Viterbi algorithm

By Suneesh Jacob

Problem:

Transition probabilities:

$$T = \begin{array}{c c} & \begin{array}{cc} Rainy & Sunny \end{array} \\ \begin{array}{c} Rainy \\ Sunny \end{array} & \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix} \end{array}$$

Emission probabilities:

$$E = \begin{array}{c c} & \begin{array}{ccc} Walk & Shop & Clean \end{array} \\ \begin{array}{c} Rainy \\ Sunny \end{array} & \begin{bmatrix} 0.1 & 0.4 & 0.5 \\ 0.6 & 0.3 & 0.1 \end{bmatrix} \end{array}$$

Initial probabilities:

$$\begin{array}{c c} & \begin{array}{cc} Rainy & Sunny \end{array} \\ \pi = & \begin{bmatrix} 0.6 & 0.4 \end{bmatrix} \end{array}$$

Observed states for three consecutive days:

$$[Walk, Shop, Clean]$$

Hidden states for these three days = ?

Solution:

The observed states for the three consecutive days are

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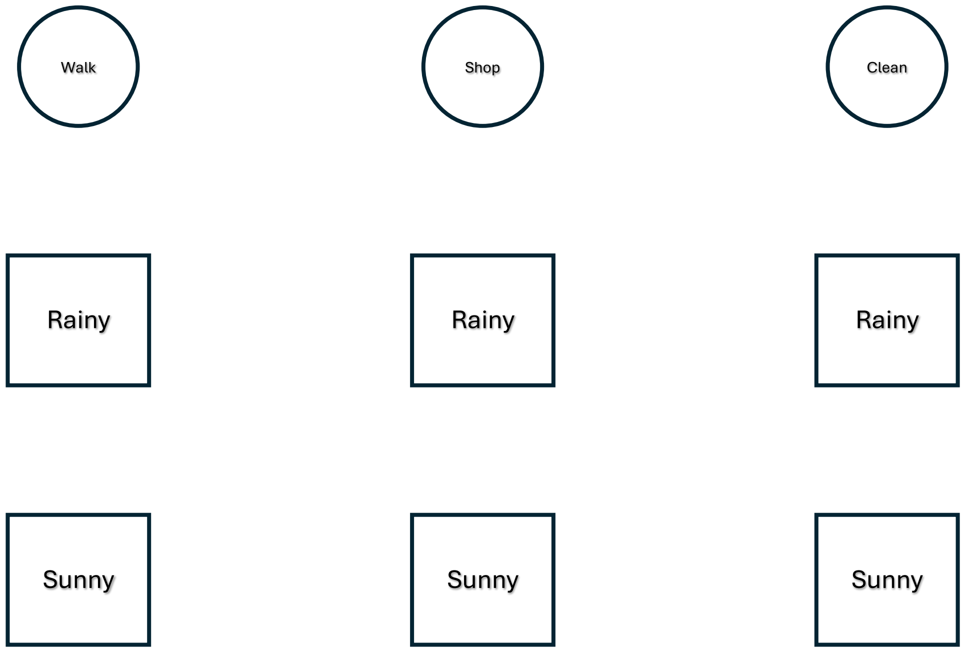
We first list both of them under each day's observed state (as shown in figure).



Solution:

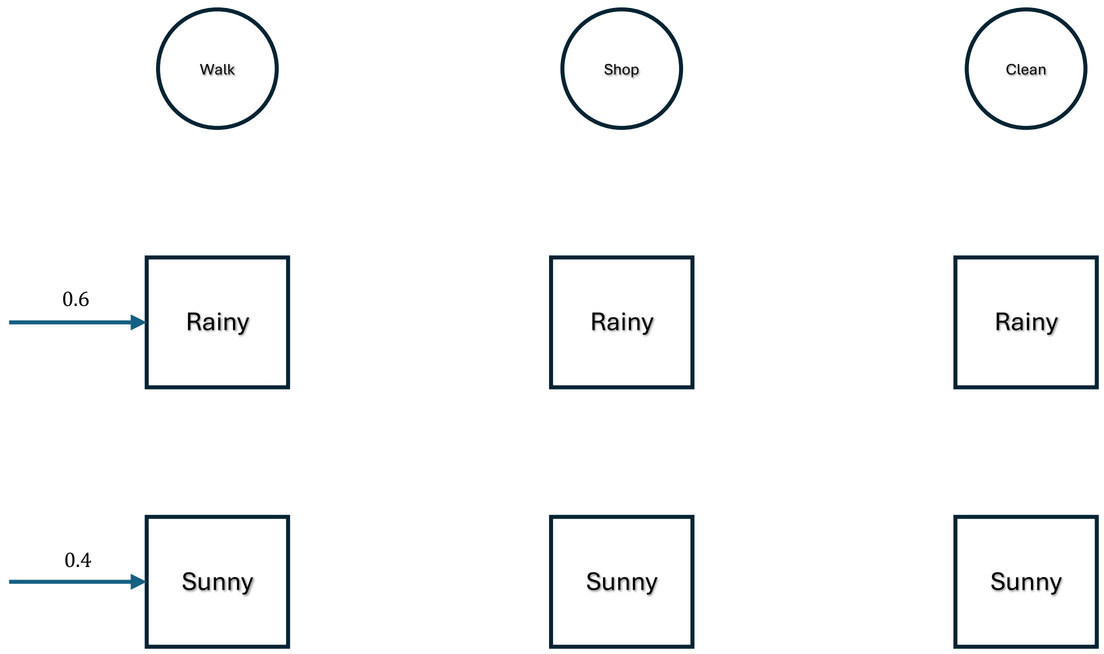
The observed states for the three consecutive days are

