MAE 789
APPLIED OPTIMAL CONTROL
PROJECT

Optimal Control of Autonomous Vehicles Approaching A Traffic Light

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A Project by

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-Any system approaching perfect self-control also approaches perfect self-frustration
-Alan Watts

Agenda

- **Problem Statement &**

  Objective of the chosen paper
- Continuous time Optimization
- Results & Discussion
- Discrete time Optimization
- Incorporation of a comparatively realistic

  dynamic model
- Results & Discussion
- Conclusion

Problem statement

- Vehicles approaching a traffic signal have a lot of issues like long arrival time, expenditure of more energy and unnecessary idling at a red signal

Objective

- To develop an optimal acceleration/velocity profile for a single autonomous vehicle to achieve a short travel time, low energy consumption and to avoid idling at a red signal by taking full advantage of the traffic signal information

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Objective function

$$J = \min \int_{t_0}^{t_p} [\rho_t + \rho_u u^2(t)]\,dt$$

System Dynamics

The dynamics of the vehicle is modelled as a double integrator,

$$\dot{x}(t) = v(t)$$
$$\dot{v}(t) = u(t)$$

Subject to constraints:

$$x(t_p) = l,$$
$$v_{min} \le v(t) \le v_{max}$$
$$u_{min} \le u(t) \le u_{max}$$
$$kT \le t_p \le kT + DT$$

Where, $\rho_t = \rho . \frac{v_{min}}{l}$

$\rho_u = \frac{(1-\rho)}{(v_{max}-v_{min}).u_{max}}$ assuming $l \ge (v_{max}-v_{min}).\, v_{min}/u_{max}$

Challenges in solving the Optimization problem

- Finding feasible green light intervals lead to Mixed-Integer Programming problem formulation

- State constraints related to speed limits. The inclusion of bounds on state variables poses a significant challenge for most optimization method

Mixed Integer Programming

- Since they are having their feasible search space in form of a set of some discrete points, so these problems are NON-CONVEX
- Optimality of the solution is not guaranteed due to the non-convexity of the problem involved with integer variable
- Usually the way to deal with these kind of non-convex problems is through enumeration, which requires us to explore all the different possible values of those discrete variables. Since the number of combinations of the possible discrete solutions grows exponentially, it is computationally expensive

Strategy adopted in paper

They have adopted a two-step method,

Step 1

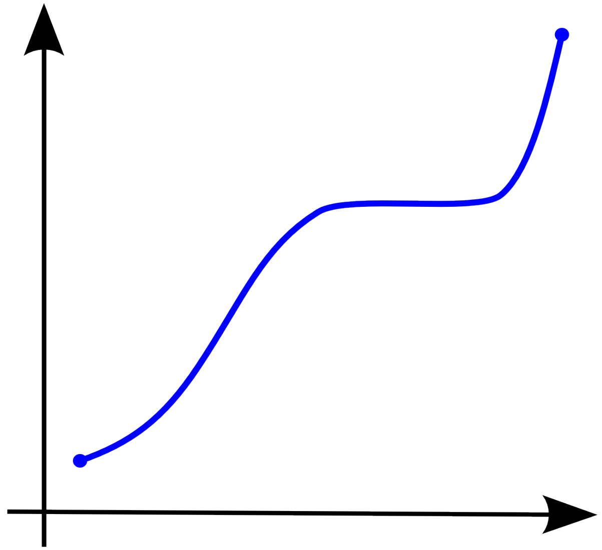
- Addressing the free terminal time Optimal control problem without traffic constraint thereby removing the mixed integer constraints and terminal time is free

Step 2

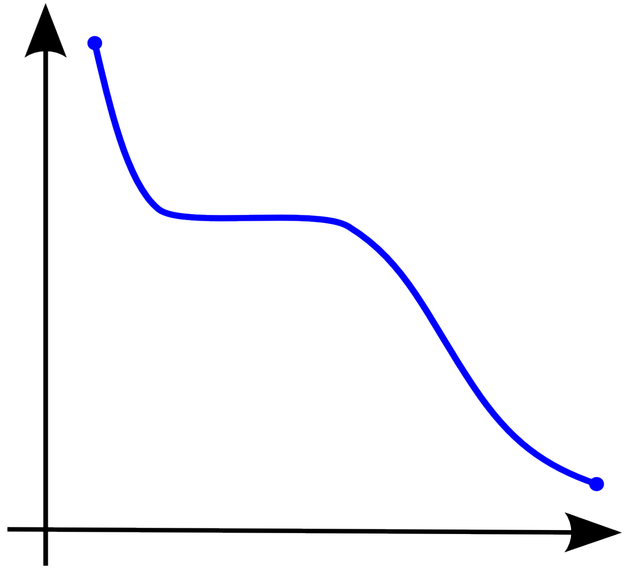
- If the terminal time is within the green light interval, the problem is solved
- If not, Optimal terminal time could be either the end of the previous green light or beginning of the next green light interval by using the monotonicity property of the objective function.

Monotonicity of a function

A function is monotonically increasing if for all x>y, f(x)>f(y) or in simple words, non decreasing.

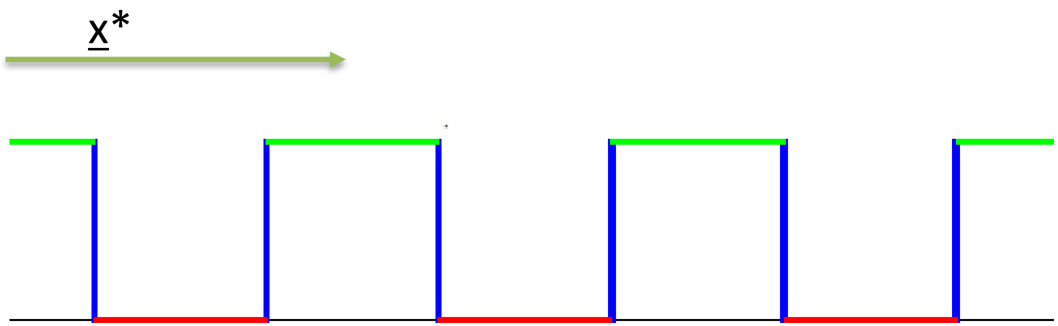


A function is monotonically decreasing if for all x>y, f(x)<f(y) or in simple words non increasing

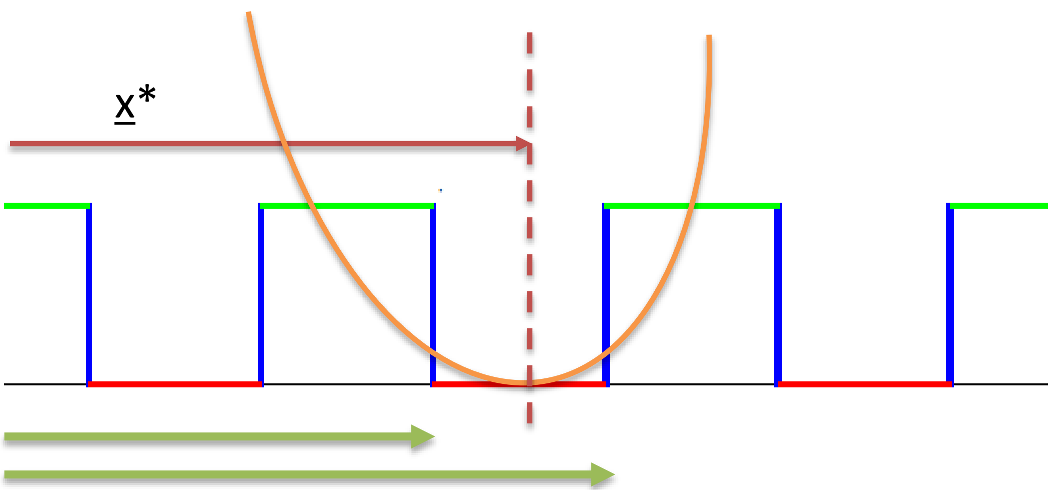


Source: Wikipedia

Monotonicity of the given problem:



Problem solved!



$\underline{x}^*$ actually lies at these points by monotonicity property if we land in the red-light zone

Solving the Continuous time Optimal control problem,

- Hamiltonian is given by:

$$\boldsymbol{\mathcal{H}} = \boldsymbol{\rho_t} + \boldsymbol{\rho_u u(t)^2} + \boldsymbol{p_1 v} + \boldsymbol{p_2 u}$$

Applying Pontryagin's minimum principle, we get the optimal switching control law,

$$u^* = -\frac{p_2}{2\rho_u} \qquad if\ 0 \geq p_2 \geq -2\ \rho_u u_{max}$$

$$u^* = u_{max} \qquad if \qquad p_2 < -2\ \rho_u u_{max}$$

So we get the following differential equations,

$$\dot{p}_1 = 0$$

$$\dot{p}_2 = -p_1$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{p_2}{2\rho_u} \qquad if\ 0 \geq\ p_2 \geq -2\ \rho_u u_{max}$$

$$\dot{x}_2 = u_{max} \qquad if \qquad p_2 < -2\ \rho_u u_{max}$$

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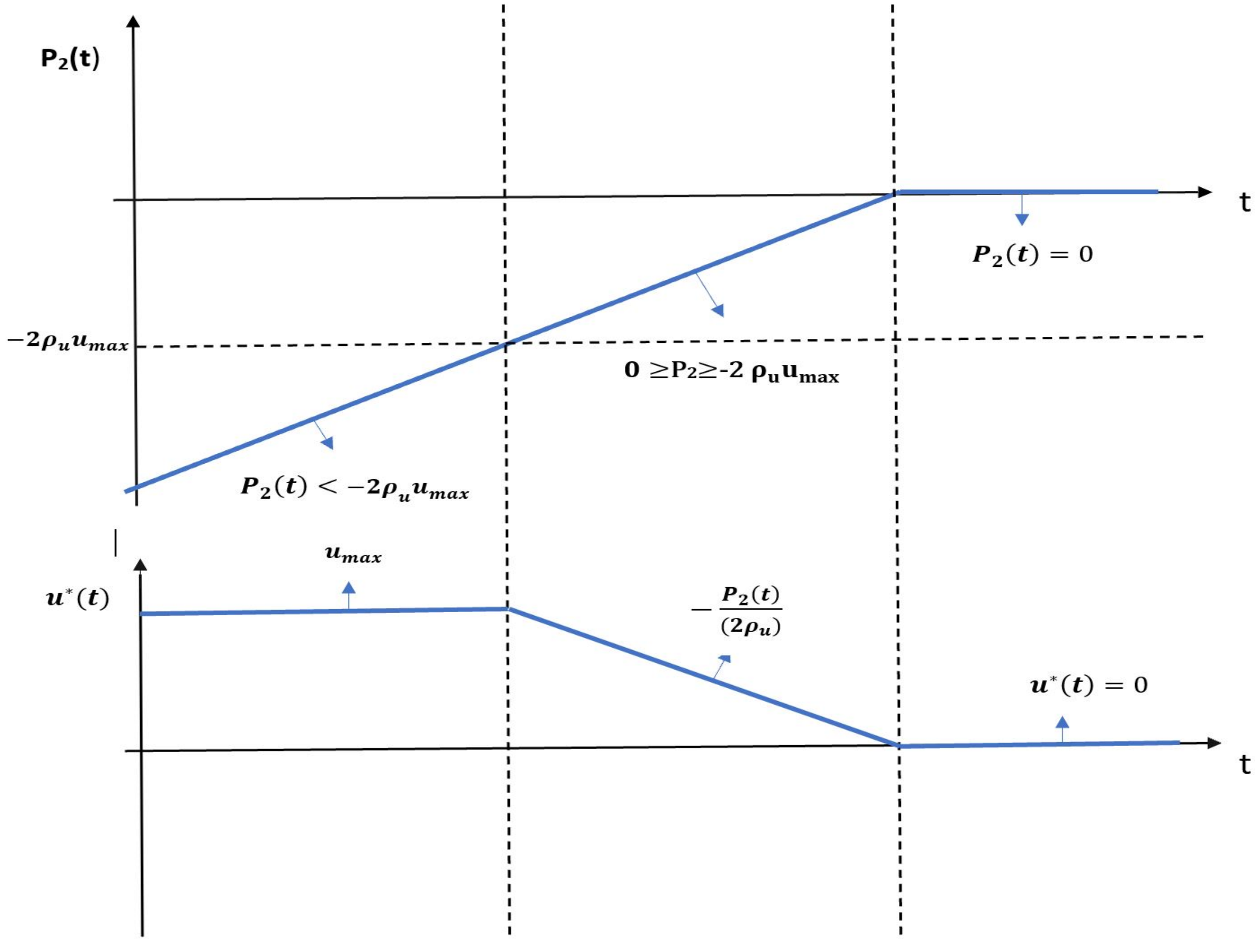
From the costate equations,

$$p_1 = c_1$$
$$p_2 = c_1 t + c_2$$



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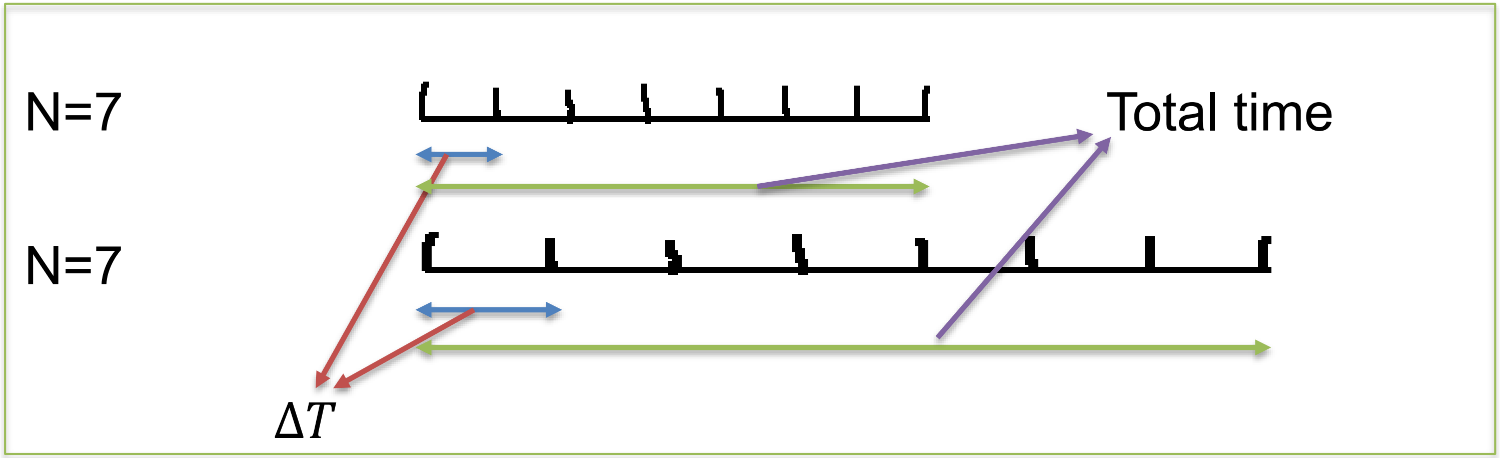
Addressing the problem of free time:

