

INDIAN INSTITUTE OF TECHNOLOGY DELHI

ELL784 Introduction to Machine Learning
Assignment-3 Report

Topic: Regression and SVMs

Submitted To:

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Polynomial Curve Fitting:

- **Objective:** Learn a Polynomial function which maps the input variable **X** to continuous target variable **Y**.

Huber Cost Function:

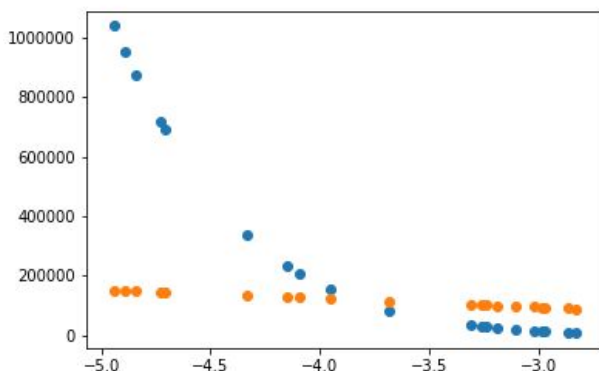
We considered Huber loss Function, which contains both mean square error and mean absolute error. Till $\delta = 50$ (error), loss function uses absolute error function, after δ becomes less than 50, it uses mean square error because absolute error function is not differentiable at global minimum.

$$loss = \begin{cases} \frac{1}{2} * (x - y)^2 & \text{if } (|x - y| \leq \delta) \\ \delta * |x - y| - \frac{1}{2} * \delta^2 & \text{otherwise} \end{cases}$$

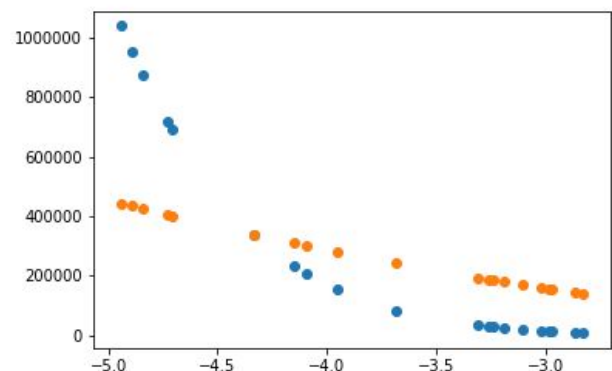
Note: Variance is Normalized

- **Curve fitting for First 20 Points:**

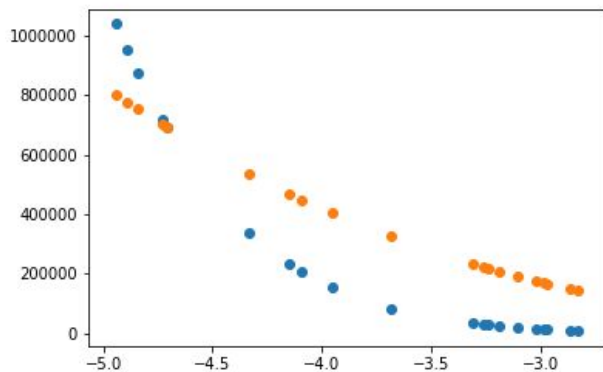
- Degree = 1, alpha = 0.01
- Noise variance = 1.1699678e-10



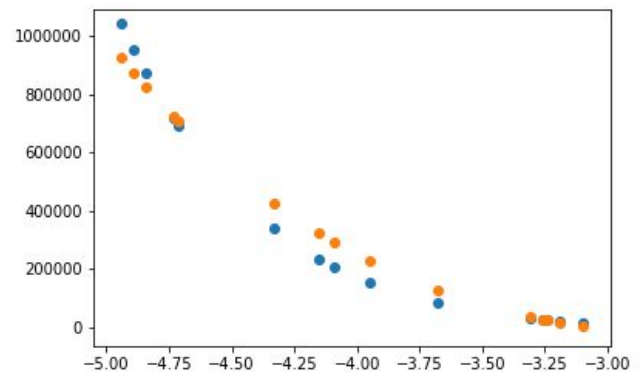
- Degree = 2 alpha = 0.01
- Noise Variance = 2.09316666e-10