$$[Walk, Shop, Clean]$$

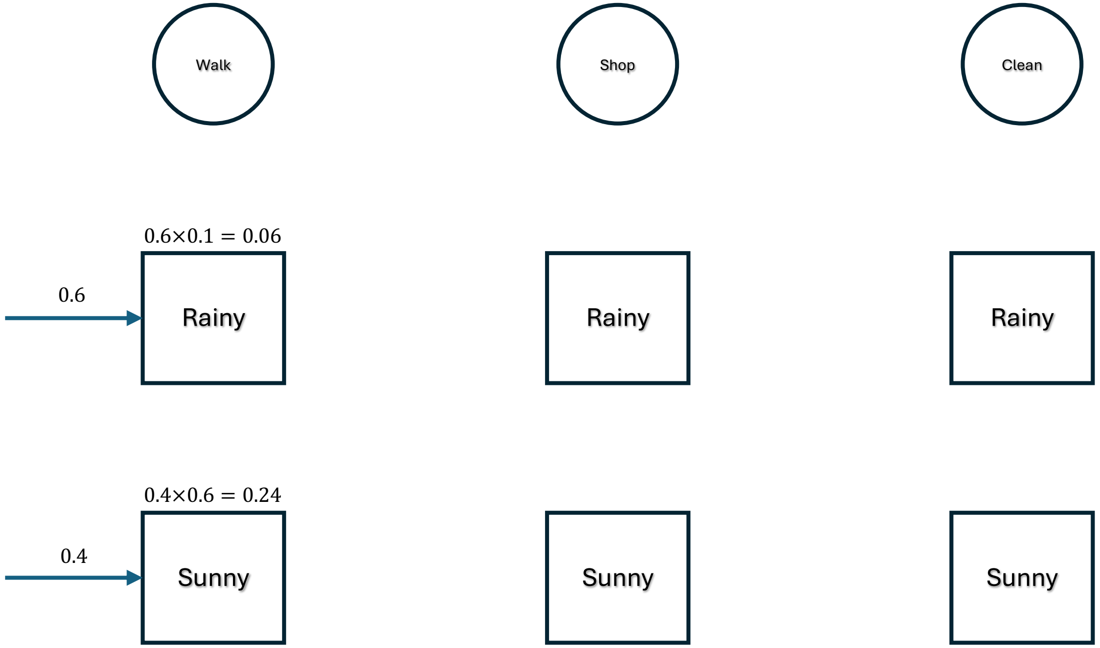
The hidden states are not known, but they must be among ‘Rainy’ and ‘Sunny’.

We first list both of them under each day’s observed state (as shown in figure).

The initial probabilities are 0.6 and 0.4 for Rainy and Sunny, respectively.

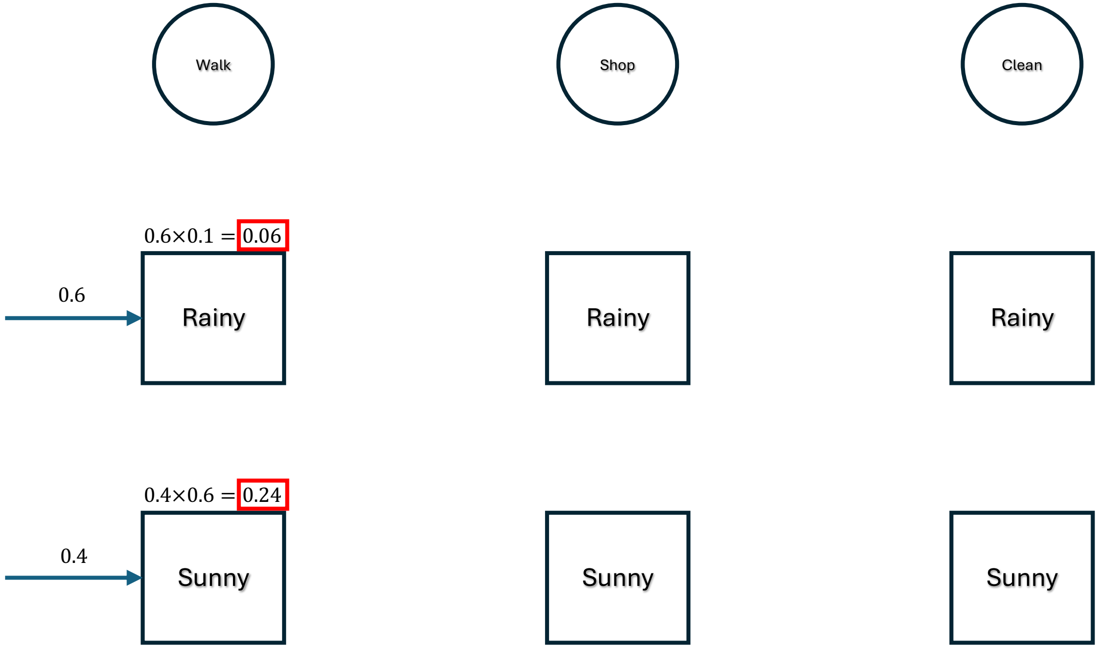


The emission probabilities for ‘Walk’ are 0.1 and 0.6 for ‘Rainy’ and ‘Sunny’, respectively.



The emission probabilities for ‘Walk’ are 0.1 and 0.6 for ‘Rainy’ and ‘Sunny’, respectively.

Since there is only one probability for each hidden state, we choose this *one* probability for each of the hidden states.



