

Construct a Network

UNIT - 3

① Activity

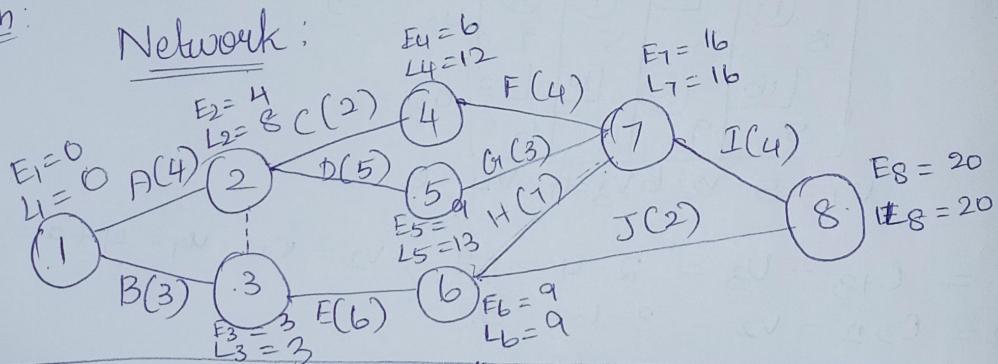
Immediate Precedence

Duration

| | | |
|---|------|---|
| A | - | 4 |
| B | - | 3 |
| C | A, B | 2 |
| D | A, B | 5 |
| E | B | 6 |
| F | C | 4 |
| G | D | 3 |
| H | F, G | 7 |
| I | F, G | 4 |
| J | E, H | 2 |

Draw the project network, critical path & corresponding project completion time.

Soln:



Forward computation

$$E_1 = 0$$

$$E_2 = E_1 + D = 0 + 4 = 4$$

$$E_3 = E_1 + D = 0 + 3 = 3$$

$$E_4 = E_2 + D = 4 + 2 = 6$$

$$E_5 = E_4 + D = 4 + 5 = 9$$

$$E_6 = E_3 + D = 3 + 6 = 9$$

$$E_7 = \max \{ E_4 + D, E_5 + D, E_6 + D \}$$

$$= \max \{ 10, 12, 16 \} = 16$$

Backward computation

$$L_8 = 20$$

$$L_7 = L_8 - 4 = 20 - 4 = 16$$

$$L_6 = \min \{ L_7 - 3, L_8 - 2 \}$$

$$= \min \{ 13, 18 \} = 13$$

$$L_5 = L_7 - 3 = 16 - 3 = 13$$

$$L_4 = L_7 - 4 = 16 - 4 = 12$$

$$L_3 = L_6 - 6 = 13 - 6 = 7$$

②

$$E_8 = \max$$

$$= \max$$

$$E_8 = 2$$

Possible

$$1 - 2 - 4$$

$$1 - 2 - 5$$

$$\boxed{1 - 3 -}$$

$$1 - 3 -$$

∴ Critical

2 its

$$E_8 = \max \{ E_7 + D, E_6 + D \}$$

$$= \max \{ 20, 11 \}$$

$$E_8 = 20.$$

$$L_2 = \max \{ L_4 - 2, L_5 - 5 \}$$

$$= \max \{ 10, 8 \} = 8$$

$$L_4 = \max \{ L_2 - 4, L_3 - 3 \}$$

$$= \max \{ 4, 0 \} = 0$$

$$L_4 = 0.$$

$$(a, i)$$

$$-D = 10 - 1 = 9$$

$$\max \{ L_2 - D, L_3 - D \}$$

$$\max \{ 9 - 4, 1 - 1 \}$$

$$\max \{ 5, 0 \} = 5$$

$$0.$$

| Possible Path | Duration |
|--------------------------|--|
| 1 - 2 - 4 - 7 - 8 | $4 + 2 + 4 + 4 = 14$ |
| 1 - 2 - 5 - 7 - 8 | $4 + 5 + 3 + 4 = 16$ |
| <u>1 - 3 - 6 - 7 - 8</u> | <u>$3 + 6 + 7 + 4 = 20$</u> |
| 1 - 3 - 6 - 8 | $3 + 6 + 2 = 11$ |

