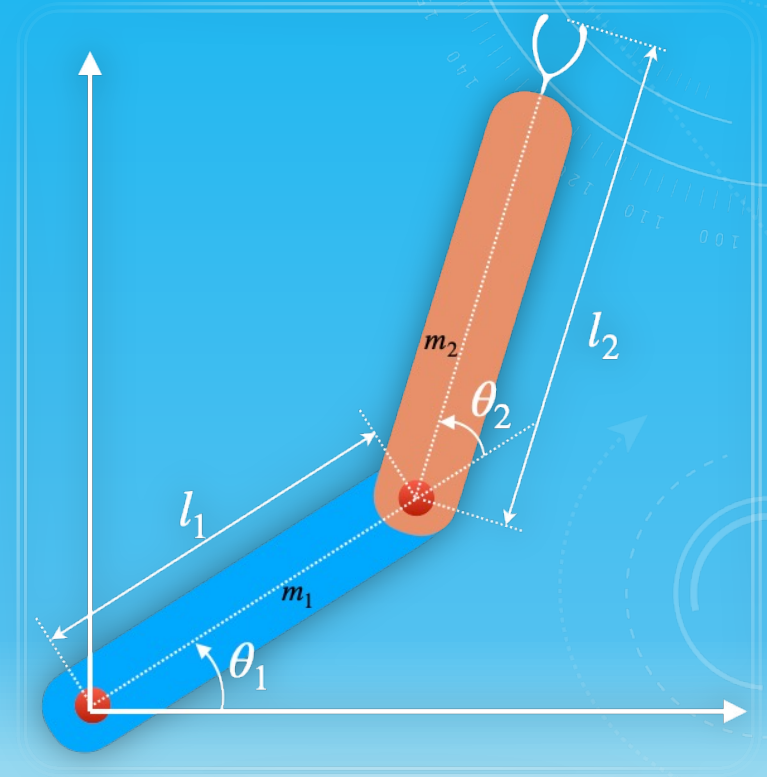
The background is a solid blue color with a subtle, light blue graphic overlay. This graphic consists of several concentric circular arcs and dashed lines, some of which are marked with degree values such as 150, 160, 170, 180, 190, 200, 210, 220, 230, 240, 250, and 260. There are also small arrows indicating a clockwise direction of movement or rotation.

ROBOTICS (OPTIMAL CONTROL)

SUNEESH JACOB A.

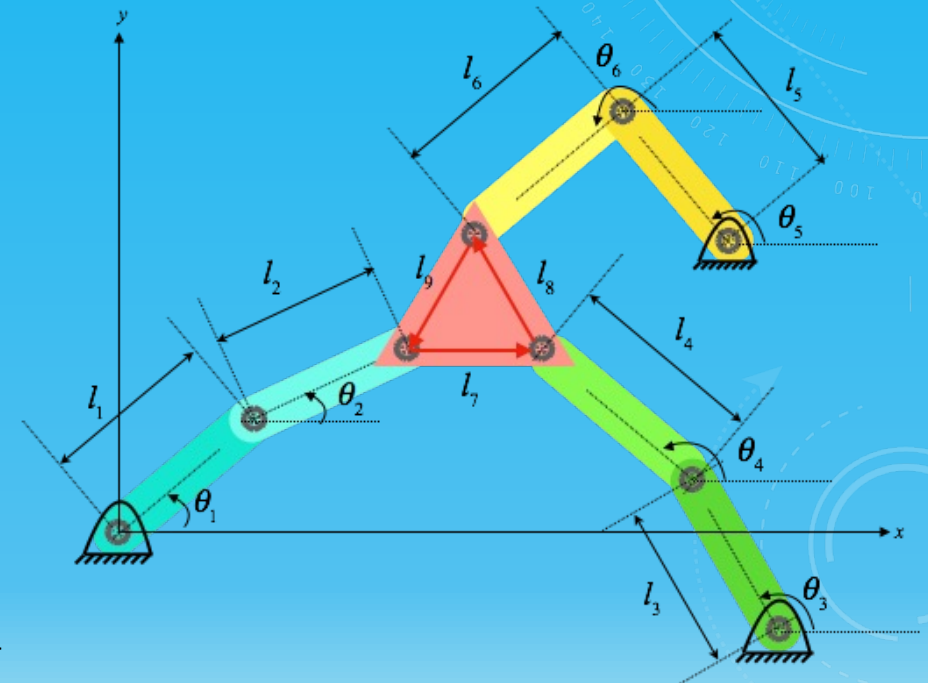
TWO-LINK ROBOT KINEMATICS

- $$\vec{p} = \begin{Bmatrix} l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2) \\ l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2) \end{Bmatrix}$$



EIGHT-LINK PARALLEL ROBOT KINEMATICS

- $\vec{p}_1 = \begin{Bmatrix} l_1 \cos(\theta_1) + l_2 \cos(\theta_2) \\ l_1 \sin(\theta_1) + l_2 \sin(\theta_2) \end{Bmatrix}$
- $\vec{p}_2 = \begin{Bmatrix} l_3 \cos(\theta_3) + l_4 \cos(\theta_4) \\ l_3 \sin(\theta_3) + l_4 \sin(\theta_4) \end{Bmatrix}$
- $\vec{p}_3 = \begin{Bmatrix} l_5 \cos(\theta_5) + l_6 \cos(\theta_6) \\ l_5 \sin(\theta_5) + l_6 \sin(\theta_6) \end{Bmatrix}$



- $\vec{o}_2 = \vec{o}_1 + \begin{Bmatrix} l_1 \cos(\theta_1) + l_2 \cos(\theta_2) + l_7 \cos(\theta_7) - l_4 \cos(\theta_4) - l_3 \cos(\theta_3) \\ l_1 \sin(\theta_1) + l_2 \sin(\theta_2) + l_7 \sin(\theta_7) - l_4 \sin(\theta_4) - l_3 \sin(\theta_3) \end{Bmatrix}$
- $\vec{o}_3 = \vec{o}_2 + \begin{Bmatrix} l_3 \cos(\theta_3) + l_4 \cos(\theta_4) + l_8 \cos(\theta_8) - l_6 \cos(\theta_6) - l_5 \cos(\theta_5) \\ l_3 \sin(\theta_3) + l_4 \sin(\theta_4) + l_8 \sin(\theta_8) - l_6 \sin(\theta_6) - l_5 \sin(\theta_5) \end{Bmatrix}$
- $\vec{o}_1 = \vec{o}_3 + \begin{Bmatrix} l_5 \cos(\theta_5) + l_6 \cos(\theta_6) + l_9 \cos(\theta_9) - l_2 \cos(\theta_2) - l_1 \cos(\theta_1) \\ l_5 \sin(\theta_5) + l_6 \sin(\theta_6) + l_9 \sin(\theta_9) - l_2 \sin(\theta_2) - l_1 \sin(\theta_1) \end{Bmatrix}$

WHAT IS MOTOR CONTROL

Control: Computing appropriate torque to bring the robot's actuating joint from one state to another state

Drive: Converting the small current/voltage signals to real-time (large) signals that can be fed to motor

WHY DO WE NEED MOTOR CONTROL

- Input \rightarrow force
- Output \rightarrow state (position and velocity)

DYNAMICS

- Dynamic equations

TWO-LINK ROBOT DYNAMICS

- $\vec{p}_1 = \begin{Bmatrix} \frac{l_1}{2} \cos(\theta_1) \\ \frac{l_1}{2} \sin(\theta_1) \end{Bmatrix}$

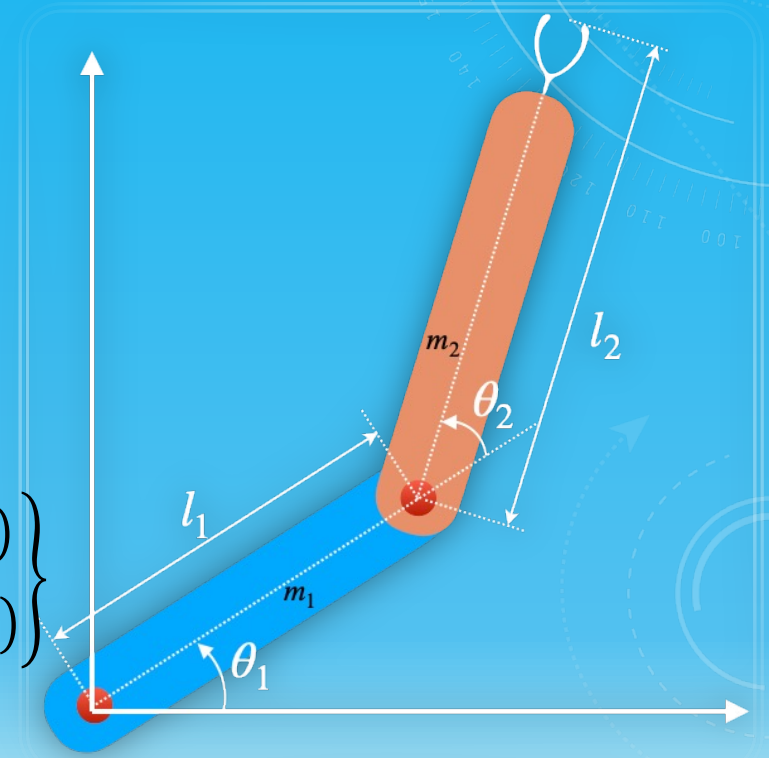
- $\vec{p}_2 = \begin{Bmatrix} l_1 \cos(\theta_1) + \frac{l_2}{2} \cos(\theta_1 + \theta_2) \\ l_1 \sin(\theta_1) + \frac{l_2}{2} \sin(\theta_1 + \theta_2) \end{Bmatrix}$

