AEEM 5117/6117 Intelligent Robotics

Fuzzy Logic Systems

Feb 21, 2019

Desired Attributes from an Intelligent System

- Robustness

  The ability to handle uncertainty and dynamic changes

  - Sensor noise
  - Ambiguity in developing accurate situational awareness
  - Modeling errors
  - Dynamic changes such as environmentally related or caused by a non-cooperative (hostile) agent
- Adaptability
  - Learn from data
  - Learn from past experience

Classical logic vs Fuzzy logic

- **Boolean logic:** A statement is either entirely true or entirely false
- **Fuzzy logic:** Any statement can be fuzzy.
- The major advantage that fuzzy reasoning offers is the ability to reply to a yes-no question with a not-quite-yes-or-no answer.
- Humans do this kind of thing all the time (think how rarely you get a straight answer to a seemingly simple question), but it is a rather new trick for computers.

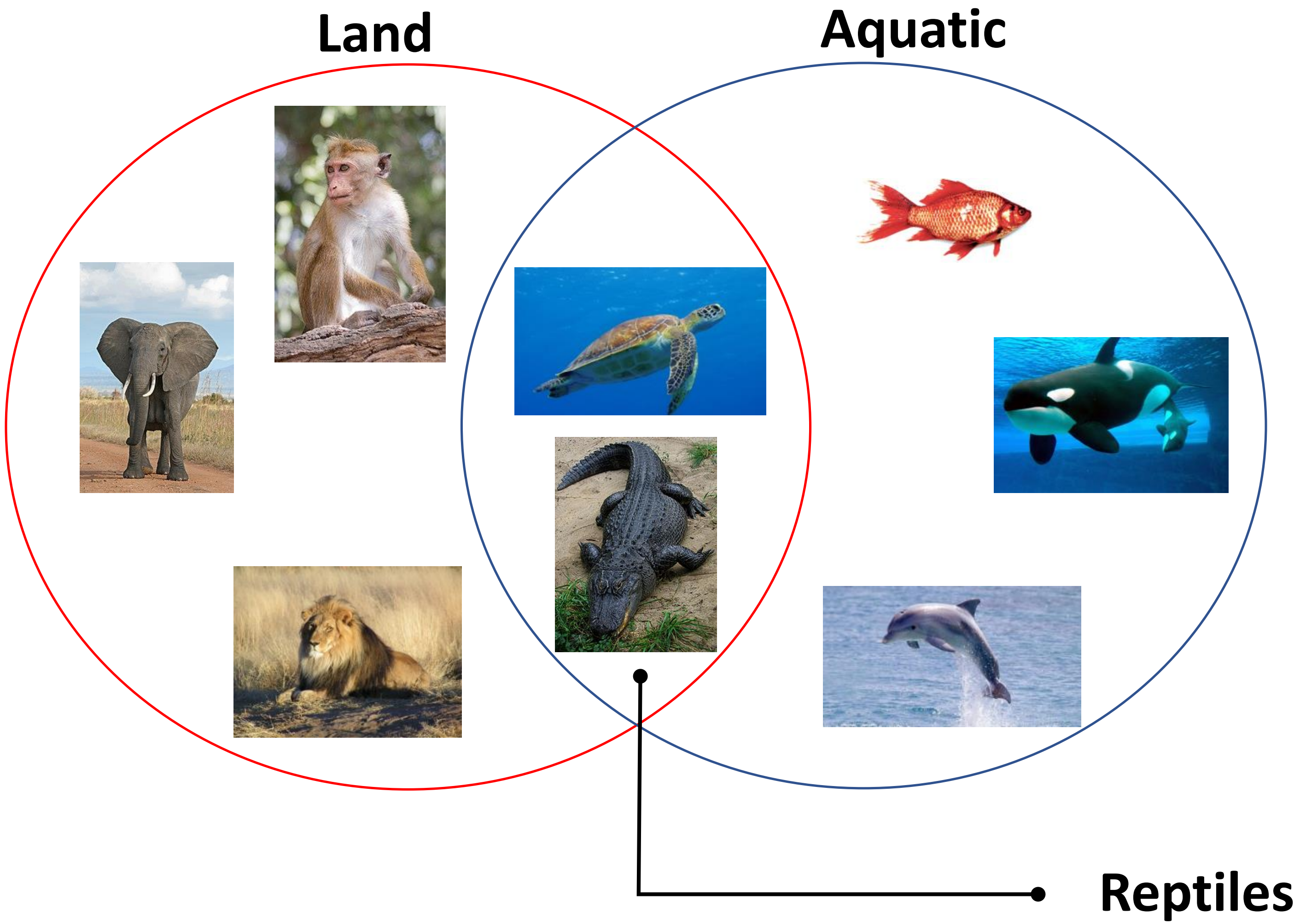
Classical logic vs Fuzzy logic

- How does it work?
- Reasoning in fuzzy logic is just a matter of generalizing the familiar yes-no (Boolean) logic.
- If you give true the numerical value of 1 and false the numerical value of 0, this value indicates that fuzzy logic also permits in-between values like 0.2, 0.7453 etc.

