determine a stable system's transfer function, e.g. $H(s)$, one generates a complex function of a complex variable s. The stable system's frequency response is given by $H(s)|_{j\omega} = H(j\omega)$, a complex function of a real variable. And as a function of a real variable, frequency response can be characterized by easily-visualized plots of magnitude and phase versus ω.

It is tempting to try to formalize these easily understood frequency response plots, and focus on connecting plot properties to guarantees about system performance. We will take an alternative approach, and instead focus on using frequency response as a tool for suggesting design alternatives. In particular, we will examine the notion of phase margin , show that good controllers have large phase margins, and show how to design candidate controllers that increase phase margin. But, while good controllers have large phase margins, having a large phase margin DOES NOT guarantee a good controller, so our candidate controllers must be verified.

Design is different from analysis, and is often more about generating worthwhile alternatives than providing a-priori guarantees. Assuming, of course, that one has the tools to verify any final design.

3) Matlab Tools for CT and Frequency Response.

You can use matlab to manipulate continuous-time transfer functions. First, you have to tell matlab to use s as the CT transform variable, by typing

```
>> s = tf('s')
```

After that, you can define a CT transfer function. For example, suppose your system is described by a set of differential equations

$$\frac{dv(t)}{dt} = x(t), \quad \frac{dw(t)}{dt} = v(t), \quad \frac{dy(t)}{dt} = -0.1y(t) + w(t),$$

then assuming $y(t) = Ye^{st}$ and $x(t) = Xe^{st}$,

$$Y = \frac{1}{s+0.1}\frac{1}{s}\frac{1}{s}X = H(s)X,$$

where $H(s)$ is the CT transfer function given by

$$H(s) = \frac{1}{(s+0.1)s^2}.$$

In matlab, if you type

```
>> H = (1/(s+0.1))*(1/s)*(1/s)
```

Matlab will response with

```
H =

       1
  -------------
  s^3 + 0.1 s^2
```

Then if you type

```
>> pole(H)
```

matlab will respond with poles (or natural frequencies),

```
ans =

        0
        0
  -0.1000
```

as you would expect since in CT, two of your poles are at zero.

We will make extensive use of the *step, feedback,* and *bode* commands. You can find information about any matlab command by typing, for example,

```
help step
```

to find out a little about the *step* command, or

```
doc step
```

to launch a documentation page with comprehensive information.

If you wanted a Bode plot of $H(j\omega)$ based on the H described above, you could type

```
bode(H)
```

As a more complete example, consider a system with input $x(t)$ and output $y(t)$ described by a first-order differential equation,

$$\frac{dy(t)}{dt} = -y(t) + 1000x(t),$$

where y is the output and x is the input. Suppose we know that $x(t) = \cos(\omega t) = \cos(2\pi f t)$ where ω is the frequency in radians per second and f is the frequency in cycles per second. Approximately what is $y(t)$ in the limit as $t \to \infty$?

In this case, the system function is $H(s) = \frac{1000}{s+1}$. We say $x(t)$ is equation to a unit step if $x(t) = 0$ for $t < 0$, and $x(t) = 1$ for $t \geq 0$. To plot the response to the unit step using matlab, in the command window type (be sure you have defined s to matlab using $s = tf('s')$).

```
H = 1000/(s+1)
```

to define the system function to matlab, and then

```
step(H)
```

to plot $y(t)$ given $x(t)$ is a unit step. Note that $y(t)$ rises to 1000 with a time constant of one second (about two-thirds of the rise in one second).

The frequency response of this first-order system can be plotted using

```
bode(H)
```

A Bode plot which labels the unity-gain ($|H(j\omega)| = 1$) frequency, and its associated phase margin, can be plotted using

```
margin(H)
```

If we use feedback with this first order system, so that

$$X = K(s)(Y_{desired} - Y),$$

then we can apply Black's formula to determine the closed-loop system function, $G(s)$,

$$G(s) = \frac{K(s)H(s)}{1 + K(s)H(s)}.$$

If we set $K(s) = 1$, then

$$G(s) = \frac{1000}{s + 1001}.$$

We can also use matlab to determine the system function with the feedback command,

```
G = feedback(1000/(s+1),1)
```

In order to plot the locus of poles of $G(s)$ as a function of K_0 of

$$G(s) = \frac{K_0 K(s)H(s)}{1 + K(s)H(s)} = \frac{1000K_0}{s + 1001},$$

you can use the root locus command

```
rlocus(1000/(s+1))
```

Root locus plots can be used to select a good controller. For example, for third-order system given by

$$H(s) = \frac{1}{(s + 20)(s^2 + 2s + 1)},$$

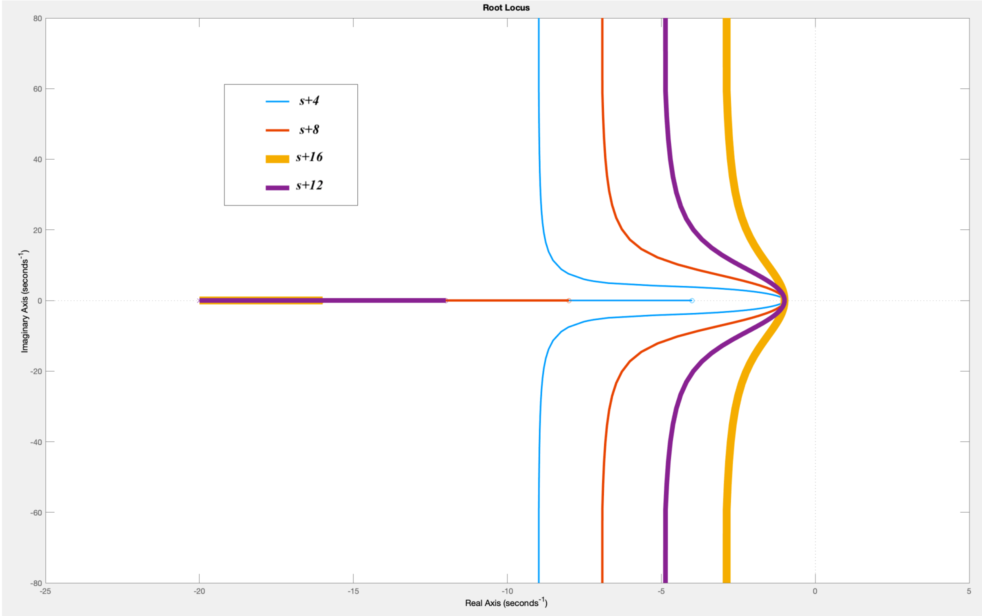
we could use a controller with a single zero (or a proportional-derivative). To determine the best zero location, for example,

$$K(s) = (s + 4), \quad K(s) = (s + 8), \quad K(s) = (s + 12), \quad K(s) = (s + 16),$$

we could use the following matlab commands to plot root loci for each of the controllers.

```
rlocus((s+4)/((s+20)*(s*s+2*s+1)))
hold on
rlocus((s+8)/((s+20)*(s*s+2*s+1)))
rlocus((s+12)/((s+20)*(s*s+2*s+1)))
rlocus((s+16)/((s+20)*(s*s+2*s+1)))
```

The above commands will generate a plot like the following (where we have recolored the lines and added a legend). Note that the controllers zeros at -4, -8, -12, and -16 generate root loci that have progressively less negative real parts, for worsening stability.



To plot the step response of the feedback system,

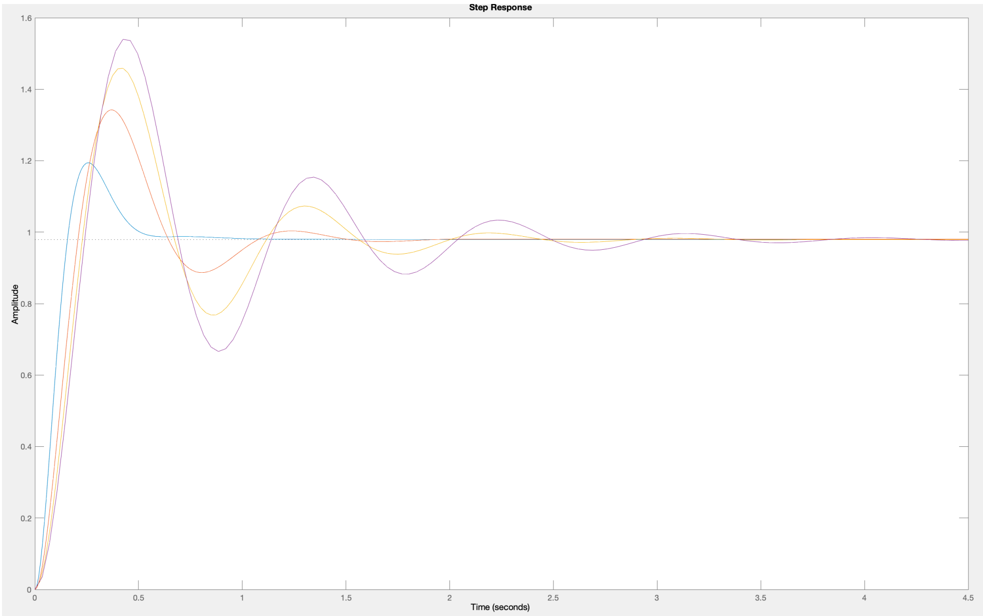
```
step(G)
```

The frequency response also shows the impact of including the first-order system in a closed loop, as can be seen in the plot generated by

```
bode(G)
```

