(1) Transportation problem:

5	4	2	6	20
8	3	5	7	30
5	9	4	6	50
10	40	20	30.	100

Total Supply = Total Demand

∴ The table is balanced.

Step 1: To find the IBFS by VAM method.

5	4 [10]	2 [10]	6	$20-10=10-10=0$	(2) (2) (2)(4)
8	3 [30]	5	7	$30-30=0$	(2) (2)
5 [10]	9	4	6.	$50-10=40$	(1) (2) (2)(2)
$10-10=0$	$40-30=10-10=0$	$20-10=10$	30		
(3)	(1)	(2)	(1)		
	(1)	(2)	(1)		
	(5)	(2)	(0)		
		(2)	(0)		

4 [10]	6 [30]	$40-30=10$	(2)
10	$30-30=0$		
(4)	(0)		

Final table:

5	4 [10]	2 [10]	6
8	3 [30]	5	7
5 [10]	9	4 [10]	6 [30]

No of allocation = m+n-1

6 = 3+4-1

6 = 7-1

6 = 6

∴ It is degenerated.

~~Th~~ Total cost = (4×10) + (2×10) + (3×30) + (5×10) + (4×10) + (6×30).

= 40 + 20 + 90 + 50 + 40 + 180.

= 420/-

Step 2: To find the optimal solution by MODI method.

(2) 5	$\boxed{10}$ 4	$\boxed{10}$ 2	(2) 6	$u_1 = -2$
(6) 8	$\boxed{30}$ 3	(1) 5	(4) 7	$u_2 = -3$
$\boxed{10}$ 5	(3) 9	$\boxed{10}$ 4	$\boxed{30}$ 6	$u_3 = 0$
$v_1 = 5$	$v_2 = 6$	$v_3 = 4$	$v_4 = 6$	

$u_1 = c_{13} - v_3 = 2-4 = -2$, $v_2 = c_{12} - u_1 = 4+2 = 6$

$u_2 = c_{22} - v_2 = 3-6 = -3$.

For unallocated cell:

$d_{11} = c_{11} - (u_1+v_1) = 5 - (-2+5) = 5-3 = 2$

$d_{14} = c_{14} - (u_1+v_4) = 6 - (-2+6) = 6-4 = 2$

$d_{21} = c_{21} - (u_2+v_1) = 8 - (-3+5) = 8-2 = 6$

$d_{23} = c_{23} - (u_2+v_3) = 5 - (-3+4) = 5-4 = 1$

$d_{24} = c_{24} - (u_2+v_4) = 7 - (-3+6) = 7-3 = 4$.

$d_{32} = c_{32} - (u_3+v_4) = 9 - (0+6) = 9-6 = 3$.

∴ ~~It attains~~ The solution is optimal and unique.

② Transportation problem:

19	30	50	10	7
70	30	40	60	9
40	8	70	30	18
5	8	7	14	34

Total supply = Total demand

∴ The table is balanced.

Step 1: To find the IBFS by VAM method.

19 $\boxed{5}$	30	~~50~~	~~10~~ $\boxed{2}$	7-5 = 2-2 = 0 (9) (9) (40)
70	30	40	60	9 (10) (20) (20) (20)
40	8 $\boxed{8}$	~~70~~	~~30~~ $\boxed{10}$	18-8 = 10-10 = 0 (22) (10) (40) (40)

5-5 = 0   8-8 = 0   7   14-2 = 12-10 = 2.

(21) (22) (10) (20)

(21) (10) (20)

(10) (20)

(30) (30)

40 $\boxed{7}$	60 $\boxed{2}$	9-2 (20) = 7
7.	2-2 = 0	
(40)	(60)	

∴ Final table.

19 $\boxed{5}$	30	50	10 $\boxed{2}$
70	30	40 $\boxed{7}$	60 $\boxed{2}$
40	8 $\boxed{8}$	70	30 $\boxed{10}$

No. of allocation = m+n-1

6 = 3+4-1

6 = 7-1

6 = 6.

∴ It is degenerated.