\therefore critical path is

1 - 3 - 6 - 7 - 8

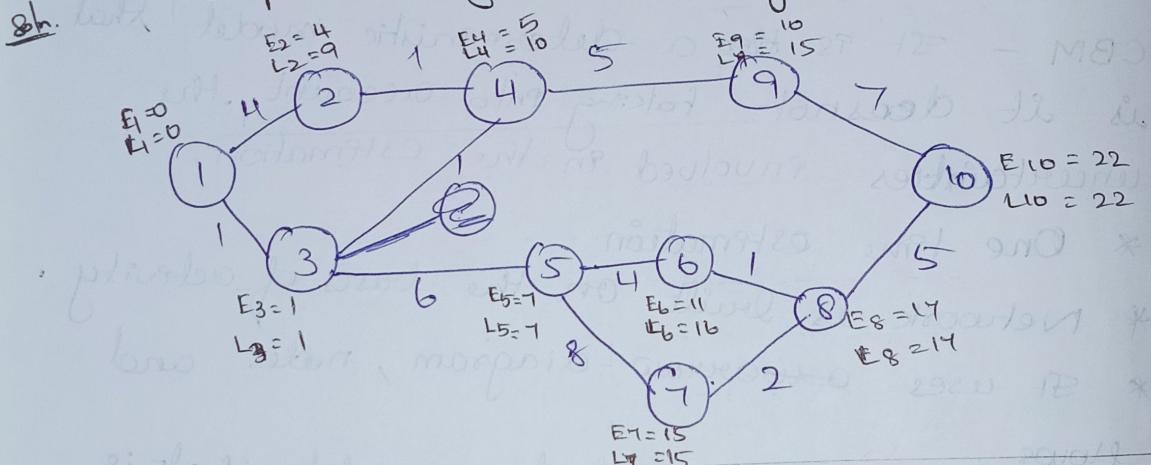
2 its project duration is 20 weeks.

5
6
5
8
0
5
5
4
5
6

Q) Construct a Network for the Project whose activities and their precedence relationship are given below.

| | | | | | | | | | | | |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|------|
| Activity | 1-2 | 1-3 | 2-4 | 3-4 | 3-5 | 4-5 | 5-6 | 5-7 | 6-8 | 7-8 | 8-10 |
| Time(hrs) | 4 | 1 | 1 | 1 | 6 | 5 | 4 | 8 | 1 | 2 | 5 |

- (i) Construct a network
- (ii) Compute Earliest & latest time for each event.
- (iii) Find critical path & min completion time of project.
- (iv) Compute total float for each activity.



Forward Computation

$$E_1 = 0$$

$$E_2 = E_1 + D = 0 + 4 = 4$$

$$E_3 = E_1 + D = 0 + 1 = 1$$

$$E_4 = \max\{E_2 + D, E_3 + D\}$$

$$= \max\{4+1, 1+1\}$$

$$= \max\{5, 2\} = 5$$

$$E_5 = E_3 + D = 1 + 6 = 7$$

$$E_6 = E_5 + D = 7 + 4 = 11$$

$$E_7 = E_5 + D = 7 + 8 = 15$$

$$E_8 = \max\{E_6 + D, E_7 + D\}$$

$$= \max\{11+1, 15+2\}$$

$$= \max\{12, 17\} = 17$$

Backward Computation

$$L_{10} = 22$$

$$L_9 = L_{10} - D = 22 - 7 = 15$$

$$L_8 = L_{10} - D = 22 - 5 = 17$$

$$L_7 = L_8 - D = 17 - 2 = 15$$

$$L_6 = L_8 - D = 14 - 1 = 13$$

$$L_5 = \min\{L_6 - D, L_7 - D\}$$

$$= \min\{13-4, 15-8\}$$

$$= \min\{12, 7\}$$

$$= 7$$

$$L_4 = L_9 - D = 15 - 5 = 10$$

$$L_3 = \min\{L_4 - D, L_5 - D\}$$

$$= \min\{10-1, 7-4\}$$

| Activity | Time (earliest) duration | E |
|----------|--------------------------|---|
| 1-2 | 4 | |
| 1-3 | 1 | |
| 2-4 | 1 | |
| 3-4 | 1 | |
| 3-5 | 6 | |
| 4-9 | 5 | |
| 5-6 | 4 | |
| 5-7 | 8 | |
| 6-8 | 1 | |
| 7-8 | 2 | |
| 8-10 | 5 | |
| 9-10 | 7 | |

