# Math 215 - Assignment 3

Introduction to Statistics (Athabasca University)

# Assignment 3

## Overview

**Total marks:** / 75

This assignment covers content from Unit 3 of the course. It assesses your knowledge of random variables, types of random variables and various types of probability distributions, along with their means and standard deviations.

## Instructions

- Show all your work and justify all of your answers and conclusions, except for the TRUE/FALSE questions.
- Keep your work to 4 decimals, unless otherwise stated.

**(4 marks)**

1. Circle True (T) or False (F) for each of the following statements:

T (F) The following table, which lists values of $x$ and their probabilities, represents a valid probability distribution:

| $x$ | $P(x)$ |
|---|---|
| 3 | 0.32 |
| 4 | 0.54 |
| 5 | 0.24 |

$\Sigma P(x) = 1.1$ (X)

- Although all P(x) values follow the $0 \leq P(x) \leq 1$ characteristic;
- The $\Sigma P(x)$ value should equal 1 and in this example it does not (1.1)

T (F) The following table, which lists values of $x$ and their probabilities, represents a valid probability distribution:

| $x$ | $P(x)$ |
|---|---|
| 0 | −.09 |
| 1 | 0.28 |
| 2 | 0.42 |
| 3 | 0.39 |

$\Sigma P(x) = 1$

- all values of P(x) do not fit into $0 \leq P(x) \leq 1$ characteristic

(T) F The speed of a car travelling on the Queen Elizabeth Highway is an example of a continuous variable. This variable can assume any value contained in one or more intervals

T (F) The binomial distribution can be used only when the probabilities of the two possible outcomes are equal.

The probabilities of the two outcomes does not have to be equal, just need to remain constant throughout experiment

**(16 total marks)**

2. The following table lists the frequency distribution of the number of vehicles owned per household from a sample of 200 households:

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $f$ | 33 | 106 | 45 | 10 | 4 | 2 |

**(4 marks)**

a. Construct a probability distribution table for the number of vehicles owned per household.

| #vehicles | freq | rel. freq | P(x) | x P(x) | $x^2$ | $x^2 P(x)$ |
|---|---|---|---|---|---|---|
| 0 | 33 | 0.165 | 0.165 | $0 \times 0.165 = 0$ | 0 | 0 |
| 1 | 106 | 0.53 | 0.53 | $1 \times 0.53 = 0.53$ | 1 | .53 |
| 2 | 45 | 0.225 | 0.225 | $2 \times 0.225 = 0.45$ | 4 | .9 |
| 3 | 10 | 0.05 | 0.05 | $3 \times 0.05 = 0.15$ | 9 | .45 |
| 4 | 4 | 0.02 | 0.02 | $4 \times 0.02 = 0.08$ | 16 | 0.32 |
| 5 | 2 | 0.01 | 0.01 | $5 \times 0.01 = 0.05$ | 25 | 0.25 |
| | $\Sigma f = 200$ | $= 1$ | $\Sigma P(x) = 1$ | $\Sigma x P(x) = 1.26$ | | $\Sigma x^2 P(x) = 2.45$ |

rel freq = P(x)

**(2 marks)**

b. Calculate the mean of this probability distribution. Hint: Consider adding the appropriate column to the table created in part a.

$\mu = \Sigma x P(x)$

$\mu = 1.26$

**(4 marks)**

c. Calculate the standard deviation of this probability distribution. Hint: Consider adding the appropriate columns to the table created in part a.

$$\sigma = \sqrt{\Sigma x^2 P(x) - \mu^2}$$
$$\sigma = \sqrt{(2.45) - 1.26^2}$$
$$\sigma = \sqrt{2.45 - 1.5876}$$
$$\sigma = \sqrt{0.8624}$$
$$\boxed{\sigma = 0.9287}$$

**(4 marks)**

d. Give a brief interpretation (one or two sentences each) of the values of the mean and the standard deviation.