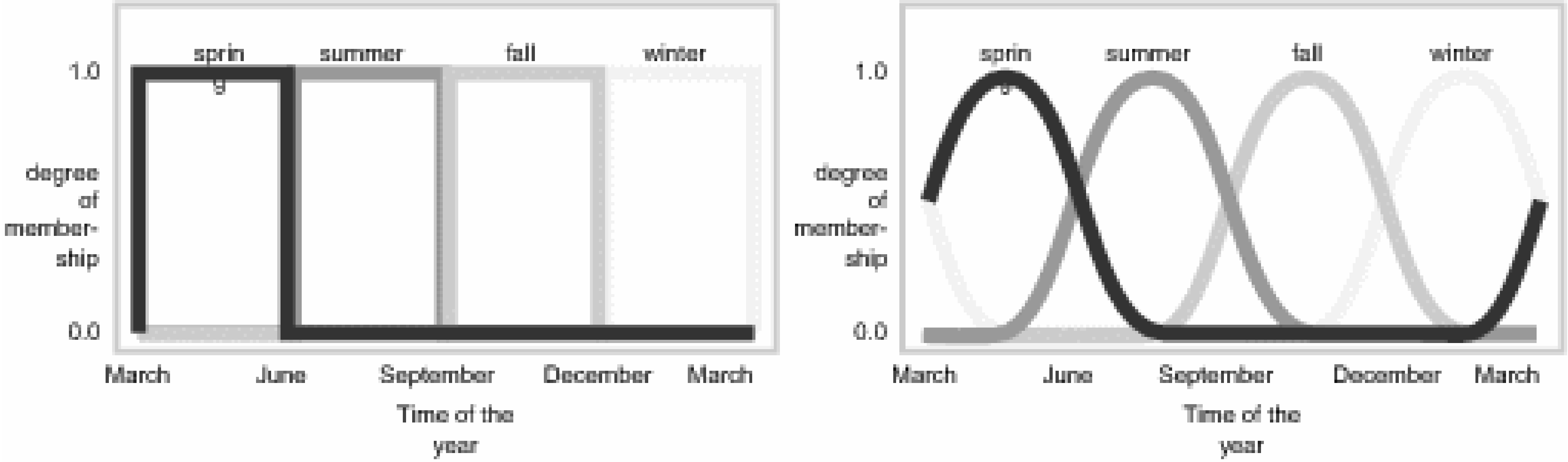
Fuzzy Sets

- Sets that are fuzzy, or multivalent, break the law of the excluded middle- to some degree.
- Items belong only partially to a fuzzy set.
- They may also belong to more than one set. Even to just one individual, the air may feel cool , just right and warm to varying degrees.
- Whereas the boundaries of standard sets are exact, those of fuzzy sets are curved or taper off, and this curvature creates partial contradictions.
- The air can be 20 percent cool-and at the same time, 80 percent warm.

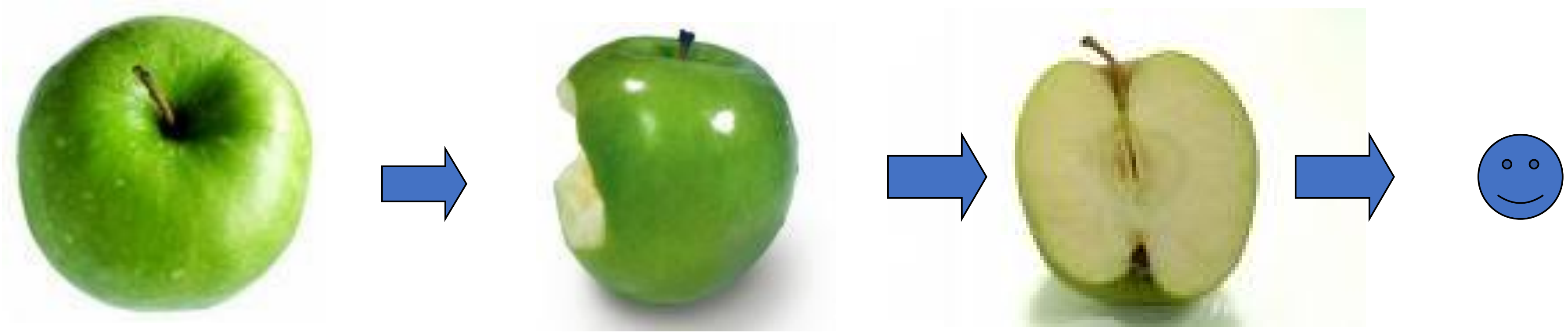
Land vs aquatic creatures



Defining seasons



Fuzziness is Greyness



- The apple changes from a thing to nothing.
- When you hold half an apple, the apple is as much there as not.
- The half apple is a fuzzy apple, the gray between the black and the white.

