ECE/CS 584: Embedded and cyberphysical system verification Fall 2019

Dawei Sun, daweis2 *Homework 3: Temporal logics and composition*

Due October 17^{th}

Typeset your solutions using LaTeX zip your writeup (.pdf) and code (.py) in a single file called `nedid-584-F19.zip` *and upload this file through Compass.*

Problem 1. CTL reductions (20 points) Convert the following CTL formulas to equivalent formulas that use only **E**, **X**, and **U**:

- $\mathbf{AF}\ f_1$ (infinitely often)
- $\mathbf{AG}\ f_1$ (invariance)
- $\mathbf{AFAG}\ f_1$ (stabilization)
- $\mathbf{A} f_1\ \mathbf{U}\ f_2$,

Solution

(a) $\mathbf{AF}\ f_1 \equiv \neg\mathbf{EG}\ \neg f_1$

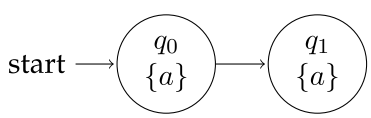
(b) $\mathbf{AG}\ f_1 \equiv \neg\mathbf{E}(\text{true}\mathbf{U}\neg f_1)$

(c) $\mathbf{AFAG}\ f_1 \equiv \neg\mathbf{EG}\ \neg(\mathbf{AG} f_1) \equiv \neg\mathbf{EG}\ \mathbf{E}[\text{true}\mathbf{U}\neg f_1]$

(d) $\mathbf{A} f_1\ \mathbf{U}\ f_2 \equiv \neg\mathbf{E}[(\neg f_2)\mathbf{U}(\neg(f_1 \vee f_2))] \vee \mathbf{EG}\neg f_2$

Problem 2. CTL to automata (8 points) Draw a finite automaton with labeled states that satisfies the CTL formula: $\mathbf{AF}(a \wedge \mathbf{AX}a)$.

Solution



Problem 3. CTL model checking (32 points) Consider the following automaton $= \langle Q, Q_0, T, L\rangle$. The set of states $Q = \{s_0, \ldots, s_4\}$, initial states $Q_0 = \{s_0, s_3\}$, the set of atomic propositions $AP = \{a, b\}$, transitions T, and the state labels L are shown in the figure.

Consider the following CTL formulas:

1. $\phi_1 = \mathbf{A}(a\mathbf{U}b) \vee \mathbf{EX}(\mathbf{AG}b)$
2. $\phi_2 = \mathbf{AGA}(a\ \mathbf{U}\ b)$

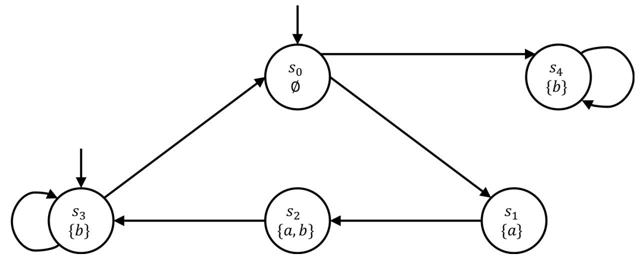


Figure 1: Automaton with state $Q = \{s_1, \ldots, s_4\}$. State labels (atomic propositions) are shown under each state.

3. $\phi_3 = (a \wedge b) \Rightarrow \mathbf{E\,G\,E\,X\,A}(a\,\mathbf{U}\,b\ \vee\ \mathbf{G}\,a)$

4. $\phi_4 = \mathbf{A\,G\,E\,F}\phi_3$.

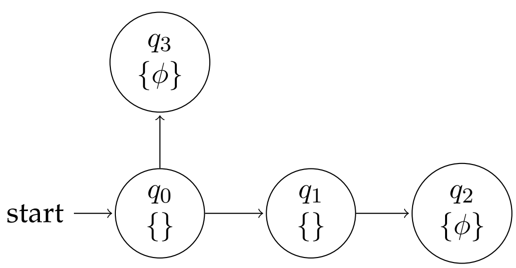
For each formula ϕ_i, determine the set of states that satisfy it, and state whether satisfies it. (Problem 6.3 from [**?**])

Problem 4. CTL equivalences (30 points) Let ϕ, ψ be arbitrary CTL formulas. Which of the following equivalences for CTL formulas are correct. Either give a proof or a counterexample.

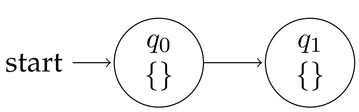
1. $\mathbf{AXAF}\ \phi \equiv \mathbf{AFAX}\ \phi$
2. $\neg\mathbf{A}(\ \phi\ \mathbf{U}\ \psi) \equiv \mathbf{E}(\ \phi\ \mathbf{U}\ \neg\psi)$
3. $(\phi \Rightarrow \mathbf{AX}\phi) \wedge (\psi \Rightarrow \mathbf{AX}\psi) \equiv (\phi \wedge \psi) \Rightarrow \mathbf{AX}(\phi \wedge \psi)$

Solution

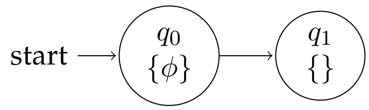
1. False. This is a counterexample satisfying the LHS while not satisfying the RHS.



False. This is a counterexample satisfying the LHS while not satisfying the RHS.



False. This is a counterexample satisfying the RHS while not satisfying the LHS.



Problem 5. Composition (10 points) Give an example of a pair of compatible HIOAs whose composition is a not an HIOA.