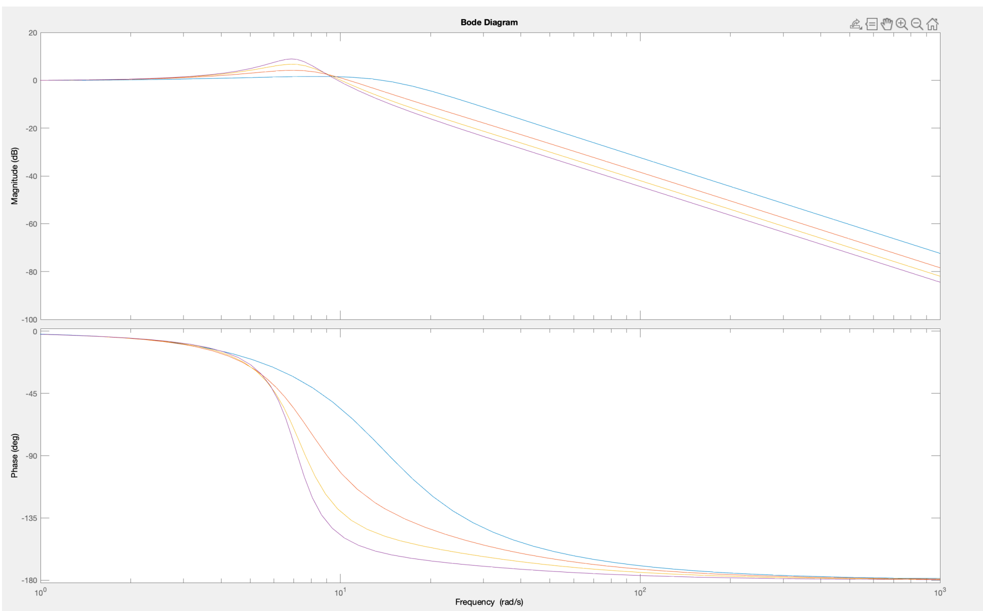
For our third-order example, the step responses for the closed-loop systems can be compared, where notice that we have set a scale factor for each controller so that they all have the same steady-state value, $K(j\omega)H(j\omega)$ for $\omega = 0$.

```
step(feedback((240*(s+4)/((s+20)*(s*s+2*s+1))),1))
hold on
step(feedback((120*(s+8)/((s+20)*(s*s+2*s+1))),1))
step(feedback((80*(s+12)/((s+20)*(s*s+2*s+1))),1))
step(feedback((60*(s+16)/((s+20)*(s*s+2*s+1))),1))
```



In the above plot, the step response for the $s+4$ controller has the smallest overshoot, unsurprising given the root locus plot. And what about the closed loop frequency response?

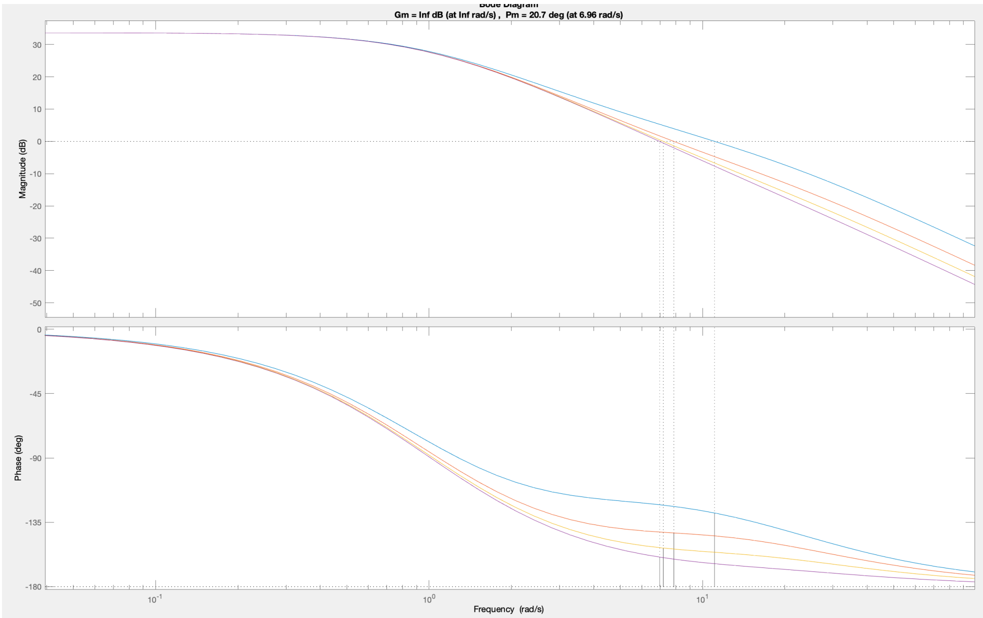
```
bode(feedback((240*(s+4)/((s+20)*(s*s+2*s+1))),1))
hold on
bode(feedback((120*(s+8)/((s+20)*(s*s+2*s+1))),1))
bode(feedback((80*(s+12)/((s+20)*(s*s+2*s+1))),1))
bode(feedback((60*(s+16)/((s+20)*(s*s+2*s+1))),1))
```



And the phase margin

```
margin(240*(s+4)/((s+20)*(s*s+2*s+1)))
hold on
margin(120*(s+8)/((s+20)*(s*s+2*s+1)))
```

```
margin(80*(s+12)/((s+20)*(s*s+2*s+1)))
margin(60*(s+16)/((s+20)*(s*s+2*s+1)))
```



Notice that the $s+16$ controller resulted in a system whose step response had the most overshoot, whose frequency response had the most peaking, and whose phase margin was the smallest. Step response, closed-loop frequency response, and phase margin, they all they all tell the same story. The $s+4$ controller is more stable.

4) Example problems

- exercises: Lead Compensation (Due Apr 04, 2024; 10:30 PM)
- exercises: Bode and Steps (Due Apr 04, 2024; 10:30 PM)
- exercises: Arm Like Example (Due Apr 04, 2024; 10:30 PM)
- exercises: Phase Margin (Due Apr 04, 2024; 10:30 PM)

Lead Compensation

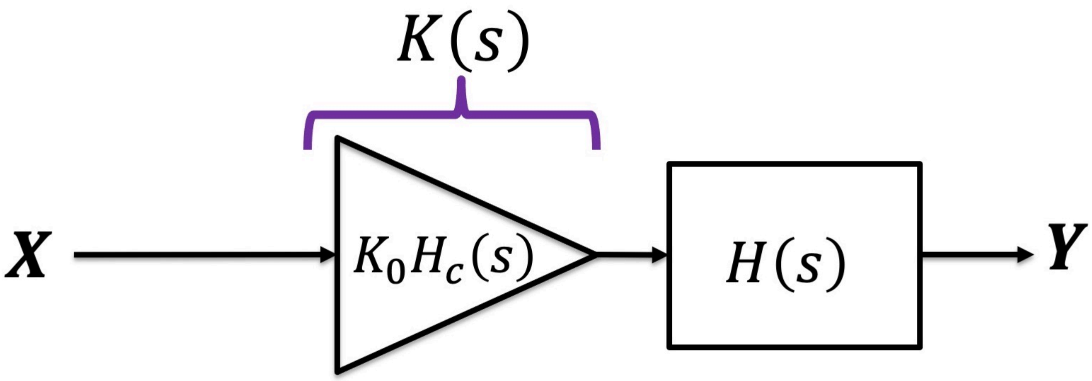
The questions below are due on Thursday April 04, 2024; 10:30:00 PM.

1) Compensator Design Using Frequency Response.

What follows is a very brief discussion of lead compensators, with some short questions interspersed. We are trying to convey a sense of "why is this useful" and the "how to design with it", and not focus on formal guarantees. In a few weeks, we will switch to state-space, a better setting in which to be formal.

2) Notation and Reminders

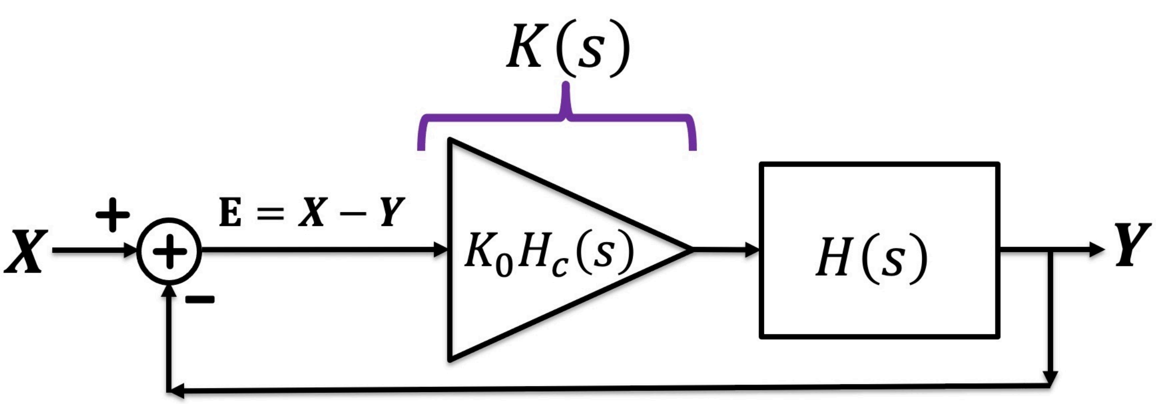
We have usually divided the forward path of our archetype feedback system in to the controller (what we design), with transfer function $K(s)$, and the "plant" (what we are controlling), with transfer function $H(s)$. We will deviate slightly from the $K(s)$ and $H(s)$ notation, and further divide the controller in to the product of a constant gain K_0, and a normalized compensator transfer function, $H_c(s)$. In this way, candidate compensators can be more easily compared. A good compensator is not the one with the highest innate gain, but the one that, *after* optimizing over K_0, yields a feedback system with the smallest tracking errors and highest disturbance rejection.



More specifically, suppose we are given the above open-loop system,

$$Y(s) = K_0 H_c(s) H(s) X(s).$$

Then suppose we "close the loop" around the system, as below



where we can use what should be the comfortably-familiar Black's formula to derive

$$Y(s) = \frac{K_0 H_c(s) H(s)}{1 + K_0 H_c(s) H(s)} X(s).$$

When we are referring to the closed-loop transfer function, we will denote it as $G(s)$, where $G(s)$ is given

$$G(s) = \frac{K_0 H_c(s) H(s)}{1 + K_0 H_c(s) H(s)}.$$

If $x(t) = e^{j\omega t}$ for all $-\infty \leq t \leq \infty$ (a mathematical idealization), then in closed-loop,

$$y(t) = G(j\omega) e^{j\omega t}.$$

and open-loop,

$$y(t) = K_0 H_c(j\omega) H(j\omega) e^{j\omega t}.$$

The closed-loop frequency response is the magnitude and angle of $G(j\omega)$ as a function of ω, and the open-loop frequency response is the magnitude and angle of $K_0 H_c(j\omega) H(j\omega)$ as a function of ω.