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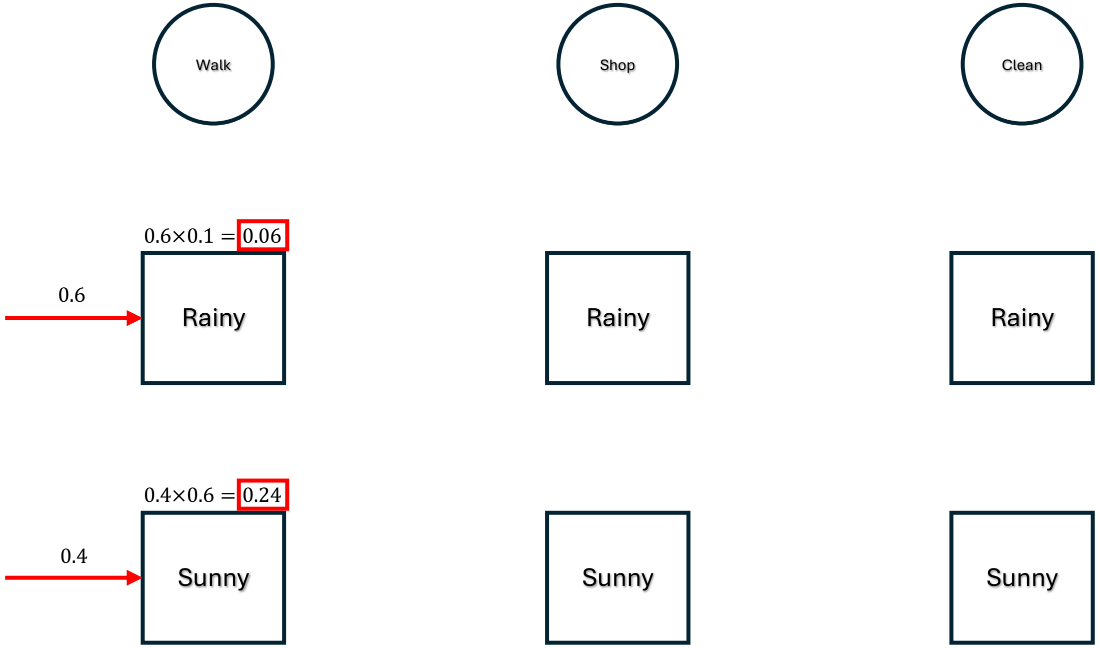
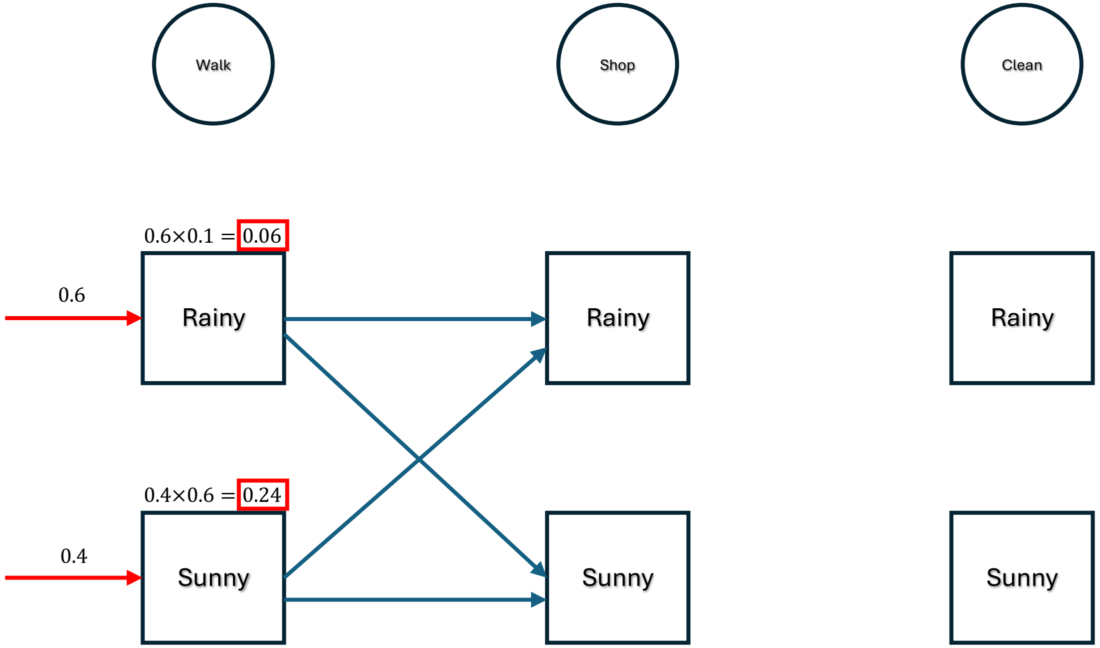
Hence, we also mark both the paths.

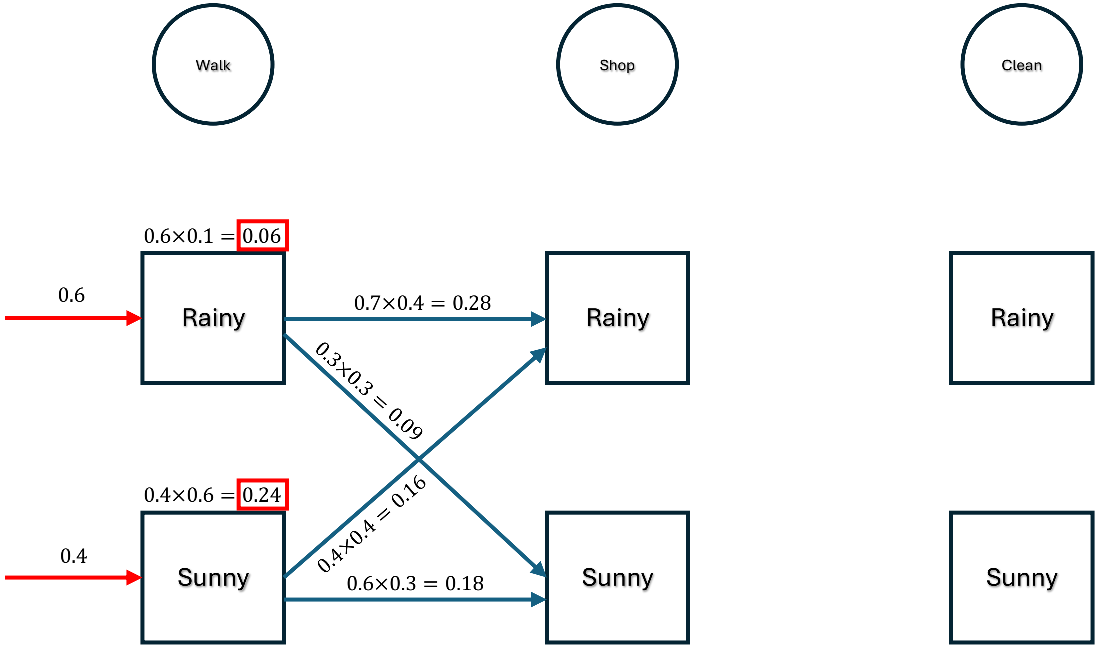


The next day's observation is 'Shop', and its emission probabilities are 0.4 and 0.3 for Rainy and Sunny, respectively.



The next day’s observation is ‘Shop’, and its emission probabilities are 0.4 and 0.3 for Rainy and Sunny, respectively.

Now, from first day to second day, the transition possibilities (along with their respective probabilities) are multiplied with the emission probabilities of the second day’s observation (i.e., shop).



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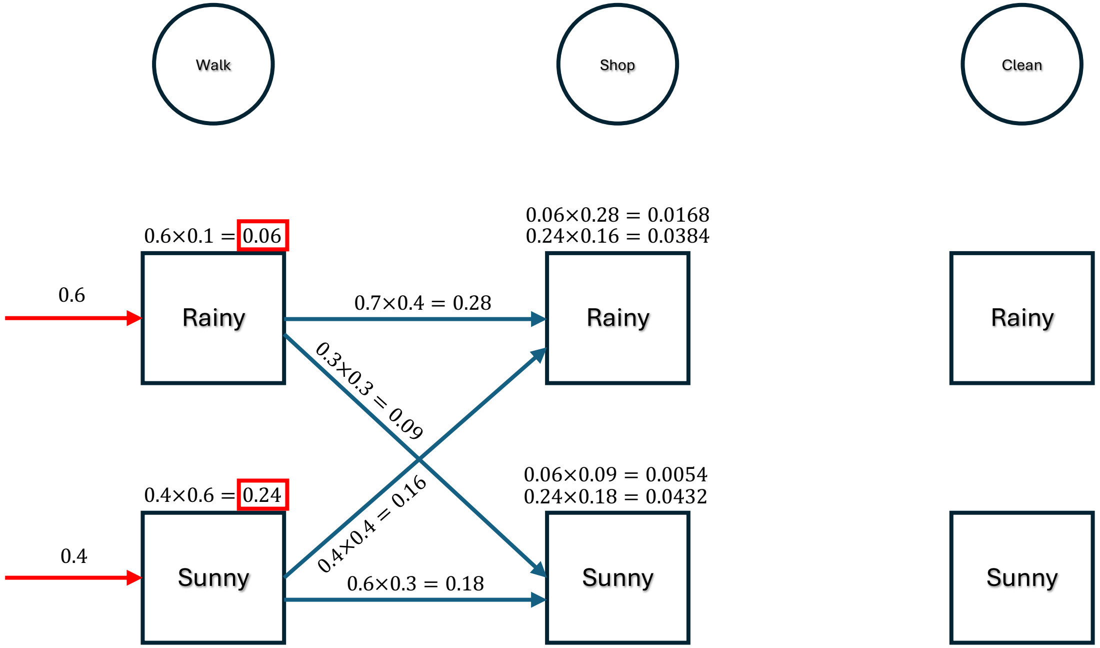
The probabilities of these paths till the second day are computed and shown.



