Problem 1

Solution

We denote the ReLU function by σ. The neural network is $\tilde{f}(x) = \sigma(a_2\sigma(a_1x + b_1) + b_2)$, where $a_1 = -1/\epsilon$, $b_1 = 1$, $a_2 = -1$ and $b_2 = 1$.

Problem 2

Solution

We denote the ReLU function by σ. First, we construct a neural network $\tilde{f}(x)$ to mimic $\mathbb{I}_{x\geq 0}$ except in the region $x \in [-\epsilon, 0]$. Simillar to Problem 1, $\tilde{f}(x) = \sigma(a_2\sigma(a_1x + b_1) + b_2)$, where $a_1 = -1/\epsilon$, $b_1 = 0$, $a_2 = -1$ and $b_2 = 1$.
Then, the final neural network is $4\sigma(\sum_{i=1}^4 \tilde{f}(z_i)-3.5)-1$, where $z_1 = 1-(x_1+x_2)$, $z_2 = 1-(x_1-x_2)$, $z_3 = 1-(-x_1+x_2)$, $z_4 = 1-(-x_1-x_2)$. This neural network mimics the groud truth except for $\{x|\ 1 \leq |x_1| + |x_2| \leq 1+\epsilon\}$. For $x \in \{x|\ |x_1| + |x_2| < 1\}$, we must have $z_i > 0$ and $\tilde{f}(z_i) = 1$ for all i. Thus, the output is 1. For $x \in \{x|\ |x_1| + |x_2| > 1+\epsilon\}$, at least one $z_i < -\epsilon$ and $\tilde{f}(z_i) = 0$. Thus, $\sum_{i=1}^4 \tilde{f}(z_i) \leq 3$, and the output is -1.