If a system is stable, we can measure the frequency response by applying unit amplitude sinusoidals, waiting until the output achieves sinusoidal steady state, and measuring the amplitude and phase shift of the output. Even if a system is not stable, the above construction can be evaluated mathematically, but not measured practically.

3) Gain and Phase Margin

What happens if the denominator in the expression for the closed-loop frequency response,

$$G(j\omega) = \frac{K_0 H_c(j\omega) H(j\omega)}{1 + K_0 H_c(j\omega) H(j\omega)},$$

is zero? That is, one might be concerned about the possibility that

$$1 + K_0 H_c(j\omega) H(j\omega) = 0$$

for some $\omega = \omega_0$. Should that be the case, then $G(j\omega_0) = \infty$. And since G describes a tracking feedback system, an important design objective is for $G(j\omega) = 1$ for all ω (or for as wide a range of frequencies as possible). Unfortunately, ∞ is infinity far from that objective.

In fact, $K_0 H_c(j\omega_0) H(j\omega_0)$ need only be close to -1 to cause problems, and the closer it is to -1, the further $G(j\omega_0)$ will be from unity. One can make a more formal link to closed-loop stability, but for now, it is sufficient to note that IT IS BAD FOR

$$K_0 H_c(j\omega_0) H(j\omega_0) \text{ to be near } -1$$

for any ω_0, and WE SHOULD DESIGN OUR COMPENSATORS SO THAT -1 IS AVOIDED.

For $K_0 H_c(j\omega_0) H(j\omega_0)$ to equal -1, an angle and a magnitude condition must each be satisfied:

- Magnitude condition: $|K_0 H_c(j\omega_0) H(j\omega_0)|$ must be equal to 1.
- Angle Condition: $\angle \left(K_0 H_c(j\omega_0) H(j\omega_0)\right)$ must equal $+\pi$ or $-\pi$ radians or $+180$ or -180 degrees.

Since being exactly equal to -1 is unlikely, but being near is also bad, we need to define metrics for how close we are to the dreaded -1. To that end, there are three terms commonly used to describe the closeness:

- *Unity gain frequency* (ω_{unity}): the value of ω such that $|K_0 H_c(j\omega_{unity}) H(j\omega_{unity})| = 1$. It is possible to have multiple unity gain frequencies, though it is uncommon.

- *Phase Margin* : How far the $\angle\left(K_0 H_c(j\omega_{unity})H(j\omega_{unity})\right)$ is from $-\pi$ or $+\pi$ radians or $+180$ or -180 degrees when the magnitude is unity. Phase margin is usually specified in degrees, and is (in degrees):

$$180 - |\angle(K_0 H_c(j\omega_{unity})H(j\omega_{unity}))|$$

- *Gain Margin* : For those values of ω for which $\angle\left(K_0 H_c(j\omega)H(j\omega)\right)$ equals either $+180$ or -180, the gain margin (in dB) is defined as -1 times the least positive $20\log|(K_0 H_c(j\omega)H(j\omega))|$. If the gain margin is negative, then

$$|(K_0 H_c(j\omega)H(j\omega))| > 1$$

  for all $\omega's$ for which the phase is 180 degrees.

Phase and gain margins can be positive or negative, but they are only guaranteed to indicate a problem when they are close to zero. To better understand what we mean by "problem", consider the three-pole example in the next section.

4) A Three Pole Example

Consider the three-pole example

$$H(s) = \frac{1000}{(s+1)(s+10)(s+100)}$$

with a null compensator. That is,

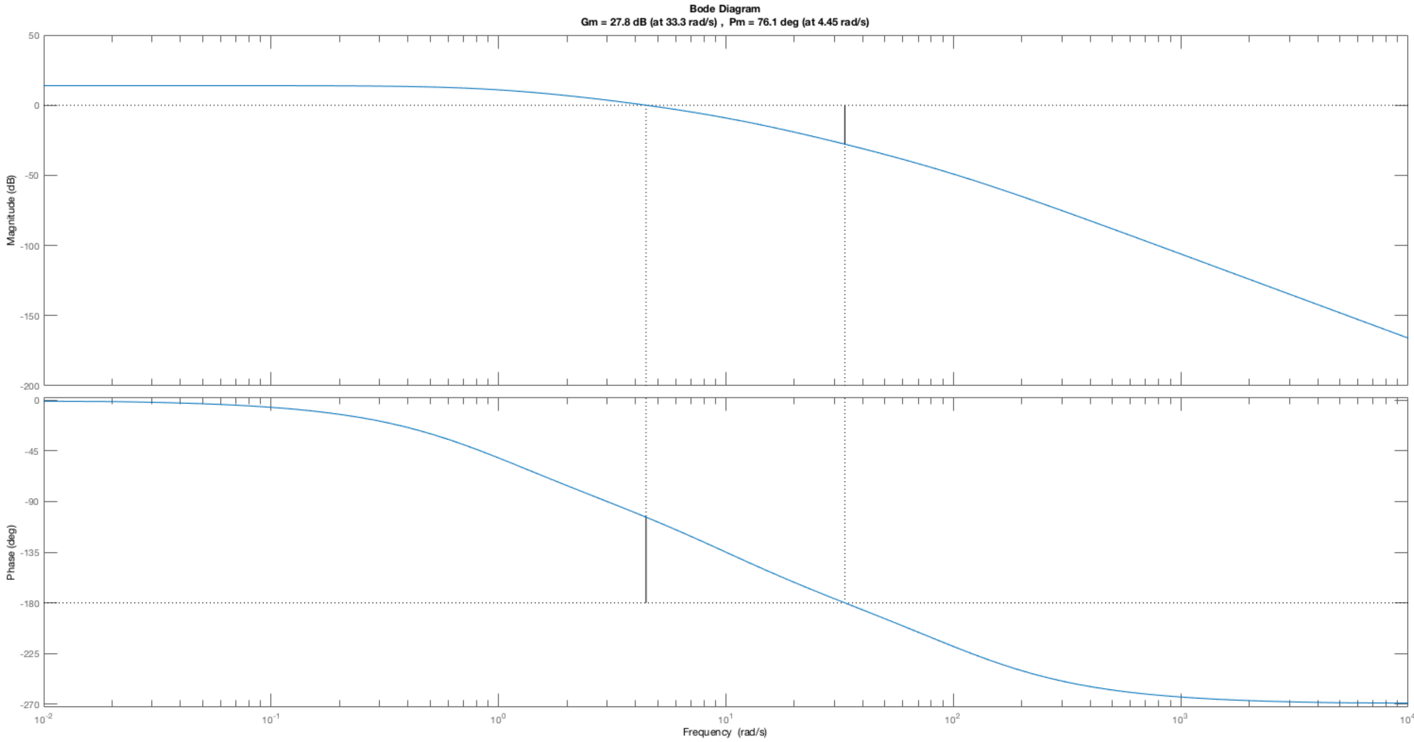
$$H_c(s) = 1.$$

Since $H(0) = 1$, the plant is normalized, and any gain will come from $K_0 > 1$ in

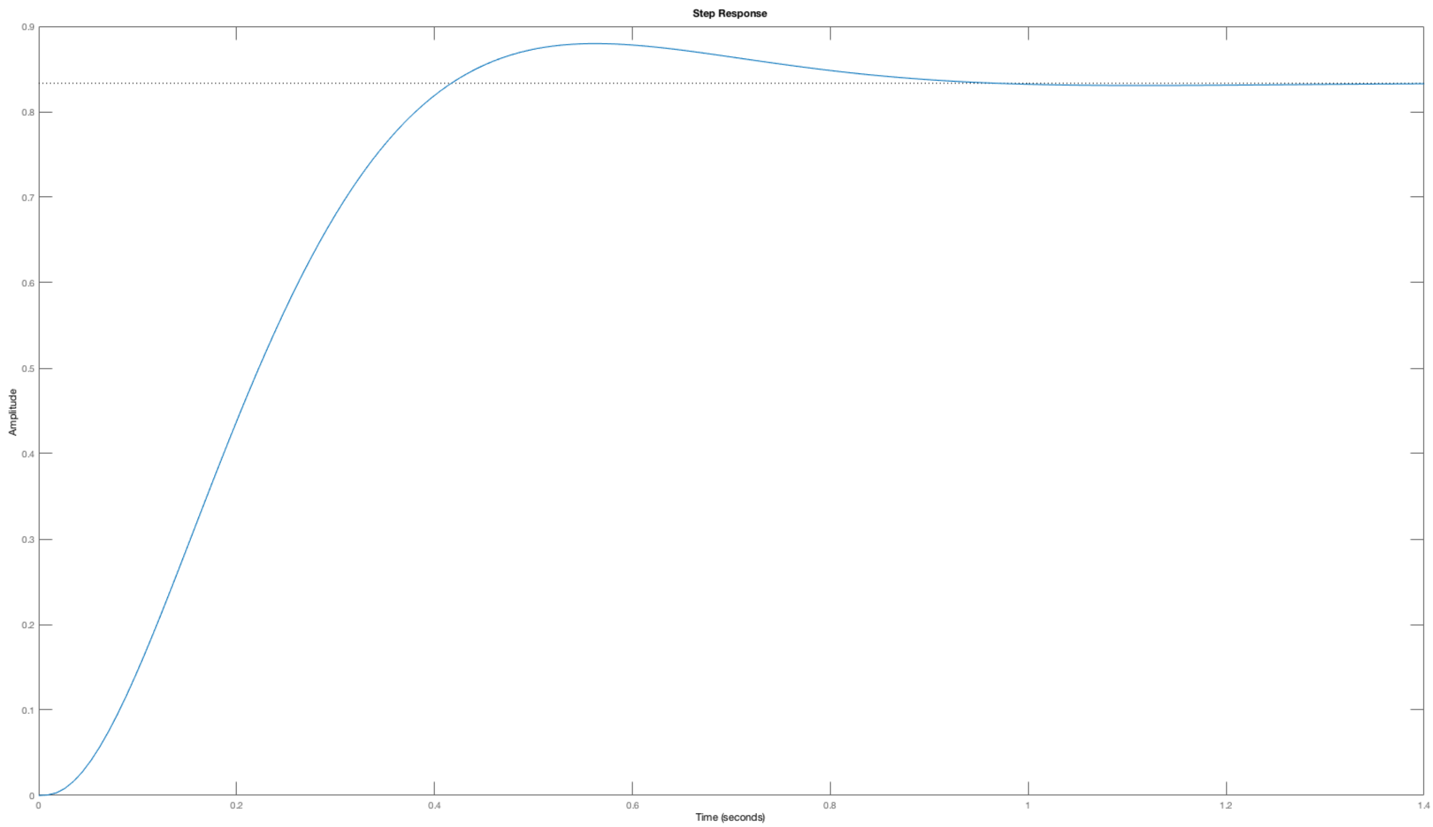
$$K_0 H_c(j\omega)H(j\omega)) = K_0 \frac{1000}{(s+1)(s+10)(s+100)}$$

.