- This is done by treating the time step as a decision variable.
- It makes the algorithm perform poorly in some scenarios, because the dynamics also depend on time step and the dynamics become more and more inaccurate with an increasing time step



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- Discretization of the model using Forward-Euler approximation
- Setting up the Optimization problem for SQP:

$$\text{Minimize } J(\underline{u}, x(0), \Delta T) = \sum_{0}^{N-1}[\Delta T + u(k)^2\ \Delta T] + J_f$$

Subject to,

$$\underline{v}(k+1) = \underline{v}(k) + \frac{\Delta T}{m}\left(u(k) - C_{rr}mg - 0.5\rho v(k)^2 C_d A_{ref}\right.$$

$$x(k+1) = x(k) + v(k)T$$

Subject to,

$$v_{min} \leq v(k) \leq v_{max}$$

$$u_{min} \leq u(k) \leq u_{max}$$

$$x(t_p) = l$$

Making sure the problem is well posed

- Since we are treating the time step as a decision variable, this problem could easily become ill posed or poorly posed if the objective is minimizing time. (Since the algorithm could just return a negative value or a zero value to minimize time).
- To resolve this, we need to add appropriate terminal constraints to make sure the optimized time is not zero (poorly posed) or tending to zero (ill posed).
- We also need to add a constraint making the time step always positive so that the algorithm does not return a large negative value (in some cases it is possible to reach the terminal constraints even with negative time)

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Parameters for solving the Optimization problem using SQP (fmincon)

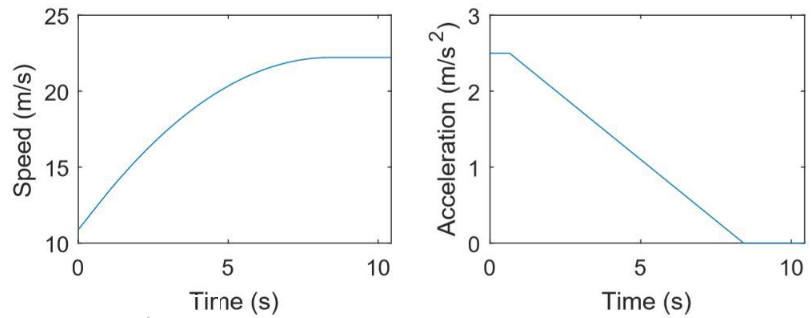
Mass of the vehicle,	m	= 1000 kg,
Acceleration due to gravity,	g	= 9.8m/s2 ,
Coefficient of Rolling resistance,	C_{rr}	= 0.01,
Density of air,	ρ	= 1.2 kg/m3 ,
Coefficient of Drag,	C_d	= 0.4
Reference Area.,	Aref	= 5 m^2
Weight,	ρ	= 0.9549
Weight on time,	ρ	= 0.013273
Weight on control input,	ρ	= 0.000928
Distance,	L	= 200 m & 2203 m

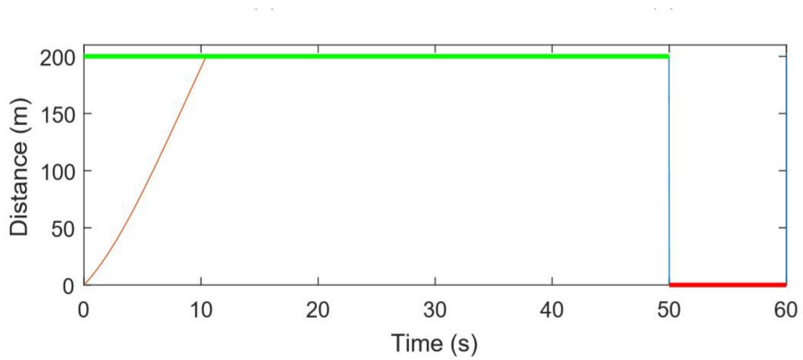
Initial velocities and the total distance are different for different simulation cases

CASE 1 – As per dynamic model from paper

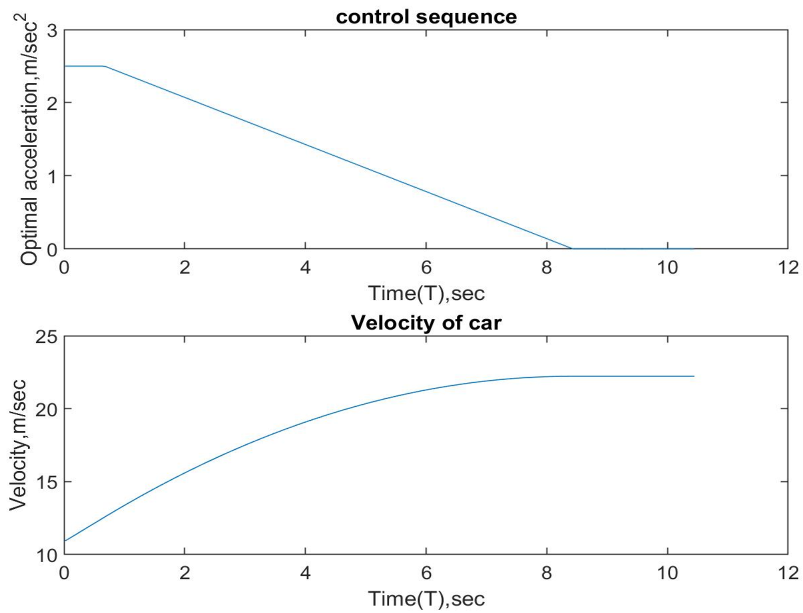
Initial Velocity, v_0 = 10.8869 m/s
Distance, L = 200 m