EST -

EFT -

LFT -

LST -

TF =

FF -

IF -

$$E_9 = E_4 + D = 5 + 5 = 10$$

$$\begin{aligned} E_{10} &= \max\{E_9 + D, E_8 + D\} \\ &= \max\{10+7, 11+5\} \\ &= \max\{17, 22\} \end{aligned}$$

$$E_{10} = 22$$

$$\begin{aligned} &= \min(9, 1) \\ &= 1 \\ L_2 &= L_1 - D = 10 - 1 = 9 \\ L_1 &= \min\{L_2 - D, L_3 - D\} \\ &= \min\{9 - 4, 1 - 1\} \\ &= \min\{5, 0\} = 0 \end{aligned}$$

$$L_1 = 0$$

| Activity | Time (months) duration | EST | EFT = EST + D | LFT | LST = LFT - D | Total Float TF = LFT - EFT (or) LST - EST |
|----------|------------------------|------|---------------|-----|---------------|---|
| 1-2 | 4 | E1=0 | 4 | 9 | 5 | 5 |
| 1-3 | 1 | 0 | 1 | 1 | 0 | 0 |
| 2-4 | 1 | 4 | 5 | 10 | 9 | 5 |
| 3-4 | 1 | 1 | 2 | 10 | 9 | 8 |
| 3-5 | 6 | 1 | 7 | 7 | 1 | 0 |
| 4-9 | 5 | 5 | 10 | 15 | 10 | 5 |
| 5-6 | 4 | 7 | 11 | 16 | 12 | 5 |
| 5-7 | 8 | 7 | 15 | 15 | 11 | 4 |
| 6-8 | 1 | 11 | 12 | 17 | 16 | 5 |
| 7-8 | 2 | 15 | 17 | 17 | 15 | 2 |
| 8-10 | 5 | 14 | 22 | 22 | 17 | 0 |
| 9-10 | 7 | 10 | 17 | 22 | 15 | 5 |

EST - Earliest Starting time

EFT - Earliest Finishing time

LFT - Latest Finishing time

LST - Latest Starting time

TF = Total float

FF - Free float

IF - Independent float

$$FS = EF - ES$$

| | | | | |
|-----|-------------------------------|-----|--------------------------|----------|
| | | CO3 | BTL-1 | Remember |
| 8. | D.C. | | | |
| 9. | Possible Path | | Duration | |
| 10. | 1 - 2 - 4 - 9 - 10 | | $4 + 1 + 5 + 7 = 17$ | |
| 11. | 1 - 3 - 4 - 9 - 10 | | $1 + 1 + 5 + 7 = 14$ | |
| 12. | 1 - 3 - 5 - 6 - 8 - 10 | | $1 + 6 + 4 + 1 + 5 = 17$ | |
| 1. | <u>1 - 3 - 5 - 7 - 8 - 10</u> | | $1 + 6 + 8 + 2 + 5 = 22$ | |