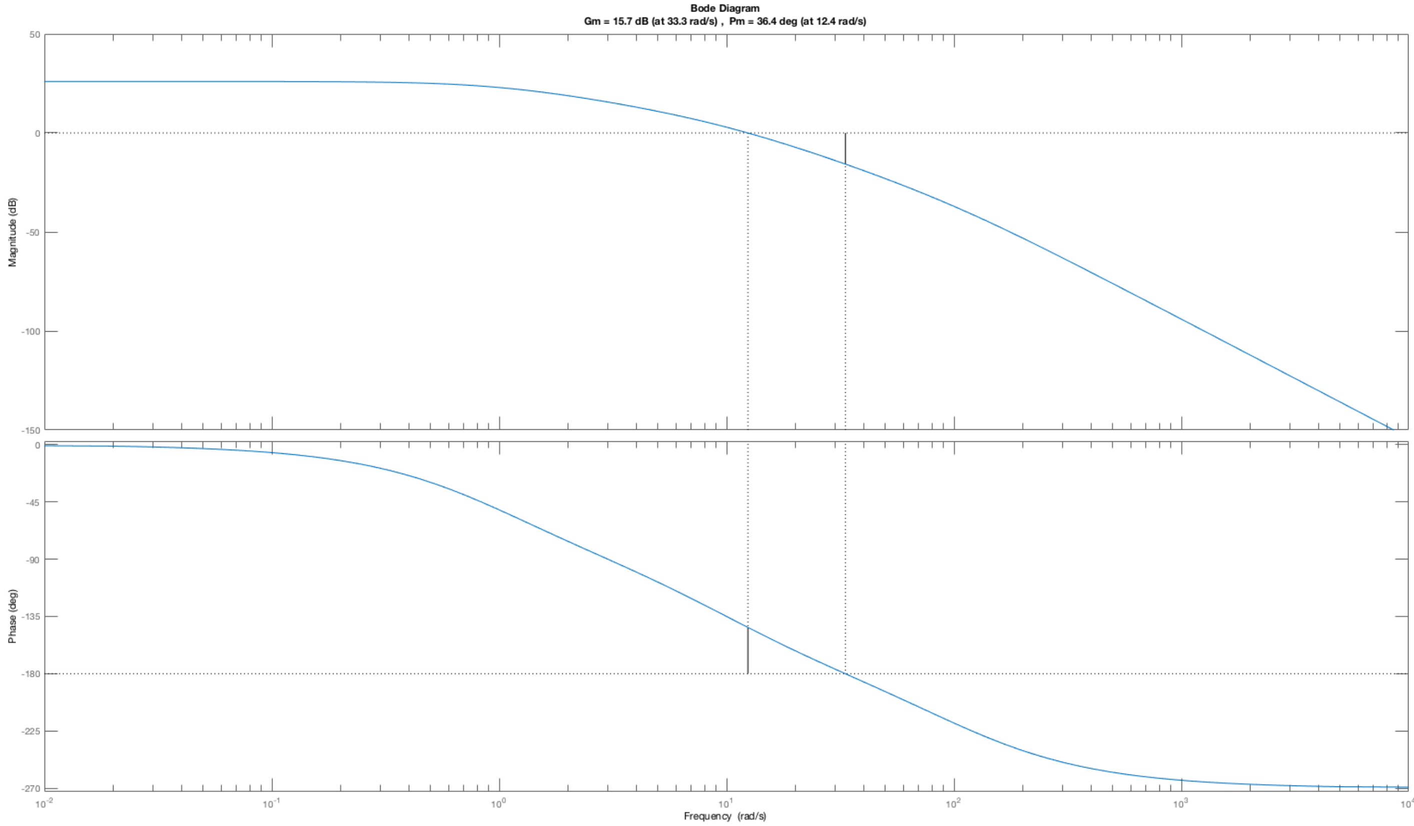
Consider the open-loop margin plot and the closed-loop step response for the cases $K_0 = 5, 20,\ and\ 50$ below. As is clear from the plots, the phase margin decreases with higher and higher gains (values of K_0), and the step response becomes more oscillatory. The low phase margin was a correct indication of a problem, as the oscillation in the step response at $K_0 = 50$ is certainly a problem.



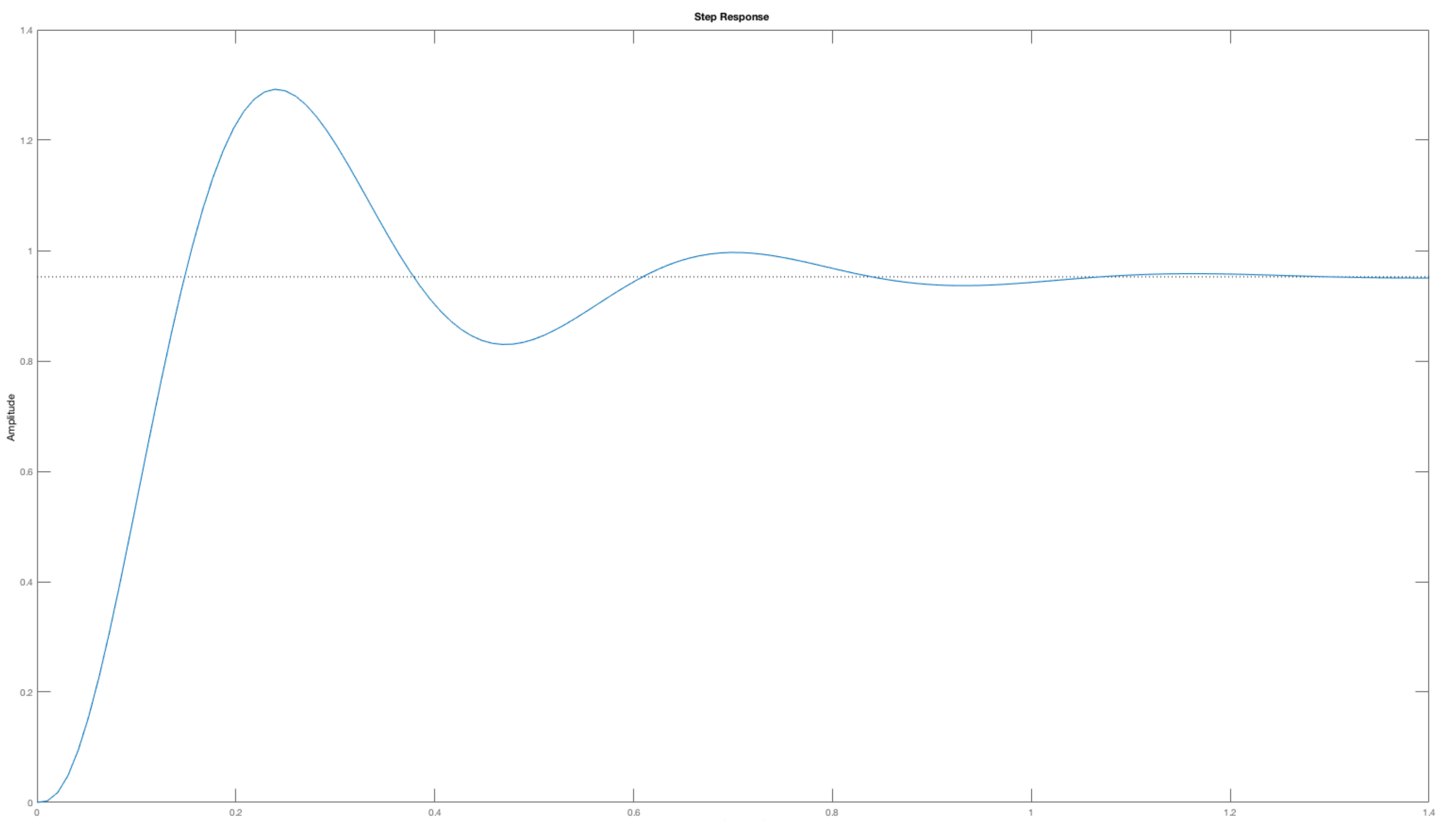
Bode Plot For $K_0 = 5$ showing gain and phase margins.