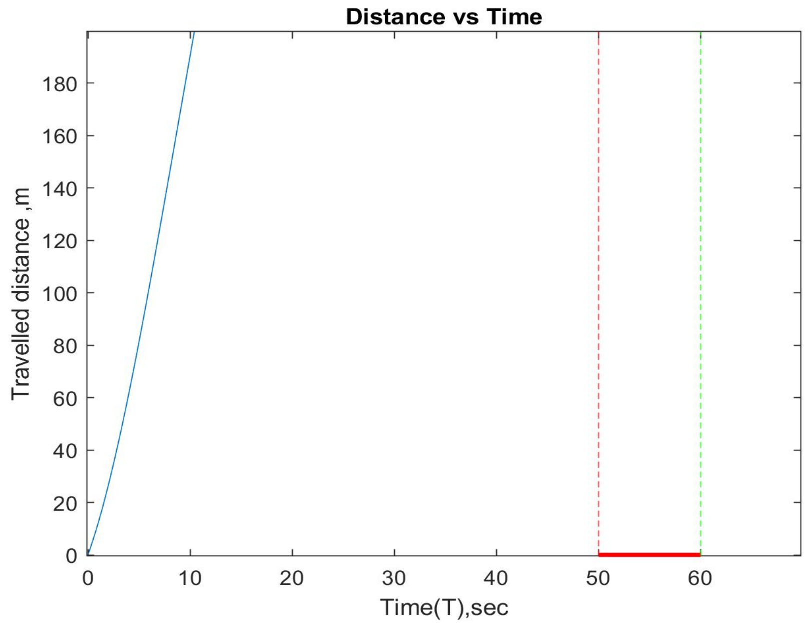
Results from paper





Our Results

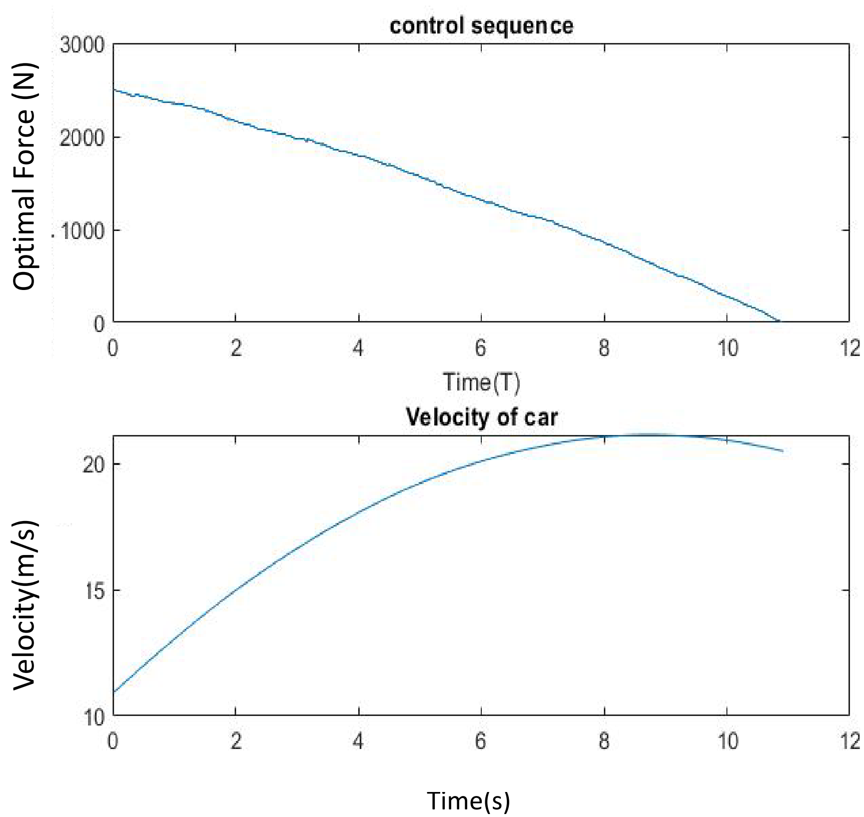


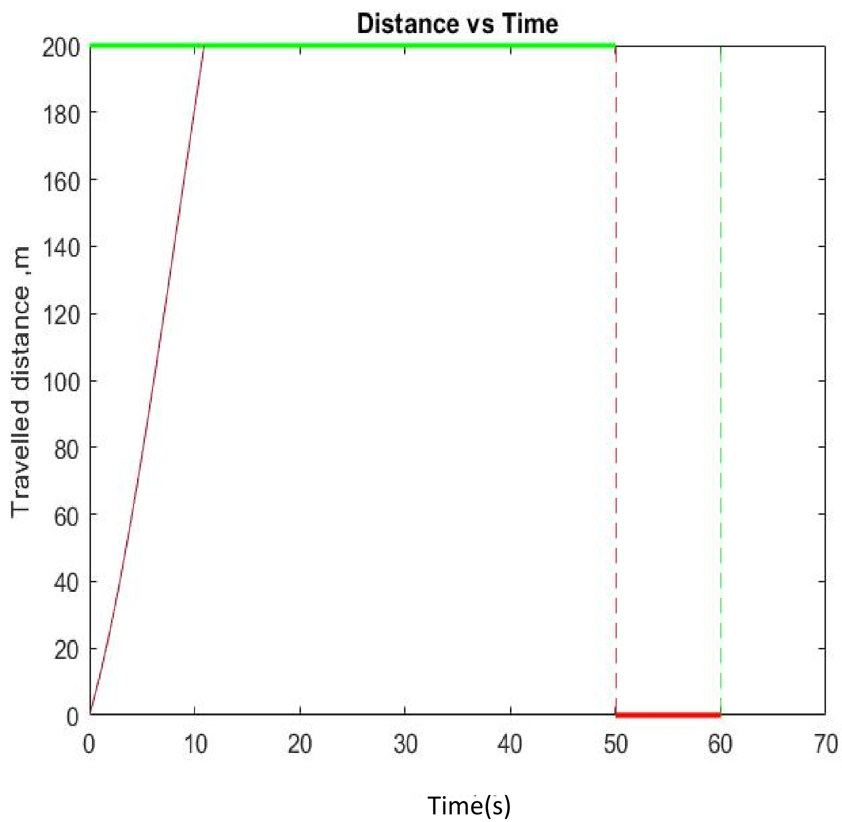


CASE 1 – As per realistic dynamic model

Our Results

Initial Velocity, v_0 **= 10.8869 m/s**
Distance, L **= 200 m**

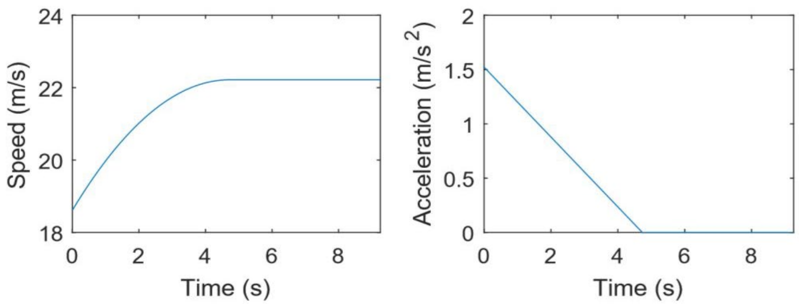


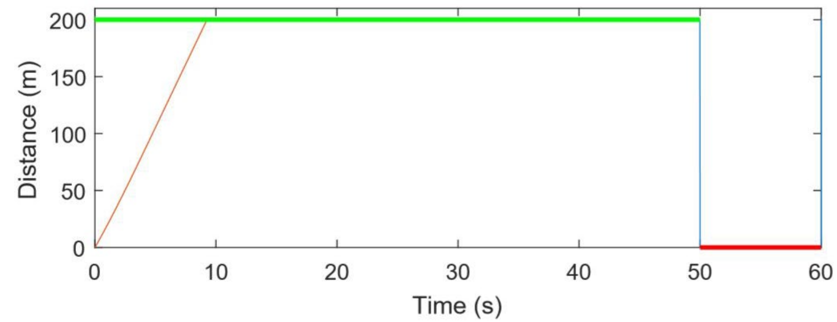


CASE 2 – As per dynamic model from paper

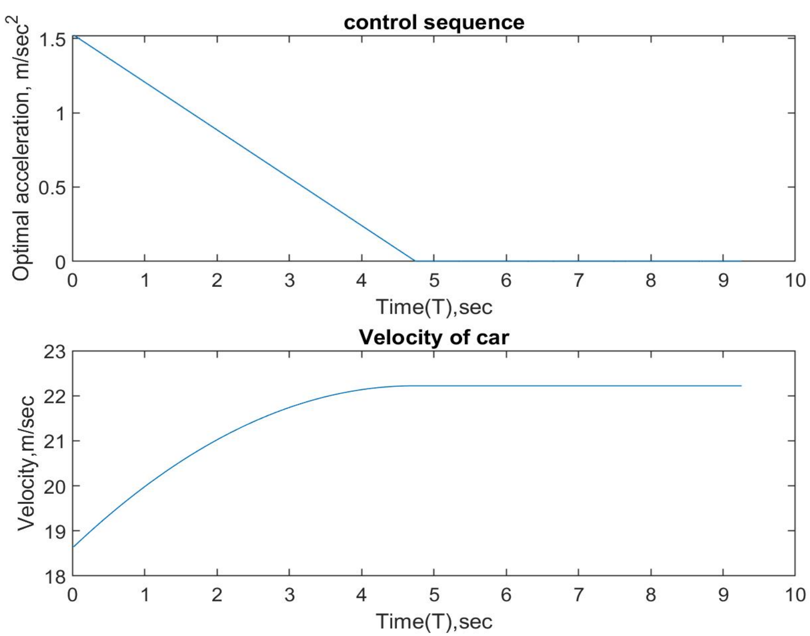
Initial Velocity, v_0 **= 18.6182 m/s**
Distance, **L** **= 200 m**

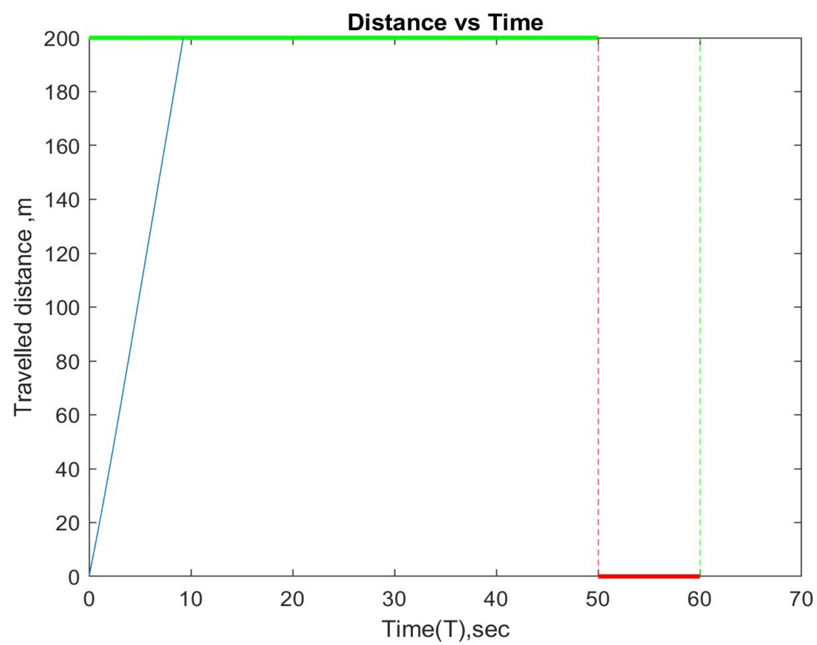
Results from paper





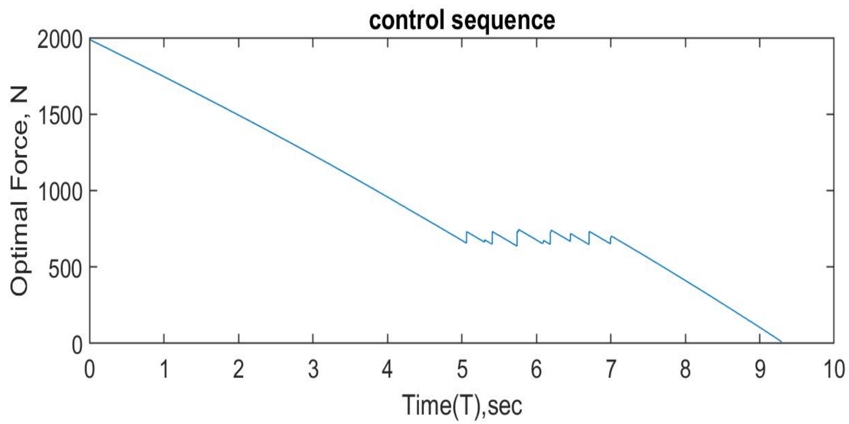
Our Results

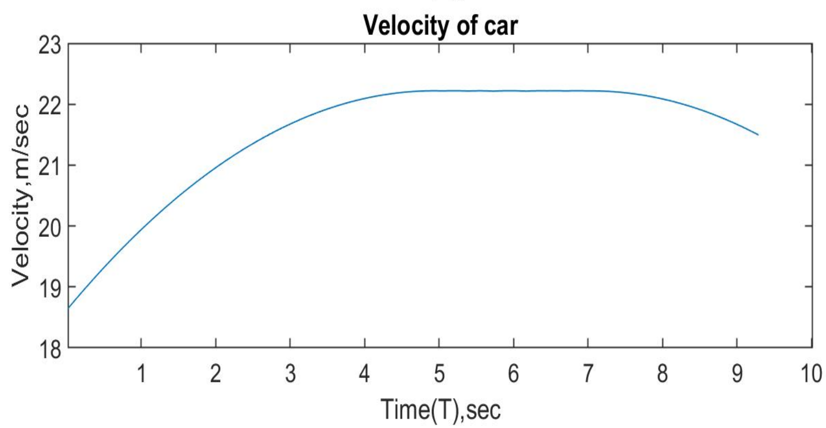


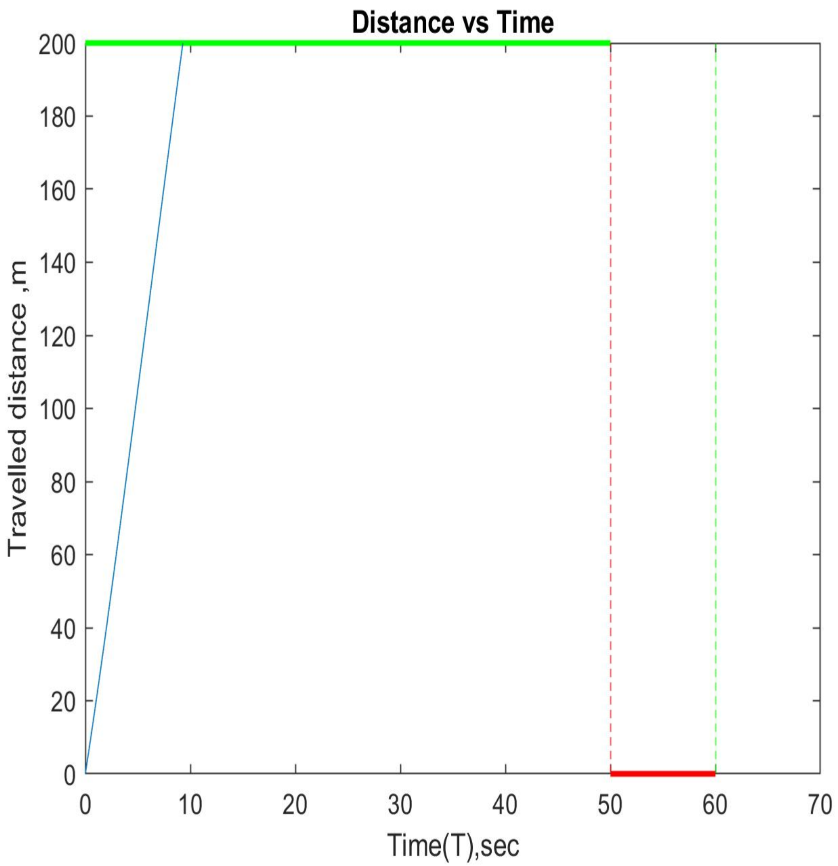


CASE 2 – As per realistic dynamic model

Our Results



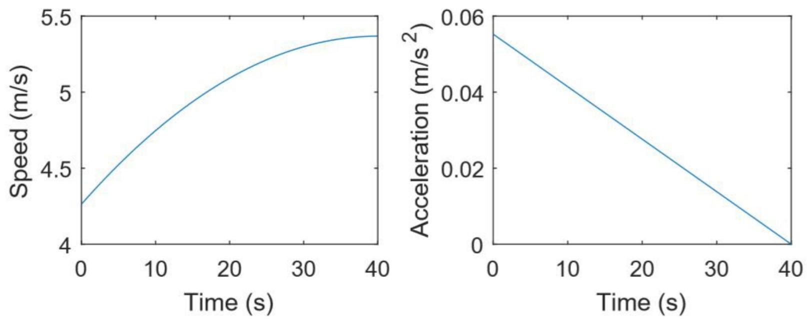


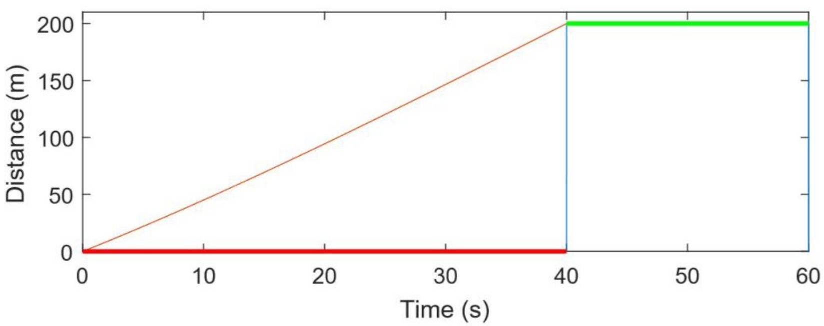


CASE 3 – As per dynamic model from paper

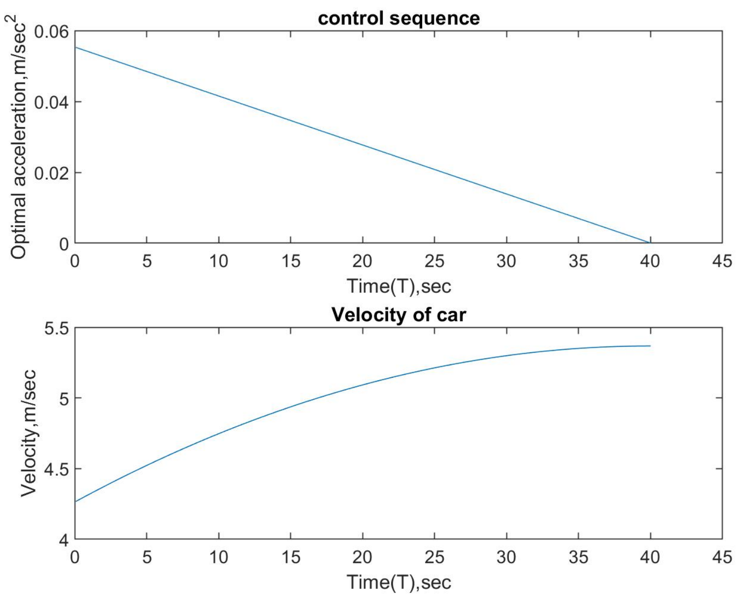
Initial Velocity, v_0 **= 4.2634 m/s**
Distance, L **= 200 m**