- $\vec{v}_1 = \begin{Bmatrix} -\frac{l_1}{2} \sin(\theta_1) \dot{\theta}_1 \\ \frac{l_1}{2} \cos(\theta_1) \dot{\theta}_1 \end{Bmatrix}$

- $\vec{v}_2 = \begin{Bmatrix} -l_1 \sin(\theta_1) \dot{\theta}_1 - \frac{l_2}{2} \sin(\theta_1 + \theta_2) (\dot{\theta}_1 + \dot{\theta}_2) \\ -l_1 \cos(\theta_1) \dot{\theta}_1 + \frac{l_2}{2} \cos(\theta_1 + \theta_2) (\dot{\theta}_1 + \dot{\theta}_2) \end{Bmatrix}$

- $\omega_1 = \dot{\theta}_1$

- $\omega_2 = \dot{\theta}_1 + \dot{\theta}_2$



TWO-LINK ROBOT DYNAMICS

- $\vec{p}_1 = \begin{Bmatrix} \frac{l_1}{2} \cos(\theta_1) \\ \frac{l_1}{2} \sin(\theta_1) \\ 0 \end{Bmatrix}$

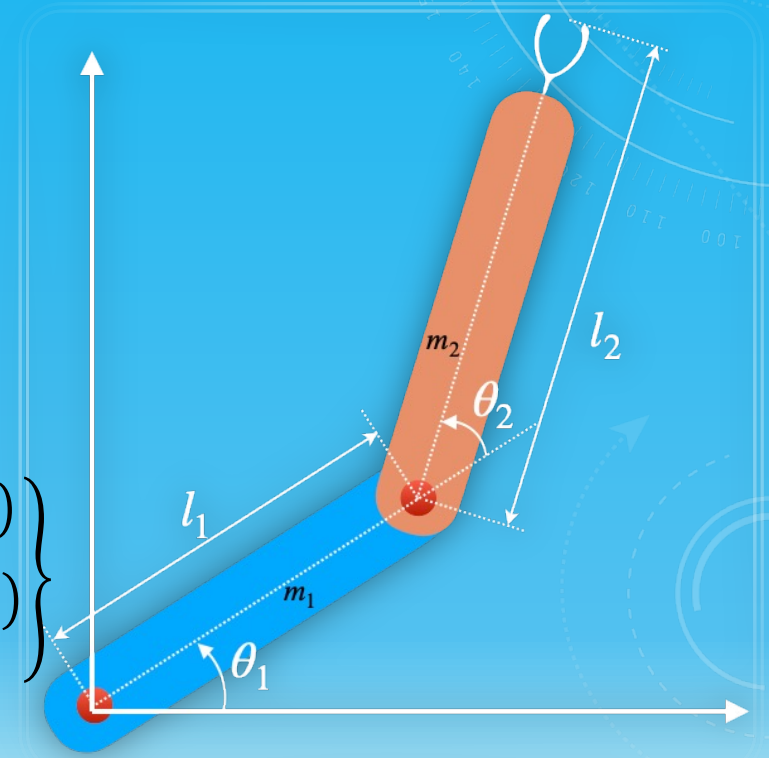
- $\vec{p}_2 = \begin{Bmatrix} l_1 \cos(\theta_1) + \frac{l_2}{2} \cos(\theta_1 + \theta_2) \\ l_1 \sin(\theta_1) + \frac{l_2}{2} \sin(\theta_1 + \theta_2) \\ 0 \end{Bmatrix}$

- $\vec{v}_1 = \begin{Bmatrix} -\frac{l_1}{2} \sin(\theta_1) \dot{\theta}_1 \\ \frac{l_1}{2} \cos(\theta_1) \dot{\theta}_1 \\ 0 \end{Bmatrix}$

- $\vec{v}_2 = \begin{Bmatrix} -l_1 \sin(\theta_1) \dot{\theta}_1 - \frac{l_2}{2} \sin(\theta_1 + \theta_2) (\dot{\theta}_1 + \dot{\theta}_2) \\ -l_1 \cos(\theta_1) \dot{\theta}_1 + \frac{l_2}{2} \cos(\theta_1 + \theta_2) (\dot{\theta}_1 + \dot{\theta}_2) \\ 0 \end{Bmatrix}$

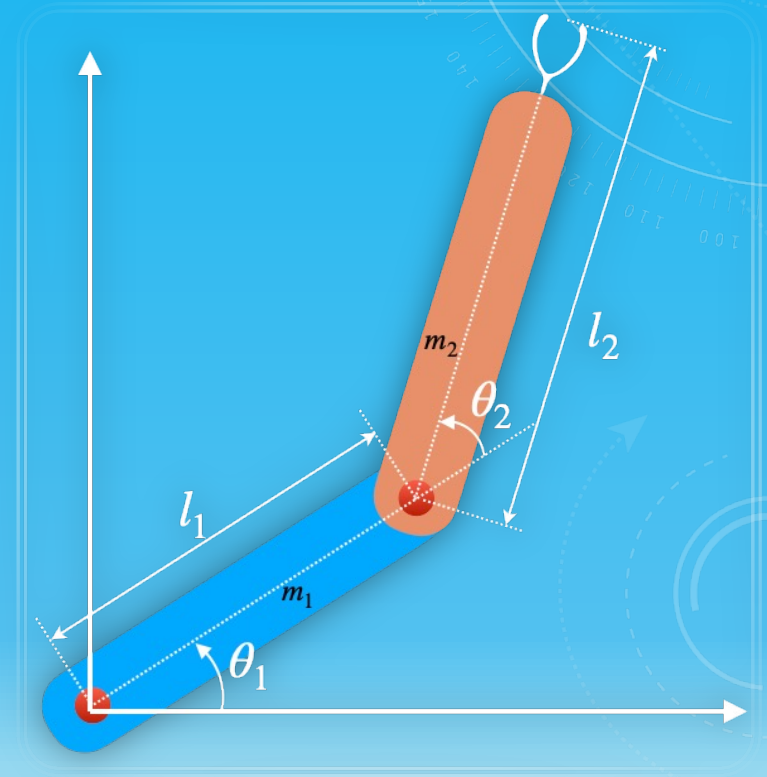
- $\vec{\omega}_1 = \begin{Bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{Bmatrix}$

- $\vec{\omega}_2 = \begin{Bmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 \end{Bmatrix}$



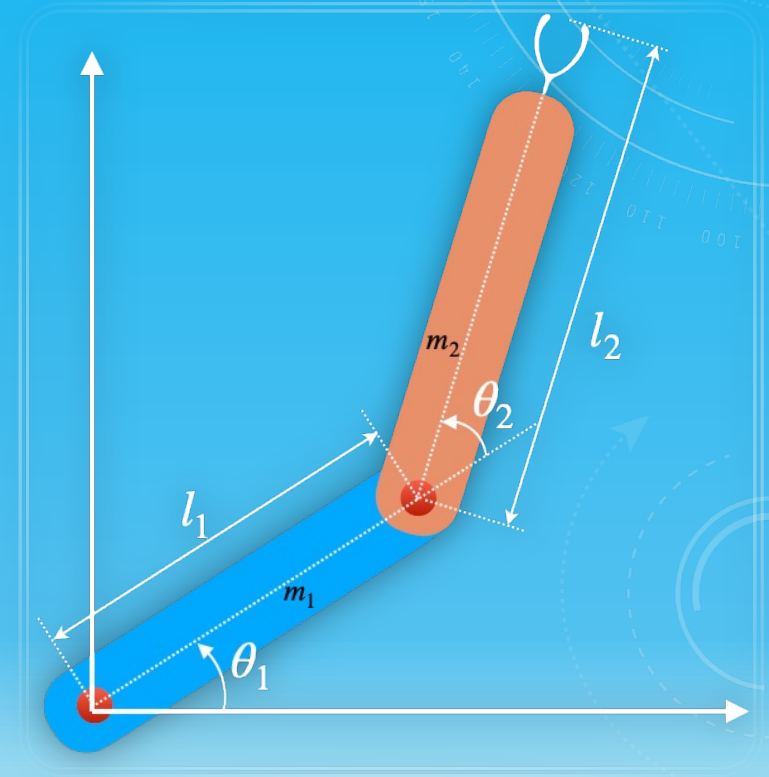
TWO-LINK ROBOT DYNAMICS

- $T = \frac{1}{2}m_1(\vec{v}_1 \cdot \vec{v}_1) + \frac{1}{2}m_2(\vec{v}_2 \cdot \vec{v}_2) + \frac{1}{2}I_1(\vec{\omega}_1 \cdot \vec{\omega}_1) + \frac{1}{2}I_2(\vec{\omega}_2 \cdot \vec{\omega}_2)$
- $V = 0$
- $L = T - V$
- $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}_1} \right) - \frac{\partial L}{\partial \theta_1} = \tau_1$
- $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}_2} \right) - \frac{\partial L}{\partial \theta_2} = \tau_2$