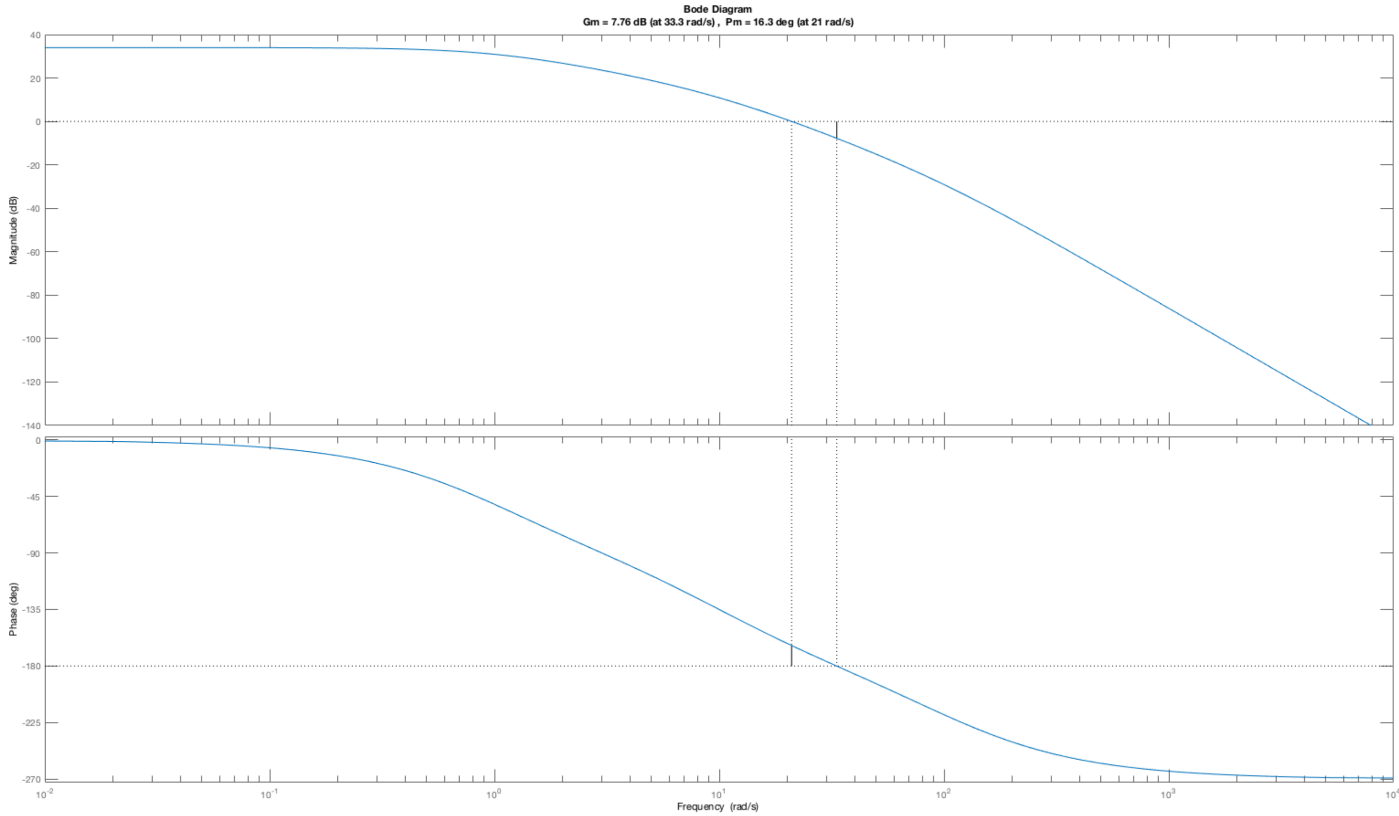
Closed-Loop step response for the $K_0 = 5$ case.



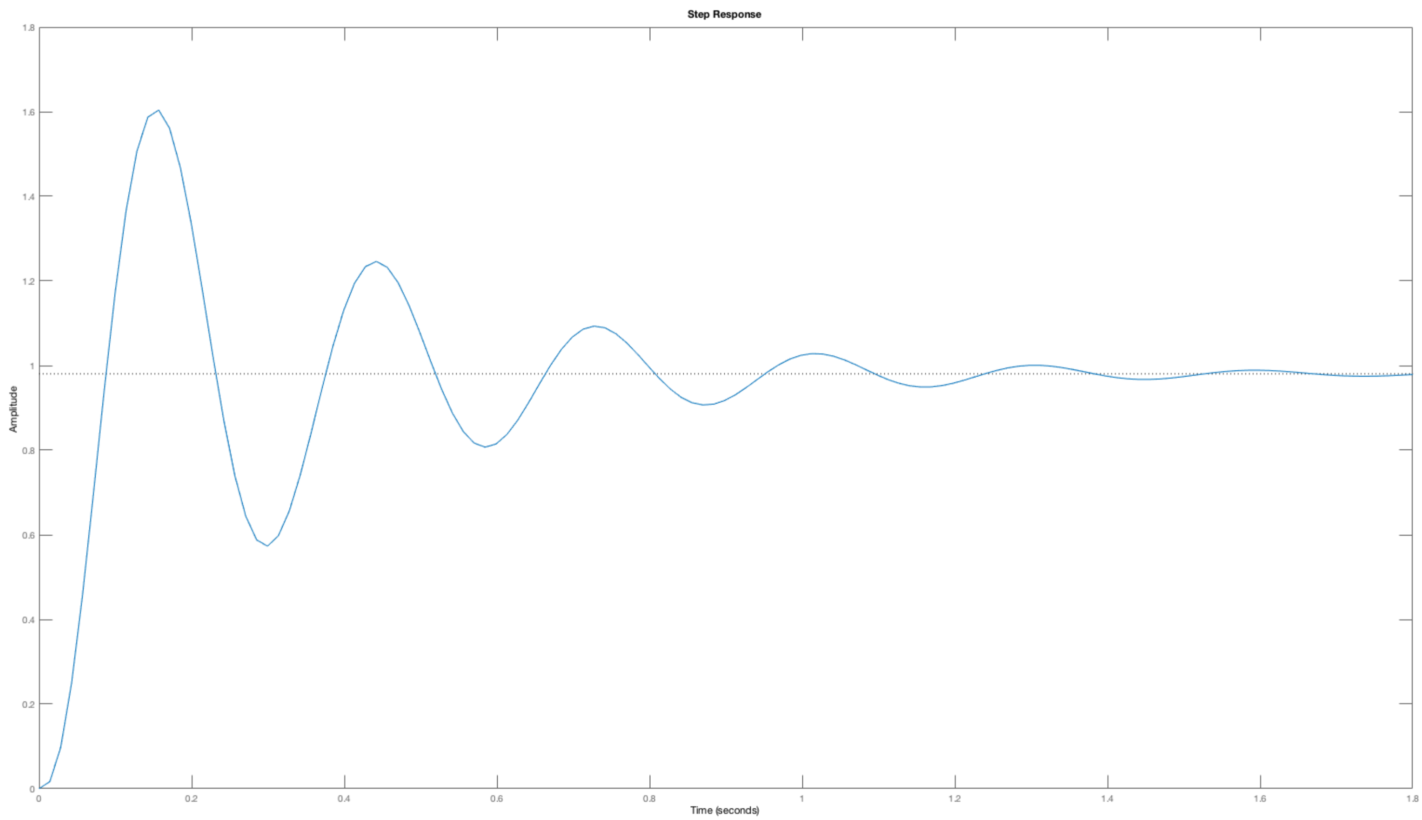
Bode Plot For $K_0 = 20$ showing gain and phase margins.



Closed-Loop step response for the $K_0 = 20$ case.



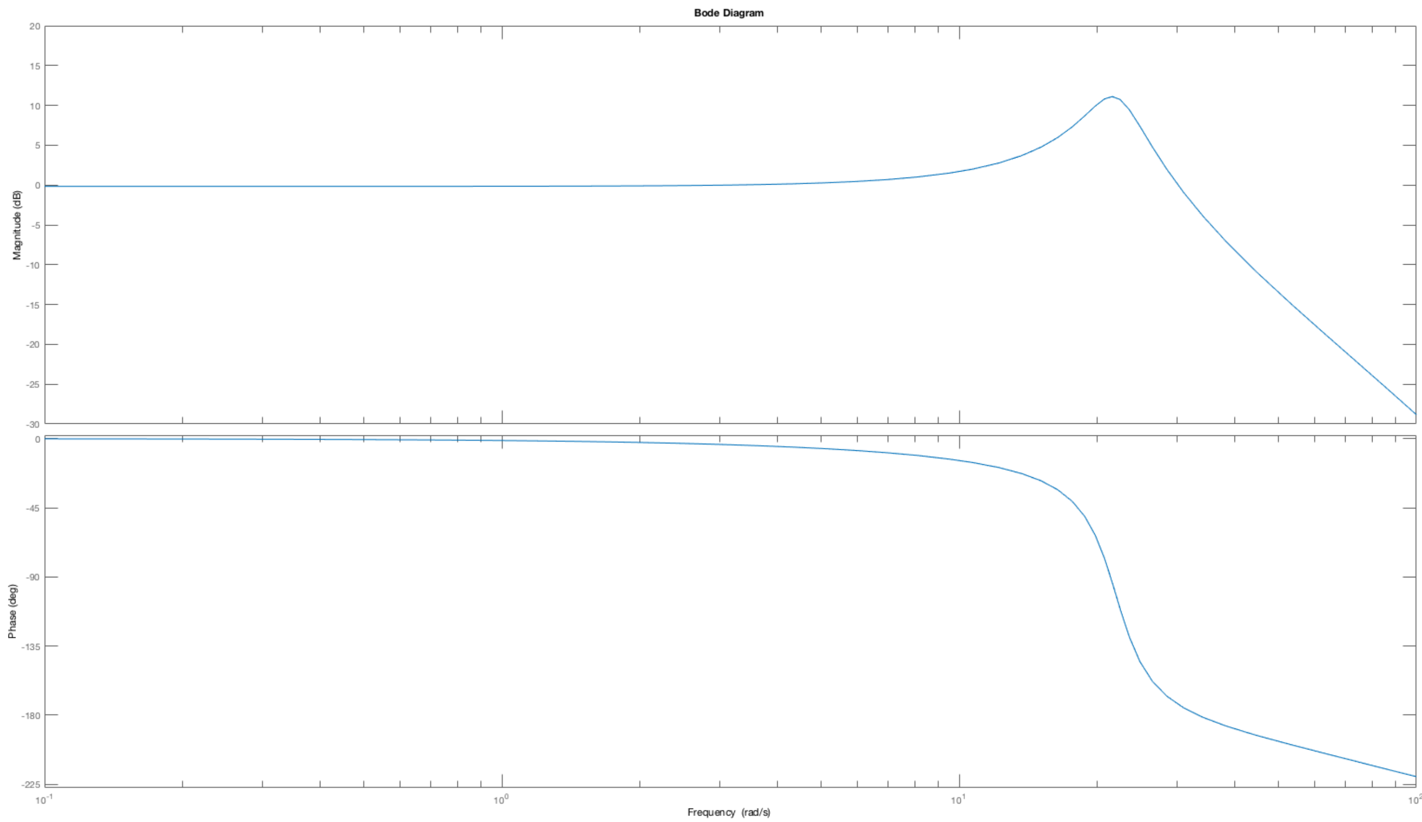
Bode Plot For $K_0 = 50$ showing gain and phase margins.