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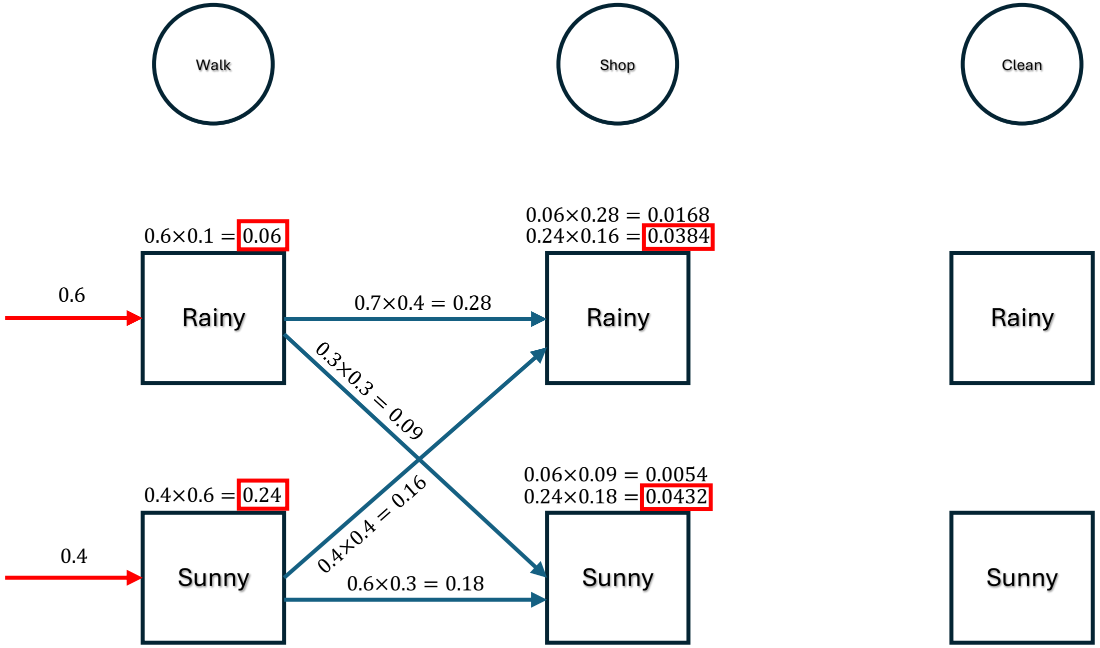

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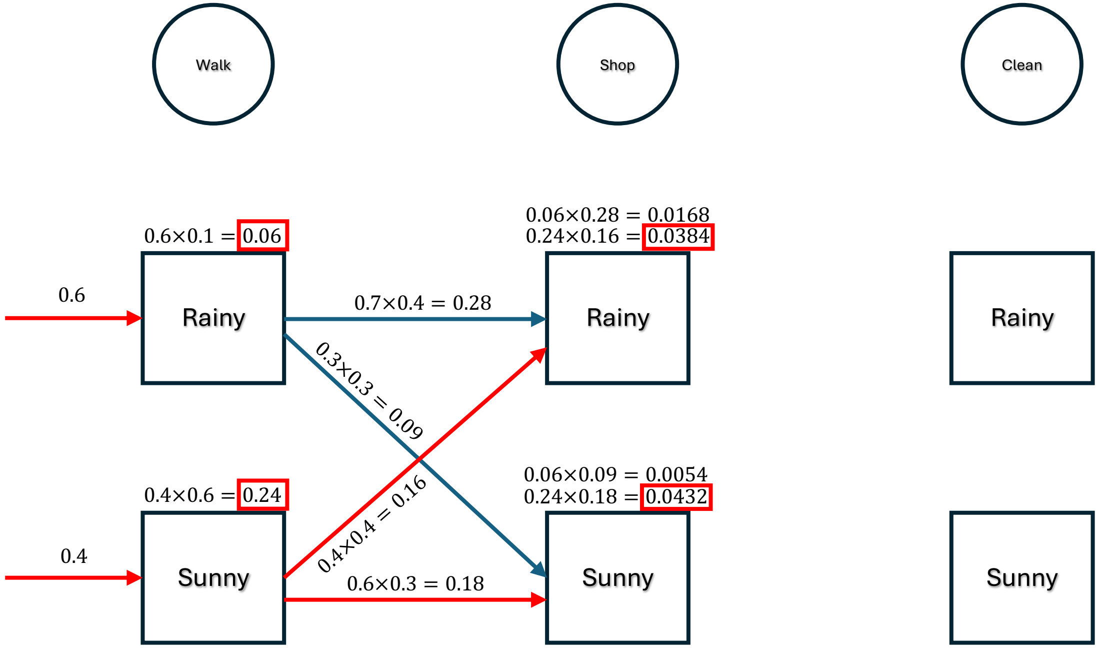
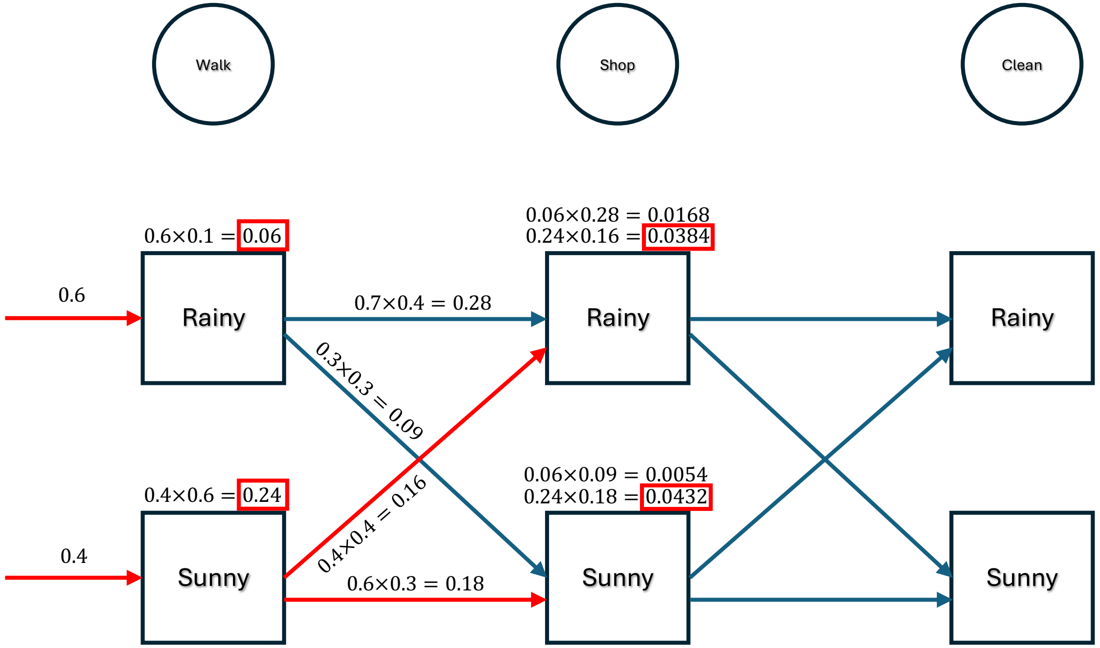
The corresponding paths are chosen.