Total cost = (19×5) + (10×2) + (40×7) + (60×2) + (8×8) + (30×10).

= 95 + 20 + 280 + 120 + 64 + 300

= Rs. 879/-

Step 2: To find the optimal solution by MODI method.

19 $\boxed{5}$	30 (42)	50 (60)	10 $\boxed{2}$	$u_1 = 10$
70	30	40 $\boxed{7}$	60 $\boxed{2}$	$u_2 = 60$
40	8 $\boxed{18}$	70	30 $\boxed{10}$	$u_3 = 30$
$v_1 = 9$	$v_2 = -22$	$v_3 = -20$	$v_4 = 0$	

The maximum allocated is column 4

$v_4 = 0$.

Since $v_4 = 0$

$\Rightarrow u_1 = 10, \; u_2 = 60, \; u_3 = 30$

To find v_1, $c_{11} = u_1 + v_1$

$v_1 = c_{11} - u_1 = 19 - 10 = 9$

$\boxed{v_1 = 9}$

To find v_2, $c_{32} = u_3 + v_2$.

$v_2 = c_{32} - u_3 = 8 - 30$

$\boxed{v_2 = -22}$

To find v_3, $c_{23} = u_2 + v_3$

$v_3 = c_{23} - u_2 = 40 - 60 = -20$.

To find for ~~non~~ unallocated cell.

$d_{ij} = c_{ij} - (u_i + v_j)$.

$d_{12} = c_{12} - (u_1 + v_2) = 30 - (10 - 22) = 30 - (-12)$
$= 30 + 12 = 42$

$d_{13} = c_{13} - (u_1 + v_3) = 50 - (10 - 20) = 50 - (-10) = 50 + 10$
$= 60$

$d_{21} = c_{21} - (u_2 + v_1) = 70 - (60 + 9) = 70 - 69 = 1$

$d_{22} = c_{22} - (u_2 + v_2) = 30 - (60 - 22) = 30 - 38 = -8$.

$d_{31} = c_{31} - (u_3 + v_1) = 40 - (30 + 9) = 40 - 39 = 1$

$d_{33} = c_{33} - (u_3+v_3) = 70 - (30-20) = 70 - 10 = 60.$

$\therefore$ The solution is not optimal and not unique.

Step 3: Looping :

19 [5]	30	50	10 [2]
70	30 [5]	40 [1 4]	60 [2] +
40	8 [8]	70 + [2]	30 [8] − [10]

(i) Select an unallocated cell which should pass only the allocated way.

(ii) Starting & ending are same (cycle).

(iii) give the least value of allocated to the unallocated cell.

~~(iv) Subtract the leas~~

19 [5]	30	50	10 [2]	$u_1 = -20$
70	30 ~~8~~	40 [5]	60 [4]	$u_2 = -30$
40	8 [8]	70 [2]	30 [10]	$u_3 = 0$
$v_1 = 39$	$v_2 = 8$	$v_3 = 70$	$v_4 = 30$	

$u_1 = c_{14} - v_4$
$= 10 - 30 = -20$
$u_1 = -20$
$u_2 = c_{2\cancel{3}} - v_{\cancel{2}}$
~~$= 60 - 30 = 30$~~
$= \cancel{30}$
$v_1 = c_{11} - u_1$
$= 19 + 20 = 39$

To find for unallocated cells.

$d_{ij} = c_{ij} - (u_i + v_j)$

$d_{12} = c_{12} - (u_1+v_2) = 30 - (-20+8) = 30 - (-12)$
$= 30+12 = 42$

$d_{13} = c_{13} - (u_1+v_3) = 50 - (-20+70) = 50 - 50 = 0$

$d_{21} = c_{21} - (u_2+v_1) = 70 - (-30+39) = 70 - \cancel{69} = 61$

$d_{22} = c_{22} - (u_2+v_2) = 30 - (-30+8) = 30 - \cancel{22} = 8$

$d_{31} = c_{31} - (u_3+v_1) = 40 - (0+39) = 40 - 39 = 1$

$\therefore$ The Solution is optimal ..