Given a group of 200 families one can expect that they will own 1.26 vehicles each with a standard deviation of 0.9287 vehicles.

**(2 marks)**

e. What is the probability that a household selected at random will have at least two vehicles?

$$P(\text{@least 2}) = P(2) + P(3) + P(4) + P(5)$$
$$= (0.225) + (0.05) + (0.02) + (0.01)$$
$$\boxed{P(\text{@least 2}) = 0.305}$$

**(13 total marks)**

3. When transferring a goldfish to a new water source, such as a different fish tank, there is an 8% chance that the goldfish will die within the first week.

   If we select at random 5 goldfish that have been transferred to a new water source, what is the probability:

**(3 marks)**

a. that exactly one of them will die within the first week?

$${}_5C_1 = \frac{5!}{1!(5-1)!} = \frac{120}{(1)(24} = \frac{120}{24} = 5$$

$$P(x) = {}_nC_x P^x q^{n-x}$$

$$= {}_5C_1 (0.08)^1 (1-0.08)^{5-1}$$

$$= {}_5C_1 (0.08)(0.92)^4$$

$$= (5)(0.08)(0.92)^4$$

$$= (5)(0.08)(0.71639296)$$

$$P(x) = 0.2866$$

**(6 marks)**

b. that fewer than three of them will die within the first week?

See extra sheet for P(x) of each value

$$P(x<3) = P(0) + P(1) + P(2)$$

$$= 0.659081523 + 0.286557184 + 0.049836032$$

$$P(<3) = 0.9962$$

**(2 marks)**

c. that at least one of them will die within the first week?

See extra sheet for P(x) of each.

$$P(1 \leq 5) = 1 - P(0)$$

$$= 1 - .659081523$$

$$P(1 \leq 5) = 0.3409$$

**(2 marks)**

d. Circle True (T) or False (F) for each of the following statements:

(T) F If we randomly select 6 goldfish that have been transferred instead of 5, the experiment continues to satisfy the conditions for a binomial experiment.

(T) F John transfers each goldfish to the same bowl. In this case, the chance that a goldfish will die goes up by 1% for each additional goldfish that is selected. This new experiment continues to satisfy the conditions for a binomial experiment.

**(11 total marks)**

4. Thirty percent of students graduate from high school before they reach the age of 18. In a random sample of 16 high-school graduates, what is the probability that: [Hint: Use binomial table.]

n = 16 P(.30)
X = TBD

* see extra paper for Probabilities for all questions

**(3 marks)**

a. More than 10 of them graduated before they were 18 years old?

$P(x>10) = P(11) + P(12) + P(13) + P(14) + P(15) + P(16)$

$= 0.0015 + 0.0002 + 0 + 0 + 0 + 0$

$P(x>10) = 0.0015$

**(3 marks)**

b. At most 4 of them graduated before they were 18?

$P(x \leq 4) = P(0) + P(1) + P(2) + P(3) + P(4)$

$= (0.0033) + (0.0208) + (0.0732) + (0.1465) + (0.2040)$

$P(x \leq 4) = 0.4498$

**(3 marks)**

c. Fewer than 7 of them graduated after they turned 18?

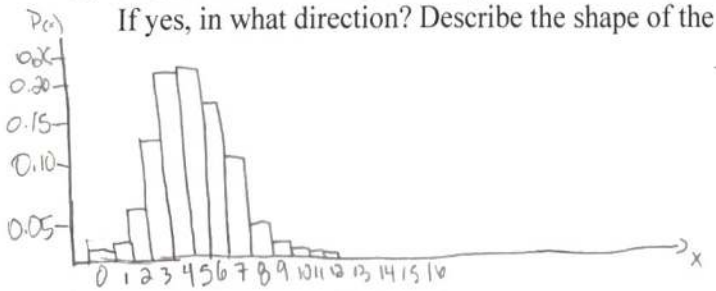
$P(x<7) = P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6)$

$= 0.4498 + 0.2099 + 0.1649$ (see above)

$P(x<7) = 0.8246$

**(2 marks)**

d. Would the binomial probability distribution representing the sample in Question 4a be skewed? If yes, in what direction? Describe the shape of the distribution in context of the study.