The next day's observation is 'Clean', and its emission probabilities are 0.5 and 0.1 for Rainy and Sunny, respectively.



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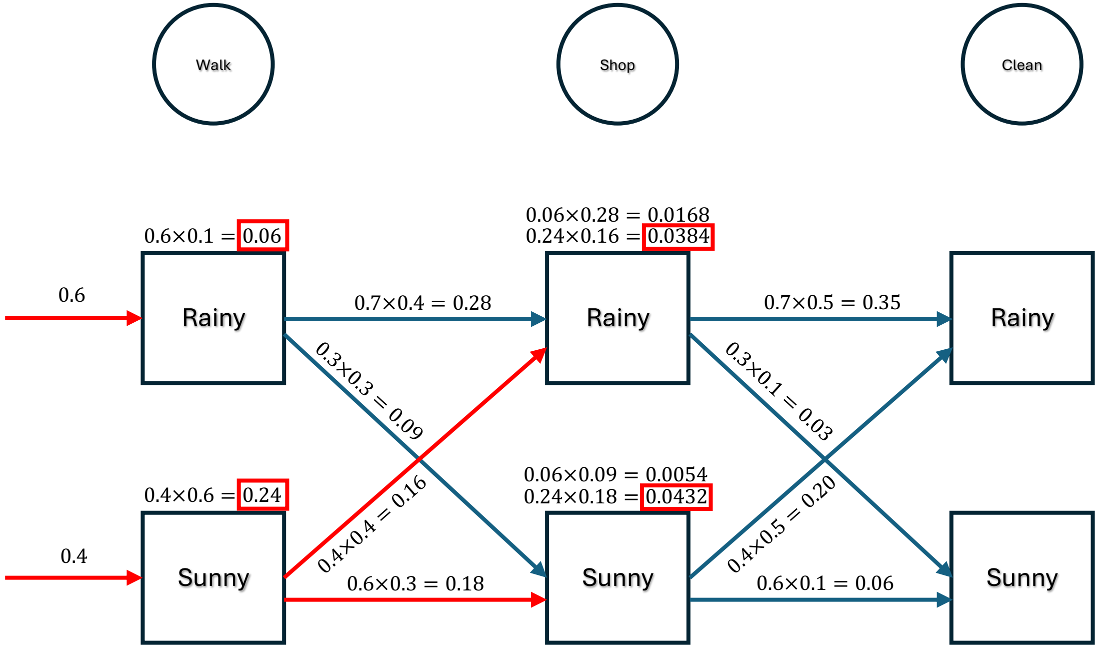

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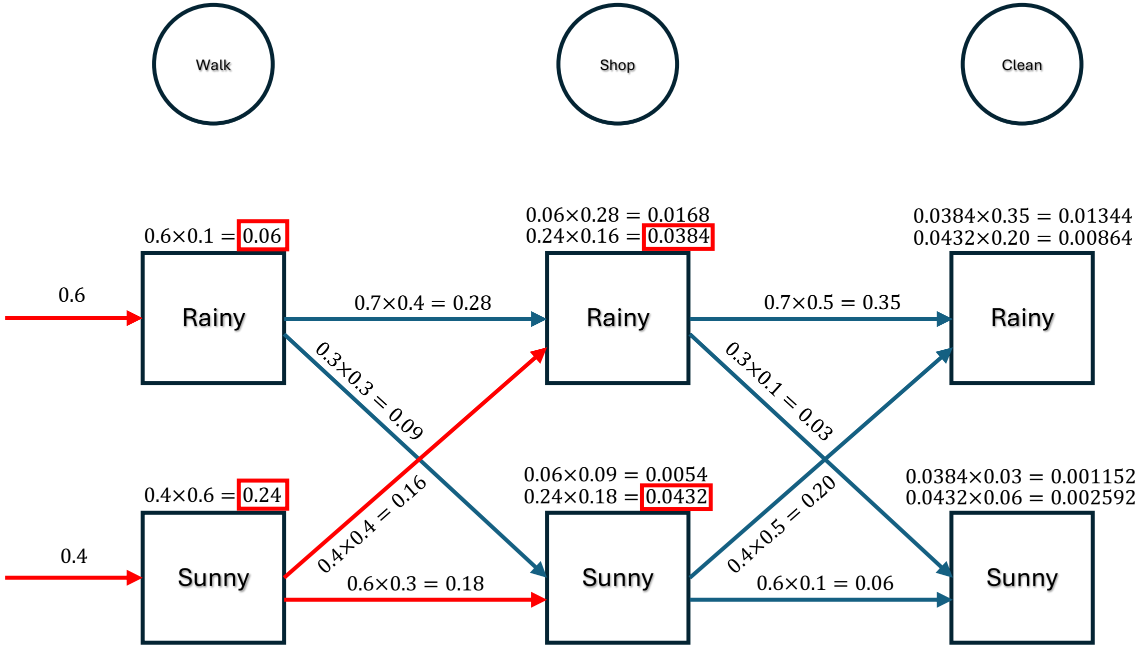