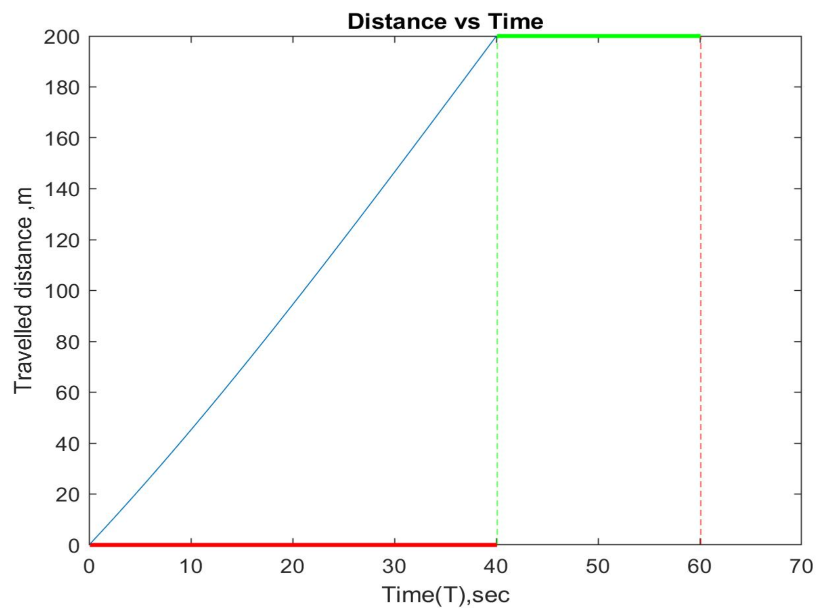
Results from paper





Our Results

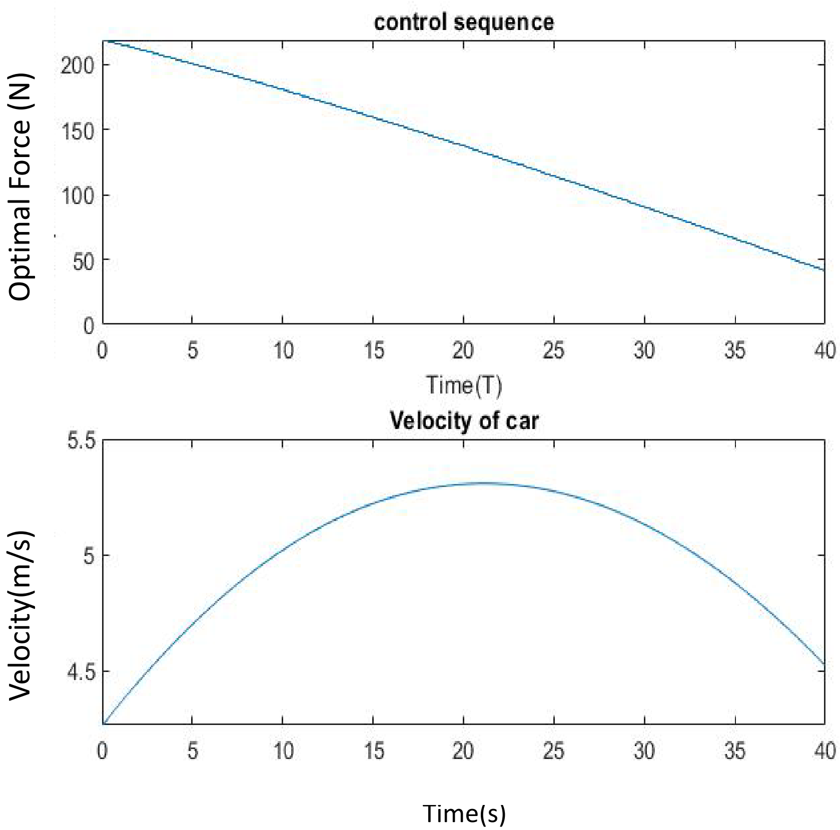


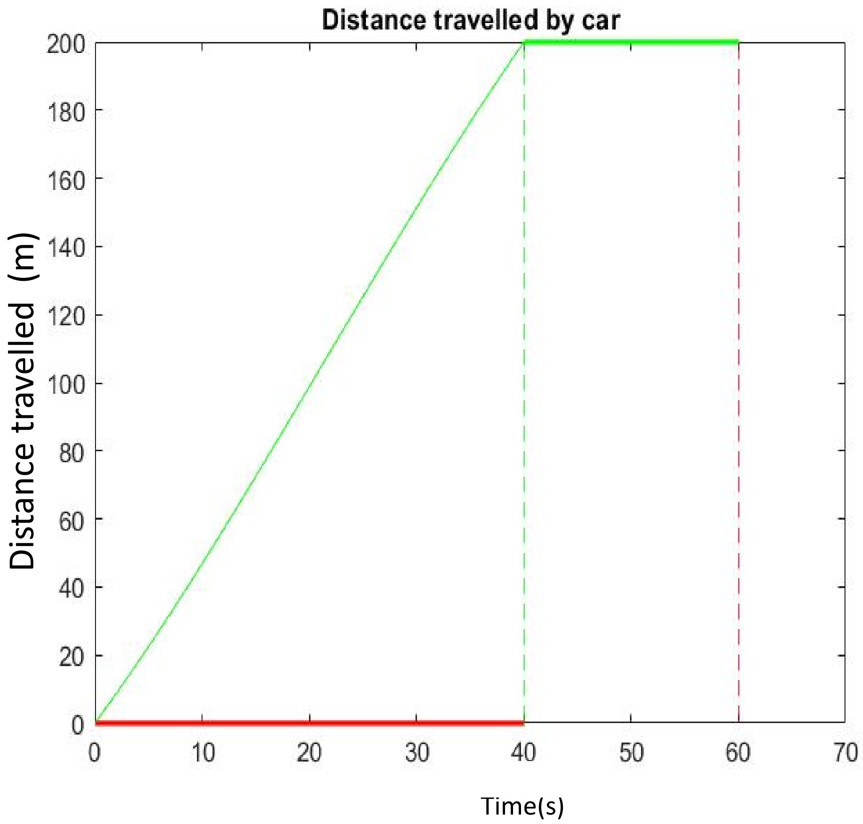


CASE 3 – As per realistic dynamic model

Results

Initial Velocity, v_0 **= 4.2634 m/s**
Distance, **L** **= 200 m**

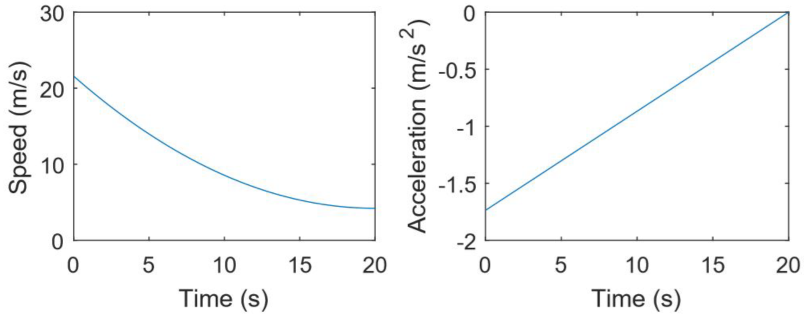


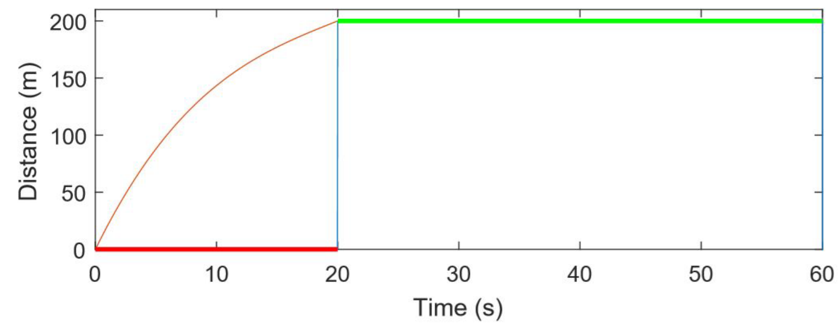


CASE 4 – As per dynamic model from paper

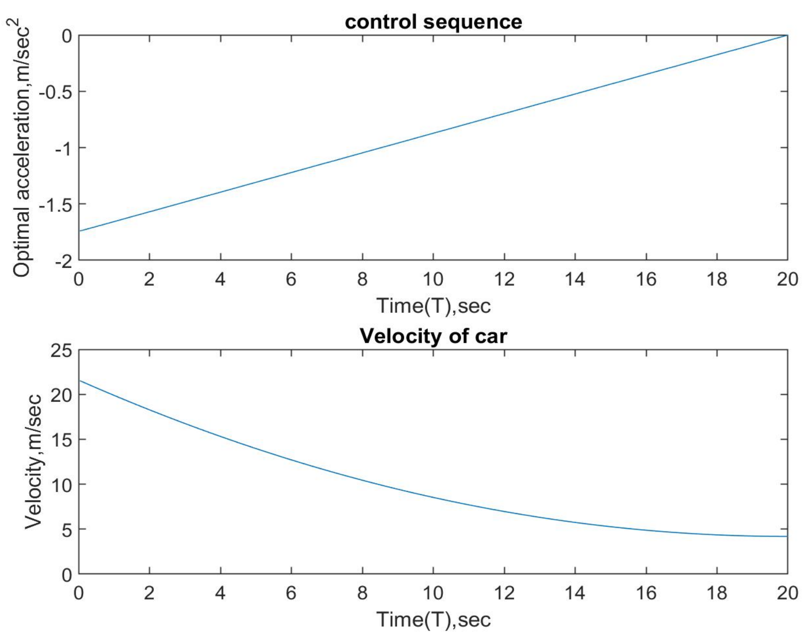
Initial Velocity, v_0 = 21.5791 m/s
Distance, L = 200 m