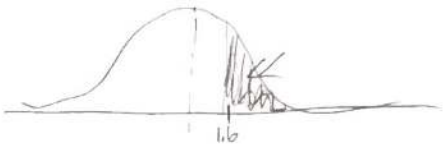

This distribution is skewed to the right which means that the p falls below .5

**(5 total marks)**

5. Use the standard normal distribution table to find:

**(2 marks)**

a. $P(z > 1.60)$



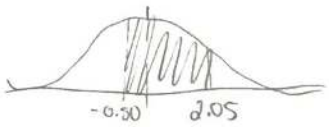

$P(z) = 1 - (z\ 1.60)$

$P_{z(c)} = 1 - .9452$

$P(z>1.6) = 0.0548$

**(3 marks)**

b. $P(-0.50 < z < 2.05)$




$P(z) = 0.9798 - 0.6915$

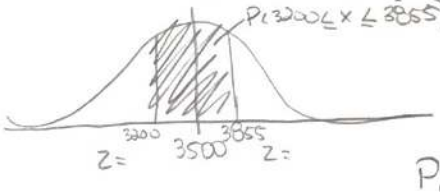
$P(z) = 0.2883$

**(11 total marks)**

6. The daily milk production of a dairy cow is normally distributed with a mean of 3,500 milliliters and a standard deviation of 250 milliliters. $\mu = 3{,}500$ ml, $\sigma = 250$ ml

**(4 marks)**

a. What is the probability that a cow selected at random will produce between 3,200 and 3,855 milliliters of milk per day?




Determine z value

$z = \frac{x - \mu}{\sigma} = \frac{3200 - 3500}{250}$

$z(3200) = -1.2$

$z(3855) = \frac{3855 - 3500}{250}$

$z(3855) = 1.145$

use SNDT

$P(3200 \leq x \leq 3855) = z(3855) - z(3200)$

$= .9265 - 0.1151$

$P(3200 \leq x \leq 3855) = 0.8114$

**(4 marks)**

b. What is the probability that a cow selected at random will produce 3,900 milliliters of milk or more?

$z = \frac{3900 - 3500}{250}$

$z = 1.6$

$P(z>1.60) = 1 - .9452$


$P(x>3900) = 0.0548$

**(3 marks)**

c. Forty percent (40%) of cows will likely produce less than what amount of milk, in milliliters?

Using stand normal dist. table $p(.40) = .3974$

$z = -0.26$

$$z = \frac{x - \mu}{\sigma}$$

$$(250)(-0.26) = \frac{x - 3500}{250}$$

$$+3500 \quad -65 = x - 3500 \quad (+3500)$$

$x = 3435$

**(4 marks)**

7. The time taken to assemble a car in a certain plant is a **normal** random variable having a mean of 20 hours. If 6.3% of the cars assembled take longer than 25 hours to assemble, what is the standard deviation of assembly time?


Let $\mu = 20$

$x = 25$

$z = 1 - 0.063$

inv z value of $.0937 = 1.53$

$$z = \frac{x - \mu}{\sigma} \qquad \frac{(\sigma)1.53}{1.53} = \frac{25 - 20}{\sigma}$$

$$\sigma = \frac{5}{1.53}$$

$\sigma = 3.2680$

**(11 total marks)**

8. A national poll found that 60% of Canadians believe that life exists on other planets. In a randomly selected sample of 300 Canadians:

$np > 5 \quad nq > 5$

Let $n$ = total canadians samples (300) Let $x =$

Let $p$ = probability of believing (.60) $\mu = np = (300)(.60) = 180$

Let $q$ = probality of no (0.40) $\sigma = \sqrt{npq} = \sqrt{(300)(.60)(.4)} = \sqrt{72}$