Boolean & Fuzzy Logic Operations

Boolean Logic - Standard truth tables

AND

A	B	A and B
0	0	0
0	1	0
1	0	0
1	1	1

OR

A	B	A or B
0	0	0
0	1	1
1	0	1
1	1	1

NOT

A	not A
0	1
1	0

Fuzzy Logic – Truth tables using “min” and “max”

AND

A	B	min(A,B)
0	0	0
0	1	0
1	0	0
1	1	1

OR

A	B	max(A,B)
0	0	0
0	1	1
1	0	1
1	1	1

NOT

A	1 - A
0	1
1	0

Boolean & Fuzzy Logic Operations

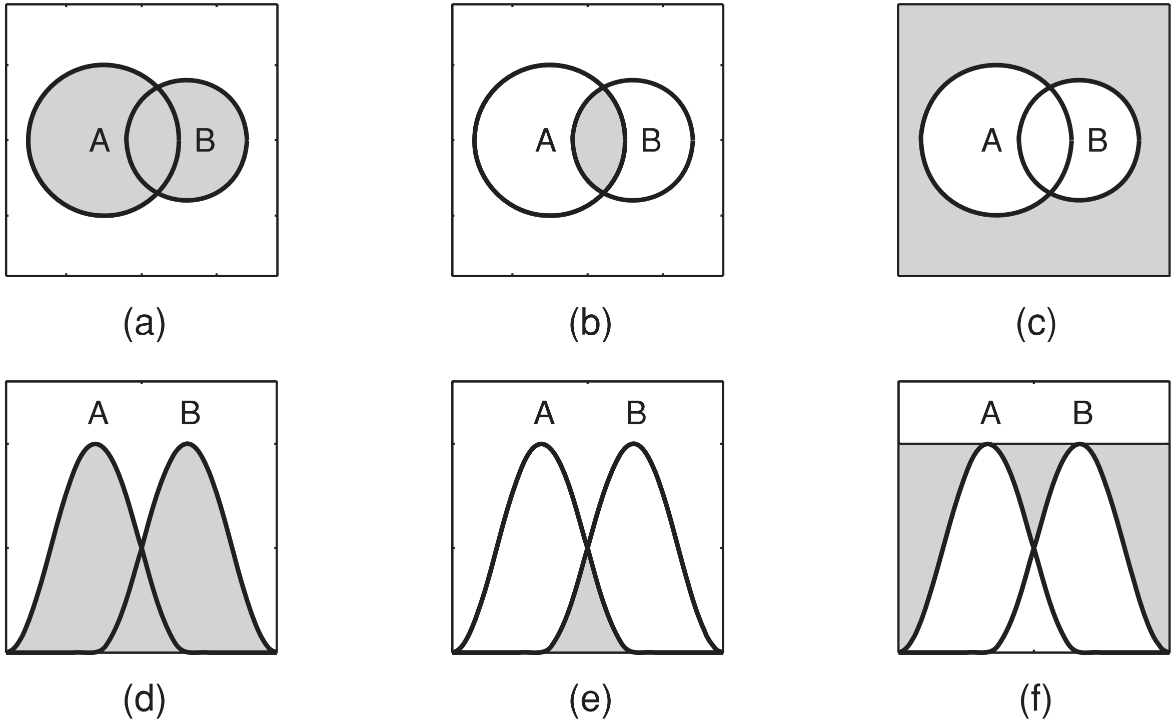


Figure 2.4 Set operations. The top row shows classic Venn diagrams; the universe is represented by the points within the rectangle, and sets by the interior of the circles. The bottom row shows their fuzzy equivalents; the universal set is represented by a horizontal line at membership $\mu = 1$, and sets by membership functions. The shaded areas are: union $A \cup B$ (a, d), intersection $A \cap B$ (b, e), and complement $\overline{A \cup B}$ (c, f). (figvenn2.m)

Source: Foundations of Fuzzy Control, Jan Jantzen, Wiley 2013

The fuzzy union $\mathcal{A} \cup \mathcal{B}$ is

$$\mu_{\mathcal{A} \cup \mathcal{B}}(x) \equiv \max(\mu_{\mathcal{A}}(x), \mu_{\mathcal{B}}(x))$$

The fuzzy intersection $\mathcal{A} \cap \mathcal{B}$ is

$$\mu_{\mathcal{A} \cap \mathcal{B}}(x) \equiv \min(\mu_{\mathcal{A}}(x), \mu_{\mathcal{B}}(x))$$

The fuzzy complement $\overline{\mathcal{A}}$ of $\mathcal{A}$ is

$$\mu_{\overline{\mathcal{A}}}(x) \equiv 1 - \mu_{\mathcal{A}}(x)$$

Example Problem

$$\underset{\sim}{A} = \left\{ \frac{1}{2} + \frac{0.5}{3} + \frac{0.3}{4} + \frac{0.2}{5} \right\} \quad \text{and} \quad \underset{\sim}{B} = \left\{ \frac{0.5}{2} + \frac{0.7}{3} + \frac{0.2}{4} + \frac{0.4}{5} \right\}.$$

We can now calculate several of the operations just discussed (membership for element 1 in both $\underset{\sim}{A}$ and $\underset{\sim}{B}$ is implicitly 0):

Complement

$$\overline{\underset{\sim}{A}} = \left\{ \frac{1}{1} + \frac{0}{2} + \frac{0.5}{3} + \frac{0.7}{4} + \frac{0.8}{5} \right\}.$$

$$\overline{\underset{\sim}{B}} = \left\{ \frac{1}{1} + \frac{0.5}{2} + \frac{0.3}{3} + \frac{0.8}{4} + \frac{0.6}{5} \right\}.$$

Union

$$\underset{\sim}{A} \cup \underset{\sim}{B} = \left\{ \frac{1}{2} + \frac{0.7}{3} + \frac{0.3}{4} + \frac{0.4}{5} \right\}.$$