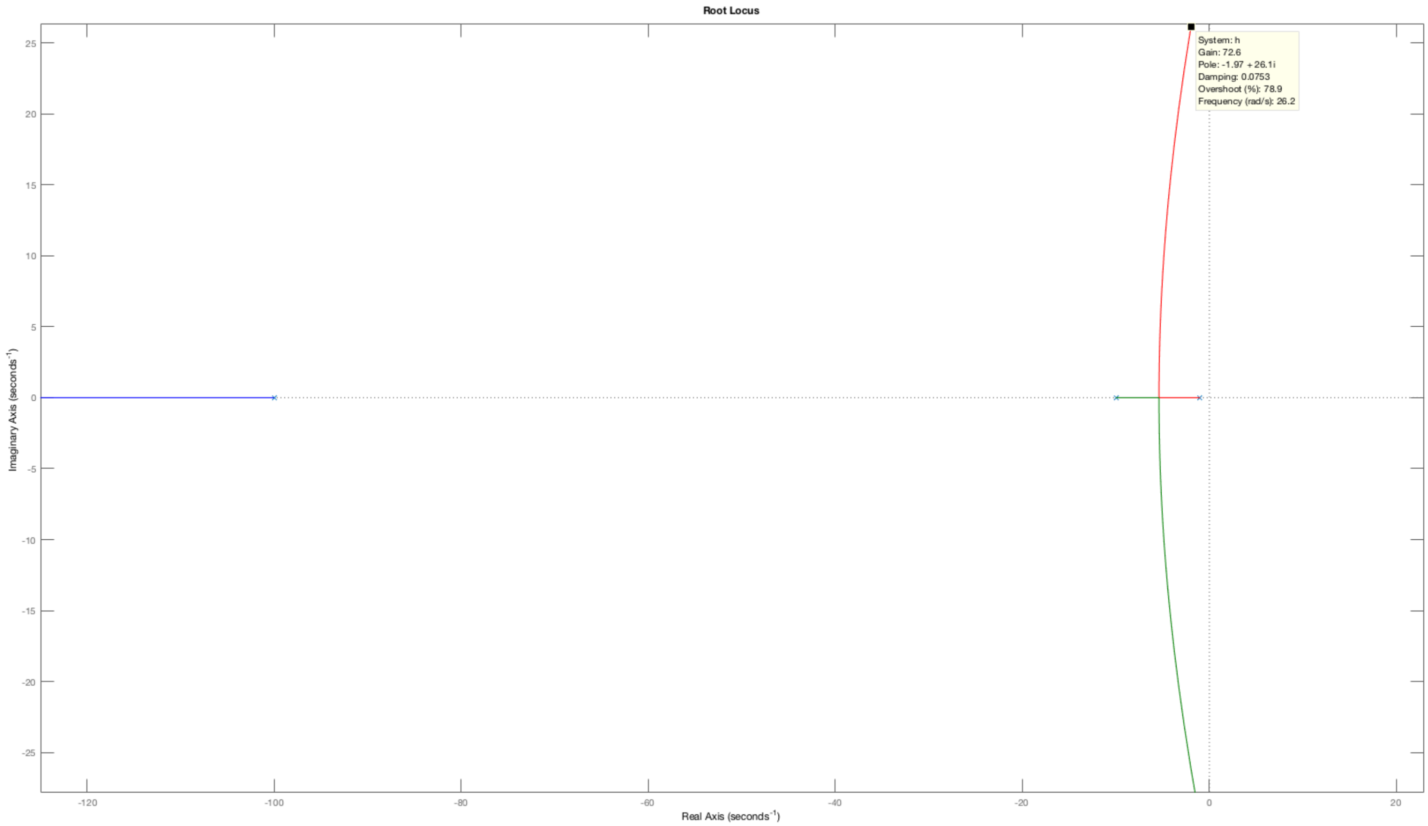
Closed-Loop step response for the $K_0 = 50$ case.

One might expect that an open-loop system with a small phase margin will likely result in a closed-loop system with an oscillatory step response, such as the particularly pronounced $K_0 = 50$ case. But phase- margin-based predictions are not quantitative. The oscillation amplitude of a closed-loop system, including its oscillation frequency, is far more readily predicted by considering the Bode plot for the closed-loop system with $K_0 = 50$ (see figure below). As the Bode plot for the closed loop system shows, the frequency response has a large peak, exactly at the frequency of the oscillating step response. Or, we could have considered the root-locus plot for this system, which would have indicated that at $K_0 = 50$, the poles of the closed-loop system move close to the imaginary axis (the second figure below). Finally, when $K_0 = 5$, the gain is too low to accurately track the input, and the steady state value for the step response (which should be one) is off by more than ten percent. Phase and gain margin tell us little about this low gain problem.

So, why bother with gain and phase margin?



Bode Plot of the CLOSED-LOOP response for the $K_0 = 50$ case, showing the peaking of the response at ≈ 22 radians/second.



The root-locus plot for $H(s) = \frac{1000}{(s+1)(s+10)(s+100)}$, showing a pair of poles moving towards the imaginary axis as the gain increases (the highest gain shown is $K_0 \approx 70$).

5) Compensators Based on Increasing Phase Margin

We know that a small phase margin is bad, but as noted above, we have other tools that can provide a far more sophisticated analysis of the kinds of problems indicated by a low phase margin. On the other hand, if we do have low phase margin, there is an easy way to design a compensator to increase it.

The key insight to increasing phase margin is that the angle for a product of two transfers functions is the sum of the two angles. That is,

$$\angle\left(H_1(j\omega)H_2(j\omega)\right) = \angle\left(H_1(j\omega)\right) + \angle\left(H_2(j\omega)\right).$$

To see how to use this additive property, suppose $K_0 H(s)$ has a small phase margin. For systems dominated by poles (as most physical systems are), the angle of the frequency response will be negative. Then since the phase margin is small, the angle at unity gain must be close to $-\pi$, or in degrees,

$$\angle(K_0 H(j\omega_{unity})) \approx -180.$$

If we can design a compensator whose angle is positive at ω_{unity}, but has a magnitude of one, then we can use the compensator to increase the phase margin. And since the magnitude of the product of the transfer functions (which equals the product of the magnitudes) will be unchanged, we can guarantee that there will be NO change in the unity gain frequency.

But not so fast! We can not exactly enforce the unit magnitude constraint, while still adding phase, but we can come close. And, we can come close with a transfer function that is easily implemented (in continuous time with a circuit, or approximately in discrete-time, as we shall set later).

Consider a normalized compensator transfer function:

$$H_c(s) = \frac{s_p}{s_z}\frac{s - s_z}{s - s_p}.$$

where s_p is the typically-negative pole location and s_z is the typically-negative zero location (yes, z is an unfortunate choice of subscript, we have NOT stepped back into discrete-time).

What is magnitude of the normalized compensator at zero frequency?

Check Formatting Submit View Answer

You have 30 submissions remaining.

In terms of s_p and s_z, what is the magnitude of the compensator in the limit as $\omega \rightarrow \infty$? Use s_p to denote s_p and s_z to denote s_z,

$H(j\infty)$

Check Syntax Submit View Answer

You have 10 submissions remaining.

Can we use this zero-over-pole compensator to add phase at the unity gain frequency? And can we come close to keeping the magnitude of the compensator near one, so we do not shift the unity gain frequency?

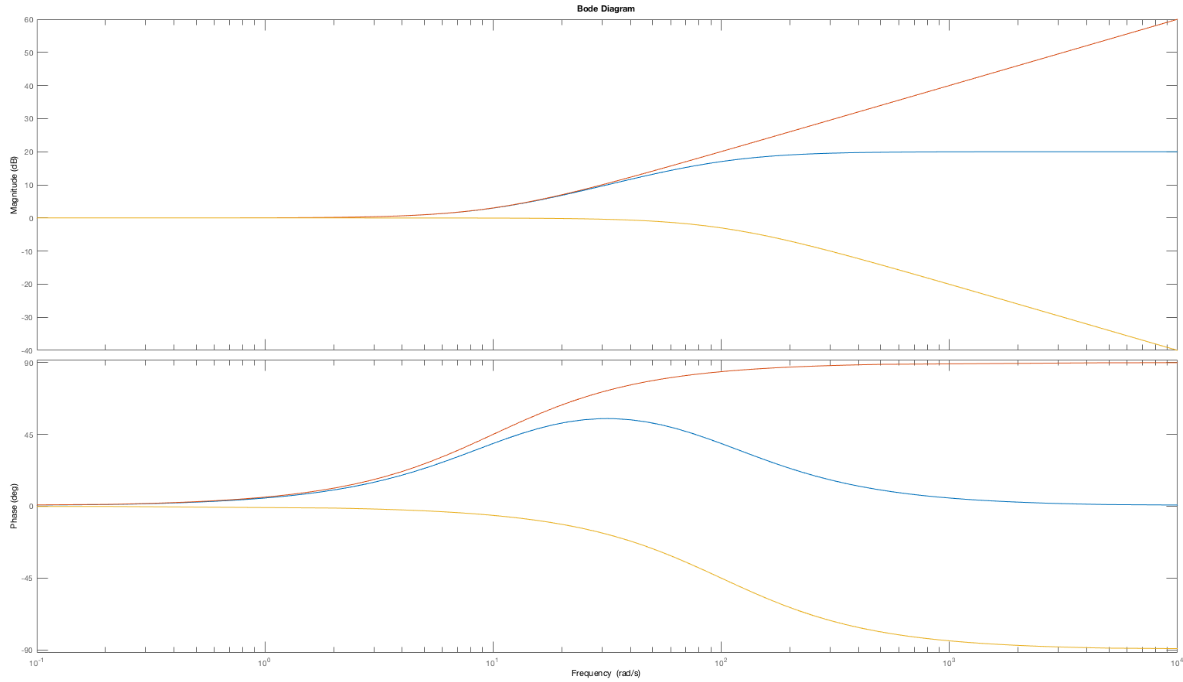
Consider the example of a compensator whose zero is $s_z = -10$, and whose pole is $s_p = -100$. With this pole and zero, the resulting compensator is of the form

$$H_c(s) = 10\frac{s+10}{s+100} = (0.1(s+10))\left(\frac{100}{s+100}\right)$$

In the figure below, there are three plots. One is a plot of $H_c(s)$, whose magnitude is level, and then monotonically increases, before leveling off a factor of ten higher. Its angle increases, but visually, it seems to start increasing at a lower frequency than its magnitude increases. Perhaps we can use this to approximate our ideal phase margin increaser.