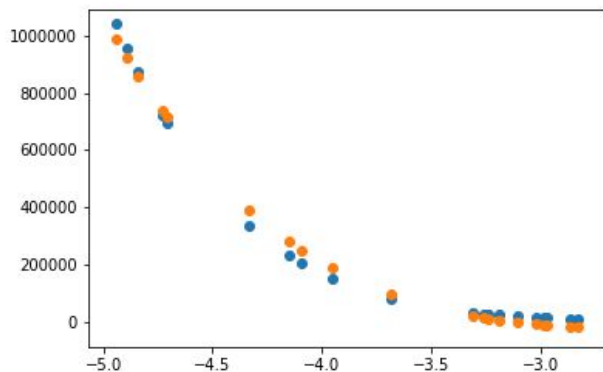
- Degree = 3 alpha = 0.01
- Noise Variance=1.0078693869e-09



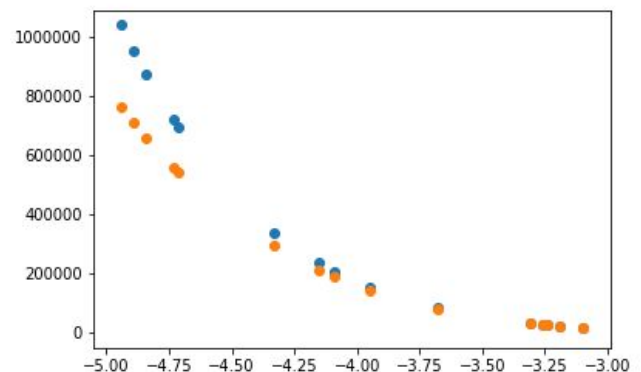
- Degree = 4 alpha = 0.001
- Noise Variance = 4.608632249e-09



- Degree = 5 alpha = 0.0001
- Noise Variance = 9.90292212e-09



- Degree = 6 alpha = 0.0001
- Noise Variance = 3.614914722e-08



- From The above results, Noise variance is increasing, so we can conclude that model is being over fitted.
- We got The Training error at 4rd degree polynomial = 0.30876 and test error is 0.27097
- We got The training error at 5th degree polynomial = 0.25809 and test error is 0.26302
- Since the test error increased at 5th degree polynomial, model started being over fitted.

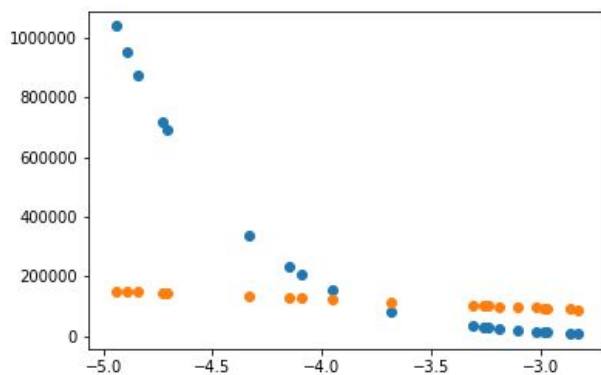
Regularization: Regularization in machine learning is the process of regularizing the parameters that constrain, regularizes, or shrinks the coefficient estimates towards zero. It avoids the model to become overfitted.

We took **L1 Regularization** because the target values are very large(in millions), If we take L2 regularization, then it will add more bias, so the model will go in under fitting.

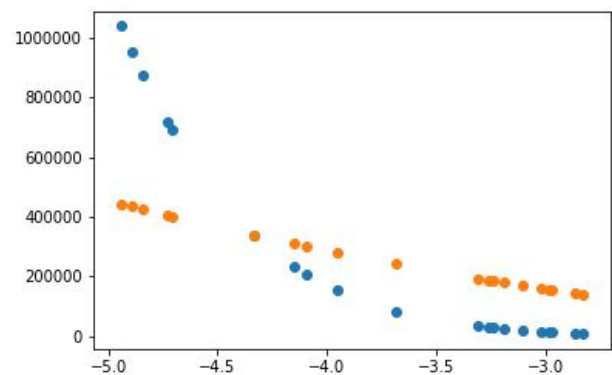
Regularization Parameter: We took **LAMBDA = 0.05**, due to the large error value. If we take greater lambda value, then it will add more bias.

$$\text{Cost Function} = 0.5 * (Y_TRUE - Y_PRED)^2 - \text{delta} * (Y_TRUE - Y_PRED) - 0.5 * \text{delta}^2 + \text{Lambda} * \text{Sigma}(\text{Theta})$$

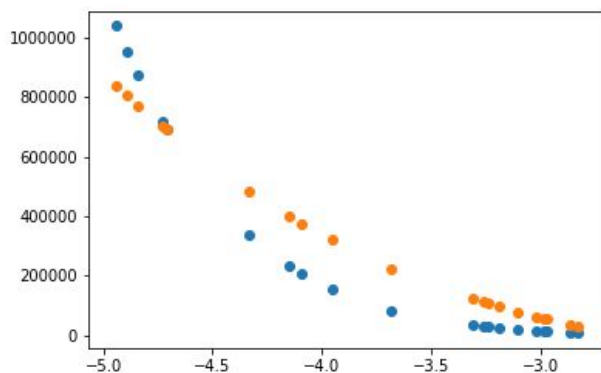
- Degree = 1, alpha = 0.01
- Noise variance = 1.161222955e-10