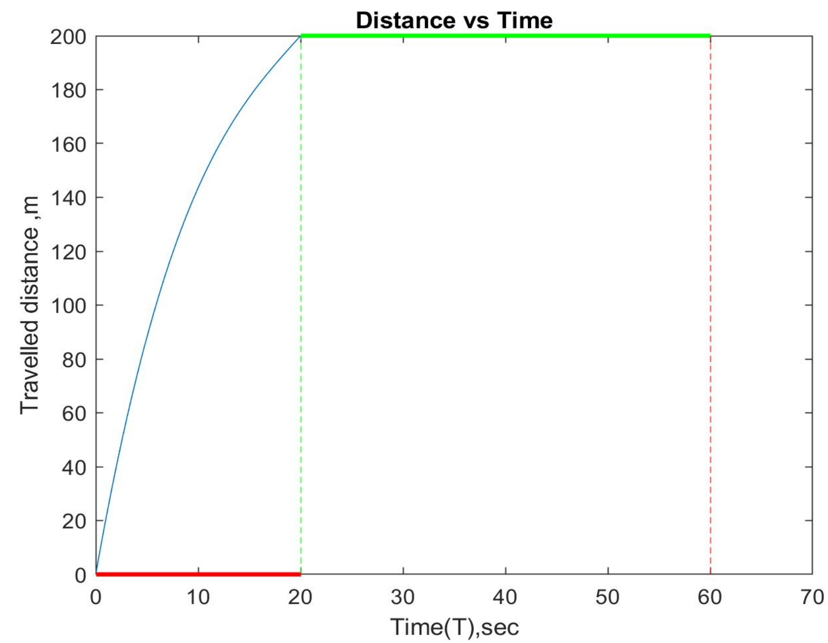
Results from paper





Our Results

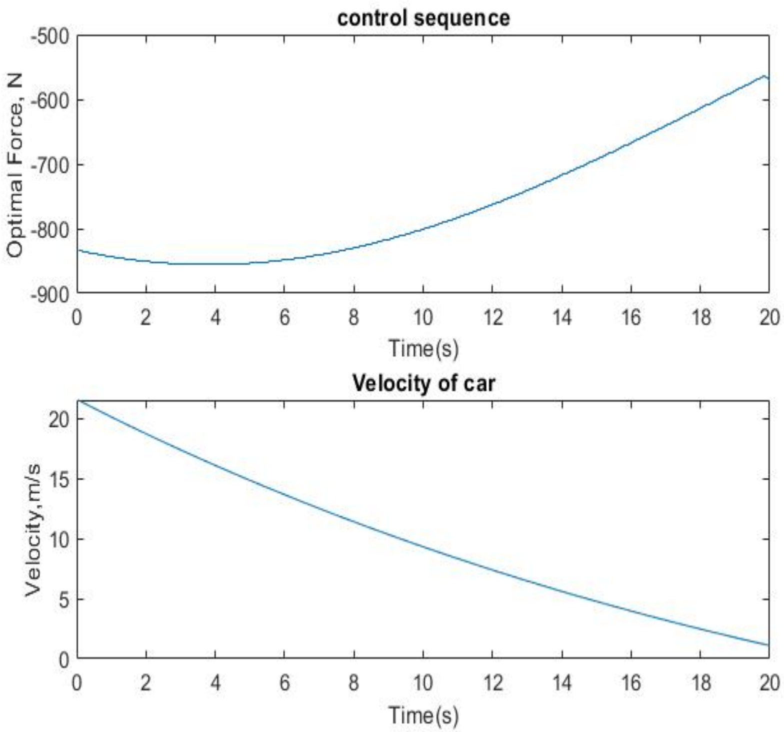


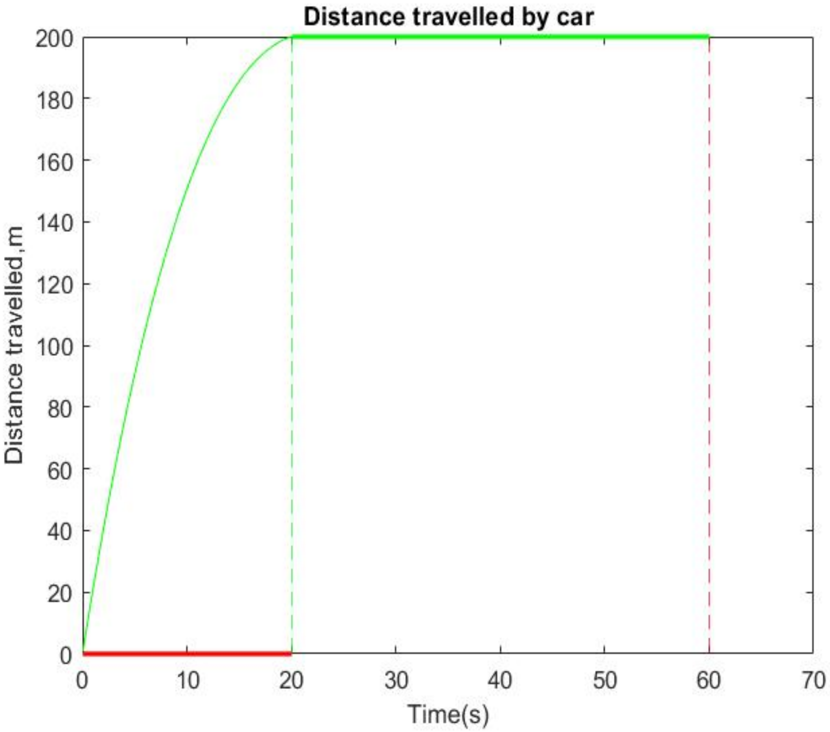


CASE 4 – As per realistic dynamic model

Results

Initial Velocity, v_0 = 21.5791 m/s
Distance, L = 200 m

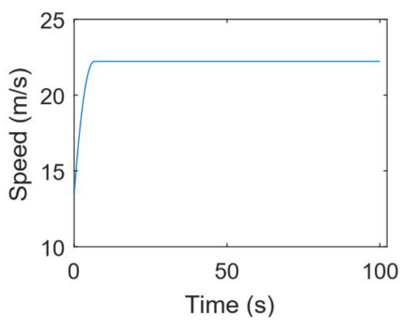


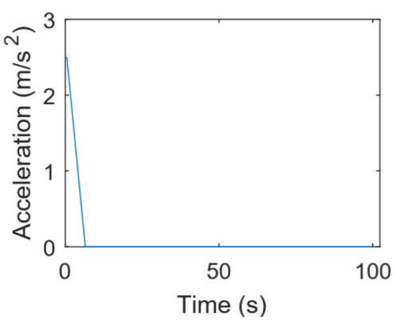


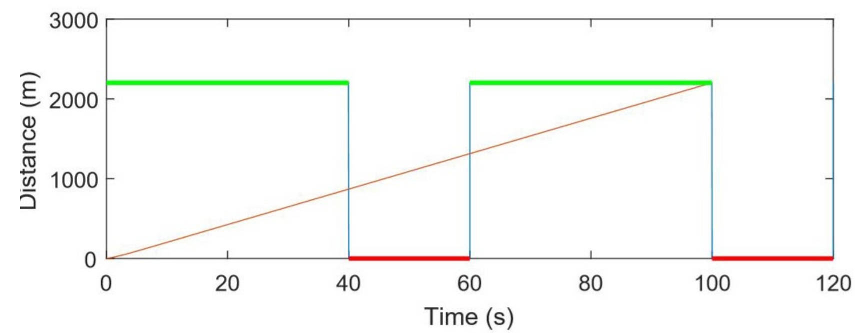
CASE 5 – As per dynamic model from paper

Initial Velocity, v_0 = 13.4875 m/s
Distance, L = 2203 m

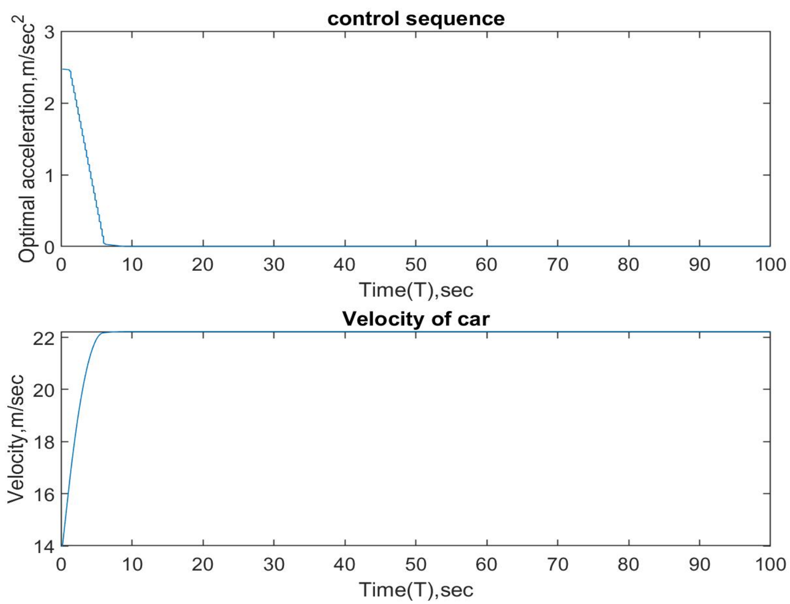
Results from paper

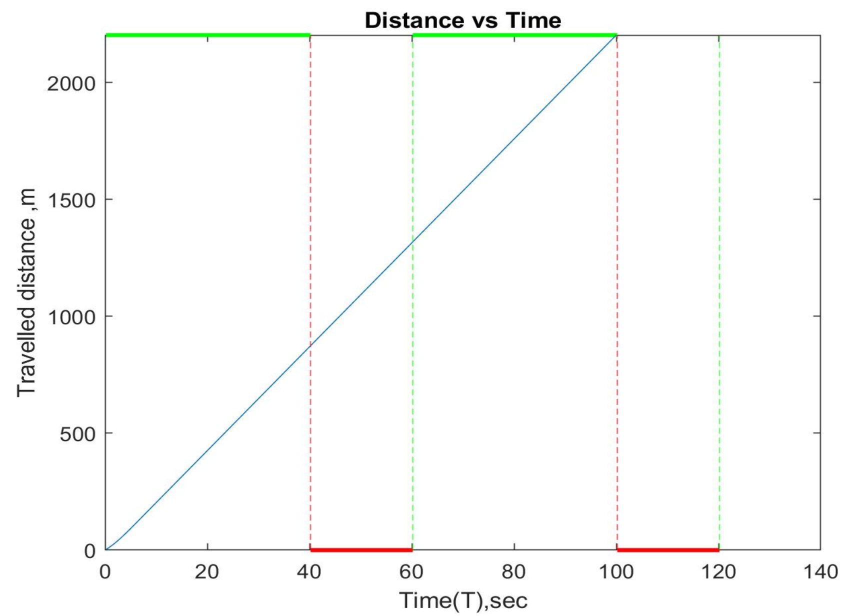






Our Results

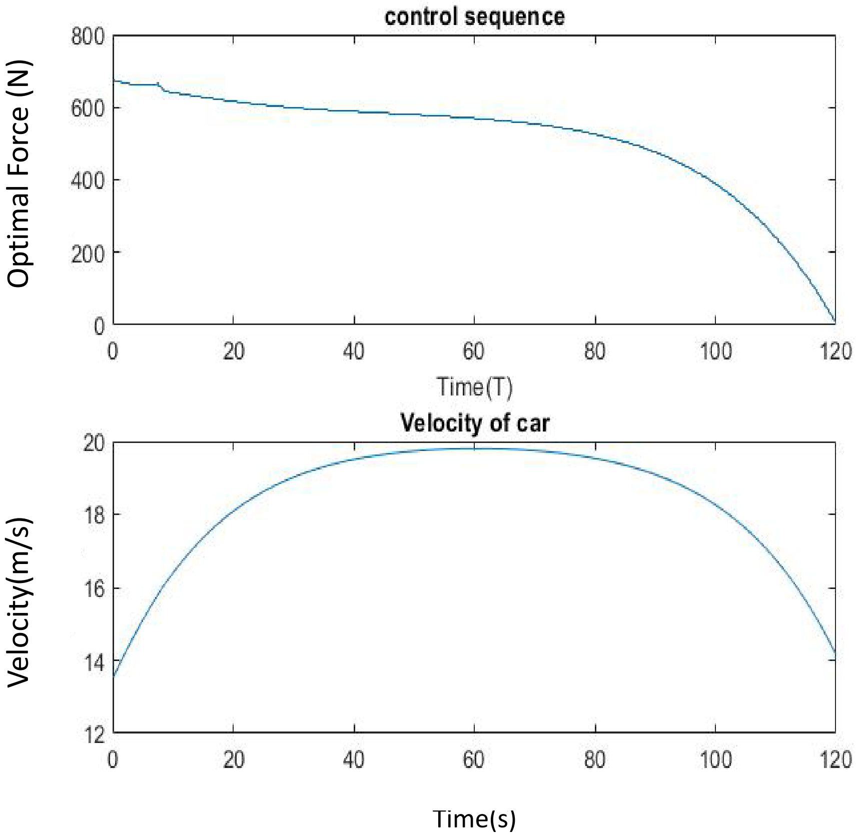


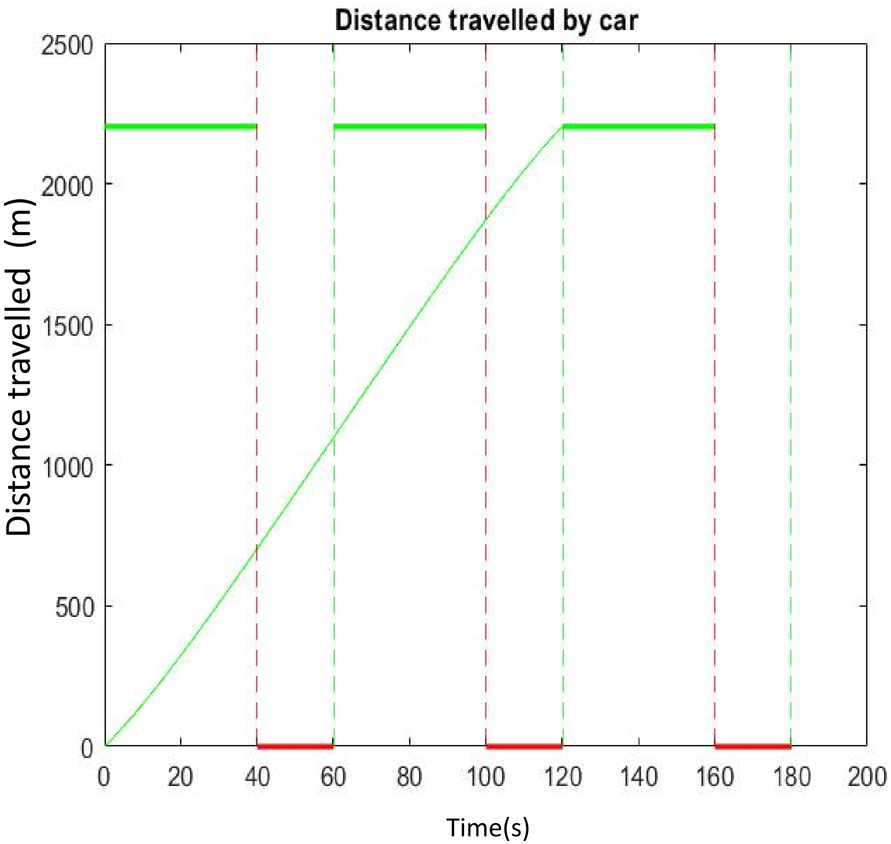


CASE 5 – As per realistic dynamic model

Results

Initial Velocity, $\mathbf{v_0}$ **= 13.4875 m/s**
Distance, **L** **= 2203 m**



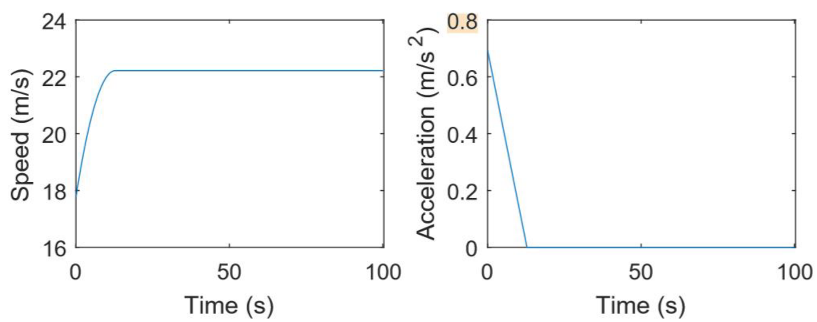


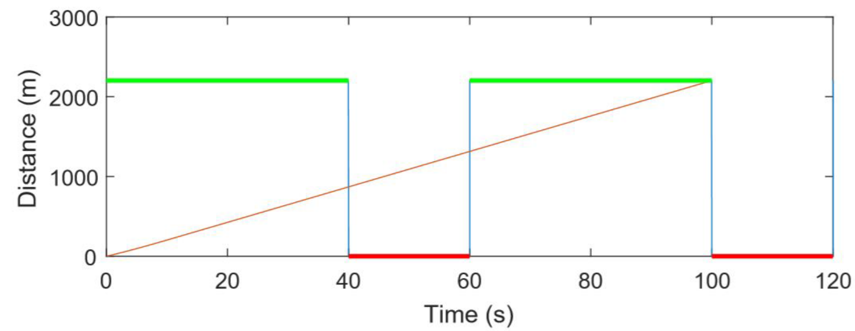
CASE 6 – As per dynamic model from paper