Intersection

$$\underset{\sim}{A} \cap \underset{\sim}{B} = \left\{ \frac{0.5}{2} + \frac{0.5}{3} + \frac{0.2}{4} + \frac{0.2}{5} \right\}.$$

Difference

$$\underset{\sim}{A} | \underset{\sim}{B} = \underset{\sim}{A} \cap \overline{\underset{\sim}{B}} = \left\{ \frac{0.5}{2} + \frac{0.3}{3} + \frac{0.3}{4} + \frac{0.2}{5} \right\}.$$

$$\underset{\sim}{B} | \underset{\sim}{A} = \underset{\sim}{B} \cap \overline{\underset{\sim}{A}} = \left\{ \frac{0}{2} + \frac{0.5}{3} + \frac{0.2}{4} + \frac{0.4}{5} \right\}.$$

De Morgan's principles

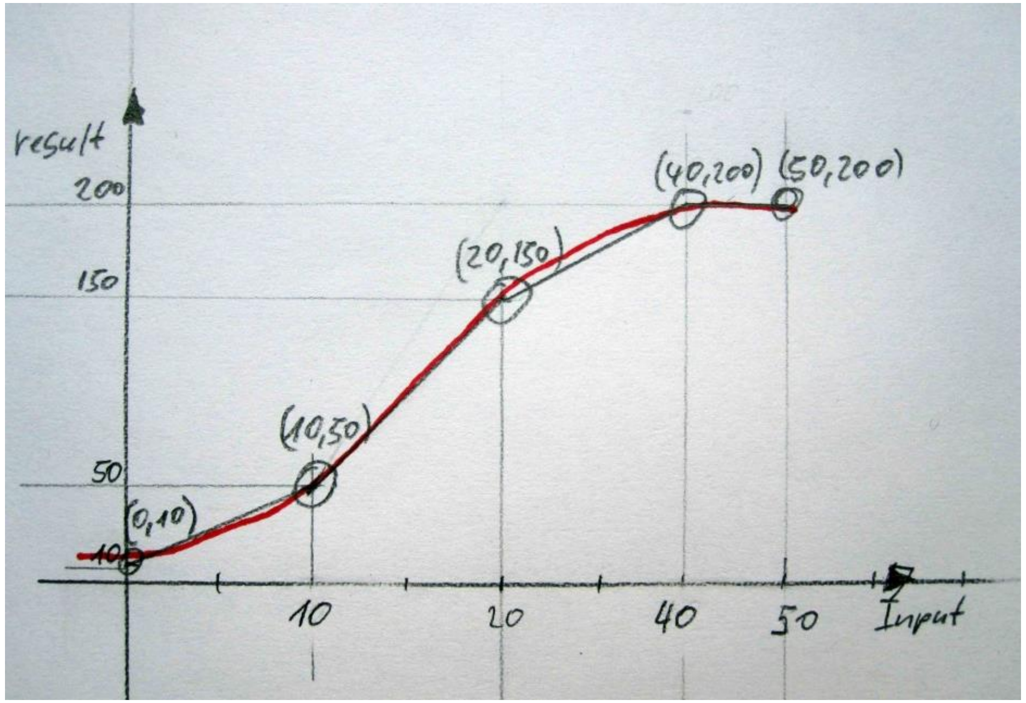
$$\overline{\underset{\sim}{A} \cup \underset{\sim}{B}} = \overline{\underset{\sim}{A}} \cap \overline{\underset{\sim}{B}} = \left\{ \frac{1}{1} + \frac{0}{2} + \frac{0.3}{3} + \frac{0.7}{4} + \frac{0.6}{5} \right\}.$$

$$\overline{\underset{\sim}{A} \cap \underset{\sim}{B}} = \overline{\underset{\sim}{A}} \cup \overline{\underset{\sim}{B}} = \left\{ \frac{1}{1} + \frac{0.5}{2} + \frac{0.5}{3} + \frac{0.8}{4} + \frac{0.8}{5} \right\}.$$

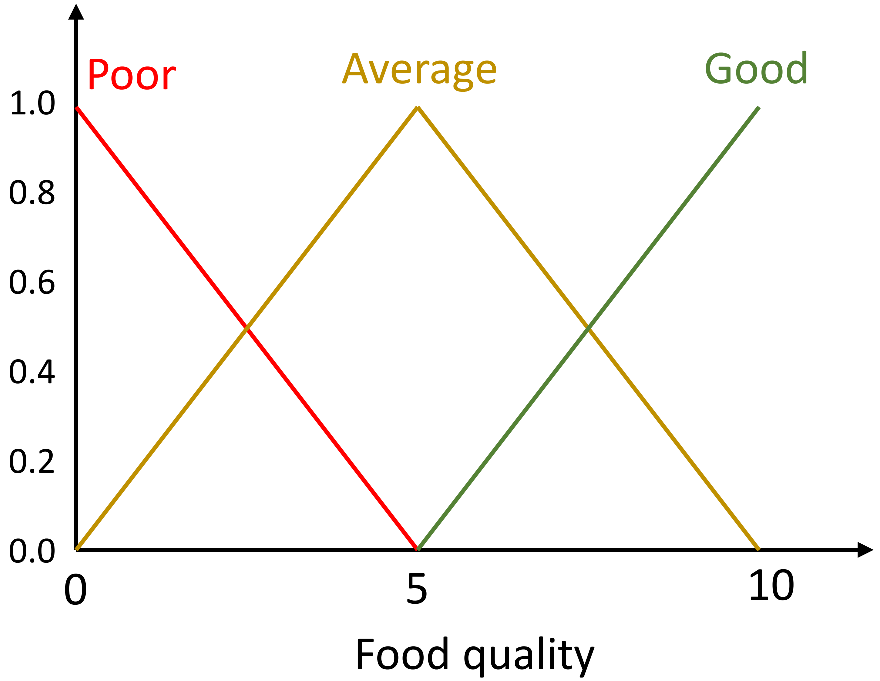
Fuzzy Logic System (FLS) as a Universal Approximator