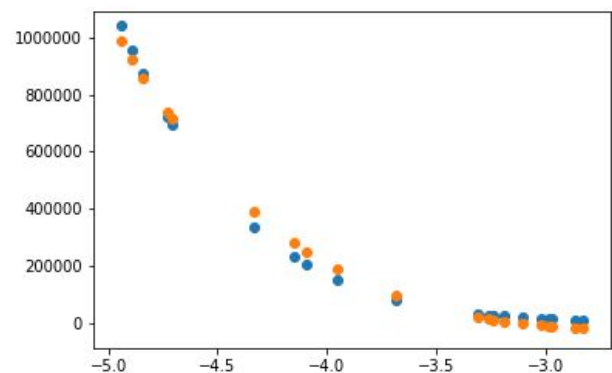
- Degree = 2, alpha = 0.01
- Noise variance = 2.092954732e-10



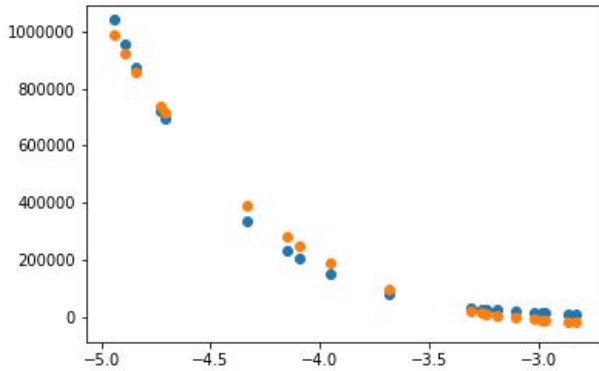
- Degree = 3, alpha = 0.01
- Noise variance = 1.007569989e-09



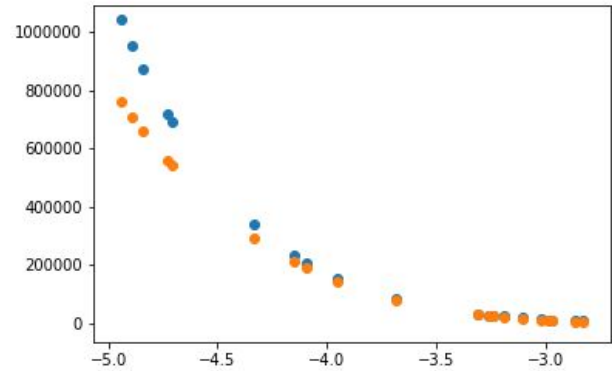
- Degree = 4, alpha = 0.001
- Noise variance = 4.60812061e-09



- Degree = 5, $\alpha = 0.0001$
- Noise variance = $9.902486995e-09$



- Degree = 6, $\alpha = 0.00001$
- Noise variance = $3.614904521e-09$



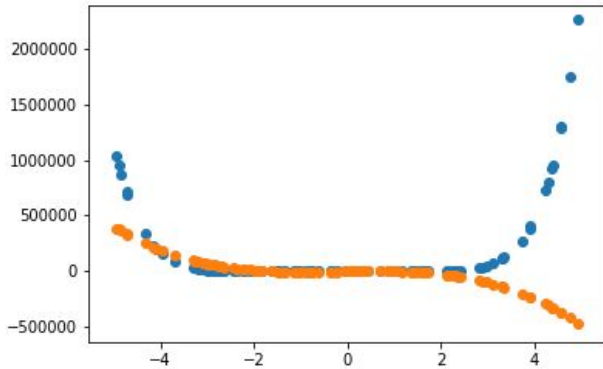
- Training data is 0 to 14 (-4.94 to -3.19) and testing data is from 15 to 19 (-3.10 to 2.83)
- The noise variance which we obtained without regularization is high compared to with regularization. SO we can confirm that regularization term helped the model to not become overfitted.

