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To Whom It May Concern:

I am happy to submit this letter on behalf of Dr. Sundar Krishnamurthy. My opinion is based on a professional judgment of his work as I do not know him personally and I have never directly worked with him. However, I have followed some of the outstanding research work of Dr. Krishnamurthy in the field of network information theory and wireless communications, more specifically, multi-user interference management in communication systems, as elaborated below.

First, allow me to provide some background on myself. I am currently a Professor in the department of Electrical Engineering at Stanford University. I am credited with pioneering the invention of MIMO. MIMO is fundamental to all current and next generation wireless systems. I have authored over 400 research papers, two text books and am a co-inventor of 63 US patents. I have won over a dozen awards, notably the Marconi Prize and Fellowship, 2014 and the IEEE Alexander Graham Bell Medal, 2011. I am a Fellow of eight National Scientific Academies including those in US, China, India and Sweden. I am a Fellow of IEEE and AAAS. My books are: Introduction to Space-Time Wireless Communications, A Paulraj, R. Nabar and D. Gore, Cambridge Univ. Press, May 2003, Reprinted Chinese Ed. 2004, Reprinted Russian Ed. 2007, Electronic Version 2008, and MIMO Wireless Communications, E. Biglieri, A. Constantinides, R. Calderbank, A. Goldsmith, A. Paulraj and V. Poor, Cambridge Univ. Press, 2007, Reprinted Japanese Ed. 2008. In 1999, I founded Iospan Wireless Inc., which developed and established MIMO-OFDMA (orthogonal frequency division multiple access) wireless as the core 4G technology. Iospan was acquired by Intel Corporation in 2003. In 2004 I co-founded Beceem Communications Inc. The company became the market leader in 4G-WiMAX (Worldwide Interoperability for Microwave Access) semiconductor and was acquired by Broadcom Corp. in 2010. During my 30 years in the Indian Navy (1961-1991), I founded three national level laboratories in India and headed one of India's most successful military R&D projects – APSOH (Advanced Panoramic Sonar Hull) sonar. I have received over a dozen awards (many at the national level) in India including the Padma Bhushan, one of India's highest awards. A copy of my curriculum vitae is attached for your reference.

I know Dr. Krishnamurthy primarily through his research work on the Degrees of Freedom (DoF) of Interference networks with Multiple Input Multiple Output (MIMO) Rank deficient channels. MIMO has been the fundamental revolutionary technology that allowed us to tremendously improve capacity in a plethora of communication systems, both in theory and in practice. Ever since the advent of Interference Alignment, notably after the groundbreaking work of Professor Syed Jafar, Dr. Krishnamurthy's Ph.D. Advisor, a large section of information theory community has investigated DoF, a first-order approximation of capacity, for a range of interference networks. While capacity is not easily tractable for many networks in the information theoretic sense, DoF characterization is a good first step in understanding the fundamental nature of different multi-user interference networks. The

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paradigm of Interference Alignment has facilitated characterization of DoF for various interference networks wherein novel techniques, frameworks, principles, and design guidelines have been developed in the past decade. In this context, Dr. Krishnamurthy has made significant contributions to the understanding of MIMO systems. Rank deficiency is a fundamental phenomenon in MIMO systems arising due to spatial correlation in the MIMO coefficients due to presence of channel conditions such as poor scattering and strong line-of-sight components. In order to understand the fundamental aspects of MIMO interference networks, it is natural to consider rank-deficient channels wherein the impact of each signal dimension on the total DoF or capacity could be explored.

In his journal article entitled "Degrees of Freedom of Rank deficient MIMO Interference Networks", published in IEEE Transactions on Information Theory, Dr. Krishnamurthy presented his research results on the optimal DoF of Two-, Three- and K-user MIMO rank deficient interference networks with different ranks for the direct and the cross channels. Interestingly, Dr. Krishnamurthy identified an analytical expression for the optimal DoF of such MIMO networks with different channel ranks. To this end, he had developed novel interference alignment schemes and zero-forcing techniques, and showed that the achievable DoF are also optimal through information theoretic outer bounds. It was indeed intriguing to understand how such spatial dependencies, which arise due to rank deficient channels, could impact the total DoF. Rank deficiency of the MIMO channels changed the opportunities for both zero-forcing and interference alignment, and it was not clear whether it was ultimately beneficial or not. Dr. Krishnamurthy's works shed light on this conundrum and delineated when and how the rank deficiency could hurt or improve the total DoF in a MIMO interference network. He showed that the rank deficiency in the cross channels could not hurt DoF and could only improve, while the rank deficiency in the direct channels could not improve DoF and could only damage it. These impressive research results would be fundamental in designing MIMO systems of the future, since designs considering channel rank deficiency are highly essential in practice. He had addressed many challenging problems that involve substantial highly specialized knowledge of how signal subspaces overlap in the presence of rank deficiencies. Another fundamental contribution of Dr. Krishnamurthy was how he showed that at least half the signal dimensions can be used for transmission of the desired signal free of any interference, despite the cross channels being rank deficient, as long as the direct channels are of sufficient rank. He showed this by extending the asymptotic interference alignment scheme considering rank deficient channels, with arbitrary number of users.

Dr. Krishnamurthy also found the DoF of Multi-hop Multi-flow Rank-deficient Interference channels, described in his paper entitled "Rank Matching for Multihop Multiflow" published in IEEE Transactions on Information Theory. He considered the 2x2x2 Interference channel with M antennas at all nodes and showed that the optimal total DoF of 2M are achievable as long as the channel ranks in the first hop match those in the second hop. To this extent, Dr. Krishnamurthy had also developed novel alignment techniques over multiple hops and established the DoF penalty to be the mismatch in the channel ranks. This rank-matching phenomenon is interestingly similar to Impedance matching in Circuit theory, which tells us how the impedance mismatch affects the maximum power transfer in circuits. He had also developed novel information theoretic outer bounds to show optimality in all his works.

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I find the research contributions of Dr. Krishnamurthy to be innovative, and fundamental I'm confident that many of his scholarly works can have important applications on the design of future communication systems. In my professional opinion, Dr. Krishnamurthy is an extraordinarily talented researcher who is one of few at the top of his field. He has demonstrated his ability to make fundamental and significant research contributions and I am confident that he will continue to do so in the years to come.

Sincerely,

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