(4) Transportation Problem :

40	25	22	33	100
44	35	30	30	30
38	38	28	30	70
40	20	60	30	150 / 200

Total Supply ≠ Total demand.

∴ The table is unbalanced.

40	25	22	33	0	100
44	35	30	30	0	30
38	38	28	30	0	70
40	20	60	30	50	200.

The table is balanced

Step 1: To find the IBFS by VAM method.

40	$\boxed{20}$ 25	22	33	0	100-20=80 (22) (22) (3)
44	35	30	30	$\boxed{30}$ 0	30-30=0 (30)
38	38	28	30	$\boxed{20}$ 0	70-20=50 (28) (28) (2)
40	20-20 =0.	60	30	50-30=20-20=0	

(2) (10) (6) (3) (0)

(2) (13) (6) (3) (0)

(2) (13) (6) (3)

40	60 22	33		$80-\cancel{60}=20$ (11) (7)
38	28	30 30		$50-30=20$ (2) $\underline{(8)}$
40	$\cancel{60}$ 60 $=0$	$30-30=0$		
(2)	(6)	(3)		
(2)		(3)		

20 40	$20-20=0$	(40)
20 38	20	(38)
$40-20=20$		
(2)		

Final table.

20 40	20 25	60 22	33	0
44	35	30	30	30 0
20 38	38	28	30 30	20 0.

No. of allocation $= m+n-1$

$7 = 3+5-1$

$7 = 8-1$

$7 = 7.$

$\therefore$ It is degenerated.

Total cost $= (40\times20)+(25\times20)+(22\times60)$
$+(0\times30)+(0\times20)+(38\times20)+(30\times30)$

$= 800+500+1320+0+0+760+900$

$=$ Rs. 4280/-

Step 2: To find the optimal solution by MODI method.

40 $\boxed{20}$	25 $\boxed{20}$	22 $\boxed{60}$	33 (1)	0 (-2)	$u_1 = 0$
44 (6)	35 (12)	30 (10)	30 (0)	0 $\boxed{30}$	$u_2 = -2$.
38 $\boxed{20}$	38 (15)	28 (8)	30. $\boxed{30}$	0 $\boxed{20}$	$u_3 = -2$
$v_1 = 40$	$v_2 = 25$	$v_3 = 22$	$v_4 = 32$	$v_5 = 2$	

The max allocated is row 1 & row 3.

~~u=~~ $u_1 = 0$.

Hence $u_1 = 0$

$\Rightarrow v_1 = 40,\ v_2 = 25,\ v_3 = 22$

To find u_3, $c_{31} = u_3 + v_1 \Rightarrow u_3 = c_{31} - v_1$
$= 38 - 40 = -2$

To find v_4, $c_{34} = u_3 + v_4 \Rightarrow v_4 = c_{34} - u_3$
$= 30 + 2 = 32$.

To find $v_5 \Rightarrow c_{35} = u_3 + v_5 \Rightarrow v_5 = c_{35} - u_3$
$= 0 - (-2) = 2$.

To find $u_2 \Rightarrow c_{25} = u_2 + v_5$
$u_2 = c_{25} - v_5 = 0 - 2 = -2$.

For unallocated cells,

$d_{ij} = c_{ij} - (u_i + v_j)$.

$d_{14} = c_{14} - (u_1 + v_4) = 33 - (0 + 32) = 33 - 32 = 1$

$d_{15} = 0 - (0 + 2) = 0 - 2 = -2$

$d_{21} = c_{21} - (u_2 + v_1) = 44 - (-2 + 40) = 44 - 38 = 6$

$d_{22} = c_{22} - (u_2 + v_2) = 35 - (-2 + 25) = 35 - 23 = 12$

$d_{23} = c_{23} - (u_2 + v_3) = 30 - (-2 + 22) = 30 - 20 = 10$

$d_{24} = c_{24} - (u_2 + v_4) = 30 - (-2 + 32) = 30 - 30 = 0$

$d_{32} = c_{32} - (u_3 + v_2) = 38 - (-2 + 25) = 38 - 23 = 15$

$d_{33} = c_{33} - (u_3 + v_3) = 28 - (-2+22) = 28 - 20 = 8.$

$\therefore$ The solution is optimal but not unique.