• RESULT For 1st 20 Points:-

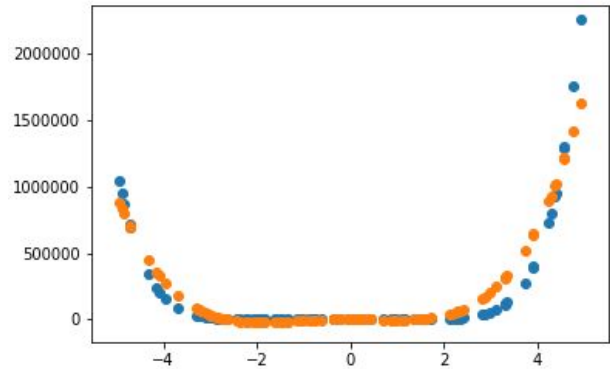
- Therefore 4rd Degree Polynomial is Best fit
- Underfit model is 1st degree(linear)
- Overfit model is having degree greater than equal 5

• Curve Fitting for All 100 Points:

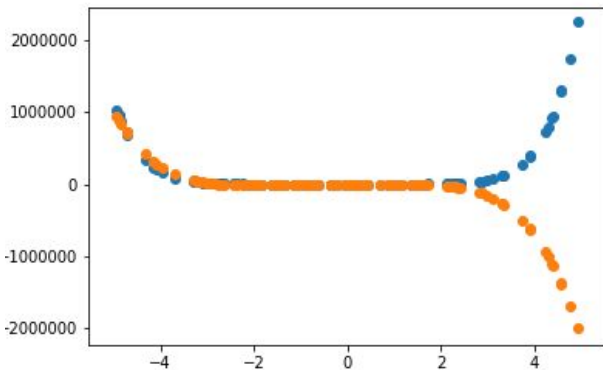
- Degree = 3, $\alpha = 0.001$
- Noise Variance = $9.553190821\text{e-}11$



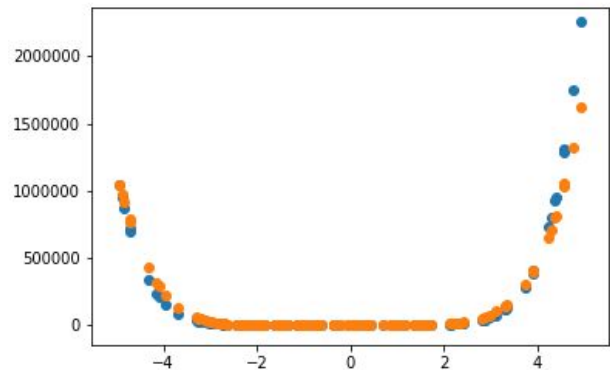
- Degree = 4, $\alpha = 0.001$
- Noise Variance = $9.678592684\text{e-}10$



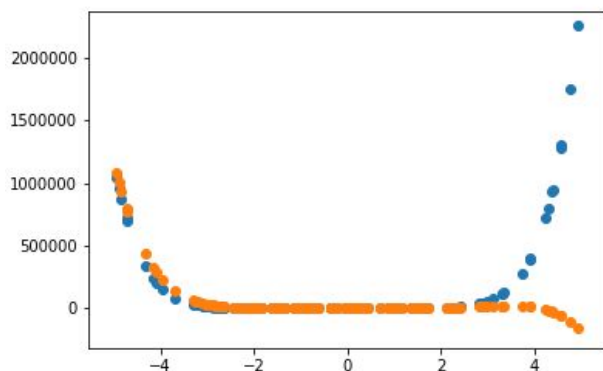
- Degree = 5, $\alpha = 0.0001$
- Noise Variance = $1.613132593\text{e-}09$



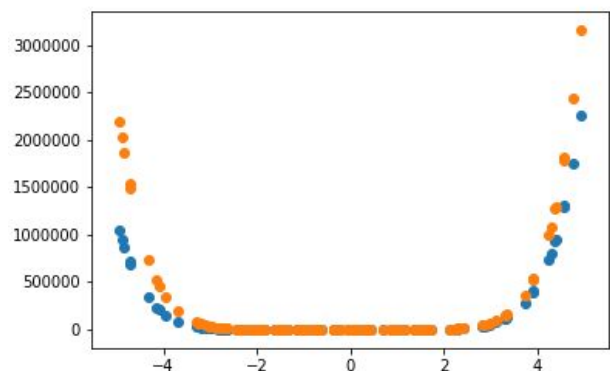
- Degree = 6, $\alpha = 0.00001$
- Noise Variance = $2.87540784\text{e-}09$