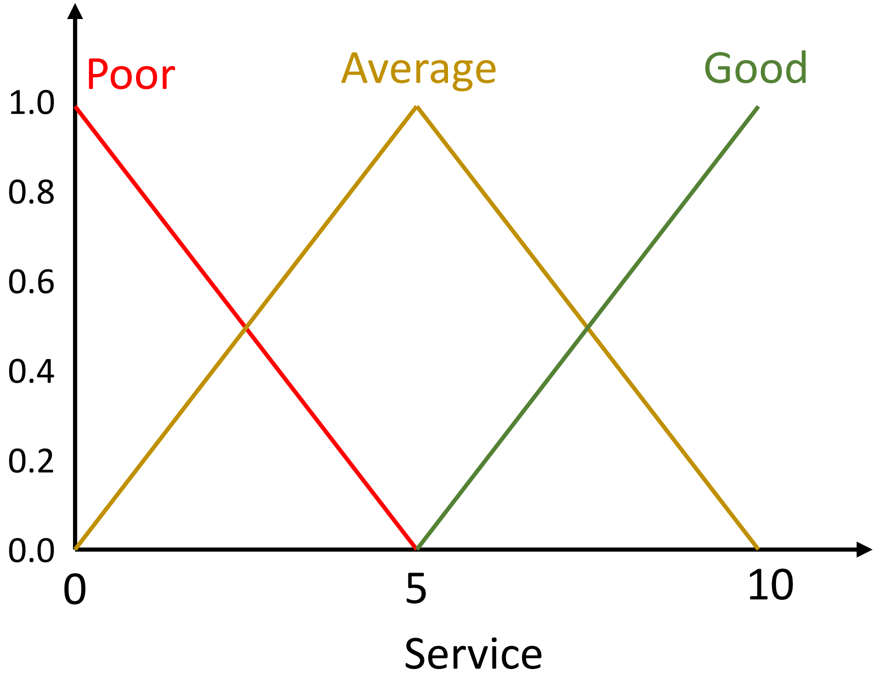
The ability to approximate any arbitrary non-linear mapping to any arbitrary degree of accuracy

- Has the **potential** to deliver near optimal control
- Assumptions (such as those associated with linear system theories) and biases do not **apriori** exclude any portions of the **solution space**