TWO-LINK ROBOT DYNAMICS

- $\tau_1 = \left(\frac{1}{3}m_1l_1^2 + m_2 \left(l_1^2 + \frac{1}{3}l_2^2 + l_1l_2 \cos(\theta_2) \right) \right) \ddot{\theta}_1 + \left(\frac{1}{3}m_2l_2^2 + \frac{1}{2}m_2l_1l_2 \cos(\theta_2) \right) \ddot{\theta}_2 - m_2l_1l_2 \sin(\theta_2) \left(\dot{\theta}_1\dot{\theta}_2 + \frac{1}{2}\dot{\theta}_2^2 \right)$
- $\tau_2 = m_2 \left(\frac{1}{3}l_2^2 + \frac{1}{2}l_1l_2 \cos(\theta_2) \right) \ddot{\theta}_1 + \left(\frac{1}{3}m_2l_2^2 \right) \ddot{\theta}_2 + \left(\frac{1}{2}m_2l_1l_2 \sin(\theta_2) \right) \dot{\theta}_1^2$



TWO-LINK ROBOT DYNAMICS

- $\tau_1 = \left(\frac{1}{3}m_1l_1^2 + m_2 \left(l_1^2 + \frac{1}{3}l_2^2 + l_1l_2 \cos(\theta_2) \right) \right) \ddot{\theta}_1 + \left(\frac{1}{3}m_2l_2^2 + \frac{1}{2}m_2l_1l_2 \cos(\theta_2) \right) \ddot{\theta}_2 - m_2l_1l_2 \sin(\theta_2) \left(\dot{\theta}_1\dot{\theta}_2 + \frac{1}{2}\dot{\theta}_2^2 \right)$
- $\tau_2 = m_2 \left(\frac{1}{3}l_2^2 + \frac{1}{2}l_1l_2 \cos(\theta_2) \right) \ddot{\theta}_1 + \left(\frac{1}{3}m_2l_2^2 \right) \ddot{\theta}_2 + \left(\frac{1}{2}m_2l_1l_2 \sin(\theta_2) \right) \dot{\theta}_1^2$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix} + \begin{Bmatrix} E \\ F \end{Bmatrix} = \begin{Bmatrix} \tau_1 \\ \tau_2 \end{Bmatrix}$$

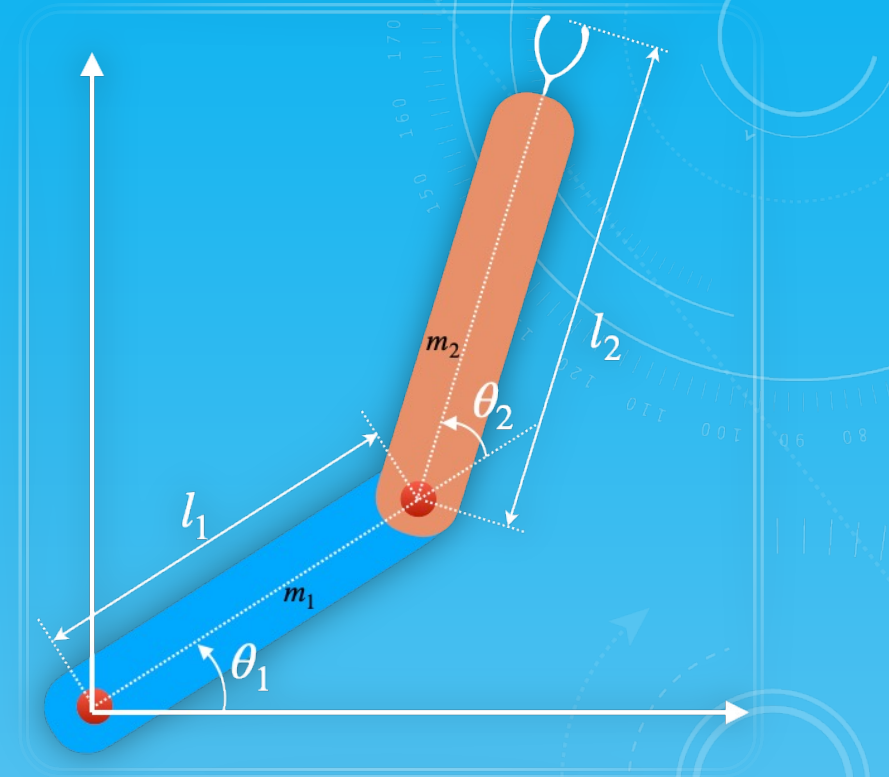
$$\Rightarrow \begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \left(\begin{Bmatrix} \tau_1 \\ \tau_2 \end{Bmatrix} - \begin{Bmatrix} E \\ F \end{Bmatrix} \right)$$

$$\ddot{\theta}_1 = \frac{D\tau_1 - B\tau_2 + BF - DE}{AD - BC}$$

$$\ddot{\theta}_2 = \frac{-C\tau_1 + A\tau_2 + EC - AF}{AD - BC}$$

$$\ddot{\theta}_1 = f_1(\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2, \tau_1, \tau_2)$$

$$\ddot{\theta}_2 = f_2(\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2, \tau_1, \tau_2)$$



$$A = \frac{1}{3}m_1l_1^2 + m_2 \left(l_1^2 + \frac{1}{3}l_2^2 + l_1l_2 \cos(\theta_2) \right)$$

$$B = C = \frac{1}{3}m_2l_2^2 + \frac{1}{2}m_2l_1l_2 \cos(\theta_2)$$

$$D = \frac{1}{3}m_2l_2^2$$

$$E = -m_2l_1l_2 \sin(\theta_2) \left(\dot{\theta}_1\dot{\theta}_2 + \frac{1}{2}\dot{\theta}_2^2 \right)$$

$$F = \left(\frac{1}{2}m_2l_1l_2 \sin(\theta_2) \right) \dot{\theta}_1^2$$

TWO-LINK ROBOT DYNAMICS

- $\tau_1 = \left(\frac{1}{3}m_1l_1^2 + m_2 \left(l_1^2 + \frac{1}{3}l_2^2 + l_1l_2 \cos(\theta_2) \right) \right) \ddot{\theta}_1 + \left(\frac{1}{3}m_2l_2^2 + \frac{1}{2}m_2l_1l_2 \cos(\theta_2) \right) \ddot{\theta}_2 - m_2l_1l_2 \sin(\theta_2) \left(\dot{\theta}_1\dot{\theta}_2 + \frac{1}{2}\dot{\theta}_2^2 \right)$
- $\tau_2 = m_2 \left(\frac{1}{3}l_2^2 + \frac{1}{2}l_1l_2 \cos(\theta_2) \right) \ddot{\theta}_1 + \left(\frac{1}{3}m_2l_2^2 \right) \ddot{\theta}_2 + \left(\frac{1}{2}m_2l_1l_2 \sin(\theta_2) \right) \dot{\theta}_1^2$

$$\ddot{\theta}_1 = \frac{D\tau_1 - B\tau_2 + BF - DE}{AD - BC}$$

$$\ddot{\theta}_2 = \frac{-C\tau_1 + A\tau_2 + EC - AF}{AD - BC}$$

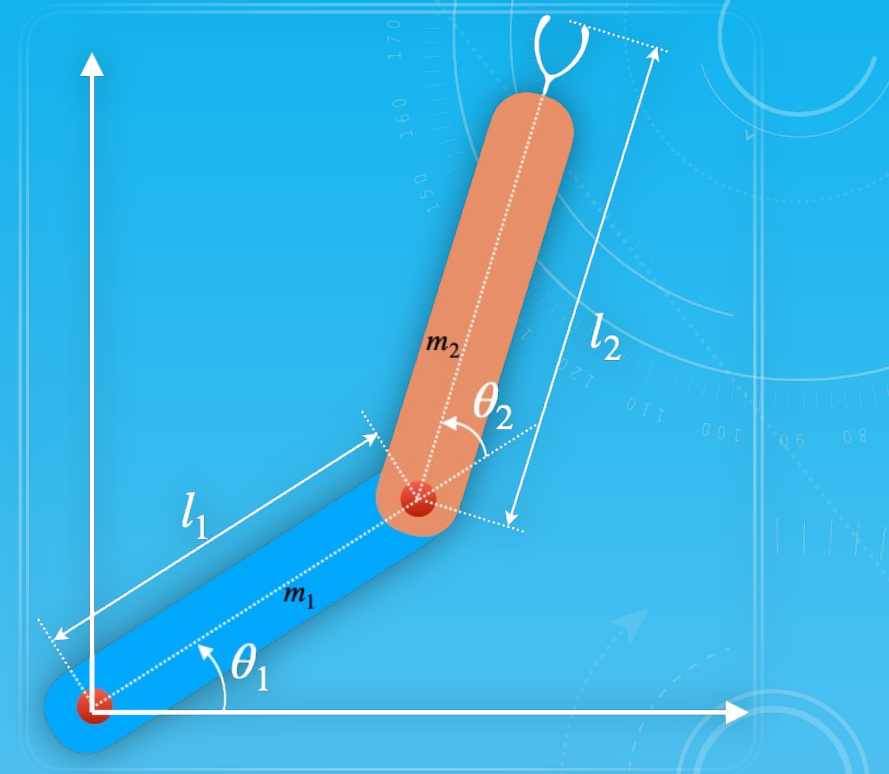
$$z_1 = \theta_1, z_2 = \theta_2, z_3 = \dot{\theta}_1, z_4 = \dot{\theta}_2$$