Initial Velocity, v_0 = 17.7745 m/s

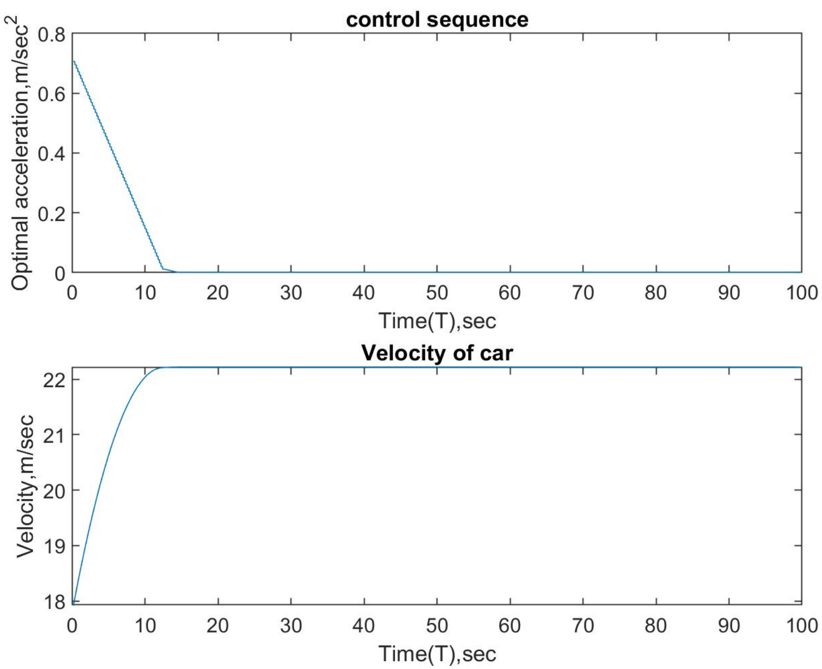
Distance, L = 2203 m

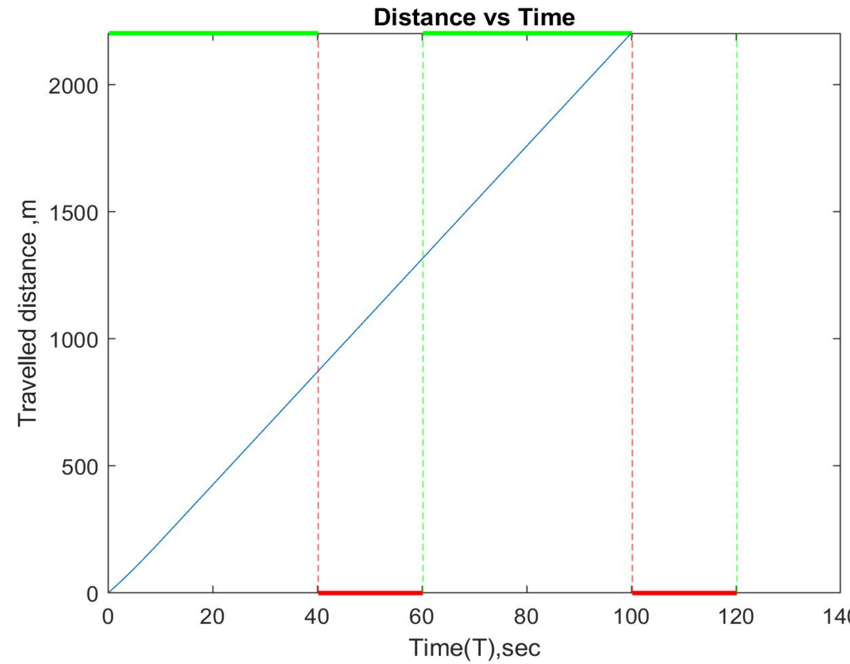
Results from paper





Our Results



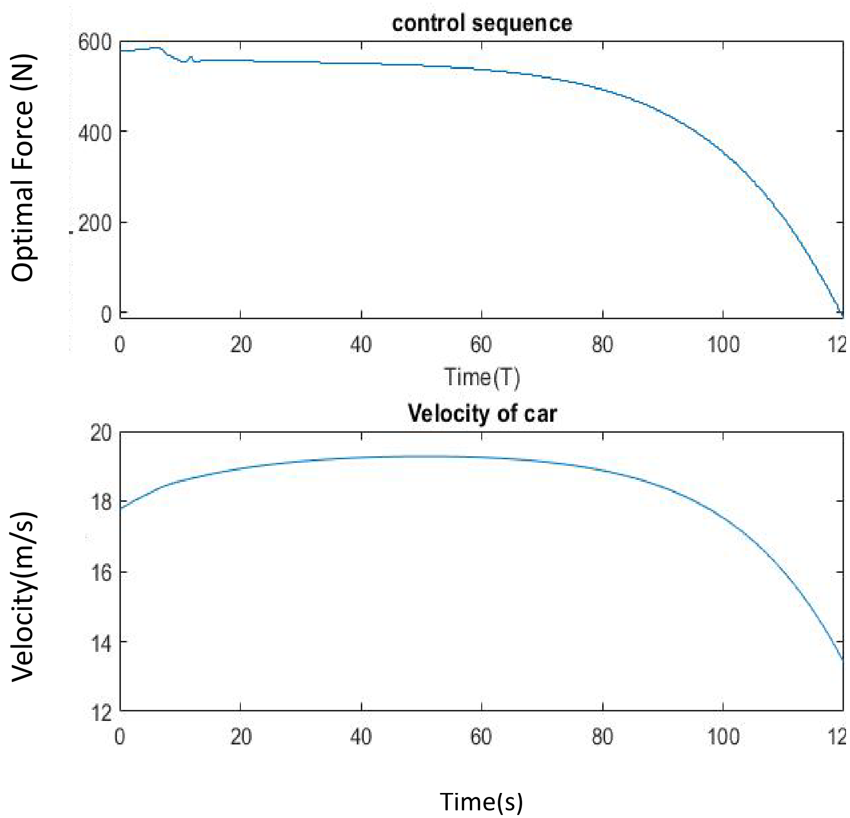


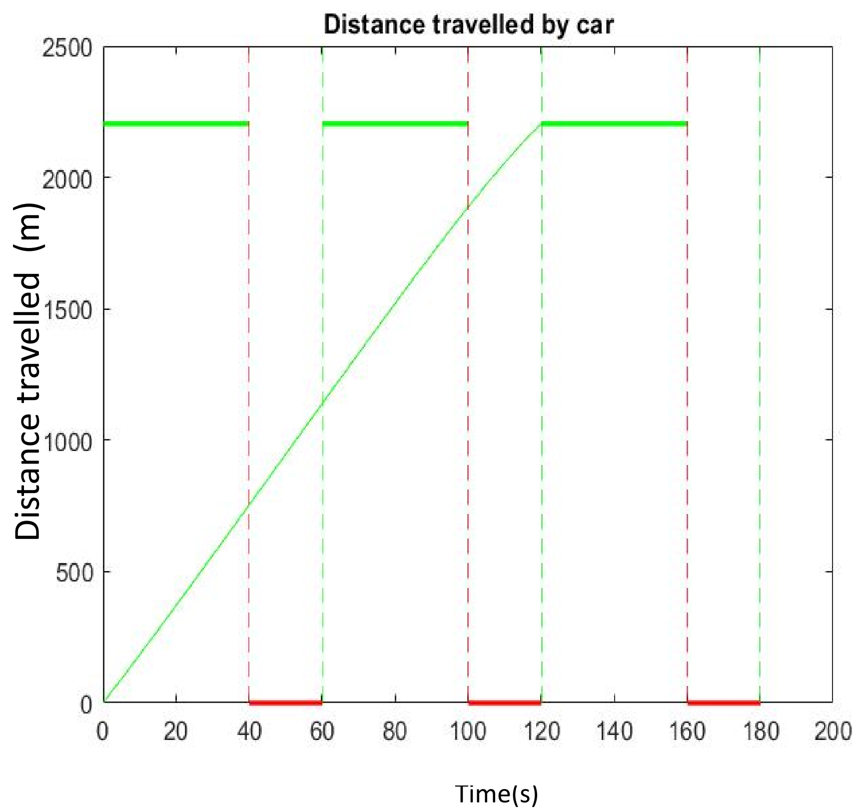
CASE 6 – As per realistic dynamic model

Results

Initial Velocity, v_0 = 17.7745 m/s

Distance, L = 2203 m

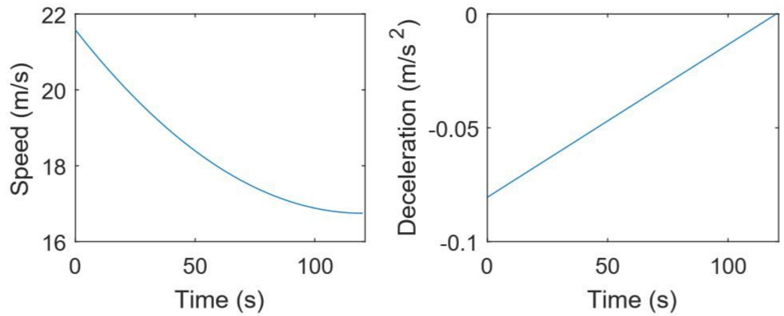


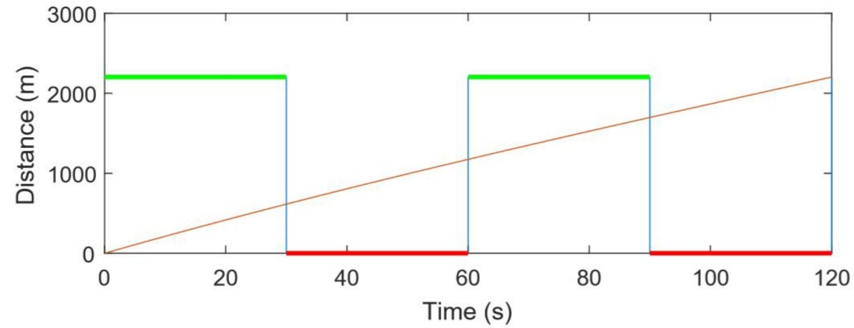


CASE 7 – As per dynamic model from paper

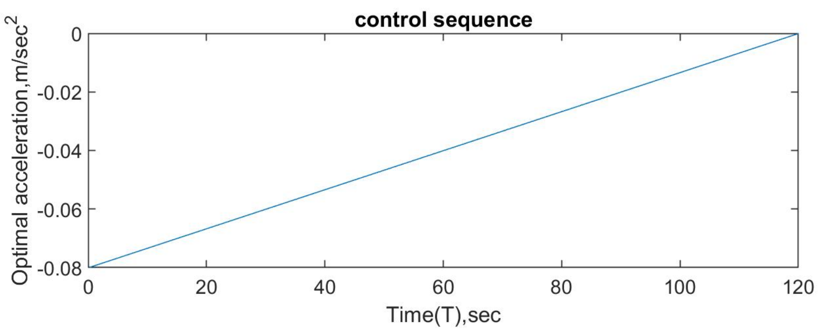
Initial Velocity, v_0 **= 21.5791 m/s**
Distance, **L** **= 2203 m**

