Unit - 1

Part - B

1. Max $Z = 100x_1 + 80x_2$ Sub to

$5x_1 + 20x_2 \leq 50,$

$8x_1 + 2x_2 \geq 16,$

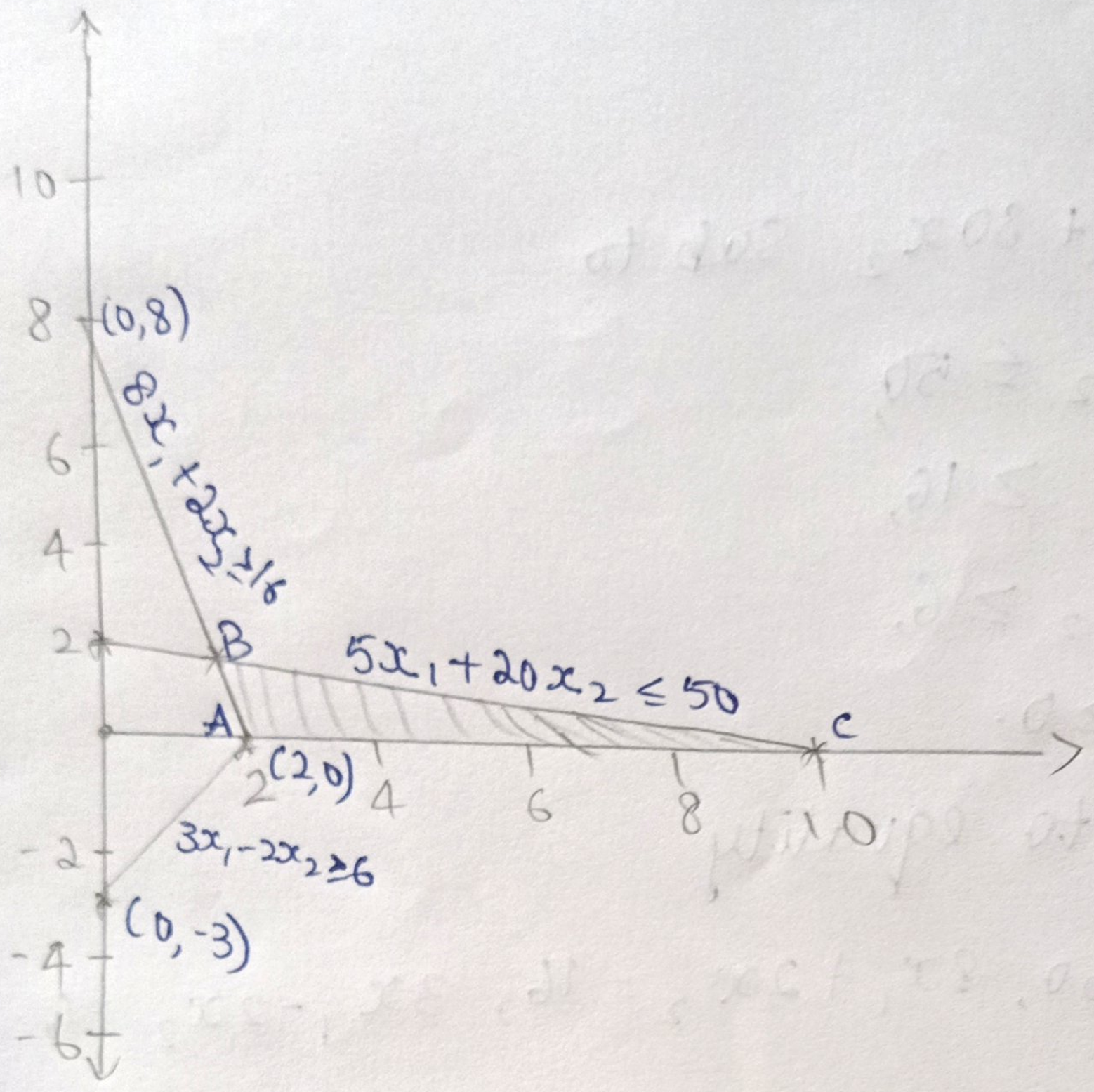
$3x_1 - 2x_2 \geq 6,$

$x_1, x_2 \geq 0.$

convert inequality to equality

$5x_1 + 20x_2 = 50,\ 8x_1 + 2x_2 = 16,\ 3x_1 - 2x_2 = 6$

$5x_1 + 20x_2 = 50$	$8x_1 + 2x_2 = 16$	$3x_1 - 2x_2 = 6$
Put $x_1 = 0$	Put $x_1 = 0$	Put $x_1 = 0$
$20x_2 = 50$	$2x_2 = 16$	$-2x_2 = 6$
$x_2 = 2.5$	$x_2 = 8$	$x_2 = -3$
$(0, 2.5)$	$(0, 8)$	$(0, -3)$
Put $x_2 = 0$	Put $x_2 = 0$	Put $x_2 = 0$
$5x_1 = 50$	$8x_1 = 16$	$3x_1 = 6$
$x_1 = 10$	$x_1 = 2$	$x_1 = 2$
$(10, 0)$	$(2, 0)$	$(2, 0)$



To find B

$8x_1 + 2x_2 = 16$ x 10

$5x_1 + 20x_2 = 50$

$80x_1 + 20x_2 = 160 \rightarrow ①$

$5x_1 + 20x_2 = 50 \rightarrow ②$

(-) (-) (-)

$75x_1 = 110$

$x_1 = \frac{110}{75}$

$x_1 = 1.5$

Sub $x_1 = 1.5$ in eqn ①

$8(1.5) + 2x_2 = 16$

$12 + 2x_2 = 16$

$2x_2 = 16 - 12 = 4$

$\boxed{x_2 = 2}$

B (1.5, 2)

Corner pts	Max $Z = 100x_1 + 80x_2$
A (2, 0)	$100(2) + 0 = 200$
B (1.5, 2)	$100(1.5) + 80(2) =$ $150 + 160 = 310$
C (10, 0)	$100(10) + 0 = 1000$

Max $Z = 100x_1 + 80x_2$

C (10, 0)