$$\dot{z}_1 = \dot{\theta}_1 = z_3$$

$$\dot{z}_2 = \dot{\theta}_2 = z_4$$

$$\dot{z}_3 = \ddot{\theta}_1 = \frac{D\tau_1 - B\tau_2 + BF - DE}{AD - BC}$$

$$\dot{z}_4 = \ddot{\theta}_2 = \frac{-C\tau_1 + A\tau_2 + EC - AF}{AD - BC}$$



$$A = \frac{1}{3}m_1l_1^2 + m_2 \left(l_1^2 + \frac{1}{3}l_2^2 + l_1l_2 \cos(\theta_2) \right)$$

$$B = C = \frac{1}{3}m_2l_2^2 + \frac{1}{2}m_2l_1l_2 \cos(\theta_2)$$

$$D = \frac{1}{3}m_2l_2^2$$

$$E = -m_2l_1l_2 \sin(\theta_2) \left(\dot{\theta}_1\dot{\theta}_2 + \frac{1}{2}\dot{\theta}_2^2 \right)$$

$$F = \left(\frac{1}{2}m_2l_1l_2 \sin(\theta_2) \right) \dot{\theta}_1^2$$

TWO-LINK ROBOT DYNAMICS

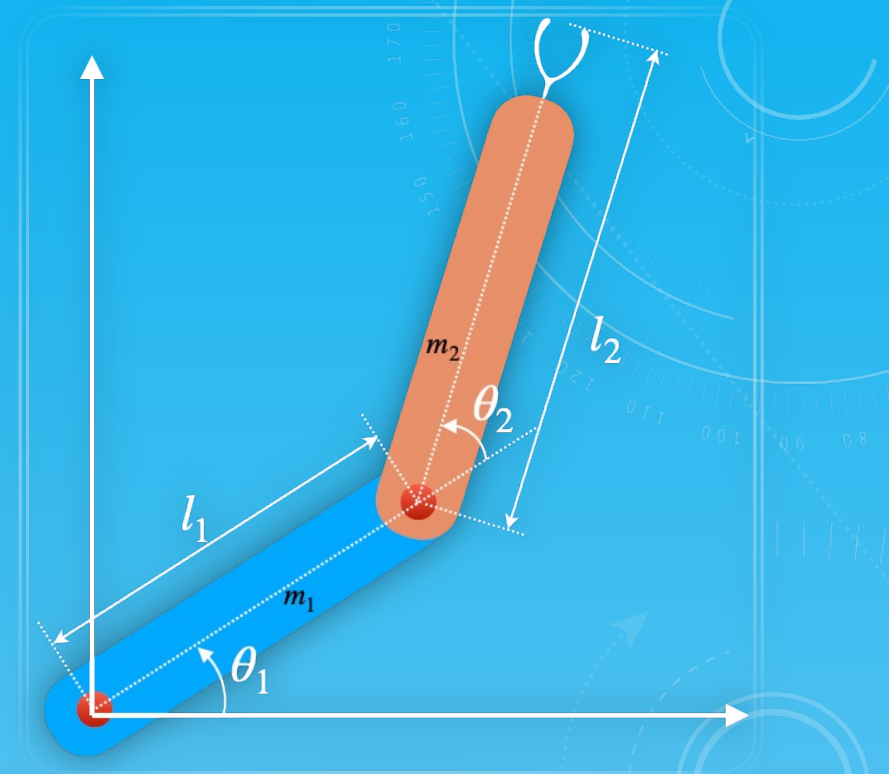
- $\tau_1 = \left(\frac{1}{3}m_1l_1^2 + m_2 \left(l_1^2 + \frac{1}{3}l_2^2 + l_1l_2 \cos(\theta_2) \right) \right) \ddot{\theta}_1 + \left(\frac{1}{3}m_2l_2^2 + \frac{1}{2}m_2l_1l_2 \cos(\theta_2) \right) \ddot{\theta}_2 - m_2l_1l_2 \sin(\theta_2) \left(\dot{\theta}_1\dot{\theta}_2 + \frac{1}{2}\dot{\theta}_2^2 \right)$
- $\tau_2 = m_2 \left(\frac{1}{3}l_2^2 + \frac{1}{2}l_1l_2 \cos(\theta_2) \right) \ddot{\theta}_1 + \left(\frac{1}{3}m_2l_2^2 \right) \ddot{\theta}_2 + \left(\frac{1}{2}m_2l_1l_2 \sin(\theta_2) \right) \dot{\theta}_1^2$

$$\ddot{\theta}_1 = \frac{D\tau_1 - B\tau_2 + BF - DE}{AD - BC}$$

$$\ddot{\theta}_2 = \frac{-C\tau_1 + A\tau_2 + EC - AF}{AD - BC}$$

$$z_1 = \theta_1, z_2 = \theta_2, z_3 = \dot{\theta}_1, z_4 = \dot{\theta}_2$$

$$\begin{Bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{Bmatrix} = \begin{Bmatrix} z_2 \\ z_4 \\ \frac{D\tau_1 - B\tau_2 + BF - DE}{AD - BC} \\ \frac{-C\tau_1 + A\tau_2 + EC - AF}{AD - BC} \end{Bmatrix} = f(t, \mathbf{z}, \boldsymbol{\tau})$$



$$A = \frac{1}{3}m_1l_1^2 + m_2 \left(l_1^2 + \frac{1}{3}l_2^2 + l_1l_2 \cos(\theta_2) \right)$$

$$B = C = \frac{1}{3}m_2l_2^2 + \frac{1}{2}m_2l_1l_2 \cos(\theta_2)$$

$$D = \frac{1}{3}m_2l_2^2$$

$$E = -m_2l_1l_2 \sin(\theta_2) \left(\dot{\theta}_1\dot{\theta}_2 + \frac{1}{2}\dot{\theta}_2^2 \right)$$

$$F = \left(\frac{1}{2}m_2l_1l_2 \sin(\theta_2) \right) \dot{\theta}_1^2$$

SOLUTION

- Euler's method
 - $\frac{dz}{dt} = f(t, z)$
 - $\frac{z_{t+\Delta t} - z_t}{\Delta t} = f(t, z_t)$
 - $z_{t+\Delta t} = z_t + \Delta t f(t, z_t)$

SOLUTION

- Runge-Kutta method
 - $\frac{dz}{dt} = f(t, z)$
 - $z_{t+\Delta t} = z_t + \frac{\Delta t}{6} (k_1 + 2k_2 + 2k_3 + k_4)$
 - $k_1 = f(t, z_t)$
 - $k_2 = f\left(t + \frac{\Delta t}{2}, z_t + \Delta t \frac{k_1}{2}\right)$
 - $k_3 = f\left(t + \frac{\Delta t}{2}, z_t + \Delta t \frac{k_2}{2}\right)$
 - $k_4 = f(t + \Delta t, z_t + \Delta t k_3)$

CONTROL STRATEGIES

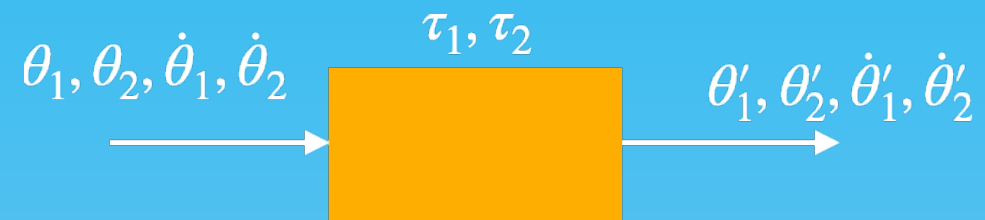
- Open-loop controllers
- Closed-loop controllers
- Fuzzy logic controllers
- ML-based controllers
- ...