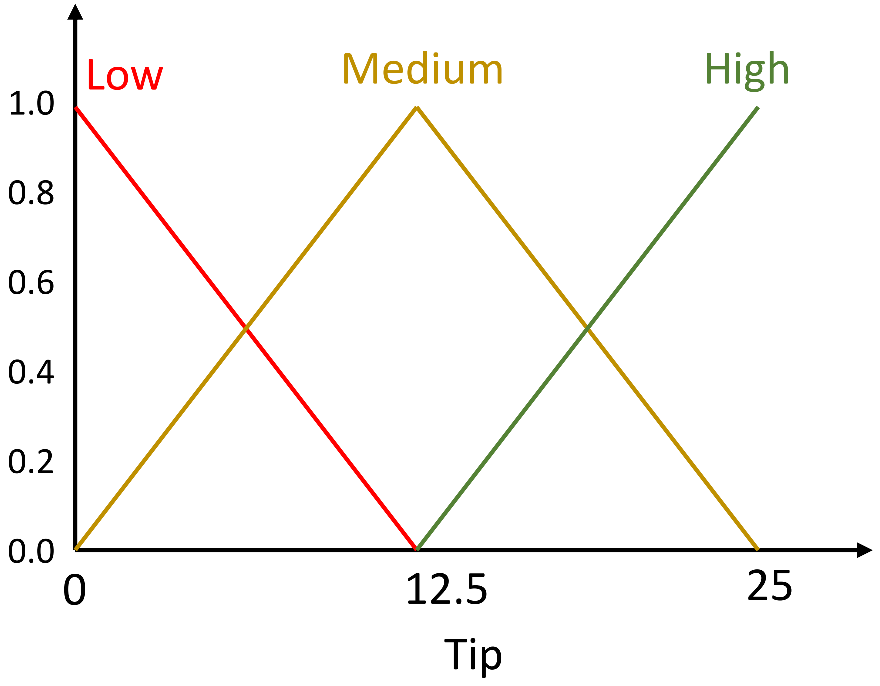


Tipper problem: Inputs





Tipper Problem

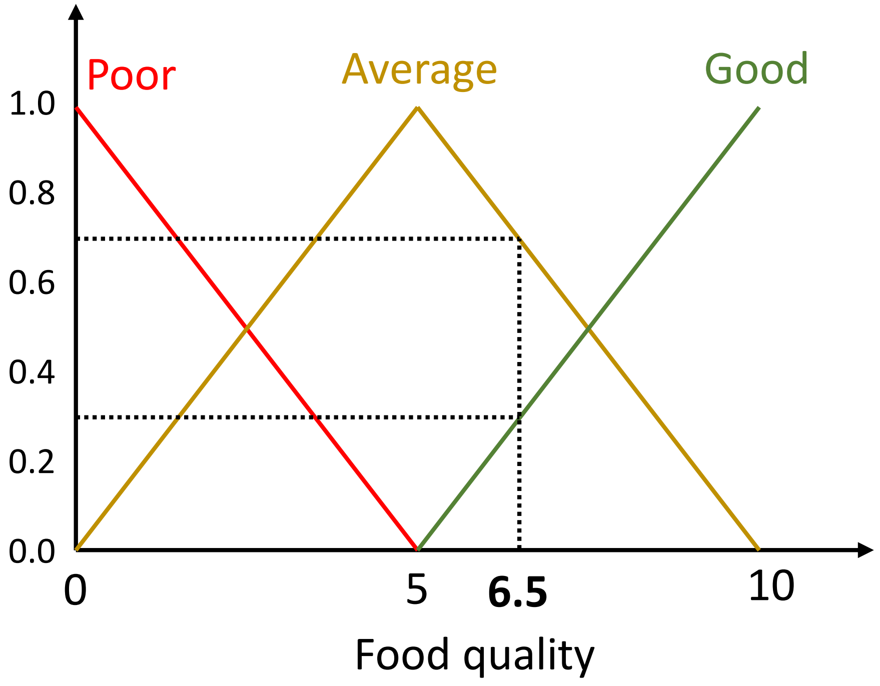


- Rulebase:
  - If the food is poor AND the service is poor, then the tip will be low
  - If the service is average, then the tip will be medium
  - If the food is good OR the service is good, then the tip will be high.

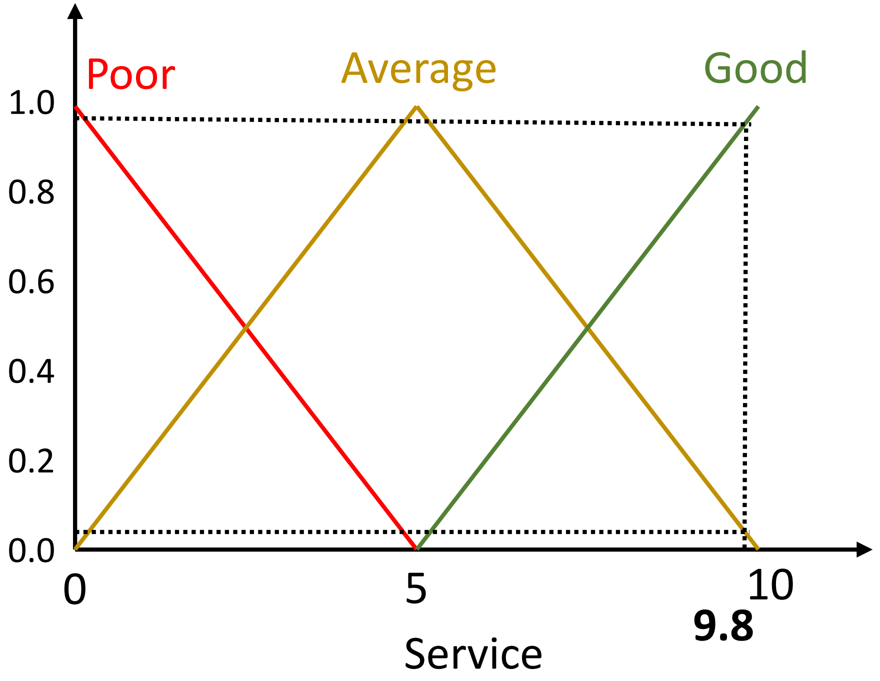
Inputs: Food quality = 6.5
Service = 9.8

Output = ??