Overall, the angle of the compensator rises from an angle of 0, achieves a maximum, and then returns to an angle of 0. We also show the seperate components in the plot below. There is the angle and magnitude for just the numerator zero, which is flat until the zero frequency and then the angle and magnitude increase monotonically. There is also the angle and magnitude for the denominator pole, which is also flat until the pole frequency, and then starts monotonically decreasing in angle and magnitude.

NOTE: we refer to this compensator as a *LEAD* compensator because the zero leads the pole.



Suppose $H_c(s) = \frac{s_p}{s_z}\left(\frac{s-s_z}{s-s_p}\right)$, with real s_z, $s_p < 0$. In terms of s_p and s_z, what is the value of $\omega_{extreme}$, the frequency where the angle of $H_c(j\omega)$ achieves its extreme (either maximum or minimum)? Hint: recall that the angle of H_c is the angle of the zero minus the angle of the pole. So differentiate the difference with respect to ω, and find where the derivative is zero. Use s_p to denote s_p and s_z to denote s_z.

$\omega_{extreme}$

Check Syntax | Submit | View Answer

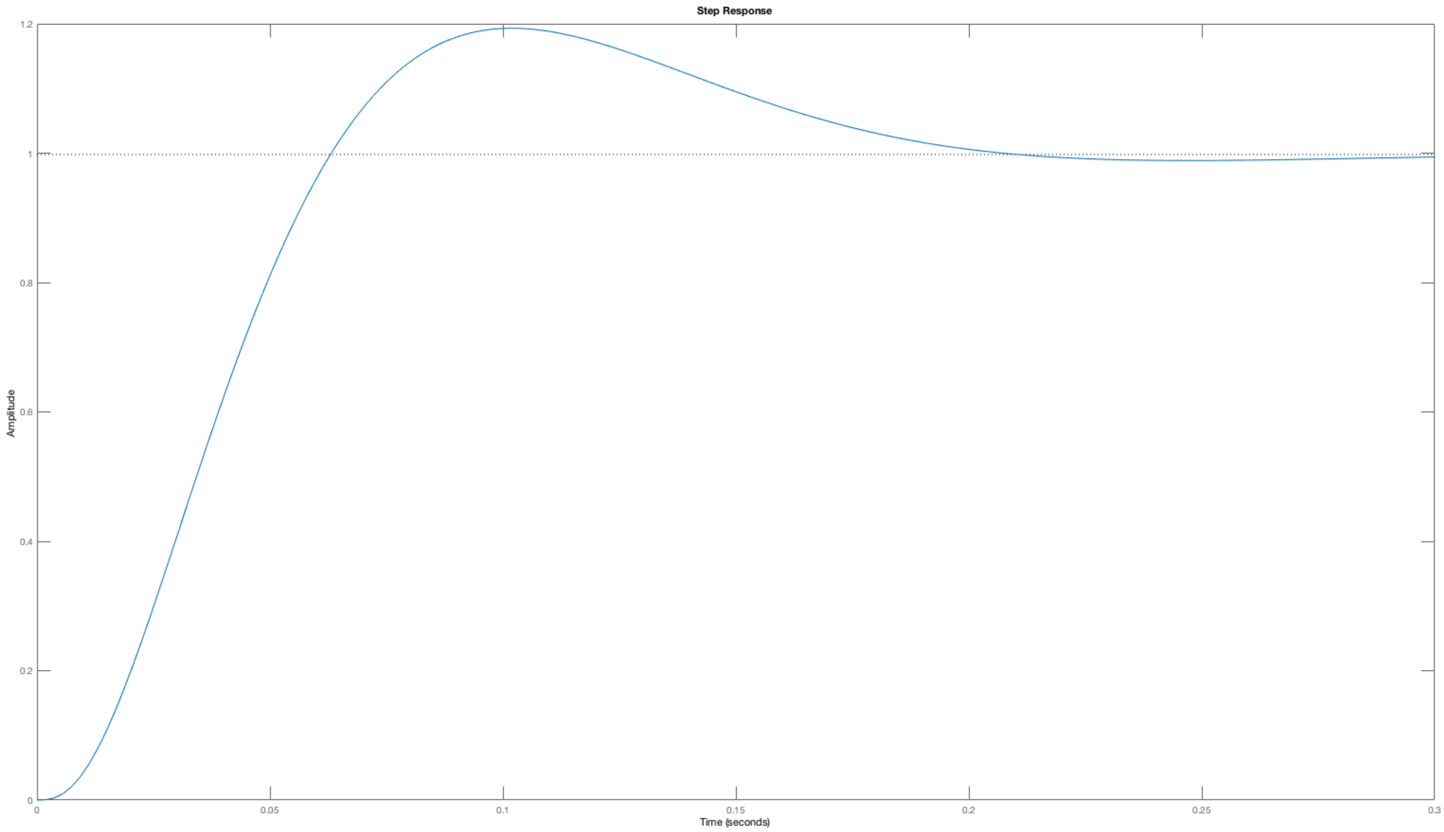
You have 10 submissions remaining.

Let us return to the worst example from above, $H(s) = \frac{1000}{(s+1)(s+10)(s+100)}$ with $K_0 = 50$. The phase margin is quite low, roughly 15 degrees, and the step response oscillates. Suppose we place a normalized compensator with a zero at approximately the unity gain frequency ($s_z = -20$) and a pole at a ten times higher frequency ($s_p = -200$)? What is the new phase margin (you will need a bode plotter, use either matlab or python). To check yourself, we have plotted the step response of the compensated system (which is QUITE an improvement).

Phase Margin:

Check Formatting | Submit | View Answer

You have 30 submissions remaining.


Step Response
Amplitude
Time (seconds)
0
0.2
0.4
0.6
0.8
1
1.2
0.05
0.1
0.15
0.2
0.25
0.3

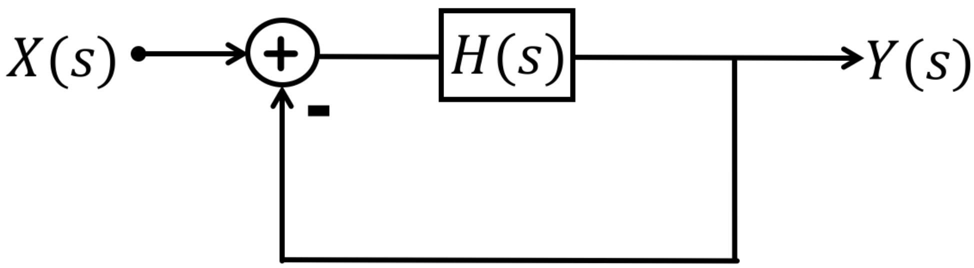
Bode and Steps

The questions below are due on Thursday April 04, 2024; 10:30:00 PM.

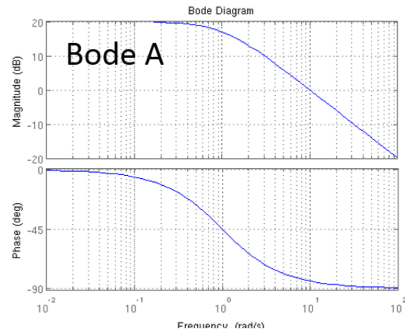
To review the relationship between open-loop frequency response and closed-loop step responses, consider the following Bode plots of open-loop systems, $H(s)$, and the step responses of the closed-loop systems,

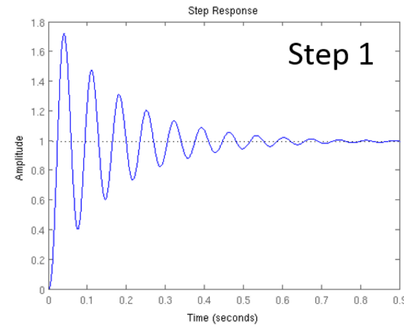
$$Y = \frac{H(s)}{1+H(s)}\,X$$

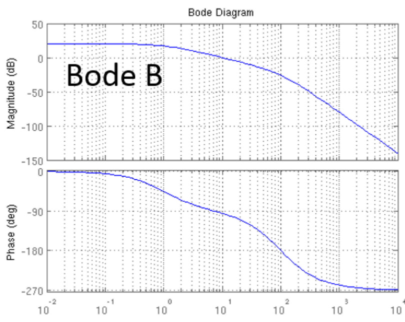
as shown in the feedback block diagram below.

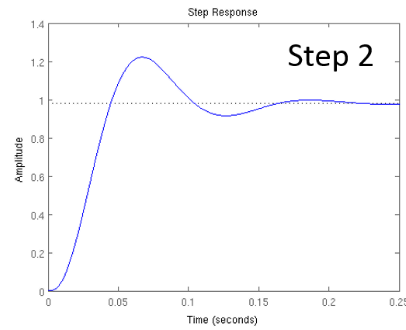


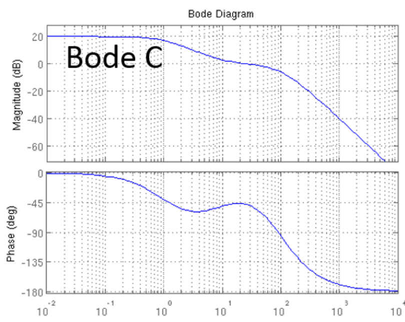
Please match the bode plot of $H(s)$ with the step response of $\frac{H(s)}{1+H(s)}$. To find matches, you can work backward from the closed-loop system step response. Closed-loop step response features such as overshoot and oscillation period indicate the closed-loop natural frequencies, from which one can infer the closed-loop system frequency response. Given the closed-loop system frequency response, one can estimate the frequency response of the open-loop system. You can also work forward from the open-loop frequency response, with the key being the phase margin of the open-loop system. A small phase margin results in highly oscillatory closed-loop step responses.