CLOSED-LOOP CONTROLLERS

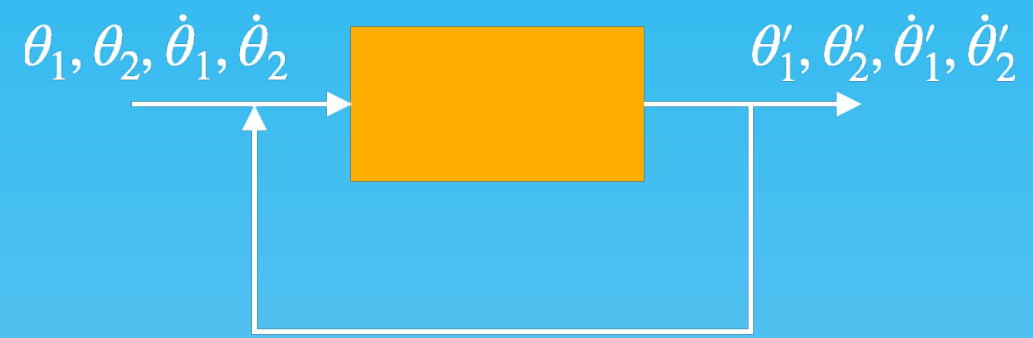
- PID controller

SOLUTION

- Open loop control



ERROR

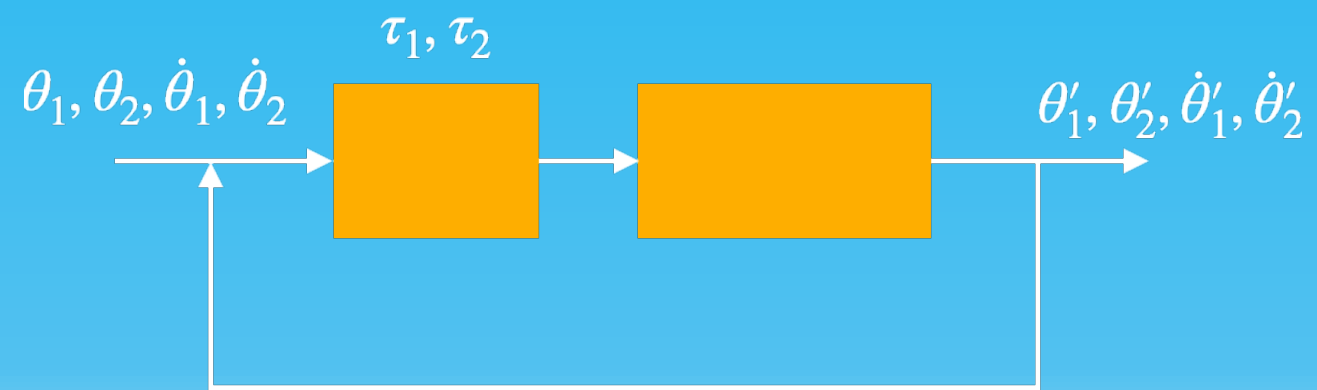


ERROR

- $e_1 = \theta_1 - \theta'_1$
- $e_2 = \theta_2 - \theta'_2$
- $\dot{e}_1 = \dot{\theta}_1 - \dot{\theta}'_1$
- $\dot{e}_2 = \dot{\theta}_2 - \dot{\theta}'_2$



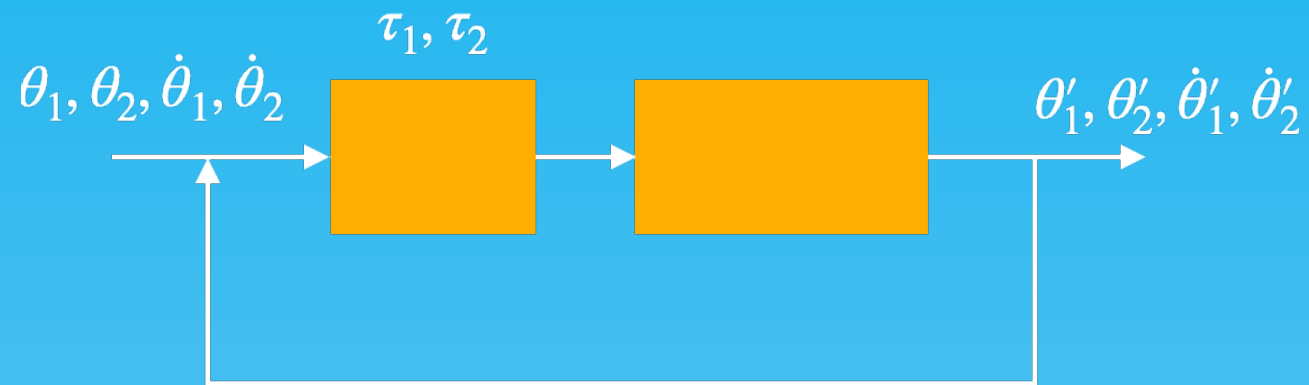
ERROR



ERROR

- $\tau_1 = F_1(\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2)$
- $\tau_2 = F_2(\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2)$

- $\tau_1 = K_p e_1 + K_d \dot{e}_1 + K_i \int_0^t e_1 dt$



CONTROL STRATEGIES

- Open-loop controllers
- Closed-loop controllers
- Fuzzy logic controllers
- ML-based controllers
- ...

OPEN-LOOP CONTROLLERS

STEPPER-MOTOR

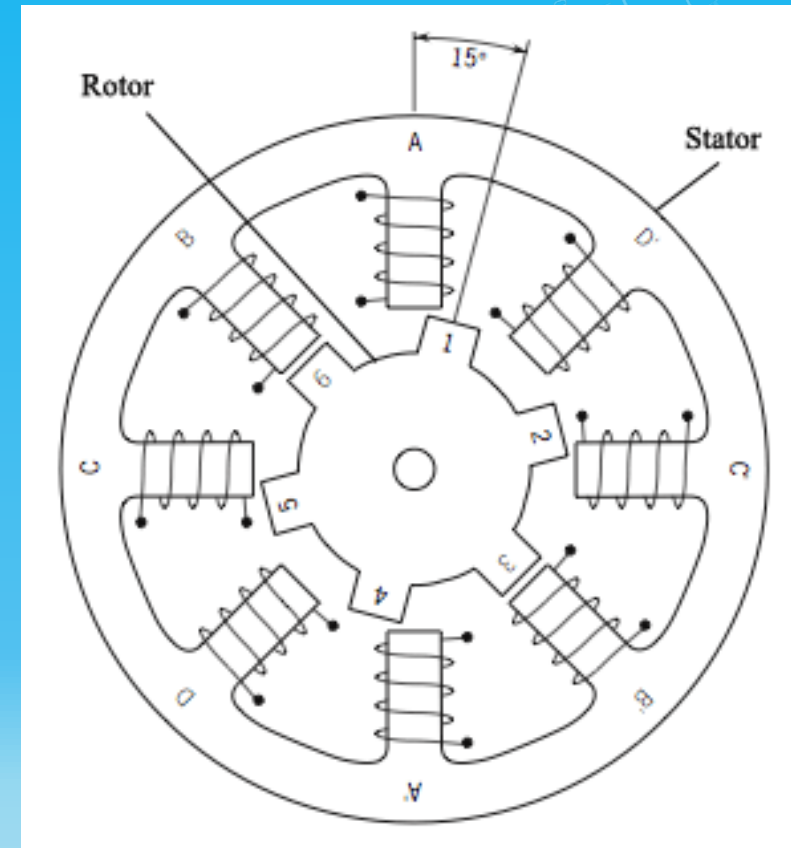


Image source:

<https://circuitdigest.com/sites/default/files/inlineimages/u/Stepper-Motor-Internal-Structure.png>

FUZZY LOGIC CONTROL

- Rules-based

$$\ddot{\theta} = \frac{g \sin \theta + \cos \theta \left(\frac{-F - ml\dot{\theta}^2 \sin \theta}{M + m} \right)}{l \left(\frac{4}{3} - \frac{m \cos^2 \theta}{M + m} \right)}$$

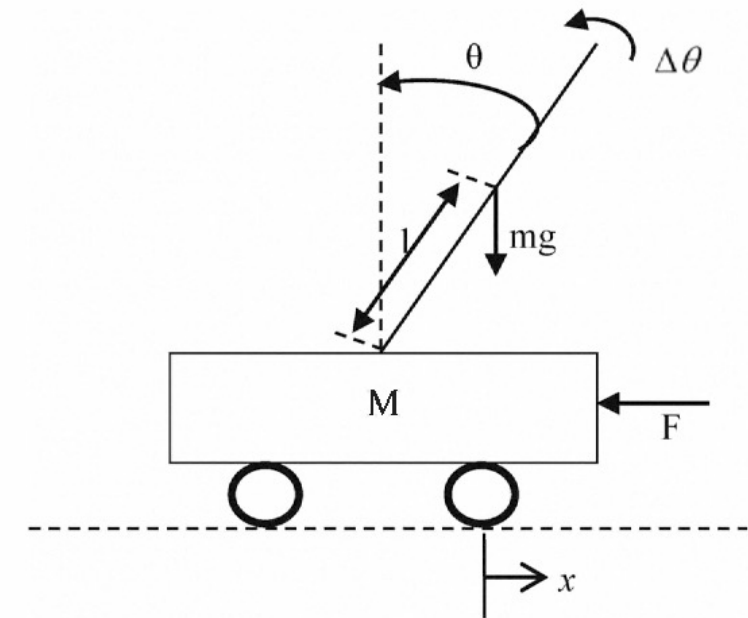


Figure 2. Cart-pole typed inverted pendulum system

Image source:

<https://www.semanticscholar.org/paper/Fuzzy-logic-controller-for-an-inverted-pendulum-Shill-Akhand/e6cf50180f74be11ffd7d9d520b36dd1650aae6c/figure/1>