Critical Path

1 - 3 - 5 - 7 - 8 - 10

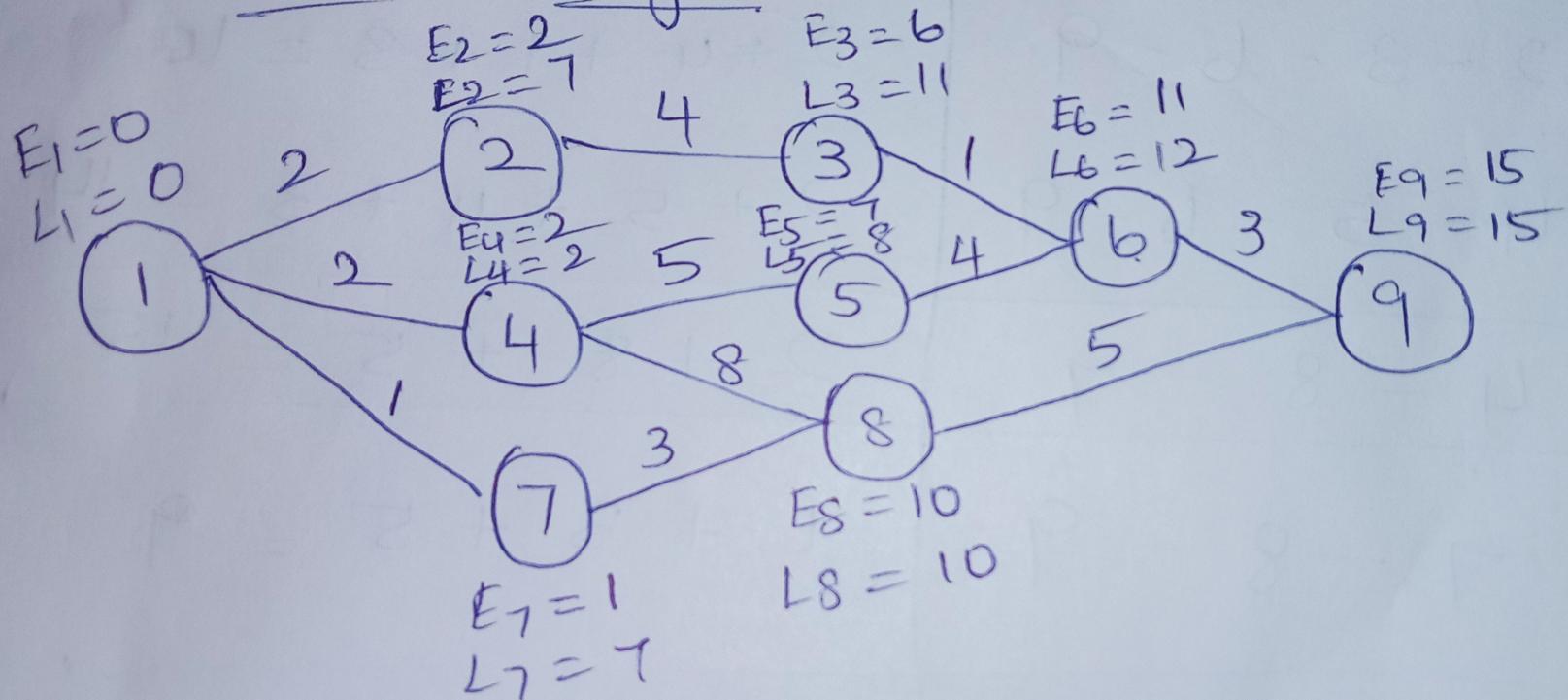
& its project duration 22 days.

- ② The total activity data, for a network is given below. Determine total float, free float, & identify the critical path.

| | | | | | | | | | | | | |
|----------|-----------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 3 | Activity | 1-2 | 1-4 | 1-7 | 2-3 | 3-6 | 4-5 | 4-8 | 5-6 | 6-9 | 7-8 | 8-9 |
| | Duration | 2 | 2 | 1 | 4 | 1 | 5 | 8 | 4 | 3 | 3 | 5 |

Sln:

Network diagram:



Forward computation

$$\begin{aligned}
 E_1 &= 0 \\
 E_2 &= E_1 + D = 0 + 2 = 2 \\
 E_3 &= E_2 + D = 2 + 4 = 6 \\
 E_4 &= E_1 + D = 0 + 2 = 2 \\
 E_5 &= E_4 + D = 2 + 5 = 7 \\
 E_6 &= \max\{E_3 + D, E_5 + D\} \\
 &= \max\{6 + 1, 7 + 4\} \\
 &= \max\{7, 11\} = 11 \\
 E_7 &= E_1 + D = 0 + 1 = 1 \\
 E_8 &= \max\{E_4 + D, E_7 + D\} \\
 &= \max\{2 + 8, 1 + 3\} \\
 &= \max\{10, 4\} = 10 \\
 E_9 &= \max\{E_6 + D, E_8 + D\} \\
 &= \max\{11 + 3, 10 + 5\} \\
 &= \max\{14, 15\} = 15 \\
 E_9 &= 15
 \end{aligned}$$