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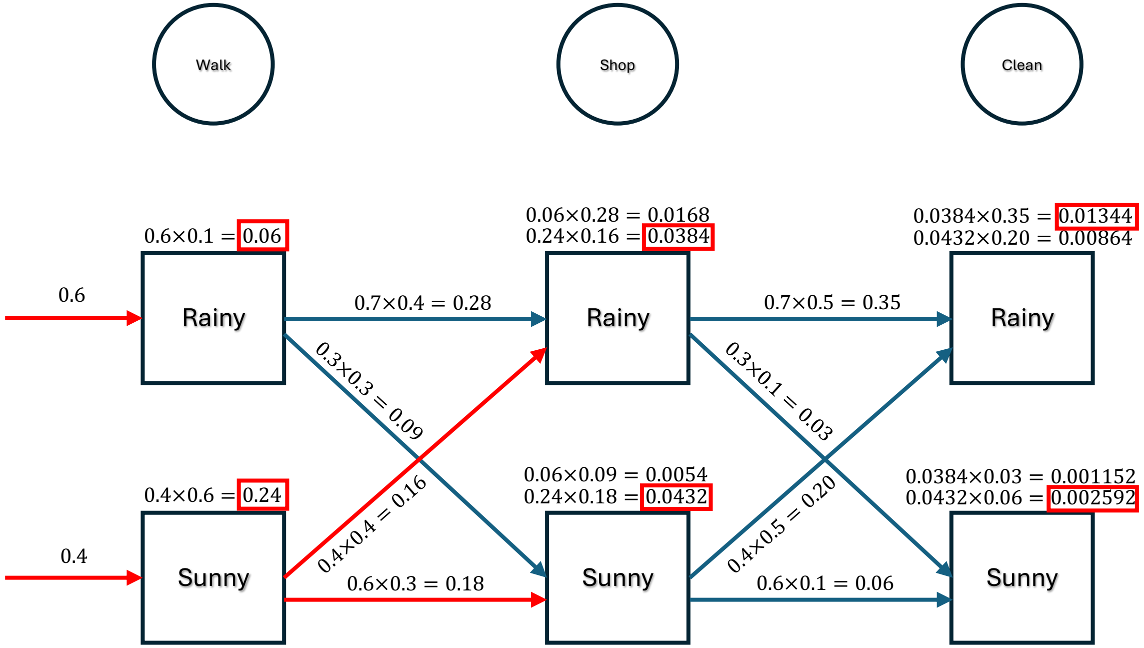

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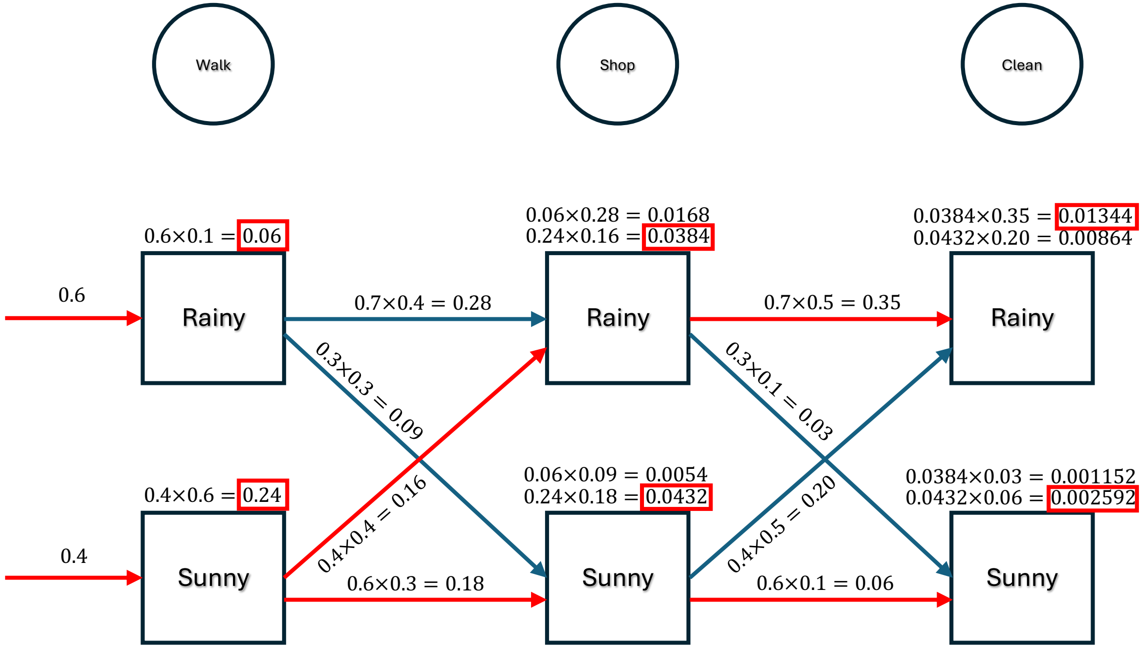
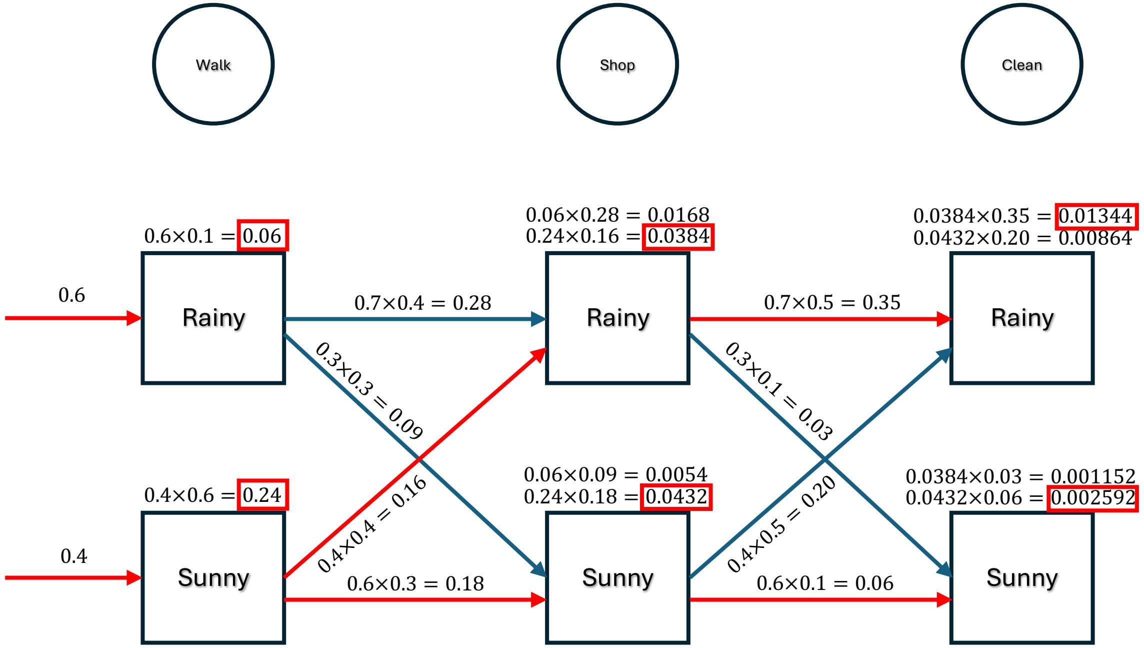
The corresponding paths are chosen.