FUZZY LOGIC CONTROL

- Rules-based

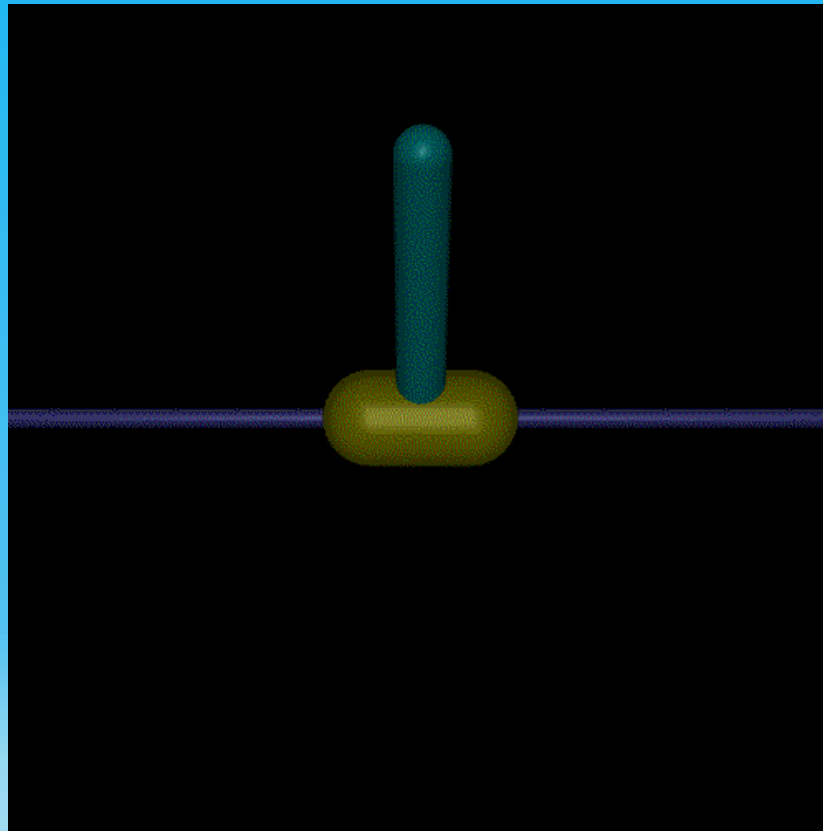


Image source: https://mgoulao.github.io/gym-docs/_images/inverted_pendulum.gif

$$\ddot{\theta} = \frac{g \sin \theta + \cos \theta \left(\frac{-F - ml\dot{\theta}^2 \sin \theta}{M + m} \right)}{l \left(\frac{4}{3} - \frac{m \cos^2 \theta}{M + m} \right)}$$

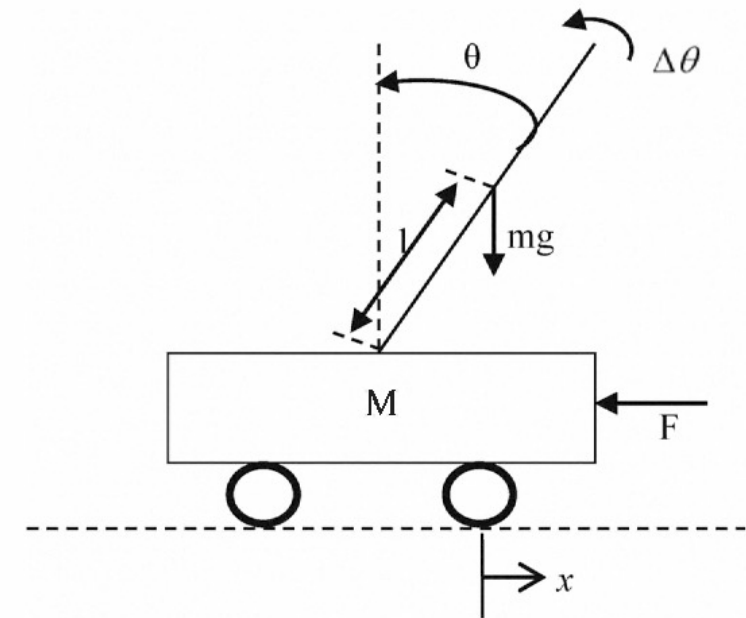


Figure 2. Cart-pole typed inverted pendulum system

Image source:

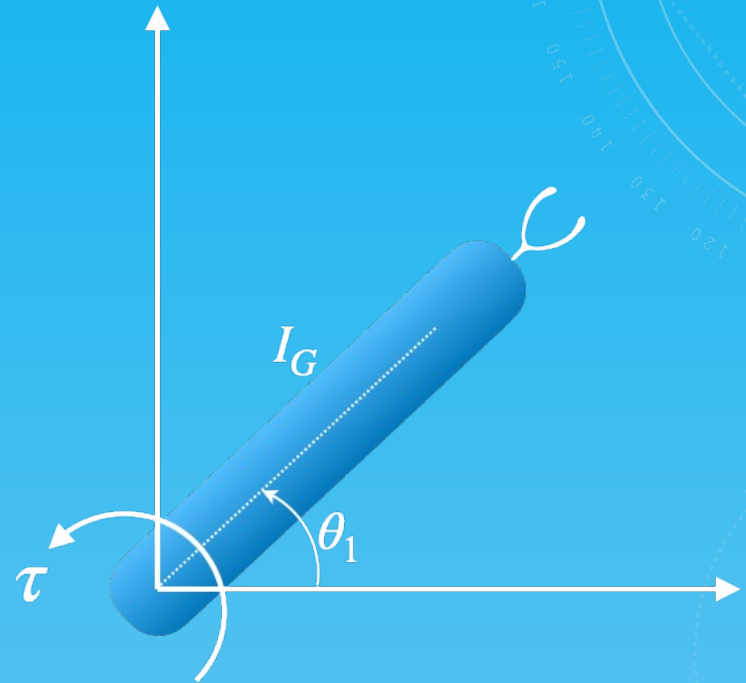
<https://www.semanticscholar.org/paper/Fuzzy-logic-controller-for-an-inverted-pendulum-Shill-Akhand/e6cf50180f74be11ffd7d9d520b36dd1650aae6c/figure/1>

MACHINE LEARNING

- Data-driven controlling

MACHINE LEARNING

- Data-driven controlling
 - Joint flexibility
 - Link flexibility
 - Base flexibility



MACHINE LEARNING

- Data-driven controlling
 - Joint flexibility
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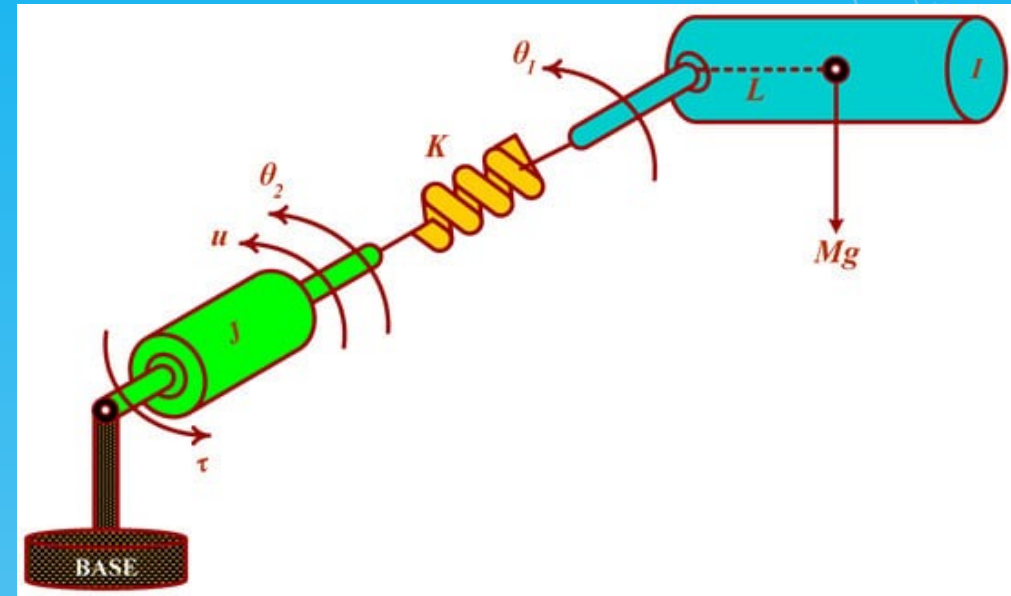


Image source: https://www.mdpi.com/sensors/sensors-21-03252/article_deploy/html/images/sensors-21-03252-g001-550.jpg

MACHINE LEARNING

- Data-driven controlling
 - Joint flexibility
 - Link flexibility
 - Base flexibility

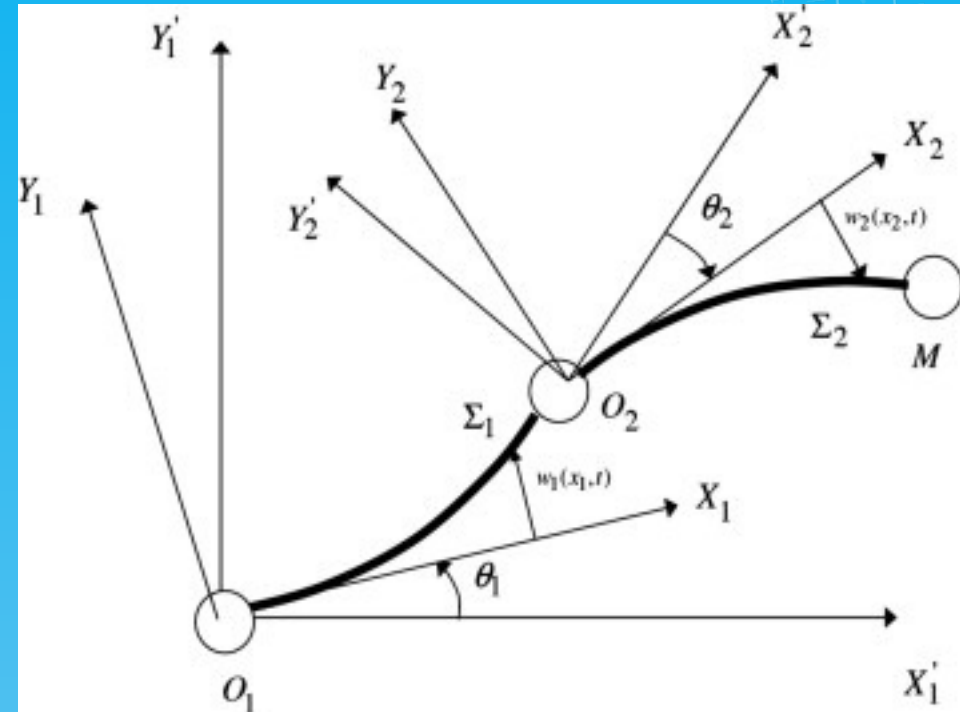


Image source: <https://ars.els-cdn.com/content/image/1-s2.0-S0307904X09000183-gr2.jpg>