Backward computation

$$\begin{aligned}
 L_9 &= 15 \\
 L_8 &= L_9 - 5 = 15 - 5 = 10 \\
 L_7 &= L_8 - 3 = 10 - 3 = 7 \\
 L_6 &= L_9 - 3 = 15 - 3 = 12 \\
 L_5 &= L_6 - 4 = 12 - 4 = 8 \\
 L_4 &= \min\{L_5 + 5, L_8 - 8\} \\
 &= \min\{8 - 5, 10 - 8\} \\
 &= \min\{3, 2\} = 2 \\
 L_3 &= L_6 - 1 = 12 - 1 = 11 \\
 L_2 &= L_3 - 4 = 11 - 4 = 7 \\
 L_1 &= \min\{L_2 - 2, L_4 - 2\} \\
 &= \min\{7 - 2, 2 - 2\} \\
 &= \min\{5, 0\} = 0 \\
 L_1 &= 0
 \end{aligned}$$

critical
and pts
compute. π

| Activity | Duration | ES |
|----------|----------|----|
| 1-2 | 2 | |
| 1-4 | 2 | |
| 1-7 | 1 | |
| 2-3 | 4 | |
| 3-6 | 1 | |
| 4-5 | 5 | |
| 4-8 | 8 | |
| 5-6 | 4 | |
| 6-9 | 3 | |
| 7-8 | 3 | |
| 8-9 | 5 | |

Possible Path

Duration

| | |
|-----------|----------------|
| 1-2-3-6-9 | $2+4+1+3 = 10$ |
| 1-4-5-6-9 | $2+5+4+3 = 14$ |
| 1-4-8-9 | $2+8+5 = 15$ |
| 1-7-8-9 | $1+3+5 = 9$ |

Critical path : 1-4-8-9

and its project duration is 15.

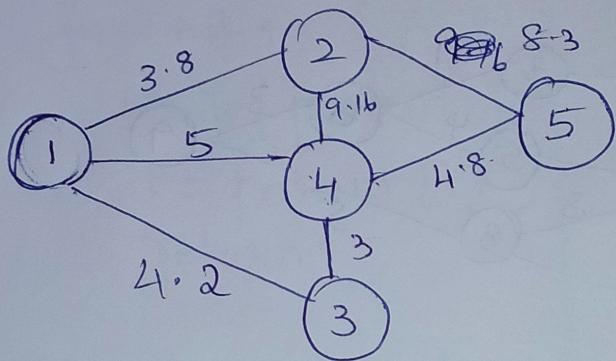
Compute TF, FF, IF.

| Activity | Duration | EST | EFT = EST + D | LFT | $LST = LFT - D$ | TF $LFT - EFT$ | FF $TF - (LFT - EFT)$ | IF $TF - (LST - EST)$ |
|----------|----------|-----|---------------|-----|-----------------|-------------------|--------------------------|--------------------------|
| 1-2 | 2 | 0 | 2 | 7 | 5 | 5 | 0 | 0 |
| 1-4 | 2 | 0 | 2 | 2 | 0 | 0 | 0 | 0 |
| 1-7 | 1 | 0 | 1 | 7 | 6 | 6 | 0 | 0 |
| 2-3 | 4 | 2 | 6 | 11 | 7 | 5 | 0 | 0 |
| 3-6 | 1 | 6 | 7 | 12 | 11 | 5 | 0 | 0 |
| 4-5 | 5 | 2 | 7 | 8 | 3 | 1 | 0 | 0 |
| 4-8 | 8 | 2 | 10 | 10 | 2 | 0 | 0 | 0 |
| 5-6 | 4 | 7 | 11 | 12 | 8 | 1 | 0 | 0 |
| 6-9 | 3 | 11 | 14 | 15 | 12 | 1 | 0 | 0 |
| 7-8 | 3 | 11 | 14 | 10 | 7 | 6 | 0 | 0 |
| 8-9 | 5 | 10 | 15 | 15 | 10 | 0 | 0 | 0 |