Tipper Problem: Fuzzification



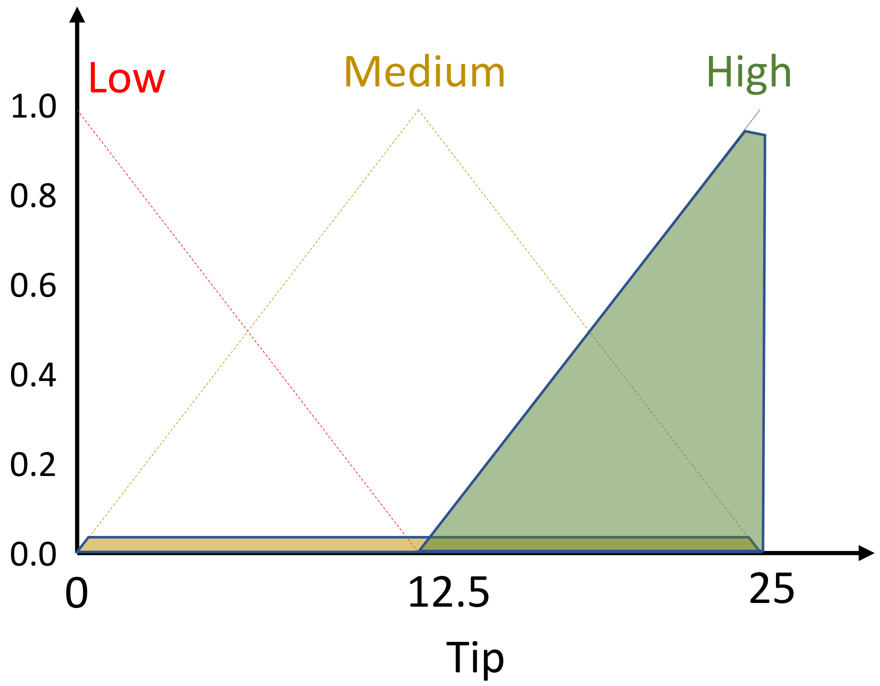
Food quality (6.5) = {0, 0.7, 0.3}



Service (9.8) = {0, 0.04, 0.96}

Tipper Problem: Evaluating the rules and defuzzification

- Rulebase:
  - If the food is poor (0) AND the service is poor (0), then the tip will be low (0)
  - If the service is average (0.04), then the tip will be medium (0.04)
  - If the food is good (0.3) OR the service is good (0.96), then the tip will be high (0.96).

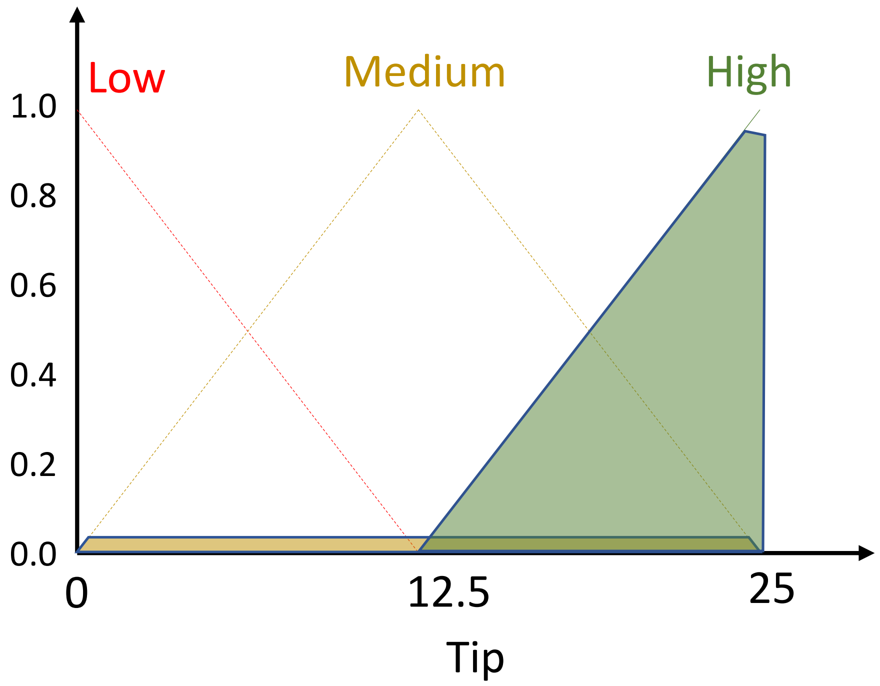


Centroid defuzzification

$$z^* = \frac{\int \mu_{\underset{\sim}{B}}(z) \cdot z \, dz}{\int \mu_{\underset{\sim}{B}}(z) \, dz}$$

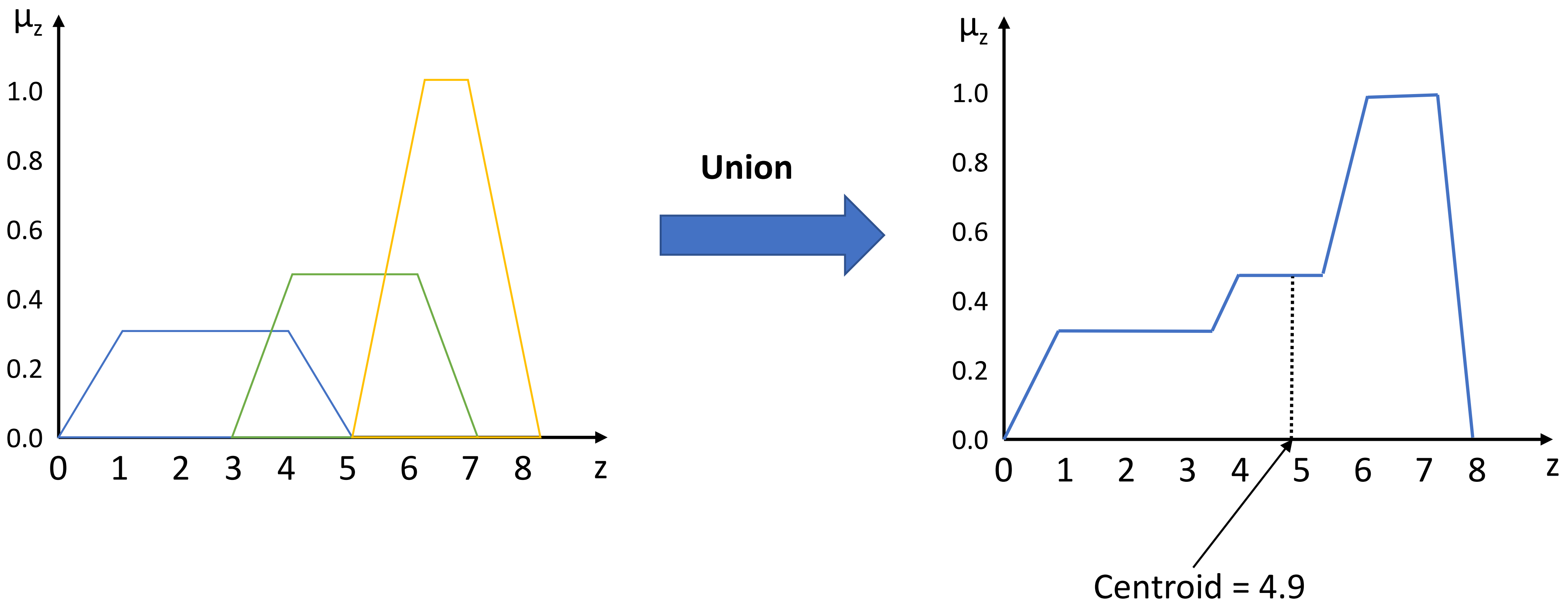
Tipper Problem: Evaluating the rules and defuzzification

