Solutions to Problems in Perkins, *Introduction to High Energy Physics*, Third Edition

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Chapter 1

Problem 1.1

> An electron of energy 20 GeV is deflected through an angle of $5°$ in an elastic collision with a stationary proton. What is the value of the square of the 4-momentum transfer, q^2, and down to what approximate distance does such a collision probe the internal structure of the proton? (The mass of the electron can be neglected compared with the energies involved. The proton mass Mc^2 is 0.938 GeV.)

First let's define terms: $p_i = 20$ GeV/c is the magnitude of the electron's initial momentum. p_e and p_p are the final electron and proton momenta. $\theta_e = 5°$ and θ_p are the electron and proton scattering angles.

Neglecting the electron mass, energy conservation requires

$$p_i c + M_p c^2 = p_e c + \left[p_p^2 c^2 + M_p^2 c^4\right]^{1/2}$$

which can be rearranged to give

$$p_p^2 = q^2 + 2qM_p c, \qquad q = p_i - p_e. \tag{1}$$

Momentum conservation requires

$$\begin{aligned} p_i &= p_e \cos\theta_e + p_p \cos\theta_p \quad (2)\\ 0 &= p_e \sin\theta_e + p_p \sin\theta_p. \quad (3) \end{aligned}$$

From the second of these we obtain

$$\sin\theta_p = -\frac{p_e}{p_p}\sin\theta_e$$

so

$$\cos\theta_p = \left[1 - \frac{p_e^2}{p_p^2}\sin^2\theta_e\right]^{1/2}.$$

With this we can eliminate θ_p from (1):

$$p_i = p_e\cos\theta_e + p_p\left[1 - \frac{p_e^2}{p_p^2}\sin^2\theta_e\right]^{1/2}$$

or

$$\begin{aligned}(p_i - p_e\cos\theta_e)^2 &= p_p^2 - p_e^2\sin^2\theta_e\\ p_i^2 + p_e^2 - 2p_ip_e\cos\theta_e &= p_p^2.\end{aligned}$$

Plugging (1) into this gives

$$\begin{aligned}p_i^2 + p_e^2 - 2p_ip_e\cos\theta_e &= q^2 + 2qM_pc\\ &= p_i^2 + p_e^2 - 2p_ip_e + 2p_iM_pc - 2p_eM_pc\end{aligned}$$

so

$$2p_ip_e\left[1-\cos\theta_e\right] + 2p_eM_pc = 2p_iM_pc.$$

With this we find

$$\begin{aligned}p_e &= \frac{p_iM_pc}{p_i\left[1-\cos\theta_e\right]+M_pc}\\ &= \frac{(20\,\mathrm{GeV/c})\cdot(0.938\,\mathrm{GeV/c})}{(20\,\mathrm{GeV/c})[1-\cos 5°]+(0.938\,\mathrm{GeV/c})}\\ &= 18.5\,\mathrm{GeV/c}.\end{aligned}$$

This gives a q value (three-momentum transfer) of $20 - 18.5 = 1.5$ GeV/c. The four-momentum transfer is the square root of the difference between this and the energy transfer, which is just p_pc. Using (1) we have

$$\begin{aligned}\text{square of four-momentum transfer} &= p_p^2c^2 - q^2c^2\\ &= 2qM_pc^3\\ &= 2\cdot(1.5\,\mathrm{GeV/c})\cdot c\cdot(0.938\,\mathrm{GeV}) = 2.81\,\mathrm{GeV}^2.\end{aligned}$$

To turn an energy into a length you divide $\hbar c$ by it:

$$\begin{aligned}\text{distance probed} &\approx \frac{\hbar c}{\text{four-momentum transfer}}\\ &= \frac{(6.6\cdot 10^{-25}\,\mathrm{Gev\ s})\cdot(3\cdot 10^{23}\,\mathrm{fm/s})}{1.68\,\mathrm{GeV}}\\ &= .12\quad\mathrm{fm}\quad.\end{aligned}$$

This does not agree with Perkins' answer. What am I doing wrong?

Problem 1.2

> The flux of primary cosmic rays averaged over the Earth's surface is approximately 1 cm^-2 s^-1, and their average kinetic energy is 3 GeV. Show that the power transferred to the Earth from cosmic rays is about 2.5 gigawatt. (Earth radius=6400 km).

Earth surface area $= 4\pi(6400\,\mathrm{km})^2 = 5.1 \cdot 10^{18}\,\mathrm{cm}^2$

Cosmic ray particle flux $\approx$ 1 per cm^2 per second
$\rightarrow\approx 5.1 \cdot 10^{18}$ cosmic ray particles per second impinging on Earth's surface

Average particle energy 3 GeV
$\rightarrow\approx 1.5 \cdot 10^{28}$ eV/s power delivered to earth surface

$$1.5 \cdot 10^{28}\,\mathrm{eV/s} \cdot \frac{1.6 \cdot 10^{-19}\,\mathrm{Joule}}{1\,\mathrm{eV}} = 2.4 \cdot 10^{9}\,\mathrm{Joule/s} \rightarrow 2.4\,\mathrm{Gw}$$