(5.)

| Activity | 1-2 | 1-3 | 1-4 | 2-4 | 2-5 | 3-4 | 4-5 |
|----------|-----|-----|-----|-----|-----|-----|-----|
| a | 2 | 3 | 4 | 8 | 6 | 2 | 2 |
| m | 4 | 4 | 5 | 9 | 8 | 3 | 5 |
| b | 5 | 6 | 6 | 11 | 12 | 4 | 7 |

(i) Network diagram :



(6.)

| Activity | a | m | b | Duration $E = \frac{a+4m+b}{6}$ | Variance $V = \left(\frac{b-a}{6}\right)^2$ |
|----------|---|---|----|------------------------------------|--|
| 1-2 | 2 | 4 | 5 | 3.8 | 0.25 |
| 1-3 | 3 | 4 | 6 | 4.2 | 0.25 |
| 1-4 | 4 | 5 | 6 | 5 | 0.1 |
| 2-4 | 8 | 9 | 11 | 9.1b | 0.25 |
| 2-5 | 6 | 8 | 12 | 8.3 | 1 |
| 3-4 | 2 | 3 | 4 | 2.3 | 0.1 |
| 4-5 | 2 | 5 | 7 | 4.8 | 0.69 |

(6.)

Soln:

| Possible path | Duration |
|---------------|--|
| 1 - 2 - 5 | $3 \cdot 8 + 8 \cdot 3 = 12 \cdot 1$ |
| 1 - 2 - 4 - 5 | $5 + 9 \cdot 16 + 8 \cdot 3 = 22 \cdot 46$ |
| 1 - 4 - 5 | $5 + 4 \cdot 8 = 9 \cdot 8$ |
| 1 - 3 - 4 - 5 | $4 \cdot 2 + 3 + 4 \cdot 8 = 12$ |

Critical path : 1 - 2 - 4 - 5
2 hrs. Project duration is 23

Expected variance length of given critical path
 $= 0.25 + 0.25 + 0.69$.

$$= 1.19 \approx 1.2$$

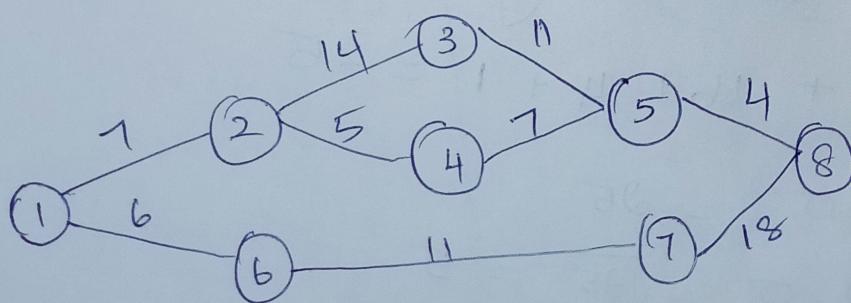
$$\sigma^2 = 1.2$$

$$\Rightarrow \sigma = \sqrt{1.2} = 1.095$$

$$\therefore \boxed{\sigma = 1.095}$$

| Job | 1-2 | 7-8 | 2-3 | 3-5 | 5-8 | 6-7 | 4-5 | 2-4 | 1-6 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| a | 3 | 4 | 6 | 5 | 1 | 3 | 3 | 2 | 2 |
| b | 15 | 28 | 30 | 14 | 7 | 27 | 15 | 8 | 14 |
| m. | 6 | 19 | 12 | 11 | 4 | 9 | 6 | 5 | 5 |