MODEL PREDICTIVE CONTROL

Optimal Control

- Minimise

$$J = \mathbf{e}_1^2 + \mathbf{e}_2^2 + \cdots + \mathbf{e}_N^2 + \boldsymbol{\tau}_0^2 + \boldsymbol{\tau}_1^2 + \cdots + \boldsymbol{\tau}_{N-1}^2$$

- subject to

$$\mathbf{z}_{k+1} = \mathbf{z}_k + \Delta t \mathbf{f}(t, \mathbf{z}_k, \boldsymbol{\tau}_k) \quad \text{for } k = 0, 1, 2, \dots, N-1$$

- where

$$\mathbf{e}_i = \mathbf{z}_i - \mathbf{z}_d$$

$$\boldsymbol{\tau}_i = \begin{Bmatrix} \tau_1 \\ \tau_2 \end{Bmatrix} \text{ at } i^{\text{th}} \text{ step}$$

MODEL PREDICTIVE CONTROL

Optimal Control

- Minimise

$$J = (\mathbf{z}_1 - \mathbf{z}_d)^2 + (\mathbf{z}_2 - \mathbf{z}_d)^2 + \cdots + (\mathbf{z}_N - \mathbf{z}_d)^2 + \boldsymbol{\tau}_0^2 + \boldsymbol{\tau}_1^2 + \cdots + \boldsymbol{\tau}_{N-1}^2$$

- subject to

$$\mathbf{z}_1 = \mathbf{z}_0 + \Delta t \mathbf{f}(t, \mathbf{z}_0, \boldsymbol{\tau}_0)$$

$$\mathbf{z}_2 = \mathbf{z}_1 + \Delta t \mathbf{f}(t, \mathbf{z}_1, \boldsymbol{\tau}_1)$$

$$\mathbf{z}_3 = \mathbf{z}_2 + \Delta t \mathbf{f}(t, \mathbf{z}_2, \boldsymbol{\tau}_2)$$

\vdots

$$\mathbf{z}_N = \mathbf{z}_{N-1} + \Delta t \mathbf{f}(t, \mathbf{z}_{N-1}, \boldsymbol{\tau}_{N-1})$$

MODEL PREDICTIVE CONTROL

Optimal Control

- Minimise

$$(\mathbf{z}_1 - \mathbf{z}_d)^T(\mathbf{z}_1 - \mathbf{z}_d) + (\mathbf{z}_2 - \mathbf{z}_d)^T(\mathbf{z}_2 - \mathbf{z}_d) + \cdots + (\mathbf{z}_N - \mathbf{z}_d)^T(\mathbf{z}_N - \mathbf{z}_d) + \boldsymbol{\tau}_0^T \boldsymbol{\tau}_0 + \boldsymbol{\tau}_1^T \boldsymbol{\tau}_1 + \cdots + \boldsymbol{\tau}_{N-1}^T \boldsymbol{\tau}_{N-1}$$

- subject to

$$\mathbf{z}_1 = \mathbf{z}_0 + \Delta t \mathbf{f}(t, \mathbf{z}_0, \boldsymbol{\tau}_0)$$

$$\mathbf{z}_2 = \mathbf{z}_1 + \Delta t \mathbf{f}(t, \mathbf{z}_1, \boldsymbol{\tau}_1)$$

$$\mathbf{z}_3 = \mathbf{z}_2 + \Delta t \mathbf{f}(t, \mathbf{z}_2, \boldsymbol{\tau}_2)$$

$$\vdots$$

$$\mathbf{z}_N = \mathbf{z}_{N-1} + \Delta t \mathbf{f}(t, \mathbf{z}_{N-1}, \boldsymbol{\tau}_{N-1})$$

MODEL PREDICTIVE CONTROL

Optimal Control

- Minimise

$$(\mathbf{z}_1 - \mathbf{z}_d)^T \mathbf{Q}(\mathbf{z}_1 - \mathbf{z}_d) + (\mathbf{z}_2 - \mathbf{z}_d)^T \mathbf{Q}(\mathbf{z}_2 - \mathbf{z}_d) + \cdots + (\mathbf{z}_N - \mathbf{z}_d)^T \mathbf{Q}(\mathbf{z}_N - \mathbf{z}_d) + \boldsymbol{\tau}_0^T \mathbf{R} \boldsymbol{\tau}_0 + \boldsymbol{\tau}_1^T \mathbf{R} \boldsymbol{\tau}_1 + \cdots + \boldsymbol{\tau}_{N-1}^T \mathbf{R} \boldsymbol{\tau}_{N-1}$$

- subject to

$$\mathbf{z}_1 = \mathbf{z}_0 + \Delta t \mathbf{f}(t, \mathbf{z}_0, \boldsymbol{\tau}_0)$$

$$\mathbf{z}_2 = \mathbf{z}_1 + \Delta t \mathbf{f}(t, \mathbf{z}_1, \boldsymbol{\tau}_1)$$

$$\mathbf{z}_3 = \mathbf{z}_2 + \Delta t \mathbf{f}(t, \mathbf{z}_2, \boldsymbol{\tau}_2)$$

$$\vdots$$

$$\mathbf{z}_N = \mathbf{z}_{N-1} + \Delta t \mathbf{f}(t, \mathbf{z}_{N-1}, \boldsymbol{\tau}_{N-1})$$

MODEL PREDICTIVE CONTROL

Optimal Control

- Minimise

$$J = \sum_{i=0}^{N-1} (\mathbf{z}_{i+1} - \mathbf{z}_d)^T \mathbf{Q} (\mathbf{z}_{i+1} - \mathbf{z}_d) + \boldsymbol{\tau}_i^T \mathbf{R} \boldsymbol{\tau}_i$$

- subject to

$$\mathbf{z}_{k+1} = \mathbf{z}_k + \Delta t \mathbf{f}(t, \mathbf{z}_k, \boldsymbol{\tau}_k) \quad \text{for } k = 0, 1, 2, \dots, N-1$$

Decision variables: $\boldsymbol{\tau}_i$

REINFORCEMENT-BASED CONTROL

DDPG

- Bellman's equation

$$Q(\mathbf{s}, \mathbf{a}) = R(\mathbf{s}, \mathbf{a}) + \gamma \max_{\mathbf{s}'} Q(\mathbf{s}', \mathbf{a}')$$

- Reward function

$$R(\mathbf{s}, \mathbf{a}) = -(\mathbf{s} - \mathbf{s}_d)^T (\mathbf{s} - \mathbf{s}_d) - \lambda_{\text{torque}} \mathbf{a}^T \mathbf{a}$$

REINFORCEMENT-BASED CONTROL

DDPG

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$$\mathbf{s} = \begin{Bmatrix} \mathbf{z} \\ \mathbf{z}_d \end{Bmatrix}, \mathbf{a} = \boldsymbol{\tau}$$

REINFORCEMENT-BASED CONTROL

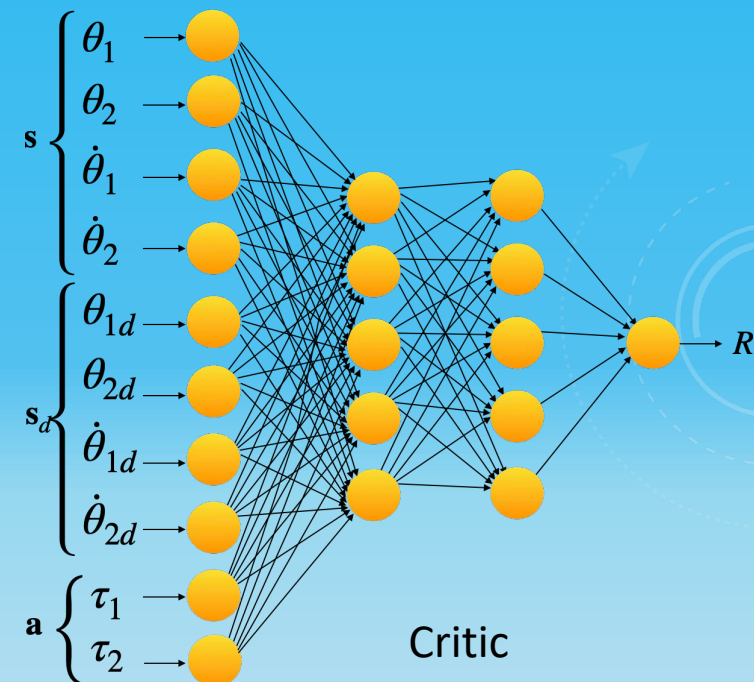
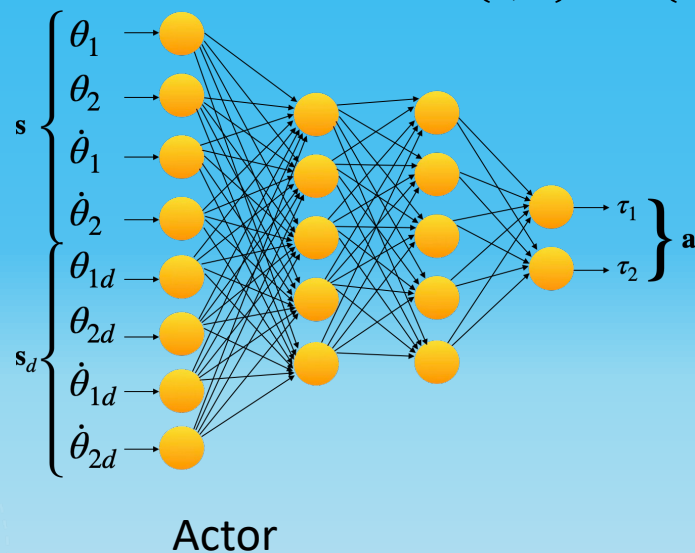
DDPG