$\sigma = 8.4853$

**(3 marks)**

a. what is the probability that fewer than 200 people in the sample believe in extraterrestrial life?

correction for continuity = $P(x < 199.5)$

$$z = \frac{199.5 - 180}{8.4853}$$

$z = 2.2980$

$P(x < 199.5) = 0.9890$

**(4 marks)**

b. what is the probability that at least 160 people in the sample believe in extraterrestrial life?

correction for continuity = 159.5 $P(x \geq 160)$

$$1 - z = \frac{x - \mu}{\sigma}$$

$$= \frac{159.5 - 180}{8.4853}$$

$= -2.4159$

$= 1 - .0080$

$P(160 < x < 300) = 0.992$

**(4 marks)**

c. what is the probability that exactly 190 Canadians in the sample believe in extraterrestrial life?

continuity correction = $P(x = 189.5)$

$$Z = \frac{X - \mu}{\sigma}$$

$$Z = \frac{189.5 - 180}{8.4853}$$

$$Z = 1.1196$$

$P(x = 189.5) = .8665$

These extra pages are for additional calculations. If you need them for your solutions, please reference them in the appropriate place in the questions.

question #3

$P(\ ) = {}_nC_x\, p^x q^{n-x}$ $\quad {}_nC_x = \frac{n!}{x!(n-x)!}$

| n | x | P(0.08) |
|---|---|---|
| 5 | 0 | $P(0) = {}_5C_0\,(0.08)^0(1-0.08)^{5-0}$<br>$= (1)(1)(.92)^5$<br>$= 0.659081523$<br>${}_5C_0 = \frac{5!}{0!\,5!} = \frac{120}{(1)(120)} = 1$ |
| 5 | 1 | See question 3a for equation<br>$= 0.286557184$ |
| 5 | 2 | $P(2) = {}_5C_2(0.08)^2(.92)^3$<br>$= (10)(0.0064)(0.778688)$<br>$= 0.049836032$<br>${}_5C_2 = \frac{5!}{(2!)(3!)} = \frac{120}{12} = 10$ |
| 5 | 3 | $P(3) = {}_5C_3\,(.08)^3(.92)^2$<br>$= (10)(0.000512)(0.8464)$<br>$= 0.043335568$<br>${}_5C_3 = \frac{5!}{3!\,2!} = \frac{120}{12} = 10$ |
| 5 | 4 | $P(4) = {}_5C_4\,(.08)^4(.92)^1$<br>$= (5)(0.0000040960)(.92)$<br>$= 0.000188416$<br>${}_5C_4 = \frac{5!}{4!\,1!} = \frac{120}{24} = 5$ |
| 5 | 5 | $P(5) = {}_5C_5\,(.08)^5(.92)^{5-5}$<br>$P(5) = (1)(0.0000032768)(1)$<br>$= 0.0000032768$<br>${}_5C_5 = \frac{5!}{5!\,0!} = \frac{120}{(120)(1)} = 1$ |

quest #4 - using binomial table Pg. 30

| n | x | |
|---|---|---|
| 16 | 0 | 0.0033 |
| 16 | 1 | 0.0228 |
| 16 | 2 | 0.0732 |
| 16 | 3 | 0.1465 |
| 16 | 4 | 0.2040 |
| 16 | 5 | 0.2099 |
| 16 | 6 | 0.1649 |
| 16 | 7 | 0.1010 |
| 16 | 8 | 0.0487 |
| 16 | 9 | 0.0185 |
| 16 | 10 | 0.0056 |
| 16 | 11 | 0.0013 |
| 16 | 12 | 0.0002 |
| 16 | 13 | 0.0000 |
| 16 | 14 | 0.0000 |
| 16 | 15 | 0.0000 |
| 16 | 16 | 0.0000 |