Sln: (i) Network diagram



| Activity | a | m | b | $E = \frac{a+4m+b}{6}$ | $V = \left(\frac{b-a}{6}\right)^2$ | Duration |
|----------|---|----|----|------------------------|------------------------------------|-----------------|
| 1-2 | 3 | 6 | 15 | 7 | 4 | $E_1 = 0$ |
| 7-8 | 4 | 19 | 28 | 18 | 16 | $E_2 = E_1 + 1$ |
| 2-3 | 6 | 12 | 30 | 14 | 16 | $E_3 = E_2 + 1$ |
| 3-5 | 5 | 11 | 17 | 11 | 4 | $E_4 = E_2 + 1$ |
| 5-8 | 1 | 4 | 7 | 4 | 1 | $E_5 = m$ |
| 6-7 | 3 | 9 | 27 | 11 | 16 | $= \max$ |
| 4-5 | 3 | 6 | 15 | 7 | 4 | $= \max$ |
| 2-4 | 2 | 5 | 8 | 5 | 1 | $E_6 = E_1$ |
| 1-6 | 2 | 5 | 14 | 6 | 4 | $E_7 = E_6 - 1$ |

Possible path

1-2-3-5-8

1-2-4-5-8

1-6-7-8

Duration.

$$7 + 14 + 11 + 4 = 36$$

$$7 + 5 + 7 + 4 = 23$$

$$6 + 11 + 18 = 35$$

Critical path: 1-2-3-5-8

∴ project duration is 36.

Expected variance of critical path

$$= 4 + 16 + 4 + 1 = 25$$

$$V = \sigma^2 = 25$$

$$\Rightarrow \sigma = \sqrt{25} = 5$$

$$\boxed{\sigma = 5}$$

Activity

1-2

7-8

2-3

3-5

5-8

6-7

4-5

2-4

1-6

Foward computation

$$E_1 = 0$$

$$E_2 = E_1 + D = 0 + 7 = 7$$

$$E_3 = E_2 + D = 7 + 14 = 21$$

$$E_4 = E_2 + D = 7 + 5 = 12$$

$$\begin{aligned} E_5 &= \max\{E_3 + D, E_4 + D\} \\ &= \max\{21 + 11, 12 + 5\} \\ &= \max\{32, 17\} = 32. \end{aligned}$$

$$E_6 = E_1 + D = 0 + 6 = 6$$

$$E_7 = E_6 + D = 6 + 11 = 17$$

$$\begin{aligned} E_8 &= \max\{E_5 + D, E_7 + D\} \\ &= \max\{32 + 4, 17 + 18\} \\ &= \max\{36, 35\} = 39 \end{aligned}$$

$$\boxed{E_8 = 36}$$

Backward computation

$$L_8 = 36$$

$$L_7 = L_8 - 18 = 36 - 18 = 18$$

$$L_6 = L_7 - 11 = 18 - 11 = 7$$

$$L_5 = L_8 - 4 = 36 - 4 = 32$$

$$L_4 = L_5 - 7 = 32 - 7 = 25$$

$$L_3 = L_5 - 11 = 32 - 11 = 21$$

$$L_2 = \min\{E_3 - 14, E_4 - 5\}$$

$$= \min\{21 - 14, 17 - 5\}$$

$$= \min\{7, 12\} = 7$$

$$L_1 = \min\{L_2 - 4, L_6 - 6\}$$

$$= \min\{7 - 4, 7 - 6\}$$

$$= \min\{3, 1\} = 0.$$

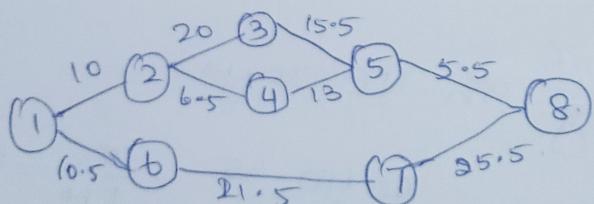
| Activity | Duration | EST | EFT = EST + D | LFT | LST = LFT - D | TF = EFT - EST |
|----------|----------|-----|---------------|-----|---------------|----------------|
| 1-2 | 7 | 0 | 7 | 7 | 0 | 0 |
| 7-8 | 18 | 17 | 35 | 36 | 18 | 1 |
| 2-3 | 14 | 7 | 21 | 21 | 7 | 0 |
| 3-5 | 11 | 21 | 32 | 32 | 21 | 0 |
| 5-8 | 4 | 32 | 36 | 36 | 32 | 0 |
| 6-7 | 11 | 6 | 17 | 18 | 7 | 1 |
| 4-5 | 7 | 12 | 19 | 32 | 25 | 13 |
| 2-4 | 5 | 7 | 12 | 25 | 20 | 13 |
| 1-6 | 6 | 0 | 6 | 7 | 1 | 1 |