$\boxed{Z = 1660}$

4) 2nd. Lesson

VAM.

11	20	7	8	50
21	16	20	12	40
8	12	8	9	70
30	25	35	40	130 | 160

Total Supply ≠ Total Demand.

The given problem is unbalanced.

11	20	7	8	0	50
21	16	20	12	0	40
8	12	8	9	0	70
30	25	35	40	30	160

The table is balanced

By using VAM method,

11	20	7	8	0	50	(7)
21	16	20	12	0 [30]	40-30=10	(12)
8	12	8	9	0	70	(8)
30	25	35	40	30-30=0		
(3)	(4)	(1)	(1)	(0)		

11	20	7 [35]	8	$50^{-35=15}$ (1) (1) (1) (1)
21	16	20	12 [10]	$10^{10=0}$ (4)
8 [30]	[25] 12	8	9	$70^{-25=45-30=15}$ (1) (1) (1) (1)
30-30 =0	25-25 =0	35-35 =0	40-10 =30	
(3)	(4)	(1)	(1)	
(3)	(8)	(1)	(1)	
(3)		(1)	(1)	
		(1)	(1)	

8	15(8)
9 [15]	15(9) -15 =0
30-15=15 (1)	

8 [15]

Final Table.

11	20	7 [35]	8 [15]	0	50
21	16	20	12 [10]	0 [30]	40
8 [30]	12 [25]	8	9 [15]	0	70
30	25	35	40	30	

No. of allocation $= m+n-1$

$= 3+5-1$

$= 8-1 = 7$.

$7 = 7$

∴ It is degenerated.

Total cost $= (7\times35)+(8\times15)+(12\times10)+(0\times30)$

$+ (8\times30)+(12\times25)+(9\times15)$

$= 245+120+120+0+240+300$

$+135$

$= 1160/-$

Step 2: To find the optimal solution by MODI Method.

(4) 11	(9) 20	$\boxed{35}$ 7	$\boxed{15}$ 8	(4) 0	50	$u_1 = -1$
(10) 21	(1) 16	(9) 20	$\boxed{10}$ 12	$\boxed{30}$ 0	40	$u_2 = 3$
$\boxed{30}$ 8	$\boxed{25}$ 12	(0) 8	$\boxed{15}$ 9	(3) 0	70	$u_3 = 0$
30	25	35	40	30		

$v_1 = 8$ $v_2 = 12$ $v_3 = 8$ $v_4 = 9$ $v_5 = -3$.

for allocated cells.

$c_{ij} = u_i + v_j$.

$c_{31} = u_3 + v_1$
$v_1 = 8 - 0$
$v_1 = 8$.

