1. The lattice records a value for each variable. Suppose we have n variables $v_1, \ldots, v_n$ in the program. Then an example element of the lattice is $[v_1 \to \{+, 0\}, \ldots, v_n \to \{0, -\}]$.

2. Forward analysis.

3. $x_1 \vee x_2 = [x_1[v_1] \cup x_2[v_1], \ldots, x_1[v_n] \cup x_2[v_n]]$.

4. Here, c is a constant. $f_{x=c}(V) = \begin{cases} V[x \to \{+\}] \text{ if } c \text{ is positive,} \\ V[x \to \{-\}] \text{ if } c \text{ is negative,} \\ V[x \to \{0\}] \text{ if } c = 0 \end{cases}$.

5. Here, c is a constant. The transfer function is shown as following

x \ c	-	0	+
{+}	{-,0,+}	{+}	{+}
{0,+}	{-,0,+}	{0,+}	{+}
{0}	{-}	{0}	{+}
{-,0}	{-}	{-,0}	{-,0,+}
{-}	{-}	{-}	{-,0,+}
{-,+}	{-,0,+}	{-,+}	{-,0,+}
{-,0,+}	{-,0,+}	{-,0,+}	{-,0,+}
ϕ	ϕ	ϕ	ϕ

6. $f_{x=c}(V)$ is distributive.

$$\begin{aligned} f_{x=c}(V_1 \vee V_2) = f_{x=c}([&V_1[x_1] \cup V_2[x_1], \ldots, V_1[x] \cup V_2[x], \ldots, V_1[x_n] \cup V_2[x_n]]) \\ &= [V_1[x_1] \cup V_2[x_1], \ldots, sgn(c), \ldots, V_1[x_n] \cup V_2[x_n]] \end{aligned}$$

$$\begin{aligned} &f_{x=c}(V_1) \vee f_{x=c}(V_2) \\ &= [V_1[x_1], \ldots, sgn(c), \ldots, V_1[x_n]] \vee [V_2[x_1], \ldots, sgn(c), \ldots, V_2[x_n]] \\ &= [V_1[x_1] \cup V_2[x_1], \ldots, sgn(c), \ldots, V_1[x_n] \cup V_2[x_n]] = f_{x=c}(V_1 \vee V_2) \end{aligned}$$

$f_{x=x+c}(V)$ is distributive. By enumerating all the possible value of $V_1[x]$, $V_2[x]$, an c, we can prove that $f_{x=x+c}(V_1) \vee f_{x=x+c}(V_2) = f_{x=x+c}(V_1 \vee V_2)$ always holds.

7. Yes. Since F is distributive, it is monotone. Also, the lattice is finite, and thus the algorithm always terminates.

8. Yes. According to [Kildall, 1973], since F is distributive, MOP = MFP.