- Degree = 7, $\alpha = 0.00001$
- Noise Variance = $9.613132593\text{e-}08$

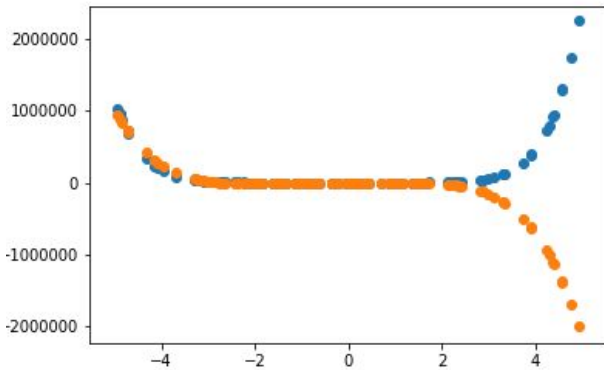


- Degree = 8, $\alpha = 0.00001$
- Noise Variance = $2.29424178\text{e-}08$

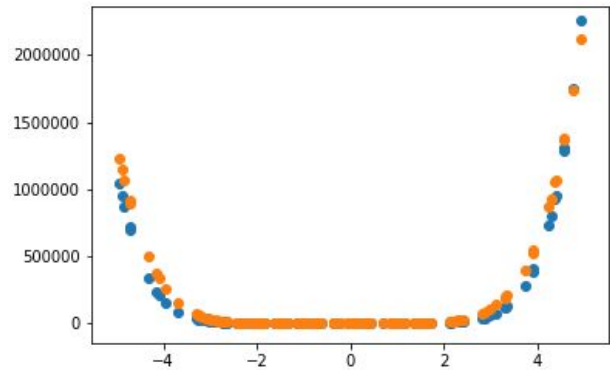


- **After Adding Regularization Parameter:**

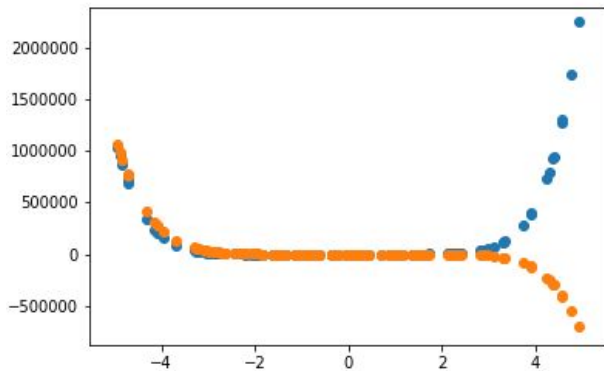
- Degree = 5, alpha = 0.0001
- Noise Variance = 0.993086395e-09



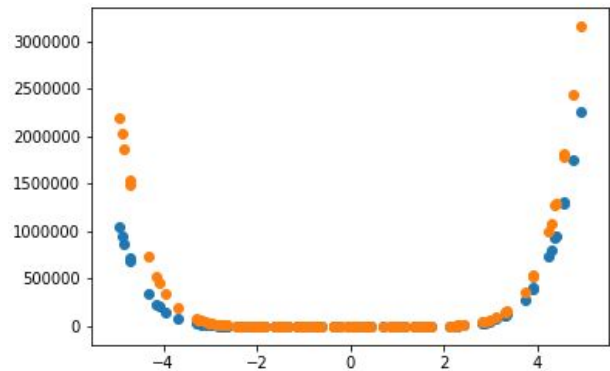
- Degree = 6, alpha = 0.00001
- Noise Variance = 2.01263485e-09



- Degree = 7, alpha = 0.00001
- Noise Variance = 7.057548211e-08



- Degree = 8, alpha = 0.00001
- Noise Variance = 1.437918596e-08



- Training data is 0 to 79 and testing data is from 80 to 99
- The noise variance which we obtained without regularization is high compared to with regularization. SO we can confirm that regularization term helped model to not become overfitted.

- **RESULT For All The Points:-**

- Therefore 6th Degree Polynomial is Best fit
- Underfit model is 1st to 3rd degree(linear)
- Overfit model is having degree greater than equal 7