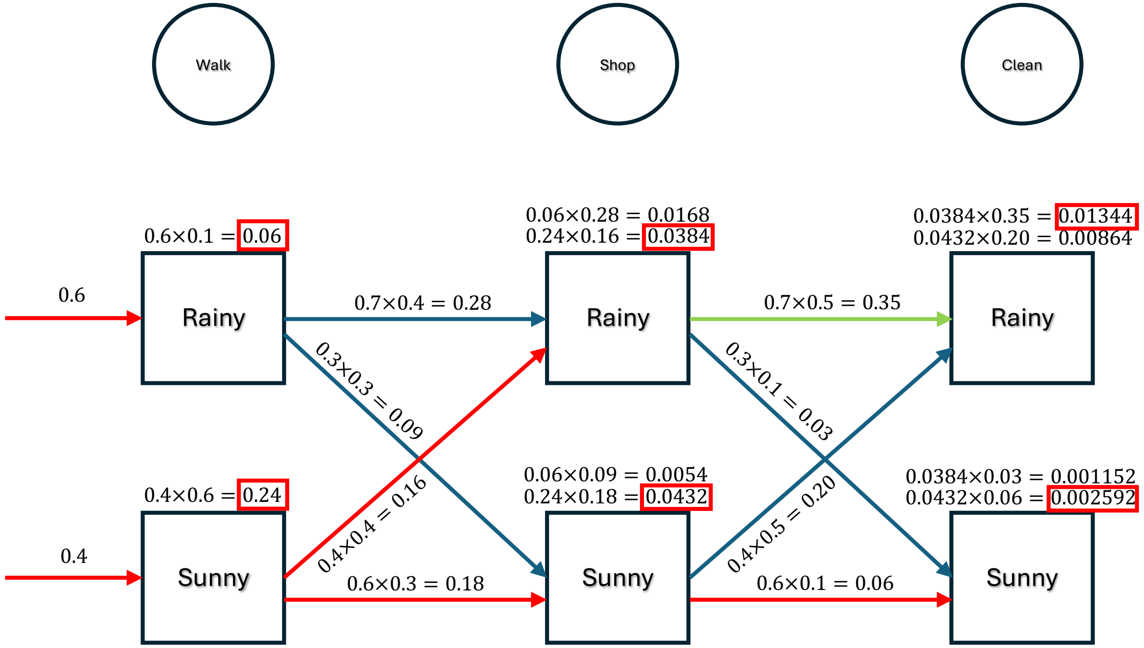


Now that we have computed all the probabilities, we will do backward recursion to find the sequence of. hidden states



Now that we have computed all the probabilities, we will do backward recursion to find the sequence of. hidden states

For the last day (third day), the two (already chosen) probabilities are 0.01344 and 0.00864 for 'Rainy' and 'Sunny', respectively. The maximum of these probabilities is 0.01344 which corresponds to 'Rainy'. Hence the hidden state for the last day is 'Rainy'. Hence, the red arrow pointing towards 'Rainy' of third day is chosen (shown in green colour).



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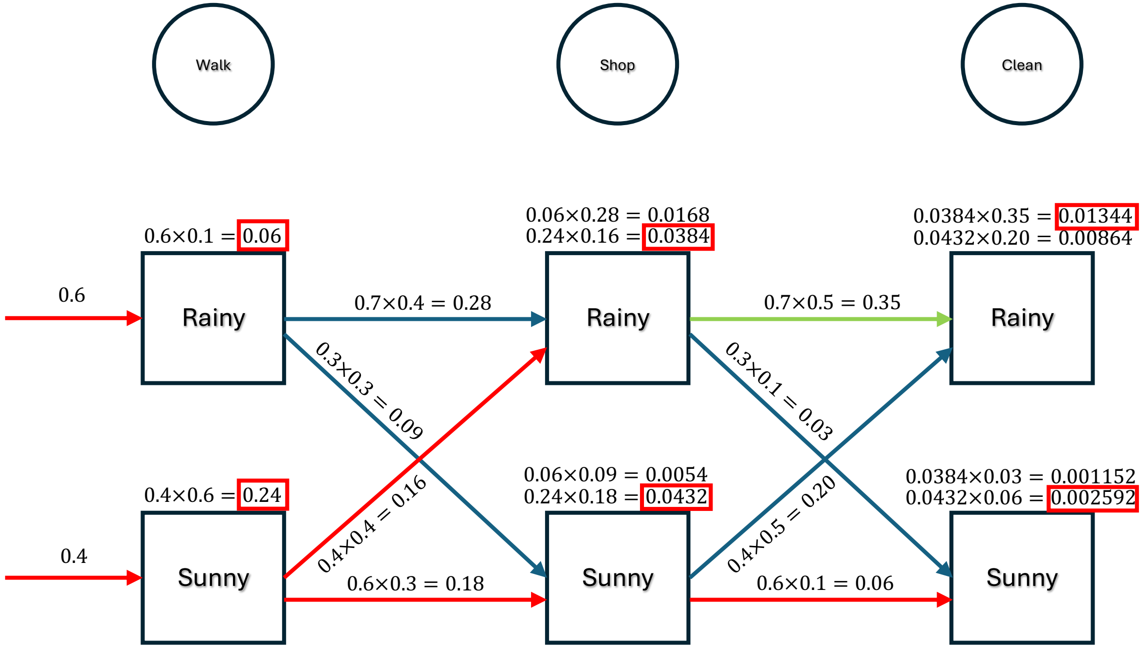