- Rulebase:
  - If the food is poor (0) AND the service is poor (0), then the tip will be low (0)
  - If the service is average (0.04), then the tip will be medium (0.04)
  - If the food is good (0.3) OR the service is good (0.96), then the tip will be high (0.96).

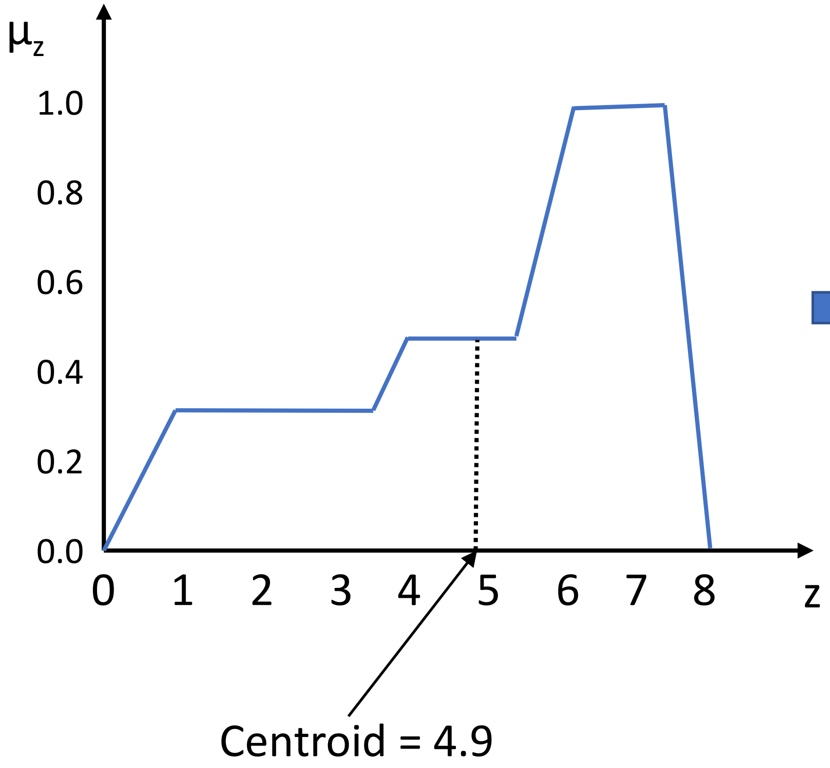


Tip = 20.2% (centroid defuzzification)

Defuzzification example



Defuzzification example



$$
\begin{aligned}
z^* &= \frac{\int \mu_{\underset{\sim}{B}}(z)\cdot z\,\mathrm{d}z}{\int \mu_{\underset{\sim}{B}}(z)\,\mathrm{d}z} \\
&= \left[\int_0^1 (0.3z)z\,\mathrm{d}z + \int_1^{3.6} (0.3)z\,\mathrm{d}z + \int_{3.6}^{4}\left(\frac{z-3.0}{2}\right) z\,\mathrm{d}z + \int_4^{5.5} (0.5)z\,\mathrm{d}z \right.\\
&\quad \left. + \int_{5.5}^{6} (z-5)z\,\mathrm{d}z + \int_6^7 z\,\mathrm{d}z + \int_7^8 (8-z)\,z\,\mathrm{d}z\right] \\
&\quad \div \left[\int_0^1 (0.3z)\,\mathrm{d}z + \int_1^{3.6} (0.3)\,\mathrm{d}z + \int_{3.6}^{4}\left(\frac{z-3.6}{2}\right)\mathrm{d}z + \int_4^{5.5} (0.5)\,\mathrm{d}z \right.\\
&\quad \left. + \int_{5.5}^{6}\left(\frac{z-5.5}{2}\right)\mathrm{d}z + \int_6^7 \mathrm{d}z + \int_7^8 \left(\frac{7-z}{2}\right)\mathrm{d}z\right] \\
&= 4.9\ \mathrm{m},
\end{aligned}
$$