-:SVM:-

Solution 2A

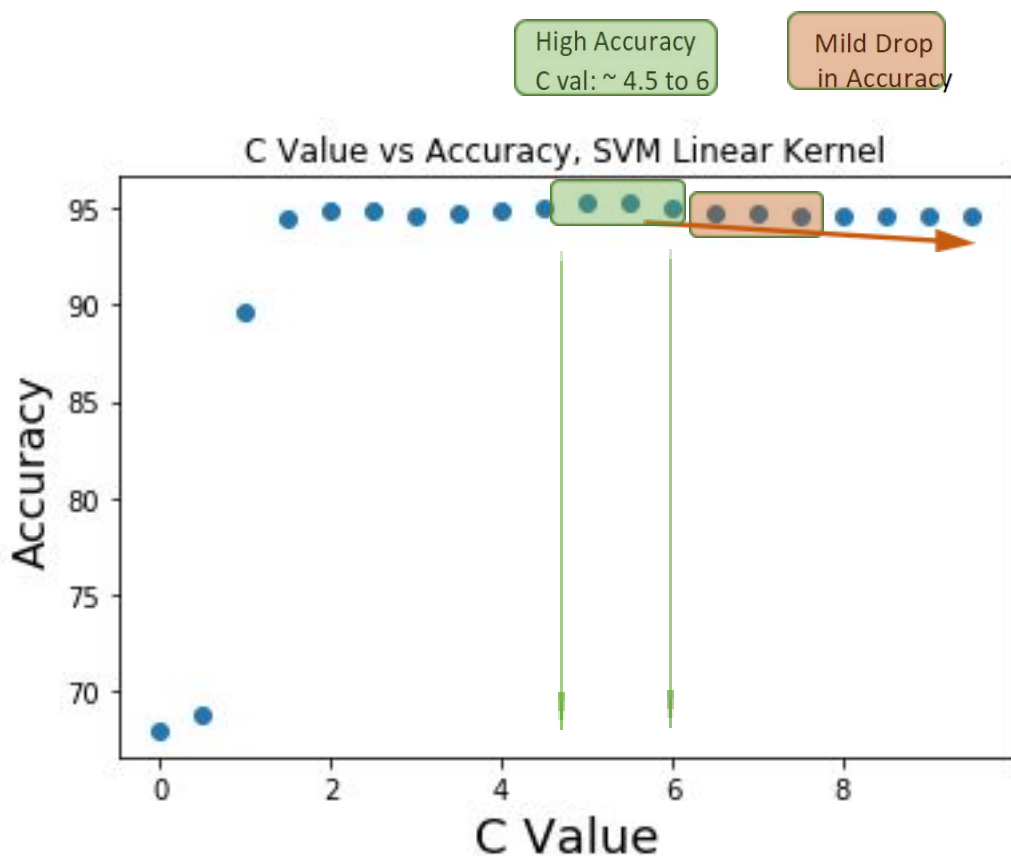
- For the first part of the question a Linear kernel is used.
- The 'libsvm' is installed using 'pip' command and the libsvm version used is 3.23.0.4
- The C Value is varied from $C > 0$, which is 0.01 till 10 in steps of 0.3:
C is picked from a set 0.01, 0.31, 0.61.... 9.91 (< 10)
- The number of iterations will be approximately $(9.91 - 0.01) / 0.3 \approx 33$
- This can be configured before running the application here :

Observation 1:

- High accuracy : 95.25423728813558, C : 5.01

- C = 5.01, Accuracy = 95.254%
- For C Values

```
start = 0.01  
stop = 10  
step = 0.5
```



Observation 2:

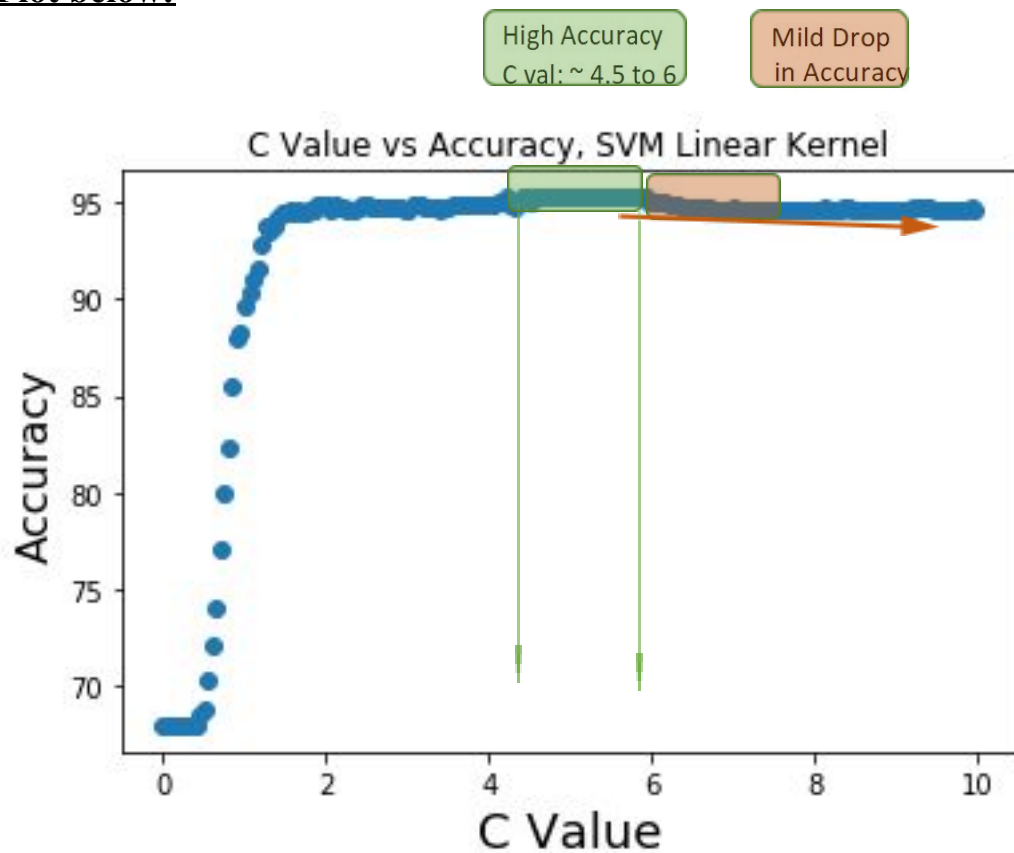
```
High accuracy : 95.25423728813558, C : 4.21
```