$c_{14} = u_1 + v_4$
$u_1 = c_{14} - v_4$
$u_1 = 8 - 9$
$\boxed{u_1 = -1}$

$u_2 = c_{24} - v_4$
$= 12 - 9$
$u_2 = 3$

$c_{13} = u_1 + v_3$
$v_3 = c_{13} - u_1$
$v_3 = 7 + 1$
$v_3 = 8$

$c_{25} = u_2 + v_5$
$v_5 = c_{25} - u_2$
$v_5 = 0 - 3$
$\boxed{v_5 = -3}$

For unallocated cells,

$d_{ij} = c_{ij} - (u_i + v_j)$ $\quad d_{ij} \geq 0$

$d_{11} = c_{11} - (u_1 + v_1) \Rightarrow 11 - (-1 + 8) = 11 - 7 = 4$

$d_{12} = c_{12} - (u_1 + v_2) \Rightarrow 20 - (-1 + 12) = 20 - 11 = 9$

$d_{15} = c_{15} - (u_1 + v_5) \Rightarrow 0 - (-1 - 3) = -(-4) = 4$

$d_{21} = c_{21} - (u_2 + v_1) \Rightarrow 21 - (3 + 8) = 21 - (11) = 10$

$d_{22} = c_{22} - (u_2 + v_2) \Rightarrow 16 - (3 + 12) = 16 - 15 = 1$

$d_{23} = c_{23} - (u_2 + v_3) \Rightarrow 20 - (3 + 8) = 20 - 11 = 9$

$d_{33} = c_{33} - (u_3 + v_3) \Rightarrow 8 - (0 + 8) = 8 - 8 = 0$

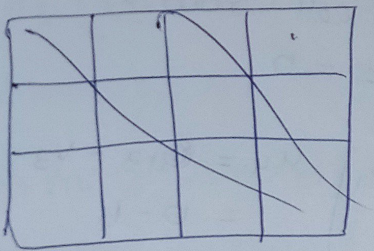
$d_{35} = c_{35} - (u_3 + v_5) \Rightarrow 0 - (0 - 3) = -(-3) = 3.$

$\therefore$ It is optimal

(6) unbalanced TP

5	1	7	10
6	4	6	80
3	2	5	15
75	20	50	145 / 105

Not balanced.



5	1	7	10
6	4	6	80
3	2	5	15
0	0	0	40
75	20	50	

Balanced.

By VAM,

5	1 [10]	7	10-10 =0	(4)	(4)	
6	4	6	80	(2)	(2)	(2)
3 [15]	2	5	15-15 =0	(1)	(1)	(1)
0	0	0 [40]	40-40 =0	(0)		
75-15 =60	20-10 =10	50-40 =10				
(3)	(1)	(5)				
(2)	(1)	(1)				
(3)	(2)	(1)				

6 [60]	4 [10]	6 [10]	80 -60 = 20-10 = 10-10 =0	(2)
60-60 =0	10-10 =0	10-10 =0		
(6)	(4)	(6)		

Final table.

5	1 [10]	7
6 [60]	4 [10]	6 [10]
3 [15]	2	5
0	0	0 [40]

No. of allocation $= m+n-1$

$= 4+3-1$

$= 7-1 = 6$

$6 = 6.$

∴ It is degenerated

Total cost $= (1\times10)+(6\times60)+(4\times10)+(6\times10)+(3\times15)+(0\times40)$

$= 10+360+40+60+45+0 = 515/-$

Step 2: To find the optimal sol by modi method.

(2) 5	$\boxed{10}$ 1	(4) 7	$u_1 = -3$
$\boxed{60}$ 6	$\boxed{10}$ 4	$\boxed{10}$ 6	$u_2 = 0$
$\boxed{15}$ 3	(1) 2	(2) 5	$u_3 = -3$
(0) 0	(2) 0	$\boxed{40}$ 0	$u_4 = -6$

$v_1 = 6 \quad v_2 = 4 \quad v_3 = 6$

The max allocated cell row 2

$\therefore u_2 = 0$.

For allocated cell,

$u_1 = c_{12} - v_2 = 1 - 4$, $u_1 = -3$

$u_3 = c_{31} - v_1 = 3 - 6$, $u_3 = -3$

$u_4 = c_{43} - v_3 = 0 - 6$, $u_4 = -6$.

For unallocated cells, $d_{ij} \geq 0$

$d_{11} = c_{11} - (u_1 + v_1) \Rightarrow 5 - (-3+6) = 5 - 3 = 2$

$d_{13} = c_{13} - (u_1 + v_3) \Rightarrow 7 - (-3+6) = 7 - 3 = 4$

$d_{32} = c_{32} - (u_3 + v_2) \Rightarrow 2 - (-3+4) = 2 - 1 = 1$

$d_{33} = c_{33} - (u_3 + v_3) \Rightarrow 5 - (-3+6) = 5 - 3 = 2$

$d_{41} = c_{41} - (u_4 + v_1) \Rightarrow 0 - (-6+6) = 0$

$d_{42} = c_{42} - (u_4 + v_2) \Rightarrow 0 - (-6+4) = -(-2) = 2$.