| Job | 1-2 | 1-6 | 2-3 | 2-4 | 3-5 | 4-5 | 6-7 | 5-8 | 7-8 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| a | 1 | 2 | 2 | 2 | 7 | 5 | 5 | 3 | 8 |
| b | 7 | 5 | 14 | 5 | 10 | 5 | 8 | 3 | 17 |
| m | 13 | 14 | 26 | 8 | 19 | 17 | 29 | 9 | 32 |

Sh:

| Activity | a | m | b | $E = \frac{a+4m+b}{6}$ | $V = \sigma^2 = \left(\frac{b-a}{6}\right)^2$ |
|----------|---|----|----|------------------------|---|
| 1-2 | 1 | 13 | 7 | 10 | 1 |
| 1-6 | 2 | 14 | 5 | 10.5 | 0.25 |
| 2-3 | 2 | 26 | 14 | 20 | 4 |
| 2-4 | 2 | 8 | 5 | 6.5 | 0.25 |
| 3-5 | 7 | 19 | 10 | 15.5 | 0.25 |
| 4-5 | 5 | 17 | 5 | 13 | 0 |
| 6-7 | 5 | 29 | 8 | 21.5 | 0.25 |
| 5-8 | 3 | 9 | 3 | 5.5 | 0 |
| 7-8 | 8 | 32 | 17 | 25.5 | 2.25 |

Network.



Possible Path

Duration

1-2-3-5-8

$$10 + 20 + 15.5 + 5.5 = 51$$

1-2-4-5-8

$$10 + 6.5 + 13 + 5.5 = 35$$

1-6-7-8

$$10.5 + 21.5 + 25.5 = 57.5$$

Critical path : 1-6-7-8.

Project Duration = 57.5 days.

Expected variance of critical path.

$$V = 0.25 + 0.25 + 2.25 = 2.75$$

$$V = \sigma^2 = 2.75$$

$$\Rightarrow \sigma = \sqrt{2.75} = 1.66$$

$$\boxed{\sigma = 1.66}$$

For Cr

$$E_1 = 0$$

$$E_2 = E_1 + D = 0 + 10 = 10$$

$$E_3 = E_2 + D = 10 + 20 = 30$$

$$E_4 = E_3 + D = 30 + 6.5 = 36.5$$

$$E_5 = \max\{45.5, 29.5\} \\ = 45.5$$

$$E_6 = E_5 + D = 45.5 + 10.5 = 56$$

$$E_7 = E_6 + D = 56 + 21.5 = 77.5$$

$$E_8 = \min\{57, 57.5\}$$

$$E_8 = 57.5$$

Bas

$$L_8 = 57.5$$

$$L_7 = L_8 - D \\ = 57.5 - 25.5 = 32$$

$$L_6 = L_7 - D = 32 - 21.5 = 10.5$$

$$L_5 = 57.5 - 10.5 = 47$$

$$L_4 = 47 - 13 = 34$$

$$L_3 = 34 - 15.5 = 18.5$$

$$L_2 = \min\{16.5, 32.5\} \\ = 16.5$$

$$L_1 = \min\{6.5, 0\} = 0$$

$$\boxed{L_1 = 0}$$

Prob of completing in 40 days.

$$Z = \frac{T_8 - T_E}{\sigma} = \frac{40 - 35}{1.66} = \frac{5}{1.66} = 3.01$$

35 days.

$$Z = \frac{35 - 35}{1.66} = \frac{0}{1.66} = 0$$