C = 4.21, Accuracy = 95.254%

For C values :

```
start = 0.01  
stop = 10  
step = 0.05
```

Plot below:



- The final predicted results are written to 'SVM_Linear.txt' that will be in the same folder. A result sheet for Observation 1 is attached here.
- **SVM_Linear.txt**

SOLUTION 2B:

- The SVM kernel used here is a Gaussian kernel with 5 fold cross validation.
- This is achieved by using both C and Gamma parameters of the libsvm.
- Gamma here is $1 / (2 (\sigma)^2)$

Plot for the final set of 5 fold cross validation is show below for :

- All the graphs are plotted together for C value vs Accuracy. The gamma vs Accuracy is explained on the same diagram.

Observation 1:

C Value Range:

```
cStart = 0.01  
cStop = 10  
cStep = 0.5
```

Gamma Range:

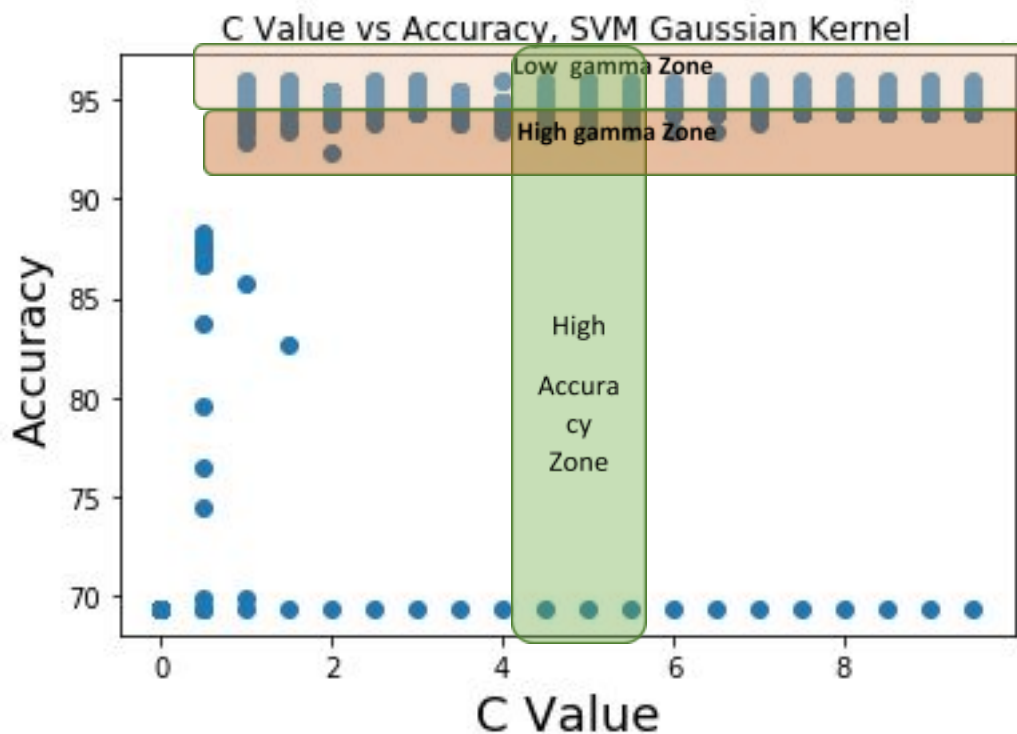
```
gammaStart = 0.01  
gammaStop = 10  
gammaStep = 0.5
```

Plot for C value vs Accuracy for :

Cross validation train set Fold {4,0, 1, 2} , and Test set Fold 3

Plot for other validation sets are also provided when the code is run.

```
(C_Val[4], Gamma[4], Accuracy[4]) = SVM_CV_Train(CV_Set_X_Train_4012, CV_Set_y_Train_4012, CV_Set_X_Test_3, CV_Set_y_Test_3, f0jbec
```