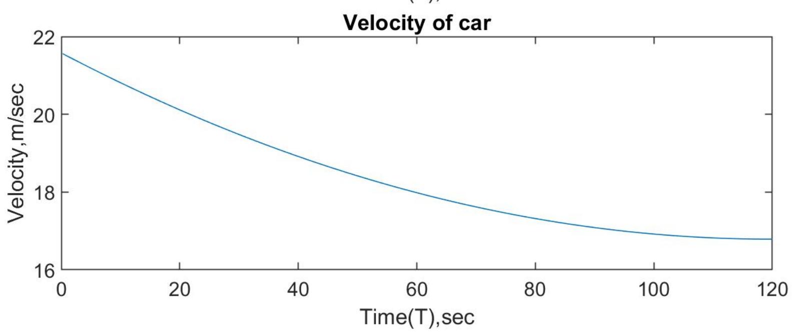
Results from paper

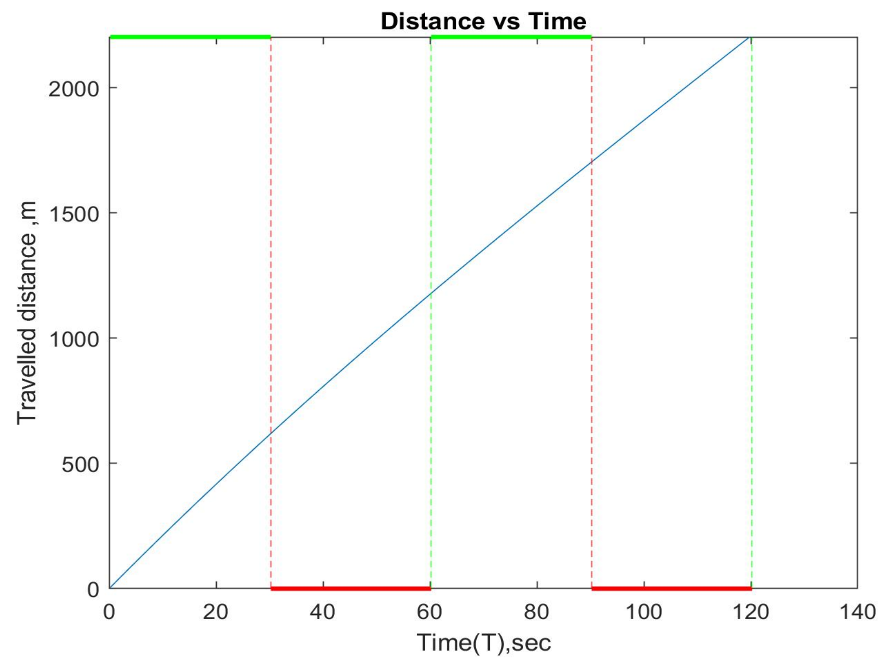




Our Results

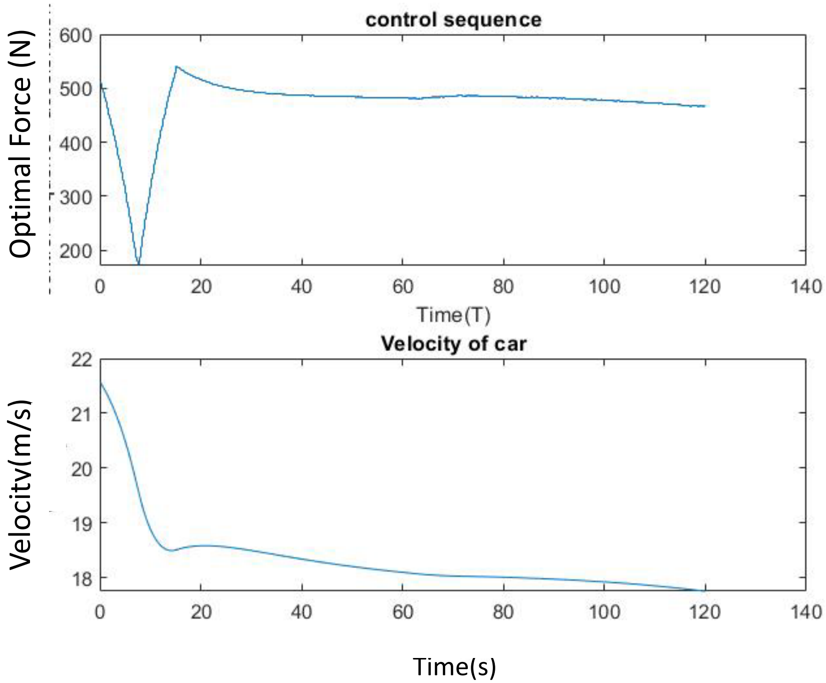


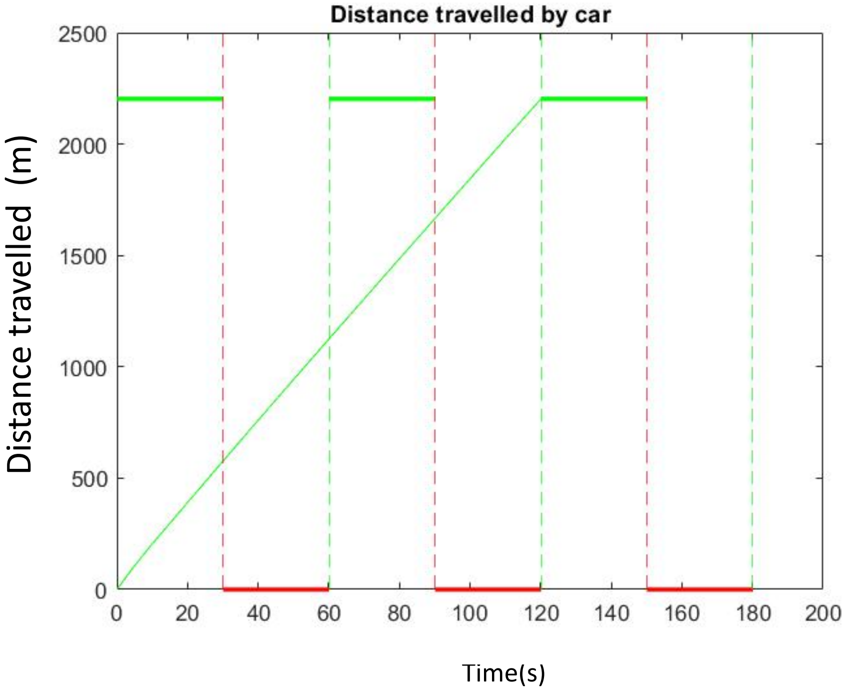




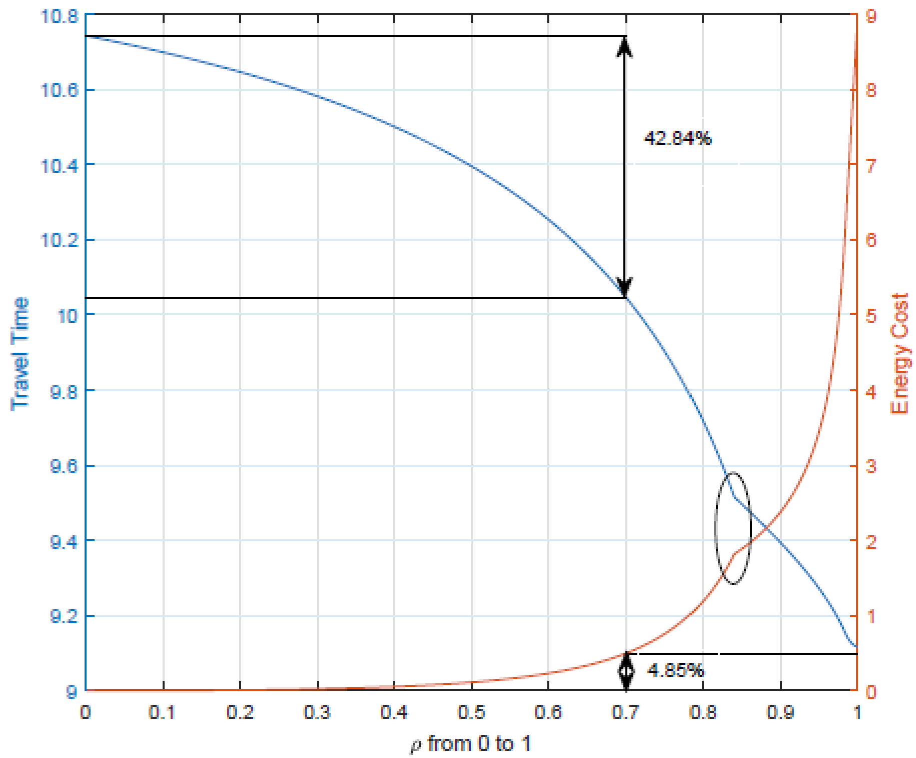
CASE 7 – As per realistic dynamic model

Results





Understanding the trade off between ρ_u and ρ_t

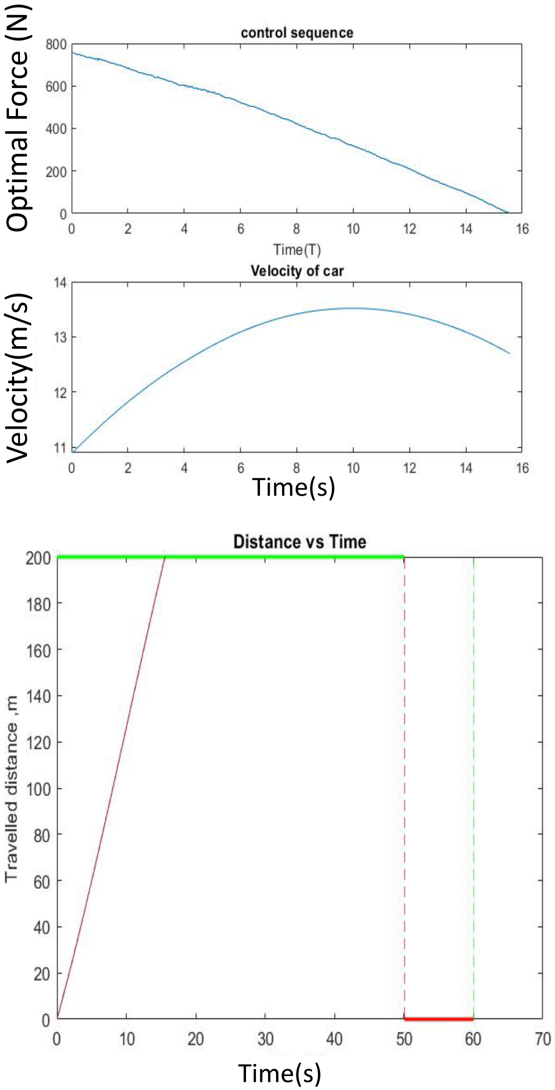


Drawn for initial velocity =18.6182m/s

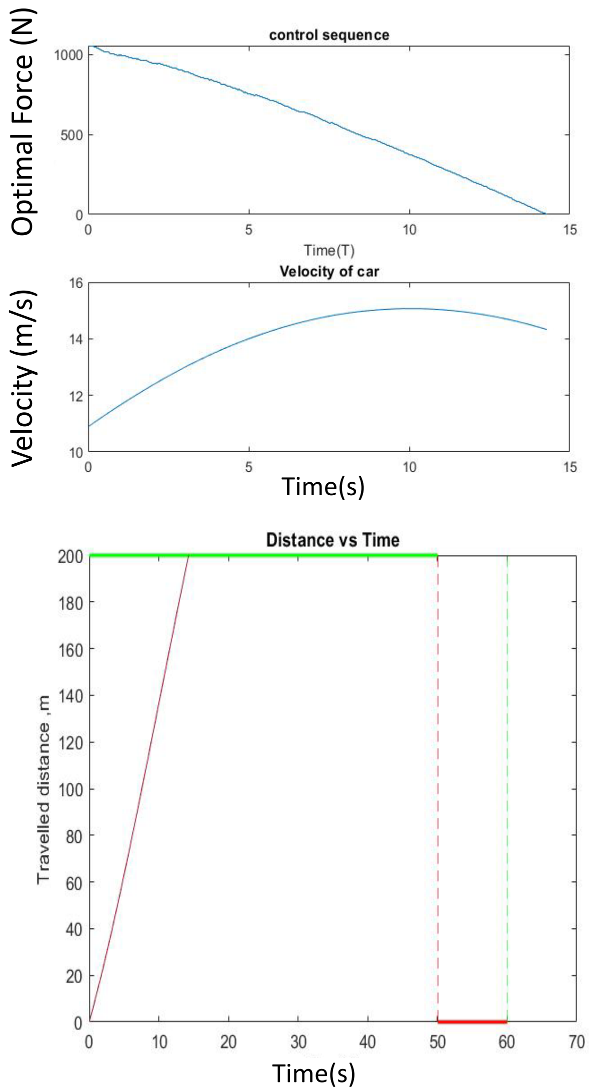
Image taken from the reference paper*

Initial Velocity, v_0 = 10.8869 m/s
Distance, L = 200 m

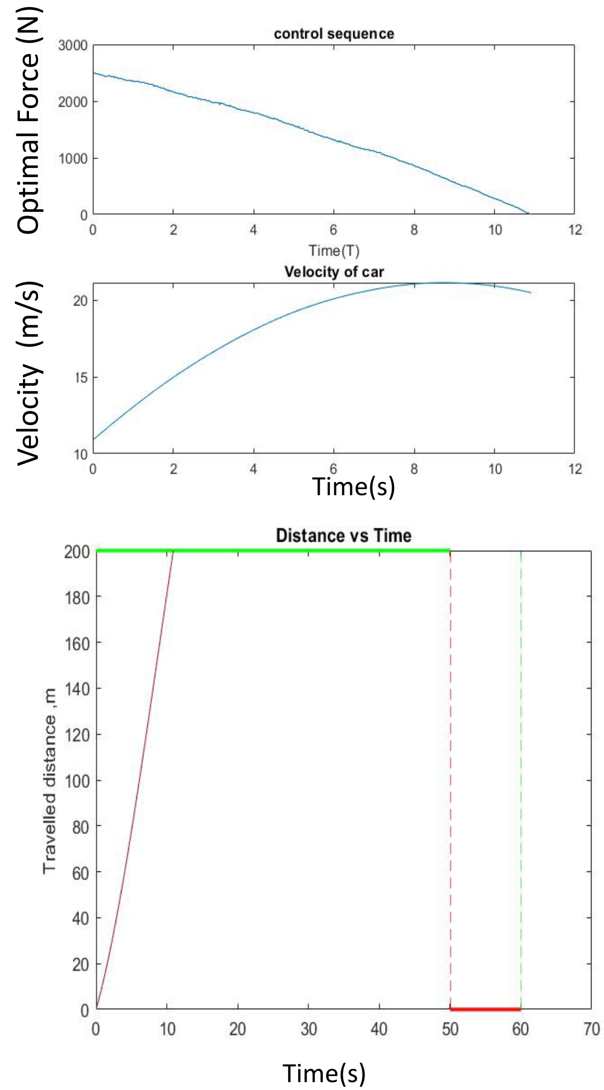
$\rho = 0.7$



$\rho = 0.8$



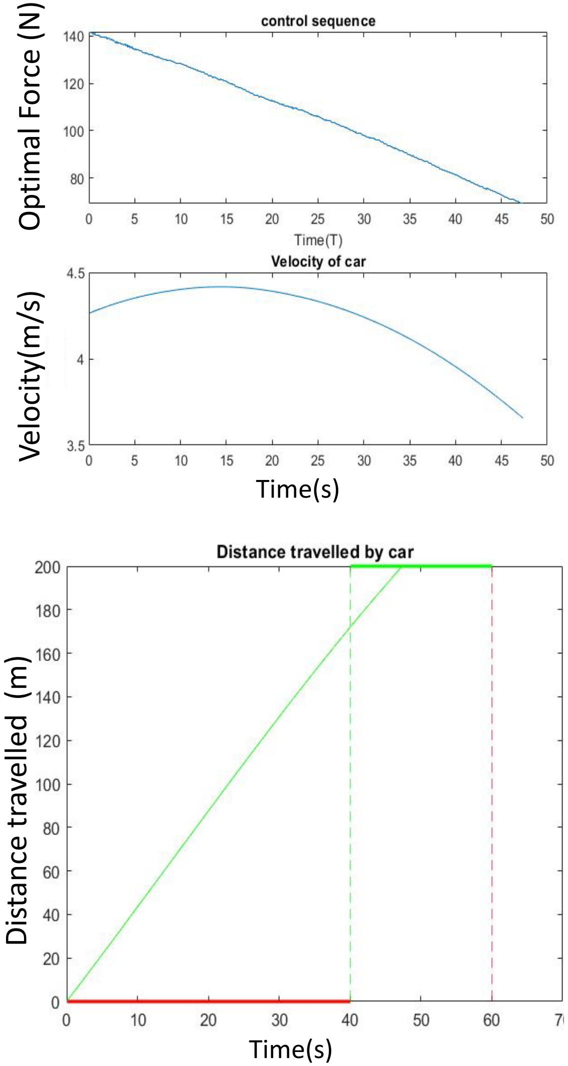
$\rho = 0.9549$



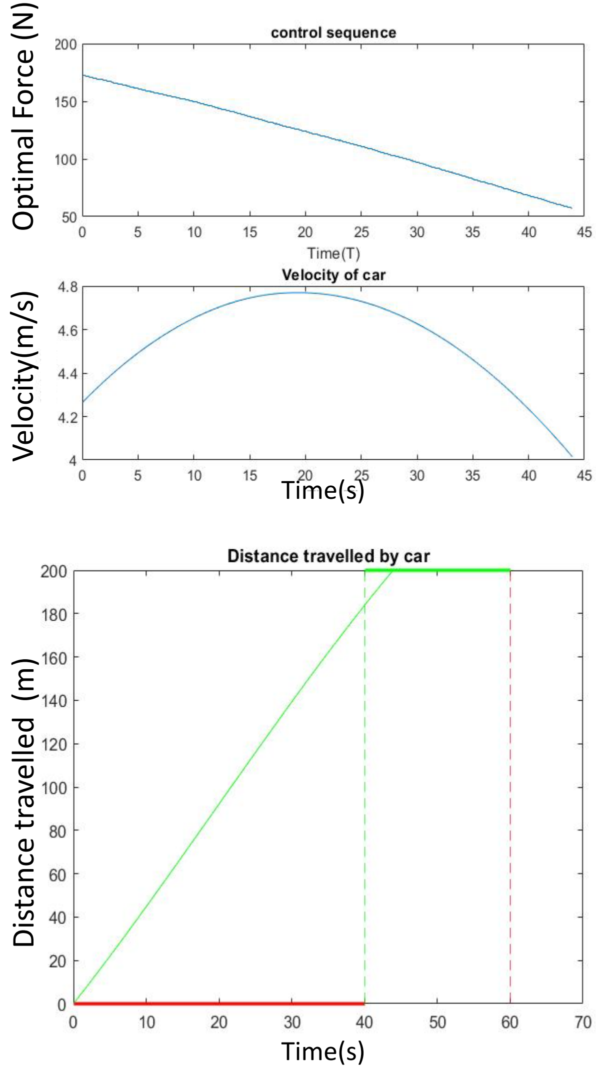
Initial Velocity, $\mathbf{v_0}$ = 4.2634 m/s

Distance, L = 200 m

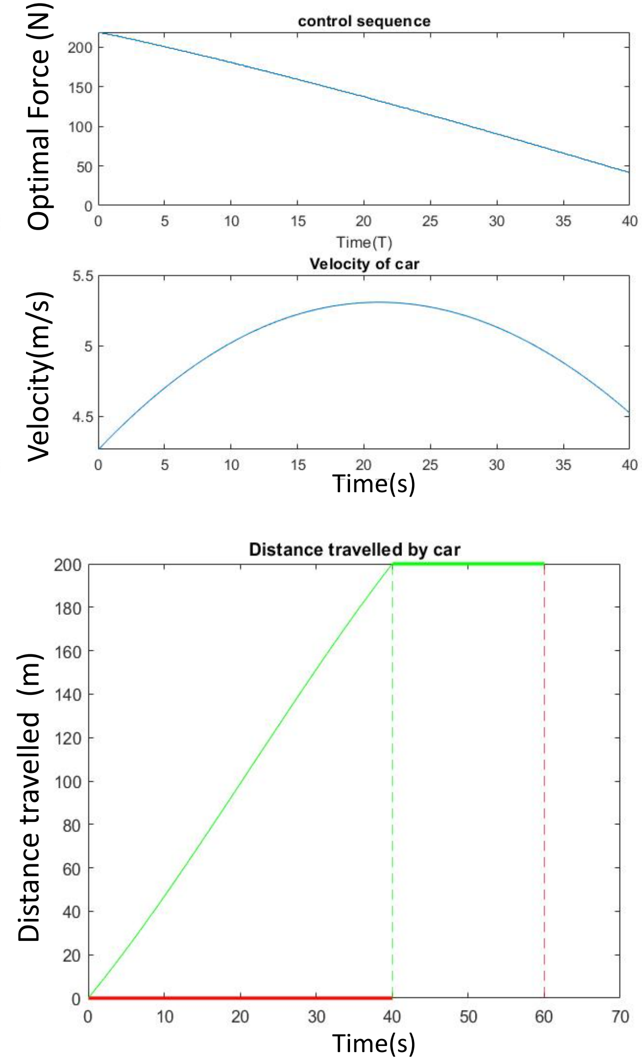
$\rho = 0.5$



$\rho = 0.83$

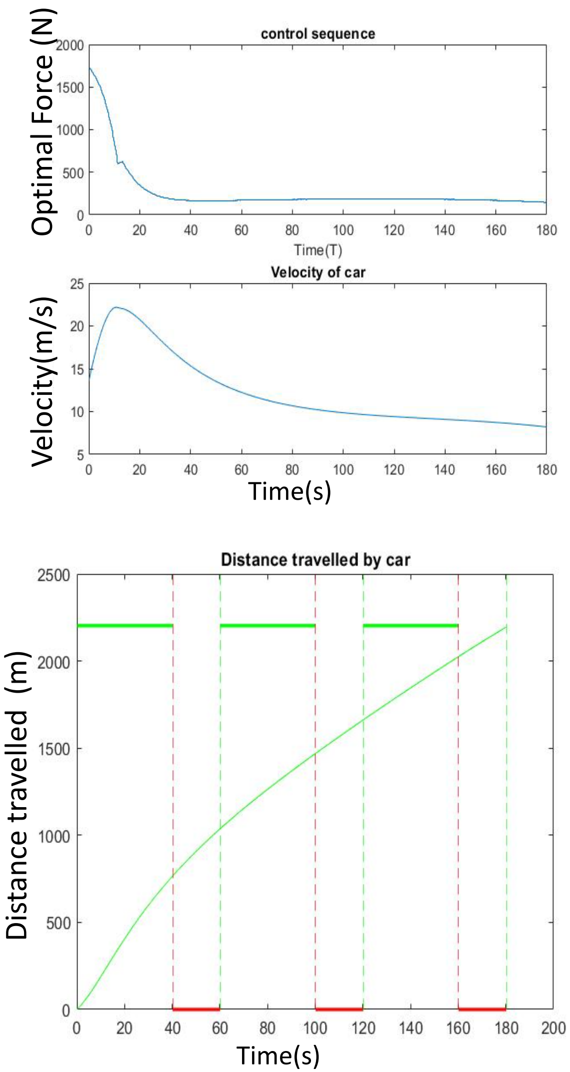


$\rho = 0.9549$

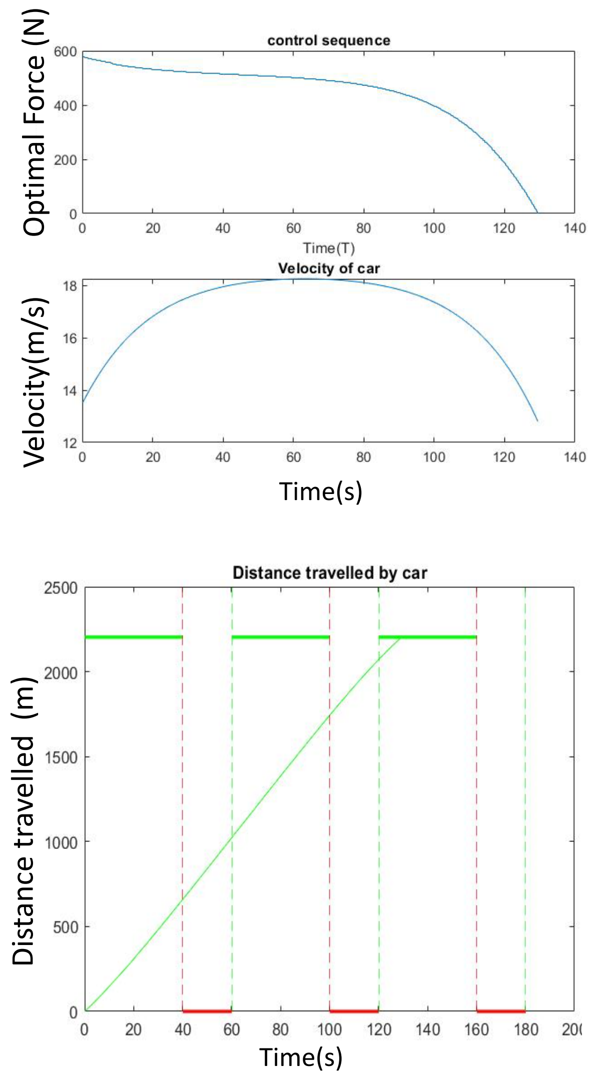


Initial Velocity, $\mathbf{v_0}$ **= 13.4875 m/s**
Distance, **L** **= 2203 m**

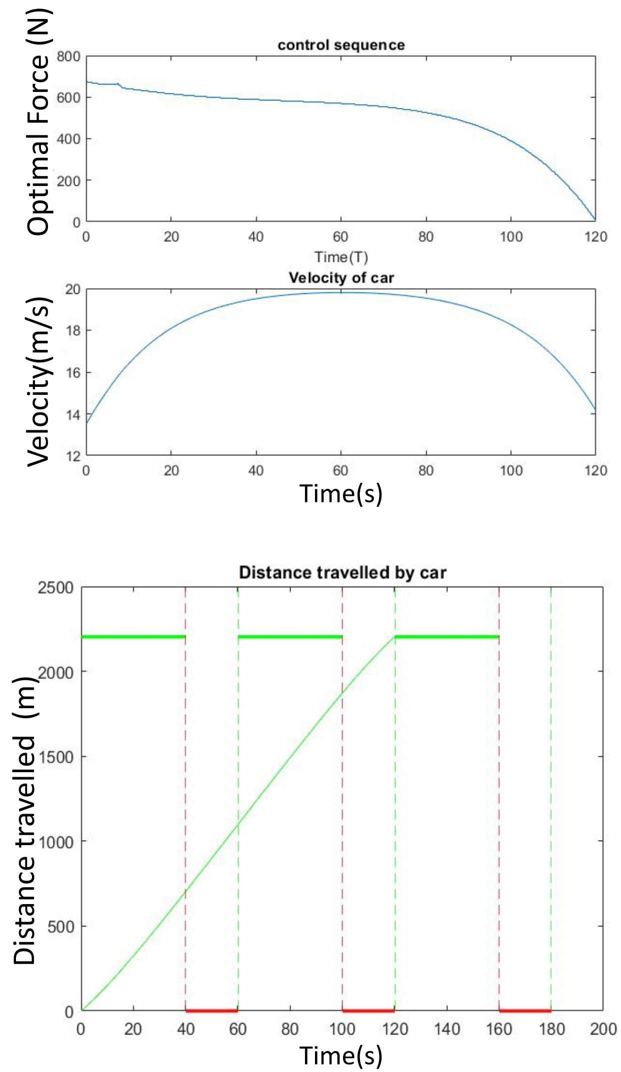
$\rho = 0.8$



$\rho = 0.9049$



$\rho = 0.9549$



Agenda

- Problem Statement & Objective

  of the chosen paper
- Continuous time Optimization
- Results & Discussion
- Discrete time Optimization
- Incorporation of a comparatively

  realistic dynamic model
- Results & Discussion
- **Conclusion**

Conclusion

- The proposed algorithm offered better efficiency in terms of travel time and energy consumption, which has been verified through extensive simulations.
- The simulation results showed that the algorithm achieved substantial performance improvement in arrival time and the energy expenditure of the autonomous vehicle avoiding the idling time at the red signal, compared to a human driver.

Few Takeaways:

- The optimization depends greatly with the number of decision variables and we had to do a lot of tuning to determine number of decision variables for each case. Research to relate a free time discrete time problem with the number of decision variables for best result would alleviate this problem.
- Normalizing constraints was a key in getting correct results.
- Solving first for continuous time gave us a general idea of how our control signal should look and we observed a similar trend for discrete time