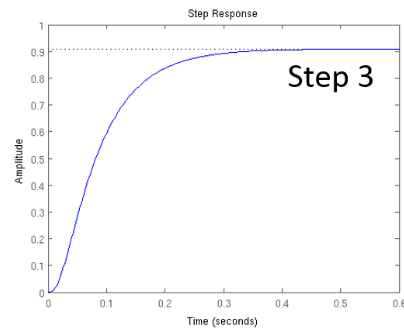

Bode Diagram
Bode A
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

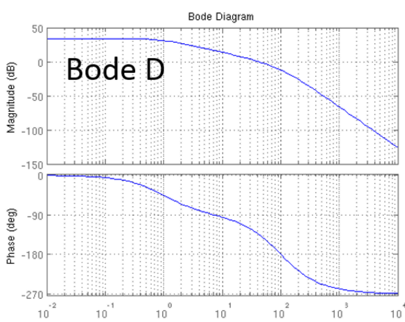

Step Response
Step 1
Amplitude
Time (seconds)

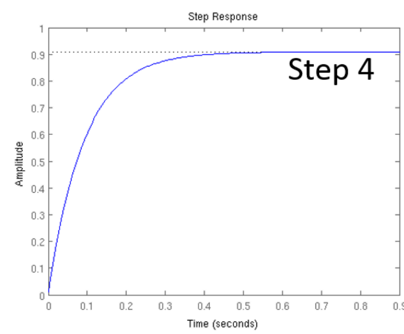

Bode Diagram
Bode B
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

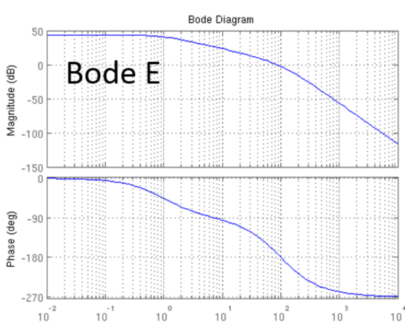

Step Response
Step 2
Amplitude
Time (seconds)

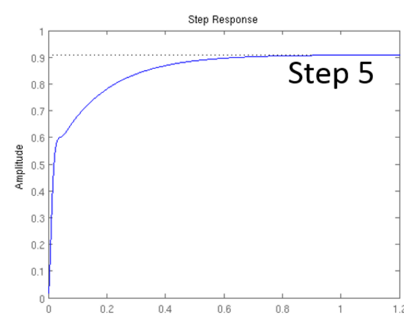

Bode Diagram
Bode C
Magnitude (dB)
Phase (deg)
Frequency (rad/s)


Step Response
Step 3
Amplitude
Time (seconds)


Bode Diagram
Bode D
Magnitude (dB)
Phase (deg)
Frequency (rad/s)


Step Response
Step 4
Amplitude
Time (seconds)


Bode Diagram
Bode E
Magnitude (dB)
Phase (deg)
Frequency (rad/s)


Step Response
Step 5
Amplitude
Time (seconds)

Bode plot A corresponds to step response: --

Save Submit View Answer

You have infinitely many submissions remaining.

Bode plot B corresponds to step response: --

Save Submit View Answer

You have infinitely many submissions remaining.

Bode plot C corresponds to step response: --

Save Submit View Answer

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Bode plot D corresponds to step response: --

Save Submit View Answer

You have infinitely many submissions remaining.

Bode plot E corresponds to step response: --

Save Submit View Answer

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Arm Like Example

The questions below are due on Thursday April 04, 2024; 10:30:00 PM.

1) An example similar to the Copter Arm.

When we included the thrust-buildup in the model for the copter-arm, the transfer function was not too different from

$$H(s) = \frac{200}{(s+10)s^2}$$

where we have assumed a $\gamma = 20$ and a continuous time thrust build-up pole of -10.

Is there any K_0 for which $K_0 H(s)$ has a positive phase margin?

Is there any K_0 for which K_0*H(s) has a positive phase margin? --

Save Submit View Answer

You have 30 submissions remaining.

Suppose we have three possible compensators:

$$K_A(s) = 10\frac{s+1}{s+10}$$

$$K_B(s) = 10\frac{s+5}{s+50}$$

$$K_C(s) = 10\frac{s+10}{s+100}$$

which compensator would produce the best phase margin for the plant described above? Enter a single compensator (such as `K_D` to indicate $K_D(s)$) or multiple compensators separated with commas.

Submit View Answer

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Does using $K_C(s)$ give you a positive phase margin?

Does using $K_C(s)$ give you a positive phase margin? --

Save Submit View Answer

You have 30 submissions remaining.

Is there any $\beta > 1$ for which $10\frac{s+10\beta}{s+100\beta}$ gives you a postive phase margin. (Hint: with $\beta > 1$, the leading zero in the compensator is more negative than the most negative pole in the plant).

Is there a $\beta > 1$ for which using $10 * (s + 10\beta)/(s + 100\beta)$ gives a positive phase margin? --

Save Submit View Answer

You have 30 submissions remaining.

In a lead compensator, the zero "leads" the pole. That is, the compensator's zero is less negative than the compensator's pole. Which of the following statements about lead compensators is true?

"If a lead compensator is applied to a stable three real pole system, and the compensator's zero is as negative, or more negative, than the most negative system pole, then the compensator will NOT improve the system's phase margin."

Hint: What about $H(s) = \frac{60}{(s+1)*(s+2)*(s+3)}$?

No positive phase margin with three stable real poles? --

Save Submit View Answer

You have 30 submissions remaining.

"If a lead compensator is applied to a system with one negative real pole and two poles at 0, and the compensator's zero is as negative, or more negative, than the most negative system pole, then the compensated system will NEVER have a positive phase margin."

No positive phase margin with two poles at 0, the other negative real? --

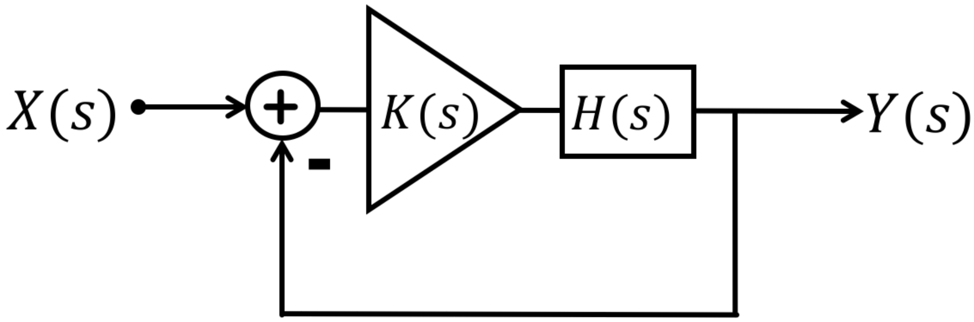
Save Submit View Answer

You have 30 submissions remaining.

Phase Margin

The questions below are due on Thursday April 04, 2024; 10:30:00 PM.

Consider again the feedback system below,



where $H(s)$ has the three-pole transfer function

$$H(s) = \frac{10 * 1000 * 1000}{(s+10)(s+1000)(s+1000)}$$

and $K(s) = K$ is a constant gain factor. From Black's formula, the closed loop transfer function is

$$G(s) = \frac{KH(s)}{1+KH(s)}.$$

You may find it easiest to do this problem by trial and error using MATLAB's bode and/or margin plots, though you should be able to intelligently tune K. Also, when looking at overshoot, you can right click the step response in Matlab and have it display the peak amplitude and the final steady-state (settled) value.

Determine a value for K so that $KH(s)$ has a phase margin that is close (within 1 degree) to 90 degrees.

K

Check Formatting Submit View Answer

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What is the peak overshoot, delta above steady-state, in the unit step response of the closed loop system when its loop transfer function has 90 degrees of phase margin? Give an answer in the form of (peak value - final value). If there is no overshoot, enter zero.

Check Formatting Submit View Answer

You have infinitely many submissions remaining.

Where are the poles of the closed loop system with 90 degrees of phase margin?

- Left-half of s-plane
- Right-half of s-plane
- On the $s = jw$ axis

Save Submit View Answer

You have infinitely many submissions remaining.

Now set K to yield about 60 degrees of phase margin. What is the peak overshoot, delta above steady-state, in the unit step response of the closed loop system when its loop transfer function has 60 degrees of phase margin? Give an answer in the form of (peak value - final value). If there is no overshoot, enter zero.

Check Formatting Submit View Answer

You have infinitely many submissions remaining.

Where are the poles of the closed loop system with 60 degrees of phase margin?

- Left-half of s-plane
- Right-half of s-plane
- On the $s = jw$ axis

Save Submit View Answer

You have infinitely many submissions remaining.

Now set K to yield 30 degrees of phase margin. What is the peak overshoot, delta above steady-state, in the unit step response of the closed loop system when its loop transfer function has 30 degrees of phase margin? Give an answer in the form of (peak value - final value). If there is no overshoot, enter zero.

Check Formatting Submit View Answer

You have infinitely many submissions remaining.

Where are the poles of the closed loop system with 30 degrees of phase margin?

- Left-half of s-plane
- Right-half of s-plane
- On the $s = jw$ axis

Save Submit View Answer

You have infinitely many submissions remaining.

Now set K to yield 0 degrees of phase margin. One pole will be in the left-half of the s-plane. Where are the remaining conjugate pair poles of the closed loop system?

- ○ Left-half of s-plane
- ○ Right-half of s-plane
- ○ On the $s = jw$ axis

Save Submit View Answer

You have infinitely many submissions remaining.

Now set K to yield -60 degrees of phase margin (phase drops below 180 before gain drops below 1). Where are the poles of the closed loop system? Solve numerically for the exact locations (for your own learning) and then indicate generally where they are below.

- ○ Left-half of s-plane
- ○ Right-half of s-plane
- ○ On the $s = jw$ axis

Save Submit View Answer

You have infinitely many submissions remaining.

In the case with -60 degrees of phase margin, is the system stable or unstable?

- ○ Stable
- ○ Unstable

Save Submit View Answer

You have infinitely many submissions remaining.