$Z = 1000$

3. Min $Z = 20x_1 + 10x_2$

Sub to

$$x_1 + 2x_2 \leq 40 \quad \rightarrow (1)$$

$$3x_1 + x_2 \geq 30 \quad \rightarrow (2)$$

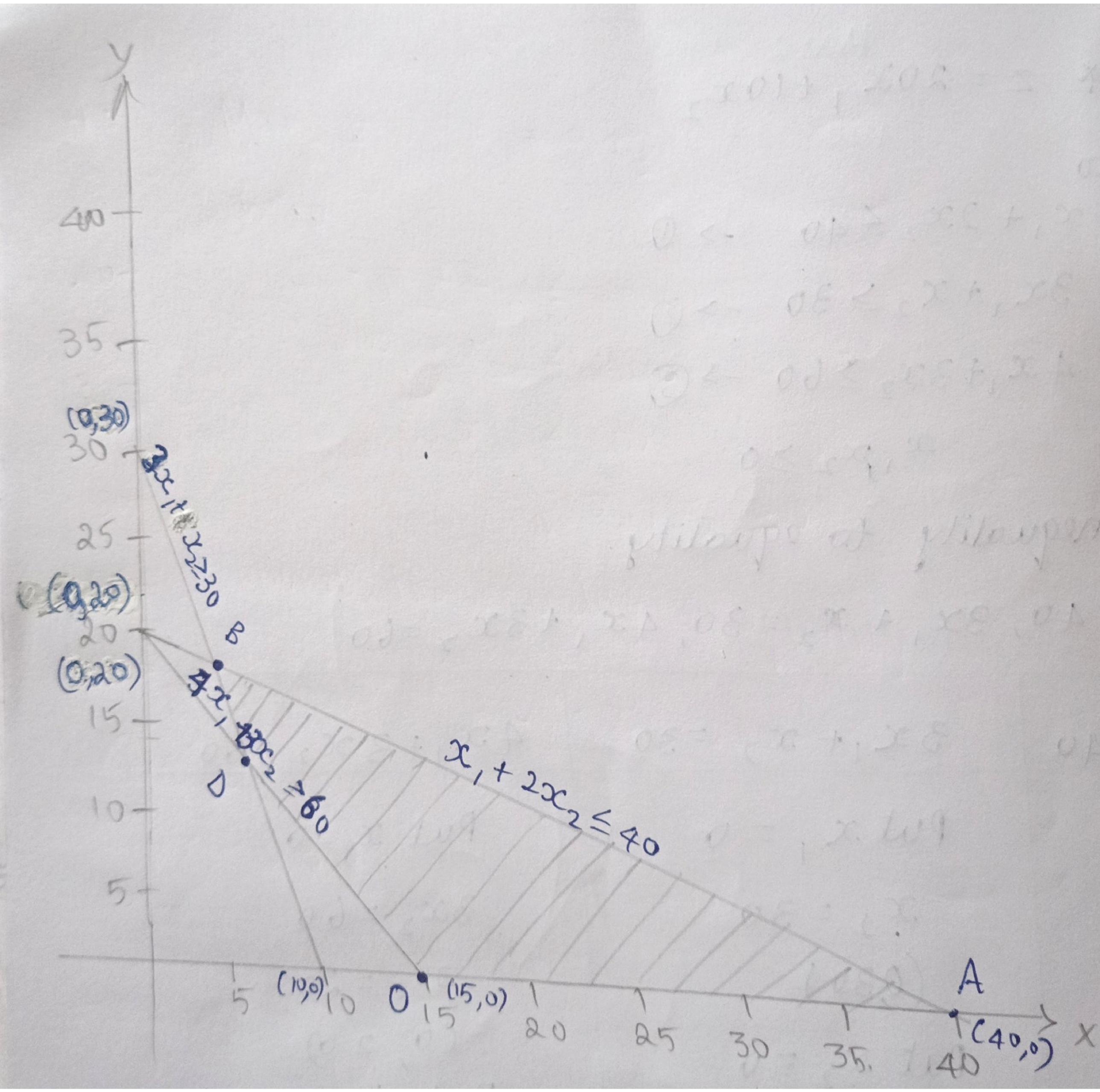
$$4x_1 + 3x_2 \geq 60 \quad \rightarrow (3)$$

$$x_1, x_2 \geq 0$$

Convert inequality to equality.

$x_1 + 2x_2 = 40$, $3x_1 + x_2 = 30$, $4x_1 + 3x_2 = 60$

$x_1 + 2x_2 = 40$	$3x_1 + x_2 = 30$	$4x_1 + 3x_2 = 60$
Put $x_1 = 0$	Put $x_1 = 0$	Put $x_1 = 0$
$2x_2 = 40$	$x_2 = 30$	$3x_2 = 60$
$x_2 = 20$	$(0, 30)$	$x_2 = 20$
$(0, 20)$	Put $x_2 = 0$	$(0, 20)$
Put $x_2 = 0$	$3x_1 = 30$	Put $x_2 = 0$
$x_1 = 40$	$x_1 = 10$	$4x_1 = 60$
$(40, 0)$	$(10, 0$	$x_1 = 15$
		$(15, 0)$


40
35
(0,30)
30
25
(0,20)
20
(0,20)
15
10
5
$3x_1 + 2x_2 \geq 30$
B
C
$x_1 + 3x_2 \geq 60$
D
$x_1 + 2x_2 \leq 40$
(10,0)
10
O
(15,0)
15
20
25
30
35
A
(40,0)
40
x

To find B

$x_1 + 2x_2 = 40$

$3x_1 + x_2 = 30$ $\times 2$

$x_1 + 2x_2 = 40 \rightarrow (1)$

$6x_1 + 2x_2 = 60 \rightarrow (2)$

(-) (-) (-)

$-5x_1 = -20$

$x_1 = 4$

Sub $x_1 = 4$ in (3)

$4x_1 + 2x_2 = 40$

$4(4) + 2x_2 = 40$

$2x_2 = 40 - 4 = 36$

$x_2 = \frac{36}{2}$

$x_2 = 18$

To find D

$3x_1 + x_2 = 30$ $\times 3$

$4x_1 + 3x_2 = 60$

$4x_1 + 3x_2 = 60$

$9x_1 + 3x_2 = 90$

(-) (-) (-)