- Bellman's equation

$$Q(\mathbf{s}, \mathbf{a}) = R(\mathbf{s}, \mathbf{a}) + \gamma \max_{\mathbf{s}'} Q(\mathbf{s}', \mathbf{a}')$$

- Reward function

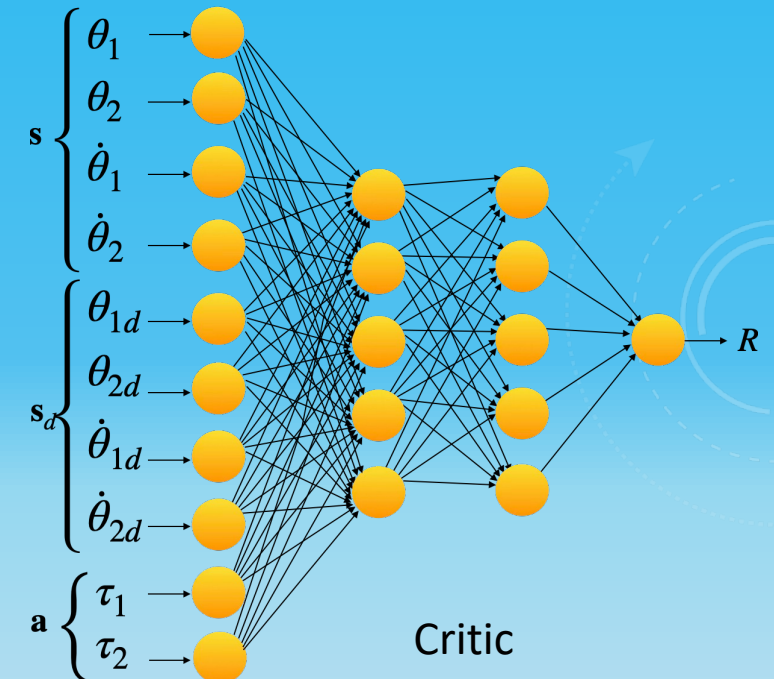
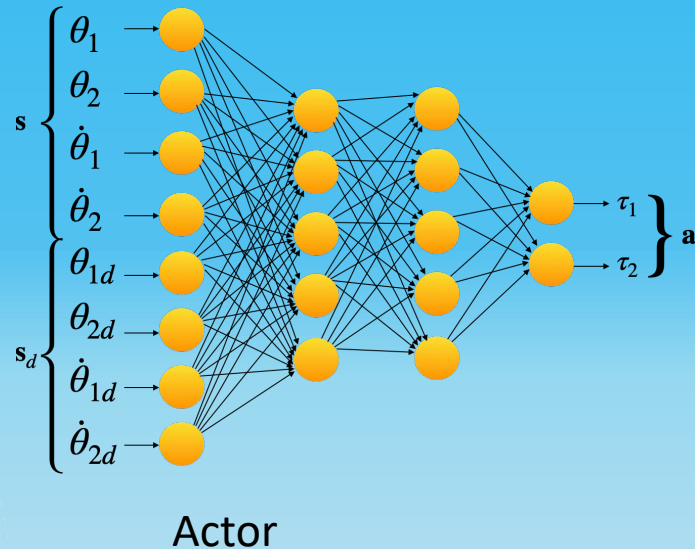
$$R(\mathbf{s}, \mathbf{a}) = -(\mathbf{s} - \mathbf{s}_d)^T (\mathbf{s} - \mathbf{s}_d) - \lambda_{\text{torque}} \mathbf{a}^T \mathbf{a}$$



REINFORCEMENT-BASED CONTROL

DDPG

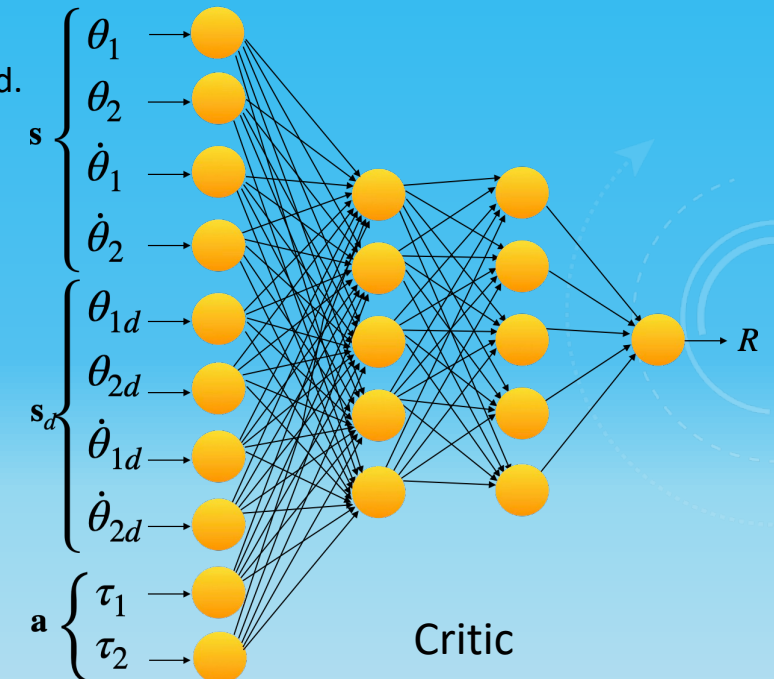
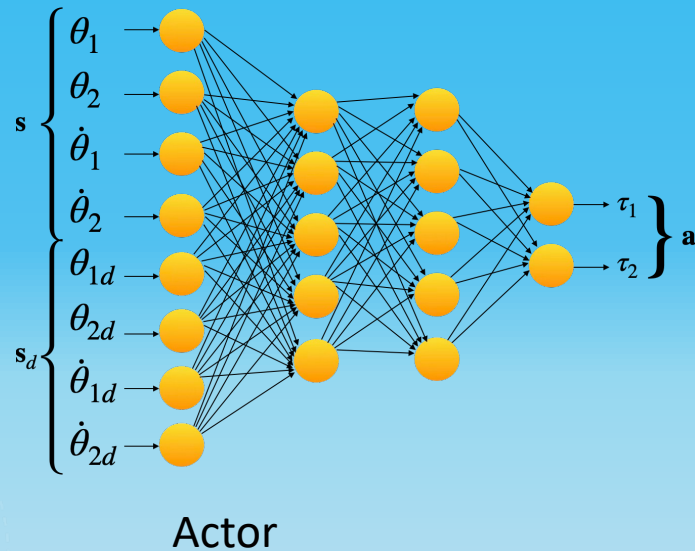
- Actor loss function: $-Q(\mathbf{s}, \mathbf{s}_d, \mathbf{a})$
- Critic loss function: $R(\mathbf{s}, \mathbf{a}) + \gamma \max_{\mathbf{s}'} Q(\mathbf{s}', \mathbf{a}') - Q(\mathbf{s}, \mathbf{a})$
- Replay buffer row: $[\mathbf{s} \quad \mathbf{a} \quad \mathbf{s}' \quad \mathbf{R} \quad \text{done}]$
- ϵ -greedy policy for actor network actions vs randomised actions



REINFORCEMENT-BASED CONTROL

DDPG

- Generation of actions
 - After training, use the actor network with the initial state and the desired state to generate actions for the next step.
 - Find the next state using differential equation and use this as the new input state.
 - Again find the actions using the first step, and repeat until the goal state is reached.



The background is a solid blue color. In the top right corner, there is a large, faint, circular graphic resembling a radar or a clock face, with concentric circles and radial lines. In the bottom left corner, there is a smaller, faint, circular graphic with concentric circles. A horizontal gradient bar, transitioning from blue to a lighter blue, is positioned across the middle of the image. The word "Queries?" is centered in the middle of the image, below the gradient bar.

Queries?

The background is a solid blue gradient, lighter at the bottom. It features several faint, light-blue geometric patterns. In the top-left corner, there is a small circular arc with an arrow pointing clockwise. In the top-right corner, there is a large, complex circular pattern with concentric arcs and radial lines, resembling a stylized sun or a technical diagram. In the bottom-left corner, there is a circular arc with an arrow pointing counter-clockwise. In the bottom-right corner, there is a circular pattern with concentric arcs and an arrow pointing clockwise.

Thank you!