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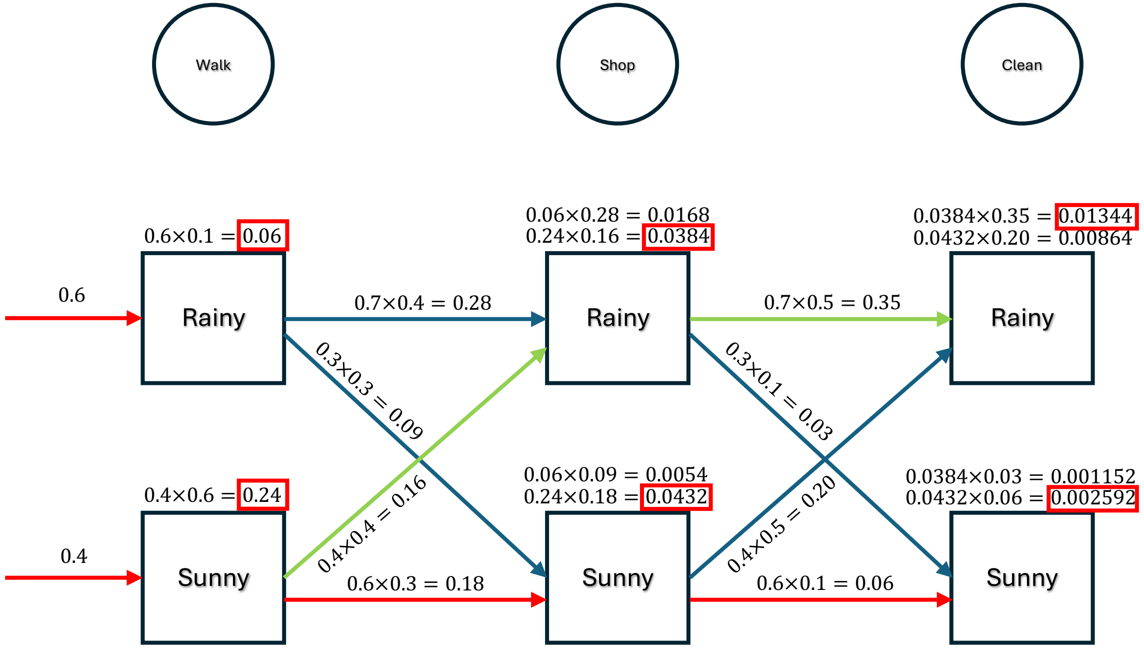

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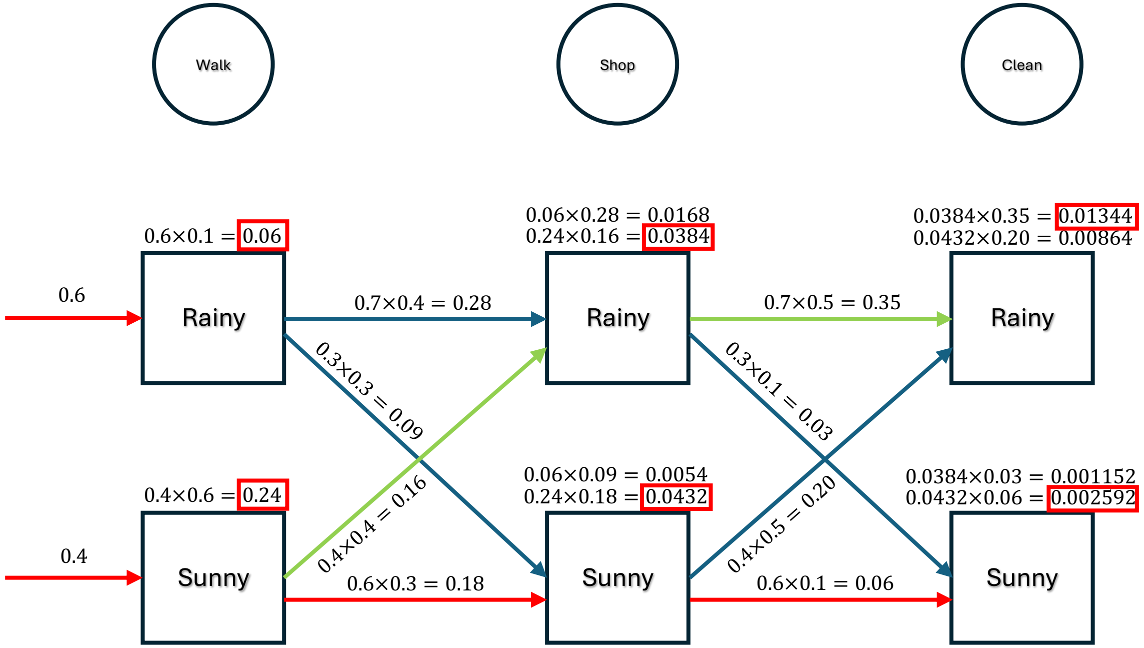
Again, this new arrow starts from ‘Sunny’ of the first day.



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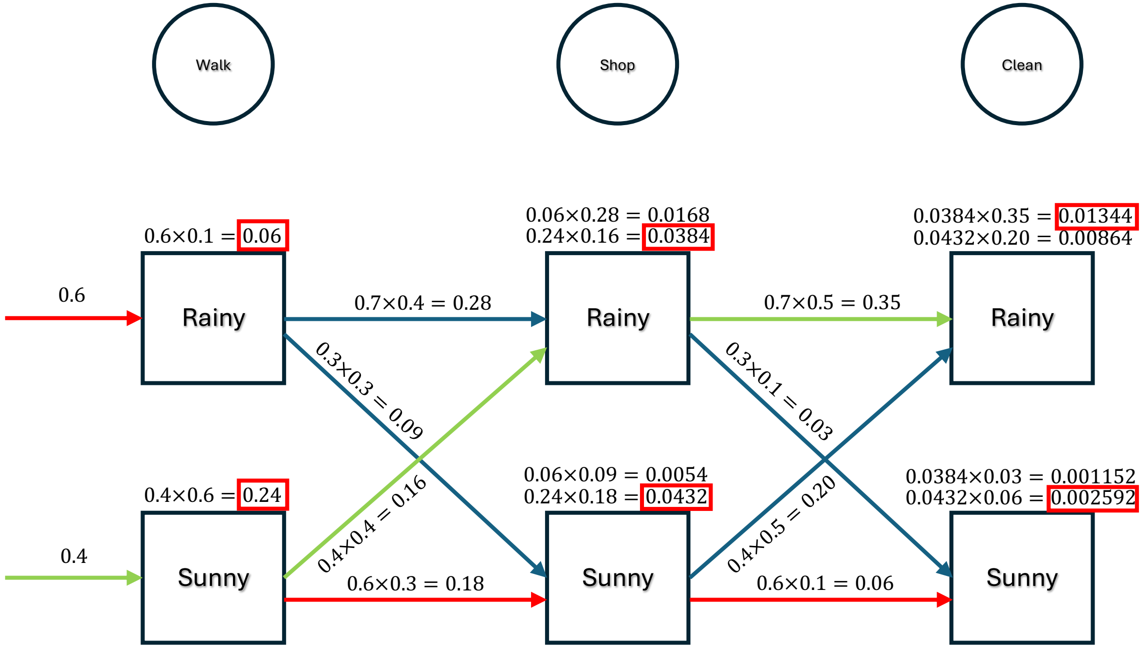
Again, this new arrow starts from ‘Sunny’ of the first day. Hence, the hidden state for the first day is ‘Sunny’.



Hence, the hidden state sequence is

[Sunny, Rainy, Rainy]