$-5x_1 = -30$

$x_1 = \frac{30}{5}$

$x_1 = 6$

Sub $x_1 = 6$ in (2)

$3(6) + x_2 = 30$

$18 + x_2 = 30$

$x_2 = 12$

Feasible region are

OABD

corner pts	objective fn $(20x_1 + 10x_2)$
O (15, 0)	20(15) + 0 = 300
A(40, 0)	40(15) + 0 = 800
B(4, 18)	20(4) + 10(18) = 260
D (6, 12)	20 (6) + 10(12) = 240

Min $Z = 20x_1 + 10x_2$

D (6, 12)

$Z = 240$

4. Max $Z = 3x_1 + 5x_2$

Sub to

$x_1 + 2x_2 \leq 2000$

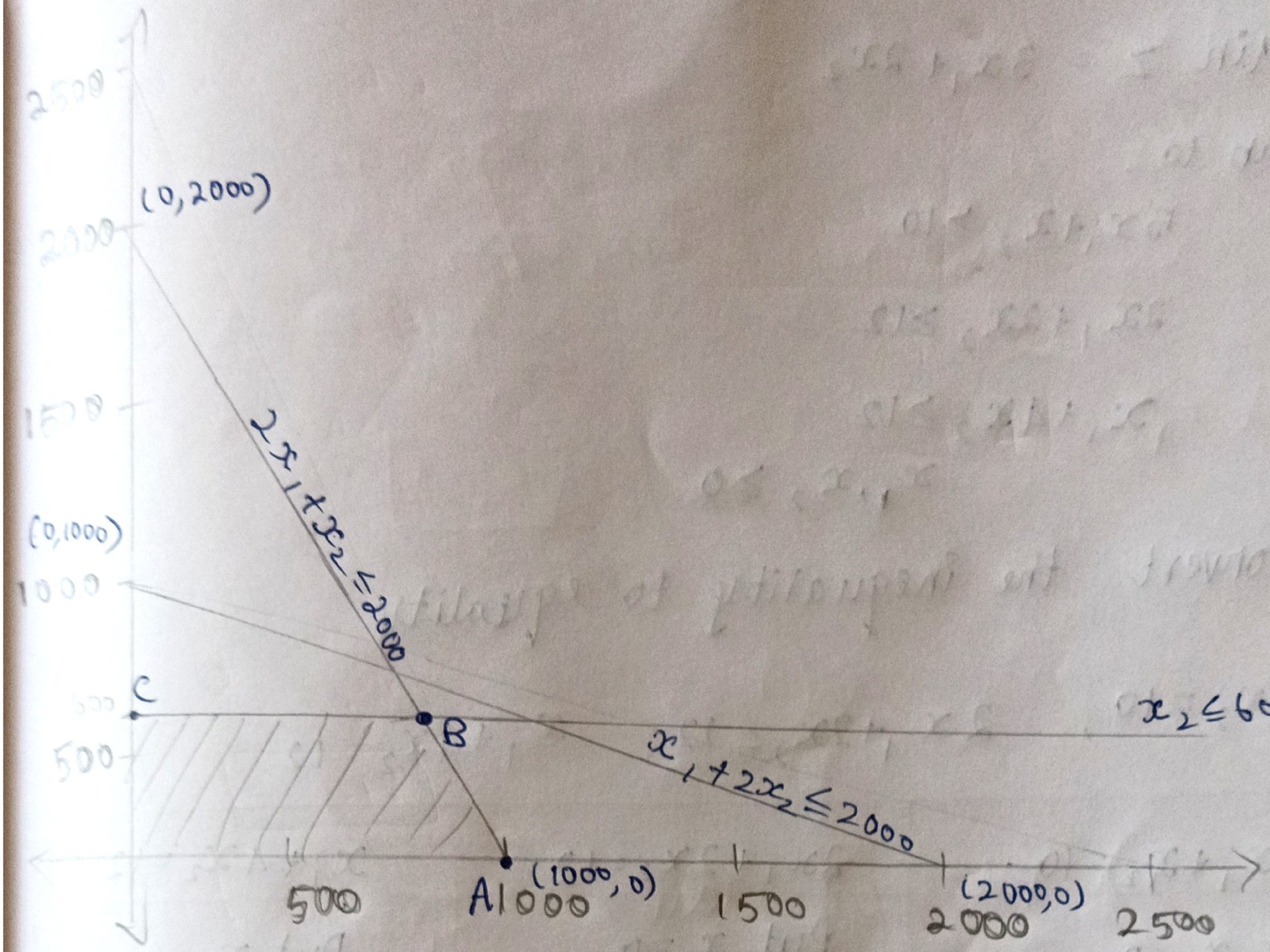
$2x_1 + x_2 \leq 2000$

$x_2 \leq 600$

$x_1, x_2 \geq 0$

Convert the equality to equality

$x_1 + 2x_2 = 2000$	$2x_1 + x_2 = 2000$	$x_1 = 60$
Put $x_1 = 0$ $2x_2 = 2000$ $x_2 = 1000$ $(0, 1000)$ Put $x_2 = 0$ $x_1 = 2000$ $(2000, 0)$	Put $x_1 = 0$ $x_2 = 2000$ $(0, 2000)$ Put $x_2 = 0$ $2x_1 = 2000$ $x_1 = 1000$ $(1000, 0)$	$x_2 = 600$



Feasible region -> O A B C

To find B

$2x_1 + x_2 = 2000$

$\boxed{x_2 = 600}$

$2x_1 + 600 = 2000$

$2x_1 = 1400$

$\boxed{x_1 = 700}$

B(700, 600)

Corner pts	Max $z = 3x_1 + 5x_2$
O (0,0)	0
A (1000, 0)	$3(1000) + 0 = 3000$
B (700, 600)	$3(700) + 5(600)$ $2100 + 3000 = 5100$
C (0, 600)	$0 + 5(600) = 3000$

Optimal Soln

Max $z = 3x_1 + 5x_2$

B (700, 600)

$\boxed{z = 5100}$

5)