Cross validation accuracy with C Value and Gamma for 5 fold cross validation:

```
Cross Validation C_value : [4.01 7.01 9.51 7.51 4.01]
Cross Validation Gamma   : [0.51 0.51 0.51 2.51 1.01]
Cross Validation Accuracy : [94.89795918 98.46938776 93.87755102 94.3877551 95.91836735]
Cross Validation Accuracy mean : 95.51020408163268
Begin SVM with 5 fold cross validation hyper paramter MEAN data for C, Gamma: 6.409999999999999
1.01
```

C value 6.40, Gamma value 1.01 => Sigma value = 0.7035

Accuracy = 95.510

- Accuracy is slightly improved compared to Linear kernel

Observation 2:

For C Value range:

```
cStart = 0.01
cStop = 10
cStep = 1
```

For Gamma value range:

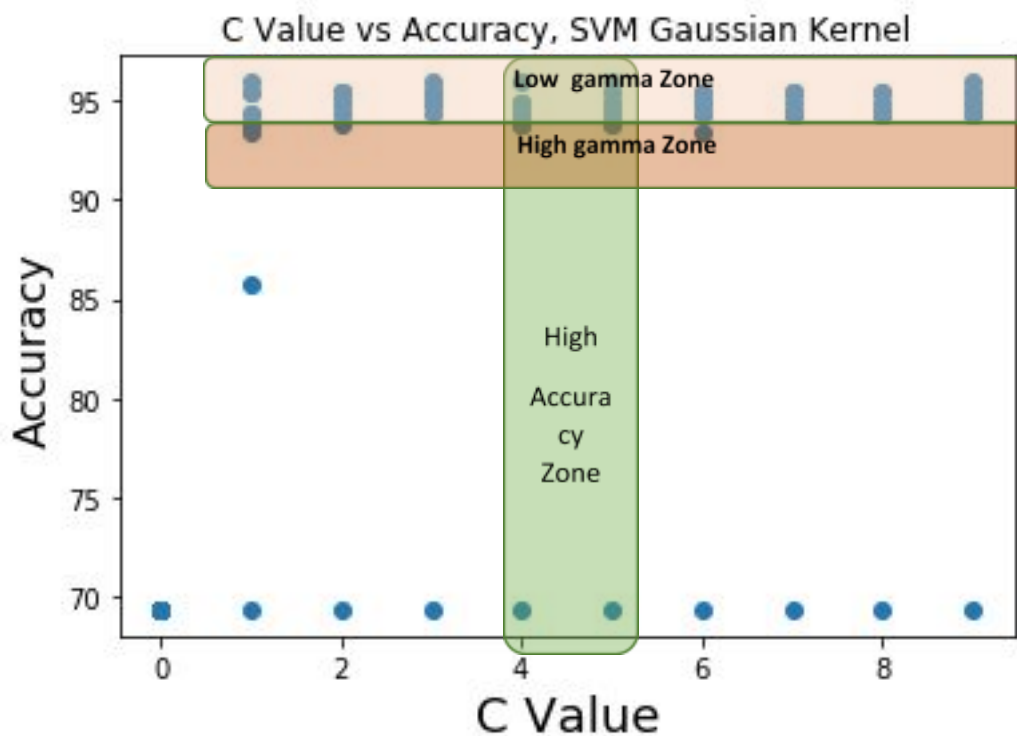
```
gammaStart = 0.01
gammaStop = 10
gammaStep = 1
```

Plot for C value vs Accuracy for :

Cross validation train set Fold {4,0, 1, 2} , and Test set Fold 3

```
(C_Val[4], Gamma[4], Accuracy[4]) = SVM_CV_Train(CV_Set_X_Train_4012, CV_Set_y_Train_4012, CV_Set_X_Test_3, CV_Set_y_Test_3, f0jbec
```

Plot for other validation sets are also provided when the code is run:



C value 5.20, Gamma value 1.01 => Sigame value = 0.7035

Accuracy : 95.102

```
Cross Validation C_value : [8.01 2.01 6.01 6.01 4.01]
Cross Validation Gamma   : [5.01 1.01 1.01 1.01 1.01]
Cross Validation Accuracy : [94.89795918 97.44897959 93.36734694 93.87755102 95.91836735]
Cross Validation Accuracy mean : 95.10204081632654
Begin SVM with 5 fold cross validation hyper paramter MEAN data for C, Gamma: 5.209999999999999 1.8099999999999998
```

- The predicted values are printed to SVM_Gaussian.txt.
- A sample for observation 2 is uploaded here.
- **SVM_Gaussian.txt**