Min $Z = 3x_1 + 2x_2$

Sub to

$5x_1 + x_2 \geq 10$

$2x_1 + 2x_2 \geq 12$

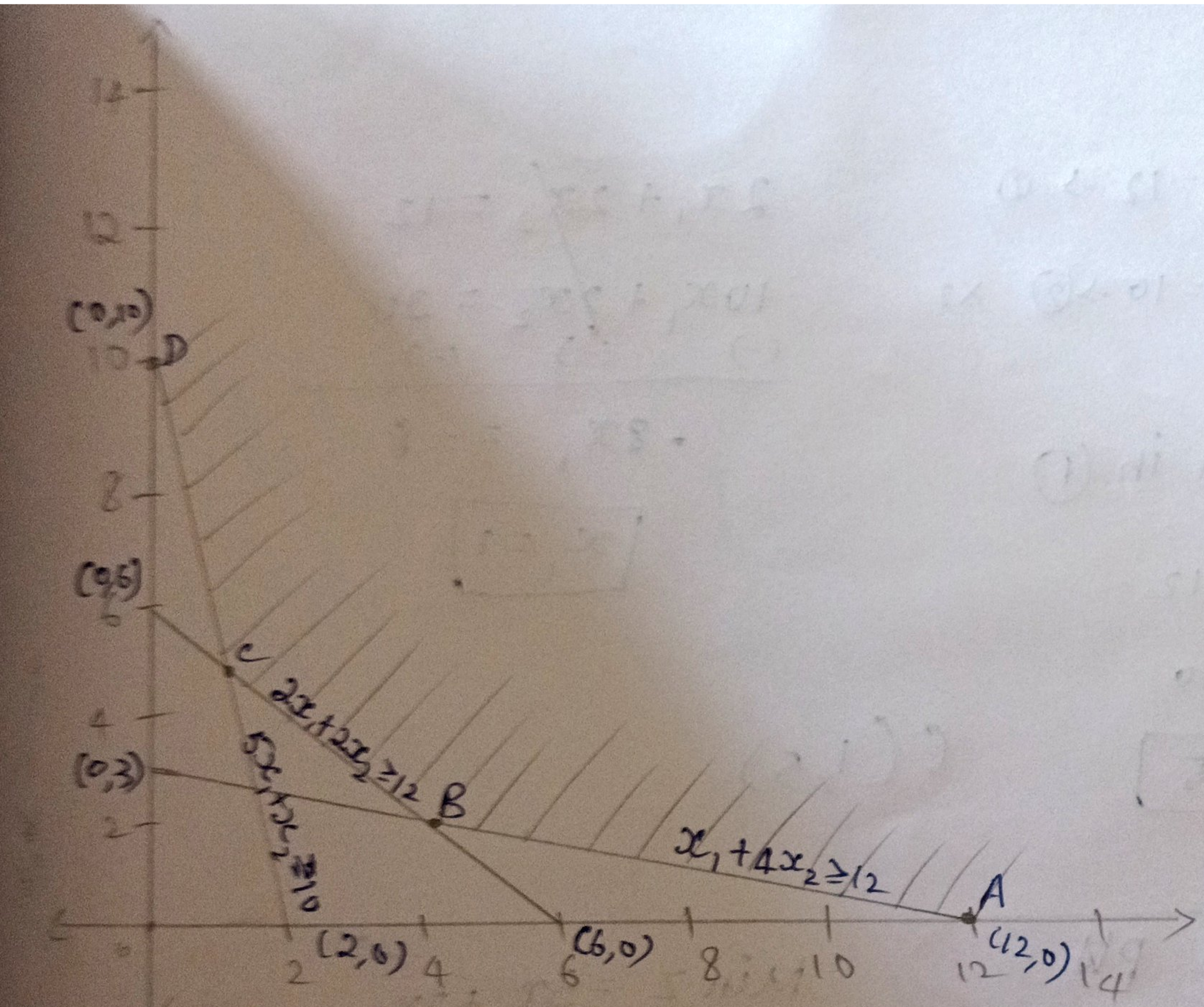
$x_1 + 4x_2 \geq 12$

$x_1, x_2 \geq 0$

Convert the inequality to equality

$5x_1 + x_2 = 10$, $2x_1 + 2x_2 = 12$, $x_1 + 4x_2 = 12$

$5x_1 + x_2 = 10$	$2x_1 + 2x_2 = 12$	$x_1 + 4x_2 = 12$
Put $x_1 = 0$ $x_2 = 10$ $(0, 10)$ Put $x_2 = 0$ $5x_1 = 10$ $x_1 = 2$ $(2, 0)$	Put $x_1 = 0$ $2x_2 = 12$ $x_2 = 6$ $(0, 6)$ Put $x_2 = 0$ $2x_1 = 12$ $x_1 = 6$ $(6, 0)$	Put $x_1 = 0$ $4x_2 = 12$ $x_2 = 3$ $(0, 3)$ Put $x_2 = 0$ $x_1 = 12$ $(12, 0)$



To find B

$x_1+4x_2=12 \rightarrow ① \times 2$

$2x_1+2x_2=12 \rightarrow ②$

$$\begin{array}{rrr} 2x_1 & +8x_2 & =24 \\ 2x_1 & +2x_2 & =12 \\ (-) & (-) & (-) \\ \hline & 6x_2 & =12 \end{array}$$

$\boxed{x_2=2}$

Sub $x_2=2$ in ①

$x_1+4(2)=12$

$x_1+8=12$

$\boxed{x_1=4}$

B(4,2)

To find C

$2x_1 + 2x_2 = 12 \rightarrow ①$

$5x_1 + x_2 = 10 \rightarrow ② \quad \times 2$

$2x_1 + 2x_2 = 12$

$10x_1 + 2x_2 = 20$

(-) (-) (-)

$-8x_1 = -8$

$x_1 = 1$

Sub $x_1 = 1$ in ①

$2 + 2x_2 = 12$

$2x_2 = 10$

$x_2 = 5$

C (1, 5)

Corner pts	Min $Z = 3x_1 + 2x_2$
A (12, 0)	$3(12) + 0 = 36$
B (4, 2)	$3(4) + 2(2) = 16$
C (1, 5)	$3(1) + 2(5) = 13$
D (0, 10)	$0 + 2(10) = 20$

Min $Z = 3x_1 + 2x_2$

C (1, 5)

$Z = 13$